ENPM 667

CONTROL OF ROBOTIC SYSTEMS

FINAL PROJECT

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UNDER THE GUIDANCE OF **PROF. WASEEM MALIK**

Q1. Consider a crane that moves along a one-dimensional track. It behaves as a frictionless cart with mass M actuated by an external force F that constitutes the input of the system. There are two loads suspended from cables attached to the crane. The loads have mass m1 and m2, and the lengths of the cables are l1 and l2, respectively. The following figure depicts the crane and associated variables used throughout this project.

Given:

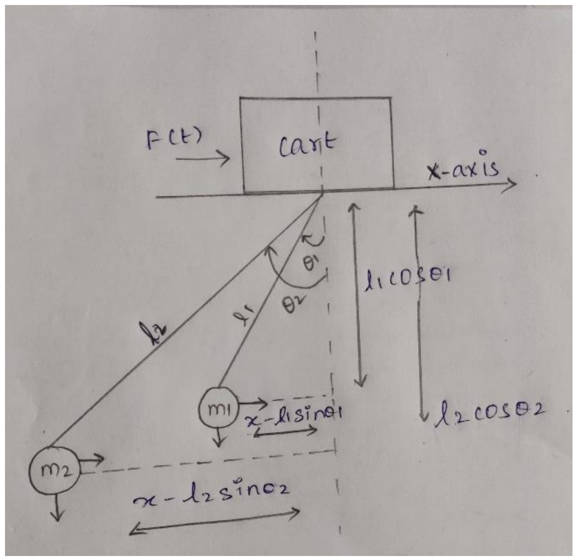


Fig 1 : Free body diagram

Question A: Obtain the equations of motion for the system and the corresponding nonlinear state-space representation.

Solution A:

Kinematics:

The distance travelled by the center of mass of the pendulum due to the angular rotation in the pendulum is $l_1\, sin\, \theta_1\ and\ l_2 \sin \theta_2$.

Similarly, the vertical distance travelled by the pendulum is given by $l_1\, cos\, \theta_1\ and\ l_2 \cos \theta_2$. Now,

$$Xm_1 = x - l_1\, sin\, \theta_1\ , \dot{X}m_1 = \dot{x} - l_1\dot{\theta}_1\, cos\, \theta_1$$

$$Xm_2 = x - l_2\, sin\, \theta_2,\ \dot{X}m_2 = \dot{x} - l_2\dot{\theta}_2\, cos\, \theta_2$$

$$y_{m1} = l_1\, cos\, \theta_1\ , \dot{y}_{m1} = -l_1\dot{\theta}_1\, sin\, \theta_1\ ; y_{m2} = l_2\, cos\, \theta_2, \dot{y}_{m2} = -l_2\dot{\theta}_2\, sin\, \theta_2$$

Potential energy:

The potential energy is given by V = mgY_m

The cart is along the flat surface, so the potential energy won't change. Therefore,

For mass 1 => $V_1 = m_1 g_1 l_1 \cos\theta_1$ and for mass 2 => $V_2 = m_2 g_2 l_2 \cos\theta_2$. The net potential energy is given as

$$V = -m_1 g_1 l_1 \cos\theta_1 - m_2 g_2 l_2 \cos\theta_2$$

Kinetic energy:

The kinetic energy is given by T = $\frac{1}{2}mv^2$, where v is the net velocity of the system.

$T = \frac{1}{2}M\dot{x}^2 + \frac{1}{2}M_1(\dot{x}_{m1}^2 + \dot{y}_{m1}^2) + \frac{1}{2}M_2(\dot{x}_{m2}^2 + \dot{y}_{m2}^2)$

$T = \frac{1}{2}M\dot{x}^2 + \frac{1}{2}m_1\left(\dot{x} - l_1\dot{\theta}_1 \cos\theta_1\right)^2 + \frac{1}{2}m_1\left(-l_1\dot{\theta}_1 \sin\theta_1\right)^2 + \frac{1}{2}m_2\left(\dot{x} - l_2\dot{\theta}_2 \cos\theta_2\right)^2 + \frac{1}{2}m_2\left(-l_2\dot{\theta}_2 \sin\theta_2\right)^2$

Langrage's equation:

According to langrage's equation,

$$\frac{\partial}{dt}\left(\frac{dL}{\partial\dot{x}}\right) - \left(\frac{dL}{dx}\right) = F$$

L = Kinetic Energy – Potential Energy => L = T-V

Therefore,

$L = \frac{1}{2}M\dot{x}^2 + \frac{1}{2}m_1\left(\dot{x} - l_1\dot{\theta}_1 \cos\theta_1\right)^2 + \frac{1}{2}m_1\left(-l_1\dot{\theta}_1 \sin\theta_1\right)^2 + \frac{1}{2}m_2\left(\dot{x} - l_2\dot{\theta}_2 \cos\theta_2\right)^2 + \frac{1}{2}m_2\left(-l_2\dot{\theta}_2 \sin\theta_2\right)^2 - (-m_1 g_1 l_1 \cos\theta_1 - m_2 g_2 l_2 \cos\theta_2)$

$L = = \frac{1}{2}M\dot{x}^2 + \frac{1}{2}m_1\left(\dot{x} - l_1\dot{\theta}_1 \cos\theta_1\right)^2 + \frac{1}{2}m_1\left(-l_1\dot{\theta}_1 \sin\theta_1\right)^2 + \frac{1}{2}m_2\left(\dot{x} - l_2\dot{\theta}_2 \cos\theta_2\right)^2 + \frac{1}{2}m_2\left(-l_2\dot{\theta}_2 \sin\theta_2\right)^2 + m_1 g_1 l_1 \cos\theta_1 + m_2 g_2 l_2 \cos\theta_2$

$$\frac{dL}{\partial\dot{x}} = M\dot{x} + m_1\dot{x} - m_1 l_1\dot{\theta}_1 \cos\theta_1 + m_2\dot{x} - m_2 l_2\dot{\theta}_2 \cos\theta_2$$

⇨ $(M + m_1 + m_2)\,\dot{x} - m_1 l_1\dot{\theta}_1 \cos\theta_1 - m_2 l_2\dot{\theta}_2 \cos\theta_2$

$$\frac{dL}{dx} = 0$$

$\frac{\partial}{dt}\left(\frac{dL}{\partial \dot{x}}\right) - 0 \Rightarrow (M + m_1 + m_2)\ddot{x} - m_1 l_1 \cos\theta_1 \ddot{\theta}_1 + m_1 l_1 \sin\theta_1 \dot{\theta}_1^2 - m_2 l_2 \cos\theta_2 \ddot{\theta}_2 + m_2 l_2 \sin\theta_2 \dot{\theta}_2^2 = \mathrm{F}$

For θ_1:

$$\frac{dL}{d\theta_1} = -m_1 l_1 \dot{x}\cos\theta_1 + m_1 l_1^2 \dot{\theta}_1$$

$$\frac{d}{dt}\left(\frac{dl}{d\dot{\theta}_1}\right) = -m_1 l_1 \ddot{x}\cos\theta_1 + m_1 l_1 \dot{x}\sin\theta_1 \ddot{\theta}_1 + m_1 l_1^2 \ddot{\theta}_1$$

$$\frac{dl}{d\theta_1} = m_1\left(\dot{x} - l_1\dot{\theta}_1 \cos\theta_1\right)(-l_1 \cos\theta_1) + m_1(l_1\dot{\theta}_1 \sin\theta_1)(l_1 \sin\theta_1)$$

$$\frac{d}{dt}\left(\frac{dl}{d\dot{\theta}_1}\right) - \frac{dl}{d\theta_1} \Rightarrow m_1 l_1 \dot{x}\sin\theta_1 \ddot{\theta}_1 - m_1 l_1 \ddot{x}\cos\theta_1 + m_1 l_1^2 \ddot{\theta}_1 - m_1 l_1^2 \dot{\theta}_1 + m_1 l_1 \dot{x}\cos\theta_1 = 0$$

For θ_2:

$$\frac{dL}{d\theta_2} = -m_2 l_2 \dot{x}\cos\theta_2 + m_2 l_2^2 \dot{\theta}_2$$

$$\frac{d}{dt}\left(\frac{dl}{d\dot{\theta}_2}\right) = -m_2 l_2 \ddot{x}\cos\theta_2 + m_2 l_2 \dot{x}\sin\theta_2 \ddot{\theta}_2 + m_2 l_2^2 \ddot{\theta}_2$$

$$\frac{dl}{d\theta_1} = m_1\left(\dot{x} - l_1\dot{\theta}_1 \cos\theta_1\right)(-l_1 \cos\theta_1) + m_1\left(l_1\dot{\theta}_1 \sin\theta_1\right)(l_1 \sin\theta_1)$$

$$\frac{d}{dt}\left(\frac{dl}{d\dot{\theta}_2}\right) - \frac{dl}{d\theta_2} \Rightarrow m_2 l_2 \dot{x}\sin\theta_2 \ddot{\theta}_2 - m_2 l_2 \ddot{x}\cos\theta_2 + m_2 l_2^2 \ddot{\theta}_2 - m_2 l_2^2 \dot{\theta}_2 + m_2 l_2 \dot{x}\cos\theta_2 = 0$$

The equations are as follows:

$$l_1\ddot{\theta}_1 = \ddot{x}\cos\theta_1 - g\sin\theta_1$$

$$l_2\ddot{\theta}_2 = \ddot{x}\cos\theta_2 - g\sin\theta_2$$

Substituting the values in this equation $(M + m_1 + m_2)\ddot{x} - m_1 l_1 \cos\theta_1 \ddot{\theta}_1 + m_1 l_1 \sin\theta_1 \dot{\theta}_1^2 - m_2 l_2 \cos\theta_2 \ddot{\theta}_2 + m_2 l_2 \sin\theta_2 \dot{\theta}_2^2 = \mathrm{F}$

We get,

$$\ddot{x} = \frac{F - m_1 l_1 \dot{\theta}_1^2 \sin\theta_1 - m_2 l_2 \dot{\theta}_2^2 \sin\theta_2 - m_1 g \cos\theta_1 \sin\theta_1 - m_2 g \cos\theta_2 \sin\theta_2}{M + m_2\, sin\, \theta_2^2 + m_1\, sin\, \theta_1^2}$$

From the above equations we got the states as,

$$\begin{bmatrix} \dot{x}(t) \\ \ddot{x}(t) \\ \dot{\theta}_1(t) \\ \ddot{\theta}_1(t) \\ \dot{\theta}_2(t) \\ \ddot{\theta}_2(t) \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \frac{F - m_1 l_1 \dot{\theta}_1^2 \sin\theta_1 - m_2 l_2 \dot{\theta}_2^2 \sin\theta_2 - m_1 g \cos\theta_1 \sin\theta_1 - m_2 g \cos\theta_2 \sin\theta_2}{M + m_2\, sin\, \theta_2^2 + m_1\, sin\, \theta_1^2} \\ \dot{\theta}_1 \\ \frac{\ddot{x}\cos\theta_1 - g\sin\theta_1}{l_1} \\ \dot{\theta}_2 \\ \frac{\ddot{x}\cos\theta_2 - g\sin\theta_2}{l_2} \end{bmatrix}$$

Question B:

Obtain the linearized system around the equilibrium point specified by x = 0 and $\theta_1 = \theta_2 = 0$. Write the state-space representation of the linearized system.

