Pricing and Risk in Sovereign Green Debt: Evidence from Chile*

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Abstract

We study the pricing of sovereign green bonds using Chile's pioneering green bond program and its cross-design issuance. Employing a panel of Chilean U.S.-dollar bonds, we estimate no-arbitrage pricing kernels for green and conventional bonds. The results reveal a declining greenium across maturities, driven by the higher interest-rate risk exposure of green bonds. We find no evidence of investor segmentation or liquidity differences between green and conventional bonds. Instead, we explain the observed pricing patterns through a representative-agent asset-pricing model in which investors derive nonpecuniary benefits from the real value of their green bond holdings. During high-inflation periods, as observed in our sample, the real value of green bond portfolios deteriorates, making the convenience service they provide scarcer and more valuable. This positive correlation between green convenience yield and inflation generates a risk premium that compresses the greenium especially at longer maturities, producing a downward-sloping greenium term structure.

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1. Introduction

The market for sovereign green bonds, debt instruments directed towards green projects, has expanded rapidly in recent years. Existing work documents a modest and positive greenium, the yield differential between conventional and green bonds, often attributing it to a static convenience yield arising from investor taste. However, if investors' taste for green assets rises and falls with macro conditions, the greenium reflects not only average convenience but also compensation for bearing that state-dependent risk. Treating the greenium as a constant convenience yield may mismeasure issuance costs by overlooking important risk exposure.

This paper presents facts and theory for the term-structure of greenia for Chilean sovereign bonds, a pioneer in green bond issuance. We show that Chile's sovereign greenium is largest at short maturities and declines with maturity. This downward slope reflects not only differences in expected future greenium, but importantly different priced risk. Liquidity or investor segmentation does not explain this differential pricing. Instead, we rationalize it through a representative-investor model with preferences over real green bond portfolio. In states of high inflation, as we see in our sample, green convenience becomes scarce and therefore more valuable; this positive correlation commands a risk-premium that depresses the greenium especially at long maturities.

We find an average short greenium of 40 basis points (bps), which decreases monotonically to zero at the 20 year maturity. Our empirical setting leverages Chile's dollar-denominated sovereign green bond program, which provides variation for green bonds at different maturities. The measurement strategy estimates a no-arbitrage, exponentially-affine term-structure model that prices green and conventional coupons. This structure allows us to decompose each yield into a path of short rates and compensation for loading on risk factors; estimating it jointly for green and conventional bonds allows us to recover this same decomposition for the greenium.

Our theoretical contribution shows that green preferences typically used in climate finance models generate a different risk exposure for green bonds when accounting for sources of macroeconomic risk. We provide a general decomposition of the greenium term-structure, whose key driver is the covariance between the investor's stochastic discount factor and a stochastic convenience yield driven by taste for green assets. Given the prominence of inflation as a risk factor in our sample, we model investors' green convenience as the real value of the green portfolio. Our parsimonious calibration shows that inflation risk can explain 10% of the estimated greenium term-structure slope. The mechanism is as follows: high inflation erodes the real value of green wealth, leading to high marginal convenience. This implies a negative correlation with the investor's nominal stochastic discount factor, commanding a risk-premium that reduces the greenium, more so for long maturities.

Sovereign green finance appears materially cheaper at short maturities but carries a different risk profile, which has direct implications for issuance strategy and policy design. We use our estimated stochastic discount factor (SDF) to evaluate alternative green bond structures, focusing on sustainability-linked bonds (SLB), whose coupons step up if environmental targets are missed. Leveraging Chile's pioneering sovereign SLB, we price these securities with our SDF and find that markets value them much like standard use-of-proceeds green bonds that earmark funds for specific projects. This suggests that investors reward the green signaling of Chile's SLBs, implying that SLBs can deliver cheaper financing without ring-fencing proceeds. We then quantify the strength of the embedded incentives: once discounting is accounted for, the annualized financing cost implied by a potential step-up is roughly one fifth of the headline step-up. For instance, a 50 basis point (bps) potential step-up translates to about a 10 bps increase in effective annual financing costs. This suggests that SLB's design have the potential to embed even stronger coupon penalties, in order to produce higher financial incentives.

Related literature. Our paper develops a no-arbitrage term-structure model for sovereign green bonds and uses it to measure the maturity profile of the greenium, the yield discount of green relative to otherwise identical conventional bonds. We build on an empirical literature that typically finds small green premia but uses different identification strategies across markets (Giglio et al. (2021) and Hong and Shore (2023)). For U.S. municipals and corporates, Baker, Bergstresser, Serafeim and Wurgler (2018) document a premium for green bonds and interpret it through investor nonpecuniary utility; we take the same taste channel to the sovereign term-structure and show how such a theory implies different risk-exposure for green and non-green bonds that finds support in the data through our estimated no-arbitrage model. Pástor, Stambaugh and Taylor (2022) show how realized green returns can move with flows and shifts in tastes, implying that simple averages can mislead about expected premia; our framework carries this insight to the sovereign term-structure and uses a no-arbitrage model to measure expected excess returns on green and non-green bonds. Using matching and two-step regressions, Zerbib (2019) estimates a greenium of 2bp while Larcker and Watts (2020) argue that once risk and contractual features are tightly controlled the premium is near zero. Our baseline estimate finds greenium that is relatively larger for Chilean sovereign green bonds, of around 40 bps in the short end, but declining in maturity, with no statistically significant greenium beyond the 10 year maturity. Nonetheless, our estimate of the short-greenium is admittedly noisy due to the lack of green bonds with less than 5 year maturity in our sample. Still, this is generally true for the broader universe of green bonds, which are typically issued at longer horizons. For this reason, our finding of declining greenium with maturity is not only more robust but also more empirically relevant. In line with our findings, Fatica, Panzica and Rancan (2021) find a greenium for supranationals and corporates; Caramichael and Rapp (2024) show an issuance premium for U.S./euro corporate green bonds. Flammer (2021) documents real effects around corporate green issuance. The magnitude of the greenium in these papers serve as external discipline on the magnitude of the greenia our model recovers. We build on them by showing how imposing additional structure, through the exponentially-affine form, allows us to measure greenia across comparable but not identical sets of securities.

A growing body of work targets sovereign green bonds specifically. Roch, Ando, Fu and Wiriadinata (2023) provide a broad cross-country assessment of sovereign greenium magnitudes (smaller in advanced economies and larger in EMs); we complement their cross-sectional lens with a structural time-series approach that recovers maturity-specific sovereign green premia. Twin-bond programs, most prominently Germany's, create near-perfect green/conventional comparisons and an observable green yield curve at multiple maturities. Closest to our paper is D'Amico, Klausmann and Pancost (2023) that leverages this setting to estimate a dynamic term-structure model to isolate a "benchmark greenium" purged of non-environmental factors. Our approach is closely related in spirit but we emphasize the term-structure, showing it is informative about the different risk-properties of green bonds and convenience yields more generally. They also find a downward sloping greenium curve, for a different albeit overlapping time period and using German bonds. We propose an asset pricing theory based on preferences for real value of green bonds that can rationalize this term-structure fact. We also show that no-arbitrage term-structure models can be used more broadly beyond the twin bond setting. Our measurement strategy shows how sovereigns and investors can evaluate pricing counterfactuals even in the absence of identical securities, which can be informative for policy makers when designing green bond issuance.

Our notion of a green convenience yield relates to a broader literature that views certain assets as delivering nonpecuniary services, typically from safety or liquidity services, that lower required returns. In the Treasury market, Krishnamurthy and Vissing-Jorgensen (2012) quantify these services, using debt-supply variation to estimate an average convenience yield on the order of tens of basis points. Nagel (2016) show that liquidity premia for near-money assets comove positively with short-term interest rates. In our sample, short-term greenia correlates positively with inflation, which commands a risk-premium when inflation corresponds to bad states of the world for investors. More recent work demonstrates that these premia can flip sign when intermediation frictions dominate: during March 2020, dealer balance-sheet constraints generated inconvenience yields with Treasury–OIS spreads turning positive He, Nagel and Song (2022); post-GFC, a regime shift in which dealers became net long Treasuries helps rationalize negative swap spreads and their connection to funding stresses Du, Hébert and Li (2023); and at the front end, T-bill yields have at times exceeded other risk-free benchmarks when dealer balance-sheet constraints bind Klingler and Sundaresan (2023). Consistent with this, we estimate time-varying greenium term-structure, especially at shorter maturities.

