Math Modeling Contests

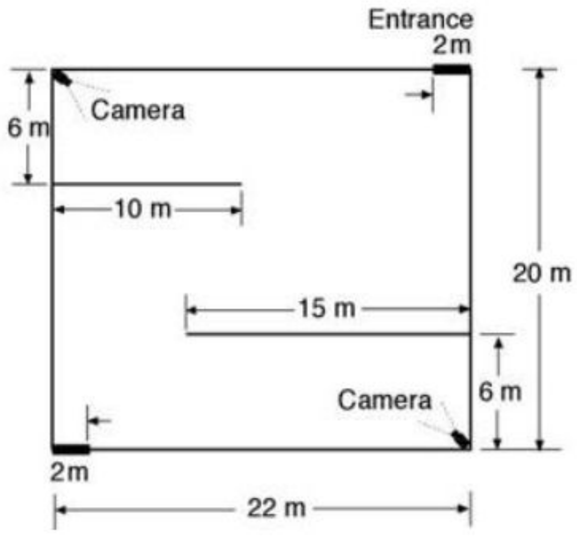
Lauren Shareshian

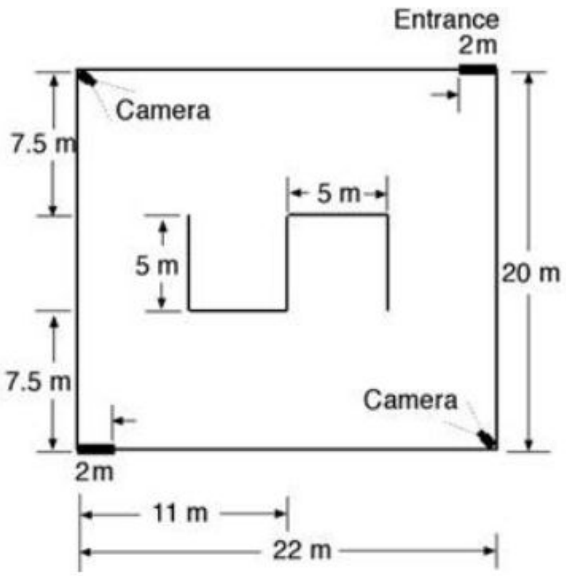
HiMCM & M3C

- High School Mathematical Contest in Modeling (HiMCM)
  - 11 day contest period in November (changed from 36 hrs)
  - Open to HS students or younger
  - The registration fee is $100 per team.
  - Access to all winning papers for $29
- Mathworks Math Modeling (M3) Challenge (formerly Moody's)
  - Teams work during a weekend in March
  - Open to juniors/seniors only
  - No participation fees
  - Monetary prizes
  - Free access to all winning papers

HiMcM 2004 Problem: The Art Gallery

An art gallery exhibit needs to hold 50 paintings. Two security cameras with 30 degree scan beams rotate back and forth at opposite ends of the room. The outer walls and portable walls have particular measurements, and the layout should be aesthetically pleasing. Where should you place the paintings in order to maximize security?



