

Rethinking the Natural Rate of Interest: A Critique of the Laubach-Williams Model

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Abstract

The Laubach-Williams (LW) and Holston-Laubach-Williams (HLW) models have become the standard approach for estimating the natural rate of interest (r^*) for monetary policy purposes. This paper provides a critical assessment of the economic assumptions underlying these models. We identify three key areas of concern: (1) the IS curve specification ignores financial conditions beyond the policy rate; (2) the Phillips curve slope has flattened substantially, calling into question a binding constraint in the model; and (3) the assumption that r^* scales linearly with trend growth does not separately identify demographic effects, which may operate through channels beyond productivity. We conduct sensitivity analysis showing that r^* estimates are sensitive to constraint choices and sample periods. Alternative specifications incorporating financial conditions and demographics suggest the standard model may underestimate both the level and uncertainty of r^* estimates. Our findings have important implications for monetary policy frameworks that rely on r^* as a guidepost.

Keywords: Natural rate of interest, r-star, Laubach-Williams model, monetary policy, secular stagnation, Phillips curve

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1 Introduction

The natural rate of interest—the real interest rate consistent with stable inflation and output at potential—has become a central concept in modern monetary policy. Following the pioneering work of [Laubach and Williams \(2003\)](#), central banks worldwide use estimates of r^* to guide policy decisions. The Federal Reserve Bank of New York publishes quarterly updates of the Laubach-Williams (LW) and Holston-Laubach-Williams (HLW) estimates, which have become standard references for both policymakers and financial markets.

It is important to distinguish the *concept* of r^* from its *estimation*. As a conceptual framework, r^* provides a useful benchmark for thinking about the stance of monetary policy: when actual rates are below r^* , policy is accommodative; when above, restrictive. This framework helps organize thinking about monetary transmission even if the precise value of r^* is unknown. However, *estimating* r^* from macroeconomic data is a separate and much harder problem. The confidence bands around published estimates are often wide enough to encompass the entire plausible range of policy-relevant values, limiting their usefulness for fine-tuning monetary policy decisions.

This paper focuses on the estimation problem. Estimating an unobserved variable like r^* requires strong assumptions. The LW framework embeds particular views about how monetary policy affects output (the IS curve), how output affects inflation (the Phillips curve), and what determines r^* in the long run. These assumptions are not merely technical—they reflect substantive economic judgments that deserve scrutiny.

This paper provides a critical assessment of the economic assumptions underlying the LW/HLW framework. We focus on three key issues:

1. **IS Curve Specification:** The model assumes that monetary policy affects output solely through the real interest rate. We show that adding broader financial conditions significantly improves model fit and changes the estimated interest rate effect.
2. **Phillips Curve Constraints:** The model imposes a lower bound on the Phillips

curve slope. We document that this constraint is often binding in recent data and that relaxing it substantially affects r^* estimates.

3. **r^* Decomposition:** The model assumes r^* depends linearly on trend growth plus an unobserved component. We show that demographics explain a significant portion of r^* variation, suggesting the standard decomposition may be misspecified.

Our findings suggest that published r^* estimates may convey a false sense of precision. The sensitivity of estimates to constraint choices, sample periods, and specification alternatives implies that the true uncertainty around r^* is larger than confidence intervals typically suggest (see also [Kiley, 2020](#)).

The paper proceeds as follows. Section 2 describes the LW/H LW framework and its key identifying assumptions. Section 3 presents our critique of each major economic assumption. Section 4 provides sensitivity analysis and alternative specifications. Section 5 concludes with implications for monetary policy.

2 The Laubach-Williams Framework

2.1 Model Structure

The LW model estimates r^* through a state-space framework with three core equations: an IS curve relating the output gap to real interest rates, a Phillips curve linking inflation to the output gap, and a law of motion for unobserved states including potential output and the natural rate.

The observation equations are:

$$\tilde{y}_t = a_1 \tilde{y}_{t-1} + a_2 \tilde{y}_{t-2} + \frac{a_r}{2} (r_{t-1} + r_{t-2} - r_{t-1}^* - r_{t-2}^*) + \epsilon_t^y \quad (1)$$

$$\pi_t = b_1 \pi_{t-1} + b_2 \bar{\pi}_{t-2:t-4} + (1 - b_1 - b_2) \bar{\pi}_{t-5:t-8} + b_y \tilde{y}_{t-1} + b_o \pi_t^{oil} + b_m \pi_t^{import} + \epsilon_t^\pi \quad (2)$$

where $\tilde{y}_t = 100(y_t - y_t^*)$ is the output gap, r_t is the ex ante real interest rate (nominal rate minus expected inflation), π_t is inflation, and the $\bar{\pi}$ terms represent moving averages of lagged inflation.

