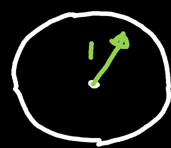
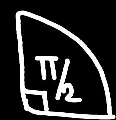

QUESTIONS

Q

Consider a square of side 4 cm. Now if a man runs at a distance of 1 cm from the sides of the square. How much distance will he travel.





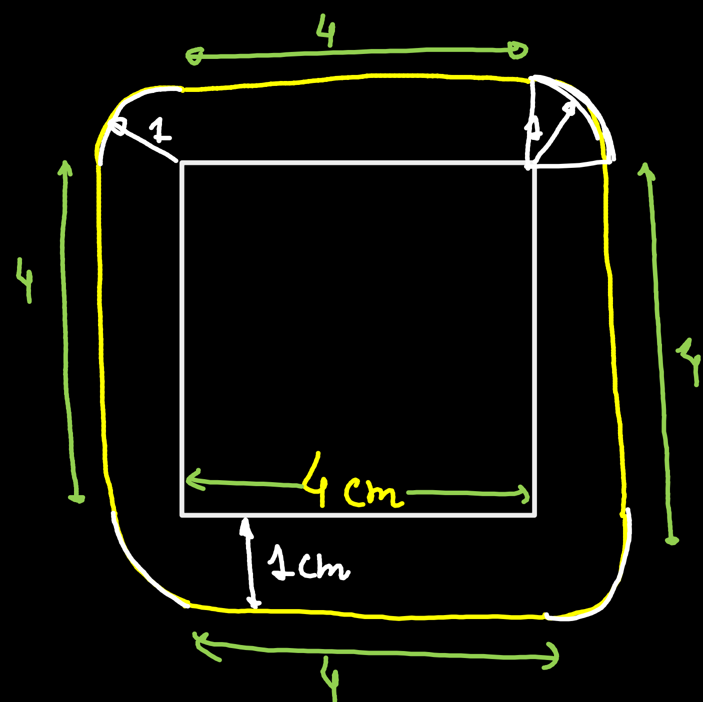
$$l = 1\left(\frac{\pi}{2}\right)\times 4 = 2\pi$$

$$\Rightarrow 4(4) + 2\pi(1)$$

$$\Rightarrow 16 + 2(3.14)$$

$$\Rightarrow 16 + 6.28$$

$$\Rightarrow \boxed{22.28\ \text{cm}}$$



Q

Let $f_k(x) = \frac{1}{k}\left(\sin^k x + \cos^k x\right)$ where $x \in R$ and $k \geq 1$.

Then $f_4(x) - f_6(x)$ equals $\frac{1}{4} - \frac{1}{6}$

(a) $\frac{1}{4}$ (b) $\frac{1}{12}$ (c) $\frac{1}{6}$ (d) $\frac{1}{3}$

[JEE M 2014]

$f_4(x) - f_6(x)$

$\Rightarrow \frac{1}{4}\left(\sin^4 x + \cos^4 x\right) - \frac{1}{6}\left(\sin^6 x + \cos^6 x\right)$

$\Rightarrow \frac{1}{4}\left(1 - 2\sin^2 x\cos^2 x\right) - \frac{1}{6}\left(1 - 3\sin^2 x\cos^2 x\right)$

$\Rightarrow \frac{1}{4} - \frac{1}{6} \Rightarrow \boxed{\frac{1}{12}}$

#NVStyle

$\boxed{M\text{-}2}$ $\boxed{x = 0^\circ}$

$f_k(x) = \frac{1}{k}(1) = \frac{1}{k}$

For any $\theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ the expression

$3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$ equals:

[JEE M 2019]

(a) $13 - 4\cos^2\theta + 6\sin^2\theta\cos^2\theta$

(b) $13 - 4\cos^6\theta$

(c) $13 - 4\cos^2\theta + 6\cos^4\theta$

(d) $13 - 4\cos^4\theta + 2\sin^2\theta\cos^2\theta$

$$3(s-c)^4 + 6(s+c)^2 + 4s^6$$

$$= 3(1-2sc)^2 + 6(1+2sc) + 4(1-c^2)^3$$

$$= 3(1+4s^2c^2-4sc) + 6 + 12sc + 4\left(1 - c^6 - 3c^2(1-c^2)\right)$$

$$= 3 + 12 s^2 c^2 - 12\cancel{sc} + 6 + 12\cancel{sc} + 4 - 4c^6 - 12c^2 + 12c^4$$

$$= 3 + 12c^2(1 - c^2) + 10 - 4c^6 - 12c^2 + 12c^4$$

$$= 3 + \cancel{12c^2} - \cancel{12c^4} + 10 - 4c^6 - \cancel{12c^2} + \cancel{12c^4}$$

$$= \underline{13 - 4\cos^6\theta}$$

Unacademy Atoms

Q

If $15\sin^4\alpha + 10\cos^4\alpha = 6$, for some $\alpha \in R$, then the value of $27\sec^6\alpha + 8\,cosec^6\alpha$ is

A. 350
B. 500
C. 400
D. 250

[JEE M 2021]

Concept $\sin^2\alpha = t = \frac{2}{5}$

$\cos^2\alpha = 1 - t = \frac{3}{5}$

$15(\sin^2\alpha)^2 + 10(\cos^2\alpha)^2 = 6$

$\Rightarrow 15(t)^2 + 10(1-t)^2 = 6$

$\Rightarrow 15t^2 + 10(1+t^2-2t) = 6$

$\Rightarrow 25t^2 - 20t + 4 = 0$

$\Rightarrow (5t-2)^2 = 0 \Rightarrow t = \frac{2}{5}$

$\frac{27}{(\cos^2\alpha)^3} + \frac{8}{(\sin^2\alpha)^3}$

$\Rightarrow \frac{27}{(3/5)^3} + \frac{8}{(2/5)^3}$

$\Rightarrow \boxed{250}$

Q

If $\boxed{\dfrac{\sin^4 x}{2}+\dfrac{\cos^4 x}{3}=\dfrac{1}{5}}$, then

MCQ

(a) $\tan^2 x = \dfrac{2}{3}$

(b) $\boxed{\dfrac{\sin^8 x}{8}+\dfrac{\cos^8 x}{27}}=\dfrac{1}{125}$

AB

[JEE Adv. 2009]

~~(c)~~ $\tan^2 x = \dfrac{1}{3}$

~~(d)~~ $\boxed{\dfrac{\sin^8 x}{8}+\dfrac{\cos^8 x}{27}}=\dfrac{2}{125}$

$\dfrac{(\sin^2 x)^2}{2}+\dfrac{(\cos^2 x)^2}{3}=\dfrac{1}{5}$

$\boxed{\begin{aligned}&\text{Let, } \sin^2 x = t\\ &\cos^2 x = 1-t\end{aligned}}$ } Concept

$\dfrac{t^2}{2}+\dfrac{(1-t)^2}{3}=\dfrac{1}{5}$

$\boxed{\begin{aligned}&\sin^2 x = 2/5\\ &\cos^2 x = 3/5\end{aligned}}$

$\tan^2 x = 2/3$

$$5\left(3t^2+2(1-t)^2\right)=6$$

$$5\left(3t^2+2t^2-4t+2\right)=6$$

$$25t^2-20t+4=0$$

$$(5t-2)^2=0$$

$$\therefore\ t=\frac{2}{5}$$

$$\boxed{\frac{\sec^4 x}{a}+\frac{\tan^4 x}{b}=c}$$

$$\left.\begin{aligned}\tan^2 x&=t\\ \sec^2 x&=1+t\end{aligned}\right\}$$

$$\frac{(\sin^2 x)^4}{8}+\frac{(\cos^2 x)^4}{27}$$

$$\Rightarrow \frac{(2/5)^4}{8}+\frac{(3/5)^4}{27}$$

$$\Rightarrow \frac{2}{5^4}+\frac{3}{5^4}$$

$$\Rightarrow \frac{5}{5^4}=\frac{1}{5^3}=\frac{1}{125}$$

Q

Find $\tan\frac{\pi}{11}+\tan\frac{2\pi}{11}+\tan\frac{4\pi}{11}+\tan\frac{7\pi}{11}+\tan\frac{9\pi}{11}+\tan\frac{10\pi}{11}$

CAST

$\tan\left(\frac{\pi}{11}\right)+\tan\left(\frac{2\pi}{11}\right)+\tan\left(\frac{4\pi}{11}\right)+\tan\left(\pi-\frac{4\pi}{11}\right)+\tan\left(\pi-\frac{2\pi}{11}\right)+\tan\left(\pi-\frac{\pi}{11}\right)$

$\cancel{\tan\left(\frac{\pi}{11}\right)}+\cancel{\tan\left(\frac{2\pi}{11}\right)}+\cancel{\tan\left(\frac{4\pi}{11}\right)}-\cancel{\tan\frac{4\pi}{11}}-\cancel{\tan\frac{2\pi}{11}}-\cancel{\tan\frac{\pi}{11}}$

$\Rightarrow \boxed{0}$

$$\boxed{\star\ \tan(\underbrace{\pi-\theta}_{\text{II}})=-\tan\theta}$$

Q

Find sin 420° cos 390° + cos(-300°) sin(-330°)