Solution B:

Linearization of the system:

According to the given condition we can state that $sin\, \theta_1 = \theta_1$, $sin\, \theta_2 = \theta_2$, $\cos\theta_1 = 1, \cos\theta_2 = 1\,, \dot{\theta}_1^2 = \dot{\theta}_2^2 = 0$

Applying the conditions we get,

$$\ddot{x}(t) = \frac{F - 0 - 0 - m_1 g \theta_1(1) - m_2 g \theta_2(1)}{M}$$

$$\ddot{x}(t) = \frac{F - m_1 g \theta_1 - m_2 g \theta_2}{M}$$

$$\ddot{\theta}_1(t) = \frac{F - m_1 g \theta_1 - m_2 g \theta_2 - M g \theta_1}{Ml1}$$

$$\ddot{\theta}_2(t) = \frac{F - m_1 g \theta_1 - m_2 g \theta_2 - M g \theta_2}{Ml2}$$

Linear state space representation:

The general state space equation of the system is given as

$$\dot{X} = AX + BU$$

The Jacobian matrix for the linearized points for the states $x,\ \dot{x},\ \theta_1,\ \dot{\theta}_1,\ \theta_2, \dot{\theta}_2$ can be calculated as,

$$\begin{bmatrix} \frac{dF_1}{\partial x} & \frac{dF_1}{\partial \dot{x}} & \frac{dF_1}{\partial \theta_1} & \frac{dF_1}{\partial \dot{\theta}_1} & \frac{dF_1}{\partial \theta_2} & \frac{dF_1}{\partial \dot{\theta}_2} \\ \frac{dF_2}{\partial x} & \frac{dF_2}{\partial \dot{x}} & \frac{dF_2}{\partial \theta_1} & \frac{dF_2}{\partial \dot{\theta}_1} & \frac{dF_2}{\partial \theta_2} & \frac{dF_2}{\partial \dot{\theta}_2} \\ \frac{dF_3}{\partial x} & \frac{dF_3}{\partial \dot{x}} & \frac{dF_3}{\partial \theta_1} & \frac{dF_3}{\partial \dot{\theta}_1} & \frac{dF_3}{\partial \theta_2} & \frac{dF_3}{\partial \dot{\theta}_2} \\ \frac{dF_4}{\partial x} & \frac{dF_4}{\partial \dot{x}} & \frac{dF_4}{\partial \theta_1} & \frac{dF_4}{\partial \dot{\theta}_1} & \frac{dF_4}{\partial \theta_2} & \frac{dF_4}{\partial \dot{\theta}_2} \\ \frac{dF_5}{\partial x} & \frac{dF_5}{\partial \dot{x}} & \frac{dF_5}{\partial \theta_1} & \frac{dF_5}{\partial \dot{\theta}_1} & \frac{dF_5}{\partial \theta_2} & \frac{dF_5}{\partial \dot{\theta}_2} \\ \frac{dF_6}{\partial x} & \frac{dF_6}{\partial \dot{x}} & \frac{dF_6}{\partial \theta_1} & \frac{dF_6}{\partial \dot{\theta}_1} & \frac{dF_6}{\partial \theta_2} & \frac{dF_6}{\partial \dot{\theta}_2} \end{bmatrix}$$

Now, the value of A and B matrix is given as,

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-gm_1}{M} & 0 & \frac{-gm_2}{M} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{-M_g-m_1g}{Ml_1} & 0 & \frac{-gm_2}{Ml_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{-gm_1}{Ml_2} & 0 & \frac{-Mg-m_2g}{Ml_2} & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ \frac{1}{M} \\ 0 \\ \frac{1}{Ml_1} \\ 0 \\ \frac{1}{Ml_2} \end{bmatrix}$$

Writing in the general state space format we get,

$$\begin{bmatrix} \dot{x}(t) \\ \ddot{x}(t) \\ \dot{\theta}_1(t) \\ \ddot{\theta}_1(t) \\ \dot{\theta}_2(t) \\ \ddot{\theta}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-gm_1}{M} & 0 & \frac{-gm_2}{M} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{-M_g - m_1 g}{Ml_1} & 0 & \frac{-gm_2}{Ml_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{-gm_1}{Ml_2} & 0 & \frac{-Mg - m_2 g}{Ml_2} & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\theta}_1 \\ \ddot{\theta}_1 \\ \dot{\theta}_2 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{M} \\ 0 \\ \frac{1}{Ml_1} \\ 0 \\ \frac{1}{Ml_2} \end{bmatrix} u$$

Similarly, the general output equation is given as,$Y = CX + DU$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } D = 0$$

Question C:

Obtain conditions on M, m1, m2, l1, l2 for which the linearized system is controllable.

Solution:

Condition for controllability,

Qc = $[B \;\; AB \;\; A^2B \;\; A^3B \;\; A^4B \;\; A^5B \;]$

To check if the system is controllable or not, we can take the determinant of Qc and check if its equal to or not. We calculated this matrix equation in MATLAB. The determinant of Qc is given as

$$|Qc| = \frac{-(g^6(l_1 - l_2)^2)}{(Ml_1 l_2)^6}$$

This above equation gives us the condition that $l_1 \neq 0$, $l_2 \neq 0, l_1 \neq l_2$

Question D:

Choose M = 1000Kg, m1=m2= 100Kg, l1 = 20m, l2 = 10m. Check that the system is controllable and obtain an LQR controller. Simulate the resulting response to initial conditions when the controller is applied to the linearized system and also to the original nonlinear system. Adjust the parameters of the LQR cost until you obtain a suitable response. Use Lyapunov's indirect method to certify stability (locally or globally) of the closed- loop system.

Solution:

1. *Checking for controllability:*

Given that M = 1000Kg, m1=m2= 100Kg, l1 = 20m, l2 = 10m, substituting the values we get,

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.98 & 0 & -0.98 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -0.539 & 0 & -0.049 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -0.098 & 0 & -1.078 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0.001 \\ 0 \\ 0.00005 \\ 0 \\ 0.0001 \end{bmatrix}$$

To check if the system is controllable or not, we are again using the controllability formula and check the determinant. MATLAB calculations shows that the system is **controllable**.

Here the system is both controllable and oscillating in the open loop. This is because the eigen values of the open loop system is not real and hence the system will be oscillating.

To make the system stable we are going to use the full state feedback controller. The FSFC we are using here is LQR controller.

This Linear Quadratic Regulator is an optimal control regulator.

In order to design the LQR controller we need to follow the following steps:

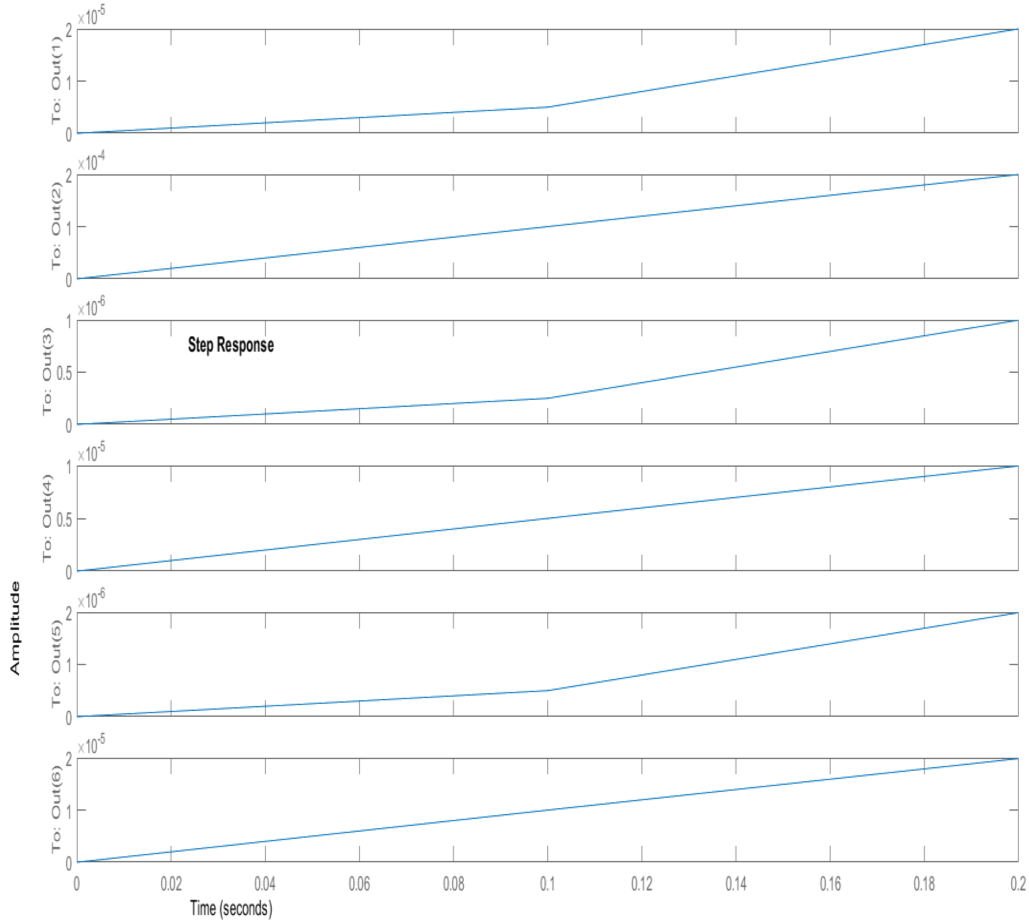
1. Develop linear model with A and B matrices.
2. Check for controllability
3. Initialize Q and R
4. Adjust Q and R
5. Find optimal gain K
6. Simulate the response

The first two steps are already done above. Now we need to initialize the values for Q and R. With the help of the graphs, we can tune the values and determine the stable system.

