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1 Sync

1.1 Sync

```

1 int main(){
2     std::ios::sync_with_stdio(false);
3     // 開始寫程式
4 }

```

2 Data Structure

2.1 Binary Search

```

1 int binary_search(int arr[maxn], int lef, int rig,
2     int target){
3     if(lef > rig) return 0x3f3f3f3f;
4     int mid = (lef + rig) >> 1;
5     if(arr[mid] == target) return mid;
6     else if(arr[mid] > target){
7         return binary_search(arr, lef, mid - 1,
8             target);
9     }
10    else{
11        return binary_search(arr, mid + 1, rig,
12            target);
13    }
14 }

```

2.2 BIT

```

1 /* BIT Binary Index Tree */
2 #define lowbit(k) (k & -k)
3 void add(vector<int> &tr, int id, int val) {
4     for (; id <= n; id += lowbit(id)) {
5         tr[id] += val;
6     }
7 }
8 int sum(vector<int> &tr, int id) {
9     int ret = 0;
10    for (; id >= 1; id -= lowbit(id)) {
11        ret += tr[id];
12    }
13    return ret;
14 }

```

2.3 BWT

```

1 /* BWT 資料轉換演算法 */
2 void BWT(){
3     for(int i = 0; i < n; ++i){
4         if(back[i] == 0)
5             mini[zero++] = i;
6         for(int i = 0; i < n; ++i)
7             if(back[i] == 1)
8                 mini[zero++] = i;
9     }
10    int ptr = mini[0];
11    for(int i = 0; i < n; ++i){
12        cout << back[ptr] << " ";
13        ptr = mini[ptr];
14    }
15    cout << endl;
16 }

```

3 Divide and Conquer

3.1 count inversions

```

1 /*逆序數對*/
2 int arr[maxn], buf[maxn];
3 int count_inversions(int lef, int rig){
4     if(rig - lef <= 1) return 0;
5     int mid = (lef + rig)/2;
6     int ans = count_inversions(lef, mid) +
7         count_inversions(mid, rig);
8     int i = lef, j = mid, k = lef;
9     while(i < mid || j < rig){
10        if(i >= mid) buf[k] = arr[j++];
11    }
12 }

```

```

10     else if(j >= rig) buf[k] = arr[i++];
11     else{
12         if(arr[i] <= arr[j]) buf[k] = arr[i++];
13         else{
14             buf[k] = arr[j++];
15             ans += mid - i;
16         }
17     }
18     k++;
19 }
20 for(int k = lef; k < rig; ++k) arr[k] = buf[k];
21 return ans;
22 }

```

4 DP

4.1 Doubling

```

1  /* 倍增 */
2  int LOG = sqrt(N); // 2^LOG >= N
3  vector<int> arr(N);
4  vector<vector<int>> dp(N, vector<int>(LOG));
5  for(int i = 0; i < N; ++i) cin >> arr[i];
6  int L, Q, a, b;
7  cin >> L >> Q;
8  for(int i = 0; i < N; ++i){
9      dp[i][0] = lower_bound(arr.begin(), arr.end(),
10         arr[i] + L) - arr.begin();
11      if(dp[i][0] == N || arr[i] + L < arr[dp[i][0]])
12         dp[i][0] -= 1;
13 }
14 for(int i = 1; i < LOG; ++i)
15     for(int j = 0; j < N; ++j)
16         dp[j][i] = dp[dp[j][i-1]][i-1];
17 for(int i = 0; i < Q; ++i){
18     cin >> a >> b;
19     a--; // 要減減是因為arr的index從0開始但題目從1開始
20     b--;
21     if(a > b) swap(a, b);
22     int ans = 0;
23     for(int i = LOG - 1; i >= 0; --i){ // 從後往回推
24         if(dp[a][i] < b){
25             ans += (1 << i);
26             a = dp[a][i];
27         }
28     }
29     cout << ans + 1 << endl;
30 }

```

4.2 LCS

```

1  /* Longest Common Subsequence */
2  int LCS(string s1, string s2) {
3      int n1 = s1.size(), n2 = s2.size();
4      int dp[n1+1][n2+1] = {0};
5      // dp[i][j] = s1的前i個字元和s2的前j個字元
6      for (int i = 1; i <= n1; i++) {
7          for (int j = 1; j <= n2; j++) {
8              if (s1[i-1] == s2[j-1]) {
9                  dp[i][j] = dp[i-1][j-1] + 1;
10             } else {
11                 dp[i][j] = max(dp[i-1][j], dp[i][j-1]);
12             }
13         }
14     }
15     return dp[n1][n2];
16 }

```

4.3 LIS

```

1  /* Longest Increasing Subsequence */
2  int LIS(vector<int> &a) {
3      vector<int> s;
4      for (int i = 0; i < a.size(); i++) {
5          if (s.empty() || s.back() < a[i]) {
6              s.push_back(a[i]);
7          } else {
8              *lower_bound(s.begin(), s.end(), a[i],
9                 [](int x, int y) {return x < y;}) = a[i];
10             }
11         }
12     return s.size();
13 }

```

4.4 LIS 2

```

1  int LIS(vector<int> &a){
2      int len[a.size()];
3      for(int i = 0; i < a.size(); ++i) len[i] = 1;
4      int maxi = -1;
5      for(int i = 0; i < a.size(); ++i)
6          for(int j = i + 1; j < a.size(); ++j)
7              if(a[i] <= a[j]) len[j] = max(len[j],
8                 len[i] + 1);
9
10     for(int i = 0; i < a.size(); ++i)
11         maxi = max(maxi, len[i]);
12     return maxi;
13 }

