

Example: Spring Potential

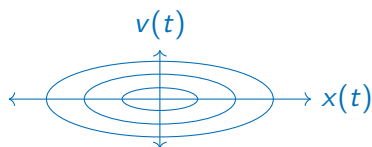
Consider a mass m at position $x(t)$ moving with speed $v(t)$ while attached to a spring with zero rest length and stiffness k :

$$\Phi(x, v) = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

Assuming no forcing or friction, energy is conserved

$$\Phi(x, v) = E_0, \quad E_0 = \text{initial energy}$$

producing motion tracing out an ellipse in (x, v) -space:



All these ellipses satisfy a DE:

$$kx + mv \frac{dv}{dx} = 0$$