CS 540-1: Introduction to Artificial Intelligence **Fall 2014-15**

Homework Assignment 2

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1 Question 1

$$f(x) = -x^5 + \frac{5}{2}x^4 + \frac{75}{3}x^3 + 5$$

First, take the derivative of this function to find the places where this function is flat.

$$f'(x) = -5x^4 + 10x^3 + 75x^2$$

Now find the values where this function is 0.

$$-5x^4 + 10x^3 + 75x^2 = 0$$
$$x(-5x^3 + 10x^2 + 75x) = 0$$
$$x(-5x^2 + 10x + 75) = 0$$
$$-5(x^2 + 2x + 15) = 0$$
$$(x - 5)(x + 3)$$

We have that

$$x = 5$$
$$x = 0$$
$$x = -3$$

The values must be between -3.5 and 4. Then, x=5 cannot be used. Then what left is x=0 and x=-3. Applying both value in the main function we have:

For x=0:

$$f(0) = -0^5 + \frac{5}{2}0^4 + \frac{75}{3}0^3 + 5$$
$$f(0) = 5$$

$$f(-3) = -(-3)^5 + \frac{5}{2}(-3)^4 + \frac{75}{3}(-3)^3 + 5$$
$$f(-3) = -224.5$$

Then the minimum global for this function between [-3.5, 4] is -224.5.

2 Question 2

For this training set, I just pick all the point and calculate their distance with the points given and select the 3 ones with smaller distance to see for which class the point will belong.

Class	Training Set Points
class1	(1, 1), (2,2), (1,2)
class2	(3,2.5), (2.7, 2.8), (1.8, 3.2)
class3	(0.9, 4), (2.8, 4.1)

2.1 a)

Classifying the points: (1.5, 1.5), (3, 2.2), (1.3, 3) using the given equation for distance:

$$\delta(x_i, x_j) = \sum_{k=1}^{d} |x_{ik} - x_{jk}|$$

Classifying (1.5,1.5):

$$\delta((1,1),(1.5,1.5)) = |1-1.5| + |1-1.5| = 1*$$

$$\delta((2,2),(1.5,1.5)) = |2-1.5| + |2-1.5| = 1*$$

$$\delta((1,2),(1.5,1.5)) = |1-1.5| + |2-1.5| = 1*$$

$$\delta((3,2.5),(1.5,1.5)) = |3-1.5| + |2.5-1.5| = 2.5$$

$$\delta((2.7,2.8),(1.5,1.5)) = |2.7-1.5| + |2.8-1.5| = 2.5$$

$$\delta((1.8,3.2),(1.5,1.5)) = |1.8-1.5| + |3.2-1.5| = 2$$

$$\delta((0.9,4),(1.5,1.5)) = |0.9-1.5| + |4-1.5| = 3.1$$

$$\delta((2.8,4.1),(1.5,1.5)) = |2.8-1.5| + |4.1-1.5| = 4.9$$

After this, we can see that the closest points to (1.5,1.5) are all Class1, which means that the point (1.5,1.5) was classify as *Class1*.

Classifying (3, 2.2):

$$\delta((1,1),(3,2.2)) = |1-3| + |1-2.2| = 3.2$$

$$\delta((2,2),(3,2.2)) = |2-3| + |2-2.2| = 1.2*$$

$$\delta((1,2),(3,2.2)) = |1-3| + |2-2.2| = 2.2$$

$$\delta((3,2.5),(3,2.2)) = |3-3| + |2.5-2.2| = 0.3*$$

$$\delta((2.7, 2.8), (3, 2.2)) = |2.7 - 3| + |2.8 - 2.2| = 0.9*$$
$$\delta((1.8, 3.2), (3, 2.2)) = |1.8 - 3| + |3.2 - 2.2| = 2.2$$

$$\delta((0.9, 4), (3, 2.2)) = |0.9 - 3| + |4 - 2.2| = 3.9$$
$$\delta((2.8, 4.1), (3, 2.2)) = |2.8 - 3| + |4.1 - 2.2| = 2.1$$

After this, we can see that the closest points to (3, 2.2) are two from Class2 and one from Class1, which means that the point (3, 2.2) was classify as *Class2*, because the majority is Class2 and the smaller value is also from Class2.