```

4.5 Minimum Edit Distance

```

1  // 利用 dfs 輸出替換字串的步驟
2  void backtracking(int i, int j){
3      if(i == 0 || j == 0){
4          while(i > 0){
5              cout << cnt++ << " Delete " << i << endl;
6              i--;
7          }
8          while(j > 0){
9              cout << cnt++ << " Insert " << i + 1 <<
10                 ", " << strB[j-1] << endl;
11              j--;
12          }
13          return;
14      }
15      if(strA[i-1] == strB[j-1]){
16          backtracking(i-1, j-1);
17      }
18      else{
19          if(dis[i][j] == dis[i-1][j-1] + 1){
20              cout << cnt++ << " Replace " << i << ", "
21                 << strB[j-1] << endl;
22              backtracking(i-1, j-1);
23          }
24          else if(dis[i][j] == dis[i-1][j] + 1){
25              cout << cnt++ << " Delete " << i << endl;
26              backtracking(i-1, j);
27          }
28          else if(dis[i][j] == dis[i][j-1] + 1){
29              cout << cnt++ << " Insert " << i + 1 <<
30                 ", " << strB[j-1] << endl;
31              backtracking(i, j-1);
32          }
33      }
34  }
35  void MED(){
36      // 由於 B 是 0，所以 A 轉換成 B
37      // 時每個字元都要被刪除
38      for(int i = 0; i <= strA.size(); ++i) dis[i][0] =
39         i;
40      // 由於 A 是 0，所以 A 轉換成 B
41      // 時每個字元都需要插入

```

```

36     for(int j = 0; j <= strB.size(); ++j) dis[0][j] =
37         j;
38     for(int i = 1; i <= strA.size(); ++i){
39         for(int j = 1; j <= strB.size(); ++j){
40             // 字元相同代表不需修改，修改距離直接延續
41             if(strA[i-1] == strB[j-1]) dis[i][j] =
42                 dis[i-1][j-1];
43             else{
44                 // 取 replace, delete, insert
45                 // 最小，選其 +1 為最少編輯距離
46                 dis[i][j] = min(dis[i-1][j-1],
47                     min(dis[i-1][j], dis[i][j-1])) +
48                     1;
49             }
50         }
51     }
52 }

```

5 Enumerate

5.1 Halfcut Enumerate

```

1  /* 折半枚舉 */
2  void dfs(set<long long int> &s, int depth, int T,
3         long long int sum){
4      if(depth >= T){
5          s.insert(sum);
6          return;
7      }
8      dfs(s, depth + 1, T, sum); // 取或不取的概念
9      dfs(s, depth + 1, T, sum + A[depth]);
10 }
11 int main(){
12     int N, T;
13     set<long long int> s1, s2;
14     cin >> N >> T;
15     for(int i = 0; i < N; ++i) cin >> A[i];
16     dfs(s1, 0, N/2, 0); // 折半枚舉
17     dfs(s2, N/2, N, 0);
18     long long int ans = 0;
19     // 題目:枚舉集合 Sx 的數字 Sxi，找出 Sy
20     // 集合內小於等於 T-Sxi 中最大的數 Syj
21     for(auto &x : s1){
22         auto it = s2.upper_bound(T - x);
23         long long int y = *(--it);
24         if(x + y <= T) ans = max(ans, x + y);
25     }
26     cout << ans << endl;
27 }

```

6 Graph

6.1 SPFA

```

1  bool SPFA(int s){
2      // 記得初始化這些陣列
3      int cnt[1000+5], dis[1000+5];
4      bool inqueue[1000+5];
5      queue<int> q;
6
7      q.push(s);
8      dis[s] = 0;
9      inqueue[s] = true;
10     cnt[s] = 1;
11     while(!q.empty()){
12         int now = q.front();
13         q.pop();
14         inqueue[now] = false;
15
16         for(auto &e : G[now]){

```

```

17             if(dis[e.t] > dis[now] + e.w){
18                 dis[e.t] = dis[now] + e.w;
19                 if(!inqueue[e.t]){
20                     cnt[e.t]++;
21                     if(cnt[e.t] > m){
22                         return false;
23                     }
24                     inqueue[e.t] = true;
25                     q.push(e.t);
26                 }
27             }
28         }
29     }
30     return true;
31 }

```

6.2 Dijkstra

```

1  /* Dijkstra 最短路徑 */
2  struct Edge{
3      int v, w;
4  };
5  struct Item{
6      int u, dis;
7      // 取路徑最短
8      bool operator < (const Item &other) const{
9          return dis > other.dis;
10     }
11 };
12 int dis[maxn];
13 vector<Edge> G[maxn];
14 void dijkstra(int s){
15     for(int i = 0; i <= m; i++){
16         dis[i] = inf;
17     }
18     dis[s] = 0;
19     priority_queue<Item> pq;
20     pq.push({s, 0});
21     while(!pq.empty()){
22         // 取路徑最短的點
23         Item now = pq.top();
24         pq.pop();
25         if(now.dis > dis[now.u]){
26             continue;
27         }
28         // 把與 now.u 相連的點都跑一遍
29         for(Edge e : G[now.u]){
30             if(dis[e.v] > now.dis + e.w){
31                 dis[e.v] = now.dis + e.w;
32                 pq.push({e.v, dis[e.v]});
33             }
34         }
35     }
36 }

```

6.3 Floyd Warshall

```

1  void floyd_warshall(){
2      for(int i = 0; i < n; i++){
3          for(int j = 0; j < n; j++){
4              G[i][j] = INF;
5          }
6          G[i][i] = 0;
7      }
8      for (int k = 0; k < n; k++){ // 嘗試每一個中繼點
9          for (int i = 0; i < n; i++){ // 計算每一個 i 點與每一個 j 點
10             for (int j = 0; j < n; j++){
11                 G[i][j] = min(G[i][j], G[i][k] + G[k][j]);
12             }
13         }
14     }
15 }