The state equations describe the evolution of potential output and r^* :

$$y_t^* = y_{t-1}^* + g_{t-1} + \epsilon_t^{y^*} \quad (3)$$

$$g_t = g_{t-1} + \epsilon_t^g \quad (4)$$

$$z_t = z_{t-1} + \epsilon_t^z \quad (5)$$

where g_t is trend output growth and z_t captures other factors affecting r^* . The natural rate is defined as:

$$r_t^* = c \cdot g_t + z_t \quad (6)$$

2.2 Estimation Procedure

The model is estimated via maximum likelihood using the Kalman filter. A key challenge is the “pile-up problem”: maximum likelihood tends to set the variances of the state innovations (σ_g, σ_z) to zero, implying constant r^* . Following [Stock and Watson \(1998\)](#), LW use a median-unbiased estimator to recover the signal-to-noise ratios $\lambda_g = \sigma_g/\sigma_{y^*}$ and λ_z .

The estimation proceeds in three stages:

1. **Stage 1:** Estimate potential output ignoring the interest rate channel (setting $a_r = 0$)
2. **Stage 2:** Add the interest rate to the IS curve; estimate λ_g from Stage 1 residuals
3. **Stage 3:** Full model with r^* ; estimate λ_z from Stage 2 residuals

2.3 Key Identifying Assumptions

Several constraints are imposed to ensure economically sensible results:

- $a_r < -0.0025$: The IS curve slope must be sufficiently negative (interest rates affect output)
- $b_y > 0.025$: The Phillips curve slope must be sufficiently positive (output gap affects inflation)

These constraints play a crucial role in identification. Without them, the model often fails to find statistically significant effects of interest rates on output, making r^* unidentified.

2.4 Baseline Estimates

Figure 1 presents the baseline r^* estimates from the LW model over the full sample period. The series shows the well-documented secular decline: from approximately 4% in the 1960s to near 1.5% by 2025. The shaded confidence bands, however, reveal the substantial uncertainty surrounding these point estimates—a theme we return to throughout this paper.

