Financial Markets: Module 4

MSc Financial Engineering

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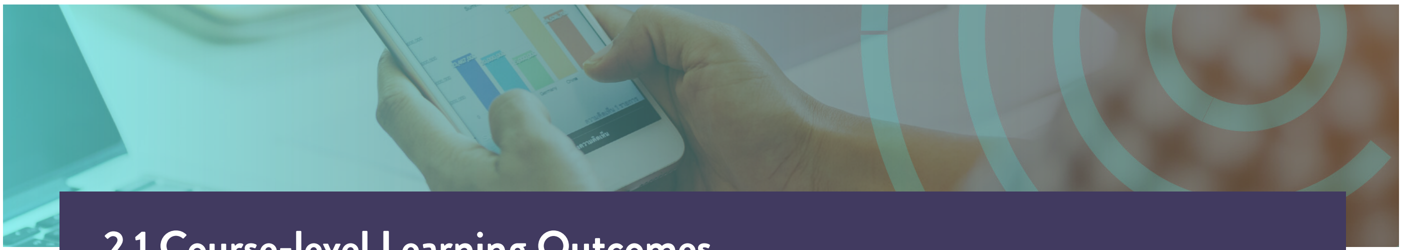
1. Brief

This document contains the core content for Module 4 of Financial Markets, entitled Fixed Income and Bond Markets. It consists of four video lecture scripts, four sets of supplementary notes, and a peer review question.

2. Course Context

Financial Markets is the first course presented in the WorldQuant University (WQU) Master of Science in Financial Engineering (MScFE) program. The course sets the tone for the wider program, providing the context for the field of financial engineering, while introducing you to the financial markets, analysis of market events and the valuations of financial instruments.



2.1 Course-level Learning Outcomes

Upon completion of the Financial Markets course, you will be able to:

1. Describe the types and components of financial markets.
2. Identify and define the key characteristics of financial instruments.
3. Evaluate the different ways in which financial instruments address risk.
4. Perform valuations of simple financial instruments (especially bonds and options).
5. Understand the impact of credit risk within financial markets.

2.2 Module Breakdown

The Financial Markets course consists of the following one-week modules:

1. Introduction to Financial Markets
2. Market Regulation
3. Interest and Money Markets
4. Fixed Income and Bond Markets
5. Stock and Equity Markets
6. Futures, Options and Derivatives
7. Market Making and Trading

3. Module 4

Fixed Income and Bond Markets

Beginning with a brief description of bond markets in general, the fourth module introduces key components such as the relationship between assets, equity and debt, as well as investments based on these distinctions. Later in the module the focus shifts to understanding risk related to fixed-income instruments, and bond valuation.

3.1 Module-level Learning Outcomes

Upon completion of the fixed income and bond markets module, you will be able to:

1. Understand the distinctions between equity- and debt-based funding.
2. Identify risks related to fixed-income investments, in particular.
3. Provide a detailed explanation of the bond valuation process.

3.2 Transcripts and Notes

3.2.1 Transcript: Introduction to Bond Markets

In this module, we will study bond markets. Bond and equity markets together constitute capital markets, which were briefly discussed in Module 1 of this course. If you have studied accounting, you might remember the well-known equation stating that Assets equal Equity plus Debt. As an example, to start a business, the founder or founders need to contribute the initial capital to start the business themselves, or they need to borrow it (or choose some combination of contributing and borrowing). From there, further money can be contributed (if, for example, the founders reinvest profits that they make) or borrowed. The well-known accounting equation reflects this fact: simply put an organization is funded by some combination of owner contribution (known as equity) and debt.

In the previous module, we discussed how businesses can obtain short-term funding from the money market. We are now discussing the long-term, more fundamental funding of the business. If we ignore short-term, money market borrowing — which changes from month and month, depending on many changing factors — we are left with the long-term debt, and the equity. In this module, we examine this long-term debt, while the equity aspect is studied in Module 5. The markets in which these two types of funding are traded are known as the bond and equity markets respectively. Together these two types of markets constitute the capital market.

A bond was defined in Module 3 as a **long-term debt instrument** – that is, a longer-term version of a bill or note. Just as in the money market, an entity borrows money by issuing a bond. This is a simply a promise to pay an amount at a future date (although later we'll consider bonds that pay several amounts at several times). As with money-market instruments, the amount to be paid is called the par value (although the terms **face value, nominal value or principal** are also used for bonds), and the future payment date is called the **maturity**. In exchange for the right to receive the par amount in the future, a bond investor pays some price — determined by supply and demand in the bond market — to the issuer. Thus, the investor lends money to the issuing entity.

An entity must be funded in some way, and so a choice must be made between issuing debt and issuing equity (both to establish the entity initially, and to raise further funds later on in its life). It is well-known to accountants that choosing debt over equity is riskier for the entity — the first set of notes in this module explains why. The lender invests their money in exchange for the promise of a future payment (the risk that the promise might not be fulfilled — that the issuer might **default** — is dealt with later). This future payment is known in advance, which is why the bond markets are also known as the **fixed-income markets**. The fact that the future payments are fixed makes the investment risk-free **in a certain sense** — the amount received is not going to vary in an unfavorable way, whereas other investments can turn out to have much less value than expected. Holding a bond is not risk-free in all senses though. An important risk is the fact that, although the future payment is fixed, the bond value at an earlier time — that is to say, the price determined by the bond market — can change.

