Fits for Dark Matter Direct Detection

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1 Introduction

The existence of dark matter (DM) is strongly evident as an explanation for various astrophysical observations, such as the flatness of galactic rotation curves as well as the early formation of galaxies [1]. While we have yet to discover DM, there have been numerous searches in the form of direct detection and indirect detection. Direct detection experiments analyze the scattering process of DM with nuclei by measuring the recoil energy of the nuclei.

One example of such an experiment is Xenon100 in which recoil energies of liquid Xenon isotopes are measured. One can describe the scattering process between DM and a Xenon isotope with an effective field theory (EFT) in which the Lagrangian potentially depends on various operators representing different couplings. These couplings are associated with unknown parameters called Wilson coefficients (WC). One can constrain these coefficients and subsequently constrain the EFT used by comparing experimental data to what is expected.

The python code that I wrote consists of various functions relevant for the *kinematics* involved in direct detection scattering processes. I would like to put emphasis on the fact that the user must supply their own differential scattering cross-section or use the one that I hardcoded into the project files which represents the simplified model explained in Sec. 2 as it pertains to the Xenon100 experiment. The differential cross-section in the code was obtained using the Mathematica packages DirectDM [4] and DMFormfactor [2], [3].

2 Procedure

One way that the experimental data from Xenon100 can be compared with theory is by minimizing a chi-squared function that relates observed and theoretical values. For my example, I chose to compare the number of observed scattering events to that derived from the EFT used. To greatly simplify matters, I took the Lagrangian to only depend on one WC, namely $C_{1,u}^{(6)}$. I then minimized the chi-squared function to get the value for $C_{1,u}^{(6)}$ that gives a best fit to the data. Ultimately, I wanted to create a function that gave the best fit value for $C_{1,u}^{(6)}$, but I thought it instructive to make seperate functions for many of the operations involved along with functions for plots.

There are two main pipelines that one can follow to obtain best-fit values for Wilson coefficents. One main pipeline is hardcoded for the model explained above as it pertains to the Xenon100 data and many of the relevant functions only require the recoil energy and DM mass as an input. The other pipeline consists of the same functions, with the exception of plot functions, generalized so that the user has more freedom in terms of models and experiments to be analyzed. Both of these pipelines have their disadvantages; see Sec. 4 for more details.

In order to get the number of predicted events I took the following steps:

1. From the differential cross-section per recoil energy found in Mathematica I obtained the differential rate per recoil energy.

For this step, one must assume a DM halo velocity distribution. For this example, I assume a Maxwell-Boltzmann velocity distribution,

$$f_E(v)d^3v = \sqrt{\frac{2}{\pi}}\frac{v^2}{v_0^3}\exp^{-\frac{v^2}{2v_0^2}}dv \tag{2.1}$$

where v_0 is a constant that is hardcoded in the project files. Following the derivations in [5], the differential rate per recoil energy is,

$$\frac{dR}{dE_R} = \sum_{i=isotope} N_{T,i}\frac{dR_{T,i}}{dE_R} = N_T n_\chi \sum_i n_i \int_{v>v_{min,i}} \left(\frac{d\sigma(v,E_R)}{dE_R}\right)_i v f_E(\vec{v})d^3v \tag{2.2}$$

where N_T is the number of target nuclei and n_χ is the local dark matter number density. n_i is the relative abundance, $(d\sigma(v,E_R)/dE_R)_i$ the differential cross section per recoil energy, and $v_{min,i}$ the minimum velocity of the i-th isotope,

$$v_{min,i} = \frac{1}{\mu_{T,i}}\sqrt{\frac{m_{T,i}E_R}{2}} \tag{2.3}$$

where $m_{T,i}$ is the target nuclei mass and $\mu_{T,i}$ is the reduced mass of the combined nuclei and dark matter system. I then integrated eq. (2.2) using scipy.integrate.quad.

