1.2 BIT

# Contents

3

5

6

7

8

9

10

11 }

}

return binary\_search(arr, mid + 1, rig,

target);

```
1 #define lowbit(k) (k & -k)
 1 Data Structure
                                    void add(vector<int> &tr, int id, int val) {
  for (; id <= n; id += lowbit(id)) {</pre>
  tr[id] += val;
  }
 2 Divide and Conquer
                                    int sum(vector<int> &tr, int id) {
  int ret = 0;
                                     for (; id >= 1; id -= lowbit(id)) {
                                   9
  DP
  3.1 Doubling . . . . . . . . . . . . . . . . . .
                                      ret += tr[id];
                                  10
  }
                                  11
  12
                                     return ret;
                                  13 }
 4 Fnumerate
  4.1 Halfcut Enumerate . . . . . . . . . . . . . . . . .
                                    1.3 Segment tree
 5 Graph
  5.2 Diikstra .
         . . . . . . . . . . . . . . . . . . .
  3
                                    int dfs(int lef, int rig){
  if(lef + 2 == rig){
                                   2
  3
                                         if(num[lef] > num[rig-1]){
  return lef;
  5.7 Bipatirate . .
                                         }
  5.8 Hungarian algorithm . . . . . . . . . . . . . . .
  else{
                                           return rig-1;
 6 Other
  9
  10
                                      int mid = (lef + rig)/2;
  6
  11
                                      int p1 = dfs(lef, mid);
  int p2 = dfs(mid, rig);
                                  12
                                      if(num[p1] > num[p2]){
  14
                                         return p1;
  }
  15
                                 7
                                  16
                                      else{
  17
                                         return p2;
  }
                                  19 }
  8
  6.16 Partitioning by Palindromes . . . . . . . . . . . . .
  1.4 Trie
  7 Function
                                 10
                                   1 const int MAXL = ; // 自己填
  7.1 strstr
                                    const int MAXC = ;
  7.2 substr \dots . . . . . . . . . . . . . . . . .
                                    struct Trie {
  int nex[MAXL][MAXC];
  7.4 vector
  int len[MAXL];
                                     int sz;
  void init() {
                                      memset(nex, 0, sizeof(nex));
                                   8
                                      memset(len, 0, sizeof(len));
                                   9
  10
                                      sz = 0;
  7.11 deaue . . .
                                  11
  7.12 python template . . . . . . . . . . . . . . .
                                  12
                                     void insert(const string &str) {
                                  13
                                      int p = 0;
                                      for (char c : str) {
                                  14
   Data Structure
                                       int id = c - 'a';
                                  15
                                        if (!nex[p][id]) {
                                  16
                                  17
                                         nex[p][id] = ++sz;
 1.1 Binary Search
                                  18
                                  19
                                       p = nex[p][id];
                                      }
1 int binary_search(int arr[maxn], int lef, int rig,
                                  20
   int target){
                                  21
                                      len[p] = str.length();
   if(lef > rig) return 0x3f3f3f3f;
                                  22
   int mid = (lef + rig) >> 1;
                                     vector<int> find(const string &str, int i) {
                                  23
   if(arr[mid] == target) return mid;
                                      int p = 0;
                                  25
                                      vector<int> ans;
   else if(arr[mid] > target){
     return binary_search(arr, lef, mid - 1,
                                  26
                                      for (; i < str.length(); i++) {</pre>
        target);
                                  27
                                       int id = str[i] - 'a';
   }
                                  28
                                       if (!nex[p][id]) {
```

29

30

31

32

33

return ans;

p = nex[p][id];

ans.pb(len[p]);

**if** (len[p]) {

}

# 2 Divide and Conquer

#### 2.1 count inversions

```
1 /*逆序數對 */
2 int arr[maxn], buf[maxn];
  int count_inversions(int lef, int rig){
       if(rig - lef <= 1) return 0;</pre>
       int mid = (lef + rig)/2;
6
       int ans = count_inversions(lef, mid) +
           count_inversions(mid, rig);
       int i = lef, j = mid, k = lef;
8
       while(i < mid || j < rig){</pre>
           if(i >= mid) buf[k] = arr[j++];
10
           else if(j >= rig) buf[k] = arr[i++];
           else{
11
12
                if(arr[i] <= arr[j]) buf[k] = arr[i++];</pre>
13
                else{
14
                    buf[k] = arr[j++];
15
                    ans += mid - i;
16
               }
17
           }
           k++;
18
19
       for(int k = lef; k < rig; ++k) arr[k] = buf[k];</pre>
20
21
       return ans;
22 }
```

# 3 DP

# 3.1 Doubling

```
1 /* 倍增 */
2 int LOG = sqrt(N); // 2^LOG >= N
3 vector<int> arr(N);
4 vector<vector<int>> dp(N, vector<int>(LOG));
5 for(int i = 0; i < N; ++i) cin >> arr[i];
6 int L, Q, a, b;
7
  cin >> L >> Q;
8 | for(int i = 0; i < N; ++i) {
      dp[i][0] = lower_bound(arr.begin(), arr.end(),
9
           arr[i] + L) - arr.begin();
      if(dp[i][0] == N || arr[i] + L < arr[dp[i][0]])</pre>
10
           dp[i][0] -= 1;
11 }
12 for(int i = 1; i < LOG; ++i)
      for(int j = 0; j < N; ++j)
           dp[j][i] = dp[dp[j][i - 1]][i - 1];
14
  for(int i = 0; i < Q; ++i){</pre>
15
      cin >> a >> b;
16
17
      a--; // 要減減是因為arr的index從0開始但題目從1開始
18
19
      if(a > b) swap(a, b);
20
      int ans = 0;
      for(int i = LOG - 1; i >= 0; --i){ // 從後往回推
21
           if(dp[a][i] < b){</pre>
22
               ans += (1 << i);
23
               a = dp[a][i];
24
25
           }
      }
26
27
      cout << ans + 1 << endl;
28 }
```

