Course: MATH6183001 – Scientific Computing

Method of Assessment: Case Study

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Class : Computer Science

Topic : Regression & Interpolation, Taylor Series, Numerical Differentiation, Numerical Integration

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1. The relationship between the average temperature on the earth's surface in odd years between 1981 - 1999, is given by the following below: (35%)

Year (y)	Temperature (x, ºC)
1981	14.1999
1983	14.2411
1985	14.0342
1987	14.2696
1989	14.197

1991	14.3055
1993	14.1853
1995	14.3577
1997	14.4187
1999	14.3438

a. Estimate the temperature in even years by linear, quadratic, and cubic interpolation order! Choose the method that you think is appropriate, and explain the difference.

Assurance of Learning: Study Case
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1.a.i. Linear Interpolation : $f(x) = y = y_0 + (x - x_0)\dfrac{(y_1 - y_0)}{(x_1 - x_0)}$

1) 1982

$$f(1982) = 14{,}1999 + (1982 - 1981)\frac{(14.2411 - 14.1999)}{(1983 - 1981)} = 14{,}2205 \Rightarrow y$$

2) 1984

$$f(1984) = 14{,}2411 + (1984 - 1983)\frac{(14{,}0342 - 14.2411)}{(1985 - 1983)} = 14.1376 \Rightarrow y$$

3) 1986

$$f(1986) = 14.0342 + (1986 - 1985)\frac{(14.2696 - 14.0342)}{(1987 - 1985)} = 14.1519 \Rightarrow y$$

4) 1988

$$f(1988) = 14.2696 + (1988 - 1987)\frac{(14.197 - 14.2696)}{(1989 - 1987)} = 14.2333 \Rightarrow y$$

5) 1990

$$f(1990) = 14.197 + (1990 - 1989)\frac{(14.3055 - 14.197)}{(1991 - 1989)} = 14.2512 \Rightarrow y$$

6) 1992

$$f(1992) = 14.197 + (1992 - 1991)\frac{(14.1853 - 14.197)}{(1993 - 1991)} = 14.2454 \Rightarrow y$$

7) 1994

$$f(1994) = 14.1853 + (1994 - 1993)\frac{(14.3577 - 141853)}{(1995 - 1993)} = 14.2715 \Rightarrow y$$

8) 1996

$$f(1996) = 14.3577 + (1996 - 1995)\frac{(14.4187 - 14.3577)}{(1997 - 1995)} = 14.3852 \Rightarrow y$$

9) 1998

$$f(1998) = 14.4187 + (1998 - 1997)\frac{(14.3438 - 14.4187)}{(1999 - 1997)} = 14.3812 \Rightarrow y$$

ii. Quadratic Interpolation : $f(x) = y = \underbrace{b_0 + b_1(x - x_0)}_{\text{Linear Interpolation}} + b_2(x - x_0)(x - x_1)$

$b_0 = f(x_0)$

$$b_1 = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$b_2 = \frac{\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} - \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}}{x_2 - x_0}$$

1) 1982

$$y = 14.2205 + \left[\frac{(14.0342 - 14.2411)}{(1985 - 1983)} - \frac{(14.2411 - 14.1999)}{(1983 - 1981)} \Big/ (1985 - 1981)\right](1982 - 1981)(1982 - 1983)$$

$= 14.2515$

2) 1984

$$y = 14.1376 + \left[\frac{(14.2696 - 14.0342)}{(1987 - 1985)} - \frac{(14.0342 - 14.2411)}{(1985 - 1983)} \Big/ (1987 - 1983)\right](1984 - 1983)(1984 - 1985)$$

$= 14.0824$

3) 1986

$$Y = 14.1519 + \left[\frac{(14.197 - 14.2696)}{(1989-1987)} - \frac{(14.2696 - 14.0342)}{(1987-1985)} \Big/ (1989-1985)\right](1986-1985)(1986-1987)$$

$= 14.1904$ //

4) 1988

$$Y = 14.2333 + \left[\frac{(14.3055 - 14.197)}{(1991-1989)} - \frac{(14.197 - 14.2696)}{(1989-1987)} \Big/ (1991-1987)\right](1988-1987)(1988-1989) = 14.2107 \text{ //}$$

