Financial Markets
Module 6

MSc Financial Engineering

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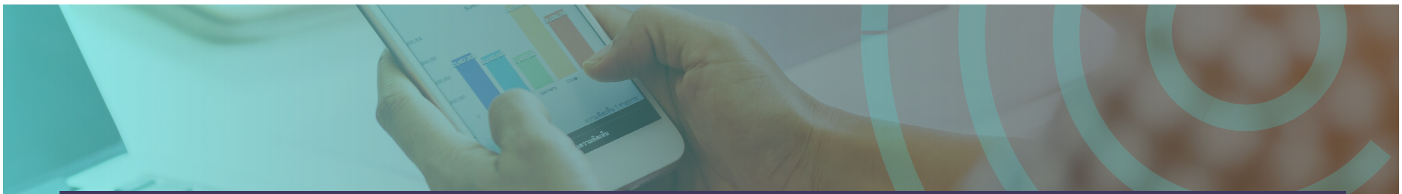
1. Brief

This document contains the core content for Module 6 of Financial Markets, entitled Futures, Options and Derivatives. It consists of four video lecture scripts and four sets of supplementary notes, and a peer review question.

2. Course Context

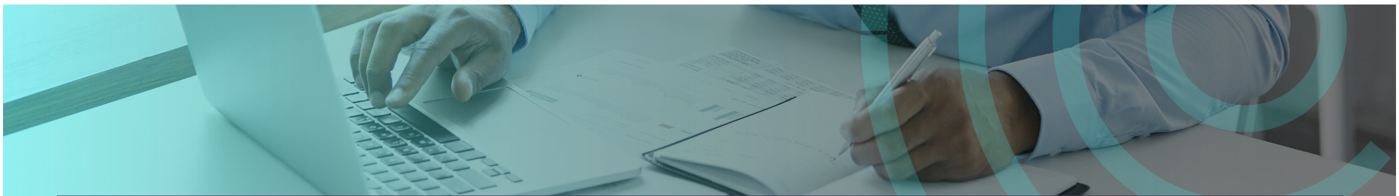
Financial Markets is the first course presented in the WorldQuant University (WQU) Master of Science in Financial Engineering (MScFE) program. The course sets the tone for the wider program, providing the context for the field of financial engineering, while introducing you to the financial markets, analysis of market events and valuations of financial instruments.



2.1 Course-level Learning Outcomes

Upon completion of the Financial Markets course, you will be able to:

1. Describe the types and components of financial markets.
2. Identify and define the key characteristics of financial instruments.
3. Evaluate the different ways in which financial instruments can address risk.
4. Perform valuations of simple financial instruments (especially bonds and options).
5. Understand the impact of credit risk within financial markets.



2.2 Module Breakdown

The Financial Markets course consists of the following one-week modules:

1. Introduction to Financial Markets
2. Market Regulation
3. Interest and Money Markets
4. Fixed Income and Bond Markets
5. Stock and Equity Markets
6. Futures, Options and Derivatives
7. Market Making and Trading

3. Module 6

Futures, Options and Derivatives

This module considers derivatives: financial instruments that depend on other assets. Common types of derivatives are considered, including their specific details and how these give rise to the functions they are suitable for. The module also considers the risks associated with derivatives, as well as derivative valuation.

3.1 Module-level Learning Outcomes

Upon completion of the Futures, Options and Derivatives module, you will be able to:

1. Define derivative instruments, and specifically define common types.
2. Understand the various features of these common derivatives, and the associated terminology.
3. Understand the functions that derivatives can serve and the risks they carry, and how these arise from the nature of the derivatives themselves.
4. Have a basic knowledge of derivative valuation.

3.2 Transcripts and Notes

3.2.1 Transcript: Introduction to Forward Contracts

As we have seen in our previous modules: market types are very closely related to the instruments traded within them, and this is equally true of derivative markets. In this module, we'll consider derivatives as an instrument in detail, rather than focus on the derivatives market itself.

As you may recall, derivatives are financial instruments that depend on other, underlying assets, and as you know, a financial instrument is a contract of a financial nature and gives rise to an asset or liability. Now traditionally, the value of this asset or liability depends on what the contract specifies, but in the case of a derivative this value depends on underlying assets. Therefore, the instrument can be said to derive its value from these assets, which are known as the derivative's underlying.

In our previous modules, we examined various types of asset that one can invest in, including money-market instruments, bonds and equity holdings. These investments can serve as the underlying asset that gives value to a derivative contract, and these derivatives in turn give rise to a new class of potential investments. One can invest in equity, and one can also invest in derivatives with equity as the underlying.

To clarify derivatives a bit more, we'll look at a type of derivative known as a forward contract. A forward contract is a contractual agreement between two parties for an asset to be traded at a specified price and at a specified future time. Similar to bonds' terms, the fixed future date of the contract is known as the maturity. However, in the case of a forward contract one party to the agreement commits to buying the asset, called the long party, whereas the party agreeing to sell the underlying is called the short party.

