

# FORMATION OF THE EARTH

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## INTRODUCTION

Probably the most fundamental problem of geology is that of understanding the physical and chemical processes and events that controlled the formation of the Earth and determined its initial state. In many ways our planet's subsequent evolution and present state were determined by this initial state, although striking differences between Earth and its sister planet Venus suggest that additional, and perhaps unpredictably stochastic, events only slightly later in their early history may have had major long-term consequences (e.g. Kaula 1990).

The traditional and well-established approach to understanding the history of the Earth is examination of the geological record. Application of this approach to the formation and earliest history of the Earth is confounded, however, by the geological activity of our planet. Hutton (1788) was nearly correct when he wrote that in the rocks of the Earth, "We find no sign of a beginning." Fortunately, the rocks do reveal some important evidence relevant to the beginning of the Earth, such as its  $4500 \pm 50$  m.y. age (Patterson 1956). Progress during the last several decades in the development of a variety of isotopic methods for measuring the age of igneous and metamorphic events has lifted the veil that previously obscured the correlation and interpretation of Precambrian events, encompassing 87% of Earth history. As a consequence, some important evolution in tectonic and petrologic styles is now apparent, even though the moderately well-preserved terrestrial record extends back only as far as about 3.8 b.y. ago (reviewed by Kröner 1985). This ancient and limited terrestrial evidence has now been extended by well-dated records of earlier geological events in the solar system obtained from less active planetary bodies: the Moon and the asteroidal sources of meteorites (Wetherill 1975, Tera et al 1981, Van Schmus 1981, Carlson & Lugmair 1988).

A complementary source of information regarding the formation and initial state of the Earth is provided by astronomical theory and observations. The formation of the Earth and other planets was a by-product of the formation of the Sun and is thereby linked to observational evidence and theoretical understanding of the processes by which stars like the Sun form. A considerable, but by no means definitive, body of theoretical work also exists regarding the manner in which residual material, remaining after the formation of the Sun, may be accumulated to form planets.

The origin and initial state of the Earth can therefore be approached from two directions: looking backward into the geological record, and working forward from the protosolar environment to planet formation. Achievement of a satisfactory understanding requires that the conclusions reached by these two approaches be at least compatible and, better yet, reinforce one another. To some extent this has occurred, but in other ways, this dual approach appears at present to lead to paradoxes that represent challenges requiring further investigation.

In this review, principal emphasis is placed on the formation of the Earth. It can be considered an update of an earlier review on the formation of the terrestrial planets (Wetherill 1980) and a more specialized version of a recent review of planet formation (Wetherill 1989). A general review of solar system formation is that of Cameron (1988). Many aspects of the subject are treated in more detail in individual chapters of the book *Protostars and Planets II* (Black & Matthews 1985). A collection of articles providing extensive theoretical treatments of many dynamical problems highly relevant to the formation of the Earth may be found in a volume assembled by Hayashi and his colleagues, mostly in Tokyo and Kyoto (Hayashi et al 1988). This publication was not available until well after the deadline for submission of this review, but an effort has been made to mention some of the important contributions contained therein. The forthcoming proceedings of the Conference on the Origin of the Earth, held in Oakland, California, in December 1988, may be expected to contain important reviews and research results, particularly with regard to chemical aspects of the formation of the Earth (Jones & Newsom 1990). An effort is made to organize the present review on a conceptual basis, rather than emphasizing adversarial positions that have been taken by various workers at various times.

### *Standard Model of Planet Formation*

A discussion of the origin of the Earth must be placed in the context of the formation of the Sun and the planets. This is done in an abbreviated way here, with references to other recent reviews. This context is provided by the emergence during the past 20 years of what can be considered to

be a standard model for the formation of the sun and planets that encompasses the work of the Moscow, Kyoto and Tokyo workers, the Tucson Planetary Science Institute, the Carnegie Institution of Washington, and a number of others. The sequential course of events that took place during the formation of the solar system, according to the standard model, is outlined briefly. The model is quite flexible, and alternate pathways occur at several junctures. Many of these alternatives are skipped over in this introduction, but, when highly relevant to Earth formation, they are considered in later sections. Although for expository purposes the model is described in a declarative manner, it must be remembered that for most of these events definitive, or even quantitative, demonstration is lacking, and this catalog of events should be thought of more as a research agenda than as a list of conclusions.

According to this model, the Sun formed, together with a number of stars, as a result of gravitational instability in a dense interstellar molecular cloud. Probably the most common mode of gravitational collapse of a rotating system of this kind leads to formation of double- and multiple-star systems. However in those cases where the rotation is slow, i.e. angular momentum of the collapsing system is relatively low, it is possible for single stars like the Sun to form. Even in these cases, a mechanism that transfers excess angular momentum away from the central star is required. Primary candidates for this mechanism are angular momentum transport associated with a viscous gaseous accretion disk (Lynden-Bell & Pringle 1974) surrounding the central star, and angular momentum transport by gravitational torques associated with asymmetries in the collapse (Larson 1984, Boss 1984). In either case, a small (2–10%) portion of the gas and dust will form a flattened disk surrounding the star that contains most of the angular momentum of the system. This disk of dust and gas is termed the “solar nebula.”

According to the standard model, planets grow by agglomeration of planetesimals that form in the solar nebula. In the vicinity of the Earth’s orbit, temperatures are expected to be low enough to permit crystallization of (Fe, Mg)-silicates and metallic iron but not ices of the volatile elements H, C, and N. Particles grow into these rocky planetesimals, either by nongravitational cohesion or by gravitational instabilities, up to diameters of a few kilometers. Larger bodies (“planetary embryos”) in the mass range of  $10^{25}$ – $10^{26}$  g ( $\sim$  Moon- or Mercury-size) form as a result of collision and merger of these small planetesimals. The rate of growth of the planetesimals into embryos is determined by their relative velocity, which in turn is determined by mutual gravitational perturbations and, therefore, by the mass distribution of the growing planetesimal swarm. The significance of this self-regulated coupling of the mass and velocity dis-

tribution was quantitatively identified by Safronov (e.g. Safronov 1969) and later developed further by others (Nakagawa et al 1983, Wetherill & Stewart 1989). During the growth of planetesimals into embryos, the growing planetesimals will be sufficiently closely packed to permit collisions between them, a necessary condition for their growth and merger. Calculations of this stage of growth indicate that bodies as large as  $10^{26}$  g can form in  $\sim 10^5$  yr.

The present total mass of the terrestrial planet region is  $1.18 \times 10^{28}$  g, of which 51% is in the Earth-Moon system. Therefore, the growth of the Earth and terrestrial planets would require the merger of  $\sim 100$   $10^{26}$ -g planetary embryos. These larger preplanetary bodies will usually be more distant from one another, the rate of growth will slow down, and this final stage of terrestrial planet growth will require  $10^7$ – $10^8$  yr for its completion, probably long after the loss of the gaseous solar nebula in  $\lesssim 3 \times 10^6$  yr. This stage of growth is likely to be characterized by the emergence, after  $\sim 5 \times 10^6$  yr, of the dominant embryos of the two large terrestrial planets, Earth and Venus, in orbits of relatively low eccentricity and inclination. As a result of gravitational perturbations associated with close encounters of the smaller  $10^{26}$ – $10^{27}$ -g embryos with Earth and Venus, the remainder of the material will probably be found in more eccentric orbits ( $e \sim 0.1$ – $0.3$ ) that span the terrestrial planet region, eliminating local “feeding zones” for each planet. Final accumulation of Earth and Venus then includes “giant impacts,” collisions with residual smaller planet-size embryos. Such impacts have been associated with special events in terrestrial planet history, such as the formation of the Moon (reviewed by Stevenson 1987), the loss of Mercury’s silicate mantle (Wetherill 1988, Cameron et al 1988, Vityazev et al 1988), and the loss of the Earth’s original atmosphere (Cameron 1983).

The same model must account, in a natural way, for the mechanism and time scale for the formation of other bodies in the solar system [i.e. Jupiter and the other giant planets (Lissauer 1987, Stevenson & Lunine 1988)], the depletion of mass in the asteroid belt (Weidenschilling 1988, Wetherill 1989, 1990a), and the formation of the source regions of long- and short-period comets (Fernandez & Ip 1981). Our understanding of all these events is, at best, rudimentary. A complete theory of Earth and terrestrial planet formation will require that the formation of these other solar system bodies be understood more adequately.

Among these other problems, the way in which Jupiter was formed and its formation time scale are most critical. Because almost all of Jupiter’s mass consists of hydrogen and helium, the gaseous solar nebula must have still been present when Jupiter (and Saturn) were formed. Observations

of pre-main-sequence stars (Walter 1986, Strom et al 1989) show that protostellar gas envelopes are removed on time scales  $< 10^7$  yr, and sometimes  $\ll 3 \times 10^6$  yr. For this reason, the  $\sim 15 M_{\oplus}$  (Earth mass) cores of Jupiter and Saturn must have formed rapidly in order to initiate the accretion of nebular gas while it was still available. These events and their timing are highly relevant to the formation of the Earth. They proscribe the nebular gas concentration during the final stages of Earth formation, which in turn determines the size and nature of Earth's primordial atmosphere (Hayashi et al 1979, Nakazawa et al 1985) and the role played by gas-embryo gravitational torques in the radial migration of planetary embryos in the inner solar system (Ward 1986, 1988). In addition, the regions occupied by Jupiter and the other giant planets, as well as the asteroid belt, may have been source regions for volatile and oxidized constituents of the Earth (Wänke 1981, Dreibus & Wänke 1989). Determination of the time scale and quantity of this transferred material requires an understanding of the formation of these bodies that lie beyond the orbit of the Earth.

The most significant variant of the standard model is the "Kyoto model" (Hayashi et al 1985) in which the loss of the nebular gas occurred *after* the formation of the Earth, in contrast to the assumption of most other workers that the loss of nebular gas at 1 AU (astronomical unit) occurred well before the final stages of the Earth's growth. Some principal differences between the outcome of the gas-free and gas-rich models are chemical in nature, inasmuch as accumulation of Earth in a nebula that had time to cool to  $\sim 300$  K would provide a major admixture of volatile compounds, perhaps similar to those found in some carbonaceous meteorites. In addition, a large ( $> 10^{26}$  g)  $H_2$ -rich primordial terrestrial atmosphere,  $\sim 10^5$  times as massive as the present atmosphere, is predicted by the gas-rich model (Hayashi et al 1979). As a result of the work of Takeda et al (1985) and Takeda (1988), which predicts a gravitationally enhanced and therefore very large gas drag coefficient for a planet-size body, it may be expected that important dynamical differences (Ohtsuki et al 1988) may also exist that have not yet been fully studied.

Some workers have termed the standard model a "paradigm." I regard this appellation to be premature. At present, when compared with what needs to be known, the model tells us rather little that can be considered at all certain. Some specific problems have been explored relatively thoroughly and quantitatively, but most are in the qualitative "scenario" stage. At present the model can be regarded as a very useful working hypothesis, one that can provide a focus for workers of varying backgrounds and experience, facilitate communication between them, and permit a ra-

tional division of labor, rather than requiring everyone to make up a personal grand model. Only time will tell if it will succeed as a unifying theory.

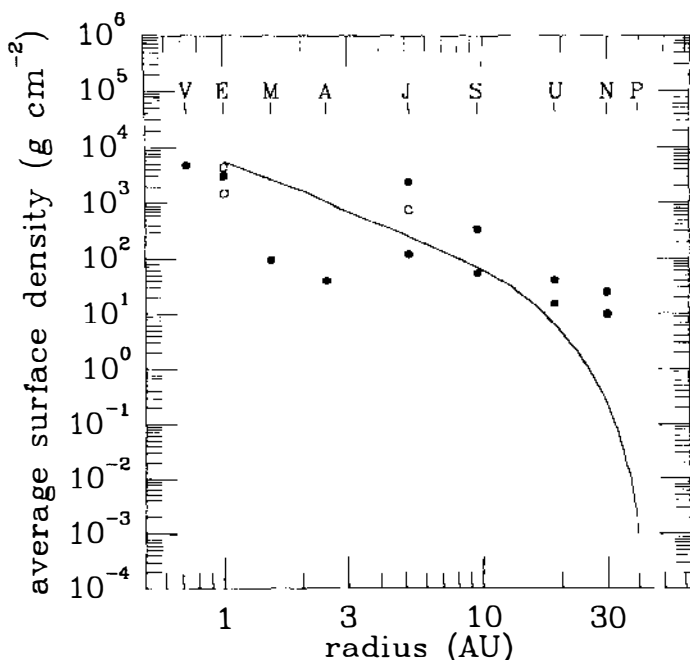
There are of course alternatives to the standard model. The most prominent of these has been the concept that planetary formation took place in parallel with the formation of the Sun as a consequence of massive gas-dust instabilities in the solar nebula, termed "giant gaseous protoplanets." Theories of this general type have a venerable history (e.g. von Weizsäcker 1944, Kuiper 1951, Cameron 1978), but their more recent development has been rather quiescent. References to these and other alternative models can be found in the conference report *The Origin of the Solar System* (Dermott 1978).

## FORMATION OF THE SUN AND SOLAR NEBULA

Radio and infrared observations show that  $\sim 1-M_{\odot}$  (solar mass) star formation occurs in cold (10 K), dense ( $10^4$ – $10^5$   $\text{H}_2$  molecules  $\text{cm}^{-3}$ ) cores found within larger dark molecular cloud complexes with masses of  $10^3$ – $10^4$   $M_{\odot}$ . Low-mass stars are also formed, along with more massive stars, in giant molecular clouds with masses up to  $10^6$   $M_{\odot}$  (Shu et al 1987). It is reasonable to use observational and theoretical work on present-day star-forming regions of this kind as models for the formation of the Sun 4.5 b.y. ago. It may also be recognized, however, that star formation in galaxies often occurs in episodic "starbursts," perhaps associated with galactic collisions and mergers (Schweizer 1986). Scalo (1988) has presented evidence that such a starburst occurred in our Galaxy about 5 b.y. ago, approximately the age of the Sun.

Calculations of the collapse of a rotating molecular cloud core with specific angular momentum  $\gtrsim 5 \times 10^{18} \text{ cm}^2 \text{ s}^{-1}$  show that a disk forms that is centrifugally supported, i.e. that does not rapidly collapse onto the central protostellar object. When the specific angular momentum is high enough ( $\gtrsim 2 \times 10^{20} \text{ cm}^2 \text{ s}^{-1}$ ) and a strong central condensation is absent, fragmentation into a binary or multiple-star system will occur. Even in the absence of turbulent viscosity, a core with intermediate specific angular momentum and/or initial strong central condensation can probably evolve into a central single star, surrounded by a circumstellar disk, as required for the solar nebula in the standard model (Boss 1989, 1990).

