

# A Two-stage Otsu's Thresholding Based Method on a 2D Histogram

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## Abstract

*We propose a fast and robust thresholding method that can overcome the shortcoming of the traditional and two-dimensional (2D) Otsu's methods. Our method takes advantage of the gradient gray level to divide the 2D histogram region, and applies the traditional Otsu's thresholding method twice on two projection histograms to separate the regions of interest from the background. The experimental results show that our method is robust against noise, and it requires less computational time than the 2D Otsu's and four other thresholding methods, which also give highly satisfactory and comparable results.*

## Index Terms

*Image segmentation, Thresholding, 2D Otsu's method, Two-dimensional histogram.*

## 1. Introduction

The objective of thresholding is to extract objects or regions of interest in an image from the background based on its gray level distribution. Many thresholding methods are exhaustively described and evaluated based on different error measures in [1]. One well-known thresholding method is Otsu's [2] which selects the optimal threshold by maximizing the between-class variance. This traditional or one-dimensional (1D) Otsu's method becomes widely used for its satisfactory results, easy computation, and broad scope of applications. However, using of only a 1D histogram of an image cannot reflect spatial information between image pixels, it is difficult to obtain satisfactory results when images contain noise. Lui *et al.* [3] thus extended using of a 1D histogram into a two-dimensional (2D) histogram, which utilizes not

only the gray level distribution of pixels, but also the average gray level distribution of their neighborhood to select the optimal threshold vector. This method gives better thresholding results than 1D Otsu's method, but it requires longer execution time. Many techniques thus were proposed to reduce time spent on computation and still maintain reasonable thresholding results. Gong *et al.* [4] proposed a fast recursive technique that can efficiently reduce computational time. Dongju *et al.* [5] proved that K-means is equivalent to the objective function of Otsu's method. K-means thus can be extended to a 2D thresholding method, which performs more efficiently than Otsu's. Ningbo *et al.* [6] presented a projection of a 2D histogram for searching of the optimal threshold. This method greatly enhances thresholding execution time, but it requires large space for three look-up tables. Chen *et al.* [7] pointed out the weakness of region division by a threshold vector in the 2D Otsu's method. They proposed the 2D Otsu's method on a gray level-gradient histogram, however, a good initialization is required.

## 2. Two-dimensional Otsu's method

Given an image  $f(x, y)$  represented by  $L$  gray levels and the total number of pixels in the image,  $N$ . Let the neighborhood average image  $g(x, y)$  represented by  $L$  gray levels.

For each pixel in the image, we can obtain a pair  $(i, j)$ , where  $i$  is the original gray level appeared in  $f(x, y)$ , and  $j$  is the neighborhood average gray level appeared in  $g(x, y)$ . Let  $c_{ij}$  denote the frequency of pair  $(i, j)$ , and its joint probability can be expressed as  $p_{ij} = \frac{c_{ij}}{N}$ , where  $\sum_{i=0}^{L-1} \sum_{j=0}^{L-1} p_{ij} = 1$ ,  $i$  and  $j = 0, 1, \dots, L-1$ .

Given an arbitrary threshold vector  $(s, t)$ . This threshold vector divide the 2D histogram into four regions as shown in Figure 1(a). The regions  $A$  and  $D$  represent the object and the background, respectively.

The regions  $B$  and  $C$  represent edges and noise. Let two classes  $C_0$  and  $C_1$  represent the object and the background, respectively; and  $\omega_i$  and  $\mu_i$  represent the probability and mean value vector of the class  $C_i$ , respectively. The probabilities of  $C_0$  and  $C_1$  can be denotes as

$$\omega_0 = \sum_{i=0}^s \sum_{j=0}^t p_{ij}, \quad \omega_1 = \sum_{i=s+1}^{L-1} \sum_{j=t+1}^{L-1} p_{ij}.$$

The mean value vectors of  $C_0$  and  $C_1$  can be expressed as

$$\begin{aligned} \mu_0 &= (\mu_{0i}, \mu_{0j})^T \\ &= \left( \sum_{i=0}^s \sum_{j=0}^t \frac{ip_{ij}}{\omega_0}, \sum_{i=0}^s \sum_{j=0}^t \frac{j p_{ij}}{\omega_0} \right)^T, \end{aligned} \quad (1)$$

$$\begin{aligned} \mu_1 &= (\mu_{1i}, \mu_{1j})^T \\ &= \left( \sum_{i=s+1}^{L-1} \sum_{j=t+1}^{L-1} \frac{ip_{ij}}{\omega_1}, \sum_{i=s+1}^{L-1} \sum_{j=t+1}^{L-1} \frac{j p_{ij}}{\omega_1} \right)^T \end{aligned} \quad (2)$$

