No target

The purpose of this document is to examine equilibria for coupled biomass-catch systems. The first system is given by[1]

$$\frac{dB}{dt} = rB\left(1 - \frac{B}{K}\right) - C \tag{1}$$

$$\begin{aligned}\frac{dC}{dt} &= kC\frac{dB}{dt} \\ &= kC\left(rB\left(1 - \frac{B}{K}\right) - C\right)\end{aligned} \tag{2}$$

Equilibrium points occur on the intersection of the biomass and catch null-clines. To obtain the non-trivial nullcline for biomass

$$\begin{aligned}rB\left(1 - \frac{B}{K}\right) - C &= 0 \\ C &= rB\left(1 - \frac{B}{K}\right)\end{aligned} \tag{3}$$

To obtain the non-trivial nullcline for catch

$$\begin{aligned}kC\left(rB\left(1 - \frac{B}{K}\right) - C\right) &= 0 \\ C &= rB\left(1 - \frac{B}{K}\right)\end{aligned} \tag{4}$$

As the nullclines (3) and (4) are equal, there exists an infinite number of equlibria for this system depending on the starting values for the two states (Figure 1). The effect of this rule is to stabilise biomass close to the starting value.

[1]Continuous analogue to difference equation for catch of $C_{t+1} = C_t(1 + k\lambda)$, where λ is the rate of change of B over some time period, above we assume that it is instantaneous.

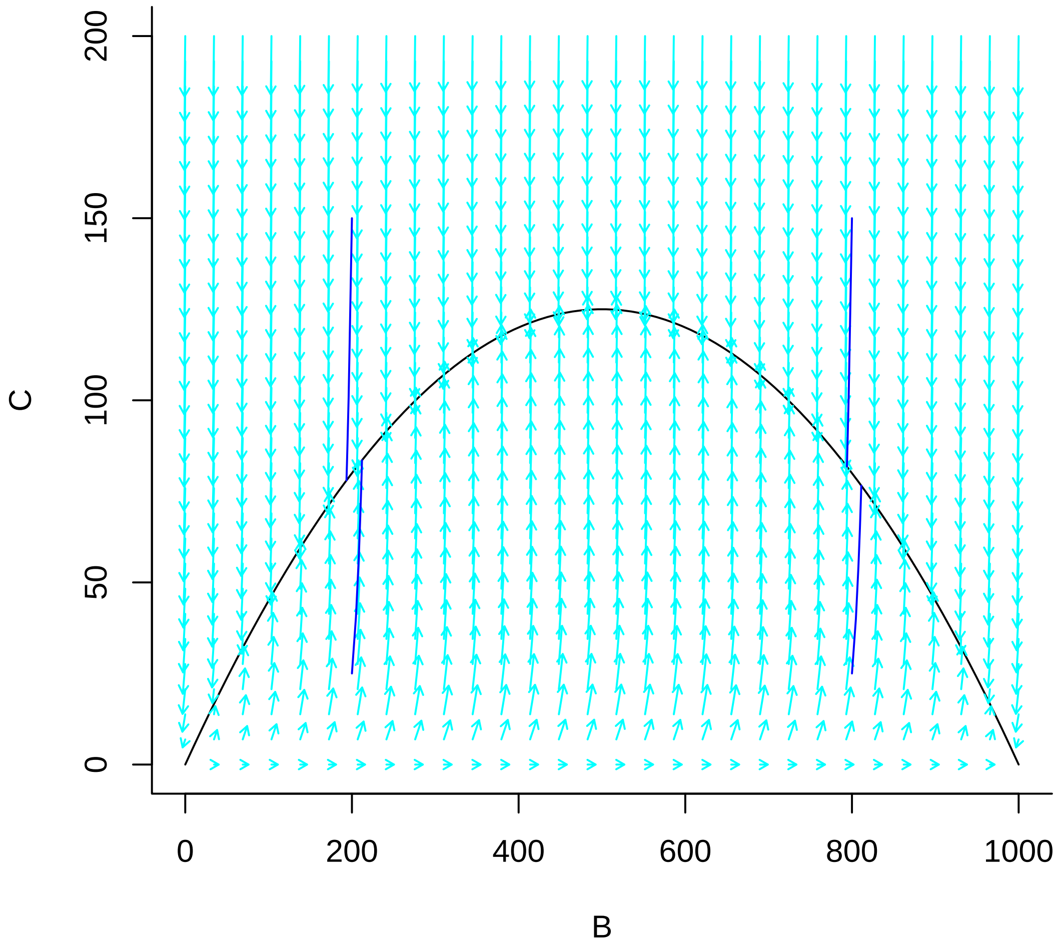

Figure 1: Directional field for no-target system given by Equations (1) and (2) with parameter set $\{r = 1/2, K = 10^3, k = 1/10\}$. Example solutions are shown in blue.

