An Evaluation of the Robustness of Indices of Productivity Used for the Management of Data Poor and Knowledge Limited Fish Species.

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Abstract

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- Life history attributes have many usages in stock assessment, e.g. ...
- In data poor situation they are used in ERA/PSA to rank species in order of productivity
- Life history attributes are correlated e.g. L_{50}, k even M with l_{max}
- In data poor situations where data are missing ad-hoc approaches are used to fill in missing values, BMI is an alternative
- Here we propose a method using BMI to develop indices of productivity and evaluate their robustness, i.e. reliability and stability

to select the single variables used for the indicator construction requires to

- analyse the correlation structure of the variables;
- normalise and weight the variables;
- aggregate the single variables to the composite indicator;
- test the robustness and the sensitivity of the composite indicator

1 Introduction

Following the adoption of the precautionary approach (PA, Garcia, 1996) by many fisheries organisations biological reference points have become central to management. Reference points are used as targets to maximise surplus production and limits to minimise the risk of depleting a resource to a level where productivity may be compromised. They must integrate biological processes such as growth, recruitment, mortality and connectivity into indices for productivity and spawning reproductive potential (Kell et al., 2015b) to provide limits and targets for exploitation. They are increasingly required for by-caught, threatened, endangered, and protected species where data and knowledge are limited, not just for the main commercial stocks, where analytical assessments are available (Sainsbury and Sumaila, 2003).

In data poor situations life history parameters, such as maximum size and size at first maturity have been used as proxies for productivity (Roff, 1984; Jensen, 1996; Caddy, 1998; Reynolds et al., 2001; Denney et al., 2002). For example in Ecological Risk Assessment (ERA), where the risk of a stock becoming overfished is evaluated using indices of susceptibility and productivity (Hobday et al., 2011). Susceptibility is derived by estimating the overlap of fishing activity and stock distribution, while life history attributes are combined and used to rank stocks, populations or species in order of productivity (e.g. Corts et al., 2010; Arrizabalaga et al., 2011).

Ranking species using a mixture of attributes is sensitive to the choice of attributes, the weights applied when combining them and the methods used in their derivation. In the case of data poor stocks, life history attributes may not be available for all species, and a variety of ad-hoc approaches have been used to handle missing data, which may introduce noise and bias (Schafer and Graham, 2002), leading to switches in rankings and influence outcomes considerably (Freyer, 2014). Robust rankings, where ranking order is stable, are normally considered to be reliable and trustworthy, and conversely non-robust rankings as unreliable and unstable (Permanyer, 2011). The robustness ranking of a composite index, however, may be due to a redundancy of attributes, e.g. if individual attributes are correlated with each other. In which case it makes little sense to combine them into an index (McGillivray, 1991). Robustness in this case can be both desirable and undesirable simultaneously.

Despite these difficulties robustness is important when providing advice. In statistics, a test is robust if it provides insight despite its assumptions being violated. While in engineering, a robust control system is one that still functions correctly in the presence of uncertainty or stressful environmental conditions (Radatz et al., 1990). To be robust an index should be both reliable and stable. A test is reliable if despite uncertainty it provides an accurate result and stable if despite random error, similar results are produced across multiple trials.

2 Material and Methods

To evaluate the reliability and stability of indices used to rank data poor species in order of productivity a data rich dataset was first compiled. Then a data poor dataset was created by removing values at random from the reference dataset. Indices were calculated using the data poor set and compared to the population parameters based on the reference set for which they are proxies. Reliability and stability were evaluated by comparing rankings based on indices of life history attributes obtained using the data poor dataset with the population parameters from the reference set.

2.1 Data

The reference set was created by selecting species of the order *teleost* from FishBase where parameters for growth, length at maturity and length/weight parameters were available for several species within a family. The resulting reference set comprised eighteen families, namely *Carangidae, Clupeidae, Cyprinidae, Engraulidae, Gadidae, Lutjanidae, Merlucciidae, Mugilidae, Mullidae, Pleuronectidae, Salmonidae, Sciaenidae, Scombridae, Scophthalmidae, Sebastidae, Serranidae, Soleidae* and *Sparidae*; 139 species in total. The data poor dataset was then generated by removing a third of the observations at random.