```

```
14 }
15 }
```

6.4 Disjoint set Kruskal

```
1 struct Edge{
2     int u, v;
3     double w;
4     bool operator < (const Edge &rhs) const{
5         return w < rhs.w;
6     }
7 }edge[maxn * maxn];
8 vector<Edge> G[maxn]; // 紀錄有哪些邊在 MST 上
9 int parent[maxn];
10 // disjoint set
11 int find(int x){
12     return x == parent[x] ? x : parent[x] =
13         find(parent[x]);
14 }
15 bool unite(int a, int b){
16     int x = find(a);
17     int y = find(b);
18     if(x == y) return false;
19     parent[x] = y;
20     return true;
21 }
22 double kruskal(){
23     m = 0; // m: 邊的數量
24     for(int i = 0; i < n; ++i)
25         for(int j = i + 1; j < n; ++j)
26             edge[m++] = (Edge){i, j, dist(i, j)};
27     sort(edge, edge + m);
28     for(int i = 0; i < n; ++i){
29         parent[i] = i;
30         G[i].clear();
31     }
32     double total = 0.0;
33     int edge_cnt = 0;
34     for(int i = 0; i < m; ++i){
35         int u = edge[i].u, v = edge[i].v;
36         double cnt = edge[i].w;
37         if(unite(u, v)){
38             G[u].push_back((Edge){u, v, cnt});
39             G[v].push_back((Edge){v, u, cnt});
40             total += cnt;
41             if(++edge_cnt == n-1) break;
42         }
43     }
44     return total;
45 }
```

6.5 Disjoint set Weight

```
1 /* 帶權並查集 + 數論 */
2 const int maxn = 20000+5;
3 int n, Q, parent[maxn], value[maxn];
4 int find(int x){
5     if(parent[x] != x){
6         int tmp = parent[x];
7         parent[x] = find(parent[x]);
8         value[x] ^= value[tmp];
9     }
10    return parent[x];
11 }
12 bool unionSet(int x, int y, int v){
13     int xParent = find(x);
14     int yParent = find(y);
15     if(xParent == yParent) return (value[x] ^
16         value[y]) == v;
17     if(xParent == n) swap(xParent, yParent);
18     parent[xParent] = yParent;
19     value[xParent] = value[x] ^ v ^ value[y];
20     return true;
21 }
```

```
20 }
21 int main(){
22     int p, q, v, k, x, Case = 1;
23     while(scanf("%d %d", &n, &Q) && n && Q){
24         printf("Case %d:\n", Case++);
25         for(int i = 0; i <= n; ++i) parent[i] = i;
26         memset(value, 0, sizeof(value));
27         char str[100+5];
28         bool flag = false;
29         int facts = 0;
30         for(int i = 0; i < Q; ++i){
31             scanf("%s", str);
32             if(str[0] == 'I'){
33                 gets(str);
34                 facts++;
35                 if(flag) continue;
36                 int cnt = sscanf(str, "%d %d %d", &p,
37                     &q, &v);
38                 if(cnt == 2){
39                     v = q;
40                     q = n;
41                 }
42                 if(!unionSet(p, q, v)){
43                     flag = true;
44                     printf("The first %d facts are
45                         conflicting.\n", facts++);
46                 }
47             }
48             else{
49                 scanf("%d", &k);
50                 int ans = 0;
51                 bool check = true;
52                 map<int, int> mp;
53                 for(int j = 0; j < k; ++j){
54                     scanf("%d", &x);
55                     if(flag) continue;
56                     int xParent = find(x);
57                     ans ^= value[x];
58                     mp[xParent]++;
59                 }
60                 if(flag) continue;
61                 map<int, int>::iterator it;
62                 for(it = mp.begin(); it != mp.end();
63                     it++){
64                     if(it->second % 2){
65                         if(it->first != n){
66                             check = false;
67                             break;
68                         }
69                         else ans ^= value[it->first];
70                     }
71                 }
72                 if(check) printf("%d\n", ans);
73                 else printf("I don't know.\n");
74             }
75         }
76     }
77     printf("\n");
78 }
```

6.6 Bipatirate 2

```
1 /* 二分圖匹配 + 最小點覆蓋 */
2 const int maxn = 1000+5;
3 int R, C, N;
4 bool arr[maxn][maxn], visitX[maxn], visitY[maxn];
5 int matchX[maxn], matchY[maxn];
6 int dfs(int x){
7     visitX[x] = true;
8     for(int y = 1; y <= C; ++y){
9         if(arr[x][y] && !visitY[y]){
10             visitY[y] = true;
11             if(matchY[y] == 0 || dfs(matchY[y])){
12                 matchX[x] = y;
13                 matchY[y] = x;
14                 return 1;
15             }
16         }
17     }
18     return 0;
19 }
```

```

15     }
16 }
17 }
18 return 0;
19 }
20 int Match(){
21     int sum = 0;
22     memset(matchX, 0, sizeof(matchX));
23     memset(matchY, 0, sizeof(matchY));
24     for(int i = 1; i <= R; ++i){
25         memset(visitX, false, sizeof(visitX));
26         memset(visitY, false, sizeof(visitY));
27         sum += dfs(i);
28     }
29     return sum;
30 }
31 int main(){
32     while(cin >> R >> C >> N && R && C && N){
33         memset(arr, false, sizeof(arr));
34         memset(visitX, false, sizeof(visitX));
35         memset(visitY, false, sizeof(visitY));
36         int row, col;
37         for(int i = 0; i < N; ++i){
38             cin >> row >> col;
39             arr[row][col] = true;
40         }
41         int cnt = Match();
42         cout << cnt;
43         memset(visitX, 0, sizeof(visitX));
44         memset(visitY, 0, sizeof(visitY));
45         for(int i = 1; i <= R; ++i){
46             if(matchX[i] == 0) dfs(i);
47         }
48         for(int i = 1; i <= R; ++i)
49             if(!visitX[i]) cout << " r" << i;
50         for(int i = 1; i <= C; ++i)
51             if(visitY[i]) cout << " c" << i;
52         cout << endl;
53     }
54 }

```

6.7 Hungarian algorithm

```

1  /* 匈牙利演算法 */
2  const int maxn = 500+5;
3  int t, N, bn, gn, match[maxn];
4  bool visited[maxn];
5  vector<vector<int>> G(maxn);
6  struct People{
7      int h;
8      string music, sport;
9      People(){
10         People(int h, string music, string sport){
11             this->h = h;
12             this->music = music;
13             this->sport = sport;
14         }
15     }lef[maxn], rig[maxn];
16     bool check(People boy, People girl){
17         if(abs(boy.h - girl.h) <= 40 && boy.music ==
18             girl.music && boy.sport != girl.sport) return
19             true;
20         return false;
21     }
22     bool dfs(int s){
23         for(int i = 0; i < G[s].size(); ++i){
24             int v = G[s][i];
25             if(visited[v]) continue;
26             visited[v] = true;
27             if(match[v] == -1 || dfs(match[v])){
28                 match[v] = s;
29                 return true;
30             }
31         }
32         return false;
33     }
34     int Hungarian(){