A calculated surface density distribution for one such case is shown in Figure 1. The mass of the initial  $1.04-M_{\odot}$  system is separated into a stellar core of mass  $0.99 M_{\odot}$  and a circumstellar disk shown with a mass of  $0.05 M_{\odot}$ . The decrease in surface density with heliocentric distance does not follow a simple power law. The calculated surface densities are considerably greater than those required to form the terrestrial planets and



*Figure 1* Calculated total (gas + solid) density distribution in a model residual nebula of  $0.05 M_{\odot}$  surrounding a  $0.99 M_{\odot}$  star (Boss 1989). The *open symbols* at the position of Earth (E) represent two estimates ( $7.5$  and  $22.4 \text{ g cm}^{-2}$ ) of the surface density solid matter required to form the Earth. The *open symbol* at the position of Jupiter (J) represents the value of the surface density at Jupiter's distance needed to form Jupiter in about  $0.5 \text{ m.y.}$  (Lissauer 1987). The *solid symbols* bracket the "reconstruction" of the gaseous solar nebula using present solid-matter densities by Weidenschilling (1977a), based on present masses and positions.

asteroids but less than those required to form Jupiter rapidly (Lissauer 1987) and Uranus and Neptune at all. This system has not yet reached centrifugal equilibrium, but gravitational torques are already transferring angular momentum rapidly outward from the inner part of the disk. It is plausible that a star-disk system with these, or not very different, parameters will evolve into a solar nebula resembling those usually assumed in theories of planet formation in about one free-fall time ( $\sim 10^5 \text{ yr}$ ). No tendency to form massive gas-dust instabilities leading to giant gaseous protoplanets was found. The midplane temperature variations in this same nebula are shown in Figure 2. These temperatures ( $\sim 1500 \text{ K}$  out to  $3 \text{ AU}$ ) are considerably higher than those found in some other solar nebula models (Cameron & Pine 1973). This difference arises from including in the three-dimensional (3D) models of Boss (1990) the heating associated

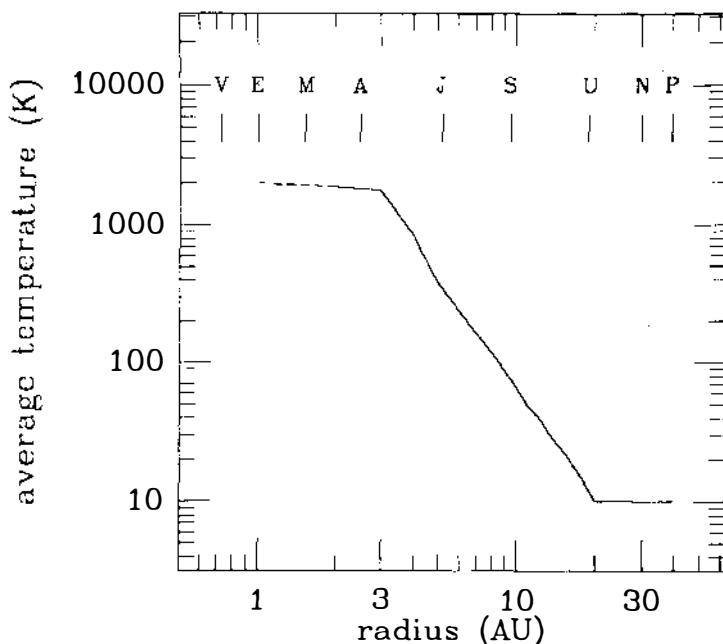


Figure 2 Variations of temperature with distance for the model of Figure 1 (Boss 1989). Temperatures in the asteroid belt reach 1500 K, high enough to vaporize solid grains of silicate and iron. The *dashed line* is an extrapolation into the terrestrial planet region, where the temperature must be at least as high as in the asteroid belt.

with compression of the collapsing gas. The time scale for cooling of this nebula is estimated to be  $\sim 10^5$  yr. These temperatures are sufficiently high to volatilize iron grains (1420 K; Pollack et al 1985) and all but the most refractory silicate and oxide minerals. It is possible that the temperature of the solar nebula was self-regulated by the opacity of iron grains.

There is considerable observational evidence for embedded infrared sources in dark cloud cores (Beichman et al 1986) that are likely to be protostars, surrounded by disks. Observations of young pre-main-sequence stars constitute observational evidence that this obscuration can be removed on a short ( $\sim 10^6$ – $10^7$  yr) time scale (Lada 1985, Strom et al 1989). Observation of residual, strongly flattened dust extending to  $\sim 500$  AU surrounding the main-sequence star  $\beta$  Pictoris (Smith & Terrile 1984) provides additional generally supporting but qualitative evidence that the picture of solar system formation outlined by the standard model has some basis in reality.



## COAGULATION OF GRAINS AND FORMATION OF SMALL (1–10 km DIAMETER) PLANETESIMALS

### *Settling of Grains to the Midplane of the Nebula and Nongravitational Coagulation*

At least on average, the interstellar grains associated with the infalling gas that formed the solar nebula were originally quite small ( $< 0.1 \mu\text{m}$ ). It is not clear to what extent the nonvolatile constituents of the Earth then experienced a stage of volatilization and recondensation or coagulated to form larger dust grains without vaporization. The  $\sim 1500 \text{ K}$  theoretically based temperatures discussed in the previous section support extensive vaporization. On the other hand, there is abundant evidence for isotopic disequilibrium in both refractory and volatile constituents of meteorites (reviewed by Anders 1988, Zinner 1988). This is not what one would expect from a well-mixed gaseous solar nebula, but continued infall of dust and gas at later times could “contaminate” a once hot solar nebula with isotopically heterogeneous interstellar matter.

In either case, formation of planetesimals, planetary embryos, and final planets required the aggregation of newly condensed grains and/or refractory residues into larger bodies. The present status of this problem of early coagulation has been reviewed by Weidenschilling (1988), Sekiya & Nakagawa (1988), and Weidenschilling et al (1989). The principal problem appears to be the need to grow fairly large ( $\gtrsim 10 \text{ m}$ ) planetesimals before the time that nebular gas is removed from the terrestrial planet region ( $\sim 10^6 \text{ yr}$ ); otherwise, the material needed to form the Earth will be removed along with the gas.

Because of gas drag, in the absence of turbulence, small grains will spiral down toward the central plane of the solar nebula. The characteristic settling time  $\tau_z$  of a small ( $\lesssim 1 \text{ cm}$ ) grain is (Weidenschilling 1988)

$$\tau_z = \frac{\rho c}{s \rho_s \Omega^2}, \quad (1)$$

where  $s$  is the particle radius,  $\rho_s$  the particle's material density,  $\Omega$  the Keplerian orbital angular velocity,  $\rho$  the gas density, and  $c$  the mean molecular speed. At 1 AU, for  $T = 1000 \text{ K}$ ,  $c = 3.1 \times 10^5 \text{ cm s}^{-1}$ ,  $\rho = 1.18 \times 10^{-9} \text{ g cm}^{-3}$ ,  $\rho_s = 1 \text{ g cm}^{-3}$ , the characteristic settling time of a  $1\text{-}\mu\text{m}$  grain will be  $2.9 \times 10^6 \text{ yr}$ . This is very long, comparable to the time scale for removal of the gas in the solar nebula. Turbulence will lengthen the settling time even more by entraining the small particles in the turbulent flow.

In order to retain solid matter in the solar nebula during the loss of

nebular gas, it is necessary that settling proceed more rapidly, presumably because the grains grew larger. For example, if chondrules of  $\sim 1$ -mm radius, as found in most stony meteorites, formed in the solar nebula (Grossman 1988), the corresponding settling time of these objects in a nonturbulent nebula will be shorter ( $\sim 3 \times 10^3$  yr). Even in the absence of chondrule formation, from the theoretical point of view it may be expected that significant coagulation of grains will result from particle-to-particle collisions during descent to the central plane, provided that any turbulence in the solar nebula has decreased sufficiently to preclude mutual collisional fragmentation of particles. For the low values of turbulent velocity calculated by Cabot et al (1987) of  $\sim 0.01$  times the sound speed, particles could grow without disruption. For these values of turbulent velocity and for 100% "sticking efficiency" at collision, coagulation during descent could possibly concentrate solid bodies as large as 10 m in the central plane of the nebula on a  $10^3$ -yr time scale without major loss of material into the Sun. Further growth can occur, primarily by sweep up of material during radial migration within the central dust layer. Weidenschilling (1988) estimates that kilometer-sized planetesimals could grow in this manner in  $\sim 10^4$  yr, but this estimate is lengthened by one or two orders of magnitude if the nebular grains are fluffy aggregates, rather than compact objects like chondrules (Donn & Meakin 1988, Weidenschilling et al 1989).

Introduction of the concepts of turbulence and porous fluffy aggregates into the discussion of formation of planetesimals gives rise to paradoxes. Both a moderate degree of turbulence and soft porous target bodies facilitate the nongravitational growth of planetesimals. On the other hand, they both delay the settling of bodies to the nebular midplane, perhaps excessively (Mizuno 1989). It has been proposed (Völk 1983) that the "best of both worlds" may have occurred: oscillation of the nebula between turbulent and nonturbulent states. Growth would occur during turbulent times, followed by falling of objects to the central plane when turbulence was absent.

In addition to turbulence-induced velocities, during the stage of growth from 1 to 10 cm up to 1–10 km, relative velocity differences between bodies of different sizes will also be caused by size-dependent gas drag. Large bodies ( $\geq 1$  m) will move in Keplerian orbits at Keplerian velocity. The nebular gas will be supported by its buoyancy, partially reducing solar gravity, and will move at a velocity about 0.2% slower than the Keplerian velocity (Whipple 1973, Adachi et al 1976, Weidenschilling 1977b). Because small bodies ( $\leq 10$  cm) move with the gas, a relative velocity between large and small bodies of magnitude  $\sim 60 \text{ m s}^{-1}$  will result. Although the physical processes are not well understood, it has been proposed that even at these velocities, and at high temperatures, com-

pressible or porous bodies could grow. The relatively high-velocity small particles may be embedded within the large bodies during impact (Leliwa-Kopystynski et al 1984), and collisions between more similar-sized bodies will be at low relative velocity, also permitting growth.

The nongravitational “coagulative” stage of growth will end when the relative velocity of the larger bodies becomes dominated by their mutual gravitational perturbations, rather than by the small residual turbulence of the nebula. The size at which this transition occurs is uncertain, primarily because of uncertainties in the estimated turbulent velocities. Planetesimal radii of 0.1–10 km correspond to plausible values of the relevant parameters.

### *Growth of Planetesimals by Gravitational Instability in the Central Dust Layer of the Nebula*

An alternative mode of growth of small bodies into kilometer-size bodies is dust-layer gravitational instability in the central plane of the nebula (Edgeworth 1949, Safronov 1969, Goldreich & Ward 1973, Sekiya 1983). This mechanism has been considered appealing in that it appears to involve no assumption regarding the sticking efficiency of colliding bodies. In fact, it now appears that “stickiness” is necessary in any case. In the presence of even modest turbulence, bodies will have to grow to  $\gtrsim 1$  m in size if they are to be concentrated sufficiently in the central plane to cause gravitational instability. On the other hand, if because of unexpectedly low turbulence,  $< 1$ -m-size bodies do concentrate in the central plane, shear between the central dust layer and the gas above and below it will produce a turbulent Ekman boundary layer that will stir the dust sufficiently to preclude the gravitational instability (Weidenschilling 1980). For this reason, acceptance or rejection of dust-layer instability must be decided on its own merits and not on the dubious basis of simplicity.

Although various authors have formulated the problem in different ways, there is general agreement that the mass of the instabilities ( $m_c$ ) will have a value of about

$$m_c = \frac{\xi \pi^5 \sigma^3 G^2}{\Omega^4}, \quad (2)$$

where  $\xi$  is a factor  $\approx 1$ ,  $\sigma$  is the surface dust density, and  $G$  is the gravitational constant. For  $\sigma = 10 \text{ g cm}^{-2}$  and  $\xi = 1$  at 1 AU,  $m_c$  has a value of  $8.6 \times 10^{17} \text{ g}$ , corresponding to a diameter of 11.8 km for a planetesimal material density of  $1 \text{ g cm}^{-3}$ . Because the angular velocity  $\Omega$  decreases with heliocentric distance, probably more rapidly than the surface density  $\sigma$ , Equation (2) shows that the mass of the planetesimals can be strongly

dependent on heliocentric distance. On the other hand, because the mechanism is purely gravitational, no major dependence on the physical properties of the aggregating material would be expected once the conditions for achieving the instability were reached.

A problem with this mechanism is the very low dust particle and turbulent velocities required for the instability to develop. Even in the absence of turbulence, dust velocities can be no more than  $\sim 10 \text{ cm s}^{-1}$  at 1 AU. This is only  $\sim 3 \times 10^{-5}$  times the sound speed. For bodies smaller than  $\sim 1 \text{ m}$ , even lower velocities are necessary, because otherwise turbulent motion will limit achieving the dust concentration required for the gravitational instability to develop. Even after allowing for our present ignorance of the extent of turbulence in the solar nebula, it would be surprising if the turbulent velocity could become so low. On the other hand, if the objects are large enough ( $\sim 1 \text{ m}$ ) they could fall toward the central plane to form a layer thin enough to possibly produce a gravitational instability (Weidenschilling 1988). In any case, nongravitational coagulation is required to produce the 1-m bodies. Purely gravitational formation of planetesimals seems very unlikely.

Because of our near-ignorance of the actual physical state of the solar nebula and of the solid grains that comprise its dust component, despite serious work by several workers, it is not really possible at present to say whether formation of  $\sim 0.1\text{--}10 \text{ km}$  planetesimals in the central plane of the nebula would be expected to proceed by gravitational instability or by continuing nongravitational agglomeration. If one simply assumes that somehow or other planetesimals must have formed or we would not be here, the problem can be set aside, and one can go on to a discussion of the way planetesimals grew into planets. There is some merit to this view, but it is excessively facile. It may be that differences in the mode of formation of planetesimals at different distances and times in the solar nebula may be the key to a number of obscure problems, such as the time scales for giant-planet formation and the depletion of the region between 1 and 5 AU, as well as that interior to Venus.

## GRAVITATIONAL ACCUMULATION OF PLANETESIMALS INTO PLANETARY EMBRYOS

### *Transition From the Nongravitational to the Gravitational Growth Regime*

Accumulation of micron-size dust grains into  $\sim 1\text{-km}$  bodies was discussed in the previous section. In that grain-size range, the relative velocities of the accumulating bodies were primarily determined by nongravitational forces, those forces resulting from coupling of small bodies to the turbulent

velocity field of the nebula, and the differential velocity resulting from the size-dependent buoyancy caused by the radial gas pressure gradient of the nebula. The mutual cohesion of particles required for bodies to grow was also of nongravitational origin, because even weak Van der Waals binding energies of  $10^3 \text{ ergs g}^{-1}$  are comparable to the gravitational binding energy of a 1-km-diameter body.

