- 1. Write a list of 3 real-world scenarios where graph coloring may be useful.
- 2. For each, discuss how graph coloring is useful and contrast it with alternative ways to address those scenarios.
- 3. In your responses to peers, discuss how much you agree with their examples and whether or not graph coloring would truly be an effective concept to implement in each case. Respond in ways that promote further conversations whenever possible such as by asking follow-up questions.

The topic this week is fun. There are many applications where graph coloring may be useful. I am sure that you will have no problem finding three of them. I think that the most famous graph theory problem is how to color **maps**. Given any map of countries, states, counties, etc., how many colors are needed to color each region on the map so that neighboring regions are colored differently? This problem can be easily converted to a vertex coloring problem. If the vertex coloring has the property that adjacent vertices are colored differently, then the coloring is called proper. Every graph has a proper vertex coloring. For example, you could color every vertex with a different color. But often you can do better. The smallest number of colors needed to get a proper vertex coloring is called the chromatic number of the graph.

The discussion board this week will expand your understanding of how graph coloring can be applied to the real-world. Make sure to post early so you have plenty of time to engage with your peers about the scenarios everyone comes up with. I look forward to your posts.

Three ways that graph coloring can be used in real world applications are:

The alternative to this method is just to traditionally solve the sudoku tables using the 1 -9 for each row and column which can be difficult.

#### 1. To assign subway rail systems

We can also use graph coloring for subway rail systems. This will help to identify paths that certain rails take verses other rails. If we need to travel from a certain area to another, we can assign different colors for each route or edges and sets of connected stops or vertices. This will allow us to determine which train quickly and easily we need to board and what stop we need to get off if we must transfer to another train. Below is a map of the New York rail system where we can clearly see graph coloring help determine routes and stops.

I can no think of alternative ways to complete this task. Using graph colors is by far the most efficient way to do this.

#### 2. Assigning seating plans

If we have a list of people attending an event where we know that certain groups of people have conflict. We can use graph coloring to make sure they are not sat together. We can assign vertices to the names to each of the guests and use edges to represent which guests do not get along. We can then assign colors to determine which tables guests will sit at and help determine the minimum number of tables will be needed to make sure there are no issues.

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https://www.washingtonpost.com/news/wonk/wp/2014/06/27/this-new-mapping-technology-will-soon-show-you-whether-global-warming-will-drown-your-town/?arc404=true

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https://fivethirtyeight.com/features/why-it-only-took-one-player-with-coronavirus-to-shutter-sports/

3 colors

Utah Jazz

Teams that shared space with or played the Utah Jazz

Teams that did not share space or play the Utah Jazz

 $\rightarrow$  3 color alternative.

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Cause of mortality: communicable, maternal, neonatal, and nutritional diseases.

Global Burden of Disease (GBD) data are freely available to the world's researchers and policymakers. All global, regional, national, and in some cases subnational estimates of the burden of diseases, injuries, and risk factors from GBD can be retrieved from IHME's catalog of the world's health and demographic data.

http://www.healthdata.org/gbd/2019

https://vizhub.healthdata.org/qbd-compare

D	$\cap$	ŧ٠

Hello everyone,

#### 1. Identify roads by potential for flooding

Graph coloring can be used to highlight storm-related flooding threat levels on Lidar (Light Detection and Ranging) maps. States commission these elevation maps and can notify communities regarding potential property damage.

I can think of no alternative for this task.

### 2. Team contact tracing

If we want to determine whether or not a team that has tested positive for Covid-19 has potentially infected other teams in the league, we can use graph coloring. The following graph illuminates the connections between the positive team (Utah Jazz) and other teams in the league. The teams are indicated by their team colors if they shared an arena with the infected team.

I think a better way to color the vertices is by the following: 1) Is the team the Utah Jazz?; 2) If not, did the team play the Utah Jazz that season already or did the team share space with the Utah Jazz during that season? This schema would elicit 3 colors for the vertices. Of course, if we wanted to adhere to proper vertex coloring, each vertex would maintain a different color than the other vertices, most likely the team color. This coloring would produce a chromatic number  $\chi(G) = 30$  (Levin, 2020).

