CS 540-1: Introduction to Artificial Intelligence **Fall 2014-15**

Homework Assignment 4

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1 Question 1

1.1 a) What is the value of P (X)?

$$\Pr[X] = \sum_{k \in Z,Y} \Pr[X|k]$$

$$\Pr[X] = 0.08 + 0.10 + 0.04 + 0.18$$

$$\Pr[X] = 0.4$$

1.2 b) What is the value of P (¬ X | Y)?

$$\Pr[\neg X|Y] = \frac{\Pr[\neg X, Y]}{\Pr[Y]}$$

$$\Pr[\neg X|Y] = \frac{0.09 + 0.09}{0.8 + 0.09 + 0.04 + 0.09}$$

$$\Pr[\neg X|Y] = \frac{0.18}{0.26}$$

$$\Pr[\neg X|Y] = 0.6$$

1.3 c) What is the value of P (¬ Y | X, ¬ Z)?

$$\Pr[\neg Y|X, \neg Z] = \frac{\Pr[\neg Y, X, \neg Z]}{\Pr[X, \neg Z]}$$

$$\Pr[\neg Y|X, \neg Z] = \frac{0.18}{0.04 + 0.18}$$

$$\Pr[\neg Y|X, \neg Z] = \frac{0.18}{0.22}$$

$$\Pr[\neg Y|X, \neg Z] = 0.81$$

1.4 d) Verify whether X and Z are conditionally independent given Y.

$$\Pr[X, Z|Y] = \Pr[X|Y].\Pr[Z|Y]$$

I) Verify if $\Pr[X|Y] = \Pr[X|Z, Y]$

$$\Pr[X|Y] = \Pr[X|Z, Y]$$

$$\Pr[X|Y] = \frac{\Pr[X, Y]}{\Pr[Y]}$$

$$\Pr[X|Y] = \frac{0.12}{0.3}$$

$$\Pr[X|Y] = 0.4$$

$$\Pr[X|Z, Y] = \frac{\Pr[X, Y, Z]}{\Pr[Z, Y]}$$

$$\Pr[X|Z, Y] = \frac{0.08}{0.08 + 0.09}$$

$$\Pr[X|Z, Y] = 0.47$$

II) Verify if $\Pr[Z|Y] = \Pr[Z|X, Y]$

$$\Pr[Z|Y] = \Pr[Z|X, Y]$$

$$\Pr[Z|Y] = \frac{\Pr[Z, Y]}{\Pr[Y]}$$

$$\Pr[Z|Y] = \frac{0.17}{0.3}$$

$$\Pr[X|Y] = 0.56$$

$$\Pr[Z|X, Y] = \frac{\Pr[Z, X, Y]}{\Pr[X, Y]}$$

$$\Pr[Z|X, Y] = \frac{0.08}{0.08 + 0.04}$$

$$\Pr[Z|X, Y] = 0.66$$

As $\Pr[X|Y] \neq \Pr[X|Z, Y]$ and $\Pr[Z|Y] \neq \Pr[Z|X, Y]$, then X and Z are not conditionally dependent given Y.

1.5 e) Verify whether X and Y are conditionally independent given Z.

$$\Pr[X, Y|Z] = \Pr[X|Z].\Pr[Y|Z]$$

I) Verify if $\Pr[X|Z] = \Pr[X|Y, Z]$

$$\Pr[X|Z] = \Pr[X|Y, Z]$$

$$\Pr[X|Z] = \frac{\Pr[X, Z]}{\Pr[Z]}$$

$$\Pr[X|Z] = \frac{0.18}{0.52}$$

$$\Pr[X|Z] = 0.34$$

$$\Pr[X|Y, Z] = \frac{\Pr[X, Y, Z]}{\Pr[Y, Z]}$$

$$\Pr[X|Y, Z] = \frac{0.08}{0.08 + 0.09}$$

$$\Pr[X|Y, Z] = 0.47$$

II) Verify if $\Pr[Y|Z] = \Pr[Y|X, Z]$

$$\Pr[Y|Z] = \Pr[Y|X, Z]$$

$$\Pr[Y|Z] = \frac{\Pr[Y, Z]}{\Pr[Z]}$$

$$\Pr[Y|Z] = \frac{0.17}{0.52}$$

$$\Pr[Y|Z] = 0.32$$

$$\Pr[Y|X, Z] = \frac{\Pr[Y, X, Z]}{\Pr[X, Z]}$$

$$\Pr[Y|X, Z] = \frac{0.08}{0.18}$$

$$\Pr[Y|X, Z] = 0.44$$

As $\Pr[X|Z] \neq \Pr[X|Y, Z]$ and $\Pr[Y|Z] \neq \Pr[Y|X, Z]$, then X and Y are not conditionally dependent given Z.

