CS 540-1 | HW #1

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Question 1

a)

$$f(x) = \ln(4 + \sin^2 x) + e^{3x} \cos x$$

Calculating the first part of the equation → $\ln(4 + \sin^2 x)$:

$$\frac{df(u)}{dx} = \ln(u) \,.\, \frac{d}{dx}(4 + \sin^2 x)$$

1)Calculating the part after multiplication, we have:

$$\frac{d}{dx}(4 + \sin^2 x) = \frac{d}{dx}(4) + \frac{d}{dx}(\sin^2 x)$$

$$= 0 + \frac{d}{du}(u^2) . \frac{d}{dx}(\sin x\ )$$

$$= \ 2u. \cos x$$

$$= 2 \sin x \,.\, \cos x$$

Going back to df(u)/dx:

$$= \frac{1}{u}\ .2 \sin x + \cos x$$

$$= \frac{1}{4 + \sin^2 x} . 2 \sin x + \cos x$$

$$= \frac{2 \sin x + \cos x}{4 + \sin^2 x}$$

This is the value for the first part of the equation.

2) Now it is necessary to calculate the second part of the equation → $e^{3x} \cos x$:

$$e^{3x} \cos x = (f.g)'$$

$$\frac{d}{dx}(e^{3x} \cos x) = \frac{d}{dx}(e^{3x}) . \cos x + \frac{d}{dx}(\cos x) . e^{3x}$$

Doing the calculus for the first part of this equation:

$$\frac{d}{dx}(e^{3x}) = \frac{d}{dx}(e^{u}) = \frac{d}{du}(3x) = 3e^{3x}$$

Now, doing the math for the second part:

$$\frac{d}{dx}(\cos x) = -\sin x$$

Substituting the values of derivatives in the equation we have:

$$3e^{3x}.\cos x - \sin \mathrm{x}\,.\,e^{3x}$$

$$e^{3x}(3\cos x - \sin x)$$

Joining all together (1 and 2):

$$f'(x) = \frac{2\sin x + \cos x}{4 + \sin^2 x} + \ e^{3x}(3\cos x - \sin x)$$

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b)

2 dices w/ 6 sides => 6x6 = 36 possibilities

6+5 and 6+6 > 11 => $\frac{2}{36}$ possibilities. Then $\frac{34}{36}$ is the probability that the sum is less than 11, which simplifying by 2, gives us the value of $\frac{17}{18} = 0.94$.

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c)

$$\lim_{x\to 0}\frac{(e^{x}-1).\sin x}{x\ln(x+1)}$$

Using L'Hospital:

$$\frac{\frac{d}{dx}(e^{x}-1).\sin x}{\frac{d}{dx}x\ln(x+1)}$$

Derivation of the top part:

$$\frac{d}{dx}(e^x - 1).\sin x + (e^x - 1).\frac{d}{dx}(\sin x)$$

$$e^x \sin x + e^x \cos x - \cos x$$

Derivation of the bottom part:

$$\frac{d}{dx}(x).\ln(\mathrm{x} + 1) + x.\frac{d}{dx}(\ln(x + 1))$$

$$\ln(x + 1) + \frac{x}{x + 1}$$

Joining both:

$$\frac{e^x \sin x + e^x \cos x - \cos x}{\ln(x + 1) + \frac{x}{x + 1}}$$

Multiplying everything by (x+1) to remove it, we stay with the following equation:

$$\frac{(x + 1).(e^x \sin x + e^x \cos x - \cos x)}{(x.\ln(x + 1) + \ln(x + 1) + x}$$

This equation still can produce 0/0, then we apply again L'Hospital:

$$\frac{\frac{d}{dx}(x + 1).(e^x \sin x + e^x \cos x - \cos x)}{\frac{d}{dx}(x.\ln(x + 1) + \ln(x + 1) + x}$$

$$\lim_{x \to 0} = \frac{\big(-1 + e^x(3 + 2x)\big)\cos x + (1 + e^x + x)\sin x}{2 + \ln(1 + x)}$$

