Assignment 3

1. a) $k=2$

n = number of students

m = number of points students can receive (0 to 100 points)

$m = 101$

$$1 < \left\lceil \frac{n}{m} \right\rceil \leq 2$$

$$1 < \frac{n}{m} \qquad \frac{n}{m} \leq 2$$

$$1 < \frac{n}{101} \qquad \frac{n}{101} \leq 2$$

$$101 < n \qquad n \leq 202$$

$$\therefore 102 \leq n \leq 202$$

b) $k=6$

n = minimum number of students

m = number of letter grade (A, B, C, D or F)

$m = 5$

$$5 < \left\lceil \frac{n}{m} \right\rceil \leq 6$$

$$5 < \frac{n}{m} \qquad \frac{n}{m} \leq 6$$

$$5 < \frac{n}{5} \qquad \frac{n}{5} \leq 6$$

$$25 < n \qquad n \leq 30$$

$$\therefore n = 26$$

$P(B_1) = 0.7$ $\quad$ $P(W|B_1) = 0.2$

$P(B_2) = 0.3$ $\quad$ $P(W|B_2) = 0.4$

W= purchase extended warranty

a. $P(B_1) = 0.7$

b. $P(B_2) = 0.3$

c. $P(W|B_1) = 0.2$

d. $P(W|B_1) = \dfrac{P(W \cap B_1)}{P(B_1)}$

$$P(W \cap B_1) = P(W|B_1) \times P(B_1)$$
$$= 0.2 \times 0.7$$
$$= 0.14$$

e. $P(B_2 \cap W) = P(W \cap B_2)$

$$= P(W|B_2) \times P(B_2)$$
$$= 0.3 \times 0.4$$
$$= 0.12$$

f. $P(W) = P(W|B_1)\,P(B_1) + P(W|B_2)\,P(B_2)$

$$= 0.2(0.7) + 0.4(0.3)$$
$$= 0.14 + 0.12$$
$$= 0.26$$

g. $P(B_1|W) = \dfrac{P(W|B_1)\,P(B_1)}{P(W)}$

$$= \frac{0.2 \times 0.7}{0.26}$$

$$= \frac{7}{13}$$

3. a. Vertices

- dot that represent a nonempty set

b. Edges

- connection between vertices to perform relationship

c. Adjacent Vertices

- vertices pairs that connected by an edge

d. Incident Edge

- edges thare share a common vertex

e. Isolated vertex

- vertex that is not incident with any edge

f. Loop

- an edge that incident on a single vertex

g. Parallel Edges

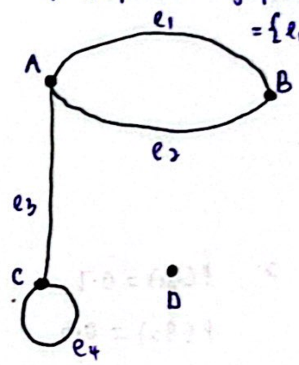
- two or more distinct edges conneited at same set of endpoints.

d. incident edges =
$= \{e_1, e_2\}, \{e_1, e_3\}, \{e_2, e_3\}$
$\{e_4\}, \{e_1, e_2, e_3\}$

e. isolated vertex = D

f. loop = e_4

g. parallel edge $= \{e_1, e_2\}$

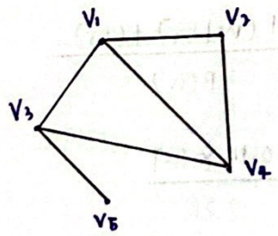


a. vertices = A, B, C, D

b. edges = e_1, e_2, e_3, e_4

c. adjacent vertices
$= \{A, B\}, \{A, C\}$

4. $G = \{V, E\}$



$d(v_1) = 3$

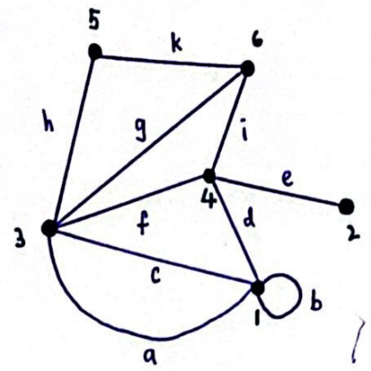
$d(v_2) = 2$

$d(v_3) = 3$

$d(v_4) = 3$

$d(v_5) = 1$

5.



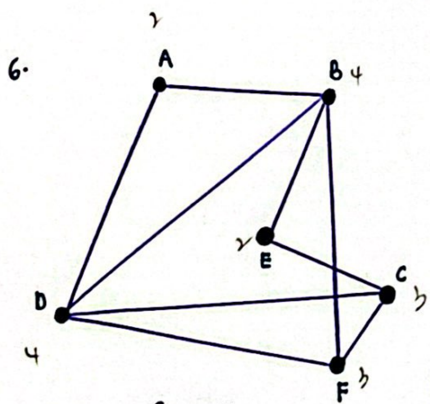
ii. adjacency matrix

$$
\begin{array}{c|cccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 \\
\hline
1 & 1 & 0 & 2 & 1 & 0 & 0 \\
2 & 0 & 0 & 0 & 1 & 0 & 0 \\
3 & 2 & 0 & 0 & 1 & 1 & 1 \\
4 & 1 & 1 & 1 & 0 & 0 & 1 \\
5 & 0 & 0 & 1 & 0 & 0 & 1 \\
6 & 0 & 0 & 1 & 1 & 1 & 0
\end{array}
$$

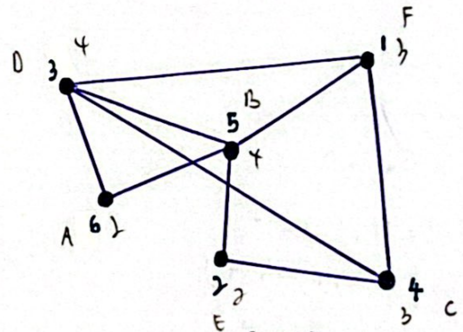
1. Incidence matrix

$$
\begin{array}{c|cccccccccc}
 & a & b & c & d & e & f & g & h & i & k \\
\hline
1 & 1 & 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
3 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\
4 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\
5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
6 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1
\end{array}
$$

6.



