



Height & Distances



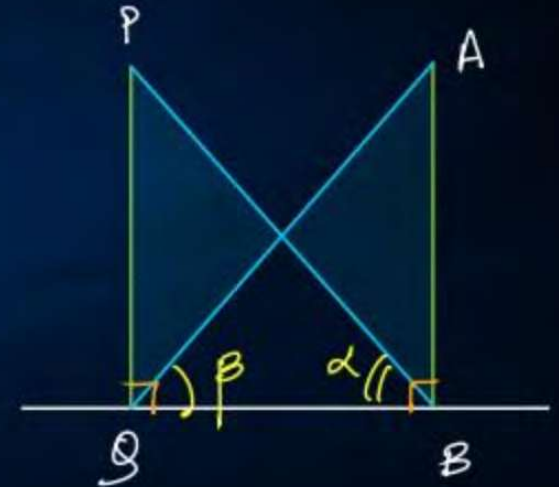
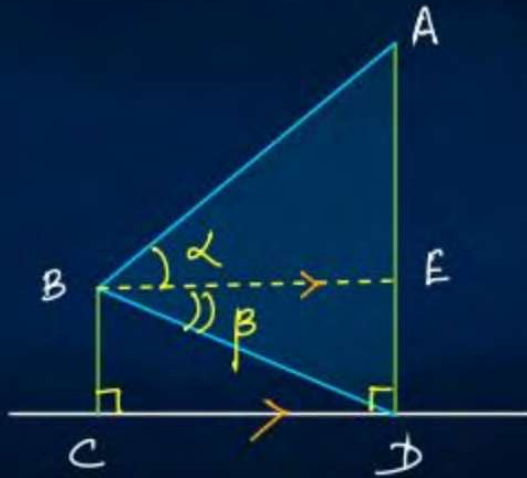
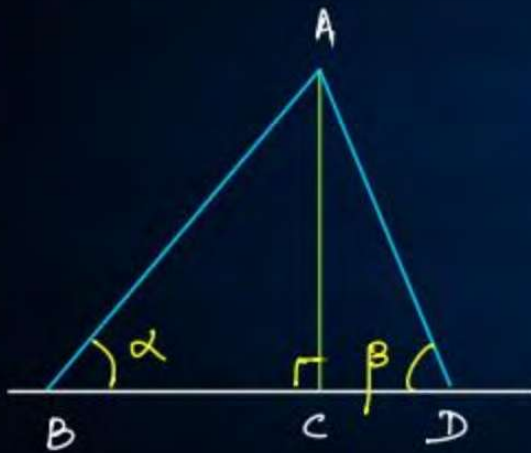
$$* \text{ Height \& Distance } = 80\% \text{ diagram} + 20\% \text{ figo}$$



Height & Distances

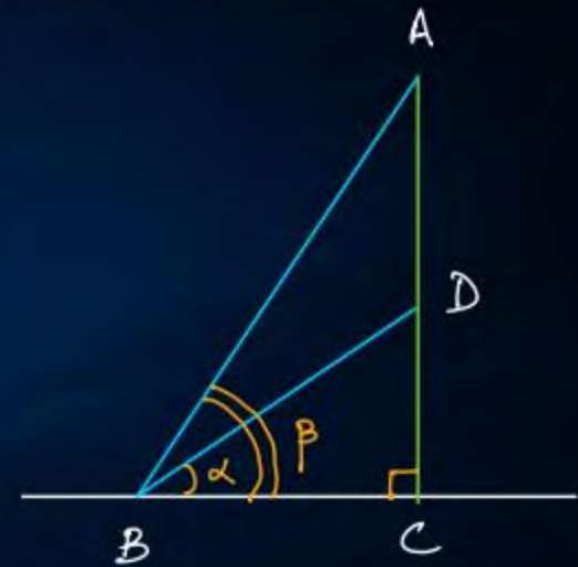
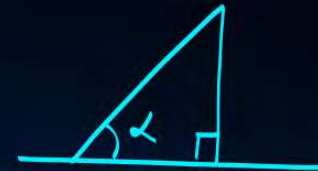
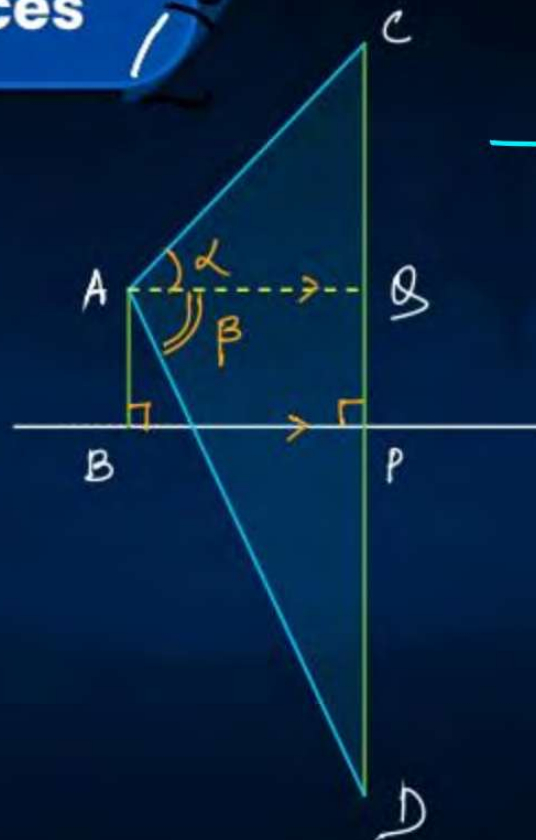
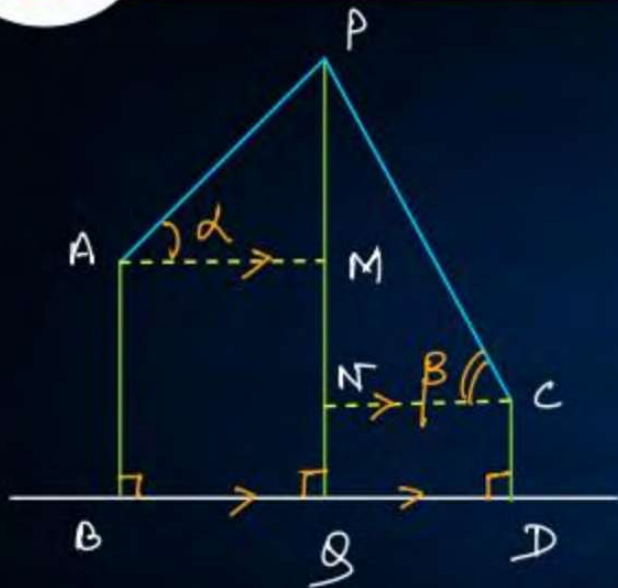


