FREQUENCY RESPONSE

Topic Outcomes

- Become acquainted with the frequency response of a BJT and FET amplifier.
- Be able to find the Miller effect capacitance at the input and output of an amplifier due to a feedback capacitor.

Introduction

- The analysis thus far has been limited to a particular frequency.
- For the amplifier, it was a frequency that normally permitted ignoring the effects of the capacitive elements, reducing the analysis to one that included only resistive elements and sources of the independent and controlled variety.
- We will now investigate the frequency effects introduced by the larger capacitive elements of the active device at high frequencies.
- Because the analysis will extend through a wide frequency range, the logarithmic scale will be defined and used throughout the analysis.

Introduction

- In addition, because the industry typically uses a decibel scale on its frequency plots, the concept of the decibel is introduced in some detail.
- The similarities between the frequency response analyses of both BJTs and FETs permit the coverage of both in the same module.

Frequency Response of an Amplifier

Frequency Response of an Amplifier

- Consider a sinusoidal signal of angular frequency represented by:

$$AV_m \sin(\omega t + \varphi)$$

- If the voltage gain of the amplifier has a magnitude A and the signal suffers a phase change θ, then the output will be:

$$AV_m \sin(\omega t + \varphi + \theta) = AV_m \sin[\omega\left(t + \frac{\theta}{\omega}\right) + \varphi]$$

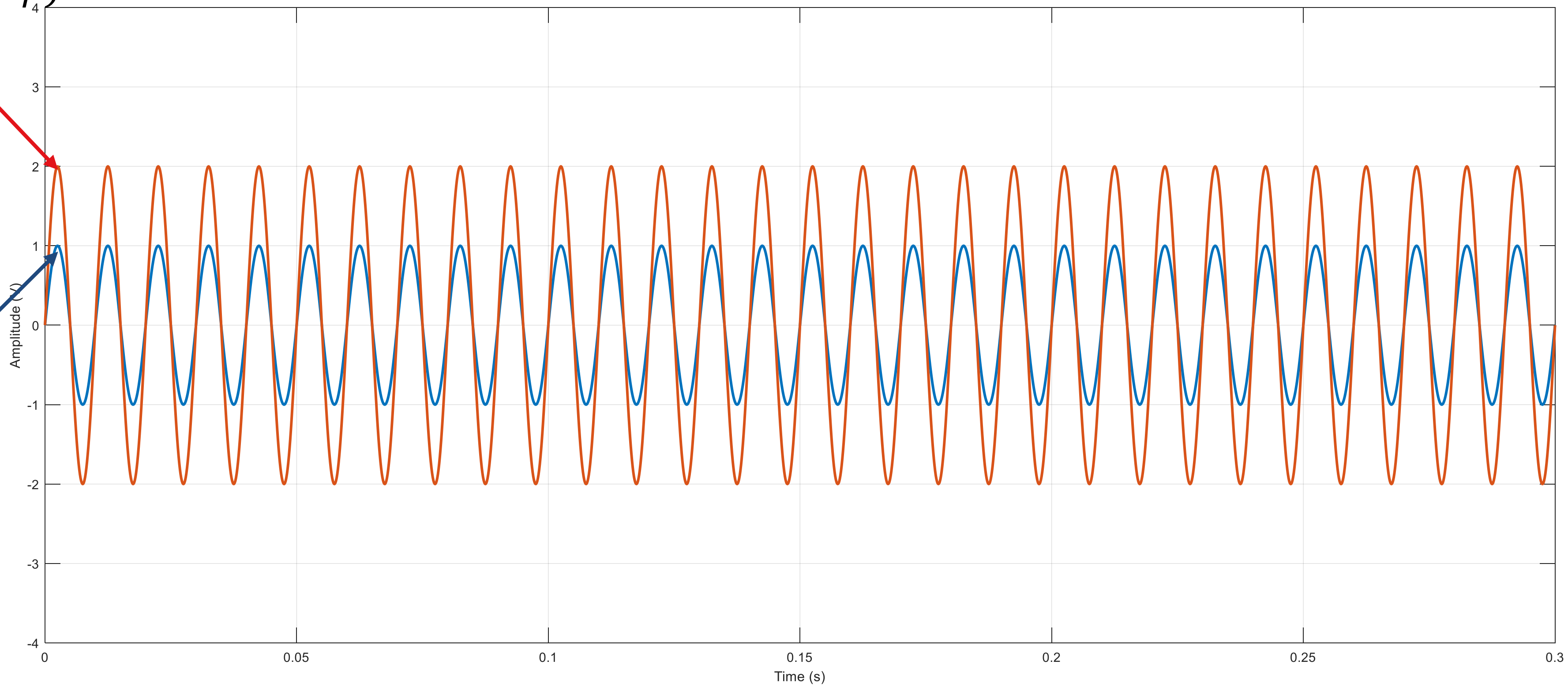
- If the amplification A is independent of frequency and if the phase shift θ is proportional to frequency then the amplifier will follow the input signal although the signal will be shifted in time by $\frac{\theta}{\omega}$. It means both amplitude and delay responses are sensitive indicators of frequency distortion.