On mechanisms, Giglio, Kelly and Stroebel (2021) surveys macro-finance models that incorporate climate concerns. Among these are models where agents have non-consequentialist preferences over green securities. Our theory builds on this class of models. In equilibrium, such preferences create a convenience yield and lower expected returns for green assets (Pástor, Stambaugh and Taylor (2021); Pedersen, Fitzgibbons and Pomorski (2021)). We also connect to recent theory linking asset returns and climate policy: Chittaro, Piazzesi, Schneider and Sena (2025) and Pedersen (2025) show how cross-sectional return wedges can mimic carbon taxes, which in our sovereign-bond context implies that a maturity-varying greenium provides a price-based incentive for sovereign green projects. Our theory also relates to Aron-Dine, Beutel, Piazzesi and Schneider (2024) who uses new survey on households portfolio choice for green assets to quantify an asset pricing model with nonpecuniary benefits and hedging demand. While we do not allow for hedging demand, our mechanism obtains with or without it. Our emphasis on the role of risk in determining the greenium term-structure echoes findings in Hong, Kubik and Shore (2025) that finds that green-asset volatility is a key determinant of decarbonization. Here, we show the existence of risk specific for green bonds, which can significantly diminish greenium in long-term bonds.

Outline. The rest of the paper is organized as follows. In the next section we provide institutional background on Chile's green bond program and outline the data sources. Section 3 presents how we measure pricing differentials and risk exposure by estimating an exponentially affine model for green and conventional bonds. In Section 4 we present facts on liquidity and holdings of Chilean sovereign green bonds, arguing they do not explain price differences between green and conventional bonds. In Section 5 we develop a parsimonious asset pricing model with non-pecuniary benefit for real green bond portfolio holding that shows how inflation risk can rationalize the downward sloping greenium term-structure.

2. Data and Setting: The Chile Green Bond Program

This paper estimates a term-structure model and establishes facts for Chilean green bonds. This section gives institutional context on the Chilean green bond program and describes the different datasets from which we draw, indicating their main purpose in the analysis.

2.1. Chile Green Bond Program

Chile's sovereign green bond program began in 2019 as a Ministry of Finance initiative to finance climate-aligned public investment while signaling policy commitment and setting a regional benchmark for sovereign sustainable finance. The government approved its Green Bond Framework in May 2019, defining eligible sectors (clean transportation, energy efficiency, renewable energy, living natural resources and protected areas, water management, and green buildings) and committing to annual allocation and impact reports to ensure transparency for investors. Implementation is overseen by an inter-ministerial Sustainable/Green Bonds Committee led by the Ministry of Finance with support from the Ministry of Environment; the committee screens expenditures, links issuances to certified project portfolios, and coordinates reporting in line with Climate Bonds Initiative and International Capital Market Association (ICMA) guidance. The program's stated policy logic is twofold: mobilize low-cost funding for Chile's transition to a low-carbon, climate-resilient economy and establish a sovereign benchmark to catalyze domestic and regional sustainable debt markets.

Issuance has been frequent and staged across currencies since the debut. We focus on green and sustainable dollar bonds. It is worth emphasizing that sovereigns that have issued green bonds typically do so in foreign currency, even in countries like Chile that traditionally has most of its outstanding debt in local currency. Up until 2022, all bonds issued were of the use-of-proceeds type, where bond issuances are linked to specific projects. In 2022, Chile became the first sovereign to issue sustainability-linked bonds (SLBs). This is a different bond design that differs by allowing general budget financing while embedding step-up coupons if predefined key performance indicators (KPIs) are missed. This structure can be preferred by sovereign treasuries because it preserves financing flexibility (no ring-fencing of proceeds), while the contractual incentives to meet economy-wide environmental targets preserves green signalling. There is a belief that such an issuance could potentially lower funding costs by also targeting investors' with a higher willingness to pay for green bonds, upon perceiving that issuers have set credible and meaningful targets. As we show in section 3.5, we find that indeed SLBs are priced as cheaply as use-of-proceeds green bonds.

2.2. Data Sources

We draw from three different data sources: Bloomberg for prices, eMaxx for bond holdings and Luxembourg Stock Exchange for green bond contractual details.

2.2.1. Prices

We obtain all prices and yields from the Bloomberg terminal, as well as other relevant financial information, such as bid-ask spreads and pricing sources. In Appendix A.1 we present detailed information on each bond used in our estimation.

2.2.2. Holdings Data

We use the eMAXX database from Thomson Reuters (now Refinitiv). It is a proprietary dataset that provides comprehensive, quarterly, security-level data on the holdings of fixed income securities by institutional investors, including insurance companies, mutual funds, and pension funds. The funds are primarily based in the United States. Holdings are aggregated from regulatory disclosures by asset managers and institutional investors. eMAXX does not cover households, banks, or governments. Sample selection is based on availability of regulatory disclosure, e.g., insurance companies' reports to the National Association of Insurance Commissioners and mutual funds' U.S. Securities and Exchange Commission (SEC) filings. The data also contains characteristics of individual bonds and firms. We use this data to track a panel of funds that hold both the green and non-green Chilean bonds.

2.2.3. Green Bonds Data

We source green bonds data from Luxembourg Stock Exchange (LuxSE), a leading venue for the listing of green bonds, hosting a wide variety of sustainable debt instruments on its exchange. The selection and classification of green bonds on LGX follow internationally recognized standards and strict eligibility criteria to ensure integrity and transparency. The data is available through the LuxSE data platform, which provides detailed information on green bond issuances, including issuance date, maturity, coupon, and other contractual characteristics, such as the eligible expenditures for use of proceeds bonds and the conditions for step-up coupon payments for the sustainability-linked bonds. The database covers more than 14000 green, social, sustainability and sustainability-linked bonds from more than 3000 issuers worldwide. For our purposes, we use only Chilean green and sustainability dollar bonds.

3. Affine Term-Structure Model for Green Bonds

Investors are willing to pay more for short- and medium-dated green exposure but not for very long maturities, which manifests as a steeper green yield curve that converges to the non-green curve at long horizons. A two-factor, no-arbitrage term-structure model with separate pricing kernels for green and non-green bonds uncovers a 40bps greenium for short maturities and reduces at the long end. This is driven by both different short-rate and different prices of risk on the level of interest rates.

3.1. Asset-Pricing Model

Setup. We price coupon Chilean sovereign bonds with two estimated SDFs, indexed by $j \in \{\text{Green}, \text{NonGreen}\}$ that share state dynamics, but differ in short rate and market prices of risk. We adopt the exponentially affine specification (Duffie and Kan (1996), Ang and Piazzesi (2003)) which allows for a parsimonious and tractable specification to clarify the different pricing properties of both set of bonds.

For an h-period cash-flow claim,

$$P_t^{(n),j} = \mathbb{E}_t\left[M_{t+1}^j P_{t+1}^{(n-1),j}\right], \qquad M_{t+1}^j = \exp\Big(-r_t^j - \tfrac{1}{2}\Lambda_t^{j\prime}\Lambda_t^j + \Lambda_t^{j\prime}\varepsilon_{t+1}\Big), \tag{1}$$

where the state shocks satisfy $\varepsilon_{t+1} \sim \mathcal{N}(0, I)$.

The market price of risk and short-rate are affine in the state,

$$\Lambda_t^j = \Lambda_0^j + \Lambda_1^j z_t \tag{2}$$

$$r_t^j = \rho_0^j + \rho_1^{j\prime} z_t \tag{3}$$

where Λ_1^j governs how market-prices of risk vary with the state of the economy. The state follows a stable VAR(1),

$$z_t = \Psi z_{t-1} + \Sigma\varepsilon_t \tag{4}$$

The estimated parameters are $\theta_j = (\rho_0^j, \rho_1^j, \Lambda_0^j, \Lambda_1^j, \Psi, \Sigma)$ for $j \in \{\text{Green}, \text{NonGreen}\}$. We first estimate the VAR(1) parameters (Ψ, Σ) and once fixing those, we estimate the SDF parameters $(\rho_0^j, \rho_1^j, \Lambda_0^j, \Lambda_1^j)$ by minimizing the pricing errors for green and conventional bonds.

Factors. We use a parsimonious two-factor state $z_t = \big(\text{Level}_t,\ \text{GreenSpread}_t\big)'$, where Level_t summarizes the level of the conventional bond term-structure and GreenSpread_t captures systematic differences between green and non-green yields. For our baseline specification, we use the first principal component of conventional bond yields as the level factor. The level factor explains more than 99% of the cross-sectional and time-series variation of non-green bonds. The time period considered in our sample features the sharp interest rate increase following the inflationary shock after Covid, which explains how the level factor explains an unusually large share of the variation of yields. For the green factor, we use the difference between a long non-green yield and long green yield.

We show in Appendix B a family of alternative specifications that yield similar results. We choose this as our baseline estimate since it shows a good fit of both green and conventional yields with a parsimonious number of factors. Alternative specifications show similar results, as we show in Appendix B. As we show below, the key driving forces for our estimate is the large increase in interest rates in the sample and the concomitant decline in the greenium during the same time period.

Under (1)-(4), bond prices and yields are exponential-affine in z_t. This structure allows us to trace the greenium directly to differences in short-rate and priced risk, holding the economic environment z_t fixed.