* Some important figures that will be encountered :-





Height & Distances



etc



Height & Distances



* To draw diagrams meaning of following must be clear :-

i) Horizontal Base (land, water, etc)	vi) AOD (Angle of depression)
ii) constructions	vii) line of sight
iii) Observer	viii) Horizontal for \angle for ^m .
iv) Object	
v) Elevation Concept	
vi) Depression Concept	
vii) AOE (Angle of Elevation)	



Height & Distances



i) Horizontal Surface/Base :-

- Whenever a question of Height & distance starts we draw a base or Horizontal surface.
- On it everything will be drawn. (As our reference).
- As talked directly/indirectly in question.
- It can be ground, road, highway, water surface (lake, pond, etc).
- As represented with a horizontal line.



Height & Distances



Examples :-

- The angle of elevation of tower from a pt. on the ground
- A tower is 100 m high
- The angle of elevation of a cloud from a pt. 'x' m above a lake

Direct Mention

hand

Indirect Mention

Water surface

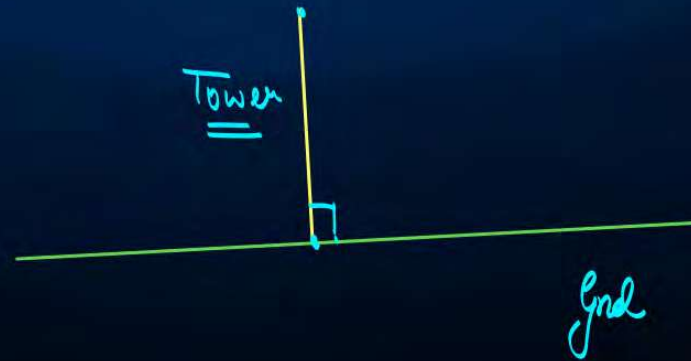


Height & Distances



Ex) Constructions :-

- In Qs, constructions like buildings, towers, cliffs, flagstaff, lighthouse, wall, etc are being talked about.
- By default they are assumed to a vertical line \perp to the horizontal base.



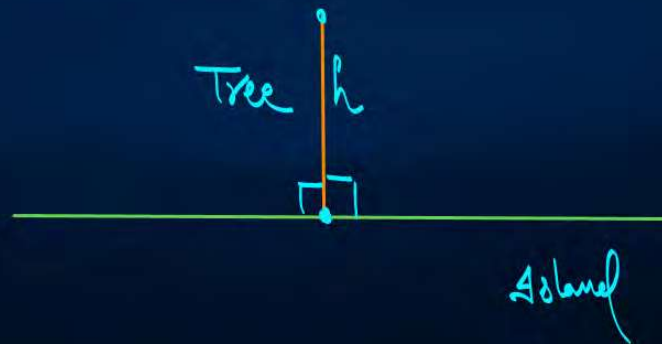
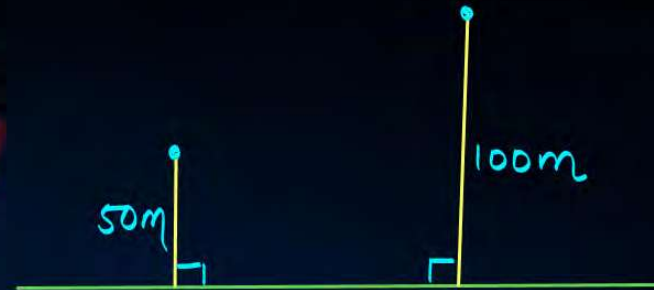


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* Examples :-

- i) There is a small tree standing on an island with height ' h '
- ii) The horizontal distance b/w 2 towers of heights 100m & 50m is



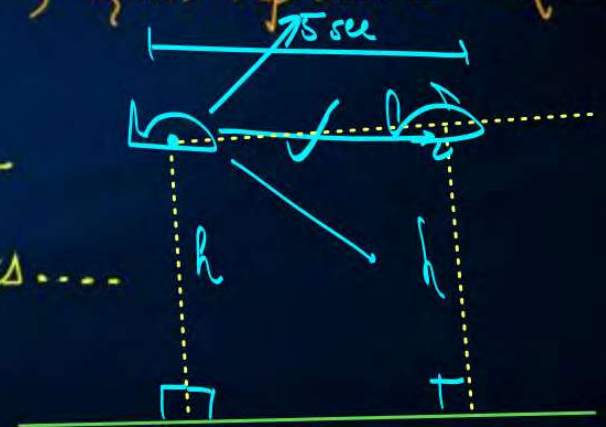


Height & Distances



→ Also, sometimes there aren't any constructions but things above the ground, floating/flying in midair. In such cases, they are always considered as lying vertically above the ground with talked height, again represented like construction.

ii) A jet plane is flying at a constant height of 'h' meters, maintaining the same height after 5 sec....





Height & Distances



iv) A cloud is there h^2 meters above a lake





Height & Distances



ii) **Observer** :-

- The one who observes, sees, views (Dikhe Wala)
- He maybe talked in Q : directly / indirectly.
- Way to represent him :-
- There can be 2 observers at 2 different positions of observers.

