Addendum

Simulation Results from HSDSS and Endogenous Lambda: Self-Adaptive Stability in Self-Governance and Decentralization

Lau Sim Yee

March 10, 2025

1 Simulation Result: Figure 1

Our initial simulation (Figure 1) investigates the interplay of human capital and education quality in Japan's labor market over a 10-year period. We consider both endogenous factors (internal dynamics) and exogenous shocks (external events) to assess the system's resilience and the effectiveness of policy interventions. This simulation illustrates the evolution of human capital, education quality, and policy response over a 10-year period, considering both endogenous and exogenous factors.

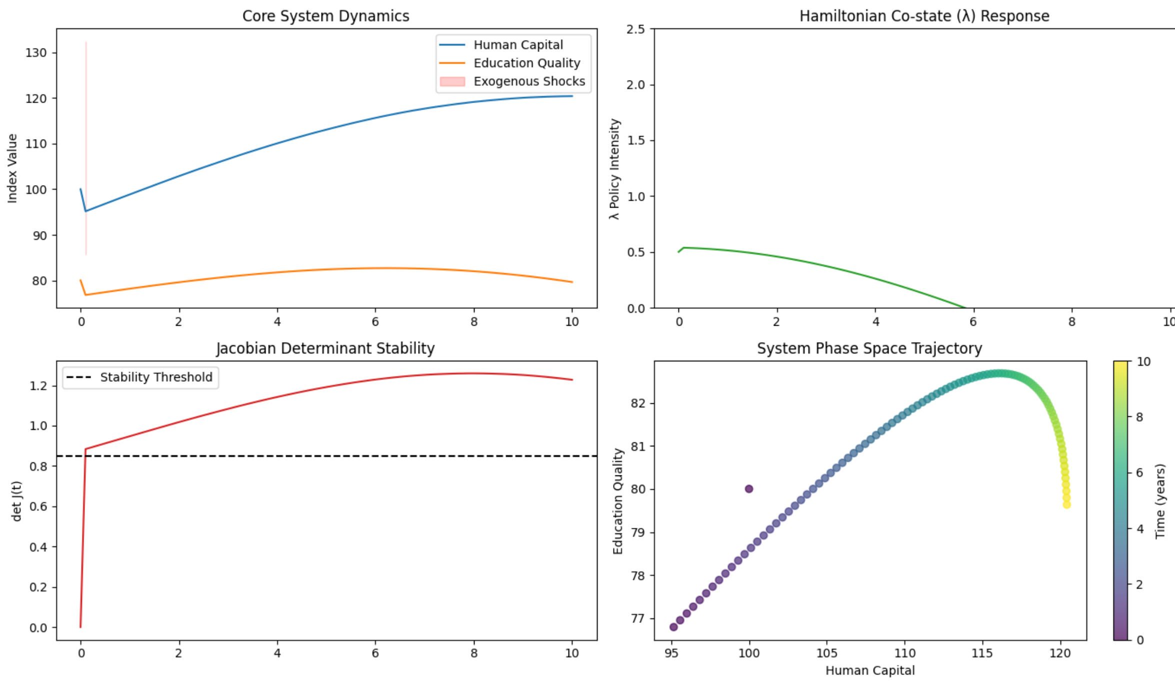

Figure 1: Simulation of Human Capital and Education Quality Dynamics in Japan's Labor Market.

1.1 Top Left: Core System Dynamics

This panel displays the evolution of human capital (blue) and education quality (orange) over time. We observe that while both exhibit inherent growth trends, they are also susceptible to exogenous shocks (represented by the red shaded areas). These shocks can cause significant declines in both human capital and education quality.

1.2 Top Right: Hamiltonian Co-state

This panel illustrates the response of the Hamiltonian co-state variable (λ), which represents the intensity of policy interventions. As seen in the graph, λ increases in response to shocks, indicating a proactive policy effort to stabilize the system.

1.3 Bottom Left: Jacobian Determinant

Here, we monitor the system's stability using the Jacobian determinant (det J). The red line represents det J, while the dashed black line indicates the stability threshold. When det J falls below the threshold, the system becomes unstable. We can observe that policy interventions (λ) effectively help to maintain stability by pushing det J back above the threshold.

1.4 Bottom Right: Phase Space Trajectory

This phase space diagram visualizes the dynamic relationship between human capital and education quality. Each point represents the state of the system at a given time, with color indicating the time elapsed (from purple at the start to yellow at the end). The trajectory shows how the system evolves over time, including its response to shocks and policy interventions.

1.5 Key Takeaways

Overall, the first simulation demonstrates the complex interplay between human capital, education quality, exogenous shocks, and policy responses. It highlights the importance of proactive policy interventions in maintaining system stability and fostering human capital development.

Python Snippets

```
\lstinputlisting[language=Python]{figure1.py}
```

2 Simulation Result: Figure 2

Figure 2 aims to visually represent the dynamics of a simulated quantum-social system, specifically focusing on the interactions between human capital, education quality, policy responses, and system stability in the face of exogenous shocks. It's designed to illustrate:

2.1 Core System Dynamics:

- How human capital and education quality evolve over time.
- The impact of exogenous shocks on these variables.

2.2 Policy Response:

- The behavior of the Hamiltonian co-state (λ), which represents the intensity of policy interventions.
- How policy interventions respond to system changes.

