Modeling road-cycling performance

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Olds, T. S., K. I. Norton, E. L. A. Lowe, S. Olive, F. Reay, and S. Ly. Modeling road-cycling performance. *J. Appl. Physiol.* 78(4): 1596–1611, 1995.—This paper presents a complete set of equations for a "first principles" mathematical model of road-cycling performance, including corrections for the effect of winds, tire pressure and wheel radius, altitude, relative humidity, rotational kinetic energy, drafting, and changed drag. The relevant physiological, biophysical, and environmental variables were measured in 41 experienced cyclists completing a 26-km road time trial. The correlation between actual and predicted times was 0.89 ($P \leq 0.0001$), with a mean difference of 0.74 min (1.73% of mean performance time) and a mean absolute difference of 1.65 min (3.87%). Multiple simulations were performed where model inputs were randomly varied using a normal distribution about the measured values with a SD equivalent to the estimated day-to-day variability or technical error of measurement in each of the inputs. This analysis yielded 95% confidence limits for the predicted times. The model suggests that the main physiological factors contributing to road-cycling performance are maximal O_2 consumption, fractional utilization of maximal O_2 consumption, mechanical efficiency, and projected frontal area. The model is then applied to some practical problems in road cycling: the effect of drafting, the advantage of using smaller front wheels, the effects of added mass, the importance of rotational kinetic energy, the effect of changes in drag due to changes in bicycle configuration, the normalization of performances under different conditions, and the limits of human performance.

biophysics; sports performance; mathematical modeling

MATHEMATICAL MODELING and computer simulation can produce precise, unambiguous, and quantitative descriptions of reality. They permit the study of multifactor interaction and dynamic system responses, which would be impossible when using purely empirical data. Furthermore, the process of simulation can often suggest further experimentation and highlight gaps in our body of knowledge. Recently, exercise scientists have become interested in developing mathematical models of sports performance. In a previous paper (27), we applied a mathematical model of cycling, derived largely from first principles, to track endurance-cycling performance in elite cyclists. In that study, there was a good correlation ($r = 0.81$) and small absolute differences (<3% of mean performance time) between model-predicted and measured performance times.

The model involved finding one expression for the energy demand of cycling, which is a function of velocity and distance, and another for physiological energy supply, which is a function of time. For a fixed distance, time (and hence velocity) can be iteratively adjusted until demand matches supply. This represents an estimate of the best possible performance time for a given set of environmental conditions. This model can be used to quantify in the common currency of performance time the effect of changes in physiological, environmental, and biophysical variables.

The present paper is an extension of that work, applying a similar model to road-cycling performance (26-km time trial) and including several modifications to the original model. We have also incorporated a method for predicting a range of performance times based on the day-to-day variability and technical errors of measurement of the values of individual model variables.

Road cycling, because of the increased variability of topological (road surface, slope, configuration) factors, would prima facie appear more difficult to model than track endurance cycling. Furthermore, track endurance riders perform at or near maximum oxygen consumption ($\dot{V}O_{2\,max}$) for the duration of their event (4–6 min). A road time trial (26 km, or 35–52 min in this study), on the other hand, must involve a fractional use of $\dot{V}O_{2\,max}$. Modeling this event, therefore, requires an estimate of fractional utilization of $\dot{V}O_{2\,max}$.

METHODS

Subjects. Thirty-two male and nine female subjects aged between 16 and 39 yr, ranging from novice to international level, agreed to participate in this study. The means and SD values for the physiological characteristics of the subjects are contained in Table 1.

Testing schedule. Subjects visited the laboratory, which was air-conditioned to 20–24°C for all tests, for two 3-h testing sessions and performed a road time trial within a 2-wk period. On the first lab visit, subjects performed a maximal test on a cycle ergometer to determine $\dot{V}O_{2\,max}$ and ventilatory threshold. After this, anthropometric measurements and the first of two mechanical efficiency tests were performed. The second mechanical efficiency test and a supramaximal test to determine maximal accumulated oxygen deficit were performed on the second visit. These tests are described below.

$\dot{V}O_{2\,max}$. A geared, wind-braked cycle ergometer with components and configuration similar to a racing bicycle was used for the $\dot{V}O_{2\,max}$, mechanical efficiency, and supramaximal tests. Cyclists used handlebars of their choice and their own pedals. The ergometer was calibrated dynamically throughout the physiological range of measurements by use of an electronic torque meter. Workloads were set using the

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TABLE 1. *Effect of changes in model variables on predicted time-trial time*

Variable	Values	Predicted Time, min	ΔTime, min	ΔTime, %
		Subject-dependent		
$\dot{V}O_{2\,max}$	5.438	38.545	−2.883	−7.0
	4.528	*41.428*		
	3.619	45.412	3.985	9.6
f	0.836	39.724	−1.704	−4.1
	0.752			
	0.668	43.464	2.036	4.9
Grad	0.0106	40.037	−1.391	−3.4
	0.01166			
	0.01272	42.746	1.318	3.2
A_p (%A_b)	18.6	40.155	−1.270	−3.1
	21.1			
	23.6	42.625	1.197	2.9
Int	0.214	40.512	−0.916	−2.2
	0.407			
	0.601	42.431	1.003	2.4
M	60.2	40.546	−0.882	−2.1
	70.3			
	80.4	42.236	0.808	1.9
Ht	166.84	41.044	−0.384	−0.9
	175.51			
	184.18	41.799	0.371	0.9
$\dot{V}O_{2\,init}$	1.180	41.400	0.029	−0.7
	0.810			
	0.440	41.457	0.029	0.7
Def_{max}	4.96	41.352	−0.076	−0.2
	3.87			
	2.79	41.467	0.039	0.1
		Subject-independent		
P	135	41.218	−0.210	−0.5
	116			
	97	41.724	0.296	0.7
T	291.8	41.210	−0.218	−0.5
	287.4			
	283.1	41.651	0.223	0.5
v_w	0.13	41.351	−0.077	−0.2
	0.77			
	1.41	41.609	0.181	0.4
PB	755	41.352	−0.076	−0.2
	759			
	764	41.503	0.075	0.2
M_b	10.2	41.402	−0.026	−0.1
	11.3			
	12.3	41.454	0.026	0.1
RH	86.5	41.417	−0.011	0.0
	73.5			
	60.5	41.439	0.011	0.0

Values are varied ±1 SD about means (indicated by italics). Change in predicted time is expressed in min and as percentage of mean predicted time. Variables are listed in descending order of sensitivity and classified into subject-dependent and subject-independent variables. See Table 2 for definitions.

variables of gearing and pedal cadence. The seat height was adjusted to match that of the cyclist's own racing bicycle. Gas analysis was performed using a breath-by-breath system (Ametek OCM2 metabolic assessment system, Applied Electrochemistry, Pittsburgh, PA), with the data being averaged over longer time periods according to the requirements of the test: 5 s for supramaximal test (oxygen deficit and $\dot{V}O_2$ kinetics) and 30 s for $\dot{V}O_{2\,max}$ and mechanical efficiency tests.

Subjects warmed up on the ergometer for 2–5 min at a workload of their own choosing. They then rested while being connected to the gas-analyzing equipment. One of three incremental protocols was chosen to elicit $\dot{V}O_{2\,max}$, according to the expected ability of the subject. The starting workloads ranged from 77 to 149 W, with 2-min increments of 30–50 W. The further increase in work rate was halted when $\dot{V}O_{2\,max}$ was identified [an oxygen uptake ($\dot{V}O_2$) difference of <0.15 l/min for successive work rates] or when the subject was unable to maintain the required pedal cadence (95–105 rpm). Subjects were allowed a 5-min recovery on the ergometer before being asked to cycle for at least 2 min at the final work rate they had achieved. The purpose of this last stage was to ensure that $\dot{V}O_{2\,max}$ had been reached. $\dot{V}O_{2\,max}$ was taken as the maximum oxygen uptake averaged over 30 s recorded during the full test. In every test, the respiratory exchange ratio exceeded 1.10 at exhaustion.

Ventilatory threshold was defined as the point at which the gradient of the curve relating minute ventilation ($\dot{V}E$) to $\dot{V}O_2$ changed dramatically. This was calculated with specifically designed software, which fits two lines to the data points sequentially (28). The intersection of the two lines for which the pooled squared residuals was least indicated the ventilatory threshold ($\dot{V}O_{2\,AT}$). The $\dot{V}O_2$ at ventilatory threshold was expressed in liters per minute, and the model variable f was defined in this study as $\dot{V}O_{2\,AT}/\dot{V}O_{2\,max}$. This estimate of f was based on data on Australian road cyclists who had competed in a 40-km time trial (N. Craig, personal communication). When lactate threshold was used as an estimate of fractional utilization in this previous work, the predicted values were within 30 s of the actual time-trial times. Heart rate and lactate measurements were taken on these cyclists during the event, which suggested that they were riding close to their thresholds throughout. In this group, there was a close match between ventilatory and lactate thresholds.

Mechanical efficiency. Each of the two mechanical efficiency tests consisted of six 5-min incremental workloads designed to elicit an estimated 40–90% of the subject's $\dot{V}O_{2\,max}$. Gears were selected to keep pedal cadence as close as possible to 100 rpm, since this is the cadence road cyclists usually maintain. It is also known that mechanical efficiency will vary with pedal cadence. Subjects were prepared as for the $\dot{V}O_{2\,max}$ test. Respiratory data were collected for the last 2 min of each stage and averaged over 30-s intervals to determine steady-state values. In all, 10–12 workloads were performed, depending on the subject's ability to complete the last stage of each test. The gradient (Grad) and intercept (Int) of the regression of $\dot{V}O_2$ on power output (in W) were used as indicators of cycling efficiency.

Supramaximal tests. In the supramaximal test, subjects were asked to cycle for as long as possible at a target work rate of 115% of $\dot{V}O_{2\,max}$. Gearing was again chosen so that pedal cadence was close to 100 rpm. Subjects completed a warm-up of their choice before being connected to the gas-analysis equipment. They were instructed to reach their target cadence as quickly as possible at the start of the test. In conjunction with the mechanical efficiency tests, this test to exhaustion was used to determine maximal accumulated oxygen deficit (Def_{max}) according to the procedures of Medbø et al. (23). The work rate required was estimated from the $\dot{V}O_2$ vs. power output relationship determined for each subject. The mean duration of these tests was 162 ± 56 s. The $\dot{V}O_2$ measured immediately preceding the start of the supramaximal test was used for the value of prerace $\dot{V}O_2$ ($\dot{V}O_{2\,init}$) in the model.