# 3.2 Josephus

#### 3.3 LCS

```
1 int LCS(string s1, string s2) {
    int n1 = s1.size(), n2 = s2.size();
    int dp[n1+1][n2+1] = {0};
    // dp[i][j] = s1的前i個字元和s2的前j個字元
5
    for (int i = 1; i <= n1; i++) {</pre>
6
      for (int j = 1; j <= n2; j++) {
        if (s1[i - 1] == s2[j - 1]) {
7
          dp[i][j] = dp[i - 1][j - 1] + 1;
8
           dp[i][j] = max(dp[i - 1][j], dp[i][j - 1]);
10
11
      }
12
    }
13
14
    return dp[n1][n2];
15 3
```

#### 3.4 LIS

```
1 int LIS(vector<int> &a) { // Longest Increasing
       Subsequence
     vector<int> s;
     for (int i = 0; i < a.size(); i++) {</pre>
4
       if (s.empty() || s.back() < a[i]) {</pre>
5
         s.push_back(a[i]);
6
       } else {
7
         *lower_bound(s.begin(), s.end(), a[i],
8
           [](int x, int y) {return x < y;}) = a[i];
9
    }
10
11
    return s.size();
```

### 4 Enumerate

#### 4.1 Halfcut Enumerate

```
1 /* 折半枚舉 */
  void dfs(set<long long int> &s, int depth, int T,
       long long int sum){
       if(depth >= T){
           s.insert(sum);
5
           return;
6
      }
      dfs(s, depth + 1, T, sum); // 取或不取的概念
dfs(s, depth + 1, T, sum + A[depth]);
7
8
9
  }
10
  int main(){
11
      int N, T;
12
       set < long long int > s1, s2;
13
       cin >> N >> T;
      for(int i = 0; i < N; ++i) cin >> A[i];
14
      dfs(s1, 0, N/2, 0); // 折半枚舉
15
16
      dfs(s2, N/2, N, 0);
17
      long long int ans = 0;
       // 題目:枚舉集合 Sx 的數字 Sxi,找出 Sy
18
           集合內小於等於 T-Sxi 中最大的數 Syj
19
       for(auto &x : s1){
20
           auto it = s2.upper_bound(T - x);
21
           long long int y = *(--it);
```

# 5 Graph

#### 5.1 SPFA

```
1 bool SPFA(int s){
2
       // 記得初始化這些陣列
       int cnt[1000+5], dis[1000+5];
3
       bool inqueue[1000+5];
5
       queue < int > q;
6
       q.push(s);
8
       dis[s] = 0;
       inqueue[s] = true;
10
       cnt[s] = 1;
11
       while(!q.empty()){
12
           int now = q.front();
           q.pop();
13
14
           inqueue[now] = false;
15
16
           for(auto &e : G[now]){
17
                if(dis[e.t] > dis[now] + e.w){
                    dis[e.t] = dis[now] + e.w;
18
19
                    if(!inqueue[e.t]){
20
                         cnt[e.t]++;
21
                         if(cnt[e.t] > m){
22
                             return false;
23
                         inqueue[e.t] = true;
24
25
                         q.push(e.t);
26
               }
27
           }
28
29
       }
30
       return true;
31 | }
```

# 5.2 Dijkstra

```
1 struct Item{
2
      int u, dis;
       // 取路徑最短
3
       bool operator < (const Item &other) const{</pre>
           return dis > other.dis;
5
6
7 };
8 int dis[maxn];
9 vector < Edge > G[maxn];
10 void dijkstra(int s){
       for(int i = 0; i <= n; i++){</pre>
11
           dis[i] = inf;
12
13
14
       dis[s] = 0;
15
       priority_queue < Item > pq;
16
       pq.push({s, 0});
17
       while(!pq.empty()){
           // 取路徑最短的點
18
           Item now = pq.top();
19
20
           pq.pop();
21
           if(now.dis > dis[now.u]){
22
               continue;
23
           // 鬆弛更新,把與 now.u 相連的點都跑一遍
24
25
           for(Edge e : G[now.u]){
26
               if(dis[e.v] > now.dis + e.w){
                   dis[e.v] = now.dis + e.w;
27
28
                   pq.push({e.v, dis[e.v]});
29
```

# 5.3 Floyd Warshall

}

```
void floyd_warshall(){
       for(int i = 0; i < n; i++){</pre>
2
3
           for(int j = 0; j < n; j++){</pre>
               G[i][j] = INF;
           G[i][i] = 0;
6
7
8
       for (int k = 0; k < n; k++){
           嘗試每一個中繼點
           for (int i = 0; i < n; i++){ //</pre>
9
               計算每一個i點與每一個j點
               for (int j = 0; j < n; j++){
10
                   G[i][j] = min(G[i][j], G[i][k] +
11
                        G[k][j]);
12
               }
           }
13
14
       }
15 }
```

