Derivation

Initial equation

$$SP_t = \sum_{\tau=1}^{\infty} \beta^{\tau} \frac{E_t\Lambda_{t+\tau}}{\Lambda_t} E_t Y_{t+\tau} \tag{1}$$

$$= \beta \frac{E_t\Lambda_{t+1}}{\Lambda_t} E_t Y_{t+1} + \sum_{\tau=2}^{\infty} \beta^{\tau} \frac{E_t\Lambda_{t+\tau}}{\Lambda_t} E_t Y_{t+\tau}. \tag{2}$$

the canoncial form of the second term is

$$\sum_{\tau=2}^{\infty} \beta^{\tau} \frac{E_t\Lambda_{t+\tau}}{\Lambda_t} E_t Y_{t+\tau} = \beta^2 \frac{E_t\Lambda_{t+2}}{\Lambda_t} E_t Y_{t+2} + \beta^3 \frac{E_t\Lambda_{t+3}}{\Lambda_t} E_t Y_{t+3} + \dots \tag{3}$$

$$= \beta \frac{E_t\Lambda_{t+1}}{E_t\Lambda_{t+1}} \frac{1}{\Lambda_t} * [\beta E_t\Lambda_{t+2} E_t Y_{t+2} + \beta^2 E_t\Lambda_{t+3} E_t Y_{t+3} + \dots] \tag{4}$$

$$= \beta \frac{E_t\Lambda_{t+1}}{\Lambda_t} \underbrace{[\beta \frac{E_t\Lambda_{t+2}}{E_t\Lambda_{t+1}} E_t Y_{t+2} + \beta^2 \frac{E_t\Lambda_{t+3}}{E_t\Lambda_{t+1}} E_t Y_{t+3} + \dots]}_{E_t SP_{t+1}} \tag{5}$$

Finally, we have

$$SP_t = \sum_{\tau=1}^{\infty} \frac{\beta^{\tau} E_t\Lambda_{t+\tau}}{\Lambda_t} E_t Y_{t+\tau} \tag{6}$$

$$= \beta \frac{E_t\Lambda_{t+1}}{\Lambda_t} E_t Y_{t+1} + \beta \frac{E_t\Lambda_{t+1}}{\Lambda_t} E_t SP_{t+1} \tag{7}$$

Steady-state values are

$$SP_{ss} sp_t = \beta \frac{\Lambda_{ss}}{\Lambda_{ss}} Y_{ss}(E_t\lambda_{t+1} - \lambda_t + E_t y_{t+1}) + \beta \frac{\Lambda_{ss}}{\Lambda_{ss}} SP_{ss}(E_t\lambda_{t+1} - \lambda_t + E_t sp_{t+1}) \tag{8}$$

$$sp_t = \beta \frac{Y_{ss}}{SP_{ss}} * (E_t\lambda_{t+1} - \lambda_t + E_t y_{t+1}) + \beta(E_t\lambda_{t+1} - \lambda_t + E_t sp_{t+1}) \tag{9}$$

where

$$SP_{ss} = \sum_{\tau=1}^{\infty} \beta^{\tau} Y_{ss} = \frac{\beta}{1-\beta} Y_{ss}. \tag{10}$$

Using this gives

$$sp_t = (1-\beta)(E_t\lambda_{t+1} - \lambda_t + E_t y_{t+1}) + \beta(E_t\lambda_{t+1} - \lambda_t + E_t sp_{t+1}) \tag{11}$$

$$sp_t = (1-\beta) E_t y_{t+1} + \beta E_t sp_{t+1} + E_t\lambda_{t+1} - \lambda_t \tag{12}$$

Corrected (?) Derivation

Equation (47) in Nakamura and Steinsson (2018a)

$$\frac{\Lambda_{t+j}}{\Lambda_t} = \frac{M_{t,t+j}}{\beta^j}\frac{P_{t+j}}{P_t} \tag{13}$$

$$\Leftrightarrow \quad M_{t,t+j} = \beta^j \frac{\Lambda_{t+j}}{\Lambda_t}\frac{P_t}{P_{t+j}}. \tag{14}$$

Initial equation

$$SP_t = \sum_{\tau=1}^{\infty} \beta^{\tau} \frac{E_t \Lambda_{t+\tau}}{\Lambda_t} \frac{P_t}{E_t P_{t+\tau}} E_t Y_{t+\tau} \tag{15}$$

$$= \sum_{\tau=1}^{\infty} E_t M_{t,t+\tau} E_t Y_{t+\tau} \tag{16}$$

$$= \beta \frac{E_t \Lambda_{t+1}}{\Lambda_t} \frac{E_t P_{t+1}}{P_t} E_t Y_{t+1} + \beta \frac{E_t \Lambda_{t+1}}{\Lambda_t} \frac{E_t P_{t+1}}{P_t} E_t SP_{t+1} \tag{17}$$

Steady-state values are

$$SP_{ss} sp_t = \beta \frac{\Lambda_{ss}}{\Lambda_{ss}} \frac{P_{ss}}{P_{ss}} Y_{ss}(E_t \lambda_{t+1} - \lambda_t + E_t y_{t+1} - E_t \pi_{t+1}) + \beta \frac{\Lambda_{ss}}{\Lambda_{ss}} \frac{P_{ss}}{P_{ss}} SP_{ss}(E_t \lambda_{t+1} - \lambda_t + E_t sp_{t+1} - E_t \pi_{t+1}) \tag{18}$$

$$sp_t = \beta \frac{Y_{ss}}{SP_{ss}} * (E_t \lambda_{t+1} - \lambda_t + E_t y_{t+1} - E_t \pi_{t+1}) + \beta(E_t \lambda_{t+1} - \lambda_t + E_t sp_{t+1} - E_t \pi_{t+1}) \tag{19}$$

where

$$SP_{ss} = \sum_{\tau=1}^{\infty} \beta^{\tau} Y_{ss} = \frac{\beta}{1-\beta} Y_{ss}. \tag{20}$$

Using this gives

$$sp_t = (1-\beta)(E_t \lambda_{t+1} - \lambda_t + E_t y_{t+1} - E_t \pi_{t+1}) + \beta(E_t \lambda_{t+1} - \lambda_t + E_t sp_{t+1} - E_t \pi_{t+1}) \tag{21}$$

$$sp_t = (1-\beta) E_t y_{t+1} + \beta E_t sp_{t+1} + E_t \lambda_{t+1} - \lambda_t - E_t \pi_{t+1} \tag{22}$$