Point Observer

If no dimensions of observer is being talked about, then its taken as point observer.
(That pt = eyes)

A
pt obs
(eyes)

If height of the observer is given, then its repⁿ by a constⁿ

Observer with height

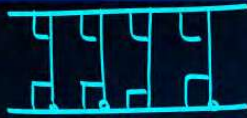
(eyes)
5m
(foot)



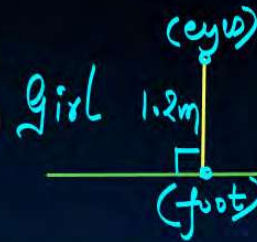
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Examples :



i) A 1.2m tall girl standing on the ground spots a balloon

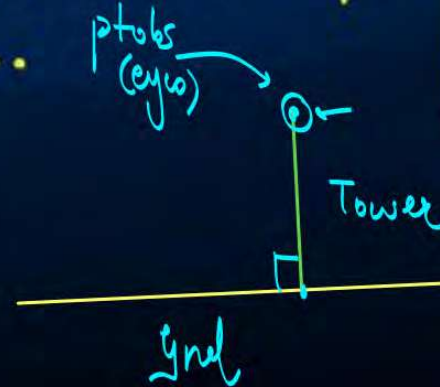
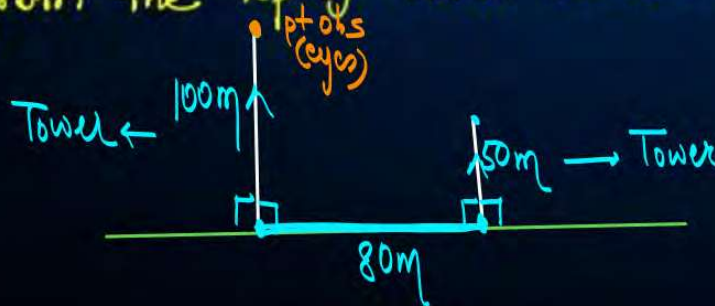


Directly Mentioned
(with height)

ii) A man standing on the top of a tower 50m high observe

Directly Mentioned
Pt. observer.

iii) Distance b/w 2 towers of 100m & 50m is 80m. As seen from the top of 100m tower



Indirectly
Pt. observer

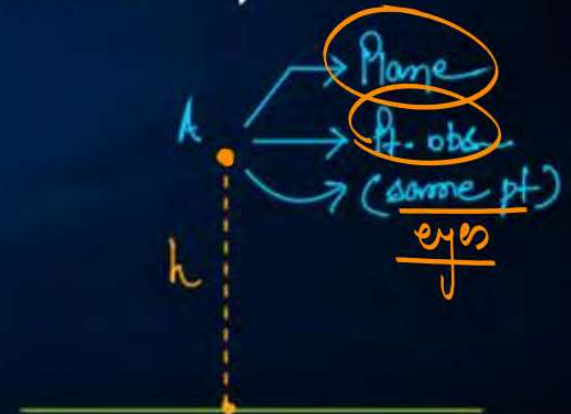
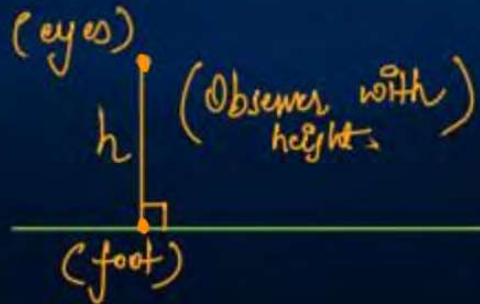
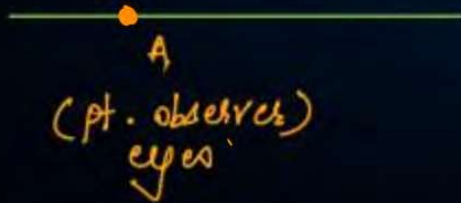


Height & Distances



Note :

- i) The AOE/AOD of from location of observer.
- ii) Anything whose dimensions are not given is represented as a point.





Height & Distances



iv) Object :-

- Things which are seen by the observer is called object.
- It can be mentioned directly / indirectly.
- Object is always a point.
- The AOE / AOD of object.
- Observer always views anything making an acute angle.
- There can be 2 objects in Q also.

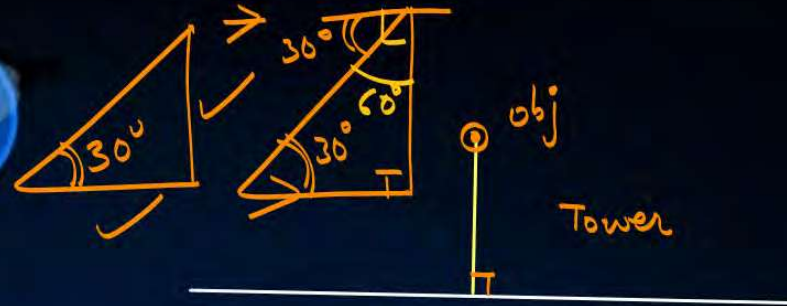


Height & Distances

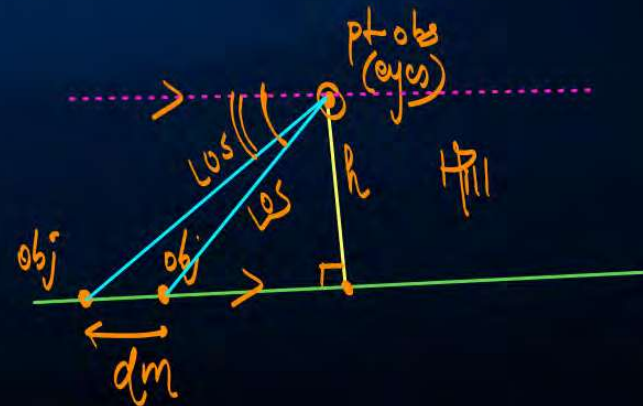


*

Examples :-



- i) The angle of elevation of top of a tower 50 m high
- ii) From a pt. on the hill with height 'h', the angle of depression of 2 km stones 'd m' apart on the same side of hill is



