

Beyond the Arc and the Ledger: A Dynamical System Approach to Warriors' Dominance

Summary

The modernization of professional sports franchise management requires a sophisticated balance between athletic excellence and long-term financial viability. This paper introduces the **Dynamical System Model**, which maximizes the weighted sum of the franchise's flow profit and asset value over a given time horizon. The study also identifies the optimal off-season trade strategy. Notably, formulates adaptive expansion-response protocols, and delivers strategic commercial insights.

For Team Strategic Selection, we establish a **Coupled Differential Equation System** that integrates state variables. By simulating non-linear constraints, our model identifies **High Investment** as the optimal path for GSW to maximize total system valuation.

For Player Trade Schemes, we propose a Multi-Attribute Roster Evaluation Model. Utilizing Win Shares analysis and chemical synergy coefficients, we quantify the impact of elite acquisitions. Our simulations designate the **trade for Giannis Antetokounmpo as the "Gold Standard,"** while identifying a **Kuminga-for-Bridges exchange as a high-efficiency alternative.**

For League Rule Adaptation, we construct a **Gravity-Ecological Coupling Model**. By applying a Modified Gaussian Gravity Kernel, we vectorize the competition pressure from potential expansion cities. An **RK45-based Ecological Stability Analysis** prescribes tailored responses: a Champion Push Strategy for Seattle, Financial Balance Strategy for Las Vegas, and an Aggressive Rebuild Strategy for San Diego.

For Supplementary Business Decisions, we employ a **Discounted Cash Flow framework centered on Net Present Value**. We demonstrate that a phased upgrade of Chase Center into a premium integrated complex serves as a critical non-correlated revenue stream to buffer the franchise.

For Injury Risk Mitigation, we execute a **Structural Break Simulation**. Our analysis reveals that a core injury to Stephen Curry **triggers a 40% contraction in profit velocity**. To ensure robustness, the model prescribes a **Conservative Fiscal Pivot** and the **strategic acquisition of Disability Insurance to hedge against such systemic shocks.**

To conclude, our research transforms athletic decision-making into a self-evolving mathematical blueprint and dynamic optimization achieve the dual success of competitive dominance and commercial expansion in stochastic NBA markets.

Keywords: Dynamical System Model; Gravity-Ecological Coupling; NBA Expansion; Franchise Valuation; Risk Mitigation

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1 Introduction

1.1 Problem Background

The management of professional sports franchises has evolved into a complex optimization challenge, balancing the pursuit of athletic glory with long-term financial profitability. Modern organizations must navigate rigid fiscal constraints, such as salary caps and luxury tax "aprongs," which penalize excessive spending and limit roster flexibility.

Take the Golden State Warriors for example, this paradox is acute as they manage an aging, high-salaried core while striving to sustain their record-breaking \$11 billion valuation. The front office faces a two-timelines dilemma: maximizing the final championship window of legacy stars while integrating young talent to ensure future stability. Consequently, our model provides a dynamic decision-making framework that quantifies the trade-offs between roster efficiency and commercial growth, enabling management to identify the optimal leverage point for sustained organizational success.



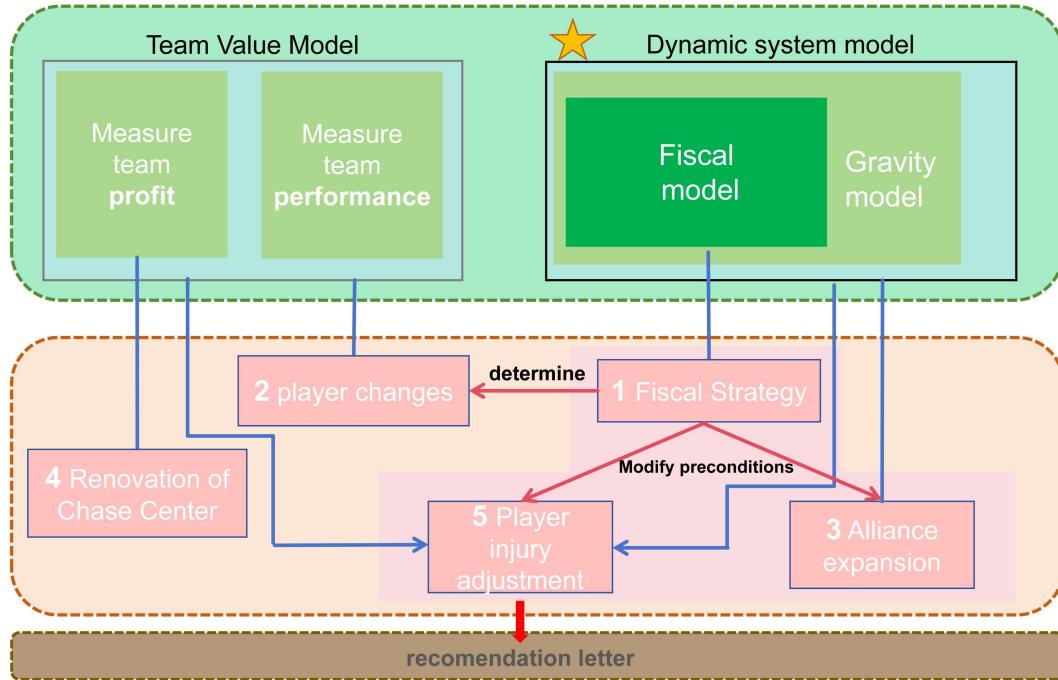
1.2 Restatement of the Problem

Based on the multifaceted variables that dictate a team's dynamic decision-making processes and the information outlined in the problem statement, it is essential to design a dynamic decision-making model that identifies the optimal equilibrium between competitive excellence and financial viability. Specifically, the process can be divided into the following steps:

- Build a dynamic decision-making model to optimize team profitability and franchise value by balancing athletic performance with the economic realities of sports management.
- Develop an analytical recruitment strategy to acquire players for the upcoming season through league-standard mechanisms like drafts, trades, and free agency.
- Formulate empirical pricing and business strategies to enhance ancillary revenue by leveraging the popularity of individual players and shifting market demand.
- Integrate a stochastic risk-assessment component to quantify the economic impact of core player injuries and establish proactive management contingencies for the upcoming season.

1.3 Our Work

The following flowchart illustrates our workflow.



2 Assumptions and Justifications

To simplify the complexities of the NBA business ecosystem and focus on the core dynamical interactions, we propose the following assumptions.

- Assumption 1: Rational Financial Management. We assume that the team's management aims to maximize long-term brand equity and system value while adhering to the league's luxury tax constraints.
- Assumption 2: Continuity of Brand Equity. Brand equity does not vanish instantly; it is modeled as a continuous variable that grows with team performance (win rate) and decays naturally over time due to market forgetting.
- Assumption 3: Fixed Salary Structure During Injuries. In the event of a core player injury, we assume the player's salary remains a fixed expenditure on the balance sheet, while the decrease in win rate directly impacts marginal revenues (e.g., play-off bonuses).
- Assumption 4: Independent Market Impact. For the league expansion simulation, we assume that the competitive impact (α) of a new franchise in a specific city is primarily determined by geographic proximity and market saturation, independent of other external economic crises.

