



**CATOLICA**  
ESCOLA SUPERIOR  
DE BIOTECNOLOGIA  
PORTO

**Processamento de Sinal e Imagem**  
1º ano, 1º Semestre, Mestrado em Engenharia  
Biomédica

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**Exercises sheet 2**  
**Continuous Signals Sampling**

1. State under what conditions a continuous signal  $x_c(t)$  can be represented by a discrete signal  $x[n]$  obtained by sampling  $x_c(t)$ .
2. Consider the continuous signal  $x_c(t) = \cos(4000\pi t)$ , which was sampled with a sampling period  $T_s = 1/6000$ s.
  - a. What is the angular frequency of the signal  $\Omega_0$
  - b. What is the digital angular frequency of the signal  $w_0$
  - c. What is the angular frequency of sampling  $\Omega_s$
  - d. Was the sampling theorem fulfilled?
  - e. What is the expression of the resulting sampled signal  $x[n]$ ?
3. The continuous signal  $x_c(t) = \sin(2\pi \cdot 100t)$  was sampled with the sampling period  $T_s = 1/400$ s. What is the expression of the resulting discrete signal  $x[n]$ ?
4. The signal discrete sequence  $x[n] = \cos(\pi n/4)$ ,  $-\infty < n < +\infty$ , was obtained per sampling the follow signal:

$$x_c(t) = \cos(\Omega_0 t) \quad -\infty < t < +\infty$$

At the sampling frequency 1000 samples/s. What are the possible positive values of  $\Omega_0$  that may have resulted in  $x[n]$ ?

5. The continuous signal  $x_c(t) = \cos(4000\pi t)$  was sampled with a sampling period  $T_a$  and it results in the follow discrete signal:

$$x[n] = \cos\left(\frac{\pi n}{3}\right)$$

- a. Determine a consistent  $T_a$ .
  - b. It is the  $T_a$ 's choice of the previous paragraph unique? If so, explain why, otherwise determine another value for  $T_a$  also consistent.
6. Consider the follow discrete signals  $x_1(t)$  e  $x_2(t)$  such that:

$$|\Omega| > \Omega_1 \Rightarrow X_1(j\Omega) = 0$$

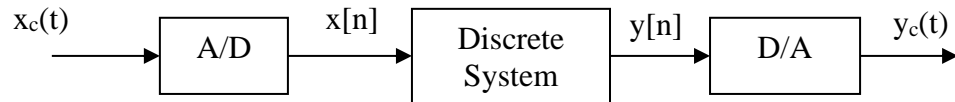
$$|\Omega| > \Omega_2 \Rightarrow X_2(j\Omega) = 0$$

$$\Omega_2 > \Omega_1$$

Determine the minimum sampling frequency required to represent  $x(t)$  in the cases:

- a.  $x(t) = x_1(t) + x_2(t)$
- b.  $x(t) = x_1(t) \cdot x_2(t)$

7. Consider the system of the figure with the discrete system being an ideal low pass filter with a cutoff frequency  $\pi/8$  rad.



- a. If  $x_c(t)$  is limited to the frequency of 5 kHz, what is the maximum value of  $T_a$  to avoid aliasing in the A / D converter?
- b. If  $1/T_a = 10$  kHz what should be the effective cutoff frequency of the filter?