Topics to be covered

1) Statistics

Introduction

Statistics is all about studying the behavior of big data. In statistics we define some parameters which help us to do that. So, here in this chapter we will be studying about two such parameters namely "Central tendency" & "Dispersion".

Statistic

- **Central Tendency**
  - $\boxed{\text{Mean}}$ → average of data
  - median ✓
  - mode ✓
- **Dispersion**
  - Mean deviation about:
    - mean
    - median
    - any given number
  - Standard deviation (σ) and variance (σ^2)

2. Measures of central tendency

We are already acquainted with these measures. We normally study 3 measures of central tendency:

(a) Mean
(b) Median
(c) Mode

2. Measures of central tendency

Let's Pick them one by one

2. Measures of central tendency

Mean:

As such finding mean is very simple

(a) $\boxed{\bar{x} = \frac{\sum x_i}{n}}$

(b) $\bar{x} = \frac{\sum f_i x_i}{N}$

N = sum of frequency.

Remark:

For class wise data (i.e. continuous data), we take mid points of classes as x_i,

2. Measures of central tendency

(a) Mean of 1, 2, 4, 6, 8 is $\frac{1+2+4+6+8}{5} =$

(b) Mean of

x_i	2	3	4	6
f_i	1	2	2	4

is

$$\frac{2 \times 1 \times 3 \times 2 \times 4 \times 2 + 6 \times 4}{1 + 2 + 2 + 4}$$

(c) Mean of

Class	f_1
0-10	2
10-20	2
20-30	4

is $\frac{2 \times 2 \times 15 \times 2 + 2 \times 25 \times 4}{2 + 2 + 4}$

x_i	f_i	$x_i f_i$
80	4	320
85	5	425
90	10	900
100	2	200
$N = 21$		$\sum x_i f_i = 1845$

$$\bar{x} = \frac{\sum x_i f_i}{N}$$

$$= \frac{\cancel{1845}\ 615}{\cancel{21}\ 7}$$

$$= \frac{615}{7}$$

Class	f_i	x_i	$x_i f_i$
0 – 10	4	5	20
10 – 20	3	15	45
20 – 30	5	25	125
30 – 40	10	35	350

$\sum f_i = 22$

$\boxed{540 = \sum x_i f_i}$

$$\bar{x} = \frac{\sum x_i f_i}{N}$$

$$\bar{x} = \frac{540}{22}$$

① $\boxed{x_i \longrightarrow x_i \pm \alpha}$ then $\boxed{\bar{x} \longrightarrow \bar{x} \pm \alpha}$

② $\boxed{x_i \longrightarrow k\,x_i}$ then $\boxed{\bar{x} \longrightarrow k\,\bar{x}}$

$$\text{old } \bar{x} = \frac{\overset{x_1+\alpha}{x_1} + \overset{x_2+\alpha}{x_2} + x_3 + \dots + \overset{x_n+\alpha}{x_n}}{n} \qquad \text{New } \bar{x} = \frac{kx_1 + kx_2 + \dots + kx_n}{n}$$

2. Measures of central tendency

Now, we need to study dependence of $\bar{x}$ on "change of origin & scale"

2. Measures of central tendency

Consider a data $x_1, x_2, ..., x_n$ with mean $\bar{x}$

(a) If $y_i = \dfrac{x_i}{n}$ then try to observe:

$\bar{y} \ \dfrac{\bar{x}}{n}$ (so, mean changes with change of scale)

(b) If $\bar{y} = \bar{x} - A$ then try to observe

$\bar{y} = \bar{x} - A$ (so, mean changes with change of origin)

2. Measures of central tendency

Result

If $\dfrac{x_i - A}{n}$ then $\bar{y} = \dfrac{1}{n}(\bar{x} - A)$

2. Measures of central tendency

Now, let's do some examples on mean

Let's Solve

Ex. A group of 10 items has mean 6. If mean of 4 of these items is 7.5, then find mean of remaining items

QUESTION [JEE MAIN 2015]

The mean of data set comprising of 16 observations is 16. If one of the observation valued 16 is deleted and 3 observations 3, 4, & 5 are added then mean of resultant data is:

(A) 16.8

(B) 16.0

(C) 15.8

(D) 14

QUESTION

The means of a set of numbers is $\bar{x}$. If each number is divided by 3, then new mean is