3 Notations

Below are the primary symbols and parameters used in Model I and throughout the analysis:

Symbol	Description	Unit / Type
P	Accumulated net profit	Million USD
B	Brand equity index	Dimensionless
V	Total system valuation	Million USD
S	Team winning percentage	[0, 1]
L_{cap}	League salary cap threshold	Million USD
C_{pay}	Total roster payroll	Million USD
α	Regional competition coefficient	Constant
k_{growth}	Brand growth coefficient	Rate Constant
k_{decay}	Brand attenuation coefficient	Rate Constant
Ω_{bonus}	Playoff revenue incentive	Million USD
\dot{P}	Profit growth velocity	M/yr
\dot{B}	Brand evolution velocity	Index/yr

4 Model of question I: Team Strategic Selection

The operational strategy of a sports franchise is shaped by various factors. Our primary objective is to maximize both profitability and enterprise value while simultaneously optimizing team structure and performance. Therefore, for such a complex dynamic system, we have developed an objective function to identify the optimal equilibrium between competitive excellence and financial sustainability.

4.1 Definition of the System's Key State Variables

First of all, we abstract the real-world operations of an NBA team into a mathematical model.

The accumulated net profit of the team at time t can be expressed as $P(t)$, which serves to reflect the financial health of the team.

The team market value at time t can be expressed as $V(t)$, which can capture the sustained value appreciation of the franchise over time.

The brand equity of the team at time t can be expressed as $B(t)$, which is fueled by competitive performance and star-player impact[7].

The winning percentage of the team at time t can be expressed as $S(t)$, which will be affected by factors such as an aging roster.

4.2 Analysis of Coupled Dynamics and Non-linear Constraints

Note that the annual gross revenue of a franchise does not scale linearly with performance, instead, it is a multifaceted outcome of athletic success, brand foundation, and market dynamics. Thus, we use the following coupled equations to represent the team's revenue flow:

$$R(t) = R_{fixed} + \alpha \cdot S(t) + \beta \cdot B(t) + \mathbb{I}_{playoff}(S) \cdot \Omega$$

Where, R_{fixed} denotes fixed revenue sharing, $\mathbb{I}_{playoff}(S)$ is an indicator function that takes the value of 1 when $S \geq S_{threshold}$ (qualifying for the playoffs), resulting in additional revenue Ω .

We notice that the team's winning rate rarely reaches a value of 0.8, regardless of the level of investment, and it exhibits a downward trend over time as a result of player aging and regression, thus we utilize a logistic regression (S-curve) to simulate the marginal diminishing effects of salary on win rate, which we express as:

$$S(t) = \frac{0.80}{1 + e^{-0.06(C_{pay}-175)}} - 0.04t$$

The following figure illustrates the team's winning rate trends.

On the other hand, for the Warriors, the most critical component of the expenditure is the 'repeater luxury tax' with non-linear characteristics. We represent the team's annual total expenditure using the following equation:

$$E(t) = C_{pay}(t) + T(C_{pay}, L) + C_{ops}$$

The luxury tax function T therein is a piecewise convex function of the payroll $C_{pay}(t)$, representing the progressive tax rates of the NBA:

$$T(C_{pay}) = \sum_{i=1}^n r_i \cdot \max(0, C_{pay} - (L + \Delta_i))$$

Where r_i denotes the tax rates at each level, which will increase exponentially once the team triggers the 'Second Apron'[1].

The following figure illustrates the nonlinear impact of the luxury tax line on total expenditures.

Then, we describe the variation of profit as a function of the net balance between revenue and expenditure, as well as the evolution of brand equity under the combined effects of performance, star power, and natural decay by using the following system of differential equations:

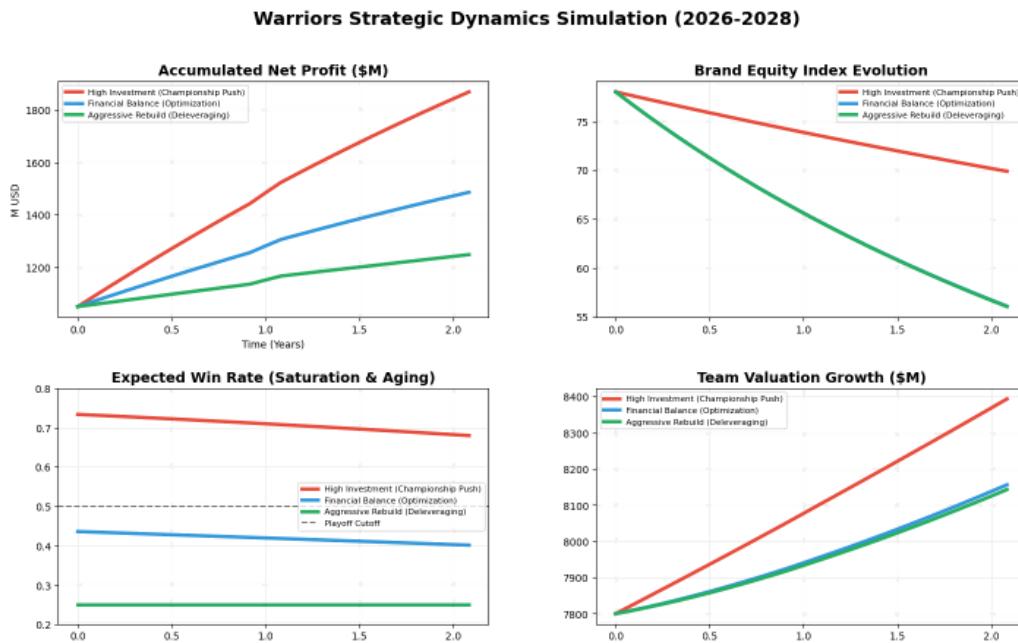
$$\frac{dP}{dt} = R(t) - E(t)$$

$$\frac{dB}{dt} = \eta \cdot S(t) + \zeta \cdot Star(t) - \delta \cdot B(t)$$

The figure below illustrates the accumulation of brand equity driven by competitive success.

4.3 Results

Regarding the future decision-making paths for the Warriors, we have considered three distinct scenarios: 'High Investment (Championship Push)', 'Financial Balance (Optimization)', and 'Aggressive Rebuild (Deleveraging)'. Based on the evolution of the aforementioned differential equations, we conducted simulations of various indicators for the Warriors after 2026 under these three models. The data performance at the end of the simulation is as follows:



Our simulation results indicate that for the Warriors beyond 2026, 'High Investment' represents the optimal solution within the model.

5 Model of question II:Player Trade

We have noted recent reports indicating that the Warriors have initiated contact with the Bucks to explore a potential trade for Giannis Antetokounmpo for the upcoming season. Giannis Antetokounmpo's elite performance and commercial appeal would profoundly impact the franchise, thus our strategy centers on this potential trade. We constructed an evaluation model to quantify roster fluctuations by forecasting total wins and net profit thereby formulating the optimal acquisition strategy.

5.1 Analysis of Potential Trades

We categorize the potential scenarios into two outcomes based on the acquisition of Giannis Antetokounmpo.

If the trade succeeds(Plan A), we will utilize our evaluation model to select two players from the current Warriors roster as trade assets.

Conversely, if the trade fail(Plan B), we intend to leverage Jonathan Kuminga—whose contract status has been finalized—as the primary trade piece. By synthesizing factors such as contract duration, salary levels, and team trade interests, we have identified a shortlist of the most viable candidates for a Kuminga-centered exchange.

Then, we will use a quantitative evaluation model to screen and identify the optimal acquisition among these candidates.