$\Rightarrow \sin 420^\circ \cos 390^\circ - \cos 300^\circ \sin 330^\circ$

$\Rightarrow \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{-1}{2}\right)$

$\Rightarrow \frac{3}{4} + \frac{1}{4}$

$\Rightarrow \boxed{1}$

$\sin 420^\circ$

$= \sin(360^\circ + 60)$

$= \sin(\underbrace{2\pi + 60^\circ}_{1^{st}})$

$= +\sin 60^\circ$

$= \frac{\sqrt{3}}{2}$

$\cos 390^\circ$ $\quad 2\pi$

$= \cos(360 + 30^\circ)$

$= +\cos 30^\circ$

$= \frac{\sqrt{3}}{2}$

$\cos 300^\circ$

$= \cos(360 - 60)$ $\quad$ IV

$= +\cos(60^\circ)$

$= \frac{1}{2}$

$\cos(270 + 30)$

$= \cos\left(\frac{3\pi}{2} + 30^\circ\right)$

$= +\sin(30^\circ) = 1/2$

$\sin 330$

$= \sin(360 - 30)$

$= -\sin 30$

$= \frac{-1}{2}$

The expression $3\left[\sin^4\left(\frac{3\pi}{2}-\alpha\right)+\sin^4(3\pi+\alpha)\right]-2\left[\sin^6\left(\frac{\pi}{2}+\alpha\right)+\sin^6(5\pi-\alpha)\right]$ is equal to

(JEE M PYQ)

$$3\left(\cos^4\alpha+\sin^4\alpha\right)-2\left(\cos^6\alpha+\sin^6\alpha\right)$$

$$=3\left(1-2\cancel{s^2c^2}\right)-2\left(1-3\cancel{s^2c^2}\right)$$

$$=\boxed{1}$$

Q

Let $\theta \in \left(0, \frac{\pi}{4}\right)$ and $t_1 = (\tan\theta)^{\tan\theta}$, $t_2 = (\tan\theta)^{\cot\theta}$,

$t_3 = (\cot\theta)^{\tan\theta}$ and $t_4 = (\cot\theta)^{\cot\theta}$, then

[JEE Adv. 2006]

(a) $t_1 > t_2 > t_3 > t_4$

(b) $t_4 > t_3 > t_1 > t_2$

(c) $t_3 > t_1 > t_2 > t_4$

(d) $t_2 > t_3 > t_1 > t_4$

$\theta \in \left(0, \frac{\pi}{4}\right)$

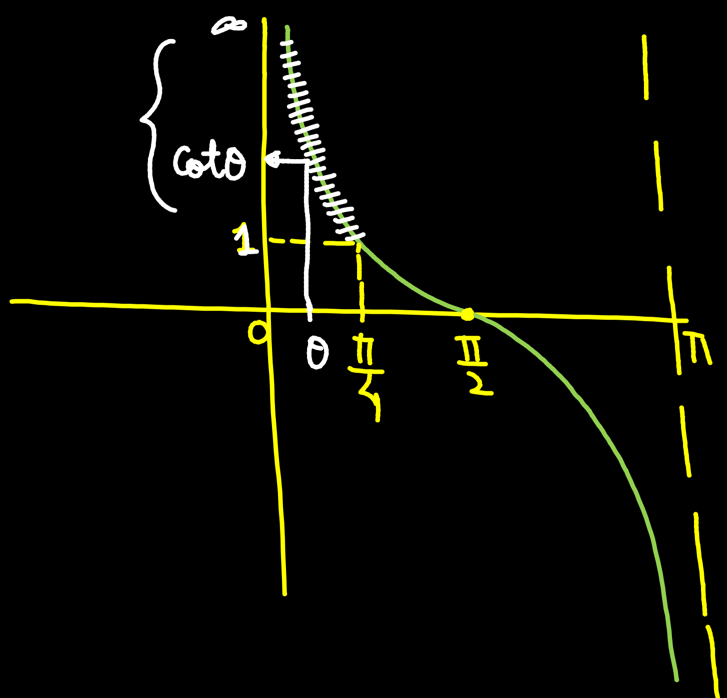
$$\boxed{\begin{array}{l} \tan\theta \in (0,1) \\ \cot\theta \in (1,\infty) \end{array}}$$

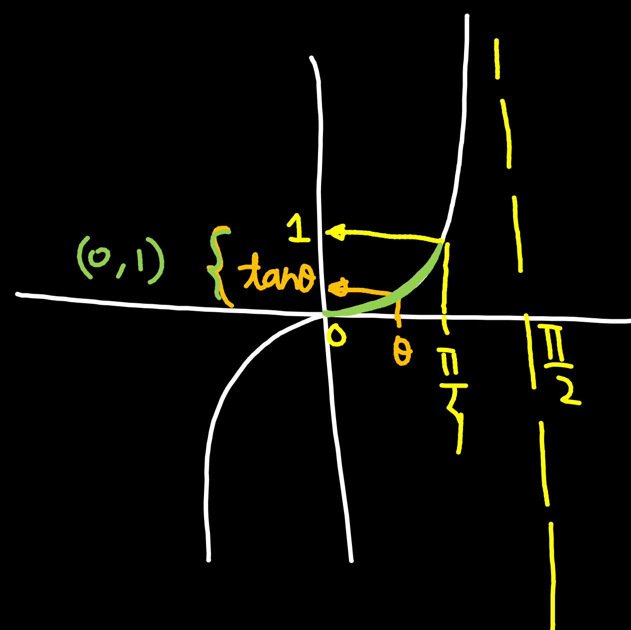
$\tan\theta = 1/2$

$\cot\theta = 2$

t_1	t_2	t_3	t_4
$\left(\frac{1}{2}\right)^{1/2}$	$\left(\frac{1}{2}\right)^{2}$	$2^{1/2}$	2^{2}
0.7	0.25	1.414	4

$$\cot\frac{\pi}{4} = 1$$





Unacademy Atoms

Q

Let α, β be such that $\pi < \alpha - \beta < 3\pi$.

If $\boxed{\sin\alpha + \sin\beta = -\frac{21}{65}}$ and $\boxed{\cos\alpha + \cos\beta = -\frac{27}{65}}$, then the value of $\cos\left(\frac{\alpha-\beta}{2}\right)$

$\underbrace{\frac{\alpha-\beta}{2}}_{2^{nd}/3^{rd}}$

S	A
T	C

[JEE M 2004]

(a) $\frac{-6}{65}$

(b) $\cancel{\frac{+3}{\sqrt{130}}}$

(c) $\cancel{\frac{6}{65}}$

(d) $-\frac{3}{\sqrt{130}}$ ✓

$$\frac{\pi}{2} < \frac{\alpha-\beta}{2} < \frac{3\pi}{2}$$

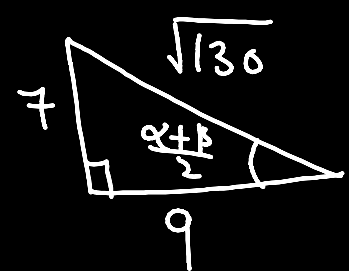
$$\cos\left(\frac{\alpha-\beta}{2}\right) \rightarrow \text{II} / \text{III}$$

$$2\left(\frac{\cancel{7}}{\sqrt{130}}\right)\cos\left(\frac{\alpha-\beta}{2}\right) = \frac{-\cancel{21}^{\,3}}{65} \quad \text{——} \; 1$$

$$\boxed{\cos\left(\frac{\alpha-\beta}{2}\right) = \frac{-3}{\sqrt{130}}}$$

$$2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) = \frac{-27}{65} \quad \text{——} \; 2$$

$$1 \div 2 \qquad \boxed{\tan\left(\frac{\alpha+\beta}{2}\right) = \frac{7}{9}}$$



$$\sin\left(\frac{\alpha+\beta}{2}\right) = \frac{7}{\sqrt{130}}$$

Q

If $\boxed{\cot\alpha = 1}$ and $\boxed{\sec\beta = -\frac{5}{3}}$, where $\underbrace{\pi < \alpha < \frac{3\pi}{2}}$

$\boxed{\tan\alpha = 1}$

and $\underbrace{\frac{\pi}{2} < \beta < \pi}$, then the value of $\underbrace{\tan(\alpha+\beta)}$ and the quadrant in which $\alpha+\beta$ lies, respectively are

[JEE M 2022]

A. $-\frac{1}{7}$ and IV^{th} quadrant

B. 7 and I^{st} quadrant

C. -7 and IV^{th} quadrant

D. $\frac{1}{7}$ and I^{st} quadrant

$\alpha \to III \qquad \beta \to II$

$$\tan(\alpha+\beta) = \frac{\boxed{\tan\alpha} + \boxed{\tan\beta}}{1 - \tan\alpha\tan\beta}$$

$$= \frac{1 - \frac{4}{3}}{1 - (1)\left(-\frac{4}{3}\right)} = -\frac{1}{7}$$

$\pi < \alpha$

$\frac{\pi}{2} < \beta$

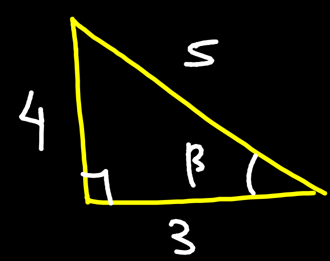
$\boxed{\frac{3\pi}{2} < \alpha + \beta}$

