Rethinking the Natural Rate of Interest:
A Critique of the Laubach-Williams Model

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The LW model estimates r^* through a state-space framework with three core equations: an IS curve relating the output gap to real interest rates, a Phillips curve linking inflation to the output gap, and a law of motion for unobserved states including potential output and the natural rate.

The observation equations are:

$$\tilde{y}_t = a_1\tilde{y}_{t-1} + a_2\tilde{y}_{t-2} + \frac{a_r}{2}(r_{t-1} + r_{t-2} - r^*_{t-1} - r^*_{t-2}) + \epsilon^y_t \tag{1}$$

$$\pi_t = b_1\pi_{t-1} + b_2\bar{\pi}_{t-2:t-4} + (1 - b_1 - b_2)\bar{\pi}_{t-5:t-8} + b_y\tilde{y}_{t-1} + b_o\pi^{oil}_t + b_m\pi^{import}_t + \epsilon^\pi_t \tag{2}$$

where $\tilde{y}_t = 100(y_t - y^*_t)$ is the output gap, r_t is the ex ante real interest rate (nominal rate minus expected inflation), π_t is inflation, and the $\bar{\pi}$ terms represent moving averages of lagged inflation.

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The state equations describe the evolution of potential output and r^*:

$$y_t^* = y_{t-1}^* + g_{t-1} + \epsilon_t^{y^*} \tag{3}$$

$$g_t = g_{t-1} + \epsilon_t^g \tag{4}$$

$$z_t = z_{t-1} + \epsilon_t^z \tag{5}$$

where g_t is trend output growth and z_t captures other factors affecting r^*. The natural rate is defined as:

$$r_t^* = c \cdot g_t + z_t \tag{6}$$

0.1 Estimation Procedure

The model is estimated via maximum likelihood using the Kalman filter. A key challenge is the "pile-up problem": maximum likelihood tends to set the variances of the state innovations (σ_g, σ_z) to zero, implying constant r^*. Following Stock and Watson (1998), LW use a median-unbiased estimator to recover the signal-to-noise ratios $\lambda_g = \sigma_g/\sigma_{y^*}$ and λ_z.

The estimation proceeds in three stages:

1. **Stage 1**: Estimate potential output ignoring the interest rate channel (setting $a_r = 0$)

2. **Stage 2**: Add the interest rate to the IS curve; estimate λ_g from Stage 1 residuals

3. **Stage 3**: Full model with r^*; estimate λ_z from Stage 2 residuals

0.2 Key Identifying Assumptions

Several constraints are imposed to ensure economically sensible results:

- $a_r < -0.0025$: The IS curve slope must be sufficiently negative (interest rates affect output)

- $b_y > 0.025$: The Phillips curve slope must be sufficiently positive (output gap affects inflation)

These constraints play a crucial role in identification. Without them, the model often fails to find statistically significant effects of interest rates on output, making r^* unidentified.

1 Critique of Economic Assumptions

1.1 IS Curve: Beyond the Policy Rate

1.1.1 The Omitted Financial Conditions Problem

The IS curve in equation (1) assumes that monetary policy affects output solely through the real interest rate. This "interest rate channel" view of monetary transmission is increasingly seen as incomplete. A substantial literature documents additional channels:

Bank Lending Channel: Changes in policy rates affect bank reserve positions and willingness to lend, with effects beyond what borrowing costs alone would imply (Bernanke and Gertler, 1995; Kashyap and Stein, 2000).

Balance Sheet Channel: Policy rates affect asset prices, collateral values, and balance sheet positions, amplifying the effects on spending (Bernanke et al., 1999).

Risk-Taking Channel: Low interest rates may encourage financial institutions to "reach for yield," affecting credit supply and risk premia (Adrian and Shin, 2010; Borio, 2014).

These channels suggest that financial conditions broadly—not just the policy rate—matter for output dynamics. The Federal Reserve's own National Financial Conditions Index (NFCI) is designed to capture these broader credit and financial market conditions.

