IDRIS — Systems Programming Meets Full Dependent Types

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Abstract

Dependent types have emerged in recent years as a promising approach to ensuring program correctness. However, existing dependently typed languages such as Agda and Coq work at a very high level of abstraction, making it difficult to map verified programs to suitably efficient executable code. This is particularly problematic for programs which work with bit level data, e.g. network packet processing, binary file formats or operating system services. Such programs, being fundamental to the operation of computers in general, may stand to benefit significantly from program verification techniques. This paper describes the use of a dependently typed programming language, IDRIS, for specifying and verifying properties of low-level systems programs, taking network packet processing as an extended example. We give an overview of the distinctive features of IDRIS which allow it to interact with external systems code, with precise types. Furthermore, we show how to integrate tactic scripts and plugin decision procedures to reduce the burden of proof on application developers. The ideas we present are readily adaptable to languages with related type systems.

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General Terms Languages, Verification

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1. Introduction

Systems software, such as an operating system or a network stack, underlies everything we do on a computer, whether that computer is a desktop machine, a server, a mobile phone, or any embedded device. It is therefore vital that such software operates correctly in all situations. Dependent types have emerged in recent years as a promising approach to ensuring the correctness of software, with high level verification tools such as Coq [8] and Agda [25] being used to model and verify a variety of programs including domain-specific languages (DSLs) [26], parsers [9], compilers [16] and algorithms [34]. However, since these tools operate at a high level of abstraction, it can be difficult to map verified programs to efficient low level code. For example, Oury and Swierstra's data description language [26] works with a list of bits to describe file

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PLPV'11, January 29, 2011, Austin, Texas, USA. Copyright © 2011 ACM 978-1-4503-0487-0/11/01...\$5.00. formats precisely, but it does not attempt to store concrete data compactly or efficiently.

This paper explores dependent type based program verification techniques for systems programming, using the IDRIS programming language. We give an overview of IDRIS, describing in particular the key features which distinguish it from other related languages and give an extended example of the kind of program which stands to benefit from type-based program verification techniques. Our example is a data description language influenced by PADS [19] and PACKETTYPES [22]. This language is an embedded domain-specific language (EDSL) [14] — that is, it is implemented by embedding in a host language, exploiting the host's parser, type system and code generator. In this EDSL, we can describe data formats at the bit level, as well as express constraints on the data. We implement operations for converting data between high level IDRIS data types and bit level data, using a foreign function interface which gives IDRIS types to C functions. This language has a serious motivation: we would like to implement verified, efficient network protocols [1]. Therefore we show two packet formats as examples: Internet Control Message Protocol (ICMP) packets, and Internet Protocol (IP) headers.

1.1 Contributions

The main contribution of this paper is to demonstrate that a high level dependently typed language is capable of implementing and verifying code at a low level. We achieve this in the following specific ways:

- We describe the distinctive features of IDRIS which allow integration of low level systems programming constructs with higher level programs verified by type checking (Section 2).
- We show how an effective Foreign Function Interface can be embedded in a dependently typed language (Section 2.6).
- We introduce a serious systems application where a programming language meets program verification, and implement it fully: a binary data description language, which we use to describe ICMP and IP headers precisely, expressing the data layout and constraints on that data (Section 3).

We show how to tackle some of the awkward problems which can arise in practice when implementing a dependently typed *application*. These problems include:

- Dealing with foreign functions which may have more specific inputs and outputs than their C types might suggest e.g. we might know that an integer may lie within a specific range.
- Satisfying proof obligations which arise due to giving data and functions precise types. As far as possible, we would like proof obligations to be solved automatically, and proof requirements should not interfere with a program's *readability*.

Type	Meaning
Int	Machine integers
Float	Floating point numbers
Char	Characters
String	Strings of characters
Ptr	External pointers

Table 1. Primitive Types

2. An Overview of IDRIS

IDRIS is an experimental functional programming language with dependent types, similar to Agda [25] or Epigram [7, 21]. It is eagerly evaluated and compiles to C via the Epic supercombinator compiler¹. IDRIS has monadic I/O in the style of Hancock and Setzer [13], and a simple foreign function interface. It is implemented on top of the IVOR theorem proving library [4], giving direct access to an interactive tactic-based theorem prover. In this section, we give an overview of the main features of IDRIS. It is not intended as a tutorial² on dependently typed programming, or even IDRIS specifically, but rather to show in particular the features which allow program verification to meet practical systems programming.

2.1 Simple Types and Functions

IDRIS data types are declared using a similar syntax to Haskell data types. However, IDRIS syntax is not whitespace sensitive and declarations must end with a semicolon. For example, natural numbers, an option type and lists are declared in the standard library:

Functions are implemented by pattern matching, again using a similar syntax to Haskell. The main syntactic difference is that IDRIS requires type declarations for any function with greater than zero arguments, using a single colon: (rather than Haskell's double colon:). For example, addition on natural numbers can be defined as follows, again taken from the standard library:

Additionally, IDRIS has a number of *primitive* types, summarised in Table 1. Note in particular that String is a primitive, rather than a sequence of characters, for efficiency reasons. Ptr is used as a simple means of referencing C values.

2.2 Dependent Types

Dependent types allow types to be predicated on values. IDRIS uses *full-spectrum* dependent types, meaning that there is no restriction on which values may appear in types. For example, *Vectors* are lists which carry their size in the type. They are declared as follows in the standard library, using a syntax similar to that for Generalised Algebraic Data Types (GADTs) in Haskell [28]:

```
infixr 5 ::;
data Vect : Set -> Nat -> Set where
   VNil : Vect a 0
   | (::) : a -> Vect a k -> Vect a (S k);
```

This declares a *family* of types. We explicitly state the type of the type constructor Vect — it takes a type and a Nat as an argument,

and returns a new type. Set is the type of types. We say that Vect is parameterised by a type, and indexed over Nats. Each constructor targets a different part of the family — VNil can only be used to construct vectors with zero length, and : : to construct vectors with non-zero length.

