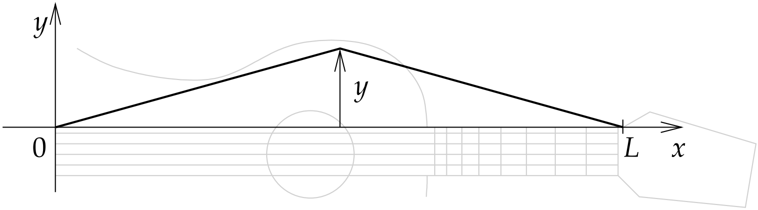
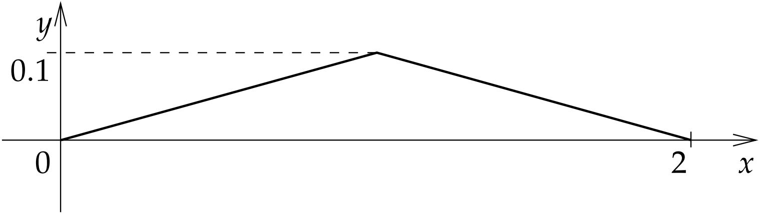
ex: Imagine a guitar string length of 2 pulled 0.1 units away from its rest position.





ex: $y_{tt} = y_{xx} \quad 0 < x < 2 \qquad y(0) = y(2) = 0,$

$$y(x,0) = \begin{cases} 0.1x & 0 < x < 1 \\ 0.1(2-x) & 1 \leq x < 2 \end{cases} \qquad y_t(x,0) = 0$$

$$y(x,t) = \sum_{n=1}^{\infty} c_n \cos\left(\frac{n\pi}{L} t\right) \sin\left(\frac{n\pi}{2} x\right)$$

c_n is obtained from an odd periodic extension of the initial condition

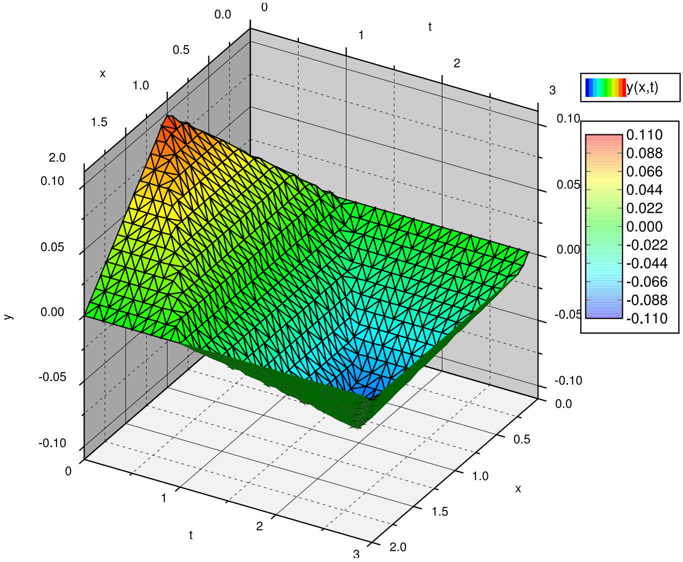
$$c_n = \frac{2}{2}\left[\int_0^1 0.1x \sin\left(\frac{n\pi}{2}x\right) dx + \int_1^2 0.1(2-x)\sin\left(\frac{n\pi}{2}x\right) dx\right]$$

$$= 0.1\left[\frac{4\sin\left(\frac{n\pi}{2}\right) - 2n\pi\cos\left(\frac{n\pi}{2}\right)}{\pi^2 n^2} + \frac{4\sin\left(\frac{n\pi}{2}\right) + 2n\pi\cos\left(\frac{n\pi}{2}\right)}{\pi^2 n^2}\right]$$

$$= \frac{0.8}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right)$$

1. https://www.wolframalpha.com/input?key=&i=integral+of+x*sin%28n*pi*x%2F2%29+from+0+to+1+assuming+n+is+an+integer
2. https://www.wolframalpha.com/input?key=&i=integral+of+%282-x%29sin%28n*pi*x%2F2%29+from+1+to+2+assuming+n+is+an+integerr

$$y(x,t)=\sum_{n=1}^{\infty}\frac{0.8}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right)\cos\left(\frac{n\pi}{L}t\right)\sin\left(\frac{n\pi}{2}x\right)$$



ex: $y_{tt} = 16 y_{xx} \quad 0 < x < 1$ $\qquad y(0) = y(1) = 0,$

$y(x,0) = 0$ $\qquad y_t(x,0) = \sin(5\pi x)$

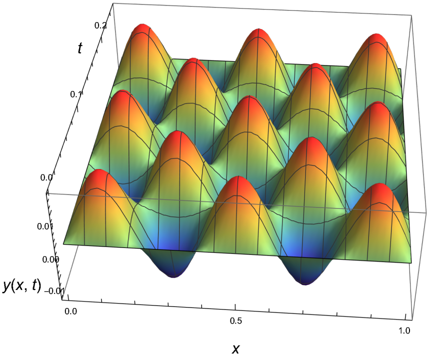
$$y(x,t) = \sum_{n=1}^{\infty} b_n \sin(n\pi x) \frac{1}{4n\pi} \sin(4n\pi t)$$

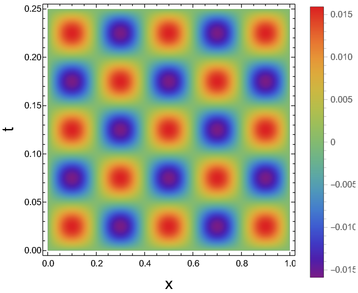
b_n is obtained from an odd periodic extension of the initial condition

$$b_n = \begin{cases} 1 & n = 5 \\ 0 & \text{otherwise} \end{cases}$$

$$y(x,t) = \frac{1}{20\pi} \sin(5\pi x) \sin(20\pi t)$$

$$y(x,t) = \frac{1}{20\pi}\sin(5\pi x)\sin(20\pi t)$$





Standing wave with 5 peaks/troughs and period of 1/20 time units.
Some positions have constant zero displacement (nodes).

Vibrational Modes - vibrations in space and time

Standing waves with a temporal period and spatial period (wavelength)

n^{th} vibrational mode:

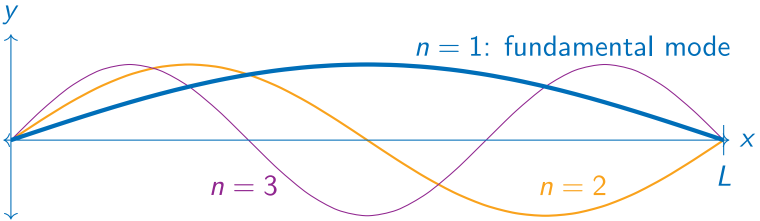
Temporal Period

$$T_n = \frac{2L}{cn}$$

Wavelength

$$\lambda_n = \frac{2L}{n}$$

Each mode has $n+1$ nodes and n antinodes (peaks/troughs).



https://www.youtube.com/watch?v=PVX4V5Adbzk

Physical manipulation of frequencies

$$c^2 = \frac{\text{Tension}}{\text{mass density}}$$

$$f_n = \frac{1}{T_n} = \frac{cn}{2L}$$

$$f_n \propto \sqrt{\text{Tension}}$$
$$f_n \propto \frac{1}{\text{Length}}$$

Tigthen a guitar string - frequency goes up (pitch gets higher)
Hold the string against the neck - frequency goes up (pitch gets higher)