$\alpha + \beta \longrightarrow ?$

II

$\sec\beta = \frac{-5}{3}$

$\underline{\tan}\,\beta = -\frac{4}{3}$



IV

If $a_1, a_2, \ldots, a_n$ are in A.P. with common difference d then the sum of series

$\sin d\,\{\sec a_1 \sec a_2 + \sec a_2 \sec a_3 + \cdots + \sec a_{n-1} \sec a_n\}$

A. $\sec a_1 - \sec a_n$

B. $\operatorname{cosec} a_1 - \operatorname{cosec} a_n$

C. $\cot a_1 - \cot a_n$

D. $\tan a_n - \tan a_1$

$$\Rightarrow \left\{ \boxed{\frac{\sin(a_2-a_1)}{\cos a_1 \cos a_2}} + \boxed{\frac{\sin(a_3-a_2)}{\cos a_2 \cos a_3}} + \cdots + \frac{\sin(a_n-a_{n-1})}{\cos a_{n-1} \cos a_n} \right\}$$

$$\tan a_2 - \tan a_1$$
$$\tan a_3 - \tan a_2$$
$$\vdots$$
$$\tan a_n - \tan(a_{n-1})$$

$$\tan(a_n) - \tan a_1$$

$$\frac{\sin(a_2 - a_1)}{\cos a_1 \cos a_2}$$

$$\Rightarrow \frac{\sin a_2 \cos a_1 - \cos a_2 \sin a_1}{\cos a_1 \cos a_2}$$

$$\Rightarrow \tan a_2 - \tan a_1$$

Q

Let $\boxed{\dfrac{\sqrt{2}\sin\alpha}{\sqrt{1+\cos 2\alpha}}=\dfrac{1}{7}}$ and $\boxed{\sqrt{\dfrac{1-\cos 2\beta}{2}}=\dfrac{1}{\sqrt{10}}}$ where $\alpha, \beta \in \left(0, \dfrac{\pi}{2}\right)$.

$\alpha, \beta \rightarrow \mathrm{I}$

Unacademy Atoms

[JEE M 2020]

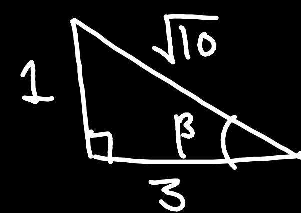
Then tan $(\alpha + 2\beta)$ is equal to

$$\frac{\sqrt{2}\sin\alpha}{\sqrt{2\cos^2\alpha}}=\frac{1}{7}$$

$$\tan\alpha=\frac{1}{7}$$

$$\sqrt{\frac{\cancel{2}\sin^2\beta}{\cancel{2}}}=\frac{1}{\sqrt{10}}$$

$$\sin\beta=\frac{1}{\sqrt{10}}$$



$$\boxed{\tan\beta=\frac{1}{3}}$$

$\tan(\alpha + 2\beta)$

$1/7$ $\quad$ $3/4$

$$\Rightarrow \frac{\boxed{\tan\alpha} + \boxed{\tan(2\beta)}}{1 - \tan\alpha \tan(2\beta)}$$

$$\Rightarrow \frac{\frac{1}{7} + \frac{3}{4}}{1 - \frac{1}{7} \times \frac{3}{4}} = \frac{25}{25} = 1$$

$$\tan 2\beta = \frac{2\tan\beta}{1 - \tan^2\beta}$$

$$= \frac{2\left(\frac{1}{3}\right)}{1 - \frac{1}{9}} = \frac{3}{4}$$

The value of

$\cos^2 10° - \cos 10° \cos 50° + \cos^2 50°$ is :

[JEE M 2019]

(a) $\frac{3}{4} + \cos 20°$

(b) 3/4

(c) $\frac{3}{2}(1 + \cos 20°)$

(d) 3/2

$2\cos^2\theta = 1 + \cos 2\theta$

$2\cos A \cos B = \cos(A+B) + \cos(A-B)$

$\Rightarrow \frac{1}{2}\left(2\cos^2 10° - 2\cos 10 \cos 50 + 2\cos^2 50°\right)$

$\Rightarrow \frac{1}{2}\left((1 + \cos 20°) - \left(\frac{1}{2} + \cos 40\right) + (1 + \cos 100°)\right)$

$\Rightarrow \frac{1}{2}\left(\frac{3}{2} + \cos 20 - \cos 40 + \cos 100\right)$

$\frac{20-40}{2}$

$$\Rightarrow \frac{1}{2}\left(\frac{3}{2} + \cancel{2}\,\cancel{\sin 30^\circ}\, \sin 10^\circ + \underline{\cos(90+10)}\right)$$

$$\Rightarrow \frac{1}{2}\left(\frac{3}{2} + \cancel{\sin 10^\circ} - \cancel{\sin 10}\right)$$

$$\Rightarrow 3/4$$

Q

The value of $\sin 10^\circ \underline{\sin 30^\circ} \underline{\sin 50^\circ} \underline{\sin 70^\circ}$ is

A. $\frac{1}{16}$

B. $\frac{1}{32}$

C. $\frac{1}{18}$

D. $\frac{1}{36}$

[JEE M 2019]

$$\Rightarrow \frac{1}{2}\left(\sin 10^\circ \sin(60-10)\sin(60+10)\right)$$

$$\Rightarrow \frac{1}{2}\left(\frac{1}{4}\sin 30^\circ\right)$$

$$\Rightarrow \frac{1}{2}\left(\frac{1}{4}\times\frac{1}{2}\right)$$

$$\Rightarrow \frac{1}{16}$$

$\sin(A)\,\underbrace{\sin(60^\circ+A)\,\sin(60-A)} = \frac{1}{4}\sin 3A$

$\cos A\,\underbrace{\cos(60^\circ+A)\,\cos(60^\circ-A)} = \frac{1}{4}\cos 3A$

$\tan A\,\tan(60+A)\,\tan(60-A) = \tan(3A)$

HINT $\boxed{\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B}$

$= \cos^2 A - \sin^2 B$

16sin (20°) sin(40°) sin (80°) is equal to :

A. √3

B. 2√3

C. 3

D. 4√3

[JEE M 2022]

$16 \sin 20 \sin(60-20) \sin(60+20)$

$\Rightarrow 16\left(\frac{1}{4}\sin 60\right)$

$\Rightarrow 4 \times \frac{\sqrt{3}}{2} \Rightarrow 2\sqrt{3}$

Unacademy Atoms

Q

If $\sin^2(10^\circ)\sin(20^\circ)\sin(40^\circ)\sin(50^\circ)\sin(70^\circ) = \alpha - 1/16\sin(10^\circ)$, then $\boxed{16 + \alpha^{-1}}$ is equal to_

[JEE M 2022]

$$\boxed{\sin^2 10^\circ\ \sin(20)\ \sin 40\ \sin 50\ \sin 70 = \alpha - \frac{1}{16}\sin 10^\circ}$$

$$\left(\sin 10^\circ \sin(60-10)\sin(60+10)\right)\left(\sin 10 \sin 20^\circ \sin 40\right)$$

(BADE MIYA)

$$\Rightarrow \frac{1}{8}\left(\frac{\sin 10\,(2\sin 20^\circ \sin 40)}{2}\right)$$

$$\boxed{2\sin A\sin B = \cos(A-B) - \cos(A+B)}$$

$$\Rightarrow \frac{1}{16}\left(\sin 10\left(\cos 20^\circ - \frac{1}{2}\right)\right)$$

$10-20$

$$\Rightarrow \frac{1}{16}\left(\frac{2\sin 10\cos 20}{2} - \frac{\sin 10}{2}\right)$$

$$2\sin A\cos B = \sin(A+B) + \sin(A-B)$$

$$\Rightarrow \frac{1}{32}\left(\frac{1}{2} + \sin(-10) - \sin 10\right)$$

$$\Rightarrow \frac{1}{32}\left(\frac{1}{2} - 2\sin 10\right)$$

$$\Rightarrow \frac{1}{64} - \frac{1}{16}\sin 10$$

$$\Rightarrow \alpha - \frac{1}{16}\sin 10$$

$$\alpha = \frac{1}{64}$$

$$16 + \frac{1}{\alpha} = 16 + 64$$

$$= 80$$

Unacademy Atoms

Q Value of $\cos^3\frac{\pi}{8}\cos\frac{3\pi}{8}+\sin^3\frac{\pi}{8}\sin\frac{3\pi}{8}$

[JEE M 2020]

A. $\frac{1}{2\sqrt{2}}$

B. $\frac{1}{\sqrt{2}}$

C. $\frac{1}{2}$

D. $-\frac{1}{2}$

$\Rightarrow \cos^3\frac{\pi}{8}\sin\frac{\pi}{8}+\sin^3\frac{\pi}{8}\cos\frac{\pi}{8}$

$\Rightarrow \cos\frac{\pi}{8}\sin\frac{\pi}{8}\left(\cos^2\frac{\pi}{8}+\sin^2\frac{\pi}{8}\right)$