5) 1990

$$Y = 14.2512 + \left[\frac{(14.1853 - 14.3055)}{(1993-1991)} - \frac{(14.3055 - 14.197)}{(1991-1989)} \Big/ (1993-1989)\right](1990-1989)(1990-1991) = 14.2798 \text{ //}$$

6) 1992

$$Y = 14.2454 + \left[\frac{(14.3577 - 14.1853)}{(1995-1993)} - \frac{(14.1853 - 14.3055)}{(1993-1991)} \Big/ (1995-1991)\right](1992-1991)(1992-1993) = 14.2088 \text{ //}$$

7) 1994

$$Y = 14.2715 + \left[\frac{(14.4187 - 14.3577)}{(1997-1995)} - \frac{(14.3577 - 14.1853)}{(1995-1993)} \Big/ (1997-1993)\right](1994-1993)(1994-1995) = 14.2854 \text{ //}$$

8) 1996

$$Y = 14.3852 + \left[\frac{(14.3438 - 14.4187)}{(1999-1997)} - \frac{(14.4187 - 14.3577)}{(1997-1995)} \Big/ (1999-1995)\right](1996-1995)(1996-1997) = 14.4022 \text{ //}$$

II. Cubic Interpolation $= f(x) = Y =$ quadratic (x) +

$$\left[\frac{\left(\dfrac{\frac{(Y_3-Y_2)}{x_3-x_2} - \frac{(Y_2-Y_1)}{x_2-x_1}}{x_3-x_1}\right) - \left(\dfrac{\frac{(Y_2-Y_1)}{x_2-x_1} - \frac{(Y_1-Y_0)}{x_1-x_0}}{x_2-x_0}\right)}{x_3-x_0}\right] \cdot (x-x_0) \cdot (x-x_1) \cdot (x-x_2)$$

1) 1982

$$Y = 14.2515 + \left[\frac{\left(\dfrac{\frac{14.2696-14.0342}{1987-1985} - \frac{14.0342-14.2411}{1985-1983}}{1987-1983}\right) - \left(\dfrac{\frac{14.0342-14.2411}{1985-1983} - \frac{14.2411-14.1999}{1983-1981}}{1985-1981}\right)}{1987-1981}\right] \cdot (1982-1981) \cdot (1982-1983) \cdot (1982-1985)$$

$= 14.2947$ //

2) 1984

$$Y = 14.0824 + \left[\frac{\left(\dfrac{\frac{14.197-14.2696}{1989-1987} - \frac{14.2696-14.0342}{1987-1985}}{1989-1985}\right) - \left(\dfrac{\frac{14.2696-14.0342}{1987-1985} - \frac{14.0342-14.2411}{1985-1983}}{1987-1983}\right)}{1989-1983}\right] \cdot (1984-1983) \cdot (1984-1985) \cdot (1984-1987)$$

$= 14.0355$ //

3) 1986

$$Y = 14.1904 + \left[\frac{\left(\dfrac{\frac{14.3055-14.197}{1991-1989} - \frac{14.197-14.2696}{1989-1987}}{1991-1987}\right) - \left(\dfrac{\frac{14.197-14.2696}{1989-1987} - \frac{14.2696-14.0342}{1987-1985}}{1989-1985}\right)}{1991-1985}\right] \cdot (1986-1985) \cdot (1986-1987) \cdot (1986-1989)$$

$= 14.221$ //

4) 1988

$$Y = 14.2107 + \left[\frac{\left(\dfrac{\frac{14.1853-14.3055}{1993-1991} - \frac{14.3055-14.197}{1991-1989}}{1993-1989}\right) - \left(\dfrac{\frac{14.3055-14.197}{1991-1989} - \frac{14.197-14.2696}{1989-1987}}{1991-1987}\right)}{1993-1987}\right] \cdot (1988-1987) \cdot (1988-1989) \cdot (1988-1991)$$

$= 14.1851$ //

5) 1990

$$Y = 14.2798 + \left[\frac{\left(\dfrac{\frac{14.3577-14.1853}{1995-1993} - \frac{14.1853-14.3055}{1993-1991}}{1995-1991}\right) - \left(\dfrac{\frac{14.1853-14.3055}{1993-1991} - \frac{14.3055-14.197}{1991-1989}}{1993-1989}\right)}{1995-1989}\right] \cdot (1990-1989) \cdot (1990-1991) \cdot (1990-1993)$$