Anthropometric and equipment measures. Before the $\dot{V}O_{2\,max}$ test, the subject's height (Ht) and nude mass (M) were measured. The subject was also weighed in racing attire, and the mass of the time-trial bicycle was determined. The combined mass of the racing attire and the bicycle constituted the variable M_b. The projected frontal area (A_p) of

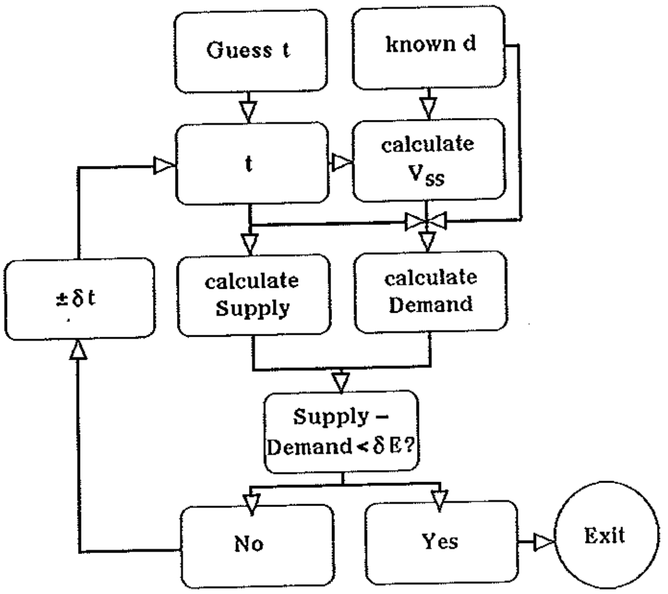

FIG. 1. Flow diagram of modeling strategy used. t, Time-trial time; v_{ss}, asymptotic bicycle speed, d, time-trial distance; Demand, energy demand of cycling; Supply, physiological energy supply; δE, arbitrarily small amount of energy, δt, arbitrarily small change in time.

the cyclist and their bicycle was assessed using the photographic weighing method. During these measures, the cyclists had precisely the clothing and equipment (including filled water bottles) that they would use during the road time trials.

Road time trials. The road time trials were conducted on a fairly smooth, flat 6.5-km course of sealed bitumen in dry weather at sea level. The course rose and fell by <0.5% over the 6.5-km course. Subjects completed four lengths of this course (a total of 26 km). Turnaround times were not included and were generally under 30 s. Temperature (T), relative humidity (RH), and wind speed (v_w) and direction (XWind) were measured on site every 5–10 min. The mean wind speeds and wind directions for each 6.5-km leg were used in the final analysis. Barometric pressure (P_B), tire pressure (P), and wheel diameter (Diam) were also recorded. Riders used geared road bicycles. No drafting was permitted, and riders were spaced sufficiently far apart (>5 min) to make this impossible. Four riders used special "aerodynamic" wheels (one used trispoke wheels and three used disk wheels). No corrections were made to the coefficient of drag (C_D) for these accessories, as there is evidence that they do not greatly assist performance (Ref. 2; see below *The effect of changed drag*). "Aero" bars were permitted, and approximately one-half the riders used them. The other riders rode in the "drops" position. It should be noted that because of the manner in which the model was derived, no correction was made for the use of aero bars as opposed to the drops position. Wind-tunnel measures of changed drag due to aero bars suggest that this may affect predicted time by ~1% (see Table 5). Tire pressures were chosen by the cyclists themselves, who inflated their tires with foot pumps before the trials began.

Description of the model. The basic model used here has been described previously (27). Figure 1 shows the general modeling strategy. The measured, derived, assumed, and iterated variables used in the model are shown in Table 2. A complete description of the model equations is given in APPENDIX.

Model modifications. The original model was modified in the following ways.

1) In the modified model, we assume that the expression of the oxygen deficit follows a monoexponential pattern with a time constant τ_{def} s, with a τ_{def} value of 0.417 min (13). The effect of varying τ_{def} from 0.08 to 1.67 min on performance time using mean values was <1 s. The portion of Def_{max} expressed [i.e., the energy derived from anaerobic sources (E_{an})] is therefore

$$E_{an} = Def_{max}\left[1 - \exp\left(\frac{-t/60}{\tau_{def}}\right)\right]CF_{E_{an}} \tag{1}$$

where t is the elapsed time (s) and CF is a correction factor relating to the oxygen deficit. It was assumed that the entire anaerobic capacity (here taken to be equivalent to the accumulated Def_{max}) would not be expressed during the time trial (30). Therefore the expression $CF_{E_{an}}$ is a factor reducing the oxygen deficit expressed according to the equation suggested by Péronnet et al. (30)

$$CF_{E_{an}} = 1 - 0.233 \ln\left(\frac{t}{420}\right) \tag{2}$$

The minimum value for the E_{an} expenditure was set at $\tau \cdot (\dot{V}O_{2\,max} \cdot f - \dot{V}O_{2\,init})$, which is equivalent to the deficit incurred in reaching steady-state during constant-load submaximal work. Simulations run with and without the correction factor yielded a difference in predicted time of 0.19 min for the mean values found in this study.

2) Corrections were made for head and crosswinds according to the model proposed by Hill (10). A more detailed analysis of the assumptions and limitations of this correction is developed in *Assumption 5: problem of crosswinds*.

3) The coefficient of rolling resistance (μ_r) was calculated using the following equation

$$\mu_r = \frac{a + \frac{b}{P}}{Diam} CF_{Surface} \tag{3}$$

where a and b are constants, P is in pounds per square inch or psi, Diam is in inches, and $CF_{Surface}$ is a correction factor for surface type. This model was based on one suggested by Whitt and Wilson (37). Data were collected from a number of studies (4, 8, 13, 17, 18, 20) to parameterize the equation. $CF_{Surface}$ was determined from data from Kyle (17) and Kyle and von Valkenburgh (20), where the same tires were tested on different surfaces. The ratio of the mean μ_r on concrete to the mean μ_r on asphalt was 0.87. The cumulated data from the above studies yielded values of $a = -0.00051$ and $b = 9.73744$ ($r = 0.69$, $P \le 0.0001$). The lack of an obvious way of quantifying surface smoothness and the lack of data relating surface characteristics to rolling resistance mean that a more sophisticated analysis of the effect of surface characteristics on rolling resistance is not yet available.

4) Air density was adjusted for relative humidity. As humidity increases, water vapor replaces other gases of greater density, and air density decreases. The pressure of water vapor in saturated air at a given temperature was determined from algorithmic approximations to table values. This pressure was then multiplied by the relative humidity as a decimal fraction to give the actual pressure of water vapor (P_{H_2O}). The partial pressures of oxygen (P_{O_2}) and nitrogen (P_{N_2}) were determined as

$$P_{O_2} = 0.2093(P_B - P_{H_2O}) \tag{4}$$

$$P_{N_2} = 0.7904(P_B - P_{H_2O}) \tag{5}$$

TABLE 2. *Measured, assumed, optional, iterated, and derived variables used in the model, along with symbols, units, and descriptions*

Symbol	Units	Description
		Measured variables
$\dot{V}O_{2\,max}$	l/min	Maximal aerobic power
$\dot{V}O_{2\,init}$	l/min	Prerace O_2 uptake
f		Fractional utilization of $\dot{V}O_{2\,max}$ (here, ventilatory threshold)
Def_{max}	l	Maximal accumulated O_2 deficit
Grad	$l \cdot min^{-1} \cdot W^{-1}$	Gradient of the $\dot{V}O_2$-$\dot{W}$ regression line, where $\dot{W}$ is work rate
Int	l/min	Intercept of the $\dot{V}O_2$-$\dot{W}$ regression line
P_B	mmHg	Barometric pressure
T	K	Ambient temperature
RH	%	Relative humidity
M	kg	Nude mass of the cyclist
Ht	cm	Height of the cyclist
M_b	kg	Mass of bicycle and accessories, including clothing
Slope		Slope of time-trial course (decimal fraction)
v_w	m/s	Wind speed
XWind	rad	Angle of wind relative to direction of travel of system
Diam	in.	Wheel diameter
P	$lb/in.^2$ (psi)	Tire pressure
Alt	m	Altitude above sea-level
Spacing	m	Distance between cyclist and pack/paceline/vehicle
d	m	Course distance
		Assumed variables
C_D		Coefficient of drag of the system (0.592)
τ	min	Time-constant for $\dot{V}O_2$ kinetics (0.667)
τ_{def}	min	Time-constant for deficit kinetics (0.417)
k	s	Time-constant for acceleration (10)
$CF_{Surface}$		Factor correcting μ_r for road surface (asphalt = 1, concrete = 0.87)
Cadence	rpm	Pedal frequency (100)
		Optional variables
δDrag	g	Relative change in drag
v_a	m/s	Airspeed at which δDrag is measured
		Iterated variable
t	s	Time
		Derived variables
Demand	J	Energy demand of cycling
E_{Rr}	J	Energy required to overcome rolling resistance
E_{air}	J	Energy required to overcome air resistance
E_{KE}	J	Energy required to impart kinetic energy to the system
E_{grade}	J	Energy required to move system vertically
Supply	J	Physiological energy supply
E_{aer}	J	Energy derived from aerobic sources
E_{an}	J	Energy derived from anaerobic sources
v_{ss}	m/s	Average velocity of the system
μ_r		Coefficient of rolling resistance of the system
ρ	kg/m^3	Air density
A_b	m^2	Body surface area
A_p	m^2	Projected frontal area of the system
P_{O_2}	Torr	Partial pressure of O_2
P_{N_2}	Torr	Partial pressure of N_2
P_{H_2O}	Torr	Partial pressure of H_2O vapor
CF_{Draft}		Factor correcting E_{air} for drafting
CF_{XWind}		Factor correcting air resistance for crosswinds
$CF_{Humidity}$		Factor correcting air density for humidity
$CF_{\dot{V}}$		Factor correcting $\dot{V}O_{2\,max}$ for altitude
CF_{Grav}		Factor correcting the acceleration due to gravity for altitude
G	m/s^2	Estimate of acceleration due to gravity used in calculating default values for P_B at altitude

Air density was multiplied by the following correction factor for humidity ($CF_{Humidity}$)

$$CF_{Humidity} = \frac{P_{O_2} \times 32 + P_{N_2} \times 28 + P_{H_2O} \times 18}{0.2093 \times 32P_B + 0.7904 \times 28P_B} \quad (6)$$

where 32 is the mol wt of O_2, 28 is the mol wt of N_2, and 18 is the mol wt of H_2O. The effect of changes in relative humidity is very small. When mean values were used, a change from 0 to 100% relative humidity decreased mean performance time by 0.08 min or 0.2% of mean predicted performance time.

