Simple Harmonic Motion

Position and Velocity of SHM

$x = A\sin(\omega t + \Phi_0)$

$v = A\omega\cos(\omega t + \Phi_0)$

$v = \omega\sqrt{A^2 - x^2}$

a) $x = 0 \rightarrow v_{max} = A\omega$

b) $x = \frac{A}{2} \rightarrow v = \frac{\sqrt{3}A\omega}{2}$
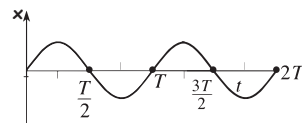
c) $x = \frac{A}{\sqrt{2}} \rightarrow v = \frac{A\omega}{\sqrt{2}}$
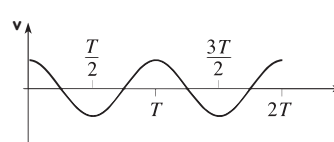
d) $x = A \rightarrow v = 0$

Graphical Representation of Position and Velocity

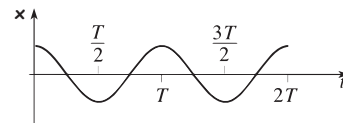Start from mean position, $\Phi_0 = 0$
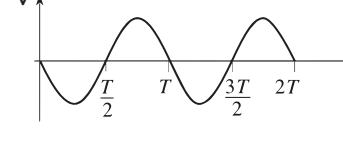
$x = A\sin(\omega t)$

$v = A\omega\cos(\omega t)$

Start from extreme position $\Phi_0 = \frac{\pi}{2}$

$x = A\cos(\omega t)$

$v = -A\omega\sin(\omega t)$

$A = \sqrt{\dfrac{v_1^2 x_2^2 - v_2^2 x_1^2}{v_1^2 - v_2^2}}$

$\omega = \sqrt{\dfrac{v_1^2 - v_2^2}{x_2^2 - x_1^2}}$

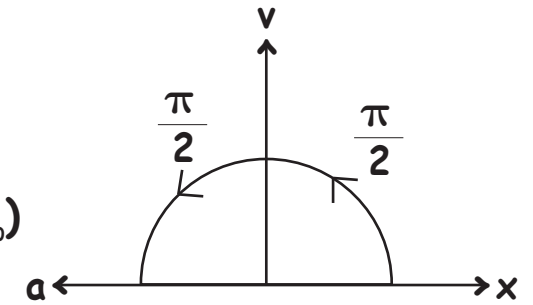## Acceleration of SHM

$x = A\sin(\omega t + \Phi_0)$

$v = A\omega\cos(\omega t + \Phi_0)$

$a = -A\omega^2\sin(\omega t + \Phi_0)$

$a_{max} = -A\omega^2$

Phase difference between x and v = $\dfrac{\pi}{2}$
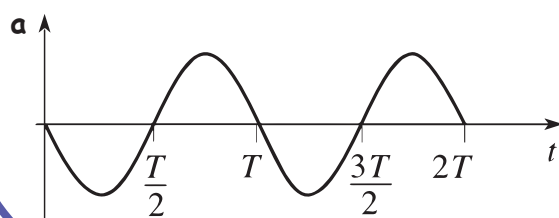
Phase difference between v and a = $\dfrac{\pi}{2}$

Phase difference between x and a = π

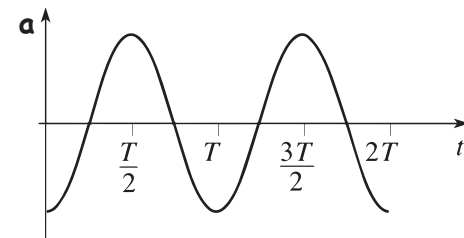# OSCILLATIONS 01

Graphical Representation of Acceleration

Start from mean position, $\Phi_0 = 0$

$a = -A\omega^2\sin(\omega t)$

Start from extreme position, $\Phi_0 = \frac{\pi}{2}$

$a = -A\omega^2\cos(\omega t)$

Calculation of Time period and amplitude

$v_{max} = A\omega$

$a_{max} = A\omega^2$

$\omega = \dfrac{a_{max}}{v_{max}}$

$A = \dfrac{v_{max}^2}{a_{max}}$
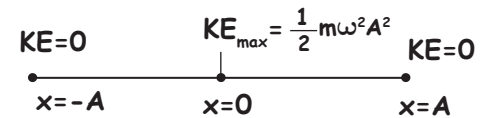
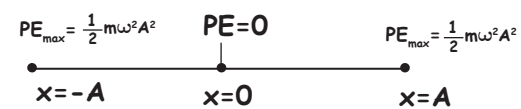$\dfrac{2\pi}{T} = \dfrac{a_{max}}{v_{max}}$

