

1. For an RG058-model coaxial cable, given with  $a=0.9$  mm,  $b=2.95$  mm,  $\epsilon_r=2.3$ , find:

(a) The characteristic impedance and wave velocity in the TEM mode.

$$Z_c = \frac{1}{2\pi} \sqrt{\frac{\mu}{\epsilon}} \ln(b/a) \approx 46.97 \Omega$$

$$v_p = \frac{c}{\sqrt{\epsilon_r}} \approx 1.98 \cdot 10^8 \text{ m/s}$$

(b) The propagation time of a 1-ns pulse in a 10-m long section of this coaxial cable.

$$t_p = 10/v_p \approx 50 \text{ nsec}$$

(c) The phase accumulation vs. frequency (the term  $kz$  in Eqs. 1a, b), in the range 0 – 6 GHz.

(d) How the results above would be changed for  $b=4$  mm and for  $\epsilon_r=4$ ?

$$\rightarrow Z_c = 44.7 \Omega \quad v_p = 1.5 \cdot 10^8 \text{ m/s}$$

$$t_p = 66 \text{ nsec}$$

from the above formulas

2. For a rectangular waveguide, given with  $a=100$  mm,  $b=50$  mm, find:

(a) The dispersion relation in the TE<sub>10</sub> mode, also in a graph as in Fig. 3.

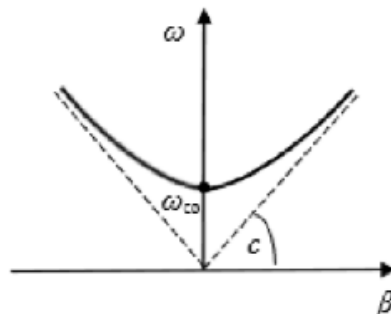


Figure 3: The dispersion relation of a rectangular waveguide in the TE<sub>10</sub> mode.

for the first mode:

$$k_{10} = \sqrt{\mu \epsilon \omega^2 - \left(\frac{\pi}{a}\right)^2}$$

$$\beta_{10} = \sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{\pi}{a}\right)^2}$$

(b) The cutoff frequency and wavelength (how the latter relates to the waveguide width?)

$$\omega_{c0} = \frac{\pi c}{a \sqrt{\epsilon_r}} = 2\pi f_{c0} \quad \begin{cases} \epsilon_r = 1 \\ c = 3 \cdot 10^8 \text{ m/s} \\ a = 0.1 \text{ m} \end{cases}$$

$$f_{c0} = 1.5 \text{ GHz}$$

$$\lambda = c/f_{c0} = 92 \text{ mm} = 200 \text{ mm}$$

(c) Repeat (a) and (b) for the second higher mode(s).

For the 2<sup>nd</sup> higher mode = TE<sub>20</sub>:  $\beta_{20} = \sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{2\pi}{a}\right)^2}$

$$\omega_{c0} = 2\pi c/a = 6\pi \cdot 10^9$$

$$f_{c0} = \frac{\omega_{c0}}{2\pi} = 3 \text{ GHz}$$

$$\lambda = \frac{c}{f_{c0}} = 0.1 \text{ m} = 100 \text{ mm}$$

(d) This waveguide is now truncated to a length of 1 m with perfect mirrors, to form a rectangular cavity (as in Fig. 4). What would be the resonance frequencies of this cavity? Where would these frequencies appear on the dispersion diagram?

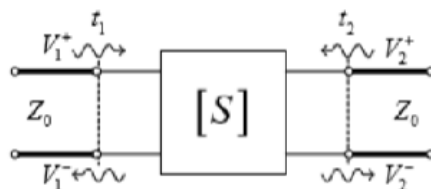
$$\omega_n = c \sqrt{\left(\frac{\pi}{a}\right)^2 + \left(n\pi/L\right)^2} \quad L = 1 \text{ m}$$

Fig 4  $\rightarrow n=3 \quad \omega_3 = 3 \cdot 10^8 \sqrt{(10\pi)^2 + (3\pi)^2} \approx 9.84 \text{ GHz}$

