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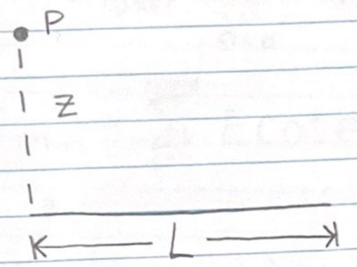
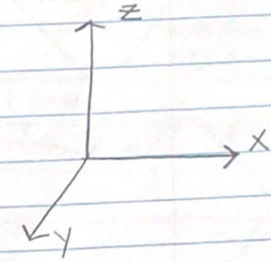
Physics 4321
Homework #2

3. Find the electric field a distance z above one end of a straight line segment of length L that carries a uniform line charge λ. Check that your formula is consistent with what you would expect for the case $z >> L$.

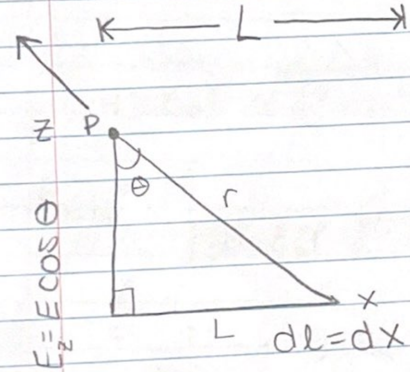


$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$



← uniform λ charge on x-axis

$$r^2 = x^2 + y^2 = x^2 + z^2 \qquad E_{TOTAL} = E_x + E_z$$



$$dE = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\lambda dx}{x^2+z^2}\left[\frac{-x}{(x^2+z^2)^{1/2}} + \frac{z}{(x^2+z^2)^{1/2}}\right]$$

$E_x = E\sin\theta$

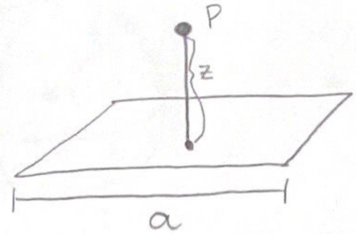
$$E = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda}{r^2}$$

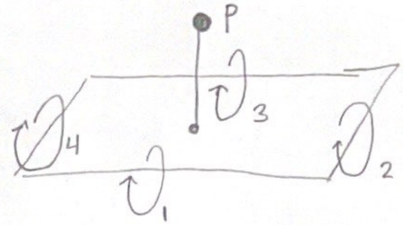
$$E = \int_0^L \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{x^2+z^2}\left[\frac{-x}{(x^2+z^2)^{1/2}} + \frac{z}{(x^2+z^2)^{1/2}}\right]dx$$

$$= \int_0^L \frac{\lambda}{4\pi\varepsilon_0}\left[\frac{-x}{(x^2+z^2)^{3/2}} + \frac{z}{(x^2+z^2)^{3/2}}\right]dx$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\left[\frac{1}{\sqrt{x^2+z^2}}\Bigg|_0^L + \frac{zx}{z^2\sqrt{x^2+z^2}}\Bigg|_0^L\right] = \frac{\lambda}{4\pi\varepsilon_0}\left[\frac{1}{\sqrt{L^2+z^2}} - \frac{1}{z} + \frac{L}{z\sqrt{L^2+z^2}} - 0\right]$$

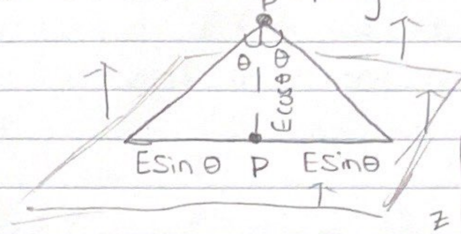
$$= \frac{\lambda}{4\pi\varepsilon_0 z}\left[\frac{z}{\sqrt{L^2+z^2}} - 1 + \frac{L}{\sqrt{L^2+z^2}}\right]$$