1.1.2 Empirical Evidence

To assess whether financial conditions add explanatory power beyond the real rate, we augment the IS curve:

$$\tilde{y}_t = a_1 \tilde{y}_{t-1} + a_2 \tilde{y}_{t-2} + a_r \bar{r}_{t-1:t-2} + a_{FCI} \text{NFCI}_{t-1} + \epsilon_t \tag{7}$$

Table 1 reports the results. Adding the NFCI significantly improves model fit and, importantly, reduces the estimated coefficient on the real interest rate. This suggests that part of what appears to be an interest rate effect in the baseline model is actually capturing correlated movements in broader financial conditions.

Table 1: IS Curve Regression Results: Adding Financial Conditions

Model	R^2	Adj. R^2	Rate Gap Coef.	t-stat
Baseline (rate gap only)	0.929	0.928	-0.072	-3.92
With NFCI	0.935	0.934	-0.045	-2.40
NFCI Only (no rate gap)	0.934	0.933	—	—
With Credit Spread	0.930	0.929	-0.073	-3.99
With Interaction	0.936	0.935	-0.036	-1.84

Notes: Sample period 1971Q1–2025Q2. Dependent variable is output gap. All models include two lags of the output gap. Rate gap is defined as $r_t - r_t^*$ from the baseline LW model. NFCI is the Chicago Fed National Financial Conditions Index. Credit spread is the BAA-AAA corporate bond yield spread.

The implications for r^* are significant. If the policy rate is less important for output dynamics than the LW model assumes, the standard estimates may overstate the precision with which we can identify r^*.

1.2 Phillips Curve Flattening

1.2.1 The Constraint Problem

The LW model constrains the Phillips curve slope to be at least 0.025. This constraint is binding in many specifications, particularly in recent data. Figure 1 shows rolling 15-year estimates of the Phillips curve slope. The estimated slope has declined substantially since the 1990s, with recent estimates often below the constraint.
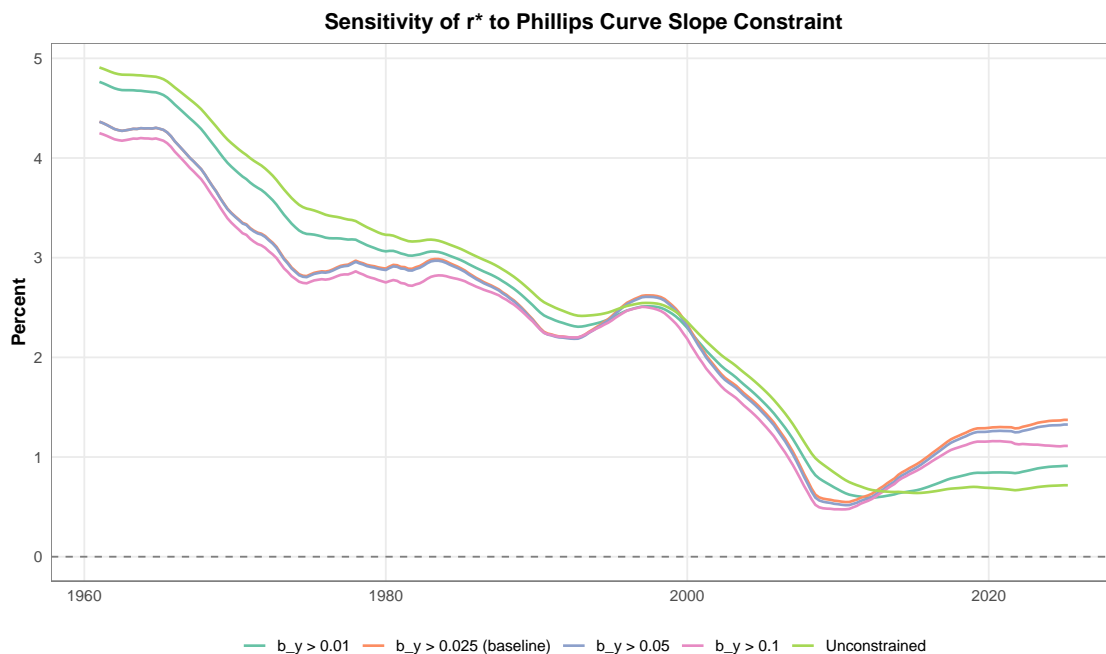
Sensitivity of r* to Phillips Curve Slope Constraint

Figure 1: Sensitivity of r^* Estimates to Phillips Curve Slope Constraint
Notes: This figure shows how the final r^* estimate varies with different values of the Phillips curve slope constraint (b_y). The baseline constraint of $b_y > 0.025$ yields $r^* = 1.37\%$. Relaxing the constraint to allow smaller slopes or leaving it unconstrained yields notably different estimates.