5) Because $\dot{V}O_2$ kinetics was not measured at submaximal levels, a time constant τ of 0.67 min was assumed, being typical of values found in the literature during submaximal work on the bicycle ergometer when a similar model is used.

The effect of varying τ in a very wide range from 0.08 to 1.67 min on performance time using mean values was only 0.24 min (0.6% of mean predicted performance time).

6) In our original model, a linear pattern of acceleration was assumed. In the present model, the acceleration pattern is assumed to be monoexponential. That is

$$v_t = v_{ss}\left[1 - \exp\left(\frac{-t}{k}\right)\right] \tag{7}$$

where v_t is the velocity at time t (s), v_{ss} is the asymptotic ground speed of the system, and k is a time constant (s). Since the change in kinetic energy (KE), the energy required to move the system vertically, and the energy required to overcome rolling resistance are independent of instantaneous velocity, only the effects of the acceleration pattern on the energy required to overcome air resistance need to be considered. The instantaneous power required to overcome air resistance (P_{air}) is $P_{air} = c \cdot v_t^3$, where c is a lumped parameter including C_D, A_p, and air density (ρ). Given that $v_t = v_{ss} \cdot [1 - \exp(-t/k)]$, the energy required to overcome air resistance (E_{air}) is

$$\int cv_t^3 dt = v_{ss}\left[t + 3k\exp\left(\frac{-t}{k}\right) - 1.5k\exp\left(\frac{-2t}{k}\right) + \frac{k}{3}\exp\left(\frac{-3t}{k}\right) - \frac{11k}{6}\right] \tag{8}$$

One effect of this assumed acceleration pattern is that the power demand during the accelerative phase is very high, mainly because of the need to impart KE to the system. To provide this extra power, energy must be generated rapidly by the anaerobic system, and τ_{def} may be quite small. Because the power demand is not constant, the rate of increase in aerobic energy supply will not necessarily mirror the rate of decrease in anaerobic energy supply. Furthermore, $\dot{V}O_2$ kinetics may be somewhat faster at the onset of exercise than predicted by the model, since the rate of rise of $\dot{V}O_2$ will be driven by very high power demands early in exercise. For the longer distances modeled in this paper, the consequences will be unimportant.

7) In the original model, $C_D \cdot A_p \cdot \rho$ was treated as a lumped parameter, and adjustments were made for varying A_p, T, and PB. In the current model, we have estimated the A_p of the subjects in the original di Prampero et al. study (4), which was not measured, using equations developed in our own laboratory. Twenty-six riders were photographed in the "drops" position, as in the di Prampero et al. study, and their A_p was determined by photographic weighing. The following equation was developed relating the A_p of the rider (m²) to the rider's body surface area (A_b)

$$A_p = 0.3176A_b - 0.1478 \; (r = 0.83, \text{RMSR} = 0.0358) \tag{9}$$

where RMSR is root mean square residual. By using this equation, the estimated mean A_p (rider only) for the subjects in the di Prampero et al. study was 0.4147 m².

Based on this estimate of A_p, the C_D for classic track or modern road bicycles was calculated to be 0.592, somewhat less than other investigators have found (2, 18). Since these investigators have also measured A_p, and derived estimates of C_D from A_p, these differences may be due to differences in measurement technique, particularly in the choice of a reference dimension when photographic methods are used. It is also unclear whether these investigators measured the total A_p (rider plus bicycle) or just the rider. The model corrects estimated A_p for the body surface area of the rider by using the following equation

$$A_p = 0.4147\frac{A_b}{1.771} + 0.1159 \tag{10}$$

where 1.771 is the estimated A_b of the subjects in the di Prampero et al. study (4) and 0.1159 m² is the mean A_p of the bicycle alone measured in our laboratory. Because the model uses this correction factor based on A_b, the differences in the relative sizes of C_D and A_p are not important. That is, even if the model underestimates C_D, it will then overestimate A_p, so that the drag area ($C_D \cdot A_p$) remains compatible with those found in other studies for riders of similar size.

Some data have recently become available on drag characteristics of aerodynamic bicycles relative to standard bicycles (2, 29). These can be incorporated into the model, although none of the cyclists in this study used aerodynamic bicycles.

8) The bicycle-rider system has not only linear KE but also rotational KE from the movement of the cranks and pedals about the bottom bracket, the pedals about the pedal spindle, the cluster about the rear axle, and both wheels about their respective axles. Also, the rider has rotational KE, from the movement of the thigh, shank, and foot about the hip joint, the movement of the shank and foot about the knee joint, and the movement of the foot about the ankle joint. Since there is no rotational KE in the system at the start of the time trial, the rotational KE in the system at the end of the time trial (as the cyclist crosses the line) requires a metabolic energy input. By using data on the fractional masses and radii of gyration of body segments, segment lengths (32), the kinematics of the relevant joints (31), measured masses, and estimates of radii of gyration of bicycle components, it is possible to estimate the total rotational KE of the system in terms of the following variables: M, M_b, Ht, v_{ss}, Diam, k, and an assumed cadence (100 rpm). The estimated masses, radii of gyration, and angular velocities, along with their derivations, are given in Table 3. From these, the energy required to impart the final angular KE ($E_{KE\,ang}$) to the system can be calculated.

9) Although not directly relevant to this study, we have included corrections for performance at altitude and the effects of drafting. A full account of the equations regarding altitude is found in Olds (26). The air resistance is adjusted using a correction factor for drafting

$$\text{CF}_{\text{Draft}} = 1 - 0.3835 + 1.25\,\text{Spacing} + 0.0405\,\text{Spacing}^2 \tag{11}$$

where Spacing is the wheel-to-wheel distance (m) between the bicycle and the preceding pack or paceline. This equation is a fit to empirical data, some in graphical form, from Kyle (16).

10) It is also possible to correct for changed drag. Wind-tunnel tests yield changes in drag when standard items are replaced by "aerodynamic" alternatives. These drag changes (δDrag) are usually reported in grams, measured at a certain air speed (v_a; m/s). The energy required to overcome the added drag at race speed ($E_{\Delta\text{Drag}}$) is then approximated by

$$E_{\Delta\text{Drag}} = d\;\delta\text{Drag}\;9.80665 \times \frac{(v_{ss} + v_w)^2}{v_a^2} \tag{12}$$

This slightly overestimates the energy required to overcome added drag because it assumes that the cyclist achieves v_{ss} from the start of the time trial. However, in longer time trials, the difference is negligible.

The equations used in the revised model are shown in APPENDIX.

Influence of variability in model inputs. The following procedure was employed to estimate the sensitivity of the predicted time to day-to-day variations in the values of model variables. For each variable in the model, either the coefficient of variation (CV) in day-to-day repeated measures or

TABLE 3. *Mass, radius of gyration, and angular velocity of each of body segments and bicycle components having angular kinetic energy in cycling*

Source	M, kg	Rad of Gyration, m	Angular Velocity, rad/s	KE, J
Hip joint	$0.3536 \cdot M$[a]	$0.39 \cdot$ Ht/170.18[b]	$0.785 \cdot$ Cadence/60[c]	7
Knee joint	$0.1286 \cdot M$[a]	$0.27 \cdot$ Ht/170.18[b]	$1.309 \cdot$ Cadence/60[c]	3
Ankle joint	$0.0276 \cdot M$[a]	$0.18 \cdot$ Ht/170.18[b]	$0.349 \cdot$ Cadence/60[c]	<1
Cranks/pedal	1.20[d]	0.14[e]	Cadence $\cdot \pi/30$[f]	1
Pedals	0.45[d]	0.03[e]	Cadence $\cdot \pi/30$[f]	<1
Cluster	0.46[d]	0.02[e]	$v_{ss} \cdot [1 - \exp(-t/k)]/(\text{Diam} \cdot 0.0127)$[g]	<1
Wheels	3.16[d]	0.23[e]	$v_{ss} \cdot [1 - \exp(-t/k)]/(\text{Diam} \cdot 0.0127)$[g]	80
Chainwheels	0.25[d]	0.08[e]	Cadence $\cdot \pi/30$[f]	<1
			Total	<95

See Table 2 for definitions. Cadence = 100 rpm; k = 10 s; Diam = 27 in.; v_{ss} = ~10.5 m/s. [a] Fractional masses: thigh = $0.1125 \cdot M$; shank = $0.505 \cdot M$; foot = $1.38 \cdot M$ (mean of male and female values). Thigh, shank, and foot rotate about the hip joint; shank and foot rotate about the knee joint; foot rotates about the ankle joint. Values are multiplied by 2 for each leg. [b] Length of thigh = 0.4137 m, lower leg = 0.3772 m, foot = 0.255 m for an individual 170.18 m tall (Ref. 32). Average length of leg about hip joint = thigh + (lower leg + foot)/2. Average length of shank about knee joint = lower leg + foot/2. Radius of gyration of thigh occurs at 0.54 of its length, of lower leg at 0.529, of foot at 0.69. [c] First number represents angular displacement of segment (rad). These are multiplied by Cadence/60 (converting rpm to Hz). [d] Measured values for typical racing bicycle. [e] Estimated values. [f] Cadence in rpm divided by 60 yields cadence in Hz. Multiply by 2π rad/cycle. [g] $v_{ss} \cdot [(1 - \exp(-t/k)]$ is final velocity of system. $2 \cdot \pi \cdot$ Diam $\cdot$ 0.0127 is diameter (m) of wheel. Therefore wheel goes through $v_{ss} \cdot [(1 - \exp(-t/k)]/(2 \cdot \pi \cdot \text{Diam} \cdot 0.0127)$ cycles/s. Each cycle represents $2 \cdot \pi$ rad.

the technical error of measurement (TEM = square root of within-subjects mean squares) for multiple measurements was determined either from our own data, from data available in the literature, or from estimates. The relevant CVs, TEMs, and references are shown in Table 4. For each cyclist, 100 simulations were performed and the predicted values recorded. The value entered for each of the variables was randomly and normally varied about the measured value at each run of the simulation, i.e.

$$V_i = V + CV \times V \times z \tag{13}$$

where V_i is the value of the variable for the ith simulation, V is the measured value, CV is the coefficient of variation or TEM associated with the variable, and z is a random normal number from a distribution where $\mu = 0$ and $\sigma = 1$. This type of analysis assumes that the variabilities are independent of each other. This is not always true. For example, because of the nature of the linear regression relating $\dot{V}O_2$ to work rate ($\dot{W}$), there will be an inverse relationship between the gradient and the intercept of the $\dot{V}O_2$-$\dot{W}$ equation. The assumption of independence in this case will increase the variability of predicted times.