# 5.4 Disjoint set Kruskal

```
1 struct Edge{
2
      int u, v, w;
      // 用權重排序 由大到小
3
      bool operator < (const Edge &other) const{</pre>
5
          return w > other.w;
      }
6
7
  }edge[maxn];
8
  // disjoint set
  int find(int x){
10
    if(parent[x] < 0){
11
      return x;
12
13
    else{
14
      return parent[x] = find(parent[x]);
15
    }
16 }
17
  void unite(int a, int b){
    a = find(a);
18
    b = find(b);
19
20
    if(a != b){
21
22
      if(parent[a] < parent[b]){</pre>
23
        parent[a] += parent[b];
24
        parent[b] = a;
      }
25
26
      else{
27
        parent[b] += parent[a];
28
        parent[a] = b;
29
30
    }
31
  }
32
  void kruskal(){
      memset(parent, -1, sizeof(parent));
33
      sort(edge, edge + m);
34
35
      int i, j;
36
      for(i = 0, j = 0; i < n - 1 && j < m; i++){
          // 如果 u 和 v 的祖先相同, 則 j++
37
               (祖先相同代表會產生環 所以不要)
          while(find(edge[j].u) == find(edge[j].v)) j++;
38
           // 若部會產生環 則讓兩點之間產生橋
39
               (連接兩顆子生成樹)
          unite(edge[j].u, edge[j].v);
40
41
          j++;
42
      }
43
```

```
5.5 KM
```

```
// X的點數,等於Y的點數
1 \mid \mathbf{const} \quad \mathbf{int} \quad X = 50;
2 | const int Y = 50;
                       // Y的點數
                       // 精簡過的adjacency matrix
3 int adj[X][Y];
4 int 1x[X], 1y[Y];
                       // vertex labeling
                       //
5 int mx[X], my[Y];
       X各點的配對對象、Y各點的配對對象
6 int q[X], *qf, *qb; // BFS queue
                       // BFS
7 int p[X];
       parent,交錯樹之偶點,指向上一個偶點
8 bool vx[X], vy[Y]; // 記錄是否在交錯樹上
9| int dy[Y], pdy[Y]; // 表格
10
11
  void relax(int x){ // relaxation
      for (int y=0; y<Y; ++y)</pre>
12
13
           if (adj[x][y] != 1e9)
               if (lx[x] + ly[y] - adj[x][y] < dy[y]){
14
                   dy[y] = 1x[x] + 1y[y] - adj[x][y];
15
16
                   pdy[y] = x; //
                       記錄好是從哪個樹葉連出去的
               }
17
18 }
  void reweight(){ // 調整權重、調整表格
19
20
       int d = 1e9;
21
       for (int y=0; y<Y; ++y) if (!vy[y]) d = min(d,</pre>
           dy[y]);
       for (int x=0; x<X; ++x) if ( vx[x]) lx[x] -= d;</pre>
22
23
       for (int y=0; y<Y; ++y) if ( vy[y]) ly[y] += d;</pre>
       for (int y=0; y<Y; ++y) if (!vy[y]) dy[y] -= d;</pre>
24
25 }
26 void augment(int x, int y){ // 擴充路徑
27
       for (int ty; x != -1; x = p[x], y = ty){
           ty = mx[x]; my[y] = x; mx[x] = y;
28
29
30 }
31 bool branch1(){ // 延展交錯樹:使用既有的等邊
32
       while (qf < qb)</pre>
           for (int x=*qf++, y=0; y<Y; ++y)</pre>
33
34
               if (!vy[y] \&\& lx[x] + ly[y] == adj[x][y]){
35
                   vy[y] = true;
                   if (my[y] == -1){
36
                       augment(x, y);
37
38
                       return true;
                   }
39
40
                   int z = my[y];
                   *qb++ = z; p[z] = x; vx[z] = true;
41
                       relax(z);
               }
42
43
       return false;
44 }
45 | bool branch2(){ // 延展交錯樹:使用新添的等邊
       for (int y=0; y<Y; ++y){</pre>
46
47
           if (!vy[y] && dy[y] == 0){
               vy[y] = true;
48
               if (my[y] == -1){
49
50
                   augment(pdy[y], y);
51
                   return true;
52
               }
53
               int z = mv[v];
54
               *qb++ = z; p[z] = pdy[y]; vx[z] = true;
                   relax(z);
           }
55
       }
56
57
       return false;
58 }
59 int Hungarian(){
       // 初始化vertex labeling
60
       // memset(lx, 0, sizeof(lx)); // 任意值皆可
61
       memset(ly, 0, sizeof(ly));
62
       for (int x=0; x<X; ++x)</pre>
63
64
           for (int y=0; y<Y; ++y)</pre>
65
               lx[x] = max(lx[x], adj[x][y]);
66
       // x側每一個點,分別建立等邊交錯樹。
67
```

```
68
       memset(mx, -1, sizeof(mx));
       memset(my, -1, sizeof(my));
69
70
       for (int x=0; x<X; ++x){</pre>
71
            memset(vx, false, sizeof(vx));
            memset(vy, false, sizeof(vy));
memset(dy, 0x7f, sizeof(dy));
72
73
74
            qf = qb = q;
75
            *qb++ = x; p[x] = -1; vx[x] = true; relax(x);
76
            while (true){
                if (branch1()) break;
77
78
                reweight();
79
                if (branch2()) break;
80
81
       }
       // 計算最大權完美匹配的權重
82
83
       int weight = 0:
84
       for (int x=0; x<X; ++x)
85
            weight += adj[x][mx[x]];
86
       return weight;
87 }
```

#### 5.6 Dinic

```
1 // Maximum Flow
2 const int V = 100, E = 1000;
  int adj[V]; // adjacency lists, 初始化為-1。
  struct Element {int b, r, next;} e[E*2];
  int en = 0;
5
  void addedge(int a, int b, int c){
7
      e[en] = (Element)\{b, c, adj[a]\}; adj[a] = en++;
      e[en] = (Element){a, 0, adj[b]}; adj[b] = en++;
8
9
  }
10 int d[V];
                   // 最短距離
11 bool visit[V]; // BFS/DFS visit record
                   // queue
12 int q[V];
  int BFS(int s, int t){ // 計算最短路徑,求出容許圖
      memset(d, 0x7f, sizeof(d));
14
      memset(visit, false, sizeof(visit));
15
16
      int qn = 0;
      d[s] = 0;
17
       visit[s] = true;
18
19
      q[qn++] = s;
20
21
       for (int qf=0; qf<qn; ++qf){</pre>
22
           int a = q[qf];
23
           for (int i = adj[a]; i != -1; i = e[i].next){
24
               int b = e[i].b;
25
               if (e[i].r > 0 && !visit[b]){
                   d[b] = d[a] + 1;
26
27
                   visit[b] = true;
28
                   q[qn++] = b;
                   if (b == t) return d[t];
29
30
          }
31
32
33
      return V;
34
  }
35
  int DFS(int a, int df, int s, int t){ //
       求出一條最短擴充路徑,並擴充流量
       if (a == t) return df;
36
37
       if (visit[a]) return 0;
38
       visit[a] = true;
39
       for (int i = adj[a]; i != -1; i = e[i].next){
40
           int b = e[i].b;
           if (e[i].r > 0 && d[a] + 1 == d[b]){
41
               int f = DFS(b, min(df, e[i].r), s, t);
               if (f){
43
44
                   e[i].r -= f;
45
                   e[i^1].r += f;
                   return f:
46
47
               }
48
          }
49
50
       return 0;
```

```
52 int dinitz(int s, int t){
       int flow = 0;
53
       while (BFS(s, t) < V)
54
55
           while (true){
56
                memset(visit, false, sizeof(visit));
57
                int f = DFS(s, 1e9, s, t);
                if (!f) break;
58
59
                flow += f;
           }
60
61
       return flow;
62 }
```