## 3. Assigning rank to infection and disease rates by country

The Institute for Health Metrics and Evaluation collected data on the global burden of disease by country and ranked the rates in each country by category (see Figure 3). If we graph the ranks (see Figure 4) we will see that the vertices do not show an identifiable color schema.

Given that  $C_7$  is a planar graph, we can defer to The Four Color Theorem (Levin, 2020) which states that "If G is a planar graph, then the chromatic number of G is less than or equal to 4," and reduce the graph colors to 4.

Thanks,

Lauren Alexandra

#### References:

Levin, O. (2020). *Discrete Mathematics: An Open Introduction*. http://discrete.openmathbooks.org/dmoi3/frontmatter.html

Figure 1. Graph of . Reprinted from The Washington Post, L. Montgomery, 2014, by 2014, . From "This new mapping technology will show whether global warming could drown your town" by L. Montgomery, 2014, *The Washington Post*. Copyright 2021 by The Washington Post.

Figure 1. A Lidar map of New Bern, N.C. shows potential flooding from a 16-foot storm surge. Reprinted from The Washington Post, by L. Montgomery, 2014, Retrieved from https://www.washingtonpost.com/news/wonk/wp/2014/06/27/this-new-mapping-technology-will-soon-show-you-whether-global-warming-will-drown-your-town/?arc404=true. Copyright 2021 by The Washington Post.

Price, D. (2018, March 23). Laziness does not exist. Medium.

https://humanparts.medium.com/laziness-does-not-exist-3af27e312d01

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Figure 1 *Lidar Map* 



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Figure 2

NBA Teams Contact with Utah Jazz

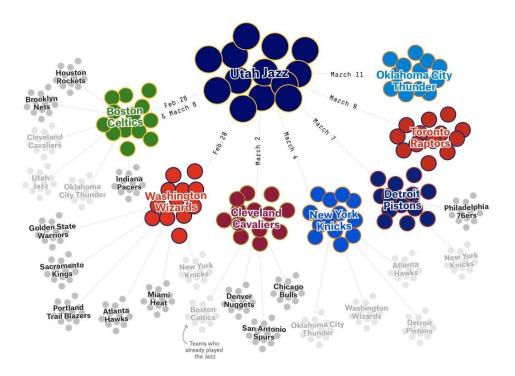


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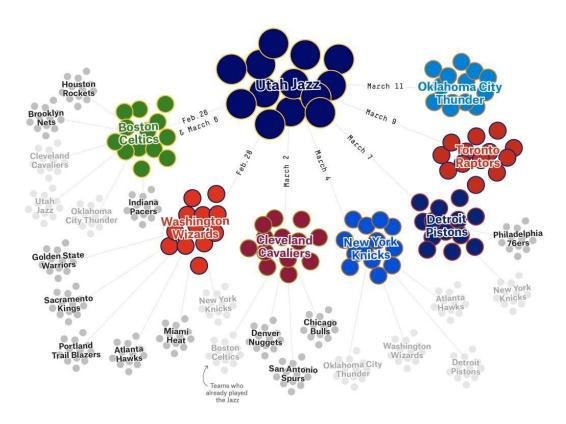


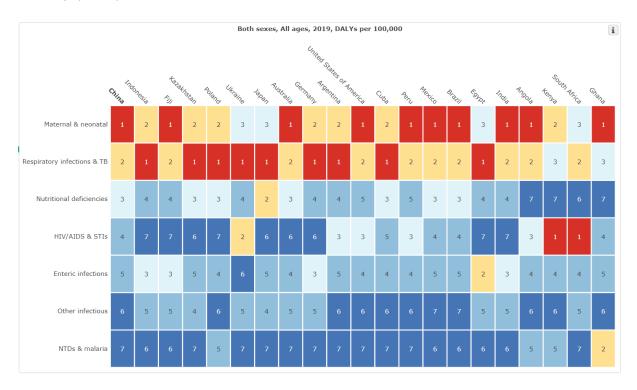
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# Communicable, Maternal, Neonatal, and Nutritional Disease and Infection Rates by Country (2019)