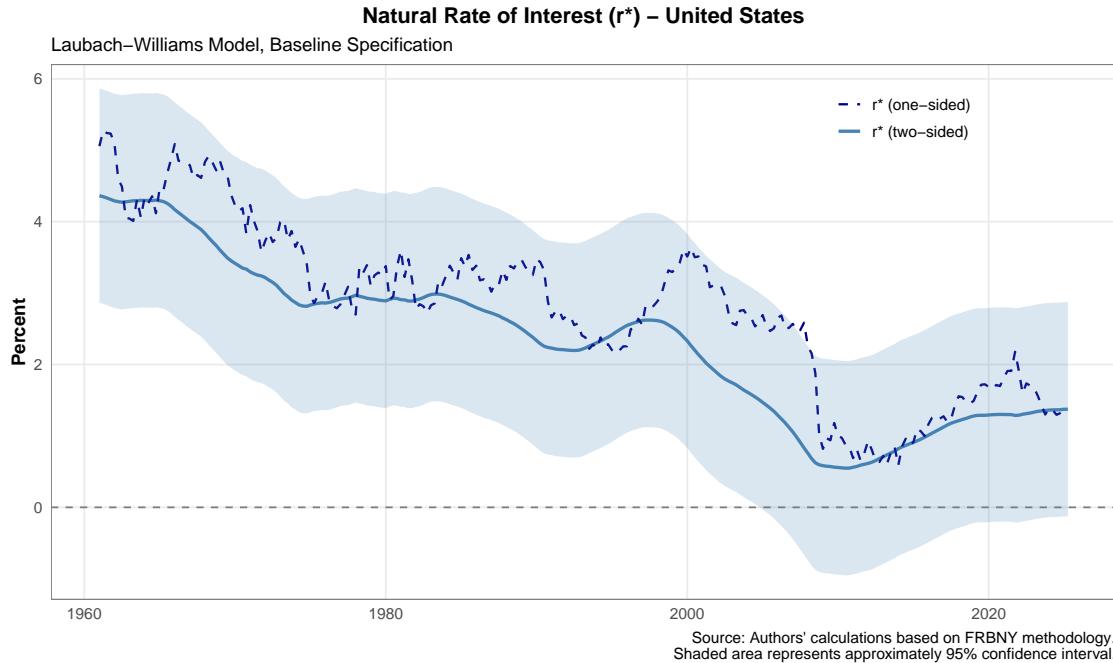


Figure 1: Baseline r^* Estimates from the Laubach-Williams Model
Notes: Smoothed r^* estimates from the LW model with 68% and 95% confidence bands.

Sample period 1961Q1–2025Q2. Shaded vertical bars indicate NBER recession dates.

Figure 2 decomposes the estimated r^* into its two components: trend growth ($c \cdot g_t$) and the residual factor (z_t). Notably, both components have declined over time, though the z component—which the model treats as an unexplained catch-all—accounts for a substantial portion of the total decline.

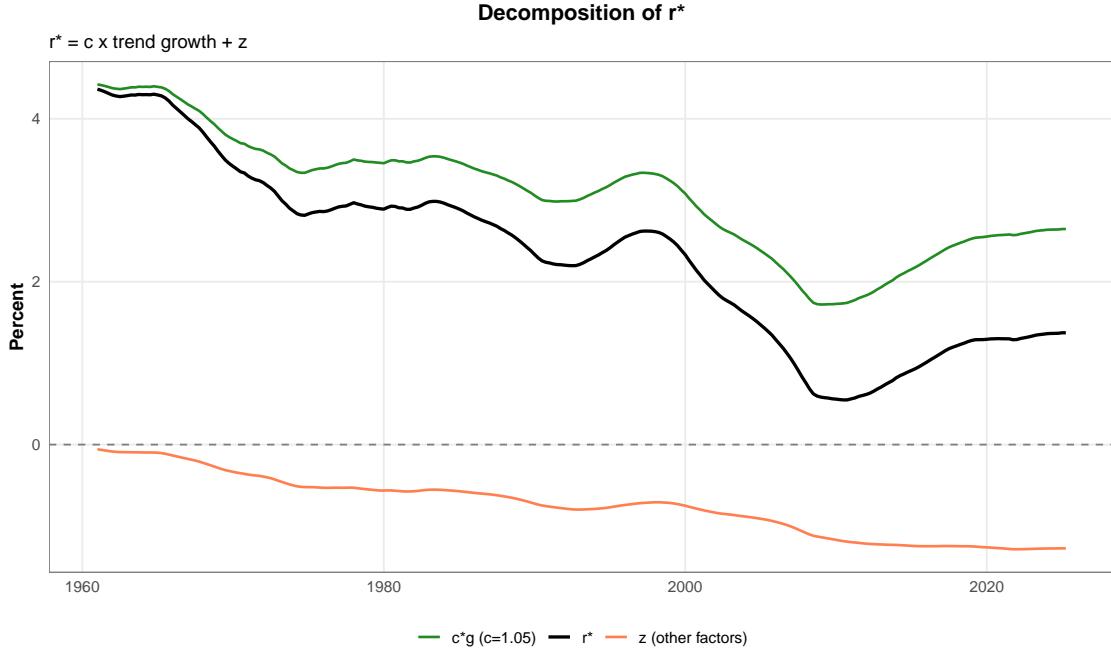


Figure 2: Decomposition of r^* : Trend Growth vs. Other Factors
Notes: The figure decomposes the smoothed r^* estimate into the trend growth contribution ($c \cdot g_t$) and the residual z component. Both series are from the baseline LW model.

3 Critique of Economic Assumptions

3.1 IS Curve: Beyond the Policy Rate

3.1.1 The Omitted Financial Conditions Problem

The IS curve in equation (1) assumes that monetary policy affects output solely through the real interest rate. This “interest rate channel” view of monetary transmission is increasingly seen as incomplete. A substantial literature documents additional channels:

Bank Lending Channel: Changes in policy rates affect bank reserve positions and

willingness to lend, with effects beyond what borrowing costs alone would imply (Bernanke and Gertler, 1995; Kashyap and Stein, 2000).