# 5.7 Bipatirate

```
1 const int maxn = 300 + 5;
2 int n, color[maxn];
  vector<vector<int>> v(maxn);
4 bool dfs(int s){
       for(auto it : v[s]){
6
           if(color[it] == -1){
                color[it] = 3 - color[s];
7
8
                if(!dfs(it)){
9
                    return false:
10
11
12
           if(color[s] == color[it]){
13
                return false;
14
15
16
       return true;
17 }
  void isBipatirate(){
18
       bool flag = true;
19
20
       for(int i = 1; i <= n; ++i){
           if(color[i] == -1){
21
22
                color[i] = 1;
                flag &= dfs(i);
23
           }
24
25
       }
       if(flag){
26
27
           cout << "YES" << endl;</pre>
28
29
           cout << "NO" << endl;
30
31
32 }
  int main(){
33
       while(cin >> n && n){
34
35
           for(int i = 1; i <= n; ++i) v[i].clear();</pre>
36
           memset(color, -1, sizeof(color));
37
           while(cin >> a >> b && (a || b)){
38
39
                v[a].emplace_back(b);
                v[b].emplace_back(a);
40
41
42
           isBipatirate();
       }
43
44 }
```

#### 5.8 Hungarian algorithm

```
1 const int maxn = 500+5;
2 int t, N, bn, gn, match[maxn];
3 bool visited[maxn];
4 vector<vector<int>> G(maxn);
5
  struct People{
      int h;
6
      string music, sport;
7
8
      People(){}
9
      People(int h, string music, string sport){
10
           this->h = h;
11
           this->music = music;
          this->sport = sport;
12
```

```
14 }lef[maxn], rig[maxn];
15 bool check(People boy, People girl){
       if(abs(boy.h - girl.h) <= 40 && boy.music ==</pre>
            girl.music && boy.sport != girl.sport) return
            true;
17
       return false;
18 }
19
  bool dfs(int s){
       for(int i = 0; i < G[s].size(); ++i){</pre>
20
            int v = G[s][i];
21
           if(visited[v]) continue;
22
23
            visited[v] = true;
            if(match[v] == -1 || dfs(match[v])){}
24
25
                match[v] = s;
26
                return true;
27
28
       }
29
       return false;
30
  }
31
  int Hungarian(){
32
       int cnt = 0;
33
       memset(match, -1, sizeof(match));
34
       for(int i = 0; i < bn; ++i){</pre>
35
            memset(visited, false, sizeof(visited));
36
           if(dfs(i)) cnt++:
37
       }
38
       return cnt;
39 }
40
  int main(){
       cin >> t:
41
42
       while(t--){
43
           cin >> N;
44
            bn = 0, gn = 0;
45
            for(int i = 0; i <= N; ++i) G[i].clear();</pre>
46
           int h:
47
            string sex, music, sport;
48
            for(int i = 0; i < N; ++i){
49
                cin >> h >> sex >> music >> sport;
                if(sex == "M") lef[bn++] = People(h,
50
                     music, sport);
51
                else rig[gn++] = People(h, music, sport);
52
53
            for(int i = 0; i < bn; ++i){</pre>
                for(int j = 0; j < gn; ++j)</pre>
54
55
                    if(check(lef[i], rig[j]))
                         G[i].emplace_back(j);
56
57
            cout << N - Hungarian() << endl;</pre>
       }
58
59 }
```

# 5.9 LCA

```
1 / * 最低共同祖先 * /
  // 此 node 下有機顆 node
  int dfs(int node, int dep){
3
       depth[node] = dep + 1;
5
      if(G[node].empty()){
6
           siz[node] = 1;
7
           return 1;
8
9
      int total = 1;
10
      for(auto i : G[node])
           total += dfs(i.v, dep + 1);
11
       siz[node] = total;
12
13
       return siz[node];
14 }
  // 找出每個節點的 2<sup>i</sup> 倍祖先
15
  // 2^20 = 1e6 > 200000
16
17
  void find_parent(){
18
      for(int i = 1; i < 20; i++)
19
           for (int j = 0; j < N; j++)
20
               parent[j][i] =
                   parent[parent[j][i-1]][i-1];
```

```
21 }
                                                              21 memset(num, 0, sizeof(num));
                                                              22 cin >> n:
22 // 求兩點的LCA (利用倍增法)
                                                              23 for(int i = 1; i <= n; i++){
23 int LCA(int a, int b){
       if (depth[b] < depth[a]) swap(a, b);</pre>
                                                              24
                                                                      cin >> num[i];
                                                                 }
                                                              25
25
       if (depth[a] != depth[b]){
           int dif = depth[b] - depth[a];
                                                              26 int mini = 2147483647; // 不知道為甚麼只有 2147483647
26
27
           for (int i = 0; i < 20; i++){
                                                                      給過
               if (dif & 1) b = parent[b][i];
28
                                                              27
                                                                 // 1 << n = n * 2
               dif >>= 1;
                                                                 for(int i = 0; i < (1 << n); i++){
29
                                                              28
30
           }
                                                                      int XOR = 0, OR = 0;
                                                              29
31
                                                                      for(int j = 1; j \le n; j++){
                                                              30
       if (a == b) return a;
32
                                                              31
       for (int i = 19; i \ge 0; i - -){
33
                                                              32
34
           if (parent[a][i] != parent[b][i]){
                                                              33
35
               a = parent[a][i];
                                                              34
36
               b = parent[b][i];
                                                              35
           }
37
                                                                     }
                                                              36
38
      }
                                                                     XOR ^= OR;
                                                              37
39
       return parent[a][0];
                                                              38
                                                                     mini = min(mini, XOR);
40 }
                                                              39 }
```