Note: $\omega = 2\pi f$

Where f = frequency in Hertz

Frequency Response of an Amplifier

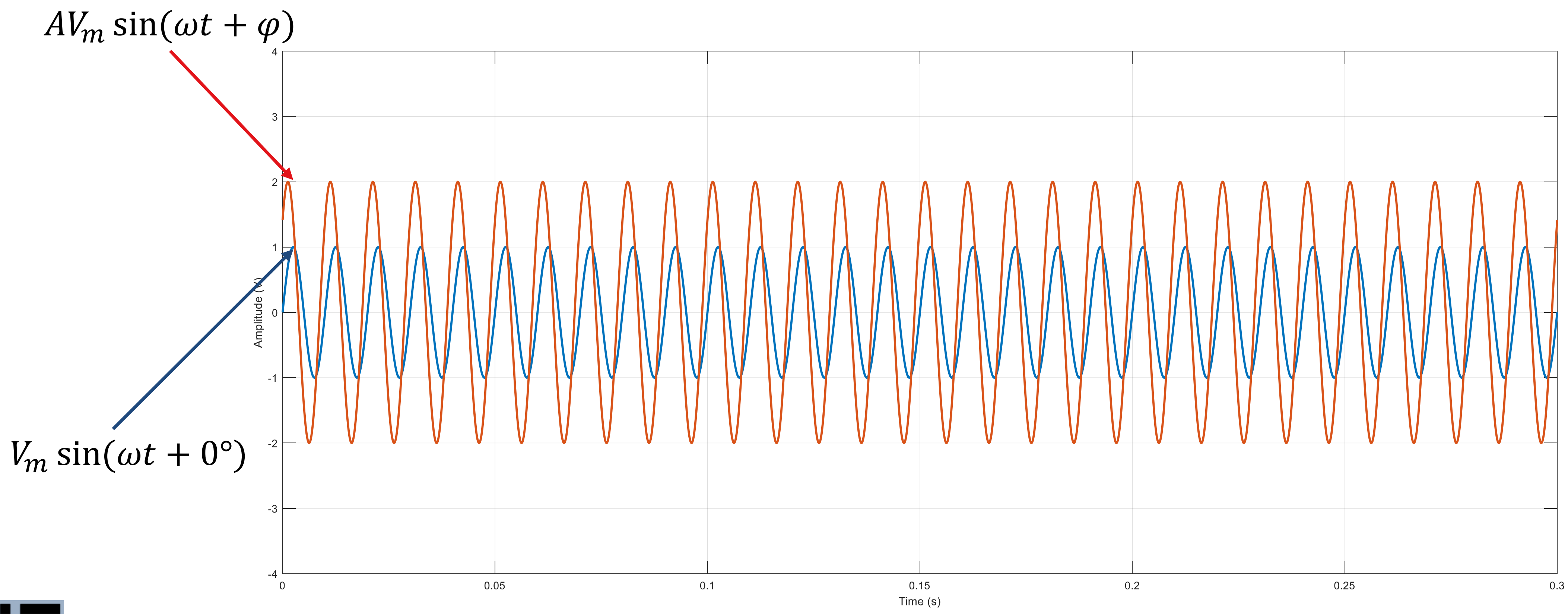
$AV_m \sin(\omega t + \varphi)$



$V_m \sin(\omega t + \varphi)$

$A = 2, V_m = 1, f = 100\ Hz, \varphi = 0$

Frequency Response of an Amplifier

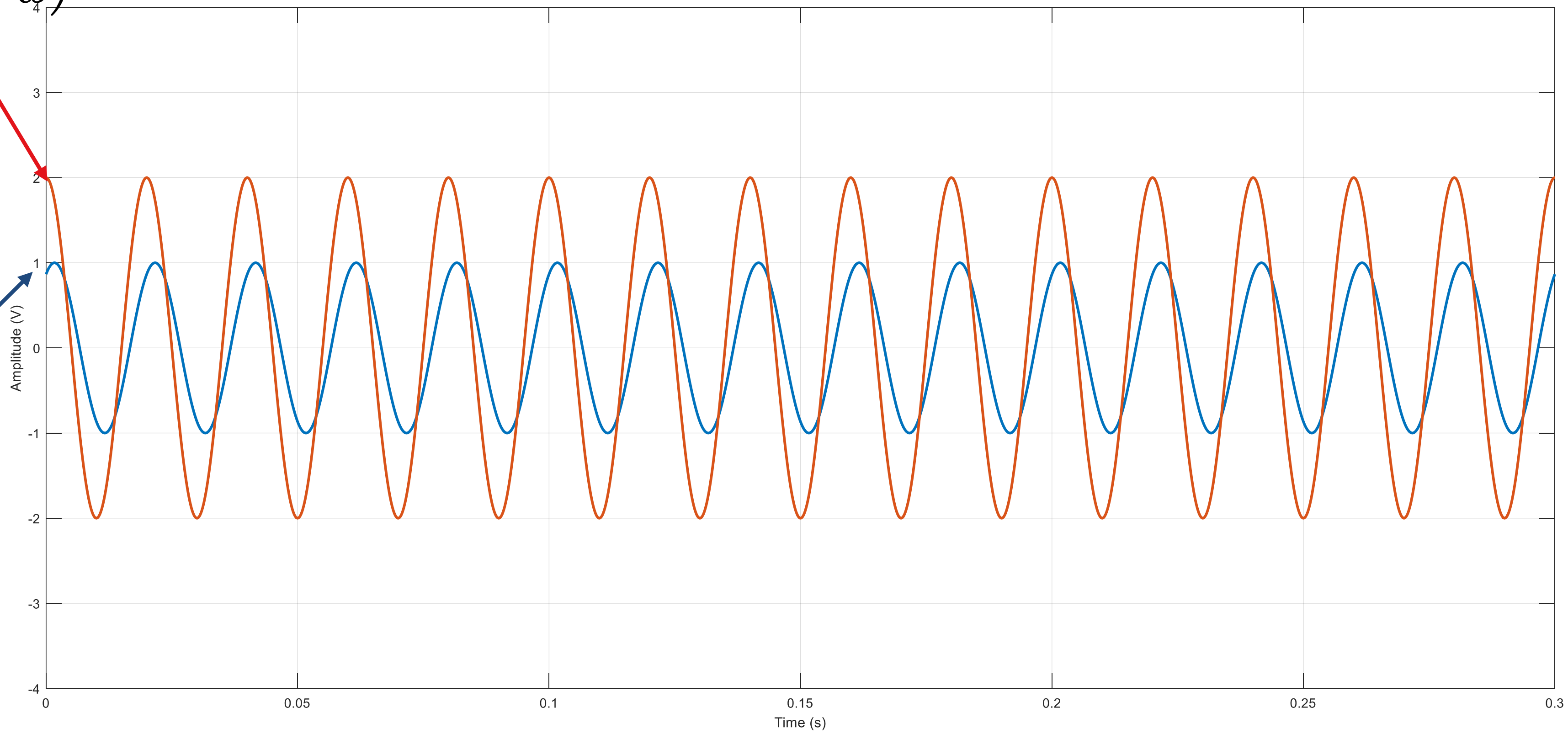


$$A = 2, V_m = 1, f = 100\ Hz, \varphi = 45°$$

Frequency Response of an Amplifier

$$AV_m \sin[\omega\left(t+\frac{\theta}{\omega}\right)+\varphi]$$

$$V_m \sin(\omega t+\varphi)$$



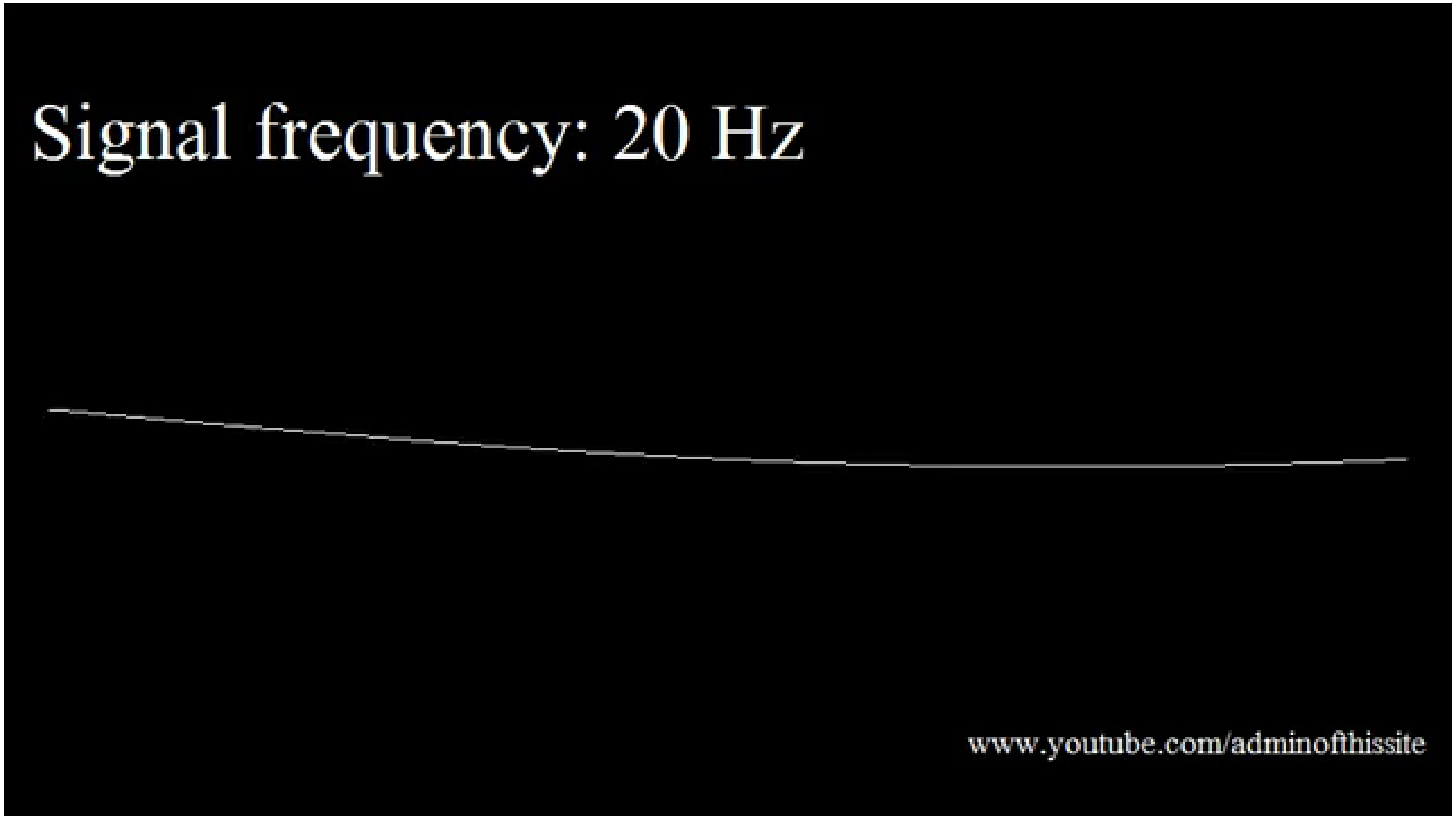
$$A=2, V_m=1, f=50\ Hz, \varphi=45°, \theta=30°$$

Frequency Response of an Amplifier

https://www.youtube.com/watch?v=VK-fZ9L-CGQ

Frequency Response of an Amplifier

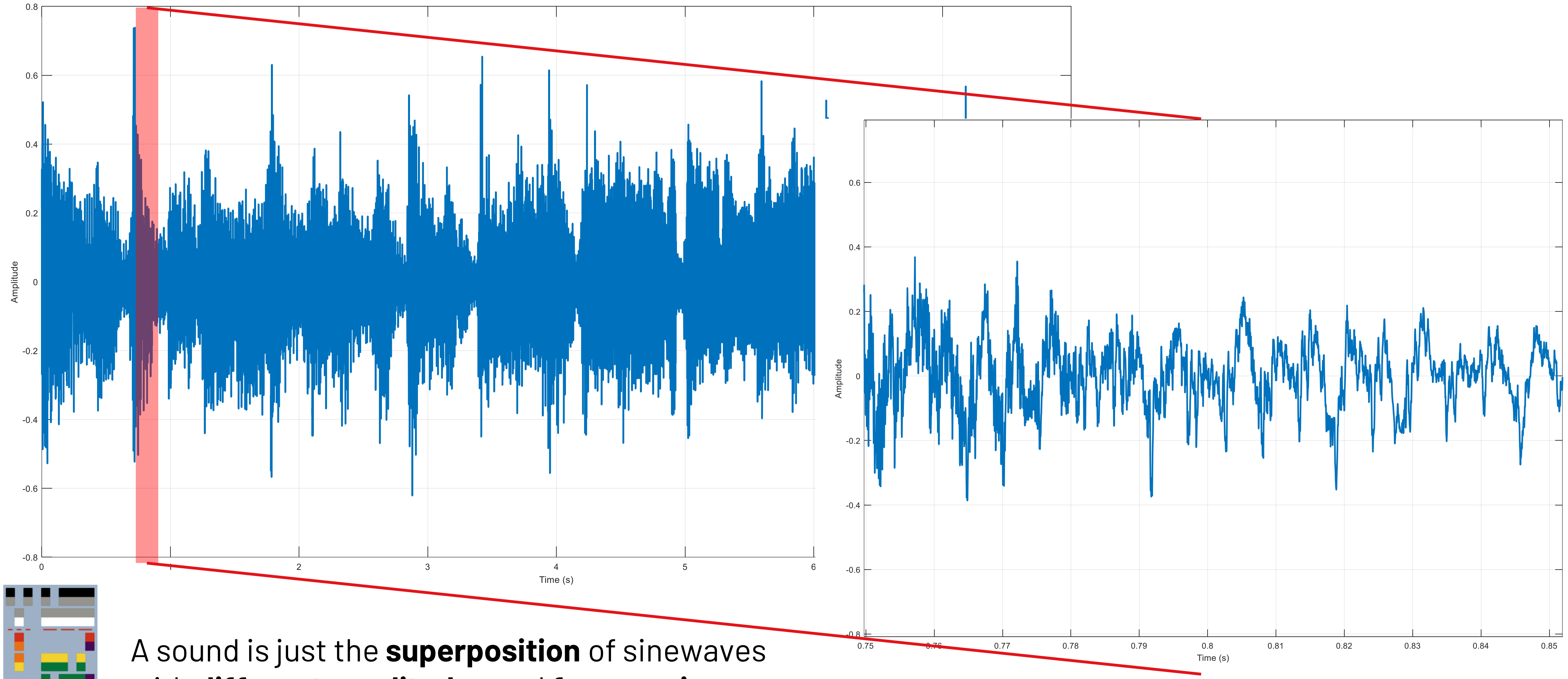
https://www.youtube.com/watch?v=qNf9nzvnd1k



Frequency Response of an Amplifier

https://www.youtube.com/watch?v=TfY8hR5frEQ

Frequency Response of an Amplifier

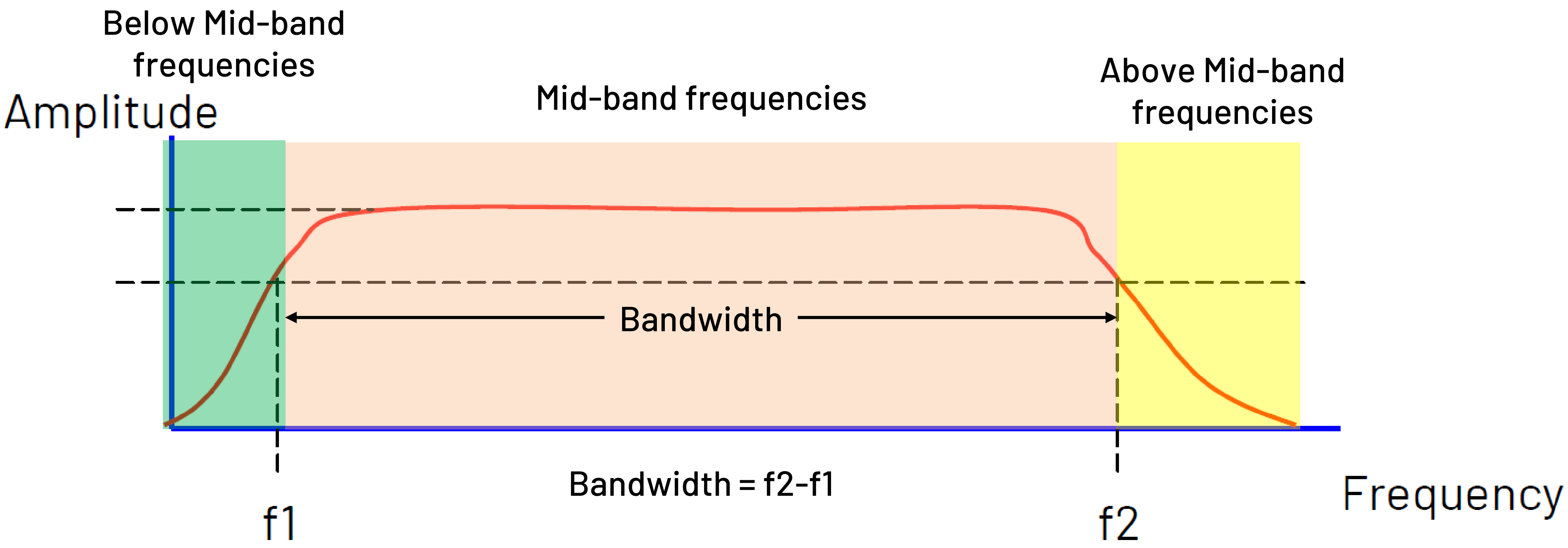


A sound is just the **superposition** of sinewaves with **different amplitudes** and **frequencies**

Frequency Response of an Amplifier

- For an amplifier stage the frequency characteristic may be divided into three regions.
  - **Mid-band frequency** : the region of frequency where the amplification and delay is reasonably constant i.e. gain is nearly equal to one.
  - **Below mid-band frequency** : the region where the active circuit may behave as a simple high pass circuit. The response decrease with decreasing frequency and output approaches to zero.
  - **Above mid-band frequency**: the circuit behaves like a low-pass circuit and the response decreases with increase in frequency.