For different values of Q and R we get different responses for the linear system

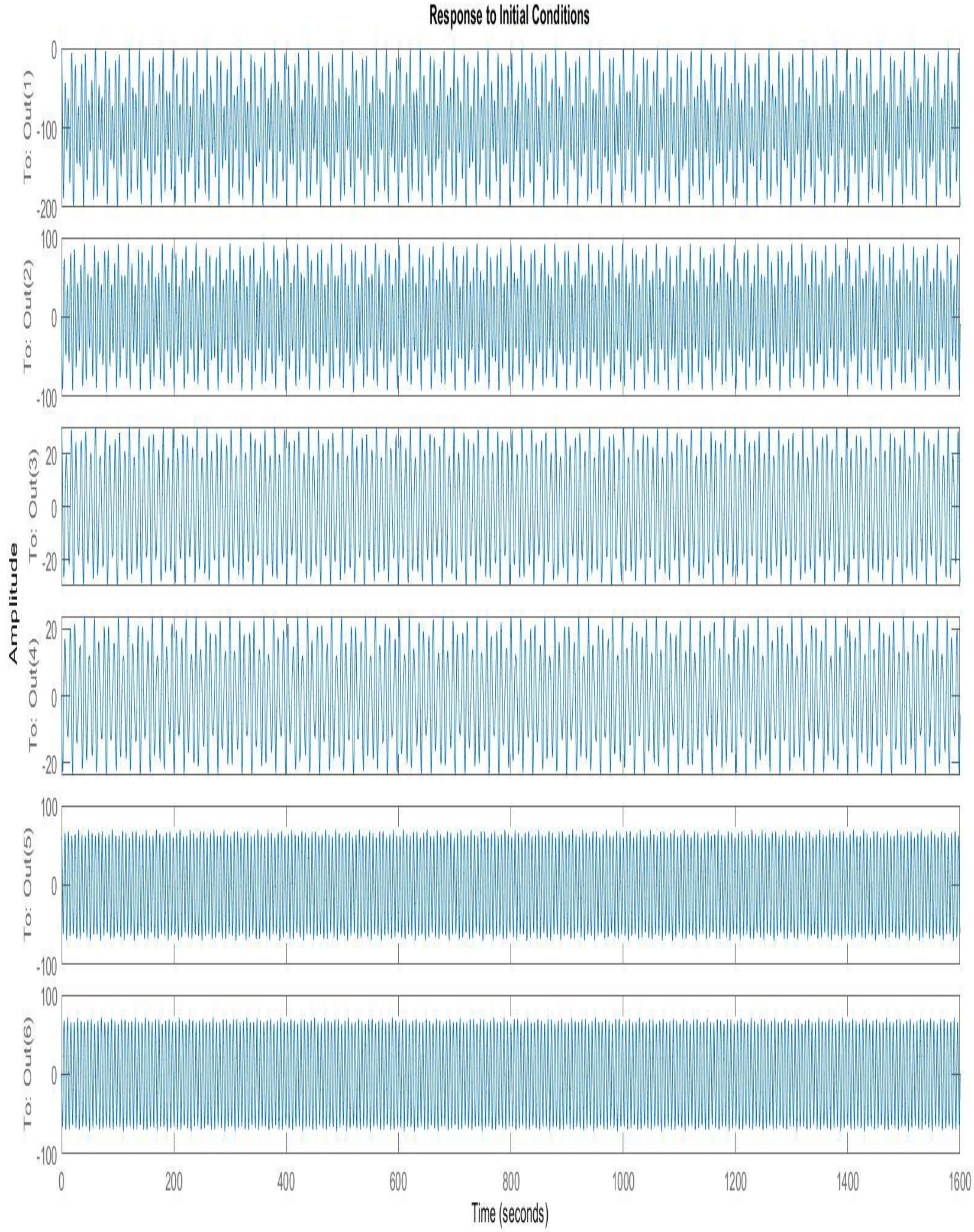
LINEAR SYSTEM RESPONSES:

RESULTS:



INITIAL RESPONSE BEFORE APPLYING LQR CONTROLLER

For different values of Q And R the responses as follows,



Response to Initial Conditions
Amplitude
To: Out(1)
To: Out(2)
To: Out(3)
To: Out(4)
To: Out(5)
To: Out(6)
0
-100
-200
100
0
-100
20
0
-20
20
0
-20
100
0
-100
100
0
-100
0
200
400
600
800
1000
1200
1400
1600
Time (seconds)

Lyapunov's indirect stability:

The closed loop gain is given as,

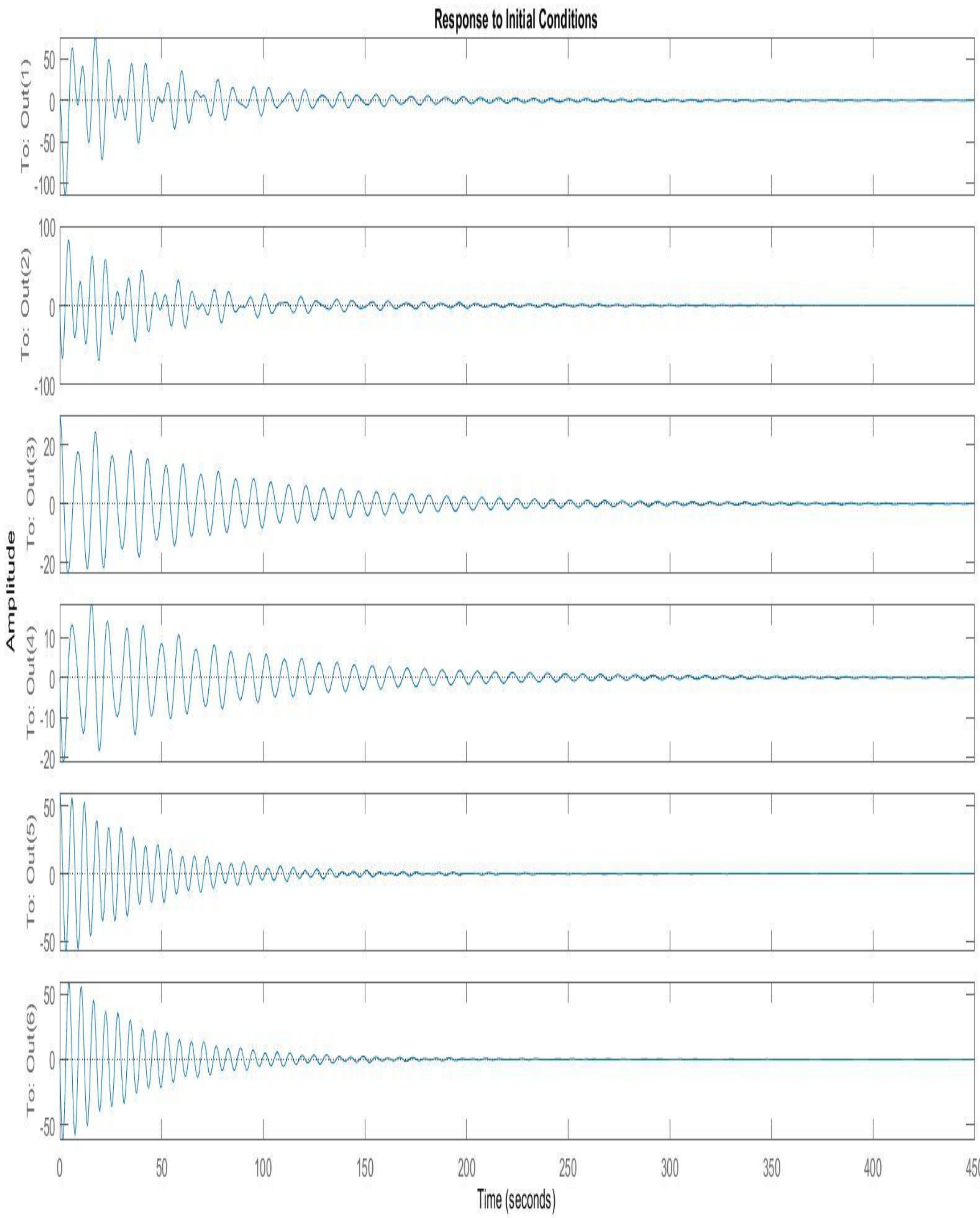
K = 1.0e+03 *[1.0000 1.8299 -0.1887 -0.9930 -0.0633 -0.5697]