3. Present the general form of a 2-port scattering matrix, and explain its various terms.

Scattering matrix may be helpful to know that as voltage and current are to electrical circuit analysis, S parameters are to microwave network analysis. S parameter are formed from ratios of reflected and incident voltage wave amplitudes.

$$[V^-] = [S] \cdot [V^+]$$



$S_{11}$  can be written as the ratio of reflected wave and incident wave.

$$S_{11} = \frac{V_1^-}{V_1^+}, V_2^+ = 0$$

Therefore we obtain  $S_{11} = \Gamma_{11}$  which is the voltage reflection coefficient at port 1.

$$S_{21} = \frac{V_2^-}{S_1^+}, V_2^+ = 0$$

With matched load at port 2 so  $V_2^+ = 0$  then  $S_{21} = \Gamma_{21}$  which is the voltage transmission coefficient from port 1 to 2.

**4. Describe the VNA functions in general, and explain its calibration procedure.**

Vector Network Analyzer (VNA) is an instrument that measures the network parameters of electrical networks. VNA commonly measure s-parameters because reflection and transmission of electrical networks are easy to measure at high frequencies, but there are other network parameter sets such as y-parameters, z-parameters, and h-parameters. Network analyzers are often used to characterize two-port networks such as amplifiers and filters.

**5. What would you expect to get as  $S_{11}$  for the cavity given in Question 2(c) above?**

Expect to get  $S_{11}=0$  since input port is matched therefore there is no reflected wave.

**6. The formulae in Eqs. (6, 7) enable the analysis of a given micro-strip line (given the dimensions and dielectric fill, one can find the effective impedance and wave velocity). However, for the design of a micro-strip line, the desired impedance and wave velocity are known, but the line dimensions and dielectric fill shall be found. Find alternative equations and graphs for the design of a micro-strip line.**

**7. Given a dielectric fill for a micro-strip line of  $\epsilon_r = 2.1$  and  $h=82$  mm, find the strip width  $w$  in order to realize a  $50 \Omega$  micro-strip line. What would be eff  $\epsilon$  in this case?**

and for  $w/h > 1$  by

$$\epsilon_{\text{eff}} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2 \cdot \sqrt{1 + 12h/w}}, \quad Z_c = \frac{120\pi}{\sqrt{\epsilon_{\text{eff}}} \left[ \frac{w}{h} + 1.393 + \frac{2}{3} \ln \left( \frac{w}{h} + 1.444 \right) \right]}$$

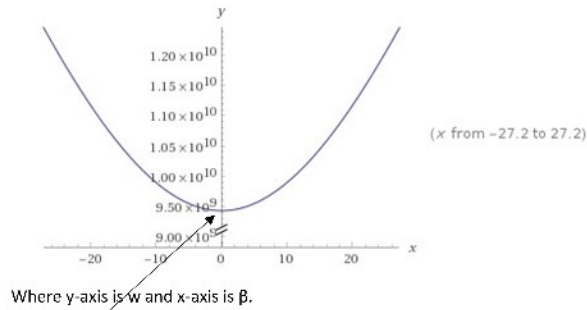
$$w = 3.208 \times 8.2 \times 10^{-3} = 0.026[m]$$

$$\epsilon_{\text{eff}} = 1.802$$

### Part 1: Rectangular waveguide simulation

Analysis:

(a) Given the cross-section dimensions of the rectangular waveguide, plot its dispersion diagram (as in Fig. 2).



9.5 GHz=resonance frequency

(b) Mark the measured resonance frequencies on the theoretically computed dispersion diagram.

$$w_0 = \frac{\pi c}{a} = \pi * 3 * 10^8 * 100 * 10^{-3} = 9.42 \text{ GHz}$$

Which corresponds to the resonance frequency found from the above graph (approximately).  
a=100mm

(c) Given the cavity length L, verify that the resulted wavenumbers  $\beta n$  satisfy the resonance condition presented in Eq. (5) for  $n = 1, 2, 3 \dots$

$$\omega_n = c \sqrt{\left(\frac{\pi}{a}\right)^2 + \left(\frac{n\pi}{L}\right)^2}$$

L = 680 mm  $\beta = n\pi/L$  (Waveguide 1)

N=1 ->  $w=9.53e9$ ,  $\beta=4.61$

N=2 ->  $w=9.82e9$ ,  $\beta=9.24$

N=3 ->  $w=1.03e10$ ,  $\beta=13.86$

We can observe that the numbers we get by calculus match our graph. The resonance condition is respected.