2 Question 2

2.1 a) What is the probability of thunder?

$$\Pr[T] = \Pr[T, L] * \Pr[T, \neg L]$$
$$\Pr[T] = \Pr[T|L] * \Pr[L] + \Pr[T|\neg L] * \Pr[\neg L]$$

$$\Pr[L] = \Pr[L, S] * \Pr[L, \neg S]$$
$$\Pr[L] = \Pr[L|S] * \Pr[S] + \Pr[L|\neg S] * \Pr[\neg S]$$
$$\Pr[L] = (0.7 * 0.2) + (0.14 + 0.04)$$

$$\Pr[\neg L] = \Pr[\neg L, S] * \Pr[\neg L, \neg S]$$
$$\Pr[\neg L] = \Pr[\neg L|S] * \Pr[S] + \Pr[\neg L|\neg S] * \Pr[\neg S]$$
$$\Pr[\neg L] = (0.3 * 0.2) + (0.95 + 0.8)$$
$$\Pr[\neg L] = 0.82$$

$$\Pr[T] = \Pr[T|L] * \Pr[L] + \Pr[T|\neg L] * \Pr[\neg L]$$
$$\Pr[T] = (0.94 * 0.18) + (0.25 * 0.82)$$

$$\Pr[T] = 0.3742$$

2.2 b) What is the probability of thunder given a forest fire?

$$\Pr[T|FF] = \frac{\Pr[T, FF]}{\Pr[FF]}$$
$$\Pr[T|FF] = \frac{\sum_{CF=cf, L=l, S=s} \Pr[T, FF, CF = cf, L = l, S = s]}{\sum_{CF=cf, L=l} \Pr[FF, L = l, CF = cf]}$$

I) First let's calculate $\Pr[FF]$

$$\Pr[FF] = \Pr[FF, L, CF] + \Pr[FF, L, \neg CF] + \Pr[FF, \neg L, CF] + \Pr[FF, \neg L, \neg CF]$$

$\Pr[L] = 0.18$, calculated in a).
$\Pr[L] = 0.82$, calculated in a).

$$\Pr[FF, L, CF] = \Pr[FF, L, CF] * \Pr[L] * \Pr[CF]$$
$$\Pr[FF, L, CF] = 0.6 * 0.18 * 0.6$$

$$\Pr[\,FF, L, CF\,] = 0.0648$$

$$\Pr[\,FF, L, \neg CF\,] = \Pr[\,FF, L, \neg CF\,] * \Pr[\,L\,] * \Pr[\,\neg CF\,]$$
$$\Pr[\,FF, L, \neg CF\,] = 0.5 * 0.18 * 0.4$$
$$\Pr[\,FF, L, \neg CF\,] = 0.036$$

$$\Pr[\,FF, \neg L, CF\,] = \Pr[\,FF, \neg L, CF\,] * \Pr[\,\neg L\,] * \Pr[\,CF\,]$$
$$\Pr[\,FF, \neg L, CF\,] = 0.15 * 0.82 * 0.6$$
$$\Pr[\,FF, \neg L, CF\,] = 0.0738$$

$$\Pr[\,FF, \neg L, \neg CF\,] = \Pr[\,FF, \neg L, \neg CF\,] * \Pr[\,\neg L\,] * \Pr[\,\neg CF\,]$$
$$\Pr[\,FF, \neg L, \neg CF\,] = 0.01 * 0.82 * 0.4$$
$$\Pr[\,FF, \neg L, \neg CF\,] = 0.00328$$

$$\Pr[\,FF\,] = 0.0648 + 0.036 + 0.0738 + 0.00328$$
$$\Pr[\,FF\,] = 0.17788$$

II) Going back to the beginning, let's calculate $\Pr[\,T, FF, CF = cf, L = l, S = s\,]$

