Study Guide: Chapter 14 - Online Algorithms

Fiat and Woeginger, LNCS #1442

1 Introduction

Chapter 14 of *Online Algorithms: The State of the Art* by Fiat and Woeginger focuses on important topics in **online algorithms**, specifically addressing competitive analysis and algorithms for dynamic problems. The chapter emphasizes algorithmic approaches in settings where decisions are made sequentially without full knowledge of future inputs.

2 Competitive Analysis of Online Algorithms

Competitive analysis compares the performance of an online algorithm against the optimal offline algorithm. The **competitive ratio** is used as a measure of performance. If $\mathrm{ALG}(I)$ is the cost of an online algorithm ALG on input I, and $\mathrm{OPT}(I)$ is the cost of the optimal offline algorithm, the competitive ratio is defined as:

$$\frac{\mathrm{ALG}(I)}{\mathrm{OPT}(I)}$$

The goal is to minimize the competitive ratio across all possible inputs.

2.1 Adversary Models

Different types of adversaries can be considered:

- **Oblivious Adversary**: The adversary generates the input sequence without knowledge of the algorithm's actions.
- **Adaptive Adversary**: The adversary adjusts its strategy based on the algorithm's previous decisions.

3 Paging and Caching Problems

Paging is a classical online problem where a limited cache must hold frequently used data. When new data is requested, a decision must be made on which item to evict. Common algorithms for paging include:

- **LRU (Least Recently Used)**: Evicts the least recently used page.
- **FIFO (First In, First Out)**: Evicts the page that was loaded the earliest.
- **LFU (Least Frequently Used)**: Evicts the least frequently used page.

The **competitive ratio** for paging problems is known to be k, where k is the cache size.

4 Dynamic Online Problems

Many online problems require dynamically updating solutions as the input evolves. This section explores algorithms that handle dynamic changes efficiently, such as:

- **Dynamic Search Problems**: Maintaining a search structure (e.g., balanced trees) under dynamic updates.
- **Reordering Problems**: Adjusting the sequence of operations to minimize access or retrieval costs.

5 Trade-offs in Online Decision Making

Online algorithms often face trade-offs between short-term decisions and long-term performance. The balance between immediate benefits and future costs is a recurring theme in this chapter, with strategies involving:

- **Greedy Algorithms**: Make locally optimal decisions at each step.
- **Lookahead**: Incorporating limited future information can improve decision quality.

6 Applications of Online Algorithms

The concepts discussed in this chapter are applied in various domains, including:

- **Network Routing**: Making routing decisions in a network with dynamically changing conditions.
- **Job Scheduling**: Allocating resources to tasks that arrive over time.
- **Financial Models**: Decision making in uncertain environments, such as stock trading or portfolio optimization.

7 Concluding Remarks

Chapter 14 highlights the challenges and strategies for designing efficient online algorithms. By utilizing competitive analysis and understanding the adversary models, one can design algorithms with provable performance guarantees in various dynamic and uncertain environments.