2.2 Methods

Life history parameters were used to parameterise Von Bertalanffy (1957) growth curves, logistic ogives for proportion mature-at-age and natural mortality-at-age vectors (Lorenzen and Enberg, 2002) for each species. These age based processes were then combined with Beverton and Holt (1993) stock recruit relationship to parameterise an equilibrium production modelSissenwine and Shepherd (1987) and a Leslie matrix (Caswell, 1989). Spawning stock biomass (SSB) was used as a proxy for stock reproductive potential (SRP Trippel, 1999), assuming that fecundity is proportional to the mass-at-age of the sexually mature portion of the population and that processes such as sexual maturity are simple functions of age (Matsuda et al., 1996) and independent of gender.

Five population parameters were then derived to validate data poor indices indices. These were i) population growth rate at low population size (r); ii) population growth rate at SSB_{MSY} ($r_{B_{MSY}}$), where SSB_{MSY} is the expected SSB at a level of fishing mortality (F_{MSY}) that provides the maximum long-term sustainable yield (MSY); iii) the ratio of SSB_{MSY} to virgin biomass (SSB_{MSY}/K); iv) the lifetime reproductive output (LRO Annett and Pierotti, 1999) of a cohort fished at F_{MSY} relative to an unfished cohort ($LRP_{F_{MSY}} : LRP_{F_0}$); and v) the size at which a year-class achieves its maximum biomass (L_{opt}).

Since r is a level of exploitation that would drive a population to extinction it is equivalent to the fisheries limit reference point F_{crash}. While $r_{B_{MSY}}$ is equivalent to F_{MSY}, which was previously a target reference point but it as now regarded as a limit since if exploitation is greater than F_{MSY} economic targets will not be met. SSB_{MSY}/K provides an indicator of how well productivity will be maintained at low population levels.

Limit reference points are also required to prevent recruitment and growth overfishing. If $LRP_{F_{MSY}} : LRP_{F_0}$ is much less than 1.0 then maternal effects may reduce spawning reproductive potential (SRP Kell et al., 2015b) and so is a measure of resilience to recruitment overfishing; while L_{opt} is a measure of resilience to growth overfishing.

Since population parameters are sensitive to the assumptions about natural mortality-at-age (Jiao et al., 2012), vulnerability of age classes to the fisheries (Brooks et al., 2009), and the relationship between stock and recruitment is difficult to estimate in practice (e.g. Szuwalski et al., 2014; Pepin, 2015). Therefore four scenarios (comprising four factors with two levels) were considered corresponding to i) the shape of the selection pattern (dome shaped or flat topped), ii) whether juveniles were vulnerable to fishing, iii) M modelled as either a single value or varied by age and iv) the steepness of the stock recruitment relationship.

2.2.1 Imputation

The data poor dataset was generated by removing a third of the data points at random. Values were k and the asymptotic length L_∞ of the the Von Bertalanffy (1957) growth equation, L_{50} the length at which 50% of individuals attain maturity for the first time and b the exponent of the length weight relationship. This procedure was repeated 100 times and Multiple imputation (MI, Rubin, 2004) used to fill in the missing entries.

Imputation involves drawing values from a posterior distribution, which reflects the uncertainty surrounding the parameters of the distribution that generated the data. It therefore simulates both the process generating the data and the uncertainty associated with the parameters. Rubin (1987) showed that if the method to create the imputations is 'proper', then the resulting inferences will be statistically valid. Multiple imputations are said to be proper if the MI estimates $\hat{Q}_{MI}$ are asymptotically normal with mean $\hat{Q}$ and a consistent variancecovariance estimate B; and within-imputation variance estimate (W) is a consistent estimate of the variancecovariance estimate (U) with variability of a lower order than $Var(\hat{Q}_{MI})$.

Species were ranked by each of the five population parameters for the 100 datasets in turn and compared to ranks from the reference set using Spearmans rank correlation coefficient (ρ Spearman, 1904). ρ is a non-parametric measure of dependence between two variables, a correlation coefficient of +1 or 1 for two variables indicates that there is a perfect monotonic relationship between them. Analysis of the imputed data is simpler than the same analysis without imputation since there is no need to bother with the missing data.

Two index were consider $index_5$ which combined k, L_{50}, L_∞, L_{50}/L_∞ and b and $index_3$ which combined k, L_{50} and L_∞. The benefit of improving data by targeted studies, was evaluated by creating four datasets where missing values (for k, L_∞, L_{50} and b) were replaced with the values from the reference.