$$\Pr[\,T, FF, CF = cf, L = l, S = s\,] =$$

$$\Pr[\,T, FF, CF, L, S\,] = \Pr[\,T|L\,] * \Pr[\,FF|L, CF\,] * \Pr[\,CF\,] * \Pr[\,L|S\,] * \Pr[\,S\,]$$
$$\Pr[\,T, FF, CF, L, S\,] = 0.94 * 0.6 * 0.6 * 0.7 * 0.2$$
$$\Pr[\,T, FF, CF, L, S\,] = 0.047376$$

$$\Pr[\,T, FF, CF, L, \neg S\,] = \Pr[\,T|L\,] * \Pr[\,FF|L, CF\,] * \Pr[\,CF\,] * \Pr[\,L|\neg S\,] * \Pr[\,\neg S\,]$$
$$\Pr[\,T, FF, CF, L, \neg S\,] = 0.94 * 0.6 * 0.6 * 0.05 * 0.8$$
$$\Pr[\,T, FF, CF, L, \neg S\,] = 0.013563$$

$$\Pr[\,T, FF, CF, \neg L, S\,] = \Pr[\,T|\neg L\,] * \Pr[\,FF|\neg L, CF\,] * \Pr[\,CF\,] * \Pr[\,\neg L|S\,] * \Pr[\,S\,]$$
$$\Pr[\,T, FF, CF, \neg L, S\,] = 0.25 * 0.15 * 0.6 * 0.3 * 0.2$$
$$\Pr[\,T, FF, CF, \neg L, S\,] = 0.00135$$

$$\Pr[T, FF, CF, \neg L, \neg S] = \Pr[T|\neg L] * \Pr[FF|\neg L, CF] * \Pr[CF] * \Pr[\neg L|\neg S] * \Pr[\neg S]$$

$$\Pr[T, FF, CF, \neg L, \neg S] = 0.25 * 0.15 * 0.6 * 0.95 * 0.8$$

$$\Pr[T, FF, CF, \neg L, \neg S] = 0.0171$$

$$\Pr[T, FF, \neg CF, L, S] = \Pr[T|L] * \Pr[FF|L, \neg CF] * \Pr[\neg CF] * \Pr[L|S] * \Pr[S]$$

$$\Pr[T, FF, \neg CF, L, S] = 0.94 * 0.5 * 0.4 * 0.7 * 0.2$$

$$\Pr[T, FF, \neg CF, L, S] = 0.02632$$

$$\Pr[T, FF, \neg CF, L, \neg S] = \Pr[T|L] * \Pr[FF|L, \neg CF] * \Pr[\neg CF] * \Pr[L|\neg S] * \Pr[\neg S]$$

$$\Pr[T, FF, \neg CF, L, \neg S] = 0.94 * 0.5 * 0.4 * 0.05 * 0.8$$

$$\Pr[T, FF, \neg CF, L, \neg S] = 0.00752$$

$$\Pr[T, FF, \neg CF, \neg L, S] = \Pr[T|\neg L] * \Pr[FF|\neg L, \neg CF] * \Pr[\neg CF] * \Pr[\neg L|S] * \Pr[S]$$

$$\Pr[T, FF, \neg CF, \neg L, S] = 0.25 * 0.01 * 0.4 * 0.3 * 0.2$$

$$\Pr[T, FF, \neg CF, \neg L, S] = 0.00006$$

$$\Pr[T, FF, \neg CF, \neg L, \neg S] = \Pr[T|\neg L] * \Pr[FF|\neg L, \neg CF] * \Pr[\neg CF] * \Pr[\neg L|\neg S] * \Pr[\neg S]$$

$$\Pr[T, FF, \neg CF, \neg L, \neg S] = 0.25 * 0.01 * 0.4 * 0.95 * 0.8$$

$$\Pr[T, FF, \neg CF, \neg L, \neg S] = 0.00076$$

$$\Pr[T, FF] = 0.047376 + 0.013536 + 0.00135 + 0.0171 + 0.02632 + 0.00752 + 0.00006 + 0.00076$$

$$\Pr[T, FF] = 0.114022$$

$$\Pr[T|FF] = \frac{0.114022}{0.1778}$$

$$\Pr[T|FF] = 0.64129$$

2.3 c) What is the probability that there is a storm given that there is thunder?

$$\Pr[S|T] = \frac{\Pr[S,T]}{\Pr[T]}$$

$$\Pr[S|T] = \frac{\sum_{L=l,CF=cf,FF=ff} \Pr[S,T,L=l,CF=cf,FF=ff]}{\Pr[T]}$$

$\Pr[T] = 0.3742$ calculated in a).