The total mean vector of the 2D histogram is

$$\mu_T = (\mu_{Ti}, \mu_{Tj})^T = \left( \sum_{i=0}^{L-1} \sum_{j=0}^{L-1} ip_{ij}, \sum_{i=0}^{L-1} \sum_{j=0}^{L-1} j p_{ij} \right)^T. \quad (3)$$

In most cases, the probability that is away from the diagonal can be negligible. It can easily verify the following relations:  $\omega_0 + \omega_1 \approx 1$ , and  $\mu_T \approx \omega_0 \mu_0 + \omega_1 \mu_1$ . The between-class discrete matrix is defined as

$$S_b = \sum_{k=0}^1 [(\mu_k - \mu_T)(\mu_k - \mu_T)^T]. \quad (4)$$

The trace of discrete matrix could be expressed as

$$Tr S_b(s, t) = \sum_{k=0}^1 (\omega_k [(\mu_{ki} - \mu_{Ti})^2 + (\mu_{kj} - \mu_{Tj})^2]). \quad (5)$$

The optimal threshold is thus defined as

$$(s^*, t^*) = \arg \max_{0 \leq s, t \leq L-1} Tr S_b(s, t). \quad (6)$$

Note that pixels that satisfy conditions  $f(x, y) \leq s^*$  and  $g(x, y) > t^*$ ; and  $f(x, y) > s^*$  and  $g(x, y) \leq t^*$  are ignored and are set to either 0 or 1. In this paper, we set to 1 in the experiments.

### 3. Proposed method

We observe that pixels of the object, the background, edges, and noise have their own unique characteristics. Interior pixels of the object have their original gray

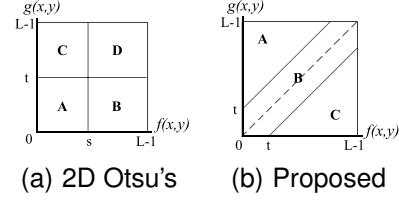


Figure 1. Region division of the 2D histogram

levels close to their average gray levels; and this characteristic is also applied with interior pixels of the background. However, pixels representing edges and noise have their original gray levels far different from their average gray levels.

We use the gradient gray level to reflect this gray level dissimilarity values. It can be noticed that in the 2D histogram, the gradient magnitude values along the diagonal are zero, and are increased where coordinates  $(i, j)$  are far away from the diagonal. In the same way, the small gradient magnitude values likely appears on pixels representing the object and the background while the large gradient magnitude values likely appears on pixels representing edges and noise. Therefore, the probability of classes near the diagonal can be approximated to 1, while the probability of classes far away from the diagonal is approximated to 0. It is obvious that the region division of the traditional 2D's Otsu method does not suitably separate the region. We can thus take advantage from the observed characteristics such that the 2D histogram can be divided into regions that separate the object and background from edges and noise. We can compute a gradient gray level histogram from the 2D histogram by

$$H_{gradient}(x) = \sum_{x=|i-j|} p_{ij}, \quad (7)$$

where  $i$  and  $j = 0, 1, \dots, L-1$ .

In the next step, we apply the traditional Otsu's method on the 1D histogram,  $H_{gradient}$ , to get an optimal threshold of gradient gray level,  $t$ , for region division of the 2D histogram. We apply  $t$  to generate two lines that are parallel to the diagonal so that the 2D histogram can be divided into three regions as shown in Figure 1(b). The region  $B$  contains coordinates that have lower gradient values than  $t$ , so it is the area close along the diagonal (dashed line). The coordinates in the region  $B$  highly represent either the object or the background. The regions  $A$  and  $C$  contain coordinates that have higher gradient values than  $t$ , so they are far away from the diagonal. The coordinates in these two regions possibly represent either edges or noise.

We then project the 2D histogram based on each region's property to generate a new 1D histogram.

Coordinates in the region  $B$  are projected by their original gray levels. Coordinates in the regions  $A$  and  $C$  are projected by their average gray levels instead of their original gray levels, since they are possibly either edge or noise pixels and ignoring them causes unsatisfactory results. The new 1D histogram can be thus expressed as

$$H_{new}(y) = \sum_{|y-j| \leq t} p_{yj} + \sum_{|i-y| > t} p_{iy}, \quad (8)$$

where  $y$ ,  $i$  and  $j = 0, 1, \dots, L-1$ . The first term represents the projection by original gray levels. The second term represents the projection by average gray levels.

