Research interests

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Typical example ($\Omega \subset \mathbb{R}^N$)

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-\Delta u + V(x)u = f(u) \quad & \text{in } \Omega, \qquad & \nabla u = (u_{x_1}, \dots, u_{x_N}) \\
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whose solutions are critical points ($\Phi'(u) = 0$) of

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Nonlinear Functional Analysis (differential calculus in ∞-dim spaces)
Critical Point Theory, Algebraic Topology, Differential Geometry

Recent publications ✷ Calc. Var. PDEs, 63 (2024) (S. Liu & K. Perera)

$$-\mathrm{div}(|\nabla u|^{p-2}\nabla u + |\nabla u|^{q-2}\nabla u) = \lambda h(x)|u|^{r-2}u + g(x)|u|^{p^*-2}u \quad \text{in } \mathbb{R}^N.$$

Arbitrary m pairs of solutions are found for $\lambda \gg 1$.

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A simple new proof of Gagliardo-Nirenberg-Sobolev Inequality

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$$-\Delta u - u\Delta(u^2) = k(x)|u|^{q-2}u - h(x)|u|^{s-2}u \quad \text{in } \mathbb{R}^N.$$

Quasilinear, the exponent s can be supercritical.

Thank you!

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