STATISTICAL INFERENCE - Peer Assignment

Version: V00

Date: 21-dec-2014

GitHub Repository:
https://github.com/A6111E/datasciencecoursera/tree/master/Statistical_Inference/Assignment_Part1

Introduction:

Exponential Distribution: is the probability distribution that describes the time between events in a Poisson process, i.e. a process in which events occur continuously and independently at a constant average rate.

The exponential distribution occurs naturally when describing the lengths of the inter-arrival times in a homogeneous Poisson process. Describes the time for a continuous process to change state.

In real scenarios, the assumption of a constant rate (or probability per unit time) is rarely satisfied.

For example, the rate of incoming phone calls differs according to the time of day. But if we focus on a time interval during which the rate is roughly constant, such as from 2 to 4 p.m. during work days, the exponential distribution can be used as a good approximate model for the time until the next phone call arrives.

Examples which yield approximately exponentially distributed variables:

- Time until a radioactive particle decays, or the time between clicks of a geiger counter.
- Time it takes before your next telephone call

Exponential variables can also be used to model situations where certain events occur with a constant probability per unit length, such as the distance between mutations on a DNA strand, or between roadkills on a given road.

Expected Value: the mean or expected value of an exponentially distributed random variable X with rate parameter $lambda$ is given by:

$E[X] = 1/lambda = beta$

Variance: $Var[X] = 1/lambda^2$

Standard Deviation (is equal to the mean): $sd[X] = 1/lambda = beta$

Mean Standard Error: $se[X] = sd/sqrt(1/n)$

Sample Mean: $Sample_Mean[X] = 1/sample.size(sum(xi))$

t Confidence Interval: $tConfidenceInterval = E[X] +/- t(n-1) \times sd/sqrt(n)$

Partial Source: http://en.wikipedia.org/wiki/Exponential_distribution

Data Processing:

Data Set

Table: Summary Simulation Random Exponential Distribution

Simulations = 1000 - Sample Size = 40 - lambda = 0.2

```
colNames <- c("Trial_1", "Trial_100", "Trial_500", "Trial_750", "Trial_10
00")
pr1 <- DT[ , colNames, with = FALSE]

xt <- xtable(summary(pr1))
print(xt, type = "html", floating = FALSE)
```

Trial_1	Trial_100	Trial_500	Trial_750	Trial_1000
Min. : 0	Min. : 0	Min. : 0.2	Min. : 0.1	Min. : 0.0
1st Qu.: 2	1st Qu.: 2	1st Qu.: 1.0	1st Qu.: 0.4	1st Qu.: 1.5
Median : 4	Median : 3	Median : 2.3	Median : 1.7	Median : 3.7
Mean : 6	Mean : 5	Mean : 4.0	Mean : 2.7	Mean : 4.9
3rd Qu.: 6	3rd Qu.: 6	3rd Qu.: 5.5	3rd Qu.: 3.3	3rd Qu.: 7.2
Max. :32	Max. :37	Max. :16.9	Max. :16.1	Max. :21.0

Table: Mean Value for each Simulation

```
xt <- xtable(DT1[1:10, ], digits = 2)
print(xt, type = "html", include.rownames = FALSE, hline.after=c(1), floa
ting = FALSE)
```

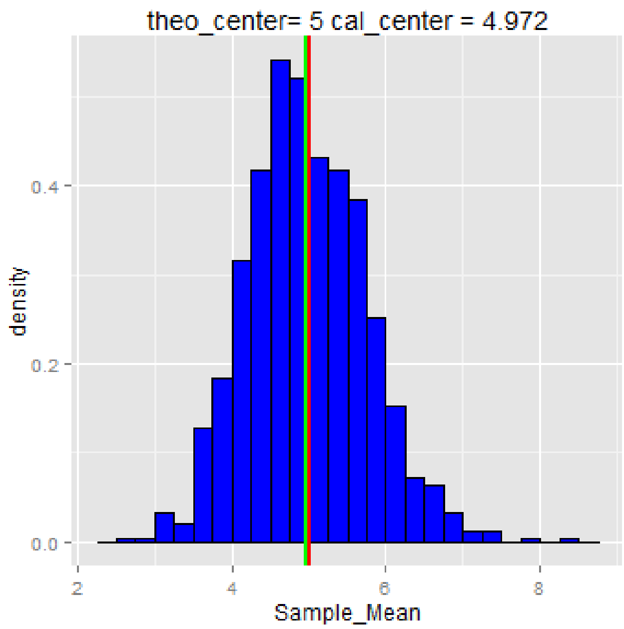
Trial	Sample_Mean
1	5.55
2	4.67
3	4.26
4	4.61
5	5.30
6	3.63
7	6.23

8	3.68
9	4.84
10	4.98

Graphic: Random Exponential Distribution

```
theo_center <- mu                # distribution theoretical center
cal_center <- mean(p)            # distribution calculated center

p1 <- ggplot(DT1, aes(x = Sample_Mean))
        p1 <- p1 + geom_histogram(fill = "blue",
                                  binwidth = 0.25, aes(y = ..density..), co
lour = "black")
        p1 <- p1 + geom_vline(xintercept = theo_center, size = 0.75, colo
ur = "red")
        p1 <- p1 + geom_vline(xintercept = cal_center, size = 0.75, colou
r = "green")
        mse <- round(cal_center, 3)
        p1 <- p1 + labs(title = paste('theo_center = ', theo_center, ' ca
l_center = ', mse))
grid.arrange(p1, ncol = 1)
```



Remarks: Red Vertical Line: distribution theoretical center / Green Vertical Line: distribution calculated center

Q1 - Results:

The calculated center from 1000 simulations for 40 exponential distributions with $lambda = 0.2$, is approximately 5, which corresponds to theoretical center of distribution ($E[X] = 1/lambda = 5$).