This methos suggests us to check the stability by checking the eigen values of

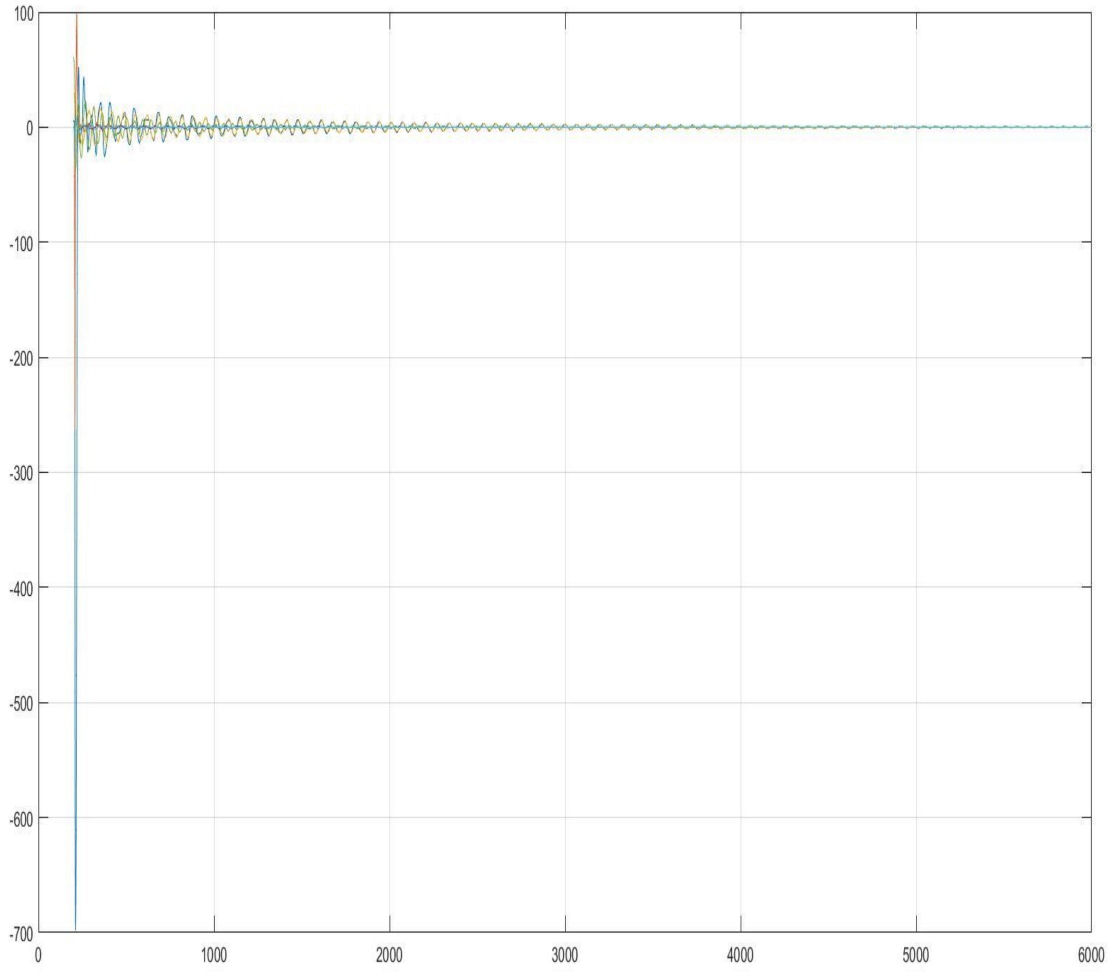
Ac = (A-B) * k

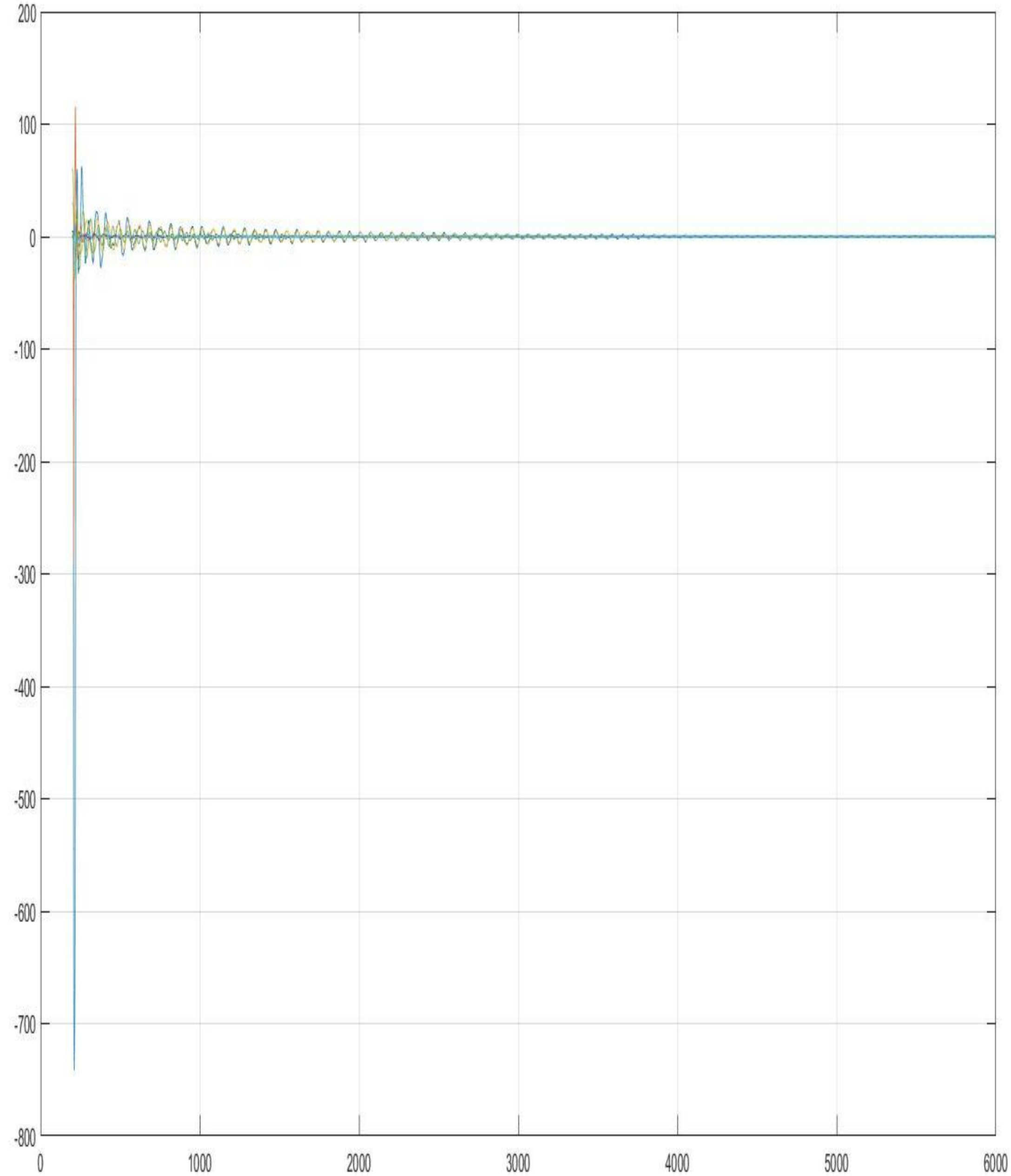
$$\begin{bmatrix} 0.8199 + 0.5294i \\ -0.8199 - 0.5294i \\ -0.0290 + 1.0058i \\ -0.0290 - 1.0058i \\ -0.0127 + 0.7055i \\ -0.0127 - 0.7055i \end{bmatrix}$$

The eigen values has both real and imaginary parts. Thus we can consider that the system is stable even though there is some oscillations.


Response to Initial Conditions
Amplitude
To: Out(1)
To: Out(2)
To: Out(3)
To: Out(4)
To: Out(5)
To: Out(6)
Time (seconds)
0
50
100
150
200
250
300
350
400
450
-100
-50
-20
-10
10
20

NON-LINEAR SYSTEM:
Different responses for different values of Q and R.





200
100
0
-100
-200
-300
-400
-500
-600
-700
-800
0
1000
2000
3000
4000
5000
6000

Question E: Suppose that you can select the following output vectors: x(t), $(\theta_1(t), \theta_2(t))$, (x(t), $\theta_2(t)$) or (x(t), $\theta_1(t), \theta_2(t)$). Determine for which output vectors the linearized system is observable.

Solution:

To check the observability of the system, we need to check for its rank. There are 4 cases for which we have to check for the observability.

1. x(t)
2. $\theta_1(t), \theta_2(t)$
3. x(t), $\theta_2(t)$
4. x(t), $\theta_1(t), \theta_2(t)$

$$OB = \begin{bmatrix} C \\ CA \\ CA^2 \\ CA^3 \\ CA^4 \\ CA^5 \end{bmatrix}$$

The rank of OB should be 6 for the system to be stable. We are calculating using MATLAB

Case 1: x(t)

For this case, we take C1 = [1 0 0 0 0 0]

In MATLAB the rank of obsv(A, C1) is 6 which means the system is **observable**.

Case 2: $\theta_1(t), \theta_2(t)$:

For this case, we take C2 = $\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

In MATLAB the rank of obsv(A, C2) is 4 which means the system is **not observable** in this case.

Case 3: x(t), $\theta_2(t)$:

For this case, we take C3 = $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

In MATLAB the rank of obsv(A, C2) is 6 which means the system is **observable** in this case.

Case 4: x(t), $\theta_1(t), \theta_2(t)$:

For this case, we take C4 = $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

In MATLAB the rank of obsv(A, C2) is 6 which means the system is **observable** in this case.

Question F:

Obtain your "best" Luenberger observer for each one of the output vectors for which the system is observable ad simulate it response to initial conditions and unit step input. The simulation should be done for the observer applied to both the linearized system and the original nonlinear system.

Solution:

Since we are trying to estimate all the states of the system, we need an observer or an estimator. Here, we are going to design a Luenberger observer. The dynamics are same as that of the general system. The linear state space system is represented as,