Frequency Response of an Amplifier



Frequency Response of an Amplifier

Amplifier with normal response:

Amplifier with poor high frequency response:

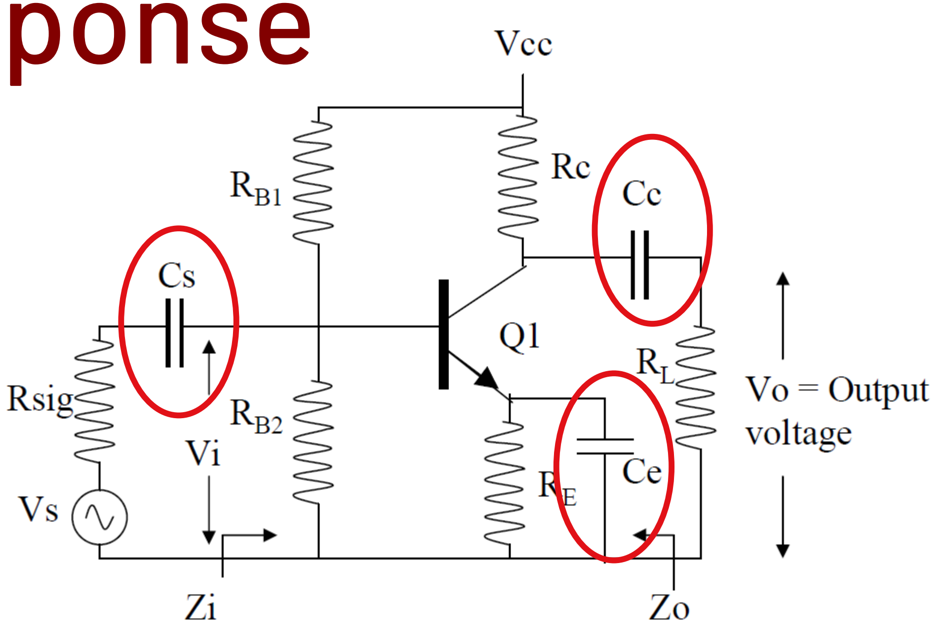
Amplifier with poor low frequency response:

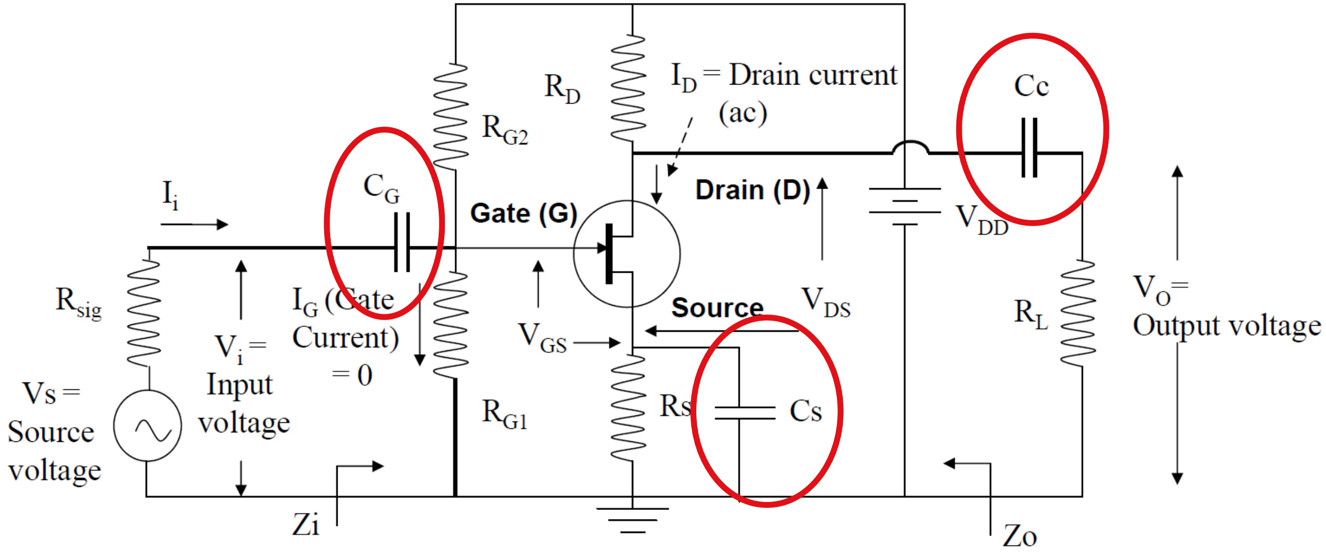
Amplifier with poor low and high frequency response:

Low Frequency Response

Low Frequency Response

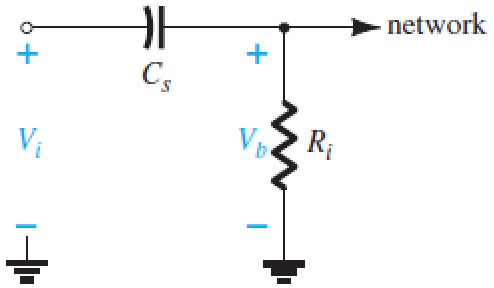
- For the low frequency region of the single-stage BJT or FET amplifier, the RC combinations formed by the coupling capacitors at the input and output and the bypass capacitors at R_E and/or R_S , and the network's resistive parameters determine the cutoff frequencies.



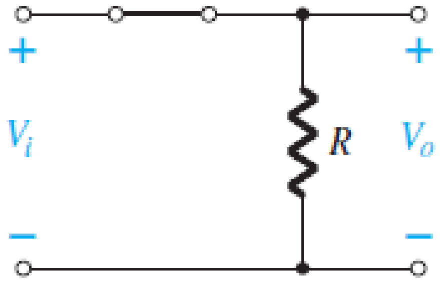


Low Frequency Response

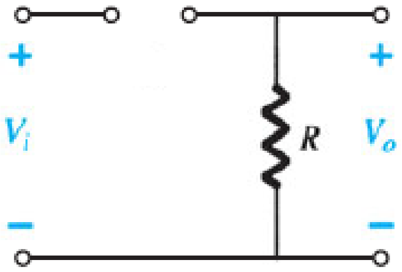
- At low frequencies, the equivalent circuit is a high-pass filter
  - A high-pass filter passes only frequencies higher than the cutoff frequency
- An RC network similar to the figures can be established for each capacitive element and the frequency at which the output voltage drops to 0.707 of its maximum value determined.
- Once the cutoff frequencies due to each capacitor are determined, they can be compared to establish which will determine the low cutoff frequency for the system.



RC circuit that will define a low cutoff frequency



RC circuit at very high frequencies



RC circuit at low Frequencies (i.e. f=0)

Low Frequency Response

- At DC circuits, the resistance of a capacitor is $\infty\Omega$.
- At AC circuits this is called reactance, X.
- X_C or the capacitor's reactance is determined by the equation

$$X_C = \frac{1}{2\pi f C}$$

Where:

j = imaginary unit/unit imaginary number = $\sqrt{-1}$

f= frequency in Hz (Hertz)

C = Capacitance in F (Farad)

- This shows that the capacitor's reactance changes according to frequency and capacitance.
- At constant capacitance, and increasing frequency, the reactance becomes smaller.
- At constant capacitance, and decreasing frequency, the reactance becomes larger.
- In AC circuits, the capacitor's impedance is

$$Z_C = -jX_C$$

Low Frequency Response

$$V_o = V_b = V_i \frac{R}{R + Z_C}$$

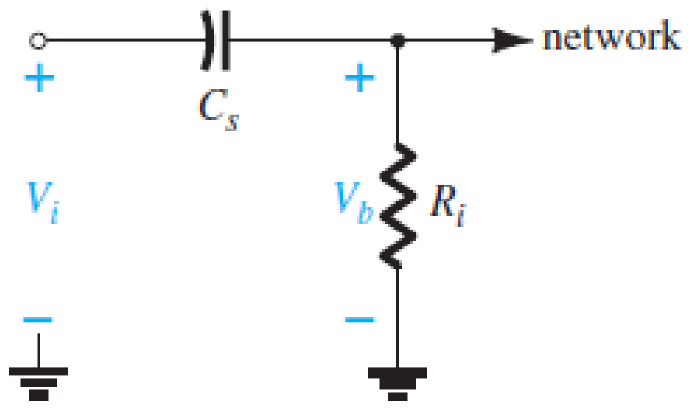
$$A_V = \frac{V_o}{V_i} = \frac{R}{R - jX_C}$$

$$A_V = \frac{1}{1 - \frac{jX_C}{R}}$$

$$A_V = \frac{1}{1 - j\left(\frac{1}{2\pi fRC}\right)}$$

Let $f_1 = \frac{1}{2\pi RC}$

$$A_V = \frac{1}{1 - j\left(\frac{f_1}{f}\right)}$$



Low Frequency Response

$$A_V = \frac{1}{1 - j\left(\frac{f_1}{f}\right)}$$

In magnitude and phase form:

$$A_V = \frac{1}{\sqrt{1 + \left(\frac{f_1}{f}\right)^2}} \angle \left(\tan^{-1}\left(\frac{f_1}{f}\right)\right) = \frac{1}{\sqrt{1 + \left(\frac{f_1}{f}\right)^2}} e^{j\left(\tan^{-1}\left(\frac{f_1}{f}\right)\right)}$$

$$|A_V| = \frac{1}{\sqrt{\left(1+\frac{f_1}{f}\right)^2}}$$

$$\arg(A_v) = \tan^{-1}\left(\frac{f_1}{f}\right)$$

Low Frequency Response

The magnitude when $f_1 = f$

$$|A_V| = \frac{1}{\sqrt{1 + \left(\frac{f}{f}\right)^2}} = \frac{1}{\sqrt{2}} = 0.707$$

In logarithmic form, the gain in dB (decibel) is

$$A_{V(dB)} = 20 \ \log_{10}\left(\frac{1}{\sqrt{1 + \left(\frac{f_1}{f}\right)^2}}\right)$$

$$A_{V(dB)} = 20 \log_{10} 0.707 = -3 \ dB$$

Low Frequency Response

- This means that when the frequency is at the lower cutoff frequency, the gain at that point is 0.707 times the gain at mid-band frequencies
- This also means that it is -3dB lower than the gain at the mid-band frequencies.

