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Collaborators : None

1) a) $A = \sum_{x=1}^{n}\sum_{y=1}^{m} b(x,y) = 15$

$$\bar{x} = \frac{1}{A}\sum_{x=1}^{n}\sum_{y=1}^{m} x\, b(x,y) =$$

$$= \frac{4(2)+5(2)+6(3)+7(1)+8(3)+9(1)+10(3)}{15}$$

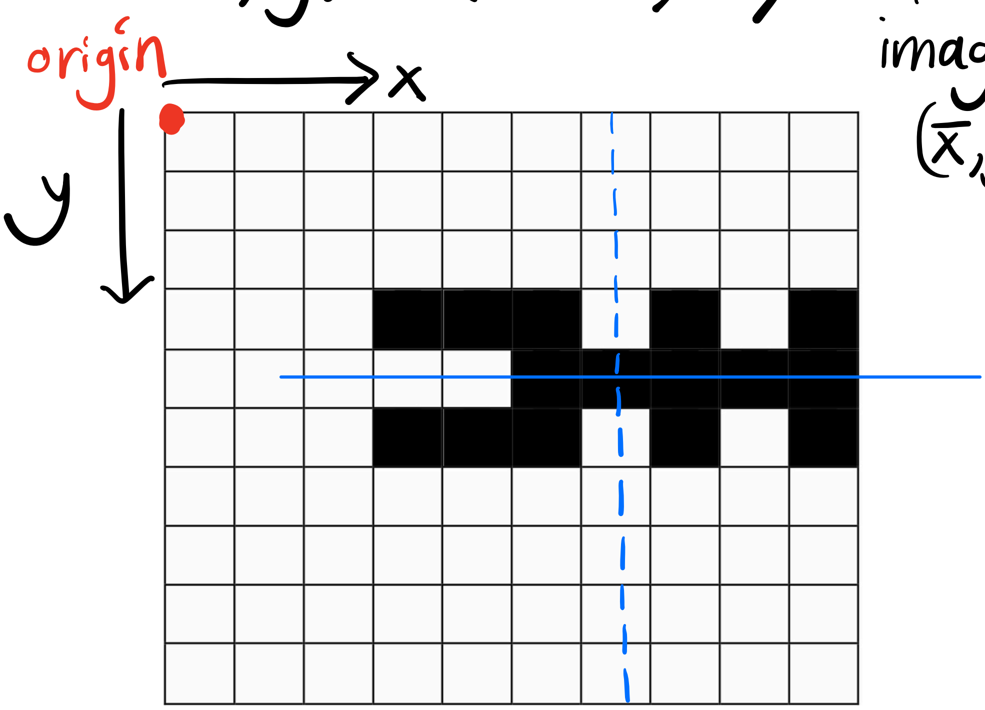
$$= \frac{106}{15} = 7.067$$

$$\bar{y} = \frac{1}{A}\sum_{x=1}^{n}\sum_{y=1}^{m} y\, b(x,y) = \frac{4(5)+5(5)+6(5)}{15} = \frac{75}{15} = 5$$

$$(\bar{x},\bar{y}) = (7.067, 5)$$

*if you want to index the image (take floor) $(\bar{x},\bar{y}) = (7,5)$

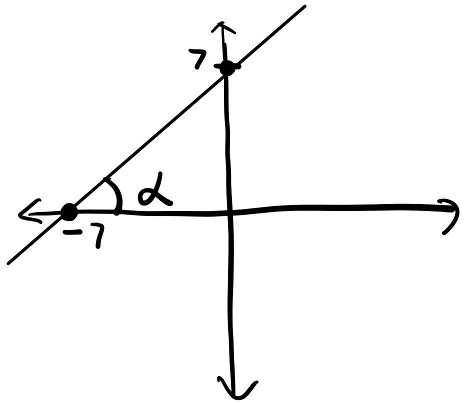
b)