3.2. Estimation and Fit

Method. We estimate the model by pricing the cashflows from every coupon bond and minimizing the squared fitting errors made by the model. In Figure 1 we report the distribution of pricing errors, pooling across bond and time-period. We highlight the different distribution of pricing errors for green and conventional bonds, showing that both are similar in magnitude and distribution. In appendix B we show the time-series fit for green and conventional bonds. Our sample consists of 5 green bonds and 6 conventional dollar denominated Chilean sovereign bonds, from 2019 to 2024. The model is estimated at the monthly frequency, and prices are aggregated from daily data as end of month prices. Table **??** summarizes the bonds in our sample.

	Green	Non-Green
Number of Bonds	5	6
Short Maturity	Jan/2027	Mar/2025
Long Maturity	Jan/2050	Jun/2047
Average Issue Bn USD	1.8	1.5

Regularity Constraints Dynamic exponentially-affine term-structure models are plagued by identification and estimation challenges (Collin-Dufresne et al. (2008), Bauer et al. (2012), Hamilton and Wu (2012)). Our choice of using observable factors allow factor dynamics and market price of risk parameters to be estimated separately, which makes the estimation procedure more tractable (Joslin, Singleton and Zhu (2011)). Apart from requiring pricing errors to be minimized, we also impose the following constraints in our estimation that that help ensure convergence of the estimator as well obtaining an estimated SDF that is economically plausible.

In the spirit of Cochrane and Saa-Requejo (2000) and Jiang, Lustig, Van Nieuwerburgh and Xiaolan (2024), we impose "good deal bounds" on our SDF, requiring that its standard deviation is below 2. We also penalize short-rate estimates that lead to green short-rate that are above or below 1 percentage point from the non-green short-rate. This can be viewed as an empirically plausible regularization of short-rate parameters, taking heed on the prior literature estimates for the literature. Since we do not observe short maturity green bonds, these restrictions are required to discipline the short-end of the green yield term-structure. Note we do not impose any restriction on the sign of the short greenium.

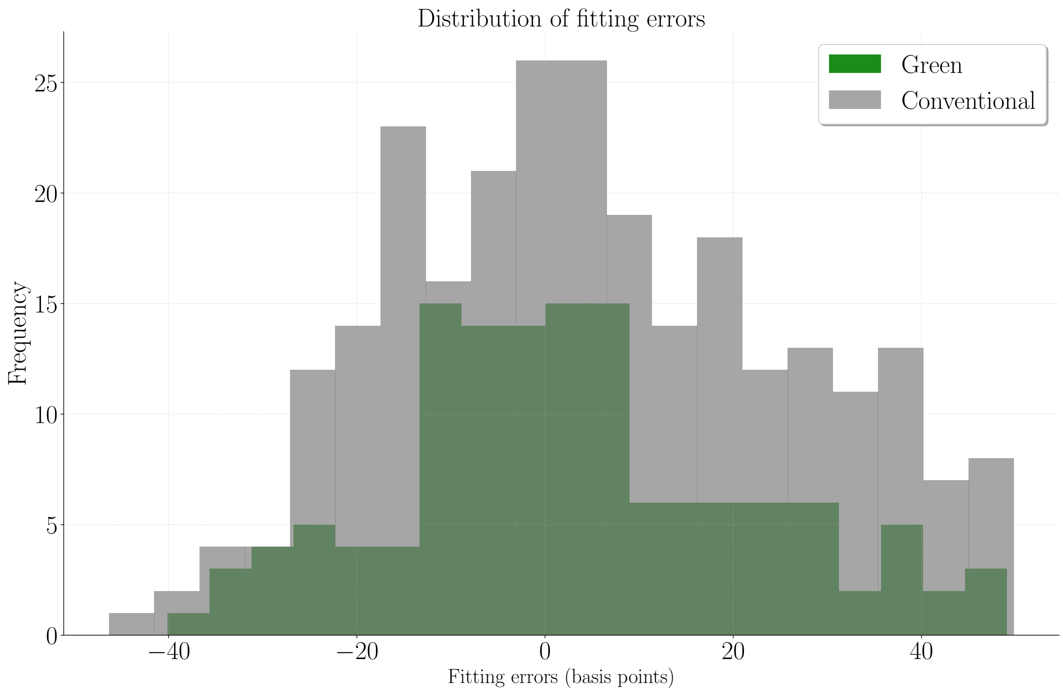


Figure 1: Distribution of pricing errors pooled across bonds and time. Errors are expressed in basis points subtracting observed yields from model implied yields (model minus market price).

3.3. Results: Shape and Magnitude of the Greenium

Yield curves. In Figure 2 we plot the model-implied average term-structures (over the time-series) for green and conventional bonds. We see that the green curve lies below the conventional sovereign bond curve, but with steeper slope. This means that while green bonds are cheaper for the issuer, the expected excess returns are higher for green bonds. Eventually, at around the 20 year maturity, both bonds are equally priced. Figure 3 shows the implied greenium term-structure with 95% confidence intervals obtained from 1000 bootstrap samples, following the method described in Hamilton and Wu (2012).

3.4. What Drives the Results? Different Green Bond Risk-Exposure

Evidence from matched bonds. To gauge drivers of the greenium term-structure, it is useful to inspect the model-implied risk-exposure of green and conventional yields. A good feature of exponentially-affine models is that in spite of the nonlinearity of pricing equations, it leads to

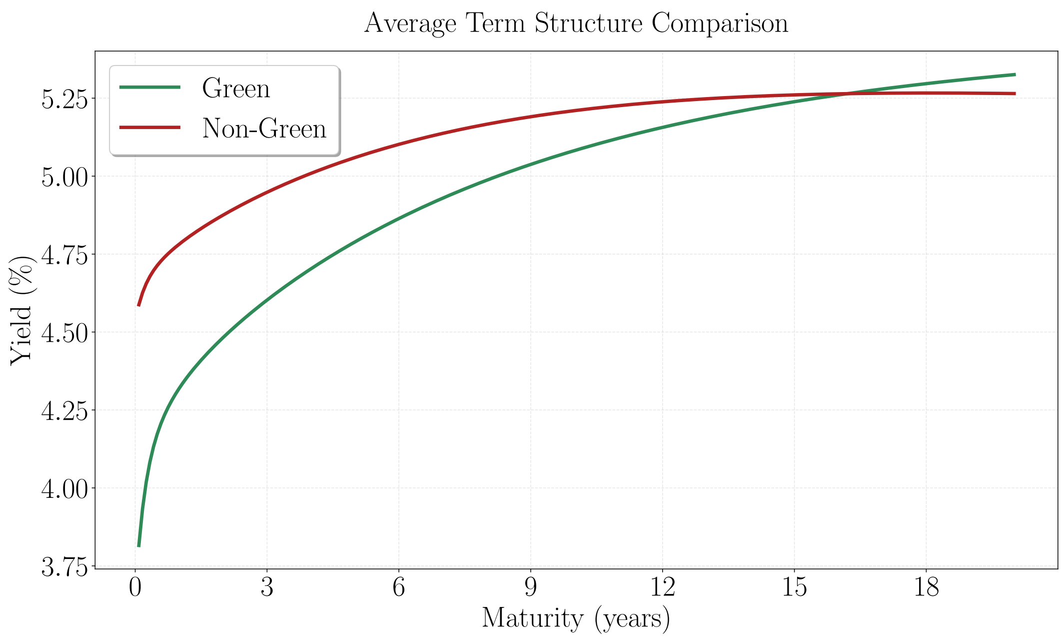


Figure 2: Model-implied average term-structures (over the time-series) for green and conventional bonds.

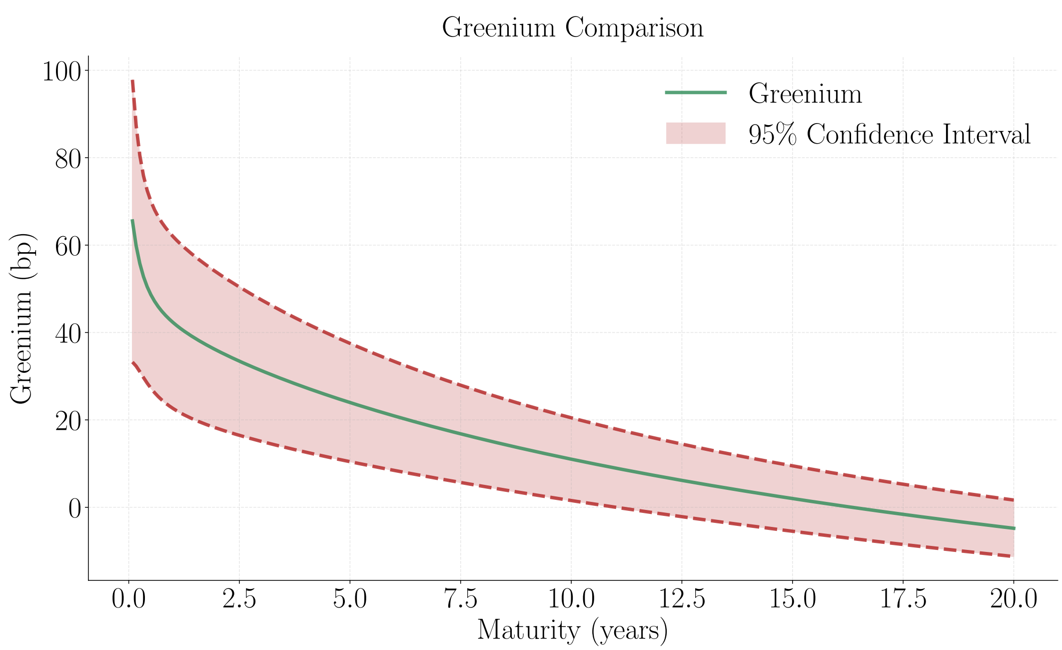


Figure 3: Implied greenium term-structure with 95% confidence intervals obtained from 1000 bootstrap samples.