$= 14.3124$ //

6) 1992

$$Y = 14.2088 + \left[\frac{\left(\frac{\frac{14.4187-14.3577}{1997-1995} - \frac{14.3577-14.1853}{1995-1993}}{1997-1993}\right) - \left(\frac{\frac{14.3577-14.1853}{1995-1993} - \frac{14.1853-14.3055}{1993-1991}}{1995-1991}\right)}{1997-1991}\right] \cdot (1992-1991) \cdot (1992-1993) \cdot (1992-1995) = 14.1836$$

7) 1994

$$Y = 14.2854 + \left[\frac{\left(\frac{\frac{14.3438-14.4187}{1999-1997} - \frac{14.4187-14.3577}{1997-1995}}{1999-1995}\right) - \left(\frac{\frac{14.4187-14.3577}{1997-1995} - \frac{14.3577-14.1853}{1995-1993}}{1997-1993}\right)}{1999-1993}\right] \cdot (1994-1993) \cdot (1994-1995) \cdot (1994-1997) = 14.2839$$

Berdasarkan 3 jenis interpolasi, QUADRATIC INTERPOLATION merupakan metode yang terbaik. Hal ini dikarena kan, terdapat 10 titik data yang sifatnya tidak terlalu kompleks. Dengan begitu, QUADRATIC Interpolation dapat menunjukkan perkiraan yang lebih akurat jika dibandingkan dengan Linear Interpolation.

Typed Conclusion:

Berdasarkan 3 jenis interpolasi, QUADRATIC INTERPOLATION merupakan metode yang terbaik. Hal ini dikarenakan, terdapat 0 titik data yang sifatnya tidak terlalu kompleks maupun tidak sederhana. Dengan begitu, QUADRATIC INTERPOLATION dapat menunjukkan perkiraan yang lebih akurat jika dibandingkan dengan Linear dan Cubic Interpolation.

b. Perform a least-square regression of the above data to estimate the temperature in even years.

1.b.

	X	Y	X^2	Y^2	XY
1	1981	14.1999	3924361	201,63716	28130
2	1983	14.2411	3932289	202,8089292	28240,1
3	1985	14.0342	3940225	196,9587696	27857,89
4	1987	14.2696	3948169	203,6214842	28353,7
5	1989	14.197	3956121	201,554809	28237,83
6	1991	14.3055	3964081	204,6473303	28482,25
7	1993	14.1853	3972049	201,2227361	28271,3
8	1995	14.3577	3980025	206,1435493	28643,61
9	1997	14.4187	3988009	207,8989097	28794,14
10	1999	14.3438	3996001	205,7445984	28673,26
Σ	19900	142.5528	39601330	2032,238276	283684.08

$$\beta_0 = \frac{(\Sigma Y \cdot \Sigma X^2) - (\Sigma X \cdot \Sigma XY)}{(n \cdot x^2) -}$$

$$= \frac{(142.5528 \cdot 39601330) - (19900 \cdot 283684.08)}{(10 \cdot 39601330) - 396010000}$$

$$= -9.934677576 \approx -9,9347$$

$$\beta_1 = \Sigma XY - (\Sigma X \cdot \Sigma Y) / (n \cdot \Sigma x^2) -$$

$$= \frac{283684.08 - (19900 \cdot 142,5528)}{(10 \cdot 3961330) - 396010000}$$

$$= 0,012155758 = 0,0122$$

LEAST SQUARE REGRESSION

$LSQ = \beta_0 + \beta_1 (x)$ $\therefore \beta_0 = -9,934677576$; $\beta_1 = 0,012155758$

1) 1982 = −9,934677576 + 0,012155758 (1982) = 14,15803394 = 14,158
2) 1984 = −9,934677576 + 0,012155758 (1984) = 14,18234545 = 14,1823
3) 1986 = −9,934677576 + 0,012155758 (1986) = 14,20665697 = 14,2066
4) 1988 = −9,934677576 + 0,012155758 (1988) = 14,23096848 = 14,231
5) 1990 = −9,934677576 + 0,01255758 (1990) = 14,25528 = 14,2553
6) 1992 = −9,934677576 + 0,01255758 (1992) = 14,27959151 = 14,2796
7) 1994 = −9,934677576 + 0,01255758 (1994) = 14,30390303 = 14,3039
8) 1996 = −9,934677576 + 0,01255758 (1996) = 14,32821455 = 14,3282
9) 1998 = −9,934677576 + 0,01255758 (1998) = 14,35252606 = 14,3525

c. Perform an analysis of the difference between the results of the regression and interpolations you can above, explain based on the theoretical basis you have learned.