$T = 2\pi\dfrac{v_{max}}{a_{max}}$

Energy of SHM

$$K.E = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2(A^2 - x^2)$$

KE=0 $KE_{max} = \frac{1}{2}m\omega^2A^2$ KE=0

x=-A   x=0   x=A

$$P.E = \frac{1}{2}m\omega^2x^2$$

$PE_{max} = \frac{1}{2}m\omega^2A^2$ PE=0 $PE_{max} = \frac{1}{2}m\omega^2A^2$

x=-A   x=0   x=A

Total mechanical energy, $E = \frac{1}{2}m\omega^2A^2 = $ Constant

1) $x = \frac{A}{\sqrt{2}}$

$$K.E = P.E = \frac{E}{2}$$

2) $x = \frac{A}{2}$

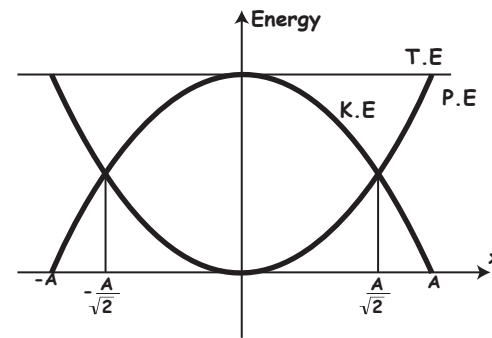$$K.E = \frac{3E}{4}, \quad P.E = \frac{E}{4}$$

3) x=0

$$K.E = E \qquad P.E = 0$$

4) x=A

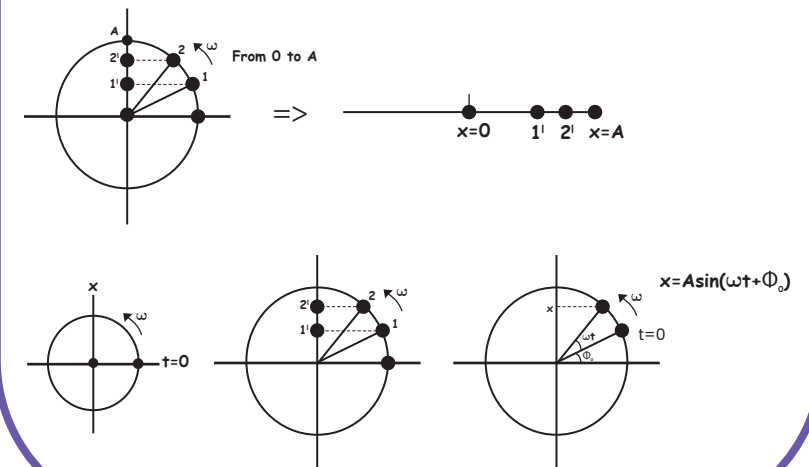$$K.E = 0 \qquad P.E = E$$

Energy vs x graph showing T.E, K.E, P.E

Total mechanical energy(E) $= \frac{1}{2}m\omega^2A^2$

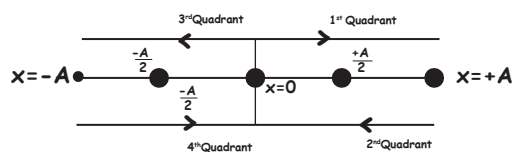Note: In SHM, if particle oscillates with frequency ω, then the K.E & P.E oscillate with 2ω