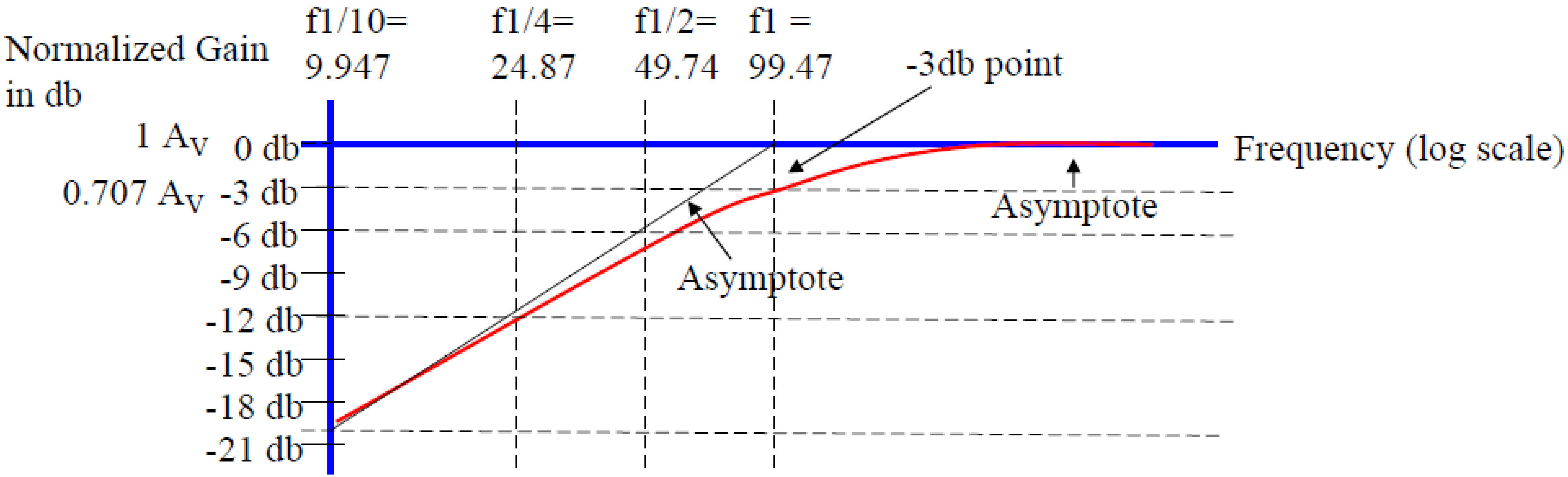
Low Frequency Response

- Example: A circuit has a coupling capacitor of 0.2 microfarad and a load resistor of 8 kΩ, what is the cut-off frequency?

$$f_c = \frac{1}{2\pi R_L C} = \frac{1}{2\pi(8000\ \Omega)(0.2 \times 10^{-6} F)}$$

$$f_c = 99.47 Hz$$

Low Frequency Response



As a general rule of thumb:

At $f = f_1$ $\quad A_v = -20\log_{10} 1 = 0\ dB$

At $f = \frac{f_1}{2}$ $\quad A_v = -20\log_{10} 2 = -6dB$

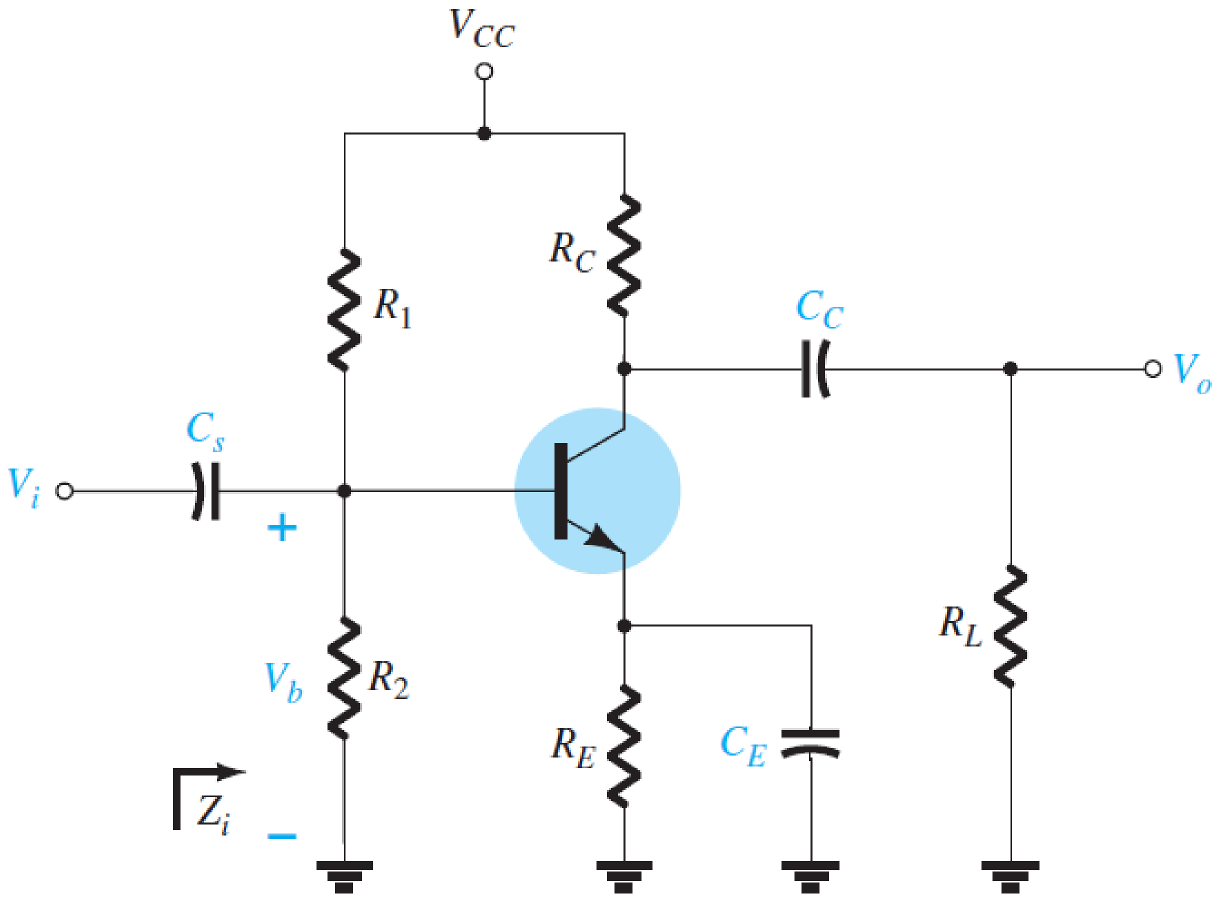
At $f = \frac{f_1}{4}$ $\quad A_v = -20\log_{10} 4 = -12\ dB$

At $f = \frac{f_1}{10}$ $\quad A_v = -20\log_{10} 10 = -20\ dB$

Low Frequency Response-BJT

Low Frequency Response-BJT

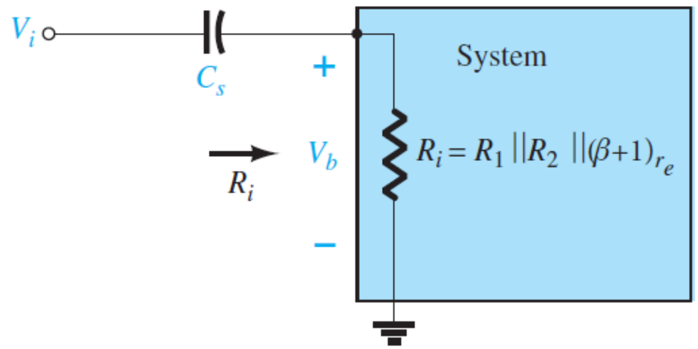
- For any BJT configuration, it will simply be necessary to find the appropriate equivalent impedances (input, output impedances, etc.) for the RC combination with the capacitors C_C , C_E, and C_S to determine the low-frequency response of a network.



Low Frequency Response-BJT

- The equivalent circuit for the input side is shown.
- Thus, the low frequency response for the input is:

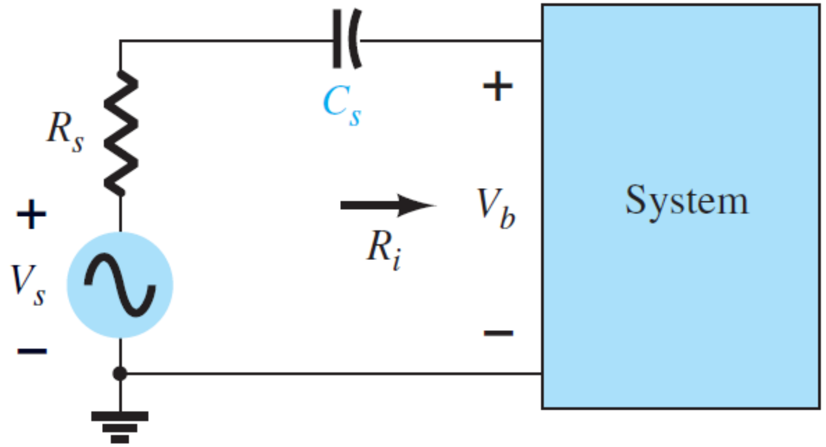
$$f_{L_s} = \frac{1}{2\pi R_i C_s}$$



Low Frequency Response-BJT

- When the source resistance is considered, the total resistance is just the series of the input resistance and the signal source resistance that is:

$$f_{L_s} = \frac{1}{2\pi(R_s + R_i)C_s}$$



Low Frequency Response-BJT

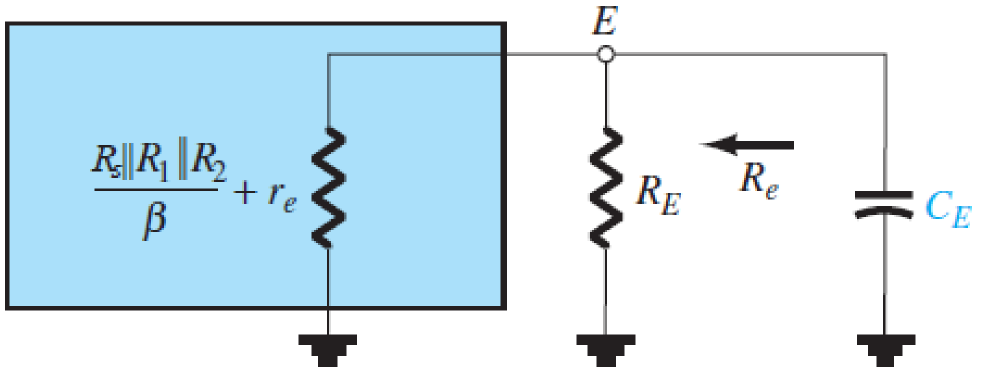
- For the emitter capacitor, the cutoff frequency is determined by:

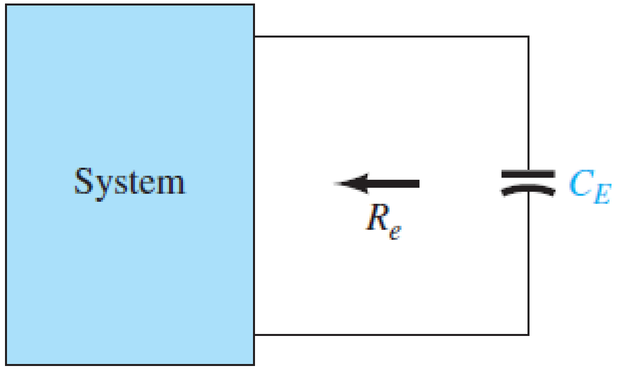
$$R_e = R_E \parallel \left(\frac{R_s'}{\beta} + r_e\right)$$

