**CS 540-1: Introduction to Artificial Intelligence** **Fall 2014-15**

# Homework Assignment 3

*Name:* Lauro Tremea Culau
*Wisc User:* tremeaculau **Date: Oct 24, 2014**

# 1 Question 1

Given that G(z) = sign(z), we have that

$$z = w0 * 1 + A * w1 + B * w2$$
$$sign(z) = \begin{cases} \text{1, z} \geq 0 \\ \text{0, otherwise} \end{cases}$$

Then, for this problem, using $w_A = w_B =$ -1 ( which satisfies $w_A = w_B =$ — 1 — ), and setting $w_0 = 0.5$, we get:

$$w_A = w_B = -1, w_0 = 0.5$$
$$z = w0 * 1 + A * w1 + B * w2$$

$$(0.5 * 1 + 0 * (-1) + 0 * (-1) = 0.5$$
$$(0.5 * 1 + 0 * (-1) + 1 * (-1) = -0.5$$
$$(0.5 * 1 + 1 * (-1) + 0 * (-1) = -0.5$$
$$(0.5 * 1 + 1 * (-1) + 1 * (-1) = -1.5$$

This satisfies the NOR, where only when both bits are 0 the output is 1. As it is possible to see, only in the first line the value of G(z) $\geq$ 0.

# 2 Question 2

## 2.1 a)

$$\Pr[Face|Diamond] = \frac{\Pr[Diamonds, Face]}{\Pr[Diamond]}$$

$$\Pr[Face|Diamond] = \frac{\frac{3}{52}}{\frac{13}{52}}$$

$$\Pr\left[\,Face|Diamond\,\right] = \frac{3}{52} * \frac{52}{13}$$
$$\Pr\left[\,Face|Diamond\,\right] = \frac{3}{13}$$

## 2.2 b)

First pick:

$$\Pr\left[\,Blue\,\right] : \frac{3}{9}$$
$$\Pr\left[\,Green\,\right] : \frac{4}{9}$$
$$\Pr\left[\,Red\,\right] : \frac{2}{9}$$

Second pick:

$$\Pr\left[\,Blue\,\right] : \frac{2}{8}$$
$$\Pr\left[\,Green\,\right] : \frac{3}{8}$$
$$\Pr\left[\,Red\,\right] : \frac{1}{8}$$

Putting the two picks together:

$$\Pr\left[\,Blue\,\right] : \frac{3}{9} * \frac{2}{8} = \frac{6}{72}$$
$$\Pr\left[\,Green\,\right] : \frac{4}{9} * \frac{3}{8} = \frac{12}{72}$$
$$\Pr\left[\,Red\,\right] : \frac{2}{9} * \frac{1}{8} = \frac{2}{72}$$

Summing all options,the probability of picking up two marbles of the same color from the bag one after another without replacement is:

$$\Pr\left[\,Marbles\,\right] = \frac{(6+12+2)}{72} = \frac{20}{72}$$

# 3 Question 3

## 3.1 a)

Mapping from $\Re^2$ to $\Re^3$:

$$K(x,y) = 2\|x\|\|y\|$$
$$K(x,y) = 2 * (\sqrt{x^T * x}) * (\sqrt{y^T * y})$$
$$K(x,y) = 2 * \sqrt{x^2} * \sqrt{y^2}$$
$$K(x,y) = 2xy$$

$$\phi(x) = \{x, y, 2xy\}$$

Points in the new space:

P(-4,-4) = (-4,-4,32)

P(-3,-3) = (-3,-3,18)

P(-1,-1) = (-1,-1,2)

