Midterm Exam Question 2

November 1, 2022

Part A

The workspace of this manipulator can be created if we find the maximum length it can reach, which is $l_1 + l_2$ and the minimum length which is $l_1 - l_2$. The minimum length is somewhat incorrect as if you look at the manipulator the prismatic join will hit the first joint, however since i don't know the radius of the prismatic joint this is a decent approximation. So now we have a minimum and maximum length, which can be thought of as the min and max radius we can calculate the size of the workspace. This is, $d_4\pi(l_1 + l_2)^2 - d_4\pi(l_1 - l_2)^2$ which simplifying is $16\pi d_4 l_1 l_2$.

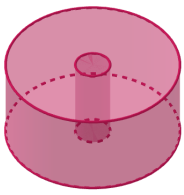

Figure 1: Workspace of our manipulator

Part B

These should be slightly different since I'm missing the height of joint 2 so what these parameters really represent is impossible in reality, but good enough for simulation.

Part C

To compute this first we take the homogeneous transformation matrix, which is this

Table 1: DH Parameters

Joints	θ_i (deg)	α_i (deg)	a_i(cm)	d_i(cm)
1	θ_1	0	60	0
2	θ_2	0	40	0
3	0	180	0	$-d_4$
4	θ_3	0	0	0

$$\begin{bmatrix} \cos\theta & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & \alpha_i\cos\theta_i \\ \sin\theta & -\sin\theta_i\cos\alpha_i & -\sin\theta_i\cos\alpha_i & \alpha_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

So we calculate this for each joint then multiply it together. This gives us our final result,

$$\begin{bmatrix} \cos\theta_1+\theta_2-\theta_3 & -\sin\theta_1+\theta_2-\theta_3 & 0 & 60\cos\theta_1+40\cos\theta_1+\theta_2 \\ \sin\theta_1+\theta_2-\theta_3 & \cos\theta_1+\theta_2-\theta_3 & 0 & 60\sin\theta_1+40\sin\theta_1+\theta_2 \\ 0 & 0 & -1 & -d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ We now use the forth column to calculate our position of $62.3, 68.6, -14$