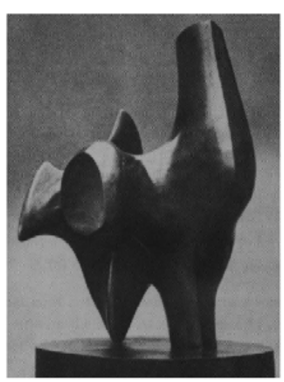
Edge Detection

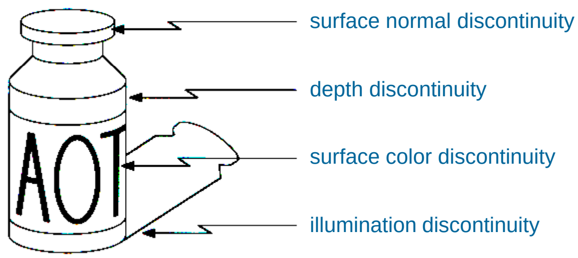
Slides from Cornelia Fermüller and Marc Pollefeys

Edge detection

- Convert a 2D image into a set of curves
  - Extracts salient features of the scene
  - More compact than pixels

Origin of Edges



- Edges are caused by a variety of factors

Edge detection

1. Detection of short linear edge segments (edgels)
2. Aggregation of edgels into extended edges
3. Maybe parametric description

Edge is Where Change Occurs

- Change is measured by derivative in 1D
- Biggest change, derivative has maximum magnitude
- Or 2nd derivative is zero.

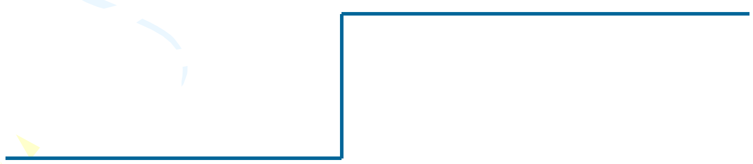
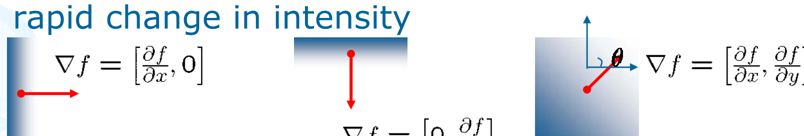


Image gradient

- The gradient of an image:

$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

- The gradient points in the direction of most rapid change in intensity



- The gradient direction is given by:

$$\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$$

  – Perpendicular to the edge
- The *edge strength* is given by the magnitude

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

How discrete gradient?

- By finite differences
  - $f(x+1,y) - f(x,y)$
  - $f(x, y+1) - f(x,y)$

The Sobel operator

- Better approximations of the derivatives exist
  - The *Sobel* operators below are very commonly used

$$\frac{1}{8}\begin{array}{|c|c|c|}\hline -1 & 0 & 1 \\ \hline -2 & 0 & 2 \\ \hline -1 & 0 & 1 \\ \hline \end{array} \qquad \frac{1}{8}\begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 0 & 0 & 0 \\ \hline -1 & -2 & -1 \\ \hline \end{array}$$

$$s_x \qquad\qquad\qquad s_y$$

  - The standard defn. of the Sobel operator omits the 1/8 term
    - doesn't make a difference for edge detection
    - the 1/8 term **is** needed to get the right gradient value, however

Gradient operators

(a)

Δ_1: $\begin{matrix} 0 & 1 \\ -1 & 0 \end{matrix}$ Δ_2: $\begin{matrix} 1 & 0 \\ 0 & -1 \end{matrix}$

(b)

Δ_1: $\begin{matrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{matrix}$ Δ_2: $\begin{matrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{matrix}$

(c)

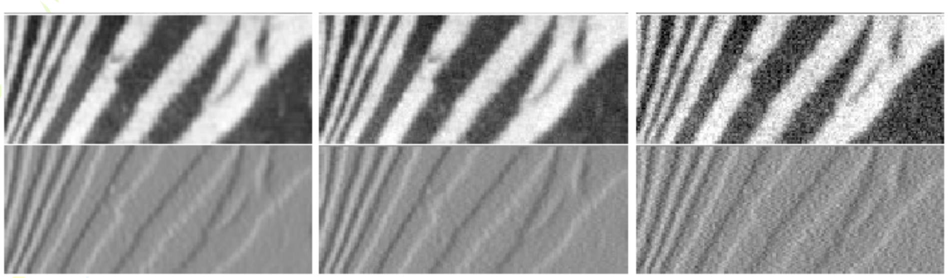
Δ_1: $\begin{matrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{matrix}$ Δ_2: $\begin{matrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{matrix}$