2)

$$(-1,1)^T \vec{x} - 7 = 0$$

a)



b)

$$-x + y - 7 = 0$$

$$\tan\alpha = \frac{y}{x} = \frac{7}{-7} = -1$$

$$\tan^{-1}(-1) = \frac{3\pi}{4}$$

$$x \sin\frac{3\pi}{4} - y\cos\frac{3\pi}{4} + g = 0$$

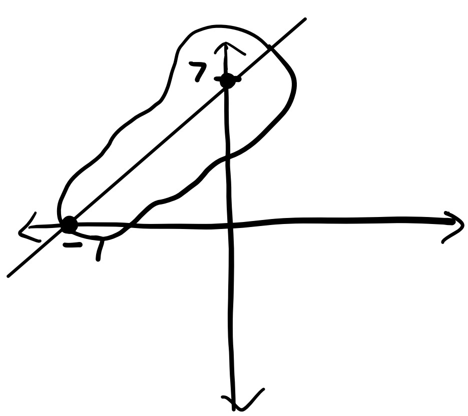
$$0\left(\frac{\sqrt{2}}{2}\right) - 7\left(-\frac{\sqrt{2}}{2}\right) + g = 0$$

$$\frac{7\sqrt{2}}{2} + g = 0$$

$$g = -\frac{7\sqrt{2}}{2}$$

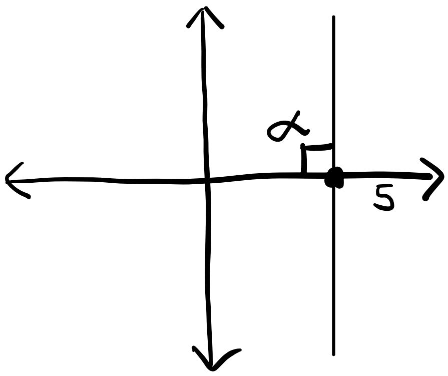
$$\boxed{x\sin\frac{3\pi}{4} - y\cos\frac{3\pi}{4} - \frac{7\sqrt{2}}{2} = 0}$$

c)



$(1,0)^T \vec{x} - 5 = 0 \qquad x - 5 = 0$

a)



b)

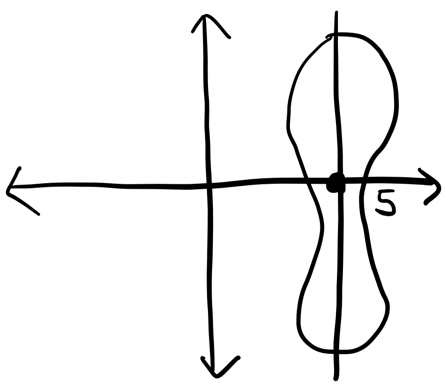
$$x \sin\frac{\pi}{2} - y\cos\frac{\pi}{2} + g = 0$$

$$5 + g = 0$$

$$g = -5$$

$$\boxed{x \sin\frac{\pi}{2} - y\cos\frac{\pi}{2} - 5 = 0}$$

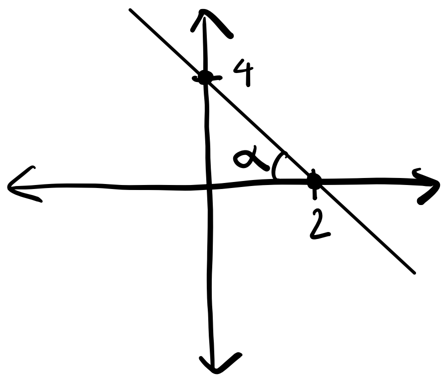
c)



$(2,1)^T\vec{x} - 4 = 0$

$2x + y - 4 = 0$

a)



b) $\tan\alpha = \frac{y}{x} = \frac{4}{2} = 2$

$$\tan^{-1}(2) = 63.435^\circ$$

$$x\sin(63.435^\circ) - y\cos(63.435^\circ) + g = 0$$

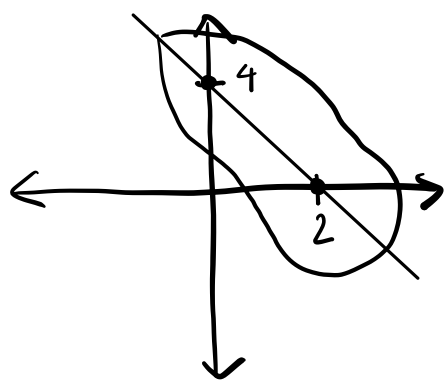
$$(2)\sin(63.435^\circ) - (0)\cos(63.435^\circ) + g = 0$$

$$1.789 + g = 0$$

$$g = -1.789$$

$$\boxed{x\sin(63.435^\circ) - y\cos(63.435^\circ) - 1.789 = 0}$$

c)