When the bodies become larger than 10 km diameter, this situation changes. For bodies of this size, mutual gravitational perturbations associated with mutual close encounters, balanced with collisions, are sufficient to maintain a steady-state velocity comparable to their escape velocity of  $13 \text{ m s}^{-1}$ . In comparison, the turbulence-induced velocity of a body this size will be lower,  $\sim 2 \text{ m s}^{-1}$  (Völk et al 1980) at a Mach number of 0.05. The gravitational binding energy will be  $\geq 5 \times 10^5 \text{ ergs g}^{-1}$ , approaching the mechanical strength of coherent but weak bodies. For this reason, a theoretical discussion of the accumulation of bodies larger than this must include gravitational effects.

This transition is not sudden, however, and the regions dominated by gravitational effects overlap those in which nongravitational effects are of primary importance. For example, it cannot be ruled out that collective dust-layer gravitational instability could allow agglomeration of  $\sim 1$ -m-diameter objects into larger bodies to occur in the central dust layer. On the other hand, as long as a significant fraction of the material encountered is in the  $\lesssim 1$ -m size range, possibly as a result of continuing infall, impacts with 10-km bodies will occur at the relative velocity of the gas ( $\sim 60 \text{ m s}^{-1}$ ), far above the escape velocity of these larger bodies. Depending on the mechanical properties of the colliding bodies, this could lead either to nongravitational growth by embedding of the small particles in their targets or to "sandblasting" of the planetesimal's surface, inhibiting growth. If the 10-km body is growing by accumulation from high-velocity small particles entrained in such a "head wind," the associated angular momentum transfer will tend to make the orbits of the larger bodies circular and coplanar.

### *Runaway vs. Nonrunaway Gravitational Growth: General Considerations*

Considerable work has been done on the gravitational accumulation of initially 1–10 km diameter bodies into much more massive  $\sim 4000$ -km objects. For the terrestrial planets to be assembled from  $\sim 10$ -km-diameter ( $\sim 10^{18} \text{ g}$ ) planetesimals,  $\sim 10^{10}$  initial bodies are required. Following the orbital evolution of so large a swarm of objects by the methods of celestial mechanics is not feasible. The alternative of treating the evolution of the swarm by the methods of gas dynamics has had significant success. In this

approach, the individual planetesimals are the analogues of gas molecules in the kinetic theory of gases. This technique has been used quite successfully in stellar dynamics (Chandrasekhar 1942). From this point of view, the planetesimals constitute a swarm of bodies moving in nearly circular and coplanar concentric heliocentric orbits. In a reference frame rotating with the objects at a given heliocentric distance, the relative velocities of the bodies will be very low in comparison with their circular Keplerian velocity of  $30 \text{ km s}^{-1}$  at 1 AU. Their mutual gravitational perturbations will cause the orbits to have small but nonzero eccentricities and inclinations. This introduces small random and disordered components of velocity relative to that associated with the velocity characteristic of the near-circular Keplerian motion. In the rotating reference frame, this disordered motion is analogous to the random thermal motion of gas molecules confined in a container with walls at a fixed temperature. For this reason, this approach to planetesimal growth is often termed “particle in a box,” to distinguish it from a treatment in which the celestial mechanical orbits are explicitly introduced.

In the simplified case where a planetesimal grows by sweeping up bodies considerably smaller than itself, the rate of growth of its mass ( $M$ ) will be

$$\frac{dM}{dt} = \pi R^2 \bar{V}_{\text{rel}} \rho F_g \quad (3)$$

where  $R$  is the radius of the growing body,  $\bar{V}_{\text{rel}}$  the average relative velocity between the large and small bodies,  $\rho$  the mass density of small bodies per unit volume of the nebula, and  $F_g$  a factor representing the extent to which the gravitational field of the growing body enhances its effective cross section above the geometric value of  $\pi R^2$ . When the growth results from nongravitational cohesion, as considered in the previous section,  $F_g = 1$ . In the approximation that the encounter between the bodies can be treated as a two-body interaction between the colliding bodies, rather than as a three-body interaction in which the solar gravity appears as a perturbation during the encounter, the gravitational enhancement factor assumes the familiar form

$$F_g = (1 + V_e^2 / \bar{V}_{\text{rel}}^2) = (1 + 2\vartheta) \quad (4)$$

where  $V_e$  is the surface escape velocity of the larger body, and  $\vartheta$  is the dimensionless Safronov number,  $V_e^2 / 2\bar{V}_{\text{rel}}^2$ . This factor arises from the ratio of the distance of close approach to the asymptotic unperturbed impact parameter in a two-body hyperbolic encounter (illustrated by Wood 1961). In this approximation, any object that in the absence of gravity would

have a distance of closest approach  $RF_g^{1/2}$  will be gravitationally perturbed to an approach distance  $< R$ , i.e. it will collide.

The factor  $F_g$  is of considerable conceptual importance because its value determines whether the growth of a planetesimal swarm is either to be "orderly," in which case the mass distribution evolves smoothly and continuously as the bodies grow, or "runaway," in which case a single largest body in a local region of the swarm grows so rapidly relative to the others that the mass distribution becomes grossly discontinuous, with eventually most of the mass being concentrated in the largest body. In the more detailed discussion later in this section, this distinction between orderly and runaway growth appears as a bifurcation in the numerical solutions of the system of coupled nonlinear equations that determines the mass and velocity evolution of a swarm consisting of bodies of different mass and with mass-dependent relative velocities. The physical basis for this bifurcation can be illustrated by consideration of a simpler case, however.

When the surface escape velocity  $V_e$  is expressed in terms of its radius, i.e.

$$V_e^2 = \frac{2GM}{R} = \frac{8}{3} \pi \rho_p R^2, \quad (5)$$

where  $\rho_p$  is the material density of the planetesimal, Equation (3) then becomes

$$\frac{dM}{dt} = \pi R^2 \bar{V}_{\text{rel}} \rho \left( 1 + \frac{8}{3} \frac{\pi \rho_p R^2}{V_{\text{rel}}^2} \right). \quad (6)$$

When  $V_{\text{rel}}$  is large compared with  $V_e$ , the growth rate is approximately proportional to  $R^2$ ; when  $V_{\text{rel}}$  is small compared with  $V_e$ ,  $dM/dt$  is nearly proportional to  $R^4$ . For any given intermediate value of  $V_{\text{rel}}/V_e$ , the dependence of  $dM/dt$  on  $R$  will vary continuously between  $R^2$  and  $R^4$ . The curve describing this dependence will have a varying slope, but at any point its slope can be represented locally by a power-law dependence on  $R$ :

$$\frac{dM}{dt} = kR^q, \quad (7)$$

where the exponent  $q$  varies between 2 and 4, depending on the value of  $R$ . At what value of  $q$  does the bifurcation take place?

Consider two large bodies with masses  $M_1$ ,  $M_2$  and radii  $R_1$ ,  $R_2$  ( $M_1 > M_2$ ,  $R_1 > R_2$ ). The rate of change of the ratio of their masses will be

$$\frac{d}{dt} \left( \frac{M_1}{M_2} \right) = \frac{1}{M_2} \left( \frac{dM_1}{dt} - \frac{M_1}{M_2} \frac{dM_2}{dt} \right). \quad (8)$$

If we express  $dM_1/dt$  and  $dM_2/dt$  in terms of Equation (7), make use of the  $R_1^{-3}$ -dependence of  $M_1$ , and factor out positive-definite terms, Equation (8) becomes

$$\frac{d}{dt} \left( \frac{M_1}{M_2} \right) = \frac{kM_1}{M_2^2} R_2^q \left[ \left( \frac{R_1}{R_2} \right)^{q-3} - 1 \right]. \quad (9)$$

When  $q > 3$ ,  $d/dt(M_1/M_2)$  is positive and  $M_1$  will become even larger relative to  $M_2$ . When  $q < 3$ ,  $d/dt(M_1/M_2)$  is negative and  $M_2$  will tend to "catch up" with the larger body. When  $q = 3$  (i.e. the growth rate is proportional to the mass of the growing body)  $d/dt(M_1/M_2)$  is zero and the ratio of the two masses neither decreases nor increases. This is the boundary between orderly and runaway growth for this simple case.

### *Safronov's Solution of the Coagulation Equation*

Safronov (1962, 1969) obtained an analytical solution for the integro-differential coagulation equation that represents the extension of Equation (3) to a continuous distribution in which the masses of all the bodies are growing by collisions with those smaller than themselves, subject to the condition that the growth rate is proportional to the mass [i.e. obeys Equation (8) with  $q = 3$ , the limiting value for nonrunaway growth]. In that work, the initial mass distribution was assumed to be of the form

$$n(m) = \frac{N_0 e^{-m/m_0}}{m_0}, \quad (10)$$

where  $n(m)dm$  is the number density of bodies between  $m$  and  $m + dm$ ,  $N_0$  is the total number density at the initial time, and  $m_0$  is the mean mass of the initial mass distribution. After an initial transient, a solution in dimensionless form is found to be

$$n(m, \tau) = \frac{(1 - \tau)}{2\pi^{1/2}\tau^{3/4}} m^{-3/2} e^{-(1 - \tau^{1/2})^2}, \quad (11)$$

where  $\tau$  is a dimensionless timelike variable relating the total number density of bodies  $N$  to the initial number density  $N_0$ , i.e.

$$\tau = 1 - N/N_0, \quad (12)$$

and where  $n$  is in units of  $N_0$  and  $m$  is in units of  $m_0$ .

Under the circumstances considered by Safronov, the ratio  $(N/N_0)$  decays exponentially:



$$\left(\frac{N}{N_0}\right) = (1 - \tau) = e^{-\eta}. \quad (13)$$

The dimensionless scaled time  $\eta$  is given by

$$\eta = A_1 N_0 m_0 t \quad (14)$$

where  $t$  is actual time, and  $A_1$  is the constant coagulation coefficient that represents the assumed proportionality of the growth rates to the combined masses, in accordance with the value  $q = 3$  in Equation (9). At the initial time  $t = 0$ , it follows that  $N = N_0$  and thus that  $\tau = 0$ . At later times, the number of bodies decreases as a result of coagulation and  $\tau \rightarrow 1$  for very long times.

Safronov's result is plotted in cumulative form in Figure 3, including the initial transient. These curves describe the archetypal form of nonrunaway growth. Although some differences are found when the assumptions are varied, all calculations of orderly growth have the general morphology of

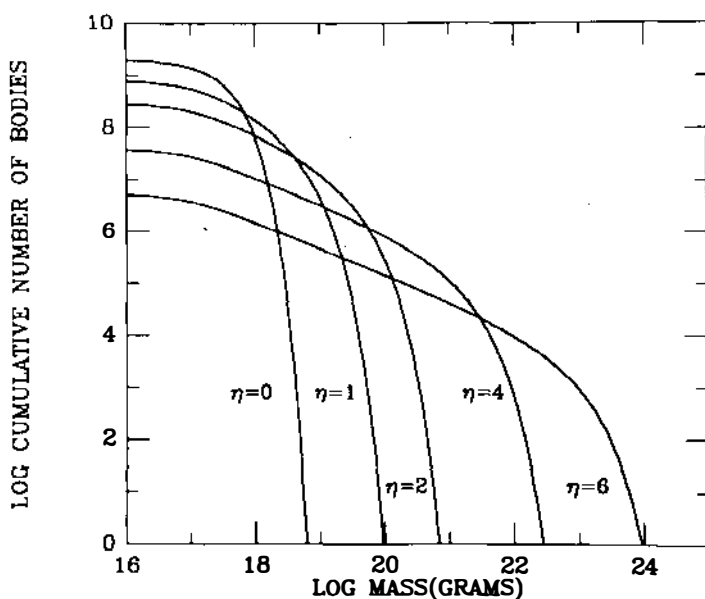


Figure 3 Solution of the coagulation equation with linear kernel (growth rate proportional to sum of masses) found by Safronov (1962, 1969). The initial conditions correspond to a reasonable planetesimal swarm in which the total mass of  $6 \times 10^{26}$  g is distributed over a 0.02-AU-wide zone centered at 1 AU, and the mean mass of the initial planetesimals is  $3 \times 10^{17}$  g. The equivalence to actual time of the dimensionless time variable  $\eta$  is model dependent, but  $\eta = 1$  approximately corresponds to  $3 \times 10^5$  yr. With the passage of time, the initial steep distribution evolves into a power law, as described in the text.

the results plotted in this figure. These results are presented for several values of  $\eta$  and for a reasonable choice of  $m_0$  of  $3 \times 10^{17}$  g and a total mass of  $6 \times 10^{26}$  g in a local zone 0.02 AU in width centered at 1 AU. Although the decay in  $N$  is not actually exponential in real physical time as a consequence of the thickness of the swarm expanding with time, roughly  $\eta = 1$  corresponds to  $t = 3 \times 10^5$  yr.

The initial very steep mass distribution of Equation (10) evolves into a rather flat power-law mass dependence ( $dn/dm \propto m^{-3/2}$ ) truncated at the high-mass end by an exponential remnant of the original steep distribution. With the passage of time, the upper end of the distribution “marches” forward in mass, but with diminishing amplitude. At the same time the power-law portion of the distribution includes an increasing range of masses. In this way an initial steep exponential distribution evolves into a power-law distribution in which most of the mass is concentrated in the largest bodies. Power-law distributions of this kind, with varying exponents, form the basis for much of Safronov’s treatment of planetary growth.

Although supported by other theoretical calculations, orderly growth was “built in” to Safronov’s result by the assumption that  $q = 3$ . This constraint that the growth rate is proportional to the mass was dropped in the subsequent work on this same problem by Nakagawa et al (1983). Consistent with the earlier studies of the Kyoto “school” [see Hayashi et al (1985) for references], this work differed from that of Safronov in that the velocity damping that balances the gravitationally induced increase in velocity was assumed to result from gas drag rather than from mutual collisions. Both of these phenomena can be expected to play a role in decreasing the velocity of the swarm. Safronov did not include gas drag, whereas Hayashi and his associates did not include collisional damping. The final effect of this considerable difference in assumptions is not as great as one might guess. The reason is that the steady-state velocity is approximately proportional to only the fourth root of the coefficients of the expressions for the energy changes associated with collisional and gas drag damping of velocity. Omission of one or the other of these terms will require only a somewhat higher steady-state velocity to establish the steady-state balance. It is sometimes stated that Hayashi and colleagues erred by considering gas drag to be important, whereas “it is well-known” that gas drag is of minor importance for large bodies. This criticism is unjustified, however. It is true that for the same relative velocity with respect to the gas, the effect of the gas drag force, proportional to  $R^2$ , falls off as  $1/R$  when the acceleration is obtained by dividing the force by the mass. In the absence of gravitational enhancement of the collisional cross-section factor  $F_g$  in Equation (3), this is also true for the acceleration

caused by collisional damping. In both cases the system will regulate itself by increasing the velocity  $v$  of the large bodies, thereby offsetting this dependence of  $dv/dt$  on the radius. When gravitational focusing causes  $dM/dt$  to be proportional to  $R^3$ , this  $1/R$ -dependence is no longer present for collisional damping, but the fourth-root dependence of the steady-state velocity requires a velocity difference of only a factor of two relative to the unfocused case when the radius of the body increases from 100 km to lunar size. For this reason, the basic results of Nakagawa et al (1983) are not very different from those of Safronov.