Principal Component Analysis (PCA, Dunteman, 1989) was used to summarise the reference set life history and population parameters. PCA assumes that components with larger variance correspond to the interesting dynamics and lower ones to noise. The first principal axis is the one which maximizes the variance, as reflected by its eigenvalue. The second component is orthogonal to the first and maximizes the remaining variance.

3 Results

The life history parameters (k, L_{50}, L_∞, L_{50}/L_∞ and b)) are summarised in **Figure** 1 using PCA. The first two components account for about 70% of the variance and so yield a good approximation of the original variables; ellipses show the 0.95% normal probability densities for there contrasting families Clupeidae (blue), Scombridae (red) and Sebastidae (green). The first component contrasts large and small species, i.e. those that reach a large size (L_∞) with smaller species that are fast growing (k) and mature early (L_{50}/L_∞). The second component summarises body shape (b). For example *Clupeidae* are fast growing, early maturing and thin; in contrast to *Sebastidae* which are late maturing slow growing and have a compact body shape.

The population parameters derived from the reference set are summarised in **Figure** 2 using PCA. The first component contrasts population growth rate at B_{MSY} (rc) with the shape of the production function (sk); i.e. fish that are resilient to exploitation (e.g. *Clupeidae*) with those with low population growth rates that can easily become depleted (e.g. *Sebastidae*). The second component summarises species which are susceptible to growth overfishing with those that can be harvested at small size. r and life time reproductive output at B_{MSY} lro are orthogonal, which implies that population growth rate is not necessarily related to total egg production.

In the absence of data on egg production and other processes it is difficult to fully interpret the population parameters. Therefore scenarios were run to evaluate the sensitivity of the results to parameters that are difficult to measure, namely the shape of the selection pattern (dome shaped or flat topped), whether adults or juveniles were subjected to fishing, if M varied by age and the steepness of the stock recruitment relationship. Figure 3 showed that the selection pattern (age classes exploited by the fishing) had little effect when juveniles were considered but had an effect when mature fish were considered. Reducing steepness redcued population growth rates, assuming M was constant at age increased population growth rates and reduced L_{opy}.

Figure 5 summarises the Spearman rank correlations (ρ) between the data poor indices and the life history parameters singularly (columns) and the reference set population parameters (rows). A reliable index is one that shows a value of ρ close to 1, while a stable index is one where the variability is small. The boxplot hinges correspond to the inter quartile range, while the whiskers extends from the hinges to the values that are within 1.5 times the interquartile. An index that is robust to uncertainty about processes will not vary by scenario. The scenarios are shown within a panel; tirst eight scenarios are for for an M vector that does not vary by age and the last eight for the Gislason form. Odd scenarios are for a steepness of 0.75 and even for a steepness of 0.9.

The benefit of reducing uncertainty by collecting more data is evaluated in Figure 4, and combines all the scenarios into a single box plot.

4 Discussion and Conclusions

Uncertainty In fisheries science and management uncertainties are pervasive due to imperfect information, the natural variability of aquatic ecosystems and lack of perfect control over fisheries Peterman (2004). To ensure that the risk of failing to meet management objectives is low requires a consideration of uncertainty (Kell et al., 2015a).

Risk Risk is an uncertainty that matters. What matters depends on management and conservation objectives, and whether objectives are achieved depends on the management framework. Depending on the level of uncertainty different procedures may be used to derive reference points for use in management (Reuter et al., 2010).

Management Frameworks In data rich cases where an analytical stock assessment is available and a clear relationship between recruitment and spawning stock biomass is evident and growth, natural mortality (M), maturity and selectivity are known then maximum sustainable yield (MSY) based reference may be used. If the stock recruitment relationship is uncertain then per-recruit reference points are used instead. While for information and data poor situations life history attributes are combined in to index and used to rank in order of productivity.

Robustness An aim of the study was to evaluate the robustness of productivity indices used in data poor situations. An index is robust if it provides insight despite the assumptions being violated. Therefore to evaluate robustness we generated a data poor dataset from a data rich reference set; alternative data poor indices were then calculated and compared them to reference points and population parameters derived from the reference set. This was done for a range of scenarios to evaluate the sensitivity of the indices to parameters that are difficult to measure such as natural mortality and the relationship between recruitment and stock.