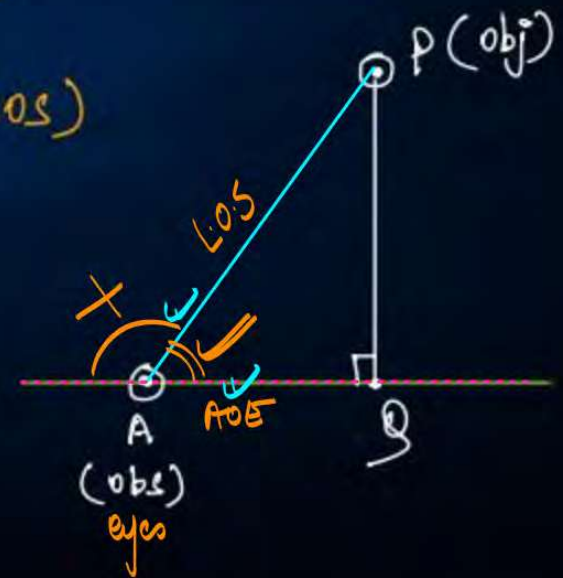


Height & Distances



ex) line of sight :-

- ✓ The line segment joining eyes of observer & object is called line of sight.
- ✓ It's essential to draw AOE / AOD.
- ✓ A fig. may have more than 1 line of sight. (LOS)



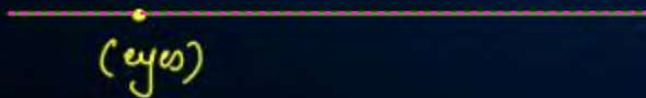


Height & Distances



vi) Horizontal for angle formation

- Straight horizontal line passing through eyes of the observer.
- It is either coincident / parallel to the horizontal base.
- It is required to draw AOE / AOD.



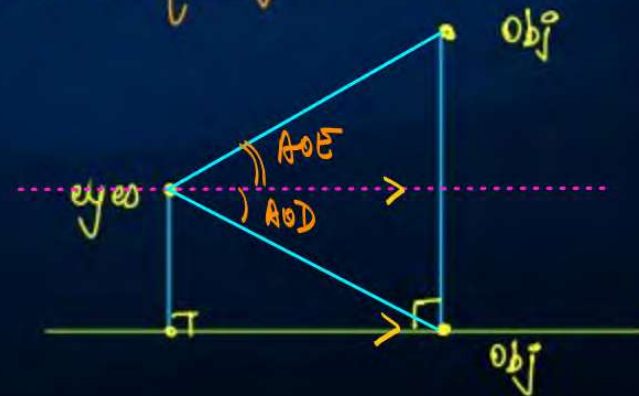
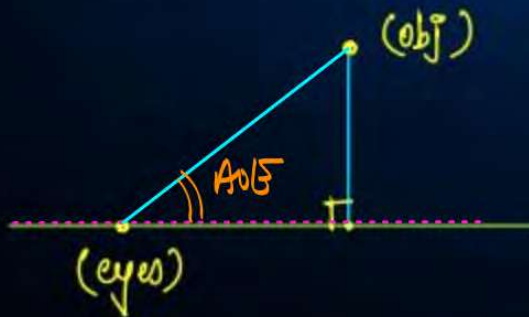


Height & Distances



2011) $\frac{AOE}{AOD} :-$

- It's the acute angle b/w LOS & horizontal for angle form.
- AOE is inside the right Δ & AOD is outside.
- You may get to inside angle via either of 2 methods.





Height & Distances



→ AOE (Angle of elevation)

When eyes below $\&$ obj is placed above.

→ AOD (Angle of depression)

When eyes above $\&$ obj below.

QUESTION

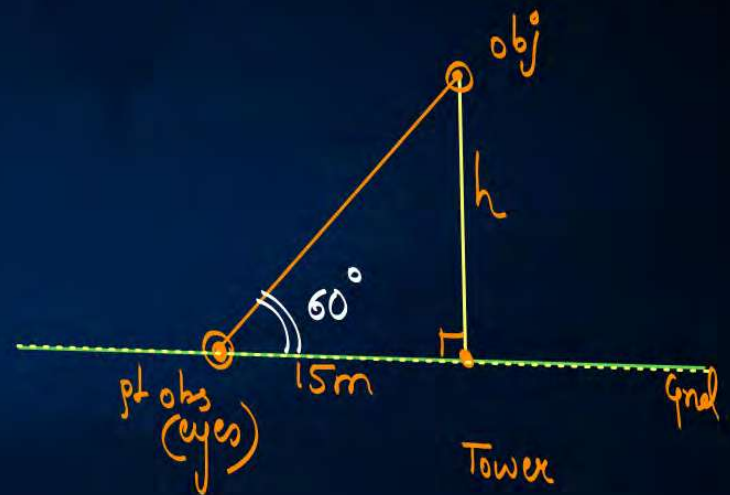
A tower stands vertically on the ground. From a point on the ground, which is 15 m away from the foot of the tower, the angle of elevation of the top of the tower is found to be 60° . Find the height of the tower.