$\Rightarrow \frac{2\cos\frac{\pi}{8}\sin\frac{\pi}{8}}{2}(1)$

$\Rightarrow \frac{\sin\frac{\pi}{4}}{2}\Rightarrow \frac{1}{2\sqrt{2}}$

$\cos\frac{3\pi}{8}$

$=\cos\left(\frac{4\pi-\pi}{8}\right)$

$=\cos\left(\frac{\pi}{2}-\frac{\pi}{8}\right)$

$=\sin\frac{\pi}{8}$

Let $f:[0,2] \longrightarrow \mathbb{R}$ be the function defined by

$$f(x) = (3 - \sin(2\pi x))\sin\left(\pi x - \frac{\pi}{4}\right) - \sin\left(3\pi x + \frac{\pi}{4}\right).$$

If $\alpha, \beta \in [0,2]$ are such that $\{x \in [0,2] : f(x) \geq 0\} = [\alpha, \beta]$, then the value of $\beta - \alpha$ is ______

[JEE Adv. 2020]

$$\left(3 - \sin(\overbrace{2\pi x}^{2\theta+\frac{\pi}{2}})\right)\sin(\overbrace{\pi x - \tfrac{\pi}{4}}^{\theta}) - \sin(\overbrace{3\pi x + \tfrac{\pi}{4}}^{\pi+3\theta}) \geq 0$$

$$\left(3 - \sin\left(\frac{\pi}{2} + 2\theta\right)\right)\sin\theta - \sin(\pi + 3\theta) \geq 0$$

$$(3 - \cos 2\theta)\sin\theta + 3\sin\theta - 4\sin^3\theta \geq 0$$

$$\pi x - \frac{\pi}{4} = \theta$$

$$\Rightarrow \pi x = \theta + \frac{\pi}{4}$$

$$\Rightarrow \boxed{2\pi x = 2\theta + \frac{\pi}{2}}$$

$$\Rightarrow 3\pi x = 3\theta + \frac{3\pi}{4}$$

$$\Rightarrow 3\pi x + \frac{\pi}{4} = 3\theta + \pi$$

$$\sin\theta\,(3-\cos 2\theta+3-4\sin^2\theta)\geq 0$$

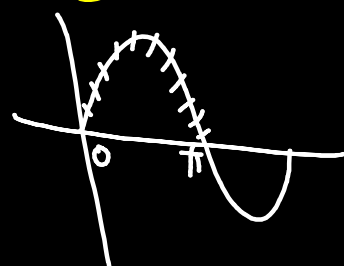
$$\sin\theta\,(6-(1-2\sin^2\theta)-4\sin^2\theta)\geq 0$$

$$\underbrace{\sin\theta}_{+}\underbrace{(5-2\overset{[0,1]}{\boxed{\sin^2\theta}})}_{+}\underbrace{\geq 0}_{+}$$

$$\Rightarrow \boxed{\sin\theta\geq 0}$$

$$0\leq\theta\leq\pi$$

$$0\leq\pi x-\frac{\pi}{4}\leq\pi$$



$\alpha=\frac{1}{4}\quad \beta=\frac{5}{4}$

$$\beta-\alpha=1$$

$\sin^2\theta\in[0,1]$

$$\frac{\cancel{\pi}}{4}\leq\cancel{\pi}x\leq\frac{5\cancel{\pi}}{4}$$

$$\frac{1}{4}\leq x\leq\frac{5}{4}$$

$$x\in\left[\frac{1}{4},\frac{5}{4}\right]$$

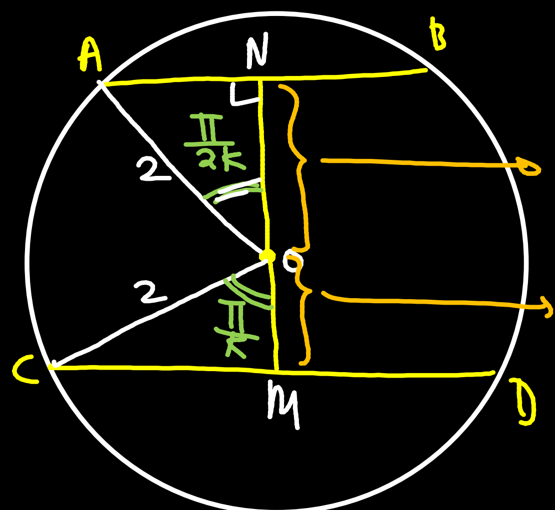
Q

Two parallel chords of a circle of radius 2 are at a distance $\sqrt{3}+1$ apart. If the chords subtend at the center, angles of $\frac{\pi}{k}$ and $\frac{2\pi}{k}$, where $k > 0$, then the value of $[k]$ is

[JEE Adv. 2010]

$r = 2$

$NM = (\sqrt{3}+1)$



$\rightarrow 2\cos\left(\frac{\pi}{2k}\right)$

$\rightarrow 2\cos\left(\frac{\pi}{k}\right)$

$\frac{\pi}{2k} \rightarrow$ Acute

$$2\cos\left(\frac{\pi}{2K}\right) + 2\cos\left(\frac{\pi}{K}\right) = \sqrt{3}+1 \qquad \frac{\pi}{2K} = \theta \Rightarrow \boxed{\frac{\pi}{K} = 2\theta}$$

$$2\cos\theta + 2\cos 2\theta = \sqrt{3}+1$$

$$2\cos\theta + 2\left(2\cos^2\theta - 1\right) = \sqrt{3}+1$$

$$4\cos^2\theta + 2\cos\theta - (3+\sqrt{3}) = 0$$

$$\cos\theta = \frac{-2 \pm 2\sqrt{1 + (4)(3+\sqrt{3})}}{8}$$

$$\cos\theta = \frac{-1 \pm \sqrt{13+4\sqrt{3}}}{4}$$

$$13+4\sqrt{3}$$

$$= (2\sqrt{3})^2 + (1)^2 + 2(2\sqrt{3})(1)$$

$$= (2\sqrt{3}+1)^2$$

$$= \frac{-1 \pm (2\sqrt{3}+1)}{4}$$

$$= \frac{\cancel{-1}+2\sqrt{3}+\cancel{1}}{4} \quad \text{or} \quad \cancel{\frac{-1-2\sqrt{3}-1}{4}}$$

$$= \sqrt{3}/2$$

$$\cos\theta = \frac{\sqrt{3}}{2}$$

$$\theta = \frac{\pi}{6} = \frac{\pi}{2K}$$

$$\therefore \boxed{K=3}$$

Unacademy Atoms

Let α and β be real numbers such that $-\frac{\pi}{4} < \beta < 0 < \alpha < \frac{\pi}{4}$. If $\sin(\alpha+\beta) = \frac{1}{3}$ and $\cos(\alpha-\beta) = \frac{2}{3}$, then the greatest integer less than or equal to

$$\left(\frac{\sin\alpha}{\cos\beta} + \frac{\cos\beta}{\sin\alpha} + \frac{\cos\alpha}{\sin\beta} + \frac{\sin\beta}{\cos\alpha}\right)^2 \Rightarrow \left\lfloor \frac{16}{9} \right\rfloor$$

is ______.

[JEE Adv 2022]

$$\Rightarrow 1$$

$$\Rightarrow \frac{\cos(\alpha-\beta)}{\sin\beta\cos\beta} + \frac{\cos(\alpha-\beta)}{\sin\alpha\cos\alpha}$$

$$\Rightarrow \frac{2}{3}\left(\frac{1\times 2}{2\sin\beta\cos\beta} + \frac{1\times 2}{2\sin\alpha\cos\alpha}\right)$$

$$\Rightarrow \frac{4}{3}\left(\frac{1}{\sin 2\beta} + \frac{1}{\sin 2\alpha}\right)$$

$$2\sin A \sin B = \cos(A-B) - \cos(A+B)$$

$$\Rightarrow \frac{2\times 4}{3}\left(\frac{\boxed{\sin 2\alpha + \sin 2\beta}}{2\sin 2\alpha \sin 2\beta}\right)$$

$$\Rightarrow \frac{8}{3}\left(\frac{2\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)}{\boxed{\cos(2\alpha-2\beta)} - \boxed{\cos(2\alpha+2\beta)}}\right)$$

$$\Rightarrow \frac{32}{27}\left(\frac{1}{\left(2\cos^2(\alpha-\beta)-1\right) - \left(1-2\sin^2(\alpha+\beta)\right)}\right)$$

$$\Rightarrow \frac{32}{27}\left(\frac{1}{\left(2\left(\frac{2}{3}\right)^2-1\right)-\left(1-2\left(\frac{1}{3}\right)^2\right)}\right)$$

$$\Rightarrow \frac{32}{27}\left(\frac{1}{\left(-\frac{1}{9}\right)-\left(\frac{7}{9}\right)}\right)$$

$$\Rightarrow \frac{\cancel{32}^{\,4}}{\cancel{27}_{\,3}} \times \frac{\cancel{9}}{-\cancel{8}} \Rightarrow \left(-\frac{4}{3}\right)^2 = \frac{16}{9}$$