Now, to understand how the value of the underlying affects the value of such a forward contract, we'll look at an example. Imagine you've entered a forward contract as the long party and have agreed to buy a barrel of oil at $50 in a month's time. Let's suppose that upon maturity, the oil price has increased drastically to $90 per barrel. Luckily, since you've entered into a forward contract, it entitles you to buy something that is worth $90 for only $50. At this point, the contract gives you a benefit of $40, and therefore is an asset for you, and a liability to the short party, who has to sell an asset worth $90 for the relatively low price of $50. Now as we can see this $40 value ascribed to the forward contract depends entirely on the value of the underlying, and therefore the forward is an example of a derivative.

Forwards are settled in different ways; sometimes they are physically settled, whereby the short party has to physically deliver the underlying to the long party; or more commonly, a derivative is cash settled, which the two parties make a cash transfer that has the same financial effect the exchange of the underlying would.

If your contract from the example was physically settled, the short party would have to produce a barrel of oil at the maturity date, and you as the long party would pay the $50 (known as the delivery price). If it were cash settled, the short party would simply pay the long party $40, which we've just seen to be the net value of the contract, in favor of the long party in this case.

Derivatives, such as forwards, can be very useful, particularly at mitigating some degree of uncertainty. Continuing with the example of the forward based on oil, suppose you knew that you would have to buy oil in the future. You of course don't know what it's future price is going to be. Therefore, entering into the forward contract as the long party fixes the price of this future purchase, and addresses the risk of oil prices increasing.

We will discuss the financial risk implications of derivatives in more detail later, but even now we can see that the forward has addressed the risk that the oil price increases. Many businesses need to purchase oil in order to run their operations, and they face the risk that the oil price increases — therefore businesses may enter long forward contracts, which fixes the price of their future oil purchases, which greatly mitigates this risk. The short party would enter the contract because they also obtain some benefit — a company that produces oil, for example, knows that they will need to sell oil in the future, and can fix the price of these sales with short forward contracts.

3.2.2 Notes: Fundamental Derivatives: Forward Contracts

Derivatives are financial instruments; which means they are contractual agreements of a financial nature in the same way that bills, bonds, and stocks are. However, derivatives differ from most other instruments covered in previous modules as these contracts, and the financial implications they specify, are based on some other asset. Derivatives are so-named because they derive their value from this other asset, which is called the derivative's underlying.

Derivatives give rise to a whole new type of investment position — in the same way you can invest in various assets, you can also take on derivative positions (i.e. enter derivative contracts) that are based on these assets. In principle, and occasionally in practice, you can encounter compound derivatives, which is a derivative whose underlying is another derivative. This illustrates how derivative contracts expand the class of investments and broaden the scope financial positions available.

Forward contracts

A forward contract is a fundamental type of a derivative, and knowing its definition is essential: it is an agreement to trade an asset at a specified future time and for a specified price. The specified price of the trade is known as the delivery price, and the specified future time of the trade is known as the maturity (or the delivery date).

Accordingly there are two parties to the agreement; one agreeing to buy the asset (for the delivery price at maturity) and one agreeing to sell the asset for the delivery price. As discussed in our video these two parties are called the long and short parties, respectively.

The forward contract itself will also specify details about how the agreed trade will take place. For instance, it can specify the settlement conditions of the contract, which may be either physically settled or cash settled. The former meaning that the short party must produce the underlying asset at the maturity date, while the latter means the net value of the contract would be paid.

If physically settled, additional details will be given about precisely how this is to be done. In the case of a forward written on oil, the exact type of oil that needs to be delivered will be specified (e.g. Brent Crude Oil).

Conversely the contract might specify a cash settlement, in which case the underlying asset does not need to be delivered, but cash flows must be made that are equivalent to the exchange of the asset. At first this might seem a bit strange, since a forward is, by definition, an agreement to trade an asset. So why would the trade not need to take place? The answer is that the trade itself is not as important as the eventual financial effect of the trade, and this effect can be achieved with a suitable payment between the parties. Let us consider this effect in detail.

Positions in forward contracts

The forward contract has two different effects on the long party (who has agreed to buy the asset): they receive the underlying asset, and they must pay the pre-agreed delivery price. The long position is worth the net total of these two effects — mathematically, this is given by: exchange. The initial process

$$S_T - K \tag{1}$$

where S_T is the value of the underlying asset at the maturity date and K is the delivery price (which is given a negative sign in Equation (1) because the long party needs to pay this amount). Here, T indicates the maturity date — S_t is a common symbol for the asset price at any time, and here we consider the maturity date in particular. The short party will have the exact opposite position — they will receive the delivery price, but will need to deliver the underlying asset, which in total has a value of:

$$K - S_T \tag{2}$$

Types of forward contract settlements

In the case of physical settlements, the short party receives the delivery price and must deliver an asset of stipulated value at the delivery time. Equation (2) gives the net total of this position. Note, that if the short party owned the underlying asset they could have potentially sold it to the market at large, and they forfeit this sale when delivering the underlying. This is why Equation (2) applies to the short party regardless of whether they own the underlying and gives the value of the short party's position. You can consider the oil-written forward discussed in our previous video as an example of these value equations.