For money market instruments, this concern is minor, because the fixed payment of the par value will occur in the near future. Suppose you hold a bond that matures in 15 years, however, and suppose the price drops suddenly. The fixed nature of the future payment is not necessarily comforting, because there is a very long time to wait until it will occur. This risk — that the market price of a bond can drop — is known as **market risk.** This can also be called **interest-rate risk,** because — as we've seen in the previous module and will see again here — interest rates are linked to prices, as a change to interest rates causes a change to bond prices as a direct consequence.

3.2.2 Notes: The Bond Markets

The relationship between assets, equity and debt

We begin here by elaborating on the accounting equation: **assets** equal **equity** plus **debt.** One view of this equation is that all of the entity's assets, or the entity itself (since it may be viewed as consisting of these assets) need to be purchased in one of two ways. Purchases can be conducted with either money contributed by the owners, or money that the owners have borrowed.

The owners' contributions — the equity — does not just consist of the initial investment of capital the owners began with, but also includes profits that have been reinvested in the entity. This reinvestment of capital includes any money the entity produces or that belongs to the owners (if the obligations associated with the borrowings are accounted for) since it represents additional equity that they are effectively contributing.

Another view is that the equity of the business — the part that belongs to the owners — is simply the assets of the business minus any liabilities. These assets include, amongst other things, all the accumulated profits in the business's bank account. Typically, the most significant liability is the long-term debt of the business. According to this view, we have taken the debt to the other side of the equation: we have assets minus debt equals equity.

Since the entity's assets are ultimately funded by a combination of debt and equity, the debt holders and the equity holders are often described as the primary stakeholders of the entity. The debt holders are the lenders to the business, which could be a bank that has extended a loan or a collection of bond investors. The equity holders are the owners of the business, whether the sole founder of a small business or the millions of shareholders of a large corporation.

These primary stakeholders have a vested interest in the business (due to either a loan or an equity contribution), which they put forward in the hope of future rewards. These rewards may be either the repayment of the loan with compensating interest, or increased value derived from their equity, which follows from an increased share price or dividends received from the shares.

From the perspective of the entity itself, the equity holders and debt holders both represent liabilities or obligations: the entity must meet its debt obligations, and, because it is owned by its equity holders, it owes its eventual profits (or losses) to its owners. It is important to note that there is a crucial difference between these two obligations: the amount due to lenders is fixed, while the amount due to equity holders is flexible and depends on how much profit is made.

This flexibility makes the equity a less risky obligation from the perspective of the entity. This is because the equity holders are entitled to precisely what is available for them, even if that amount is negative. Therefore, if the entity makes a loss, this loss is just transferred over the equity holders. The fixed nature of the debt obligation makes it risky — the debt holders are entitled to a certain amount, whether or not the entity has this money available. If the entity does not have the required money, they will default on their debt obligations, with dire consequences. This can potentially include the declaration of bankruptcy and the entity being wound up (the operations being terminated, and the individual assets sold so that the lenders recover some of their stake).

To illustrate this, suppose it takes 100 dollars to start a business. In scenario A, an individual contributes this 100, and becomes the sole stakeholder in the new business. In scenario B, the individual borrows 50 and contributes the remaining 50 needed. A year later, suppose the business in both scenarios has been equally profitable and is now worth 130 (net profits of 30 have been earned and thus added to the initial value).

In scenario A, this new, increased value belongs to the owners, who has made a good return on his investment (of 30%, the increase to the initial contribution). In scenario B, the new value of 130 does not only belong to the owner — the lender is entitled to the loan of 50, and an interest payment of 5. So, the owner's contribution grows from 50 to 75, since 75 = 130-55 (which is a 50% increase/return). If the business makes no profit, there is no return in scenario A, but in scenario B, the owner in fact makes a loss — the owner's stake has shrunk from 50 to 45, since 45=100-55, noting that the lender is still entitled to their repayment with interest. The following table shows these outcomes as well as others, which you should verify by calculating the returns and comparing the two scenarios.

	Scenario A (pure equity)	Scenario B (debt and equity)
Profits of 30	Return of 30% for the owner	Return of 50% for the owner
Profits of 10	Return of 10% for the owner	Return of 10% for the owner
No profits	Return of 0% for the owner	Return of -10% for the owner
Loss of 30	Return of -30% for the owner	Return of -70% for the owner
Loss of 50	Return of -50% for the owner	Default and possible bankruptcy

Note how the introduction of debt in scenario B increases the range of possible returns. One way to think about this is that, in the presence of debt, the equity holders don't have to share profits with the lenders — the profits, generated in part with the borrowed money, belong only to the equity holders. Conversely, they cannot share any losses with the lenders — all of the losses are theirs to bear. In general, debt increases the variation possible in returns on equity. A high debt-to-equity ratio is risky in this sense.