Imputation was used to create a database of life history attributes, filling in missing values, before calculating the indices. Imputation is usually the most challenging step since it must account for the process that created the missing data. Namely the data must be missing completely at random. Analysis of the imputed data is simpler than the same analysis without imputation since there is no need to bother with missing data. The final step pooling consists of computing statics. In this case the statistic used was Spearman correlation to compare the reliability of ranks based on the indices and imputation used to evaluate the stability of the ranks.

Lessons for data poor case studies Knowledge about the structure is helpful in identify the mechanism which generated the missing data and for aiding in selection of an appropriate imputation method to estimate missing values (Templ et al., 2012). For example estimates of L_∞ and k are often correlated owing to a lack of large old fish in samples, and so only the product of the two can be estimated with reasonable precision (Gislason et al., 2010).

Lessons for data rich case studies An Understanding of population rynamics is important for stock assessment, since data rich stock assessments often rely upon life history parameters for deriving priors for difficult to estimate population parameters (Lee et al., 2011, 2012; Jiao et al., 2012; Simon et al., 2012). Life history parameters are also a major input into ecological risk assessments (ERA) used to prioritise management action (Suter II, 2006; Corts et al., 2010) and ecological models used to help develop an ecosystem based approach to fisheries management (Thorson et al., 2012).

https://www.ramas.com/CMdd.htm

5 Conclusions

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6 Acknowledgement

This study does not necessarily reflect the views of ICCAT and in no way anticipates the Commission's future policy in any area.

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7 Figures

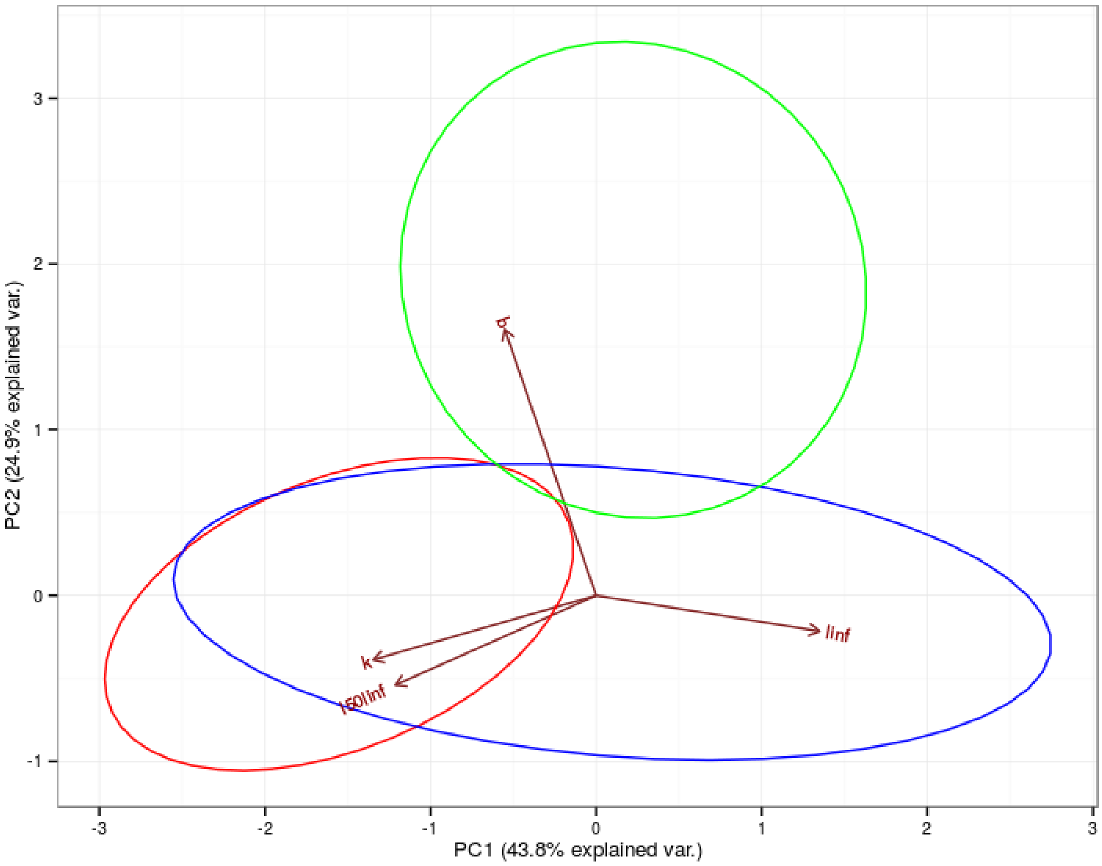


Figure 1: Biplots of the first two principal component of the reference set life history parameters, 95% normal probability densities are shown for Clupeidae (blue), Scombridae (red) and Sebastidae (green).