Classifying (1.3, 3):

$$\delta((1, 1), (1.3, 3)) = |1 - 1.3| + |1 - 3| = 2.3$$
$$\delta((2, 2), (1.3, 3)) = |2 - 1.3| + |2 - 3| = 1.7$$
$$\delta((1, 2), (1.3, 3)) = |1 - 1.3| + |2 - 3| = 1.3*$$

$$\delta((3, 2.5), (1.3, 3)) = |3 - 1.3| + |2.5 - 3| = 2.2$$
$$\delta((2.7, 2.8), (1.3, 3)) = |2.7 - 1.3| + |2.8 - 3| = 1.6$$
$$\delta((1.8, 3.2), (1.3, 3)) = |1.8 - 1.3| + |3.2 - 3| = 0.7*$$

$$\delta((0.9, 4), (1.3, 3)) = |0.9 - 1.3| + |4 - 3| = 1.4*$$
$$\delta((2.8, 4.1), (1.3, 3)) = |2.8 - 1.3| + |4.1 - 3| = 2.6$$

After this, we can see that the closest points to (1.3, 3) are one each class. Then I selected the smaller distance to set the class, which means that the point (1.3, 3) was classify as *Class2*, because smaller value (0.7) is from a point of Class2.

2.2 b)

Adding a class2 point (2.5,2.7) to the training set and calculate everything again.

Classifying (1.5,1.5):

$$\delta((1, 1), (1.5, 1.5)) = |1 - 1.5| + |1 - 1.5| = 1*$$
$$\delta((2, 2), (1.5, 1.5)) = |2 - 1.5| + |2 - 1.5| = 1*$$
$$\delta((1, 2), (1.5, 1.5)) = |1 - 1.5| + |2 - 1.5| = 1*$$

$$\delta((3, 2.5), (1.5, 1.5)) = |3 - 1.5| + |2.5 - 1.5| = 2.5$$
$$\delta((2.7, 2.8), (1.5, 1.5)) = |2.7 - 1.5| + |2.8 - 1.5| = 2.5$$
$$\delta((1.8, 3.2), (1.5, 1.5)) = |1.8 - 1.5| + |3.2 - 1.5| = 2$$

$$\delta((2.5, 2.7), (1.5, 1.5)) = |2.5 - 1.5| + |2.7 - 1.5| = 2.2$$

$$\delta((0.9, 4), (1.5, 1.5)) = |0.9 - 1.5| + |4 - 1.5| = 3.1$$
$$\delta((2.8, 4.1), (1.5, 1.5)) = |2.8 - 1.5| + |4.1 - 1.5| = 4.9$$

After this, we can see that the closest points to (1.5,1.5) are all Class1, which means that the point (1.5,1.5) was classify as *Class1*.

Classifying (3, 2.2):

$$\delta((1, 1), (3, 2.2)) = |1 - 3| + |1 - 2.2| = 3.2$$
$$\delta((2, 2), (3, 2.2)) = |2 - 3| + |2 - 2.2| = 1.2$$
$$\delta((1, 2), (3, 2.2)) = |1 - 3| + |2 - 2.2| = 2.2$$

$$\delta((3, 2.5), (3, 2.2)) = |3 - 3| + |2.5 - 2.2| = 0.3*$$
$$\delta((2.7, 2.8), (3, 2.2)) = |2.7 - 3| + |2.8 - 2.2| = 0.9*$$
$$\delta((1.8, 3.2), (3, 2.2)) = |1.8 - 3| + |3.2 - 2.2| = 2.2$$
$$\delta((2.5, 2.7), (3, 2.2)) = |2.5 - 3| + |2.7 - 2.2| = 1*$$

$$\delta((0.9, 4), (3, 2.2)) = |0.9 - 3| + |4 - 2.2| = 3.9$$
$$\delta((2.8, 4.1), (3, 2.2)) = |2.8 - 3| + |4.1 - 2.2| = 2.1$$

After this, we can see that the closest points to (3, 2.2) are all from Class2, which means that the point (3, 2.2) was classify as *Class2*.