Note that the ratio between debt and equity is referred to as the capital structure of an entity. Also note that the distinction between debt and equity is not necessarily so clear in practice. Preference shares, for example, are a type of equity with certain debt-like characteristics.

Debt investments versus Equity investments

Now consider the lender's perspective. An investor must decide whether to lend their money to some bond issuer, or to invest elsewhere, such as in an equity stake (in other words, an investor can buy bonds, or can buy other assets). Bonds are unusual assets in that their future values are known since the par value is to be paid at maturity is specified. This is unlike the majority of other assets, since the future value of a share or a house, for example, is not known in advance. It is important to note that we are overlooking the possibility of default for the moment.

The fixed future cash flows add some predictability to the bond investment but does not make it completely free of risk. The current price is not fixed and can change for better or worse. Variation of this nature means that bond investments exhibit market risk, or because prices are linked to interest rates, interest-rate risk. This idea is much more relevant when terms are long, and maturity is far away, since there is greater distance between the variable current price and the fixed maturity value (we will express this idea mathematically later on).

It is important to note that there is a possible confusion between a bond's price (at a time before its maturity) and its par value (the value it pays at maturity). These are of course different quantities, but if someone states, "I will invest in a million dollars' worth of bonds", it is not clear which quantity they are referring to. One meaning — probably the more conventional one amongst financial practitioners — is that they intend to purchase bonds with a par value of a million, which means that the value of the investment today will be less (possibly much less, especially if interest rates are high and if there is a long time until maturity). But another reasonable interpretation is that they are investing a million today, which will purchase bonds with a par value larger than a million. You should be aware of this ambiguity, and, more generally, of the terminology we have introduced and how it is applied to a bond investment.

3.2.3 Transcript: Fixed-Income Risks

Having described the market risk inherent in a bond investment, we now consider the other risks that are present. A very important one is the risk of default — or the credit risk — which was overlooked in the previous set of notes.

We have seen that a bond is a method for money to be lent from one party (the bond investor or holder) to another (the bond issuer). The nature of issuer is greatly relevant to their ability to repay the loan — that is, to fulfil the promise to pay the par value at the maturity date. As was discussed in the previous module, a large government has a much greater chance of meeting their debt obligations than a new business does. So, default risk depends greatly on the borrowing, or the issuing, entity. Note also that default risk is **asymmetric**: it applies only to the lender, who is the only party that can face a loss in the event of default. A default — that is, a failure to fulfil an obligation at the maturity date — does not necessarily imply that the investor receives nothing; it just implies that they don't receive the full par value amount. Any amount that the investor does receive is known as a **recovery** amount.

Recall from the previous module that the forces of supply and demand will cause the possibility of default to be factored into bond prices — investors will pay less for a bond if there is a greater chance of default, all other things being equal. So, the prices of credit-risky bonds are influenced by at least two types of factors: the bond-market forces that determine the prices of all bonds including government bonds, and the forces of supply and demand that determine what additional discount is needed to compensate investors for the possibility of default. The term **discount** is used in many ways in finance — note its use here, and ensure you don't confuse it with a **discount rate**, **a discount factor,** or the **discounting of cash flows** from a later to an earlier date.

This introduces an interesting dynamic to bond prices. When a bond is issued, the market will determine a price based on the market's collective desire to hold fixed-income assets (i.e., on market interest rates, which reflect this desire) and on their collective view of how likely a default is (as well as how much recovery will be possible in that event). Recall from Module 1 that the initial issuance of the bond is known as a primary market trade, whereas subsequent trading of the bonds — which does not affect or concern the issuing entity — takes place in the secondary market. The bond price will continue to evolve as it is traded secondarily, as the forces of supply and demand evolve.

For example, a new regulation or tax code might induce investors to increase their bond holdings, which would force the bond prices upward. This would be the first type of factor. For an example of the second type, suppose that a bond issuer releases new financial statements that reveal it is in financial distress. The prices of its bonds trading on the secondary market are likely to drop, as a future default now appears more likely.

In general, financial markets tend to incorporate new information into prices virtually immediately — put differently, financial markets tend to be efficient. This efficiency means that a default doesn't need to take place for there to be price effect — it just needs to be perceived as possible. For instance, the market might expect that financial statements will reveal bad financial news for the borrowing entity, in which case the bond prices would have already dropped when this expectation was formed. If the statements turn out to be better than expected, the prices will bounce back up to some extent, as the market continues to update its outlook. In conclusion, the mere possibility of default and the perceived likelihood of the default cause a non-trivial effect on bond prices (or, equivalently, on the interest rates offered by bonds).

In the next set of notes, we will elaborate on some of these points and develop a comprehensive list of the risks involved in bond investing.