(A) $\bar{x}$

(B) $\bar{x} + 3$

(C) $3\bar{x}$

(D) $\frac{\bar{x}}{3}$

QUESTION

The mean of a set of observations is $\bar{x}$. If each observation is divided by α & then is increased by 10, then new mean is

(A) $\frac{\bar{x}}{\alpha}$

(B) $\frac{\bar{x}+10}{\alpha}$

(C) $\frac{\bar{x}+10\alpha}{\alpha}$

(D) $\alpha\bar{x}+10$

$$\frac{\bar{x}}{\alpha}+10$$

$$=\boxed{\frac{\bar{x}+10\alpha}{\alpha}}$$

Median

(i) Median of an individual series:

Eg ① 2, 8, 19, 35, 100.

median

Eg ② 2, 8, 19, 35, 100, 101

T_3 T_4

median $= \frac{19+35}{2} = \frac{54}{2} = 27$

① If no. of data are odd.
then median is $\left(\frac{n+1}{2}\right)^{th}$ observation

② If 'n' is even then.
median is average of $\left(\frac{n}{2}\right)^{th}$ & $\left(\frac{n}{2}+1\right)^{th}$ term.

Median

(i) Median of an individual series:

Step 1: Arrange the data in ascending or descending order.

Step 2:

(i) If n is odd, then

Median = value of the $\frac{1}{2}(n+1)^{\text{th}}$ observation

(ii) If n is even, then

Median = mean of the $\left(\frac{n}{2}\right)$th and $\left(\frac{n}{2}+1\right)$th bservation

(ii) Median of a discrete series

x_i	f_i	c.f.
3	4 ✓	4
10	2 ✓	6
✓ 19	5	11
35	1	12
49	3	$\boxed{15}$

M-I median is $\left(\frac{15+1}{2}\right)^{th}$ observation

$= 8^{th}$ observation $= \boxed{19}$ ans.

M-II median is the observation whose c.f is just greater than or equal to $\frac{N}{2} = \frac{15}{2} = 7.5$

(ii) Median of a discrete series

(ii) Median of a discrete series:

Step 1: Arrange the values of the variate in ascending or descending order

Step 2: Prepare a cumulative frequency table

Step 3: Median is the observation whose cumulative frequency is equal to or just greater than N/2, where N is sum of frequencies.

(iii) Median of a continuous series:

Step 1: Prepare the cumulative frequency table

Step 2: Find the median class, i.e. the class in which the $\left(\frac{n}{2}\right)^{th}$ observation lies

the class whose c.f is either equal to or just greater than $\frac{N}{2}$

Step 3: The median value is given by the formula n 2 Cf

$$\text{Median} = l + \left(\frac{\left(\frac{n}{2} - Cf\right)}{f}\right) \times h$$

Where

l = lower limit of the median class

n = total frequency

f = frequency of the median class

h = width of the median class

Cf = cumulative frequency of the class preceding the median class

Let's do an example on finding median of continuous data, that is the only case whose finding median is sometimes challenging

QUESTION [JEE Main 2024 /30 Jan. – Shift-1]

Let M denote the median of the following frequency distribution.

Class	0–4	4–8	8–12	12–16	16–20
Frequency	3	9	10	8	6
c. f	3	12	22	30	36

Then 20M is equal to:

$\frac{N}{2} = \frac{36}{2} = 18$

(A) 416

(B) 104

(C) 52

(D) 208

c.f.

$\text{median} = \overset{8}{\cancel{l}} + \frac{\left(\overset{18}{\frac{N}{2}} - \overset{12}{\cancel{cf}}\right)}{\underset{10}{\cancel{f}}} \times \cancel{h}\,4 = 8 + \frac{6\times 4}{10} = 8 + 2.4 = 10.4$

$20m = 20 \times 10.4$
$= 208$

Mode

The mode is that value in a series of observations which occurs with greatest frequency.

x_i	f_i
5	1
13	1
7	4
11	15
35	100.

mode

1, 1, 1, 13, 13, 13, 13, 2, 11, $\underbrace{7, 7, 7, 7, 7}_{\text{mode}}$

Mode

The mode is that value in a series of observations which occurs with greatest frequency.

(i) In case of individual series, the mode is the value which occurs more frequently.

Mode

The mode is that value in a series of observations which occurs with greatest frequency.

(ii) In case of discrete series, quite often mode can be determined just by inspection i.e. by looking to that value of variable around which the items are most heavily concentrated.