TABLE 4. *CV for day-to-day repeated measures or TEM values for variables used in predicting time-trial times*

Variable	Description	CV or TEM, %	Ref.
$\dot{V}O_{2\,max}$	Maximal aerobic power	3.1	11
$\dot{V}O_{2\,init}$	Pretrial $\dot{V}O_2$	3.8	11
f	Ventilatory threshold	7.5	3
Grad	Gradient of $\dot{V}O_2$-$\dot{W}$ equation	6.5	Own data†
Int	Intercept of $\dot{V}O_2$-$\dot{W}$ equation	33.0	Own data†
Def_{max}	Maximal O_2 deficit	10.7	7
PB	Barometric pressure	0.1	Own data‡
T	Temperature	0.1	Own data‡
RH	Relative humidity	0.5	Own data‡
M	Mass of the cyclist	0.9	15
Ht	Height of cyclist	0.2	Own data‡
M_b	Mass of bicycle	2.0	Own data‡
v_w	Windspeed	23.1	Own data§
XWind	Wind direction	37.3	Own data§
Diam	Wheel diameter	1.0	Own data‡
P	Tire pressure	5.0	Estimate
d	Time-trial distance	0.3	Own data‡
τ	Time-constant for $\dot{V}O_2$ kinetics	26.6	11
τ_{def}	Time-constant for deficit kinetics	10.0	Estimate
k	Acceleration time-constant	20.0	Estimate

TEM, technical error of measurement. * Weighted mean of studies reported. † Based on repeated measures with 5–6 data points per subject (*day 1* vs. *day 2* mechanical efficiency tests). Coefficient of variation (CV) is based on 5–6 data points and may therefore be less when 12 data points are used. ‡ Intertester technical error of measurement. § Intratester technical error of measurement.

Statistical analysis. Standard descriptive statistics (means, SD values, medians, and interquartile ranges) are used throughout. The strength of the association between predicted and actual times was quantified using the Pearson product-moment correlation coefficient. Student's paired t-test was used to compare mean predicted and mean actual times. Factorial analysis of variance was used to determine whether the mean differences between actual and predicted values differed among the various subgroups (i.e., recreational, club, state-, and national-level cyclists). Fisher's paired least significant difference test was used in post hoc testing. An alpha level of 0.05 was adopted in all tests.

RESULTS

Table 1 shows the means and SD values for each of the model variables. The mean difference between actual and predicted times was 0.74 ± 2.07 min or 1.73% of the mean actual time (range: 5.56 to −3.15 min). The 95% confidence limits for the mean difference were 0.08–1.40 min. This difference was therefore significant at the 0.05 level. The mean absolute difference between actual and predicted times was 1.65 ± 1.44 min or 3.87% of mean actual time (range: 0.01–5.56 min). The 95% confidence limits for the mean absolute difference were 1.20–2.10 min. There was a significant correlation between predicted and actual times (r = 0.89, $P \le 0.0001$, RMSR = 2.02 min; see Fig. 2). The 95% confidence limits of the correlation coefficient were 0.81–0.94. The equation relating actual times (t_{actual}) to predicted times (t_{pred}) was

$$t_{actual} = 1.17\, t_{pred} - 6.22$$

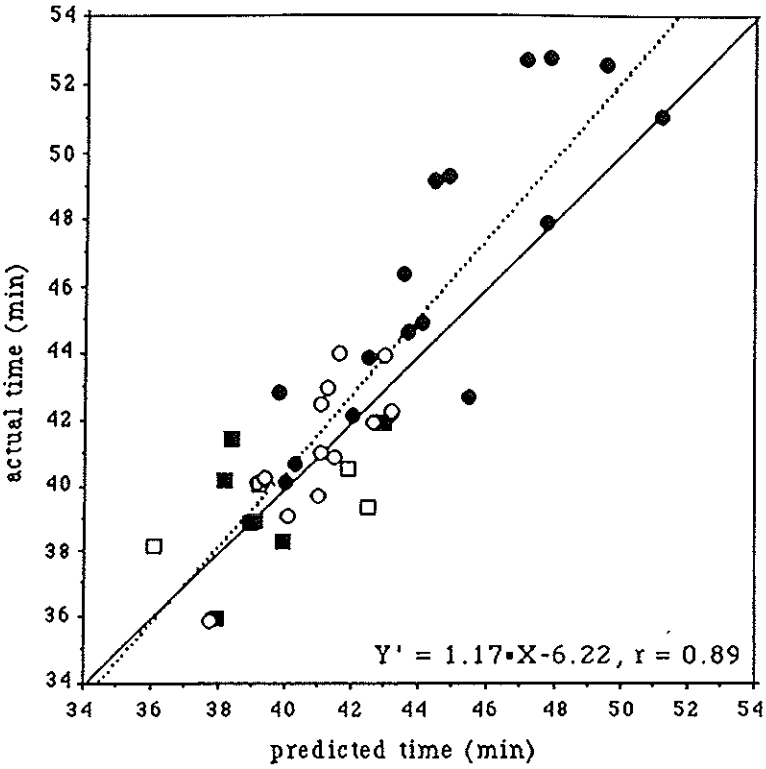

FIG. 2. Scatterplot showing relationship between predicted times and actual time-trial times. ●, Recreational; ○, club; □, state; ■, (inter)national-level cyclists.

The 95% confidence limits of the slope and intercept did not exclude the identity line.

Cyclists were divided into the following subgroups according to their competitive level: recreational (n = 16), club (n = 13), state (n = 4), and national or international (n = 8). Analysis of variance showed that there were significant differences between the mean differences between actual and predicted times among the groups (F = 3.66, P = 0.03). Post hoc analysis showed that the mean difference between actual and predicted times for recreational cyclists (+1.90 min) was different from those of club (−0.09 min), state (+0.38 min), and national (−0.01) -level cyclists.

Figure 3 shows the mean values and two SDs about the mean for multiple tests when the values for model inputs were randomly and normally varied. The predicted values were approximately normally distributed, with a slight positive skew. This skewness was due to the nonlinear relationship between supply and demand, on the one hand, and performance time on the other. As expected, there was no difference between the mean time predicted from multiple simulations and the predicted time when single measures were entered as inputs. The coefficient of variation for multiple simulations was 3.5 ± 1.1%. This suggests that, if a cyclist were tested repeatedly and performance times were predicted on the basis of those tests, the predicted times would vary within 7% of the mean time for 95% of predictions. In 7 of 41 cases, the actual performance time did not fall within two SDs of the mean predicted time. In six of these cases, the actual performance time was greater than the predicted range, and in only one case was the performance time less than the predicted range.

DISCUSSION

Model accuracy. As with track endurance cycling in our earlier investigation, the model used here predicted actual road time trial time quite well. The model accounted for >80% of the variability in time-trial performance in a relatively homogeneous group ($\dot{V}O_{2\,max}$ = 65.2 ± 8.4 $ml \cdot kg^{-1} \cdot min^{-1}$; CV for time-trial performance = 10%). The small mean difference (0.73 ± 2.07 min or 1.73%) and mean absolute difference (1.65 ± 1.44 min or 3.87%) between actual and predicted times suggest that the model is an accurate predictor of road as well as track performance.

The model predicts the cyclist's best possible performance time given values for physiological and environmental variables. Psychological factors of a cognitive and emotive nature (e.g., pacing, focus, motivation) are also presumably important but difficult to quantify, especially if the "first principles" nature of the model is adhered to. The model predicted worst for recreational cyclists (where it underestimated actual time-trial time by a mean of 1.90 min), who might be expected to have the worst sense of pacing. The accuracy was better for experienced club, state, and national cyclists. This interpretation is confirmed by the positive correlation between predictive error and time-trial time (r = 0.68, P ≤ 0.0001), showing that the model underpredicts actual time for cyclists with less ability.

Assumptions and possible limitations. A number of other investigators have used modeling to examine performance in cycling and other sports (e.g., Refs. 4, 30). Some have used purely statistical methods (24) such as multiple or stepwise multiple regression. Others have used first principles modeling but have focused on ei-

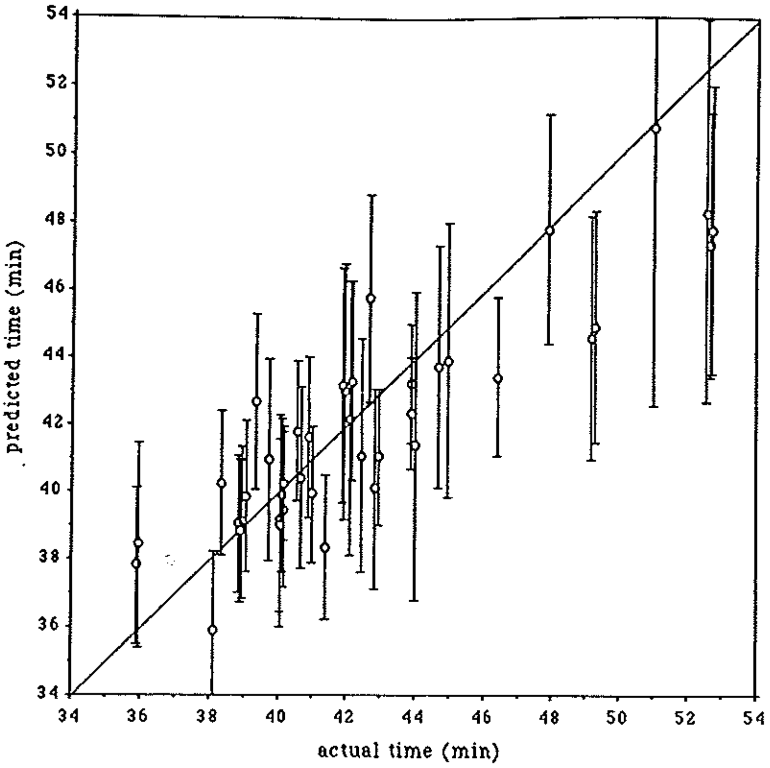

FIG. 3. Mean predicted times plotted against actual performance time. Error bars show ± 2 SDs about the mean based on multiple simulations. Line shown is the identity line.

ther the supply or the demand (4, 12) side of the equation, have used simplified algorithms, or have used default values (13, 29) for model variables. This study is, to our knowledge, the first to have measured nearly all the relevant supply and demand variables in a large group of road cyclists, to have employed a complete description of the energetics of cycling, and to have compared individual measured and predicted times.