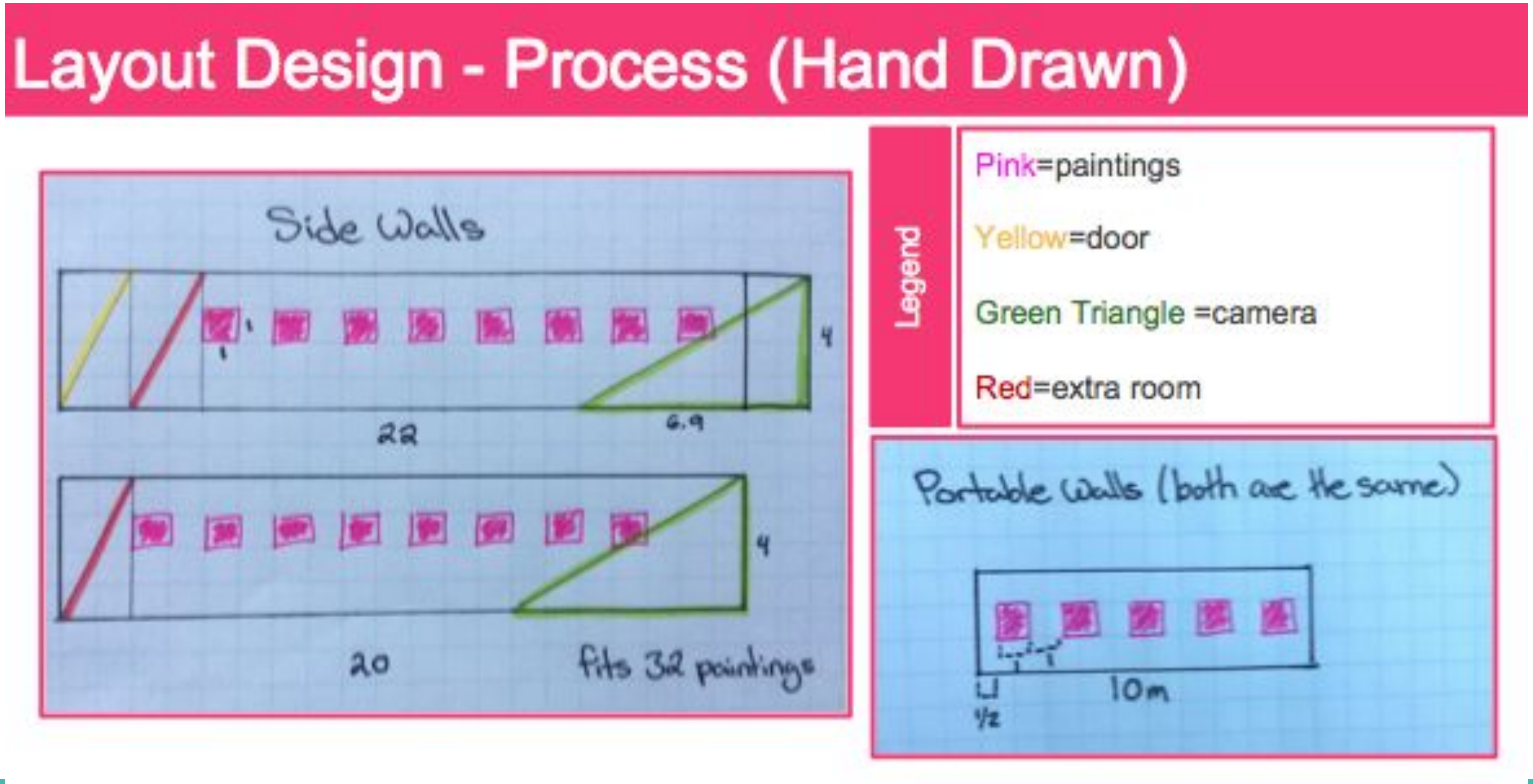


These problems allow for differentiation

- Students use whatever math they are most comfortable with.
- Leads to a variety of solutions, even among students in the same class.
- The next set of student slides all comes from the same modeling class.