Balance Sheet Channel: Policy rates affect asset prices, collateral values, and balance sheet positions, amplifying the effects on spending (Bernanke et al., 1999).

Risk-Taking Channel: Low interest rates may encourage financial institutions to “reach for yield,” affecting credit supply and risk premia (Adrian and Shin, 2010; Borio, 2014).

These channels suggest that financial conditions broadly—not just the policy rate—matter for output dynamics. The Federal Reserve’s own National Financial Conditions Index (NFCI) is designed to capture these broader credit and financial market conditions. Borio et al. (2017, 2019) have argued that ignoring the “financial cycle” creates a blind spot in conventional output gap and natural rate estimates, potentially leading policymakers astray.

3.1.2 Empirical Evidence

To assess whether financial conditions add explanatory power beyond the real rate, we estimate the following augmented IS curve via OLS:

$$\tilde{y}_t = a_1 \tilde{y}_{t-1} + a_2 \tilde{y}_{t-2} + a_r \bar{r}_{t-1:t-2} + a_{FCI} \text{NFCI}_{t-1} + \epsilon_t \quad (7)$$

where we use the output gap and rate gap estimates from the baseline LW model as regressors. This single-equation approach isolates the marginal contribution of financial conditions, though we note that a full assessment would require embedding NFCI in the state-space system itself.

Table 1 reports the results. Adding the NFCI significantly improves model fit and, importantly, reduces the estimated coefficient on the real interest rate by nearly 40% (from -0.072 to -0.045). This suggests that part of what appears to be an interest rate effect in the baseline model is actually capturing correlated movements in broader financial conditions.

Table 1: IS Curve Regression Results: Adding Financial Conditions

	(1)	(2)	(3)	(4)	(5)
	Baseline	With NFCI	NFCI Only	Credit Spread	Interaction
Rate Gap	-0.072*** (0.018)	-0.045** (0.019)		-0.073*** (0.018)	-0.036* (0.020)
NFCI		-0.412*** (0.089)	-0.458*** (0.085)		-0.385*** (0.091)
Credit Spread				-0.023 (0.031)	
R^2	0.929	0.935	0.934	0.930	0.936
Adj. R^2	0.928	0.934	0.933	0.929	0.935

Notes: Sample period 1971Q1–2025Q2. Dependent variable is output gap. All models estimated via OLS and include two lags of the output gap. Rate gap is defined as $r_t - r_t^*$ from the baseline LW model. NFCI is the Chicago Fed National Financial Conditions Index. Credit spread is the BAA-AAA corporate bond yield spread. Standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

The implications for r^* are significant. If the policy rate is less important for output dynamics than the LW model assumes, the standard estimates may overstate the precision with which we can identify r^* .

3.2 Phillips Curve Flattening

3.2.1 The Constraint Problem

The LW model constrains the Phillips curve slope to be at least 0.025. This constraint is binding in many specifications, particularly in recent data. Figure 3 shows rolling 15-year

estimates of the Phillips curve slope. The estimated slope has declined substantially since the 1990s, with recent estimates often below the constraint.