Solⁿ

$$\tan 60^\circ = \frac{h}{15}$$

$$\Rightarrow \sqrt{3} = \frac{h}{15}$$

$$\Rightarrow \boxed{h = 15\sqrt{3} \text{ m}}$$



QUESTION

$$\tan 60^\circ = \frac{3.7}{d} \quad \left| \quad \sqrt{3}d = 3.7 \right. \quad \left. \frac{3.7\sqrt{3}}{3} \text{ m} \right.$$

$$\Rightarrow \sqrt{3} = \frac{3.7}{d} \quad \left| \quad d = \frac{3.7}{\sqrt{3}} \text{ m} \right.$$

An electrician has to repair an electric fault on a pole of height 5 m. She needs to reach a point 1.3 m below the top of the pole to undertake the repair work. What should be the length of the ladder that she should use which, when inclined at an angle of 60° to the horizontal, would enable her to reach the required position? Also, how far from the foot of the pole should she place the foot of the ladder? (You may take $\sqrt{3} = 1.73$)

Solⁿ

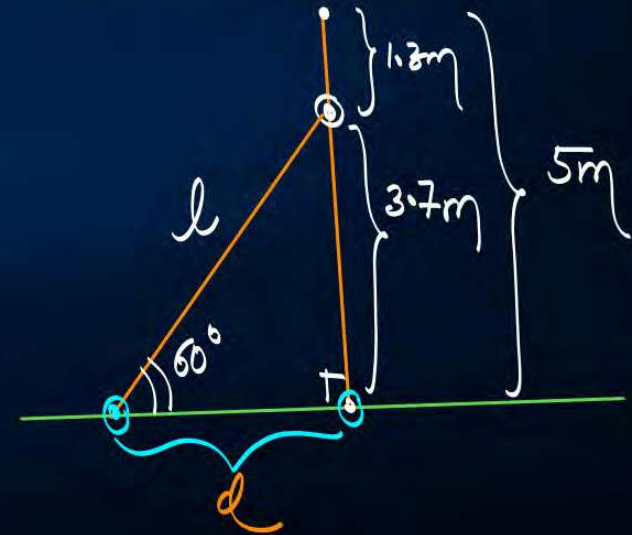
$$\sin 60^\circ = \frac{3.7}{l}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{3.7}{l}$$

$$\Rightarrow \sqrt{3}l = 7.4$$

$$\Rightarrow l = \frac{7.4}{\sqrt{3}} \text{ m}$$

$$\frac{7.4}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{7.4\sqrt{3}}{3} \text{ m}$$



QUESTION

An observer 1.5 m tall is 28.5 m away from a chimney. The angle of elevation of the top of the chimney from her eyes is 45° . What is the height of the chimney?

Solⁿ

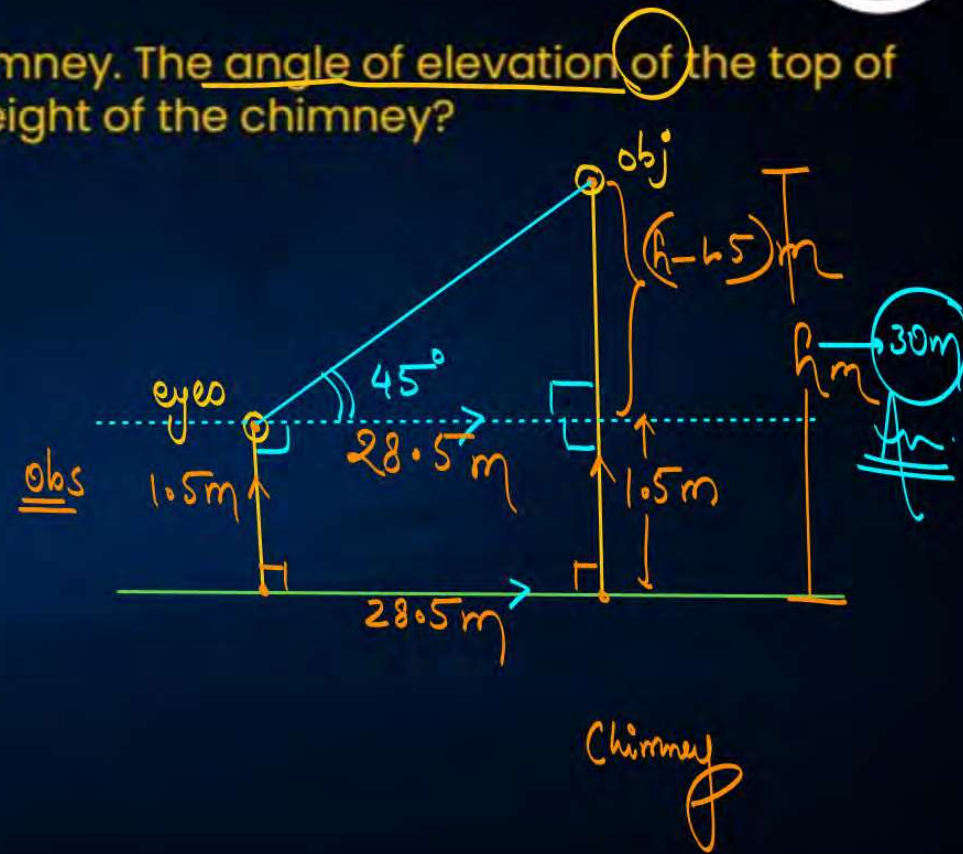
$$\tan 45^\circ = \frac{h - 1.5}{28.5}$$