affine relationship between yields and risk-factors, in this case,

$$y_t^{(n),j} = -\frac{1}{n}\left(A_j(n) + B_{1,j}(n)\text{Level}_t + B_{2,j}(n)\text{GreenSpread}_t\right),\ j \in \{\text{Green}, \text{Conventional}\} \quad (5)$$

where the coefficients $A_j(n), B_{1,j}(n), B_{2,j}(n)$ are non-linear transformations of the underlying estimated parameters, that we specify in Appendix B. The parameters $B_{i,j}$ measures the risk-exposure of bond type j to the i-th factor, that is, how much yields change when for example, the level factor increases. The parameter $A_j(n)$ is the average yield. In Figure 4 we plot these estimated parameters for green and conventional bonds. We see that the constant coefficients

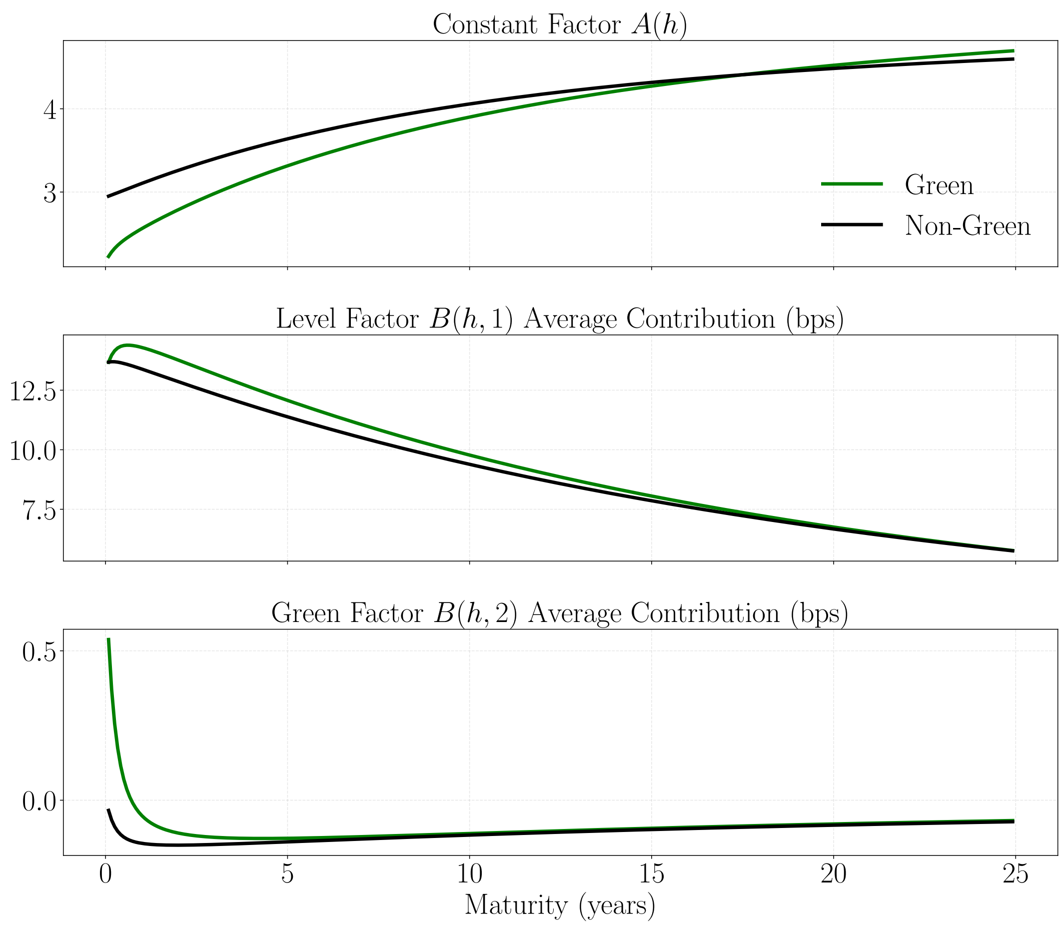


Figure 4: Estimated risk-exposure of green and conventional bonds to the level and green spread factors.

mimic the estimated term-structure shape, which means that on average the greenium term-structure is downward sloping. Further, we see that green bonds are more exposed to interest rate risk, here represented by the level factor. When interest rates increase, yields on green bonds increase by more than conventional bonds, especially at the short end. This means that as interest rates increase, which is the case in our sample, the greenium term-structure flattens. We use as a measure of short-rate an index of short-term interest rates for corporates rated A, which is the same credit rating as Chile.

Evidence from matched bonds. To further see the raw features in the data that lead to these estimates, we construct approximate greenium by matching green and non-green bonds of similar maturity and taking yield differences. This is the conventional greenium measure in the literature, albeit here it is imperfect due to the maturity mismatch. The plot shows that short-term greenium decreases when interest rates rise, while long-term greenium hovers around zero, in line with the estimated exposures in Figure 4.

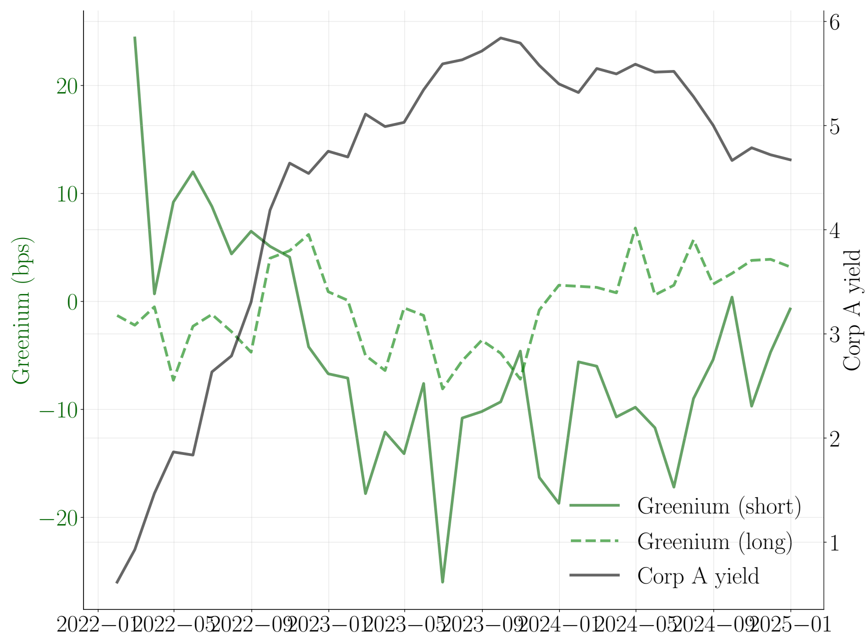


Figure 5: Approximate greenium term-structure constructed by matching green and non-green bonds of similar maturity.

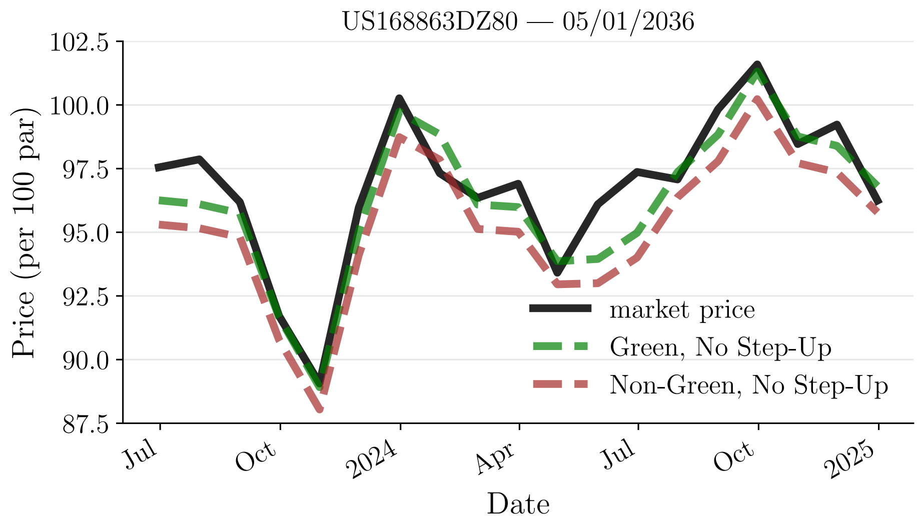
3.5. Do SLB's provide cheaper financing and meaningful financial incentives?