2. From the differential rate per recoil energy I obtained the total number of scattering events.
For this step, one must assume a detector efficiency for given recoil energies. For a detector efficiency, $G_i(E)$, in some energy band, $[E_{min,i}, E_{max,i}]$, the total rate per detector for the i-th energy band is found as,

$$R_i = \int \frac{dR}{dE_R}G_i'(E)dE_R \tag{2.4}$$

For this example, I took the detector effieciency data from [5]. The total number of scattering events for the i-th energy band is then,

$$N_i = R_i * t_{\mathrm{exp}} \tag{2.5}$$

where t_{exp} is the relevant exposure time.

3. From the Xenon100 data and the number of predicted events I obtained $C_{1,u}^{(6)}$ that minimizes the chi-squared function.
For this step, one must input the relevant experimental data or use the data coded for Xenon100 taken from [5]. Given the number of observed events for n energy bins and the corresponding error, $\vec{N}_0$ and $\delta\vec{N}_0$, the number of observed events for a given theory, $\vec{N}(\hat{C})$, and the number of background events, $\vec{N}_b$, the chi-squared function is found as,

$$\chi^2(\hat{C}) = \sum_{i=1}^{n}\frac{[N_{0,i} - (N_{b,i} + N_i(\hat{C}))]^2}{\delta N_{0,i}^2} \tag{2.6}$$

I then minimized eq. (2.6) using scipy.optimize.minimize to find $C_{1,u}^{(6)}$ that gave a best fit to the data.

3 Results

There are very many results that one can obtain with this code. The user is able to obtain separate calculations for most of the relevant functions for finding the minimum chi-squared, as well as for plotting many of these functions. Some example plots obtained for the Xenon100 experiment are shown in Figs. 1 and 2. From the Xenon100 pipeline, I found the value of $C_{1,u}^{(6)}$ that gave the best fit to the data to be $C_{1,u\,\mathrm{best}}^{(6)} = 7.9 * 10^{-9}$ with a chi-squared per d.o.f. of $\chi^2_{\mathrm{min}}/\mathrm{d.o.f} = 41.2$.

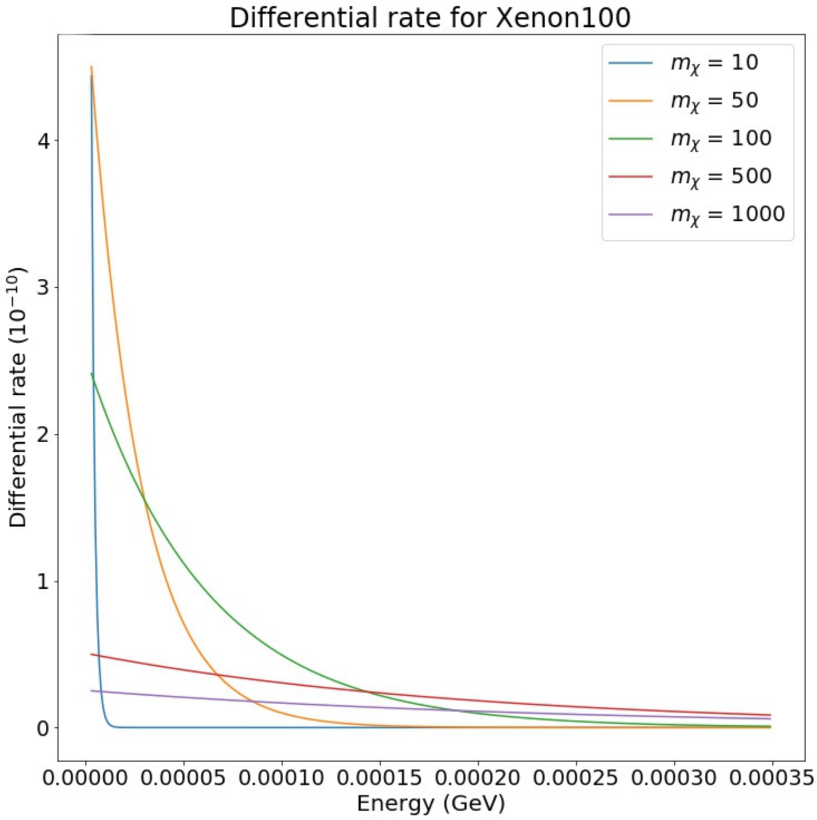

Figure 1: Differential rate per recoil energy for various DM masses - Xenon100.