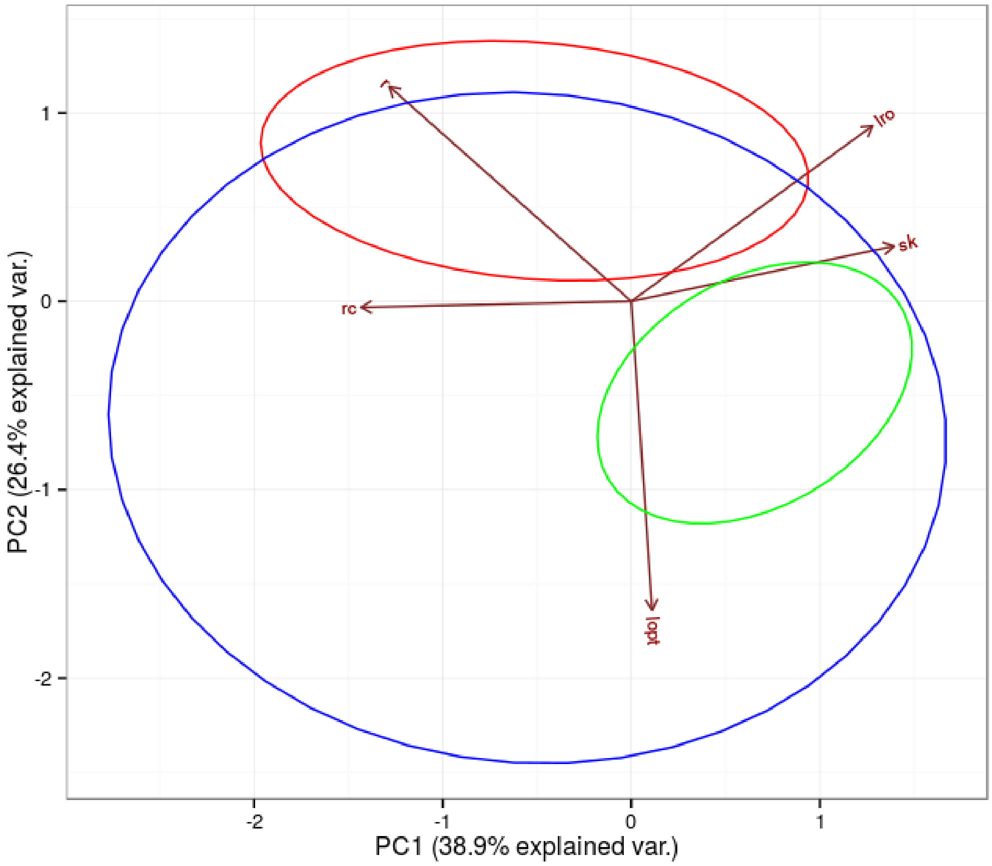


Figure 2: Biplots of the first two principal component of the reference set population parameters, 95% normal probability densities are shown for Clupeidae (blue), Scombridae (red) and Sebastidae (green).

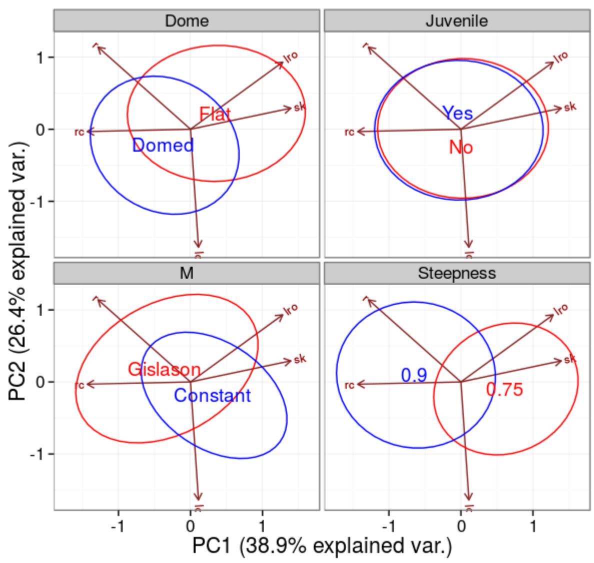


Figure 3: Biplots of the first two principal component of the reference set population parameters with 30% normal probability densities for scenario factors and levels.