3.2.4 Notes: Fixed-Income Risks

Why government bonds seem free of risk

In the last video we mentioned that large governments have virtually no credit risk. One reason for this is that governments are much more well-established and stable entities than new businesses. In our example, the government has a wider and more diverse set of income sources with which to meet its obligations, as governments usually do. A second and more important reason is that governments have the unique ability to have their central bank print money that can be used to repay their debts. You may recall from Module 3 that borrowers are strongly incentivized to meet their obligations, if at all possible, so that lenders will agree to lend to them in the future. Although government defaults are possible, governments will usually resort to printing additional money, if necessary, so they can meet their debt obligations. A government default would signal a lack of credibility of the government. As a result, future borrowing will be expensive and perhaps impossible for the government, in addition to other, far-reaching consequences for the economy.

For these reasons, government-issued bonds are often viewed as free of (or approximately free of) default risk. This makes them a useful point of comparison. If, for example, a government bond is trading at a price of 800, and a corporate bond (a bond issued by a company), with the same par value and maturity date, is trading at 750, the difference of 50 can be ascribed to default risk. If someone tried to sell their corporate bond for 800, no one would buy it — any rational, informed investor would prefer to buy the government bond which is available at this same price. Instead, the market settles on a lower price, which can be seen as compensation to the investors for being vulnerable to the possible default of the company and therefore the default risk the investors face.

In the next video, we put this price comparison in terms of interest rates — here the 800 and 750 depend greatly on the par value and maturity date of these bonds. Therefore, the difference of 50 can only be interpreted in light of these additional details. Interest rates account for different sizes of par value and for the time until maturity. Hence, the difference between the two bonds' interest rates is a more easily interpreted, more convenient way to express how the market perceives the default risk of the company.

Default risk in addition to interest-rate risk

The way that the bond markets incorporate default risk into bond prices makes the dynamics of defaultable bonds (i.e., bonds that exhibit a non-negligible default risk) more complicated. The variation of a defaultable bond price can be viewed as two layers of variation: the variation in the default-free bond prices, and the variation in the adjustment the market makes for default. Using the government bonds as a benchmark allows us to separate these two layers and to gain a better understanding of how bond investments work (as we can compare the two bond prices, as done in the previous section).

A corporate bond price can change because the prices of all bonds change (i.e., market interest rates change — this is the first layer mentioned above), or because the market view of the default risk of this particular bond changes (the second layer). This second layer relies on perceptions — the market adjusts prices to account for new information as they collectively perceive it. Suppose, for example, the release of a company's financial statements causes its bond prices to drop — the bond markets have updated their perception of the company's ability to repay their debt. You might agree with the market that the financial statements do contain negative information, but perhaps you think that the drop was too significant relative to how bad the new information was. Therefore, you agree the bonds should now be worth less, but not quite as little as the market has decided. Seeing this overcorrection (or undervaluation) you probably would want to purchase these bonds since the bonds are cheap relative to your view, and this cheapness is probably enough to induce you to bear the risk of default (which you view differently to the market).

Recall that the way the market updates prices as new information is revealed is known as the efficiency of the market. This does not necessarily assume that the Efficient Markets Hypothesis (which we dealt with in Module 1) is true. The Efficient Markets Hypothesis is a very specific and formal statement about how efficient markets are. Whether this hypothesis truly holds, or which precise version of the hypothesis holds, is an open academic matter, but it is definitely true that markets are usually very efficient. In the normal course of events, modern financial markets do in fact incorporate new information (whether formal financial news or informal speculation about prospects for the future) very rapidly, as investors compete to adjust to any new information about potential investments. In our previous example of an investor — who thinks the market reaction to the financial statements was too severe — buying the cheap bond is market efficiency in action.

Accordingly, the increased demand will pressure the bond price to increase, and it is the aggregation of all competing investor's actions that result in efficient, continually-updating market prices. We have discussed how bonds exhibit market risk (their prices can change before maturity), and, if the issuer is liable to default, credit risk.

A comprehensive view of risk in bond markets

As we have seen, credit risk is reflected in market prices, so credit risk and interest-rate risk seem related. One approach is to define market risk more inclusively, whereby it includes both layers of price variation described above. It is an essentially equivalent approach to view market/interest-rate risk as the variation in non-defaultable bond prices (or interest rates), and to view credit risk as something separate. In either approach, the conceptual separation is important (and comparing to government bonds helps to make this separation).

The crucial feature of bond investments is the fixed nature of their future payments — although one cannot know the value before maturity, or be sure that the issuer won't default, one can be sure of the par value amount. However, one cannot be sure how much this amount will purchase at the future maturity date, because general prices tend to increase due to inflation. If prices increase faster than expected, the par value will have less purchasing power, exposing the bond-holder to added risk and uncertainty known as inflation risk. We can phrase this using economic jargon by saying that the par value is known in nominal terms, but not in real terms (we know the amount itself, but not how much it will actually be able to purchase).

A bond investment can also exhibit liquidity risk, meaning that it can be difficult to convert one's investment to cash. The main way this can arise is if there are a relatively low number of participants in bond markets. Bonds issued by the United States government have so many potential investors that one can always find a buyer if one wishes to liquidate an investment. Bonds issued by a small company might not attract the interest of many investors, and one might be unable to sell these bonds at a price that suitably accounts for the involved market, credit and inflation risk. You might have to lower the price further (to induce a buyer to enter the market) or wait until buyers present themselves. Both of these are manifestations of liquidity risk. The longer the term of bonds (relative to money-market instruments), the more relevant this factor will be: if waiting until maturity is a matter of years rather than months, an investor might have to face an illiquid market if they need money in the near term.