Classifying (1.3, 3):

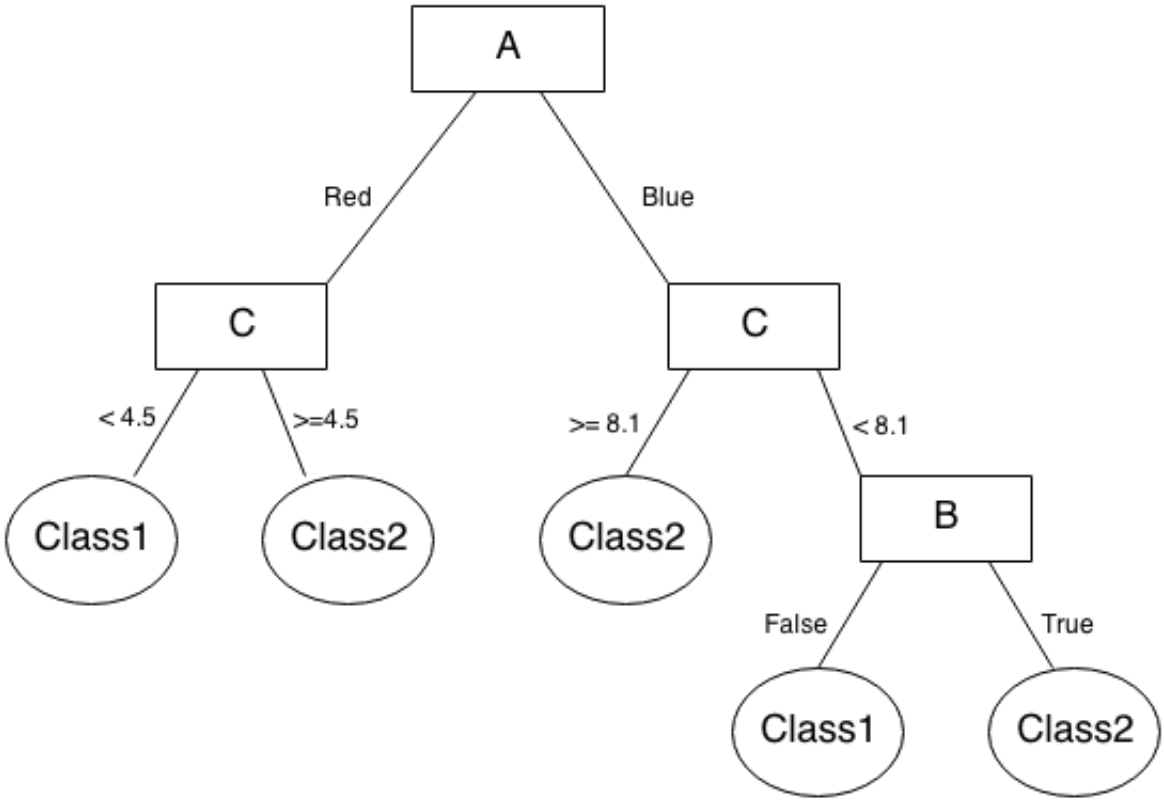
$$\delta((1, 1), (1.3, 3)) = |1 - 1.3| + |1 - 3| = 2.3$$
$$\delta((2, 2), (1.3, 3)) = |2 - 1.3| + |2 - 3| = 1.7$$
$$\delta((1, 2), (1.3, 3)) = |1 - 1.3| + |2 - 3| = 1.3*$$

$$\delta((3, 2.5), (1.3, 3)) = |3 - 1.3| + |2.5 - 3| = 2.2$$
$$\delta((2.7, 2.8), (1.3, 3)) = |2.7 - 1.3| + |2.8 - 3| = 1.6$$
$$\delta((1.8, 3.2), (1.3, 3)) = |1.8 - 1.3| + |3.2 - 3| = 0.7*$$
$$\delta((2.5, 2.7), (1.3, 3)) = |2.5 - 1.3| + |2.7 - 3| = 1.5$$

$$\delta((0.9, 4), (1.3, 3)) = |0.9 - 1.3| + |4 - 3| = 1.4*$$
$$\delta((2.8, 4.1), (1.3, 3)) = |2.8 - 1.3| + |4.1 - 3| = 2.6$$

We can see that the closest points to (1.3, 3) are one each class. Then I selected the smaller distance to set the class, which means that the point (1.3, 3) was classify as *Class2*, because smaller value (0.7) is from a point of Class2.