$$\dot{X}(t) = AX(t) + BU(\mathrm{t})$$

$$\mathrm{Y(t)} = CX(t) + DU(\mathrm{t})$$

After applying the luenbergers observer, the system is represented as,

$$\dot{\tilde{X}}\ (t) = A\tilde{X}(t) + BU(t) + L(y(t) - \tilde{y}(t)))$$

$$\tilde{Y}(t) = C\tilde{X}(t) + DU(t)$$

L -observer gain

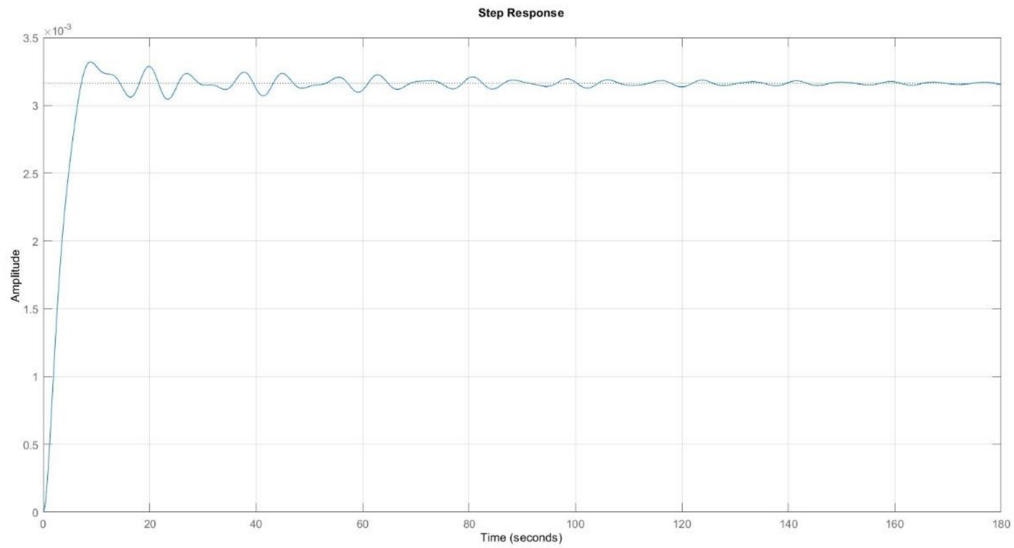
$\tilde{X}(t) -$ Estimated state

Y(t) -state output

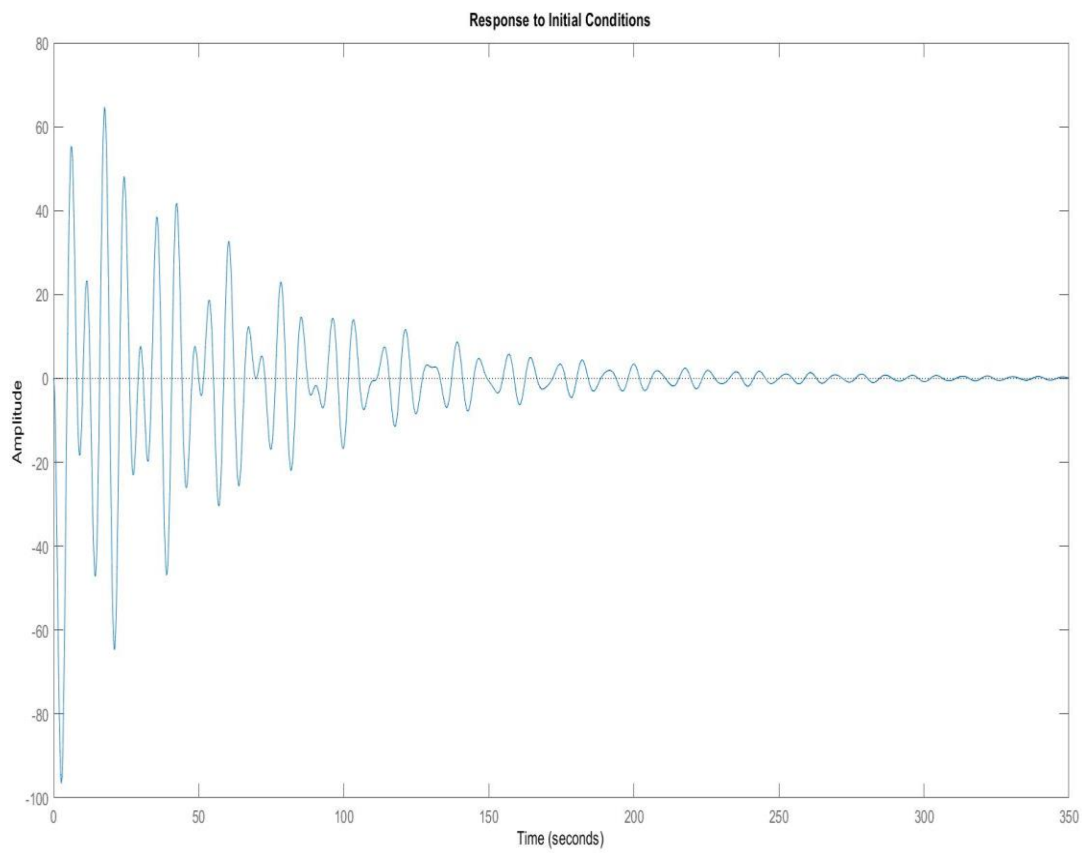
We can find L by placing the poles for A-LC. Here the error should converge to zero therefore we can put it far from eigen values of A-BK, where K is our feedback gain.

RESULTS:

CASE 1:

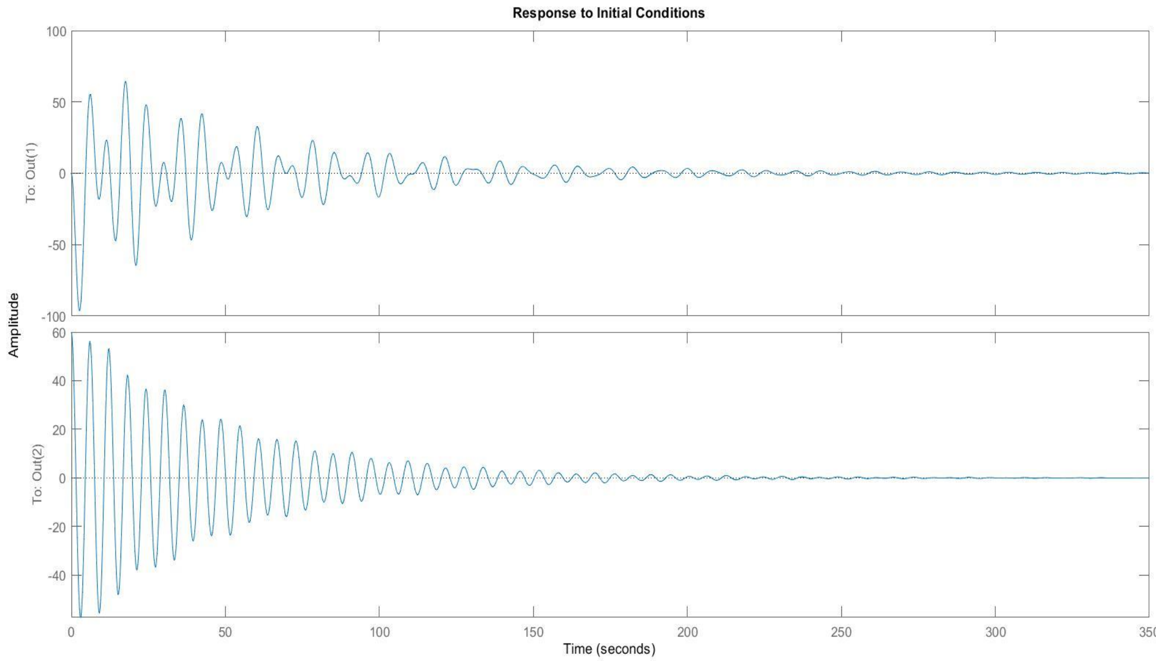


Step Response
×10^-3
Amplitude
Time (seconds)

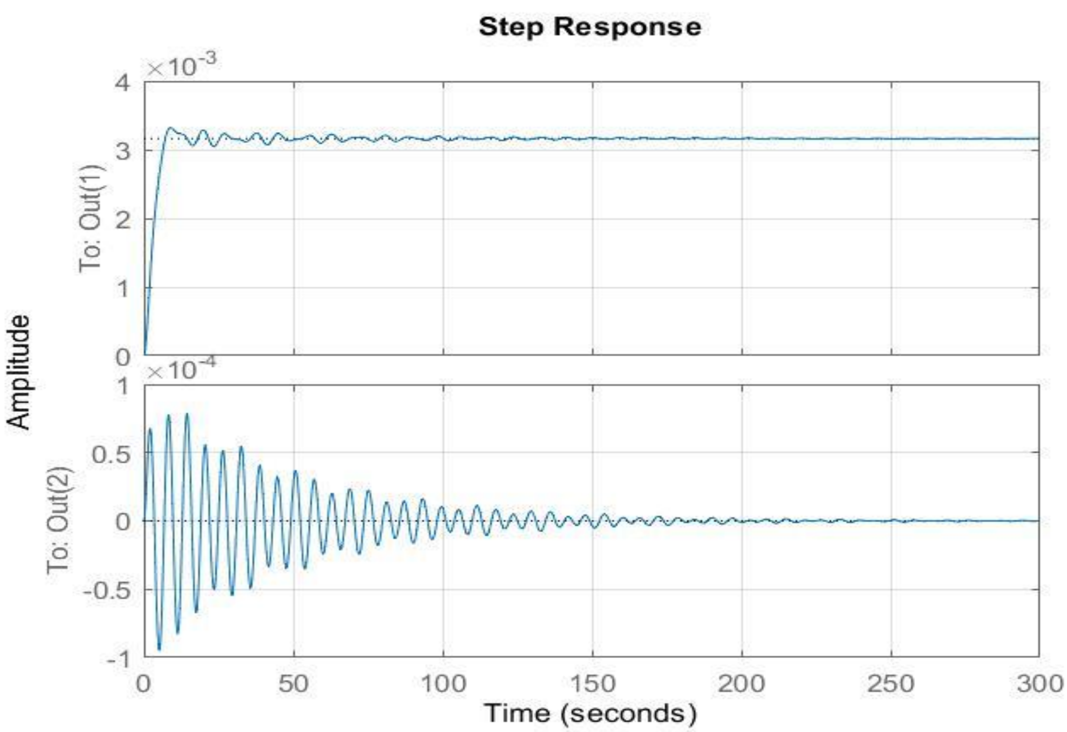


Response to Initial Conditions
Amplitude
Time (seconds)

CASE 2:

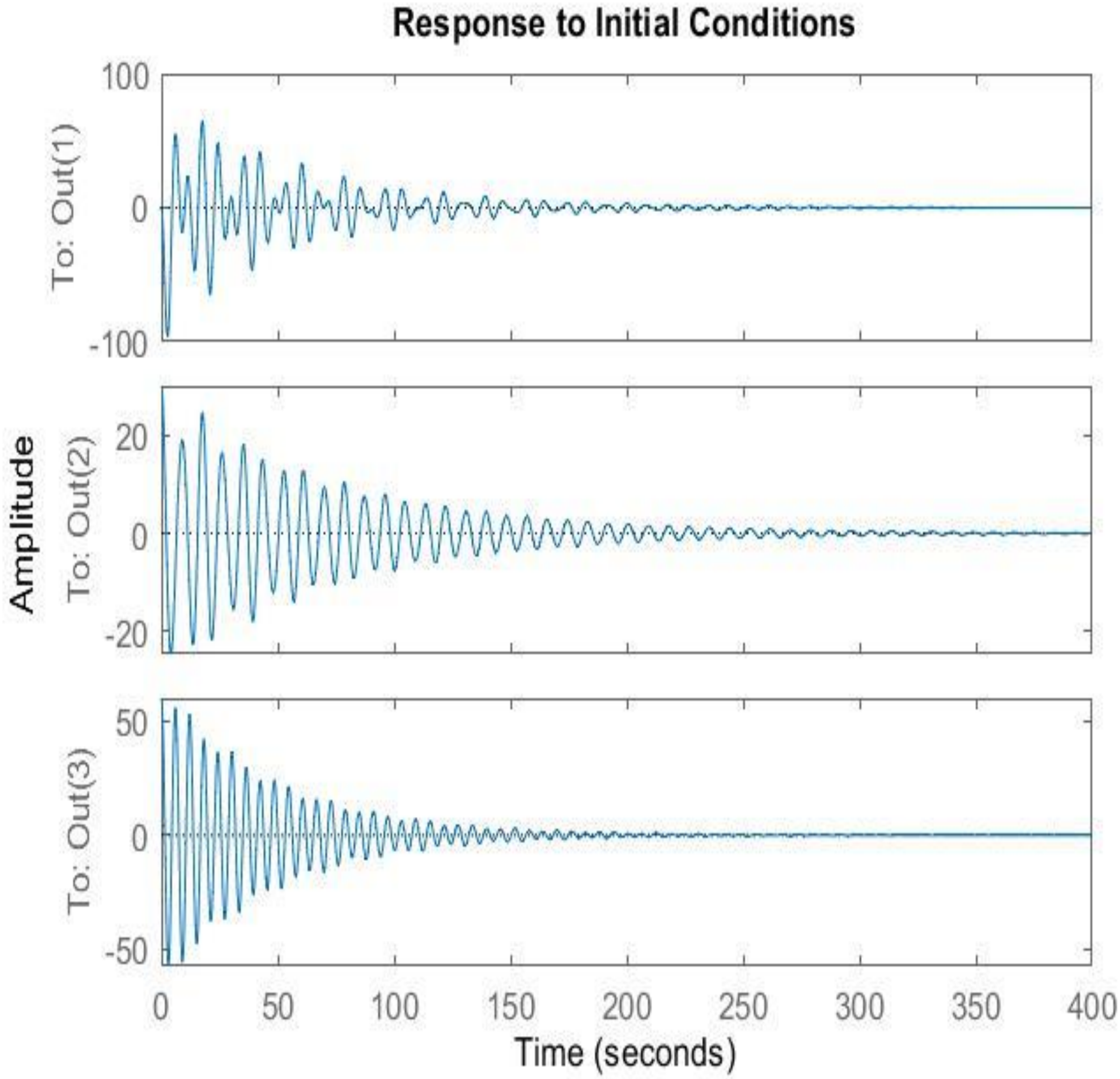


Response to Initial Conditions
Amplitude
To: Out(1)
To: Out(2)
Time (seconds)