### *Equipartition of Energy in a Swarm Consisting of Bodies With Unequal Masses*

The effect of the swarm containing bodies of different mass is more important than differences associated with the relative importance of collisional and gas drag damping of velocity. In both the Tokyo-Kyoto and Moscow work, as well as the tutorial illustrative calculations of Wetherill (1980), the expressions for the gravitational “pumping up” of velocities were always positive. This is actually true only when the bodies in the swarm are equal in mass. Even if the initial masses were all equal, stochastic dispersion of the swarm’s mass distribution would quickly occur, if only because of combinatorial statistics. Stewart & Kaula (1980) showed that when the bodies were unequal in mass, the effect of their mutual gravitational perturbations could decrease their disordered components of velocity as well as increase them. This effect arises from the tendency of gravitational perturbations to equipartition energy between bodies of different size, analogous to the familiar nongravitational equipartition of energy in the kinetic theory of gases. This phenomenon is related to the “dynamical friction” introduced by Chandrasekhar (1943) in stellar dynamics. Its effect is to introduce additional terms into the theoretical expressions describing the gravitationally induced change of velocity of a body of mass  $m_1$  and velocity  $v_1$  by perturbations due to a second body of mass  $m_2$  and  $v_2$ .

These terms are of the form

$$\frac{dv_1}{dt} \propto (m_2 v_2^2 - m_1 v_1^2). \quad (15)$$

If  $v_1$  is approximately equal to  $v_2$ , and  $m_1 > m_2$ , these terms tend to cause the velocity  $v_1$  to decrease. The analogous terms for  $dv_2/dt$  will have the opposite sign. Thus, a dispersion in the mass distribution will lead to velocity changes that decrease the average velocity of the larger bodies of the swarm, increase the average velocities of the smaller bodies, and leave relatively unchanged the velocity of bodies intermediate in mass.

The work of Stewart & Kaula (1980), Stewart (1980), and Hornung et al (1985) has been extended by Stewart & Wetherill (1988) using Boltzmann and Fokker-Planck equations to describe the velocity distribution of an assemblage of bodies in near-Keplerian orbits. Expressions for  $d\mathbf{v}/dt$  as a function of the masses and velocities of the interacting bodies were obtained that include mutual gravitational perturbations, collisions, and gas drag. These expressions were then used in numerical calculations in which the evolution of the coupled velocities and masses of a swarm of bodies was followed in successive time steps, during each of which bodies grew in accordance with an expression equivalent to Equation (3).

Because both the changes in mass and in velocity during a single time step are nonlinear functions of all the other masses and velocities, a general solution of these coupled equations cannot be obtained. On the other hand, it is found that for physically realistic initial conditions, the numerical solutions bifurcate into two qualitatively distinct types: runaway vs. orderly growth. These calculations are more complex analogues of the oversimplified system described by Equation (9), in which two non-interacting large masses both grew by sweeping up many much smaller bodies at a constant value of the parameter  $q$  and the Safronov number  $\vartheta$  [Equation (4)]. In the general case the mass distribution is nearly continuous, and  $\vartheta$  is neither a free parameter nor constant with either time or mass but rather is determined by the evolving mass and velocity distribution.

### *Numerical Calculations Corresponding to Runaway and Nonrunaway Growth*

In the earlier work of the Moscow and Kyoto groups, their solutions followed the branch of orderly growth (Safronov 1969, Nakagawa et al 1983). These results were reproduced by the calculations of Wetherill & Stewart (1989) when the equipartition-of-energy terms [Equation (15)] were omitted, in accordance with the formulation of the problem by these workers. For orderly growth,  $\sim 10^{25}$ -g sublunar-size bodies formed at 1 AU in about  $10^6$  yr (Figure 4). In contrast, when these terms were included, the same initial conditions used by these authors led to a rapid runaway in which objects larger than the Moon formed in  $\sim 10^5$  yr, and a marked discontinuity of two orders of magnitude appeared between the masses of the large runaway body and those of the remainder of the swarm (Figure 5). The difference between these two calculations was caused by the much lower velocities of the larger bodies attributable to the equipartition-of-energy terms. As shown in Figure 6, after only  $3 \times 10^4$  yr, the velocities of all the bodies greater than about  $10^{21}$  g in mass have dropped below their initial values, whereas those of the smaller bodies have increased. As a consequence, the relative velocities of the largest body and bodies of

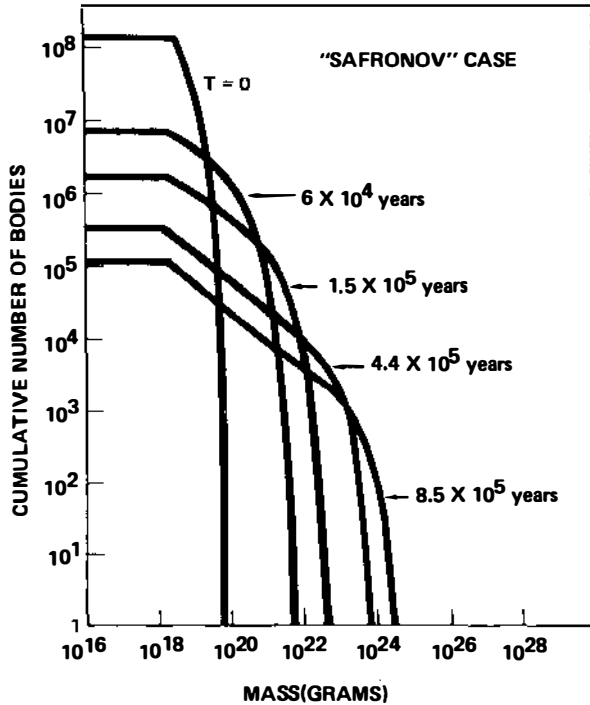


Figure 4 Numerical calculation of the evolution of the mass distribution of a swarm of planetesimals, for which the velocity distribution is determined entirely by the balance between positive-definite gravitational "pumping up" of velocity and collisional damping (Wetherill & Stewart 1989). The initial mass of the swarm was  $6 \times 10^{26}$  g, distributed between 0.99 and 1.01 AU with mean initial mass of  $3 \times 10^{18}$  g. The growth is found to be "orderly" and generally similar to the analytical result of Safronov, shown in Figure 3. No runaway is found; the evolution leads to a final mass distribution in which most of the mass is concentrated in  $10^{24}$ – $10^{25}$  g bodies at the upper end of the distribution.

intermediate mass will decrease, and the gravitational focusing factor  $F_g$  [Equation (4)] will become larger, increasing the rate at which the largest body sweeps up bodies of intermediate mass. This faster increase in the mass and, consequently, in the escape velocity of the largest body causes it to grow even more rapidly. This leads to the runaway instability seen in Figure 5. Because these calculations refer only to local zones 0.02 AU in width, it may be anticipated that many similar runaways lead to an assemblage of runaway bodies in concentric circular orbits around the Sun.

Similar runaways were reported by Greenberg et al (1978) under circumstances for which Wetherill & Stewart (1989) found no runaway. Those familiar with the computer code used by Greenberg et al believe

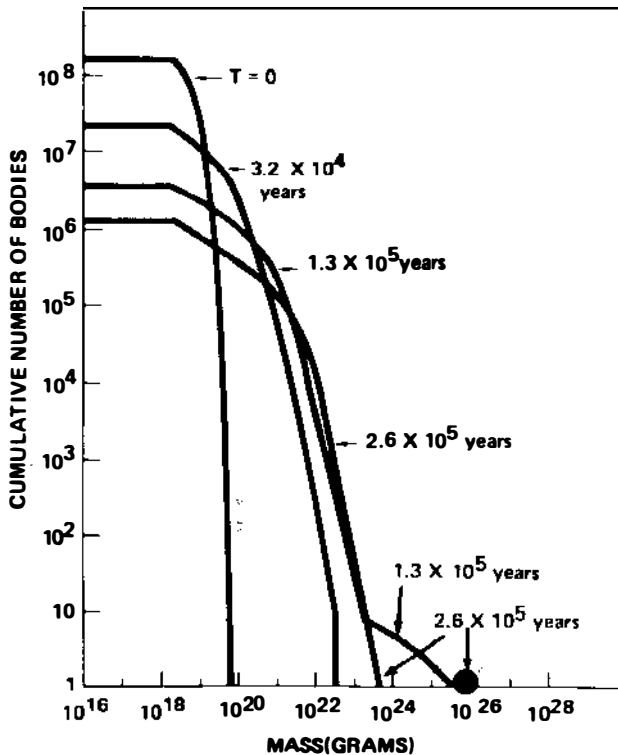


Figure 5 Effect of including the equipartition-of-energy terms [Equation (15)] in the calculation of the evolution of the same initial swarm as that in Figure 4 (Wetherill & Stewart 1989). The tendency toward equipartition of energy results in a velocity dispersion in which the velocity (with respect to a circular orbit) of the massive bodies falls below that of the swarm. After about  $10^5$  yr, a "multiple runaway" appears as a bulge in the mass distribution in the mass range  $10^{24}$ – $10^{25}$  g. After  $2.6 \times 10^5$  yr, a single largest body of mass  $\sim 10^{26}$  g (large solid circle) has swept up the bulge, and the continuous distribution of remaining bodies have masses  $< 10^{24}$  g.

that numerical inaccuracies in the calculations of Greenberg et al are the most likely explanation of this difference (S. Weidenschilling; C. Patterson, D. Spaute, private communication, 1989). This difficulty is likely to be related to considerations of mass-binning in numerical solutions of coagulation equations discussed by Ohtsuki et al (1989).

### *Other Factors That are Likely to Influence Runaway Accumulation*

Both the model and the theoretical expressions used in all of these calculations are obviously oversimplified. In an actual circumstellar nebula,

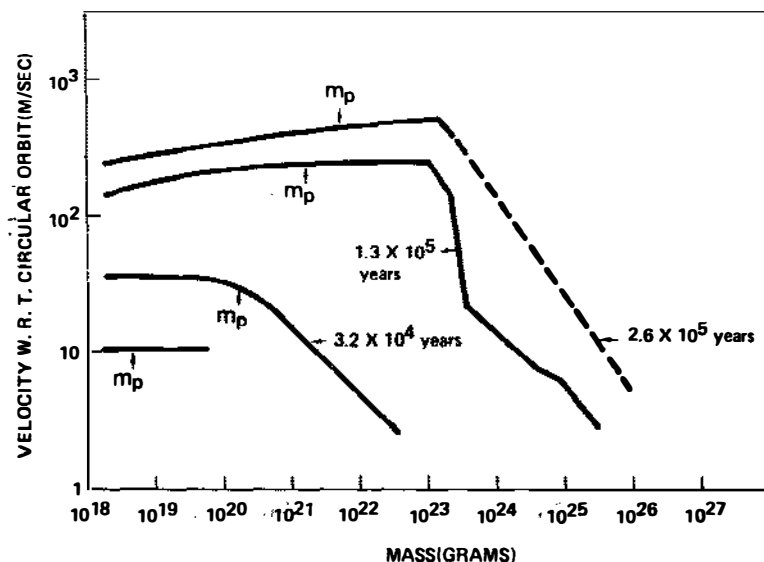


Figure 6 Velocity distribution resulting from inclusion of equipartition-of-energy terms (Wetherill & Stewart 1989). After  $3 \times 10^4$  yr, the relative velocities of the largest bodies drop well below the midpoint mass  $m_p$ , the value for which half the mass of the swarm is in smaller bodies. This leads to a rapid growth of the largest bodies and ultimately to a runaway.

significant initial discontinuities in the mass distributions may have existed. Deviations from the two-body expression [Equation (4)] for gravitational focusing are found at low values of  $\bar{V}_{\text{rel}}/V_e$  by numerical integration (Nishida 1983, Wetherill & Cox 1985, Ida & Nakazawa 1989, Greenzweig & Lissauer 1990). The smaller bodies of the swarm have relative velocities so large that they will fragment when they collide; this feature was introduced into such calculations for the first time by Greenberg et al (1978). The rate of growth of the runaway body may also increase because the body's gravitational field concentrates nebular gas in its vicinity, increasing gas drag and lowering velocity (Takeda et al 1985, Ohtsuki et al 1988).

The complications introduced by inclusion of these additional effects are formidable, and they have not been considered in anything like an exact way. Nevertheless, it is reasonably clear that all the phenomena listed above will increase both the likelihood and the rate of runaway rather than decrease them. Approximate inclusion of several of these effects by Wetherill & Stewart (1989) indicates that at 1 AU, bodies as large as  $1.2 \times 10^{26}$  g (about two lunar masses) may grow in less than  $10^5$  yr. Such rapid growth encroaches on the settling and growth time scales

of grains in the solar nebula, and these could turn out to be the rate-limiting factors during this first stage of planetesimal growth.

Are there also factors that may limit the runaway, or inhibit it and lead to orderly growth? Three such phenomena are now discussed, the last two of which seem unlikely to be of much significance for the formation of the Earth at 1AU, even though they may be important for the formation of asteroids.

1. As the planetesimals grow into embryos, the supply of available smaller bodies necessary for their continued growth is exhausted, either because these bodies are beyond the "gravitational reach" of the embryos or because neighboring embryos accumulated them first.

During the orderly planetesimal growth depicted in Figures 3 and 4 and the early stage of that shown in Figure 5, the relative velocities are found to be high enough for the gravitational interactions of the bodies to be dominated by two-body close encounters of bodies in orbits that cross one another. Collisions leading to growth represent a subset of these close encounters that are unusually close. Under these circumstances, it was valid to use two-body dynamics and simply consider the evolution of the planetesimal swarm by the methods of the kinetic theory of gases, with each planetesimal being equivalent to a "gas molecule" moving in a gas comprised of other planetesimals. The encounter and growth rates are determined by the local density of the planetesimal "gas." At later times, much of the mass of the swarm will be concentrated in larger ( $\sim 10^{25}$ -g) bodies in neighboring heliocentric zones, and continued growth requires that these larger bodies still collide and merge with one another. When the growth is orderly, the relative velocities of even the largest bodies remain high enough to permit the orbits of neighboring bodies to cross. Under these circumstances, there is little tendency for the embryos to isolate themselves from their neighbors.