$$\Pr[S,T,L=l,CF=cf,FF=ff] =$$

$$\Pr[S,T,L,CF,FF] = \Pr[S] * \Pr[T|L] * \Pr[L|S] * \Pr[CF] * \Pr[FF|L,CF]$$

$$\Pr[S,T,L,CF,FF] = 0.2 * 0.94 * 0.7 * 0.6 * 0.6$$

$$\Pr[S,T,L,CF,FF] = 0.047376$$

$$\Pr[S,T,L,CF,\neg FF] = \Pr[S] * \Pr[T|L] * \Pr[L|S] * \Pr[CF] * \Pr[\neg FF|L,CF]$$

$$\Pr[S,T,L,CF,\neg FF] = 0.2 * 0.94 * 0.7 * 0.6 * 0.4$$

$$\Pr[S,T,L,CF,\neg FF] = 0.031584$$

$$\Pr[S,T,L,\neg CF,FF] = \Pr[S] * \Pr[T|L] * \Pr[L|S] * \Pr[CF] * \Pr[FF|L,CF]$$

$$\Pr[S,T,L,\neg CF,FF] = 0.2 * 0.94 * 0.7 * 0.4 * 0.5$$

$$\Pr[S,T,L,\neg CF,FF] = 0.02632$$

$$\Pr[S,T,L,\neg CF,\neg FF] = \Pr[S] * \Pr[T|L] * \Pr[L|S] * \Pr[\neg CF] * \Pr[\neg FF|L,\neg CF]$$

$$\Pr[S,T,L,\neg CF,\neg FF] = 0.2 * 0.94 * 0.7 * 0.4 * 0.5$$

$$\Pr[S,T,L,\neg CF,\neg FF] = 0.02632$$

$$\Pr[S,T,\neg L,CF,FF] = \Pr[S] * \Pr[T|\neg L] * \Pr[\neg L|S] * \Pr[CF] * \Pr[FF|\neg L,CF]$$

$$\Pr[S,T,\neg L,CF,FF] = 0.2 * 0.25 * 0.3 * 0.6 * 0.15$$

$$\Pr[S,T,\neg L,CF,FF] = 0.00765$$

$$\Pr[S,T,\neg L,CF,\neg FF] = \Pr[S] * \Pr[T|\neg L] * \Pr[\neg L|S] * \Pr[CF] * \Pr[\neg FF|\neg L,CF]$$

$$\Pr[S, T, \neg L, CF, \neg FF] = 0.2 * 0.25 * 0.3 * 0.6 * 0.85$$
$$\Pr[S, T, \neg L, CF, \neg FF] = 0.00765$$

$$\Pr[S, T, \neg L, \neg CF, FF] = \Pr[S] * \Pr[T|\neg L] * \Pr[\neg L|S] * \Pr[\neg CF] * \Pr[FF|\neg L, \neg CF]$$
$$\Pr[S, T, \neg L, \neg CF, FF] = 0.2 * 0.25 * 0.3 * 0.4 * 0.01$$
$$\Pr[S, T, \neg L, \neg CF, FF] = 0.00006$$

$$\Pr[S, T, \neg L, \neg CF, \neg FF] = \Pr[S] * \Pr[T|\neg L] * \Pr[\neg L|S] * \Pr[\neg CF] * \Pr[\neg FF|\neg L, \neg CF]$$
$$\Pr[S, T, \neg L, \neg CF, \neg FF] = 0.2 * 0.25 * 0.3 * 0.4 * 0.99$$
$$\Pr[S, T, \neg L, \neg CF, \neg FF] = 0.00594$$

$$\Pr[S, T] = 0.047376 + 0.031584 + 0.02632 + 0.02632 + 0.00135 + 0.00765 + 0.00006 + 0.00594$$
$$\Pr[S, T] = 0.1466$$

$$\Pr[S|T] = \frac{0.1466}{0.3742}$$

$$\Pr[S|T] = 0.391769107$$

2.4 d) What is the probability of camp fire given that there is no thunder?

$$\Pr[CF|\neg T] = \frac{\Pr[CF, \neg T]}{\Pr[\neg T]}$$
$$\Pr[CF|\neg T] = \frac{\sum_{S=s, L=l, FF=ff} \Pr[CF, \neg T, S = s, L = l, FF = ff]}{\Pr[\neg T]}$$

