Duration Immunization of a 5-Year Liability

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Executive Summary

Objective: Match the present value and interest-rate sensitivity of a $10,000 liability due in 5 years using two coupon bonds.

Approach: Price each bond at base yield i, compute Macaulay duration, solve allocations to match PV and duration (T years), and test ±100 bps shocks.

Key results: PV(Liability) = $8219.27, allocations A1 = $2634.74, A2 = $5584.53, portfolio duration ≈ 5, Asset−Liability @ +100 bps = $1.60, @ −100 bps = $1.79.

Conclusion: The two-bond mix immunizes small rate moves. Residual differences reflect convexity.

Set-up & Data

• Liability: L = $10000, term T = 5 years, base yield i = 0.04.

• Bonds:

  – Bond 1: Face $100, coupon rate 0.03, maturity 3 years.

  – Bond 2: Face $100, coupon rate 0.06, maturity 7 years.

• Shocks: ±100 bps around i.

• Assumption: Annual coupons, flat yield for discounting.

Method

Pricing: $P = \Sigma v^t CF_t$

Macaulay duration: $D = \frac{\Sigma t v^t CF_t}{\Sigma v^t CF_t}$

Immunization conditions:

  1) PV match: $A_1 + A_2 = PV_L$

  2) Duration match: $T = \frac{(A_1 \cdot D_1 + A_2 \cdot D_2)}{(A_1 + A_2)}$

Closed-form allocations: $A_1 = \frac{PV_L \cdot (T - D_2)}{(D_1 - D_2)}$, $A_2 = PV_L - A_1$

Results

Table 1 — Bond measures @ i

Bond	Price	Macaulay Duration
3y, 3%	$97.22	2.91
7y, 6%	$112.00	5.99

Table 2 — Allocation & checks (base)

PV(Liability)	$8219.27
A1 (Bond 1 dollars)	$2634.74
A2 (Bond 2 dollars)	$5584.53
Portfolio PV (A1 + A2)	≈ $8219.27
Portfolio Duration	≈ 5.00
Notes	Matches PV and duration. Small residuals are expected due to convexity.

Table 3 — Asset vs. Liability under ± shock

Scenario	Asset PV	Liability PV	Asset − Liability
i + 100 bps	$7836.86	$7835.26	+1.60
i − 100 bps	$8627.88	$8626.09	+1.79

Interpretation: ~$1–$2 residuals on ~$8.2k PV are minor and reflect convexity. Duration matching handles small parallel moves well.

Limitations & Next Steps

• Duration is a first-order (linear) measure, larger shocks or non-parallel moves will create mismatches.

• Flat curve assumption. A term structure would refine pricing.

• Next steps: add a third bond to increase convexity, test non-parallel shifts, and compare modified vs. Macaulay duration.