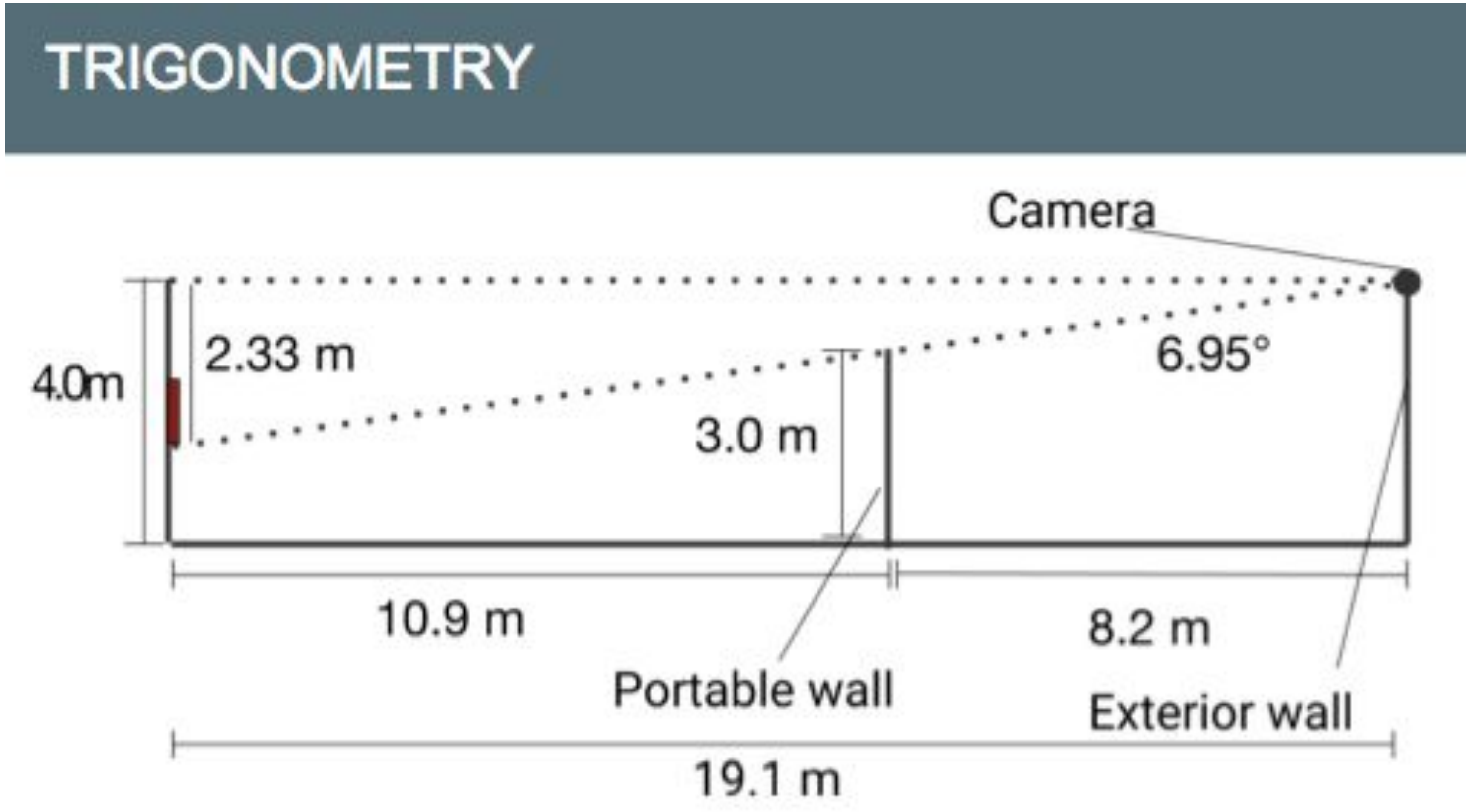
Differentiation – Art Gallery #1

- Low-tech but effective (cardboard box and pencil and paper)



Differentiation – Art Gallery #2

- Another group made such detailed trigonometric diagrams that you could put them in a precalculus textbook!