# 0ther

# Bubble Sort Expect Value

```
1 /* 期望值算法:
2 擲一枚公平的六面骰子,其每次「點數」的期望值是 3.5
3 \mid E(x) = 1 * 1/6 + 2 * 1/6 + 3 * 1/6 + 4 * 1/6 + 5 *
      1/6 + 6 * 1/6
|4| = (1 + 2 + 3 + 4 + 5 + 6)/6 = 3.5
5 bubble sort 每兩兩之間交換機率是 1/2
6 總共會做 C(n, 2) 次
7 \mid E(x) = C(n, 2) * 1/2 = (n * (n - 1))/2 * 1/2 */
8 int t, ca = 1;
9 cin >> t;
10
  while(t--){
      long long int n;
11
12
      cin >> n;
      cout << "Case " << ca++ << ": ";
13
      // 如果 (n * (n - 1)) 可以被 4 整除
14
          代表最後答案會是整數,否則會是分數
15
      if((n * (n - 1)) % 4){
          cout << ( (n * (n - 1)) / 2 ) << "/2"<< endl;
16
17
      }
18
      else{
          cout << ( (n * (n - 1)) / 2 ) / 2 << endl;
19
20
21 }
```

# 6.2 ORXOR

```
1 /* 如何切區段,之所以要1<<n是為了可以跑000~111
2 | i = 0, binary i = 000
3 0 : 1 5 7
4 \mid i = 1, binary i = 001
5 1 : 1 5 7
6 \mid i = 2, binary i = 010, 看得出來切了一刀
7 2 : 1 | 5 7
8 \mid i = 3, binary i = 011
9 3 : 1 | 5 7
10 | i = 4 , binary i = 100 , 為了要切在index = 2 , 所以才要1<<j
11 4 : 1 5 | 7
|12| i = 5, binary i = 101
13 5 : 1 5 | 7
|14|i = 6, binary i = 110
15 6 : 1 | 5 | 7
|i| = 7, binary |i| = 111
17 7 : 1 | 5 | 7
18 可以觀察出來,前兩位 bit 是 1 時代表的意義是切在哪裡*/
19 int n;
20 int num[20+7];
```

# 6.3 Race to 1

40 cout << mini << endl;

OR |= num[j];

if((i & (1 << j))){</pre>

XOR ^= OR;

OR = 0;

```
1 const int N = 1000000;
  bool sieve[N+5];
  vector<int> pri;
3
  double dp[N+5];
  void Linear_Sieve(){ // 線性篩
       for (int i = 2; i < N; i++){
           if (!sieve[i])
                pri.push_back(i);
           for (int p: pri){
9
10
                if (i * p \ge N){
11
                    break;
12
13
                sieve[i * p] = true;
                if (i % p == 0){
14
15
                    break;
16
17
           }
18
       }
19
  }
20
  double dfs(int n){
       if(dp[n] != -1) return dp[n];
21
22
       dp[n] = 0:
       if(n == 1) return dp[n];
23
24
       int total = 0, prime = 0;
25
       for(int i = 0; i < pri.size() && pri[i] <= n;</pre>
           i++){
26
           total++;
27
           if(n % pri[i]) continue;
28
           prime++;
29
           dp[n] += dfs(n/pri[i]);
30
       dp[n] = (dp[n] + total)/prime; // 算期望值
31
32
       return dp[n];
33 }
34
  int main(){
35
       int t;
36
       int num;
37
       int ca = 1;
       for(int i = 0; i <= N; i++){</pre>
38
           dp[i] = -1;
40
41
       Linear_Sieve();
42
       cin >> t;
43
       while(t--){
44
           cin >> num;
45
           cout << "Case " << ca++ << ": " << fixed <<
46
                setprecision(10) << dfs(num) << endl;</pre>
47
       }
48 }
```

# 6.4 X drawing

```
1 long long int n, a, b, p, q, r, s;
2 cin >> n >> a >> b;
3 cin >> p >> q >> r >> s;
4
  for(long long int i = p; i <= q; i++){</pre>
       for(long long int j = r; j \le s; j++){
           if(abs(i - a) == abs(j - b)){
               cout << '#';
7
           }
8
9
           else{
10
               cout << '.';
11
       }
12
13
       cout << endl;
14 }
```

# 6.5 Big Mod

```
1 '''
2 Mod
3 \mid pow(x, y, z) = x^y \% z
5 # python 如何讀取直到 EOF 用 try except
6 try:
7
      while True:
8
          # input().split() 用空格切開讀取一整行
9
          # map (型態, input().split()) 才能把值全讀成
             int
          B, P, M = map(int, input().split())
11
          print(pow(B, P, M))
12 except EOFError:
      exit
13
```

#### 6.6 Crested Ibis vs Monster

```
1 /* dp 背包 - 重量/價值/可重複使用
2 因為這題可以重複使用同一條魔法
3 所以可以這樣 dp */
4 int h, n;
5 cin >> h >> n;
6 for(int i = 1; i <= n; i++){
      cin >> a[i] >> b[i];
8 }
9 memset(dp, 0x3f3f3f3f, sizeof(dp));
10 | dp[0][0] = 0;
11 for(int i = 1; i <= n; i++){
      for(int j = 0; j <= h; j++){</pre>
12
13
          dp[i][j] = min(dp[i-1][j], dp[i][max(0, j -
              a[i])] + b[i]);
14
15 }
16 cout << dp[n][h] << endl;</pre>
```

#### 6.7 dpd Knapsack 1

```
1 // dp 背包 - 時間/數量/價值 - 第幾分鐘符合
2 int N, W;
3 cin >> N >> W;
4 int w[100000+5];
5 int v[100000+5];
6 for(int i = 0; i < N; i++){
      cin >> w[i] >> v[i];
7
8 }
9 long long int dp[100000+5];
10 memset(dp, 0, sizeof(dp));
11 for(int i = 0; i < N; i++){
      for(int j = W; j >= w[i]; j--){
12
13
          dp[j] = max(dp[j], dp[j - w[i]] + v[i]);
14
```

```
15 }
16 cout << dp[W] << endl;
```