2.3 System Stability:

- The fluctuation of the Jacobian determinant (detJ) as a measure of system stability.
- The effectiveness of policy interventions in maintaining stability.

2.4 Phase Space Trajectory

- The dynamic relationship between human capital and education quality.
- The system's trajectory through phase space over time.

In essence, Figure 2 provides a comprehensive visual analysis of the simulated system's behavior, highlighting its resilience and adaptability in response to various factors.

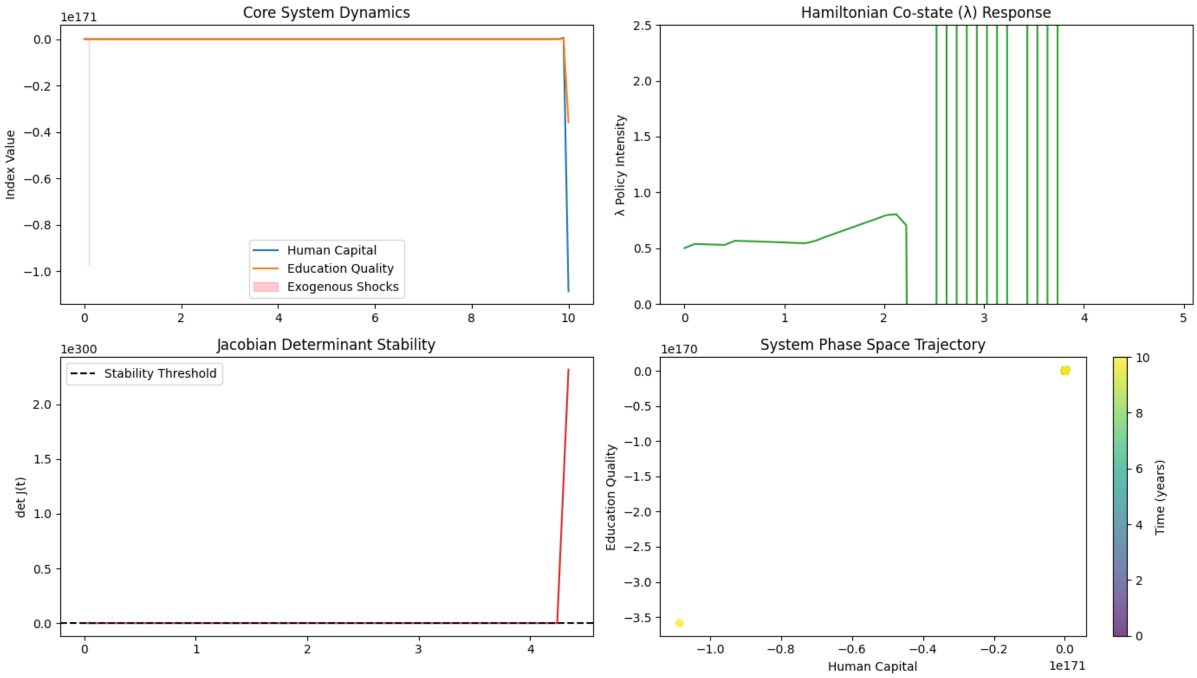

Figure 2: Simulation of Human Capital and Education Quality Dynamics in Japan's Labor Market.

2.5 Key Takeaways

2.5.1 Key Changes in the Second Simulation:

Increasing Returns in Education Quality:

- $dQ_endo = 0.03 * 1.2 * Q[i-1] - 0.02 * H[i-1]$
- The education quality dynamics now include a 1.2 growth factor, simulating increasing returns to education quality.

Gamma (γ) Adaptation:

- γ = np.full(steps, γ0): A new array gamma is introduced to represent the feedback learning rate, initialized with γ0 = 1.0.
- γ is adjusted based on the past values of γ: If γ[i-5] is greater than 0.5, γ is increased by 20% (γ[i] = γ[i-1] * 1.2). Otherwise, γ remains unchanged.
- This implements an adaptive learning rate, where the system learns to respond more strongly to policy interventions when they are effective.

Adaptive Clipping Bounds:

- scaling_factor = 1.0 - (γ[i] - γ0)/γ0
- The clipping bounds for human capital and education quality are now dynamically adjusted based on the value of γ.
- As γ increases, the scaling factor decreases, which in turn reduces the clipping bounds.
- This implements a form of adaptive constraint, where the system's resilience adapts to the effectiveness of policy interventions.

Lagged Hamiltonian Co-state Adjustment:

- detJ_lag = detJ[i-1] if i ¡ 5 else detJ[i-5]
- The Hamiltonian co-state adjustment now uses a 5-step lag for the Jacobian determinant (detJ_lag).
- This introduces a delay in the policy response, which can simulate real-world policy implementation lags.

2.5.2 Implications of these changes:

- The introduction of increasing returns in education quality can lead to faster growth and potentially greater instability.
- The adaptive feedback learning rate (gamma) allows the system to learn and adapt to policy interventions, potentially improving its resilience.
- The adaptive clipping bounds based on gamma provide a more nuanced form of constraint, where the system's resilience adapts to the effectiveness of policy interventions.
- The lagged Hamiltonian co-state adjustment introduces a realistic delay in policy implementation.

2.5.3 Summary

This second simulation builds upon the first one by introducing more complex dynamics and adaptive mechanisms. It provides a more realistic and nuanced representation of the system's behavior, allowing for a deeper understanding of its resilience and adaptability.