$$\Rightarrow 1 = \frac{h - 1.5}{28.5}$$

$$\Rightarrow h - 1.5 = 28.5$$

$$\Rightarrow h = 28.5 + 1.5$$

$$\boxed{h = 30\text{m}}$$



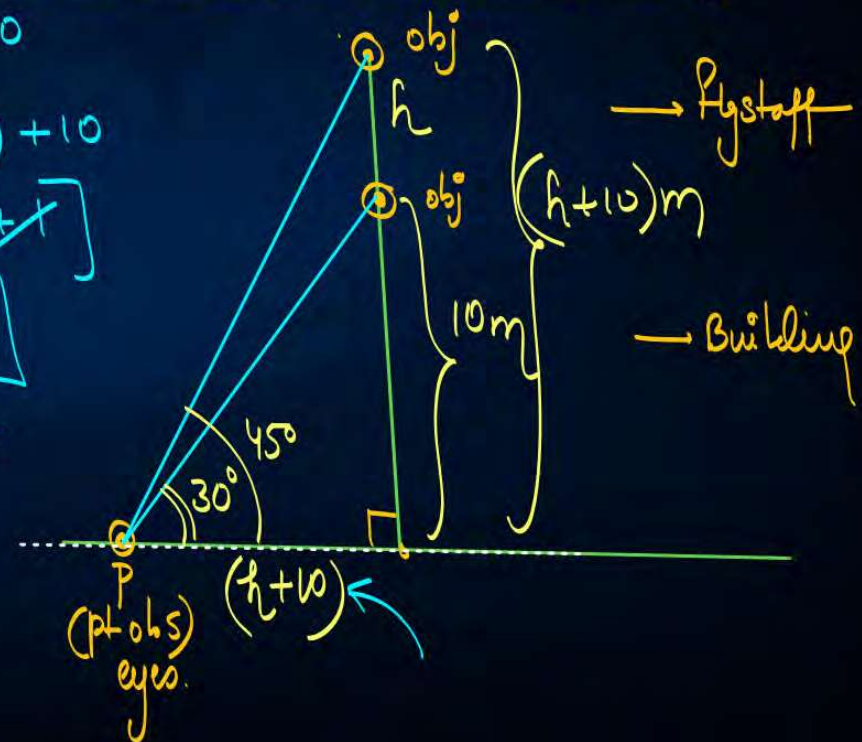
QUESTION

From a point P on the ground the angle of elevation of the top of a 10 m tall building is 30° . A flag is hoisted at the top of the building and the angle of elevation of the top of the flagstaff from P is 45° . Find the length of the flagstaff and the distance of the building from the point P.

Solⁿ

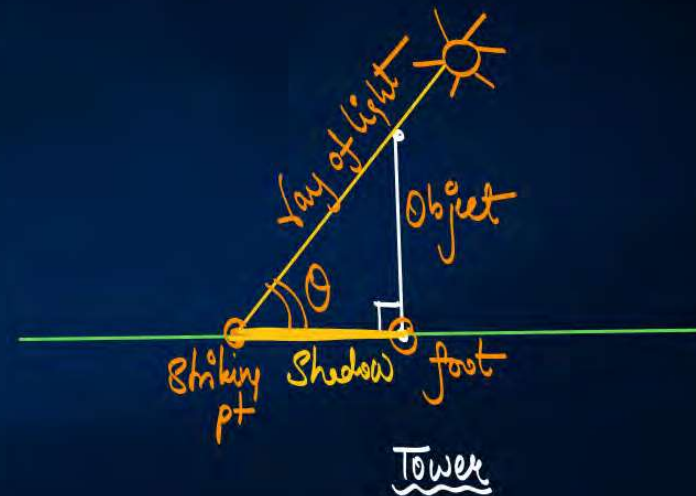
$$\begin{aligned} \tan 30^\circ &= \frac{10}{h+10} \\ \Rightarrow \frac{1}{\sqrt{3}} &= \frac{10}{h+10} \\ \Rightarrow h+10 &= 10\sqrt{3} \\ \Rightarrow h &= (10\sqrt{3}-10)\text{m} \\ \boxed{h} &= \boxed{10(\sqrt{3}-1)\text{m}} \end{aligned}$$

$$\begin{aligned} \text{dist} &= h+10 \\ &= 10(\sqrt{3}-1)+10 \\ &= 10[\sqrt{3}-1+1] \\ &= \boxed{10\sqrt{3}\text{ m}} \\ &\underline{\underline{A}} \end{aligned}$$



* Shadow Concept

- i) How to draw?
- ii) Variation of θ with \angle .

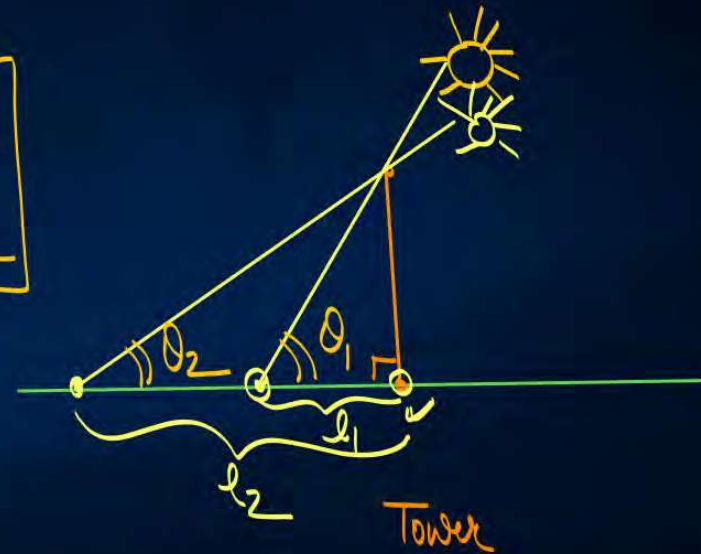


* Shadow Concept

- i) How to draw?
- ii) Variation of ' θ ' with ' l '.

$$\theta \propto \frac{1}{l}$$

$$\begin{aligned} \theta_1 &> \theta_2 \\ l_1 &< l_2 \end{aligned}$$



QUESTION

The shadow of a tower standing on a level ground is found to be 40 m longer when the Sun's altitude is 30° than when it is 60° . Find the height of the tower.

$$\Rightarrow 3h = 40\sqrt{3} + h$$

$$\Rightarrow \cancel{2h} = \frac{40\sqrt{3}}{\cancel{2}} \Rightarrow h = 20\sqrt{3} \text{ m}$$

Solⁿ

$$\tan 60^\circ = \frac{h}{x} \quad \left\{ \quad \tan 30^\circ = \frac{h}{(40+x)} \right.$$

