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1.1 Lévy stable distribution and fat tails

The only partial agreement of the EMH (i.e., the random walk with independent increments) with the recently discovered empirical evidence, suggested the needs for more accurate, though more complex, probability distribution function (PDF) than the Gaussian as models for price fluctuations or return.

Instead of considering the absolute price change $\triangle P = P_{t+1} - P_t$ it is common practice to use one of the following definitions of *relative change* or *return* [1]:

- *simple net return* R_t , on the asset between dates *t*-1 and *t*, $R_t = P_t / P_{t-1} - 1$
- *simple gross return* is just $1 + R_t$
- *continuously compounded return* or *log return* is the logarithm of the gross return, $r_t = log(1 + R_t) = log P_t / P_{t-1} = log P_t - log P_{t-1}$.

In 1963 Mandelbrot proposed the Lévy stable distribution as model for the distribution returns [11]. He proposed a power law form $\propto x^{-(1+\alpha)}$, $\alpha \leq 2$ for intermediate and large *x*, but rounded peak at $x \rightarrow 0$. In statistics, it arises from the generalization of the central limit theorem to a wider class of distributions which do not have a finite second moment. In particular it is the asymptotic distribution of a random variable $P_n = \sum_{i=1}^{n} x_i$ for $n \rightarrow \infty$ where the variables x_i are distributed as $P(x) \sim x^{-(1+\alpha)}$.

The Lévy stable probability density function (PDF) does not have an analytic closed form but can be expressed by its characteristic function [10] or in term of its Fourier transforms [6]. This distribution function is *leptokurtic* (positive excess kurtosis $\kappa = \mu_4 / \sigma^4 - 3$ where μ_4 is the fourth central moment and σ is the standard deviation), that is, it has more probability mass in its tails and center than a Gaussian. The advantage over the Gaussian is that the Lévy PDF with its fat tails accounts for *a higher probability of extreme events*.
Recent empirical studies conducted on asset prices of different markets have shown *partial* agreement with the Lévy distribution [13, 6]. In fact the excess kurtosis of daily returns ranges between 2 and 50, and is even higher for intra-day data [2],.

Although Lévy distribution are stable, that is, the sum of two independent random variables characterized by the same Lévy distribution is itself characterized by the same Lévy distribution, the problem is that the resulting distribution have infinite second and higher moments. This is in contrast with empirical data which have finite second moments. The central part appear to be well fit by a Lévy distribution but the asymptotic behaviour of the distribution of returns shows faster decay then predicted by a Lévy distribution. For this purpose a *truncated Lévy* distribution (TL) has been proposed [9, 12, 13]. Recently the *scale-invariant truncated Lévy* distribution (STL) has been proposed to model the *scale invariance* observed in empirical data [11, 13].

The STL distribution is defined as where , , is related to the size of the truncation of the Lévy distribution and the exponential pre-factor ensures a smooth truncation. The parameter A determines the ``spread'' in the central region [12, 15].

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