The alternative to physical settlements are cash settlements which are made possible by the previous two equations. This is because a financial equivalent to the underlying can be determined using the two equations to calculate the positions' total values. A cash settlement may be preferable if it would be more convenient for the parties to make exchanges of cash flow to the same financial effect as the physical trade. Instead of receiving the asset and paying the delivery price, the long party would simply receive the amount given in Equation (1) and if this amount is negative, then they need to pay the amount to the short party, not receive it.

The reason making the cash flow is often more convenient than making the physical trade is because many expenses and potential disputes can be avoided. If the long party really does want the physical asset, they can use the cash flow to help to make the purchase from the market.

Because Equations (1) and (2) give the financial effect of the contract, whether it is physically settled, or cash settled, we can calculate these equations to know the financial features of the forward contract rather than concern ourselves with the settlement details.

Why long and short?

Equations (1) and (2) explain why we describe the buying party as long and the selling party as short. Equation (1) demonstrates a positive sensitivity to the underlying asset and accordingly the equation takes on a larger value when the asset's value is large. This positive relationship describes the exposure that the long party has to the underlying asset. 'The long party', in equity investment for instance, is simply a way of describing the buying of an equity investment where the party has a positive exposure to the value of the asset, and where they hope the asset's value increases. Inversely, 'the short party' describes the selling position, which has the opposite exposure. The short party would prefer the underlying asset's value to be low (so that Equation (2)'s value is large), which is often described as a short position on the underlying asset.

Equations (1) and (2) can also suggest why forwards can be so useful. In our previous video we saw that a forward contract can be useful to someone who will buy an asset a future time because the contract helps address the risk of an adverse price change. Equation (1) shows how a long forward contract offsets this risk: the forward contract becomes more valuable if the price of the underlying is large, which can be used to make up the large expense of buying the asset. More generally, however, Equation (1) shows us that a long forward contract is suitable for anyone seeking a positive exposure to the underlying asset (this includes those needing to buy it in the future, but many others as well). Equation (2) shows how a short forward position provides a negative exposure, which can be useful for many reasons that we will describe further below.

Many interesting questions follow from this introduction of forward contracts. One is how the delivery price should be set so that the contract is fair to both parties (looking at Equations (1) and (2), you should be able to see that the long party would prefer a small delivery price, and that the short party would prefer a large delivery price). One simple but important idea here is supply and demand: the price cannot be too large, or the long party would not agree to the contract and cannot be too small or the short party would not agree. We will return to this point later.

Another question you may have is about whether contracts that depend on underlying assets in more complicated ways can be written, which we will consider in our next video.

3.2.3 Transcript: Introduction to Futures and Options

In today's lesson we'll expand on the fundamental types of derivative contracts, like the forward contract discussed in our previous lesson. To begin, let's look at a similar contract called a futures contract.

Futures

Now, in order to understand a futures contract, it is essential to first understand forward contracts, as well as the distinction between over-the-counter (OTC) and exchange-based trades. This is because a futures contract is simply an exchange-traded version of a forward contract. In most regards, futures contracts are very similar to forward contracts — there is a long party, who gets a positive exposure to the underlying asset, and there is a short party who gets a negative (or a short) exposure.

A key difference between them is that a forward contract is an over-the-counter trade, which means the agreement takes place directly between the relevant parties. Accordingly, the long party will make an agreement directly with the short party. This differs drastically from a futures contract which is exchange-traded, meaning the long party and short party do not deal with each other directly, and instead they deal with a centralized exchange which connects willing buyers to willing sellers.

The exchange-traded nature of the futures contract introduces some important differences. The first is that futures contracts will be far more standardized than forwards. When you are dealing directly with a counterparty, you can customize your forward agreement to precisely match your requirements since you have room to negotiate over all details of the contract. However, when dealing with the exchange, you cannot expect to customize the agreement, because the exchange has to offer a contract to the whole market at large.

For instance, if you can find a willing counterparty, you can agree to a forward based on the exact quantity of the underlying asset you are interested in — for example, 75 barrels of oil. However, futures contracts will likely only be available in certain standardized quantities, such as 100 barrels per futures contract. Sacrificing customizability of OTC forward contracts does have some benefits, as we will see in the following set of notes.

Options

Call options

The third type of derivative contract we'll look at is known as a call option, which is a variation on a long forward contract. In a long forward, the long party commits to buying the underlying asset at a specified time and price; whereas in a call option, the contract gives the long party the right, but not the obligation, to the buy the underlying asset at a specified time and price.

To examine the function of a call option let's reexamine our previous example of a forward written on a barrel of oil, with a delivery price of $50. If a barrel of oil turns out to be worth $90, the long party to the forwards makes a net profit of $40, but if the oil price turned out to be $35, the long party would instead make a net loss of $15 (as they would have to pay $50 for something worth only $35).