Finally, an investor with any international concerns can face currency risk when investing in bonds. The known par value is given in a certain currency, but how this will convert to other currencies is uncertain. A government might have the authority to have their own currency printed, but they certainly cannot do so in foreign currencies (and indeed, a government default, or a country's economic difficulties, can make their currency depreciate). An investor might not be concerned with the value of their investment in other currencies, but if they are (risk can depend greatly on the context), currency risk can be a relevant factor in a bond investment.

3.2.5 Transcript: Bond Valuation I

Previously, we discussed the various qualitative concerns that affect a bond issuer and a bond investor. We now address how the prices that result from these concerns can be handled quantitatively.

We first need to summarize the valuation concepts introduced in Module 3. An interest rate is the return that a lender will earn on the amount they invested, or in other words money borrowed from them, assuming the promised repayments are met. If there is a large difference between a bond's purchase price and its par value, there is a large amount of interest that the holder will earn from buying the bond and holding it until maturity. If a bond's price is low relative to its par value, its interest rate (or yield, which is the bond-market term for an interest rate) is high, and vice versa. An interest rate or yield expresses the rate at which the loan value increases over its life, and a rate is a scale-free way of expressing an increase per unit of time.

The specific, mathematical way in which the interest rate expresses this rate of increase (or rate of return on investment) can vary; the various ways are known as different **interest-rate conventions** or **compounding conventions**. In the following set of notes, we will list the different interest rate conventions that were defined in Module 3 and are needed in this module as well. Understanding these interest-rate conventions allows you to move from a bond price to an interest rate, or from an interest rate to a bond price. When valuing a bond in this way, the value is not produced from thin air, but is based on an interest rate that must be taken as an input.

Module 3 concludes with two factors that complicate the valuation of debt instruments. The first is that interest rates are not necessarily the same for loans of differing terms (and therefore for bonds of differing times until maturity). The phenomenon is known as the t**erm structure of interest rates.** So, while interest rates adjust for loan size and term, loans of differing terms do not typically have interest rates that coincide exactly.

The second factor is that default risk affects an interest rate. Because default risk causes bond prices to decrease, and because bond prices and interest rates have an inverse relationship, greater default risk is reflected by a higher interest rate. This stands to reason: lenders demand a larger interest to compensate them for the possibility that the borrower defaults, and the lender does not end up earning the promised interest. The difference between a yield on a defaultable bond (i.e. its interest rate) and the risk-free interest rate is known as the spread (or credit spread). This is an extremely important use of risk-free, or virtually risk-free, government bonds as benchmarks. Of course, when comparing defaultable-bond and government-bond yields to determine spreads, one should compare bonds of equal maturity; otherwise the term structure of interest rates makes the comparison unfair.

The following set of notes formalizes and elaborates on these ideas, and also considers **coupon-bearing bonds** (or just **coupon bonds**): bonds that make regular payments, called coupons, throughout their life, in addition to the payment of the par value at the end of its life. The simpler bonds we have been considering, with one payment at one date, can be called **zero-coupon bonds.**

3.2.6 Notes: Bond Valuation I

In Module 3, we defined the following interest rate types/conventions: the effective interest rate, the annual effective interest rate, the n-compounded annual interest rate, the n-compounded annual discount rate, the simple annual interest rate and the simple annual discount rate.

Each interest rate describes how the initial loan value (i.e., bond price) increases to its final value (i.e., the par value). Suppose you pay 93 for a 100-nominal zero-coupon bond (i.e., a bond with a par value, or a principal, of 100), which matures in two years. If we work in terms of annual (effective) interest rates (which are the same as once-compounded annual interest rates), the equation that expresses the rate of growth from the price of 93 to the principal of 100 is:

$$93(1+r)^2 = 100 \tag{1}$$

This is the return – expressed in annual effective terms – that one gets from investing in the bond and holding it until maturity. It is also the interest rate paid by the borrower on the loan that the bond gives rise to (this assumes they are issuing the bond now – it may have been issued earlier at a different price, as we have not said whether this is a primary or secondary trade). A different convention causes Equation (1) to take on a slightly different form. For example, if using the annual rate compounded quarterly (i.e., the 4-compounded annual interest rate), one would write:

$$93(1+\frac{r^*}{4})^{4\times 2} = 100 \tag{3}$$

and, after rearranging, get a slightly different interest rate (which is why we have given it a different symbol (r^* instead of r). The various interest-rate formulae given in Module 3 arise from rearranging various versions of Equation (1). Every version – every interest-rate convention – is a language for describing how the initial value (93 in the above example) increases to the final par value (here, 100). If we were considering buying two of the above bonds, Equation (1) would be rewritten as:

$$186(1+r)^2 = 200 \tag{4}$$

Equation (4) shows how both the price (of the total investment) and the (total) par value would double. Notice that this will not change the value for r that solved the equation — the multiplication by two (or by any other scaling factor) cancels away. Thus, interest rates are scale-free.