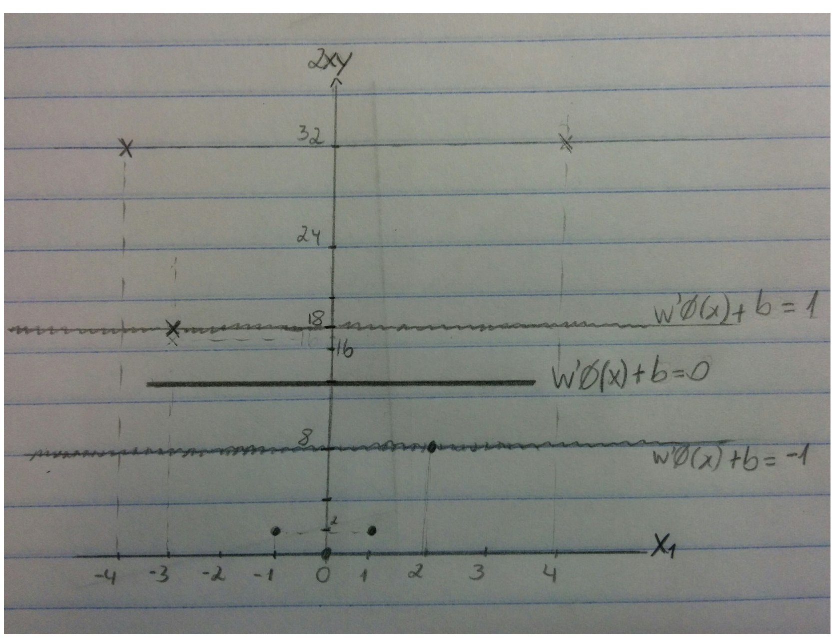
P(0,0) = (0,0,0)

P(1,1) = (1,1,2)

P(2,2) = (2,2,8)

P(4,4) = (4,4,32)

## 3.2 b)



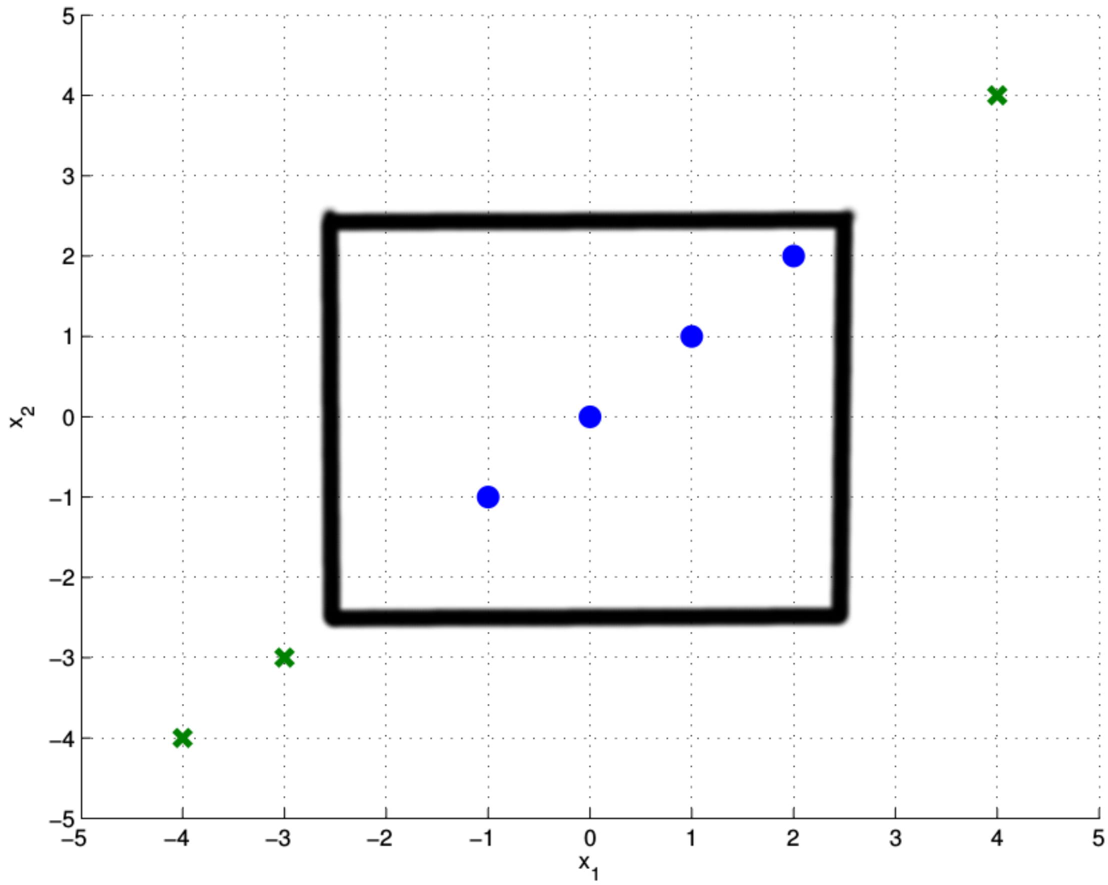
The image (in 2D for better visualization, but with the new dimension in vertical axis, so it is possible to see the division of data) shows the boundary line. Points (2,2,8) and (-3,-3,18) are Support Vector because they are in the lines which are the boundaries.