$$R_s' = R_s \parallel R_1 \parallel R_2$$

$$r_e = \frac{26mV}{I_E}$$

$$f_{L_E} = \frac{1}{2\pi R_e C_E}$$



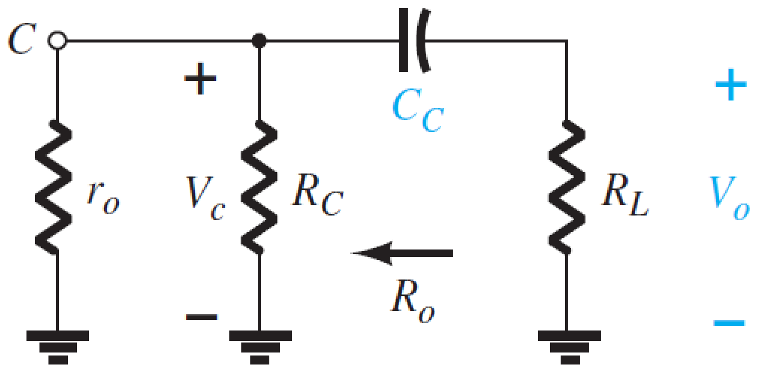


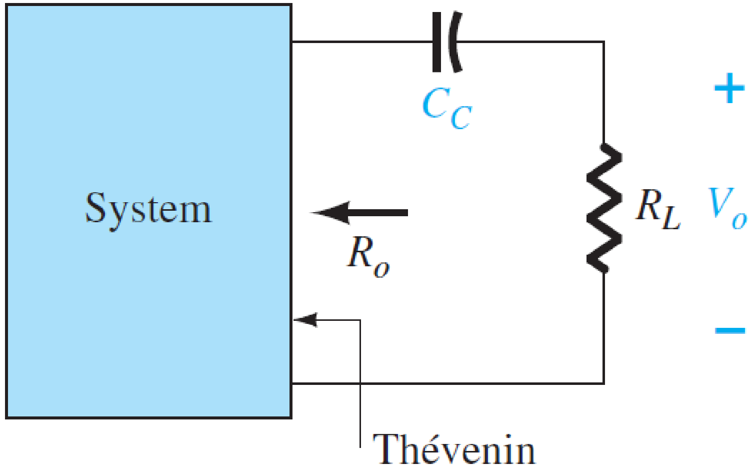
Low Frequency Response-BJT

- For the output part of the circuit, the RC configuration that determines the frequency response involves the output impedance R_0 and the load resistance R_L.
- In this case:

$$R_O = r_o \parallel R_C$$

$$f_{L_C} = \frac{1}{2\pi(R_O + R_L)C_C}$$





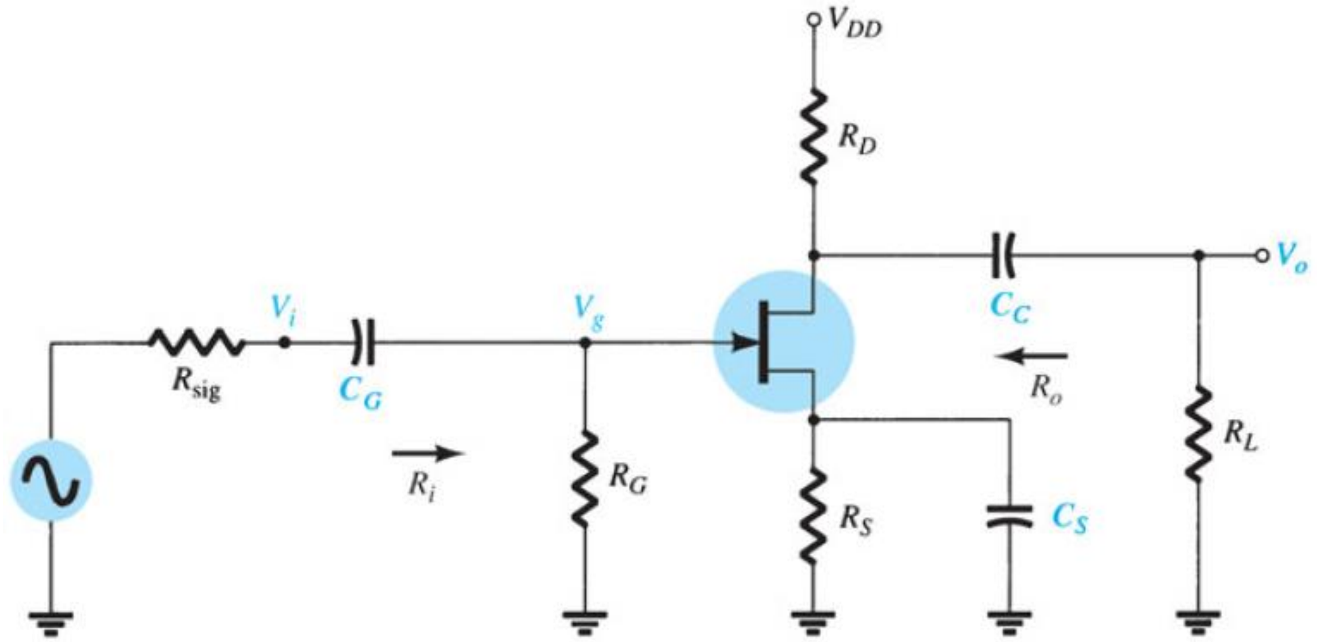
Low Frequency Response-BJT

- For the 3 solved cut-off frequencies, the circuit's low cutoff frequency is the **highest** cut-off frequency solved.
- For example, $f_{L_S} = 3.688\ Hz, f_{L_C} = 13.263\ Hz, f_{L_E} = 225.822\ Hz,$
  - f_{L_E} will predominantly affect the cutoff frequency of the whole amplifier circuit.

Low Frequency Response-FET

Low Frequency Response-FET

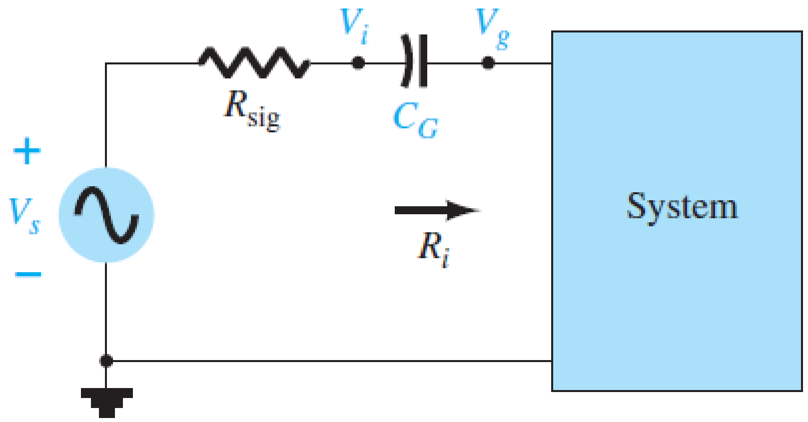
- For any FET configuration, it will simply be necessary to find the appropriate equivalent impedances (input, output impedances, etc.) for the RC combination with the capacitors C_G , C_S, and C_C to determine the low-frequency response of a network.



Low Frequency Response-FET

- The equivalent circuit for the input side is shown.
- Thus, the low frequency response for the input is:

$$f_{LG} = \frac{1}{2\pi(R_{sig} + R_i)C_G}$$



Low Frequency Response-FET

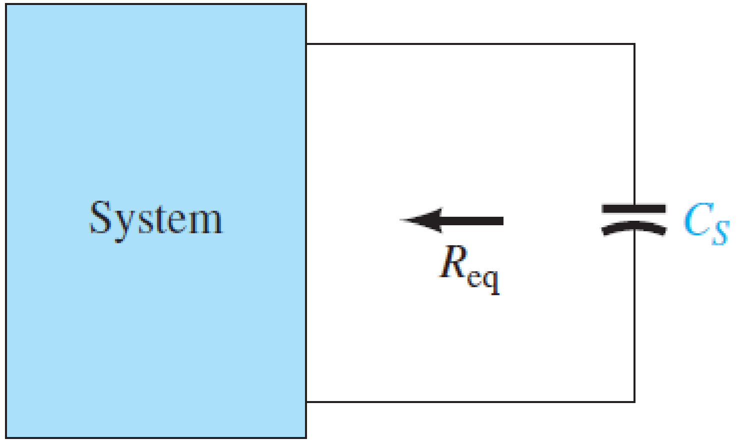
- For the source capacitor, the cutoff frequency is determined by:

$$R_{eq} = \frac{R_S}{1 + \frac{R_S(1 + g_m r_d)}{(r_d + R_D||R_L)}}$$

If $r_d \cong \infty\Omega$

$$R_{eq} = R_S \parallel \frac{1}{g_m}$$

$$f_{L_S} = \frac{1}{2\pi R_{eq} C_S}$$

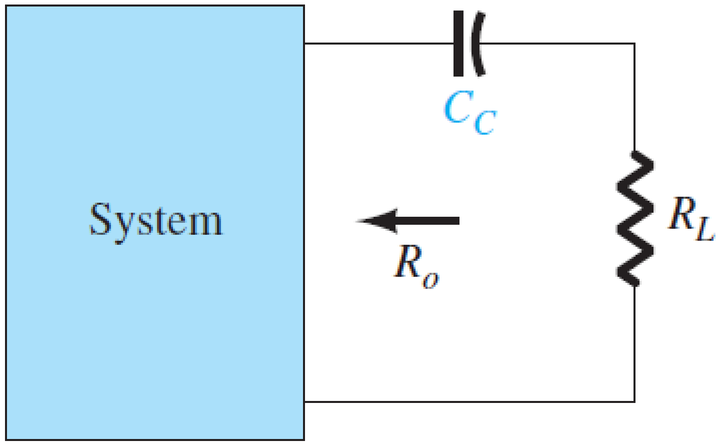


Low Frequency Response-FET

- For the output part of the circuit, the RC configuration that determines the frequency response involves the output impedance R_0 and the load resistance R_L.
- In this case:

$$R_O = r_D \parallel R_D$$

$$f_{L_C} = \frac{1}{2\pi(R_O + R_L)C_C}$$



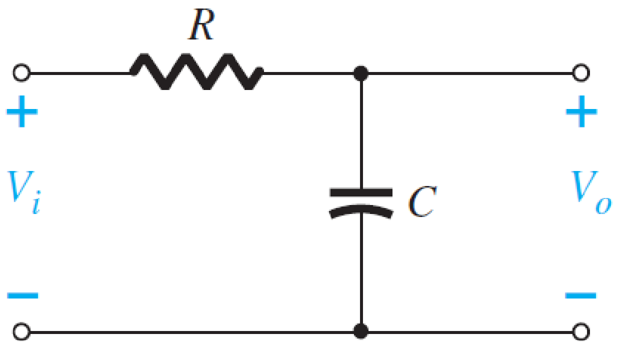
High Frequency Response

High Frequency Response

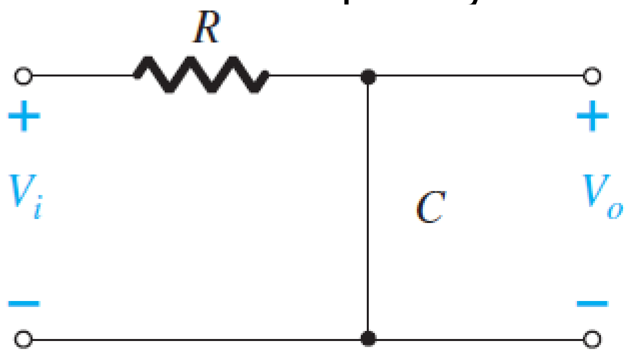
- For the high frequency region of the single-stage BJT or FET amplifier, there are two factors that will define the -3 dB point: the network capacitance (parasitic and introduced) and the frequency dependence of h_{fe} (β).

High Frequency Response

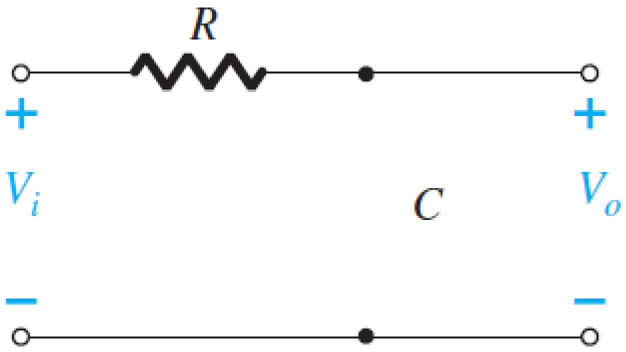
- At high frequencies, the equivalent circuit is a low-pass filter
  - A low-pass filter passes only frequencies lower than the cutoff frequency
- An RC network similar to the figures can be established for each capacitive element and the frequency at which the output voltage drops to 0.707 of its maximum value determined.
- Once the cutoff frequencies due to each capacitor are determined, they can be compared to establish which will determine the low cutoff frequency for the system.