Instead of thinking about the increase from 93, forward in time, to 100, one can think of the decrease, backward in time, from the par value 100 to the current price 93. Mathematically, one could rewrite Equation (1) as:

$$100(1+r)^{-2} = 93 \tag{5}$$

In this case the multiplicative factor that causes the decrease (namely, $100(1+r)^{-2}$) which is called a discount factor, as the process of decreasing from a future financial value to a current one is known as discounting (this is not related to a discount rate — one can express the discount factor in terms of an annual interest rate, as in Equation (5), or in terms of, for instance, an annual discount rate). Zero-coupon bonds are sometimes known as discount bonds (or pure discount bonds) — the whole instrument is based on the simple idea of a future value being discounted to some current value.

Before we contrast this with coupon-bearing bonds, let's summarise by noting that we have created a link between prices and interest rates. In the above example, we have deduced the (annual effective) interest rate associated with a certain price; one could use the link the other way around and calculate a bond price based on a given interest rate. So, we cannot value bonds out of thin air — we can only do so based on given or assumed interest rates. This is still useful, because one can apply the market interest rate (which is usually observable to the whole market) to a particular bond that one is considering. Recalling that interest rates depend on their term, one must find the market interest rate for the appropriate term (by, for example, calculating the return on a government bond of the same term) and then apply it using the equations like the above.

Consider a bond like the above one — par value of 100 and maturity in two years — but one that pays coupons of 3 units at the end of each year. Instead of just discounting the 100 par-value payment like in Equation (5), we must include the two coupons (and their corresponding timings):

$$Price = 3(1+r^{(1)})^{-1} + 103(1+r^{(2)})^{-2} \tag{6}$$

Notice firstly that each cash flow (the first coupon, paid in a year's time; and the second coupon combined with the par value, paid in two years' time) is reflected and discounted separately. Indeed, we are treating the coupon-bearing bond like a portfolio of (combination of) two zero-coupon bonds that collectively give the same cash flows. This hypothetical portfolio behaves the same as the coupon-bearing bond, and so must command the same price. In summary, coupons are treated additively (they are considered individually, and the individual prices added). Secondly, note how Equation (6) accounts for the term structure of interest rates — it uses different rates (denoted, in this case, $(r^{(1)})$ and $(r^{(2)})$ to discount cash flows/loans of different terms.

If the coupon bond considered in Equation (6) were government-issued (or were issued by an entity with negligible default risk) one could determine the bond's price by substituting prevailing risk-free interest rate values in for $(r^{(1)})$ and $(r^{(2)})$ (using one-year and two-year rates, respectively).

If the issuer had non-negligible default risk, the price reflected by this would be too high — one could then estimate what a suitable price would be, in order to compensate an investor for the possibility of default. For example, if the price suggested by the risk-free interest rate is 98, but the bond is available for 94, one must decide whether the discount of 4 is sufficient to offset the default risk. Another approach would be to add a spread to the risk-free interest rates before substituting them into Equation (6) — recall that the spread is the difference between risk-free interest rates and the interest rate implied by a defaultable bond price (so that adding it onto the risk-free rate gives a suitable rate to use in the pricing of the defaultable bond).

Recall that we cannot produce valuations from nothing — we are always using interest rate information (obtained elsewhere) to imply prices, or vice versa. But again, this can still be useful, because the spread reflects the degree of default risk. Suppose we know that entity A exhibits about the same default risk as entity B (maybe they are companies with a similar standing and financial position), and that we also know that the spreads of entity A's bond are about 1% (which one could calculate from comparing entity A's bond prices to government bond prices). It would then make sense to add 1% to risk-free interest rates, and use these adjusted rates to price bonds issued by entity B. If one thought that entity B has more default risk than entity A, then using entity A's spread would not give a good price estimate for entity B's bonds, but it would give a good estimate for the upper bound one should pay for these bonds (because an additional discount would be warranted for the greater default risk).

3.2.7 Transcript: Bond Valuation II

We will now consider some more advanced bond-valuation concepts. The first is a bond's yield-to-maturity.

To price a coupon-bearing bond, one requires several interest rates, as each of the bonds' cash flows must be discounted with an interest rate appropriate for its term. Once its price is established, consider the single interest rate that discounts the cash flows to attain the bond's price. In other words, forget about the term structure of interest rates, and then calculate the interest rate implied by the price. This rate is called the yield-to-maturity. The reason one would use only one interest rate in this way is that it gives the return earned over the whole investment. The individual interest rates give the return associated with the individual cash flows, assuming you hold the bond until they are paid and assuming the issuer does not default. The overall return is also relevant, and is made up by the various individual returns. This idea can be used to compare the investments offered by two different bonds, which can be difficult if the bonds have different maturities and coupons.