Unacademy Atoms

Q

The value of 2sin(12°) - sin(72°) is :

A. $\frac{\sqrt{5}\left(1-\sqrt{3}\right)}{4}$

B. $\frac{1-\sqrt{5}}{8}$

C. $\frac{\sqrt{3}\left(1-\sqrt{5}\right)}{2}$

D. $\frac{\sqrt{3}\left(1-\sqrt{5}\right)}{4}$

[JEE M 2022]

$\Rightarrow \sin 12 + \underbrace{\sin 12 - \sin 72}$

$\Rightarrow \sin 12^\circ + 2\cos 42 \sin(-30^\circ)$

$\Rightarrow \sin 12 - \cos 42$

$$\Rightarrow \sin(12) - \sin(48)$$

$$\Rightarrow 2\cos 30 \sin(-18^\circ)$$

$$\Rightarrow -\cancel{2} \times \left(\frac{\sqrt{3}}{\cancel{2}}\right)\left(\frac{\sqrt{5}-1}{4}\right)$$

$$\Rightarrow \underbrace{\frac{\sqrt{3}}{4}\left(1-\sqrt{5}\right)}$$

Unacademy Atoms

Q

$\alpha = \sin 36°$ is a root of which of the following equation

[JEE M 2022]

A. $10x^4 - 10x^2 - 5 = 0$

B. $16x^4 + 20x^2 - 5 = 0$

C. $16x^4 - 20x^2 + 5 = 0$

D. $16x^4 - 10x^2 + 5 = 0$

$\alpha = \frac{\sqrt{10 - 2\sqrt{5}}}{4}$

$16\alpha^2 = 10 - 2\sqrt{5}$

$8\alpha^2 = 5 - \sqrt{5}$

$(\sqrt{5})^2 = (5 - 8\alpha^2)^2$

$5 = 25 + 64\alpha^4 - 80\alpha^2$

$64\alpha^4 - 80\alpha^2 + 20 = 0$

$\boxed{16\alpha^4 - 20\alpha^2 + 5 = 0}$

Prove that

$$\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{3\pi}{15}\cos\frac{4\pi}{15}\boxed{\cos\frac{5\pi}{15}}\cos\frac{6\pi}{15}\cancel{\cos\frac{7\pi}{15}}=\frac{1}{128}$$

$-\cos\left(\frac{8\pi}{15}\right)$

$$-\left[\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\right]\left[\cos\frac{3\pi}{15}\cos\frac{6\pi}{15}\right]\left[\cos\frac{\pi}{3}\right]$$

$$-\left[\frac{\sin\left(2^4\cdot\frac{\pi}{15}\right)}{2^4\sin\frac{\pi}{15}}\right]\left[\frac{\sin\left(2^2\cdot\frac{3\pi}{15}\right)}{2^2\sin\left(\frac{3\pi}{15}\right)}\right]\left[\frac{1}{2}\right]$$

$$\Rightarrow -\left[\frac{\sin\left(\frac{16\pi}{15}\right)}{2^4\sin\frac{\pi}{15}}\right]\cdot\left[\frac{\sin\left(\frac{12\pi}{15}\right)}{2^2\sin\left(\frac{3\pi}{15}\right)}\right]\left(\frac{1}{2}\right)$$

$\cos\left(\frac{7\pi}{15}\right)$

$=\cos\left(\pi-\frac{8\pi}{15}\right)$

$=-\cos\frac{8\pi}{15}$

$$\Rightarrow -\frac{\boxed{\sin\left(\pi+\frac{\pi}{15}\right)}}{2^4 \sin\left(\frac{\pi}{15}\right)} \cdot \frac{\sin\left(\pi-\frac{3\pi}{15}\right)}{2^2 \sin\left(\frac{3\pi}{15}\right)} \cdot \frac{1}{2}$$

$$\Rightarrow \cancel{-}\, \frac{\left(\cancel{-}\sin\cancel{\frac{\pi}{15}}\right)}{2^4 \cancel{\sin\left(\frac{\pi}{15}\right)}} \cdot \frac{\cancel{\sin\left(\frac{3\pi}{15}\right)}}{2^2 \cancel{\sin\left(\frac{3\pi}{15}\right)}} \cdot \frac{1}{2}$$

$$\Rightarrow \frac{1}{128}$$

Q

Prove that $\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\boxed{\sin\frac{7\pi}{14}}=\frac{1}{8}$

$$\Rightarrow \cos\left(\frac{\pi}{2}-\frac{\pi}{14}\right)\cos\left(\frac{\pi}{2}-\frac{3\pi}{14}\right)\cos\left(\frac{\pi}{2}-\frac{5\pi}{14}\right)\times 1$$

$$\Rightarrow \cos\left(\frac{6\pi}{14}\right)\cos\left(\frac{4\pi}{14}\right)\cos\left(\frac{2\pi}{14}\right)$$

$$\Rightarrow \cos\left(\frac{\pi}{7}\right)\cos\left(\frac{2\pi}{7}\right)\cos\left(\frac{3\pi}{7}\right)$$

$$\Rightarrow -\cos\left(\frac{\pi}{7}\right)\cos\left(\frac{2\pi}{7}\right)\cos\left(\frac{4\pi}{7}\right)$$

Product

\# Cosine

\# ~~Sine series~~

$$\sin\theta = \cos\left(\frac{\pi}{2}-\theta\right)$$

$$\cos\left(\frac{3\pi}{7}\right)$$

$$= \cos\left(\pi-\frac{4\pi}{7}\right)$$

$$= -\cos\left(\frac{4\pi}{7}\right)$$

$$\Rightarrow -\frac{\sin\left(2^3 \cdot \frac{\pi}{7}\right)}{2^3 \sin\left(\frac{\pi}{7}\right)}$$

$$\Rightarrow \frac{\cancel{\boxed{-\sin\left(\pi + \frac{\pi}{7}\right)}}}{2^3 \cancel{\sin\left(\frac{\pi}{7}\right)}}$$

$$\Rightarrow \frac{1}{8}$$

$$\sin\left(\pi + \frac{\pi}{7}\right)$$

$$= -\sin\left(\frac{\pi}{7}\right)$$

Q

Prove that $\sin\frac{\pi}{5}\sin\frac{2\pi}{5}\sin\frac{3\pi}{5}\sin\frac{4\pi}{5}=\frac{5}{16}$

H.W.

Cos Cos

$$2\sin\left(\frac{\pi}{22}\right)\sin\left(\frac{3\pi}{22}\right)\sin\left(\frac{5\pi}{22}\right)\sin\left(\frac{7\pi}{22}\right)\sin\left(\frac{9\pi}{22}\right)$$

is equal to $2\cos\left(\frac{\pi}{2}-\frac{\pi}{22}\right)\cos\left(\frac{\pi}{2}-\frac{3\pi}{22}\right)\cos\left(\frac{\pi}{2}-\frac{5\pi}{22}\right)\cos\left(\frac{\pi}{2}-\frac{7\pi}{22}\right)\cos\left(\frac{\pi}{2}-\frac{9\pi}{22}\right)$

[JEE M 2022]

A. $\frac{3}{16}$

B. $\frac{1}{16}$

C. $\frac{1}{32}$

D. $\frac{9}{32}$

$$\cos\left(\frac{5\pi}{11}\right) = \cos\left(\pi - \frac{6\pi}{11}\right) = -\cos\frac{6\pi}{15}$$

$$2\cos\left(\frac{10\pi}{22}\right)\cos\left(\frac{8\pi}{22}\right)\cos\left(\frac{6\pi}{22}\right)\cos\left(\frac{4\pi}{22}\right)\cos\left(\frac{2\pi}{22}\right)$$

$$\Rightarrow 2\cos\left(\frac{\pi}{11}\right)\cos\left(\frac{2\pi}{11}\right)\underline{\cos\left(\frac{3\pi}{11}\right)}\cos\left(\frac{4\pi}{11}\right)\underline{\cos\left(\frac{5\pi}{11}\right)}$$

$$\Rightarrow 2\left(\cos\frac{\pi}{11}\cos\frac{2\pi}{11}\cos\frac{4\pi}{11}\right)\left(-\cos\left(\frac{3\pi}{11}\right)\cos\left(\frac{6\pi}{11}\right)\right)$$

$$\sin\left(\frac{12\pi}{11}\right) = \sin\left(\pi + \frac{\pi}{11}\right) = -\sin\frac{\pi}{11}$$

$$\Rightarrow -2\left(\frac{\sin\left(2^3\cdot\frac{\pi}{11}\right)}{2^3\sin\left(\frac{\pi}{11}\right)}\right)\left(\frac{\sin\left(2^2\cdot\frac{3\pi}{11}\right)}{2^2\sin\left(\frac{3\pi}{11}\right)}\right)$$

$$\Rightarrow \cancel{-}2\left(\frac{\cancel{\sin\left(\frac{3\pi}{11}\right)}}{2^3\cancel{\sin\left(\frac{\pi}{11}\right)}}\right)\left(\frac{\cancel{-}\cancel{\sin\frac{\pi}{11}}}{2^2\cancel{\sin\frac{3\pi}{11}}}\right) = \frac{1}{16}$$

Q

The value of

$$\underbrace{\cos\frac{\pi}{2^2}\cos\frac{\pi}{2^3}\ldots\cos\frac{\pi}{2^{10}}}\,\underbrace{\sin\frac{\pi}{2^{10}}}$$

A. $\frac{1}{512}$

B. $\frac{1}{1024}$

C. $\frac{1}{256}$

D. $\frac{1}{2}$

[JEE M 2019]

$$\Rightarrow \underbrace{\cos\left(\frac{\pi}{2^{10}}\right)\cos\left(\frac{\pi}{2^{9}}\right)\ldots\ldots\cos\left(\frac{\pi}{2^{3}}\right)\cos\left(\frac{\pi}{2^{2}}\right)}\cdot\sin\left(\frac{\pi}{2^{10}}\right)$$

$$\Rightarrow \frac{\boxed{\sin\left(2^9\cdot\frac{\pi}{2^{10}}\right)}}{2^9\cancel{\sin\left(\frac{\pi}{2^{10}}\right)}}\cdot\cancel{\sin\left(\frac{\pi}{2^{10}}\right)} \Rightarrow \frac{1}{512}$$

Find the sum of series $\cos\frac{\pi}{11}+\cos\frac{3\pi}{11}+\cos\frac{5\pi}{11}+\cos\frac{7\pi}{11}+\cos\frac{9\pi}{11}$.