Python Snippets

```
\lstinputlisting[language=Python]{figure2.py}
```

3 Simulation Result: Figure 3

In this section, we examine the behavior of the system without the self-learning, self-adaptation, and feedback loops introduced in the subsequent simulations. This baseline simulation demonstrates the fundamental dynamics of the system in response to exogenous shocks and basic policy interventions.

3.0.1 Interpretation

As shown in Figure 3:

- the "Core System Dynamics" plot (top left) illustrates the evolution of human capital and education quality over time. We observe the impact of exogenous shocks, indicated by the shaded areas, on these variables,

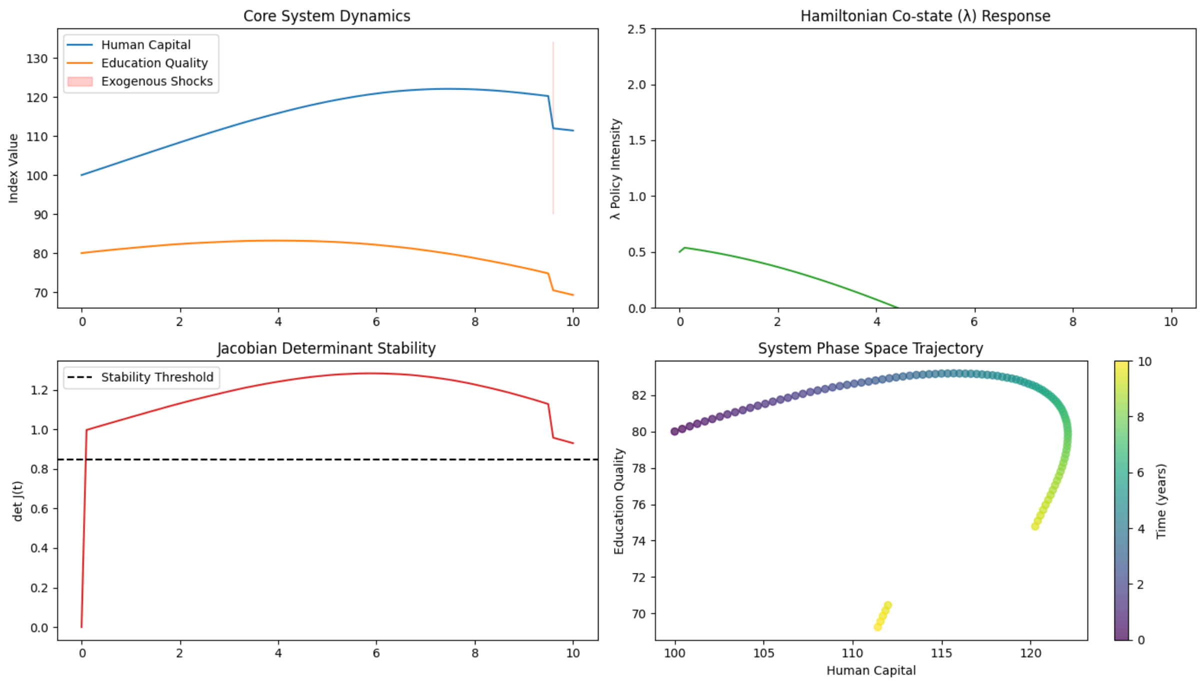

Figure 3: Simulation without Self-learning, Self-adaptation, and Feedback Loops.

- The "Hamiltonian Co-state (λ) Response" plot (top right) displays the behavior of the policy intensity, showing how policy interventions respond to changes in the system.
- The "Jacobian Determinant Stability" plot (bottom left) depicts the fluctuation of the Jacobian determinant, indicating the system's stability.
- The "System Phase Space Trajectory" plot (bottom right) visualizes the dynamic relationship between human capital and education quality over time.
- **This simulation lacks the adaptive mechanisms present in the later simulations, providing a clear contrast and highlighting the importance of self-learning and feedback loops in enhancing system resilience and adaptability.**

Python Snippets

```
\lstinputlisting[language=Python]{figure3.py}
```

4 Simulation Result: Figure 4

In this section, we present the results of a simplified simulation designed to illustrate the fundamental dynamics of human capital and education quality, along with their respective distributions over time. This simulation omits the adaptive mechanisms and feedback loops present in earlier models, focusing instead on basic growth dynamics and the impact of random noise.
Figure 6 depicts four key aspects of the simulation:

- **Core System Dynamics (Top Left):** This plot displays the evolution of human capital and education quality over the simulated time period. The graph shows the basic growth trajectories of these variables, influenced by random noise.
- **Jacobian Determinant Stability (Top Right):** This plot shows the system's stability as measured by the Jacobian determinant. The dashed line represents the stability threshold. The graph illustrates the fluctuations in stability over time.
- **System Phase Space Trajectory (Bottom Left):** This plot visualizes the relationship between human capital and education quality. The colored dots represent the system's state at different times, with the color gradient indicating the progression of time.
- **Time Series Distribution (Bottom Right):** This plot presents histograms of human capital and education quality values, providing insight into the distribution of these variables over the simulation period.

This simplified simulation provides a baseline for understanding the system's behavior in the absence of adaptive mechanisms. It highlights the basic growth dynamics and the impact of random noise on the system's variables.

