Optimization Project in Energy
PGE 306

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Hydro valley

Indices

- Set of dams $\mathcal{I}$
- Set of rivers $\mathcal{E} \subset \mathcal{I} \times \mathcal{I}$:
  $$(i, j) \in \mathcal{E} \iff \text{river flows from dam } i \text{ to dam } j.$$
- Set of time intervals: $\{1, ..., T\}$.

Optimization variables

- $q_{i,t}$: water level of dam i at the beginning of time interval $t \in \{1, ..., T+1\}$
- $x_{i,t}$: amount of water exploited at dam i during time interval $t \in \{1, ..., T\}$
- $y_{(i,j),t}$: amount of water transported over the river (i, j) during the time interval $t \in \{1, ..., T\}$
- $z_{i,t}$: amount of water exploited at dam i during the time interval $t \in \{1, ..., T\}$, not transported to any other dam.

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Parameters

- $P_{i,t}$: precipitation at i, during the time interval t
- Q_i: storage capacity of dam i
- K_i: initial level of dam i
- D_t: electricity demand during the time interval t

Functions

- $f_i : x \mapsto f_i(x)$: exploitation cost on a given time interval at dam i, as a function of the amount of exploited water x.
- $g_i : x \mapsto g_i(x)$: electricity production as a function of the amount of exploited water at dam i.

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Cost function

$$\min_{q,x,y,z} \sum_{t=1}^{T} \sum_{i \in I} f_i(x_{i,t}).$$

Constraints

- Nonnegativity of the variables:

$$q_{i,t} \geq 0, \quad x_{i,t} \geq 0, \quad y_{(i,j),t} \geq 0, \quad z_{i,t} \geq 0.$$

- Bounds:

$$q_{i,t} \leq Q_i, \quad \forall t \in \{1, ..., T+1\}.$$

- Initial condition:

$$q_{1,i} = K_i.$$

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- Demand satisfaction:

$$\sum_{i\in\mathcal{I}} g_i(x_{i,t}) = D_t, \quad \forall t \in \{1, ..., T\}.$$

- Evolution of the water level in each dam:

$$q_{i,t+1} = q_{i,t} + P_{i,t} - x_{i,t} + \sum_{\substack{j\in\mathcal{I} \\ (j,i)\in\mathcal{E}}} y_{(j,i),t}, \quad \forall t \in \{1, ..., T\}.$$

- Amount of exploited water:

$$x_{i,t} = \sum_{\substack{j\in\mathcal{I} \\ (i,j)\in\mathcal{I}}} y_{(i,j),t} + z_{i,t}, \quad \forall t \in \{1, ..., T\}.$$

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Dynamic programming.

We parametrize the problem by

- the initial time interval t
- the initial level of water in every dam $q \in \mathbb{R}^{\mathcal{I}}$.

Let $V(t, q)$ denote the corresponding optimal cost.

We have $V(T + 1, q) = 0$.

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Dynamic programming principle

Let $t \in \{1, \dots, T\}$ and let $q \in \prod_{i \in \mathcal{I}} [0, Q_i]$. Then

$$V(t,q) = \inf_{\substack{q' \in \mathbb{R}^{\mathcal{I}}, x \in \mathbb{R}^{\mathcal{I}}, \\ y \in \mathbb{R}^{\mathcal{E}}, z \in \mathbb{R}^{\mathcal{I}}}} \left(\sum_{i \in \mathcal{I}} f_i(x_i) \right) + V(t+1, q'), \qquad (DP(t,q))$$

subject to:

- Non-negativity: $q'_i \geq 0$, $x_i \geq 0$, $y_{(i,j)} \geq 0$, $z_i \geq 0$.
- Bounds: $q'_i \leq Q_i$.
- Demand: $\sum_{i \in \mathcal{I}} g_i(x_i) = D_t$.
- Conservation: $q'_i = q_i + P_{i,t} - x_i + \sum_{\substack{j \in \mathcal{I} \\ (j,i) \in \mathcal{E}}} y_{(j,i)}$.
- Exploitation : $x_i = \sum_{\substack{j \in \mathcal{I} \\ (i,j) \in \mathcal{I}}} y_{(i,j)} + z_i$.

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Remarks.

- Why does it work?
  → The level of water in the dams at time t is a **sufficient information** to take optimal decisions from t until the end of the optimization process.
  → Knowing the level of water in the dams at time t, one can completely **forget** what happened in the past.