We can put limit in both terms:

$$\frac{\lim_{x \to 0}\big(-1 + e^x(3 + 2x)\big)\cos x + (1 + e^x + x)\sin x}{\lim_{x \to 0} 2 + \ln(1 + x)}$$

Applying some math with the limit and using properties such as: limit of a constant is the constant and limit of a sum is a sum of limits, we have:

$$\frac{(-1 + (\lim_{x\to 0} e^x)(\lim_{x\to 0}(3+2x))(\lim_{x\to 0} \cos x) + \lim_{x\to 0}(1+e^x+x)\sin x}{2 + \lim_{x\to 0}(\ln(x+1)}$$

The $(\lim_{x\to 0} e^x)$ = 1 and $(\lim_{x\to 0}(3+2x))$ = 3 when approx. to 0, then we can rewrite it:

$$\frac{(-1 + 1.3.(\lim_{x\to 0} \cos x)) + \lim_{x\to 0}(1+e^x+x)\sin x}{2 + \lim_{x\to 0}(\ln(x+1)}$$

The $\lim_{x\to 0} \cos x$ = 1, when it approx. 0. Then, applying the multiplication of limits in the sum:

$$\frac{2 + ((1 + \lim_{x\to 0}(e^x) + \lim_{x\to 0} x). \lim_{x\to 0} \sin x)}{2 + \lim_{x\to 0}(\ln(x+1)}$$

The $(\lim_{x\to 0} e^x)$ = 1 and $(\lim_{x\to 0}(x))$ = 0 when approx. to 0 and a Limit of a log is a log of a Limit, then we can rewrite it

$$\frac{2 + 2 \lim_{x\to 0} \sin x}{2 + \ln(\lim_{x\to 0}(x+1)}$$

The $\lim_{x\to 0} \sin x$ = 0 and $\ln(x+1)$ = 1 when x approx. to 0.

$$\frac{2 + 2.0}{2 + \ln(1)}$$

Ln(1) = 0, then:

$$\lim_{x\to 0} = \frac{2}{2} = 1$$

Question 2:

CITY	*MA*	*MI*	*AP*	*CH*	*GC*
MA	0	78.7	105.8	216.1	223.8
MI	78.7	0	107.2	283.1	291.5
AP	105.8	107.2	0	222.4	239.3
CH	216.1	283.1	222.4	0	46.0
GC	223.8	291.5	239.3	46.0	0

CH GC MA MI AP

First Iteration:

- 5 clusters
- Smaller distance is between CH and GC.
- CH GC MA MI AP

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  CH-GC MA MI AP now we have 4 clusters

CITY	*MA*	*MI*	*AP*	*CH-GC*
MA	0	78.7	105.8	216.1
MI	78.7	0	107.2	283.1
AP	105.8	107.2	0	222.4
CH-GC	216.1	283.1	222.4	0

Second Iteration:

- 4 clusters
- Smaller distance is between MA and MI.
- CH GC MA MI AP

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  CH-GC MA-MI AP now we have 3 clusters

CITY	*MA-MI*	*AP*	*CH-GC*
MA-MI	0	105.8	216.1
AP	105.8	0	222.4
CH-GC	216.1	222.4	0

Third Iteration:

- 3 clusters
- Smaller distance is between MA-MI and AP.

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-   CH   GC    MA     MI     AP
      \    /          \    /        /
    CH-GC        MA-MI  /     now we have 2 clusters
                       \ /       /
                  MA-MI-AP
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CITY	*MA-MI-AP*	*CH-GC*
MA-MI-AP	0	216.1
CH-GC	216.1	0

Fourth Iteration:

- 3 clusters
- Smaller distance is between MA-MI and AP.

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-   CH   GC     MA    MI      AP
      \    /          \    /          /
    CH-GC        MA-MI    /        now we have 1 cluster
      |                 \ /        /
      |            MA-MI-AP
      |            /
       \         /
  MA-MI-AP-CH-GC
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CITY	*MA-MI-AP- CH-GC*
MA-MI-AP-CH-GC	0