

1. For an RG058-model coaxial cable, given with $a=0.9$ mm, $b=2.95$ mm, $\epsilon_r=2.3$, find:

(a) The characteristic impedance and wave velocity in the TEM mode.

$$Z_c = \frac{1}{2\pi} \sqrt{\frac{\mu}{\epsilon}} \ln(b/a) \approx 46.97 \Omega$$

$$v_p = \frac{c}{\sqrt{\epsilon_r}} \approx 1.98 \cdot 10^8 \text{ m/s}$$

(b) The propagation time of a 1-ns pulse in a 10-m long section of this coaxial cable.

$$t_p = 10/v_p \approx 50 \text{ nsec}$$

(c) The phase accumulation vs. frequency (the term kz in Eqs. 1a, b), in the range 0 – 6 GHz.

(d) How the results above would be changed for $b=4$ mm and for $\epsilon_r=4$?

$$\rightarrow Z_c = 44.7 \Omega \quad v_p = 1.5 \cdot 10^8 \text{ m/s}$$

$$t_p = 66 \text{ nsec}$$

from the above formulas

2. For a rectangular waveguide, given with $a=100$ mm, $b=50$ mm, find:

(a) The dispersion relation in the TE₁₀ mode, also in a graph as in Fig. 3.

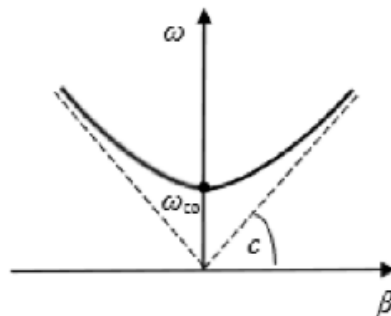


Figure 3: The dispersion relation of a rectangular waveguide in the TE₁₀ mode.

for the first mode:

$$k_{10} = \sqrt{\mu \epsilon \omega^2 - \left(\frac{\pi}{a}\right)^2}$$

$$\beta_{10} = \sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{\pi}{a}\right)^2}$$

(b) The cutoff frequency and wavelength (how the latter relates to the waveguide width?)

$$\omega_{c0} = \frac{\pi c}{a \sqrt{\epsilon_r}} = 2\pi f_{c0} \quad \begin{cases} \epsilon_r = 1 \\ c = 3 \cdot 10^8 \text{ m/s} \\ a = 0.1 \text{ m} \end{cases}$$

$$f_{c0} = 1.5 \text{ GHz}$$

$$\lambda = c/f_{c0} = 92 \text{ mm} = 200 \text{ mm}$$

(c) Repeat (a) and (b) for the second higher mode(s).

For the 2nd higher mode = TE₂₀: $\beta_{20} = \sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{2\pi}{a}\right)^2}$

$$\omega_{c0} = 2\pi c/a = 6\pi \cdot 10^9$$

$$f_{c0} = \frac{\omega_{c0}}{2\pi} = 3 \text{ GHz}$$

$$\lambda = \frac{c}{f_{c0}} = 0.1 \text{ m} = 100 \text{ mm}$$

(d) This waveguide is now truncated to a length of 1 m with perfect mirrors, to form a rectangular cavity (as in Fig. 4). What would be the resonance frequencies of this cavity? Where would these frequencies appear on the dispersion diagram?

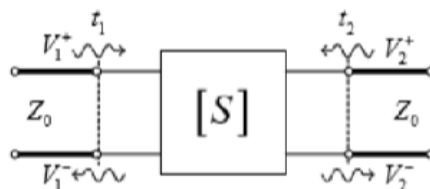
$$\omega_n = c \sqrt{\left(\frac{\pi}{a}\right)^2 + \left(n\pi/L\right)^2} \quad L = 1 \text{ m}$$

Fig 4 $\rightarrow n=3 \quad \omega_3 = 3 \cdot 10^8 \sqrt{(10\pi)^2 + (3\pi)^2} \approx 9.84 \text{ GHz}$

3. Present the general form of a 2-port scattering matrix, and explain its various terms.

Scattering matrix may be helpful to know that as voltage and current are to electrical circuit analysis, S parameters are to microwave network analysis. S parameter are formed from ratios of reflected and incident voltage wave amplitudes.

$$[V^-] = [S] \cdot [V^+]$$



S_{11} can be written as the ratio of reflected wave and incident wave.

$$S_{11} = \frac{V_1^-}{V_1^+}, V_2^+ = 0$$

Therefore we obtain $S_{11} = \Gamma_{11}$ which is the voltage reflection coefficient at port 1.

$$S_{21} = \frac{V_2^-}{S_1^+}, V_2^+ = 0$$

With matched load at port 2 so $V_2^+ = 0$ then $S_{21} = \Gamma_{21}$ which is the voltage transmission coefficient from port 1 to 2.

4. Describe the VNA functions in general, and explain its calibration procedure.

Vector Network Analyzer (VNA) is an instrument that measures the network parameters of electrical networks. VNA commonly measure s-parameters because reflection and transmission of electrical networks are easy to measure at high frequencies, but there are other network parameter sets such as y-parameters, z-parameters, and h-parameters. Network analyzers are often used to characterize two-port networks such as amplifiers and filters.

5. What would you expect to get as S_{11} for the cavity given in Question 2(c) above?

Expect to get $S_{11}=0$ since input port is matched therefore there is no reflected wave.

6. The formulae in Eqs. (6, 7) enable the analysis of a given micro-strip line (given the dimensions and dielectric fill, one can find the effective impedance and wave velocity). However, for the design of a micro-strip line, the desired impedance and wave velocity are known, but the line dimensions and dielectric fill shall be found. Find alternative equations and graphs for the design of a micro-strip line.

7. Given a dielectric fill for a micro-strip line of $\epsilon_r = 2.1$ and $h=82$ mm, find the strip width w in order to realize a 50Ω micro-strip line. What would be eff ϵ in this case?

and for $w/h > 1$ by

$$\epsilon_{\text{eff}} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2 \cdot \sqrt{1 + 12h/w}}, \quad Z_c = \frac{120\pi}{\sqrt{\epsilon_{\text{eff}}} \left[\frac{w}{h} + 1.393 + \frac{2}{3} \ln \left(\frac{w}{h} + 1.444 \right) \right]}$$

$$w = 3.208 \times 8.2 \times 10^{-3} = 0.026[m]$$

$$\epsilon_{\text{eff}} = 1.802$$