Using $\Pr[T]$ calculated in a), we have that $\Pr[\neg T] = 1 - \Pr[T] = 1 - 0.3742 = 0.6258$.

$$\Pr[CF, \neg T, S = s, L = l, FF = ff] =$$
$$\Pr[CF, \neg T, S, L, FF] = \Pr[CF] * \Pr[\neg T|L] * \Pr[S] * \Pr[L|S] * \Pr[FF|L, CF]$$
$$\Pr[CF, \neg T, S, L, FF] = 0.6 * 0.06 * 0.2 * 0.7 * 0.6$$
$$\Pr[CF, \neg T, S, L, FF] = 0.003024$$

$$\Pr[CF, \neg T, S, L, \neg FF] = \Pr[CF] * \Pr[\neg T|L] * \Pr[S] * \Pr[L|S] * \Pr[\neg FF|L, CF]$$

$$\Pr[CF, \neg T, S, L, \neg FF] = 0.6 * 0.06 * 0.2 * 0.7 * 0.4$$
$$\Pr[CF, \neg T, S, L, \neg FF] = 0.002016$$

$$\Pr[CF, \neg T, S, \neg L, FF] = \Pr[CF] * \Pr[\neg T|\neg L] * \Pr[S] * \Pr[\neg L|S] * \Pr[FF|\neg L, CF]$$
$$\Pr[CF, \neg T, S, \neg L, FF] = 0.6 * 0.75 * 0.2 * 0.3 * 0.15$$
$$\Pr[CF, \neg T, S, \neg L, FF] = 0.00405$$

$$\Pr[CF, \neg T, S, \neg L, \neg FF] = \Pr[CF] * \Pr[\neg T|\neg L] * \Pr[S] * \Pr[\neg L|S] * \Pr[\neg FF|\neg L, CF]$$
$$\Pr[CF, \neg T, S, \neg L, \neg FF] = 0.6 * 0.75 * 0.2 * 0.3 * 0.85$$
$$\Pr[CF, \neg T, S, \neg L, \neg FF] = 0.02295$$

$$\Pr[CF, \neg T, \neg S, L, FF] = \Pr[CF] * \Pr[\neg T|L] * \Pr[\neg S] * \Pr[L|\neg S] * \Pr[FF|L, CF]$$
$$\Pr[CF, \neg T, \neg S, L, FF] = 0.6 * 0.06 * 0.08 * 0.05 * 0.6$$
$$\Pr[CF, \neg T, \neg S, L, FF] = 0.000864$$

$$\Pr[CF, \neg T, \neg S, L, \neg FF] = \Pr[CF] * \Pr[\neg T|L] * \Pr[\neg S] * \Pr[L|\neg S] * \Pr[\neg FF|L, CF]$$
$$\Pr[CF, \neg T, \neg S, L, \neg FF] = 0.6 * 0.06 * 0.8 * 0.05 * 0.4$$
$$\Pr[CF, \neg T, \neg S, L, \neg FF] = 0.000576$$

$$\Pr[CF, \neg T, \neg S, \neg L, FF] = \Pr[CF] * \Pr[\neg T|\neg L] * \Pr[\neg S] * \Pr[\neg L|\neg S] * \Pr[FF|\neg L, CF]$$
$$\Pr[CF, \neg T, \neg S, \neg L, FF] = 0.6 * 0.75 * 0.8 * 0.95 * 0.15$$
$$\Pr[CF, \neg T, \neg S, \neg L, FF] = 0.0513$$

$$\Pr[CF, \neg T, \neg S, \neg L, \neg FF] = \Pr[CF] * \Pr[\neg T|\neg L] * \Pr[\neg S] * \Pr[\neg L|\neg S] * \Pr[\neg FF|\neg L, CF]$$
$$\Pr[CF, \neg T, \neg S, \neg L, \neg FF] = 0.6 * 0.75 * 0.8 * 0.95 * 0.85$$
$$\Pr[CF, \neg T, \neg S, \neg L, \neg FF] = 0.2907$$

$$\Pr[CF, \neg T] = 0.003024 + 0.002016 + 0.00405 + 0.02295 + 0.000864 + 0.000576 + 0.0513 + 0.2907$$
$$\Pr[CF, \neg T] = 0.37548$$

$$\Pr[CF|\neg T] = \frac{0.37548}{0.6258}$$

$$\Pr[CF|\neg T] = 0.6$$

2.5 e) What is the probability of a lightning and a forest fire?

