No extensions will be available for this assignment due to the late due date.

Instructions for handing in. Please...

1. Assignments may be typed in a word processor of your choice, or handwritten neatly.
2. Set answers out in order of the questions. Do NOT jump between questions.
3. There is no need to copy the questions out in your submission.
4. If answering a question using R, all commands that you enter should be provided as part of the answer, along with all relevant output. Include your ID number on output wherever possible, e.g. as part of the title of a graph.

Submission: Assignments must be submitted using Canvas as a single PDF file, before the due date and time. Handwritten assignments will need to be scanned. Prepare your assignments well in advance of the deadline in case of technical issues.

Notes:

- Summarising, analysing and communicating information is an important part of Operations Research. For this reason you will be expected to write answers which clearly communicate your thoughts. The mark you receive will be based on your written English as well as your technical work – review the relevant section in the Course Outline.
- **We encourage working together.** Discussing assignments and methods of solution with other students or getting help in understanding from staff and students is acceptable and encouraged. You must **write up your final assignment individually, in your own words.**
- By submitting this assignment, you confirm that you understand the University's policies on cheating, plagiarism and group work; that your submission is entirely your own work and you have not allowed access to any part of the assignment to any other person. See the appropriate sections in the Course Outline for more details.
- This assignment consists of **THREE** questions, and is marked out of **50 marks**.

---

1. (17 marks) An analyst for Black Moon Ltd, an airline, is doing some simulation models of the main risks faced by the company. One of the more important unknowns is the future price of jet fuel. The analyst decides to model the average jet fuel price J over the next financial year, in dollars per barrel, as a random variable with the pdf

$$f(x) = \begin{cases} \frac{x}{7200} & , 0 \le x < 60 \\ \frac{1}{120} & , 60 \le x < 100 \\ \frac{1}{120} e^{(100-x)/50} & , 100 \le x \end{cases}$$

(a) Devise a procedure for generating random variates according to the pdf f.

(b) Use your procedure to generate 100000 random variates according to the distribution, and draw a histogram with the vertical axis scaled so as to give an estimate of the pdf. (*Hint.* R has a function called `ifelse` that may be useful.)

(c) Find the exact expected values of *(i)* J; *(ii)* $\max(J, 50)$.

(d) Use a simulation to estimate the above expected values. Give the estimates in the form of 95% confidence intervals.

2. (17 marks) Black Moon is known for carefully adjusting the prices of its tickets in the final days and hours before a flight departs. One of its favourite tricks is to increase the asking price of a ticket by 10% after each ticket sale. For example, if the fourth-to-last seat on a flight is sold for \$100, then the third-to-last will be priced at \$110, the second-to-last at \$121, and the last remaining seat at \$133.10.

Suppose this strategy is adopted for a flight departing in exactly 120 hours' time. At the beginning of this period there are 13 unsold seats with an advertised price of \$250 (although in accordance with the pricing strategy, only the first of these will actually be sold at that price). Potential buyers are assumed to arrive as an inhomogeneous Poisson process with rate $\lambda(t) = \frac{16}{140-t}$, for $0 \le t \le 120$. The amounts that the buyers are willing to spend are distributed uniformly between \$150 and \$800, and independent of each other and of the arrival times.

(a) What is the expected number of *potential* buyers of these seats?

(b) Devise a method for simulating the potential buyers. Use your method to generate one realization, and list that realization's potential buyers in order of their arrival, including their arrival times and the amounts they are willing to spend. Use the last four digits of your student ID as the random seed.

(c) Perform a simulation with 5000 realizations to estimate how many of the 13 seats will be sold. Make a histogram of this quantity, and give a 95% confidence interval for its expectation. You should use the `sim.revenue` function from `simulation.R.txt`.

(d) Black Moon is experimenting with an alternative strategy in which the ticket price is increased by 10% every 10 hours, regardless of how the ticket sales are going. Re-do the histogram and confidence interval for this case.

3. (16 marks) The Black Moon customer service counter at Auckland Airport is typically staffed by one person. We model the arrival of customers at the counter as a Poisson process of rate 0.3 per minute, and the times taken to serve customers as independent exponentially-distributed random variables with rate 0.5 per minute. If necessary, customers form a first-come-first-served queue.

(a) What is the distribution of the number of customers present when this system is operating in steady state? What is the mean number of customers present? For what fraction of the time are there no customers present?

(b) Use the queueing-simulation web page to perform a simulation of the customer-service counter. Make estimates, with 95% confidence intervals, of *(i)* the mean number of customers present; *(ii)* the fraction of the time for which there are no customers present.

(c) An alternative model of the counter, meant to reflect particularly busy periods, has a customer arrival rate of 1.1 per minute and models the customer service times using a lognormal distribution instead of an exponential distribution (though the mean and standard deviation remain the same). With this model, it is clearly not possible for a single employee to operate the counter. How many staff are needed, and what will happen if there are fewer than that?

(d) Repeat the simulation of part (b) for the alternative model, using the minimum possible number of staff and assuming that waiting customers form themselves into a single queue. What now is the mean number of customers present?