```

```

33     int cnt = 0;
34     memset(match, -1, sizeof(match));
35     for(int i = 0; i < bn; ++i){
36         memset(visited, false, sizeof(visited));
37         if(dfs(i)) cnt++;
38     }
39     return cnt;
40 }
41 int main(){
42     cin >> t;
43     while(t--){
44         cin >> N;
45         bn = 0, gn = 0;
46         for(int i = 0; i <= N; ++i) G[i].clear();
47         int h;
48         string sex, music, sport;
49         for(int i = 0; i < N; ++i){
50             cin >> h >> sex >> music >> sport;
51             if(sex == "M") lef[bn++] = People(h,
52                 music, sport);
53             else rig[gn++] = People(h, music, sport);
54         }
55         for(int i = 0; i < bn; ++i){
56             for(int j = 0; j < gn; ++j)
57                 if(check(lef[i], rig[j]))
58                     G[i].emplace_back(j);
59         }
60         cout << N - Hungarian() << endl;
61     }
62 }

```

6.8 LCA

```

1  /*最低共同祖先*/
2  // 此 node 下有幾顆 node
3  int dfs(int node, int dep){
4      depth[node] = dep + 1;
5      if(G[node].empty()){
6          siz[node] = 1;
7          return 1;
8      }
9      int total = 1;
10     for(auto i : G[node])
11         total += dfs(i.v, dep + 1);
12     siz[node] = total;
13     return siz[node];
14 }
15 // 找出每個節點的 2^i 倍祖先
16 // 2^20 = 1e6 > 200000
17 void find_parent(){
18     for(int i = 1; i < 20; i++){
19         for(int j = 0; j < N; j++){
20             parent[j][i] =
21                 parent[parent[j][i-1]][i-1];
22         }
23     }
24     // 求兩點的LCA (利用倍增法)
25     int LCA(int a, int b){
26         if (depth[b] < depth[a]) swap(a, b);
27         if (depth[a] != depth[b]){
28             int dif = depth[b] - depth[a];
29             for (int i = 0; i < 20; i++){
30                 if (dif & 1) b = parent[b][i];
31                 dif >>= 1;
32             }
33         }
34         if (a == b) return a;
35         for (int i = 19; i >= 0; i--){
36             if (parent[a][i] != parent[b][i]){
37                 a = parent[a][i];
38                 b = parent[b][i];
39             }
40         }
41         return parent[a][0];
42     }
43 }

```

6.9 Trie

```

1  /* Trie 字典樹 */
2  struct Tire{
3      int path;
4      map<string, int> G[maxn];
5      void init(){
6          path = 1;
7          G[0].clear();
8      }
9      void insert(string str){
10         int u = 0;
11         string word = "";
12         for(int i = 0; i < str.size(); ++i){
13             if(str[i] == '\\'){
14                 if(!G[u].count(word)){
15                     G[u].clear();
16                     G[u][word] = path++;
17                 }
18                 u = G[u][word];
19                 word = "";
20             }
21             else word += str[i];
22         }
23     }
24     void put(int u, int space){
25         for(auto i = G[u].begin(); i != G[u].end(); ++i){
26             for(int j = 0; j < space; ++j){
27                 cout << " ";
28             }
29             cout << i->first << endl;
30             put(i->second, space + 1);
31         }
32     }
33 }tree;

```

7 Math

7.1 Hash

```

1  /* 建議搭配 Other - Stammering_Aliens 食用 */
2  #define ull unsigned long long int
3  const int maxn = 40000+5;
4  const ull seed = 131;
5  ull pw[maxn], hhash[maxn], hhash2[maxn];
6  char str[maxn];
7  void init(){
8      hhash[0] = 0;
9      for(int i = len-1; i >= 0; --i)
10         hhash[i] = (hhash[i+1] * seed + str[i]);
11 }

```

7.2 Math

- Inversion:
 $aa^{-1} \equiv 1 \pmod{m}$. a^{-1} exists iff $\gcd(a, m) = 1$.
- Linear inversion:
 $a^{-1} \equiv (m - \lfloor \frac{m}{a} \rfloor) \times (m \bmod a)^{-1} \pmod{m}$
- Fermat's little theorem:
 $a^p \equiv a \pmod{p}$ if p is prime.
- Euler function:
 $\phi(n) = n \prod_{p|n} \frac{p-1}{p}$
- Euler theorem:
 $a^{\phi(n)} \equiv 1 \pmod{n}$ if $\gcd(a, n) = 1$.
- Extended Euclidean algorithm:
 $ax + by = \gcd(a, b) = \gcd(b, a - \lfloor \frac{a}{b} \rfloor b) = bx_1 + (a - \lfloor \frac{a}{b} \rfloor b)y_1 = ay_1 + b(x_1 - \lfloor \frac{a}{b} \rfloor y_1)$

- Divisor function:
 $\sigma_x(n) = \sum_{d|n} d^x$. $n = \prod_{i=1}^r p_i^{a_i}$.
 $\sigma_x(n) = \prod_{i=1}^r \frac{p_i^{(a_i+1)x} - 1}{p_i^x - 1}$ if $x \neq 0$. $\sigma_0(n) = \prod_{i=1}^r (a_i + 1)$.
- Chinese remainder theorem:
 $x \equiv a_i \pmod{m_i}$.
 $M = \prod m_i$. $M_i = M / m_i$. $t_i = M_i^{-1}$.
 $x = kM + \sum a_i t_i M_i$, $k \in \mathbb{Z}$.