(d)

Δ_1: $\begin{matrix} -3 & -1 & 1 & 3 \\ -3 & -1 & 1 & 3 \\ -3 & -1 & 1 & 3 \\ -3 & -1 & 1 & 3 \end{matrix}$ Δ_2: $\begin{matrix} 3 & 3 & 3 & 3 \\ 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 \\ -3 & -3 & -3 & -3 \end{matrix}$

(a): Roberts' cross operator (b): 3x3 Prewitt operator
(c): Sobel operator (d) 4x4 Prewitt operator

Finite differences responding to noise

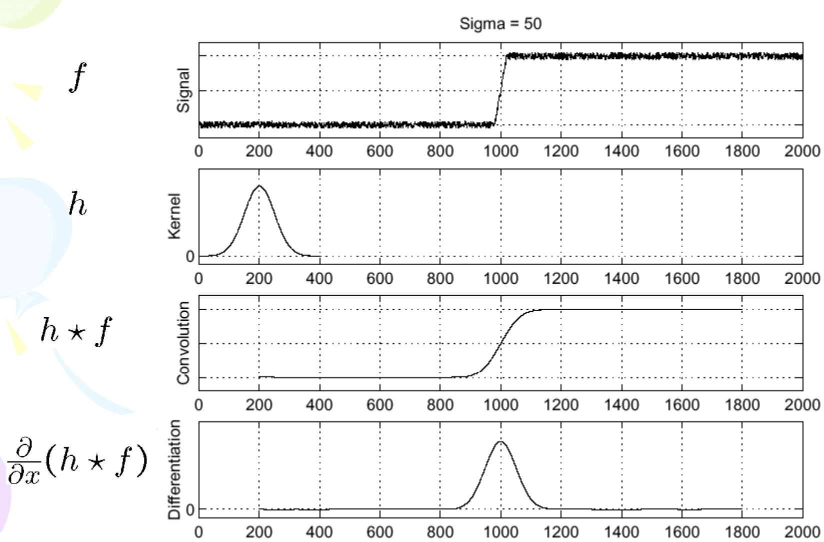


Increasing noise ->
(this is zero mean additive gaussian noise)

Smoothing reduces noise

- Generally expect pixels to "be like" their neighbours
  - surfaces turn slowly
  - relatively few reflectance changes
- Generally expect noise processes to be independent from pixel to pixel
- Implies that smoothing suppresses noise, for appropriate noise models
- Scale
  - the parameter in the symmetric Gaussian
  - as this parameter goes up, more pixels are involved in the average
  - and the image gets more blurred
  - and noise is more effectively suppressed

Solution: smooth first



- Look for peaks in $\frac{\partial}{\partial x}(h \star f)$

Derivative theorem

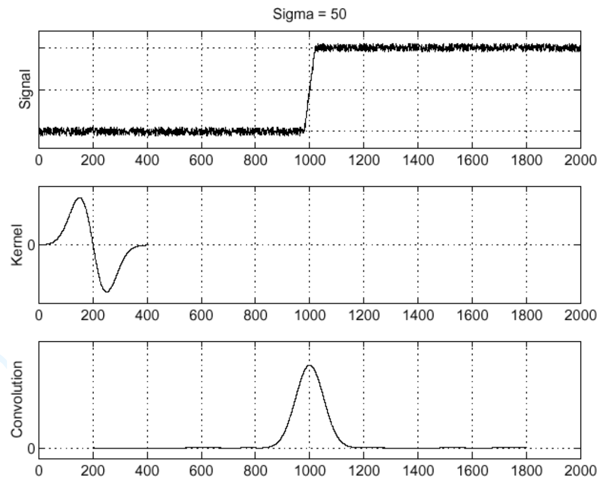
$$\frac{\partial}{\partial x}(h \star f) = (\frac{\partial}{\partial x}h) \star f$$

- This saves us one operation:

f

$\frac{\partial}{\partial x}h$

$(\frac{\partial}{\partial x}h) \star f$



Second derivative zero

- How to find second derivative?
- f(x+1, y) – 2f(x,y) + f(x-1,y)
- In 2D
- What is an edge?
  - Look for zero crossings
  - With high contrast