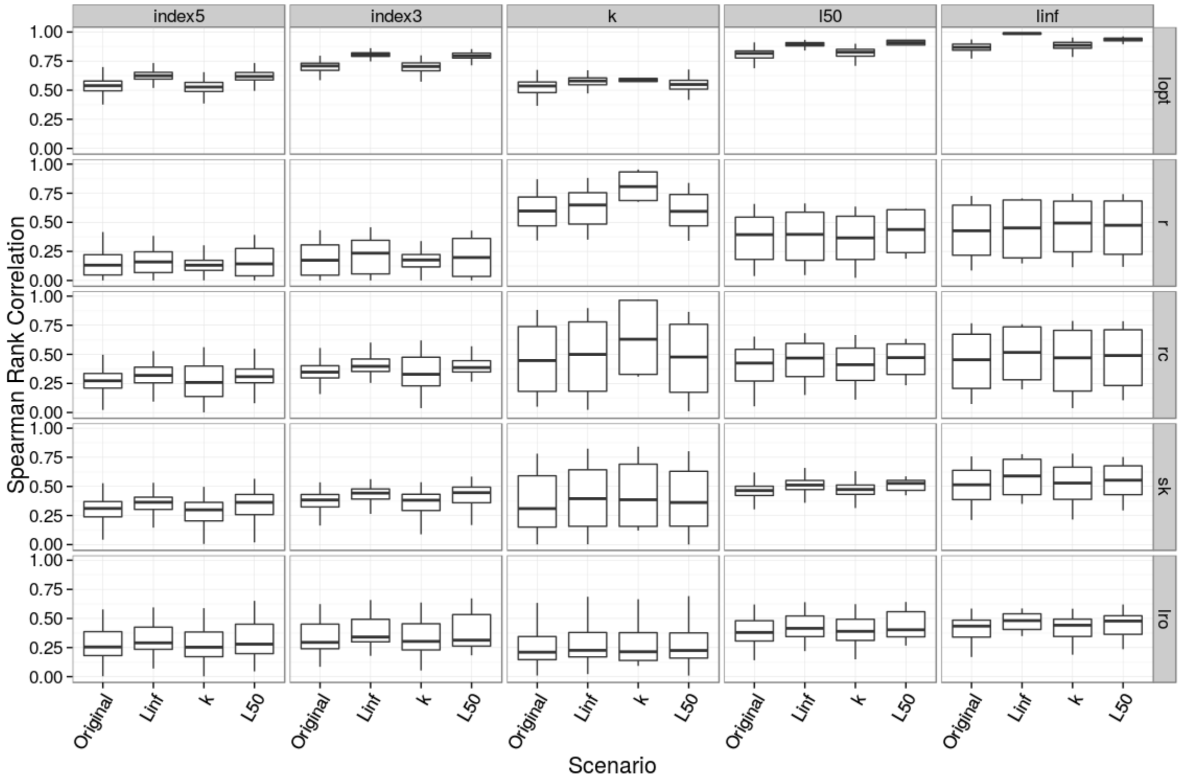
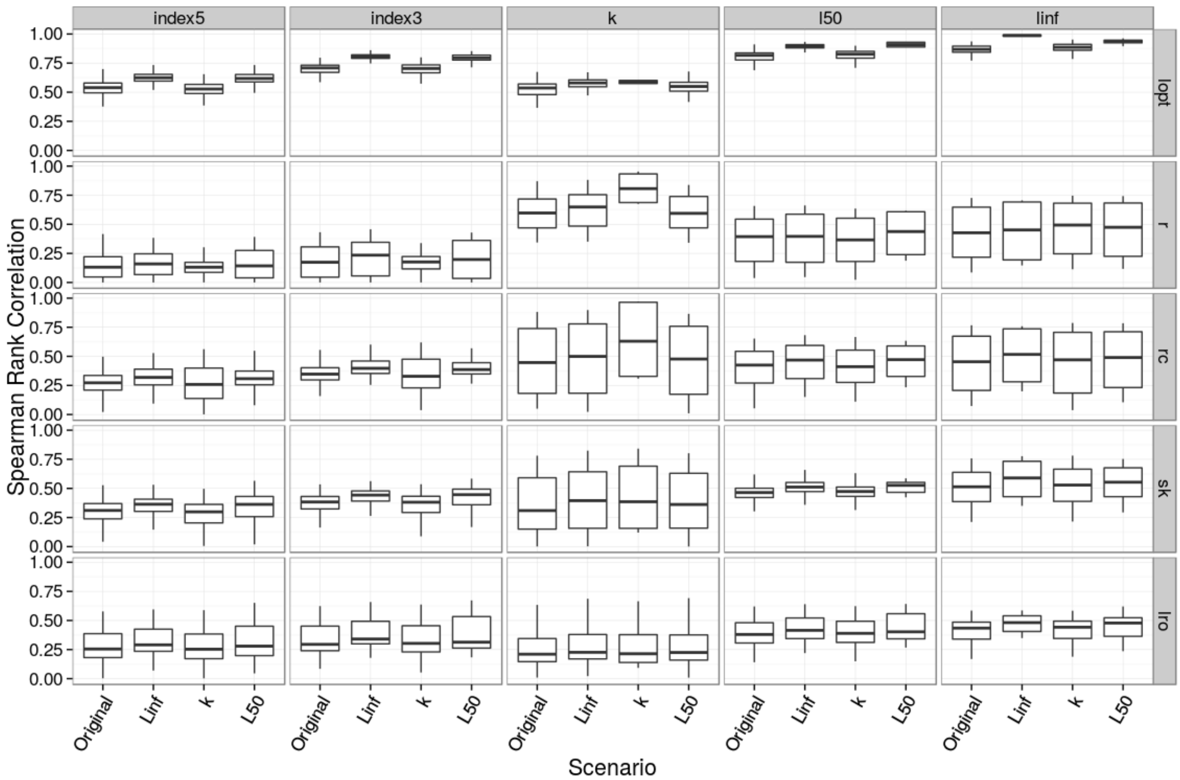


