Constants and conversions that may or may not be helpful:

- Boltzmann constant: k = 1.38×10^{-16} erg K^{-1}
- 1 Joule (J) = 10^7 erg
- sound speed: $c_s = \sqrt{\frac{\gamma P}{\rho}}$; $\gamma = \frac{5}{3}$
- ideal gas law: $P = nkT = \frac{\rho kT}{m_u \mu}$
- solar mass: $M_\odot = 2 \times 10^{33}$ g
- solar radius: $R_\odot = 6.96 \times 10^{10}$ cm
- gravitational constant: $G = 6.67 \times 10^{-8}$ cm^3 g^{-1} s^{-2}
- gravitational force: $\vec{F}_g = m\vec{g} = \frac{GMm}{r^2}\hat{r}$
- pressure scale height: $H = \frac{kt}{m_u g}$

1. This question addresses the concepts and calculations of the scale height.

   (a) **2 points:** In one or two sentences, explain qualitatively what the scale height is.

   (b) **4 points:** § 2.1 gives the pressure scale height as $H_P = c^2/(\gamma g)$. Show that this is equal to $H = kT/m_u g$. State any assumptions you make.

   (c) **5 points:** Using the expression given for scale height ($H_P = c^2/(\gamma P)$) and the values given at the bottom of the second column of page 2, calculate the scale height at the photosphere (the visible surface of the sun). If you don't know g, use 10^4. Does your answer make sense? Why or why not?

1. *Solutions*

   (a) **2 points:** In one or two sentences, explain qualitatively what the scale height is.

   Answer: The scale height is the distance over which a quantity decreases by a factor of $1/e$. This was in the paper!

   (*1 point for something about a quantity decreasing over a distance, 1 point for knowing the dropoff is exponential.*)

   (b) **4 points:** § 2.1 gives the pressure scale height as $H_P = c^2/(\gamma g)$. Show that this is equal to $H = kT/m_u g$. State any assumptions you make.

   Answer: Using the sound speed $c_s = \sqrt{\frac{\gamma P}{p}}$, and the ideal gas law $P = \frac{\rho k T}{m_u}$ (assuming the chemical abundance, μ, is equal to 1 in the photosphere):

$$\begin{aligned} H_P &= \frac{c_s^2}{\gamma g} \\ &= \frac{(\gamma P/\rho)}{\gamma g} \\ &= \frac{P}{\rho g} \\ &= \frac{\rho k T}{m_u \rho g} \\ &= \frac{kT}{m_u g} \end{aligned}$$

   (*2 points for utilizing the correct equations (even if conversion errors led to wrong expression), 1 more for deriving correct expression, 1 for assuming $\mu = 1$.*)

   (c) **5 points:** Using the expression given for scale height ($H_P = c^2/(\gamma P)$) and the values given at the bottom of the second column of page 2, calculate the scale height at the photosphere (the visible surface of the sun). If you don't know g, use 10^4. Does your answer make sense? Why or why not?

   Answer: g can be calculated from the equation for gravitational force:

$$\begin{aligned} mg &= \frac{GMm}{r^2} \\ g &= \frac{GM}{r^2} \\ &= \frac{GM_\odot}{R_\odot^2} \\ &= \frac{(6.96 \times 10^{-8})(2 \times 10^{33})}{(6.96 \times 10)^2} \\ &= 27442.78 \text{ cm s}^{-2} \end{aligned}$$

c_s is given as 7.2 km s^{-1} = 7.2×10^5 cm s^{-1}. Now solve for H_P:

$$\begin{aligned} H_P &= \frac{c^2}{\gamma g} \\ &= \frac{(7.2)^2}{(5/3)27442.78} \\ &= \sim 113.34 \times 10^5 \text{ cm} \\ &= \sim 113 \text{ km} \end{aligned}$$

The sharp discontinuity at the photosphere indicates that the density, and therefore pressure, falls off quite rapidly, so one would expect a 'relatively' low scale height. (Compare this to the radius of the Earth, at $\sim$ 6300 km).

(*2 points for correctly using the gravitational force to get g, 2 points for calculating H_P correctly, 2 points for explaining why the answer does or does not make sense.*)