Berdasarkan perhitungan yang dilakukan di atas, kita dapat melihat bahwa terdapat perbedaan antara hasil dari perhitungan dengan metode interpolasi dan metode regresi. Metode interpolasi adalahnya metode untuk menghitung estimasi suhu yang dilakukan dengan menghitung nilai-nilai suhu antara titik data yang sudah diberikan. Di sisi lain, metode regresi adalah metode untuk mengetahui estimasi suhu agar dapat menunjukkan garis lurus terbaik berdasarkan data yang ada.

Perhitungan di atas menunjukkan bahwa perhitungan regresi linear menunjukkan nilai titik-titik yang cukup akurat untuk membuat pola linear atau garis lurus. Selain itu, perhitungan interpolasi di atas juga menunjukkan titik-titik estimasi.

d. Make a plot that describes the relationship between Temperature (y) and Year (x) as informatively as possible for the reader, based on the results of your analysis using Python library.

Python Script:

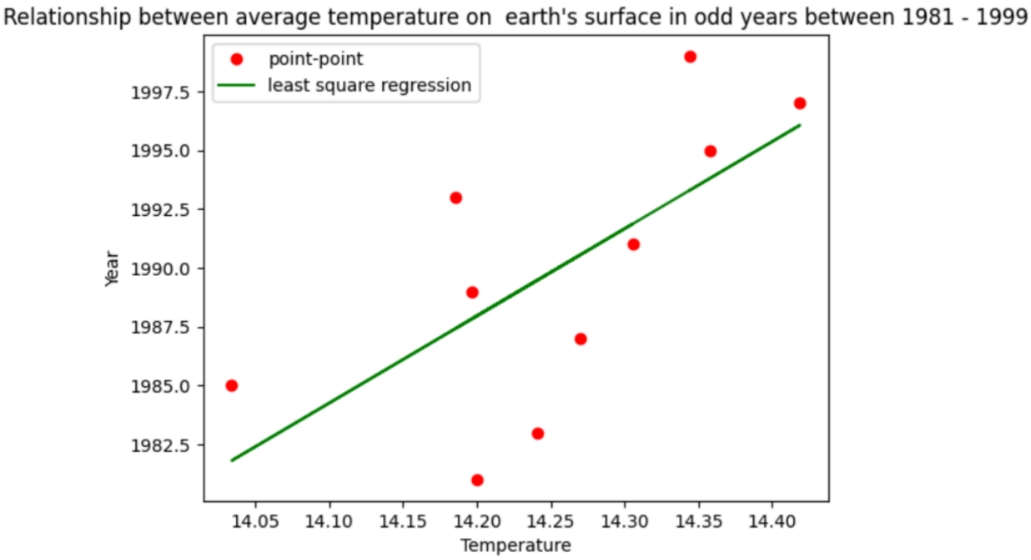
```
import numpy as np
```

```
import matplotlib.pyplot as plt

y = np.array([1981, 1983, 1985, 1987, 1989, 1991, 1993, 1995, 1997, 1999])
x = np.array([14.1999, 14.2411, 14.0342, 14.2696, 14.197, 14.3055, 14.1853, 14.3577,
    14.4187, 14.3438])

A = np.vstack((x, np.ones(len(x)))).T

y = y[:, np.newaxis]

pinv = np.linalg.pinv(A)
gama = np.dot(pinv, y)
m = gama[0]
c = gama[1]
print("m = ", gama[0])
print("c = ", gama[1])

plt.title("Relationship between average temperature on  earth's surface in odd years
    between 1981 - 1999")
plt.xlabel("Temperature")
plt.ylabel("Year")

plt.plot(x, y, "ro", label = "point-point")
plt.plot(x, m * x + c, "g", label = "least square regression")
plt.legend()
plt.show()
```

Output:

m = [37.07496648]

c = [1461.48597183]



2. Compute the fourth order Taylor expansion for sin(x) and cos(x) and sin(x)cos(x) around 0. (30%)

   a. Write down your manual calculation AND Python script to answer above's question

Manual Calculation:

2.a. ·) $\sin(x)$

$f(0) = \sin(0) = 0$

$f'(0) = \cos(0) = 1$

$f''(0) = -\sin(0) = 0$

$f'''(0) = -\cos(0) = -1$

$f''''(0) = \sin(0) = 0$

·) $\cos(x)$

$f(0) = \cos(0) = 1$

$f'(0) = -\sin(0) = 0$

$f''(0) = -\cos(0) = -1$

$f'''(0) = \sin(0) = 0$

$f''''(0) = \cos(0) = 1$

·) $\sin(x)\cos(x)$

$f(0) = \sin(0)\cos(0) = 0$

$f'(0) = \cos^2(0) - \sin^2(0) = 1$

$f''(0) = -2\sin(0)\cos(0) = 0$

$f'''(0) = -2(\cos^2(0) - \sin^2(0)) = -2$

$f''''(0) = 4\sin(0)\cos(0) = 0$

$$f(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \frac{f''''(0)x^4}{4!}$$

$$\sin(x) = x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040} \qquad \cos(x) = 1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \frac{x^8}{40320}$$

$$\sin(x)\cos(x) = x - \frac{2x^3}{3} + \frac{2x^5}{15} - \frac{4x^7}{315}$$

Python Script:

```
import sympy as sp


x = sp.Symbol('x')
sinx = sp.sin(x)
cosx = sp.cos(x)
```

```
sinxcosx = sinx * cosx


taylor_sinx = sinx.series(x, 0, 5).removeO()
taylor_cosx = cosx.series(x, 0, 5).removeO()
taylor_sinxcosx = sinxcosx.series(x, 0, 5).removeO()


print("Fourth-order Taylor expansion untuk sin(x): ", taylor_sinx)
print("Fourth-order Taylor expansion untuk cos(x): ", taylor_cosx)
print("Fourth-order Taylor expansion untuk sin(x)cos(x): ", taylor_sinxcosx)
```

Output:

Fourth-order Taylor expansion untuk sin(x): -x**3/6 + x

Fourth-order Taylor expansion untuk cos(x): x**4/24 - x**2/2 + 1

Fourth-order Taylor expansion untuk sin(x)cos(x): -2*x**3/3 + x

b. Which produces less error for x=π/2: computing the Taylor expansion for sin and cos separately then multiplying the result together, or computing the Taylor expansion for the product first then plugging in x?

Python Script:

```
import math as mp


x = mp.pi / 2 # x = n / 2


sinx = sp.sin(x)
cosx = sp.cos(x)
value = sinx * cosx


# separate counting (taking the function from 2A)
sin_x = -x**3/6 + x
cos_x = x**4/24 - x**2/2 + 1
separate_counting = sin_x * cos_x
```

```
# counting sin(x).cos(x) using taylor expansion
sin_x_cos_x = -2*x**3/3 + x

error1 = separate_counting - value
error2 = sin_x_cos_x - value

print("Error 1 : ", error1)
print("Error 2 : ", error2)
```

The output:

Error 1 : 0.0184679357262632

Error 2 : -1.01306006323009

From the result above, we could see that SEPARATE COUNTING is the accurate method. It's because the error number on error1 is very small. Not like the error2 that has minus value. Minus value indicates that the method or approximations are not accurate. That's why the separate counting method is the best method for counting the number of errors.

c. Use the same order of Taylor series to approximate cos (π/4) and determine the truncation error bound. You may include either your manual calculation OR Python script for this question

Python Script:

```
from sympy.functions.elementary.trigonometric import cos
import math as mp

x = mp.pi / 4 # x = n / 4

# approach 1
sin_x = -x**3/6 + x
cos_x = x**4/24 - x**2/2 + 1
approach1 = sin_x * cos_x

# approach 2
```

```
approach2 = -2*x**3/3 + x

# result
result = mp.sin(x) * mp.cos(x)

# count error
error1 = abs(approach1 - result)
error2 = abs(approach2 - result)

print(f"Error Approach 1: {error1}")
print(f"Error Approach 2: {error2}")
```

The Output:

Error Approach 1: 0.0015081339491602175

Error Approach 2: 0.037583885355674806

3. Given that *f(x) = x**3 - 0.3*x**2 - 8,56*x + 8,448*. (35%)

   a. Approximate with 20 evenly-spaced grid points over the whole interval using Riemann Integral, Trapezoid Rule, and Simpson's Rule. Explain the difference behind each of the method.