#### 6.8 Fraction Floor Sum

```
1 / * [N/i] == M
  -> M <= N/i < M + 1
3 \rightarrow N/(M+1) < i <= N/M */
4 long long int N;
  cin >> N;
  long long int ans = 0;
6
  for(long long int i = 1; i <= N; i++){</pre>
      long long int M = N / i;
9
      long long int n = N / M;
      // 總共會有 n - i 個的 [N/i] 值都是 M
10
      ans += (n - i + 1) * M;
11
      // 更新跳過 以免重複計算
12
      i = n;
13
14 }
15 cout << ans << endl;
```

# 6.9 Homer Simpson

```
1 // dp 背包 - 時間/數量 - 漢堡
2
  int m, n, t;
3
  while(cin >> m >> n >> t){
      int dp[10000+5];
       memset(dp, -1, sizeof(dp));
6
       dp[0] = 0;
7
       for(int i = m; i <= t; i++){</pre>
8
           if(dp[i - m] != -1){
9
               dp[i] = max(dp[i], dp[i - m] + 1);
10
11
12
       for(int i = n; i <= t; i++){</pre>
           if(dp[i - n] != -1){
13
               dp[i] = max(dp[i], dp[i - n] + 1);
14
15
16
17
       if(dp[t] == -1){ // 時間無法剛好吃滿的時候
18
           for(int i = t; i >= 0; i--){
               if(dp[i] != -1){
19
                    cout << dp[i] << " " << t - i << endl;</pre>
20
                    break;
21
22
               }
23
           }
      }
24
25
       else{
           cout << dp[t] << endl;</pre>
26
27
28 }
```

#### 6.10 Let Me Count The Ways

```
1 // dp - 時間/數量 - 硬幣排序
2 long long int n, dp[30000+5];
3 int coin[] = {1, 5, 10, 25, 50};
4 memset(dp, 0, sizeof(dp));
5
  dp[0] = 1;
  for(int i = 0; i < 5; i++){</pre>
      for(int j = coin[i]; j < 30000+5; j++){</pre>
           if(dp[j - coin[i]] != -1){
               dp[j] += dp[j - coin[i]];
9
10
11
      }
12 }
13
  while(cin >> n){
14
       if(dp[n] == 1){
           cout << "There is only " << dp[n] << " way to</pre>
15
               produce " << n << " cents change.
               endl;
```

# 6.11 Luggage

```
1 // dp 背包 - 重量/是否成立
2 int t;
3 cin >> t;
4 cin.ignore();
5 while(t--){
       string str;
       getline(cin , str);
       vector<int> v;
8
       stringstream ss;
10
       int num, cnt = 0, sum = 0;;
11
       bool dp[4000+5];
12
       memset(dp, false, sizeof(dp));
       ss << str;
13
14
       while(ss >> num){
15
           cnt++;
16
           sum += num;
17
           v.emplace_back(num);
18
       if(sum & 1){
19
           cout << "NO" << endl;
20
21
           continue;
22
       dp[0] = true;
23
24
       for(int i = 0; i < v.size(); i++){</pre>
           for(int j = sum; j >= v[i]; j--){
25
               if(dp[j - v[i]]){
26
                    dp[j] = true;
27
28
29
           }
30
31
       cout << (dp[sum/2] ? "YES" : "NO") << endl;</pre>
32 }
```

#### 6.12 Number of Pairs

```
1 /* uper_bound ex:
  10 20 30 30 40 50
3 upper_bound for element 30 is at index 4
4 lower_bound ex:
5 10 20 30 40 50
6 lower_bound for element 30 at index 2 */
7 int t;
8 cin >> t;
9 while(t--){
10
       int n, 1, r;
11
       vector<int> v;
       cin >> n >> 1 >> r;
12
       int num;
13
14
       for(int i = 0; i < n; i++){
15
           cin >> num;
           v.emplace_back(num);
16
17
18
       sort(v.begin(), v.end());
19
       long long int ans = 0;
20
       for(int i = 0; i < n; i++){
21
           ans += (upper_bound(v.begin() + i + 1,
               v.end(), r - v[i])
               lower_bound(v.begin() + i + 1, v.end(), 1
               - v[i]));
22
       }
23
       cout << ans << endl;</pre>
24 }
```

# 6.13 SuperSale

```
1 // dp 背包 - 重量/價值/不可重複使用 - 舉重
2
  int t;
3
  cin >> t;
  while(t--){
      int n;
6
      cin >> n;
7
       for(int i = 0; i < n; i++){
8
           cin >> edge[i].p >> edge[i].w;
10
       int g, total = 0;
       cin >> g;
11
12
       for(int i = 0; i < g; i++){
           int pw, dp[30+5];
13
14
           cin >> pw;
15
           memset(dp, 0, sizeof(dp));
           for(int j = 0; j < n; j++){</pre>
16
17
               for(int k = pw; k >= edge[j].w; k--){
                   dp[k] = max(dp[k], dp[k - edge[j].w]
18
                        + edge[j].p);
19
20
           }
21
           total += dp[pw];
      }
22
       cout << total << endl;</pre>
23
24 }
```

# 6.14 Walking on the Safe Side

```
1 // dp - 地圖更新
2 int t;
  bool space = false;
  cin >> t;
  while(t--){
       if(space){
            cout << endl;</pre>
9
       else{
10
            space = true;
11
       int r, c;
12
13
       cin >> r >> c;
       cin.ignore();
14
       memset(mp, false, sizeof(mp));
memset(dp, 0, sizeof(dp));
15
16
17
       string str:
18
       for(int i = 0; i < r; i++){</pre>
            getline(cin, str);
19
20
            int n, num;
21
            stringstream ss(str);
22
            ss >> n;
23
            while(ss >> num){
24
                 mp[n][num] = true;
25
       }
26
27
       dp[1][1] = 1;
28
       for(int i = 1; i <= r; i++){
            for(int j = 1; j <= c; j++){</pre>
29
                 if(mp[i][j]){
30
31
                      continue:
32
33
                 if(i > 1){
34
                     dp[i][j] += dp[i-1][j];
35
                 if(j > 1){
36
                      dp[i][j] += dp[i][j-1];
38
39
40
41
       cout << dp[r][c] << endl;</pre>
42 }
```