Finally, we apply the 1D Otsu's method on  $H_{new}$  to select the optimal threshold,  $s$ . Currently, we have two thresholds,  $s$  and  $t$ , for image binarization. The binary image  $t(x, y)$  can be obtained by

$$t(x, y) = \begin{cases} 0 & \text{if } |f(x, y) - g(x, y)| \leq t \text{ and } f(x, y) \leq s, \\ 0 & \text{if } |f(x, y) - g(x, y)| > t \text{ and } g(x, y) \leq s, \\ 1 & \text{otherwise.} \end{cases} \quad (9)$$

#### 4. Experimental Results

We conducted all experiments on a personal computer with 2.0 GHz Intel(R) Core(TM)2 Duo CPU and 4 GB DDR II memory. We implemented the proposed method in Visual C++ with OpenCV, and used Scilab to generate noised added images for noise tolerant tests including Salt&Pepper noise and Gaussian noise.

Salt&Pepper noise is represented by noise density ( $\delta$ ), the probability of swapping a pixel. Gaussian noise is represented by mean ( $\mu$ ) and variance ( $\sigma^2$ ). In our experiments, we used only  $\mu = 0$ . We compare our method with the 1D Otsu's method, 2D Otsu's methods for both traditional and recursive ones, K-means based method, and Ningbo's method because they are based on Otsu's. For each experiment that the ground truth is available, we use misclassification error (ME) to reflect the number of background pixels wrongly assigned to the foreground and vice versa; and we use modified Hausdorff distance (MHD) to measure the shape distortion of the thresholded image compared with the ground truth. ME and MHD are defined in [1].

In the first experiment, we demonstrate the robustness of each method in the presence of noise. We selected an image as our test image from *Segmentation evaluation database* [8]. Figure 2(a) and 2(b) show the test image and its ground truth, respectively. We added noise to the test image to generate new 51 images with

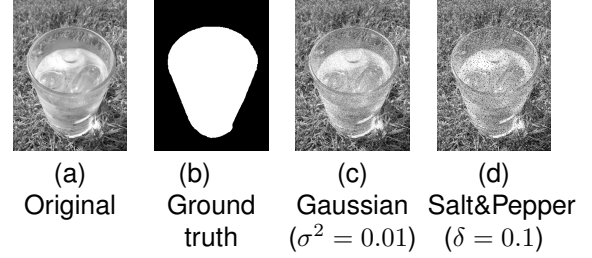
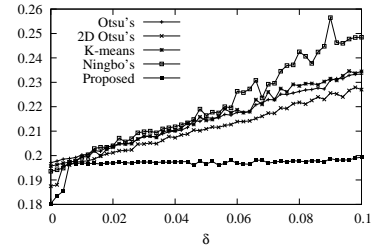
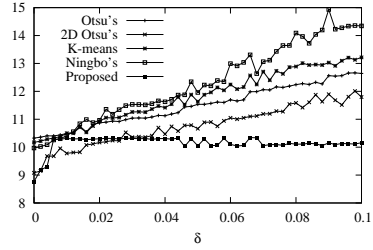


Figure 2. The image set with sample noise added images .



(a) ME

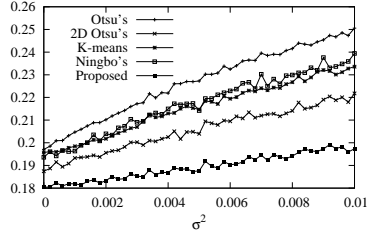


(b) MHD

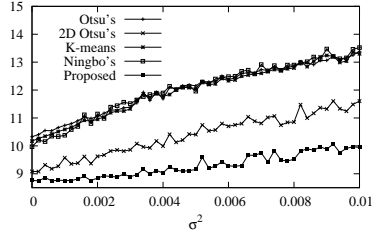
Figure 3. Comparison of ME and MHD of the test images with Salt&Pepper noise added at various  $\delta$  .

Salt&Pepper noise using  $\delta$  that are vary from 0 to 0.1, and the other 51 images with Gaussian noise using  $\sigma^2$  that are vary from 0 to 0.01. Figure 2(c) and 2(d) show example noise added images. For the test image itself, it can be challenged to segment such that some background pixels present similar gray levels as of the object. We segmented these 102 noise added images.