3 Question 3



4 Question 4

4.1 a)

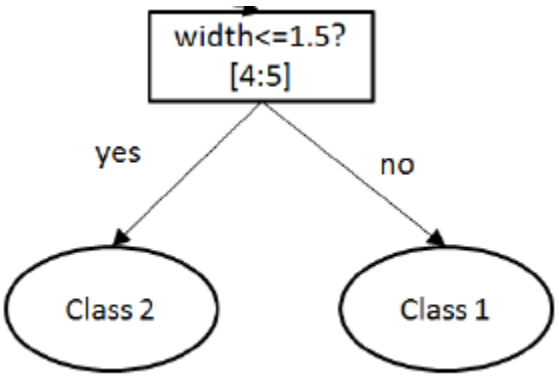
Using the training set in the tree. There are 5 examples, 3 were wrong and 2 right, then the accuracy is 2/5 - 40%.

Width	Length	Class	Result
1.55	2.2	class 2	Wrong, predicted class1
0.5	1.4	class 1	Right
1.8	0.99	class 1	Wrong, predicted class2
1.6	1.8	class 2	Wrong, predicted class1
1.4	1.5	class 1	Right

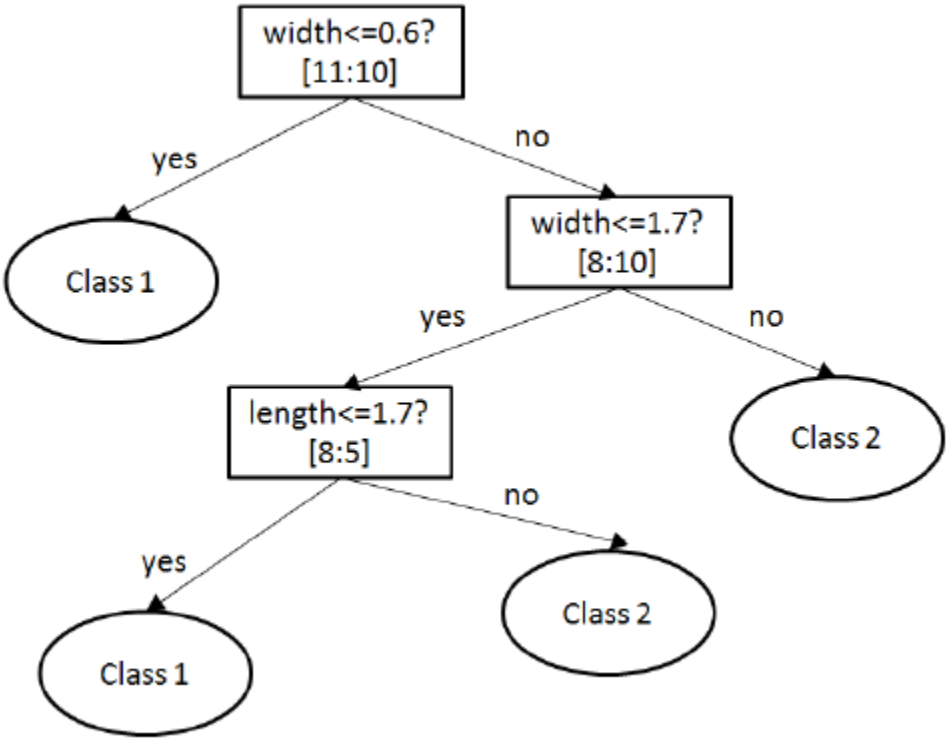
4.2 b)

Assuming that the values n:n in the value represent class1:class2, e.g 4:5, 4 class1 and 5 class2.

I started pruning the node with only leafs as children.



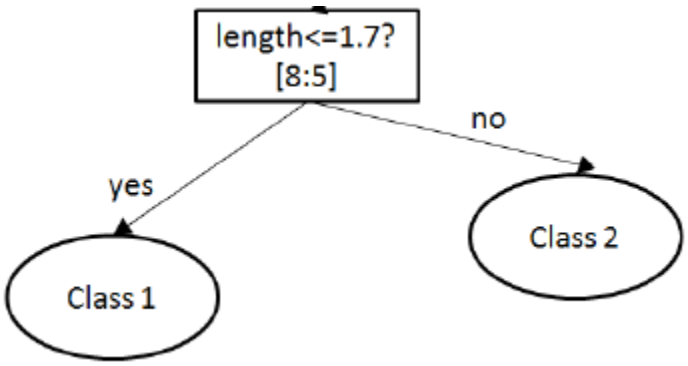
Using the assumption that the number inside the node represent class1:class2, the majority in this node are class2. Thus, the tree will be like this:



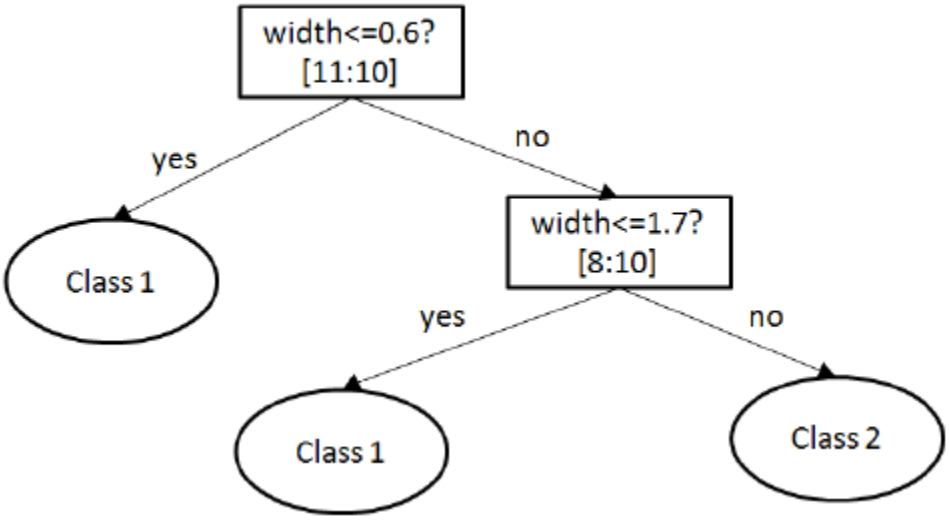
Using the examples to find the accuracy I got: 5 examples, 1 were wrong and 4 right, then the accuracy is 4/5 - 80%, accuracy increased.

Width	Length	Class	Result
1.55	2.2	class 2	Right
0.5	1.4	class 1	Right
1.8	0.99	class 1	Wrong, predicted class2
1.6	1.8	class 2	Right
1.4	1.5	class 1	Right

Continue pruning, now the node also with only leafs as children:



The majority in this node are class1. Thus, the tree will be like this:

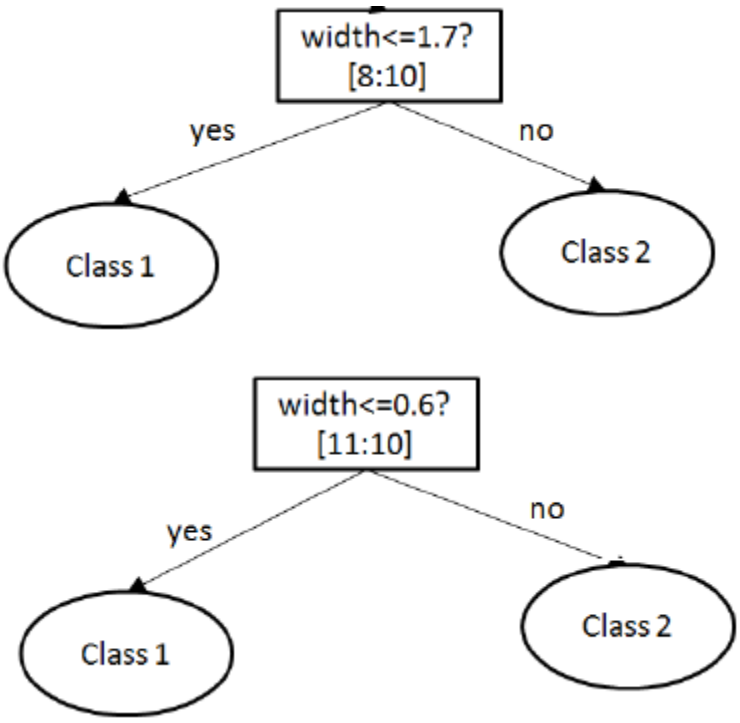


Using the examples to find the accuracy I got: 5 examples, 3 were wrong and 2 right, then the accuracy is 2/5 - 40%, accuracy decreased.