Differentiation - Art Gallery #3: Calculus

Another group used calculus concepts to quantify security

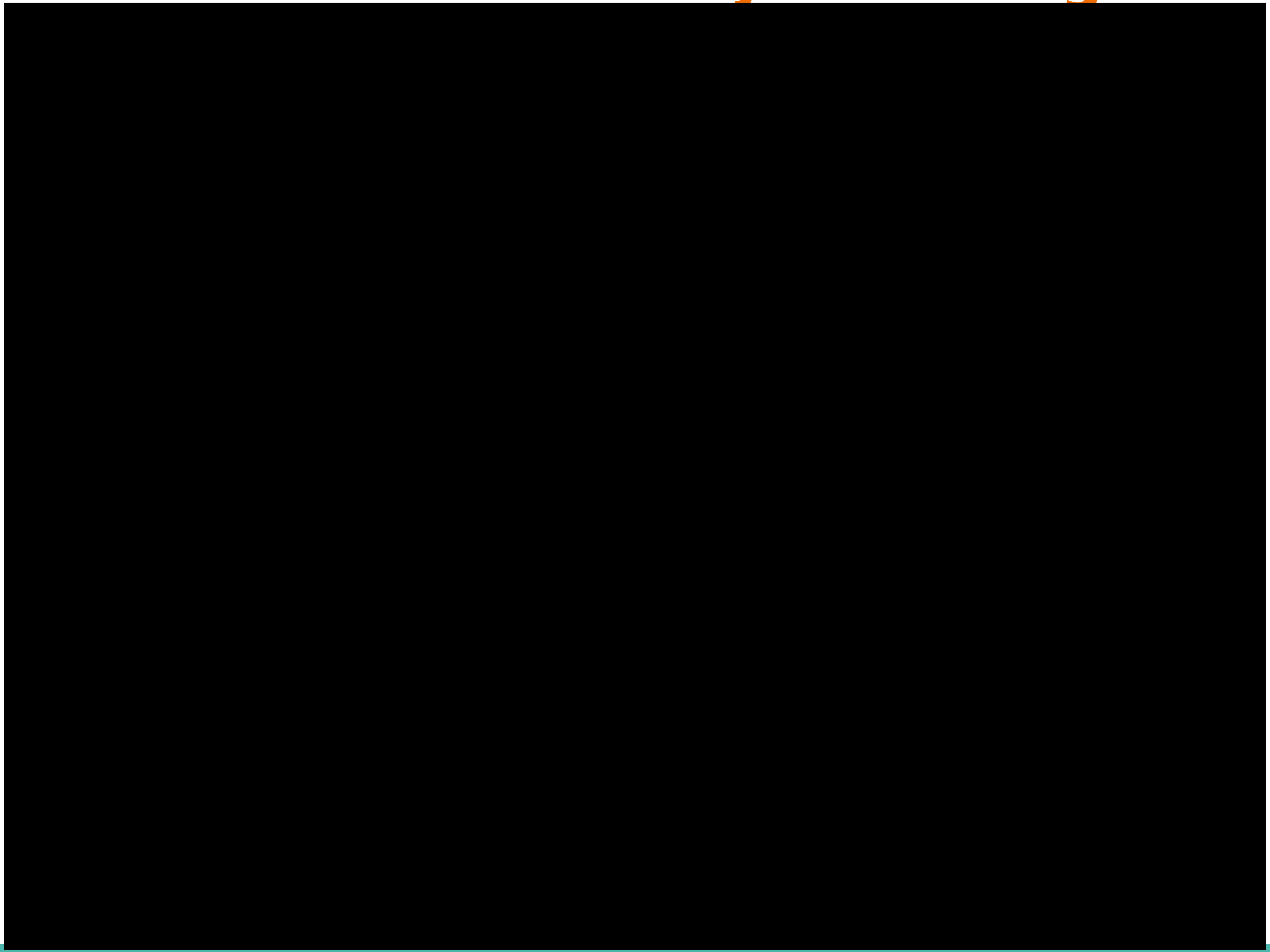
Let R be a given room, and C be a given timing for the cameras

- $V_{R,C}(t)$ is visible painting count at time t given R and C
- Security is therefore

$$S_R = \lim_{t_{max}\to\infty} \max_{C} \frac{1}{t_{max}} \int_0^{t_{max}} V_{R,C}(t)dt$$

- This is the average number of paintings seen per second as time goes to infinity

