Algoritmi si Structuri de Date

October 3, 2012

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"Algorithms + Data Structures = Programs"

- Niklaus Wirth, 1976

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- Masura a costurilor
  - timp (viteza)
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2 Mari Probleme

Sortarea

- **Input** Un vector $(a_1, a_2, \cdots, a_n)$
- **Output** $(a'_1, a'_2, \cdots, a'_n)$ a.i. $a'_1 \leq a'_2 \leq \cdots \leq a'_n$

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Cautarea

- **Input** O 'multime' $\{a_1, a_2, \cdots, a_n\}$ si o 'valoare' x
- **Output** Este $x \in \{a_1, a_2, \cdots, a_n\}$ **?** **Yes** sau **No**

Sortarea

Sortarea prin insertie

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Sortarea prin insertie (directa)

```
proc InsDir (A)
/sortare prin insertie directa a vectorului A[1..n]
for i = 2 to n do
    x := A[i]
    /se caută locul valorii x în destinație
    j := i − 1
    while (j > 0) and (x < A[j]) do
        A[j + 1] := A[j]
        j := j − 1
    endwhile
    /inserarea lui x la locul lui
    A[j + 1] := x
endfor
endproc
```

Sortarea prin insertie

Comparatii

- $C_{min} = 1 + 1 + \cdots + 1 = n - 1$
- $C_{max} = 2 + 3 + \cdots + n = n \cdot (n + 1)/2 - 1 = (1/2)n^2 + (1/2)n - 1$
- $C_{mediu} = (1/2)(2+3+\cdots+n) = (1/2)C_{max} = (1/4)n^2 + (1/4)n - 1/2$

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Mutari

- $M_{min} = 2 + 2 + \cdots + 2 = 2(n - 1)$
- $M_{max} = 3 + 4 + \cdots + (n + 1) = (n + 1) \cdot (n + 2)/2 - 3 = (1/2)n^2 + (3/2)n - 2$
- $M_{mediu} = C_{mediu} + 2(n - 1) = (1/4)n^2 + (9/4)n - 5/2$

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- la pasul/iteratia i, avem $M_i = C_i + 1$.

Performanta

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- Exista un timp **minim** de rulare
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- Comparatii + Mutari
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- Fiecare linie de cod - in timp cst...

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Timp de rulare $T(n)$ in cazul c. m. nefavorabil

- $T(n) = aC_{max} + b$ (exista cst $a, b > 0$)
- $T(n) = \Theta(n^2)$

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$$\mathcal{O}(g) := \{f \mid \exists c > 0, \exists n_0 \text{ a.i. } 0 \leq f(n) \leq cg(n) \text{ for all } n \geq n_0\}$$

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Properties:

$\Theta(g) = \mathcal{O}(g) \cap \Omega(g)$

Tranzitivitate

$f = \Theta(g), g = \Theta(h) \Longrightarrow f = \Theta(h)$
$f = \mathcal{O}(g), g = \mathcal{O}(h) \Longrightarrow f = \mathcal{O}(h)$
$f = \Omega(g), g = \Omega(h) \Longrightarrow f = \Omega(h)$

Reflexivitate

$f = \Theta(f)$
$f = \mathcal{O}(f)$
$f = \Omega(f)$

Simetrie

$f = \Theta(g)$ daca si numai daca $g = \Theta(f)$

Simetrie transpusa

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    if $n = 1$ sortat
    MergeSort(A, 1, n/2)
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MergeSort - ecuatia de recurenta pentru $T(n)$

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MergeSort - $T(n)$

- $T(n) = \Theta(nlogn)$

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- n 'mare' ?
- pentru o aplicatie concreta, ce domeniu de valori are n?