3)

a) circularity ratio $= \dfrac{E_{min}}{E_{max}}$

$$E = \sum_{i=1}^{n}\sum_{j=1}^{m} (x'\sin\theta - y'\cos\theta)^2 b(x',y')\,dx'dy'$$

where $x' = x - \bar{x}$
$y' = y - \bar{y}$

b) smaller, since sickle cells are more elongated

c) Object circularity can be used to detect when a person opens and closes their eyes since when a person blinks the ratio will be close to 0 with some threshold and open otherwise.

4)

a) $E = \dfrac{a+c}{2} \pm \dfrac{\sqrt{b^2+(a-c)^2}}{2}$

The negative side of the $\pm$ corresponds to the minimum second moment

$\therefore$ if we show $E_{min} \geq 0$ we have proven $E \geq 0$

$$\frac{a+c}{2} \geq \frac{\sqrt{b^2+(a-c)^2}}{2}$$

$$a+c \geq \sqrt{b^2+(a-c)^2}$$

$$(a+c)^2 \geq b^2+(a-c)^2$$

$$(a+c)(a+c)-(a-c)(a-c) \geq b^2$$

$$\cancel{a^2}+2ac+\cancel{c^2}-(\cancel{a^2}-2ac+\cancel{c^2}) \geq b^2$$

$$4ac \geq b^2$$

$$\cancel{4}\left[\left(\sum_{i=1}^{n}\sum_{j=1}^{m} i^2 b(i,j)\right)\left(\sum_{i=1}^{n}\sum_{j=1}^{m} j^2 b(i,j)\right) \geq \left(\cancel{2}\sum_{i=1}^{n}\sum_{j=1}^{m} ij\, b(i,j)\right)^2\right.$$

we can reduce this to the Cauchy-Schwarz inequality

$\therefore\ E \geq 0$

b) $E=0$ when $E_{min}=0$ or the object is a line.

5) we know:

$$E = \iint_I r^2 b(x,y)\, dx\, dy$$

$$E = \iint_I (x\sin\alpha - y\cos\alpha + \rho)^2 b(x,y)\, dx\, dy$$

$$\frac{\partial E}{\partial \rho} = \iint_I 2(x\sin\alpha - y\cos\alpha + \rho)\, b(x,y)\, dx\, dy$$

To find min (least second moment) we set $\frac{\partial E}{\partial \rho} = 0$

$$\iint_I 2(x\sin\alpha - y\cos\alpha + \rho)\, b(x,y)\, dx\, dy = 0$$

$$\iint_I x\sin\alpha\, b(x,y)\, dx\, dy - \iint_I y\cos\alpha\, b(x,y)\, dx\, dy + \iint_I \rho\, b(x,y)\, dx\, dy = 0$$

Divide and multiply by $A = \iint_I b(x,y)\, dx\, dy$

$$A\left(\frac{1}{A}\iint x\sin\alpha\, b(x,y)\, dx\, dy - \frac{1}{A}\iint_I y\cos\alpha\, b(x,y)\, dx\, dy \right.$$

$$+\frac{1}{A}\iint_{F} p\,b(x,y)\,dx\,dy\Big)\Big) =$$

$$A(\bar{x}\sin\alpha - \bar{y}\cos\alpha + p) = 0$$

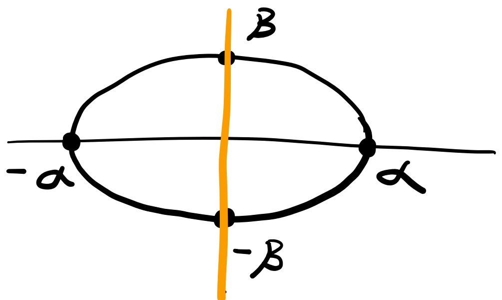
$\therefore$ $\bar{x}$ and $\bar{y}$

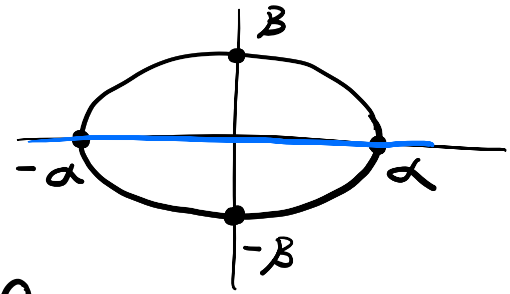
6)

$$\left(\frac{x}{\alpha}\right)^2 + \left(\frac{y}{\beta}\right)^2 = 1$$

major minor

a)