Target

There are many possible targets, here the difference between the true F_{MSY} and current F is used as an example. Consider the system

$$\frac{dB}{dt} = rB\left(1 - \frac{B}{K}\right) - C \tag{5}$$

$$\frac{dC}{dt} = kC\left(\frac{r}{2} - \frac{C}{B}\right) \tag{6}$$

Two of the nullclines for this model are given by

$$C = rB\left(1 - \frac{B}{K}\right) \tag{7}$$

$$C = B\frac{r}{2} \tag{8}$$

These intersect at point $(K/2, rK/4)$. However the phase plane for this system does not always equilbrate to the MSY values (Figure 2).

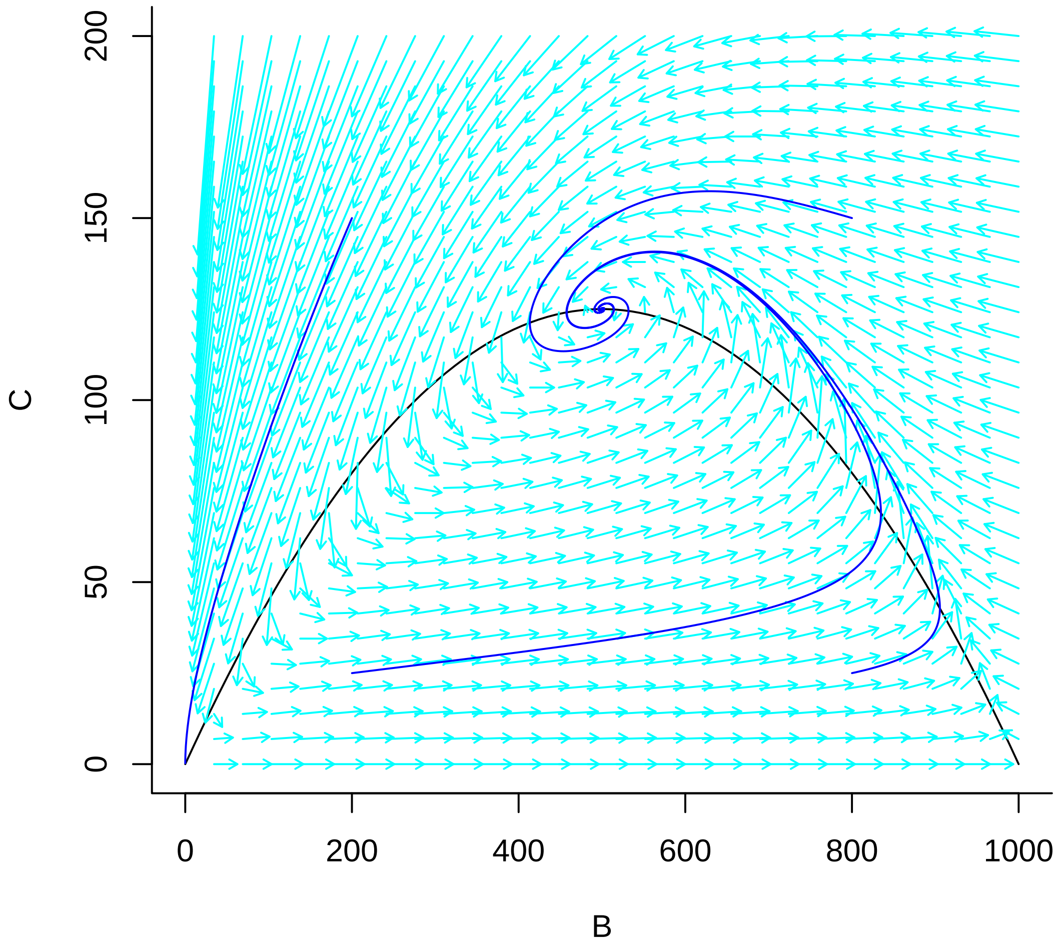

Figure 2: Directional field for target system given by Equations (5) and (6) with parameter set $\{r = 1/2, K = 10^3, k = 1/10\}$. Example solutions are shown in blue.