Differentiation – Art Gallery #4: Geogebra



Differentiation – Art Gallery #5: GSP

Differentiation – Art Gallery #6: Google Sketchup

HiMcM 2014 Problem: – The Next Plague

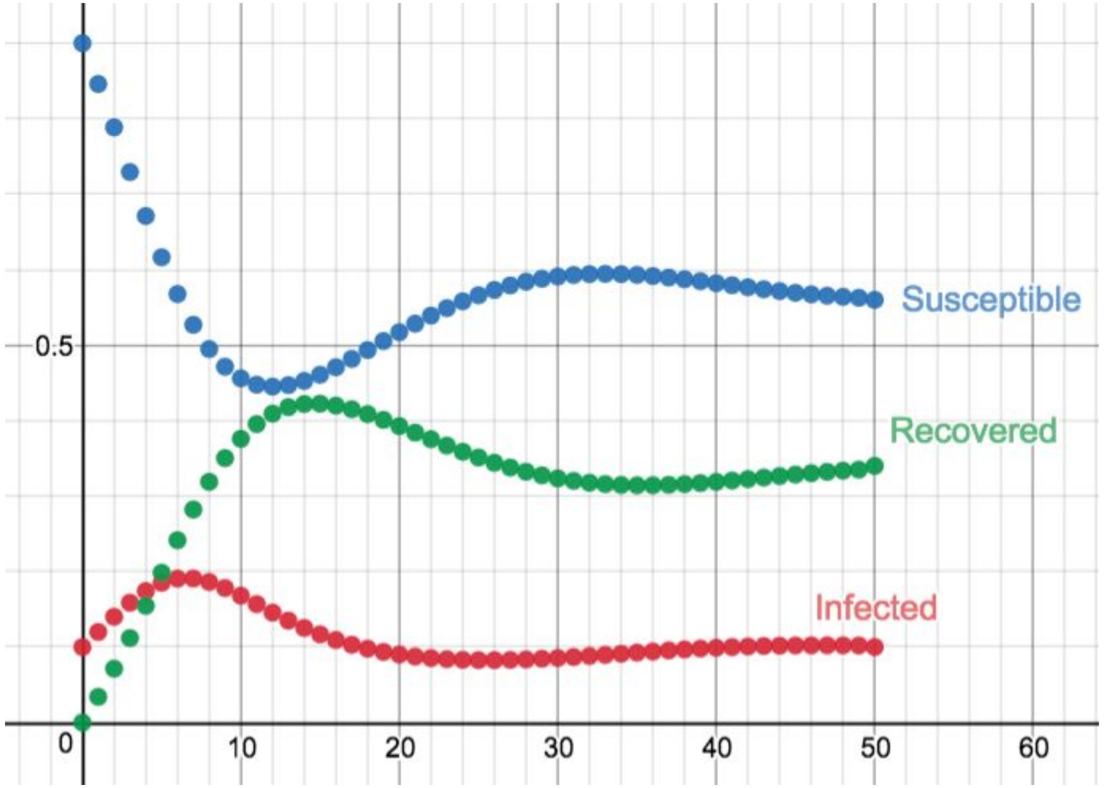
In a small island village almost half of its 300 inhabitants are showing similar symptoms of illness. In the past week, 15 of the “infected” have died. Your modeling team works for the International World Health Organization.

Determine and classify the type and severity of the spread of the disease. What initial recommendations does your team have for your country’s center for disease control?

Technology Options

- Students use whatever mathematical tools they have.
- Simple to advanced use of technology and mathematical content.
- From spreadsheets and Desmos...