3. $f(x) = x^3 - 0{,}3x^2 - 8{,}56x + 8{,}448$

a. $\int_0^{2\pi} x^3 - 0{,}3x^2 - 8{,}56x + 8{,}448$

↳ $\Delta x = \frac{2\pi - 0}{20} = \frac{\pi}{10}$ => Riemann Integral

- $i = 0 \rightarrow x = 0 + 0.(\pi/10) = 0 \rightarrow f(x) = 0^3 - 0{,}03(0)^2 - 8{,}56(0) + 8{,}448 = 8{,}448$
- $i = 1 \rightarrow x = 0 + 1.(\pi/10) = \pi/10 \rightarrow f(x) = (\pi/10)^3 - 0{,}03(\pi/10)^2 - 8{,}56(\pi/10) + 8{,}448$
- $i = 2 \rightarrow x = 0 + 2.(\pi/10) = 2\pi/10 = \pi/5 \rightarrow f(x) = (\pi/5)^3 - 0{,}03(\pi/5)^2 - 8{,}56(\pi/5) + 8{,}448$
- $i = 3 \rightarrow x = 0 + 3(\pi/10) = 3\pi/10 \rightarrow f(x) = (3\pi/10)^3 - 0{,}03(3\pi/10)^2 - 8{,}56(3\pi/10) + 8{,}448$
- $i = 4 \rightarrow x = 0 + 4(\pi/10) = 4\pi/10 = 2\pi/5 \rightarrow f(x) = (2\pi/5)^3 - 0{,}03(2\pi/5)^2 - 8{,}56(2\pi/5) + 8{,}448$
- $i = 5 \rightarrow x = 0 + 5(\pi/10) = 5\pi/10 = \pi/2 \rightarrow f(x) = (\pi/2)^3 - 0{,}03(\pi/2)^2 - 8{,}56(\pi/2) + 8{,}448$
- $i = 6 \rightarrow x = 0 + 6(\pi/10) = 6\pi/10 \rightarrow f(x) = (6\pi/10)^3 - 0{,}03(6\pi/10)^2 - 8{,}56(6\pi/10) + 8{,}448$
- $i = 7 \rightarrow x = 0 + 7(\pi/10) = 7\pi/10 \rightarrow f(x) = (7\pi/10)^3 - 0{,}03(7\pi/10)^2 - 8{,}56(7\pi/10) + 8{,}448$
- $i = 8 \rightarrow x = 0 + 8(\pi/10) = 8\pi/10 \rightarrow f(x) = (8\pi/10)^3 - 0{,}03(8\pi/10)^2 - 8{,}56(8\pi/10) + 8{,}448$
- $i = 9 \rightarrow x = 0 + 9(\pi/10) = 9\pi/10 \rightarrow f(x) = (9\pi/10)^3 - 0{,}03(9\pi/10)^2 - 8{,}56(9\pi/10) + 8{,}448$
- $i = 10 \rightarrow x = 0 + 10(\pi/10) = \pi \rightarrow f(x) = (\pi)^3 - 0{,}03(\pi)^2 - 8{,}56(\pi) + 8{,}448$
- $i = 11 \rightarrow x = 0 + 11(\pi/10) = 11\pi/10 \rightarrow f(x) = (11\pi/10)^3 - 0{,}03(11\pi/10)^2 - 8{,}56(11\pi/10) + 8{,}448$
- $i = 12 \rightarrow x = 0 + 12(\pi/10) = 12\pi/10 \rightarrow f(x) = (12\pi/10)^3 - 0{,}03(12\pi/10)^2 - 8{,}56(12\pi/10) + 8{,}448$
- $i = 13 \rightarrow x = 0 + 13(\pi/10) = 13\pi/10 \rightarrow f(x) = (13\pi/10)^3 - 0{,}03(13\pi/10)^2 - 8{,}56(13\pi/10) + 8{,}448$
- $i = 14 \rightarrow x = 0 + 14(\pi/10) = 14\pi/10 \rightarrow f(x) = (14\pi/10)^3 - 0{,}03(14\pi/10)^2 - 8{,}56(14\pi/10) + 8{,}448$
- $i = 15 \rightarrow x = 0 + 15(\pi/10) = 3\pi/2 \rightarrow f(x) = (3\pi/2)^3 - 0{,}03(3\pi/2)^2 - 8{,}56(3\pi/2) + 8{,}448$
- $i = 16 \rightarrow x = 0 + 16(\pi/10) = 16\pi/10 \rightarrow f(x) = (16\pi/10)^3 - 0{,}03(16\pi/10)^2 - 8{,}56(16\pi/10) + 8{,}448$
- $i = 17 \rightarrow x = 0 + 17(\pi/10) = 17\pi/10 \rightarrow f(x) = (17\pi/10)^3 - 0{,}03(17\pi/10)^2 - 8{,}56(17\pi/10) + 8{,}448$
- $i = 18 \rightarrow x = 0 + 18(\pi/10) = 18\pi/10 \rightarrow f(x) = (18\pi/10)^3 - 0{,}03(18\pi/10)^2 - 8{,}56(18\pi/10) + 8{,}448$
- $i = 19 \rightarrow x = 0 + 19(\pi/10) = 19\pi/10 \rightarrow f(x) = (19\pi/10)^3 - 0{,}03(19\pi/10)^2 - 8{,}56(19\pi/10) + 8{,}448$
- $i = 20 \rightarrow x = 0 + 20(\pi/10) = 2\pi \rightarrow f(x) = (2\pi)^3 - 0{,}03(2\pi)^2 - 8{,}56(2\pi) + 8{,}448$