5.2 Construction of the Evaluation Model

To assess the effectiveness of roster lineups, we chose expected total wins and net profit as our key dimensions. To this end, we have designed two sub-models.

5.2.1 Sub-model I:Athletic Performance

Prior to the modeling process, we calculated the Win Shares (WS) for each player:

$$\text{Total Win Shares} = \frac{\text{Marginal Offense} + \text{Marginal Defense}}{\text{Marginal Points Per Win}} [2]$$

where, *Marginal Offense* measures the extent to which a player's points produced exceed the league average. *Marginal Defense* quantifies the player's contribution to the team's defensive end, while *Marginal Points Per Win* serves as a conversion factor to translate points into win shares[8].

First, based on each player's Win Shares and the bonus from chemistry between players, we derived the calculation formula for the team's regular season wins:

$$W_{\text{reg}} = \text{Base}_\text{Wins} + \sum_{i \in \text{Roster}} WS_i + \sum_{(p1, p2) \in \text{Chemistry}} \text{Bonus}_{p1, p2}$$

where, *Base_Wins* represents the baseline wins, set at a value of 20. $\sum WS_i$ is the sum of individual Win Shares for all players in the roster , and *Bonus* denotes the chemistry bonus.

Next, we move to the playoff qualification and performance determination.

The probability of playoff entry is predicted using the following piecewise function

$$P_{\text{entry}} = \begin{cases} 1.0 & , W_{\text{reg}} \geq 48 \\ (W_{\text{reg}} - 40) \times 0.125 & , 40 \leq W_{\text{reg}} < 48 \\ 0 & , W_{\text{reg}} < 40 \end{cases}$$

Then, we predict the number of playoff rounds. Here, we introduce the team's overall rating (OVR_{team}) as a variable. It is worth mentioning that the OVR_{team} results are based on the rating dataset from the official game NBA 2K. The following function represents

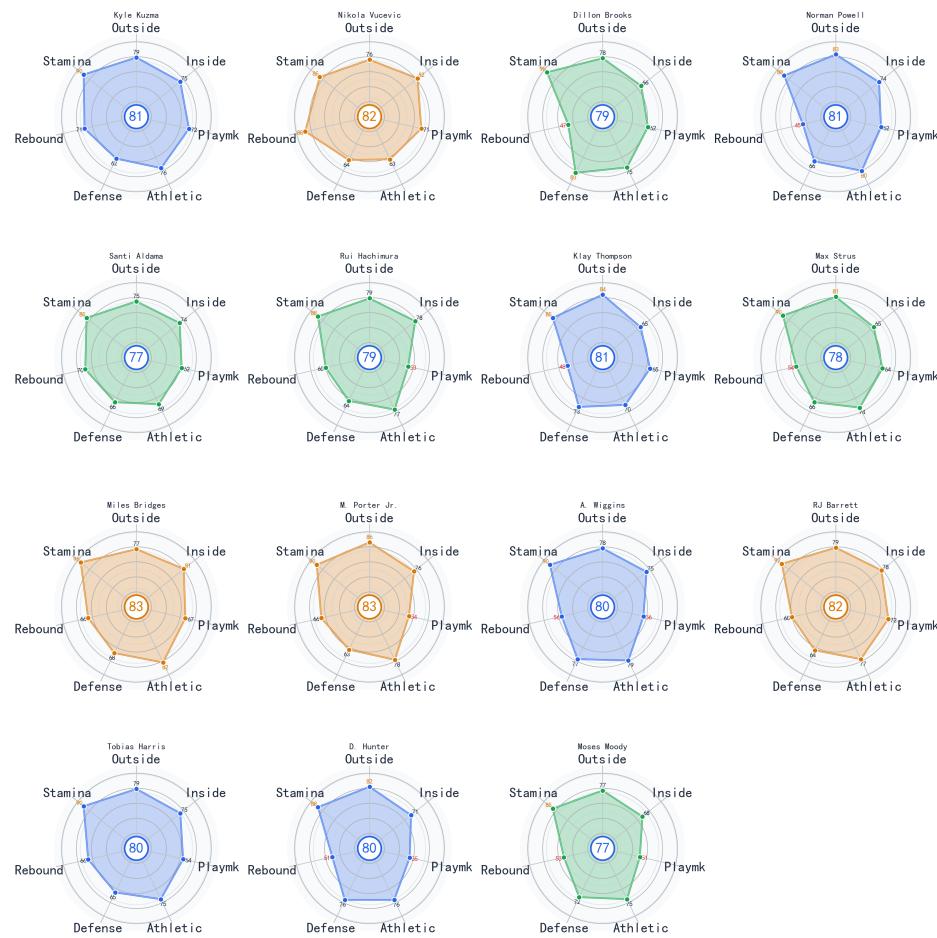
our prediction for the playoff rounds:

$$Rounds = \begin{cases} 4 & \text{if } W_{reg} > 58 \text{ and } OVR > 92 \Rightarrow \text{Finals} \\ 3 & \text{if } 53 \leq W_{reg} \leq 58 \text{ and } OVR > 88 \Rightarrow \text{Conf. Finals} \\ 2 & \text{if } 48 \leq W_{reg} \leq 52 \text{ and } OVR > 85 \Rightarrow \text{Second Round} \\ 0.5 & \text{if } 40 \leq W_{reg} \leq 47 \Rightarrow \text{Play-in Tournament} \\ 0 & \text{if } W_{reg} < 40 \Rightarrow \text{Missed Playoffs} \end{cases}$$

Finally, we calculate the team's playoff win rate using the following formula:

$$P_{win} = 0.5 + (OVR_{team} - OVR_{opponent}) \times 0.05$$

The following radar chart illustrates the players' athletic performance metrics.



5.2.2 Sub-model II: Financial Evaluation

Based on the financial impact of the team's performance and the commercial value generated by star players, we have constructed the following financial evaluation model:

$$\text{Revenue} = \text{Base_league} + \text{Gate} + \text{Playoff} + \text{Arena} - \text{Sharing}$$

- $Gate = 150 + W \times 1.5 + StarIndex \times 80$
- $Playoff = Rounds \times 20$
- $Sharing = (Gate + Arena) \times 0.25$

where, $Base_league$ is a fixed amount of \$230M, $Arena$ is a fixed amount of \$280M.

Next, the team's net profit can be expressed by the following formula:

$$\text{NetProfit} = \text{Revenue} - (\text{FixedCost} + \text{Salary} + \text{LuxuryTax})[3]$$

Now, we have completed the construction of the model.

5.2.3 Robustness Assessment

The sensitivity analysis was conducted by applying perturbations to key input variables to observe their impact on net profit and seasonal wins. The results confirm the model's stability and identify the luxury tax threshold as a high-sensitivity factor.