- The dynamic programming principle characterizes globally optimal solutions, even if the original problem is non convex.

- In the practical implementation of the method, one needs to **discretize** the variable q. The number of discretizetion points grows **exponentially** with the number of dams. This phenomenon is called **curse of dimensionality**.

ON/OFF devices

Context:

- Set of $\{1, \dots, I\}$ production units (e.g. nuclear power plants).
- These plants can be put into standby and reactived after some time, at some time to be optimized.

Constraints on disactivation and activation (unit i):

- Maximal activity duration: X_i^{on} days.
- Minimal standby duration: X_i^{off} days.

Decision variables on time interval t, associated with unit i:

- Production $u_i(t)$ (if i is activated).
- Activation/Disactivation/No change.

ON/OFF devices

States set:
The set $\mathcal{X}_i$ describes of possible states of the unit i.
It is defined by

$$\mathcal{X}_i = \{(0,x) \,|\, x = 0, ..., X_i^{\text{off}}\} \cup \{(1,x) \,|\, x = 0, ..., X_i^{\text{on}}\}.$$

Interpretation:

- $(0,x)$: unit i has been OFF for x days,
- $(1,x)$: unit i has been ON for x days.

If the unit i has been OFF for strictly more that X_i^{off}, the state is represented by $(0, X_i^{\text{off}})$.

ON/OFF devices

Transitions:

We denote $\mathcal{E}_i \subset \mathcal{X}_i \times \mathcal{X}_i$ the set of possible transitions from one state to the other. We have

Transition e	Coût $f(e)$
$(1,x) \to (1,x+1), \quad x = 0,\dots,X_i^{\text{on}} - 1$	0
$(1,x) \to (0,0), \quad x = 0,\dots,X_i^{\text{on}} - 1$	0
$(0,x) \to (0,x+1), \quad x = 0,\dots,X_i^{\text{off}} - 1$	0
$(0,X_i^{\text{off}}) \to (0,X_i^{\text{off}})$	0
$(0,X_i^{\text{off}}) \to (1,0)$	$C_i > 0$

The only transition with non-zero cost is the one corresponding to the activation of unit i.

ON/OFF devices

Other parameters:

- $D(t)$: electricity demand on interval t
- U_i: maximal production of unit i (if the unit is activated).

Optimization variables:

- $x_i(t)$: state of the unit i at the beginning of the time interval t
- $u_i(t)$: production of unit i over the interval t.

Production constraint: $(x_i(t), u_i(t)) \in \mathcal{C}_i$, where

$$\mathcal{C}_i := \{((0,x),0) \,|\, x = 1, ..., X_i^{\text{off}}\} \cup_{u \in [0,U_i]} \{((1,x),u) \,|\, x = 1, ..., X_i^{\text{on}}\}.$$

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Problem:

$$\min_{\substack{x_i(t),\, t=1,\dots,T+1 \\ u_i(t),\, t=1,\dots,t}} \quad \sum_{t=1}^{T} \sum_{i=1}^{I} \ell_i(u_i(t)) + f_i(x_i(t), x_i(t+1)),$$

$$\text{subject to:} \quad \begin{cases} (x_i(t), u_i(t)) \in \mathcal{C}_i, \\ (x_i(t), x_i(t+1)) \in \mathcal{E}_i, \\ \sum_{i=1}^{I} u_i(t) = d(t), \\ x_i(1) = y_i. \end{cases}$$

ON/OFF devices

Dynamic programming.

We parametrize the problem by

- initial state t
- state of the units $x \in \prod \mathcal{X}_i$.

Let $V(t, x)$ denote the corresponding cost.

ON/OFF devices

Dynamic programming principle

Let $t \in \{1, \ldots, T\}$ and let $x \in \prod \mathcal{X}_i$. It holds:

$$V(t,x) = \begin{cases} \inf\limits_{\substack{x' \in \prod \mathcal{X}_i, \\ u \in \mathbb{R}^I}} \left(\sum_{i=1}^{I} \ell_i(u_i) + f_i(x_i, x_i') \right) + V(t+1, x') \\ \\ \text{subject to:} \quad \begin{cases} (x_i, u_i) \in \mathcal{C}_i, \\ (x_i, x_i') \in \mathcal{E}_i. \\ \sum_{i=1}^{I} u_i = d. \end{cases} \end{cases}$$

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Remarks.

- Originally a combinatorial problem.
- Curse of dimensionality.