(d) Compare the S11 and S21 curves, and discuss their similarities and differences.

For f->flow, we can observe that S21 is highly negative dB while S12 remains at 0dB.

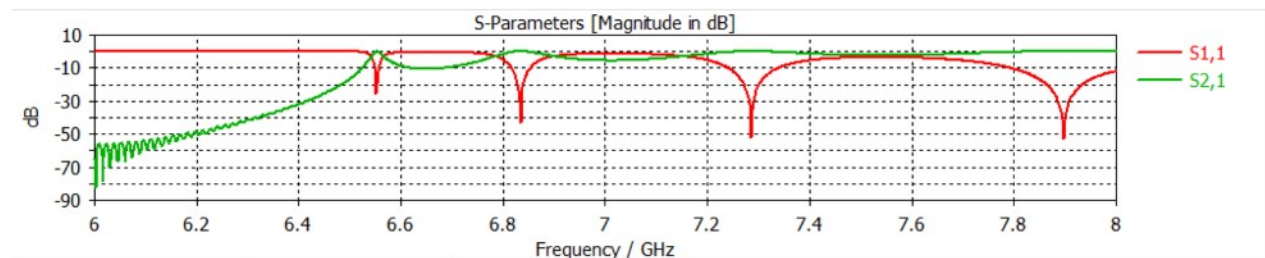
At some point,  $f=6.55\text{GHz}$ , S21 reaches 0db. This means that reflections are eliminated.

The S parameters are measured as the ratio between the incident wave to any port.

\*S21=TRANSMISSION coefficient

\*S11=REFLECTION coefficient

Looking at the curve, once we get some significant transmission, 0dB is 1 so full transmission (no reflection), However we can still observe some resonance frequencies where the reflection coefficient gets some negative value in dB, this means the reflection loss gets low at specific frequencies=resonance frequencies.



Hollow\_rect.cst:

$$\text{If } \epsilon_{\text{psilon}} = 1, f_{c,mn} = \frac{1}{2\pi} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

-First mode:  $m = 1, n = 0 \rightarrow f = 6.56\text{GHz}$

-Second mode:  $m = 0, n = 1 \rightarrow f = 14.73\text{GHz}$

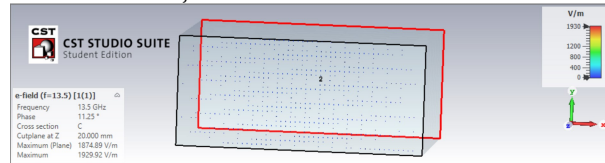
-Third mode:  $m = n = 1 \rightarrow f = 16.16\text{GHz}$



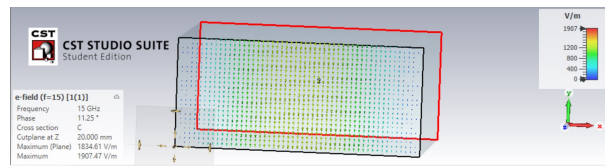
electric field cross section 2d wave

-second mode:  $m=0, n=1 \rightarrow f = 14.750116$

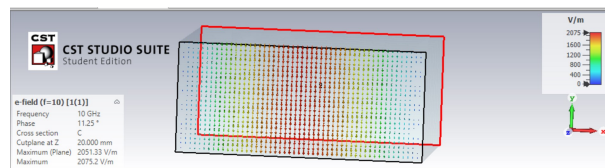
-Third mode:  $m=n=1 \rightarrow f = 16.16\text{GHz}$



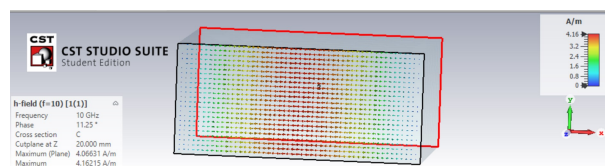
Electric field cross section 2<sup>nd</sup> mode  
(vertical alignment for  $\vec{E}$ )