① sum
② A.P. } ✓

$\alpha = \frac{\pi}{11}$ $\quad \beta = \frac{2\pi}{11} \Rightarrow \frac{\beta}{2} = \frac{\pi}{11}$

$\boxed{n = 5}$

$$\Rightarrow \frac{\sin\left(n\frac{\beta}{2}\right)}{\sin\left(\frac{\beta}{2}\right)} \cos\left(\alpha + (n-1)\frac{\beta}{2}\right)$$

$$\Rightarrow \frac{\sin\left(5\cdot\frac{\pi}{11}\right)}{\sin\left(\frac{\pi}{11}\right)} \cos\left(\frac{\pi}{11} + 4\cdot\frac{\pi}{11}\right)$$

$$\Rightarrow \frac{\boxed{2\sin\left(\frac{5\pi}{11}\right)\cos\left(\frac{5\pi}{11}\right)}}{2\sin\left(\frac{\pi}{11}\right)}$$

$$\Rightarrow \frac{\sin\left(\frac{10\pi}{11}\right)}{2\sin\left(\frac{\pi}{11}\right)} = \frac{\cancel{\sin\left(\pi-\frac{\pi}{11}\right)}}{2\cancel{\sin\left(\frac{\pi}{11}\right)}} = \frac{1}{2}$$

Q

The value of $\cos\left(\frac{2\pi}{7}\right)+\cos\left(\frac{4\pi}{7}\right)+\cos\left(\frac{6\pi}{7}\right)$ is equal to :

[JEE M 2022]

A. -1

B. $-\frac{1}{2}$

C. $-\frac{1}{3}$

D. $-\frac{1}{4}$

① sum
② A.P.

$\boxed{\alpha = \frac{2\pi}{7}}$ $\boxed{\frac{\beta}{2} = \frac{\pi}{7}}$ $n = 3$

$\sin\left(\frac{3\pi}{7}\right)$
$= \sin\left(\pi - \frac{4\pi}{7}\right)$
$= +\sin\frac{4\pi}{7}$

$$\frac{\sin\left(\frac{3\pi}{7}\right)\cos\left(\frac{2\pi}{7}+\frac{2\pi}{7}\right)}{\sin\left(\frac{\pi}{7}\right)} = \frac{\sin\left(\frac{3\pi}{7}\right)\cos\left(\frac{4\pi}{7}\right)}{\sin\left(\frac{\pi}{7}\right)}$$

$$\Rightarrow \frac{\boxed{2\sin\left(\frac{4\pi}{7}\right)\cos\left(\frac{4\pi}{7}\right)}}{2\sin\left(\frac{\pi}{7}\right)}$$

$$\Rightarrow \frac{\sin\left(\pi+\frac{\pi}{7}\right)}{2\sin\left(\frac{\pi}{7}\right)}$$

$$\Rightarrow \frac{-\cancel{\sin\left(\frac{\pi}{7}\right)}}{2\cancel{\sin\left(\frac{\pi}{7}\right)}} = -1/2$$

Q

The value of $\sum_{k=1}^{13} \frac{1}{\sin\left(\frac{\pi}{4}+\frac{(k-1)\pi}{6}\right)\sin\left(\frac{\pi}{4}+\frac{k\pi}{6}\right)}$ is equal to

[JEE Adv. 2016]

(a) $3-\sqrt{3}$

(b) $2\left(3-\sqrt{3}\right)$

(c) $2\left(\sqrt{3}-1\right)$

(d) $2\left(2-\sqrt{3}\right)$

$B - A = \frac{\pi}{6}$

$$\sum_{k=1}^{13} 2\,\frac{\sin\left(\left(\frac{\pi}{4}+\frac{k\pi}{6}\right)-\left(\frac{\pi}{4}+\frac{(k-1)\pi}{6}\right)\right)}{\sin\Big(\underbrace{\frac{\pi}{4}+\frac{(k-1)\pi}{6}}_{A}\Big)\sin\Big(\underbrace{\frac{\pi}{4}+\frac{k\pi}{6}}_{B}\Big)}$$

$$\frac{\sin(B-A)}{\sin A \sin B} = \frac{\sin B \cos A - \cos B \sin A}{\sin A \sin B} = \cot A - \cot B$$

$$2\sum_{K=1}^{13} \cot\left(\frac{\pi}{4}+\frac{(K-1)\pi}{6}\right) - \cot\left(\frac{\pi}{4}+\frac{K\pi}{6}\right)$$

$$\cot\left(\frac{\pi}{4}\right) - \cot\left(\frac{\pi}{4}+\frac{\pi}{6}\right)$$

$$\cot\left(\frac{\pi}{4}+\frac{\pi}{6}\right) - \cot\left(\frac{\pi}{4}+\frac{2\pi}{6}\right)$$

$$\vdots$$

$$\cot\left(\frac{\pi}{4}+\frac{12\pi}{6}\right) - \cot\left(\frac{\pi}{4}+\frac{13\pi}{6}\right)$$

$$S = 2\left(\cot\frac{\pi}{4} - \cot\left(\frac{3\pi+26\pi}{12}\right)\right)$$

$$S = 2\left(1 - \cot\left(2\pi + \frac{5\pi}{12}\right)\right)$$

$$S = 2\left(1 - \cot\frac{5\pi}{12}\right)$$

$\frac{5\pi}{12} = 75^\circ$

$\tan(45-30)$

$= \_\_\_\_\_\_$

$S = 2(1 - \cot 75^\circ)$

$= 2(1-(2-\sqrt{3}))$

$= 2(\sqrt{3}-1)$

$\cot 75^\circ$

$= \tan 15^\circ$

$= \boxed{2-\sqrt{3}}$

Unacademy Atoms

Q

Let $S=\left\{\theta\in\left(0,\frac{\pi}{2}\right):\sum_{m=1}^{9}\sec\left(\theta+(m-1)\frac{\pi}{6}\right)\sec\left(\theta+\frac{m\pi}{6}\right)=-\frac{8}{\sqrt{3}}\right\}$

Then

[JEE M 2022]

A. $S=\left\{\frac{\pi}{12}\right\}$

B. $S=\left\{\frac{2\pi}{3}\right\}$

C. $\sum_{\theta\in S}\theta=\frac{\pi}{2}$

D. $\sum_{\theta\in S}\theta=\frac{3\pi}{4}$

$\sum\theta=\frac{\pi}{6}+\frac{\pi}{3}=\frac{\pi}{2}$

$$\sum_{m=1}^{9} \frac{2\,\sin\left[\overbrace{\left(\theta+\frac{m\pi}{6}\right)}^{B}-\overbrace{\left(\theta+\frac{(m-1)\pi}{6}\right)}^{A}\right]}{\cos\underbrace{\left(\theta+\frac{(m-1)\pi}{6}\right)}_{A}\cos\underbrace{\left(\theta+\frac{m\pi}{6}\right)}_{B}}$$

$$B-A=\frac{\pi}{6}$$

$$\frac{\sin(B-A)}{\cos A\cos B}$$

$$=\frac{\sin B\cos A-\cos B\sin A}{\cos A\cos B}$$

$$=\tan B-\tan A$$

$$\Rightarrow 2\sum_{m=1}^{9}\tan\left(\theta+\frac{m\pi}{6}\right)-\tan\left(\theta+\frac{(m-1)\pi}{6}\right)$$

$$\cancel{\tan\left(\theta+\frac{\pi}{6}\right)}-\tan(\theta)$$

$$\cancel{\tan\left(\theta+\frac{2\pi}{6}\right)}-\cancel{\tan\left(\theta+\frac{\pi}{6}\right)}$$

$$\vdots$$

$$\tan\left(\theta+\frac{9\pi}{6}\right)-\cancel{\tan\left(\theta+\frac{8\pi}{6}\right)}$$

$$S = 2\left(\tan\left(\theta + \frac{3\pi}{2}\right) - \tan\theta\right)$$

$$\Rightarrow 2\left(+\cot\theta + \tan\theta\right) = +\frac{8}{\sqrt{3}}$$

$$\left(\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}\right) = \frac{4}{\sqrt{3}}$$

$$\frac{1\times 2}{2\sin\theta\cos\theta} = \frac{4}{\sqrt{3}}$$

Agla

$$\boxed{\sin 2\theta = \frac{\sqrt{3}}{2}}$$

$$2\theta = \frac{\pi}{3}, \pi - \frac{\pi}{3}$$

$$\boxed{\theta = \frac{\pi}{6}, \frac{\pi}{3}}$$

Illustratio

$\sin^2 A - \sin^2 B = \sin(A+B)\sin(A-B)$

$y = \sin^2\left(\frac{15\pi}{8} - 4x\right) - \sin^2\left(\frac{17\pi}{8} - 4x\right)$. *Find range of y.*