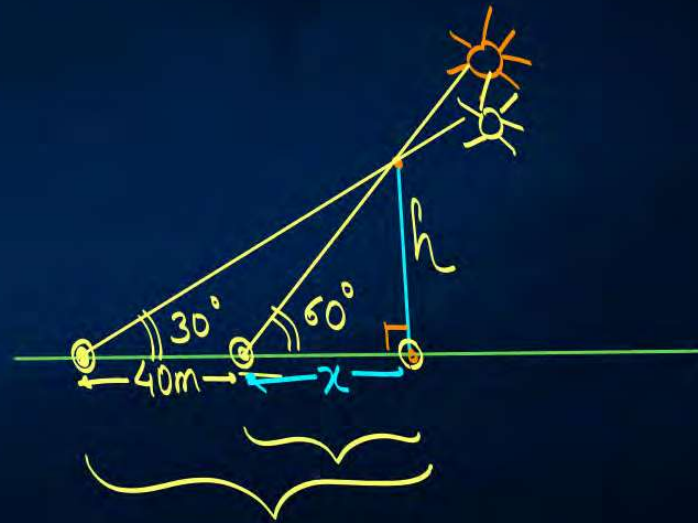
$$\Rightarrow \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow x = \frac{h}{\sqrt{3}}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{(40+x)}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{\left(40 + \frac{h}{\sqrt{3}}\right)}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{\sqrt{3}h}{(40\sqrt{3} + h)}$$



QUESTION

The angles of depression of the top and the bottom of an 8 m tall building from the top of a multi-storeyed building are 30° and 45° , respectively. Find the height of the multi-storeyed building and the distance between the two buildings.

Soln

$$\tan 30^\circ = \frac{h-8}{h}$$

$$\frac{1}{\sqrt{3}} = \frac{h-8}{h}$$

$$\Rightarrow \sqrt{3}h - 8\sqrt{3} = h$$

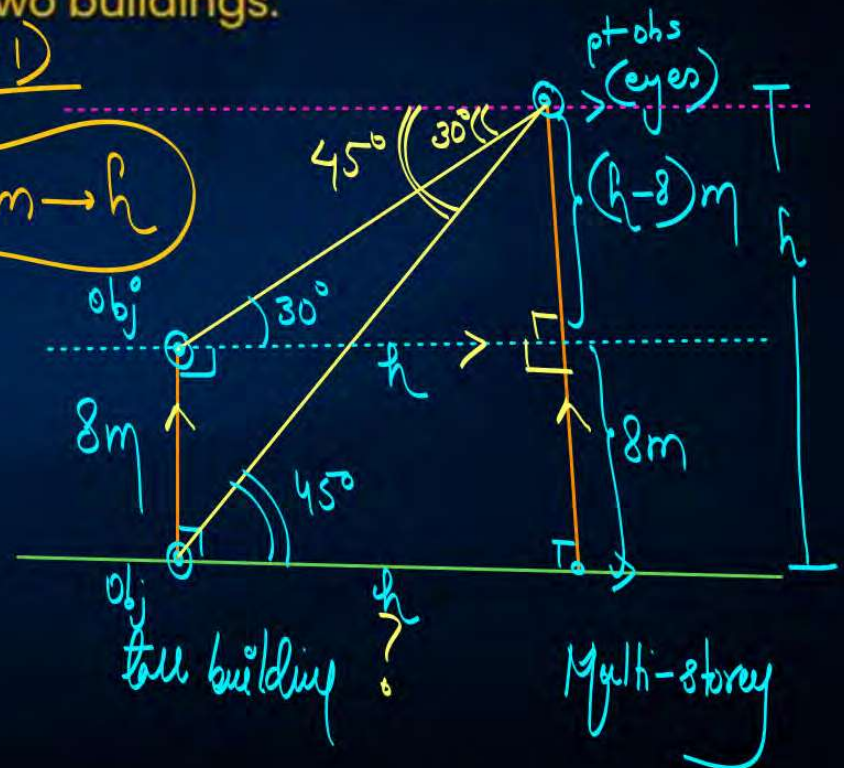
$$\Rightarrow \sqrt{3}h - h = 8\sqrt{3}$$

$$\Rightarrow h(\sqrt{3}-1) = 8\sqrt{3}$$

$$h = \frac{8\sqrt{3}}{(\sqrt{3}-1)} \text{ m}$$

$$\frac{8\sqrt{3}(\sqrt{3}+1)}{2} = 4(3+\sqrt{3}) \text{ m} \rightarrow h$$

dist



QUESTION

$\uparrow \theta, b \downarrow$

From a point on a bridge across a river, the angles of depression of the banks on opposite sides of the river are 30° and 45° , respectively. If the bridge is at a height of 3 m from the banks, find the width of the river.

Soln

$$\tan 30^\circ = \frac{3}{x}$$

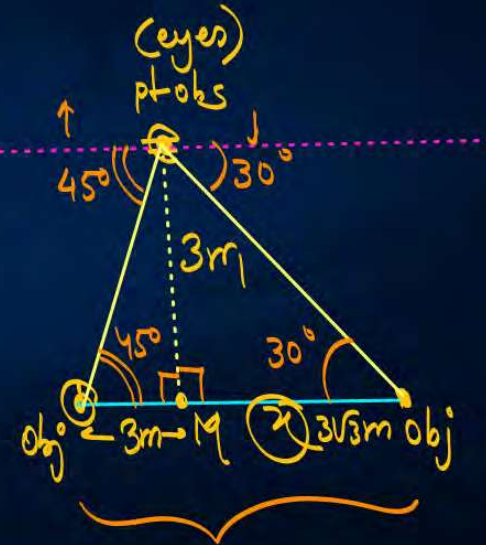
$$\frac{1}{\sqrt{3}} = \frac{3}{x}$$

$$x = 3\sqrt{3} \text{ m}$$

Width of river

$$= (3 + 3\sqrt{3}) \text{ m}$$

$$= 3(1 + \sqrt{3}) \text{ m}$$



QUESTION



$$h = x + y = \left(\frac{16\sqrt{3}}{3} + 8\sqrt{3} \right) \text{m}$$



A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.