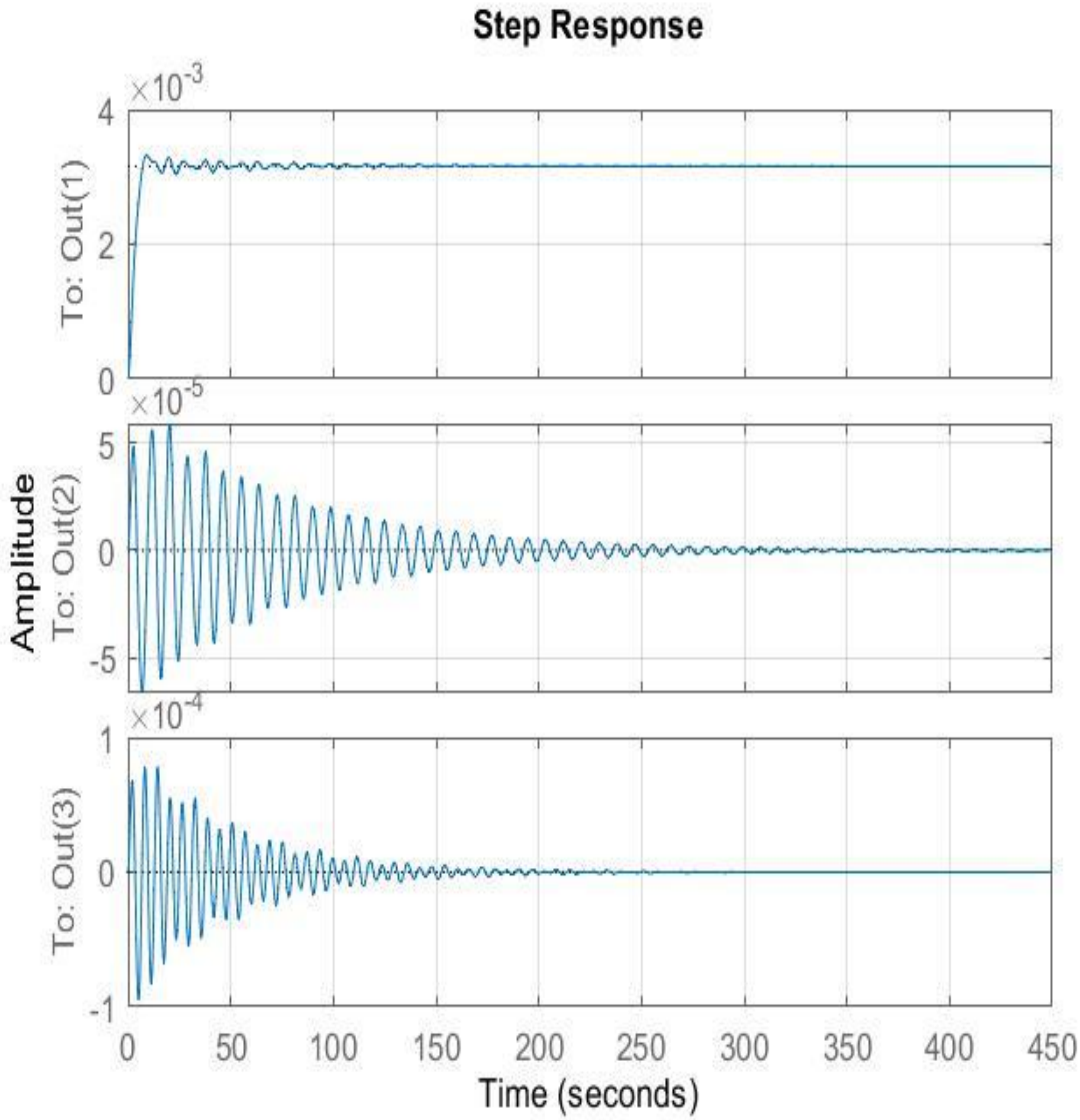


Step Response
Amplitude
To: Out(1)
To: Out(2)
$\times 10^{-3}$
$\times 10^{-4}$
Time (seconds)

CASE 3:



Response to Initial Conditions
100
0
-100
To: Out(1)
20
0
-20
To: Out(2)
Amplitude
50
0
-50
To: Out(3)
0
50
100
150
200
250
300
350
400
Time (seconds)



K =1.0e+03 *[1.0000 1.8299 -0.1887 -0.9930 -0.0633 -0.5697]

L1 =1.0e+03 *[0.0210 0.1734 -2.9329 0.0792 2.2176 -1.4496]

$$L3 = \begin{bmatrix} 13.0743 & -0.8243 \\ 56.2564 & -8.4778 \\ -89.1734 & 19.7841 \\ -20.0624 & 10.9530 \\ 0.3520 & 7.9257 \\ 3.4792 & 13.2136 \end{bmatrix} \qquad L4 = \begin{bmatrix} 8.5631 & -0.8851 & 0.0000 \\ 17.5219 & -4.9474 & -0.9800 \\ -0.9140 & 9.4369 & -0.0000 \\ -4.1173 & 20.9390 & -0.0491 \\ 0.0000 & -0.0000 & 3 \\ 0.0000 & -0.0980 & 0.9220 \end{bmatrix}$$

Question G:

Design an output feedback controller for your choice of the "smallest" output vector. Use the LQG method and apply the resulting output feedback controller to the original nonlinear system. Obtain your best design and illustrate its performance in simulation. How would y reconfigure your controller to asymptotically track a constant reference on x? Will your design reject constant force disturbances applied on the cart?

Solution:

X(t) – Minimum system vector. We are adding some gaussian noise here which are also called as system disturbances or measurement noises. To estimate the state with system disturbances we are designing a Kalman filter here.

The state space model with the measurement noises can be written as,

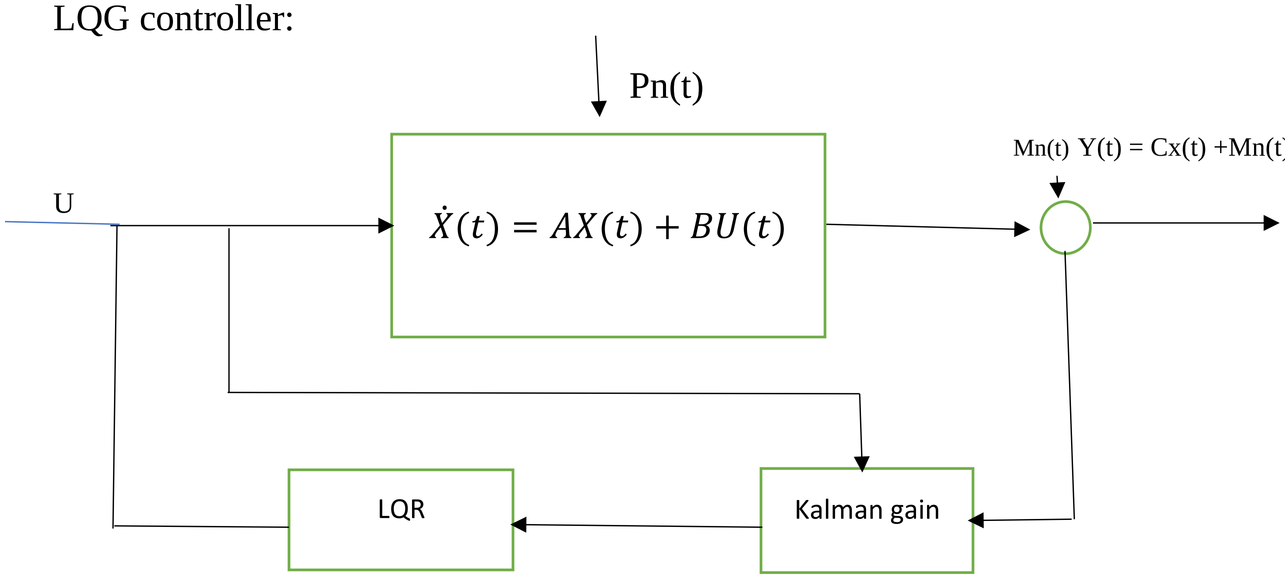
$$\dot{X}(t) = AX(t) + BU(t) + Pn(t)$$

$$Y(t) = CX(t) + DU(t) + Mn(t)$$

Pn(t)= process noise, Mn(t)= measurement noise.

When we design the Kalman filter we need to find the Kalman gain Kf.

LQG controller:



Linear system:

kk1 = [2.6090 3.1534 -0.6519 0.8021 -0.7397 0.3854]

kk2 = [2.3513 -0.0234

2.5146   -0.2950

-0.9350   -0.2049

0.5269   -0.0702

-0.0234   0.9490

0.2177   0.2006]