Note also that we have defined an infix operator, ::, and declared it as right associative with precedence 5. Functions, data constructors and type constructors may all be given infix operators as names.

We can define functions on dependent types such as Vect in the same way as on simple types such as List and Nat above, by pattern matching. The type of a function over Vect will describe what happens to the lengths of the vectors involved. For example, vappend appends two Vects, returning a vector which is the sum of the lengths of the inputs:

Implicit Arguments

In the definition of vappend above, we have used undeclared names in the type (e.g. a and n in Vect a n). The type checker still needs to infer types for these names, and add them as *implicit* arguments. The type of vappend could alternatively be written as:

The braces {} indicate that the arguments a, n and m can be omitted when applying vappend. In general, if an undeclared name appears in an index to a type (i.e. a, n and m in vappend) IDRIS will automatically add it as an implicit argument, and attempt to infer its type.

using notation

If the ordering of implicit arguments is important (e.g. if one depends on another) it is necessary to declare the implicit arguments manually. The Elem predicate, for example, is a type which states that x is an element of a vector xs:

We give the implicit arguments x and y because they depend on another implicit argument a, and xs because it depends on two implicit arguments a and n. IDRIS will add a and n automatically before the arguments which were given manually.

To avoid excessive repetition (as with x, y and xs above) and to improve the clarity when reading type declarations, IDRIS provides the using notation:

```
using (x:a, y:a, xs:Vect a n) {
  data Elem : a -> Vect a n -> Set where
    here : Elem x (x :: xs)
  | there : Elem x xs -> Elem x (y :: xs);
}
```

The notation using (x:a) { ... } means that in the block of code, if x appears in a type it will be added as an implicit argument with type a.

Termination Checking

In order to ensure termination of type checking (and therefore its decidability), we must distinguish terms for which evaluation definitely terminates, and those for which it may not. We take a simple

¹http://www.idris-lang.org/epic/

² A tutorial is available at http://www.idris-lang.org/tutorial/

but effective approach to termination checking: any functions that do not satisfy a simple syntactic constraint on recursive calls will not be reduced by the type checker. The constraint is that each recursive call must have an argument that is structurally smaller than the input argument in the same position, and that these arguments must belong to a strictly positive data type. We check for *totality* by additionally ensuring that the patterns cover all possible cases.

2.3 Syntactic sugar

IDRIS has syntactic sugar for some of the more common data types, and allows the user to extend syntax using syntax macros.

Sugar for data types

One common data type is the Pair: data Pair a b = mkPair a b;

Pair types can instead be abbreviated to (a & b), with pair values written (a, b). *Dependent* pairs, where the type of the second value depends on the first, are declared as follows:

```
data Sigma : (A:Set) -> (P:A -> Set) -> Set where
Exists : {P:A -> Set} -> (a:A) -> P a -> Sigma A P;
```

Dependent pair types can be abbreviated to (x : a ** P x), which stands for Sigma a $(\x => P x)$. A value in a dependent pair is written <| a, b |>. Often, the first value can be inferred from the type of the second, so the value is written <| _, b |>.

IDRIS also allows lists to be written with a Haskell-like notation, [a, b, c, ...] instead of Cons a (Cons b (Cons c ...)).

Syntax Macros

Syntax macros allow more complex syntactic forms to be given simpler syntax, and take the following form:

```
syntax f x1 x2 ... xn = e
```

This defines a macro f with n arguments x1 to xn. Wherever an f appears in a program (whether a pattern or an expression), with n arguments, it will be replaced with e, substituting x1 to xn in e. Macros are type checked at the point they are expanded. It is an error to apply the macro with fewer than n arguments.

There are two main uses of syntax macros. Firstly, since they are applied in patterns as well as programs, they can be used as *pattern synonyms*. Secondly, when implementing embedded languages they can be used to define a clearer syntax, taking advantage of the fact that they are not type checked until they are expanded. In particular, they are useful for hiding proof obligations as we will see in Section 2.8.

2.4 The with rule

Very often, we need to match on the result of an intermediate computation. IDRIS provides a construct for this, the with rule, modelled on a similar construct in Epigram [21] and Agda [25]. It takes account of the fact that matching on a value in a dependently typed language can affect what we know about the forms of other values. In its simplest form, the with rule adds another argument to the function being defined, e.g. the vector filter function:

If the intermediate computation itself has a dependent type, then the result can affect the forms of other arguments — we can learn the form of one value by testing another. For example, a Nat is either even or odd. If it is even it will be the sum of two equal Nats. Otherwise, it is the sum of two equal Nats plus one:

```
data Parity : Nat -> Set where
   even : Parity (plus n n)
| odd : Parity (S (plus n n));
```

We say Parity is a *view* of Nat. Its *covering function* tests whether its input is even or odd and constructs the predicate accordingly.

```
parity : (n:Nat) -> Parity n;
```

We can use this to write a function which converts a natural number to a list of binary digits (least significant first) as follows:

The value of the result of parity k affects the form of k, because the result of parity k depends on k. Therefore as well as the patterns for the result of the intermediate computation (even and odd) we also state how the results affect the other patterns. Note that there is a function in the resulting patterns (plus) and repeated occurrences of j — this is permitted since the form of these patterns is determined by the form of parity k.