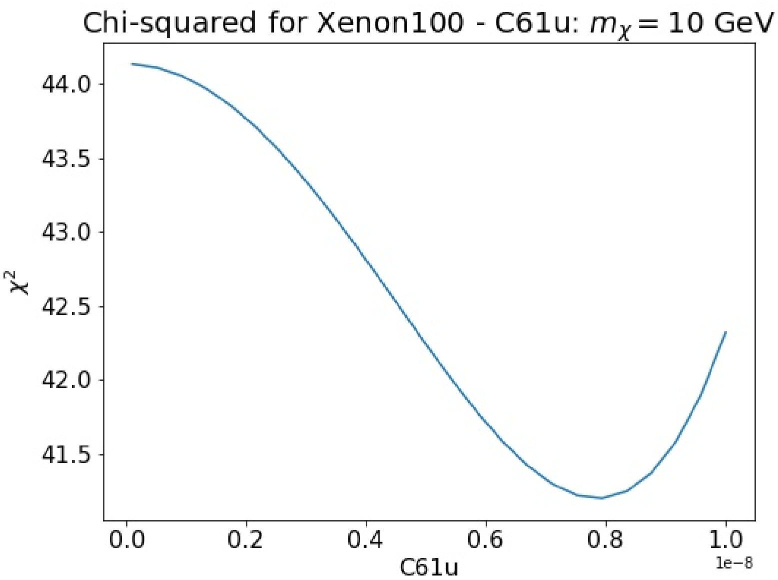

Figure 2: Chi-squared function for $m_\chi = 10$ GeV - Xenon100.

4 Future

This code could be significantly improved. First, some of the functions hardcoded for the Xenon100 data tend to be very slow. Employing Numba seemed to help with speed a bit, however, there is a lot of loop usage that could undoubtedly be coded more efficiently. Second, while the general pipeline tends to be much faster because of the lack of loops, the relevant functions require very many input parameters. This could partly be fixed by making some of the parameters variable and coding defaults. Third, there are numerous DM halo velocity distributions that have been found from simulations which could be added to the project files. Finally, more Wilson coefficients can be added to the fit for the Xenon100 experiment.

5 Conclusion

Using my python code, I obtained a best-fit value for the Wilson coefficient $C_{1,u}^{(6)}$ to be $C_{1,u\,\mathrm{best}}^{(6)} = 7.9*10^{-9}$ with a chi-squared per d.o.f. of $\chi^2_{\mathrm{min}}/\mathrm{d.o.f} = 41.2$. This result is relevant for the Xenon100 experiment assuming a Lagrangian which only depends on this Wilson coefficient. While the code was written mainly to obtain the previous results, one can also obtain the relevant functions in a stand-alone manner for general models and experiments or for the specific example outlined in this paper. In finding the best fit values for the Wilson coefficients specific to a given model, one can constrain the EFT used and potentially better understand the nature of DM.

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References

[1] M. Tanabashi et al. (Particle Data Group), Phys. Rev. D. **98**, 030001 (2018).

[2] A.L. Fitzpatrick, W. Haxton, E. Katz, N. Lubbers, and Y. Xu (2012) e-print: 1203.3542.

[3] N. Anand, A. L. Fitzpatrick, and W.C. Haxton (2013) e-print: 1308.6288.

[4] F. Bishara, J. Brod, B. Grinstein, and J. Zupan (2017) e-print: 1708.02678.

[5] XENON collaboration and B. Farmer (2017) e-print: 1705.02614.

[6] M. Maltoni and T. Schwetz (2013) e-print: 0304176.