(iii) In case of continuous series,

$$\boxed{\text{Mode} = l + \frac{f_1 - f_0}{2f1 - f_2 - f_0} \times h}$$

Where l = the lower limit of the modal class i.e. the class having maximum frequency;

f_1 = frequency of the modal class;

f_0 = frequency of the class preceding the modal class;

f_2 = frequency of the class succeeding the modal class and

h = width of the modal class

Remark

Relation between mean, Median and mode i.e.

Mode = 3 median – 2 mean

Now, before we pick measures of dispersion Let's see an example to understand its need

Consider scores of two batsmen in three matches

Batsman A: 0, 150, 0

Batsman B: 48, 50, 52

[In real world data will be big. Here, its taken small, just for easy calculations]

Try to observe average score of both is same i.e. 50, but it's very evident that they have completely different playing styles. That thing which is making you think like that is "dispersion". So, to select a batsman from these two batsmen, mean alone is not giving complete indication of nature of player, "dispersion" together with "mean" tells you more about the batting style & hence facilitates better selection.

3. Measure of dispersion

We primarily have 5 measures of dispersion, namely

(1) Mean deviation about
  (a) Mean
  (b) Median
  (c) Any given number

(2) Standard deviation (σ)

(3) Variance (σ^2)

(4) Range

(5) Coefficient of variation

	$\bar{x}$	dispersion
College A	Same	Low
College B	Same	High.

Mean deviation

Mean deviation about any number 'A' is:

(α) $\frac{1}{n}\sum |x_i - A|$

(b) $\frac{1}{N}\sum f_i |x_i - A|$

For mean deviation about "Mean" we take $\bar{x}$ in place of A For mean deviation about "Median" we take median in place of A

Remark

Formula is very simple

Only way it can be complicated is by giving continuous data & asking mean deviation about median

QUESTION

Calculate the mean deviation about median for the following data:

Class:	0-10	10-20	20-30	30-40	40-50	50-60
Frequency:	6	7	15	16	4	2

Note:

Mean division is minimum about median.

QUESTION [JEE Main 2022 (25 June – Shift 2)]

If the mean deviation about the mean of the numbers $\underbrace{1, 2, 3, \ldots, n}$, where n is odd, is $\boxed{5(n + 1)/n}$, then n is equal to ______

x_i	1	2	3			$\frac{n-1}{2}$	$\frac{n+1}{2}$	$\frac{n+3}{2}$		n
$\lvert x_i - \bar{x}\rvert$	$\left(\frac{n-1}{2}\right)$	$\frac{n-3}{2}$	$\frac{n-5}{2}$		2	1	0	1		$\left(\frac{n-1}{2}\right)$

Sum of data $= 1+2+3+\ldots+n$

$= \frac{n(n+1)}{2}$

$$\bar{x} = \frac{\frac{n(n+1)}{2}}{n} = \frac{n+1}{2}$$

$$\frac{\sum\left|(x_i - \bar{x})\right|}{n} = \boxed{\frac{2}{\cancel{n}}\left(1+2+3+\ldots+\frac{n-1}{2}\right) = \frac{5(n+1)}{\cancel{n}}}$$

p

$$\frac{\cancel{2}\left(\frac{n-1}{2}\right)\left(\frac{n-1}{2}+1\right)}{\cancel{2}} = 5(n+1) \Rightarrow \left(\frac{n-1}{2}\right)\left(\frac{n+3}{2}\right) = 5(n+1)$$

QUESTION [JEE Main 2022 (25 June – Shift 2)]

If the mean deviation about the mean of the numbers 1, 2, 3, ..., n, where n is odd, is 5(n + 1)/n, then n is equal to ______.

$$n^2-1=20(n+1)$$

$$n^2-20n-21=0$$

$$(n-21)(n+1)=0$$

$$n=-1,\ 21$$

Sum of data $= 1+2+3+\dots+n$

$$= \frac{n(n+1)}{2}$$

$$\bar{x} = \frac{\frac{n(n+1)}{2}}{n} = \frac{n+1}{2}$$

$$\frac{\cancel{2}\left(\frac{n-1}{2}\right)\left(\frac{n-1}{2}+1\right)}{\cancel{2}} = 5(n+1) \Rightarrow \left(\frac{n-1}{2}\right)\left(\frac{n+1}{2}\right) = 5(n+1)$$

Standard Deviation

Formula ①

$$\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{N}} \quad \text{OR} \quad \sigma = \sqrt{\frac{\sum f_i (x_i - \bar{x})^2}{N}}$$