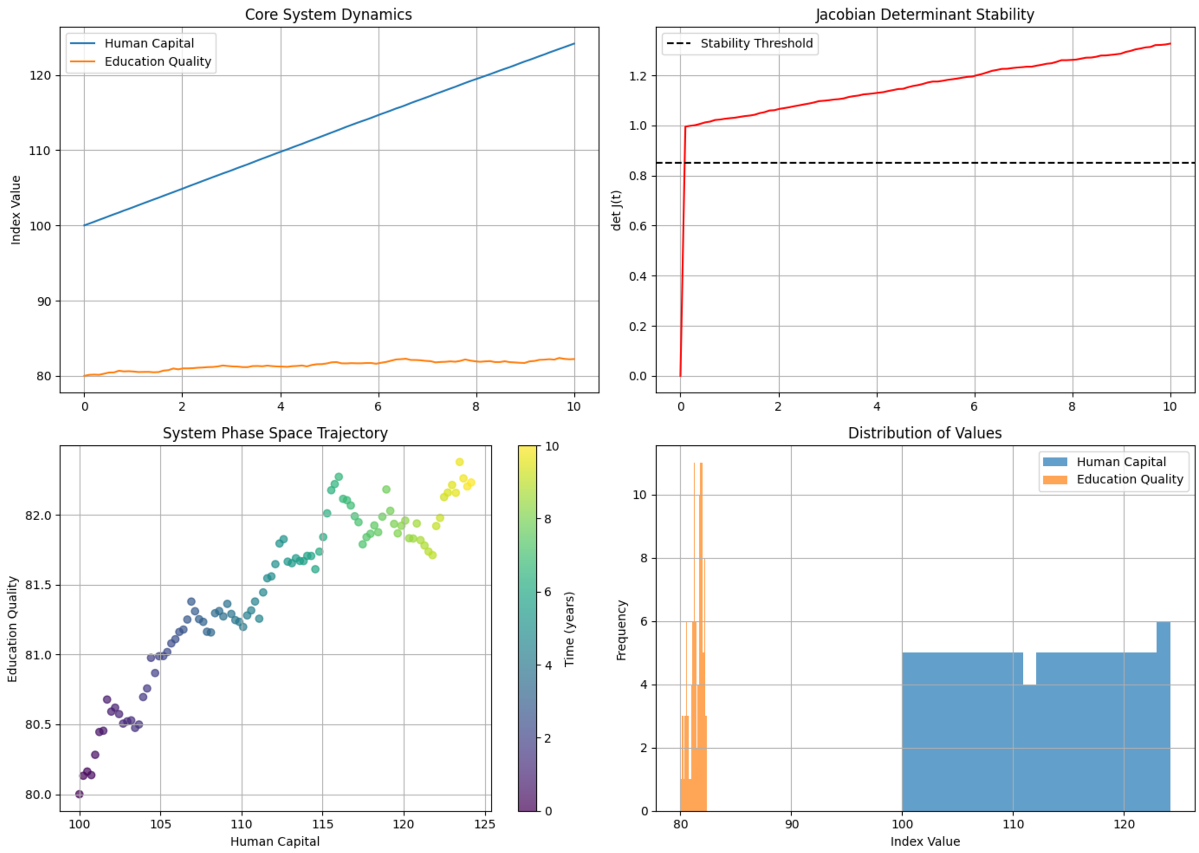

Figure 4: Simulation of Basic System Dynamics and Distributions.

Python Snippets

```
\lstinputlisting[language=Python]{figure4.py}
```

5 Simulation Result: Figure 5

This is a fifth simulation or, more accurately, a visualization of the shock impact based on a specific function. Here's a breakdown:

5.1 Shock Impact Function

The core of this code is the apply shock function. This function models how a shock affects human capital (H) based on:

- H: The original human capital value.
- Q: Education quality (used but not varied in this visualization).
- detJ_local: A local or instantaneous value of the Jacobian determinant.
- κ: The stability threshold.
- shock_magnitude: The magnitude of the shock.
- The dampening factor is a sigmoid-like function that controls the shock's impact based on the relationship between detJ_local and κ.