2.5 Evaluation and Compilation

IDRIS consists of an interactive environment with a read-evalprint loop (REPL) and a compiler. The interactive environment allows inspection of types and values and provides an interface to the theorem proving tools in IVOR[4]. For example, at the IDRIS prompt we can evaluate:

```
$ idris test.idr
Idris> natToBin (intToNat 6)
[False, True, True] : List Bool
```

With the :t command, we can check types:

```
Idris> :t Elem 0
Vect Nat y0 -> Set
```

It is particularly important in systems programming to understand the internal representation of values. With the :d command (standing for "definition"), we can inspect the internal form of a function or data structure. Using this, we can see how *erasure* transformations such as forcing and collapsing [3, 6] and optimisations such as partial evaluation [5] affect the compiled representation. These transformations are applied to each definition independently. Vectors, for example, need not store their length:

```
Idris> :d Vect
Vect constructors:
   VNil
   (::) a (Vect a k)
```

A proof that an item is an element of a list reduces to a natural number (which has an optimised representation) since it corresponds to the index at which the item appears:

```
Idris> :d Elem
Elem constructors:
    0
    S (Elem x xs)
```

Some types, such as the following "less than" predicate LT on Nats, carry no run-time information at all. Such *collapsible* types have no run-time representation:

```
data LT : Nat -> Nat -> Set where
    1t0 : LT 0 (S y)
    | 1tS : LT x y -> LE (S x) (S y);

Idris> :d LT
LT constructors:
```

The interactive system also provides commands for compiling to an executable (:c), and theorem proving (:p).

2.6 I/O system and Foreign Functions

Input and output in IDRIS, like Haskell, is achieved using an IO monad. This is implemented in the style of Hancock and Setzer [13], where an I/O operation consists of a command followed by a continuation that defines how to process the response:

In this way, we preserve purity in that a program which performs I/O does not do so directly, but rather generates an I/O tree which describes the actions to be executed when the program is run. Commands include a number of primitive operations, for example:

```
data Command : Set where
   PutStr : String -> Command
   | GetStr : Command
   ... :
```

A Response to a command is provided by the run-time system. Executing each Command gives a response of an appropriate type:

```
Response : Command -> Set;
Response (PutStr s) = ();
Response GetStr = String;
...
```

We define the usual bind and return operations:

```
(>>=) : IO a -> (a -> IO b) -> IO b;
(>>=) (IOReturn x) k = k x;
(>>=) (IODo c p)  k = IODo c (\y => (bind (p y) k));
return : a -> IO a;
return x = IOReturn x;
```

do-notation

Rather than using the (>>=) operator to sequence I/O operations, IDRIS provides do-notation, like Haskell, which expands to the (>>=) and return functions by default. For example:

Unlike Haskell, however, we do not (yet) provide a Monad type class for more general do-notation. Instead, we allow do-notation to be rebound locally. For example, we can write a bind operation for Maybe as follows:

```
maybeBind : Maybe a -> (a -> Maybe b) -> Maybe b;
maybeBind Nothing mf = Nothing;
maybeBind (Just x) mf = mf x;
```

For the return operation, we can use Just. We can use these inside a do block with a do using declaration, which takes the bind and return operations as parameters. For example, a function

which adds two Maybe Ints, using do-notation, could be written as follows:

This is, however, a temporary solution. We plan to implement type classes for overloading functions following the style of Sozeau and Oury in Coq [33].

Foreign Functions

When an I/O program is compiled, the compiler generates code for each of the Commands using appropriate standard C library functions. Conveniently, using this approach to I/O we can describe foreign functions directly in IDRIS without introducing *any* language extensions, deferring details of how the functions are executed (and how values are marshaled to and from C) to the run-time system.

We begin by defining a *universe* of types FType which can be passed to foreign functions, and a decoding function i_ftype:

```
data FType = FUnit | FInt | FStr | FPtr | FAny Set;
i_ftype : FType -> Set;
i_ftype FUnit = ();
i_ftype FInt = Int;
i_ftype FStr = String;
i_ftype FPtr = Ptr;
i_ftype (FAny ty) = ty;
```

Each of these types has a representation in C; these are int, char* and void* (or in practice, any pointer type) for FInt, FStr and FPtr respectively. For FAny, we use the run-time system's internal representation of values. FUnit is used as the return type of a void function. Then a foreign function has an external name, a list of argument types, and a return type:

```
data ForeignFun = FFun String (List FType) FType;
```

To call a foreign function, we will need a concrete list of arguments corresponding to the expected argument types. We will also find it convenient to append foreign argument lists:

For example, consider the C function fopen:

```
FILE* fopen(char* filename, char* mode);
```

This is represented as follows:

```
fopen_fun : ForeignFun;
fopen_fun = FFun "fopen" [FStr, FStr] FPtr;
```

In order to run such functions, we need to extend the Command type, and correspondingly the Response function, with foreign function descriptions. A foreign function call takes the function description and a concrete argument list, and the response gives us a value of the declared return type:

Finally, we give foreign functions a higher level IDRIS type and definition. We calculate high level types from foreign function descriptions:

For example, mkFType fopen_fun gives String -> String -> IO Ptr. We generate an executable definition which applies the Foreign command by using mkForeign, detailed in Figure 1.

Figure 1. Generating Foreign Functions

Since the foreign function descriptions are *statically* known, IDRIS evaluates mkForeign at compile-time. This leaves just an application of the Foreign constructor in the code. IO describes a domain-specific language for executing I/O operations. Conceptually, the compiled code *executes* programs written in this DSL by interpreting an I/O tree. In practice, for efficiency, an I/O tree is compiled to direct execution of the I/O operations — the only overhead associated with calling foreign functions is the marshaling between C and IDRIS values.