This possibility of loss is what the call option is designed to avoid: the call option gives the right to buy asset, so you can make the profit of $40 if oil is worth $90, but it does not obligate you to buy the asset if it were worth $35. If the oil price is less than the pre-agreed price, the holder of the call option does not need to face a loss — in the terminology of options, they will choose not to exercise their option (that is, they will choose not to use their right to buy the underlying, if doing so has a net negative effect). We'll discuss more related terminology in the following notes.

Put options

Finally, there is another type of derivative that functions similar to a call options, which is known as a put option. The key difference is that the put option entails the right, but not the obligation, to sell an asset at a future time and price, rather than purchase it. This means that it is a variation on a short forward — instead of committing to sell an asset at a pre-agreed time and price, a put option contract gives you the option of doing so.

3.2.4 Notes: Derivatives: Futures Contracts and Call and Put options

The difference between over-the-counter (OTC) and exchange-based trades is an important concept in finance and comparing a forward contract (or simply a forward) to a futures contracts (a futures) is a textbook example of this difference. As described in our previous video: a forward is a direct agreement between counterparties, while a futures contract is an agreement that is supervised by an exchange, the effects of which we'll need to examine in more detail.

Futures contracts

Let's start by comparing a futures contract to a forward: when you are the long party in a forward contract your counterparty is the short party with whom you have signed a contract. However, in the case of a long futures contract, your counterparty is the exchange, not the short party. This is because the exchange stands between all of the long parties and all of the short parties. Now, although the exchange is a counterparty to every participant in the exchange's contracts, the exchange ensures that all of its positions cancel out. If someone enters a long futures contract at a particular exchange, the exchange will ensure that someone else enters a short futures contract next.

They will ensure this by adjusting the futures price — this corresponds to the delivery price of a forward (and recall that changing the delivery price changes whether the contract is more attractive to the long or short part). Therefore, even though the exchange is a party to many contracts, the implications of the various contracts all cancel out. The exchange may have entered 20000 long futures contracts on oil with a certain maturity date and other details, but they would have ensured that had entered 20000 short contracts too. This means that they take on no risk and have no potential to make any profit or loss— the only profit the exchange can make is in the form of fees (on each of the 20000 long and 20000 short contracts, for example).

By dealing with all of the parties, the exchange effectively connects long and short parties, so that forward-like contracts can be made (meaning agreements to trade assets at fixed future times and prices). Futures contracts are like forwards, but they are not exactly the same, because the exchange introduces some important economic differences to the contract.

OTC versus Exchange

Sometimes the over-the-counter/exchange-based distinction is applied to the market as a whole, meaning that the idea of 'a derivative market' is abandoned in favor of typifying the forward market as an OTC market and the futures market as an exchange-based market. The futures market is a lot more formal and concrete than the forward market. It is made up of the various exchanges that offer futures contracts, as well as the participants and potential participants that the exchange deals with. Whereas the forward market is a distributed network of counterparties and potential counterparties.

So, what are the implications of not dealing directly with a counterparty, and instead dealing with the exchange who in effect connects you to a counterparty? We have already seen a negative implication is that one cannot expect the exchange to offer customized contracts to each of its counterparties (especially considering it wants it long and short positions to exactly cancel out). However, two positive implications of exchange-traded futures are that they have better credit-risk and liquidity characteristics than forwards, but we are not quite ready to explain the details of these features.

Vanilla options

Futures are one type of variation of forwards; another important variation is that of an option. Call options and put options — collectively known as vanilla options — are like forwards (long forwards and short forwards, respectively), but involve the right or the option to trade the underlying asset, without the commitment or the obligation to trade. The option is said to be exercised if the right/option is used.

Other terminology related to options helps to distinguish them from forwards. The agreed date of trade (or, in fact, potential trade) is known as the expiry, instead of the maturity. The agreed price of potential trade is known as the strike price (or the exercise price), instead of the delivery price. If the strike price is equal to 100, the option can be described as "struck at 100". If the strike price is equal to the value of the underlying asset, the option is said to be "at-the-money".

The terminology around the different parties is more difficult to internalize, and the reasons for this will become clearer momentarily. In the case of a call option, the party with the right to buy the asset is called the long party. Theircounterparty, the short party, must deliver the asset if the long party exercises their option. Alternatively, and more commonly in practice, the call option can be cash settled, in which case the asset is not exchanged but an equivalent cash flow takes place. We will formally describe these cash flows shortly.

In the case of a put option, the terminology is reversed, whereby the party with the right to sell the asset is called the long party. This seems inconsistent with forwards, where it was the short party who was selling the asset; here, however, the term long describes the fact that the party whom has bought the option (it just happens to be an option to sell something).

The short party to the put option is the seller of the right to the long party; that is, they must buy the underlying asset if the long party wishes to sell it. Whether an option is long or short indicates who the option — the right to decide — belongs to. Whether an option is a call, or a put indicates the right is a right to buy or a right to sell. It is important to notice the complex terminology that arises when simple ideas are combined: entering a short put option means selling the right for someone to sell an asset at a fixed time and price.