The situation is different when runaway growth occurs. Because of equipartition of energy, the orbits of the larger bodies that form the "bulge" in Figure 5 are found to become nearly circular; as a consequence, the bodies tend to become isolated, in the sense that as long as their orbits remain unchanged, close encounters and collisions cannot take place. Nevertheless, as shown by Dole (1962) and Giuli (1968), objects in apparently isolated concentric circular orbits can under some circumstances collide with one another. This is possible because at the low relative velocities associated with concentric circular orbits, gravitational interaction continues for a long time as bodies at slightly different heliocentric distances slowly pass one another as a consequence of their circular Keplerian motion. In these cases, the integrated effect of the gravita-



tional perturbations caused by the prolonged, relatively distant approach turns out to be sufficient to change the isolated orbits into crossing orbits, permitting growth to continue. On the other hand, if noncrossing orbits are too distant from one another, the mutual perturbations will be insufficient to end their isolation.

Birn (1973) found that isolation will continue if the semimajor axes of the concentric circular orbits are separated by more than about

$$\delta = 2\sqrt{3}r_H. \quad (16)$$

The quantity  $r_H$  (the "Hill sphere radius") is given by

$$r_H = a(m/3M_\odot)^{1/3}, \quad (17)$$

where  $a$  is the heliocentric radius, and  $m$  is the sum of the masses of the two planetesimals or embryos. At 1 AU,  $r_H$  will have values of about  $10^{-3}$  AU and  $10^{-2}$  AU for  $10^{25}$  g and  $1-M_\oplus$  bodies, respectively. A similar result was found by Graziani & Black (1981) and by Artymowicz (1987). Although the coefficient of  $r_H$  in Equation (16) depends somewhat on the ratio of the masses of the interacting bodies, as well as on their eccentricities and inclinations, similar results are found for small deviations from circularity (Wetherill & Cox 1984, 1985, Artymowicz 1987, Ida & Nakazawa 1989, Greenzweig & Lissauer 1990). The smaller bodies of the residual swarm will have eccentricities of  $\sim 0.007$ , and therefore their orbits will span the positions of several larger bodies in the bulge, at least as long as these larger bodies avoid colliding with the largest body. A possible quasi-steady-state outcome of this stage of growth would be a series of embryos in concentric circular orbits, separated by the minimum distance  $\delta$  [Equation (17)], each embryo having a mass

$$M_e = m/2 = 2\pi a \sigma \delta, \quad (18)$$

where  $\sigma$  is the mass surface density of the annulus. Equations (16) and (17) can be substituted into (18), which can then be solved for the mass of the embryo:

$$\begin{aligned} M_e &= 8\sqrt{2} \times 3^{1/4} \pi^{3/2} a^3 \sigma^{3/2} M_\odot^{-1/2} \\ &= 6.21 \times 10^{24} a^3 \sigma^{3/2}, \end{aligned} \quad (19)$$

where in the second expression  $a$  is expressed in astronomical units and  $\sigma$  in grams per square centimeter. If this configuration develops, at 1 AU, runaway embryos will grow to masses of  $1.1 \times 10^{26}$  g to  $5.6 \times 10^{26}$  g for the range of surface densities ( $7\text{--}20$  g cm $^{-2}$ ) required to supply the mass of the present terrestrial planets, depending on the range of heliocentric distance from which their material was obtained. Although the upper end

of the mass range is similar to the mass of Mars, it is only 10% of the mass of the Earth, and merger of a fairly large number of embryos will be required to complete the growth of the Earth. It is also possible that equipartition of energy will inhibit the merger of the bodies in the bulge until the smaller bodies are nearly all accumulated. This would effectively reduce the coefficient of  $r_H$  in Equation (16) and result in a larger number of smaller embryos. Merger of runaway embryos is the final stage in the formation of the Earth and is discussed in the next section.

2. Another factor that under some circumstances may inhibit or even prevent runaway growth is the size distribution of the initial planetesimals. The equipartition-of-energy terms that are responsible for the runaway [Equation (15)] require that the swarm contains bodies that differ in mass in order to establish the differences in velocity seen in Figure 5. Even if all the bodies in the swarm had identical masses at the outset, the mass distribution will disperse simply as a result of stochastic variations in the collision rates of the very large number of bodies in the initial swarm. For example, for a swarm of originally equal-mass bodies, as soon as two bodies collide and merge, the mass of the largest body will be twice that of those comprising the rest of the swarm. If the initial swarm contains  $10^{10}$  bodies, about 8 of them will collide and merge 12 times by the time most of the bodies have collided once. The ratio of the mass of the single largest body to that of the mean mass of the swarm will increase as the number of bodies in the swarm increases, and decrease when the number of bodies decreases, simply as a consequence of combinatorial probabilities. If the initial swarm consists of a small enough number of large bodies, the dispersion will be so small that the runaway will develop so slowly that the largest bodies will still be of similar mass when they grow to about lunar size.

It is not expected that this phenomenon will be important for the Earth and terrestrial planets. The initial masses for the runaway case shown in Figure 5 were about those expected by the expression of Goldreich & Ward (1973) for the dust-layer gravitational instability. As discussed by Weidenschilling et al (1989), gravitational instability is not particularly likely, and the first-formed planetesimals in the central plane of the solar nebula would probably be smaller (0.1–1.0 km), leading to greater dispersion and a more rapid runaway. The Goldreich-Ward bodies expected at the distance of Jupiter would be  $\sim 80$  km in diameter. It is conceivable, but not likely, that a nearly uniform-size population of such large planetesimals could form at 5 AU and greatly inhibit the runaway growth of Jupiter's core. But in the terrestrial planet region, this is even more implausible.

3. The calculations described in the foregoing discussion were all made

under the assumption that, apart from solar gravity, the only forces acting on the planetesimals were those associated with their mutual gravitational perturbations, collisions, and gas drag. It is also possible that long-range perturbations by massive, distant, external perturbers could increase the velocities of the growing planetesimals. If these perturbations could offset sufficiently the decrease in velocity of the incipient runaway embryos caused by equipartition of energy, the runaway might be aborted. It is also possible that external perturbations could increase the velocities of the smaller bodies of the swarm so much that fragmentation, rather than growth, would dominate.

The principal candidate external perturbers in the early solar system are Jupiter (and Saturn) and perturbations associated with loss of the gas of the residual solar nebula (Heppenheimer 1979, Ward 1980). Because the time scale for runaway growth of embryos at 1 AU is so short ( $\sim 10^5$  yr), it does not seem likely that Jupiter would exist at that time, nor that the gas would be lost so soon, inasmuch as the gas is required to form Jupiter and Saturn. For this reason, external perturbations are not expected to be of major importance in inhibiting runaways in the terrestrial planet region, even though they could be of great importance in the outer solar system and the asteroid belt.

## FINAL STAGE OF ACCUMULATION: EMBRYOS TO PLANETS

### *The Onset of the Final Stage of Accumulation*

Runaway growth of planetesimals is expected to yield planetary embryos in the mass range of  $10^{26}$ – $10^{27}$  g, separated from one another by 0.01–0.02 AU on a time scale of about  $10^5$  yr. In accordance with Equations (16) and (19) the spacing and mass of the embryos increase with heliocentric distance. Collision and merger of about 30 such embryos, distributed between about 0.7 and 1.1 AU, can provide the mass, energy, and angular momentum of the present terrestrial planets. A less likely alternative is that for some unexpected reason, runaway growth of embryos did not take place. In this case, orderly growth from planetesimals to embryos would be expected to produce a larger number (e.g. 1000) of smaller bodies with sublunar masses of about  $10^{25}$  g.

Actual formation of the Earth and other terrestrial planets requires that in either case these embryos can make close encounters with one another; otherwise, collisions and growth will not occur. For the case of orderly growth this is not a problem because the calculated eccentricities of the embryos are well above the values necessary to allow their orbits to cross one another. For the case of runaway growth the situation is more com-

plicated and less well studied. Nevertheless, the onset of the final stage of growth can be discussed in a semiquantitative way.

In the early stages of runaway growth, when the incipient runaway body has a mass of  $\sim 10^{24}$  g, almost all the mass of the swarm is still in small,  $\lesssim 10^{21}$ -g bodies (Figure 5). Under these circumstances, equipartition of energy with these smaller bodies is found to maintain the incipient runaways in very nearly circular and coplanar orbits. As the runaway embryos grow into the  $\sim 10^{26}$ – $10^{27}$  g range, the total mass in the residual swarm will be depleted, reducing the constraint imposed by equipartition of energy. At the same time, the masses of adjacent embryos, separated by 0.01–0.02 AU, will have become larger, increasing the strength of the long-range gravitational perturbations between embryos. When the long-range perturbations begin to dominate, the changes in eccentricities caused by these perturbations will average between about 0.005 and 0.01, approximately that required to permit the orbits of adjacent embryos to cross. The magnitude of these perturbations often exceeds the average by more than a factor of two, ensuring orbital crossing (Greenzweig & Lissauer 1990). Once orbital crossing of embryos is achieved, at least after nebular gas has been removed from the terrestrial planet region, close encounters between embryos will increase their relative velocities to values comparable to their escape velocities of about  $3 \text{ km s}^{-1}$ . Such velocities correspond to eccentricities of about 0.1, which are sufficient to permit every embryo to cross the orbits of several other embryos. In addition, as embryos approach a mass of  $\sim 10^{26}$  g, radial migration resulting from gravitational interaction between the embryos and any nebular gas still present (Ward 1986) can also be expected to lead to mutual orbital crossing, particularly if there are fairly large stochastic variations in the masses of adjacent embryos. For these reasons, it is not expected that the runaway embryos will become gravitationally or collisionally isolated from one another. If anything, it seems more likely they will begin to interact with one another somewhat before the runaway is completed.

### *Multiplanet vs. Single-Planet Accumulation*

The task of understanding the formation of the Earth and terrestrial planets then becomes that of understanding the subsequent dynamical evolution of these embryos moving in crossing orbits. Because the masses of the embryos are comparable to the masses of Mercury and even Mars, this final stage of accumulation will involve collisions of very massive bodies with one another. As orbital evolution proceeds, the number of embryos will decrease and their semimajor axes will become more widely separated. At some point, determined by their eccentricities, the bodies will become permanently gravitationally isolated from one another, and

the evolution will cease. The remaining embryos can then be designated "final planets."

Thus, during this final stage of planetary growth, the formation of the Earth necessarily becomes a problem in multiplanet, rather than single-planet, accumulation. In some investigations this fact has been set aside and the accumulation of the Earth was formally allowed to proceed as if at any early stage a single embryo was designated "Earth" and the remaining planetesimals were assumed not to grow larger, at least within a designated "feeding zone" surrounding the planet. Calculations of this kind have contributed much to our understanding of planetary formation. The very important pioneering investigations of Safronov (1969) and the Kyoto-Tokyo group (e.g. Nakagawa et al 1983) fall into this category.

To some extent, the successes of these contributions are attributable to the use of positive-definite expressions for the mutual gravitational perturbations of the embryos, i.e. they omitted the equipartition-of-energy terms introduced by Stewart & Wetherill (1988). The resulting orderly growth has the consequence that the distinction between the earlier stage during which planetesimals grow into embryos and the later one of embryos growing into planets is weakened. At all times, the planetesimals/embryos are crossing the orbits of many neighbors, and relative velocities remain near the escape velocity of the larger bodies throughout their growth. Appropriate long time scales ( $10^7$ – $10^8$  yr) for complete accumulation of the Earth are found. Within the scope of single-planet formation theories of this kind, it is even possible to infer the possible occurrence of giant impacts (Wetherill 1976), although it must be admitted that the theory provides no way of distinguishing between a giant impactor and an isolated second largest planet.

This situation is quite different in the case of runaway growth of embryos, now recognized to be physically plausible. Oversimplified continuation of such calculations leads to a single Earth-sized planet in  $\sim 10^5$  yr. Of course, a single-planet formation theory can tell nothing about the final number, heliocentric distances, or relative masses of the final planets. With the exception of the work of Horedt (1985), all studies of multiplanet accumulation have been numerical. Treatment of the orbital evolution of embryos into final planets is in one way difficult and in another way relatively easy. One difficulty stems from inapplicability of the kinetic theory approximation to systems containing only from four to several hundred bodies, distributed over a wide range of heliocentric distance. A greater difficulty is the present uncertainty regarding the possible importance of nebular gas during the beginning of the final stage of accumulation. The relative ease is due to the possibility of following the orbital history of each individual body.

### *Effect of Nebular Gas During the Final Stage of Accumulation*

The question of whether or not nebular gas played an important role during the final stage of accumulation is a vexing one. It involves two questions: whether or not a major fraction of the nebular gas was still present at 1 AU at the time the planetary embryos were formed, and if so, whether or not this may be expected to make any difference either to the dynamics of accumulation or to the chemical nature of the final planets. It is not possible to give a clear answer to either of these questions at present.

Before the likelihood of runaway formation of embryos was established, it seemed reasonably safe to ignore the complexities of embryo-nebula interaction and to simply assume that nebular gas would be lost within the first million years of the  $10^7$ – $10^8$  yr accumulation time of the final planets from embryos. Gas drag forces on  $>10^{25}$ -g embryos were expected to be small compared with other forces that can decrease relative velocities, i.e. collisions and equipartition of energy (“dynamical friction”). This view fits in well with the idea that initially well-collimated bipolar flows from pre-main-sequence stars may continually erode the nebular gas at the edges of the more or less conical boundaries of the flow. Removal of this gas, proposed to be responsible for the collimation, then permits the cone to open further and eventually evolve into a more isotropic stellar wind (Boss 1987). It might be expected that in the course of this evolution of the outflow, a “hole” nearly devoid of gas would develop in the innermost part of the nebula on a time scale that is short (e.g.  $\sim 10^5$  yr at 1 AU) compared with the time scale for removal of all of the nebular gas. Thus, gas-free final accumulation of the terrestrial planets might proceed before the giant planets had completed their growth.

Now, however, it seems likely that runaway embryos are likely to form quite rapidly, in  $\sim 10^5$  yr, and this places more strain on this hypothesized chain of events. In addition, it has also been proposed (Takeda et al 1985) that for planetary embryos the gas drag coefficient may be greatly enhanced for a gravitating body at low values of the Reynolds number ( $Re \sim 20$ ). Ohtsuki et al (1988) have pursued the consequences of this enhanced gas drag. They find, among other things, that gas drag could then be the dominant “damping” force during the coalescence of embryos; that gas-drag-induced radial migration of large embryos may be important, in addition to that caused by tidal density wave effects (Ward 1986); and that relative velocities during the final stage of accumulation may be significantly lower than in the more popular gas-free case. Provided that all these inadequately studied effects can actually produce a system of

terrestrial planets similar to that observed, it is also quite likely that the time scale for the final growth of the Earth could be as short as  $10^7$  yr, which suggests that capture of a very massive ( $> 10^{26}$  g) and very hot ( $\sim 3000$  K) primitive atmosphere (Hayashi et al 1979) might then be expected if the nebular gas were still present at 1 AU for as long as  $10^7$  yr.