7.3 Math

```

1  // 中国剩余定理
2  LL CRT(int k, LL* a, LL* r) {
3      LL n = 1, ans = 0;
4      for (int i = 1; i <= k; i++) n = n * r[i];
5      for (int i = 1; i <= k; i++) {
6          LL m = n / r[i], b, y;
7          exgcd(m, r[i], b, y); // b * m mod r[i] = 1
8          ans = (ans + a[i] * m * b % n) % n;
9      }
10     return (ans % n + n) % n;
11 }
12
13 // 模意义下取幂
14 long long binpow(long long a, long long b, long long m) {
15     a %= m;
16     long long res = 1;
17     while (b > 0) {
18         if (b & 1) res = res * a % m;
19         a = a * a % m;
20         b >>= 1;
21     }
22     return res;
23 }
24
25 //
26 a^2=b^2+c^2-2bc\cos\{90\}^\circ=b^2+c^2

```

7.4 Modular Multiplicative Inverse

```

1  // 乘法逆元
2  // c++
3  void exgcd(int a, int b, int& x, int& y) {
4      if (b == 0) {
5          x = 1, y = 0;
6          return;
7      }
8      exgcd(b, a % b, y, x);
9      y -= a / b * x;
10 }
11
12 // python
13 def exgcd(a, b):
14     if b == 0:
15         x = 1
16         y = 0
17         return x, y
18     x1, y1 = exgcd(b, a % b)
19     x = y1
20     y = x1 - (a // b) * y1
21     return x, y

```

8 Function

8.1 CHAR

```

1  isdigit()
2  isalnum() // 判斷字母 // 數字
3  isalpha()

```

```

4 islower()
5 isupper()
6 isblank() // 判斷 即 space 和 \t
7 toupper()
8 tolower()

```

8.2 string

```

1 int main(){
2     string str;
3     while(cin >> str){
4         // substr 取 str idx 2~4 的值
5         cout << str.substr(2, 4) << endl;
6         // substr 取 str idx 2 以後的所有值
7         cout << str.substr(2) << endl;
8
9         string subst;
10        cin >> subst;
11        // str.append 連接字串
12        cout << str.append(subst) << endl;
13
14        char s[100], ss[100];
15        cin >> s >> ss;
16
17        char *p;
18        // strstr 回傳在s裡找到ss後的整個字串(從 ss
19        //     idx 0 到結束)
20        p = strstr(s, ss);
21        cout << p << endl;
22        // strstr 也可以單純用來找字串
23        if(p != NULL) cout << "yes" << endl;
24        else cout << "no" << endl;
25    }
}

```

8.3 setprecision

```

1 double cnt = 3.5555;
2 cout << fixed << setprecision(3) << cnt ;

```

8.4 GCD LCM

```

1 int gcd(int a, int b){
2     return (b == 0 ? a : gcd(b, a % b));
3 }
4 int lcm(int a, int b){
5     return a * b / gcd(a, b);
6 }
7
8 /* 輾轉相除法 - 求兩數是否互質
9 如果兩數互質 最終結果其中一方為0時 另一方必為1
10 若兩數有公因數 最終結果其中一方為0時 另一方必不為1 */
11 while ( ( num1 % num2 ) != 0 && ( num2 % num1 ) != 0 );

```

8.5 reverse

```

1 int a[10] = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9};
2 reverse(a, a + 5);
3
4 vector<int> v;
5 reverse(v.begin(), v.end());
6
7 string str = "123";
8 reverse(str.begin(), str.end());
9 cout << str << endl; //321

```

8.6 sort

```

1 priority_queue<int, vector<int>, less<int>> // 大到小
2 priority_queue<int, vector<int>, greater<int>> //
3     小到大
4 int arr[] = {4, 5, 8, 3, 7, 1, 2, 6, 10, 9};
5 sort(arr, arr+10);
6
7 vector<int> v;
8 sort(v.begin(), v.end()); //小到大
9
10 int cmp(int a, int b){
11     return a > b;
12 }
13 sort(v.begin(), v.end(), cmp); //大到小

```

8.7 map

```

1 int main(){
2     map<string, string> mp;
3     map<string, string>::iterator iter;
4     map<string, string>::reverse_iterator iter_r;
5
6     mp.insert(pair<string, string>("r000", "zero"));
7
8     mp["r123"] = "first";
9
10    for(iter = mp.begin(); iter != mp.end(); iter++){
11        cout<<iter->first<<" "<<iter->second<<endl;
12    }
13    for(iter_r = mp.rbegin(); iter_r != mp.rend();
14        iter_r++){
15        cout<<iter_r->first<<"
16        "<<iter_r->second<<endl;
17    }
18
19    iter = mp.find("r123");
20    mp.erase(iter);
21
22    iter = mp.find("r123");
23    if(iter != mp.end())
24        cout<<"Find, the value is
25        "<<iter->second<<endl;
26    else
27        cout<<"Do not Find"<<endl;
28
29    mp.clear();
30    mp.erase(mp.begin(), mp.end());
31 }

```

9 Other

9.1 Ants Colony

```

1 /* LCA 最低共同祖先 */
2 const int maxn = 1e5 + 5;
3 struct Edge{
4     int v;
5     int w;
6 };
7 int N;
8 vector<Edge> G[maxn];
9 int parent[maxn][20+5];
10 int depth[maxn], siz[maxn];
11 // 此 node 下有幾顆 node
12 int dfs(int node, int dep){
13     depth[node] = dep + 1;
14     if(G[node].empty()){
15         siz[node] = 1;
16         return 1;
17     }
18     int total = 1;

