

Optimization Project in Energy PGE 306

Laurent Pfeiffer

Inria and Ecole Polytechnique, Institut Polytechnique de Paris

Ensta-Paris
Institut Polytechnique de Paris
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1 Introduction

2 Deterministic model

Introduction

- **Main goal:** programming numerical methods (Energy Management System) for a very simple model of a microgrid.
- **Microgrid:** a set consisting of the following elements:
 - a small-size **electrical load** (a building, a few houses)
 - a source of **renewable energy** (solar panels, a wind turbine)
 - a **storage** device (Energy Storage System, a battery)
 - a **macrogrid** (an access to a large-scale energy network).

Introduction

- **Time scale:** the decisions are taken **every hour** during the day.
- **Optimization variables** (at each time step) :
 - the amount of energy to be **stored** or **withdrawn** from the battery
 - the amount of energy **bought** or **sold** to the network.
- **Constraints:**
 - nonnegativity of variables
 - evolution of the battery
 - storage capacity of the battery.
- **Cost function:**
 - Cost of the energy bought on the network...
 - minus the cost of the energy sold on the network.

Introduction

■ Main challenge:

- The electrical load and the production of renewable energy are **random**.
- No available probabilistic model, instead the history of electrical load and solar production.

■ Tools and mathematical concepts:

- Dynamic programming (temporal aspect for decision process).
- Autoregressive processes.

■ Optimization is useful for...

- minimizing the management costs
- modelling random processes
- describing some functions, for which no analytical expression is available (interpolation).

Introduction

■ Philosophy:

- **Compromise** between the complexity of stochastic modelling and solvability of the problem.
- Emphasis on the **mathematical approach**. Maths concepts useful in other application contexts.
- We will work with models of **increasing** complexity.

■ Warning:

- (Very) simplified models, with artificial data.
- The purpose is not to conclude on the relevance of the introduction of such or such technology.

Introduction

■ References :

- Le Franc, Carpentier, Chancelier, de Lara. EMSx: an Energy Management System numerical benchmark. Energy System, 2021.
- Hafiz, Awal, de Queiroz, Husain. Real-time Stochastic Optimization of Energy Storage Management using Rolling Horizon Forecasts for Residential PV Applications, 2019.
- Olivares et al. Trends in microgrid control. IEEE Transactions on smart grid, 2014.

Introduction

Organisation:

- 6 units: January 11, January 18, January 25, January 26, February 1, February 2.
- It is highly recommended to participate to all units.
- Work in pairs (please form the groups by next week).
- Programming with Matlab.
- Evaluation: programming exercises to solve + work in class.

1 Introduction

2 Deterministic model

Deterministic model

- Horizon: 24 hours, stepsize: 1 hour.
Optimization over $T = 24$ intervals.
- Optimisation variable :
 - $x(s)$: state of charge of the battery at time s , $s = 1, \dots, T + 1$
 - $a(s)$: amount of electricity bought on the network ($s = 1, \dots, T$).
 - $v(s)$: amount of energy sold on the network ($s = 1, \dots, T$).
- Parameters:
 - $d(s)$: net demand of energy (load minus solar production) at time s , $s = 1, \dots, T$.
 - $P_a(s)$: unitary buying price of energy at time s
 - $P_v(s)$: unitary selling price of energy at time s
 - x_{\max} : storage capacity of the battery.

Remark: the demand is supposed to be deterministic (that is to say, known in advance), for the moment.

Deterministic model

■ Constraints:

- $x(s+1) = x(s) - d(s) + a(s) - v(s), \forall s = 1, \dots, T$
- $x(1) = 0$
- $a(s) \geq 0, \forall s = 1, \dots, T$
- $v(s) \geq 0, \forall s = 1, \dots, T$
- $0 \leq x(s) \leq x_{\max}, \forall s = 1, \dots, T + 1.$

■ Cost function to be minimized:

$$J(x, a, v) = \sum_{s=1}^T \left(P_a(s)a(s) - P_v(s)v(s) \right).$$

The buying and selling prices P_a and P_v depend on time. It holds: $P_a(s) > P_v(s)$, so that it is useless to try to buy and sell electricity on the network at the same time!

Deterministic model

Exercise 1

- 1 Write the optimization problem in a form that is compatible with the function `linprog` of Matlab.
- 2 Write a Matlab program that solves the optimization problem corresponding to the deterministic model.

Deterministic model

Main idea behind **dynamic programming**:

- We **parametrize** the problem to be solved \rightsquigarrow a sequence of problems of increasing complexity.
- We look for a **relation** (“dynamic programming principle”) between the optimal values of the different problems.

Parameters :

- Initial time $t \in \{1, \dots, T + 1\}$.
- Initial state-of-charge of the battery $y \in [0, x_{\max}]$.

We are interested in the problem with $t = 1$ and $y = 0$.

