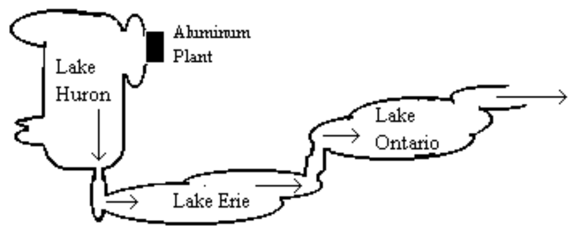
## PROBLEM 1: GREAT LAKES

### Exploring Pollution in the Great Lakes

In the Great Lakes in the United States, water flows from Lake Huron into Lake Erie, and from Lake Erie to Lake Ontario. From Lake Ontario, water flows through the St. Lawrence Seaway to the ocean. Each year, 11% of the water in Lake Huron flows into Lake Erie, while 36% of the water in Lake Erie flows into Lake Ontario, and 12% of the water in Lake Ontario flows out to the sea.




Map of the Great Lakes

In years past, there were aluminum factories on each of the lakes, pumping a pollutant into the lakes. Last year, all of the plants closed down except for one plant on Lake Huron that continues to pump 25 tons of the pollutant into the lake each year. Presently, there are 3500, 1800, and 2400 tons of this pollutant dissolved uniformly in the water in the three lakes, respectively.

1. Write recursive equations to help you model the flow of pollution through the three lakes.

2. Use a spreadsheet program to get data for the next 100 years.

3. Simulate the amount of pollution over time using a Python for loop and lists. Then plot the pollution in each lake over time.

    Hint: Create three lists that have the initial values in them.
    Then, inside a for loop, append new values to each list.
    To reference previous things in each list, you might want to an index of "i-1".
    Once your lists are populated, you can use matplotlib to create a scatterplot.

*Source: Intermath: Four Sample Problems, COMAP, Inc., Lexington, MA, 1992.*

**PROBLEM 2: Infectious disease**

The SIR model (susceptible-infected-recovered) with limited immunity models the spread of a disease in which there is limited immunity, meaning a certain fraction, $\mu$, of the "recovered" population loses immunity and becomes susceptible again. The differential equations that model the populations S(t), I(t), and R(t) are:

$$\frac{dS}{dt} = -\alpha S(t)I(t) + \mu R(t)$$

$$\frac{dI}{dt} = \alpha S(t)I(t) - \lambda I(t)$$

$$\frac{dR}{dt} = \lambda I(t) + \mu R(t)$$

Through a change of variables we can reduce this to two equations for the percentage of susceptible and infected, s(t) and i(t), and then you can always obtain removed by subtracting 1 – s(t) - i(t). (This derivation is optional homework that you can read more about in the document sirslab longer handout). Here are the equations:

$$\frac{ds}{dt} = -\beta s(t)i(t) + \mu(1 - s(t) - i(t))$$

$$\frac{di}{dt} = \beta s(t)i(t) - \lambda i(t)$$

Assuming the initial conditions s(0)=0.9 and i(0)=0.10 (for example, 10% of students came back after winter break with the flu and 90% are susceptible), model the spread of this disease over the next 100 days using a step size of $\Delta t = 1$ days and $\mu = 0.1, \beta = 0.6, \lambda = 0.34$.

1. Write Euler's equations to help you model this disease.

2. Use a spreadsheet program to get data for the next 100 days.

3. Simulate the spread of the disease over time using a Python for loop and lists. Then plot the susceptible, infected, and removed over time.

> Hint: Create four lists (for time and each population) that have the initial values in them. Then, inside a for loop, append new values to each list.
> To reference previous things in each list, you might want to use an index of "i-1".
> Once your lists are populated, you can use matplotlib to create a scatterplot.
> In addition to the time plot, you should also create a phase plane plot of the number of susceptible versus the number of infected.

*(Source: Duke University Calculus Lab Manual)*