0	–1	0
–1	4	–1
0	–1	0

–1	–1	–1
–1	8	–1
–1	–1	–1

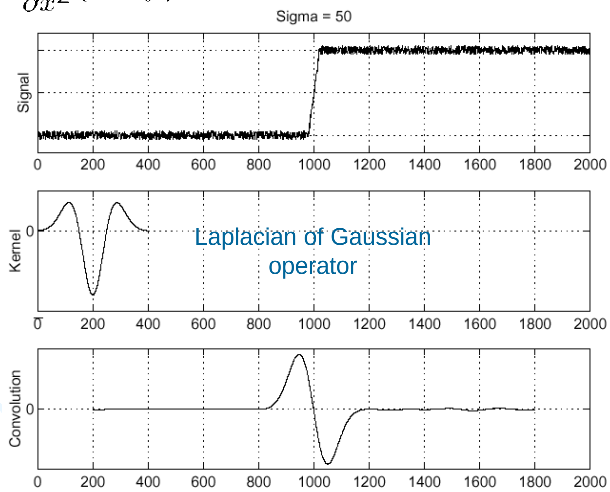
Laplacian of Gaussian: Canny

- Consider $\frac{\partial^2}{\partial x^2}(h \star f)$

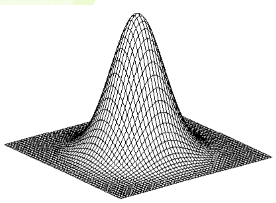
f

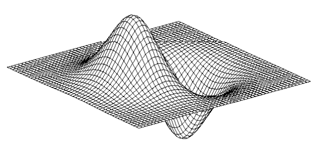
$\frac{\partial^2}{\partial x^2}h$

$(\frac{\partial^2}{\partial x^2}h) \star f$

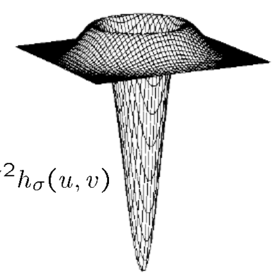


2D edge detection filters





Laplacian of Gaussian



Gaussian

$$h_\sigma(u,v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$

derivative of Gaussian

$$\frac{\partial}{\partial x} h_\sigma(u,v)$$

$$\nabla^2 h_\sigma(u,v)$$

- ∇^2 is the **Laplacian** operator:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

Optimal Edge Detection: Canny

- Assume:
  - Linear filtering
  - Additive Gaussian noise
- Edge detector should have:
  - Good Detection. Filter responds to edge, not noise.
  - Good Localization: detected edge near true edge.
  - Single Response: one per edge
- Detection/Localization trade-off
  - More smoothing improves detection
  - And hurts localization.

The Canny edge detector

- original image (Lena)

The Canny edge detector

- norm of the gradient

The Canny edge detector

- Thresholding (thick edges)

The Canny edge detector

- thinning

Effect of σ (Gaussian kernel size)

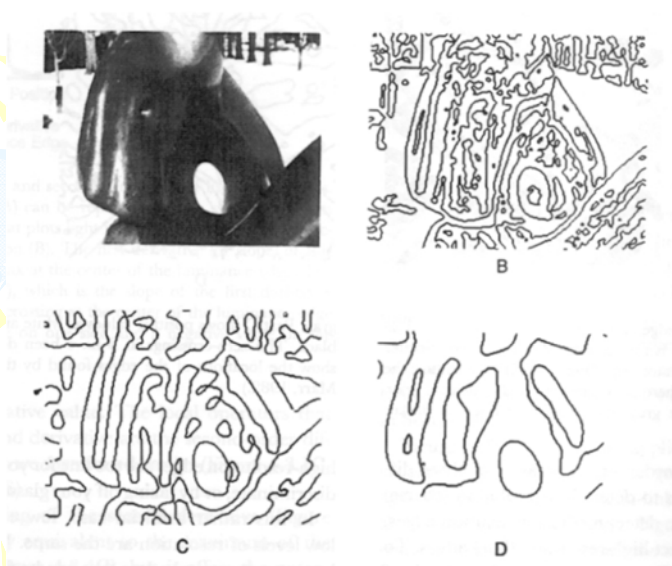
original

Canny with $\sigma = 1$

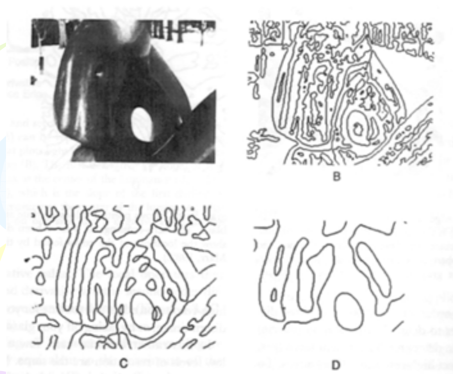
Canny with $\sigma = 2$

- The choice depends what is desired σ
  - large σ detects large scale edges
  - small σ detects fine features