RC circuit that will define a high cutoff frequency



RC circuit at very high frequencies

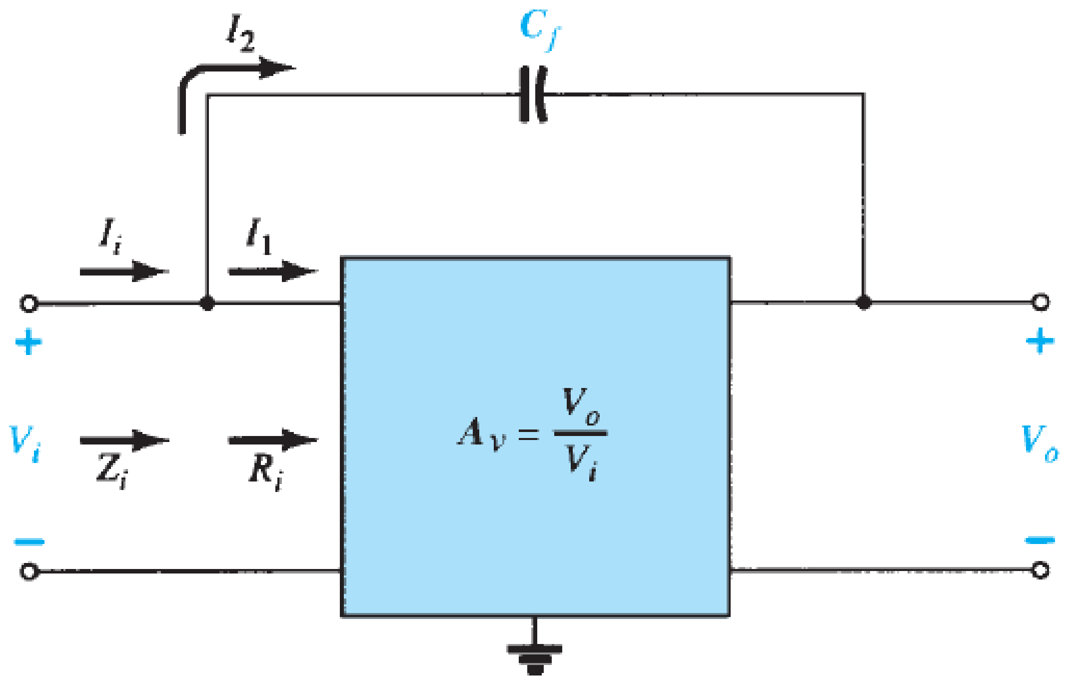


RC circuit at low Frequencies (i.e. f=0)

Miller Effect Capacitance

Miller Effect Capacitance

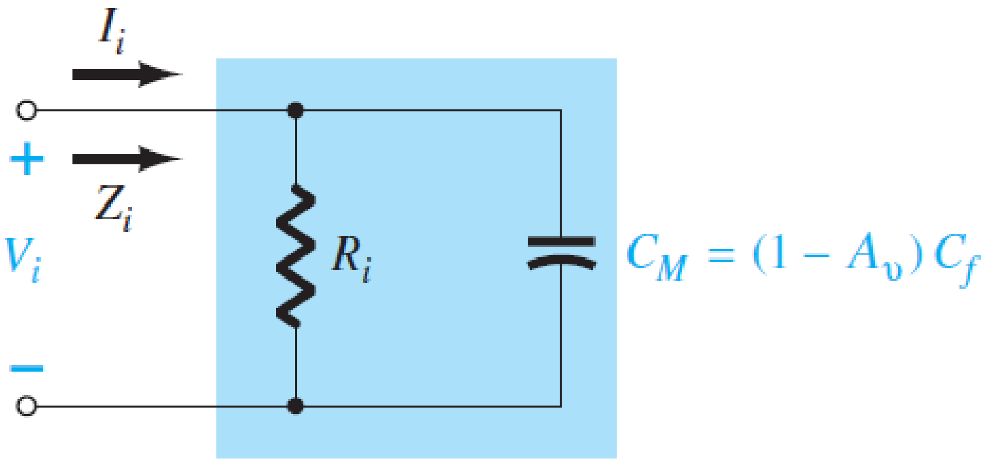
- The diagram shows the existence of a **feedback capacitance** whose reactance becomes **significantly low at high frequencies**
- This affects the performance of an amplifier.
- The input and output capacitance are **increased** by a capacitance level sensitive to the **interelectrode (between terminals) capacitance (C_f)** between the input and output terminals of the device and the gain of the amplifier.
- **Because of C_f**, an **equivalent capacitance**, called **Miller capacitance** is produced at the **input** and **output**.



Miller Effect Capacitance

- For the Miller effect input capacitance:

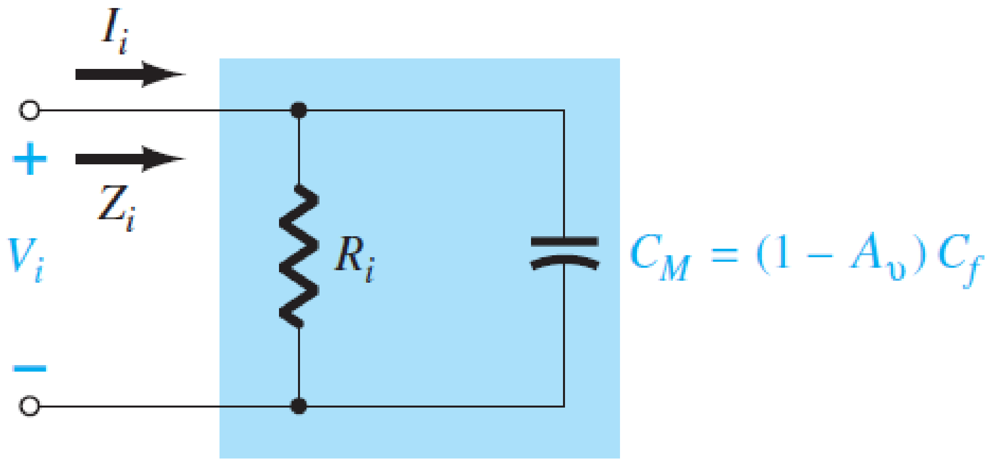
$$C_{Mi} = (1 - A_V)C_f$$



Miller Effect Capacitance

- For the Miller effect output capacitance:

$$C_{MO} = \left(1 - \frac{1}{A_V}\right) C_f$$



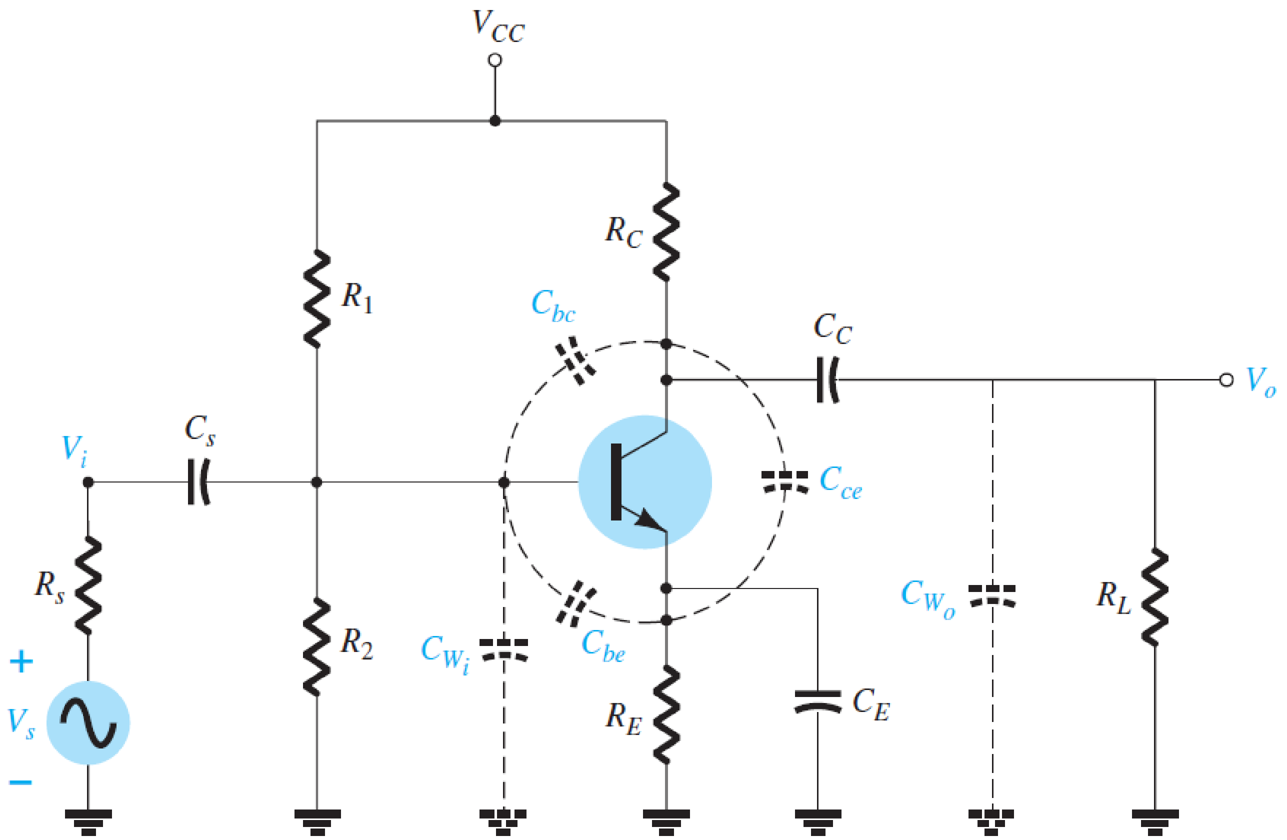
Miller Effect Capacitance

- The equations show that for any **inverting amplifier (negative A_v, with phase reversal)**, the Miller Effect capacitance is **positive**.
- If the **voltage gain is positive** (no phase reversal), Miller Effect capacitance is **negative**.