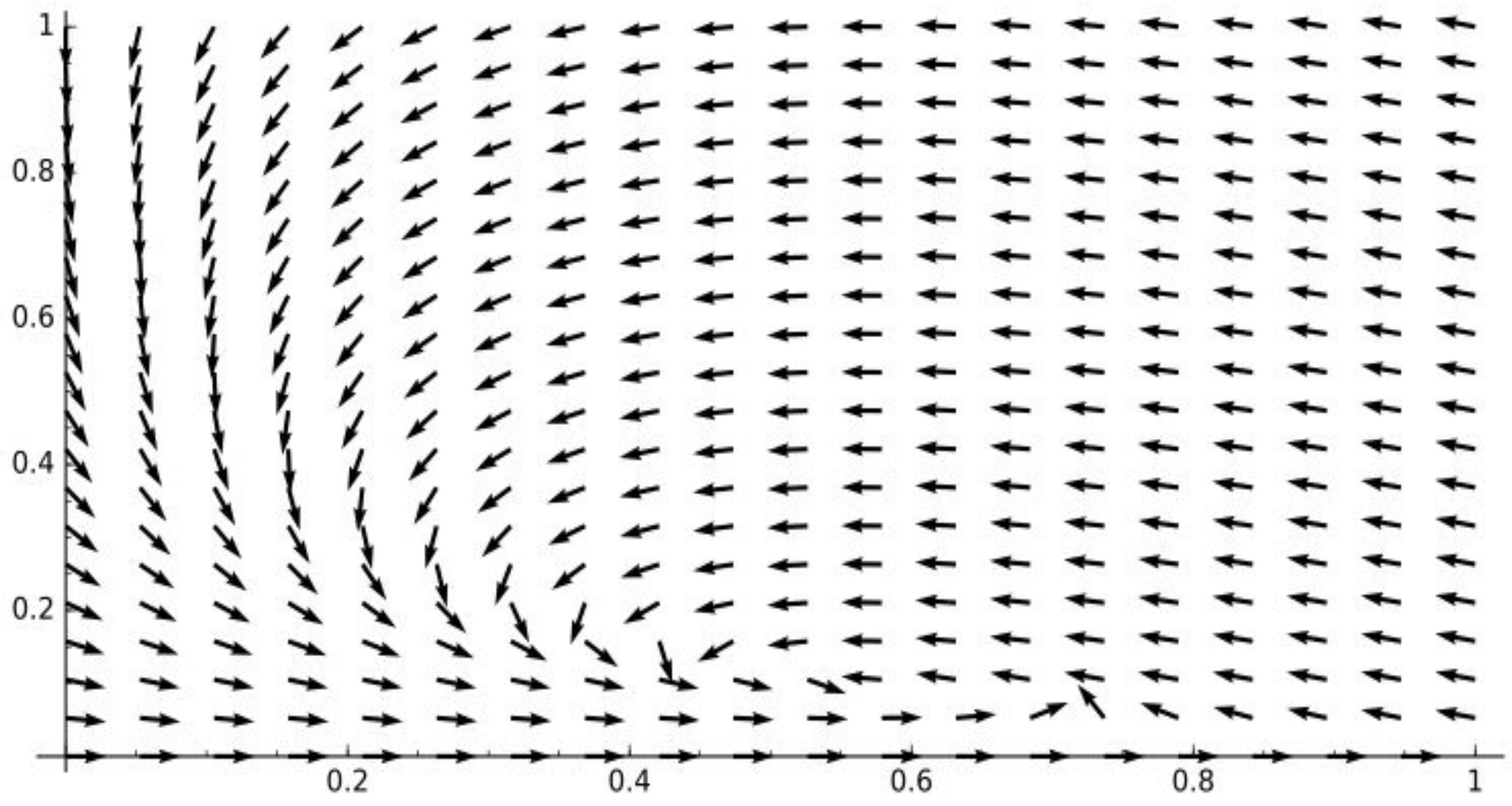
A	B	C	D
time	susceptible	infected	recovered
0	0.9	0.1	0
1	0.846	0.12	0.034
2	0.788488	0.140112	0.0714
3	0.72934202	0.1587599	0.11189808
4	0.67105767	0.17425569	0.15468664
5	0.61636496	0.18517013	0.19846491
6	0.56773203	0.19069171	0.24157626
7	[illegible]	[illegible]	[illegible]