Formula ②

$$\sigma = \sqrt{\frac{\sum x_i^2}{N} - \left(\frac{\sum x_i}{N}\right)^2} \quad \text{OR} \quad \sigma = \sqrt{\frac{\sum f_i x_i^2}{N} - \left(\underbrace{\frac{\sum x_i f_i}{N}}_{\bar{x}}\right)^2}$$

$$= \sqrt{\operatorname{avg}(x_i^2) - (\bar{x})^2}$$

$$\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{N}}$$

$$= \sqrt{\frac{\sum_{i=1}^{n} (x_i^2 - 2\bar{x}x_i + \bar{x}^2)}{N}}$$

$$= \sqrt{\frac{\sum x_i^2}{N} - \frac{\sum 2x_i\bar{x}}{N} + \frac{\sum \bar{x}^2}{N}}$$

$$-2\bar{x}\sum\frac{x_i}{N} \rightarrow \bar{x}$$

$$\rightarrow \bar{x}^2 \frac{\sum 1}{N} \rightarrow N$$

$$\sqrt{\frac{\sum x_i^2}{N} - 2(\bar{x})^2 + (\bar{x})^2}$$

$$= \sqrt{\frac{\sum x_i^2}{N} - (\bar{x})^2}$$

$$= \sqrt{\text{avg}(x_i^2) - (\bar{x})^2}$$

$$\boxed{\begin{aligned}\text{Variance} &= \sigma^2 \\ S.D &= \sigma\end{aligned}}$$

$x_i + K \qquad \bar{x} + K$

$$\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{N}}$$

$$\sigma = \sqrt{\frac{\sum (Kx_i - K\bar{x})^2}{N}}$$

① $x_i \longrightarrow x_i \pm K \Rightarrow$ No change in σ

② $x_i \longrightarrow Kx_i \Rightarrow \boxed{\sigma \rightarrow K\sigma}$

$(S.D \longrightarrow K\,S.D)$

$$\boxed{\sigma^2 \longrightarrow K^2\sigma^2}$$

Variance $\longrightarrow K^2$ variance.

Q If the variance of $x_1, x_2, x_3 \ldots x_n$ is 5 then.
what is variance of $2x_1+3, 2x_2+3, 2x_3+3, \ldots, 2x_n+3$.

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Standard Deviation

Given a data, its S.D. (denoted by σ) is found by using following formula

$$\sigma = \sqrt{\frac{1}{n}\sum (x_i - \bar{x})^2} \text{ or } \sqrt{\frac{1}{N}\sum f_i(x_i - \bar{x})^2}$$

$$\text{Also, } \sigma = \sqrt{\frac{1}{n}\sum f_i^2 - \left(\frac{\sum x_i}{n}\right)^2} \text{ or } \sqrt{\frac{1}{N}\sum f_i^2 - \left(\frac{\sum f_i x_i}{N}\right)^2}$$

Note:

Variance = σ^2

QUESTION

$1^2+2^2+3^2+\dots+n^2 = \frac{n(n+1)(2n+1)}{6}$

The variance of first 50 even natural numbers is

$2, 4, 6, 8, \dots, 100$

(A) $\frac{833}{4}$

(B) 833

(C) 437

(D) $\frac{437}{4}$

$\sigma^2 = \text{avg.}(x_i^2) - (\bar{x})^2$

$= \frac{2^2+4^2+6^2+\dots+100^2}{50} - \left(\frac{2+4+6+\dots+100}{50}\right)^2$

$= 4\left(\frac{1^2+2^2+3^2+\dots+50^2}{50}\right) - \left(\frac{2(1+2+3+\dots+50)}{50}\right)^2$

$\cancel{4}^{2}\left(\frac{\cancel{(50)}\cancel{(51)}^{17}(101)}{\cancel{6}_{\cancel{3}}\times\cancel{50}}\right) - \left(\frac{\cancel{2}\cancel{(50)}(51)}{\cancel{2}\times\cancel{50}}\right)^2$

$= 2\times17\times101 - (51)^2$

$3434 - 2601$

$= 833$

We need to study the dependence of on change of origin & scale, as it is going to provide step deviation method to find "σ" which makes involved calculations easier.