Figure 4: Spearman rank correlation coefficients between life history parameters and indices and population parameters for the data poor set; boxplots by scenarios. The upper and lower boxplot hinges correspond to the first and third quartiles (the 25^{th} and 75^{th} percentiles), while the whiskers extends from the hinge to the value that is within 1.5 times the interquartile range of the hinge.

Figure 5: Spearman rank correlation coefficients between life history parameters and indices and population parameters for the data poor set; boxplots by augmented dataset. The upper and lower boxplot hinges correspond to the first and third quartiles (the 25^{th} and 75^{th} percentiles), while the whiskers extends from the hinge to the value that is within 1.5 times the interquartile range of the hinge.



8 Appendix

8.1 Population Growth Rate

The maximum theoretical rate of increase of a population in the absence of density-dependent regulation is given by

$$r = \frac{dN}{dt}\frac{1}{N} \tag{1}$$

where the intrinsic population growth rate (r) is a function of population size (N) and $\frac{dN}{dt}$ the instantaneous rate of increase of the population.

In ecology Leslie matrices (Leslie, 1945) are widely used to estimate r (Picard et al., 2009) and have been used in fisheries to develop Bayesian priors for r for use in stock assessment (see McAllister et al., 2001). The Leslie Matrix (A) is a transition matrix that models age dynamics. Each age-class is described by a vector (B_t) of length p equal to the terminal age. Entries in the matrix are fecundity (f_i) (the quantity of age zero females produced per unit of mature biomass by each age-class) and the survival (and growth if biomass) of an age-class (s_i) in each time step i, i.e.

$$\begin{aligned} n_1 &= f_2 n_2 + ... + f_3 n_p \\ n_2 &= s_1 n_1 \\ &... \\ n_p &= s_{p-1} n_{p-1} + s_p n_p \end{aligned} \tag{2}$$