$y = \sin\left(\left(\frac{15\pi}{8} - 4x\right) + \left(\frac{17\pi}{8} - 4x\right)\right) \cdot \sin\left(\left(\frac{15\pi}{8} - \cancel{4x}\right) - \left(\frac{17\pi}{8} - \cancel{4x}\right)\right)$

$y = \underbrace{\sin(4\pi - 8x)}\ \sin\left(-\frac{\pi}{4}\right)$

$= -\sin(8x) \quad + \sin\left(\frac{\pi}{4}\right)$

$\boxed{y = \frac{\boxed{\sin 8x}}{\sqrt{2}}}$ $\boxed{\sin 8x} \rightarrow [-1, 1]$

$\left[\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$

Q

If the equation $\cos^4\theta + \sin^4\theta + \lambda = 0$ has real solutions for θ, then λ lies in interval

[JEE M 2020]

(A) $\left[-1, -\frac{1}{2}\right]$

(B) $\left[-\frac{3}{2}, -\frac{5}{4}\right]$

(C) $\left(-\frac{1}{2}, -\frac{1}{4}\right]$

(D) $\left(-\frac{5}{4}, -1\right)$

$$\lambda = -(\sin^4\theta + \cos^4\theta)$$

$$\lambda = -\left(1 - \frac{4\sin^2\theta\cos^2\theta}{2}\right)$$

$$\boxed{\lambda = \frac{\boxed{\sin^2 2\theta}}{2} - 1}$$

$0 \le \sin^2 2\theta \le 1$

$$\lambda_{min} = \frac{0}{2} - 1 = -1$$

$$\lambda_{max} = \frac{1}{2} - 1 = -\frac{1}{2}$$

Q

The maximum value of the expression

$$\frac{1 \times 2}{2\left(\sin^2\theta + 3\sin\theta\cos\theta + 5\cos^2\theta\right)} \text{ is}$$

[JEE Adv. 2010]

$$\Rightarrow \frac{2}{1 - \cos 2\theta + 3\sin 2\theta + 5(1 + \cos 2\theta)}$$

$$\Rightarrow \frac{2}{\boxed{3\sin 2\theta + 4\cos 2\theta} + 6}$$

$$\Rightarrow \frac{2}{[-5, 5] + 6} = \frac{2}{-5+6} = 2$$

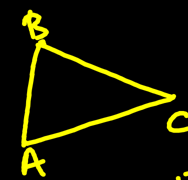
$$3\sin 2\theta + 4\cos 2\theta$$

$$-\sqrt{3^2+4^2}, \sqrt{3^2+4^2}$$

$$[-5, 5]$$

Conditional Identities

idea



$\boxed{A+B+C=\pi}$

(1) $\sin 2A+\sin 2B+\sin 2C=\underline{4\sin A\sin B\sin C}$

(2) $\cos 2A+\cos 2B+\cos 2C=-1-4\cos A\cos B\cos C$

(3) $\sin A+\sin B+\sin C=4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

(4) $\cos A+\cos B+\cos C=1+4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

(5) $\tan A+\tan B+\tan C=\tan A\tan B\tan C$

(6) $\tan\frac{A}{2}\tan\frac{B}{2}+\tan\frac{C}{2}\tan\frac{B}{2}+\tan\frac{C}{2}\tan\frac{A}{2}=1$

$$A + B = \pi - C$$

$$\sin(A+B) = \sin(\pi - C)$$

$$= \sin C$$

$$\underbrace{\sin(2A) + \sin(2B)} + \sin(2C)$$

$$\Rightarrow 2\,\underline{\sin(A+B)}\cos(A-B) + 2\sin C\cos C$$

$$\Rightarrow 2\sin C\cos(A-B) + 2\sin C\cos C$$

$$\Rightarrow 2\sin C\left(\cos(A-B) + \cos(C)\right)$$

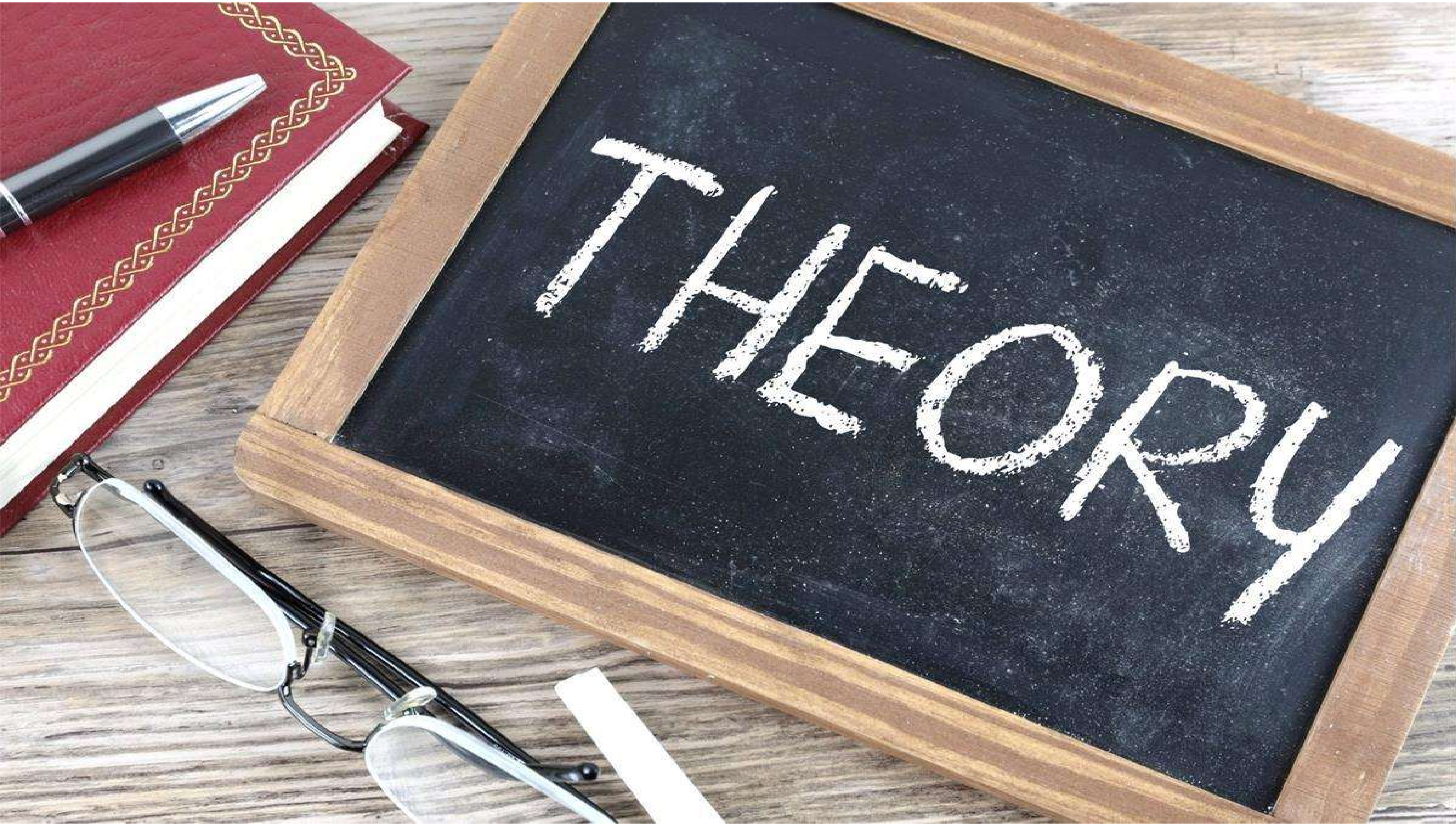
$$\Rightarrow 2\sin C\left(\cos(A-B) + \cos(\pi - (A+B))\right)$$

$$(\cancel{cc} + ss) - (\cancel{cc} - ss)$$

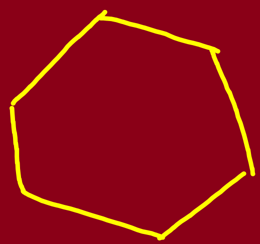
$$\Rightarrow 2\sin C\left(\cos(A-B) - \cos(A+B)\right)$$

$$\Rightarrow 2\sin C\left(2\sin A\sin B\right)$$

$$\Rightarrow \underline{4\sin A\sin B\sin C}$$


THEORY

$\underbrace{\text{Poly}}_{\text{(many)}}\underbrace{\text{nomial}}_{\text{(terms)}}$



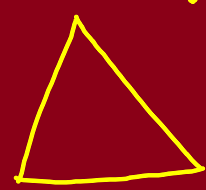
$\underbrace{\text{Poly}}_{\text{(many)}}\underbrace{\text{gon}}_{\text{(sides)}}$

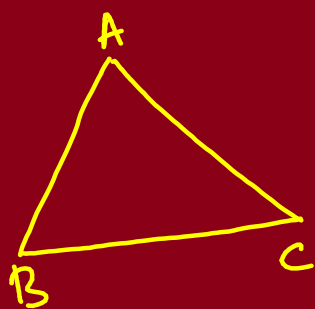
$\underbrace{\text{Trigon}}\underbrace{\text{ometry}}$

$\downarrow$

$\underbrace{\text{Trigon}}_{\text{(3-sides)}} + \underset{\text{(measurement)}}{\text{Metron}}$

(Triangle)





* sides
* Angles $\angle A$ $\angle B$ $\angle C$
* Area

most imp. (Units)

Sides : cm , m , ...