Graph Y



Graph Z

1. Both have 6 vertices and 9 edges
2. Both have 2 vertices with 2 degrees, 2 vertices with 3 degrees and 2 vertices with 4 degrees

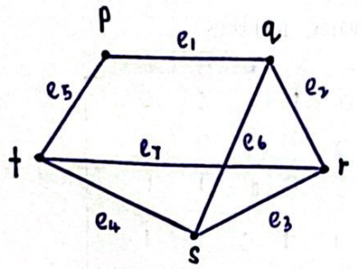
$$
A_Y = \begin{array}{c|cccccc}
 & A & B & C & D & E & F \\
\hline
A & 0 & 1 & 0 & 1 & 0 & 0 \\
B & 1 & 0 & 0 & 1 & 1 & 1 \\
C & 0 & 0 & 0 & 1 & 1 & 1 \\
D & 1 & 1 & 1 & 0 & 0 & 1 \\
E & 0 & 1 & 1 & 0 & 0 & 0 \\
F & 0 & 1 & 1 & 1 & 0 & 0
\end{array}
$$

$$
A_Z = \begin{array}{c|cccccc}
 & 6 & 5 & 4 & 3 & 2 & 1 \\
\hline
6 & 0 & 1 & 0 & 1 & 0 & 0 \\
5 & 1 & 0 & 0 & 1 & 1 & 1 \\
4 & 0 & 0 & 0 & 1 & 1 & 1 \\
3 & 1 & 1 & 1 & 0 & 0 & 1 \\
2 & 0 & 1 & 1 & 0 & 0 & 0 \\
1 & 0 & 1 & 1 & 1 & 0 & 0
\end{array}
$$

3. $f(A_{GY}) = f(6_{GZ})$
$f(B_{GY}) = f(5_{GZ})$
$f(C_{GY}) = f(4_{GZ})$
$f(D_{GY}) = f(3_{GZ})$
$f(E_{GY}) = f(2_{GZ})$
$f(F_{GY}) = f(1_{GZ})$

∴ Graph Y and Z are isomorphic.

7.



i. paths from vertex p to vertex t (no repeated vertex & edge)

= (p, e_5, t)

= $(p, e_1, q, e_6, s, e_4, t)$

= $(p, e_1, q, e_2, r, e_7, t)$

= $(p, e_1, q, e_2, r, e_3, s, e_4, t)$

= $(p, e_1, q, e_6, s, e_3, r, e_7, t)$

ii. trails from vertex p to vertex t. (no repeated edge)

= (p, e_5, t)

= $(p, e_1, q, e_6, s, e_4, t)$

= $(p, e_1, q, e_2, r, e_7, t)$

= $(p, e_1, q, e_2, r, e_3, s, e_4, t)$

= $(p, e_1, q, e_6, s, e_3, r, e_7, t)$

= $(p, e_5, t, e_7, r, e_3, s, e_4, t)$

= $(p, e_5, t, e_4, s, e_3, r, e_7, t)$

= $(p, e_5, t, e_4, s, e_6, q, e_2, r, e_7, t)$

iii. path = shortest = (p, e_5, t)

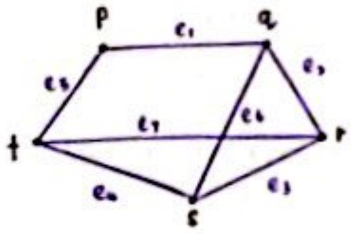
longest = $(p, e_1, q, e_2, r, e_3, s, e_4, t)$

= $(p, e_1, q, e_6, s, e_3, r, e_7, t)$

iv. trail = shortest = (p, e_5, t)

longest = $(p, e_5, t, e_4, s, e_6, q, e_2, r, e_7, t)$

7.



i. paths from vertex p to vertex t (no repeated vertex & edge)

$= (p, e_5, t)$

$= (p, e_1, q, e_6, s, e_4, t)$

$= (p, e_1, q, e_2, r, e_7, t)$

$= (p, e_1, q, e_2, r, e_3, s, e_4, t)$

$= (p, e_1, q, e_6, s, e_3, r, e_7, t)$

ii. trails from vertex p to vertex t. (no repeated edge)

$= (p, e_5, t)$

$= (p, e_5, t, e_7, r, e_2, q, e_6, s, e_4, t)$

$= (p, e_1, q, e_6, s, e_4, t)$

$= (p, e_1, q, e_2, r, e_7, t)$

$= (p, e_1, q, e_2, r, e_3, s, e_4, t)$

$= (p, e_1, q, e_6, s, e_3, r, e_7, t)$

$= (p, e_5, t, e_7, r, e_3, s, e_4, t)$

$= (p, e_5, t, e_4, s, e_3, r, e_7, t)$

$= (p, e_5, t, e_4, s, e_6, q, e_2, r, e_7, t)$

iii. path - shortest $= (p, e_5, t)$

longest $= (p, e_1, q, e_2, r, e_3, s, e_4, t)$

$= (p, e_1, q, e_6, s, e_3, r, e_7, t)$

iv. trail - shortest $= (p, e_5, t)$

longest $= (p, e_5, t, e_4, s, e_6, q, e_2, r, e_7, t)$

$= (p, e_5, t, e_7, r, e_2, q, e_6, s, e_4, t)$