OSCILLATIONS 02

PROJECTION OF CIRCULAR MOTION

Projection/shadow of uniform circular motion on y axis is SHM

From 0 to A

$x = A\sin(\omega t + \Phi_0)$

Two particles executing SHM meet at $x = \frac{\sqrt{3}A}{2}$

for particle A

$$X = \frac{\sqrt{3}A}{2}, \ t=0 \Rightarrow \frac{\sqrt{3}A}{2} = A\sin\Phi_A$$

$$\Phi_A = 60^0 = \frac{\pi}{3}$$

for particle B

$$X = \frac{\sqrt{3}A}{2}, \ t=0 \Rightarrow \frac{\sqrt{3}A}{2} = A\sin\Phi_B$$

$$\Phi_B = 90^0 + 30^0$$

$$\frac{\pi}{2} + \frac{\pi}{6} = \frac{2\pi}{3}$$

Phase difference between particles

$$\Phi = \frac{2\pi}{3} - \frac{\pi}{3} = \frac{\pi}{3} = 60^0$$

INITIAL PHASE FROM POSITION & DIRECTION

Eg: Particle is at $x = \frac{A}{2}$ [t=0] and moves towards A
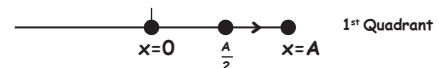
1st Quadrant

1st Quadrant

1) Particle is at $x = \frac{A}{2}$ [t=0] and moves towards A

$x = \frac{A}{2}, t=0 \Rightarrow \frac{A}{2} = A\sin\Phi_0$

$\Phi_0 = 30^0 = \frac{\pi}{6}$

$x = A\sin(\omega t + \frac{\pi}{6})$

2) Particle is at $x = \frac{A}{\sqrt{2}}$ [t=0] and moves towards A

$x = \frac{A}{\sqrt{2}}, t=0 \Rightarrow \frac{A}{\sqrt{2}} = A\sin\Phi_0$
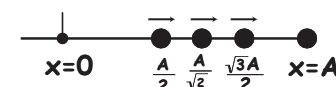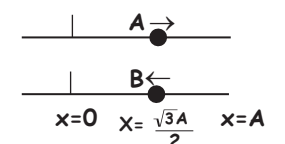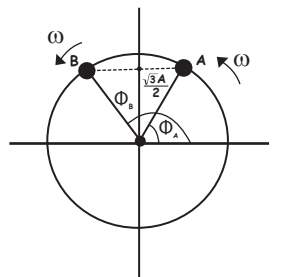
$\Phi_0 = 45^0 = \frac{\pi}{4}$

$x = A\sin(\omega t + \frac{\pi}{4})$

3) Particle is at $x = \frac{\sqrt{3}A}{2}$ [t=0] and moves towards A

$x = \frac{\sqrt{3}A}{2}, t=0 \Rightarrow \frac{\sqrt{3}A}{2} = A\sin\Phi_0$

$\Phi_0 = 60^0 = \frac{\pi}{3}$

$x = A\sin(\omega t + \frac{\pi}{3})$

2nd Quadrant

1) Particle is at $x = \frac{A}{2}$ [t=0] and moves towards O

$x = \frac{A}{2}, t=0 \Rightarrow \frac{A}{2} = A\cos\Phi_0$

$\Phi_0 = 90^0 + 60^0 = \frac{\pi}{2} + \frac{\pi}{3} = \frac{5\pi}{6}$

$x = A\cos(\omega t + \frac{5\pi}{6})$

2) Particle is at $x = \frac{A}{\sqrt{2}}$ [t=0] and moves towards O

$x = \frac{A}{\sqrt{2}}, t=0 \Rightarrow \frac{A}{\sqrt{2}} = A\cos\Phi_0$

$\Phi_0 = 90^0 + 45^0 = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$

$x = A\cos(\omega t + \frac{3\pi}{4})$

Calculation of time

$$\Delta t = \frac{\Delta\theta}{\omega} = \left(\frac{\theta_1 - \theta_2}{2\pi}\right)T$$