Example — File Handling via C

Using foreign functions involves creating a description with FFun and converting it to an IDRIS function with mkForeign. In practice, instead of using external pointer types directly we prefer to wrap them in a higher level abstract data type. For a small file handling library, we define a type for holding external file handles:

```
data File = FHandle Ptr;
```

We add descriptions of external functions for opening, closing and reading files. freadStr is defined in the run-time system to simplify reading a line from a file safely:

User level functions simply manage the wrapping and unwrapping of the external pointer in the file handle.

2.7 Metavariables and theorem proving

Sooner or later when programming with dependent types, the need to prove a theorem arises. This typically happens when using an indexed data type, if a value's index does not match the required type but is nevertheless provably equal. Since IDRIS is built on a tactic based theorem proving library, IVOR, we are able to provide access to the tactic engine. Suppose we would like to prove the following theorem about plus:

```
plusReduces0 : (n:Nat) -> (n = plus n 0);
```

We can declare just the type, and prove the theorem *interactively* in the IDRIS environment. The :p command enters the interactive proof mode and displays the current *Goal*:

```
Idris> :p plusReduces0
------
HO ? (n : Nat) -> n = plus n 0
```

In proof mode, we are given a list of premisses (initially empty) above the line, and the current goal (named H0 here) below the line. At the prompt, we can enter *tactics* to direct the construction of a proof. Here, we use the intro tactic to introduce the argument n as a premiss, followed by induction on n.

plusReducesO> intro

The resulting goal $0 = plus \ 0 \ 0$ can be solved by reflexivity (the refl tactic), since plus $0 \ 0$ normalises to 0. This leaves the inductive case:

If we reduce this goal to normal form (using the compute tactic) we can apply the inductive hypothesis ih, and complete the proof by reflexivity.

Finally, entering qed will verify the proof and output a log of the proof script which can be pasted directly into the source file.

```
plusReducesO proof {
   %intro; %induction n; %refl; %intro k,ih;
   %compute; %rewrite ih; %refl; %qed;
};
```

Proof by Pattern Matching

It is often more convenient to write proofs using pattern matching functions than to write a tactic based proof (c.f. the Equations extension to Coq [32]). Suppose for example we wish to prove that appending two lists is associative:

```
app_assoc :
    (xs:List a) -> (ys:List a) -> (zs:List a) ->
    (xs ++ (ys ++ zs) = (xs ++ ys) ++ zs);
```

(++) is implemented in the library by pattern matching and recursion on its first argument. A good approach to proving a function's properties is often to follow the structure of the function itself. It is therefore easier to break this down by writing down a pattern matching definition, and using the theorem prover to add the final details. We can do this by leaving holes (or *metavariables*) in the proof, marked by ?name:

In the Cons case we have gained access to an induction hypothesis by making a recursive call corresponding to the recursive call in the definition of (++). When we invoke IDRIS, it will report which proof obligations are to be satisfied:

```
Proof obligations:
[app_assocNil,app_assocCons]
```

These can be proved using the tactic engine, and the scripts pasted in as before. In practice, it is often clearer to write definitions with metavariables as above, which give a "sketch" of a proof, and leave the remaining proof scripts with the precise details to the end of a program file. In some ways, this resembles a paper which gives the outline of a proof and postpones the details until an appendix or an accompanying technical report.

2.8 Plugin Decision Procedures

Many proof obligations which arise in practice are solved by straightforward applications of simple rewriting rules (e.g. applications of list associativity) or by applying known decision procedures such as a Presburger arithmetic solver [31]. In these cases, we would like to avoid writing proofs by hand and where appropriate we might prefer to hide from the programmer the fact that a proof was required at all. IDRIS provides two basic mechanisms for achieving this: firstly, a decide tactic, which applies a decision procedure implemented in IDRIS; and secondly, an embedded domain specific language for constructing user defined tactics.

decide tactic

The decide tactic, when given a goal of the form T a b c, and a function f: (a:A) -> (b:B) -> (c:C) -> Maybe (T a b c), will apply f to a b c. If the result is Just p, it will solve the goal with p. For example, we could write a function which determines whether a value is an element of a list and returns a proof of the Elem predicate if so:

```
isElem : (x:a) -> (xs:Vect a n) -> Maybe (Elem x xs);
```

If the relevant values are statically known — which is not unlikely when implementing an EDSL for example [5] — the decide tactic could fill in required proofs of Elem automatically using isElem.

Tactic EDSL

```
data Tactic : Set where
   Fill : {a:Set} -> a -> Tactic
| Refine : String -> Tactic
| Trivial : Tactic
| Decide : {a:Set} -> Maybe a -> Tactic
| SearchContext : Tactic
| Try : Tactic -> Tactic -> Tactic
| Fail : String -> Tactic
```

Figure 2. EDSL for Tactic Construction

More generally, when the relevant values are *not* statically known, a simple decision procedure will not be suitable. In such cases, we can dynamically construct a tactic script using an EDSL for tactic construction similar to Coq's Ltac language [8], a fragment of which is shown in Figure 2. Each constructor corresponds to a tactic, or a means of combining tactics. Try, for example, applies the first tactic, and if that fails, applies the second.

The decide tactic, when given a goal of the form T a b c, and a function f: (a:A) -> (b:B) -> (c:C) -> Tactic, will apply f to a, b and c and execute the resulting tactic. For example, we can write a slightly extended version of a tactic to search for proofs of vector membership as follows:

This tactic first tries to apply the isElem proof search. If it succeeds, it uses the resulting proof. If it fails (or if it simply does not have enough information) the tactic uses the searchcontext tactic which searches the local and global contexts for a term which will solve the goal directly. If searching the context fails, it will use the fail tactic to report an appropriate, domain-specific error.

