$$\partial_x A^2 = -P' \underbrace{\frac{2x x_R^2}{(x^2+x_R^2)^2}}_{f(x)} \underbrace{\left[1 - \frac{2r^2 x_R^2}{w_0^2(x^2+x_R^2)}\right]}_{g(x)} \underbrace{\exp\left(-\frac{2r^2 x_R^2}{w_0^2(x^2+x_R^2)}\right)}_{h(x)}$$

$$f(x) = f(a) + f'(a)(x-a) + \frac{1}{2!} f''(a)(x-a)^2 + \dots$$

$$f'(x) = \frac{(x^2+x_R^2)\,2x_R^2 - 8x^2x_R^2}{(x^2+x_R^2)^3}$$

$$g'(x) = \frac{8r^2x_R^2 x}{w_0^2(x^2+x_R^2)^2}$$

$$h'(x) = g'(x)h(x)$$

vakio pois

$$f''(x) = -\frac{8x_R^2 x}{(x^2+x_R^2)^5} - \frac{16x x_R^2(x^2+x_R^2) - 48x^3x_R^2}{(x^2+x_R^2)^3}$$

$$g''(x) = \frac{8r^2x_R^2(x^2+x_R^2) - 32r^2x_R^2x^2}{w_0^2(x^2+x_R^2)^3}$$

$$h''(x) = g''(x)h(x) + g'(x)^2 h(x)$$

$$\partial_x A^2 = \partial_x A(a) + (x-a)\left[f'(a)g(a)h(a) + g'(a)f(a)h(a) + g'(a)h(a)g(a)f(a)\right]$$

$$+ \frac{1}{2!}\underbrace{(x-a)^2}_{\tilde{x}^2}\Big[f''(a)g(a)h(a) + f'(a)g'(a)h(a) + f'(a)g(a)h'(a)$$

$$+ g''(a)f(a)h(a) + g'(a)f'(a)h(a) + g'(a)^2 f(a)h(a)$$

$$+ g''(a)h(a)f(a)g(a) + g'(a)^2 h(a)f(a)g(a) + g'(a)^2 h(a)f(a)$$

$$+ g'(a)h(a)g(a)f'(a)\Big] + \dots$$

minimi pois

koord muutos $\to$ $x \to x - x_0$

Saadaan $p \approx -m\Omega^2 x + \dots$

$$V = \frac{1}{2}\hat{\Omega}^2 m \tilde{x}^2 \qquad \hat{\Omega}^2 = \frac{1}{m}\frac{\partial^2 V}{\partial x^2}\Big|_{x=x_0}$$

Tarkista plotatessa $V = \frac{1}{2}\hat{\Omega}^2 m \tilde{x}^2$ että saa oikean muodon