```

```

19     for(auto i : G[node])
20         total += dfs(i.v, dep + 1);
21     siz[node] = total;
22     return siz[node];
23 }
24 // 找出每個節點的 2^i 倍祖先
25 // 2^20 = 1e6 > 200000
26 void find_parent(){
27     for(int i = 1; i < 20; i++){
28         for (int j = 0; j < N; j++){
29             parent[j][i] =
                parent[parent[j][i-1]][i-1];
30     }
31 // 求兩點的LCA (利用倍增法)
32 int LCA(int a, int b){
33     if (depth[b] < depth[a]) swap(a, b);
34     if (depth[a] != depth[b]){
35         int dif = depth[b] - depth[a];
36         for (int i = 0; i < 20; i++){
37             if (dif & 1) b = parent[b][i];
38             dif >>= 1;
39         }
40     }
41     if (a == b) return a;
42     for (int i = 19; i >= 0; i--){
43         if (parent[a][i] != parent[b][i]){
44             a = parent[a][i];
45             b = parent[b][i];
46         }
47     }
48     return parent[a][0];
49 }
50 long long int dist[maxn];
51 // 從 0 開始到每個點的距離
52 void distance(){
53     for (int u = 0; u < N; ++u){
54         for(int i = 0; i < G[u].size(); ++i){
55             dist[G[u][i].v] = dist[u] + G[u][i].w;
56         }
57 }
58 int main(){
59     while(cin >> N && N){
60         memset(dist, 0, sizeof(dist));
61         memset(parent, 0, sizeof(parent));
62         memset(depth, 0, sizeof(depth));
63         memset(siz, 0, sizeof(siz));
64         for(int i = 0; i <= N; ++i){
65             G[i].clear();
66         }
67         for(int i = 1; i < N; ++i){
68             int u, w;
69             cin >> u >> w;
70             G[u].push_back({i, w});
71             parent[i][0] = u;
72         }
73         find_parent();
74         dfs(0, 0);
75         distance();
76         int s; cin >> s;
77         bool space = false;
78         for(int i = 0; i < s; ++i){
79             int a, b;
80             cin >> a >> b;
81             int lca = LCA(a, b);
82             if(space) cout << " ";
83             space = true;
84             cout << (dist[a] + dist[b]) - (dist[lca]
                * 2);
85         }
86         cout << endl;
87     }

```

9.2 Binary codes

1 /* BWT 資料轉換演算法 */

```

2 void BWT(){
3     for(int i = 0; i < n; ++i){
4         if(back[i] == 0){
5             mini[zero++] = i;
6         }
7         for(int i = 0; i < n; ++i){
8             if(back[i] == 1){
9                 mini[zero++] = i;
10            }
11            int ptr = mini[0];
12            for(int i = 0; i < n; ++i){
13                cout << back[ptr] << " ";
14                ptr = mini[ptr];
15            }
16            cout << endl;
17        }
18    }
19 }
20 int main(){
21     cin >> n;
22     for(int i = 0; i < n; ++i){
23         cin >> back[i];
24     }
25     zero = 0;
26     BWT();
27 }

```

9.3 Disk Tree

```

1 /* Trie 字典樹 */
2 const int maxn = 50000+5;
3 struct Tire{
4     int path;
5     map<string, int> G[maxn];
6     void init(){
7         path = 1;
8         G[0].clear();
9     }
10    void insert(string str){
11        int u = 0;
12        string word = "";
13        for(int i = 0; i < str.size(); ++i){
14            if(str[i] == '\\'){
15                if(!G[u].count(word)){
16                    G[path].clear();
17                    G[u][word] = path++;
18                }
19                u = G[u][word];
20                word = "";
21            }
22            else word += str[i];
23        }
24    }
25    void put(int u, int space){
26        for(auto i = G[u].begin(); i != G[u].end(); ++i){
27            for(int j = 0; j < space; ++j)
28                cout << " ";
29            cout << i->first << endl;
30            put(i->second, space + 1);
31        }
32    }
33 }tree;
34 int main(){
35     int n;
36     string str;
37     while(cin >> n && n){
38         tree.init();
39         for(int i = 0; i < n; ++i){
40             cin >> str;
41             str += '\\';
42             tree.insert(str);
43         }
44         tree.put(0, 0);
45         cout << endl;
46     }
47 }

```


10 DP

10.1 Crested Ibis vs Monster

```

1  /* dp 背包 - 重量/價值/可重複使用
2  9 3
3  8 3
4  4 2
5  2 1
6  0 3 3 3 3 3 3 3 6
7  0 2 2 2 2 3 3 3 5
8  0 1 1 2 2 3 3 3 4
9  因為這題可以重複使用同一條魔法
10 所以可以這樣 dp */
11 int a[10000+5], b[10000+5];
12 int dp[10000+5][10000+5];
13 int main(){
14     int h, n;
15     cin >> h >> n;
16     for(int i = 1; i <= n; i++)
17         cin >> a[i] >> b[i];
18     memset(dp, 0x3f3f3f3f, sizeof(dp));
19     dp[0][0] = 0;
20     for(int i = 1; i <= n; i++)
21         for(int j = 0; j <= h; j++)
22             dp[i][j] = min(dp[i-1][j], dp[i][max(0, j
23                 - a[i])] + b[i]);
24     cout << dp[n][h] << endl;

```

10.2 dpd Knapsack 1

```

1  /* dp 背包 - 時間/數量/價值 - 第幾分鐘符合
2  w[i]: 3
3  陣列每一格代表的意義是最大上限為 index
4  時可以放入的最大 value
5  0 0 0 30 30 30 30 30 30
6  w[i]: 4
7  0 0 0 30 50 50 50 80 80
8  w[i]: 5
9  0 0 0 30 50 60 60 80 90 */
10 int main(){
11     int N, W;
12     cin >> N >> W;
13     int w[100000+5], v[100000+5];
14     for(int i = 0; i < N; i++)
15         cin >> w[i] >> v[i];
16     long long int dp[100000+5];
17     memset(dp, 0, sizeof(dp));
18     for(int i = 0; i < N; i++)
19         for(int j = W; j >= w[i]; j--)
20             dp[j] = max(dp[j], dp[j - w[i]] + v[i]);
21     cout << dp[W] << endl;

```

11 Math

11.1 Big Mod

```

1  '''
2  Mod
3  pow(x, y, z) = x^y % z
4  '''
5  # python 如何讀取直到 EOF 用 try except
6  try:
7      while True:
8          # input().split() 用空格切開讀取一整行
9          # map(型態, input().split()) 才能把值全讀成
10             B, P, M = map(int, input().split())