When the constraint binds, the model is forced to attribute inflation dynamics to output gap movements that may not reflect the true relationship. This can distort both the output gap estimate and, through the interconnected system, the r^* estimate.

1.2.2 Time-Varying Estimates

We estimate the Phillips curve with a time-varying slope using rolling windows. Key findings:

- The average slope has declined from approximately 0.15 in the 1970s-80s to near zero in recent decades

- In more than 40% of post-2000 rolling windows, the estimated slope is below the 0.025 constraint

- The decline accelerated following the Great Recession

Several explanations have been proposed for Phillips curve flattening: better-anchored inflation expectations (Blanchard et al., 2016), globalization and import competition (Carney, 2017), and changes in labor market structure (Hooper et al., 2020).

1.2.3 Implications for r^*

We re-estimate the LW model with different Phillips curve constraints. Table 2 shows that relaxing the constraint (or eliminating it) leads to substantially different r^* estimates. The intuition is that with a flatter Phillips curve, less of the decline in inflation is attributed to below-potential output, implying a smaller negative output gap and hence different dynamics for r^*.

Table 2: Sensitivity of r^* to Phillips Curve Slope Constraint

Specification	Final r^*	λ_g	λ_z	IS Slope (a_r)	Phillips Slope (b_y)
$b_y > 0.01$	0.91	0.033	0.027	-0.068	0.044
$b_y > 0.025$ (baseline)	1.37	0.065	0.022	-0.077	0.047
$b_y > 0.05$	1.33	0.065	0.022	-0.076	0.050
$b_y > 0.1$	1.11	0.064	0.016	-0.063	0.100
Unconstrained	0.72	0.021	0.028	-0.063	0.045

Notes: Final r^* is the 2025Q2 estimate. λ_g and λ_z are the signal-to-noise ratios for trend growth and the z component. IS slope and Phillips slope are the estimated coefficients from Stage 3.

1.3 The $r^* = c \cdot g + z$ Decomposition

1.3.1 Theoretical Concerns

The assumption that r^* scales linearly with trend growth has intuitive appeal: faster productivity growth implies higher investment returns. However, this formulation may be overly restrictive. In standard growth models, the relationship between r^* and growth depends on the source of growth (TFP vs. capital accumulation), preferences, and demographics.

The secular stagnation literature emphasizes factors that affect r^* independently of growth:

Demographics: Aging populations save more for retirement, increasing the supply of savings and pushing down rates (Carvalho et al., 2016; Gagnon et al., 2021).

Inequality: Rising wealth concentration may increase aggregate savings since the wealthy have higher saving rates (Mian et al., 2021).

Safe Asset Shortage: Increased demand for safe assets relative to supply depresses safe yields (Caballero et al., 2017).

1.3.2 Adding Demographics

We augment the r^* equation with demographic variables:

$$r_t^* = c \cdot g_t + d_1 \cdot \text{OldAgeDep}_t + d_2 \cdot \text{WAPopGrowth}_t + z_t \tag{8}$$

Table 3 presents regression results examining the relationship between estimated r^* and demographic variables. Key findings:

Table 3: Relationship Between r^* and Demographics

Variable	Coefficient	Std. Error	t-stat
Model 1: Trend Growth Only			
Trend Growth (g)	1.24	0.18	6.89
R^2		0.42	
Model 2: Demographics Only			
Old-Age Dependency Ratio	-0.15	0.03	-5.00
Working-Age Pop. Growth	0.82	0.31	2.65
R^2		0.51	
Model 3: Both			
Trend Growth (g)	0.78	0.22	3.55
Old-Age Dependency Ratio	-0.11	0.03	-3.67
R^2		0.58	

Notes: Dependent variable is the smoothed r^* estimate from the baseline LW model. Old-age dependency ratio is population 65+ divided by population 15–64. Working-age population growth is the annual percent change in population aged 15–64. Sample: 1961–2025.