Width	Length	Class	Result
1.55	2.2	class 2	Wrong, predicted class1
0.5	1.4	class 1	Right
1.8	0.99	class 1	Wrong, predicted class2
1.6	1.8	class 2	Wrong, predicted class1
1.4	1.5	class 1	Right

Continue pruning, now the node also with only leafs as children. The majority in this node are class2. Then we will stay with a tree only with the root node and the leafs.

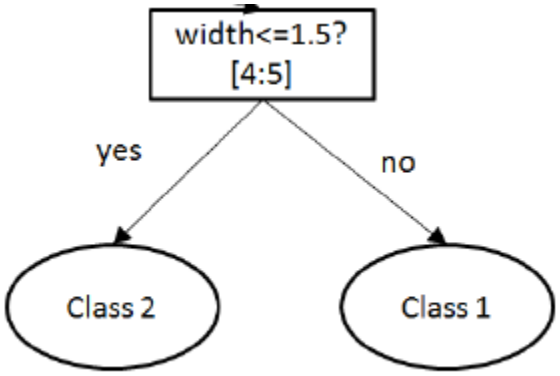
Using the examples to find the accuracy I got: 5 examples, 2 were wrong and 3 right, then the accuracy is 3/5 - 60%, accuracy increased.



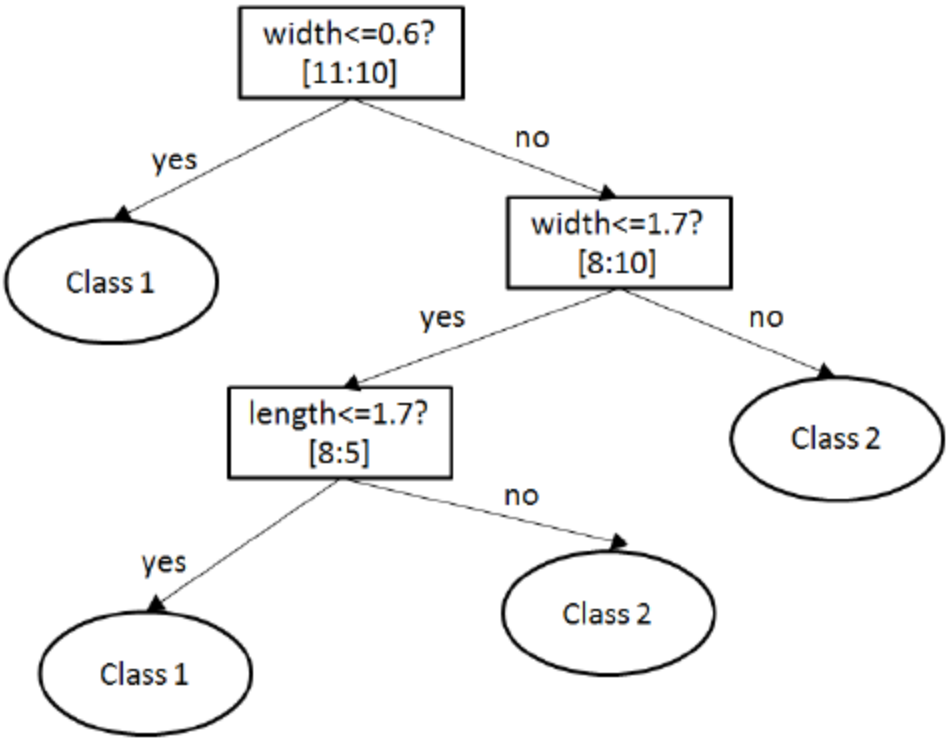
Width	Length	Class	Result
1.55	2.2	class 2	Right
0.5	1.4	class 1	Right
1.8	0.99	class 1	Wrong, predicted class2
1.6	1.8	class 2	Right
1.4	1.5	class 1	Wrong, predicted class2

4.3 b)

By looking at the results, the best node to be pruned first is the node:



With this node pruned the accuracy of the test increased from 40% to 60%, and the tree will look like this:


width<=0.6?
[11:10]
yes
no
Class 1
width<=1.7?
[8:10]
yes
no
length<=1.7?
[8:5]
Class 2
no
yes
Class 2
Class 1