The SLB bond design has emerged that attempt to align issuers environmental incentives by requiring bond coupon payments to be contingent on environmental targets, but differently from a standard green bond it does not tie the proceeds to specific projects. Chile is a pioneer in this area, having issued the first sovereign SLB in 2022. Can such an instrument reduce a sovereign's borrowing cost? Do current Chile's SLB's provide cheap financing like use-of-proceeds bonds? How big are the financial incentives in the state contingent coupon payments? In this section we use our estimated SDF to inform these questions. We show that SLB's are indeed priced as cheaply as regular use-of-proceeds bonds, especially at short maturities, echoing our term-structure finding. This is in spite of the current coupon penalties being relatively small, as we also show.

These findings suggest that Chile's signalling of environmental commitment being the main driver of the greenium, not the type of instrument.

3.5.1. Are SLB's priced like regular green bonds?

Yes. To reach this conclusion, we use our estimated SDF's to price SLB's cash-flows and compare them with market prices. Figure



We repeat this exercise for the two other SLB bonds. Since these are longer maturities, we find no material price difference when pricing with either the green of conventional SDF. This should not be surprising at this point given that we find that discount rates for both SDF's converge for long maturities. This therefore echoes the previous finding; SLB's are cheaper here when issued at short maturities. For the SLB design, this also means that setting short-maturity step ups also provide larger financial incentives.

3.5.2. How large are SLB coupon incentives?

Lower than the headline coupon step-up increase. To reach this conclusion, we use the estimated SDF to price SLB's cash-flows under each step-up scenario. We then translate price differences from each step-up scenario to the yield equivalent change. For this, we compute what is the yield change under the baseline cash-flows (no step-up) that leads to the same price obtained when coupons step-up. This quantity is the answer to the following question: the step-up option value is a subsidy to the issuer since it makes the bond more expensive today; what is this subsidy expressed in yield-equivalent terms? Put differently, it gives the financial penalty if the bond indeed steps up in terms of annual financing costs.

Table 1: SLB Step-Up Yield Subsidy (in bps)

ISIN	Step-up scenario (bps)	Maturity	Step-up yield equivalent (bps)
US168863DY16	12	July-2042	4
US168863DY16	25	July-2042	9
US168863DZ80	25	May-2036	4
US168863DZ80	50	May-2036	8
US168863EA21	5	May-2054	2
US168863EA21	10	May-2054	5

Table 1 below summarizes this quantity for the three sovereign dollar SLB bonds issued by Chile. The main takeaway is that discounting significantly reduces the financial incentives in the SLB state-contingent coupon payments. While this means that financial incentives are currently small, it also suggests that risk in coupon payments are likely to be small, which can be useful for sovereign issuance if environmental signalling is the main objective with SLB issuance.

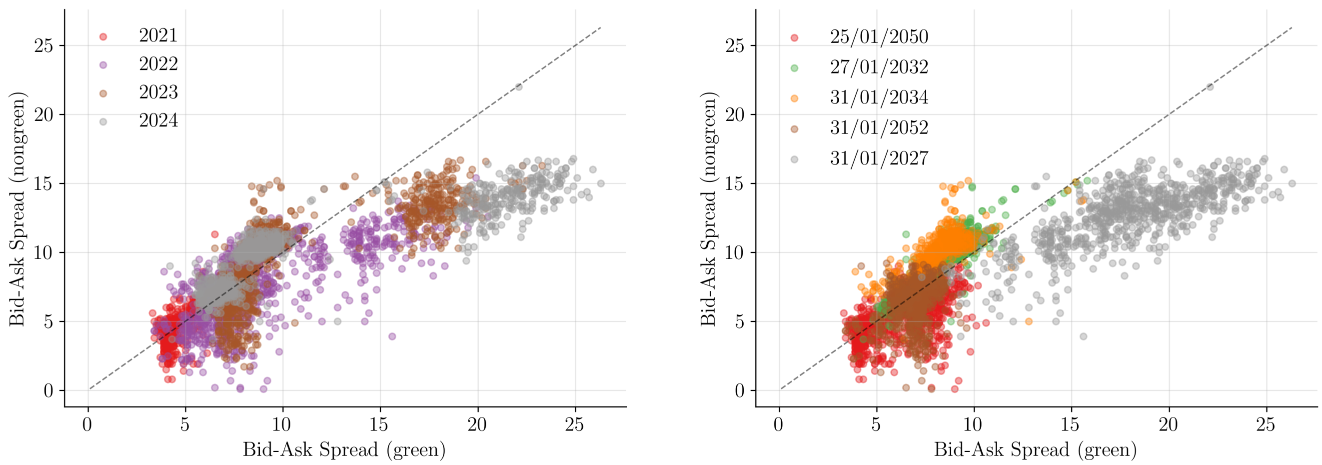
4. Facts on Green Bond Liquidity and Holders

We state two factors related to liquidity and holdings of green and conventional bonds. These facts serve as motivation for interpreting our stochastic discount factor estimation and that motivates our theoretical setup of a representative-agent model with preference for real green bond portfolio.

Liquidity on green and non-green bonds is similar Green and non-green bonds exhibit similar liquidity. We show this in Figure 6, by plotting the scatter of bid-ask spreads for matched green and conventional bonds. Ideally we would have identical green and non-green bonds in terms of maturity; since this is not the case we pair bonds with maturities that are similar. The maximum discrepancy we find between pairs is of 3 years.

The plot shows that bid-ask spreads are of around 15bps and lines up close to the 45-degree line across maturities and years. We interpret this as evidence that liquidity of green and conventional bonds of similar maturities are similar. Panel 6a color codes by observation year, while Panel 6b color codes by maturity of the green bond in the pair. We see bid-ask spread variation both over time and bond maturity, where the most significant illiquidity arises for the short-term green bonds, especially in 2024. While the difference is not big, of around 5-10bps extra bid-ask spread for the green bond pair, the presence of liquidity premia in green yields would lead us to underestimate the greenium for short-maturity. Since we find relatively larger greenia, this leads us to conclude that liquidity cannot solely explain the magnitudes we find. For the main fact of this paper, of a downward sloping greenium term-structure, this is even more true since bonds of longer maturities are more alike.

In summary, because bid-ask spreads are a standard proxy for trading frictions, this evidence implies that liquidity is unlikely to be the channel behind observed price differences between green and conventional bonds.

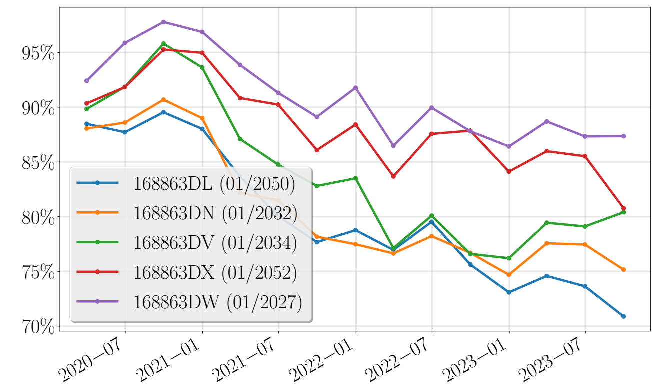


(a) Panel A: By year

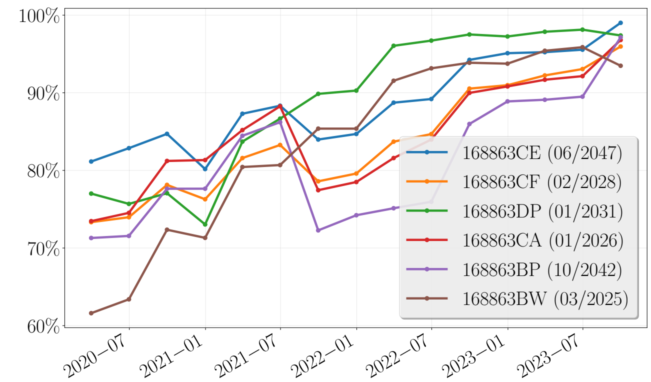
(b) Panel B: By bond maturity

Figure 6: Bid-ask spreads for green and non-green bonds by year (left) and by bond maturity (right).