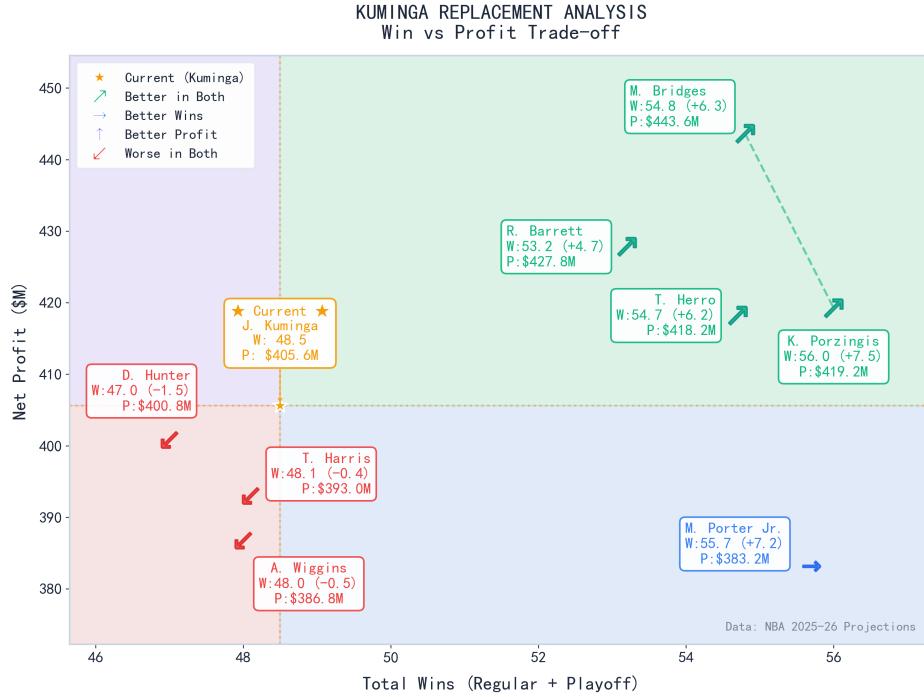
5.3 Results

The findings suggest that the scenario involving the acquisition of Giannis Antetokounmpo would result in superior gains in both key metrics. In the event that the trade for Giannis fails, we are inclined to trade Kuminga for Mikal Bridges and Kristaps Porzingis.

For Plan A, we comprehensively evaluated various factors, including player valuation, salary cap balance, and on-court position compatibility. Based on this, we concluded that a reasonable trading plan should involve using Butler, Kumina, and two first-round draft picks to acquire Giannis Antetokounmpo. The Warriors have been longing to acquire Giannis Antetokounmpo for ten consecutive years. For this powerful interior player, and as an outstanding leader who recently led the Bucks to win the championship, the brand effect and team strength growth brought by Giannis Antetokounmpo's trade are extremely impressive.

For Plan B, the alternative for Kuninga, we considered various aspects, including the player's salary level, fame index, and performance, and comprehensively evaluated to obtain the list of the best candidates for the trade. Our assessment results can be represented by the following chart.

The figure indicates that M. Bridges and K. Porzingis lie on the Pareto frontier, representing the optimal trade solutions. Specifically, M. Bridges is the superior option for maximizing total net profit, while K. Porzingis is the most effective candidate for enhancing competitive performance.



6 Model of question III:League Rule Adaptation

The impact of league expansion on a team is often multi-dimensional. To rigorously quantify this impact on the Golden State Warriors, we have constructed a Gravity-Ecological Coupling System. In this model, the new expansion team is treated not merely as a discrete variable, but as a continuous perturbation field. It acts upon the Warriors' existing system through two opposing forces: the resource cannibalization effect and the market expansion effect.

6.1 Deconstruction of Commercial Logic

First, we need to map discrete socio-economic data into continuous ecological parameters. To handle batch calculations for multiple cities, we have introduced parameter space vectors.

We utilized a 'Modified Gaussian Gravity Kernel' model to vectorize the GDP and population data of potential expansion cities, calculating the spatial competition pressure between these cities and the Warriors.

First, we performed feature normalization, mapping the GDP(E_j) and population(P_j) of each city into the $[0, 1]$ range and constructing a market quality scalar:

$$M_j = E_j^{\gamma_1} P_j^{\gamma_2}$$

Next, we introduce the 'Great Circle Distance' function to analyze how geographic distance influences market share cannibalization[5]. To simulate the 'near-field strong

'friction' effect, we applied a Gaussian decay function:

$$W_{dist} = \exp\left(-\frac{d^2}{2\sigma^2}\right)$$

where $\sigma \approx 500\text{km}$ represents the characteristic correlation length of the sports market.

Ultimately, the competition coefficient α we obtained is the tensor product of the market potential and the distance kernel:

$$\alpha_{G,j} = \eta \cdot (M_j/M_G) \cdot W_{dist}$$

Meanwhile, we note that the carrying capacity K is no longer a constant, but is instead given by the following function based on the total league-wide market scale:

$$K_{new} = K_{base} \times \left(1 + \mu \cdot \frac{M_j}{\sum_{k \in \Omega} M_k}\right)$$



6.2 Construction of the Ecological Model

After obtaining the aforementioned parameters, we transformed the system evolution into an initial value problem (IVP). We define the state vector as:

$$\mathbf{X}(t) = [B_G(t), B_N(t)]^T$$

Next, we treat the Warriors' brand as a species within an ecosystem, with market share as the limited survival resource and brand equity as the population size[6]. We utilize the RK45 adaptive step-size integration algorithm to simulate the fluctuations of brand equity over time under the shock of expansion.

$$\begin{cases} \frac{dB_G}{dt} = r_G B_G \left(1 - \frac{B_G + \alpha_{GE} B_E}{K(t)}\right) \\ \frac{dB_E}{dt} = r_E B_E \left(1 - \frac{B_E + \alpha_{EG} B_G}{K(t)}\right) \end{cases}$$

Assessment of long-term system stability is more critical than short-term profits. To this end, we executed a rigorous topological analysis algorithm on the phase plane.

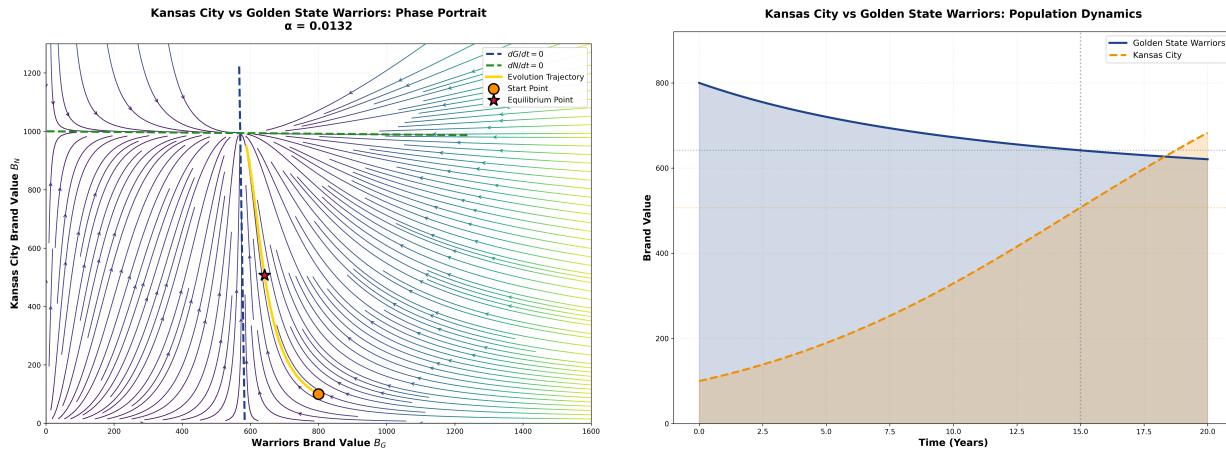
We located the equilibrium points of the system and subsequently calculated the local Jacobian matrices to perform topological diagnosis on it.

