

Doc: continuous models

Laurent

$$m(\mathbf{x}) \frac{\partial^2}{\partial t^2} \theta(\mathbf{x}, t) + d(\mathbf{x}) \frac{\partial}{\partial t} \theta(\mathbf{x}, t) = P(\mathbf{x}, t) + \nabla \cdot [\mathbf{b}(\mathbf{x}) \circ \nabla \theta(\mathbf{x}, t)], \quad (1)$$

Discretizing in time and space, updated state is computed as

$$\begin{aligned} \theta_{i,j}(t + \Delta t) = & \left[2\chi_{ij} - \frac{\Delta t^2 m_{ij}^{-1} \chi_{ij}}{\Delta x^2} \left(b_{i,j|i+1,j} + b_{i-1,j|i,j} + b_{i,j|i,j+1} + b_{i,j-1|i,j} \right) \right] \theta_{i,j}(t) \\ & - \xi_{ij} \theta_{i,j}(t - \Delta t) + \frac{\Delta t^2 \chi_{ij} m_{ij}^{-1}}{\Delta x^2} \left[b_{i-1,j|i,j} \theta_{i-1,j}(t) + b_{i,j|i+1,j} \theta_{i+1,j}(t) \right. \\ & \left. + b_{i,j-1|i,j} \theta_{i,j-1}(t) + b_{i,j|i,j+1} \theta_{i,j+1}(t) \right] + \Delta t^2 \chi_{ij} m_{ij}^{-1} P_{i,j}, \end{aligned} \quad (2)$$

where $\chi_{i,j} = \left[1 + \frac{\gamma_{i,j} \Delta t}{2} \right]^{-1}$, $\xi_{i,j} = \left[1 - \frac{\gamma_{i,j} \Delta t}{2} \right] \chi_{ij}$, with $\gamma_{i,j} = d_{i,j}/m_{i,j}$.

The previous equation follows from

$$\nabla \cdot [\mathbf{b}(\mathbf{x}) \circ \nabla \theta(\mathbf{x}, t)] = \partial_x B_x \partial_x \theta + B_x \partial_x^2 \theta + \partial_y B_y \partial_y \theta + B_y \partial_y^2 \theta, \quad (3)$$

where

$$B_x \approx \frac{b_{i,j|i+1,j} + b_{i-1,j|i,j}}{2}, \quad (4)$$

$$\partial_x B_x \approx \frac{b_{i,j|i+1,j} - b_{i-1,j|i,j}}{\Delta x}, \quad (5)$$

$$\partial_x \theta \approx \frac{\theta_{i+1,j} - \theta_{i-1,j}}{2\Delta x}, \quad (6)$$

$$\partial_x^2 \theta \approx \frac{\theta_{i-1,j} - 2\theta_{i,j} + \theta_{i+1,j}}{\Delta x^2}, \quad (7)$$

So, finally

$$\partial_x B_x \partial_x \theta + B_x \partial_x^2 \theta \approx \quad (8)$$

$$\frac{b_{i-1,j|i,j} \theta_{i-1,j} - (b_{i,j|i+1,j} + b_{i-1,j|i,j}) \theta_{i,j} + b_{i,j|i+1,j} \theta_{i+1,j}}{\Delta x^2}, \quad (9)$$

Steady state is obtained iteratively as

$$\theta_{i,j}^{(n+1)} = \frac{\Delta x^2 P_{i,j} + b_{i-1,j|i,j} \theta_{i-1,j}^{(n)} + b_{i,j|i+1,j} \theta_{i+1,j}^{(n)} + b_{i,j-1|i,j} \theta_{i,j-1}^{(n)} + b_{i,j|i,j+1} \theta_{i,j+1}^{(n)}}{b_{i,j|i+1,j} + b_{i-1,j|i,j} + b_{i,j|i,j+1} + b_{i,j-1|i,j}} \quad (10)$$

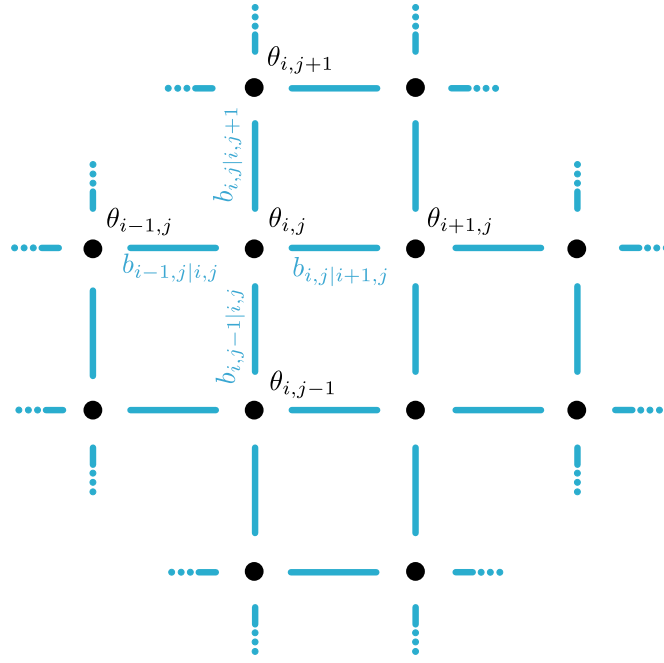


FIG. 1. Lattice with the "easy to read for a human" labels

B_x
B_y

11	11	12	12	13	13	14	14	15	15		
	11	12	12	13	13	14	14	15	15		
21	21	22	22	23	23	24	24	25	25		
	21	22	22	23	23	24	24	25	25		
31	31	32	32	33	33	34	34	35	35		
	31	32	32	33	33	34	34	35	35		
41	41	42	42	43	43	44	44	45	45		
	41	42	42	43	43	44	44	45	45		
	51	52	52	53	53	54	54	55	55		
	61	62	62	63	63	64	64	65	65		
	71	72	72	73	73	74	74	75	75		

FIG. 2. How it is implemented.