Deterministic model

Parameterized problem:

$$V(t, y) = \inf_{\substack{x(t), x(t+1), \dots, x(T+1) \\ a(t), a(t+1), \dots, a(T) \\ v(t), v(t+1), \dots, v(T)}} \sum_{s=t}^T P_a(s)a(s) - P_v(s)v(s) \quad (P(t, y))$$

under the constraints:

- $x(s+1) = x(s) - d(s) + a(s) - v(s), \forall s = t, \dots, T$
- $x(t) = y$
- $a(s) \geq 0, \forall s = t, \dots, T$
- $v(s) \geq 0, \forall s = t, \dots, T$
- $0 \leq x(s) \leq x_{\max}, \forall s = t, \dots, T+1.$

The function V is called **value function**; it plays a crucial role, in particular in the treatment of the stochastic version of the problem.

Deterministic model

Theorem [Dynamic programming principle]

The following holds true:

$$V(t, y) = \inf_{(z, a, v) \in \mathbb{R}^3} P_a(t)a - P_v(t)v + V(t+1, z),$$
$$\text{s.t.:} \quad \begin{cases} a \geq 0 \\ v \geq 0 \\ z = y - d(t) + a - v \\ 0 \leq z \leq x_{\max} \end{cases}$$

(DP(t, y))

for all $t \in \{1, \dots, T\}$ and $y \in [0, x_{\max}]$ and

$$V(T+1, y) = 0.$$

Deterministic model

Computation of the value function

- Let us suppose that the function $y \mapsto V(t+1, y)$ is known, with $t \in \{1, \dots, T\}$. By solving $DP(t, y)$ for all possible values of y , we can compute $y \mapsto V(t, y)$.
- In practice, we can only solve $DP(t, y)$ for a sample of values of $y \in [0, x_{\max}]$. We evaluate $V(t, y)$ for those values of y ; then we interpolate.
- In that way, we can compute (an approximation of) the value function, in a recursive and backward fashion (from $T+1$ to 1).

Deterministic model

Solving the initial problem.

- Let (z, a, v) denote a solution of $DP(1, 0)$.
Let us set: $\bar{x}(2) = z$, $\bar{a}(1) = a$, $\bar{v}(1) = v$.
- Let (z, a, v) be a solution of $DP(2, \bar{x}(2))$.
Let us set: $\bar{x}(3) = z$, $\bar{a}(2) = a$, $\bar{v}(2) = v$.
- Let (z, a, v) be a solution of $DP(3, \bar{x}(3))$.
Let us set: $\bar{x}(4) = z$, $\bar{a}(3) = a$, $\bar{v}(3) = v$.
- ... and so on, until the resolution of $DP(T, \bar{x}(T))$.

Deterministic model

- Let $y \in \mathbb{R}^J$ and let $z \in \mathbb{R}^J$. We consider a function f such that $z_j = f(y_j)$, for all $j = 1, \dots, J$.
- We interpolate f with a second-order polynomial, by solving

$$\inf_{\alpha \in \mathbb{R}^3} \sum_{j=1}^J (\alpha_1 + \alpha_2 y_j + \alpha_3 y_j^2 - z_j)^2.$$

- Let $\bar{\alpha}$ be the solution. We get the approximation:

$$f(y) \approx \bar{\alpha}_1 + \bar{\alpha}_2 y + \bar{\alpha}_3 y^2.$$

Exercise 2

Write a function `interpolate` implementing the interpolation with a second-order polynomial.

Inputs: $J, y \in \mathbb{R}^J, z \in \mathbb{R}^J$. Outputs: $\alpha \in \mathbb{R}^3$.

Deterministic model

Exercise 3

Write a function `DP_solve` with output the solution (z, a, v) of problem $DP(t, y)$.

We assume that the function $V(t+1, \cdot)$ is approximated by a second-order polynomial, described by a coefficient $\alpha \in \mathbb{R}^3$.

Input: $t \in \{1, \dots, T\}$, $y \in [0, x_{\max}]$, $\alpha \in \mathbb{R}^3$.

Deterministic model

Exercise 4

Write a function `DP_backward` which compute a polynomial approximation of V , in the form of a matrix $\alpha \in \mathbb{R}^{(T+1) \times 3}$, that is to say:

$$V(t, y) \approx \alpha_{t,1} + \alpha_{t,2}y + \alpha_{t,3}y^2.$$

We will proceed in a recursive fashion:

- Given $\alpha_{t+1,1}$, $\alpha_{t+1,2}$, $\alpha_{t+1,3}$, evaluate $V(t, y_j)$ for all $j = 1, \dots, J$, where $y_j = (j - 1)/(J - 1)x_{\max}$.
- Calculate $\alpha_{t,1}$, $\alpha_{t,2}$, and $\alpha_{t,3}$ by interpolating the values of $V(t, \cdot)$.

Deterministic model

Exercise 5

Write a function `DP_forward` computing the solution to the original problem. We will first make use of `DP_backward` to get an approximation of the value function.