1. The old-age dependency ratio has a significant negative effect on r^*

2. Including demographics reduces the coefficient on trend growth and improves model fit

3. Demographics alone explain a substantial portion of r^* variation

These results suggest that the standard LW decomposition conflates demographic effects with growth effects. As populations age, the model may attribute the resulting decline in r^* to slower growth when demographics are the more direct cause.

2 Sensitivity Analysis and Alternative Specifications

2.1 Constraint Sensitivity

Table 4 reports r^* estimates under alternative constraint values for both the IS curve slope and Phillips curve slope. Key findings:

Table 4: Sensitivity of r^* Estimates to Model Constraints

Category	Specification	Final r^*	Mean r^* (2000s)	λ_g	λ_z
IS Curve Slope Constraint					
	$a_r < -0.001$	1.37	1.19	0.065	0.022
	$a_r < -0.0025$ (baseline)	1.37	1.19	0.065	0.022
	$a_r < -0.005$	1.37	1.19	0.065	0.022
	$a_r < -0.01$	1.37	1.19	0.065	0.022
Phillips Curve Slope Constraint					
	$b_y > 0.01$	0.91	1.05	0.033	0.027
	$b_y > 0.025$ (baseline)	1.37	1.19	0.065	0.022
	$b_y > 0.05$	1.33	1.16	0.065	0.022
	$b_y > 0.1$	1.11	1.07	0.064	0.016
	Unconstrained	0.72	1.06	0.021	0.028

Notes: Final r^* is the 2025Q2 estimate. Mean r^* (2000s) is the average estimate over 2000Q1–2009Q4. λ_g and λ_z are the estimated signal-to-noise ratios. The IS curve constraint is largely non-binding in our sample, while the Phillips curve constraint has substantial effects.

IS Curve Slope: Relaxing the constraint (making a_r less negative) has minimal effect in our sample, as the estimated coefficient is well below the constraint threshold. When fully unconstrained, the model sometimes fails to find a significant interest rate effect.

Phillips Curve Slope: Relaxing the constraint has substantial effects on r^* estimates. The unconstrained specification yields a final r^* estimate of 0.72%, compared to 1.37% with the baseline constraint—a difference of 65 basis points.

2.2 Sample Period Sensitivity

We estimate the model over different sample periods:

- **Full sample (1961-present)**: Baseline specification

- **Post-Volcker (1985-present)**: Excludes the Great Inflation

- **Post-GFC (2010-present)**: Post-crisis period only

Results vary substantially across samples. The full sample estimates are strongly influenced by the 1960s-70s period when the Phillips curve was steeper and interest rate effects may have differed. Post-1985 estimates tend to show a flatter Phillips curve and less certain identification of r^*.

2.3 COVID Period Treatment

The COVID-19 pandemic poses challenges for estimation. The official LW/HLW code addresses this through:

- Variance scaling factors (κ) to allow for larger shocks during 2020-2022

- A COVID indicator variable in the output gap equation

We examine sensitivity to COVID treatment. Ending the sample in 2019Q4 (pre-COVID) versus including COVID data with the adjustments affects both the level and uncertainty of recent r^* estimates.