Kk3 = [

1.8056   -0.1488   -0.0435

1.3921   -0.5962   -0.4008

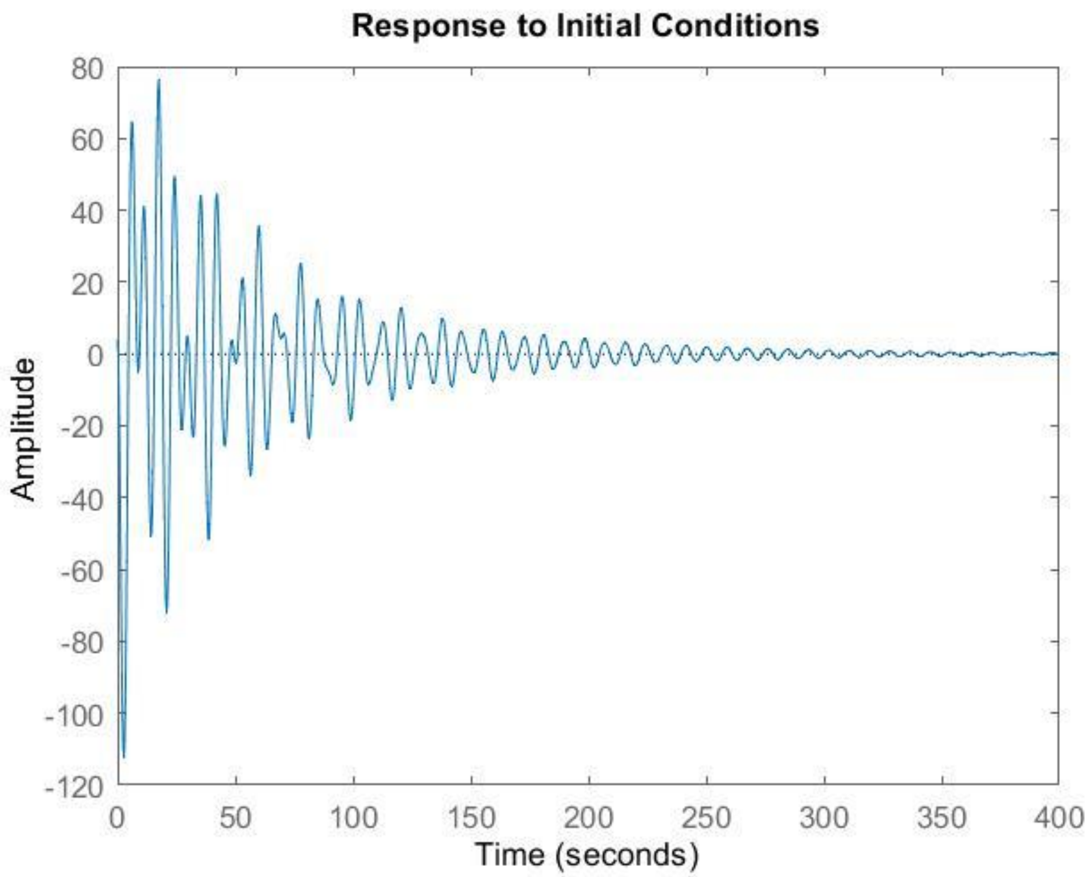
-0.1488   1.0704   -0.0459

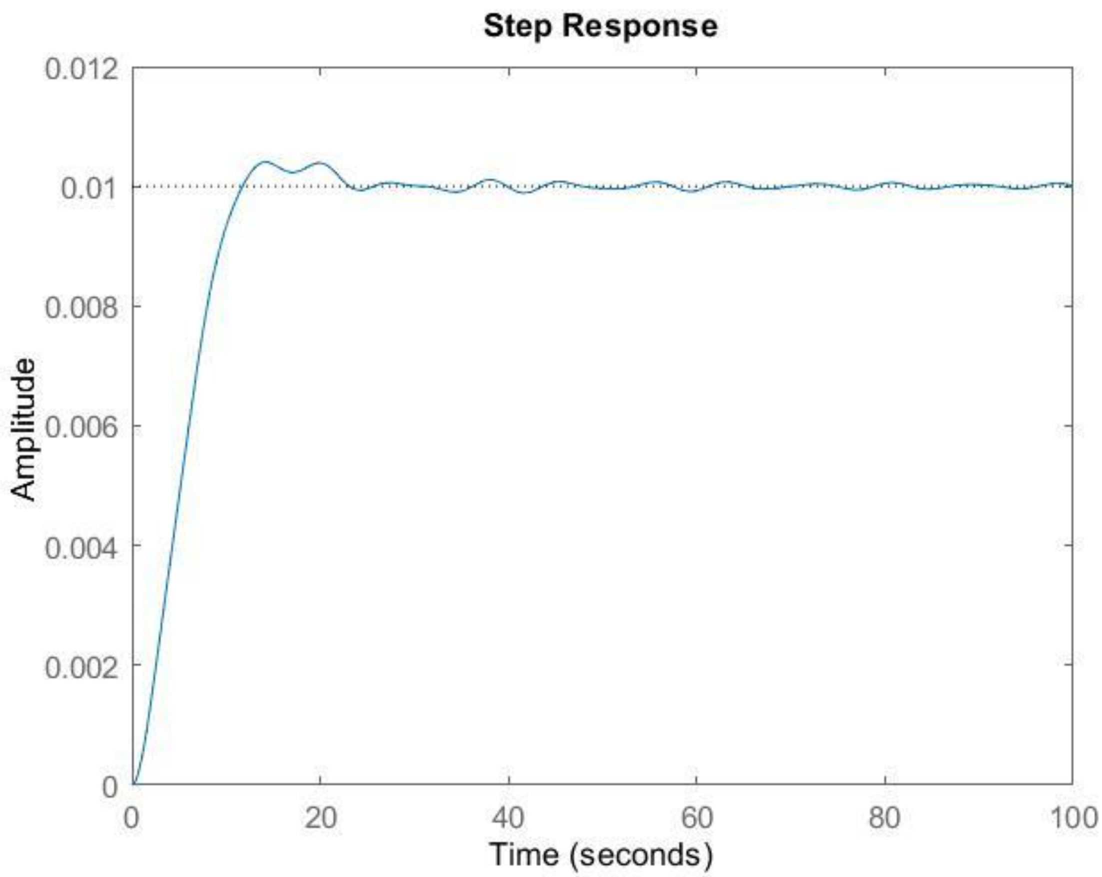
0.1703   0.3350   -0.0425

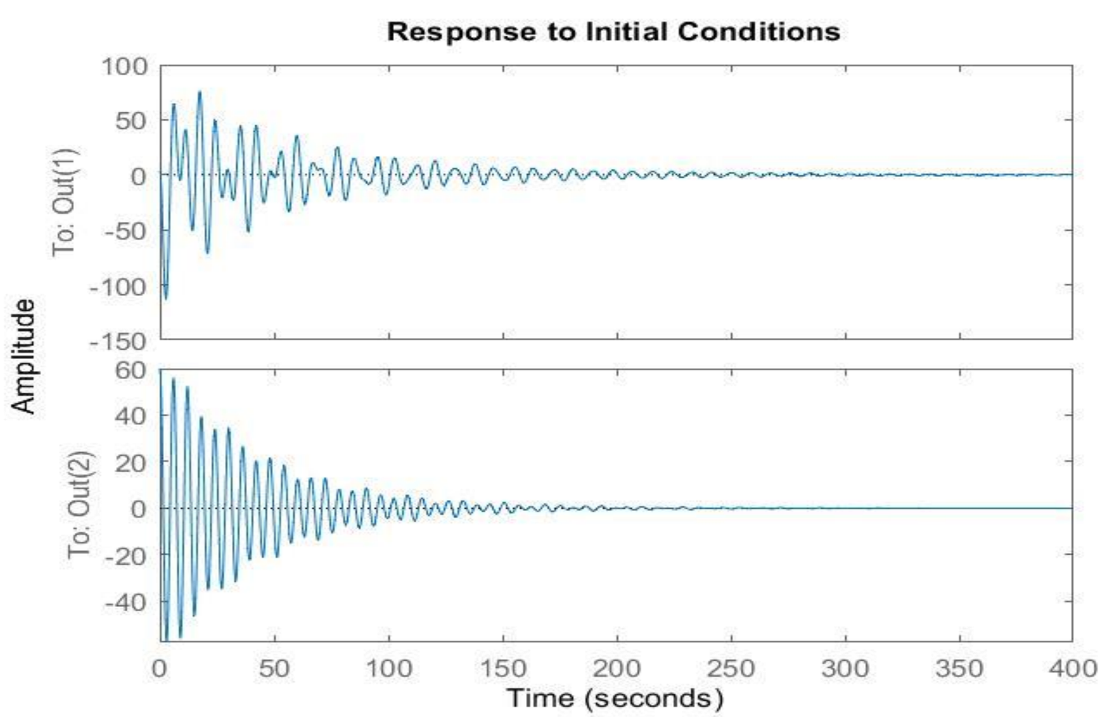
-0.0435   -0.0459   0.9264

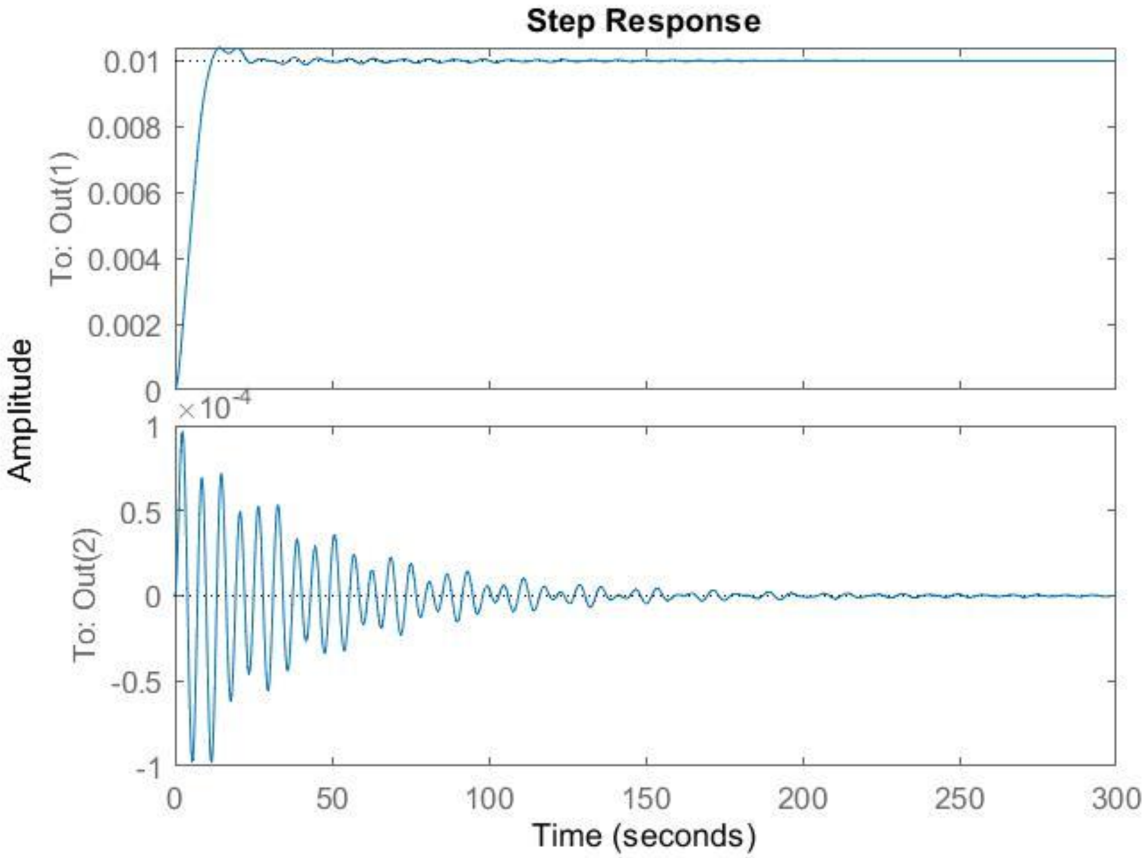
0.2888   -0.0427   0.1811]