$x = \alpha r\cos\theta$ $\quad y = \beta r\sin\theta$

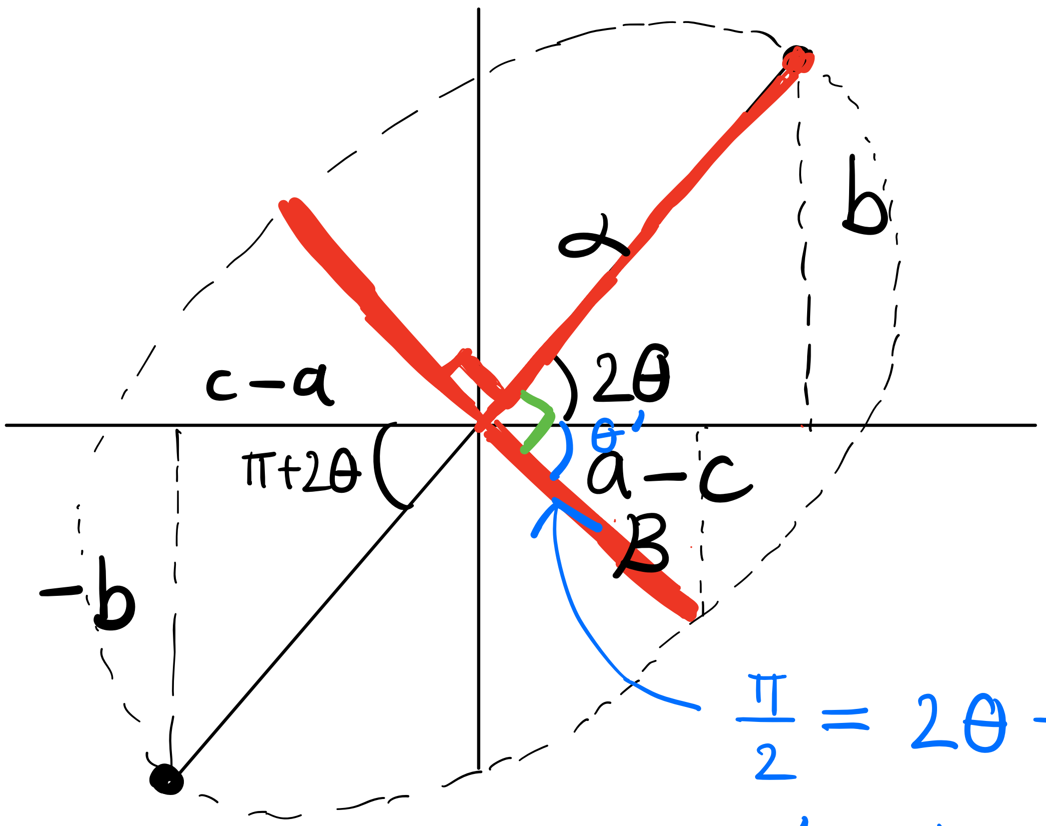
$$E_{min} = \int_0^{2\pi}\int_0^1 (\beta r\sin(\theta))^2\,\alpha\beta\,r\,dr\,d\theta$$

$$= \frac{\pi\alpha\beta^3}{4}$$

$$E_{max} = \int_0^{2\pi}\int_0^1 (\alpha r\cos(\theta))^2\,\alpha\beta\,r\,dr\,d\theta$$

$$= \frac{\pi\alpha^3\beta}{4}$$

b)



$$\frac{\pi}{2} = 2\theta + \alpha$$

$$\theta' = \frac{\pi}{2} - 2\theta$$

$$\cos\left(\frac{\pi}{2} - 2\theta\right) = \frac{a-c}{\beta}$$

$$\boxed{\begin{aligned} \beta &= \frac{a-c}{\cos\left(\frac{\pi}{2}-2\theta\right)} \\ \alpha &= \sqrt{b^2 + (a-c)^2} \end{aligned}}$$