### 3.3 c)

The function form of the corresponding decision boundary in the original space is:

$$2xy \geq 13$$

The decision boundary can be seen in the image:




## 4 Question 4

### 4.1 a)

$x_1 = 0$

$x_2 = 1$

Calculate $i_1$:

$$w'i_1 = (w_0 + w_1 * x_1 + w_2 * x_2)$$
$$w'i_1 = (-0.5 + (-2.5) * 0 + 2 * 1)$$
$$w'i_1 = 1.6$$

$$i_1 = \frac{1}{1 + e^{(-w'i_1)}}$$
$$i_1 = \frac{1}{1 + e^{-1.6}}$$
$$i_1 = \frac{1}{1 + 0.201896518}$$
$$i_1 = \frac{1}{1.201896518}$$

$$i_1 = 0.832018385$$

Calculate $i_2$:

$$w'i_2 = (w_0 + w_1 * x_1 + w_2 * x_2)$$
$$w'i_2 = (1.5 + (-1) * 0 + (-3) * 1)$$
$$w'i_2 = -1.5$$

$$i_2 = \frac{1}{1 + e^{(-w'i_2)}}$$
$$i_2 = \frac{1}{1 + e^{1.5}}$$
$$i_2 = \frac{1}{1 + 4.48168907}$$
$$i_2 = \frac{1}{5.48168907}$$

$$i_2 = 0.182425524$$

Calculate O:

$$w'O = (w_0 + w_1 * i_1 + w_2 * i_2)$$
$$w'O = (0.5 + 1 * 0.832018385 + 2.5 * 0.182425524)$$
$$w'O = (0.5 + 0.832018385 + 0.45606381)$$
$$w'O = 1.788082195$$

$$O = \frac{1}{1 + e^{(-w'O)}}$$

$$O = \frac{1}{1 + e^{1.788082195}}$$
$$O = \frac{1}{1 + 5.977976872}$$
$$O = \frac{1}{6.977976872}$$

$$O = 0.143308013$$

## 4.2 b)

Calculating the Error of the Output

$$\delta_O = (O - y) * O * (1 - O)$$

$$\delta_O = (0.143 - 1) * 0.143 * (1 - 0.143)$$
$$\delta_O = -0.105$$

Back Propagating one step to find out the new values of weights:

For $w_0$ from bias:

$$\triangle w_0 = \alpha * h_0 * \delta_O$$
$$\triangle w_0 = 0.1 * 1 * (-0.105)$$
$$\triangle w_0 = -0.0105$$

$$w_0 new = w0 - \triangle w_0$$
$$w_0 new = 1 - (-0.0105)$$
$$w_0 new = 1.0105$$

For $w_1$ from $i_1$:

$$\triangle w_1 = \alpha * h_1 * \delta_O$$
$$\triangle w_1 = 0.1 * 0.832 * (-0.105)$$
$$\triangle w_1 = -0.0087$$

$$w_1 new = w1 - \triangle w_1$$
$$w_1 new = 1 - (-0.0087)$$
$$w_1 new = 1.0087$$

For $w_2$ from $i_2$:

$$\triangle w_2 = \alpha * h_2 * \delta_O$$
$$\triangle w_2 = 0.1 * 0.182 * (-0.105)$$

$$\triangle w_2 = -0.0019$$

$$w_2new = w2 - \triangle w_2$$
$$w_2new = 2.5 - (-0.0019)$$
$$w_2new = 2.5019$$