Roll No. ☐☐☐☐☐☐☐☐☐☐☐☐ **Total No. of Pages :03**

Total No. of Questions :18

B.Tech.(Agriculture Engg./Automation & Robotics/Automation Engg./Civil Engg./Computer Science & Engg./Electrical & Electronics Engg./Electrical Engg./Electronics & Communication Engg./Electronics & Electrical Engg./Information Technology/Mechanical Engg.)/ B.Tech. (CSE/ECE) (PIT)(Sem.–1)

MATHEMATICS-I

Subject Code : BTAM-101-18

M.Code : 75353

Date of Examination : 01-07-22

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B &C have FOUR questions each.**
3. **Attempt any FIVE questions from SECTION B& C carrying EIGHT marks each.**
4. **Select atleast TWO questions from SECTION - B &C.**

SECTION-A

Solve the following :

1) What is maximum value of function $f(x) = -x^2$.

2) Find the equation of normal line to the surface $xyz = 6$ at (1, 2, 3).

3) Show that the function $f(x,y) = \begin{cases} \dfrac{xy}{2y^2+x^2}; & (x,y) \neq (0,0) \\ 0; & (x,y) = (0,0) \end{cases}$ is discontinuous at (0, 0).

4) Evaluate $\lim_{x \to \frac{\pi}{2}} \dfrac{\log\left(x - \frac{\pi}{2}\right)}{\tan x}$.

5) Calculate approximate value of $\sqrt{10}$ to two decimal places by Taylor's theorem.

6) Evaluate $\int_0^1\int_1^a \frac{1}{xy}\,dydx$.

7) Examine the nature of the series $1+\frac{1}{2}+\frac{1}{4}=\frac{1}{8}--------\infty$.

8) Define orthogonal matrices with example.

9) Show that (1, 1, 2) is an eigen vector of the matrix $A=\begin{bmatrix}3 & 1 & -1\\ 2 & 2 & -1\\ 2 & 2 & 0\end{bmatrix}$ corresponding to the eigen value 2.

10) Verify Cayley Hamilton theorem for the matrix $A=\begin{bmatrix}1 & 2\\ 2 & -1\end{bmatrix}$.

SECTION-B

11) a) Verify Cauchy's mean value theorem for $f(x) = \log x, g(x) = \frac{1}{x}$ in $[1,e]$.

b) Apply Maclaurin's theorem with Lagrange's remainder to function $f(x) = \cos x$.

12) Discuss the convergence of the following improper integral

a) $\int_0^\infty \frac{1}{b^2x^2+a^2}\,dx$ b) $\int_1^2 \frac{x+1}{\sqrt{x-1}}\,dx$.

13) Show that the rectangular solid of maximum volume that can be inscribed in a given sphere is a cube.

14) a) Evaluate by changing the order of integration of $\int_0^1\int_x^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2-y^2}}\,dy\,dx$.

b) Find the volume enclosed between the cylinders $x^2 + y^2 = 2ax$ and $z^2 = 2ax$.

SECTION-C

15) a) Discuss the convergence or divergence of the series $\sum \frac{n^p}{(n+1)^q}$.

b) Test the convergence of $\sum_{n=2}^{\infty} \frac{1}{[\log(\log n)]^n}$.

16) a) Test the convergence of $1+\frac{(1+\alpha)}{(1+\beta)}+\frac{(1+\alpha)(1+2\alpha)}{(1+\beta)(1+2\beta)}+\frac{(1+\alpha)(1+2\alpha)(1+3\alpha)}{(1+\beta)(1+2\beta)(1+3\beta)}+\ldots\ldots.$

b) Discuss the convergence or divergence of the series $\sum_{n=2}^{\infty} \frac{n+\sqrt{n}}{n^2-n}$.

17) a) Use Gauss Jordan method to find the inverse of a matrix $\begin{bmatrix} 2 & 4 & 3 & 2 \\ 3 & 6 & 5 & 2 \\ 2 & 5 & 2 & -3 \\ 4 & 5 & 14 & 14 \end{bmatrix}$.

b) Find a matrix B which transforms $A=\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ into a diagonal form.

18) Determine the values of a and b for which the system $\begin{bmatrix} 3 & -2 & 1 \\ 5 & -8 & 9 \\ 2 & 1 & a \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} b \\ 3 \\ -1 \end{bmatrix}$ has

a) a unique solution,

b) no solution,

c) infinitely many solutions.

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.