Multi-Scale Edge Detection



Multi-Scale Edge Detection



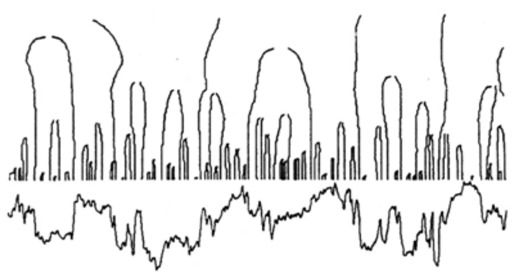
- Edges in coarser level do not disappear in finer levels
- New edges are added
- Coarser level edges are most important
- Advances like a hierarchy

Scale Integration

- Different resolution images in different levels
- How do we know where the coarser level edges are in the finer edge detected image
- Seems very complex yet eye does it easily

Witkin's Explanation

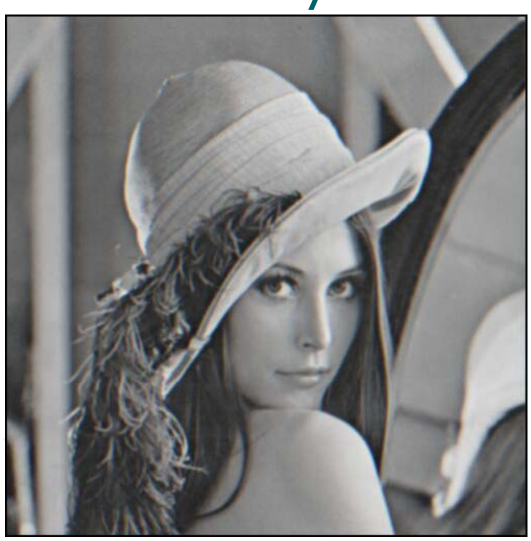
- If we do a continuous subsampling
  - Not possible in digital domain
- Edges are retained, new edges are added with refinement



Edge detection by subtraction

original

Edge detection by subtraction



smoothed (5x5 Gaussian)

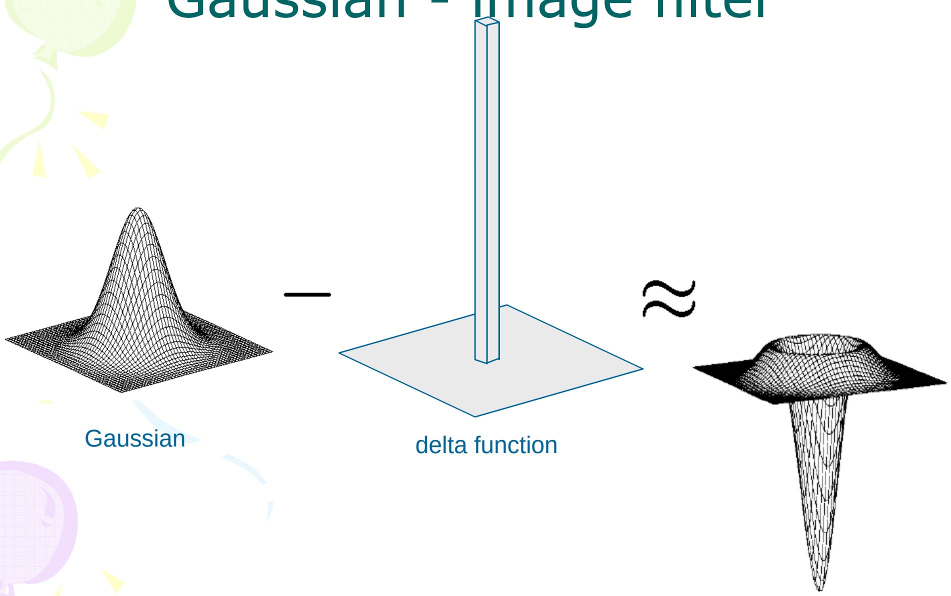
Edge detection by subtraction

smoothed – original
(scaled by 4, offset +128)

Why does this work?