Aside — Parameters

To compute a proof that a value is an element of a vector, we need to construct equality proofs between two elements of a type. Rather than pass around a function which does so, we parameterise a block of code over such a function:

```
params (eq:(x:a) -> (y:a) -> (Maybe (x = y))) {
   ...
}
```

This is similar to Coq's Section mechanism. is Elem and is Elem Tac are parameterised over eq.

Example — Safe Access Control Lists

Consider a security policy which requires a user's ID to appear in an access control list before the user is given access to a resource. Using a dependent type based approach [24], we could require a proof (using Elem) that a user is in a list of allowed users before being able to read a resource:

To use read_ok we also need to provide a proof that the user is allowed access to the resource. Rather than provide this directly, we will leave a metavariable and provide a proof separately:

```
answers = read_ok edwin exams ?;
```

Alternatively, we can invoke isElemTac directly, which will either construct a proof if possible, or search through the context for an existing proof if not, using the following syntax:

The [proof ...] syntax allows a tactic based proof to be inserted directly into a program. In practice, this is most useful when combined with syntax macros. We can define syntax for reading a resource, statically constructing a proof that access is permitted:

```
syntax read u r
= read_ok u r [proof %intro; %decide isElemTac; %qed];
```

Wherever read u r appears, the system tries to construct a proof of Elem u r using isElemTac, and reports a compile-time error if the tactic fails. An alternative syntax [tryproof ...] does not report an error if the tactic fails, but instead leaves the metavariable unsolved. This can be useful for implementing partial decision procedures.

Using this approach, we can construct functions with *domain-specific* correctness requirements, solved by *domain-specific* tactics, without the notational overhead of writing explicit metavariables and invoking tactics manually.

3. Extended Example: Binary Data Formats

In this section we present an extended example, an EDSL for describing, marshaling and unmarshaling binary data, which applies many of the techniques described in Section 2. We have chosen this example because it is a component of a real research project, rather than a contrived example, although we have omitted some of the details due to space restrictions. We are developing this embedded data description language as part of a project to describe and verify network protocols using dependent types [1].

Oury and Swierstra describe how a similar language could be implemented in principle [26]. In this section, we should how such a language can be implemented in *practice*, if it is to be implemented efficiently, and show some realistic data formats which it can encode.

3.1 Primitive Binary Chunks

We are interested in parsing, manipulating and generating data to be transmitted across networks through verified network protocols. This involves manipulating data at the bit level. For example, implementing a TCP/IP stack would involve dealing with IP packets [30], the header of which is illustrated in Figure 3. To begin, we define Chunk, a universe of primitive components of binary data:

```
data Chunk : Set where
  bit : (width: Int) -> so (width>0) -> Chunk
  | Cstring : Chunk
  | Lstring : Int -> Chunk
  | prop : (P:Prop) -> Chunk;
```

Primitive data can be a bit field, of a defined length greater than 0 (bit), a null-terminated C-style string (Cstring), a string with an explicit length (Lstring) or a *proposition* about the data. Propositions are defined as follows, covering the basic logical connectives and relations, as well as a generic equality test p_bool:

```
data Prop : Set where
    p_lt : Nat -> Nat -> Prop
| p_eq : Nat -> Nat -> Prop
| p_bool : Bool -> Prop
| p_and : Prop -> Prop -> Prop
| p_or : Prop -> Prop -> Prop;
```

Chunk and Prop each have a decoding to IDRIS types. In packet descriptions, we often need to work with numbers of a specific bit width (bit n), so we create a type based on machine integers, carrying a proof that the number is within required bounds.

```
data Bounded : Int -> Set where
BInt : (x:Int) -> so (x<i) -> Bounded i;
```

An instance of the so data type used above can only be constructed if its index is True. Effectively, this is a static proof that a dynamic check has been made:

```
data so : Bool -> Set where
     oh : so True;
```

Rather than insert such proofs by hand, we construct a tactic which, like isElemTac in Section 2.8, tries to construct a proof (oh) which succeeds if the bounds are statically known. If this fails, the SearchContext tactic searches for a proof which already exists in the context:

We define a syntax macro for bounded numbers which automatically invokes this tactic. Since the tactic only does a basic proof search, we use the tryproof construct, so that if the tactic fails then the programmer is free to construct a more complex proof by hand:

The decoding function for Chunk either gives a bounded number of the appropriate bit width (in the case of bit), a primitive String (in the case of CString or LString) or decodes a proposition.

```
chunkTy : Chunk -> Set;
chunkTy (bit w p) = Bounded (1 << w);
chunkTy Cstring = String;
chunkTy (Lstring i) = String;
chunkTy (prop P) = propTy P;</pre>
```

Similarly, decoding propositions gives an appropriate IDRIS type:

```
propTy : Prop -> Set;
propTy (p_lt x y) = LT x y;
propTy (p_eq x y) = x=y;
propTy (p_bool b) = so b;
propTy (p_and s t) = (propTy s & propTy t);
propTy (p_or s t) = Either (propTy s) (propTy t);
```

Given a proposition, we can also write a function which decides whether than proposition is true, suitable for use by the decide tactic, or for run-time checking. This proceeds structurally over a Prop:

```
testProp : (p:Prop) -> Maybe (propTy p);
```