For both calls and puts, the long party can also be called the buyer or holder of the option, and the short party can be called the seller. If the long party to a call option decides to exercise their option, they buy the underlying at the strike price, which — using the same reasoning as with the long forward — gives them a value of:

$$S_T - X \tag{3}$$

where S_T is the value of the underlying asset at the expiry date and X is the strike price. Note that the strike price is just some fixed number, and the symbol K, which we used for the delivery price, could also be used. Equation (3) can tell us when the long party would — or at least should — exercise their option: they should do so if the value is positive. If $S_T > X$ (if the underlying at expiry is worth more than the strike), then it makes sense to buy it at the strike price, as you benefit by the difference given in Equation (3). If Equation (3) is negative, it does not benefit you, and you should simply not exercise your right to do so. Letting the option "expire unexercised" is better than making a net loss by exercising it. Considering this possibility, the value of a call option to the long party is:

$$max(S_T - X, 0) \tag{4}$$

where $max(\cdot, \cdot)$ is a mathematical operator that chooses the larger of the two numbers given in brackets. If the call option is cash settled, the short party will simply pay the amount given by Equation (4) to the long party. The value of a call option to the short party is therefore:

$$-max(S_T - X, 0) \tag{5}$$

Note that a long call option gives a positive exposure to the underlying — Equation (4) tends to be larger if S_T is larger — but in a different way mathematically than the long forward. The exposure is still positive, but its nature is different. Ensure you can explain why the value of a long put option is given by:

$$max(X - S_T, 0) \tag{6}$$

and why a long put option gives a negative exposure to the underlying asset. Equation (6) is known as the payoff of a put option (the value to the long party at the expiry date). Different derivatives have different payoff structures, like we see when comparing the form of the payoff for a call option —in Equation (4) — and for a forward — in Equation (1).

3.2.5 Transcript: The Functions and Risks of Derivatives

Now that we have covered a few fundamental derivative types, we are in a position to consider the functions or uses that derivatives serve, as well as the risks they expose parties to.

A forward contract serves the simple and specific function of fixing the price of a future trade. This can help someone who will need to trade an asset in the future but does not want to face the risk that the asset's price changes. The basic function of a forward, therefore, is to manage this type of risk. We have seen that long forwards give a positive exposure to the underlying asset – long forwards are therefore not only suitable to someone who will buy an asset in the future, they are also suitable for anyone who wants a positive exposure to the underlying.

The individual who will buy oil next year faces the risk that the oil price increases and might enter a long forward to offset this risk. Another individual could be worried about oil price increases for another reason – perhaps because they think their investments will struggle if general fuel prices are high. This other individual should also consider entering a long forward contract on oil, because if the oil price does turn out to be large, the profit they make from the forward can be used to offset the other losses or lack of profits they were concerned about in the first place. Both individuals would be using the forward market for hedging, in other words they would enter a position in order to offset risk associated with their existing positions. A hedge, in finance, is a position intended to behave in the opposite way to your existing positions, so that movements in the hedge and in your positions cancel out, leaving less total variation and risk. We have seen how forwards can provide hedge positions, and indeed one critical function of the forward market is to enable hedging.

The other essential function is that of speculation, which is the attempt to profit based on a certain view of the future. If, for instance, you think the oil price will drop, it can make sense – depending how confident you are – to enter a short forward on oil. The short forward will become valuable if your view turns out to be correct (because short forwards offer a negative exposure to their underlying).

Hedging and speculation are the two key uses of derivatives. We have seen how these functions can be served by forwards; the following set of notes considers hedging and speculating with more complicated derivatives, like options. These primary functions are the typical reasons one would enter a derivative; let us now consider the risks that accompany derivative positions.

The most important risk of a derivative is the risk that it inherits from its underlying. Forwards and other derivatives written on oil depend, by definition, on the oil price. To put this in more general terms, derivatives carry market risk, because the market price of the underlying can change, which will change the value of the derivative. In terms of our example, the uncertainty in the oil price is a type of risk, and that same risk is present in oil derivatives, although the derivative may change the exact nature of that risk. For instance, a short forward creates a negative exposure, and reverses the effect of oil price movements compared to a long forward.

When a derivative is being used to hedge, the concept of market risk needs to be regarded carefully. Consider again the individual who wants to hedge the risk that oil prices increase, and who therefore enters a long forward - remember that the forward is being used to hedge; that is, to offset the pre-existing position. The long forward itself carries market risk, because the value of the long forward depends on the market price of oil. However, this market risk is the right kind of market risk for this individual, whose other positions depend on the oil price in the opposite way. The individual's original position and the long forward both carry market risk, but when these two are put together, the total carries less market risk, because some has been cancelled. This is the essence of hedging.

In our following set of notes we'll examine this concept in more detail and also describe the credit risk that can be associated with derivatives.