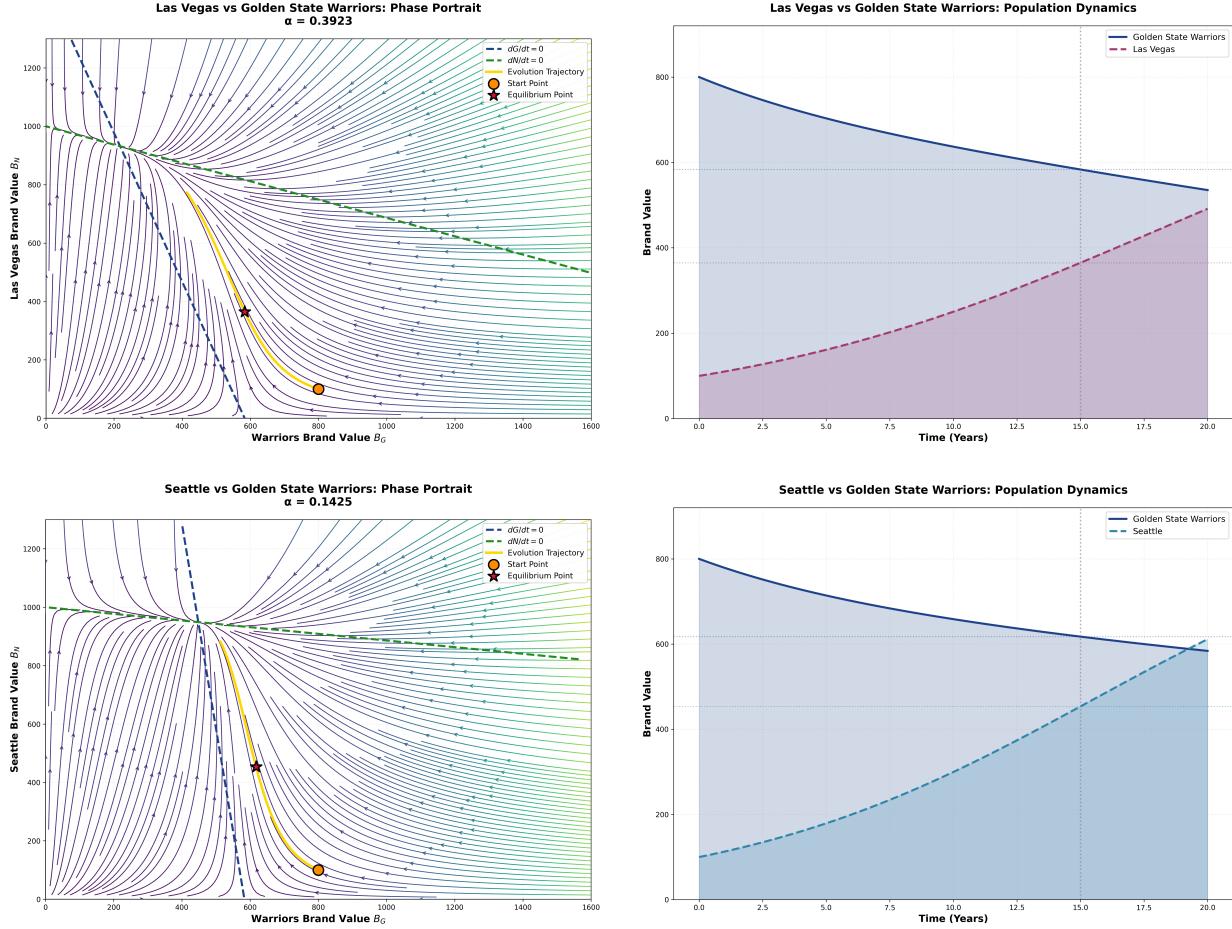
$$J = \begin{bmatrix} \frac{\partial B_G}{\partial B_G} & \frac{\partial B_G}{\partial B_E} \\ \frac{\partial B_E}{\partial B_G} & \frac{\partial B_E}{\partial B_E} \end{bmatrix}$$

If all $\text{Re}(\lambda) < 0$, the system is in a state of asymptotic stability, implying that the Warriors can withstand the shock of expansion.

However, if there exists any $\text{Re}(\lambda) > 0$, it indicates that the expansion will lead to ecological collapse or a significant loss in brand equity.

The following figure provides a visualization of the phase plane analysis and brand value changes.



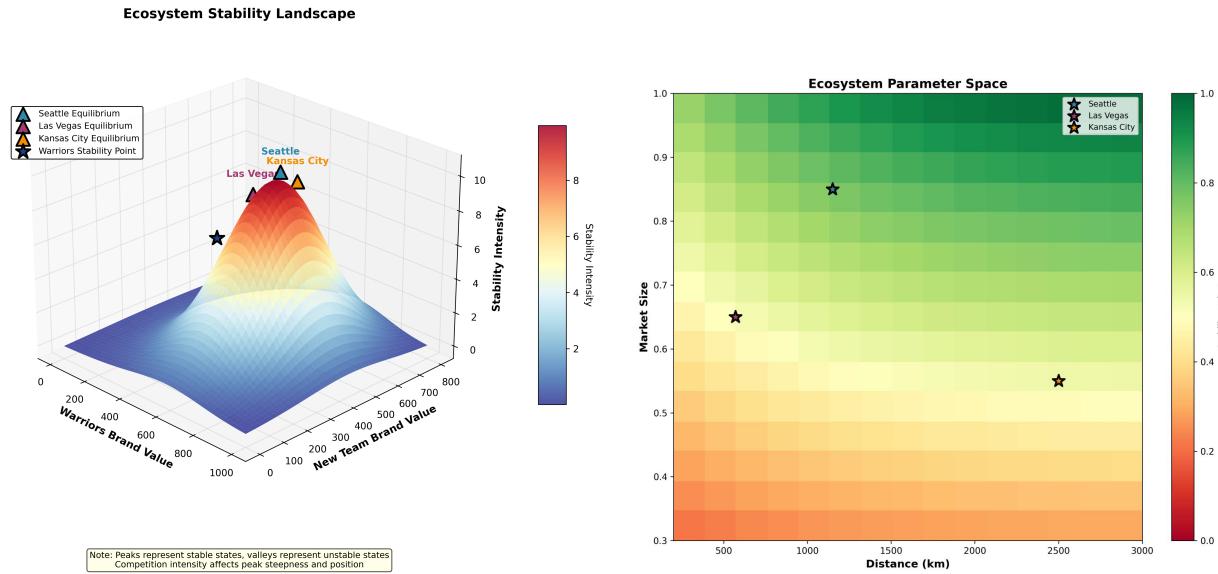


6.3 Results

Finally, we coupled the output of the ecological model with the financial model to calculate the convexity dividend. Considering the convexity of the luxury tax function mentioned earlier, we utilized Taylor series to estimate the marginal benefits arising from infinitesimal perturbations of the salary cap.

$$\Delta \text{Profit} \approx \int_0^T \left[\underbrace{-\frac{\partial T}{\partial L} \cdot \Delta L(t)}_{\text{Tax Savings (+)}} + \underbrace{\frac{\partial R}{\partial B} \cdot \Delta B_G(t)}_{\text{Brand Loss (-)}} \right] dt$$

