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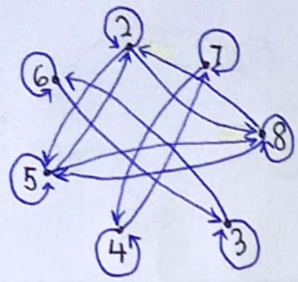
Assignment 2

1.

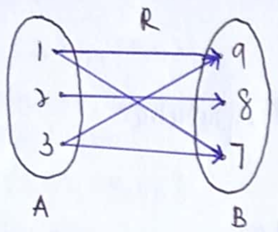


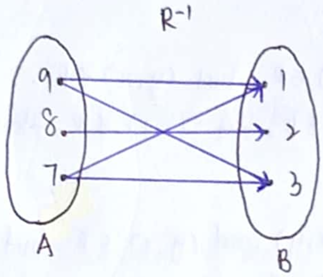
$R = \{(2,2), (2,5), (2,8), (3,3), (3,6), (4,4), (4,7), (5,2), (5,5), (5,8), (6,3), (6,6), (7,4), (7,7), (8,2), (8,5), (8,8)\}$

2. $A = \{1,2,3\}$
$B = \{9,8,7\}$

a. $R = \{(1,9), (1,7), (2,8), (3,9), (3,7)\}$
$R^{-1} = \{(9,1), (7,1), (8,2), (9,3), (7,3)\}$

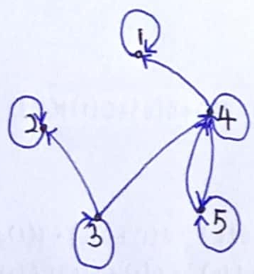
b.





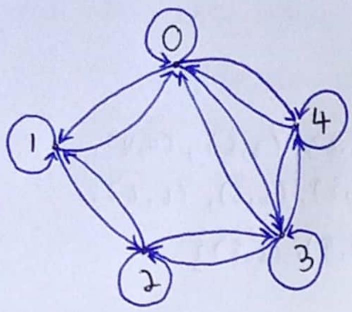
c. R^{-1} is the inverse function of R. R^{-1} : B to A, for all $(b,a) \in B \times A$, $bRa \leftrightarrow b+a = \text{even}$.

3.
$$\begin{array}{c c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$



	1	2	3	4	5
In-degree	2	2	1	3	2
Out-degree	1	1	3	3	2

4. $A = \{0,1,2,3,4\}$



R is reflexive.
Because $\forall x \in A$, $(x,x) \in R$. $(0,0)$, $(1,1)$, $(2,2)$, $(3,3)$, $(4,4)$ are each in R.

R is symmetric.
Because $\forall x,y \in A$, $(x,y) \in R \rightarrow (y,x) \in R$.
For example, $(0,1)$ and $(1,0)$ both element of R. All (x,y) and (y,x) are existed.

R is not transitive.
Because (x,y) and $(y,z) \in R$ then $(x,z) \notin R$ which $(0,1)$ and $(1,2) \in R$ but $(0,2) \notin R$.

5. $R = \{(x,y) : 3x - y = 0\}$
$R = \{(1,3), (2,6), (3,9), (4,12)\}$

a. R is not reflexive.
Because R above is $(x,y) \in R$, $x \neq y$, $x,y \in A$ but reflexive is $(x,x) \in R$, $x \in A$.
Relation above only exists $(1,3)$ but no $(1,1)$ or $(3,3)$, thus it is not reflexive.

b. R is not symmetric.
Because R above only exists $(x,y) \in R$, but $(y,x) \notin R$.
Relation above only exists $(1,3) \in R$, but $(3,1) \notin R$, thus it is not symmetric.

c. R is not transitive.
Because R above only exists (x,y) and $(y,z) \in R$ but $(x,z) \notin R$.
Relation above is $(1,3)$ and $(3,9) \in R$, but $(1,9) \notin 9$, thus it is not transitive.

