CS 540-1: Introduction to Artificial Intelligence
Homework Assignment # 3

Assigned: 10/10
Due: 10/24 before class

Hand in your homework:

This homework includes only a written portion. Please type the written portion and hand in the file in pdf format and name it as **WrittenPart.pdf**. The first page of the pdf must include a header with: **your name, Wisc username, class section, HW# , date and, if handed in late, how many days late it is**. Please create a folder named as <**Wisc username**>_**HW#**, put **WrittenPart.pdf** into the folder and compress it as <**Wisc username**>_**HW#.zip**. This final zip file should then be uploaded to the appropriate place on the course Moodle website.

Late Policy:

All assignments are due at the beginning of class on the due date. One (1) day late, defined as a 24-hour period from the deadline (weekday or weekend), will result in 10% of the total points for the assignment deducted. So, for example, if a 100-point assignment is due on a Wednesday 9:55 a.m., and it is handed in between Wednesday 9:55 a.m. and Thursday 9:55 a.m., 10 points will be deducted. Two (2) days late, 25% off; three (3) days late, 50% off. No homework can be turned in more than three (3) days late. Written questions and program submission have the same deadline. A total of two (2) free late days may be used throughout the semester without penalty.

Assignment grading questions must be raised with the instructor within one week after the assignment is returned.

Collaboration Policy:

You are to complete this assignment individually. However, you are encouraged to discuss the general algorithms and ideas with classmates, TAs, and instructor in order to help you answer the questions. You are also welcome to give each other examples that are not on the assignment in order to demonstrate how to solve problems. But we require you to:

- not explicitly tell each other the answers
- not to copy answers or code fragments from anyone or anywhere
- not to allow your answers to be copied
- not to get any code on the Web

In those cases where you work with one or more other people on the general discussion of the assignment and surrounding topics, we suggest that you specifically record on the assignment the names of the people you were in discussion with.

Question 1: Perceptron [5]

Using a linear threshold unit perceptron, implement the NOR function, shown below. That is, write down weights w_0, w_A, w_B such that a LTU will produce the NOR outputs given inputs A and B. (Choose the weights such that $|w_A| = |w_B| = 1$)

A	B	NOR
0	0	1
0	1	0
1	0	0
1	1	0

Question 2: Probabilities [15]

(a) [7] Jack, Queen and King are called face cards. What is the conditional probability that a card drawn at random from a pack of 52 cards is a face card, given that the drawn card is a diamond.

(b) [8] In a bag there are 3 blue, 4 green and 2 red marbles. What is the probability of picking up two marbles of the same color from the bag one after another without replacement?

Question 3: Support Vector Machines [20]

In this question, you are provided with several points of two classes in a two-dimensional space, as showed in Fig. 1. Points of the positive class are represented as blue • while points of the negative are represented as green ×. It's apparently a linear inseparable question. Now you are required to find a linear SVM with the help following kernel:

$$K(\mathbf{x}, \mathbf{y}) = 2\|\mathbf{x}\|\|\mathbf{y}\|$$

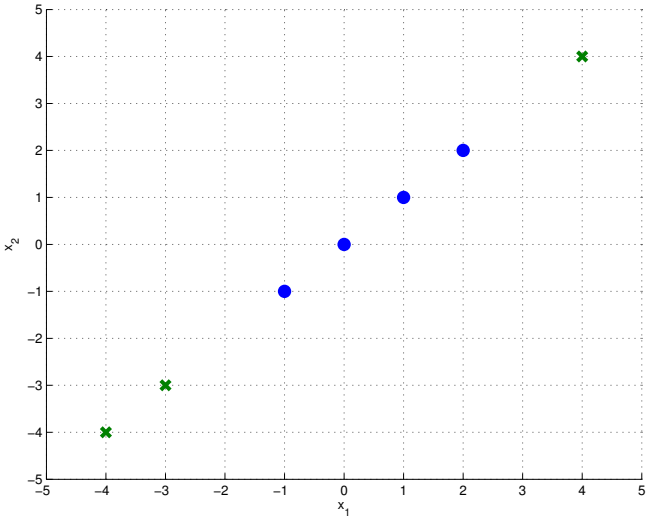


Figure 1: Points of two classes

(a) [8] Find the non-linear mapping $\phi(\cdot)$ for the given kernel K that satisfies $K(\mathbf{x}, \mathbf{y}) = \phi(\mathbf{x}) \cdot \phi(\mathbf{y})$, and then convert all original points into the new space using the mapping $\phi(\cdot)$. (The original coordinates of points are all integers)

(b) [8] Find the linear SVM for the transformed space and identify the support vectors on the margin.

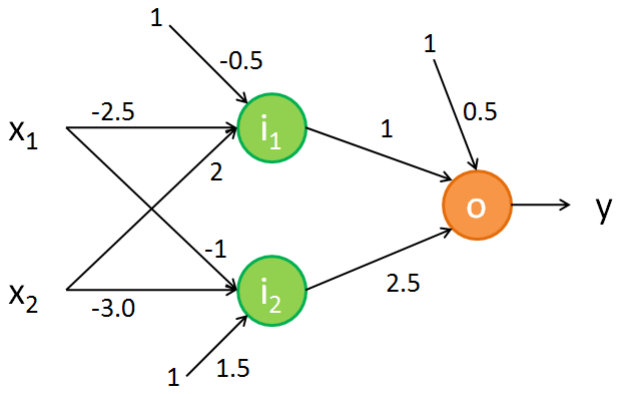
(c) [6] Write the function form of the corresponding decision boundary in the original space.

Question 4: Neural Networks [20]

The following is a representation of a neural network with hidden layer nodes colored green and the output node colored orange. x_1 and x_2 are the input variables. For the following questions, assume that the learning rate $\alpha = 0.1$. Each node also has a bias input of value 1. Also, assume that there is a sigmoid activation function at the hidden layer nodes and at the output layer node. A sigmoid activation function takes the form:

$$\sigma(\mathbf{p}) = \frac{1}{1 + e^{-(\mathbf{w}^T \mathbf{p})}}$$

where $\mathbf{p}$ is the input vector and $\mathbf{w}$ is the weight vector for that particular node.



(a) [10] Calculate the output values of nodes i_1, i_2 and o of this network for the input $\{x_1 = 0, x_2 = 1\}$. Show all the step of your calculation.

(b) [10] Now you are going to compute one step of the backpropagation algorithm. The input for the training instance is $\{x_1 = 0, x_2 = 1\}$ and the output of this training instance is $y = 1$. Please compute the updated weights for the output layer (the three incoming weights to the orange node) by performing ONE step of gradient descent. Show all steps of your calculation.