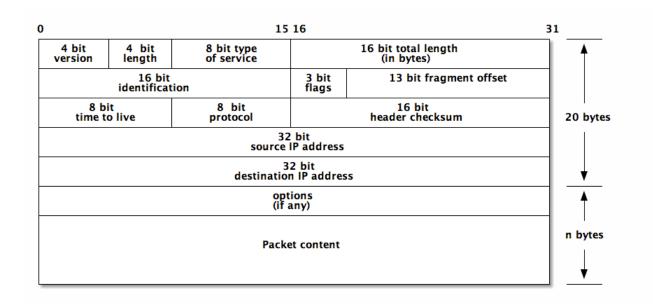


Figure 3. IP Header

3.2 Packet Descriptions

Packet formats are composed of combinations of binary Chunks. We define a language of combinators, PacketLang, in Figure 4. This is an *inductive-recursive* definition — the data type is defined simultaneously with its decoding function. This is a standard technique [10], and is particularly effective for implementing embedded languages [20]. The decoding function mkTy is shown in Figure 5.

```
mkTy : PacketLang -> Set;

data PacketLang : Set where
    CHUNK : (c:Chunk) -> PacketLang
| IF : Bool -> PacketLang -> PacketLang -> PacketLang
| (//) : PacketLang -> PacketLang -> PacketLang
| LIST : PacketLang -> PacketLang
| LISTN : (n:Nat) -> PacketLang -> PacketLang
| BIND : (p:PacketLang) -> PacketLang;
```

Figure 4. Packet Descriptions

```
mkTy : PacketLang -> Set;
mkTy (CHUNK c) = chunkTy c;
mkTy (IF x t e) = if x then (mkTy t) else (mkTy e);
mkTy (1 // r) = Either (mkTy 1) (mkTy r);
mkTy (LIST x) = List (mkTy x);
mkTy (LISTN i x) = Vect (mkTy x) i;
mkTy (BIND c k) = (x ** mkTy (k x));
```

Figure 5. Decoding Packets

At their simplest, packet formats are a sequence of Chunks, combined with a BIND operator to allow the form of later chunks to depend on values in earlier chunks:

We overload do-notation to use BIND and CHUNK instead of (>>=) and return, and provide syntax macros for decluttering the syntax, in particular for automatically inserting proofs that bit widths are greater than 0:

```
syntax bits n = CHUNK (bit n mk_so);
syntax check n = CHUNK (prop (p_bool n));
syntax lstring n = CHUNK (Lstring n);
```

For example, a data format containing an 8-bit number, guaranteed to be greater than zero, followed by a string of that length would be described as follows, where value is a function which extracts an integer from a Bounded number:

The decoding function for string_format yields nested dependent pairs, containing a length (with bounds proof), a proof that the length is greater than zero, and the string itself. We always write this type as mkTy string_format, and to work with it, we use a slightly different syntax x # y, which is more appropriate in this context in that we think of the # as a field separator:

```
syntax (#) x y = <| x , y |>;
showString : mkTy string_format -> String;
showString (len # prf # str) = str;
```

For greater flexibility, the language also includes a number of compound constructs. IF expressions allow alternative packet formats depending on a boolean value (e.g. computed from the form of earlier data):

```
IF : Bool -> PacketLang -> PacketLang -> PacketLang
```

We allow alternatives; a // b describes a packet format which can be either a or b:

```
infixl 5 // ;
(//) : PacketLang -> PacketLang -> PacketLang
```

Finally, there are two list constructs. LIST describes lists of arbitrary length, and LISTN describes lists of a specific length, perhaps computed from earlier data:

```
LIST : PacketLang -> PacketLang
LISTN : (n:Nat) -> PacketLang -> PacketLang
```

3.3 Marshaling and Unmarshaling

Given a data format described in a PacketLang, we would like to read and write concrete data in the format. Since there is no easy way to represent bit level data directly in IDRIS we represent it in C, as a pointer to a block of 32 bit words. In IDRIS, we wrap this pointer, and the length of the block, in the RawPkt type:

Packet descriptions can be sent and received over a network socket using the following foreign functions:

```
send : Socket -> RawPkt -> IO ();
recv : Socket -> IO (Maybe Recv);
```

Top level functions marshal and unmarshal then use packet descriptions to convert between the raw packet data and the high level representation of the packet type. We can think of these functions as "interpreters" for the packet language, whose semantics is to convert data from one form to another.

```
marshal : (p:PacketLang) -> mkTy p -> RawPkt;
unmarshal : (p:PacketLang) -> RawPkt -> Maybe (mkTy p);
```

In order to implement marshal and unmarshal we will need to read and and modify the contents of raw packets. This is achieved using C functions, which access packets by a location (as a bit offset) and a length in bits:

```
void setPktBits(PACKET p, int s, int len, int val);
int getPktBits(PACKET p, int s, int len);
```

Since we have given a length in bits, there is a clearly defined limit to the size of the number which can be sent or received. We therefore give a *more precise* type on the IDRIS side which expresses this limit. First, we give the foreign function description so that IDRIS programs can access the C definitions:

```
_setPktBits = mkForeign
   (FFun "setPktBits" [FPtr, FInt, FInt, FInt] FUnit);
_getPktBits = mkForeign
   (FFun "getPktBits" [FPtr, FInt, FInt] FInt);
```

Next, we write functions for packing and unpacking the raw pointer data in a RawPkt. Setting bits in a packet *modifies* the packet contents, since we aim to avoid expensive copying operations:

```
setPktBits : RawPkt -> Int -> Int -> Int -> I0 ();
setPktBits (RPkt p 1) s b dat = _setPktBits p s b dat;
getPktBits : RawPkt -> Int -> Int -> I0 Int;
getPktBits (RPkt p 1) s b = _getPktBits p s b;
```