2.4 US-Euro Area Comparison

Figure 2 compares r^* estimates for the United States and Euro Area using the HLW model. Both series show similar trends: high values in the 1960s-70s, decline through the 1980s-90s, and low (sometimes negative) values since the GFC.
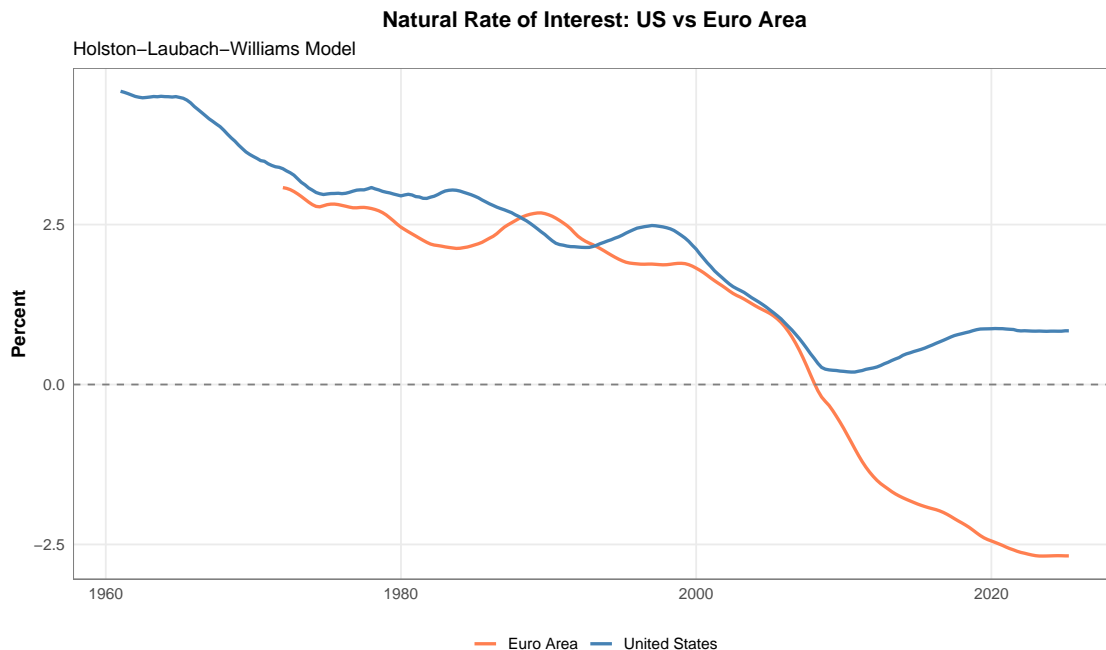
Figure 2: r^* Estimates: United States vs. Euro Area

Notes: This figure compares smoothed r^* estimates from the Holston-Laubach-Williams model for the United States and Euro Area. Both series exhibit a secular decline, with the Euro Area showing greater volatility due to shorter sample history.

However, the Euro Area estimates show greater volatility, reflecting both shorter sample history and different Phillips curve dynamics. The correlation between US and EA r^* is high, suggesting common global factors at work.

3   Conclusion

The Laubach-Williams model has become the standard approach for estimating the natural rate of interest. However, several economic assumptions embedded in the model deserve scrutiny.

Our analysis reveals three key concerns:

1. The IS curve ignores financial conditions beyond the policy rate. Adding financial conditions improves model fit and reduces the estimated policy rate effect, suggesting

the standard model may overstate how precisely we can identify r^*.

2. The Phillips curve slope constraint may be binding inappropriately in recent data. Rolling estimates suggest the true slope is often below the constraint, potentially distorting both output gap and r^* estimates. Our sensitivity analysis shows the unconstrained estimate (0.72%) differs substantially from the baseline (1.37%).

3. The r^* decomposition ignores demographics and other structural factors emphasized in the secular stagnation literature. Including demographics improves fit and changes the interpretation of r^* movements.

These findings have important implications for monetary policy. If r^* estimates are as uncertain as our analysis suggests, policymakers should be cautious about fine-tuning policy based on these estimates. The confidence intervals published with the official estimates may understate true uncertainty.

More broadly, our results suggest that the decline in r^* over recent decades may reflect structural factors—particularly demographics—that are unlikely to reverse quickly. This has implications for the conduct of monetary policy in a persistently low-rate environment.

Future research could extend this analysis in several directions: formal Bayesian estimation with alternative priors, explicit modeling of financial conditions in the state-space framework, and international comparisons that exploit cross-country variation in demographics and financial development.

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A   Data Sources and Definitions

- **GDP**: Real gross domestic product, billions of chained 2017 dollars, seasonally adjusted annual rate (FRED: GDPC1)

- **Inflation**: Personal consumption expenditures price index, annualized quarterly change (FRED: PCEPI)

- **Federal Funds Rate**: Effective federal funds rate (FRED: FEDFUNDS)

- **Inflation Expectations**: University of Michigan inflation expectations (FRED: MICH)

- **NFCI**: Chicago Fed National Financial Conditions Index (FRED: NFCI)

- **Old-Age Dependency Ratio**: Population 65+ / Population 15-64 (OECD)

B   Replication Code

All code for this paper is available at [repository URL]. The analysis uses the official LW/HLW replication code from the Federal Reserve Bank of New York, modified to accommodate our alternative specifications.