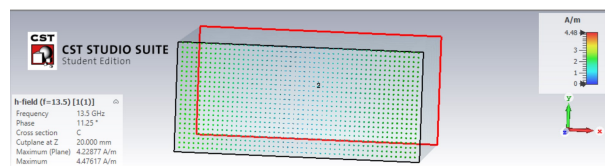
$\vec{E}$  field cross section 3<sup>rd</sup> mode



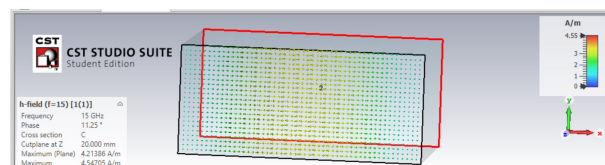
$\vec{E}$  field cross section 1<sup>st</sup> mode



$\vec{H}$  field cross section 1<sup>st</sup> mode  
(horizontal alignment for  $\vec{H}$ )



$\vec{H}$  field cross section 2<sup>nd</sup> mode



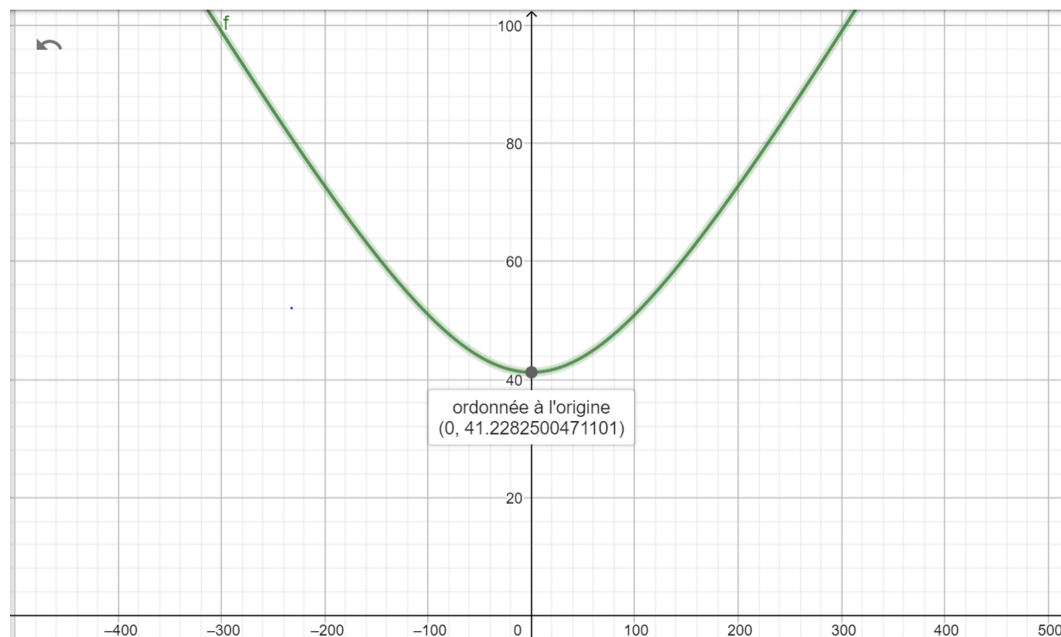
$\vec{H}$  field cross section 3<sup>rd</sup> mode -  
→ magnitude of the  $\vec{E}, \vec{H}$  fields increases  
for lowest frequencies and

Now we change the material to Teflon of dielectric constant 2.1

And we have:  $w_{co} = \frac{\pi c}{a\sqrt{\epsilon}} = 2.85e10 \rightarrow f = 4.53\text{GHz}$

(a)

In the following graph, the x axis corresponds to beta while the y axis corresponds to  $w \cdot 10^9$



From the graph we get 41.23GHz which is close to what we found in calculations.