Solⁿ

$$\tan 30^\circ = \frac{y}{8}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{y}{8}$$

$$\Rightarrow y = 8\sqrt{3} \text{ m}$$

$$\cos 30^\circ = \frac{8}{x}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{8}{x}$$

$$\Rightarrow \sqrt{3}x = 16$$

$$x = \frac{16}{\sqrt{3}} \text{ m}$$

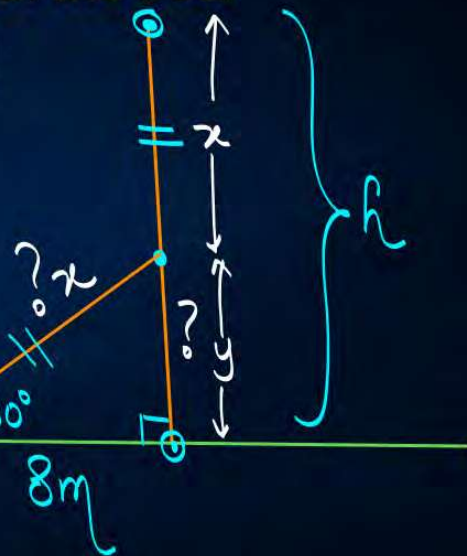
$$x = \frac{16\sqrt{3}}{3} \text{ m}$$

$$\Rightarrow \sqrt{3} \left(\frac{16}{3} + 8 \right)$$

$$\Rightarrow 8\sqrt{3} \left(\frac{2}{3} + 1 \right)$$

$$= 8\sqrt{3} \times \frac{5}{3}$$

$$= \frac{40\sqrt{3}}{3} \text{ m}$$



QUESTION

A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a slide whose top is at a height of 1.5 m, and is inclined at an angle of 30° to the ground, whereas for elder children, she wants to have a steep slide at a height of 3 m, and inclined at an angle of 60° to the ground. What should be the length of the slide in each case?

Soln

$$\sin 30^\circ = \frac{1.5}{PR}$$

$$\Rightarrow \frac{1}{2} = \frac{1.5}{PR}$$

$$\Rightarrow PR = 3 \text{ m} \quad (\text{below 5 yrs})$$

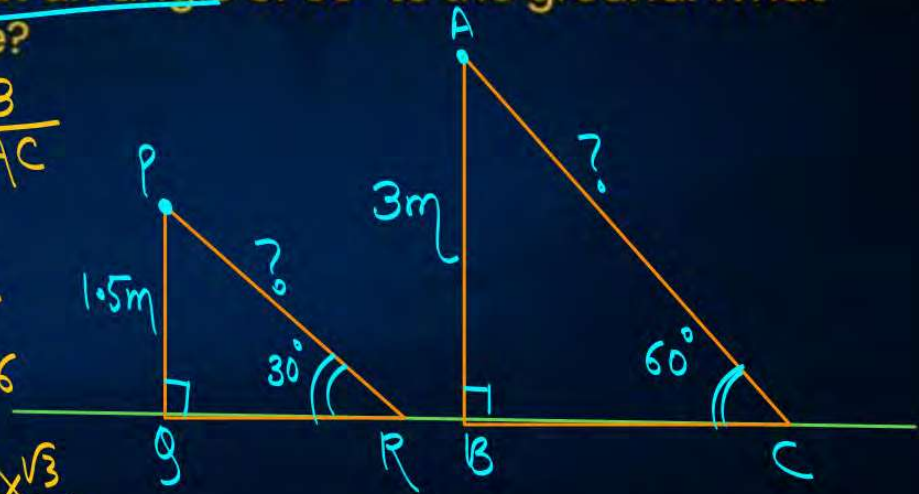
$$\sin 60^\circ = \frac{3}{AC}$$

$$\frac{\sqrt{3}}{2} = \frac{3}{AC}$$

$$\Rightarrow \sqrt{3} AC = 6$$

$$\Rightarrow AC = \frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{2\sqrt{3}}{1} \rightarrow 2\sqrt{3} \text{ m} \quad (\text{for elder ch})$$



QUESTION

A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60° . Find the length of the string, assuming that there is no slack in the string.

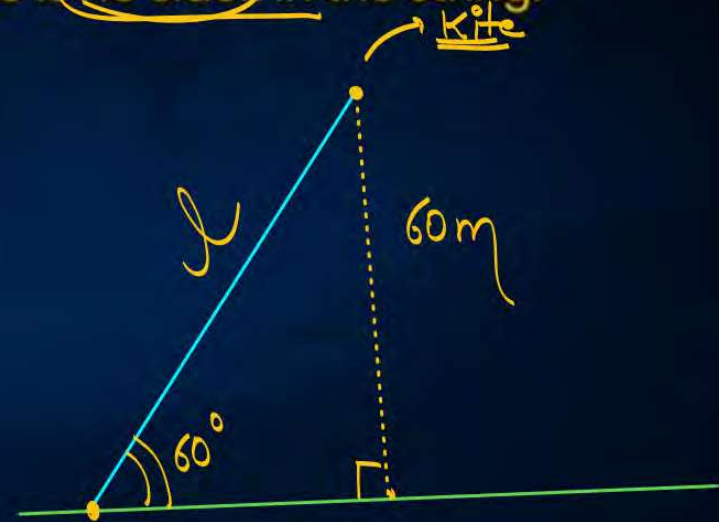
Solⁿ

$$\sin 60^\circ = \frac{60}{l}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{60}{l}$$

$$\Rightarrow \sqrt{3}l = 120$$

$$\Rightarrow l = \frac{120}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{40 \cdot \sqrt{3}}{1} = \boxed{40\sqrt{3} \text{ m}}$$



QUESTION

Speed $\rightarrow x$
time $\rightarrow t$

$$3t = 6 + t$$

$$\cancel{2t} = \cancel{6} 3$$

$$t = 3 \text{ sec}$$

A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30° , which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60° . Find the time taken by the car to reach the foot of the tower from this point.

Soln

$$\tan 60^\circ = \frac{h}{xt} \quad \left\{ \begin{array}{l} \tan 30^\circ = \frac{h}{6x + xt} \end{array} \right.$$

$$\sqrt{3} = \frac{h}{xt}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{6x + xt}$$

$$h = \sqrt{3} xt$$

$$\frac{1}{\sqrt{3}} = \frac{\sqrt{3} \cancel{xt}}{\cancel{x}(6+t)}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{\sqrt{3}t}{6+t}$$