6. a. RS

$$RS = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0(1)+0(0)+1(0)+1(0) & 0(0)+0(1)+1(1)+1(0) & 0(0)+0(0)+1(1)+1(1) & 0(1)+0(1)+1(0)+1(1) \\ 1(1)+1(0)+0(0)+0(0) & 1(0)+1(1)+0(1)+0(0) & 1(0)+1(0)+0(1)+0(1) & 1(1)+1(1)+0(0)+0(1) \\ 0(1)+0(0)+1(0)+1(0) & 0(0)+0(1)+1(1)+1(0) & 0(0)+0(0)+1(1)+1(1) & 0(1)+0(1)+1(0)+1(1) \\ 0(1)+0(0)+0(0)+1(0) & 0(0)+0(1)+0(1)+1(0) & 0(0)+0(0)+0(1)+1(1) & 0(1)+0(1)+0(0)+1(1) \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

b. $$SR = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1(0)+0(1)+0(0)+1(0) & 1(0)+0(1)+0(0)+1(0) & 1(1)+0(0)+0(1)+1(0) & 1(1)+0(0)+0(1)+1(1) \\ 0(0)+1(1)+0(0)+1(0) & 0(0)+1(1)+0(0)+1(0) & 0(1)+1(0)+0(1)+1(0) & 0(1)+1(0)+0(1)+1(1) \\ 0(0)+1(1)+1(0)+0(0) & 0(0)+1(1)+1(0)+0(0) & 0(1)+1(0)+1(1)+0(0) & 0(1)+1(0)+1(1)+0(1) \\ 0(0)+0(1)+1(0)+1(0) & 0(0)+0(1)+1(0)+1(0) & 0(1)+0(0)+1(1)+1(0) & 0(1)+0(0)+1(1)+1(1) \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

7. All functions are relations, but not all relations are function.

The difference between relation and function is relation can have many value assigned to a single value from domain but function can only have one value from domain assigned to one value from codomain. Besides, function must use all values from domain but relation can left the value from domain without pairing.

8. $A = \{2,3,4,5\}$

i) $\{(2,3),(3,4),(4,5),(5,2)\}$

function, because domain of R is equal to A and each element from domain at most one arrow pointing to element from codomain.

ii) $\{(2,4),(3,4),(5,4),(4,4)\}$

function, because domain of R is equal to A and each element from domain at most one arrow pointing to element from codomain.

iii) $\{(2,3),(2,4),(5,4)\}$

not a function, because domain of R is not equal to A which 3 and 4 does not assign to any value in codomain

iv) $\{(2,3),(3,5),(4,5)\}$

not a function, because domain of R is not equal to A which 5 is not assigned to any value in codomain

v) $\{(2,2),(2,3),(4,4),(4,5)\}$

not a function, because $(2,2),(2,3) \in R$, but $2 \neq 3$ which means element from domain has assigned to more than one value and domain of R also not equal to A which 3 and 5 does not assign to any value in codomain

9. $R = \{(x,y) \mid y = x+5, x \text{ is } Z^+ \text{ less than } 6\}$

$x = \{1,2,3,4,5\}$

$R = \{(1,6),(2,7),(3,8),(4,9),(5,10)\}$

domain $= \{1,2,3,4,5\}$

range $= \{6,7,8,9,10\}$

10. v) $f = \mathbb{R} \to \mathbb{R}$, $f(x) = 1-2x$

$f(1) = 1-2(1) = -1$ $\quad f(2) = 1-2(2) = -3$ $\quad f(3) = 1-2(3) = -5$

It is one-to-one function, because each element from domain only assigned to a single value.
It is onto function, because the codomain of function shows real number, thus each of them at least pair with one value as it is a linear equation.
Thus, it is bijective because it is both one-to-one and onto.

vi) $f = \mathbb{R} \to \mathbb{R}$, $f(x) = 5x^2-1$

$f(-2) = 5(-2)^2-1 = 19$ $\quad f(2) = 5(2)^2-1 = 19$

It is not one-to-one function, because it exists $f(x_1) = f(x_2)$ but $x_1 \neq x_2$.
It is not onto function, because the codomain of function is real number but the range is $[1, +\infty)$.
Thus, it is not bijective, because it is both not one-to-one and onto.

vii) $f = \mathbb{R} \to \mathbb{R}$, $f(x) = x^4$

$f(2) = 2^4 = 16$ $\quad f(-2) = (-2)^4 = 16$

It is not one-to-one function, because it exists $f(x_1) = f(x_2)$ but $x_1 \neq x_2$.
It is not onto function, because not all the value from codomain will pair with a value from domain, since $f(x) = x^4$, all the range will be positive, thus it is not include negative values which are also real number.
Thus, it is not bijective, because it is both not one-to-one and onto.

