Your task for this discussion is as follows:

1. Find an example of a statement that can be proven using mathematical induction from an outside resource.
2. Prove the example using mathematical induction, and make sure to explain each step carefully in your own words.
3. Reflect on your experience in elaborating the proof and discuss what steps were the most difficult for you.

Hello everyone,

1) We can prove the following statement using mathematical induction: “The sum over the first *n* Fibonacci numbers is given by: $\sum_{i=1}^{n} F_i = F_{n+2} - 1$” (Chasnov, 2019).

2) Proof:

Let P(*n*) be the statement that $\sum_{i=1}^{n} F_i = F_{n+2} - 1$. We will show that P(*n*) is true for all natural numbers *n* ≥ 1.

Base case: P(1) is the statement $F_1 = F_3 - 1$. 1 = 2 - 1, which is clearly true.

Inductive case: Let *k* ≥ 1 be a natural number. Assume (for induction) that P(*k*) is true. That means $\sum_{i=1}^{k} F_i = F_{k+2} - 1$. We will prove that P(*k*+1) is true as well. That is, we must prove that $\sum_{i=1}^{k+1} F_i = F_{k+3} - 1$. To prove this equation, start with the inductive step:

$\sum_{i=1}^{k+1} F_i = \sum_{i=1}^{k} F_i + F_{k+1}$. By the induction hypothesis:

$\sum_{i=1}^{k+1} F_i = F_{k+2} - 1 + F_{k+1} = F_{k+3} - 1$. Thus P($k$+1) is true, so by the principle of mathematical induction, P(n) is true for all natural numbers $n \geq 0$.

3) The most difficult step of the proof for me was the inductive step, but it became clear to me after I recognized that $F_{k+3} = F_{k+2} + F_{k+1}$.

Thanks,

Lauren Alexandra

References:

Chasnov, J. R. (2019). Lecture 9: Sum of Fibonacci numbers [PDF]. The Hong Kong University of Science and Technology: http://www.math.ust.hk/~machas/fibonacci.pdf

----

Context:

A partial sum represents the sum of part of a sequence. Given a sequence, the partial sum of the first N terms is described as $= \sum_{k=1}^{N} a^k$. This formula allows us to generate the partial sums sequence for each index in the Fibonacci sequence: 1, 2, 4, 7, 12, 20... S_N. Notice the difference between each pair of sums represents a number in the Fibonacci sequence. Moreover, starting at the third index, each partial sum incremented by 1 produces a number in Fibonacci.

https://www.quora.com/Whats-the-closed-form-for-the-partial-sums-of-the-Fibonacci-series

Each partial sum can be calculated as follows: $S_N = F_{n+2} - 1$. We can prove this by induction. Let *P*(*n*) be the statement that $F_1 + F_2 + F_3 + \ldots + F_n = F_{n+2} - 1$. Proof: We will show that *P*(*n*) is true for all natural numbers *n* ≥ 1. Base case: *P*(1) is the statement 1 = F_3- 1, which is clearly true. Inductive case: Let *k* ≥ 1 be a natural number. Assume (for induction) that *P*(*k*) is true. That means $S_k = F_{k+2} - 1$. We will prove that P(*k*+1) is true as well. That is, we must prove that $S_{k+1} = F_{k+3} - 1$. To prove this equation, start with the right-hand side: $F_{k+3} - 1$. Note that $F_{k+3} = F_{k+2} + F_{k+1}$. This means $F_{k+3} - 1 = F_{k+2} + F_{k+1} - 1$. By the inductive hypothesis, $F_{k+2} + F_{k+1} - 1 = S_k + F_{k+1}$.

----

1. In your responses to peers, analyze their examples and proofs. Comment on any part of their post that you agree or disagree with, and ask follow-up questions whenever possible. Also, contrast your experience in inductive proofs with those of your peers.

Agree with this part of post

Ask questions if needed

Contrast your experience with inductive proofs

Hi Tyrus,

I enjoyed your post and agree with the logic of your proof. I liked that you provided multiple examples for the mathematical statement to highlight its claim. I agree with you on practice; figuring out what to replace when manipulating the inductive case has been challenging.

Lauren

Hi Maria,

The golden ratio exists in the closed formula for the *n*th Fibonacci number, where φ represents the ratio:

This formula can be found within the statement I wanted to prove with induction:

Lauren

Partial sums: show final example

$F_{3+2} - 1$

Given our statement, P(3) = $F_{3+2} - 1 = 5 - 1 = 4$.

By the inductive hypothesis, P(3 + 1) = P(3) + $F_{3+1} = 4 + 3 = 7$.

This is equivalent to P(*k*+1): P(3 + 1) = $F_{3+3} - 1 = F_6 - 1 = 7$.