OSCILLATIONS 03

TIME PERIOD OF S.H.M

1
$$F=-kx$$
$$T=2\pi\sqrt{\frac{m}{k}}$$

2
$$F=-kx$$
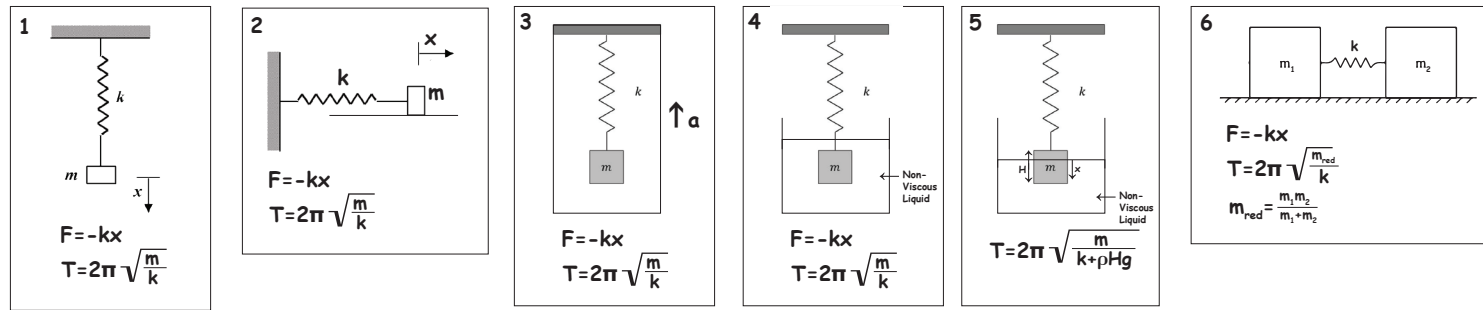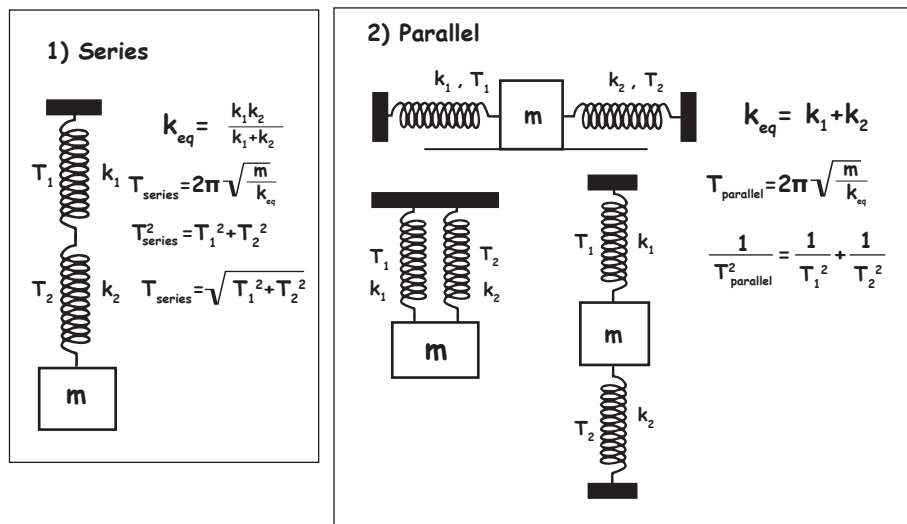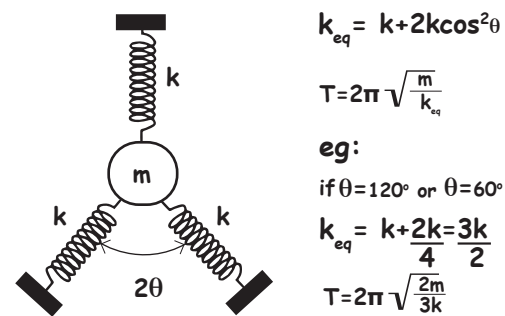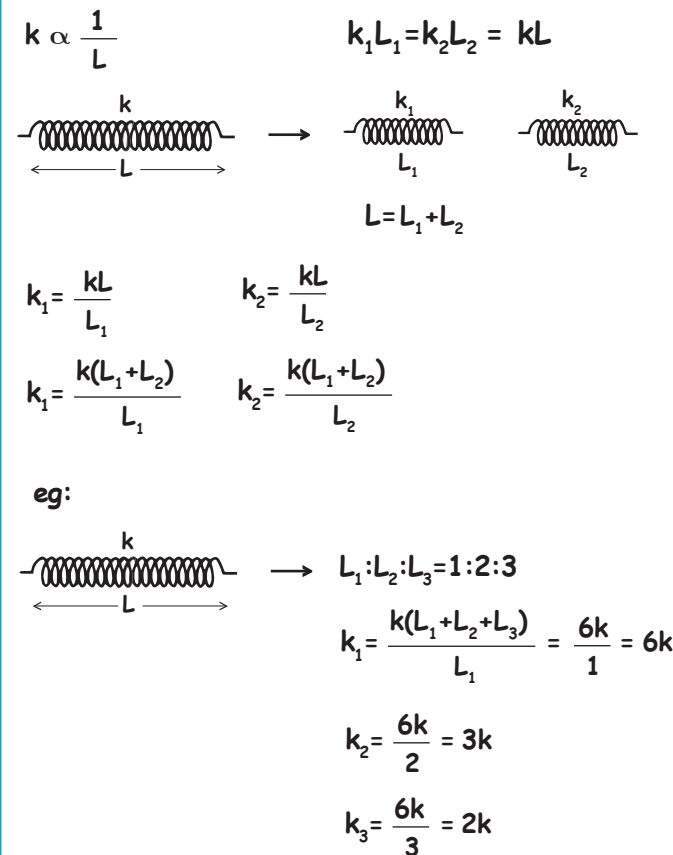$$T=2\pi\sqrt{\frac{m}{k}}$$