Common investor base The investor base for green and conventional bonds is largely the same. To show this fact, we plot in Figure 7 the share of funds that hold both green and conventional bonds in the same quarter, the time frequency of our holdings data. This share is computed over the sample of funds in our sample that hold at least one Chilean bond. The plot shows that in the beginning of the sample this share is high and increases, likely due to further issuance of new green bonds. At the end, around two-thirds of the funds in our sample hold both a green and a conventional Chilean sovereign dollar bond. This overlap rules out clientele segmentation in which for example green funds would be the primary holder of green bonds but not of the conventional bonds. Instead, we interpret our results as evidence that the same managers evaluate and price both green and conventional bonds, so differences in outcomes should be interpreted as within-investor assessments rather than cross-investor demand shifts. This interpretation guides our modelling assumption for the greenium based on investor preferences.



(a) Panel A: Conditional on holding at least one green bond



(b) Panel B: Conditional on holding at least one non-green bond

Figure 7: Share of funds that simultaneously hold a Chilean sovereign green bond and a conventional (non-green) bond in the same quarter, under two conditioning samples.

5. Rationalizing the Greenium Term-Structure

The previous section has documented new facts on the pricing of green bonds at different maturities. This section develops an asset pricing model for the term-structure of green bonds in the presence of non-pecuniary benefits for real green bond portfolio. Because inflation is a salient source of risk in our sample, the model emphasizes how inflation shocks move both discount rates and the convenience yield. However the argument that risk in convenience yields is an important determinant in the greenium, especialy at longer maturities, extends to other environments and sources of convenience yields.

5.1. Environment

A representative agent has preferences over consumption and green portfolio value

$$E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, G_t) \tag{6}$$

There are $2N$ bonds traded, where N bonds are green and N are conventional. For each type, bonds of maturities are $n \in \{1, \ldots, N\}$ are available. We take one time period to be a year. The real green bond portfolio G_t is defined as

$$G_t := \frac{\sum_{n=1}^{N} Q_t^{g,(n-1)} B_{t-1,g}^{(n-1)}}{P_t} \tag{7}$$

where P_t is the price level, $B_{t-1,g}^{(n-1)}$ are the number of bonds of maturity n bought at time $t-1$ and $Q_t^{g,(n-1)}$ is the nominal bond price of the green bond with maturity $n-1$ at time t.

The agent's budget constraint is

$$P_t C_t + \sum_{n=1}^{N} Q_{t,g}^{(n)} B_{t,g}^{(n)} + \sum_{n=1}^{N} Q_t^{(n)} B_t^{(n)} = \sum_{n=1}^{N} Q_{t,g}^{(n-1)} B_{t-1,g}^{(n-1)} + \sum_{n=1}^{N} Q_t^{(n-1)} B_{t-1}^{(n-1)} + P_t I_t \tag{8}$$

where bonds are nominal and upon maturity it pays $Q_t^{(0)} = P_{t-1}$. I_t is an exogenous source of income. We assume inflation π_{t+1} and expected inflation x_t are persistent processes:

$$x_t = \rho_x x_{t-1} + \sigma_x \varepsilon_{t,x} \tag{9}$$

$$\pi_{t+1} := \ln \frac{P_{t+1}}{P_t} = \pi_0 + x_t + \sigma_\pi \varepsilon_{t+1,\pi} \tag{10}$$

The pricing equation for conventional bonds is standard and is explicitly written in Appendix C. The novel feature here is inflation as a risk factor for green convenience. This convenience yield is determined by the marginal rate of substitution between the green bond portfolio and consumption.

$$Q_t^{g,(n)} = E_t \left[M_{t+1} \left(1 + Y_{t+1}\right) Q_{t+1}^{g,(n-1)} \right], \; Y_{t+1} := \frac{u_{G,t+1}}{u_{C,t+1}} \tag{11}$$

$$M_{t+1}^{g} := M_{t+1} \left(1 + Y_{t+1}\right) \tag{12}$$

where $M_{t+1} = \beta \frac{u_{1,t+1}}{u_{1,t}} \Pi_{t+1}^{-1}$ is the nominal pricing kernel. We define the product $M_{t+1}\left(1 + Y_{t+1}\right)$ as the green pricing kernel M_{t+1}^{g}. Differences in the estimated SDF using green and conventional bonds, as we do in our empirical SDF estimation, recover the stochastic properties of the convenience term $1 + Y_{t+1}$.

Greenium term-structure decomposition Below we state general conditions on the stochastic processes for the SDF and green convenience yields that shapes the greenium term-structure. This clarifies how the equilibrium dynamics in our model explains the downward slope of the greenium term-structure. We assume

Assumption 1 (Log-normality and homoskedastic second moments)**.**

(a) For each $j \geq 1$, (m_{t+j}, y_{t+j}) is conditionally Gaussian given $\mathcal{F}_t$.

(b) Conditional second moments are time-t functions of horizon j only

The greenium term-structure can be decomposed by

Proposition 1 (greenium term-structure decomposition)**.** *For every integer $h \geq 1$,*

$$h\, g_t^{(h)} = \sum_{j=1}^{h} \mathbb{E}_t \left[y_{t+j} \right] + \frac{1}{2} \mathrm{Var}_t\left(\sum_{j=1}^{h} y_{t+j} \right) + \mathrm{Cov}_t\left(\sum_{j=1}^{h} m_{t+j}, \sum_{j=1}^{h} y_{t+j} \right). \tag{13}$$

The proof is in Appendix C. The presence of stochastic convenience yields introduce standard asset pricing covariance terms between the stochastic discount factor and the convenience yield. All else equal, the greenium term-structure is upward sloping whenever the conditional covariance between the marginal green asset benefit and the stochastic discount factor is positive, and downward sloping when the covariance is negative.

Mechanism for a downward sloping greenium term-structure In this environment states of high inflation are states of low real green bond portfolio value. Under the standard assumption of decreasing marginal convenience yield, these are states of scarce and thus more valuable convenience. This leads to a negative covariance between the nominal stochastic discount factor and convenience yields. Here we emphasize inflation as a risk-factor for green convenience, since it was a salient source of risk in our sample.

5.2. Quantifying inflation risk in green convenience

We show that in this parsimonious asset pricing model, a quantitatively plausible inflation risk can explain around 10% of the slope of the greenium term-structure.

Calibration We calibrate the asset pricing model with the following functional form:

$$u(C_t, G_t) = \frac{1}{1-\gamma}\left(\left(C_t^{\frac{\eta-1}{\eta}} + \omega G_t^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}\right)^{1-\gamma}. \tag{14}$$

This leads to the following structural convenience function:

$$Y_{t+1} = \left(\frac{C_{t+1}}{G_{t+1}}\right)^{\frac{1}{\eta}}. \tag{15}$$

In order to asess the extent to which inflation risk can drive the greenium term-structure, we keep this as the only shock in the model. In particular, we note that contrary to a standard consumption-based model, there is no consumption risk. We calibrate the model to match the short-term rate and volatility of the data when the consumption process is constant at $\bar{C}$.

Table 2: Model Calibration Parameters

Parameter	Description	Value	Source
β	Discount factor	0.99	match short-term rate and volatility
γ	Risk aversion	2.00	external
η	Intratemporal elasticity of substitution	1.10	external
μ_π	Mean inflation	0.02	US inflation target
ρ_π	Persistence of inflation	0.95	estimated
σ_x	Std. dev. of inflation expectations shock	0.10	estimated
σ_π	Std. dev. of inflation shock (%)	0.10	estimated
ω_S	Green bond portfolio weight	0.03	match short-term greenium
$\bar{C}$	Constant consumption path	2.72	match short-term rate and volatility

We simulate the model at a yearly frequency and compute the unconditional moments of interest rates and greenium. Table 3 summarizes the main moments in the data and model.

Table 3: Moments: Data vs Model

Moment	Data	Model
Mean short rate (annual, %)	4.59	4.99
Std. short rate (annual, %)	0.91	0.76
Std. log convenience ratio	0.0372	0.0001
Mean short greenium (bps)	55.64	49.84
Greenium slope (bps/year)	-1.29	-0.16
Correlation Convenience and SDF	-0.557	-0.846

We see that inflation risk alone can drive up to 10% of the decrease in the greenium at longer maturities. Hence, while preferences can induce a greenium and therefore cheaper debt, interaction with macroeconomic source of risks can have meaningful effects in introducing risk specific to green bonds.

6. Conclusion

Green borrowing can be materially cheaper at short maturities, but come with different risk exposure. These are useful for sovereigns when planning green bond issuance and for investors when choosing between green and conventional bonds. More broadly, convenience yields on assets may change with the state of the economy, and estimating no-arbitrage term-structure models allows us to learn how this is so.

The measurement in this paper has limitations for short-greenia due to scarcity of bonds issued at very short maturities. Green bonds, nonetheless, are typically issued at longer maturities for longer-term projects. The estimation exercise here is informative about the risks specific for green bonds. Our measurement strategy can be extended for a more diverse set of issuers and allowing for a richer structure, allowing for example the measurement of green specific green bond default risk. Measuring risk in green debt can inform both sovereign issuance strategy and the design of sustainable finance instruments for cost-effective climate finance.