Correlations of −0.93 have been reported between time-trial time and ventilatory threshold (24) and of −0.94 between 1-h average power output and time-trial time (34). In this study, correlations between actual time-trial time and the major physiological factors taken individually were 0.06 for fractional utilization of $\dot{V}O_{2\,max}$, 0.43 for M, 0.59 for Ht, 0.72 for $\dot{V}O_{2\,AT}$, 0.75 for $\dot{V}O_{2\,max}$ (l/min or $ml^{-1} \cdot kg^{-1} \cdot min^{-1}$), and 0.78 for $\dot{V}O_{2\,max}/A_b$. These were all signifcantly less than the model correlation of 0.89 (95% confidence limits 0.81–0.94). Multiple regression using mechanical efficiency at ventilatory threshold, $\dot{V}O_{2\,max}$, A_b, and $\dot{V}O_{2\,AT}$ yielded a correlation coefficient of 0.90. Stepwise multiple regression retained $\dot{V}O_{2\,AT}$, mechanical efficiency at $\dot{V}O_{2\,AT}$, and $\dot{V}O_{2\,max}$ as significant predictors, with a correlation coefficient of 0.89. These correlations are similar to those of the model.

Specific predictive accuracy is not the only concern of modeling. The great advantage of first principles modeling is its generality. The model predictions will apply, if the model is complete enough, to any set of environmental, physiological, and biomechanical conditions and over any distance. First principles models allow us to perform "what if?" scenarios by changing the value of variables that may not have been measured in empirical-prediction equations. Empirical-prediction equations are limited by the choice of predictor variables and by the domain over which those variables operate in the samples from which the equations are derived. A further concern is the use of linear statistical models to describe relationships which we know to be nonlinear (such as the speed-power relationship in cycling).

The model presented here, like all models, uses a number of assumptions that sacrifice specificity for simplicity and generality. The following are some of the simplifying assumptions used.

ASSUMPTION 1: FRACTIONAL UTILIZATION IS REPRESENTED BY VENTILATORY THRESHOLD FOR TIME TRIALS OF THIS DURATION. It may be questioned whether the ventilatory and lactate thresholds coincide under the conditions that applied in this study and whether lactate threshold is an appropriate estimate of the fractional utilization of $\dot{V}O_{2\,max}$ for the performance times measured in this study.

The ventilatory threshold determined by using the $\dot{V}E$ breakpoint method has been shown to correspond to the lactate threshold under normal circumstances. Specifically, Simon et al. (33) found no difference between ventilatory and lactate thresholds in trained (but not in untrained) cyclists. There are some circumstances in which the two thresholds can be dissociated (e.g., glycogen depletion, caffeine ingestion); however, these do not apply in the present study.

The relationship between fractional utilization and time to exhaustion has been variously described by hyperbolic, monoexponential, biexponential, logarithmic, and polynomial models (1, 22). All of these curves are characterized by a rapid decline at high power outputs and a relatively flat section at power outputs near the ventilatory threshold. There have been a number of studies showing that the power output corresponding to the ventilatory threshold can be sustained for prolonged periods and is a good predictor of endurance performance. Aunola et al. (1) expressed time to exhaustion in cycling as a function of lactate threshold and $\dot{V}O_{2\,max}$. By using their equations, subjects could sustain 75–76% of their $\dot{V}O_{2\,max}$ or 106–107% of their lactate threshold for the mean performance time in this study. A power output within 10% of the power output corresponding to lactate threshold could be maintained for 34–120 min. In the present study, f represented a mean of 75.2% of $\dot{V}O_{2\,max}$. These results suggest that ventilatory threshold may be a reasonable estimate of the fractional utilization of $\dot{V}O_{2\,max}$ for the performance times measured in this study.

The range of performance times for which ventilatory threshold is a reasonable estimate of f is a subject for further study. If this model were to be applied to a 100-km time trial, for example, using ventilatory threshold as an estimate of fractional utilization, the predicted times would probably be less than the measured times. Equally, if this estimate were used to model a 10-km time trial, the predicted times would probably be greater than the measured times.

ASSUMPTION 2: CHANGES IN THE VALUES OF MODEL VARIABLES ARE LARGELY INDEPENDENT OF EACH OTHER. To simulate "what if?" scenarios, the value for any of the model variables can be changed and the change in performance time predicted. However, this process assumes that changes in one variable will not affect changes in another, other than in the ways the model specifies. We have assumed, for example, that changes in riding position resulting in changed A_p will not affect mechanical efficiency. There is disagreement as to whether this is so (2). As mentioned above, it is also assumed that the variability of any model variable is independent of the variability of all of the others.

ASSUMPTION 3: A_p OF THE CYCLIST IS PROPORTIONAL TO THE CYCLIST'S A_b AND TO THE AIR RESISTANCE THE CYCLIST ENCOUNTERS. There has been a suggestion (34, 35) that neither of these assumptions are true. A_p may constitute a smaller fraction of A_b for large cyclists than for small cyclists. Furthermore, the C_D may not be independent of body size and position. These suggestions are based both on indirect evidence (lower than expected $\dot{V}O_2$ in larger cyclists) and direct evidence (measured A_p). Swain et al. (35) found 40-km time-trial time to be strongly correlated with mean 1-h power output normalized to mass raised to the power 0.32, rather than the expected exponent of 0.67 (which would reflect surface area).

We measured A_p in 26 cyclists in three different riding positions by using the photographic weighing method, planimetry, and digitization. We found no significant relationship between M and A_p/A_b (r = 0.092, P = 0.632 for rider plus bicycle; r = 0.066, P = 0.772 for rider only). However, since we did not have wind-tunnel data, we could not measure the effect of size on drag. To try to assess the effect of these two assumptions on model accuracy, we first tried to correct A_p for body size in the manner suggested by Swain et al. (35). For this, we used a CF based on the data of Swain et al. to adjust estimated A_p for body size and reran our simulations. The correction was established as follows. Swain et al. reported that A_p values expressed as a percentage of A_b for his small and large subjects were 18.9 and 17.8, respectively. The linear equation relating A_p (%A_b) to A_b was

$$A_p(\%A_b) = 23.06 - 2.46A_b \tag{14}$$

The mean A_b for the cyclists in the di Prampero et al. (4) study was 1.771 m^2, which corresponds to a predicted A_p (%A_b) of 18.7%. Therefore, the predicted A_p for cyclists was adjusted using the following correction factor (CF_{Swain})

$$CF_{Swain} = \frac{A_p(\%A_b)}{18.7} A_p \tag{15}$$

When the simulations were rerun with CF_{Swain}, the predictive power of the model did not improve. The correlation between actual and predicted times was r = 0.88. The mean difference was 0.82 min (as opposed to 0.75 min using the present model). The mean absolute difference was 1.79 min (as opposed to 1.63 min with the present model).

This correction factor does not take into account the possible change in the coefficient of drag with size. The difference between this study and that of Swain et al. (35) may also have to do with the different riding positions the subjects assumed or with differences in how A_p was measured.

ASSUMPTION 4: MECHANICAL EFFICIENCY CAN BE DESCRIBED BY A LINEAR RELATIONSHIP BETWEEN $\dot{V}O_2$ AND $\dot{W}$. The model uses a linear regression of $\dot{V}O_2$ on $\dot{W}$ to establish the variables Grad and Int (gradient and intercept of the $\dot{V}O_2$-$\dot{W}$ regression line, respectively), which predict the external power the cyclist can generate given a certain internal energy production. In our tests, the linear relationship was always excellent (median coefficient of determination = 0.99). Recently, however, close analyses of the $\dot{V}O_2$-$\dot{W}$ relationship (e.g., Ref. 9) have suggested that it is not truly linear. There is good theoretical rationale for assuming nonlinearity, arising from the contribution of E_{an} sources and the emergence of a slow component of $\dot{V}O_2$ at $\dot{W}$s above the ventilatory threshold. This latter consideration would mean that the predicted $\dot{V}O_2$ at a given suprathreshold $\dot{W}$ would be time dependent. Unfortunately, there is no general expression relating the amplitude, time constants, and time delays of the fast and slow exponential components to $\dot{W}$. This means that curves would need to be fitted to individual data, which is a complex procedure requiring multiple tests if satisfactory fits are to be assured. Time-dependent oxygen drift would be important in events where the cyclist worked at levels clearly above the ventilatory threshold (e.g., 10- or 15-km time trial) or in events where extreme environmental conditions affected physiological responses. In the event modeled in this study, this nonlinearity is probably less important.