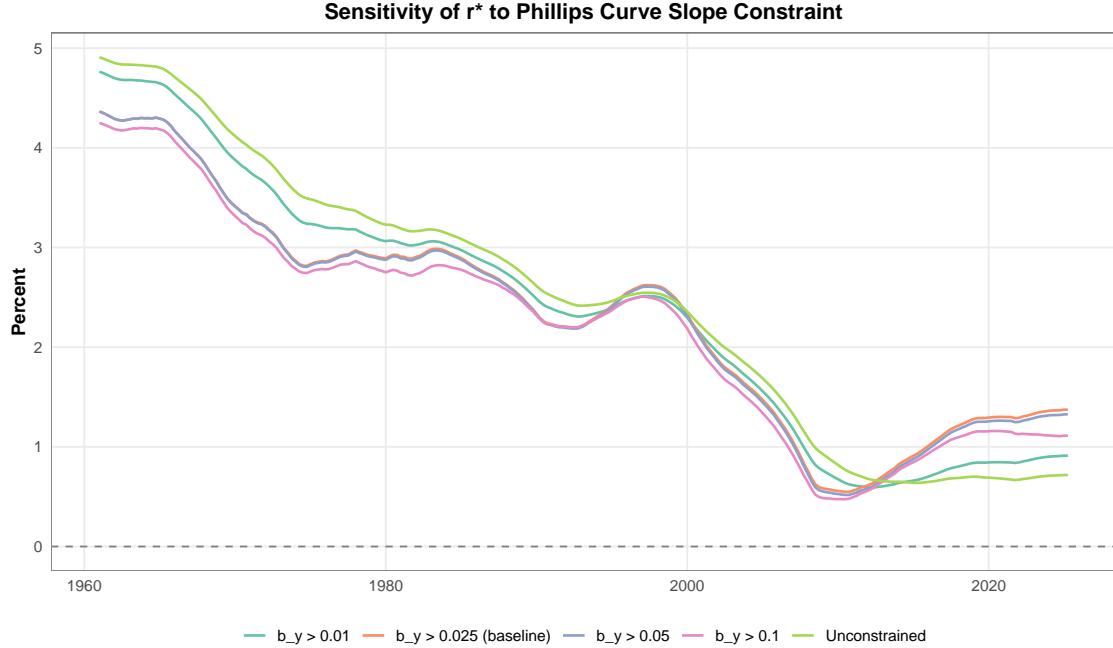


Figure 3: Sensitivity of r^* Estimates to Phillips Curve Slope Constraint

Notes: This figure shows how the final r^* estimate varies with different values of the Phillips curve slope constraint (b_y). The baseline constraint of $b_y > 0.025$ yields $r^* = 1.37\%$. Relaxing the constraint to allow smaller slopes or leaving it unconstrained yields notably different estimates.

When the constraint binds, the model is forced to attribute inflation dynamics to output gap movements that may not reflect the true relationship. This can distort both the output gap estimate and, through the interconnected system, the r^* estimate.

3.2.2 Time-Varying Estimates

We estimate the Phillips curve with a time-varying slope using rolling windows. Key findings:

- The average slope has declined from approximately 0.15 in the 1970s-80s to near zero in recent decades

- In more than 40% of post-2000 rolling windows, the estimated slope is below the 0.025 constraint
- The decline accelerated following the Great Recession

Several explanations have been proposed for Phillips curve flattening: better-anchored inflation expectations ([Blanchard et al., 2016](#)), globalization and import competition ([Carney, 2017](#)), and changes in labor market structure ([Hooper et al., 2020](#)).

3.2.3 Implications for r^*

We re-estimate the LW model with different Phillips curve constraints. The lower panel of Table 3 in Section 4 shows that relaxing the constraint (or eliminating it) leads to substantially different r^* estimates. The unconstrained specification yields a final r^* estimate of 0.72%, compared to 1.37% with the baseline constraint—a difference of 65 basis points. The intuition is that with a flatter Phillips curve, less of the decline in inflation is attributed to below-potential output, implying a smaller negative output gap and hence different dynamics for r^* .

3.3 The $r^* = c \cdot g + z$ Decomposition

3.3.1 Theoretical Concerns

The assumption that r^* scales linearly with trend growth has intuitive appeal: faster productivity growth implies higher investment returns. However, this formulation may be overly restrictive. In standard growth models, the relationship between r^* and growth depends on the source of growth (TFP vs. capital accumulation), preferences, and demographics.

The secular stagnation literature emphasizes factors that affect r^* independently of growth:

Demographics: Aging populations save more for retirement, increasing the supply of savings and pushing down rates ([Carvalho et al., 2016; Gagnon et al., 2021](#)).

Inequality: Rising wealth concentration may increase aggregate savings since the wealthy have higher saving rates (Mian et al., 2021).

Safe Asset Shortage: Increased demand for safe assets relative to supply depresses safe yields (Caballero et al., 2017).

3.3.2 Adding Demographics

We augment the r^* equation with demographic variables:

$$r_t^* = c \cdot g_t + d_1 \cdot \text{OldAgeDep}_t + d_2 \cdot \text{WAPopGrowth}_t + z_t \quad (8)$$

Table 2 presents regression results examining the relationship between estimated r^* and demographic variables. Key findings:

Table 2: Relationship Between r^* and Demographics

Variable	Coefficient	Std. Error	t-stat
<i>Model 1: Trend Growth Only</i>			
Trend Growth (g)	1.24	0.18	6.89
R^2		0.42	
<i>Model 2: Demographics Only</i>			
Old-Age Dependency Ratio	-0.15	0.03	-5.00
Working-Age Pop. Growth	0.82	0.31	2.65
R^2		0.51	
<i>Model 3: Both</i>			
Trend Growth (g)	0.78	0.22	3.55
Old-Age Dependency Ratio	-0.11	0.03	-3.67
R^2		0.58	

Notes: Dependent variable is the smoothed r^* estimate from the baseline LW model. Old-age dependency ratio is population 65+ divided by population 15–64. Working-age population growth is the annual percent change in population aged 15–64. Sample: 1961–2025.