High Frequency Response-BJT

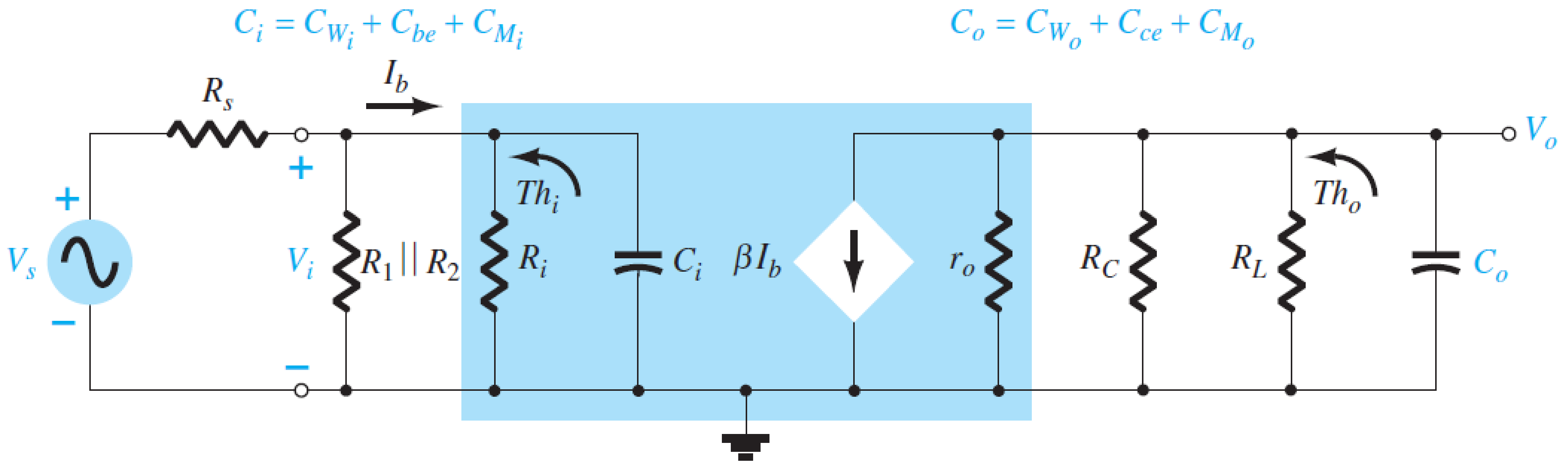
High Frequency Response-BJT

- In the figure, the various parasitic capacitance (C_{be}, C_{bc}, C_{ce}) of the transistor are included with the wiring capacitances (C_{wi} and C_{wo}) which are introduced during construction.



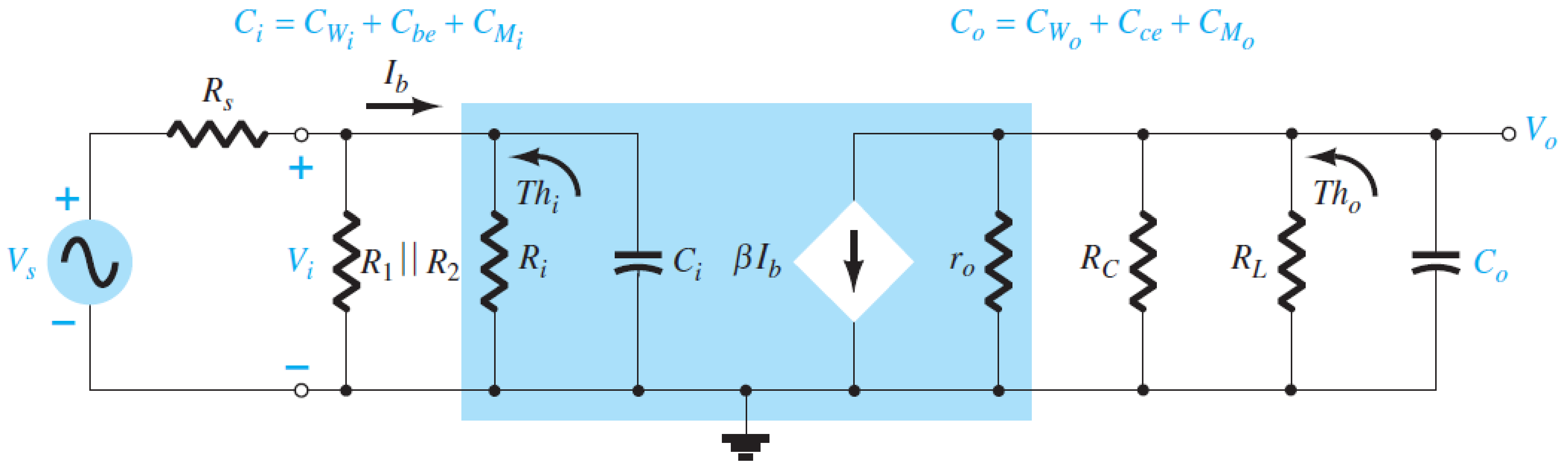
High Frequency Response-BJT

- The high frequency equivalent model for the figure in the previous slide is shown.
- C_i includes the input wiring capacitance (C_{wi}), the transition capacitance C_{be} and the input Miller capacitance (C_{mi}).



High Frequency Response-BJT

- C_o includes the output wiring capacitance (C_{wo}), the parasitic capacitance C_{ce}, and the output Miller Capacitance C_{mo}.



High Frequency Response-BJT

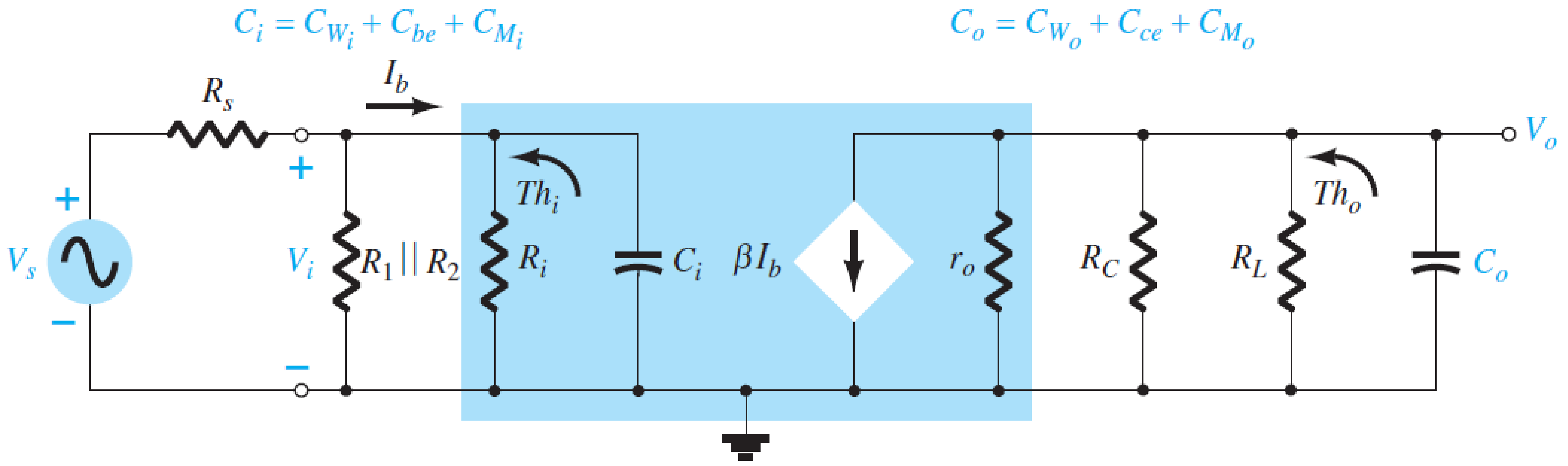
- In general, C_{be} is the larges of the parasitic capacitances with C_{ce} the smallest.
- Datasheets mostly provide the levels of C_{be} and C_{bc} and do not include C_{ce} unless it will affect the response of that transistor in a specific area of application
- C_{bc} defines the feedback capacitance to be used in the Miller effect capacitance:
  - $\mathbf{C_f = C_{bc}}$

High Frequency Response-BJT

- In general, C_{be} is the larges of the parasitic capacitances with C_{ce} the smallest.
- Datasheets mostly provide the levels of C_{be} and C_{bc} and do not include C_{ce} unless it will affect the response of that transistor in a specific area of application
- C_{bc} defines the feedback capacitance to be used in the Miller effect capacitance:
  - $\mathbf{C_f = C_{bc}}$

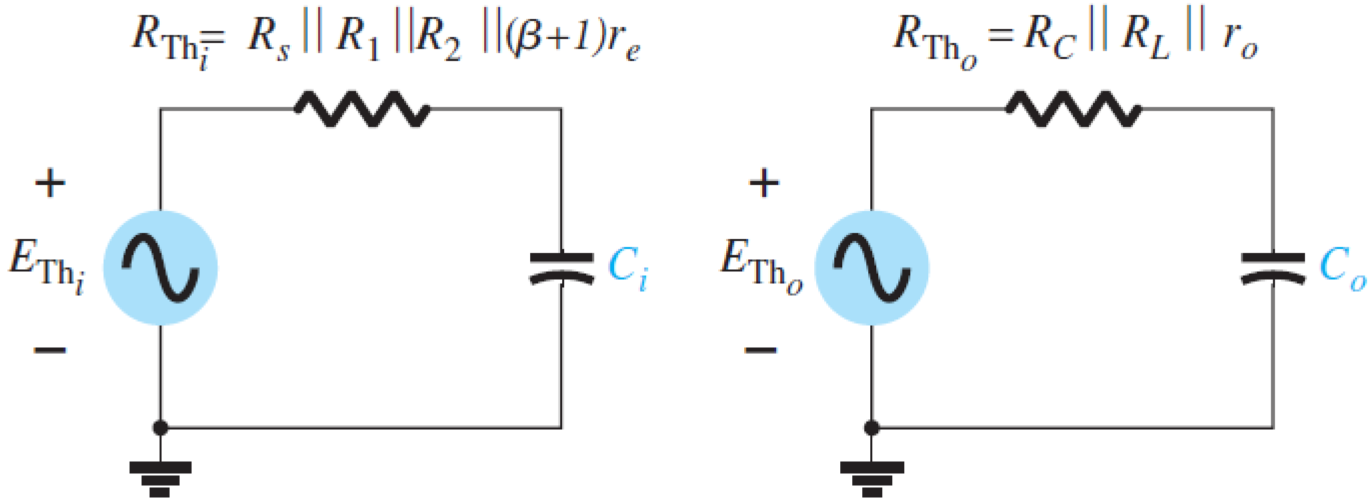
High Frequency Response-BJT

- To determine the high-cutoff frequencies, we must first convert the circuit below to its Thevenin equivalent networks for the input and the output



High Frequency Response-BJT

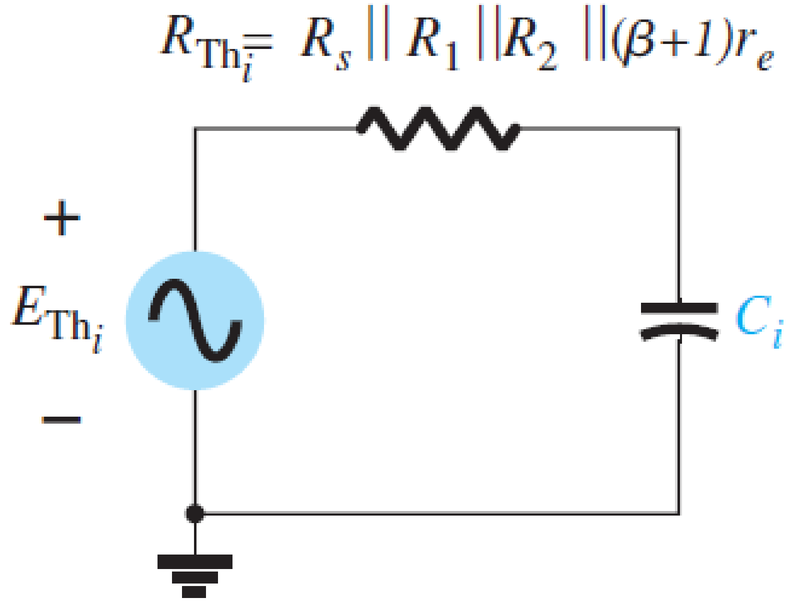
- The circuits below show the input and output Thevenin circuits.



