$u_t = \alpha u_{xx}$ for $x \in [0, L]$ with $u(0,t) = u(L,t) = 0$ <u>or</u> $u_x(0,t) = u_x(L,t) = 0$ and $u(x,0) = u_0(x)$

$$\boxed{u(x,t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} e^{-\alpha \frac{n^2\pi^2}{L^2} t}\left(a_n \cos\left(\frac{n\pi}{L}x\right) + b_n \sin\left(\frac{n\pi}{L}x\right)\right)}$$

$\underline{u(0,t) = u(L,t) = 0}$:

Dirichlet Boundary Conditions

$\underline{u_x(0,t) = u_x(L,t) = 0}$:

Neumann Boundary Conditions

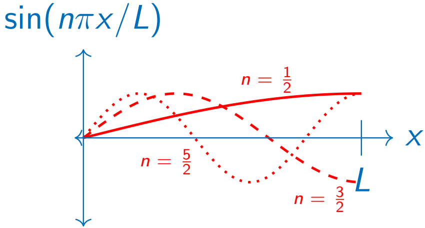
What can we do if we have mixed Neumann and Dirichlet Boundary Conditions?

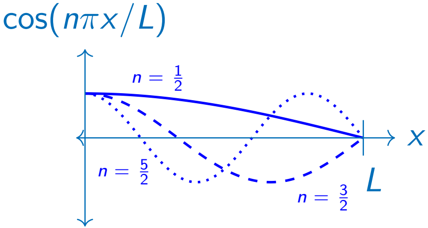
<u>ex</u>: $u(0,t) = u_x(L,t) = 0$ or $u_x(0,t) = u(L,t) = 0$

A slightly different Fourier Basis: $n =$ half integers

$u(0,t) = u_x(L,t) = 0$

$u_x(0,t) = u(L,t) = 0$





Using Sep. of Variables, we can construct solutions of the form

$$u(x,t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} e^{-\alpha \frac{(n-\frac{1}{2})^2\pi^2}{L^2} t} \left(a_n \cos\left(\frac{(n-\frac{1}{2})\pi}{L} x \right) + b_n \sin\left(\frac{(n-\frac{1}{2})\pi}{L} x \right) \right)$$

Mixed Homogeneous BCs

$u_t = \alpha u_{xx}$ for $x \in [0, L]$ with $u(0,t) = u_x(L,t) = 0$ or $u_x(0,t) = u(L,t) = 0$ and $u(x,0) = u_0(x)$

$$\boxed{u(x,t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} e^{-\alpha \frac{(n-\frac{1}{2})^2\pi^2}{L^2} t} \left(a_n \cos\left(\frac{(n-\frac{1}{2})\pi}{L} x\right) + b_n \sin\left(\frac{(n-\frac{1}{2})\pi}{L} x\right)\right)}$$

$\underline{u(0,t) = u_x(L,t) = 0}$:

Fourier sine series

$$a_n = 0$$

$$b_n = \frac{2}{L}\int_0^L u_0(x) \sin\left(\frac{(n-\frac{1}{2})\pi}{L} x\right) dx$$

$\underline{u_x(0,t) = u(L,t) = 0}$:

Fourier cosine series

$$a_n = \frac{2}{L}\int_0^L u_0(x) \cos\left(\frac{(n-\frac{1}{2})\pi}{L} x\right) dx$$

$$b_n = 0$$