The next concept is that of a yield curve. We know that interest rates differ according to the term they are based on (that is to say, we know interest rates have a term structure), so we can consider various interest rates for various different terms. Suppose we observed government bond prices for bonds with remaining terms of one, two, three years, and so on up to 20 years, and suppose we drew a graph of the rates implied by these prices (in other words, suppose, after calculating the rates, we plotted them as a function of their underlying terms). This graph is known as the yield curve — it gives the yields (in the form of a plotted curve or graph) of zero-coupon bonds of the various possible maturities. This summarizes all the interest rate information available for a particular bond issuer. A different issuer, if they had a different default risk, would have a different yield curve.
The difference between an entity's yield curve and a risk-free one is the entity's spread.
The next set of notes elaborate on this, and on why it is such a powerful idea.

Finally, we need to unpack the concept that bonds with longer terms are riskier. The primary reason for this is that there is more potential variation in a bond price if the maturity is further away (that is, longer-term bonds exhibit more market risk). We consider this idea from a mathematical perspective in the next set of notes, but one can see the intuition with a simple comparison. A one-year bond price, for instance, is determined by how the bond markets discount the par value payment in a year's time. In pricing a two-year bond, the market discounts the par value over a two-year period, which includes the one-year period of discounting involved in the first bond. There is uncertainty in how the market discounts over the one-year period — although we might observe the price now, it may change in the future. But the uncertainty in the discounting and pricing of the two-year bond is greater, as it includes the uncertain discounting over one year in addition to the uncertain discounting over the second year.

Because of the greater market risk in longer-term bonds, yield curves tend to be upward sloping — investors tend to get additional compensation (in the form of a higher yield) for bearing the additional market risk. The term of a zero-coupon bond is simply the time until its maturity, but the cash flows of a coupon bond (or a more complicated portfolio of bonds) are spread over time, and so one cannot directly compare the terms of two bond portfolios. The notes define the concept of duration, which gives the average of the terms associated with the cash flows of a bond portfolio. If the duration of portfolio A is greater than portfolio B's duration, then portfolio A, on average, has cash flows that are further away. Because longer terms induce greater market risk, portfolio A is riskier than portfolio B.

3.2.8 Notes: Bond Valuation II

In the previous set of notes, we looked at an equation (number 6) that gave the price of a certain coupon bond. In that case, there were two cash flows. Consider the following expression for the price of a general fixed-income portfolio that involves n cash flows:

$$Price = \sum_{i=1}^{n} c_i (1+r^{(t_i)})^{-t_i} \tag{7}$$

where $c_1, c_2, \ldots, c_n$ are the cash flow amounts, $t_1, t_2, \ldots, t_n$ are the respective terms of these cash flows (i.e., the time until maturity, given in years), and $r^{(t_1)}, r^{(t_2)}, \ldots, r^{(t_n)}$ are annual effective interest rates corresponding to these terms.

If we consider a fixed-income portfolio — the right to receive a set of fixed cash flows at future dates — that is comprised of a cash flow of 3 in a year's time, and of 103 in two years' time, then we have the coupon bond considered in Equation (6) (we would have $c_1 = 3$, $c_2 = 103$, $t_1 = 1$ and $t_2 = 2$). Equation can thus handle coupon-bearing bonds (where the cash flows are small and constant until the final one, c_n, is large, as it gives the par value payment as well as the final coupon), or it can handle more complicated sets of cash flows (such as the cash flows that arise for the combination of a number of coupon bonds). Just as we treated a coupon bond like a portfolio of zero-coupon bonds (by separating the individual cash flows and them adding them together), we can treat a whole fixed-income portfolio by individually treating each cash flow involved.

In order to value a fixed-income portfolio (such as a coupon bond), we need to have interest rates with which to discount each cash flow ($(1+r^{(t_i)})^{-t_i}$ is the discount factor for the ith cash flow c_i). These rates ($r^{(t_1)}, r^{(t_2)}, \ldots, r^{(t_n)}$ in Equation (7) can be read off a yield curve, which plots interest rates against terms (to find $r^{(t_1)}$, we look at the vertical-axis value corresponding to t_1 on the horizontal axis). This assumes the yield curve is given in the interest-rate convention that is used in the valuation equation — just like Equation (7) can be written in any convention, a yield curve can be given in any convention.

The yield curve is a very powerful idea. It summaries the interest-rate information of the whole market in one simple mathematical object. Referring to that object in the correct mathematical way allows you to value any fixed-income portfolio (with Equation (7), or some suitable variant). The yield curve is the most convenient and useful way to exploit the link we have created between prices and interest rates, because it is applicable to any portfolio (although credit risk can be a factor, and will be briefly discussed in the next section). The yield curve can be determined once, and then used over and over again for different portfolios.

The fact that we use different interest rates for different terms accounts for the term structure of interest rates; we must remember to use interest rates with a suitable spread included in order to account for any default risk present. So, beginning with a risk-free yield curve (one based on government bonds), one can add spreads to attain risky yield curves. Although it is not obvious how large a spread is needed for a particular entity, these ideas can still be used to make informative comparisons. If you price a defaultable coupon-bearing bond with the risk-free yield curve, you get an upper bound for the price — you can then decide what additional discount is needed for the possibility of default.