ASSUMPTION 5: PROBLEM OF CROSSWINDS. The prevailing winds during the time trials were headwinds

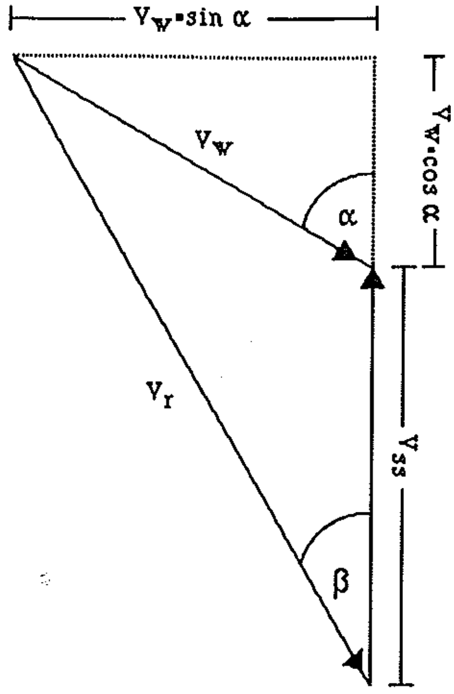

FIG. 4. Bicycle-rider system encountering a crosswind. Bicycle-rider system is moving with a velocity v_{ss}. A crosswind of velocity v_w is blowing at an angle α to the direction of travel of system. Apparent wind velocity (v_r) is the resultant of v_w and v_{ss}. β, Angle between direction of action of resistance due to apparent wind velocity and direction of travel of the system.

or tailwinds. However, in 10 of the 41 trials, there was a significant crosswind. Crosswinds proved particularly difficult to model. The appropriate procedure for crosswinds involves the vector addition of the wind velocity and the bicycle velocity (10). If the wind is blowing at an angle α rad to the direction of travel of the bicycle (Fig. 4), then the vector sum [the resultant or apparent wind velocity (v_r) in m/s] is given by the expression

$$v_r = \sqrt{v_{ss}^2 + v_w^2 + 2v_{ss}v_w \cos \alpha} \tag{16}$$

The resistance is proportional to the square of v_r. This force acts at an angle β to the direction of motion of the cyclist. From Fig. 4, it is clear that

$$\cos \beta = \frac{v_{ss} + v_w \cos \alpha}{v_r} \tag{17}$$

The component of this resistance-acting shear to the direction of travel of the cyclist (R_{head}) is proportional to $v_r^2 \cdot \cos \beta$ or

$$\begin{aligned} R_{head} &= (v_{ss}^2 + v_w^2 + 2v_{ss}v_w \cos \alpha)\,\frac{v_{ss} + v_w \cos \alpha}{v_r} \\ R_{head} &= \sqrt{v_{ss}^2 + v_w^2 + 2v_{ss}v_w \cos \alpha}\,(v_{ss} + v_w \cos \alpha) \end{aligned} \tag{18}$$

One strategy is to use *Eq. 16* to calculate the energy required to overcome air resistance. However, this would assume that the C_D and the A_p of the system is the same whether the wind is a crosswind or a head wind. In reality, one would expect both to vary according to the angle at which R_{head} acts in relation to the system. Attempts have been made to model this (14, 25). Norris (25) concludes that the ratio of drag under windy conditions to drag under windless conditions (D_x/D_0) is given by

$$\frac{D_x}{D_0} = \left(1 + \frac{v_w}{v_{ss}} \cos \alpha\right) \left(\left|1 + \frac{v_w}{v_{ss}} \cos \alpha\right| + SF\left|\frac{v_w}{v_{ss}} \sin \alpha\right|\right) \tag{19}$$

where SF is the ratio of the projected area of the system onto the sagittal plane to the projected area of the system onto the frontal plane. The interested reader is referred to Ref. 25 for the derivation of this equation. The value of SF has been estimated to be ~4 (25), but our planimetric analysis showed that it is closer to 2.5.

Unfortunately, when this method of accounting for crosswinds was used, even when the lower SF value was used, the times predicted were far greater than those actually recorded for time trials where there was a significant crosswind. For this reason, this approach was not adopted. Kyle (17) has measured the effect of crosswinds in a wind tunnel by varying the yaw angle of the system. His results show that drag does indeed increase as yaw angle increases, which is compatible with models that correct for SF. However, the magnitude of the increases he reports are not as great as those which would be predicted by Norris' model. Another possible explanation of the inconsistency between measured and expected times when crosswinds were involved is the fact that we measured crosswinds only at one part of the time-trial course. It is possible that other wind directions prevailed elsewhere or that the course was more shielded from crosswinds at other locations.

Both a priori and a posteriori considerations support the validity of the model. However, the model uses a large number of variables. Associated with each of these variables is a degree of day-to-day variability, some due to biological variation and some due to technical errors of measurement. Multiple simulations using random normal variability about measured values showed that the performance time for any one individual can only be confidently predicted within a range of times. This range is certainly too broad to be helpful to a coach or a selector in choosing between elite individuals.

Factors affecting time-trial performance. Nonetheless, the model has a number of sport-specific and general applications. The model presented here is a contribution toward understanding how physiological characteristics relate to changes in performance. It allows the sports scientist to quantify, in the common currency of time, the effect of biophysical and equipment changes on performance. Table 1 shows the effect of varying the values for each of the input variables 1 SD about the mean, while holding all other variables at the mean value. From Table 1, it is clear that among the major contributors to road-cycling performance are $\dot{V}O_{2\,max}$ (a variation of ± 1 SD in $\dot{V}O_{2\,max}$ results in a change in

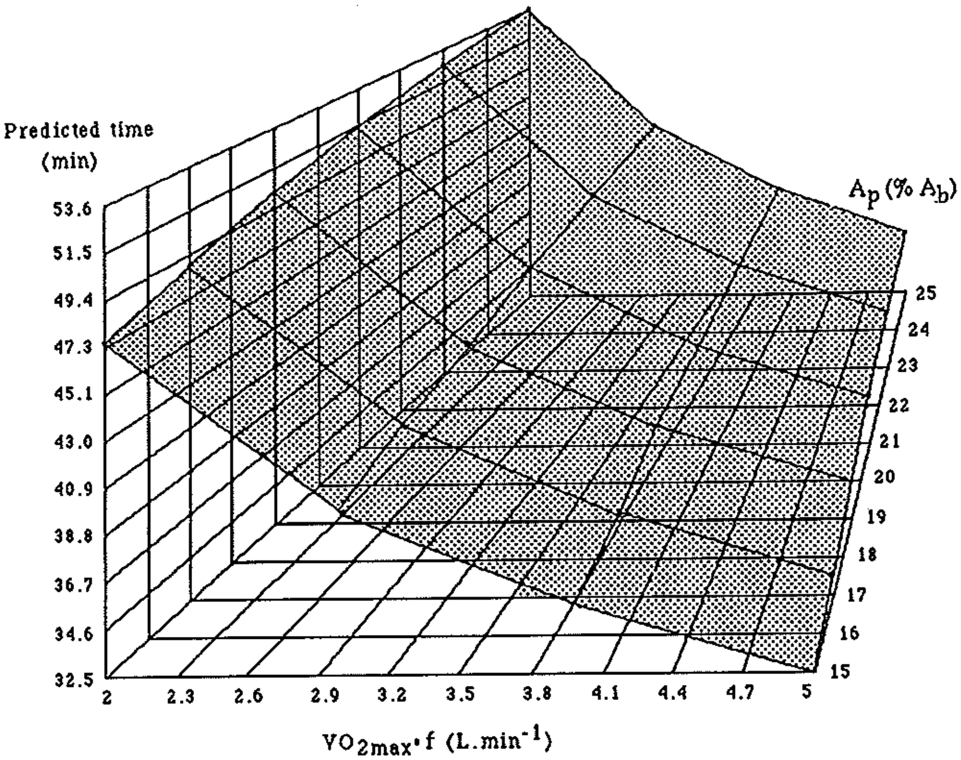

FIG. 5. Response surface for performance time as steady-state oxygen uptake ($\dot{V}O_{2\,max} \cdot f$, where $\dot{V}O_{2\,max}$ is maximal O_2 consumption and f is ventilatory threshold) and projected frontal area [A_p, %body surface area (%A_b)] vary within the limits found in this study.

predicted time of ± 7–10%), fractional utilization (± 4–5%), mechanical efficiency (combined effect of changes in Grad and Int = ± 5–6%), and A_p (± 3%). Figure 5 shows the three-dimensional response surface relating predicted performance time, the range of projected rider frontal areas (expressed as a %A_b), and the predicted steady-state $\dot{V}O_2$ ($\dot{V}O_{2\,max} \cdot f$) for the subjects in this study. Other physiological (Def$_{max}$, $\dot{V}O_{2\,INIT}$, τ, τ_{def}), environmental (PB, T, RH, v_w), and equipment (M_b, P) variables have less effect on performance time given the variability of values found in this study. The exception is Slope. The slope of the out-and-back course has a great effect on predicted performance times when the slope is large (>2% or 0.02).

Effect of drafting. The data of Kyle (16) allow us to quantify the effects of drafting behind a single bicycle or paceline on performance. The advantage from drafting depends on the spacing between the wheels of the lead and trailing bicycles. By using the mean values in this study, a wheel spacing of 0.1 m leads to a 5.89 min (14.2%) reduction in predicted time. The benefits from spacings of 0.2 m (5.85 min, 14.1%), 0.5 m (5.63 min, 13.6%), and 1 m (4.93 min, 11.9%) are also considerable. The reduction in time with a spacing of 2 m is somewhat less (−2.8 min, 6.8%), and there is no benefit with wheel spacings of ≥3 m.

Are small front wheels advantageous? Some cyclists use smaller (20-, 24-, or 26-in.) front wheels. This has the advantage of reducing the effective spacing between cyclists, allowing closer drafting, and also reduces the cyclist's A_p by tilting the rider forward. The projected area of smaller wheels is also marginally less than standard wheels. In addition, the mass of smaller front wheels is a little less. The disadvantage is that smaller wheels generally have a higher rolling resistance (*Eq. 3*). By using the model, it is possible to calculate the net effect of using smaller front wheels. A 27-in. wheel weighs ~1.4 kg. Since large and small wheels are made of the same materials, the weight of a wheel of a given diameter will be 1.4·Diam/27 kg. M_b for a bike equipped with a small wheel will then be M_b − 1.4 + 1.4·Diam/27. The wheel distance (Spacing) in drafting will decrease by (27 − Diam)/2 in. or 0.0254/2·(27 − Diam) m, 0.0254 being the factor converting inches to meters. The change in rolling resistance was modeled using *Eq. 3*. The effect of wheel size on A_p can be modeled geometrically. The lower end of the bicycle will drop 0.0127·(27 − Diam) m. The horizontal distance from the front axle to the rear axle is ~0.7 m on a standard racing bicycle. The cyclist will therefore be tilted at tan[0.0127·(27 − Diam)/0.7] rad to the horizontal. A_p was determined as part of this study. It was found that A_p values for the cyclist increase as the angle made by a line drawn from the trochanterion through C7 and the horizontal (Troch) increases, according to the formula

$$A_p = 0.26923\,A_b + 0.00404\,\text{Troch} - 0.23445 + 0.1159 \qquad (20)$$

where $r = 0.76$, $P = 0.0006$, RMSR = 0.0478, and $n = 20$.