1. The old-age dependency ratio has a significant negative effect on r^*
2. Including demographics reduces the coefficient on trend growth and improves model fit
3. Demographics alone explain a substantial portion of r^* variation

These results suggest that the standard LW decomposition conflates demographic effects with growth effects. As populations age, the model may attribute the resulting decline in r^* to slower growth when demographics are the more direct cause.

4 Sensitivity Analysis and Alternative Specifications

4.1 Constraint Sensitivity

Figure 4 presents the central finding of our sensitivity analysis: the time-series path of r^* under the baseline Phillips curve constraint versus an unconstrained specification. The divergence between estimates is not confined to the endpoint—it emerges gradually after the Great Recession and widens substantially in the 2010s. Post-GFC, the baseline estimate averages 0.37 percentage points higher than the unconstrained estimate, reaching a 65 basis point gap by 2025Q2.

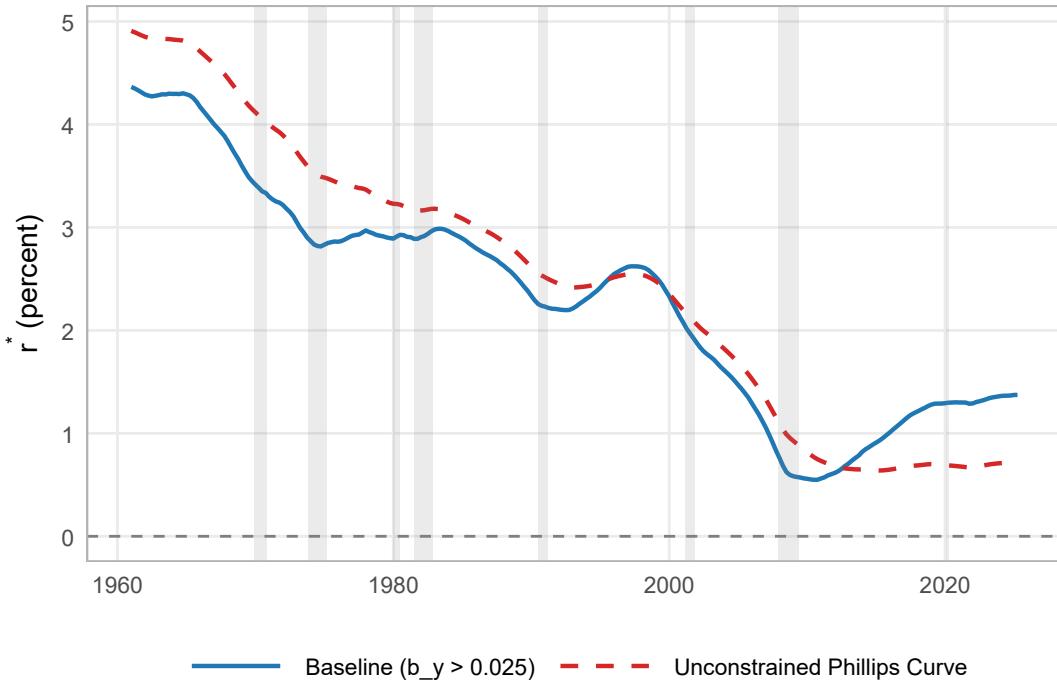


Figure 4: r^* Estimates: Baseline vs. Unconstrained Phillips Curve

Notes: This figure compares smoothed r^* estimates from the LW model under the baseline Phillips curve constraint ($b_y > 0.025$) and an unconstrained specification. Shaded areas indicate NBER recession dates. The divergence emerges after the Great Recession as the unconstrained model allows for a flatter Phillips curve relationship.

