

ex:  $u_t = \alpha u_{xx}$  for  $0 < x < L$  with  $u(x, 0) = \sin(5x/L)$   
 $u(0, t) = u(L, t) = 0$

From the BCs,

$$u(x, t) = \sum_{n=1}^{\infty} e^{-\alpha \frac{n^2 \pi^2}{L^2} t} b_n \sin\left(\frac{n\pi}{L} x\right)$$

$$\begin{aligned} b_n &= \frac{2}{L} \int_0^L \sin(5x/L) \sin\left(\frac{n\pi}{L} x\right) dx \\ &= \begin{cases} 1 & \text{if } n = 5 \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

$$u(x, t) = e^{-\alpha \frac{25\pi^2}{L^2} t} \sin\left(\frac{5\pi}{L} x\right)$$