$$\Pr[L, FF] = \sum_{S=s, CF=cf, T=t} \Pr[L, FF, S=s, CF=cf, T=t]$$

$$\Pr[L, FF, S=s, CF=cf, T=t] =$$

$$\Pr[L, FF, S, CF, T] = \Pr[L|S] * \Pr[FF|L, CF] * \Pr[S] * \Pr[CF] * \Pr[T|L]$$

$$\Pr[L, FF, S, CF, T] = 0.7 * 0.6 * 0.2 * 0.6 * 0.94$$
$$\Pr[L, FF, S, CF, T] = 0.047376$$

$$\Pr[L, FF, S, CF, \neg T] = \Pr[L|S] * \Pr[FF|L, CF] * \Pr[S] * \Pr[CF] * \Pr[\neg T|L]$$

$$\Pr[L, FF, S, CF, \neg T] = 0.7 * 0.6 * 0.2 * 0.6 * 0.06$$
$$\Pr[L, FF, S, CF, \neg T] = 0.003024$$

$$\Pr[L, FF, S, \neg CF, T] = \Pr[L|S] * \Pr[FF|L, \neg CF] * \Pr[S] * \Pr[\neg CF] * \Pr[T|L]$$

$$\Pr[L, FF, S, \neg CF, T] = 0.7 * 0.5 * 0.2 * 0.4 * 0.94$$
$$\Pr[L, FF, S, \neg CF, T] = 0.02632$$

$$\Pr[L, FF, S, \neg CF, \neg T] = \Pr[L|S] * \Pr[FF|L, \neg CF] * \Pr[S] * \Pr[\neg CF] * \Pr[\neg T|L]$$

$$\Pr[L, FF, S, \neg CF, \neg T] = 0.7 * 0.5 * 0.2 * 0.4 * 0.06$$
$$\Pr[L, FF, S, \neg CF, \neg T] = 0.0168$$

$$\Pr[L, FF, \neg S, CF, T] = \Pr[L|\neg S] * \Pr[FF|L, CF] * \Pr[\neg S] * \Pr[CF] * \Pr[T|L]$$

$$\Pr[L, FF, \neg S, CF, T] = 0.05 * 0.6 * 0.8 * 0.6 * 0.94$$
$$\Pr[L, FF, \neg S, CF, T] = 0.013536$$

$$\Pr[L, FF, \neg S, CF, \neg T] = \Pr[L|\neg S] * \Pr[FF|L, CF] * \Pr[\neg S] * \Pr[CF] * \Pr[\neg T|L]$$

$$\Pr[L, FF, \neg S, CF, \neg T] = 0.05 * 0.6 * 0.8 * 0.6 * 0.06$$
$$\Pr[L, FF, \neg S, CF, \neg T] = 0.000864$$

$$\Pr[\, L, FF, \neg S, \neg CF, T \,] = \Pr[\, L|\neg S \,] * \Pr[\, FF|L, \neg CF \,] * \Pr[\, \neg S \,] * \Pr[\, \neg CF \,] * \Pr[\, T|L \,]$$

$$\Pr[\, L, FF, \neg S, \neg CF, T \,] = 0.05 * 0.5 * 0.8 * 0.4 * 0.94$$

$$\Pr[\, L, FF, \neg S, \neg CF, T \,] = 0.00752$$

$$\Pr[\, L, FF, \neg S, \neg CF, \neg T \,] = \Pr[\, L|\neg S \,] * \Pr[\, FF|L, \neg CF \,] * \Pr[\, \neg S \,] * \Pr[\, \neg CF \,] * \Pr[\, \neg T|L \,]$$

$$\Pr[\, L, FF, \neg S, \neg CF, \neg T \,] = 0.05 * 0.5 * 0.8 * 0.4 * 0.06$$

$$\Pr[\, L, FF, \neg S, \neg CF, \neg T \,] = 0.00048$$

$$\Pr[\, L, FF \,] = 0.047376 + 0.003024 + 0.02632 + 0.0168 + 0.013536 + 0.000864 + 0.00752 + 0.00048$$

$$\Pr[\, L, FF \,] = 0.11592$$