Dependence of σ on change of origin & scale

Consider date $x_1, x_2, ... x_n$ with SD as σ_x

(a) If $y_i = x_i - A$ then $\sigma_y = \sigma_x$

i.e., SD is independent of origin

(b) If $y_i = \frac{x_i}{h}$ then $\sigma_y = \frac{\sigma_x}{h}$

Result:

If $y_i = \frac{xi - A}{h}$ then $\sigma_y = \frac{\sigma_x}{h}$

QUESTION [JEE MAIN 2013]

All the students of a class performed poorly in Mathematics. The teacher decide to give grace marks of 10 to each of the students. Which of the following statistical measures will not change even after the grace marks were given?

(A) Mean

(B) Median

(C) Mode

(D) Variance

QUESTION [JEE Main 2024 /30 Jan. – Shift-2]

$\bar{x} = \frac{176}{22} = \frac{88}{11} = 8$

The variance σ2 of the data

x_i	0	1	5	6	10	12	17	
f_i	3	2	3	2	6	3	3	22
$x_i f_i$	0	2	15	12	60	36	51	176
$(x_i-\bar{x})^2$	64	49	9	-4	4	16	81	
$f_i(x_i-\bar{x})^2$	192	98	27	8	24	48	243	Add $\boxed{6.40}$

Is ________.

110. 11

$$\sigma^2 = \frac{\sum (x_i-\bar{x})^2 f_i}{N} = \frac{640}{22} = \frac{320}{11} = 29.09$$

N 22

QUESTION [JEE Main 2024 /30 Jan. – Shift-2]

$\bar{x} = \frac{176}{22} = \frac{88}{11} = 8$

M-II

The variance σ2 of the data

x_i	0	1	5	6	10	12	17	
f_i	3	2	3	2	6	3	3	22
$x_i f_i$	0	2	15	12	60	36	51	176
x_i^2	0	1	25	36	100	144	289	
$f_i x_i^2$	0	2	75	72	600	432	867	sum 2048

Is ________.

$\text{avg}(x_i^2) = \frac{\sum f_i x_i^2}{N} = \frac{2048}{22}$

$\boxed{\sigma^2 = \frac{2048}{22} - (8)^2}$

$\sigma^2 = \text{avg}(x_i^2) - (\bar{x})^2$

QUESTION

Find the standard deviation using step deviation method of the following data:

Class-Interval	25 – 35	35 – 45	45 – 55	55 – 65	65 – 75
Frequency	21	20	16	25	18

Remark

(a) Variance is simply σ^2

(b) Range $= x_{\max} - x_{\min}$

(c) $C.V.\% = \dfrac{\sigma}{\bar{x}} \times 100$

Note:

C.V. is independent of units

QUESTION

If the variance of 20 observations is 5 & each observation is multiplied by 2 then new variance is:

(A) 5

(B) 10

(C) 20

(D) 40

QUESTION [JEE Main 2024 /01 Fab. – Shift-1]

4th observation is median.

Let the median and the mean deviation about the median of 7 observation 170, 125, 230, 190, 210, a, b be 170 and $\frac{205}{7}$ respectively. Then the mean deviation about the mean of these 7 observations is:

(A) 31

(B) 28

(C) 30

(D) 32

$$125, \underline{a}, \underline{b}, 170, 190, 210, 230.$$

mean deviation about median $= \frac{45 + (170 - a) + (170 - b) + 20 + 40 + 60}{\cancel{7}} = \frac{205}{\cancel{7}}$

$$= 505 - a - b = 205 \Rightarrow \boxed{a + b = 300}$$

$$\bar{x} = \frac{125 + \overbrace{a+b}^{300} + 170 + 190 + 210 + 230}{7} = \frac{1225}{7} = 175$$

QUESTION [JEE Main 2024 /01 Fab. – Shift-1]

4th observation is median.

Let the median and the mean deviation about the median of 7 observation 170, 125, 230, 190, 210, a, b be 170 and $\frac{205}{7}$ respectively. Then the mean deviation about the mean of these 7 observations is:

125, a, b, 170, 190, 210, 230.

(A) 31

(B) 28

(C) 30

(D) 32

M.D. about mean $= \frac{50+(175-a)+(175-b)+5+15+35+55}{7} = 30.$

300.

$\bar{x} = \frac{125+\overbrace{a+b}^{300}+170+190+210+230}{7} = \frac{1225}{7} = 175$

QUESTION [JEE Main 2024 /01 Fab. – Shift-1]

4th observation is median.