viii) $f = \mathbb{R} \to \mathbb{R}$, $f(x) = \left(\frac{x-2}{x-3}\right)$

$f(0) = \frac{0-2}{0-3} = \frac{2}{3}$ $\quad f(1) = \frac{1-2}{1-3} = \frac{-1}{-2} = \frac{1}{2}$ $\quad f(-1) = \frac{-1-2}{-1-3} = \frac{-3}{-4} = \frac{3}{4}$

$f(2) = \frac{2-2}{2-3} = 0$ $\quad f(3) = \frac{3-2}{3-3}$ (undefined)

It is a one-to-one function, because each element from domain assigned to a single value.
It is onto function, because range include negative and positive value which is the codomain of function included and each element from codomain could assigned to a value from domain.
Thus, it is bijective, because it is both one-to-one and onto.

11. ix) $f(g(x)) = f[x^2-1] = 3(x^2-1)-1 = 3x^2-4$

x	$f(g(x))$
0	$3(0)^2-4 = -4$
1	$3(1)^2-4 = -1$
2	$3(2)^2-4 = 8$
3	$3(3)^2-4 = 23$

x) $f(g(x)) = f[5x-6] = (5x-6)^2 = 25x^2-60x+36$

x	$f(g(x))$
0	$25(0)^2-60(0)+36 = 36$
1	$25(1)^2-60(1)+36 = 1$
2	$25(2)^2-60(2)+36 = 16$
3	$25(3)^2-60(3)+36 = 81$

xi) $f(g(x)) = f[x^3+1] = (x^3+1)-1 = x^3$

x	$f(g(x))$
0	$0^3 = 0$
1	$1^3 = 1$
2	$2^3 = 8$
3	$3^3 = 27$

12. (xii) $a_n = 6a_{n-1} - 9a_{n-2}$; initial conditions $a_0 = 1$ and $a_1 = 6$

$a_2 = 6a_1 - 9a_0$
$= 6(6) - 9(1)$
$= 27$

$a_3 = 6a_2 - 9a_1$
$= 6(27) - 9(6)$
$= 108$

$a_4 = 6a_3 - 9a_2$
$= 6(108) - 9(27)$
$= 405$

$a_5 = 6a_4 - 9a_3$
$= 6(405) - 9(108)$
$= 1458$

(xiii) $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$; initial conditions $a_0 = 2$, $a_1 = 5$ and $a_2 = 15$

$a_3 = 6a_2 - 11a_1 + 6a_0$
$= 6(15) - 11(5) + 6(2)$
$= 47$

$a_4 = 6a_3 - 11a_2 + 6a_1$
$= 6(47) - 11(15) + 6(5)$
$= 147$

$a_5 = 6a_4 - 11a_3 + 6a_2$
$= 6(147) - 11(47) + 6(15)$
$= 455$

(xiv) $a_n = -3a_{n-1} - 3a_{n-2} + a_{n-3}$; initial conditions $a_0 = 1$, $a_1 = -2$ and $a_2 = -1$

$a_3 = -3a_2 - 3a_1 + a_0$
$= -3(-1) - 3(-2) + 1$
$= 10$

$a_4 = -3a_3 - 3a_2 + a_1$
$= -3(10) - 3(-1) + (-2)$
$= -29$

$a_5 = -3a_4 - 3a_3 + a_2$
$= -3(-29) - 3(10) + (-1)$
$= 56$

13. $a_{n+1} = 5a_n - 3$; $a_1 = k$

k is non-zero constant

i) a_4 in terms of k

$n+1 = 4$
$n = 3$

$a_{3+1} = 5a_3 - 3$
$a_4 = 5a_3 - 3$
$= 5(25k - 18) - 3$
$a_4 = 125k - 93$

$a_{2+1} = 5a_2 - 3$
$a_3 = 5a_2 - 3$
$a_3 = 5(5k - 3) - 3$
$= 25k - 18$

$a_{1+1} = 5a_1 - 3$
$a_2 = 5a_1 - 3$
$a_2 = 5k - 3$

$\therefore a_4 = 125k - 93$

ii) $a_4 = 7$

$a_4 = 7$
$125k - 93 = 7$
$125k = 100$
$k = \frac{100}{125}$
$k = \frac{4}{5}$

$\therefore k = \frac{4}{5}$