```

```

11     print(pow(B, P, M))
12 except EOFError:
13     exit

```

11.2 How Many 0s

```

1  /* 數論 */
2  int main(){
3      long long int n, m;
4      while(cin >> n >> m && (n >= 0) && (m >= 0)){
5          long long int total1 = 0, total2 = 0;
6          long long int ten = 1, tmp = n-1;
7          while(tmp >= 10){
8              if(tmp % 10 == 0){
9                  tmp /= 10;
10                 total1 += (tmp - 1) * ten + ((n-1) %
11                     ten) + 1;
12             }
13             else{
14                 tmp /= 10;
15                 total1 += tmp * ten;
16             }
17             ten *= 10;
18         }
19         ten = 1; tmp = m;
20         while(tmp >= 10){
21             if(tmp % 10 == 0){
22                 tmp /= 10;
23                 total2 += (tmp - 1) * ten + (m % ten)
24                     + 1;
25             }
26             else{
27                 tmp /= 10;
28                 total2 += tmp * ten;
29             }
30             ten *= 10;
31         }
32         if(n == 0) total1--;
33         cout << total2 - total1 << endl;

```

11.3 ORXOR

```

1  /* bitwise operator 二進位制數論
2  如何切區段，之所以要 1<=n 是為了可以跑 000~111
3  i = 0, binary i = 000
4  0 : 1 5 7
5  i = 1, binary i = 001
6  1 : 1 5 7
7  i = 2, binary i = 010, 看得出來切了一刀
8  2 : 1 | 5 7
9  i = 3, binary i = 011
10 3 : 1 | 5 7
11 i = 4, binary i = 100, 為了要切在 index=2, 所以才要 1<<2
12 4 : 1 5 | 7
13 i = 5, binary i = 101
14 5 : 1 5 | 7
15 i = 6, binary i = 110
16 6 : 1 | 5 | 7
17 i = 7, binary i = 111
18 7 : 1 | 5 | 7
19 可以觀察出來，前兩位 bit 是 1 時代表的意義是切在哪裡
20 */
21 int main(){
22     int n; cin >> n;
23     int num[20+7];
24     memset(num, 0, sizeof(num));
25     for(int i = 1; i <= n; i++)
26         cin >> num[i];
27     // 不知道為甚麼只有 2147483647 給過
28     int mini = 2147483647;
29     // 1 <= n = n * 2

```

```

29     for(int i = 0; i < (1 << n); i++){
30         int XOR = 0, OR = 0;
31         for(int j = 1; j <= n; j++){
32             OR |= num[j];
33             if((i & (1 << j))){
34                 XOR ^= OR;
35                 OR = 0;
36             }
37         }
38         XOR ^= OR;
39         mini = min(mini, XOR);
40     }
41     cout << mini << endl;
42 }

```

12 Segement Tree

12.1 Frequent values

```

1  /* Segement Tree & RMQ (Range Sum Query)
2  idx:  1   2   3   4   5   6   7   8   9  10
3  num: -1  -1   1   1   1   1   3  10  10  10
4  fre:  2   2   4   4   4   4   1   3   3   3
5  border
6  left: 1   1   3   3   3   3   7   8   8   8
7  right:2   2   6   6   6   6   7  10  10  10 */
8  # define Lson(x) x << 1
9  # define Rson(x) (x << 1) + 1
10 const int maxn = 1e5+5;
11 struct Tree{
12     int lef, rig, value;
13 }tree[4 * maxn];
14 struct Num{
15     int lef, rig, value, fre;
16 }num[maxn];
17 // 建立 segement tree
18 void build(int lef, int rig, int x){
19     tree[x].lef = lef;
20     tree[x].rig = rig;
21     // 區塊有多長，題目詢問的重點
22     if(lef == rig){
23         tree[x].value = num[lef].fre;
24         return;
25     }
26     int mid = (lef + rig) >> 1;
27     build(lef, mid, Lson(x));
28     build(mid + 1, rig, Rson(x));
29     tree[x].value = max(tree[Lson(x)].value,
30         tree[Rson(x)].value);
31 }
32 // 查詢 segement tree
33 int query(int lef, int rig, int x){
34     // 題目所查詢的區間剛好在同個區塊上，num[lef].v
35     // == num[rig].v
36     if(num[lef].value == num[rig].value) return rig -
37         lef + 1;
38     int ans = 0;
39     // 查詢的左區間邊界切到區塊，且此區間有數個區塊
40     if(lef > num[lef].lef){
41         // 計算切到的區間大小
42         ans = num[lef].rig - lef + 1;
43         // 更新左邊界至被切區塊的右邊界加一，就不會切到邊
44         lef = num[lef].rig + 1;
45     }
46     // 查詢的右區間邊界切到區塊，且此區間有數個區塊
47     if(rig < num[rig].rig){
48         // 計算切到的區間大小，並找出最大
49         ans = max(ans, rig - num[rig].lef + 1);
50         // 更新右邊界
51         rig = num[rig].lef - 1;
52     }
53 }

```

```

50 // 如果左邊界大於右邊界，表示不需要再進行查詢直接回傳答案
51 if(lef > rig) return ans;
52 if(tree[x].lef >= lef && tree[x].rig <= rig)
53     return tree[x].value;
54 int mid = (tree[x].lef + tree[x].rig) >> 1;
55 if(lef <= mid) ans = max(ans, query(lef, rig,
56     Lson(x)));
57 if(mid < rig) ans = max(ans, query(lef, rig,
58     Rson(x)));
59 return ans;
60 }
61 int main(){
62     int n, q;
63     while(cin >> n && n){
64         cin >> q;
65         int start = 1;
66         for(int i = 1; i <= n; ++i){
67             cin >> num[i].value;
68             if(num[i].value != num[i-1].value){
69                 for(int j = start; j < i; ++j){
70                     num[j].rig = i - 1;
71                     num[j].fre = i - start;
72                 }
73                 start = num[i].lef = i;
74             }
75             else num[i].lef = start;
76         }
77         // 最後一段 [start, n]
78         for(int j = start; j <= n; ++j){
79             num[j].rig = n;
80             num[j].fre = n - start + 1;
81         }
82         build(1, n, 1);
83         int lef, rig;
84         for(int i = 0; i < q; ++i){
85             cin >> lef >> rig;
86             cout << query(lef, rig, 1) << endl;
87         }
88     }
89 }