From the evaluation results in the presence of Salt&Pepper noise shown in Figure 3, both ME and MHD values of our method are lower than those of the other methods except MHD values on test images when  $\delta$  are lower than 0.025, MHD values of our method are higher than 2D Otsu's method. It indicates that our method shows the best matching of the object and the background, and also gives the smallest amount of shape distortion. From the evaluation results in the presence of Gaussian noise shown in Figure 4, both



(a) ME



(b) MHD

Figure 4. Comparison of ME and MHD of the test images with Gaussian noise added at various  $\sigma^2$ .  
Table 1.  $\overline{ME}$ ,  $\overline{MHD}$ , and  $\overline{T}$  over 200 real images.

Method	$\overline{ME}$	$\overline{MHD}$	$\overline{T}$ (ms)
1D Otsu's	0.217102	19.601307	11.48
2D Otsu's (Traditional)	0.214391	19.641495	2977.93
2D Otsu's (Recursive)	0.214391	19.641495	23.02
K-means	0.228071	19.650166	66.89
Ningbo's	0.214173	19.690172	20.54
Proposed	0.212603	19.662943	16.62

ME and MHD values of our method on the test images are lower than those of the other methods.

In the second experiment, we tested the methods with 200 real images from *Segmentation evaluation database* [8], where the ground truth of each image has been provided. Segmentation results can be seen at <http://give.cpe.ku.ac.th/thresholding/two-stage-thresholding.php>. The average ME ( $\overline{ME}$ ) and MHD ( $\overline{MHD}$ ) values with the average computational time ( $\overline{T}$ ) over 200 images of each method are shown in Table 1. From the results, it can be seen that the average computational time of our method is lower than that of the other methods, except the 1D Otsu's method. The average ME value of our method is lower than that of the other methods, however, the average MHD value is higher than that of the other methods, except Ningbo's method. Since we replace the original gray level with the average gray level in our method; the number of pixels that wrongly assigned over the whole image is reduced, but the number of pixels that wrongly assigned on edges is slightly increased.

## 5. Conclusions

We presented an improved thresholding method to overcome the shortcoming of the 1D and 2D Otsu's methods. The method takes advantage from the characteristic of the gradient gray level to divide a 2D histogram into meaningful regions; and also uses projections of the 2D histogram to reduce the search space for the optimal threshold. The experimental results show that our method gives satisfactory results, it is robust against noise, and it requires less computational time than the other methods, which give comparable results.

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## References

- [1] M. Sezgin and B. Sankur, "Survey over image thresholding techniques and quantitative performance evaluation," *Journal of Electronic Imaging*, vol. 13, no. 1, pp. 146–168, 2004.
- [2] N. Otsu, "A threshold selection method from gray-level histograms," *Systems, Man and Cybernetics, IEEE Transactions on*, vol. 9, no. 1, pp. 62–66, 1979.
- [3] J. Liu, W. Li, and Y. Tian, "Automatic thresholding of gray-level pictures using two-dimension otsu method," in *Circuits and Systems, 1991. Conference Proceedings, China., 1991 International Conference on*, 1991, pp. 325–327 vol.1.
- [4] J. Gong, L. Li, and W. Chen, "Fast recursive algorithms for two-dimensional thresholding," *Pattern Recognition*, vol. 31, no. 3, pp. 295–300, 1998.
- [5] L. Dongju and Y. Jian, "Otsu method and k-means," in *Hybrid Intelligent Systems, 2009. HIS '09. Ninth International Conference on*, vol. 1, 2009, pp. 344–349.
- [6] Z. Ningbo, W. Gang, Y. Gaobo, and D. Weiming, "A fast 2d otsu thresholding algorithm based on improved histogram," in *Pattern Recognition, 2009. CCPR 2009. Chinese Conference on*, 2009, pp. 1–5.
- [7] Y. Chen, D.-r. Chen, Y. Li, and L. Chen, "Otsu's thresholding method based on gray level-gradient two-dimensional histogram," in *Informatics in Control, Automation and Robotics (CAR), 2010 2nd International Asia Conference on*, vol. 3, 2010, pp. 282–285.
- [8] S. Alpert, M. Galun, R. Basri, and A. Brandt, "Image segmentation by probabilistic bottom-up aggregation and cue integration," in *Computer Vision and Pattern Recognition, 2007. CVPR '07. IEEE Conference on*, 2007, pp. 1–8.