High Frequency Response-BJT

- For the input high frequency response:

$$R_{THi} = R_S \parallel R_1 \parallel R_2 \parallel (\beta + 1)r_e$$
$$C_i = C_{wi} + C_{be} + C_{Mi}$$
$$C_{Mi} = (1 - A_v)C_{bc}$$
$$f_{H_i} = \frac{1}{2\pi R_{Th_i} C_i}$$

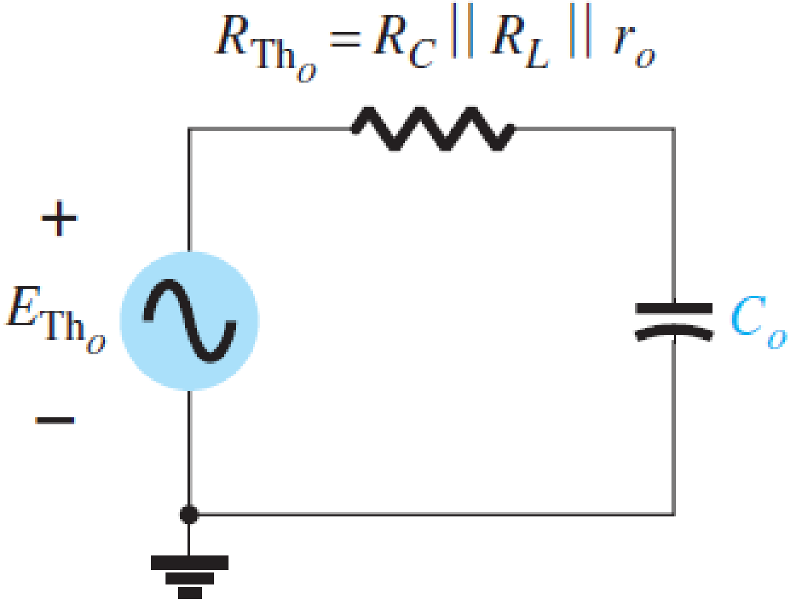


High Frequency Response-BJT

- For the output high frequency response:

$$R_{THo} = R_C \parallel R_L \parallel r_o$$
$$C_o = C_{w_o} + C_{ce} + C_{Mo}$$
$$C_{MO} = \left(1 - \frac{1}{A_v}\right) C_{bc}$$

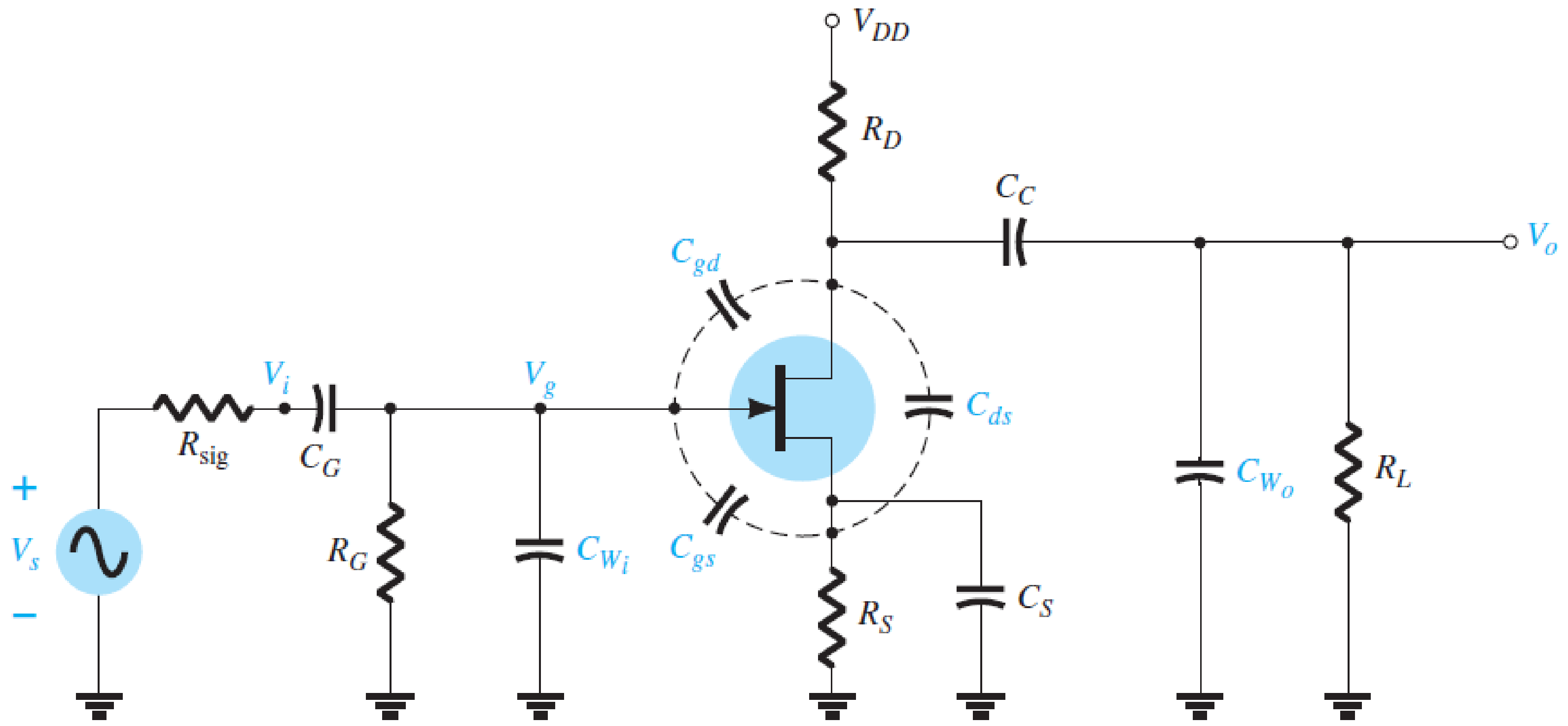
$$f_{Ho} = \frac{1}{2\pi R_{Th_o} C_o}$$



High Frequency Response-FET

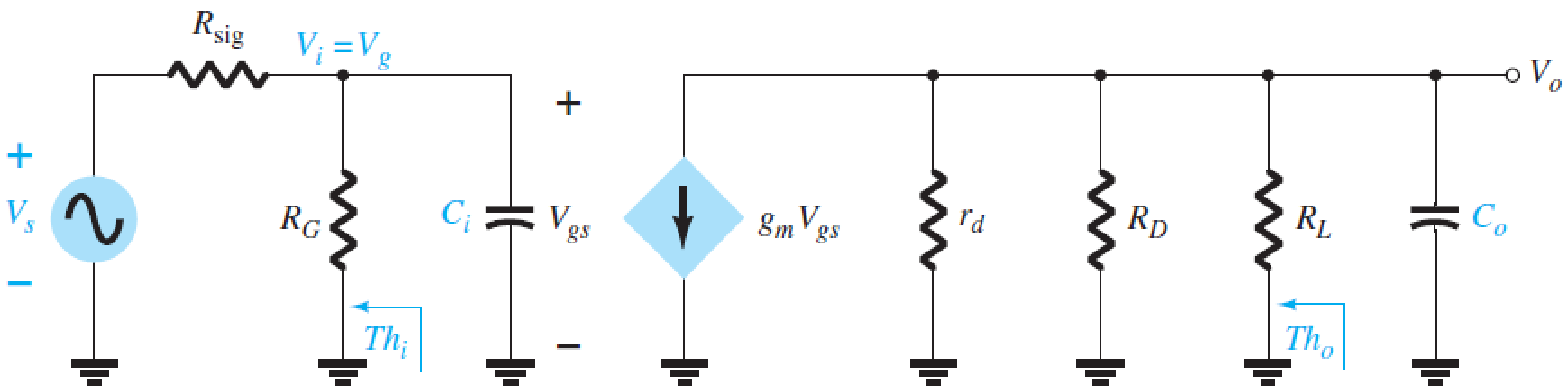
High Frequency Response-FET

- In the figure, the various parasitic capacitance (C_{gs}, C_{gd}, C_{ds}) of the transistor are included with the wiring capacitances.
- C_{gs} and C_{gd} typically vary from 1 to 10 pF while C_{ds} ranges from 0.1 to 1pF



High Frequency Response-FET

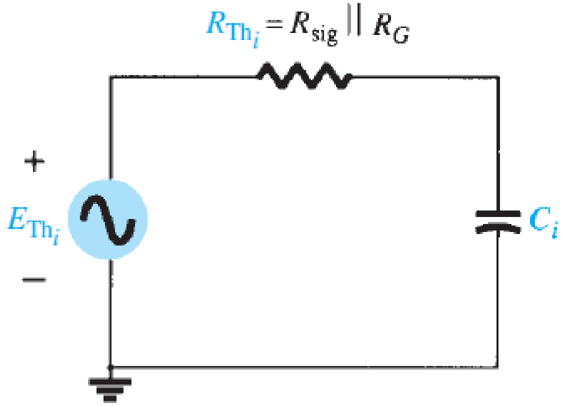
- The process of finding the high frequency response for the FET amplifier is very similar to finding the response for the BJT amplifier.
- The high frequency response AC equivalent of the FET amplifier is shown below.



High Frequency Response-FET

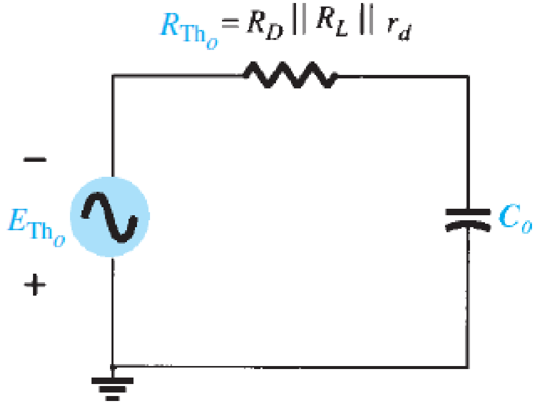
- For the input side:

$$R_{Th_i} = R_{sig} \parallel R_G$$
$$C_i = C_{wi} + C_{gs} + C_{Mi}$$
$$C_{Mi} = (1 - A_V)C_{gd}$$
$$f_{Hi} = \frac{1}{2\pi R_{Th_i} C_i}$$



- For the output side:

$$R_{Th_o} = R_D \parallel R_L \parallel r_d$$
$$C_o = C_{wo} + C_{ds} + C_{Mo}$$
$$C_{Mo} = \left(1 - \frac{1}{A_v}\right) C_{gd}$$
$$f_{Ho} = \frac{1}{2\pi R_{Th_o} C_o}$$



Summary

- The coupling and bypass capacitors of an amplifier affect the low-frequency response.
- The internal transistor capacitances affect the high-frequency response.
- The decibel is a logarithmic unit of measurement for power gain and voltage gain.
- A decrease in voltage gain to 70.7% of midrange value is a reduction of 3 dB.
- A halving of the voltage gain corresponds to a reduction of 6 dB.
- The dBm is a unit for measuring power levels referenced to 1 mW.
- Critical frequencies are values of frequency at which the RC circuits reduce the voltage gain to 70.7% of its midrange value.
- Each RC circuit causes the gain to drop at a rate of 20 dB/decade.
- For the low-frequency RC circuits, the highest critical frequency is the dominant critical frequency.

Summary

- A decade of frequency change is a ten-times change (increase or decrease).
- An octave of frequency change is a two-times change (increase or decrease).
- For the high-frequency RC circuits, the lowest critical frequency is the dominant critical frequency.
- The bandwidth of an amplifier is the range of frequencies between the dominant lower critical frequency and the dominant upper critical frequency.
- The gain-bandwidth product is a transistor parameter that is constant and equal to the unity-gain frequency.
- The dominant critical frequencies of a multistage amplifier establish the bandwidth.
- Two frequency response measurement methods are frequency/amplitude and step.

End