```

13 Dijkstra

13.1 Walk Through the Forest

```

1  /* Dijkstra + 路徑最優化 DP */
2  const int inf = 0x3f3f3f3f;
3  const int maxn = 1000+5;
4  int n, m;
5  struct Edge{
6      int v, w;
7  };
8  struct Item{
9      int u, dis;
10     bool operator < (const Item &other) const{
11         return dis > other.dis;
12     }
13 };
14 int dis[maxn];
15 long long int dp[maxn];
16 vector<Edge> G[maxn];
17 vector<int> path[maxn];
18 void dijkstra(int s){
19     for(int i = 0; i <= n; ++i){
20         dis[i] = inf;
21     }
22     dis[s] = 0;
23     priority_queue<Item> pq;
24     pq.push({s, 0});
25     while(!pq.empty()){
26         Item now = pq.top();
27         pq.pop();
28     }

```

```

29     if(now.dis > dis[now.u]){
30         continue;
31     }
32
33     for(Edge e: G[now.u]){
34         if(dis[e.v] > now.dis + e.w){
35             dis[e.v] = now.dis + e.w;
36             pq.push({e.v, dis[e.v]});
37         }
38     }
39 }
40 }
41 long long int dfs(int u){
42     // ans 是 pointer, 指向 dp[u] 的記憶體位址
43     // 對於 ans 的 value 改變會記錄在 dp[u]
44     long long int& ans = dp[u];
45     if(ans != -1) return ans;
46     if(u == 2) return ans = 1;
47     ans = 0;
48     for(int i = 0; i < path[u].size(); ++i)
49         ans += dfs(path[u][i]);
50     return ans;
51 }
52 int main(){
53     while(cin >> n && n){
54         cin >> m;
55         for(int i = 0; i <= n; ++i) G[i].clear();
56         int u, v, w;
57         for(int i = 0; i < m; ++i){
58             cin >> u >> v >> w;
59             G[u].push_back({v, w});
60             G[v].push_back({u, w});
61         }
62         dijkstra(2); // dijkstra
63         // 紀錄從終點到每個點的距離
64         memset(dp, -1, sizeof(dp));
65         for(int i = 1; i <= n; ++i){
66             path[i].clear();
67             for(int j = 0; j < G[i].size(); ++j){
68                 int v = G[i][j].v;
69                 // 如果到 v 的距離比到 i
70                 // 遠, 代表從起點經過 i 再到 v
71                 if(dis[i] > dis[v])
72                     path[i].push_back(v);
73             }
74             cout << dfs(1) << endl;
75 }

```

14 Kruskal

14.1 Qin Shi Huang Road System

```

1  /* kruskal disjoint set dfs */
2  const int maxn = 1000 + 5;
3  int n, m;
4  int x[maxn], y[maxn], p[maxn];
5  struct Edge{
6      int u, v;
7      double w;
8      bool operator < (const Edge &rhs) const{
9          return w < rhs.w;
10     }
11 }edge[maxn * maxn];
12 vector<Edge> G[maxn];
13 int parent[maxn];
14 // 計算兩點之間的距離
15 double dist(int a, int b){
16     double x2 = (x[a] - x[b]) * (x[a] - x[b]);
17     double y2 = (y[a] - y[b]) * (y[a] - y[b]);
18     return sqrt(x2 + y2);
19 }
20 // disjoint set

```

```

21 int find(int x){
22     return x == parent[x] ? x : parent[x] =
23         find(parent[x]);
24 }
25 bool unite(int a, int b){
26     int x = find(a);
27     int y = find(b);
28     if(x == y) return false;
29     parent[x] = y;
30     return true;
31 }
32 double kruskal(){
33     m = 0; // m: 邊的數量
34     for(int i = 0; i < n; ++i)
35         for(int j = i + 1; j < n; ++j)
36             edge[m++] = (Edge){i, j, dist(i, j)};
37     sort(edge, edge + m);
38     for(int i = 0; i < n; ++i){
39         parent[i] = i;
40         G[i].clear();
41     }
42     double total = 0.0;
43     int edge_cnt = 0;
44     for(int i = 0; i < m; ++i){
45         int u = edge[i].u, v = edge[i].v;
46         double cnt = edge[i].w;
47         if(unite(u, v)){
48             G[u].push_back((Edge){u, v, cnt});
49             G[v].push_back((Edge){v, u, cnt});
50             total += cnt;
51             if(++edge_cnt == n-1) break;
52         }
53     }
54     return total;
55 }
56 double maxcost[maxn][maxn];
57 bool visited[maxn];
58 void dfs(int u){
59     visited[u] = true;
60     for(int i = 0; i < G[u].size(); ++i){
61         int v = G[u][i].v;
62         if(visited[v]) continue;
63         double cost = G[u][i].w;
64         maxcost[u][v] = maxcost[v][u] = cost;
65         // 更新 MST 樹上的點到 v 點的距離
66         for(int j = 0; j < n; ++j)
67             if(visited[j])
68                 maxcost[j][v] = maxcost[v][j] =
69                     max(maxcost[j][u], cost);
70     }
71     dfs(v);
72 }
73 void solve(){
74     double total = kruskal();
75     memset(maxcost, 0, sizeof(maxcost));
76     memset(visited, false, sizeof(visited));
77     dfs(0);
78     double ans = -1;
79     // 把所有點都遍歷一次
80     for(int i = 0; i < n; ++i)
81         for(int j = i + 1; j < n; ++j)
82             ans = max(ans, (p[i] + p[j]) / (total -
83                 maxcost[i][j]));
84     printf("%.2lf\n", ans);
85 }
86 int main(){
87     int t;
88     scanf("%d", &t);
89     while(t--){
90         scanf("%d", &n);
91         for(int i = 0; i < n; ++i)
92             scanf("%d%d%d", &x[i], &y[i], &p[i]);
93         solve();
94     }
95     return 0;
96 }

```