$f(x) = f(0) + f(1) + f(2) + f(3) + \ldots + f(20) = 248{,}47252$

↳ TRAPEZOID RULE

$\Delta x = \pi/10$ from $(2\pi - 0)/20$

$T_n = (\Delta x / 2) \cdot (f(x_0) + 2f(x_1) + 2f(x_2) + \ldots + 2f(x_{n-1}) + f(x_n))$

$x_0 = 0$ | step size $= \Delta x$	...	Menggunakan $f(x_0) - f(x_{20})$ pada Riemann Integral
$x_1 = x_0 + \Delta x$	$x_{19} = x_{18} + \Delta x$	↳ maka $T_n = 229{,}24548$
$x_2 = x_1 + \Delta x$ ...	$x_{20} = x_{19} + \Delta x$	

↳ SIMPSON RULE

$\Delta x = (2\pi - 0)/20 = \pi/10$

$S_n = (\Delta x / 3) \cdot (f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + \ldots + 4f(x_{19}) + f(x_{20}))$

$x_0 = 0$ | step size $= \Delta x$	...	Maka $S_n = 248{,}94406$
$x_1 = x_0 + \Delta x$	$x_{19} = x_{18} + \Delta x$	
$x_2 = x_1 + \Delta x$ ...	$x_{20} = x_{19} + \Delta x$	

- Riemann Integral approximations are the method where the countings are based on the area under the curve that we divide into smaller parts and count them for the approximation function between each part.
- The Trapezoid Rule shows how to count function approximations within each trapezoid by dividing the area under the curves into trapezoids.
- The Simpson Rule is a rule that gives more accurate approximations rather than the Riemann integral approximations and the trapezoid rule. This rule's calculated by fitting every three adjacent points on the curve with the

quadratic function or known as parabola. Then, we count the area under each parabola.

b. Compared to the methods above, do you think that analytical integration could be more convenient to be done?

Based on the methods above, the analytical integration could be more convenient because the method is simple and accurate. From the function, *f(x) = x**3 - 0.3*x**2 - 8,56*x + 8,448* is a polynomial function. The antiderivative of f(x) over the interval is 0,2π then subtracting the antiderivative on the lower limit from the upper limit. So, the explicit integration shows the exact value of the integral.

c. Use polynomial interpolation to compute and at $x = 0$, using the discrete data below

	-1.1	-0.3	0.8	1.9
	15.180	10.962	1.920	-2.040

3.c. POLYNOMIAL INTERPOLATION

x	f(x)
-1,1	15,18
-0,3	10,962
0,8	1,920
1,9	-2,040

$$y = L_0 f(x_0) + L_1 f(x_1) + L_2 f(x_2) + L_3 f(x_3)$$

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}(y_0) + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}(y_1) + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}(y_2)$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}(y_3)$$

$f(x) = 1,21u^3 - 0,812u^2 - 8,3929u + 8,69942$

$f'(x) = 3,63u^2 - 1,624u - 8,3929$

$f''(x) = 7,26u - 1,624$

d. Calculate the accuracy result compared to the initial

3.d. $f(x) = 1,21u^3 - 0,812u^2 - 8,3929u + 8,69942$