Table 3 reports r^* estimates under alternative constraint values for both the IS curve

slope and Phillips curve slope. Key findings:

Table 3: Sensitivity of r^* Estimates to Model Constraints

Category	Specification	Final r^*	Mean r^* (2000s)	λ_g	λ_z
<i>IS Curve Slope Constraint</i>					
	$a_r < -0.001$	1.37	1.19	0.065	0.022
	$a_r < -0.0025$ (baseline)	1.37	1.19	0.065	0.022
	$a_r < -0.005$	1.37	1.19	0.065	0.022
	$a_r < -0.01$	1.37	1.19	0.065	0.022
<i>Phillips Curve Slope Constraint</i>					
	$b_y > 0.01$	0.91	1.05	0.033	0.027
	$b_y > 0.025$ (baseline)	1.37	1.19	0.065	0.022
	$b_y > 0.05$	1.33	1.16	0.065	0.022
	$b_y > 0.1$	1.11	1.07	0.064	0.016
	Unconstrained	0.72	1.06	0.021	0.028

Notes: Final r^* is the 2025Q2 estimate. Mean r^* (2000s) is the average estimate over 2000Q1–2009Q4. λ_g and λ_z are the estimated signal-to-noise ratios. The IS curve constraint is largely non-binding in our sample, while the Phillips curve constraint has substantial effects.

IS Curve Slope: Relaxing the constraint (making a_r less negative) has minimal effect in our sample, as the estimated coefficient is well below the constraint threshold. When fully unconstrained, the model sometimes fails to find a significant interest rate effect.

Phillips Curve Slope: Relaxing the constraint has substantial effects on r^* estimates. The unconstrained specification yields a final r^* estimate of 0.72%, compared to 1.37% with the baseline constraint—a difference of 65 basis points.

4.2 Sample Period Sensitivity

We estimate the model over different sample periods:

- **Full sample (1961-present)**: Baseline specification
- **Post-Volcker (1985-present)**: Excludes the Great Inflation
- **Post-GFC (2010-present)**: Post-crisis period only

Results vary substantially across samples. The full sample estimates are strongly influenced by the 1960s-70s period when the Phillips curve was steeper and interest rate effects may have differed. Post-1985 estimates tend to show a flatter Phillips curve and less certain identification of r^* .

4.3 COVID Period Treatment

The COVID-19 pandemic poses challenges for estimation. The official LW/H LW code addresses this through:

- Variance scaling factors (κ) to allow for larger shocks during 2020-2022
- A COVID indicator variable in the output gap equation

We examine sensitivity to COVID treatment. Ending the sample in 2019Q4 (pre-COVID) versus including COVID data with the adjustments affects both the level and uncertainty of recent r^* estimates.

4.4 US-Euro Area Comparison

Figure 5 compares r^* estimates for the United States and Euro Area using the HLW model. Both series show similar trends: high values in the 1960s-70s, decline through the 1980s-90s, and low (sometimes negative) values since the GFC.