filter demo

Gaussian - image filter

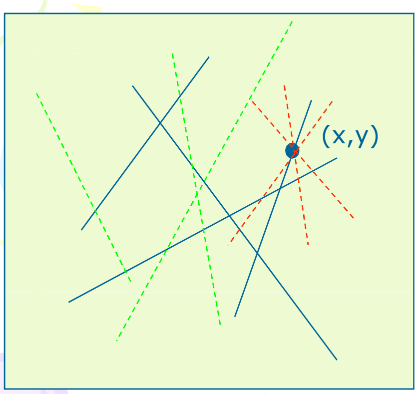


$$\text{Gaussian} - \text{delta function} \approx \text{Laplacian of Gaussian}$$

Identifying parametric edges

- Can we identify lines?
- Can we identify curves?
- More general
  - Can we identify circles/ellipses?
- Voting scheme called Hough Transform

Hough Transform

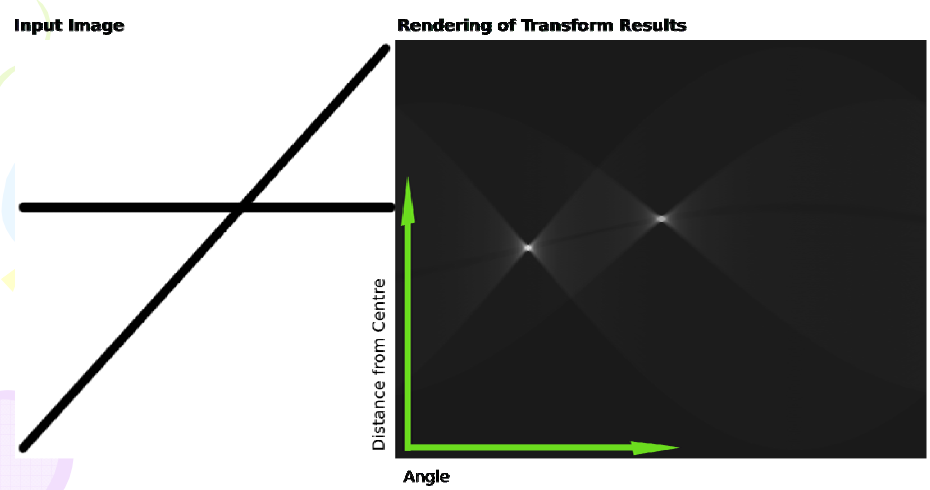


- Only a few lines can pass through (x,y)
  - $mx+b$
- Consider (m,b) space
- Red lines are given by a line in that space
  - $b = y - mx$
- Each point defines a line in the Hough space
- Each line defines a point (since same m,b)

How to identify lines?

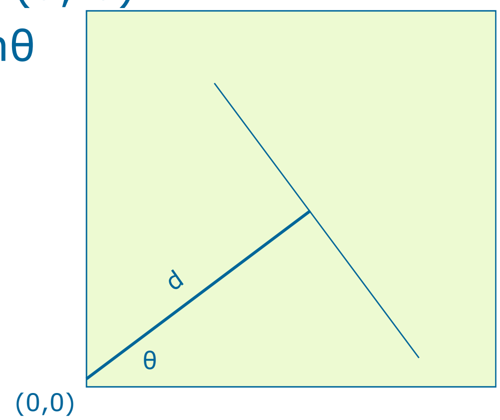
- For each edge point
  - Add intensity to the corresponding line in Hough space
- Each edge point votes on the possible lines through them
- If a line exists in the image space, that point in Hough space will get many votes and hence high intensity
- Find maxima in Hough space
- Find lines by equations y = mx+b

Example



Problem with (m,b) space

- Vertical lines have infinite m
- Polar notation of (d, θ)
- $d = x\cos\theta + y\sin\theta$



Basic Hough Transform

1. Initialize $H[d, \theta]=0$
2. for each edge point $I[x,y]$ in the image
   for θ = 0 to 180
     $H[d, \theta]$ += 1
3. Find the value(s) of (d, θ) where $H[d, \theta]$ is maximum
4. The detected line in the image is given by

Extensions

- Use the image gradient
  1. same
  2. for each edge point I[x,y] in the image
     compute unique (d, θ) based on image gradient at (x,y)
     $H[d, \theta] += 1$
  3. same
  4. same
- Give more votes for stronger edges
- Change the sampling of (d, θ) to give more/less resolution
- The same procedure can be used with circles, squares, or any other shape