**Applications of Graph Theory Concepts: Social Networks**

Lauren Alexandra

Colorado State University Global

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Dr. Wael Abushammala

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**Introduction**

Graph theory is a branch of mathematics that studies structures designed to model relationships between objects. The collection of objects or vertices are connected by lines or edges. A graph is formally defined as an ordered pair *G = (V, E)* consisting of a nonempty set *V* (vertices) and a set *E* (edges) of two-element subsets of *V* (Levin, 2021).

**Graph Characteristics**

Each edge in the following graph represents a first-degree connection which constitutes a parent-child or sibling relationship, i.e., immediate family (see Figure 1). The sum of the edges protruding from a vertex is the degree of the vertex. Each vertex within both Roddy’s and Elda’s families in addition to Sandra’s vertex has a degree of 3. My vertex, Lauren, maintains a degree of 1. This graph is not complete since all possible edges are not drawn between the vertices. If the graph showcased all familial relations, it would be complete. Further, this graph is not planar—a graph that can be drawn on a plane with no edges crossing. We can prove this with Euler’s formula where *v*, *e*, and *f* represent vertices, edges, and faces, respectively: *v* - *e* + *f* = 2 (Levin, 2021). Given our graph, this means 10 - 15 + 9 ≠ 2, therefore the graph cannot be planar.

**Figure 1**

*Social Network: First-degree Connections*

![Chart, bubble chart

Description automatically generated]()

*Note.* Graph of my social network through my mother’s branch of the family tree. Created in <https://app.diagrams.net>.

**Degrees of Separation**

The social distance between any two persons in a network can be determined by the degrees between them. The largest degree of separation between myself and another person on my mother’s side of the family is 4 (Dani or Jacob to myself). These individuals exist at vertices furthest away from each other than any other vertex, with 6 degrees of separation. The six degrees of separation theory (1967) states that the maximum degree of separation between *any* two people is only 6 degrees. The phenomenon has made an appearance in the networks of medical journal authors (Hautz et al., 2016) as well as Hollywood actors (Reynolds, 2015). With the advent of social media however, this theory has been challenged by a titan of the landscape (Daraghmi & Yuan, 2014). Facebook estimated for each person on the platform, approximately 1.59 billion active users, there is an average degree of separation of 3.57 degrees between every other person (see Figure 2).

**Figure 2**

*Degrees of Separation (Facebook)*

![Chart, histogram

Description automatically generated]()

*Figure 2.* The average degrees of separation between Facebook users. Reprinted from Facebook Research, by Bhagat, S., Burke, M., Diuk, C., Filiz, I. O., & Edunov, S., 2016, Retrieved from https://research.fb.com/blog/2016/02/three-and-a-half-degrees-of-separation. Copyright 2021 by Facebook.

**Conclusion**

Although graph theory is a relatively new area of mathematics, its applications are myriad and provide promising insights into how we relate to each other and why. With more of daily life moving to exist online, the speed, capacity, and shape of our social connections are ripe for study through the lens of this theory.

**References**

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