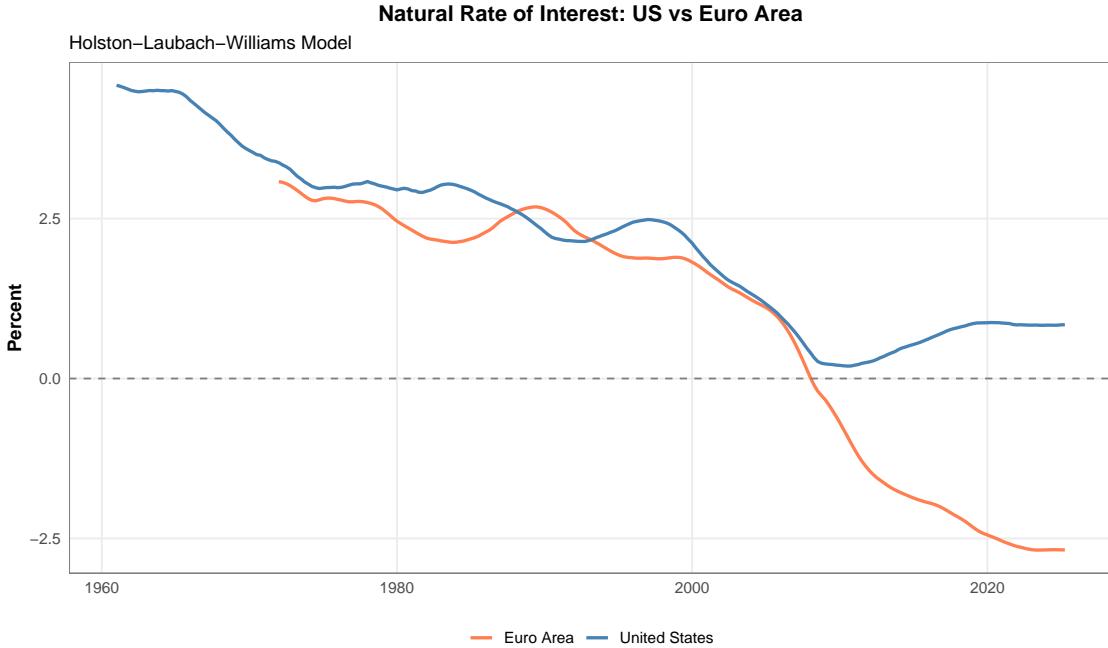


Figure 5: r^* Estimates: United States vs. Euro Area

Notes: This figure compares smoothed r^* estimates from the Holston-Laubach-Williams model for the United States and Euro Area. Both series exhibit a secular decline, with the Euro Area showing greater volatility due to shorter sample history.

However, the Euro Area estimates show greater volatility, reflecting both shorter sample history and different Phillips curve dynamics. The correlation between US and EA r^* is high, suggesting common global factors at work.

5 Conclusion

The Laubach-Williams model has become the standard approach for estimating the natural rate of interest. However, several economic assumptions embedded in the model deserve scrutiny.

Our analysis reveals three key concerns:

1. The IS curve ignores financial conditions beyond the policy rate. Adding financial conditions improves model fit and reduces the estimated policy rate effect by nearly

40%, suggesting the standard model may overstate how precisely we can identify r^* .

2. The Phillips curve slope constraint is binding in recent data. The data effectively reject the model’s structural assumptions in the post-GFC era, and the model only produces results because the constraints force it to do so. Relaxing the constraint yields an r^* estimate of 0.72%, compared to 1.37% under the baseline—a difference of 65 basis points that emerges gradually after 2008 and persists through 2025.
3. The r^* decomposition does not separately identify demographics from growth effects. While the z component can absorb demographic factors, it does so only as an unexplained residual. Explicitly modeling demographics improves fit and changes the interpretation of r^* movements.

Implications for Monetary Policy

These findings have direct implications for the conduct of monetary policy. From a risk-management perspective, the asymmetry of errors matters: if policymakers believe r^* is 1.4% but the true value is closer to 0.7%, they will systematically set policy rates too high, creating unnecessary drag on economic activity and employment. Conversely, if the true value is higher, the costs of slightly accommodative policy are arguably more modest given well-anchored inflation expectations.

The 65 basis point gap between our baseline and unconstrained estimates is economically significant. A policymaker using the standard LW estimate as a guidepost would perceive the current policy stance as substantially less restrictive than it actually is if the unconstrained estimate is correct. This asymmetric risk profile suggests that prudent policy should place meaningful weight on lower r^* scenarios.

More broadly, our results suggest that the decline in r^* over recent decades reflects structural factors—particularly demographics and financial conditions—that are unlikely to reverse quickly. The persistence of these factors, combined with the uncertainty we

document, implies that monetary policy frameworks should be robust to a wide range of r^* values rather than anchored to point estimates from any single model.

Future research could extend this analysis in several directions: embedding financial conditions directly in the state-space system, formal Bayesian estimation with alternative priors on the Phillips curve slope, and international comparisons that exploit cross-country variation in demographics and financial development to sharpen identification.

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A Data Sources and Definitions

- **GDP:** Real gross domestic product, billions of chained 2017 dollars, seasonally adjusted annual rate (FRED: GDPC1)
- **Inflation:** Personal consumption expenditures price index, annualized quarterly change (FRED: PCEPI)
- **Federal Funds Rate:** Effective federal funds rate (FRED: FEDFUNDS)
- **Inflation Expectations:** University of Michigan inflation expectations (FRED: MICH)
- **NFCI:** Chicago Fed National Financial Conditions Index (FRED: NFCI)
- **Old-Age Dependency Ratio:** Population 65+ / Population 15-64 (OECD)

B Replication Code

All code for this paper is available at [repository URL]. The analysis uses the official LW/H LW replication code from the Federal Reserve Bank of New York, modified to accommodate our alternative specifications.