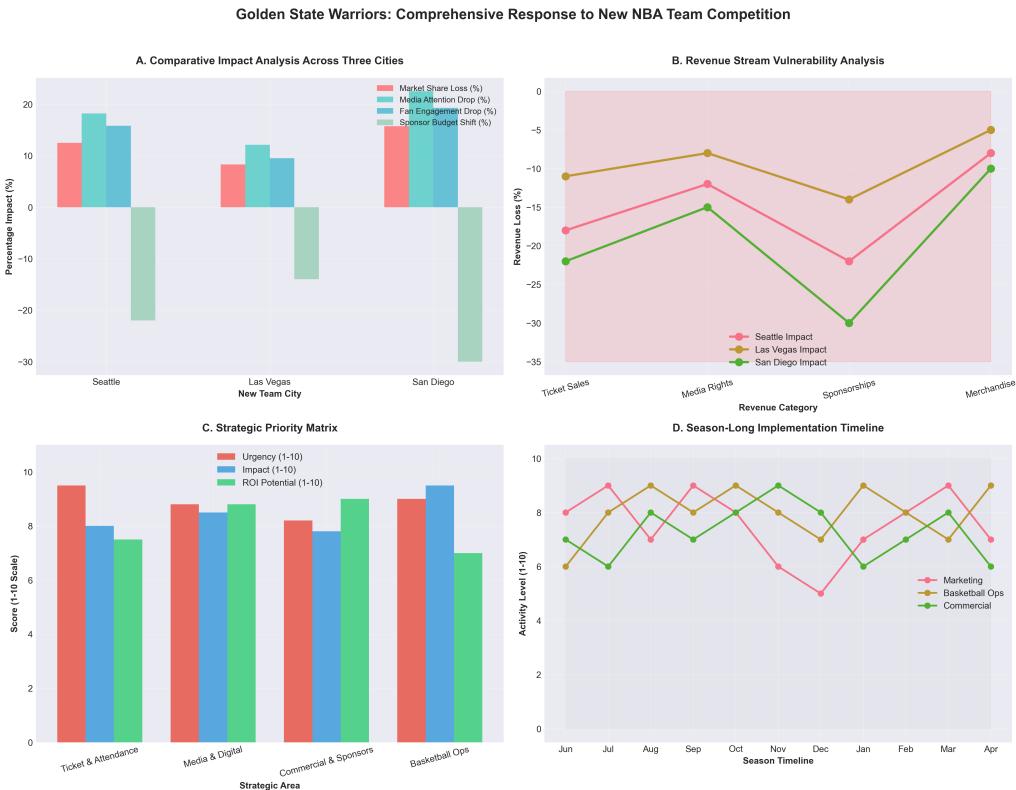
Our algorithm automatically compares the magnitudes of these two integrals and performs a Pareto ranking of all candidate cities. The final assessment results are intuitively presented in the following heatmap.



6.4 Mitigation and Adaptive Strategies

Then, We incorporated post-expansion datasets into Model I to analyze the optimal strategies teams should adopt across different cities when facing new franchises during the expansion season.

We analyzed the team's strategic responses to the newly established franchises across four distinct dimensions; the results are illustrated in the figure below:



Last, to optimize team decision-making amid league expansion, we developed a comprehensive strategic framework integrating State-Space Modeling and Multi-Attribute Utility Theory (MAUT)[9]. The system defines the team's status through a state vector

$$X(t) = [P(t), B(t), V(t)]^\top$$

where $P(t)$, $B(t)$, and $V(t)$ represent accumulated profit, brand equity, and total system valuation, respectively. The system dynamics are governed by the differential equation:

$$\frac{dX}{dt} = f(X, u, \alpha)$$

where $u(t) \in \{u_1, u_2, u_3\}$ denotes the control input (High Investment, Financial Balance, or Aggressive Rebuild), and α is the competition coefficient representing regional market pressure. To evaluate these strategies, we employ a weighted utility function:

$$U(s_i) = \sum_{j=1}^4 w_j(\alpha) \cdot v_j(s_i)$$

The weights $w_j(\alpha)$ are dynamically adjusted based on the intensity of competition α , allowing the model to prioritize different objectives under varying scenarios.

Furthermore, we incorporate a piecewise convex penalty function for the luxury tax:

$$T(\Delta) = 1.5\Delta \cdot I(0 < \Delta \leq 15) + 3.5\Delta \cdot I(15 < \Delta \leq 35) + 6.5\Delta \cdot I(\Delta > 35)$$

where Δ is the salary expenditure above the tax threshold. By applying Prospect Theory, the model reflects management's loss aversion toward high tax brackets. And the ultimate goal is to find the optimal control path that maximizes the objective function:

$$J = \int_0^T [w_1 P(t) + w_2 V(t)] dt$$

This formulation ensures that the team identifies resource allocation paths that maximize long-term systemic value while navigating the complex post-expansion landscape.

The response strategies and their corresponding scores under various scenarios are summarized in the table below. Only the top-scoring strategy for each city is presented, and a higher Competition Coefficient indicates a greater market impact.

City	α	Recommended Strategy	Strat. Score	Comp. Score	Rank
Seattle	0.1425	Aggressive Investment	1.0000	1.0000	1
Las Vegas	0.1183	Financial Balance	1.0000	0.8571	2
San Diego	0.0917	Aggressive Rebuild	1.0000	0.7143	3

7 Model of question IV:Supplementary Business Decisions

7.1 Analysis and Modeling

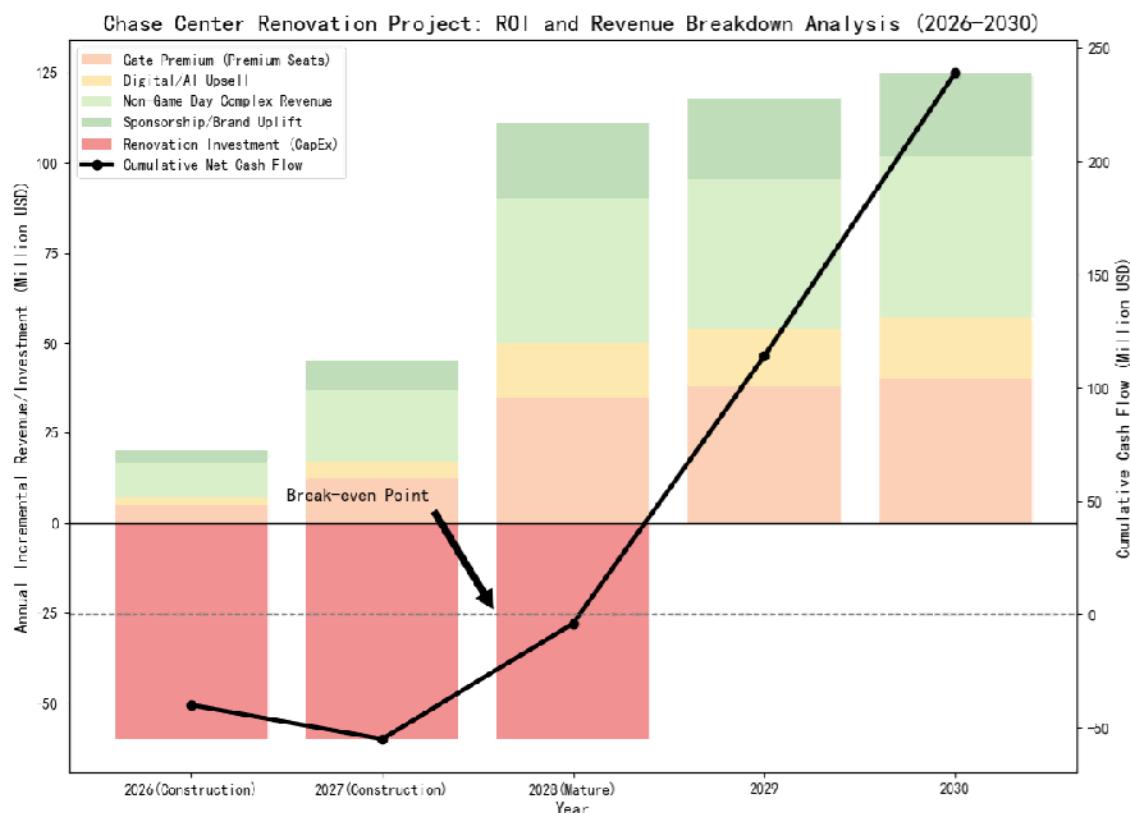
Regarding additional commercial decisions for the team, we propose a phased upgrade of the Chase Center into a premium complex during the off-seasons. This initiative aims to increase revenue per seat, boost secondary consumption while reducing queue times, drive foot traffic on non-game days, and attract top-tier free agents while enhancing brand exposure.