These functions now have high level types, but they do not express the constraints on the integers *precisely*. Therefore, we give precise types which explicitly specify the size of number which fits in the range of bits. For setField, we give a number bounded according to the number of bits b available. Calling setField is valid only if we have checked that there are enough bits available in the packet, which is a precondition expressed in the type:

For getField, we should have a guarantee that the result will fit in the number of bits available. However, this data originates in the C program, so this guarantee depends on the C function being implemented correctly. This is, unfortunately, not something we can easily check in general — all we can do is trust that the external code is implemented correctly, and make an assertion:

We have used two "unsafe" functions here: unsafePerformIO, since the C function has no side effects; and unsafeCoerce which converts the proof we can construct to the proof we need:

```
unsafeCoerce : {a:Set} -> {b:Set} -> a -> b;
```

The low level bit manipulation functions, setField and getField, are implemented externally because they require close attention to low level, possibly system specific, details. It is possible that we could optimise these operations by exploiting the instruction set of a specific architecture.

Aside — Contracts and Dynamic Checks

It is a little unsettling that we have used unsafeCoerce to create an *efficient* verified implementation. On the one hand, we argue that it is safe because we have access to the external C function's implementation, which we can verify by hand; on the other hand, there ought to be a better way. For example, what if we change the C implementation and forget to update the IDRIS type?

We plan to improve this situation in the short term by extending FType to include constraints on foreign values, for example:

```
data FType = ... | FIntP (Int -> Bool);
i_ftype (FIntP p) = (x:Int ** so (p x));
```

The predicate p is effectively a contract which the foreign value must satisfy, similar to an assert in C. Like assert we would expect to be able to switch off checking when we are certain (either through extensive testing or verification of the external code) that the predicate will never return False.

3.4 A Simple Example

To show how packet formats work in practice, we give a simple packet format containing an IP address, followed by a list of strings with explicit lengths. The address is a sequence of 4 8-bit numbers:

```
IPAddress = do { bits 8; bits 8; bits 8; bits 8; };
```

To represent strings with explicit lengths, we have an 8-bit number, followed by a string of exactly the given length. We will use a zero length to indicate the end of the list, so zero itself is an invalid length, which we will express as a constraint:

The packet format is then an IP address, followed by a string list, followed by the end marker, which must be zero:

This packet format gives us a data type (using mkTy), and conversion functions between a RawPkt and its IDRIS representation, mkTy strings. Using marshal and unmarshal to convert the data, and send and recv to transmit the data over a network connection, we can guarantee that data is formatted using the correct bit representation by the sender, and verified by the receiver. We have implemented marshal and unmarshal in such a way that when the format is *statically* known, as it typically would be in specific applications, the abstraction overhead of the generic data conversion is removed by partial evaluation [5].

High level type conversion

To work with an instance of mkTy strings, we need to know how mkTy strings is concretely represented as a high level IDRIS type. In principle, it is enough to know that two elements a and b which appear in sequence in a PacketLang description will be concretely represented as a # b. For example, from a mkTy strings we can extract the components of the IP address, the list of strings, the end marker and the proof that the marker is zero as follows:

```
readPkt : mkTy strings -> ...;
readPkt ((a # b # c # d) # xs # mark # prf) = ...;
```

In general, however, it would be much easier to work with a high level type rather than the generic types generated by mkTy. Therefore, we additionally define a high level type and conversion functions. These conversion functions will only be type correct if they do the appropriate validation when building the packet format:

One fragment of the conversion converts a 4-tuple of integers to an IP address. The format specifies that the integers must be 8-bits, however, so we must construct a proof that the integers are within range. We write a dynamic checking function which, given a Bool constructs either a proof that it is false, or a proof that it is true:

```
choose : (x:Bool) -> Either (so (not x)) (so x);
```

Using this, we convert integers to an IP address, shown in Figure 6. If the dynamic checks fail, and no proof can be constructed, the conversion fails. If they succeed, the bounded macro will retrieve the proofs generated by choose from the context.

Figure 6. IP Address Conversion

3.5 Network Packet Formats

More realistic packet formats follow the approach given above — describe the format and convert to an appropriate high level representation. Furthermore, in practice, while the PacketLang descriptions may be *longer*, they are rarely significantly more *complex*. We outline two: ICMP and IP headers.

ICMP

Figure 7 illustrates the layout of an ICMP (Internet Control Message Protocol) packet [29].

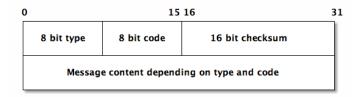


Figure 7. ICMP message

Valid codes, stored in bits 8–15, depend on the type, in bits 0–7. The 16-bit checksum depends on the content of the rest of the packet, and an invalid checksum renders the contents of the packet invalid. If we provide the checksum as part of the packet description, the unmarshaller will automatically verify the checksum, and the marshaller will automatically construct a valid checksum. The format of the message content also depends on the type and code. Figure 8 shows how this could be represented in PacketLang.

Figure 8. ICMP Packet Description

The ICMP type and code determine what form the message will take. Only certain combinations are valid:

```
validCode : Int -> Int -> Bool; validCode 0 x = x == 0; validCode 3 x = x >= 0 && x <= 13; validCode 4 x = x == 0; validCode 5 x = x >= 0 && x <= 2; ... validCode _ _ = False;
```

We could also define valid codes using the IF construct, but this check, by pattern matching, is more convenient given the number of available codes. The message format also depends on the code and type, and is calculated by ICMPformat (the details of which we have omitted).