Tilting the bicycle has much the same effect as decreasing Troch. It was therefore possible to simulate the effect of small front wheels. When no drafting was involved, and providing the forward tilt of the system does not adversely affect the cyclist's ability to exert force on the pedals, a 26-in. front wheel would result in a time reduction of 0.08 min (0.2% of the time with

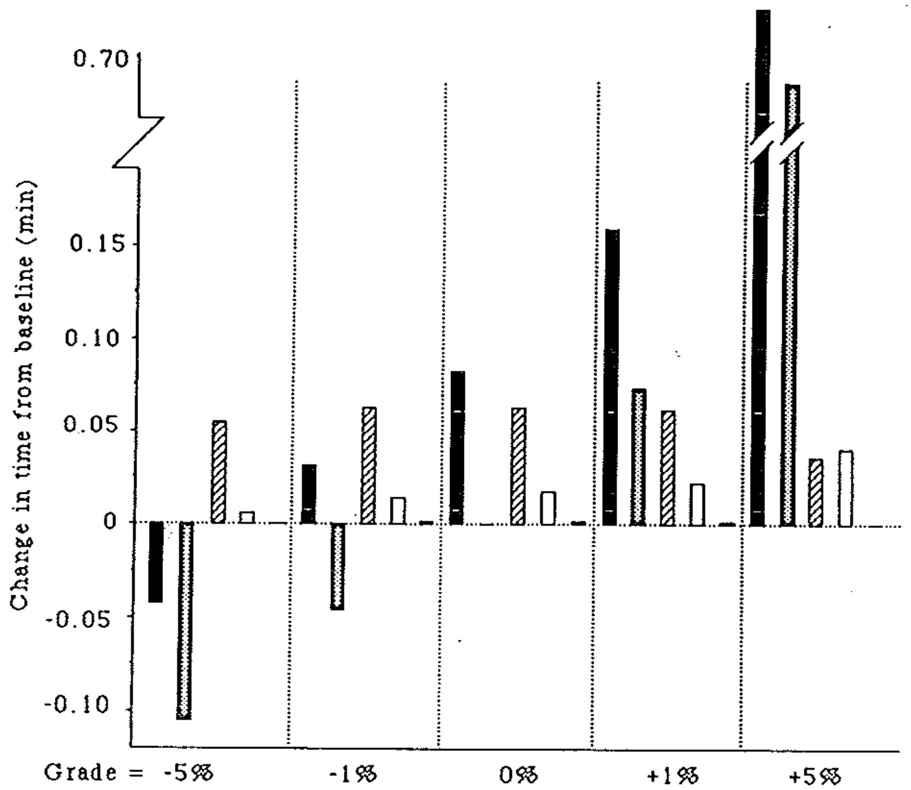

FIG. 6. Change in predicted time with addition of 1 kg of mass for a range of grades (−5% to +5%). Effects have been partitioned between change in time due to energy requirements of vertical displacement (stippled bars, E_{grade}), energy required to overcome air resistance (crossed bars, E_{air}), rolling resistance (open bars, E_{Rr}), and change in kinetic energy (dotted bars, E_{KE}). Note that bars for E_{KE} are barely visible due to small effect of changes in KE on performance time.

27-in. wheels using the mean values for other variables). A 24-in. front wheel reduced time by 0.20 min (0.5%). A 20-in. front wheel reduced time by 0.48 min (1.2%). When drafting was involved, the savings in time increased marginally, rising to 0.22 min or 0.6% for 24-in. wheels, and to 0.55 min (1.6%) with 20-in. wheels when drafting with a wheel spacing of 1 m. The gains from smaller wheels in road racing are therefore real but rather small. Ultimately, there will be a limit, when the cyclist is tilted so far past the horizontal that the A_p begins to increase. It is interesting to note that small front wheels are used in almost all world and olympic time-trial events.

Partitioning the effects of added mass. Increased mass, either of the rider or bike, can change the energy demand by affecting the rolling resistance, the energy required to ride up a grade, the KE of the system or the system's A_p and hence air resistance. On the flat, the effect of added mass is less than is commonly thought. A 1-kg increase in M resulted in a change of ~0.08 min (0.02%) for the values found in this study. However, the effect of added mass is much greater when riding up a grade. A 1-kg increase in M increased predicted time by 0.16 min (0.3%) on a 1% slope and by 0.75 min (0.9%) on a 5% slope.

The model can be used to separate the relative contribution of the effects of added mass on each of the components of energy demand [i.e., E_{grade}, E_{air}, energy to overcome rolling resistance (E_{Rr}), and E_{KE}] and to quantify these in terms of time. The net effect of added mass can be simulated by adding mass to the rider (and thereby increasing both M and A_p). When mass is added to the bicycle, M_b in the equations defining E_{grade}, E_{Rr}, and E_{KE} can be selectively controlled, thus isolating the effects of each of the energy components. The results are shown in Fig. 6. In general, the main effects of added mass arise from changes in the energy required for vertical displacement and, to a lesser extent, from the effects of M on A_p. The effects of changes in rolling resistance and KE are quite small. When the course is downhill, added mass can be advantageous if the slope is steep enough. An added M of 1 kg will be detrimental if the slope is −1% but advantageous if the slope is −5%. This is because the positive effect of the change in E_{grade} outweighs the negative effects of E_{air}, E_{Rr}, and E_{KE}. The benefit of added weight on downhill courses is far outweighed by the detriment of the same added mass on uphill courses, a consideration pertinent to out-and-back courses.

Rotational KE and the effect of the distribution of mass. Some coaches and cyclists feel that the location of extra mass is a significant factor in performance. A mass added centrally increases rolling resistance, the energy required to ride up a grade, the linear KE of the system (and the energy required to overcome air resistance, if the extra mass results in increased A_p). A weight added to the wheel rims, to the distal ends of the lower limbs, or to the distal ends of the cranks, however, increases all of the above energy demands and also adds to the rotational KE of the system. The rotational KE of the body segments is very small (amounting to <10 J using the mean values in this study; see Table 3). The rotational KE of the components (e.g., wheels) is somewhat greater (~80 J) because of their higher angular velocities. However, the total KE (linear plus angular) is ~4,570 J for the mean

M, M_b, and final velocity in this study. The angular component therefore constitutes ~2% of the total KE. The addition of 1 kg to the rims of each wheel would result in an additional rotational KE requirement of ~215 J. The effect on predicted times for the 26-km time trial would be negligible (<1-s increase). It remains possible, however, that an altered distribution of mass will affect mechanical efficiency.

Effect of changed drag. There are several reports of changes in drag from wind-tunnel or tractive tests when nonstandard components and accessories are used on bicycles. From these, the effects on performance time can be calculated. Table 5 summarizes some of these reports and the predicted change in performance time.

It should be noted that in the present study no correction was made for cyclists who used aerodynamic wheels and aero bars. It is possible, using available wind-tunnel data (Table 5), to make some estimates of the effect of these accesories. The reduced drag from disk wheels and aero bars would each be sufficient to reduce predicted 26-km time-trial time by ~1.3% of the mean measured time in this study. When predicted times were reduced for those cyclists using these accessories, the correlation between predicted and actual times was nonsignificantly higher (r = 0.92 vs. r = 0.89 for the original model). The mean difference was reduced to 0.72 min (0.75 min in the original model), and the mean absolute difference became 1.50 min (vs. 1.63 min).

Using the model to normalize performances. It is often difficult to compare performances that have occurred under different environmental conditions and using different equipment. The model allows performances to be compared in the following manner. The time trial is run, and the t_{actual} is recorded. A t_{pred} is

TABLE 5. *δDrag (g) when standard equipment is replaced by nonstandard equipment in wind-tunnel tests, v_a (m/s) under which tests were conducted, and Δt_{pred} (min) in 26-km time-trial performance using mean values recorded in this study*

Equipment Item	δDrag, g	v_a, m/s	Δt_{pred}, min	Ref.
Aero frame	−227	13.33	−0.858	19
	−130	13.33	−0.487	18
Aero bars	−90	13.33	−0.336	18
Bald head	−140	13.33	−0.525	18
Best legal helmet	−111	13.33	−0.415	18
Short hair	−8	13.33	−0.030	18
Long hair (loose)	+104	13.33	+0.380	18
Worst legal helmet	+140	13.33	+0.511	18
16-Spoke wheel	−150	13.33	−0.563	6
Wheel cover	−150	13.33	−0.563	6
18-Spoke wheel	−120	13.33	−0.449	6
Disk wheel	−40	13.33	−0.149	18
	−150	13.33	−0.563	6
No water bottle or cage	−40	13.33	−0.149	6
Skinsuit	<−40	13.33	−0.149	18
Aero water bottle	−35	13.33	−0.130	18
Clipless pedals	−20	13.33	−0.074	6
Aero cranks	−18	13.33	−0.063	17, 18
Shaved legs	−10	13.33	−0.037	6
Rain jacket	+732	13.33	+2.545	5

TABLE 6. *Environmental and equipment variables for three aspirants to female national road team*

Rider	A	B	C
Diam, in.	27	27	27
P, psi	160	150	160
M_b, kg	11.1	9.6	10.4
XWind, rad	0	90	0
PB, mmHg	762	752	758
T, K	283.3	294.6	285.9
v_w, m/s	0.62	0.67	0.62
RH, %	60	88	70
Actual time, min	38.95	39.33	38.83
Predicted time, min	39.08	42.47	39.10
Normalized time, min	38.49	39.59	38.81

calculated given the time-trial input variables. A second predicted time ($t_{standard}$) is then calculated, assuming standard environmental conditions and standardized equipment. The values for the relevant variables used are v_w = 0, XWind = 0, PB = 760 mmHg, T = 293.16 K, RH = 60%, Slope = 0, $CF_{Surface}$ = 1, Alt = 0, Diam = 27 in., P = 120 psi, no drafting. A normalized time (t_{norm}) is then calculated as

$$t_{norm} = t_{standard} \frac{t_{actual}}{t_{pred}}$$

Table 6 shows the results of this normalization procedure for three aspirants to the national female road squad.