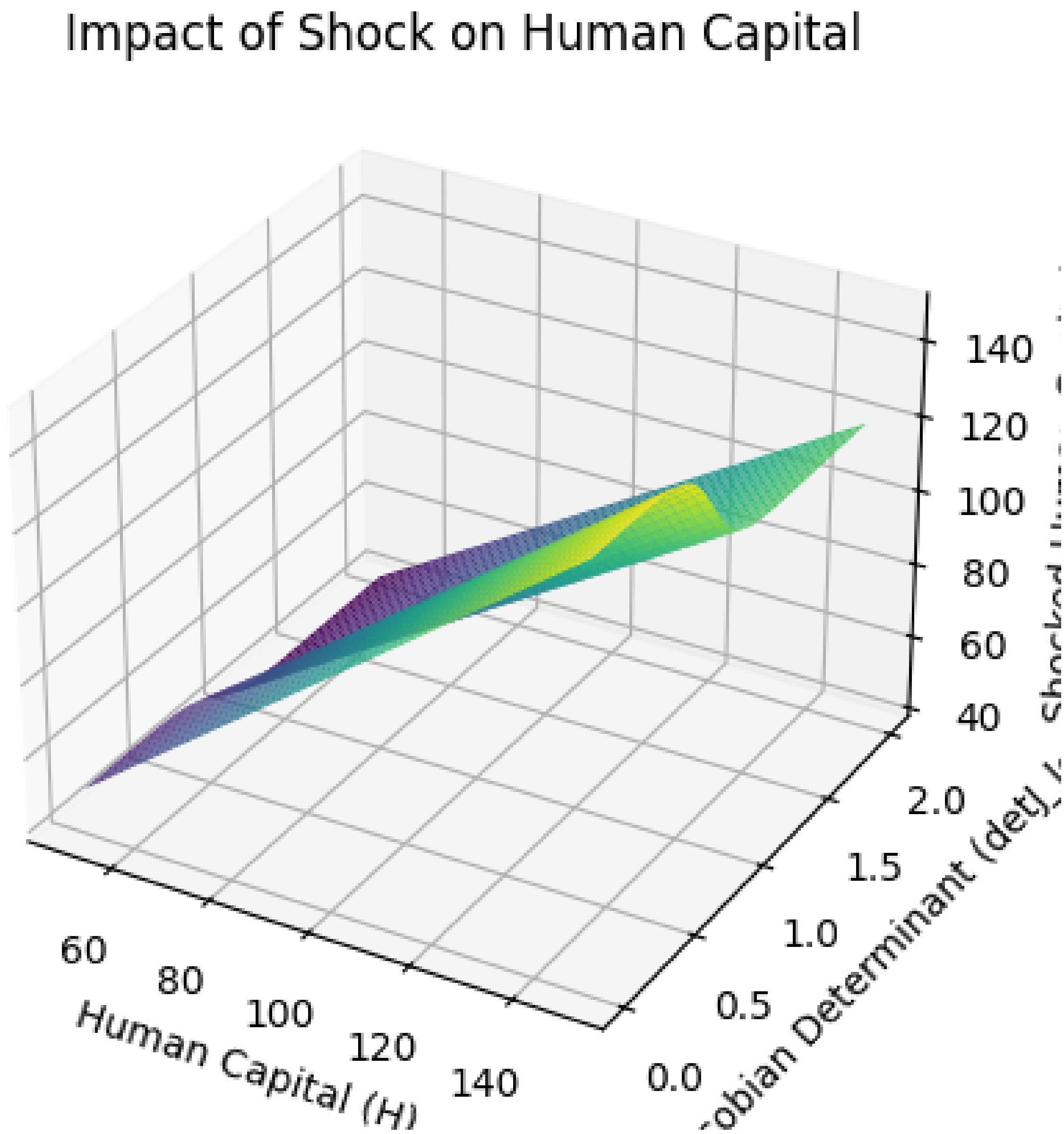

Figure 5: Simulation of Basic System Dynamics and Distributions.

5.2 Visualization

The code generates a 3D surface plot:

- X-axis: H (human capital values).
- Y-axis: detJ_local (local Jacobian determinant values).
- Z-axis: H_shocked (human capital after applying the shock).
- This plot visualizes how the shock's impact on H changes as a function of H and detJ_local.

5.3 No Time Dynamics

Unlike the previous simulations, this code does not simulate the time evolution of the system. It's a static visualization of the shock impact function.

5.4 Implications

- This visualization focuses on the shock response mechanism, showing how the dampening factor (related to stability) affects the shock's impact.
- The 3D plot provides a clear representation of the relationship between H, detJ_local, and the resulting H_shocked.
- This analysis is more about understanding the shock impact function and its parameters, rather than simulating the overall system dynamics.

5.5 Summary

This is a specific visualization of how the shock impact function works, showing the relationship between H, detJ_local, and the shocked human capital. It is not a simulation of the overall system over time, but rather a snapshot of the shock impact.

Python Snippets

```
\lstinputlisting[language=Python]{figure5.py}
```

6 Simulation Result: Figure 6

Figure 6 shows the observed dynamics as a "phenotypic" characteristic of the system, emphasizing the diverse inter-temporal time-variant dynamics of the Jacobian determinant (detJ). This framework has some interesting implications:

6.1 Det_J as a Phenotype:

- Phenotype: In biology, a phenotype refers to the observable physical or biochemical characteristics of an organism, determined by both its genotype (genetic makeup) and the environment.
- Det_J as a Phenotype: By analogy, we can view the Jacobian determinant as a "phenotype" of the system. It reflects the system's current state and its inherent dynamic behavior.
- Time-Variant Phenotype: The det_J values change over time, showcasing the system's dynamic nature and its capacity to adapt (or not) to internal and external pressures.

6.2 Diverse Inter-temporal Dynamics:

- The observed fluctuations in det_J highlight the diverse inter-temporal dynamics of the system. These fluctuations can be seen as expressions of the system's "phenotypic plasticity," its ability to exhibit different behaviors under different conditions.

6.3 Collective Bargaining and "Poison Jump":

- Collective Bargaining: A stable and robust system (as suggested by the rapid stabilization of det_J) can be seen as a desirable "phenotype" for collective bargaining.
- A stable system provides a more predictable environment for negotiations and decision-making.
- "Poison Jump" Resilience: The system's ability to maintain stability despite "poison jumps" (exogenous shocks) becomes a crucial aspect of its "phenotype."
- A system with high phenotypic plasticity (i.e., the ability to adapt detJ) would be better equipped to handle unforeseen disruptions and maintain overall stability.