Ohtsuki et al (1988) conclude that the effects described above may be overestimated because of oversimplifications in the theoretical treatment of Takeda et al (1985), e.g. the assumption of Reynolds numbers lower than those expected in the solar nebula and the neglect of solar gravity. Nevertheless, subject to much more quantitative scrutiny, an evolution of this kind may be considered to be an alternative to the more conventional gas-free final accumulation of the terrestrial planets. This alternative could have observationally distinguishable consequences, but these have not yet been worked out completely enough to be discussed in a responsible way at present.

### *Numerical Simulations of the Final Stage of Accumulation*

A number of calculations have been made of the gas-free evolution of 50 to 1000 embryos in the terrestrial planet region, both in two dimensions (Cox & Lewis 1980, Wetherill 1980, Lecar & Aarseth 1986, Ipatov 1987, Beaugé & Aarseth 1990) and in three dimensions (Wetherill 1980, 1985, 1986, 1988). All of these calculations should be considered "Monte Carlo calculations," both because of the chaotic nature of the orbital changes caused by close encounters of the embryos with one another and also because of the impossibility of completely numerically integrating, even in two dimensions, the orbits of  $\sim 100$  bodies, all perturbing one another for  $> 10^6$  orbital periods. The two-dimensional (2D) calculations raise the additional problem of interpreting the results of 2D phenomena in terms of the real, three-dimensional (3D) world. There are essential differences between the 2D and 3D situations:

1. In two dimensions crossing orbits are always intersecting orbits, whereas in three dimensions crossing orbits will not in general intersect; they can simply be looped, like links in a chain.
2. In three dimensions the ratio of close encounters to actual collisions is much greater than in two dimensions, for the simple geometrical reason that in three dimensions the number of "misses" within a distance  $R$  of collision increases with  $R^2$ , whereas in two dimensions it increases only linearly with  $R$ .

On the other hand, there are potential advantages to 2D calculations, in that some phenomena, such as the effects of distant perturbations

and possibly resonances, can be addressed in two dimensions but are computationally prohibitive in three dimensions. In this regard, Lecar & Aarseth (1986) and Beaugé & Aarseth (1990) have carried out a total of four simulations of the final stages of terrestrial planet growth in which numerical integration provides some insight regarding the importance of distant vs. close encounters.

The most extensive available investigations are those of the author, who has reported the results of 45 3D Monte Carlo simulations of the final stages of terrestrial planet growth and has also carried out more than 100 unpublished calculations, studying the effects of a wide range of initial conditions, assumptions, and physical parameters involving swarms of up to 1000 bodies. These variations include exploratory studies of the probable effects of collisional fragmentation, tidal disruption by encounters within the Roche limit, irregular mass distributions as a function of semi-major axis, and commensurability resonances with the orbital periods of Jupiter and the largest embryos. Although the techniques used to simulate these phenomena were rudimentary, it does seem that these studies show that none of these effects are likely to qualitatively modify the terrestrial planet evolution found when these phenomena are not included. Perhaps the most interesting conclusion of these studies is the general similarity of the final results, despite their other differences. In all cases the large number of initial embryos spontaneously decreased to a small number of final planet-size bodies, almost always between 3 and 5, distributed throughout the region occupied by the observed terrestrial planets. In all cases, the growth of Earth-size planets was punctuated by giant impacts, of the size proposed by Cameron & Ward (1976) and Hartmann & Davis (1975) as responsible for the formation of the Moon. Another general feature is the widespread radial migration of the smaller embryos as they are passed back and forth between the larger planetary bodies "Earth" and "Venus" and into the terrestrial planet space beyond these planets. A ubiquitous result is the extensive heating of the larger bodies. Bodies more massive than  $0.5 M_{\oplus}$  are invariably melted, as are most Mars-size final planets. All of these same phenomena are seen in the 2D calculations as well. The only obvious difference between the 3D and 2D calculations are the time scales for accumulations. In three dimensions the Earth requires about 25 m.y. to grow to half its present mass and about 100 m.y. to grow to 99% of its present mass. In two dimensions the time scales are shorter by about a factor of 100, an artifact of the geometrical difference in orbital intersections mentioned earlier.

All of these calculations, as well as all those that will be made in the foreseeable future, are inevitably subject to potentially serious limitations. Little will be gained if authors simply annihilate one another by empha-



sizing the deficiencies in the calculations of other people. A more fruitful and positive approach, whereby significance of limitations in one mode of calculation are tested by alternative approaches that involve limitations of a different kind, may be expected to sort out the serious problems from those that constitute only sophisticated nit-picking.

The Monte Carlo calculations carried out by the author represent an effort to break through the barrier presented by the overwhelming difficulty of integrating in three dimensions the orbits of hundreds of interacting bodies for more than  $10^8$  orbital periods. Even if such integration were possible, the orbital interactions in three dimensions would be determined in large part by precessional motion caused by external distant perturbers, such as the giant planets, which would also have to be included in the calculations in some way. Of course, a price must be paid for this simplification that permits the problem to be studied at all. Probably the most serious is the limitation to perturbations by bodies in crossing orbits. Comparison of the 2D calculations of Cox & Lewis (1980) and Wetherill (1980) with those of Lecar & Aarseth (1986) and Beaugé & Aarseth (1990) shows that in two dimensions neglect of distant encounters leads to premature isolation of embryos and an excessive number of final planets.

A possible deficiency of the 3D Monte Carlo calculations is the use of two-body expressions in calculating the mutual perturbations and collisions of planetary embryos. Most likely, this is not a serious problem. The calculations of Wetherill & Cox (1984, 1985) were motivated by this question. In view of the orbital complexity found by numerical integrations of close encounters at low velocity, it seemed possible that neglect of solar gravity would cause gross changes in both the collision rates and the averaged perturbations. This was found not to be the case. Individual restricted three-body trajectories deviated considerably from those found by two-body calculations when the velocity ratio

$$R_v = \frac{V}{V_e} = \frac{(e^2 + i^2)^{1/2}}{V_e} V_K \lesssim 0.35, \quad (20)$$

where  $e$  and  $i$  are, respectively, the unperturbed eccentricity and inclination of the smaller body,  $V_K$  is its circular Keplerian velocity, and  $V_e$  is the surface escape velocity of the larger body. Nevertheless, as long as  $V/V_e > 0.14$ , the averaged perturbations were within a factor of two of those found by the approximate Öpik Monte Carlo algorithm and only a modest enhancement in the collision probability was found. These relatively small effects have been confirmed by more complete investigations of the same problem (Ida & Nakazawa 1989, Greenzweig & Lissauer 1990). Approximate corrections for these effects have been introduced into

the Monte Carlo algorithms, but because the corrections are small and values of  $V/V_c < 0.1$  are rare, the effect of these changes is not noticeable. It is also sometimes stated that the use of random numbers in these calculations represents a deficiency, particularly since different choices of random numbers can lead to very different final planetary configurations. This is not a deficiency, because it is an essential characteristic of the chaotic nature of close-encounter orbits, as also demonstrated by numerical integration of planet-crossing orbits (Milani et al 1989).

Of the 45 published 3D simulations of the final stages of terrestrial planet growth, only about 10% lead to a final configuration quite similar to that of the observed terrestrial planets. Detailed illustrations of the evolution of the swarm have been presented for three such cases (Wetherill 1985, 1986, 1988). More often, the final configuration differs from the observed solar system in some significant way. For example, equivalents to one or both of the small planets (Mercury and Mars) may be absent, three  $> 2 \times 10^{27}$ -g planets may form, or a small planet may remain between the orbits of "Earth" and "Venus." It is not known to what extent these variations simply reflect the stochastic nature of planetary accumulation, or inadequacies in the model and technique of calculation. Probably both of these factors are important.

### *An Example of a 3D Simulation of the Final Stage of Accumulation*

Results of a previously unpublished calculation are presented here. This case illustrates a qualified success, in which no "Mars" was produced, even though several potential such objects were formed but later lost. The initial conditions are a variation from those presented earlier and represent only one of a large number of such variations that have been calculated (for example, see Wetherill 1990b). The initial swarm of 222 bodies extended from 0.44 to 1.32 AU, with most of the mass between 0.7 and 1.1 AU, in order to obtain a final state that matched the observed specific energy and angular momentum of the observed planets within a few percent (Figure 7a). The size distribution was that of a runaway, determined by an equation of the form of (19), except that the minimum separation of the embryos was taken as  $1.6 r_H$  instead of the value of Birn (1973) of  $2\sqrt{3} r_H$ . The time at which the local runaways reached their final masses as a function of heliocentric distance was taken to vary as  $a^{3/2}$ , to be inversely proportioned to the local surface mass density, and had a value of  $3 \times 10^4$  yr at 0.4 AU. The initial eccentricities were much lower than those of the previously published calculations, being barely those necessary to permit the orbits of the initial embryos to cross one another. Fragmentation, as described in Wetherill (1988) for initial runaway embryos, was included.

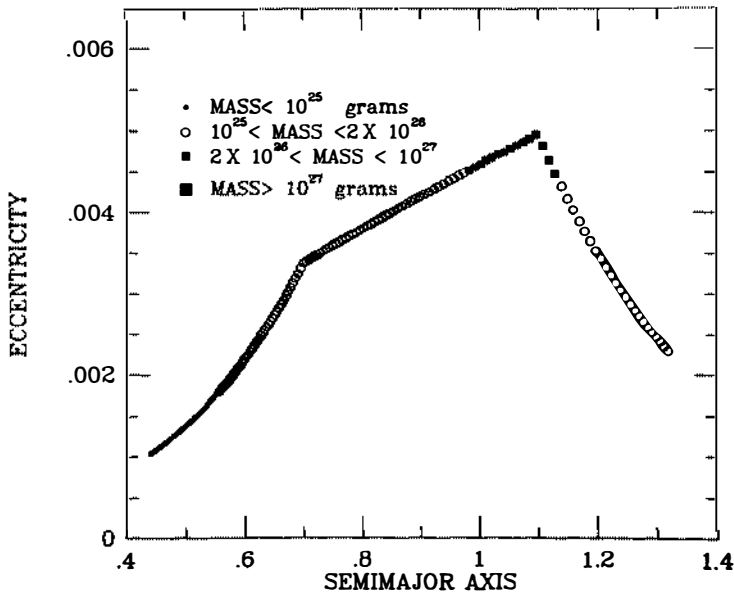
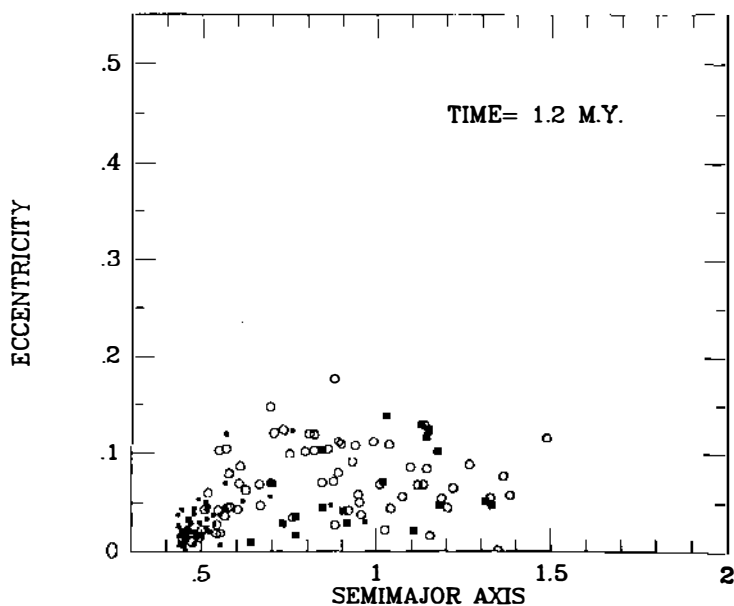
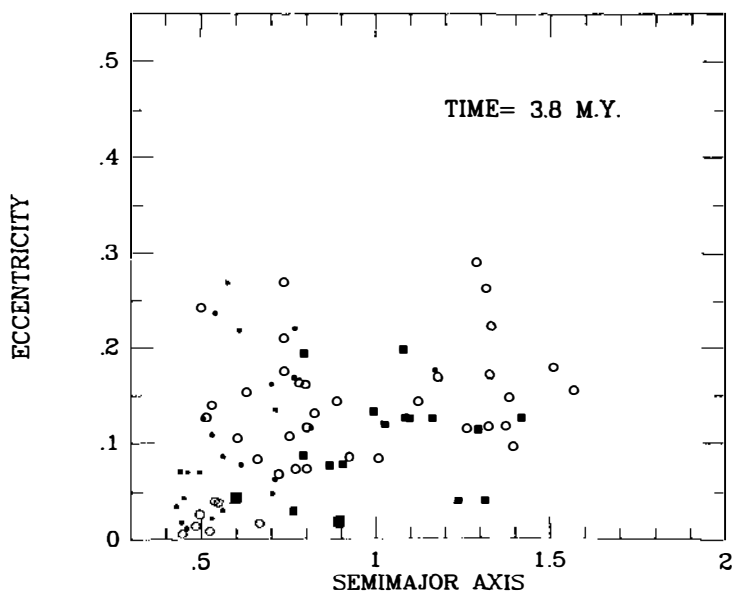


Figure 7a

*Figure 7* Simulation of the final stage of accumulation of terrestrial planets from runaway embryos. (a) Initial configuration of embryos using a surface density distribution that will evolve into one matching the present mass, energy, and angular momentum of the terrestrial planets. The symbols representing the masses of the bodies apply to Figures 7a–g. Initial eccentricities correspond to marginal orbit crossing of adjacent embryos. (b) Evolution of swarm after 1.22 m.y. Symbols same as Figure 7a. As discussed in text, radial mixing of bodies has occurred, and eccentricities have increased by an order of magnitude. (c) Swarm at 3.8 m.y. Symbols same as Figure 7a. *Large solid squares* indicate very large bodies with masses  $> 10^{27}$  g. (d) Swarm at 13.3 m.y. The large object at 0.95 AU will become the simulated “Earth.” The large body at 0.7 AU originally formed at 1.08 AU and migrated to 0.7 AU. This object will become the simulated “Venus.” (e) Swarm at 46 m.y. The growth of “Venus” continues with the merger of the two large bodies at 0.6–0.7 AU in Figure 7d. A potential “Mercury” may be seen at 0.45 AU, having a mass of  $2.0 \times 10^{26}$  g. It does not survive because of a collision at 56 m.y. with a  $2.5 \times 10^{26}$ -g body that is at 1.35 AU in this figure. (f) Swarm at 77 m.y. “Earth” has grown to 99% of its final mass, and Venus to 87%. A swarm of moderate-size collision debris now occupies Mercury’s position. After a complex chain of fragmentation and merger, some of these objects will form a final “Mercury,” completed at 178 m.y., and three potential Mars’ have  $a \gtrsim 1.5$  AU. None of these “Mars” objects survived. Two were ejected from the solar system, and the other ( $6 \times 10^{26}$  g) collided with Venus in an unusually late giant impact at 416 m.y. (g) Swarm at 879 m.y. Three final planets remain. Earth, Venus, and Mercury have masses of  $5.8 \times 10^{27}$  g,  $4.8 \times 10^{27}$  g, and  $3.2 \times 10^{26}$  g, respectively. The last impact on Earth was a 0.25-lunar-mass object at 220 m.y.