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A. Data

A.1. Chilean sovereign green and sustainability bonds

This subsection documents the Chilean sovereign bonds in our dataset that are tagged as green or sustainability. These are use-of-proceeds instruments: an amount equal to net proceeds is allocated to eligible budget expenditures under Chile's sovereign green or sustainable financing framework. Standards and reviews referenced in the tables include the Climate Bonds Initiative (CBI) Climate Bonds Standard and the International Capital Market Association (ICMA) Green Bond Principles, Social Bond Principles, and Sustainability Bond Guidelines. We report core contractual and framework information.

ISIN	US168863DL94
Bond tag (type)	Green (sovereign use-of-proceeds)
Currency	USD
Issued amount (original currency)	2,318,357,000
Total issuance in USD	2,318,357,000
Issue date	2019-06-25
Maturity date	2050-01-25
Eligible use-of-proceeds (issuer)	Clean transportation; Energy efficiency; Renewable energy; Living natural resources, land use and marine protected areas; Efficient and climate-resilient water management; Green buildings

Table 4: Chilean sovereign green/sustainability bond summary for ISIN US168863DL94.

ISIN	US168863DN50
Bond tag (type)	Green (sovereign use-of-proceeds)
Currency	USD
Issued amount (original currency)	1,500,000,000
Total issuance in USD	1,500,000,000
Issue date	2020-01-27
Maturity date	2032-01-27
Eligible use-of-proceeds (issuer)	Clean transportation; Energy efficiency; Renewable energy; Living natural resources, land use and marine protected areas; Water management; Green buildings

Table 5: Chilean sovereign green/sustainability bond summary for ISIN US168863DN50.

ISIN	US168863DV76
Bond tag (type)	Sustainability (sovereign use-of-proceeds)
Currency	USD
Issued amount (original currency)	1,500,000,000
Total issuance in USD	1,500,000,000
Issue date	2022-01-31
Maturity date	2034-01-31
Eligible use-of-proceeds (issuer)	Access to essential health services; Access to basic housing; Access to education; Clean transport; Community support through job creation; Energy efficiency; Food security; Green buildings; Living natural resources, land use and marine protected areas; Renewable energy; Support for human rights victims; Support for low-income families; Support for the elderly or people with special needs; Water management

Table 6: Chilean sovereign green/sustainability bond summary for ISIN US168863DV76.

ISIN	US168863DW59
Bond tag (type)	Sustainability (sovereign use-of-proceeds)
Currency	USD
Issued amount (original currency)	1,000,000,000
Total issuance in USD	1,000,000,000
Issue date	2022-01-31
Maturity date	2052-01-31
Eligible use-of-proceeds (issuer)	Access to essential health services; Access to basic housing; Access to education; Clean transport; Community support through job creation; Energy efficiency; Food security; Green buildings; Living natural resources, land use and marine protected areas; Renewable energy; Support for human rights victims; Support for low-income families; Support for the elderly or people with special needs; Water management

Table 7: Chilean sovereign green/sustainability bond summary for ISIN US168863DW59.

ISIN	US168863DX33
Bond tag (type)	Sustainability (sovereign use-of-proceeds)
Currency	USD
Issued amount (original currency)	1,500,000,000
Total issuance in USD	1,500,000,000
Issue date	2022-01-31
Maturity date	2027-01-31
Eligible use-of-proceeds (issuer)	Access to essential health services; Access to basic housing; Access to education; Clean transport; Community support through job creation; Energy efficiency; Food security; Green buildings; Living natural resources, land use and marine protected areas; Renewable energy; Support for human rights victims; Support for low-income families; Support for the elderly or people with special needs; Water management

Table 8: Chilean sovereign green/sustainability bond summary for ISIN US168863DX33.

B. Asset Pricing Appendix

B.1. Model Fit

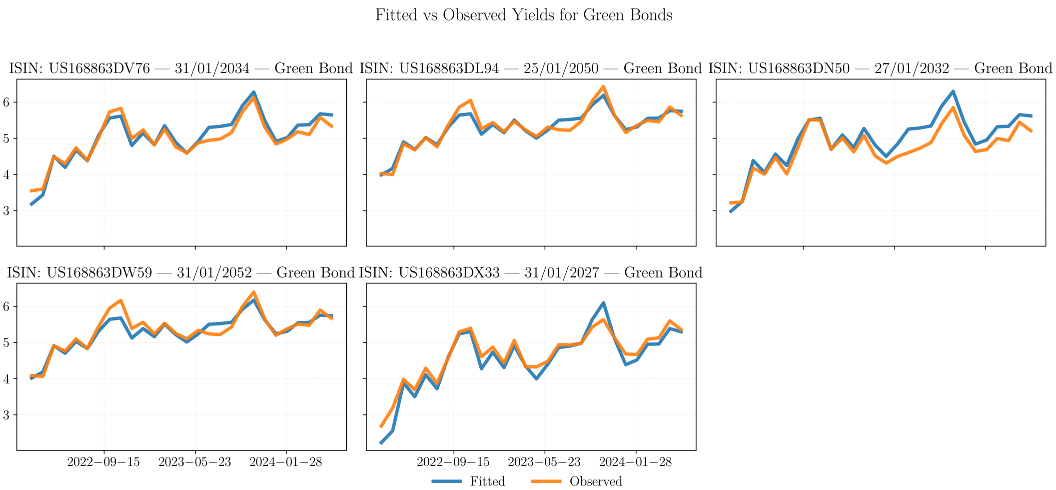

Figure 8: Model fit for green bonds: observed versus model-implied yields by maturity over time.

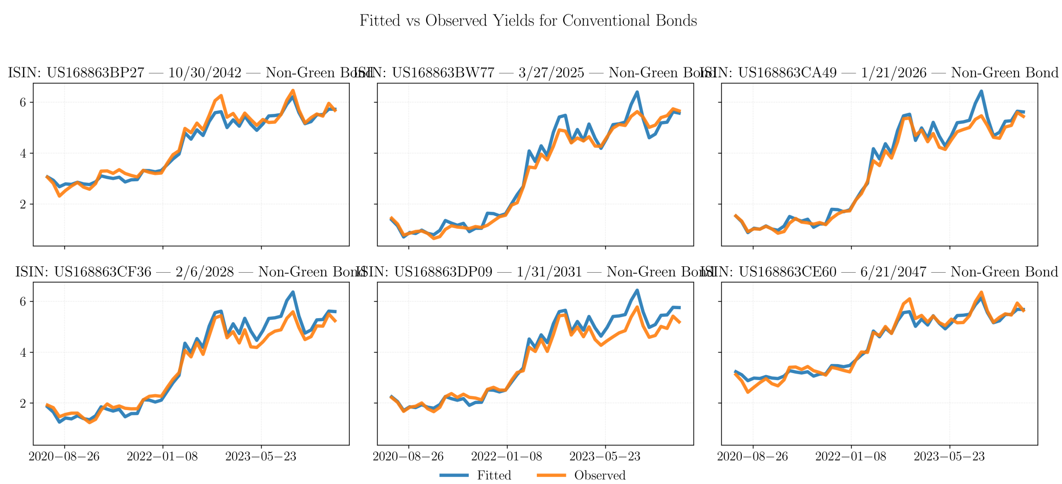

Figure 9: Model fit for conventional bonds: observed versus model-implied yields by maturity over time.

B.2. Alternative Specifications

We show here alternative specifications of the term-structure model. In particular, we show the robustness of the downward sloping greenium term-structure to different number and different choice of factors. Table 9 summarizes the results.

Table 9: Alternative factor specifications for the exponentially affine term-structure model.For the non-green factors, we use the level, slope and curvature factors from the principal components of conventional bond yields, in this order. For example, in the case without green factors and a total of two factors, we use the level and slope factors.

Green Bond Pricing Error (bps)	Conventional Bond Pricing Error (bps)	Greenium Slope	Short-term greenium mean (bps)	Short-term greenium std (bps)	N^o green factors	N^o factors total
16	16	-1.8	63	64	0	1
19	17	-1.3	24	34	0	2
17	17	-1.8	30	18	0	3
15	17	-2.3	46	14	1	2
18	20	-2.6	66	6	1	3
22	21	-1.8	11	2	2	4
20	21	-1.4	4	18	2	4

One could entertain many other combinatorial choices of factors, of which we do not report here since they typically lead to economically implausible estimated stochastic discount factors, which in particular, fit poorly both green and conventional yields.