3.2.6 Notes: Derivatives: Functions and Risks

Now that we have developed our understanding of the primary functions of derivatives, we can note the logic of why they exist at all. Some individuals are concerned about the oil price decreasing; whereas some are concerned about it increasing. Both parties will mutually benefit if they can agree to a forward contract, both parties can speculate on their views by entering a forward. In this way derivatives allow risks to be transferred around the economy, to the mutual benefit of the parties involved. If a party did not benefit, they would not choose to enter the contract in the first place.

Hedging and speculation through options

In a similar vein, options allow hedging and speculation as described in this lesson's video, however it can be done in a more customized and specific way. Compare the short forward payoff in Equation (2) (from Notes - Fundamental Derivatives: Forward Contracts), to the put option payoff in Equation (6) (from Notes - Derivatives: Futures Contracts and Call and Put options). Here we can note that the put payoff is same with the added feature that it can never be negative (reflecting that fact that an option holder would not exercise their option if it provided negative value).

Equally, put options can be used to hedge the same risk that short forwards can, namely the risk that the underlying's value decreases. If the underlying's price diminishes, the put and the short forward have the same pay off and serve the same function. If the underlying price turns out to be large (which was not the risk we were concerned about), the forward has a negative value while the option has no value.

If we were using the options-related jargon, we would say the put option lets you "hedge the downside but keep the upside". This is because the risk (the downside) is the potential for the oil price to decrease, which the put option offsets; and if the oil price increases (the upside), the put option doesn't cost anything (unlike the short forward). Therefore, a put option allows you keep any profits you are making in the upside-state of the world. Similarly, speculation with options is different from speculation with forwards — you can profit if your speculative prediction becomes true, but don't face a loss if the opposite happens.

Notice that the short party to an option, call or put, never receives money at expiry (see, for instance, Equation (5)). The best-case scenario is that the option expires unexercised. This seems too good to be true for the long party — why would anyone agree to enter a vanilla option as the short party? It is because they receive an amount, called the option premium or price, at the beginning of the option's life. A long option, call or put, gives you a right (an option) at expiry — but to have it you need to pay upfront to have the option later. Again, options will only be entered if the parties wish to enter, so the seller thinks it is worth getting the premium now in exchange for the possible payment they will have to make in the future.

Options can be combined in various different ways to achieve hedging and speculation functions in more complicated ways. More complicated derivatives — with payoffs more complicated than those in Equations (4) and (6) — are called exotic derivatives/options (in contrast to vanilla options) and can also be used for these purposes.

Describing the risk involved in derivative positions is a delicate exercise, because risk is such a complicated and context-dependent concept. The previous video explained how derivatives carry market risk, in the sense that they depend on the market price of the underlying asset. However, the type of derivative determines the way that the underlying is depended upon, and it might be a way that is suitable to hedge other positions you carry. Your positions can carry market risk, and so too can a derivative, but if these two market risks are of the opposite type, then the total market risk faced will be lowered, and this is precisely the concept behind hedging.

Risk in derivatives markets

Credit risk

Before we discuss credit risk, note that more specific risk categories can be used to analyze market risk, which is very general category of risk. For example, we shall see in the final set of notes that an option depends on the volatility of the underlying as well as its price, and so options are said to exhibit volatility risk. A derivative could be said to involve interest-rate risk, which is another way that a derivative's value can depend on market-determined quantities.

Over-the-counter derivatives exhibit credit risk. This is simply the risk that the counterparty does not meet their financial obligations (i.e. the risk that the counterparty defaults). If the short party to a forward does not deliver the underlying asset for a physical settled contract or does not pay the required cash flow for a cash settled contract, then this is the manifestation of credit risk. This would certainly spoil the function that one intended the forward to serve. One might need to sue the counterparty, which can be very expensive, or one might need to settle for some fraction of what one is owed if the counterparty has gone bankrupt. The risk of these things occurring — credit risk — is therefore relevant.

An exchange can mitigate credit risk almost entirely. An exchange, like a government, is very unlikely to default on their obligations — they are usually very large institutions, and they have reputations to maintain (if an exchange did not pay what it owed, traders would be very reluctant to trade on that exchange). This is why a futures contract has no, or virtually no, credit risk, whereas a forward does. An individual does not need to worry about the exchange defaulting.

However, the exchange does need to worry about an individual defaulting. This is the reason that futures have a different cash flow arrangement to forwards. Forwards, if cash settled, involve a cash flow at maturity (as per Equation (1) or (2)). Futures are always cash settled, but the cash flows take place each day. Instead of waiting until maturity and hoping that the counterparty does not default, the exchange requires that the profits and losses be settled every day. This way, a big profit or loss cannot accumulate and the exchange is less vulnerable to a default. Suppose you entered a long futures contract yesterday, and today the price of the underlying has decreased.

This is a bad market movement from your point of view, since you're the long party, and you want the underlying to increase. However, unlike a forward, you will need to make a payment right now to reflect this bad movement (rather than letting bad movements accumulate into a large required payment at maturity).

Liquidity risk

As discussed in our previous module, liquidity risk refers to the risk that a position cannot be easily converted to cash – and this can also be relevant to derivatives, especially derivative contracts with payoffs due after a long time. Over-the-counter derivatives are difficult to liquidate, because they involve a contract with a specific counterparty — you cannot simply cancel a contract if your counterparty does not want to.