*Figure 7b**Figure 7c*

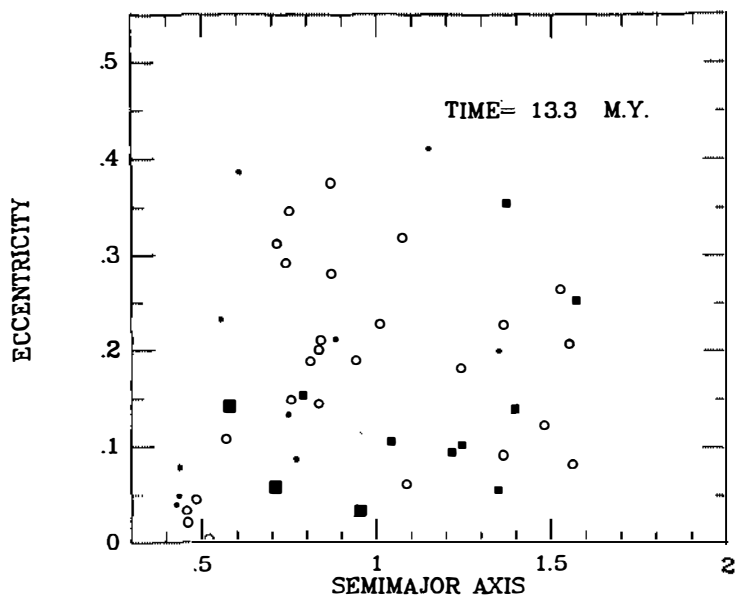


Figure 7d

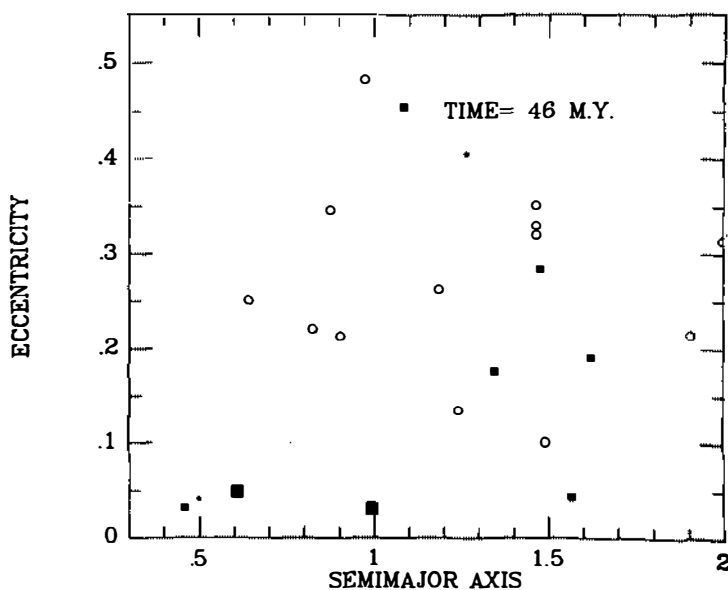


Figure 7e

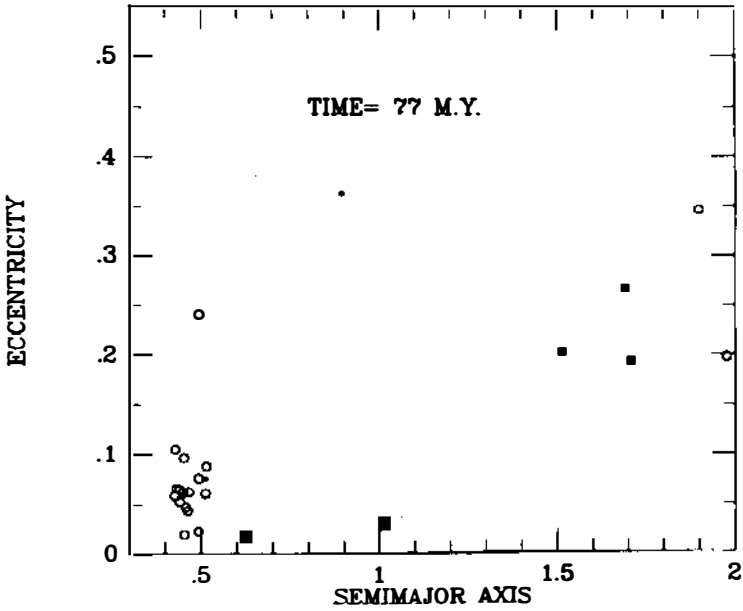


Figure 7f

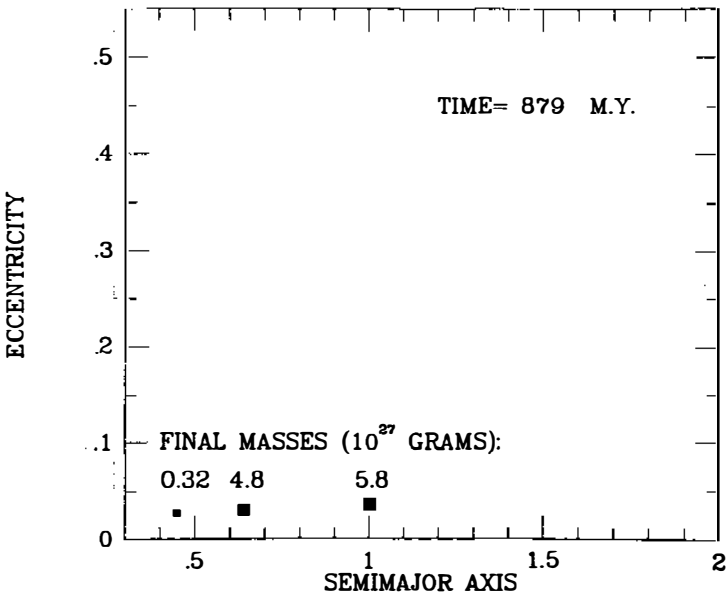


Figure 7g

These details are given in order to document what was done. In fact, it is found that the outcome of such calculations is not in any obvious way dependent on variations of this kind, and insofar as the results may be deemed unsatisfactory, they will not be significantly altered by changes of this kind.

The state of evolution of the swarm at selected times is shown in Figures 7*a-g*. The semimajor axis and eccentricity of each object is shown, with a symbol representing the mass range into which each body falls. The initial eccentricities are very low (Figure 7*a*) and were chosen to represent the situation when the small-body end of the swarm has been consumed by the runaway embryos sufficiently to relax the effect of equipartition of energy that earlier constrained the orbits of the embryos to be nearly circular. The largest initial embryos, indicated by solid squares, are between 1.13 and 0.99 AU and range in mass from  $2 \times 10^{26}$  g to  $2.8 \times 10^{26}$  g.

The mutual gravitational perturbation of the embryos caused their eccentricities to rise tenfold during the first 1.2 m.y. (Figure 7*b*). The larger initial bodies (solid squares) have been scattered between  $\sim 0.6$  AU and 1.35 AU and are well mixed with bodies that range in mass between  $10^{25}$  g and  $2 \times 10^{26}$  g. At this time the mass of the largest body is  $7.9 \times 10^{26}$  g, larger than the present mass of Mars, with a heliocentric distance of 0.64 AU. The initial mass of this body was only  $8.2 \times 10^{25}$  g, about lunar size. Its prominence is, in part, caused by the more rapid growth of runaways at smaller heliocentric distances, and in part stochastic. Six other bodies, all at greater heliocentric distances, have at least half the mass of this body.

At 3.8 m.y. (Figure 7*c*), the eccentricities have increased to values comparable to the escape velocities of the largest bodies. There are now two bodies with masses of  $\sim 1.2 \times 10^{27}$  g and with eccentricities well below the average (large solid squares). Both of these bodies have grown to this mass by collision and merger with smaller bodies. The provenance of these larger bodies spans a wide range of heliocentric distances, rather than being derived from individual feeding zones.

At 13.3 m.y. the spread in eccentricities and semimajor axis has increased further (Figure 7*d*). The massive body at 0.6 AU has increased its eccentricity and grown slightly to  $1.4 \times 10^{27}$  g. Two additional objects with masses  $> 10^{27}$  g are now found. The object near 0.95 AU was produced by the rapid growth of a body having a mass  $0.6 \times 10^{27}$  g at 3.8 m.y. into a  $3.75 \times 10^{27}$ -g body. In the course of its growth, at 11 m.y., it merged with the  $1.2 \times 10^{27}$ -g body shown at 0.9 AU on Figure 7*c*. This body is destined to become the simulated equivalent of "Earth." The other new body is at 0.7 AU with a mass of  $2.0 \times 10^{27}$  g. It is destined to become the final "Venus." This body was initially at 1.08 AU, with a mass of

$2.7 \times 10^{26}$  g. As a result of gravitational scattering by other bodies, it migrated to 0.7 AU, colliding with a number of smaller bodies during its migration.

By 46 m.y. (Figure 7e), the swarm is greatly depleted in bodies more massive than  $10^{24}$  g (Ceres size). The two  $>10^{27}$ -g bodies seen earlier around 0.6–0.7 AU have collided in a giant impact to form a single body with a mass of  $3.8 \times 10^{27}$  g. A body of mass  $2.0 \times 10^{26}$  has appeared near the position of Mercury. This body was formed by a complex series of breakup, melting, and reassemblage of objects (mostly bodies with original masses  $<10^{25}$  g) that populated the inner portion of the initial swarm (Figure 7a). A similar history is illustrated in Figure 11 of Wetherill (1988) for a body that eventually grew to become a simulated “Mercury.” In the present calculation, this body does not survive. At 56 m.y. it is disrupted by collision with the  $2.5 \times 10^{26}$  g body plotted in Figure 7e at  $a = 1.35$  AU and  $e = 0.18$ . The perihelion of this body evolved inward as a consequence of a series of chance encounters with other bodies, the most important of which were the two  $>10^{27}$ -g bodies “Earth” and “Venus.” Thus, a potential “Mars” and a potential “Mercury” were both lost by a single collision.

At 77 m.y. (Figure 7f), “Earth” at 1.0 AU has reached 99% of its final mass of  $5.83 \times 10^{27}$  g and “Venus” has grown to 87% of its final mass of  $3.84 \times 10^{27}$  g. An assemblage of melted collision fragments, mostly in the  $10^{25}$ -g size range, populate the region where  $a < 0.6$  AU. The largest of these has a fairly large mass,  $1.7 \times 10^{26}$  g. This body grew by reassemblage of smaller collision products, mostly resulting from the large collision described in the previous paragraph. It grew to a mass of  $3.1 \times 10^{26}$  before it was again disrupted at 161 m.y., after which it reassembled itself to provide the simulated “Mercury.” At 77 m.y. three potential candidates for Mars still remain between 1.5 and 1.7 AU, with masses of  $2.0 \times 10^{26}$  g,  $5.8 \times 10^{26}$  g, and  $7.6 \times 10^{26}$  g. None of these will survive. The largest and smallest of these bodies are to be ejected into solar system escape orbits, and the other object will strike “Venus” in an unusually late giant impact at 416 m.y.

At 879 m.y., three final planets remain (Figure 7g). Earth, Venus, and Mercury have final masses of  $5.8 \times 10^{27}$  g,  $4.8 \times 10^{27}$  g, and  $3.2 \times 10^{26}$  g, respectively. The last impact on “Earth” took place at 220 m.y. ( $1.9 \times 10^{25}$  g). The last impact on Venus was the giant impact at 416 m.y., and the reassemblage of “Mercury” was completed at 178 m.y. No “Mars” remains. Under different stochastically determined conditions, one of the planet-size bodies still present at  $\sim 1.6$  AU at 77 m.y. could have survived.

All three of the final planets have “melted,” in the sense that under the assumption that 75% of the center-of-mass impact energies were dissi-



pated internally as heat, the accumulated heat energy would be  $>3 \times 10^{10}$  ergs  $\text{g}^{-1}$ . This result is not sensitive to the assumption of 75% dissipation. Large planets like Earth and Venus are melted even for less than 25% dissipation, and the collision fragments that assembled to produce Mercury are also melted.

The growth of "Earth" was dominated by giant impacts: Six impacts took place with bodies intermediate in mass between Mercury and Mars, and two occurred with masses greater than Mars. The largest of these ( $1.2 \times 10^{27}$  g) occurred at 10.8 m.y. Finding so many giant impacts is unusual, but this result lies well within the range found for the more than 150 simulations that have been calculated. For the case considered here, the giant impact history of "Venus" is more typical: two impacts of bodies with masses intermediate between Mercury and Mars, and one more massive than Mars ( $1.4 \times 10^{27}$  g).

### *Comments on the Initial Distributions Used in These Simulations*

Although considerable progress has been made in understanding the processes that are likely to have accompanied the formation of the Earth, much remains to be understood. For example, attention should be drawn to the initial distribution assumed in the above simulation in which the semimajor axes of the swarm were concentrated in a fairly narrow range, narrower than that represented by the present range in heliocentric distance of the terrestrial planets. If the physical processes considered here are all that were of major significance, a restricted distribution of this kind is essential to the formation of these planets. The reason is that mass, specific energy, and specific angular momentum are nearly conserved during the growth. For bodies no larger than the Earth and so deep in the Sun's gravitational well, only  $\sim 5\%$  of the mass is lost from the system by ejection into hyperbolic solar system escape orbits. Except for angular momentum carried by escaping bodies, angular momentum is strictly conserved. Only several percent of the initial gravitational energy is dissipated as heat during the collisions responsible for the growth. For this reason, only a swarm with a mass, specific angular momentum, and specific energy nearly the same as that of the present solar system can evolve into the present planets (Wetherill 1978). This result is not only obvious but has been easily confirmed by use of inappropriate initial distributions.