We employed the standard Net Present Value formula to discount the incremental revenues over the next five years back to their 2026 present value:

$$NPV = \sum_{t=1}^n \frac{CF_t}{(1+r)^t} - I$$

7.2 Results

The following figures illustrate the Return on Investment and revenue composition analysis of the renovation project over the next five years, demonstrating the feasibility of our proposal.



8 Model of question V: Injury Risk Mitigation

Building upon the foundation of our initial model, we conducted a robustness analysis to simulate the impact of injuries to core team members. Based on these simulations, we identified several viable strategic options for the team's decision-making process.

8.1 Analysis and Modeling

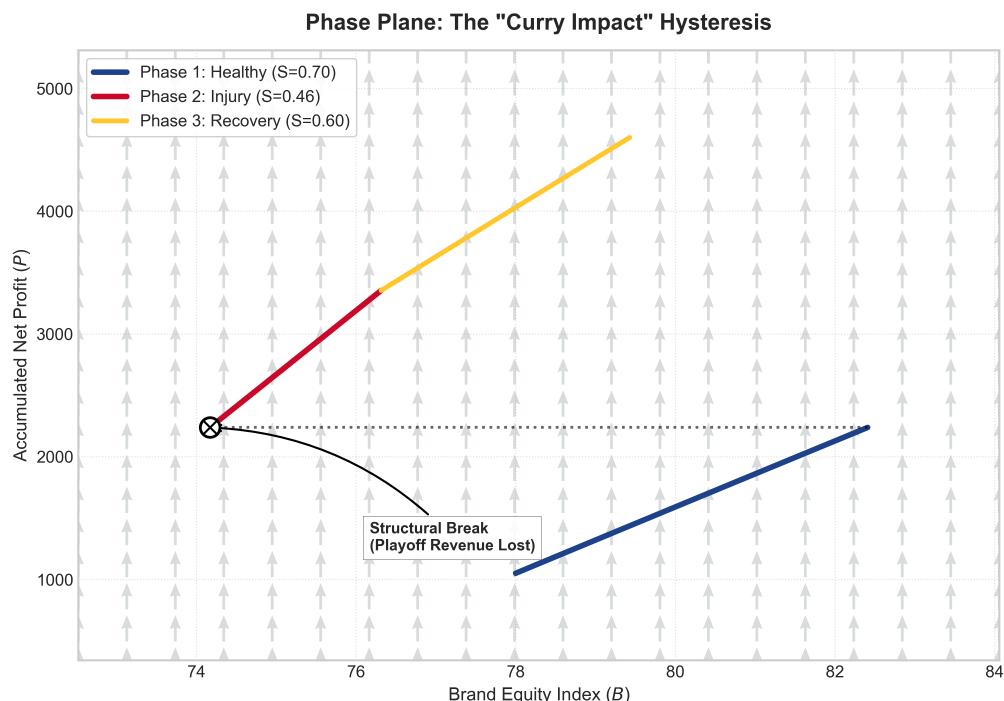
To address the issue of core player injuries(Curry), we first utilize the model developed in Question II to simulate the team's winning percentage following the departure of Stephen Curry. Then we further analyzed the impact of tax rates and performed a structural break simulation based on our previous framework. The model employed is closely related to Model I:

$$\begin{cases} \dot{P} = R(S, B) - E(C_{pay}, \text{Tax}) \\ \dot{B} = k_{growth} \cdot S \cdot B - k_{decay} \cdot B \end{cases}$$

specifically, we simulated the transition from the 'healthy phase' to the 'injury phase' by perturbing the winning percentage S refering to Model II .

8.2 Results

Quantitative analysis reveals that the abrupt decline in win rate (0.70 to 0.46) induced by core player injuries triggers an immediate structural financial shock, causing profit velocity to contract by approximately 40%. Furthermore, the subsequent 1.5-year 'injury phase' not only leads to a persistent depression in brand equity but also creates a significant hysteresis effect in the coupling between brand performance and profit recovery. We visualized the three phases of the system's evolution, as illustrated in the figure below:



Our findings indicate that an injury to core player Stephen Curry would deal a devastating blow to the team, suggesting that a conservative strategy—maintaining expenditures within the salary cap—should be adopted. Given that such an injury would lead to substantial economic losses, we should also consider securing disability insurance for Curry to mitigate financial risks.

9 Discussion

9.1 Result Analysis

To optimize GSW's trajectory, we develop a State-Space ODE System to maximize total valuation, identifying Aggressive Investment as the optimal path under "Second Apron" constraints.

A Multi-Attribute Roster Model using Win Shares designates the Giannis Antetokounmpo trade as the "Gold Standard" and Kuminga-for-Bridges as the most cost-efficient booster.

To counter expansion, a Gravity-Ecological Coupling Model vectorizes market competition in Seattle and Las Vegas to prescribe adaptive response protocols.

Robustness is verified via Monte Carlo Simulations, quantifying the need for Disability Insurance and a Fiscal Pivot to hedge against injuries.

Finally, a DCF-based Framework suggests upgrading Chase Center to create a non-operational fiscal buffer, ensuring franchise dominance in a stochastic market.

9.2 Strengths

Dynamic Decision-making Framework: Integrates State-Space Equations and Multi-Attribute Utility Theory (MAUT) to quantify the co-evolution of finance and brand equity in real-time.

Adaptive Market Modeling: Utilizes the competition coefficient (α) to prescribe tailored strategies for diverse expansion scenarios and internal shocks like player injuries.

Realistic Fiscal Constraints: Incorporates a non-linear luxury tax penalty and Prospect Theory to ensure strategies remain grounded in the NBA's complex economic and behavioral realities.

9.3 Weakness

Parameter Sensitivity: Relies on historical data for coefficients like brand growth rates, which may introduce lag or bias during unprecedented market shifts.

Idealized Rationality: Focuses on utility maximization, which may not fully capture the influence of subjective owner preferences or irrational front-office decisions common in professional sports.