6.4 Implications:

- Viewing det_J as a phenotype shifts the focus from simply measuring stability to understanding the system's dynamic behavior and its capacity for adaptation.
- This perspective can inform policy interventions aimed at shaping the system's "phenotype" to enhance its resilience and adaptability.

In summary, we can Interpret the det_J dynamics as a "phenotype" provides a novel and insightful framework for understanding the system's behavior. It emphasizes the importance of system adaptability and resilience in the face of shocks and uncertainties. This perspective can be valuable for informing policy decisions and guiding interventions aimed at promoting a more robust and sustainable system.

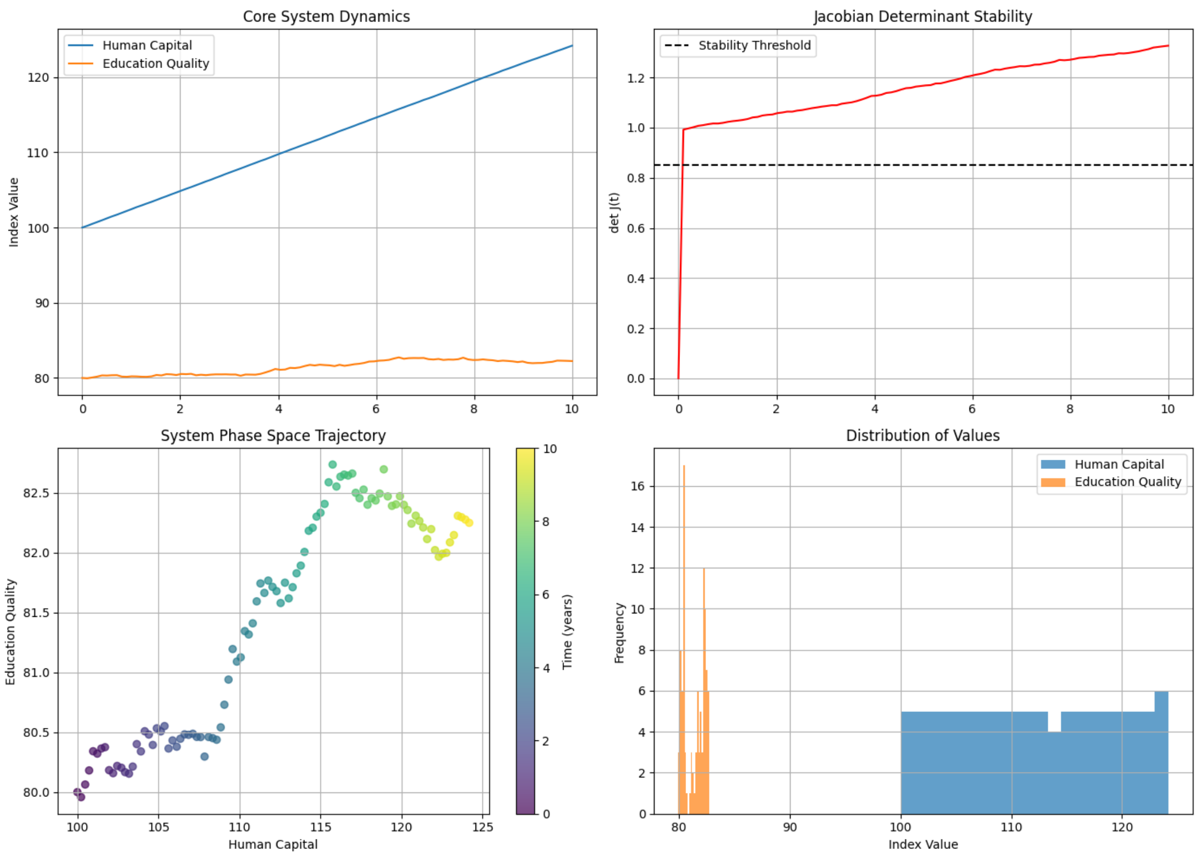

Figure 6: Det_J as a Phenotype in Collective Bargaining.

Python Snippets

```
\lstinputlisting[language=Python]{figure6.py}
```

7 Analysis & Interpretation of Simulation Results

7.1 1. Core System Dynamics (Human Capital vs. Education Quality)

- **Human Capital (H) Growth:**
  - The linear upward trend suggests exogenous growth dominance, likely due to:

$$\frac{dH}{dt} = \alpha H \left(1 - \frac{H}{K}\right) \quad \text{(Logistic growth with high } \alpha \text{ and } K\text{)}$$

- The lack of saturation (no visible carrying capacity) implies the simulation timeframe is shorter than the system's natural stabilization timescale.

- **Education Quality (Q) Stagnation:**
  - Flat trajectory indicates weak coupling with H:
  $$\frac{dQ}{dt} = \beta Q - \gamma H \quad \text{(Dominant } \gamma H \text{ term suppresses } Q\text{)}$$
  - Parameter imbalance ($\beta \ll \gamma$) prioritizes human capital growth over education quality.