3
$$F=-kx$$
$$T=2\pi\sqrt{\frac{m}{k}}$$

4
$$F=-kx$$
$$T=2\pi\sqrt{\frac{m}{k}}$$

5
$$T=2\pi\sqrt{\frac{m}{k+\rho Hg}}$$

6
$$F=-kx$$
$$T=2\pi\sqrt{\frac{m_{red}}{k}}$$
$$m_{red}=\frac{m_1 m_2}{m_1+m_2}$$

COMBINATIONS OF SPRINGS

1) Series
$$k_{eq}=\frac{k_1 k_2}{k_1+k_2}$$
$$T_{series}=2\pi\sqrt{\frac{m}{k_{eq}}}$$
$$T_{series}^2=T_1^2+T_2^2$$
$$T_{series}=\sqrt{T_1^2+T_2^2}$$

2) Parallel
$$k_{eq}=k_1+k_2$$
$$T_{parallel}=2\pi\sqrt{\frac{m}{k_{eq}}}$$
$$\frac{1}{T_{parallel}^2}=\frac{1}{T_1^2}+\frac{1}{T_2^2}$$

SPECIAL CASE

$$k_{eq}=k+2k\cos^2\theta$$
$$T=2\pi\sqrt{\frac{m}{k_{eq}}}$$

eg:
if $\theta=120°$ or $\theta=60°$
$$k_{eq}=k+\frac{2k}{4}=\frac{3k}{2}$$
$$T=2\pi\sqrt{\frac{2m}{3k}}$$

CUTTING OF SPRINGS

$$k\propto\frac{1}{L}$$
$$k_1 L_1=k_2 L_2=kL$$
$$L=L_1+L_2$$

$$k_1=\frac{kL}{L_1}\qquad k_2=\frac{kL}{L_2}$$
$$k_1=\frac{k(L_1+L_2)}{L_1}\qquad k_2=\frac{k(L_1+L_2)}{L_2}$$

eg:
$$L_1:L_2:L_3=1:2:3$$
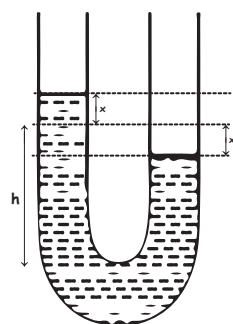$$k_1=\frac{k(L_1+L_2+L_3)}{L_1}=\frac{6k}{1}=6k$$
$$k_2=\frac{6k}{2}=3k$$
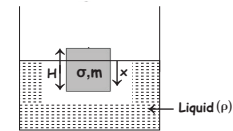$$k_3=\frac{6k}{3}=2k$$