C. Model Appendix

C.1. Computing Conditional Moments

Given our estimates and the exponentially affine structure, we can compute conditional moments analytically. We recall the stochastic discount factor notation

$$m_{t+1} = -\delta_0 - \delta_1' z_t - \frac{1}{2}\Lambda_t' \Lambda_t - \Lambda_t' \varepsilon_{t+1} \tag{16}$$

$$m_{t+1}^g = -\delta_0^g - \delta_1^{g'} z_t - \frac{1}{2}\Lambda_t^{g'} \Lambda_t^g - \Lambda_t^{g'} \varepsilon_{t+1} \tag{17}$$

Define $\|\Lambda_t\|^2 = \Lambda_t' \Lambda_t$.
From this, we have

$$\begin{aligned} y_{t+1} &= -(\delta_0^g - \delta_0) - (\delta_1^g - \delta_1)' z_t - \frac{1}{2}(\|\Lambda_t^g\|^2 - \|\Lambda_t\|^2) - (\Lambda_t^g - \Lambda_t)' \varepsilon_{t+1} & (18) \\ &= -\delta_0^y - \delta_1^{y'} z_t - \frac{1}{2}(\|\Lambda_t^g\|^2 - \|\Lambda_t\|^2) - \Lambda_t^{y'} \varepsilon_{t+1} & (19) \\ &= -\delta_0^y - \delta_1^{y'} z_t - \frac{1}{2}(\|\Lambda_t^y\|^2 + 2\Lambda_t' \Lambda_t^y) - \Lambda_t^{y'} \varepsilon_{t+1} & (20) \end{aligned}$$

where the second line is definitional.
The short interest rate is

$$\begin{aligned} i_t^{(1)} &= -E_t[m_{t+1}] - \frac{1}{2} Var_t(m_{t+1}) & (21) \\ &= \delta_0 + \delta_1' z_t + \frac{1}{2}\Lambda_t' \Lambda_t - \frac{1}{2}\Lambda_t' \Lambda_t & (22) \\ &= \delta_0 + \delta_1' z_t & (23) \end{aligned}$$

The conditional expectation of the convenience term is

$$\begin{aligned} y_{t+2} &= -\delta_0^y - \delta_1^{y'} z_{t+1} - \frac{1}{2}(\|\Lambda_{t+1}^g\|^2 - \|\Lambda_{t+1}\|^2) - \Lambda_{t+1}^{y'} \varepsilon_{t+2} & (24) \\ \implies E_{t+1}[y_{t+2}] &= -\delta_0^y - \delta_1^{y'} z_{t+1} - \frac{1}{2}(\|\Lambda_{t+1}^g\|^2 - \|\Lambda_{t+1}\|^2) & (25) \\ E_t[y_{t+2}] &= -\delta_0^y - \delta_1^{y'} \Psi z_t - \frac{1}{2} E_t[\|\Lambda_{t+1}^g\|^2 - \|\Lambda_{t+1}\|^2] & (26) \end{aligned}$$

The conditional variance is

$$Var_{t+1}(y_{t+2}) = \Lambda_{t+1}^{y'} \Lambda_{t+1}^y \tag{27}$$

The conditional covariance with the SDF

$$Cov_{t+1}(m_{t+2}, y_{t+2}) = Cov_{t+1}(-\Lambda_t' \varepsilon_{t+2}, -\Lambda_{t+1}^{y'} \varepsilon_{t+2}) \tag{28}$$
$$= \Lambda_{t+1}' \Lambda_{t+1}^{y} \tag{29}$$

For the expected greenium we have

$$E_t[g_{t+1}^{(1)}] = E_t[y_{t+2}] + \frac{1}{2} E_t[\Lambda_{t+1}^{y'} \Lambda_{t+1}^{y}] + E_t[\Lambda_{t+1}' \Lambda_{t+1}^{y}] \tag{30}$$
$$= -\delta_0^y - \delta_1^{y'} \Psi z_t - \frac{1}{2} E_t[\|\Lambda_{t+1}^{g}\|^2 - \|\Lambda_{t+1}\|^2] + \frac{1}{2} E_t[\Lambda_{t+1}^{y'} \Lambda_{t+1}^{y}] + E_t[\Lambda_{t+1}' \Lambda_{t+1}^{y}] \tag{31}$$
$$= -\delta_0^y - \delta_1^{y'} \Psi z_t - \frac{1}{2} E_t[(\|\Lambda_{t+1}^{y}\|^2)] - E_t[\Lambda_{t+1}' \Lambda_{t+1}^{y}] + \frac{1}{2} E_t[\Lambda_{t+1}^{y'} \Lambda_{t+1}^{y}] + E_t[\Lambda_{t+1}' \Lambda_{t+1}^{y}] \tag{32}$$
$$= -\delta_0^y - \delta_1^{y'} \Psi z_t \tag{33}$$

C.2. Greenium term-structure decomposition

Let $Q_t^{(h)}$ and $Q_{g,t}^{(h)}$ denote, respectively, the prices at time t of a non-green and a green zero-coupon bond with h periods to maturity. Let

$$m_{t+1} := \log M_{t+1}, \qquad y_{t+1} := \log(1 + Y_{t+1}), \qquad m_{t+1}^{g} := m_{t+1} + y_{t+1},$$

and write the green pricing recursion for $h \geq 2$ as

$$Q_{g,t}^{(h)} \;=\; \mathbb{E}_t\Big[M_{t+1}(1 + Y_{t+1})\, Q_{g,t+1}^{(h-1)}\Big] \;=\; \mathbb{E}_t\Big[\exp\big(m_{t+1} + y_{t+1}\big)\, Q_{g,t+1}^{(h-1)}\Big]. \tag{34}$$

The non-green recursion is identical with $y \equiv 0$. Define yields and the greenium

$$i_t^{(h)} := -\frac{1}{h} \log Q_t^{(h)}, \qquad i_{g,t}^{(h)} := -\frac{1}{h} \log Q_{g,t}^{(h)}, \qquad g_t^{(h)} := i_t^{(h)} - i_{g,t}^{(h)}.$$

The one-period prices and greenium are

$$Q_t^{(1)} = \mathbb{E}_t[e^{m_{t+1}}], \quad Q_{g,t}^{(1)} = \mathbb{E}_t[e^{m_{t+1}+y_{t+1}}], \quad g_t^{(1)} = \mathbb{E}_t\left[y_{t+1}\right] + \tfrac{1}{2}\operatorname{Var}_t\left(y_{t+1}\right) + \operatorname{Cov}_t\left(m_{t+1},\, y_{t+1}\right).$$

For $1 \leq a \leq b$ define

$$\bar{m}_{a:b} := \sum_{j=a}^{b} m_{t+j}, \qquad \bar{y}_{a:b} := \sum_{j=a}^{b} y_{t+j}.$$

Lemma 1 (unrolled prices)**.** *For each $h \geq 1$,*

$$Q_{g,t}^{(h)} = \mathbb{E}_t\Big[\exp\Big(\bar{m}_{1:h} + \bar{y}_{1:h}\Big)\Big], \qquad Q_t^{(h)} = \mathbb{E}_t\Big[\exp\Big(\bar{m}_{1:h}\Big)\Big]. \tag{35}$$

Proof. For $h = 1$ the identities follow from the one-period formulas above. For $h \geq 2$, use (34) and the tower property:

$$Q_{g,t}^{(h)} = \mathbb{E}_t\left[e^{m_{t+1}+y_{t+1}}\,\mathbb{E}_{t+1}\left[e^{\sum_{j=2}^{h}(m_{t+j}+y_{t+j})}\right]\right] = \mathbb{E}_t\left[e^{\sum_{j=1}^{h}(m_{t+j}+y_{t+j})}\right].$$

The non-green case is the same with $y \equiv 0$. □

Theorem 1 (multi-horizon identity)**.** *For every integer $h \geq 1$,*

$$h\,g_t^{(h)} = \sum_{j=1}^{h}\mathbb{E}_t\left[y_{t+j}\right] + \frac{1}{2}\,\mathrm{Var}_t\left(\sum_{j=1}^{h}y_{t+j}\right) + \mathrm{Cov}_t\left(\sum_{j=1}^{h}m_{t+j},\ \sum_{j=1}^{h}y_{t+j}\right). \tag{36}$$

Proof. By Lemma 1 and the log-normal moment formula, for any (conditionally) Gaussian X, $\mathbb{E}_t[e^X] = \exp\{\mathbb{E}_t[X] + \frac{1}{2}\,\mathrm{Var}_t(X)\}$, we have

$$\begin{aligned}\log Q_{g,t}^{(h)} &= \mathbb{E}_t\left[\bar{m}_{1:h} + \bar{y}_{1:h}\right] + \tfrac{1}{2}\,\mathrm{Var}_t(\bar{m}_{1:h} + \bar{y}_{1:h}),\\ \log Q_t^{(h)} &= \mathbb{E}_t[\bar{m}_{1:h}] + \tfrac{1}{2}\,\mathrm{Var}_t(\bar{m}_{1:h}).\end{aligned}$$

Since $g_t^{(h)} = -(1/h)\log Q_t^{(h)} + (1/h)\log Q_{g,t}^{(h)}$, subtracting the two yields (36). □