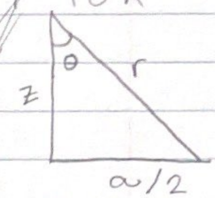


$j = \theta = 0 : \theta = 360$

4. Find the electric field a distance z above the center of a square loop (side a) carrying uniform line charge λ.



SOH
CAH
TOA



$$E_{TOTAL} = \sum_{\substack{i=1\\ \theta=0}}^{\substack{\theta=360\\ n=4}} E_{n,\theta}$$

$$E_{TOTAL} = \sum_{i=1}^{n=4} E\cos\theta = 4E\cos\theta$$

$\cos\theta = z/r$

$\sin\theta = a/2r$

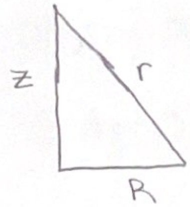
$r = \sqrt{z^2 + (a/2)^2}$

$$E = \frac{1}{4\pi\varepsilon_0 r} \cdot \frac{2\lambda L}{\sqrt{z^2+L^2}}\,\hat{z} \qquad L = a/2$$

$$E_{TOTAL} = \cancel{4}\left[\frac{1}{\cancel{4}\pi\varepsilon_0 r} \cdot \frac{\cancel{2}\lambda\, a/\cancel{2}}{\sqrt{z^2+(a/2)^2}}\right] \cdot \frac{z}{\sqrt{z^2+(a/2)^2}}$$

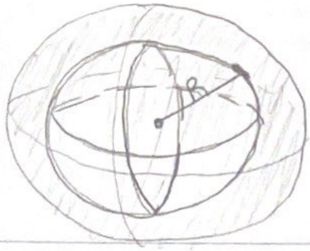
$$= \frac{1}{\pi\varepsilon_0} \cdot \frac{\lambda a}{\sqrt{z^2+a^2/4}} \cdot \frac{z}{\sqrt{z^2+a^2/4}} \cdot \frac{1}{\sqrt{z^2+a^2/4}}$$

$$= \boxed{\frac{1}{\pi\varepsilon_0}\ \frac{z\lambda a}{(z^2+a^2/4)(z^2+a^2/4)^{1/2}}}$$



7. Find the electric field a distance z from the center of a spherical surface of radius R, that carries a uniform charge density σ. $z < R$ inside, $z > R$ outside. Express your answers in terms of the total charge q on the sphere.

$(R-z) = \sqrt{R^2 + z^2 - 2Rz}$ if $R > z$ $\qquad E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}$



density·area = $\frac{\text{mass}\cdot\text{area}}{\text{volume}}$

$\sigma A = Q_{enclosed}$

11. Use Guass's law to find the electric field inside and outside a spherical shell of radius R that carries a uniform surface charge density σ. Compare your answer to problem 2.7.

$$\oint E\cdot da = \frac{1}{\varepsilon_0} Q_{enclosed}$$

Area $= 4\pi R^2$

density·area $= \sigma 4\pi R^2$

Inside the shell: $r < R$

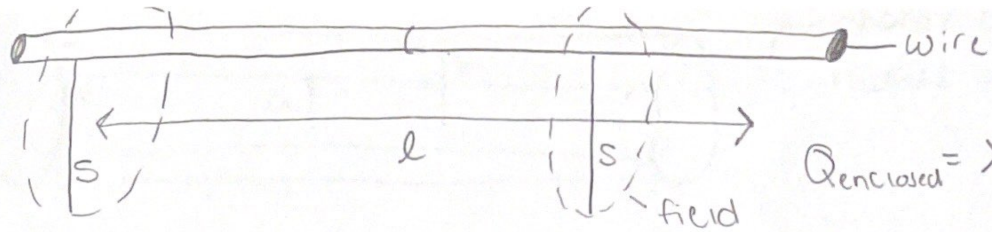
$\Sigma Q_{r<R} = Q_{enclosed} = 0$

Outside the shell: $r > R$

$$\oint E\cdot da = \frac{1}{\varepsilon_0} Q_{enclosed} = \frac{1}{\varepsilon_0}(\sigma\cdot\text{area}) = \frac{1}{\varepsilon_0}\sigma 4\pi R^2$$