Limits of human performance: 1-h record. The model permits a number of speculations regarding the limits of human performance in cycling. In cycling, one of the major tests of endurance is the distance covered in 1 h. This record has been broken a number of times recently by cyclists adopting innovative riding positions and new equipment. The record at sea level currently (July 1994) stands at 52.27 km. We simulated a performance by a rider having the physiological characteristics of a current Australian and world track endurance champion (M = 67 kg, $\dot{V}O_{2max}$ = 92 $ml \cdot kg^{-1} \cdot min^{-1}$). We used the best measured values in our study for Def_{max} (5.7 liters), mechanical efficiency (Grad = 0.0089 $l \cdot min^{-1} \cdot W^{-1}$, Int = 1.04 l/min for a national-level road cyclist), and f (0.9, an international-level road cyclist). We assumed standard environmental conditions (PB = 760 mmHg, T = 293 K, RH = 60%, v_w = 0 m/s), optimal equipment and riding surface (M_b = 9 kg, P = 160 psi, aerodynamic bicycle, $CF_{Surface}$ = 0.87), and a technique that minimizes A_p (A_p for rider only = 16% A_b, the lowest measured in the present study, for an international-level road cyclist). Under these conditions, our rider would cover 56.9 km in 1 h. At the altitude of Mexico City (PB = 580 mmHg), the athlete would cover 58.7 km.

While mathematical models of sports performance can be a very rich means of analyzing complex systems, their simplifying assumptions require constant verification before the "tribunal of experience." Some areas touched upon by the current model that invite further exploration are the interaction of riding position and mechanical efficiency, more sophisticated methods of determining A_p, the effect of body size on cycling performance, the drag characteristics of aerodynamic accessories, the rate and pattern of expression of the anaerobic capacity, and second-by-second acceleration patterns. Further empirical testing where variables are experimentally manipulated would be of value in refining this model.

APPENDIX

A Mathematical Model of Cycling

Energy demand of cycling. The energy demand (Demand) is the sum of the energy required to overcome rolling resistance (E_{Rr}), the energy required to overcome air resistance (E_{air}), the energy required to ride up or down a hill (E_{grade}), and the energy required to impart kinetic energy to the system (E_{KE})

$$\text{Demand} = E_{Rr} + E_{air} + E_{grade} + E_{KE} \qquad (A1)$$

$$E_{Rr} = \text{d}\ \mu_r \cos(\arctan \text{S})\ \text{CF}_{Grav}\ 9.80665\ (\text{Mass} + \text{M}_b) \qquad (A2)$$

$$\mu_r = \frac{a + \dfrac{b}{\text{P}}}{\text{Diam}}\ \text{CF}_{Surface} \qquad (A3)$$

where $a = -0.00051$ and $b = 9.73744$.

$$E_{grade} = \text{d}\ (M + M_b)\ \text{CF}_{Grav}\ 9.80665 \sin(\arctan \text{S}) \qquad (A4)$$

CF_{Grav} is a factor correcting weight force for altitude (Ref. 26)

$$\text{CF}_{Grav} = \left(\frac{6{,}378}{6{,}378 + \dfrac{\text{Alt}}{1{,}000}}\right)^2 \qquad (A5)$$

where 6,378 is the assumed radius of the Earth (in km).

$$E_{air} = 0.5\ \text{CF}_{Draft}\ C_D A_p \rho \text{CF}_{XWind} \times v_{ss}\left[t + 3k \exp\left(\frac{-t}{k}\right) - 1.5k \exp\left(\frac{-2t}{k}\right) + \frac{k}{3}\exp\left(\frac{-3t}{k}\right) - \frac{11k}{6}\right] + \text{KE}_{ang} \qquad (A6)$$

where CF_{Draft} is a factor correcting for the effects of drafting, based on a quadratic fit to empirical data from Ref. 16

$$\text{CF}_{Draft} = 1 - 0.3835 + 1.25\ \text{Spacing} + 0.0405\ \text{Spacing}^2 \qquad (A7)$$

$$A_p = 0.4147\frac{A_b}{1.771} + 0.1159 \qquad (A8)$$

where 1.771 is the estimated mean A_b of the cyclists in Ref. 4; 0.4147 is the mean estimated A_p of cyclists of Ht and M equivalent to those in Ref. 4, using the equation predicting A_p for the cyclist only

$$A_p = 0.3176\ A_b - 0.1478$$

where 0.1159 of *Eq. A8* is the mean measured A_p (m^2) of the bicycle in determinations made in our laboratory ($n = 26$).

$$A_b = \text{Ht}^{0.725} M^{0.425} 0.007184 \qquad (A9)$$

$$\rho = 1.225\frac{\text{P}_B}{760}\frac{288.15}{\text{T}}\ \text{CF}_{Humidity} \qquad (A10)$$

where 1.225 is the density (kg/m^3) of air at sea level at the temperature of 288.15 K.

$$\text{CF}_{Humidity} = \frac{\text{P}_{O_2} \times 32 + \text{P}_{N_2} \times 28 + \text{P}_{H_2O} \times 18}{0.2093 \times 32\ \text{P}_B + 0.7904 \times 28\ \text{P}_B} \qquad (A11)$$

where 32 is the mol wt of oxygen, and 0.2093 is the fraction of oxygen in air; 28 is the mol wt of nitrogen, and 0.7904 the fraction of nitrogen in air; and 18 is the mol wt of H_2O.

$$\text{P}_{H_2O} = 15.573 \exp[0.0606(\text{T} - 291)]\frac{\text{RH}}{100} \qquad (A12)$$

This equation is an approximation predicting the pressure of H_2O vapor from temperature (fitted from tables)

$$\text{P}_{O_2} = 0.2093(\text{P}_B - \text{P}_{H_2O}) \qquad (A13)$$

$$\text{P}_{N_2} = 0.7904(\text{P}_B - \text{P}_{H_2O}) \qquad (A14)$$

CF_{XWind} is an algorithm correcting air resistance for head and crosswinds (Ref. 10).

$$\text{CF}_{XWind} = \sqrt{(v_{ss}^2 + v_w^2 + 2v_{ss}v_w \cos \text{XWind})} \times \frac{v_{ss} + v_w \cos \text{XWind}}{v_{ss}^2} \qquad (A15)$$

This represents the ratio between the drag when there is a wind blowing from a given angle XWind rad and the drag under windless conditions

$$E_{\Delta Drag} = \text{d}\ \delta\text{Drag}\ 9.80665 \times \frac{(v_{ss} + v_w)^2}{v_a^2} \qquad (A16)$$

where δDrag is the measured change in drag (g) from wind-tunnel or tractive tests, and v_a is the measured relative air speed during these tests

$$E_{KE} = 0.5 \times (M + M_b)\left\{v_{ss}\left[1 - \exp\left(\frac{-t}{k}\right)\right]\right\}^2 + E_{KE\,ang} \qquad (A17)$$

where $E_{KE\,ang}$ is derived as explained in Table 3.

Energy supply in cycling. The energy supply expressed in liters of oxygen is the sum of the energy produced aerobically (E_{aer}) and the energy produced anaerobically (E_{an}).

$$\text{Supply} = E_{aer} + E_{an} \qquad (A18)$$

This is converted to an equivalent energy (in J) with the mechanical efficiency equation relating work rate (in W) to $\dot{V}O_2$ (liters of oxygen)

$$\text{Supply} = \frac{\left[\dfrac{(E_{aer} + E_{an}) \times 60}{t} - \text{Int}\right]}{\text{Grad}} \qquad (A19)$$

$$E_{an} = \text{Def}_{max}\left[1 - \exp\left(\frac{-t/60}{\tau_{def}}\right)\right]\left(1 - 0.233 \ln\frac{t}{420}\right) \qquad (A20)$$

the last term being a correction factor reducing the deficit expressed in longer events (Ref. 30).

$$E_{aer} = \text{CF}_v\ \dot{V}O_{2\,max} f \frac{t}{60} - \tau(\text{CF}_v\ \dot{V}O_{2\,max} f - \dot{V}O_{2\,init}) \qquad (A21)$$

CF_V is a factor correcting for $\dot{V}O_{2\,max}$ altitude (Ref. 26)

$$CF_v = \frac{23.11 + 76.89\left[1 - \exp\left(\frac{253 - P_B}{169}\right)\right]}{96.171374} \quad (A22)$$

τ is corrected for altitude by multiplying the sea-level value by

$$\frac{\dot{V}O_{2\,max} - \dot{V}O_{2\,init}}{\dot{V}O_{2\,max}\,CF_v - \dot{V}O_{2\,init}}$$

(Ref. 26).

When performance is corrected for altitude, default values for PB and T are used as described by an iterative approximation method (Ref. 36). The pressure at Z m is calculated in the following manner. First, an approximation for the acceleration due to gravity is made

$$G = 9.80665\left(\frac{6{,}378}{6{,}378 + \frac{Z}{1{,}000}}\right)^2 \quad (A23)$$

The temperature at this altitude is calculated as

$$T = 288.15 - 0.0065Z$$

where Z is a small increment in altitude (the initial value of T at $Z = 0$ is 288.15 K). PB for the i+1st iteration is then given by

$$P_{B_{i+1}} = P_{B_i}\frac{T}{288.15 - 0.0065\,Z} \times \frac{G}{287 \times 0.0065}$$

An initial PB (P_{B_0}) of 101,325 Pa is used.

Equating demand and supply. An initial value for t is guessed, and the corresponding v_{ss} is calculated. Because

$$v_t = v_{ss}\left[1 - \exp\left(\frac{-t}{k}\right)\right]$$

$$d = \int v_t dt = v_{ss}\left[t + k\exp\left(\frac{-t}{k}\right) - k\right] \quad (A24)$$

and

$$v_{ss} = \frac{d}{t + k\exp\left(\frac{-t}{k}\right) - k}$$

Supply and Demand are then calculated. If the difference between Supply and Demand is greater than some value δE, then t is incremented or decremented by a small value δt until the difference $<\delta E$.

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