**Characteristics of the Fibonacci Sequence**

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**Introduction**

The Fibonacci sequence, like any sequence, is an ordered list of numbers. The sequence begins with 0, 1, 1, 2, 3, 5, 8, 13, and 21 (Chandra & Weisstein, n.d.). However, not every sequence is renowned for being “nature’s code.” Understanding the underlying characteristics of this sequence is fundamental to exploring its applications ranging across mathematics, computer programming, art, and biology (see Figure 1).

**Figure 1**

*Nature’s Code*

A picture containing flower, plant, sunflower

Description automatically generated

*Note.* Flower seed heads producing a Phi spiral pattern. From *Cleveland Design*. (<https://clevelanddesign.com/insights/the-nature-of-design-the-fibonacci-sequence-and-the-golden-ratio>). Copyright 2020 by Cleveland Design.

**Relation to Pascal’s Triangle**

Pascal’s triangle is a number pattern composed of binomial coefficients (Levin, 2020). The pattern contains a variety of notable patterns, one such being the Fibonacci sequence. We can produce the sequence by summing the binomial coefficients (Stakhov, 1998) along the diagonals of Pascal’s triangle (see Figure 2).

**Figure 2**

*Pascal’s Triangle: Fibonacci Sequence*

A picture containing honeycomb, outdoor object

Description automatically generated

**The Golden Ratio**

The golden ratio or φis a number that approximates the value of 1.618. This ratio can be found in the closed formula for the *n*th Fibonacci number. A closed formula for a sequence () where *n* ∈ *N* is a formula for using a fixed number of operations on *n* (Levin, 2020). We can define the formula as such: . The denominator of is curious until the golden ratio is written in another form. The golden ratio is equivalent to =. If we assign the value of 1 to *b*, we can solve for *a*. Ultimately, we find *a* = . This means the closed formula for the Fibonacci sequence can be written as - ) where φ is equal to (Stakhov, 1998).

**Partial sums of the Fibonacci Sequence**

A partial sum represents the sum of part of a sequence. Given a sequence, the partial sum of the first N terms is described as = . Each partial sum of the Fibonacci sequence can be calculated with = . This formula allows us to generate the partial sums sequence for each index in Fibonacci: 1, 2, 4, 7, 12, 20 + ... + . Notice the difference between each pair of sums represents a term in Fibonacci. Moreover, starting at the third index, each partial sum incremented by 1 produces a Fibonacci number.

**Negative numbers in the Fibonacci Sequence**

A recurrence relation is an equation relating a term of a sequence to the previous terms of the sequence (Levin, 2020). The formula for the recurrence relation of the Fibonacci sequence is defined as = + . To identify the negative numbers of Fibonacci, we can modify the recurrence relation to reflect its equivalent: = - . Starting with and decrementing by 1, we generate 1, -1, 2, -3 + ... +. As we decrement the *n*th term, we uncover that every other term is negative. To produce this sequence where is negative, that is, *n* < 0, we can employ the closed formula: = .

**Conclusion**

The Fibonacci numbers boast properties and relations that have captivated mathematicians for millennia: the incremented partial sums of the sequence generate the Fibonacci terms, the negative terms of Fibonacci are mirrored in the inverse of every other *n*th term, the terms are hidden in the sums of Pascal’s triangle’s binomial coefficients, and “nature’s code” is nested within the closed formula for the sequence. Organizations like the Fibonacci Association, which has since 1963 (The Fibonacci Association, n.d.) focused on the sequence and its related applications, along with novice and veteran mathematicians, will continue to probe this eminent pattern.

**References**

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