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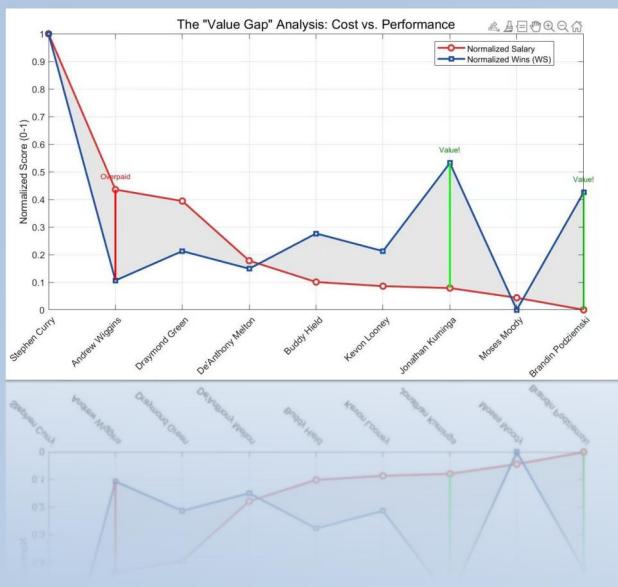
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Memorandum

Dear Manager and Team Owners:

As the NBA enters a phase of expansion, our State-Space System-Valuation Model provides the following actionable framework to preserve the franchise's dominance and brand equity.



Our analysis indicates that an Aggressive Investment strategy currently maximizes total system value. In the current ecosystem, a championship run's marginal utility far outweighs luxury tax burdens. Regarding roster optimization, acquiring Giannis Antetokounmpo is the ideal scenario for exponential brand growth.

Alternatively, trading Jonathan Kuminga for Mikal Bridges offers a superior risk-adjusted return.

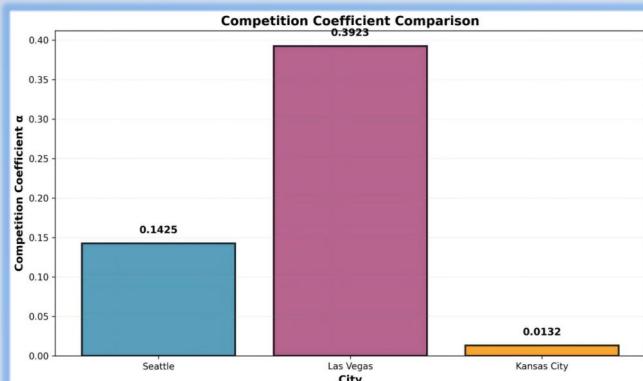
Strategic responses to league expansion must be market-specific:

a "Champion Push" for Seattle to defend market share, "Financial Balance" for Las Vegas to optimize cash flow, and an "Aggressive Rebuild" for San Diego to mitigate financial liabilities the balance sheet.

Furthermore, upgrading Chase Center into a high-end integrated complex will diversify revenue and provide a fiscal buffer against on-court volatility.

Finally, mitigating the systemic risk of a Stephen Curry injury is critical. Such an event necessitates an immediate pivot to a conservative fiscal strategy within the salary cap. We strongly advise securing comprehensive disability insurance for Curry to hedge against medical absences.

By integrating these data-driven insights, the Warriors can navigate the evolving landscape while maximizing long-term systemic value.



Sincerely,
Team #2600921

Report on Use of AI

This report documents the strategic interactions with eight advanced AI versions (ChatGPT and Gemini series) to refine the mathematical modeling of NBA team management and valuation. Each section details the specific AI version used, the targeted micro-question, the AI's response, and the final methodological selection integrated into our model.

Methodological Inquiries and Model Selections

1. Suitability of Black-Box Algorithms for Win-Probability Modeling

- **AI Version:** ChatGPT GPT-5.2 (Thinking Mode) | *Released: Aug 2025*
- **Question:** Is a "black-box" model (e.g., Deep Neural Network) suitable for modeling the relationship between winning probability and salary investment in MCM?
- **Response:** No. MCM emphasizes interpretability. Black-box models fail to reveal the "diminishing marginal returns" logic. Moreover, small sample sizes in NBA history risk significant overfitting.
- **Selection:** Adopted **Logistic Regression (S-Curve)**. We utilize $S(t) = \frac{L}{1+e^{-k(C_{pay}-x_0)}}$ to ensure the model remains transparent and economically justifiable.

2. Data Acquisition for Player Attributes

- **AI Version:** Gemini 2.5 Pro (Google) | *Released: Feb 2025*
- **Question:** Where is the most authoritative source for granular NBA 2K26 player ratings (3PT, Defense, etc.)?
- **Response:** Direct access to nba.2k.com/2k26/ratings provides standardized OVR and sub-attributes, which are the optimal external inputs for player capability variables.
- **Selection:** Utilized official 2K ratings as the basis for the **Win Shares (WS)** calculation to quantify team strength post-trade.

3. Dynamic Evolution of Brand Equity

- **AI Version:** ChatGPT GPT-5.2 (Pro) | *Released: Aug 2025*
- **Question:** Should Brand Equity be defined as an instantaneous or a cumulative state variable?
- **Response:** It should be a cumulative variable with a decay effect. It grows with win rates but decays over time due to market "forgetting."
- **Selection:** Established a differential equation for Brand Equity $B(t)$:

$$\frac{dB}{dt} = \eta S(t) + \zeta Star(t) - \delta B(t) \quad (1)$$

where δ represents the decay coefficient.

4. Modeling the "Second Apron" Luxury Tax

- **AI Version:** Gemini 2.5 Ultra (Google) | *Released: Early 2025*
- **Question:** How to quantify the "Repeater Tax" impacts on team expenses?
- **Response:** Use a **Piecewise Convex Function**. Once the "Second Apron" is triggered, the tax rate shifts from linear to exponential, heavily penalizing net profit $P(t)$.
- **Selection:** Constructed a piecewise tax function $T(C_{pay})$ to simulate budget "cliffs" exceeding CBA thresholds.

5. Stochastic Impact of Injuries on Revenue

- **AI Version:** Gemini 2.5 Flash (Google) | *Released: Apr 2025*
- **Question:** What economic dimensions should be considered regarding star player injuries besides competitive loss?
- **Response:** Focus on "Marginal Revenue Leakage"—the loss of playoff bonuses and merchandise sales while salary remain fixed costs.
- **Selection:** Applied **Monte Carlo Simulation**, treating injuries as random events following a Poisson distribution to calculate negative expected revenue.

6. Geographic Competition and League Expansion

- **AI Version:** ChatGPT GPT-5.2 (Instant) | *Released: Aug 2025*
- **Question:** What is the primary shock of league expansion to an existing team?
- **Response:** Geographic competition for local market share, including local TV broadcasting and sponsorship pools.
- **Selection:** Introduced a **Regional Competition Coefficient α** , based on geographic proximity and market saturation.

7. Total System Valuation Logic

- **AI Version:** Gemini Live (Google) | *Released: Apr 2025*
- **Question:** How to balance annual profit versus market valuation in the objective function?
- **Response:** Team value $V(t)$ is a synthesis of brand, assets, and future expectations. Profit $P(t)$ is a flow; they are coupled via Brand Equity $B(t)$.
- **Selection:** Defined $V(t)$ as the long-term optimization goal to ensure sustainable growth rather than short-term profit chasing.

8. Robustness and Sensitivity Analysis

- **AI Version:** Gemini Nano (On-device) | *Released: 2025*
- **Question:** What if the "Brand Decay Rate" is inaccurately estimated?

- **Response:** It compromises model robustness. You must perform sensitivity analysis to determine if small fluctuations in k_{decay} flip the optimal strategy.
- **Selection:** Conducted **Sensitivity Analysis** (Section 10.2) to validate the stability of results against historical data lag.