Let the median and the mean deviation about the median of 7 observation 170, 125, 230, 190, 210, a, b be 170 and $\frac{205}{7}$ respectively. Then the mean deviation about the mean of these 7 observations is:

125, a, b, 170, 190, 210, 230.

(A) 31

(B) 28

(C) 30

(D) 32

mean deviation about median $= \frac{45+(170-a)+(170-b)+20+40+60}{\cancel{7}} = \frac{205}{\cancel{7}}$

$= 505-a-b = 205 \Rightarrow \boxed{a+b=300}$

QUESTION [JEE Main 2024 /01 Fab. – Shift-2]

Consider 10 observation $x_1, x_{2,\ldots}, x_{10}$. such that $\sum_{i=1}^{10}(x_i - \alpha) = 2$ and $\sum_{i=1}^{10}(x_i - \beta)^2 = 40$, where α, β are positive integers. Let the mean and the variance of the observations be $\frac{6}{5}$ and $\frac{84}{25}$ respectively. The $\frac{\beta}{\alpha}$ is equal to:

(A) 2

(B) $\frac{3}{2}$

(C) $\frac{5}{2}$

(D) 1

QUESTION [JEE Main 2024 /27 Jan. – Shift-1]

Let $a_1, a_2, \ldots a_{10}$ be 10 observations such that $\sum_{k=1}^{10} a_k = 50$ and $\underbrace{\sum_{k<j} a_k \cdot a_j}$ $= 1100$. Then the standard deviation of $a_1, a_2, \ldots, a_{10}$ is equal to:

(A) 5

(B) $\sqrt{5}$

(C) 10

(D) $\sqrt{115}$

$$\sigma = \sqrt{\frac{\sum a_i^2}{N} - \left(\frac{\sum a_i}{N}\right)^2}$$

$$= \sqrt{\frac{300}{10} - \left(\frac{50}{10}\right)^2}$$

$$= \sqrt{30 - 25}$$

$$= \sqrt{5}$$

$$\boxed{a_1 + a_2 + a_3 + \ldots + a_{10} = 50}$$

$$(a_1 + a_2 + \ldots + a_{10})^2 = 2500$$

$$\sum_{i=1}^{10} a_i^2 + 2\overbrace{\cancel{\sum a_k a_j}}^{1100} = 2500$$

$$\Rightarrow \boxed{\sum_{i=1}^{10} a_i^2 = 300}$$

$$a_1 a_2 + a_1 a_3 + a_1 a_4 + a_2 a_4 + a_2 a_5 + a_2 a_3 \ldots = 1100$$

QUESTION [JEE Main 2024 /29 Jan. – Shift-1]

$170+110+168 = 448$

If the mean and variance of the data 65, 68, 58, 44, 48, 45, 60, α, β, 60 where α > β are 56 and 66.2 respectively, then $\alpha^2 + \beta^2$ is equal to.

$\bar{x}$

$$\text{mean} = \frac{65+68+58+44+48+45+60+\alpha+\beta+60}{10}$$

$$56 = \frac{448+\alpha+\beta}{10}$$

$$560 = 448+\alpha+\beta$$

$$\boxed{\alpha+\beta = 112}$$

$$\sigma^2 = \frac{\sum (x_i-\bar{x})^2}{10} = 66.2$$

$$= 9^2+(12)^2+(2)^2+(12)^2+(8)^2+(11)^2+(4)^2+(4)^2 + (56-\alpha)^2+(56-\beta)^2 = 662$$

$$56^2 - 2\times 56\times\alpha + \alpha^2$$

$$56^2 - 2\times 56\times\beta + \beta^2$$

QUESTION [JEE Main 2024 /29 Jan. – Shift-2]

If the mean and variance of five observations are $\frac{24}{5}$ and $\frac{194}{25}$ respectively and the mean of first four observations is $\frac{7}{2}$, then the variance of the first four observations in equal to

(A) $\frac{4}{5}$

(B) $\frac{77}{12}$

(C) $\frac{5}{4}$

(D) $\frac{105}{4}$

$$\frac{x_1+x_2+x_3+x_4+x_5}{5}=\frac{24}{5}$$

$$\boxed{x_1+x_2+x_3+x_4+x_5=24}$$

$$\frac{x_1^2+x_2^2+x_3^2+x_4^2+\cancel{x_5^2}\,10^2}{5}-\left(\frac{24}{5}\right)^2=\frac{194}{25}$$

$$\frac{x_1+x_2+x_3+x_4}{4}=\frac{7}{2}\Rightarrow \boxed{x_1+x_2+x_3+x_4=14}$$

$$x_5=10$$

770

$$\Rightarrow \frac{x_1^2+x_2^2+x_3^2+x_4^2+100}{\cancel{5}}=\frac{194+576}{\cancel{25}\,5}=\frac{770}{5}$$

$$\underbrace{x_1^2+x_2^2+x_3^2+x_4^2}_{=54}+100=154$$

QUESTION [JEE Main 2024 /29 Jan. – Shift-2]