Technology Options

- To using SageMath…

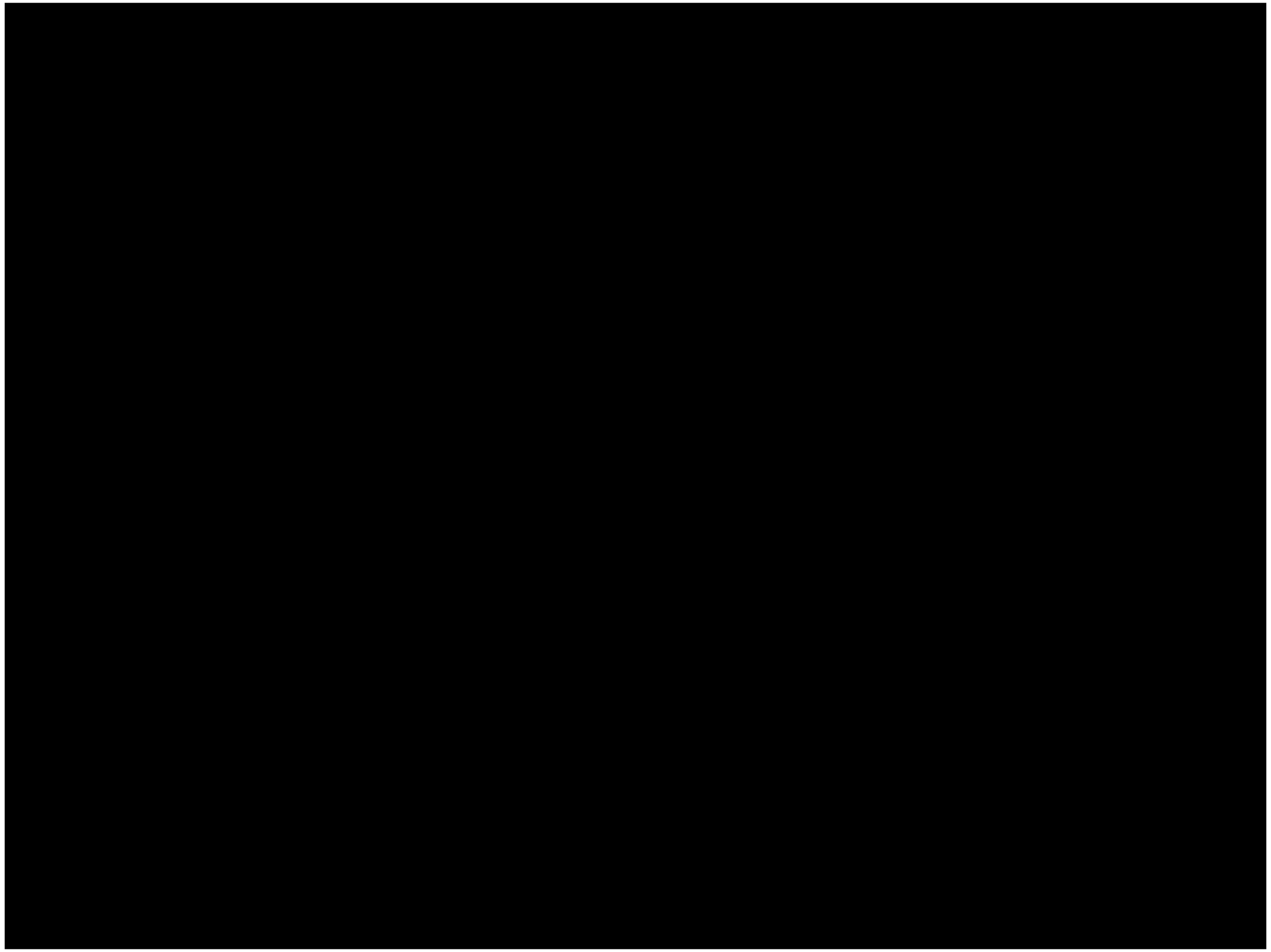
```
x,y=var('x y')
f=-0.6*x*y+0.1*(1-x-y)
g=0.6*x*y-0.34*y
VF=plot_vector_field(  (f/sqrt(f^2+g^2),   g/sqrt(f^2+g^2)),(x,0,1),(y,0,1))
plot(VF)
```



Technology Options

- To using Netlogo…

```
globals[
  k ;factor for determining arrest probability
  threshold ;by how much must illness factors overcome resistance?
]

students-own [
  resistance
  active?
  jail-term ;how many turns at home remain? if 0, student is at school
]

patches-own [ neighborhood ]

to setup
  ;; (for this model to work with NetLogo's new plotting features,
  ;; __clear-all-and-reset-ticks should be replaced with clear-all at
  ;; the beginning of your setup procedure and reset-ticks at the end
  ;; of the procedure.)
  __clear-all-and-reset-ticks

  set k 2.3
  set threshold 0.01

  ask patches[
    set pcolor gray - 1
    set neighborhood patches in-radius cough-range
  ]

  create-teachers round (initial-teacher-density * 0.01 * count patches) [
    move-to one-of patches with [not any? turtles-here]
    display-teacher
  ]

  create-students round (initial-student-density * 0.01 * count patches) [
    move-to one-of patches with [not any? turtles-here]
    set heading 0
    set resistance random-float 1.0
    if (resistance < initial-infections) [set active? true]
    if (resistance >= initial-infections) [set active? false]
    set jail-term 0
    display-student
  ]
  my-update-plots
end
```



HiMcM 2012 Problem: How much gas?