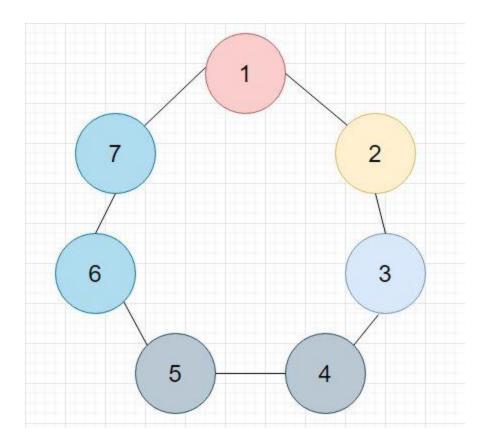
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Figure 4



Country Rankings as C

Graph



Note. Created in https://app.diagrams.net.

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**FINAL** 

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I can think of no alternative for this task.

#### 2. Team contact tracing

If we want to determine whether or not a team that has tested positive for Covid-19 has potentially infected other teams in the same league, we can create a contact tracing graph and employ graph coloring. The following graph (see Figure 2) illustrates the connections between a known positive team (Utah Jazz) and other teams in the league. The teams are indicated by their team colors if they shared an arena with the infected team.

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NBA Teams Contact with Utah Jazz

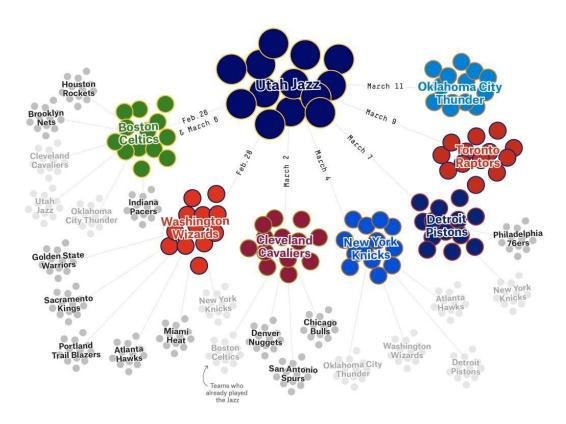


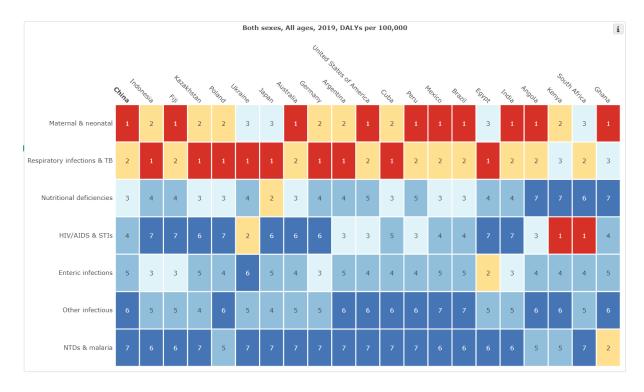
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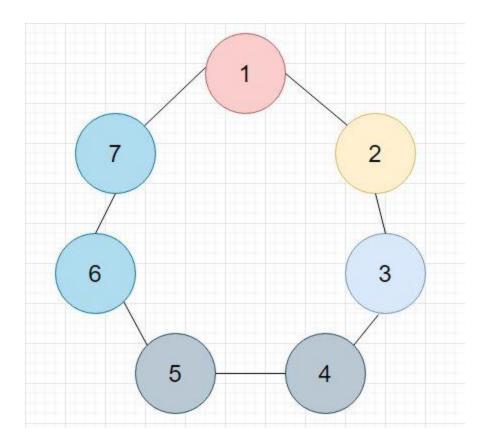
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