Clearly, this confinement of the initial embryos to a narrow band of semimajor axes requires an explanation, but a satisfactory one is not available at present. The problem is that of explaining the absence of bodies  $\gtrsim 1000$  km in diameter ( $\sim 10^{24}$  g) between the orbits of Mars and Jupiter, and the small masses of Mars and Mercury. Inside the orbit of

Venus, it is possible that condensation of solid grains was precluded by high temperatures up to the time when solar outflow removed the nebular gas and small grains from the solar system.

In the middle solar system, between the orbits of Mars and Jupiter, a taxonomy of hypothetical explanations can be given for the deficiency of material:

1. There was a gap in the density distribution in the solar nebula, in contrast to the smooth decrease shown in Figure 1. Present theoretical models of the solar nebula give no support to this hypothesis.

2. Planetesimals  $\gtrsim 10$  m in diameter failed to form in this region, and the smaller bodies were removed from the region by gas drag (Weidenschilling 1988).

3. There was no marked initial depletion of material from this region, and planetesimals at least began to form there in the same way as at 1 AU, but on a longer time scale. This time scale was proportional to the Keplerian period, varying as  $a^{3/2}$ , and to the reciprocal of the surface density. For a surface density that varies as  $a^{-3/2}$ , the time scale for growth at 3 AU would be 27 times that at 1 AU, and a runaway could require  $> 1$  m.y. to develop. If Jupiter formed before the asteroidal runaways were completed, there are several ways that the presence of Jupiter (and Saturn) and the loss of nebular gas may have aborted planet formation in this region. These are discussed by Wetherill (1990a). Although some theory of this kind may turn out to explain the evolution of the asteroid belt, at least at present it does not seem very promising for the region between 1 and 2 AU.

4. Some other physical process, not included in the 3D simulations of the kind illustrated in Figure 7, may have "compressed" the embryo swarm into the narrow band required. One such possibility has been explored numerically (Wetherill 1990b). This is transfer of angular momentum to the residual solar nebula via spiral density waves (Ward 1986, 1988) generated by resonant interactions between embryos and the gas. By using these published coefficients for radial migration, together with a model for the removal of nebular gas interior to 2 AU, it seems that the necessary compression of the embryo swarm might be achieved. It now appears, however, that as a result of further development of the theory, the coefficients that I used are a factor of 10 or more too high because the effect is nearly canceled by additional resonances not previously included (W. R. Ward, private communication, 1989). If so, this process seems unlikely to provide the answer to the problem.

Because no satisfactory quantitative theory yet exists for the processes that depleted the region between Earth and Jupiter, it is not yet possible

to properly address the important question of a possible contribution of material removed from this region to a "late veneer" of more volatile-rich and oxidized material to the Earth and other terrestrial planets, particularly Mars (Dreibus & Wänke 1989).

## INITIAL STATE AND EARLY HISTORY OF THE EARTH

### *Initial High Temperatures*

It seems most likely that the stage of terrestrial planet growth in which planetesimals coalesced to form embryos led to the formation of many more lunar- to Mars-size planetary embryos than the four terrestrial planets seen today. The difference between the outcomes of orderly and runaway growth turns out to be only one of degree rather than kind in this regard. Orderly growth leads to a larger number of embryos, which of course must therefore be on average smaller.

The final stage of accumulation consists of the collision of these embryos. Because of their large size, particularly after their further growth, these collisions represent giant impacts. These effects have been illustrated for both the case of orderly (Wetherill 1985, 1986) and runaway growth (Wetherill 1988; see also Figures 7, 8 herein). In the case of runaway formation of embryos, even without further growth the larger embryos are in the size range considered in theoretical and numerical models of lunar formation involving giant impacts (Benz et al 1986, 1987, Melosh & Sonett 1986, Kipp & Melosh 1986). Furthermore, all of the embryos are vulnerable to merger with one another, and most of the giant impacts will be between embryos that have already grown far beyond their initial mass.

A consequence of these giant impacts is that the Earth should have been extensively heated and melted during all but the earliest part of its growth. Even if the Earth and the impacting embryo were in orbits with negligible components of relative velocity, when the Earth had reached half its present mass, its gravitational attraction alone would result in impact velocities of about  $9 \text{ km s}^{-1}$ . This corresponds to a kinetic energy of about  $4 \times 10^{11} \text{ ergs g}^{-1}$  of impactor, far above the  $< 3 \times 10^{10} \text{ ergs g}^{-1}$  required for melting. Impacts between planet-size bodies must be far from elastic, and much (or even most) of this energy would be deposited and distributed within the growing Earth, as found in the numerical simulations of Benz et al (1986, 1987).

In fact, the relative velocities of the colliding bodies will probably not be small as a consequence of the embryos undergoing many more near-misses with one another than actual collisions. Both the analytical and numerical calculations discussed in the previous section show that the

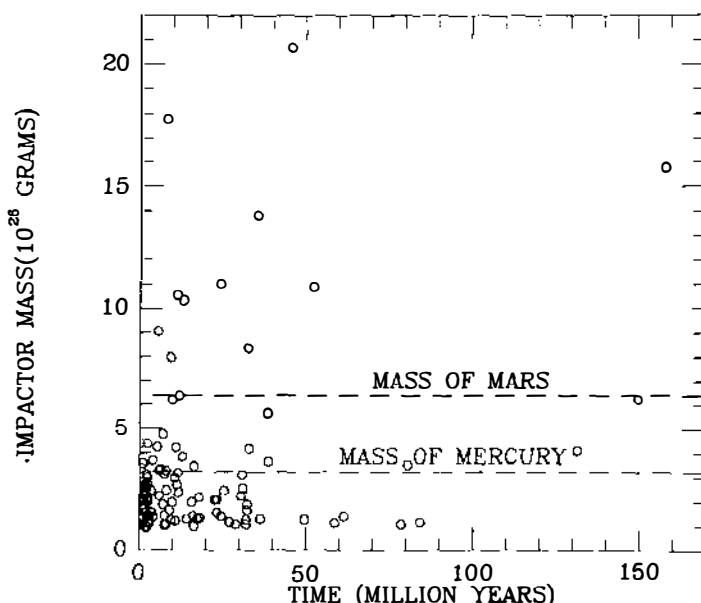


Figure 8 Distribution in size and time of giant impacts for 10 simulations of the final accumulations of runaway embryos, under the same assumptions used for the simulation shown in Figure 7. On average, each simulation is associated with one impact on "Earth" with an object more massive than the present mass of Mars and about two impacts with objects intermediate in mass between those of present-day Mercury and Mars.

random-walk accumulation of these close encounters results in relative velocities comparable to the escape velocities of the larger embryos, increasing impact heating by a factor of 2 to 4.

### *Radial Mixing of Planetary Material in the Terrestrial Planet Region*

Relative orbital velocities of  $\sim 10 \text{ km s}^{-1}$ , resulting from close encounters, are about one third of the circular Keplerian velocity at 1 AU. Therefore it should be expected that prior to their elimination by collision with a nearly formed planet, the smaller embryos will have eccentricities of  $\sim 0.3$ , sufficient to span the heliocentric distance between Earth and Venus. This causes these smaller embryos to be passed from the neighborhood of one planet to another, vitiating the concept of a chemically unique feeding zone for each final terrestrial planet. Numerical simulations of the final stage of accumulation show that this mixing does not entirely erase the memory of the initial provenance of the embryos that form the final planet, but it argues strongly against any planet gathering to itself a unique

chemical or isotopic component (Wetherill 1988). A possible, but by no means expected, exception to this conclusion regarding widespread mixing could be the gas-rich final stage of accumulation discussed by Ohtsuki et al (1988).

The chemical effects expected to accompany the growth of terrestrial planets from planetesimals have not yet been considered in any detail. Because the Sun and average solar nebula may be plausibly expected to be "chondritic," it is reasonable to suppose that the more refractory elements would be found in the Earth in nearly chondritic (or solar) proportions. On the other hand, there is no good reason to think the Earth was principally formed by the accumulation of any particular type of meteorite found in meteorite collections. It is extremely likely (e.g. Wetherill & Chapman 1988) that almost all undifferentiated meteorites are relatively recently fractured fragments of asteroidal bodies that have never been larger than about 1000 km in diameter. During formation of these asteroids, it seems likely that they experienced physical and chemical processing similar to that of the planetesimals that are supposed to have grown into the terrestrial planet embryos. Insofar as there may have been chemical and/or temperature gradients in the solar nebula, the outcome of this first stage of accumulation in the asteroid belt may be expected to have been different from that in the terrestrial planet region. In addition, the terrestrial planetary material was then subjected to the vicissitudes of accumulation into full-scale planets, accompanied by its own peculiar set of chemical and physical processes.

In view of the foregoing, is it reasonable to expect there has been some significant degree of mixing between the asteroidal meteoritical material and that of the Earth? The answer is probably yes, but so little quantitative work has been done in connection with this question that any answer may be considered to be speculative, albeit interesting. All calculations of the orbital evolution of the material from which the Earth formed show that a significant quantity of terrestrial material is transferred to the inner portion of the asteroid belt (Wetherill 1977, Ip 1989). Thus, it is plausible that to some extent the asteroid belt, greatly depleted in its own indigenous mass, was vulnerable to "pollution" with terrestrial matter. On the other hand, it is also by no means out of the question that the Earth was contaminated with large amounts of material from greater heliocentric distances—from the region of Mars, the asteroid belt, and the outer planets—as suggested on geochemical grounds (Dreibus & Wänke 1989). Chemical and isotopic differences between meteorites and the Earth demonstrate that chemical mixing of these regions was not complete. Progress in understanding these important matters from a dynamical point of view awaits replacement of uncertain preliminary calculations with serious quantitative work.

### *Theoretical vs. Observational Evidence Regarding the Initial State of the Earth*

The picture of the early 4.4–3.8 b.y. ago Earth that emerges from consideration of all even moderately quantitative theories of its formation is that of an initially extremely hot and melted planet, surrounded by a fragile atmosphere and subject to violent impacts by bodies of the size of Ceres and even the Moon, during the hundreds of millions of years that separate Earth's formation from the oldest, reasonably well-preserved geological record (about 3.8 b.y. in age). This picture is supported by the observational evidence for extensive early melting of a much smaller body, the Moon (Taylor 1982), regardless of how it was formed.

Can this picture be reconciled with the planetological, geological, and geochemical record? The answer seems to be: not easily. A proper discussion of this subject would require at least an additional review, for which I am certainly not the most prominent candidate author, and, more importantly, requires much further theoretical, observational, and experimental attention. Nevertheless, a little should be said.

There are things to be said on the positive side. The existence of the Moon, the gross difference in the inert gas composition of the atmospheres of Earth and Venus, the high I-Xe age of the Earth (Wetherill 1975, Pepin & Phinney 1976), and the large iron core of Mercury can be explained in a reasonably natural way in terms of the version of the standard model emphasized in this review. The Pb isotope age of the Earth's liquid core (Patterson 1956) and the enrichment in radiogenic  $^{143}\text{Nd}$  found in the most ancient rocks (Shirley & Hanson 1986), suggesting that the melted rocks were extracted from the Earth's mantle even earlier in its history, are at least supportive of the concept of a high-temperature primitive Earth.

Apparently serious problems arise, however, when one considers the chemical fractionation expected during the solidification of a once melted silicate mantle. Crystallization of minerals from large bodies of liquid silicate is invariably associated with some degree of major-, minor-, and trace-element chemical fractionation. The major minerals have different chemical compositions and densities from one another. The various minor and trace elements substitute with varying compatibility into the crystal structure of some of these major minerals. Differential gravitationally driven settling of major mineral grains causes the crystallization of large magma chambers to be compositionally layered. A melted Earth would constitute a magma chamber of global dimensions, and a layered terrestrial mantle would be expected to develop during the cooling and crystallization of the Earth's mantle during the first several hundred million years of Earth history.

Major-element data from samples of the Earth's mantle, combined with laboratory experimental data, have been interpreted as supporting this picture of a layered early Earth (Agee & Walker 1988). An important result of these laboratory studies is the inference that silicate melts of olivine-rich composition ( $\sim \text{Mg}_2\text{SiO}_4$ ) at upper-mantle pressure are likely to be more dense than the solid silicates crystallizing from these melts. Agee & Walker conclude that during the initial crystallization of a liquid Earth, this phenomenon may have produced a "buried magma ocean" of olivine composition, and that its subsequent crystallization, followed by convective mixing, may explain the elevated (above chondritic) Mg/Si ratio of the present upper mantle.

Trace-element data are difficult to interpret in terms of this model [but see Agee & Walker (1989) for a contrary view]. It is inferred from geochemical studies of mantle-derived rocks that a number of refractory trace elements have chondritic ratios in the mantle, e.g. Sm/Hf, Sc/Sm, Ir/Au, and Co/Ni. The principal minerals expected to form in a global crystallizing magma chamber as envisaged by Agee & Walker are olivine ( $\text{Mg}_2\text{SiO}_4$ );  $\text{MgSiO}_3$  with the dense, high-pressure perovskite structure; and the high-pressure garnet majorite. Laboratory data exist regarding the partitioning of these trace elements between these high-pressure minerals and the residual liquid that one would expect to finally crystallize in the upper mantle (Kato et al 1988a,b, 1989, McFarlane & Drake 1990). The data are interpreted by these workers as predicting observable fractionation of these elements in the upper mantle. Even though these experimental trace-element partition coefficients are not as precise as desirable, it seems unlikely that they are totally incorrect, or that observable trace-element fractionation would fail to accompany separation of at least some of these high-pressure minerals from a silicate liquid, given the differences in their atomic structures.

There are only a few general ways to resolve this paradox. One is that although the Earth was melted, the global-scale fractionation did not occur, perhaps because of mixing of residual liquid with the settling crystals (Tonks & Melosh 1990). It is also conceivable that although the fractionation did occur, it has subsequently been erased. The physical processes that accompanied the early Earth—giant and not-so-giant Moon-size impacts causing both mixing and more local partial melting and fractional crystallization, and possibly complex convective and solid-liquid separation processes—have been barely discussed. It is possible, but not necessarily expected, that the geochemical and dynamical pictures of Earth formation may converge when these processes are understood.

Finally, it is possible that the dynamical history of Earth formation described by the standard model is simply incorrect. This admission should

not be seized upon lightly. At the present time there are not even any semiquantitative theories extant that lead to an unmelted Earth. One must have faith that reliance on basic physical principles in the development of models of planet formation is the only way that the correct answer can ultimately be found. The presently available body of serious quantitative work, although quite incomplete, should not be cast aside lightly in favor of some qualitative "scenario" that may seem more pleasing. But it also should be remembered that doubt is the other side of the coin of faith, and one must clasp them both at the same time. This is natural, because in the absence of doubt there is no need for faith. It can then be replaced with certainty. We haven't reached that point yet.

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