$$E\cancel{4\pi}r^2 = \frac{1}{\varepsilon_0}\sigma\cancel{4\pi}R^2$$

$$\boxed{E = \frac{\sigma R^2}{\varepsilon_0 r^2}\,\hat{r}}$$



$Q_{enclosed} = \lambda l$

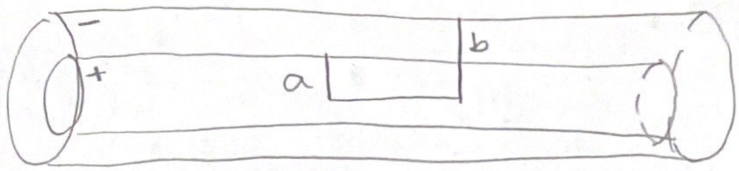
13. Find the electric field a distance s from an infinetly long straight wire that carries a uniform line charge q. Compare equation 2.9

$E = \frac{1}{4\pi\varepsilon_0} \frac{2\lambda}{z}$ $\qquad \oint E\,da = \frac{1}{\varepsilon_0} Q_{enclosed}$

Area Cylinder $= 2\pi s l + \cancel{2\pi s^2}$ ← NO field on ends (∞)

$E(2\pi s l) = \frac{1}{\varepsilon_0} Q_{enclosed}$

$E = \frac{1}{\varepsilon_0} \frac{\lambda \cancel{l}}{2\pi s \cancel{l}} = \boxed{\frac{\lambda}{2\pi\varepsilon_0 s}\,\hat{s}}$



ρ = charge density
s = radius adjusted

16. A long coaxial cable carries a uniform volume charge density ρ on the inner cylinder (radius a), and a uniform surface charge density on the outer cylindrical shell (radius b). Find the electric field in (i) $s<a$, (ii) $a<s<b$, (iii) $s>b$. Plot $|E|$ as a function of s.

(i) $\oint E\,da = \frac{1}{\varepsilon_0} Q_{enclosed}$ $\quad Q_{enclosed} = \int \rho\, d\tau = \int_0^{l}\int_0^{2\pi}\int_0^{s} \rho\, s\, ds\, d\phi\, dz$

$= \frac{s^2}{2}\cdot 2\pi \cdot \rho l$

$Q_{enclosed} = s^2 \pi \rho l$

$E(2\pi s l) = \frac{1}{\varepsilon_0}\cdot s^2 \pi \rho l = \boxed{\frac{\rho s}{\varepsilon_0 2}\hat{s}}$

(ii) $\oint E\,da = \frac{1}{\varepsilon_0} Q_{enclosed}$ $\quad Q_{enc} = \int \rho\, d\tau = \int_0^{l}\int_0^{2\pi}\int_a^{b} \rho\, s\, ds\, d\phi\, dz$

$= \rho l \cdot 2\pi \cdot \frac{(b-a)^2}{2}$

$Q_{enclosed} = (b-a)^2 \rho l \pi$

$E(2\pi s l) = \frac{1}{\varepsilon_0}(b-a)^2 \rho l \pi = \boxed{\frac{(b-a)^2 \rho}{\varepsilon_0 2 s}\hat{s}}$

(iii) $\oint E\,da = \frac{1}{\varepsilon_0} Q_{enclosed}$ $\quad Q_{enclosed} = \int \rho\, d\tau = \int_0^{l}\int_0^{2\pi}\int_{s>b}^{s>b} \rho\, s\, ds\, d\phi\, dz = 0$

$E(2\pi s l) = \frac{1}{\varepsilon_0}\cdot 0 = \boxed{0}\,\hat{s}$

Electric field as a function
of distance from centr