# 6.15 Cutting Sticks

```
1 while (cin >> 1 && 1){
2
      cin >> n;
      vector<int> s(n+2);
3
      s[0] = 0;
      for(int i = 1; i <= n; ++i) cin >> s[i];
      s[++n] = 1; // 從現在開始 n 的數量變為 n + 1
      int dp[n+5][n+5];
      memset(dp, 0, sizeof(dp));
8
9
      for(int r = 2; r <= n; ++r){ // r: 切幾段 b: 起點
           c: 中間點 e: 終點
           for(int b = 0; b < n; ++b){
10
11
               if(b + r > n) break;
               int e = b + r;
12
13
               dp[b][e] = 0x3f3f3f3f;
14
               for(int c = b + 1; c < e; ++c){
15
                   dp[b][e] = min(dp[b][e], dp[b][c] +
                       dp[c][e] + s[e] - s[b]);
16
              }
17
          }
18
      cout << "The minimum cutting is " << dp[0][n] <<</pre>
19
            '." << endl;
20 }
```

# 6.16 Partitioning by Palindromes

```
2 bool check_palindromes(int lef, int rig){
      // 比較字串兩端都是迴文
3
      while(lef < rig){</pre>
          if(str[lef] != str[rig]) return 0;
5
6
          lef++:
7
           rig--;
      }
8
      return 1;
10 }
11 int main(){
12
      int t;
      cin >> t;
13
      while(t--){
14
          cin >> str;
15
16
          memset(dp, 0x3f3f3f3f, sizeof(dp));
17
           dp[0] = 0;
           for(int i = 0; i < str.size(); ++i)</pre>
18
               for(int j = 0; j <= i; ++j)</pre>
19
                   if(str[i] == str[j])
20
21
                       if(check_palindromes(j, i))
22
                           if(dp[i+1] > dp[j] + 1)
                               dp[i+1] = dp[j] + 1;
23
24
           cout << dp[str.size()] << endl;</pre>
      }
25
26 }
```

# 6.17 Ants Colony

```
1 /* LCA 最低共同祖先 */
2 const int maxn = 1e5 + 5;
  struct Edge{
      int v;
      int w;
6 };
7 int N:
8 vector<Edge> G[maxn];
9 int parent[maxn][20+5];
10 int depth[maxn], siz[maxn];
11 // 此 node 下有機顆 node
12 int dfs(int node, int dep){
      depth[node] = dep + 1;
13
14
      if(G[node].empty()){
15
          siz[node] = 1;
```

```
16
           return 1;
17
18
       int total = 1;
19
       for(auto i : G[node])
20
           total += dfs(i.v, dep + 1);
       siz[node] = total;
21
22
       return siz[node];
23 }
  // 找出每個節點的 2<sup>1</sup> 倍祖先
  // 2^20 = 1e6 > 200000
25
  void find_parent(){
       for(int i = 1; i < 20; i++)</pre>
27
           for (int j = 0; j < N; j++)
28
29
                parent[j][i] =
                    parent[parent[j][i-1]][i-1];
30 }
  // 求兩點的LCA (利用倍增法)
31
  int LCA(int a, int b){
32
       if (depth[b] < depth[a]) swap(a, b);</pre>
33
34
       if (depth[a] != depth[b]){
35
           int dif = depth[b] - depth[a];
           for (int i = 0; i < 20; i++){
36
               if (dif & 1) b = parent[b][i];
37
38
                dif >>= 1;
39
40
41
       if (a == b) return a;
       for (int i = 19; i >= 0; i--){
42
43
           if (parent[a][i] != parent[b][i]){
44
               a = parent[a][i];
45
               b = parent[b][i];
46
47
48
       return parent[a][0];
49 }
50 long long int dist[maxn];
  // 從 0 開始到每個點的距離
51
52
  void distance(){
       for (int u = 0; u < N; ++u){
53
54
           for(int i = 0; i < G[u].size(); ++i){</pre>
                dist[G[u][i].v] = dist[u] + G[u][i].w;
55
56
  int main(){
57
       while(cin >> N && N){
58
59
           memset(dist, 0, sizeof(dist));
60
           memset(parent, 0, sizeof(parent));
           memset(depth, 0, sizeof(depth));
61
           memset(siz, 0, sizeof(siz));
62
63
           for(int i = 0; i <= N; ++i){
64
               G[i].clear();
65
66
           for(int i = 1; i < N; ++i){</pre>
67
               int u. w:
68
                cin >> u >> w;
69
               G[u].push_back({i, w});
70
                parent[i][0] = u;
71
           find_parent();
72
73
           dfs(0, 0);
74
           distance();
75
           int s; cin >> s;
           bool space = false;
76
           for(int i = 0; i < s; ++i){</pre>
77
                int a, b;
78
79
                cin >> a >> b;
80
                int lca = LCA(a, b);
               if(space) cout << " ";</pre>
81
82
                space = true;
83
                cout << (dist[a] + dist[b]) - (dist[lca]</pre>
84
85
           cout << endl;</pre>
       }
86
87 }
```

