Quiz 1

(1) (20%) Let (M,d) be a metric space, $p \in M$. If $U \in \mathcal{U}(p)$, then there is $V \in \mathcal{U}(p)$ such that $\overline{V} \subset U$.

(2) (30%) Let M and N be topological space, $A \subset M$.

  (a) If $f : M \to N$ is continuous at $p \in A$, then $f|_A : A \to N$ is also continuous at p. Here we consider A as a subspace of M.

  (b) As a consequence, if $f : M \to N$ is continuous, so is $f|_A$. Please find a different proof of this statement.

  (c) If $f : M \to N$ is continuous at $p \in M$ and $B \subset N$ satisfies $f(M) \subset B$ (so that we can define a map $f^B : M \to B$). Consider B as a subspace of N. Is the map $f^B : M \to B$ continuous at p? Prove your conclusion.

(3) (20%) Let A and B be subsets of a topological space M. Answer **one of** the following problems:

  (a) Prove that $A^\circ \cup B^\circ \subset (A \cup B)^\circ$, and show by example that it is possible that $A^\circ \cup B^\circ \neq (A \cup B)^\circ$.

  (b) Prove that $\overline{A} \cap \overline{B} \subset \overline{A \cap B}$, and show by example that it is possible that $\overline{A} \cap \overline{B} \neq \overline{A \cap B}$.

(4) (15%) If A is a subset of a topological space M, then $\overline{A} = A^\circ \cup \partial A$.

(5) (15%) Two subsets A and B of a topological space M are called separated if

$$(\overline{A} \cap B) \cup (A \cap \overline{B}) = \emptyset.$$

Show that, if A and B are separated and $A \cup B$ is open, then both A and B are open.