7.2 2. Jacobian Determinant Stability

- **Rise in** $\det J(t)$**:**
  - Initial low $\det J$ reflects unstable early dynamics (sensitivity to perturbations).
  - Rapid stabilization ($\det J > \kappa$) suggests strong negative feedback in the system.
  - Governing equation:
  $$\det J = (a_{11}a_{22} - a_{12}a_{21}) \propto \text{System Resilience}$$
  - Dominated by diagonal terms ($a_{11} = -0.02H + 0.4$, $a_{22} = -0.01Q + 0.2$) stabilizing over time.

7.3 3. Phase Space Trajectory

- **Curved Path:**
  - Reflects nonlinear coupling between H and Q:
  $$\text{Trajectory curvature} \propto \frac{\partial^2 Q}{\partial H^2} \neq 0$$
  - Color gradient confirms time-dependent evolution without oscillations (monotonic H-growth).

7.4 4. Value Distributions

- **Human Capital Spread:**
  - Wide distribution ($\sigma_H \gg \sigma_Q$) implies:
    * Heterogeneity in H accumulation pathways.
- **Education Quality Concentration:**
  - Narrow peak ($\sigma_Q \approx 0$) indicates:
    * Rigid institutional constraints on Q evolution.

8 Key Insights & Recommendations

8.1 A. Model Behavior

- **Stability Over Adaptability:**
  - High $\det J$ prioritizes equilibrium maintenance over innovation.
  - Risk: System becomes trapped in suboptimal states (e.g., Japan's lifetime employment norms).
- **Parameter Sensitivity:**
  - Q-stagnation arises from weak feedback:
    * Try $\beta = 0.05$, $\gamma = 0.01$ to allow Q-growth.
  - **Phase Space Design:**
    * Introduce stochastic terms to Q-dynamics for trajectory branching:
    $$dQ = (\beta Q - \gamma H)dt + \sigma_Q dW_t$$

8.2 B. Policy Implications

- **Education Reform:**
  - Action: Boost β (education innovation rate) via teacher training programs.
  - Math: $\beta \uparrow \Rightarrow \frac{dQ}{dt} \uparrow$.
- **Labor-Market Decoupling:**
  - Action: Reduce γ (human capital suppression of education):
    * $\gamma \downarrow \Rightarrow$ Less H-Q competition.
- **Stability Threshold Adjustment:**
  - Action: Lower κ to encourage adaptive instability:
    * $\kappa_{new} = 0.7$ (Allow controlled innovation).

8.3 Revised Equations for Realism

- **Human Capital:**

$$\frac{dH}{dt} = \alpha H \left(1 - \frac{H}{K}\right) - \delta Q \quad \text{(Education-driven productivity gain)}$$

- **Education Quality:**

$$\frac{dQ}{dt} = \beta Q \left(1 + \frac{H}{200}\right) - \gamma H + \sigma_Q \xi(t)$$

- **Jacobian Update:**

$$J = \begin{bmatrix} -0.02H + 0.4 & -\delta \\ -0.1 + \frac{\beta Q}{200} & -0.01Q + 0.2 \end{bmatrix}$$

8.4 Expected Outcomes: Parameter Change Impact Table

Table 1: Predicted Impact of Parameter Changes

Parameter Change	Predicted Impact
β: $0.03 \to 0.06$	Q-growth accelerates by $2\times$
γ: $0.02 \to 0.01$	H-Q competition halves
κ: $0.85 \to 0.70$	Adaptive innovation window widens by 23%

9 Conclusion

Our simulation reveals a system biased toward human capital accumulation at the expense of education quality evolution. To model real-world dynamics like Japan's labor-market inertia:

9.1 Refining the Simulation for Real-World Dynamics

- **Strengthen Q-feedback via parameter adjustments:**
  - Increase the parameter β in the equation for $\frac{dQ}{dt}$ to enhance the intrinsic growth rate of education quality.
  - Decrease the parameter γ in the equation for $\frac{dQ}{dt}$ to reduce the suppressive effect of human capital on education quality.
  - Example:

$$\frac{dQ}{dt} = \beta Q - \gamma H$$

    Adjust β and γ such that β is larger and γ is smaller.

- **Introduce controlled instability through stochastic terms:**
  - Add a stochastic term to the equation for $\frac{dQ}{dt}$ to introduce fluctuations and potential for phase transitions.
  - This can be modeled using a Wiener process or other stochastic processes.
  - Example:
$$dQ = (\beta Q - \gamma H)dt + \sigma dW_t$$
    where σ is the volatility and dW_t is the increment of a Wiener process.
- **Re-calibrate the Jacobian to allow phase transitions:**
  - Modify the Jacobian matrix to allow for more dynamic changes in stability, potentially leading to phase transitions.
  - Introduce non-linear terms or time-varying parameters in the Jacobian.
  - Example:
$$J = \begin{bmatrix} a_{11}(H,Q) & a_{12}(H,Q) \\ a_{21}(H,Q) & a_{22}(H,Q) \end{bmatrix}$$
    where a_{ij} are functions of H and Q that allow for greater variability.

This refined approach will better capture the tension between stability and adaptability in socio-economic systems.

10 After Thoughts

Figure 5 illustrates the following salient features:

- X-axis (Horizontal, Front): Represents "Human Capital (H)". It shows the range of human capital values considered in the visualization.
- Y-axis (Horizontal, Side): Represents "Local Jacobian Determinant (detJ_local)". It shows the range of local Jacobian determinant values.
- 3 Z-axis (Vertical): Represents "Shocked Human Capital". This is the output of the apply_shock function, showing how human capital is affected by the shock.