Exchange traded derivatives are more liquid, because the standardized environment of the exchange often allows you to convert your position to cash. This is usually done by "closing the position out", which means taking an equal but opposite position in addition to your current one, which has a net value of zero. This may happen if you buy a call option but want to leave the contract before the expiry time — if you can sell a call option with precisely the same details, the two payoffs will cancel exactly according to Equations (4) and (5). Therefore, whatever premium

premium you get for selling the call option is the cash that you have converted your long position into.

However, this can only be done if the closing-out position (the equal and opposite position) is truly equal to the one original one, and this is usually only possible in the exchange-trading environment. This is why futures are much more liquid than forwards — if you want to exit your short futures position, you simply enter a long position of equal size. The two positions cancel, and you have successfully left the position. If you are committed to a forward contract however, there might not be an easy way to exactly cancel the future exchanges you are committed to.

3.2.7 Transcript: Valuing Derivatives

The valuations of the different assets entail multiple methodologies, as we have seen in previous modules. Bonds, for example, can be valued if one has suitable interest rates to use in the valuation formula. Equities can be valued with a number of techniques, such as discounting the estimated future dividends as per the Gordon Growth Model.

In the same vein the valuation of derivatives requires its own techniques, which can be numerous and very complicated, to such an extent that they constitute a distinct discipline of their own. These techniques are studied in depth later in this program; however, here we give a brief introduction and overview.

The main idea behind the valuation of derivatives is that of a replicating portfolio. Suppose you can create a portfolio that behaves exactly like the derivative you are interested in; and thereby replicates the future behavior of the derivative. If achieved, you could conclude that the value of the derivative is the same as that of the replicated portfolio. This conclusion is based on the idea that if two portfolios have the same behavior, the market will ensure that they have the same price. The financial markets are very efficient — they tend to incorporate information, such as the information that two portfolios are behaving identically, into prices. This specific idea is known as the law of one price, and is related to the idea of arbitrage, in a way that is explained the following notes.

Valuing derivatives in this way is often described as relative valuation or relative pricing, because we are comparing the derivative relative to a replicating portfolio and determining the price relative to the price of the replicating portfolio. Many of the techniques we have seen, such as the Gordon Growth Model, are described as absolute valuation techniques. This is because when we determine the price of share according to the Gordon Growth Model, we determine a price for the share that stands on its own terms and does not rely on the price of other assets. In this sense, the Gordon Growth Model is an absolute pricing method – conversely the interest-rate valuations we used in Module 4 were relative, because they valued bonds relative to interest rates, which were taken as an input from elsewhere.

Now that we've learned the underlying logic, the remaining questions are how replicating portfolios can be calculated, and whether they can always be calculated. We'll examine this in our following notes and show how a replicating portfolio for a forward can be constructed by holding the underlying asset as well as some cash in a bank account.

According to this method a certain combination of the underlying and a bank account holding will give precisely the same payoff as a forward contract. The current value of the forward must therefore be the same as the current value of the replicating portfolio. Because one can determine this value (by, for example, looking up a share price and multiplying by the number of shares need in the replicating portfolio), we can determine a value for the forward. This value is only relative to the value of the share. If the share price changed suddenly, the value we have estimated for the forward will change suddenly.

The final set of notes for the module follows all of these points with further detail.

3.2.8 Notes: Derivative Valuations

To continue our discussion of a replicating portfolio, we'll start by exploring a related concept known as arbitrage. An arbitrage is often defined as a trade that results in a profit without any costs or risks. If two assets (or portfolios of assets) have future value that are exactly the same, the current price of the two assets must be equal. If they were not, one could create an arbitrage trade by buying/longing the relatively cheap one, selling/shorting the relatively expensive one, and profiting from the difference. This creates an arbitrage because it provides a net profit without any hidden costs and is also free of risk because the long position will cancel out the short position exactly. It is standard in derivative valuations to assume that there are no available arbitrages (which is a reasonable assumption, because of the efficiency and competitiveness of the financial markets). In this way it gives rise to the law of one price.

Now you may wonder, "What does it mean to short an asset or portfolio?" It means to do the opposite of buying it (i.e., longing it). If you already own an asset, it is easy to do this — you simply sell the asset, which is the opposite of buying. Shorting an asset or portfolio is less clear if you do not already own it, when shorting an asset can be difficult. To reiterate, this means somehow achieving the opposite position compared to buying it, which is a somewhat complicated issue. We can simply note that arrangements can often be made that result in the shorting of an asset, and also note that derivative pricing methods often make this assumption, as well as other assumptions we will mention.