If the mean and variance of five observations are $\frac{24}{5}$ and $\frac{194}{25}$ respectively and the mean of first four observations is $\frac{7}{2}$, then the variance of the first four observations in equal to

(A) $\frac{4}{5}$

(B) $\frac{77}{12}$

(C) $\frac{5}{4}$

(D) $\frac{105}{4}$

$$\text{var.} = \frac{\sum x_i^2}{4} - (\bar{x})^2$$

$$= \frac{54}{4} - \left(\frac{7}{2}\right)^2$$

$$= \frac{5}{4}$$

$$\frac{x_1+x_2+x_3+x_4}{4} = \frac{7}{2} \Rightarrow \boxed{x_1+x_2+x_3+x_4 = 14}$$

$$x_5 = 10$$

$$770$$

$$\Rightarrow \frac{x_1^2+x_2^2+x_3^2+x_4^2+100}{\cancel{5}} = \frac{194+576}{\cancel{25}\ 5} = \frac{770}{5}$$

$$\underbrace{x_1^2+x_2^2+x_3^2+x_4^2}_{54} + 100 = 154$$

QUESTION [JEE Main 2024 /30 Jan. – Shift-2]

The variance σ2 of the data

	0	1	5	6	10	12	17
	3	2	3	2	6	3	3

Is ________.

QUESTION [JEE Main 2024 /31 Jan. – Shift-2]

Let the mean and the variance of 6 observation a, b, 68, 44, 48, 60 be 55 and 194, respectively if a>b, then a+3b is:

(A) 200

(B) 190

(C) 180

(D) 210

$\bar{x}$

$\frac{a+b+68+44+48+60}{6} = 55$

$a+b+220 = 330$

$\boxed{a+b=110}$

$\sigma^2 = \frac{\sum(x_i-\bar{x})^2}{6}$

$194 = (55-a)^2+(55-b)^2+(13)^2+(11)^2+(7)^2+(5)^2$

$\Rightarrow a^2+b^2=?$

75, 35

QUESTION [JEE Main 2021 (16 March – Shift 2)]

Consider the statistics of two sets of observations as follows :

	Size	Mean	Variance
Observation I	10	2	2
Observation II	n	3	1

If the variance of the combined set of these two observations is 17/9, then the value of n is equal to :

$$\left(\sigma^2 = \frac{n_1\sigma_1^2 + n_2\sigma_2^2}{n_1+n_2} + \frac{n_1 n_2}{(n_1+n_2)^2}(\bar{x}_1 - \bar{x}_2)^2\right)$$

$$\frac{17}{9} = \frac{10(2)+n(1)}{10+n} + \frac{(10)(n)}{(10+n)^2}(3-2)^2$$

$$\frac{17(10+n)^2}{9} = (20+n)(10+n) + 10n$$

$$17(n^2+20n+100) = 9(n^2+30n+200) + 90n$$

$$8n^2 - 20n - 100 = 0$$

$$2n^2 - 5n - 25 = 0$$

$$2n^2 - 10n + 5n - 25 = 0$$

$$2n(n-5) + 5(n-5) = 0$$

$$n = 5$$

	Size	mean	variance
observation 1	n_1	$\bar{x}_1$	σ_1^2
observation 2	n_2	$\bar{x}_2$	σ_2^2

$$\boxed{\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}}$$

$$\boxed{\sigma^2 = \frac{n_1\sigma_1^2 + n_2\sigma_2^2}{n_1 + n_2} + \frac{n_1 n_2}{(n_1+n_2)^2}\left(\bar{x}_1 - \bar{x}_2\right)^2}$$

QUESTION [JEE Main 2021 (17 March – Shift 2)]

Consider a set of 3 n numbers having variance 4. In this set, the mean of first numbers is 6 and the mean of the remaining numbers is 3. A new set is constructed by adding 1 into each of first numbers, and subtracting each of the remaining n numbers. If the variance of the new set is then is equal to

HW