(b)  $a=22.86\text{mm}$

$$w_{co} = \frac{\pi c}{a\sqrt{\epsilon}} = 2.85e10 \rightarrow f = 4.53\text{GHz}$$

©  $a=22.86\text{mm}$   $L=680\text{mm}$

$$\omega_n = c \sqrt{\left(\frac{\pi}{a}\right)^2 + \left(\frac{n\pi}{L}\right)^2}$$

$L = 680\text{ mm}$   $\text{Beta} = n\pi/L$  (Waveguide 1)

$N=1 \rightarrow w=4.13e10, \text{Beta}=4.61$

$N=2 \rightarrow w=4.13e10, \text{Beta}=9.24$

$N=25 \rightarrow w=5.39e10, \text{Beta}=115.5$

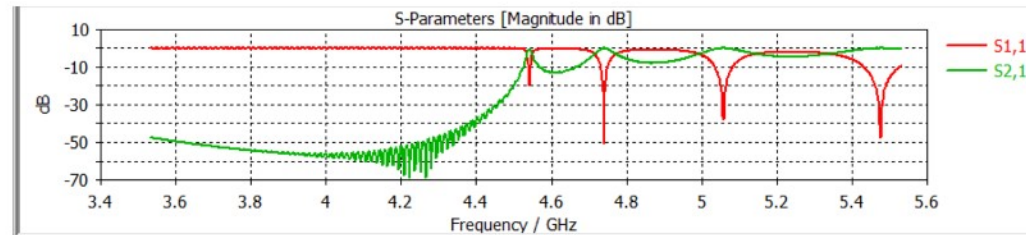
Again, we can see that the calculations match with the graph

(d)

Remind that:

\* $S_{21}$ =TRANSMISSION coefficient

\* $S_{11}$ =REFLECTION coefficient

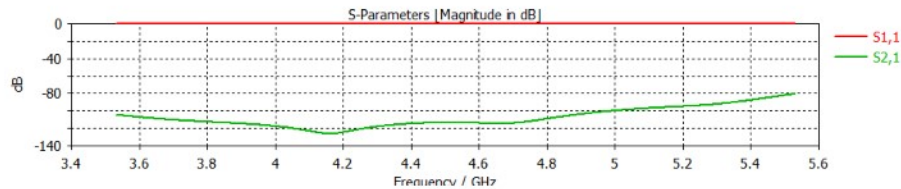


We observe the same general shape as when the rectangular was filled with vacuum but in the teflon case we can observe that as the dielectric constant affect directly  $w$  and  $f$ , the cutoff frequency changed to 4.5GHz, where the transmission coefficient gets to 1=0dB (reflection gets eliminated). And again, we have some resonance frequencies where  $S_{11}$  gets deeply negative dB values.

Now we change the stubs and probes to PEC and the rectangular waveguide interior is back to vacuum.

We can observe that now, as expected (from the fact that we are dealing with a transverse/perpendicular electric field) we'll get full reflection (constant 0dB=1) and no transmission (high negative dB).

Meaning, the input matches to the port while no radiation loss to the material (heating...).



## Part 2: Home Experiment

The microwave is displaying  $f=2.45\text{GHz}$

$$\lambda = \frac{c}{f_{co}} = 0.122$$

$$Vp = \frac{c}{\sqrt{\epsilon_r}}$$

I decided to put chocolate for the experiment.

The distance between the two adjacent melt spots is  $d=6.15\text{cm}=0.0615\text{m}$

If we do  $2 \cdot d \cdot f$  we should get  $c$ = speed of light

Let's check that:

$2 \cdot 0.0615 \cdot 2.45e9 = 3.0135e8$  which is pretty close to  $c$  !!

The small difference can be explained by experimental inaccuracies or by the fact that we might have let the chocolate melt for too long.

Why is that?

The microwave carries EM radiations (as wave) that proceed at the speed of the light. The only things that differ are the wavelength and frequency.

The distance between the two melted spots correspond to half a wavelength. This is because we prevent the plate from turning by turning it upside down. Otherwise, the melt points would distribute across the food and so the whole plate will warm up. But when not turning, the peaks are rather stable and the melt spots remain at the same place/static.

wave speed = frequency x wavelength  $\rightarrow 2.45\text{G} \cdot 0.122 = c$

The effect of such melted spots on the food heating quality in microwave is that when the plate is not turning, the rest of the food will slowly heat up from the heat of these 2 melted spots only.

That is the reason why, in microwaves the food is placed on a rotating plate, so that the peaks of heat

are distributed over the plate, and the whole food heats up.