IP

Recall Figure 3, which gave an illustration of the IP header. Figure 9 gives code to represent an IP header as a PacketLang. Reading data from a concrete IP header is then simply a matter of calling unmarshal IP_header.

The verify function, as before, verifies that the header's checksum field corresponds to a checksum calculated from the remaining fields in the header. This is managed by unmarshal and marshal; the proof, if it exists, simply has the form oh. Through the packet EDSL, reading, writing and verifying an IP header is no more complex than any either the ICMP format or the simple strings format.

```
IP_header
   = do { version <- bits 4; length <- bits 4;</pre>
          tos <- bits 8; tlength <- bits 16;
          id <- bits 16:
          rsvd <- bits 1; check (value rsvd == 0);
          df <- bits 1;
          mf <- bits 1;
          offset <- bits 13;
          ttl <- bits 8; protocol <- bits 8;
          checksum <- bits 16;</pre>
          source <- IPaddress;
          dest <- IPaddress;</pre>
          opts <- IPoptions;
          check (verify checksum version length tos
                    tlength id rsvd df mf offset
                    ttl protocol source dest opts);
};
```

Figure 9. IP Header Packet Description

4. Related Work

Xi's ATS [36] is a systems programming language with a form of dependent types. In ATS, there are separate static and dynamic components, where the static component is used for constructing and reasoning about types, and the dynamic component for evaluating programs. In contrast we allow types and values to be freely mixed. The compiler identifies the *phase distinction* between static and dynamic values, erasing static values at run-time [6]. Freely mixing types and values allows us to write *generic programs* by universe constructions [20], such as packet format descriptions.

An alternative approach could be taken using Coq and its extraction mechanism [17, 27]. Extraction generates an ML or Haskell program from a Coq proof, which we can then incorporate in a larger system which deals with the low level details (such as bit processing through foreign functions).

Our data description language is related to monadic parsers such as Parsec [15] in that we provide a set of combinators for language descriptions. Oury and Swierstra take a similar approach using Agda [26], under the assumption that there are external functions for dealing with the details of bit processing. More generally, they propose dependently typed languages as a host for embedding domain specific languages with precise type systems. In this paper, we have taken this work further: not only do we embed a domain specific language for data description, we complete the implementation for processing real bit level data and show how the language can be used in practice. We expect the embedded domain specific language approach to be applicable to other problems in systems programming, such as device driver verification [23, 35].

Our embedded data description language is inspired by previous tools such as PACKETTYPES [22] and Mirage [18] (both specifically targetting network packet formats), and more general data description languages [11] such as PADS/ML [19]. One advantage of taking a generic programming approach in a dependently typed host is that we can use features of the host language directly. We exploit this in particular to express dependencies and constraints between data. Furthermore, the framework is extensible — we are not limited to the marshal and unmarshal functions. For example, we could extend the framework with pretty printing of packets or XML conversion, directed by a PacketLang format. Of course, as well as exploiting the features of the host language, we must also work with the limitations of the host language. Disadvantages

of the generic approach, which we hope to address in future work, include the difficulty of producing good error messages and good error recovery in the parser.

5. Discussion

We have given an overview of the IDRIS programming language, including the most important language constructs and the distinctive features which make it suitable as a host language for a DSL. Our motivation is the need for systems software verification — programs such as operating systems, device drivers and network protocol implementations which are required for the correct operation of a computer system. Therefore it is important to consider not only how to verify software, but also how to do so without compromising on efficiency, and how to inter-operate with concrete data as it is represented in the machine or on a network wire. Our approach is to implement a generic data description language using a foreign function interface to access the concrete data. We use partial evaluation [5] to eliminate the abstraction overhead of the generic marshal and unmarshal functions. We have implemented some simple examples using this language, and tested them by marshaling and unmarshaling data over a real network connection.

Further Work

Our examples demonstrate the feasibility of using a dependently typed language to implement and verify systems software. To evaluate the approach fully, however, we will need significant examples with benchmarks, ideally implemented by systems programmers and network practitioners. There are some problems to be solved before we can apply our data description language in practice:

- 1. The current implementation of IDRIS uses the Boehm-Demers-Weiser conservative garbage collector [2], which is fine for most applications, but may not be suitable for low level code which may need to run in very limited memory. A much simpler solution, of which we have a prototype implementation, would be to allocate a single pool of memory which is freed on return from a top level function. This region based approach is used to good effect in Hume [12].
- In its current form PacketLang does not deal completely with bounds checking on integers. For example, we do not check that an integer is positive, or that it does not exceed an upper bound of 32 bits.
- 3. The error messages generated for incorrect EDSL programs can be hard to understand, particularly in cases where we have used syntactic sugar to make the EDSL programs more readable. To some extent we can mitigate the problem by generating more specific messages in domain-specific decision procedures but in general we will need to devise a mechanism for producing domain-specific error messages.
- 4. The path to adoption by systems programmers and network practitioners is not clear. Typically, they would use C or Java, and in order to be adopted any new approach would have to inter-operate with existing tools. A possible path to adoption would involve generating C headers for exported IDRIS functions, providing an API for accessing packet data. A good, simple, foreign function interface will be important here.

Systems software verification is vital — bugs in network and system software lead to security risks. We plan to use IDRIS and PacketLang formats to implement network applications, and generate benchmarks to compare our implementations with more conventional systems. The Domain Name Service (DNS) is a good candidate: it has a complex packet format with a built in string compression scheme which is hard to express in existing data descrip-

tion languages, and has been the source of serious bugs in DNS servers. Our hope is that language based verification techniques will help prevent such bugs in the future.

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