OSCILLATION OF LIQUID COLUMN

$$T=2\pi\sqrt{\frac{m}{2\rho gA}}$$
$$T=2\pi\sqrt{\frac{h}{g}}$$

m=mass of liquid
ρ=Density of liquid
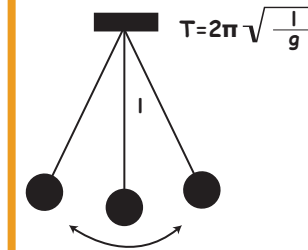A=Area of U-tube

OSCILLATION OF FLOATING BODY

$$T=2\pi\sqrt{\frac{m}{\rho gA}}$$
$$T=2\pi\sqrt{\frac{\sigma H}{\rho g}}$$

m=mass of body
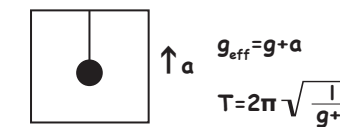σ=Density of Body
ρ=Density of liquid

SIMPLE PENDULUM

$$T=2\pi\sqrt{\frac{l}{g}}$$

• Second's pendulam
T=2 second
l=1 meter

Concept of $g_{effective}$
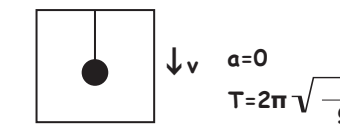$$T=2\pi\sqrt{\frac{l}{g_{eff}}}$$

Pendulum in lift

Case 1-Moving with constant upward acceleration 'a'
$$g_{eff}=g+a$$
$$T=2\pi\sqrt{\frac{l}{g+a}}$$

Case 2-Moving with constant downward acceleration 'a'
$$g_{eff}=g-a$$
$$T=2\pi\sqrt{\frac{l}{g-a}}$$
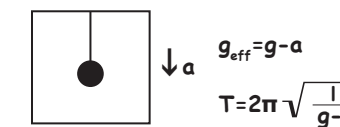
Case 3-Moving with constant velocity
$$a=0$$
$$T=2\pi\sqrt{\frac{l}{g}}$$

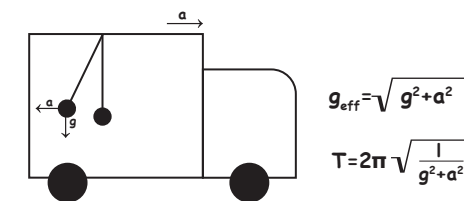Case 4-Free fall
$$a=g$$
$$g_{eff}=g-a=g-g=0 \qquad T\to\infty$$

Pendulum in a truck moving with constant acceleration
$$g_{eff}=\sqrt{g^2+a^2}$$
$$T=2\pi\sqrt{\frac{l}{g^2+a^2}}$$

Pendulum in Water
σ=Density of Bob
ρ=Density of liquid
$$g_{eff}=g\left(1-\frac{\rho}{\sigma}\right)$$
$$T=2\pi\sqrt{\frac{l}{g\left(1-\frac{\rho}{\sigma}\right)}}$$
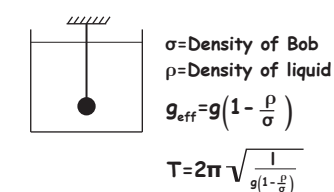
DIFFERENTIAL EQUATION OF S.H.M

$$ma=-kx$$
$$m\frac{d^2x}{dt^2}=-kx$$
$$\frac{d^2x}{dt^2}=\frac{-k}{m}x$$
$$\frac{d^2x}{dt^2}+\frac{k}{m}x=0$$
$$\frac{d^2x}{dt^2}+\omega^2 x=0$$
where, $\omega^2=\frac{k}{m}\qquad \omega=\sqrt{\frac{k}{m}}$

Solving,
$$x=A\sin(wt+\phi_0)$$
A=Amplitude of SHM
ϕ_0=Initial phase angle

Eg: $\begin{bmatrix} m=4\,Kg \\ K=320\,N/m \end{bmatrix}$

$$4\frac{d^2x}{dt^2}+320x=0$$
$$\frac{d^2x}{dt^2}+80x=0$$
$$\omega^2=80 \qquad \omega=\sqrt{80}$$
$$\frac{2\pi}{T}=\sqrt{80}$$
$$T=\frac{2\pi}{\sqrt{80}}$$

PHYSICS WALLAH