Consider now a long forward, which gives a payoff, at the maturity time T of:

$$S_T - K \tag{7}$$

where S_T is the underlying's price at maturity and K is the delivery price. Consider now a (long) investment in the underlying asset, which, at time T, is worth:

$$S_T \tag{8}$$

Consider also borrowing an amount from the bank and suppose the amount to be paid back to time T is equal to the delivery price. The value of this borrowing to you, at time T, is:

$$-K \tag{9}$$

If we combine these two positions — the long holding in the underlying, and the borrowed loan — we can see that they replicate the long forward. This is because their value at the maturity time (given by adding Equations (8) and (9) together) is exactly the same as the value of the long forward at maturity, or the payoff of the forward (given by Equation (7)). Therefore, by the law of one price, the current value of the long forward is given by the current value of the replicating portfolio.

It is important to note that we intend to find the current value of the forward, as we already know the future value, which is given in Equation (7). In order to know the value of the replicating portfolio, we must calculate the current value of a holding in the underlying, as well as the current value/cost of the aforementioned loan. If we let t denote the current time, the current value of the underlying is given by:

$$S_t \tag{10}$$

The current value of the loan depends on a few things. Let us assume that interest rates are equal to zero for the moment, in which case the current value of the loan is simply equal to the future value (because no interest is added to the loan balance):

$$-K \tag{11}$$

Note that the loan, unlike the underlying in Equation (8), is a liability, with a negative value. The current (time-t) value of the forward is then given by the sum of Equations (10) and (11):

$$S_t - K \tag{12}$$

Equation (12) gives the current value of a long forward contract, given a few assumptions, such as the assumption that interest rates are zero. There are other implicit assumptions, such as the fact that there are no fees associated with buying the underlying, and that the underlying does not involve cash costs or pay any income (if the underlying was a share that paid dividends, these dividends would need to be accounted for in the valuation).

We mentioned previously that the delivery price K needs to be set carefully, or the parties will not agree to the contract (for instance if it is too large the long party will not be incentivized to enter the forward). We can now see precisely how it must be set — K must be set so that the initial value of the forward is zero. This is the only fair initial value of the forward; if the value were non-zero, the contract would be a good deal for one party, and a relatively bad deal for the other, who would not agree to the contract. To get Equation (12) to equal zero, one must set the delivery price as:

$$K = S_t \tag{13}$$

There are many different formulae available for the fair delivery price when some of the assumptions we have made do not hold (for example, when interest rates are non-zero, or when the underlying asset pays dividends).

The basic argument underlying the valuation of the forward we have done, insisting that the value be zero, and other, more complicated valuations giving rise to the different formulae, are known as cash-and-carry arguments. This term is based on the investments in the replicating portfolio: a cash holding (which, if negative, is a loan) and the carrying of the underlying asset.

Unfortunately, replicating portfolios cannot be constructed for futures and vanilla options in this simple and transparent way (let alone for more complicated exotic derivatives). Further assumptions are needed in order for a replicating portfolio to be constructed, and this is where the theory of arbitrage and replication pricing goes beyond our scope. We can, at least, cover some very basic details.

The most important idea to be aware of is the so-called Black-Scholes (or, more correctly, Black-Scholes-Merton) model, which will be studied later on in the program. In short, the Black-Scholes-Merton model is a list of assumptions about how an underlying asset behaves and can be traded (it is called a model because a model, in this context, is a mathematical representation of something more complicated — all the complexities of the financial markets are summarized and simplified by making assumptions like the fact that trading involves no fees and that all assets can be shorted).

Although it beyond our scope to see why, if one accepts the Black-Scholes-Merton assumptions then they are able to form replicating portfolios for vanilla options. The famous Black-Scholes-Merton formulae then gives the costs of these replicating portfolios, which, in accordance with the law of one price, gives the value of the option in question.

The Black-Scholes-Merton formula for the price of a call option, at time t before the expiry T, is:

$$S_t\Phi(\frac{\ln(S_t/K)+(r+\frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}})-Xe^{-r(T-t)}\Phi(\frac{\ln(S_t/K)+(r-\frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}})$$

where S_t is the current (time-t) price of the underlying asset, X is the strike price, $\Phi(\cdot)$ is the standard normal distribution function (which can be calculated on a spreadsheet application, such as Microsoft Excel Online, for example), r is the risk-free interest rate and σ is the annual volatility of the underlying's returns. The volatility is a statistical measure of how much the underlying asset price tends to vary over time (specifically, it is the standard deviation of the annual returns offered by the underlying). Note also the mathematical constant e and the natural logarithm $\ln(\cdot)$.

The formula for a put option is:

$$Xe^{-r(T-t)}\Phi(-\frac{\ln(S_t/K)+(r-\frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}})-S_t\Phi(-\frac{\ln(S_t/K)+(r+\frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}})$$

Note how the Black-Scholes-Merton formulae are relative valuations: they price options relative to the underlying price S_t. This price is needed as an input (just like interest rates are needed as an input to a bond valuation) for a derivative valuation.

In summary, knowing the general definitions and uses of derivatives, as well as the distinctions amongst them is a critical point. As we have seen when correctly used certain derivatives can help mitigate risk, but when improperly used can expose you to undue risk. It is therefore critical to understand the particular features of the assets and instruments as discussed in our module, as well as how these features relate to each's risk characteristics and valuation methods.