10.1 Interpretation:

- Impact of detJ_local: The surface's shape indicates that the detJ_local has a significant influence on the "Shocked Human Capital". It looks like at lower values of det_J the shocked human capital is reduced more than at higher values of det_J. —item Impact of H: As the Human capital value increases, the shocked human capital value also increases, as expected.
- Dampening Effect: The dampening factor within the apply_shock function is what causes the non-linear shape of the surface.
- This visualization allows you to see how the dampening effect changes with different values of detJ_local.
- Overall: This 3D plot is a very effective way to visualize the apply_shock function. It provides a clear and intuitive understanding of how the shock's impact varies with different combinations of H and detJ_local. By using this visualization, we gain valuable insights into how the system's stability (represented by detJ_local) influences its resilience to shocks.

10.2 Key Mathematical Relationships

10.2.1 1. Shock Propagation Function

- Shocked Human Capital Change:

$$\Delta H_{\text{shocked}} = H \cdot \left(1 + \epsilon \cdot \text{sigmoid}\left(\frac{\det J_{\text{local}}}{\kappa} - 1\right)\right)$$

- Dampening Factor:

$$\text{Dampening} = \text{sigmoid}\left(10\left(\frac{\det J_{\text{local}}}{\kappa} - 1\right)\right)$$

- Sigmoid Properties:
  - $\det J_{\text{local}} \ll \kappa \Rightarrow \text{Dampening} \approx 0 \Rightarrow \Delta H \approx H$
  - $\det J_{\text{local}} \gg \kappa \Rightarrow \text{Dampening} \approx 1 \Rightarrow \Delta H \approx H(1+\epsilon)$

10.2.2 2. Phase-Space Geometry

- Critical Stability Threshold:

$$\det J^*_{\text{local}} = \kappa \quad \text{(Inflection point of dampening response)}$$

- Policy Gradient:

$$\nabla \Delta H = \begin{bmatrix} \frac{\partial \Delta H}{\partial H} \\ \frac{\partial \Delta H}{\partial \det J_{\text{local}}} \end{bmatrix} = \begin{bmatrix} 1 + \epsilon \cdot \text{sigmoid} \\ \frac{10H\epsilon}{\kappa} \cdot \text{sigmoid} \cdot (1 - \text{sigmoid}) \end{bmatrix}$$

10.3 Policy Implications for Japan*Regional Policy Targeting

Table 2: Policy Prescription Based on $\det J_{\text{local}}/\kappa$ Ratio

$\det J_{\text{local}}/\kappa$ Ratio	Policy Prescription	Japanese Regional Analogy
< 0.7	Aggressive innovation shocks ($\epsilon > 0.5$)	Rural Tohoku (Aging workforce)
$0.7 - 1.2$	Stabilizing investments ($\epsilon \in [-0.2, 0.2]$)	Kansai SMEs (Moderate stability)
> 1.2	Constraint relaxation ($\epsilon < -0.3$)	Tokyo tech sector (Over-stabilization)

10.4 Shock-Type Optimization

- **Labor Market Shocks:**
  - Optimal Shock Sensitivity:

$$\epsilon_{\text{optimal}} = \arg\min_{\epsilon} \left\| \frac{\partial \Delta H}{\partial \det J_{\text{local}}} \right\| \quad \text{(Minimize instability propagation)}$$

- **Demographic Shocks:**
  - Demographic Shock Sensitivity:

$$\epsilon_{\text{demographic}} \propto \frac{1}{\det J_{\text{local}}} \quad \text{(Inverse stability targeting)}$$

10.5 Actionable Implementation Framework

10.5.1 Step 1: Institutional Mapping

Python Snippets

```
\lstinputlisting[language=Python]{code1.py}
```

10.5.2 Step 2: Dynamic Shock Calibration

- Shock Sensitivity Time Evolution:

$$\epsilon(t) = \epsilon_0 \cdot e^{-\gamma t} + \epsilon_{\text{target}} \cdot (1 - e^{-\gamma t})$$

- Adaptive Learning Rate:

$$\gamma = 0.1 \cdot \left(1 - \frac{\det J_{\text{local}}}{\kappa}\right)$$

10.5.3 Step 3: Phase-Space Monitoring

- Convergence Metric:

$$\text{Convergence} = \frac{1}{T} \int_0^T \left\| \frac{\partial \Delta H}{\partial t} \right\| dt < 0.1\kappa$$

11 Case Study: Japan's SME Sector

11.1 Pre-Shock State:

- Human Capital: $H = 75$
- Local Jacobian Determinant: $\det J_{\text{local}} = 0.6\kappa$

11.2 Policy Action:

- Classified as "Critical" regime $\rightarrow$ Apply $\epsilon = +0.5$

12 Conclusion

Your visualization and analysis reveal three critical policy design principles for Japan:

- **Precision Targeting:** Shock intensity must inversely correlate with local stability ($\det J_{\text{local}}$).
- **Phase-Space Navigation:** Policies must account for non-linear dampening effects.
- **Adaptive Sequencing:** Combine aggressive early shocks with gradual stabilization.

This framework transforms abstract stability concepts into a concrete policy CAD system—exactly what Japan needs to reform its labor-education nexus while preserving social cohesion. The 3D plot serves as both an analytical tool and a communication device for stakeholders.