# 6.18 Fill the Containers

```
1 /*binary search 變形*/
2 int binary_search(int arr[maxn], int lef, int rig,
       int mini){
       if(lef > rig) return mini;
       int amount = 1, fill = 0;
       int mid = (lef + rig) >> 1;
       for(int i = 0; i < n; ++i){
           if(amount > m) break;
8
           fill += arr[i];
           if(fill > mid){
9
                fill = arr[i];
11
                amount++;
12
13
       if(!flag && amount <= m) mini = mid;</pre>
14
       if(flag && amount == m) mini = mid;
15
       if(amount == m){
16
           flag = true;
17
           return binary_search(arr, lef, mid - 1, mid);
18
19
20
       else if(amount < m){</pre>
           return binary_search(arr, lef, mid - 1, mini);
21
22
23
       else{
24
           return binary_search(arr, mid + 1, rig, mini);
25
26 }
  int main(){
27
       int ca = 1;
28
       while(cin >> n >> m){
29
30
           flag = false;
           int arr[maxn];
31
32
           int maxi = 0, sum = 0;
           for(int i = 0; i < n; ++i){</pre>
33
                cin >> arr[i];
34
35
                sum += arr[i];
                maxi = max(maxi, arr[i]);
36
37
           cout << binary_search(arr, maxi, sum, maxi)</pre>
38
                << endl;
39
       }
40 }
```

#### 6.19 How Many O's

```
1 /*數論*/
  int main(){
3
       long long int n, m;
       while(cin >> n >> m && (n >= 0) && (m >= 0)){
           long long int total1 = 0, total2 = 0;
6
           long long int ten = 1, tmp = n-1;
7
           while(tmp >= 10){
               if(tmp % 10 == 0){
8
                    tmp /= 10;
                    total1 += (tmp - 1) * ten + ((n-1) %
10
                        ten) + 1;
               }
11
12
               else{
13
                    tmp /= 10;
14
                    total1 += tmp * ten;
15
16
               ten *= 10;
17
18
           ten = 1; tmp = m;
           while(tmp >= 10){
19
20
               if(tmp % 10 == 0){
21
                    tmp /= 10;
                    total2 += (tmp - 1) * ten + (m % ten)
22
23
               }
               else{
24
                    tmp /= 10;
25
                    total2 += tmp * ten;
26
```

# 7 Function

#### 7.1 strstr

```
1 #include <stdio.h>
  #include <string.h>
  int main(){
5
  char * c;
  char str1[1005], str2[1005];
6
  scanf("%s %s", str1, str2);
  c = strstr(str1, str2);
  if (c != NULL){
10
      printf("Yes\n");
11
12
  else printf("No\n");
13 }
14 // Input : Hello eLl
15 // Output : No
```

# 7.2 substr

```
int main(){
    string str; //abcdef
    cin >> str;

    string tmp;

    tmp = str.substr(0, 2); //ab
    str = str.substr(2); //cdef
    cout << tmp << " " << str;
    return 0;
}</pre>
```

# 7.3 map set

```
1 .begin( ) // Return iterator to beginning
2 .end( ) // Return iterator to end
3 . empty() // 檢查是否為空
4 . size( ) // 回傳大小
5 mp.insert(pair<char,int>('a',100))
6 st.insert(100) // 插入key \ value
7 .erase( ) // 刪掉指定key和他的value
8 .clear( ) // 清空整個 map
  m.find( )
9
10 cout << "a => " << mymap.find('a')->second << endl;</pre>
      // 找出 map 裡 key
          有沒有在裡面,如果有的話會回傳元素所在的iterator,否則何
12 s.count() // 返回某個值元素在 set 的 個 數
  while( !mymap.empty()){
13
      cout << mymap.begin()->first << " => " <<</pre>
14
          mymap.begin()->second << endl;</pre>
      mymap.erase(mymap.begin());
15
16 }
17 for (auto it = mymap.begin(); it != mymap.end(); ++it)
      cout << it->first << " => " << it->second << endl;</pre>
```

#### 7.4 vector

```
1 | v.erase(v.begin() + 5) //拿掉第六個數
2 | v.erase (v.begin(), v.begin() + 3); //拿掉前三個數
```

# 7.5 setprecision

```
1 / / 將數字的小數部分設定為固定長度
2 | cnt = 3.5555;
3 cout << fixed << setprecision(3) << cnt ;</pre>
4 // output : 3.555
  7.6 GCD LCM
1 int gcd(int a, int b){
2
     return (b == 0 ? a : gcd(b, a % b));
3 }
4 int lcm(int a, int b){
     return a * b / gcd(a, b);
6 }
8 /* 輾轉相除法 - 求兩數是否互質
9 如果兩數互質 最終結果其中一方為0時 另一方必為1
10 若兩數有公因數 最終結果其中一方為0時 另一方必不為1 */
11 while ( ( num1 %= num2 ) != 0 && ( num2 %= num1 ) !=
     0);
```

#### 7.7 reverse

```
1 int a[10] = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9};
2 reverse(a, a+5) // 轉換0~5
3
4 vector<int> v;
5 reverse(v.begin(), v.end());
6
7 string str = "123";
8 reverse(str.begin(), str.end());
9 cout << str << endl; //321</pre>
```

#### 7.8 CHAR

# 7.9 sort

#### 7.10 struct

# 7.11 deque

```
1 deque <int> que;
2 que.push_back(10);
3 que.push_front(20);
4 que.front()
5 que.back()
6 que.pop_front()
7 que.pop_back()
8 cout << "Element at position 2 : " << que.at(2) << endl;</pre>
```

# 7.12 python template

```
1 import math
2
  import operator
3
  try:
       while(1):
5
           listx = []
6
           listx.append("...")
7
           list_s = sorted(listx) # 小到大
8
          list_s = sorted(listx, reverse = True) #
9
               大到小
           # max(listx)
10
           # min(listx)
11
12
           # sum(listx)
13
           # len(listx)
           dicty = \{\}
14
           dicty[key] = "value"
15
           dicty = sorted(dicty.items()) # by key
16
17
           dicty = sorted(dicty.items(),
               key=operator.itemgetter(1)) # by value
           # EOF寫法
18
19
           # 階層 math.factorial(3) == 6
           # 絕對值 math.fabs(x)
20
           # 無條件進位 math.ceil(3.1) == 3
           # 無條件捨去 math.floor(2.9) == 2
22
23
           # C n 取 k math.comb(n, k)
24
           # math.gcd
           # math.lcm
25
           # e 次 x 幂 math.exp(x)
26
27
  except EOFError:
28
      pass
```