Increasing the simulations, the calculated gets closer to theoretical center.

A. Theoretical Center: 5

B. Calculated Center: 4.97

Q2 - Results:

The calculated variance from 1000 simulations for 40 exponential distributions samples with $lambda = 0.2$ and theoretical variance $Var[X] = 1/lambda^2$ are:

A. Theoretical Variance: 25

B. Calculated Variance: 24.57

Increasing the simulations, the calculated gets near to theoretical variance, and the exponential distributions trend to be a standard normal distribution.

For example:

A. The calculated variance from 1 simulation for 40 exponential distributions random variables is 41.9 (for this simulation).

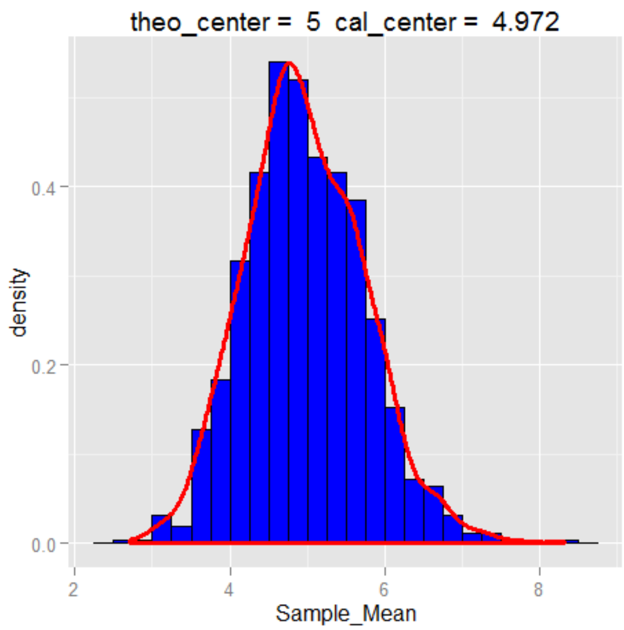
B. The theoretical variance $Var[X] = 1/lambda^2$, is 25

Decreasing the simulations, the calculated variance gets far away to theoretical variance, and the exponential distributions trend NOT to be a standard normal distribution.

Q3 - Results:

Graphic: Random Exponential Distribution ~ Standard Normal Distribution

```
p3 <- ggplot(DT1, aes(x = Sample_Mean))
        p3 <- p3 + geom_histogram(fill = "blue",
                                binwidth = 0.25, aes(y = ..density..), co
lour = "black")
        p3 <- p3 + geom_density(size = 1.25, colour = "red")
        mse <- round(cal_center, 3)
        p3 <- p3 + labs(title = paste('theo_center = ', mu, ' cal_center
= ', mse))
grid.arrange(p3, ncol = 1)
```



For a standard normal distribution, 68, 95 and 99 of the normal density, lies within 1, 2 and 3 standard deviations from the mean, respectively

The calculted percentage from the sample means of the exponential distribution ($lambda = 0.2$) are:

A. Percentage of Values between +/-1 Standard Deviation: 69.6

B. Percentage of Values between +/-2 Standard Deviation: 95.4

C. Percentage of Values between +/-3 Standard Deviation: 99.6

Both, the graphic "Random Exponential Distribution ~ Standard Normal Distribution", and the calculated percentage values, indicates that if the sample trend to be infinite, the random exponential distribution will convert to a standard normal distribution.

The confidence interval for the quantification of uncertainty in the exponential dsitribution is:

A. Confidence Interval 95% - Up: 5.01

B. Confidence Interval 95% - Down: 4.93

Session Information

```
## R version 3.1.2 (2014-10-31)
## Platform: x86_64-w64-mingw32/x64 (64-bit)
##
## locale:
## [1] LC_COLLATE=Spanish_Colombia.1252  LC_CTYPE=Spanish_Colombia.1252
## [3] LC_MONETARY=Spanish_Colombia.1252 LC_NUMERIC=C
```

```
## [5] LC_TIME=Spanish_Colombia.1252
##
## attached base packages:
## [1] grid      stats     graphics  grDevices utils     datasets  method
s
## [8] base
##
## other attached packages:
## [1] gridExtra_0.9.1  knitr_1.8        xtable_1.7-4     ggplot2_1.0.0
## [5] data.table_1.9.4
##
## loaded via a namespace (and not attached):
##  [1] chron_2.3-45     colorspace_1.2-4 digest_0.6.4     evaluate_0.5.5
##  [5] formatR_1.0      gtable_0.1.2     htmltools_0.2.6  labeling_0.3
##  [9] MASS_7.3-35      munsell_0.4.2    plyr_1.8.1       proto_0.3-10
## [13] Rcpp_0.11.3      reshape2_1.4     rmarkdown_0.3.3  scales_0.2.4
## [17] stringr_0.6.2    tools_3.1.2      yaml_2.1.13
```