**Represent a Map by Graph with Coloring**

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**Introduction**

Graph coloring is a central problem within graph theory, the study of mathematical objects comprised of vertices and edges. The total coloring of a graph entails assigning colors to the vertices of a graph such that adjacent vertices do not share the same color (Maltby & et al., 2021). The assignment of colors necessary for this outcome produces a k-colorable graph where the chromatic number of a graph *G*, *χ*(*G*), is the minimum number of colors required.

**New Hampshire**

The graph of New Hampshire’s counties (see Figure 1) only connects adjacent vertices which results in a planar graph. Every map maintains a planar graph because the edges will never cross. The four color theorem (Appel & Wolfgang, 1977) states that if a graph is a planar graph, then the chromatic number is no larger than 4. We know the New Hampshire graph can be 4-colorable or less. Additionally, we know that a graph has a chromatic number that is at most one larger than that of a subgraph with one fewer vertex (Maltby & et al., 2021). To find the chromatic number we can begin with 2 colors and defer to the 2-colorable graph theorem: if and only if a graph contains no circuits that have an odd number of vertices, then the graph is 2-colorable (Levin, 2021). Figure 1 illustrates that the graph has more than one circuit with an odd number of vertices. Next, we employ 3-color assignment and find the graph is indeed 3-colorable; the color assignment has met the aforementioned criteria. Beyond mapmaking, New Hampshire may make use of this color assignment for the scheduling of state programs across counties and perhaps even for how state resources are shared and distributed.

**Figure 1**

*New Hampshire*

![Map

Description automatically generated]()

Note*:* Graph of New Hampshire counties which share borders. Adapted from New Hampshire, In *Mapsof.Net*, n.d., Retrieved January 24, 2021, from https://www.mapsof.net/new-hampshire. Copyright 2021 by mapsof.net.

**State of Four**

Given a fictional map, we can apply graph coloring as well. Figure 2 depicts a state composed of four counties that is 3-colorable. To guarantee this color assignment, the map must surpass 2-color assignment and avoid 4-color assignment. Although maps ensure the generation of a planar graph, by which the four color theorem elicits a chromatic number as large as 4, the shared borders should only be created if, of the adjacent counties, there is only a need for 2 different vertex colors. This rule creates a graph that contains more than one circuit with an odd number of vertices, thus a map that is not 2-colorable, and instead a 3-colorable map.

**Figure 2**

*Four Counties Model*

![Diagram

Description automatically generated]()

Note*.* Map of a fictional state consisting of four counties. Created in <https://app.diagrams.net>.

**Conclusion**

Graph coloring provides us with a set of theorems for the proper coloring of maps. However, cartographers rarely determine the minimum number of colors needed for mapmaking. More frequently, graph coloring enables us to solve problems of radio interference or scheduling. Mathematicians will continue to build on these theorems, including through the proposal of new algorithms (Dey & et al., 2019) to solve emerging problems in the era of big data.

**References**

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