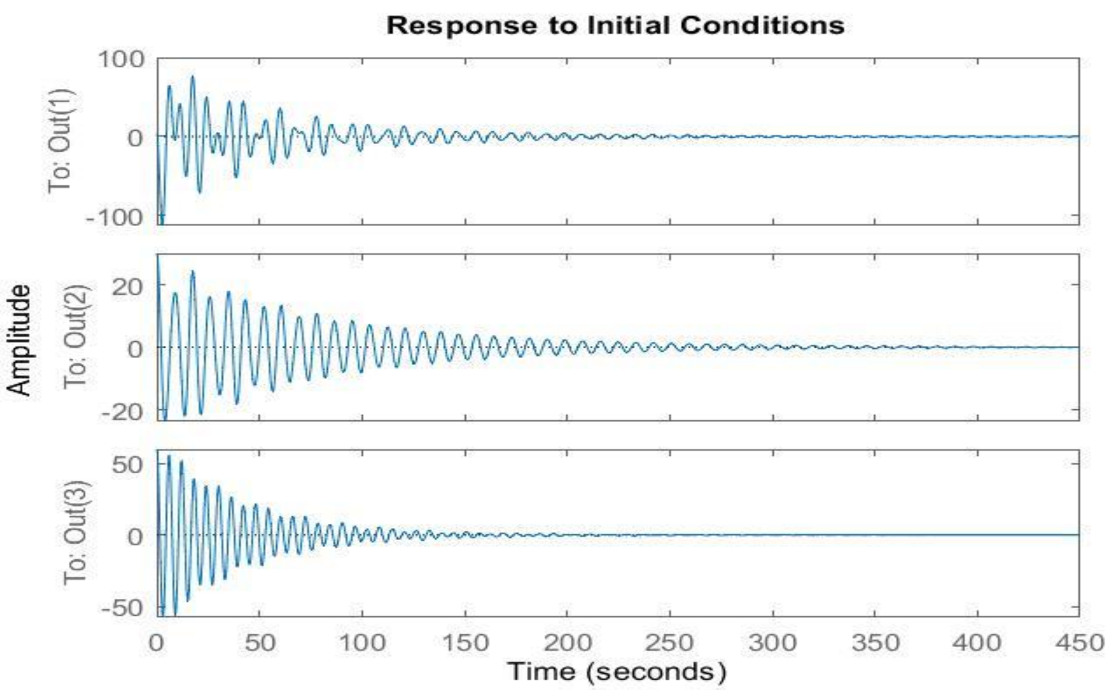
Results:

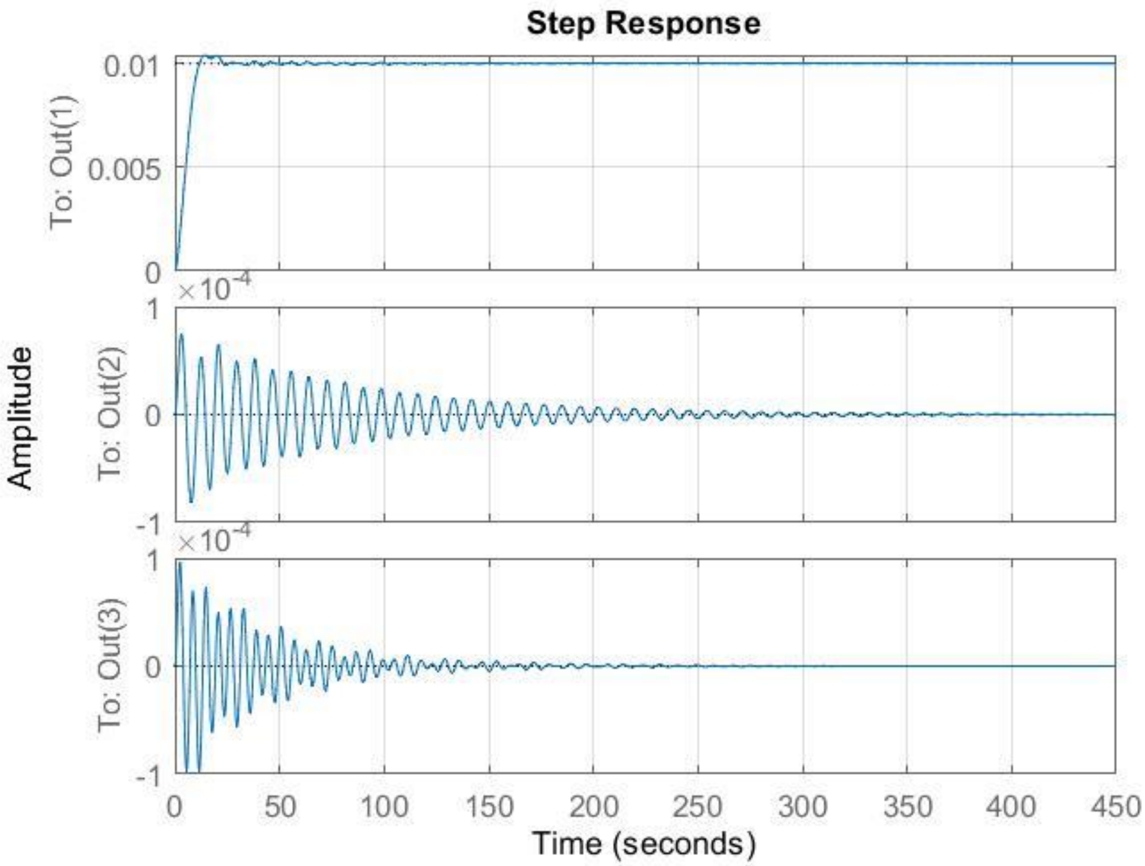

Response to Initial Conditions
Amplitude
Time (seconds)
80
60
40
20
0
-20
-40
-60
-80
-100
-120
0
50
100
150
200
250
300
350
400


Step Response
Amplitude
0.012
0.01
0.008
0.006
0.004
0.002
0
0
20
40
60
80
100
Time (seconds)


Response to Initial Conditions
Amplitude
To: Out(1)
100
50
0
-50
-100
-150
To: Out(2)
60
40
20
0
-20
-40
0
50
100
150
200
250
300
350
400
Time (seconds)

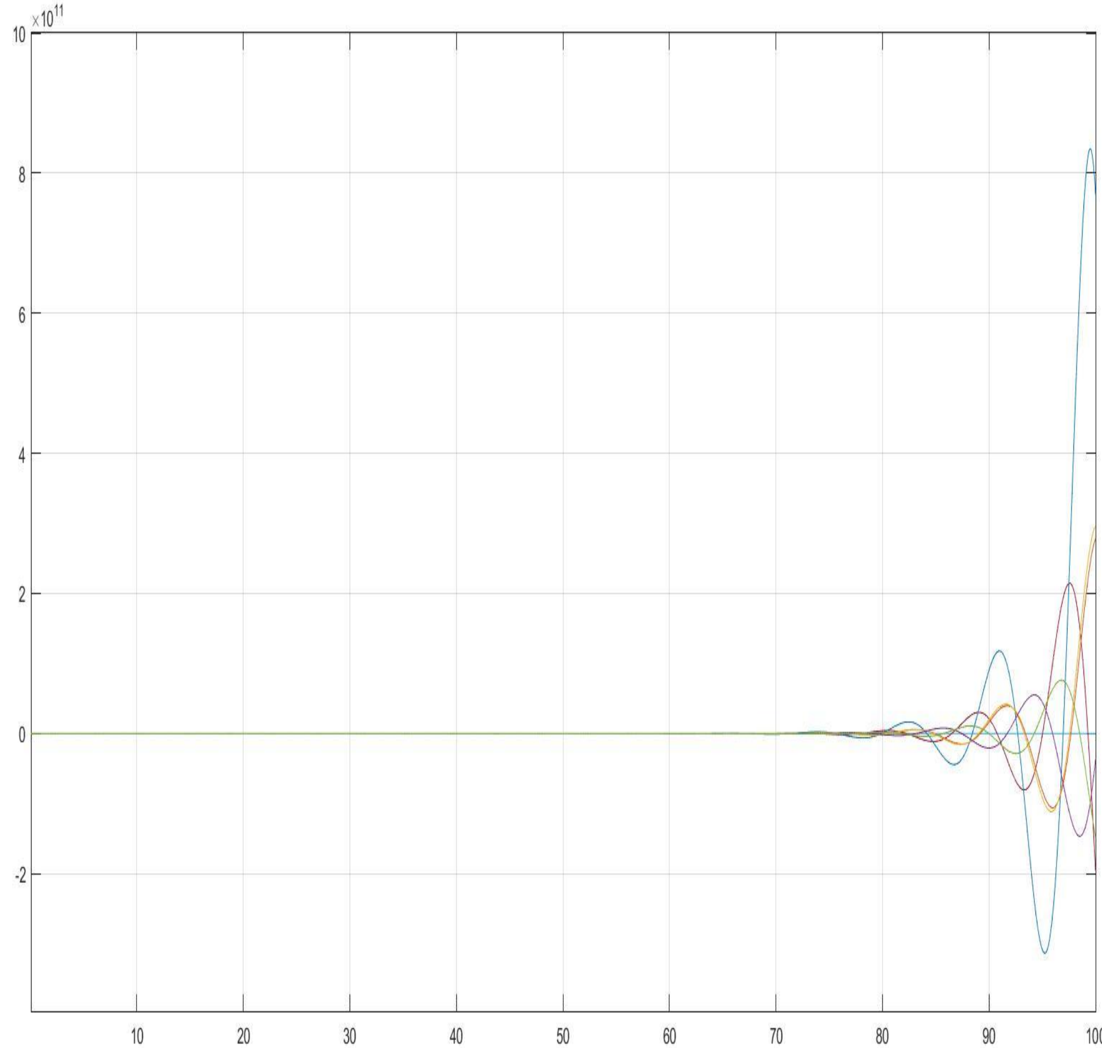

Step Response
Amplitude
To: Out(1)
0.01
0.008
0.006
0.004
0.002
0
×10^-4
1
0.5
0
-0.5
-1
To: Out(2)
0
50
100
150
200
250
300
Time (seconds)


Response to Initial Conditions
Amplitude
To: Out(1)
100
0
-100
To: Out(2)
20
0
-20
To: Out(3)
50
0
-50
0
50
100
150
200
250
300
350
400
450
Time (seconds)



None-linear system:

Results:



References:

https://www.youtube.com/watch?v=xjDIq4CK4eU

https://www.youtube.com/watch?v=H4_hFazBGxU

https://www.youtube.com/watch?v=H4_hFazBGxU

Yotube – Control Bootcamp

Githib link to code:

https://github.com/lavanyasureshkannan/Inverted-pendular-lqr-and-lqg