Gas prices fluctuate significantly from week to week. Consumers would like to know whether to fill up the tank (gas price is likely to go up in the coming week) or buy a half tank (gas price is likely to go down in the coming week).

Or related to this problem...

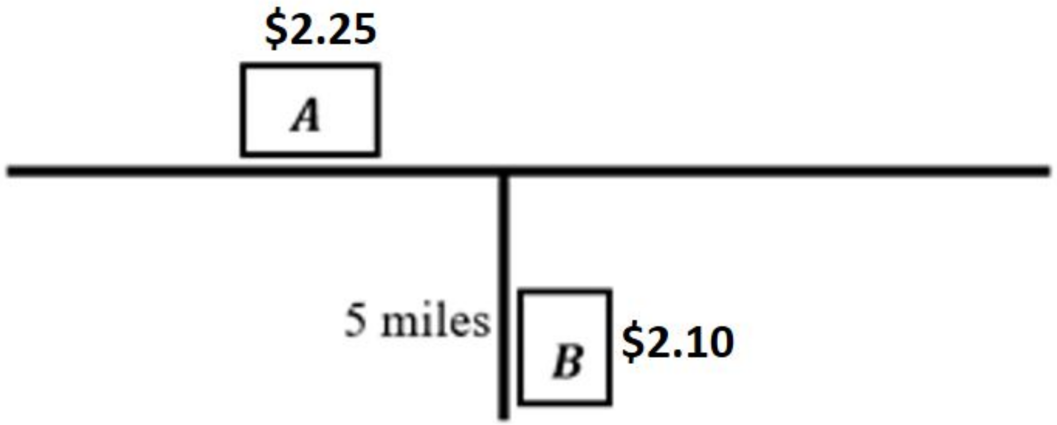
Example - Driving for Gas

A friend tells you she buys her gas at a station several miles off your normal route where the prices are cheaper. Would it be more economical for you to drive the extra distance for the less expensive gas than to purchase gas along your route?

kcd.com

Ask an easier question first or create a specific example

Station A is on your normal route and sells gas for $2.25 a gallon, while Station B is 5 miles off your route sells gas for $2.10 a gallon. Your car gets 32 miles per gallon, and your friend's car only gets 15 miles per gallon. Should either of you travel the extra distance to buy gas at Station B?



Then build up to a more general solution

- Did you oversimplify? Add more complexity step by step.

Cost of a tank of gas: $T \cdot P$

Number of useable gallons: $\left(T - \frac{2D}{M}\right)$

Cost per useable gallon: $\frac{T \cdot P}{\left(T - \frac{2D}{M}\right)}$

Our cost index:

$$I = \frac{T \cdot P}{\left(T - \frac{2D}{M}\right)} = \frac{M \cdot T \cdot P}{M \cdot T - 2D}$$

We should buy gas at the distant station if $I < P^*$.

Today's Problem: HiMcM 2013

The bank manager is trying to improve customer satisfaction by offering better service. Management wants the average customer to wait less than 2 minutes for service and the average length of the queue (length of the waiting line) to be 2 persons or fewer. The bank estimates it serves about 150 customers per day. The existing arrival and service times are given in the tables below.

Time between arrival (min.)	Probability
0	0.10
1	0.15
2	0.10
3	0.35
4	0.25
5	0.05

Table 1: Arrival times

Service Time (min.)	Probability
1	0.25
2	0.20
3	0.40
4	0.15

Table 2: Service times

(1) Build a mathematical model of the system.

(2) Determine if the current customer service is satisfactory according to the manager guidelines. If not, determine, through modeling, the minimal changes for servers required to accomplish the manager's goal.