In the previous video, we supposed that government zero-coupon bonds of many maturities were observable, in which case one simply needs to calculate the corresponding yields to form a yield curve. This process — known as bootstrapping the yield curve — can be more difficult if only coupon bonds are observable, because one coupon bond price does not imply a single interest rate. One coupon bond price implies one equation, involving many interest rates — bootstrapping the yield curve involves writing many such equations and solving for the unknown rates simultaneously.

To calculate a bond's yield-to-maturity, we must ignore the term structure of interest rates by solving the following equation, which involves just one rate, y:

$$Price = \sum_{i=1}^{n} c_i (1+y)^{-t_i} \tag{8}$$

The equation links the yield-to-maturity y with the bond's (or bond portfolio's) price. If we input the price into Equation (8), and solve for y, we find the yield-to-maturity (the total return, averaging over all cash flows) that investing in the bond/bond portfolio offers (assuming we buy at this inputted price). If we input y and solve for the price, we are determining the price that gives the total return of y.

Equation (8) might look like a naive valuation equation; one written by someone who doesn't know about the term structure of interest rates or who is simplifying by ignoring it. However, we are using Equation (8) after a proper valuation has taken place (or we are linking hypothetical valuations to yields-to-maturity). This allows us to find the average return over all aspects of a fixed-income portfolio, given a certain price for the portfolio (or to find the price that gives rise to a certain average, total return).

In the case of a zero-coupon bond, we find the price to be lower than the par value (as it is just the discounted par value). A coupon bond includes other cash flows (the coupons), which can increase the price relative to the par value. If the price is still less than the par value, the bond is said to be trading at a discount; if it is greater, the bond is said to be trading at a premium. There is an important intuition here: if a bond is trading at a premium, then the coupons are more than sufficient to cover the interest accruing, and so the buyer has to pay an extra premium to enter the bond. The loan induced by a zero-coupon bond has all the interest paid at the end; the loan induced by a coupon bond involves regular interest payments. If a bond price is equal to its par value (is trading at par or is priced at par), the coupon payments are exactly covering the interest due on the loan at each period, so that no additional payment is made at maturity (the initial loan value, the price, is just returned to the lender).

Finally, let us see how longer terms are riskier. A 100-nominal, one-year zero-coupon bond price is given by:

$$100(1+r^{(1)})^{-1} \tag{9}$$

whereas a two-year version of the same bond has a price:

$$100(1+r^{(2)})^{-2} \tag{10}$$

The exponent of -2 makes this second price more sensitive to changes in the interest rate than Equation (9). In fact, the second price is doubly sensitive — we can see this by writing Equation (10) as:

$$100(1+r^{(2)})^{-1}(1+r^{(2)})^{-1} \tag{11}$$

The discount factor $(1+r^{(1)})^{-1}$ in Equation (9) makes the first bond price risky (because it might change), the second bond price involves two such discount factors. In general, bond prices involving longer terms are more sensitive to changes in the interest rates pertaining to their cash flows. In other words, longer terms result in greater sensitivity to the yield curve. This is why interest-rate risk is not so relevant in the context of money-market debt instruments, which are characterized by short terms.

Duration is a notion of average term of a coupon bond or bond portfolio. The term of a zero-coupon bond is obvious and can be easily compared to other zero-coupon bonds; in more complicated cases, duration helps us summarize the many terms involved. It is defined with:

$$Duration=\frac{\sum_{i=1}^{n} t_i c_i (1+y)^{-t_i}}{\sum_{i=1}^{n} c_i (1+y)^{-t_i}} \tag{12}$$

This is a weighted average: it takes each term t_i, weighs it by the discounted cash flow corresponding to that term $c_i(1+y)^{-t_i}$, adds up the weighted sum, and then (in the denominator) divided by the sum of the weights used in the averaging. This is the standard procedure for taking a weighted average. For the weighted sum used by duration, the sum of the weights is the price of the portfolio. The idea is that we want an average of all of the terms, but we want to place more importance on the terms with larger cash flows (and more valuable discounted values), as these are more financially relevant. Because longer terms involve more interest-rate risk, and because duration is a measure of the terms involved in a portfolio, duration is a risk measure of a portfolio: it is a quantitative summary of the amount of interest rate risk involved.

3.3 Collaborative Review Task

In this module, you are required to complete a collaborative review task, which is designed to test your ability to apply and analyze the knowledge you have learned during the week.

Question

Suppose that a capital-markets investment fund wants to lower the amount of risk they are exposed to. They call a meeting to discuss whether they hold a suitable balance between equity and bond investments. One person thinks they should decrease their bond investments, mentioning that "a high debt-to-equity ratio is very risky". Someone else disagrees, suggesting that they decrease their equity investments because "bond investments provide fixed cash flows and are therefore free of risk".

Explain why the first statement is correctly understood and applied but is mistaken in this context. Then explain why the second person's opinion is valid, even though their statement is not quite correct (making sure to explain why it is not correct).