The matrix of this linear system is

$$\mathbf{A} = \begin{pmatrix} 0 & f_2 & ... & f_p \\ s_1 & 0 & ... & 0 \\ & ... & & \\ 0 & ... & s_{p-1} & s_p \end{pmatrix} \tag{3}$$

If the initial population is

$$\mathbf{B}^0 = \begin{pmatrix} n_1 \\ n_2 \\ ... \\ n_p \end{pmatrix} \quad (4)$$

then after time step i=1 the population is given by

$$\mathbf{B}^i = \mathbf{A}^i \mathbf{B}^0 \quad (5)$$

As i tends to infinity the system reaches equilibrium and the contribution of each age group in the population becomes stable. The population growth rate r is then derived from λ the dominant eigenvalue of A (Caswell, 1989).

To construct the Leslie matrix requires estimates of f_i and p_i. In this study these were derived by combining a stock recruitment relationship with a spawner-per-recruit (S/R) and yield-per-recruit (Y/R) analyses. The life history parameters were used to derive mass (W), proportion mature (Q), natural mortality (M) and fishing mortality (F) at age.

$$S/R = \sum_{i=0}^{p-1} e^{\sum\limits_{j=0}^{i-1} -F_j - M_j} W_i Q_i + e^{\sum\limits_{i=0}^{p-1} -F_i - M_i} \frac{W_p Q_p}{1 - e^{-F_p - M_p}} \quad (6)$$

$$Y/R = \sum_{a=r}^{n-1} e^{\sum\limits_{i=r}^{a-1} -F_i - M_i} W_a \frac{F_a}{F_a + M_a} \left(1 - e^{-F_i - M_i}\right) + e^{\sum\limits_{i=r}^{n-1} -F_n - M_n} W_n \frac{F_n}{F_n + M_n} \quad (7)$$

The second term is the plus-group, i.e. the summation of all ages from the last age to infinity.

Growth in length is modelled by the Von Bertalanffy growth equation **?**

$$L = L_\infty (1 - exp(-k(t - t_0)) \quad (8)$$

where k is the rate at which the rate of growth in length declines as length approaches the asymptotic length L_∞ and t_0 is the hypothetical time at which an individual is of zero length.

Length is converted to mass using the length-weight relationship

$$W = aL_t^b \tag{9}$$

where a is the condition factor and b is the allometric growth coefficient.

Gislason et al. (2010) showed that M is significantly related to body length, asymptotic length and k. Temperature is non-significant when k is included, since k itself is correlated with temperature. We therefore model M as

$$M = 0.55L^{1.61}L_\infty^{1.44}k \tag{10}$$

Selection pattern of the fishery was represented by a double normal (see ?)) with three parameters that describe the age at maximum selection ($a1$), the rate at which the left-hand limb increases (sl) and the right-hand limb decreases (sr) which allows flat topped or domed shaped selection patterns to be chosen.

$$f(x) = \begin{cases} 0 & \text{if } (a_{50} - x)/a_{95} > 5 \\ a_\infty & \text{if } (a_{50} - x)/a_{95} < -5 \\ \frac{m_\infty}{1.0+19.0^{(a_{50}-x)/95)}} & \text{otherwise} \end{cases} \tag{11}$$

The relationship between stock and recruitment was modelled by a Beverton and Holt stock-recruitment relationship (Beverton and Holt, 1993) reformulated in terms of steepness (h), virgin biomass (v) and $S/R_{F=0}$

$$R = \frac{0.8R_0h}{0.2S/R_{F=0}R_0(1-h) + (h-0.2)S} \tag{12}$$

where steepness is the ratio of recruitment at 20% of virgin biomass to virgin recruitment (R_0) and $S/R_{F=0}$ is the spawner per recruit at virgin biomass, i.e. when fishing mortality is zero. Steepness is difficult to estimate from stock assessment data sets as there is often insufficient contrast in biomass levels required for its estimation Pepin (2015).

S is spawning stock biomass, the sum of the products of the numbers of females, N, proportion mature-at-age, Q and their mean fecundity-at-age, F, i.e.

$$S = \sum_{i=0}^{p} N_i Q_i F_i \tag{13}$$

where fecundity-at-age is assumed proportional to biomass and the sex ratio to be 1:1. Proportion mature is 50% at the age that attains a length of $l50$, 0% below this age and 100% above.