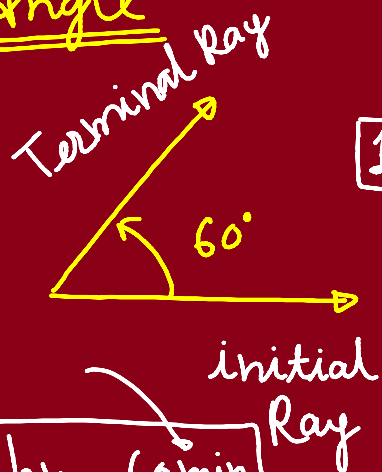
SI unit

Angle degree 60°

Radian

Grades

Angle



$\boxed{1\text{ hr} = 60\text{ min}}$

m
- cm
- dm

$\boxed{1\text{m} = 100\text{cm}}$

$$30^\circ = 30 \times 1^\circ = 30 \times 60' = \boxed{1800'}$$

System of Unit

English (degree)

$\boxed{1^\circ = 60'}$

$\boxed{1' = 60''}$

~~French (grades)~~

$\cancel{\boxed{90^\circ = 100\text{ grad}}}$

(SI)
Circular System (Radian)

$\boxed{180^\circ = \pi\text{ rad}}$

$\boxed{180^\circ = \pi^c}$

deg → Rad.

$$\boxed{\frac{180^\circ}{2} = \frac{\pi}{2}\,\text{rad}}$$

$$\frac{180^\circ}{6} = \frac{\pi}{6}\,\text{rad}$$

$$\boxed{30^\circ = \frac{\pi}{6}\,\text{rad}}$$

$$\boxed{60^\circ = \frac{\pi}{3}\,\text{rad}}$$

$$\boxed{90^\circ = \frac{\pi}{2}\,\text{rad}}$$

$$270^\circ = 9 \times 30^\circ = 9 \times \frac{\pi}{6} = \frac{3\pi}{2}$$

$$30^\circ = \_\_\_\_\_\,\text{rad}$$

$$30^\circ = \cancel{30} \times \frac{\pi}{\cancel{180}_{6}} = \frac{\pi}{6}$$

$$150^\circ = 15\cancel{0} \times \frac{\pi}{18\cancel{0}} = \boxed{\frac{5\pi}{6}}$$

$$150^\circ = 5 \times 30^\circ = \frac{5\pi}{6}$$

$$300^\circ = 10 \times 30^\circ = \frac{10\pi}{6} = \frac{5\pi}{3}$$

Remember.

$$180^\circ = \pi \text{ rad}$$

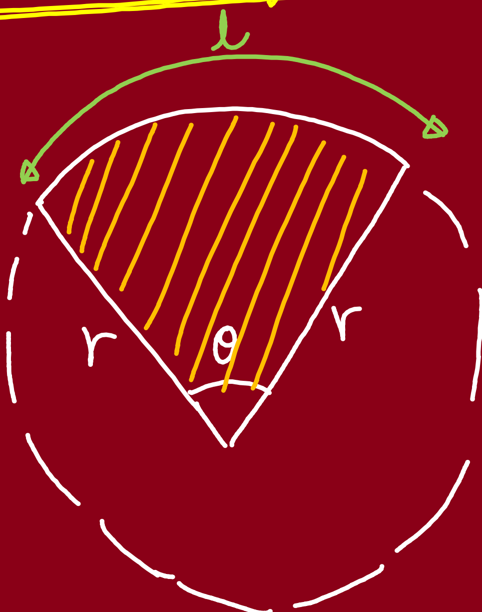
$$90^\circ = \frac{\pi}{2} \text{ rad}$$

$$45^\circ = \frac{\pi}{4} \text{ rad}$$

$$60^\circ = \frac{\pi}{3}$$

$$30^\circ = \frac{\pi}{6}$$

Arc Length



$$\boxed{l = r\theta}$$

θ (radian)

Unitary method

Angle	Arc length
2π	$2\pi r$
θ	?

$$\frac{\theta \times \cancel{2\pi} r}{\cancel{2\pi}} = l$$

$$\boxed{l = r\theta}$$

Area of Sector :-

$$\boxed{A = \frac{1}{2}\theta r^2}$$

θ → (radian)

\# $\boxed{l = r\theta}$

\# $\boxed{A = \frac{1}{2}\theta r^2}$

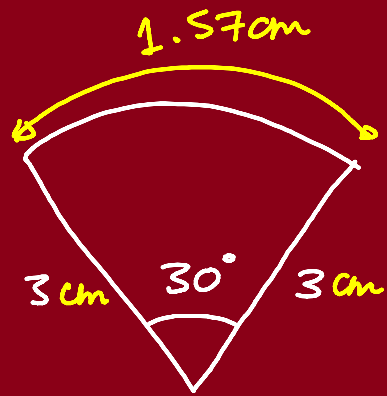
(θ → radian)

Angle	Area
2π	πr^2
θ	?

$$A = \frac{\theta \times \cancel{\pi} r^2}{2\cancel{\pi}}$$

$$\boxed{A = \frac{1}{2}\theta r^2}$$

Ex



$$\# \; l = r\theta$$

$$= 3 \times \frac{\pi}{6}$$

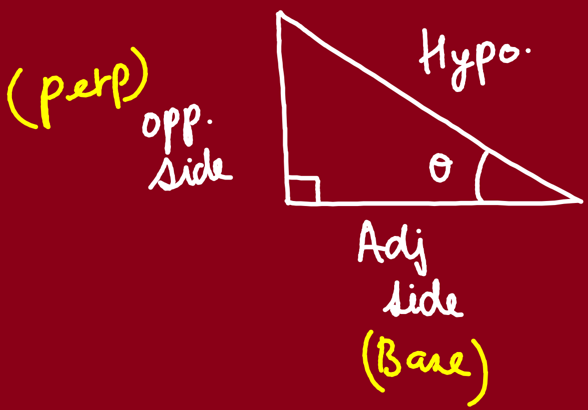
$$= \frac{\pi}{2} = \frac{3.14}{2} = \underline{1.57 \text{ cm}}$$

$$\# \; A = \frac{1}{2}\theta r^2$$

$$= \frac{1}{2}\left(\frac{\pi}{\cancel{6}_{2}}\right)(\cancel{3})(3) = \frac{3\pi}{4} = \frac{3(3.14)}{4} = \boxed{} \text{ cm}^2$$

10^{th} Trigometry :-

SOH CAH TOA



$\sin\theta = \dfrac{opp.}{Hypo} = \dfrac{O}{H}$ $\quad$ $\operatorname{cosec}\theta = \dfrac{Hypo}{opp}$

$\cos\theta = \dfrac{Adj}{Hypo} = \dfrac{A}{H}$ $\quad$ $\sec\theta = \dfrac{Hypo}{Adj}$

$\tan\theta = \dfrac{opp}{adj} = \dfrac{O}{A}$ $\quad$ $\cot\theta = \dfrac{Adj}{opp}$

$\cot\theta = \dfrac{1}{\tan\theta}$ $\quad$ $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$\# \ \sin^2\theta + \cos^2\theta = 1$

$\# \ \sec^2\theta - \tan^2\theta = 1$

$\# \ \mathrm{cosec}^2\theta - \cot^2\theta = 1$

Complimentry Angle Identity

$\sin(90^\circ - \theta) = \cos\theta$

$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$

$\cos\left(\frac{1\pi}{2} - \theta\right) = \sin\theta$

$\tan\left(\frac{1\pi}{2} - \theta\right) = \cot\theta$

$\cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$

$\mathrm{cosec}\left(\frac{\pi}{2} - \theta\right) = \sec\theta$

$\sec\left(\frac{\pi}{2} - \theta\right) = \mathrm{cosec}\,\theta$

11^{th} trigo.

$$(a+b)^3 = a^3+b^3+3ab(a+b)$$

$$a^3+b^3 = (a+b)(a^2-ab+b^2)$$

#1 $\boxed{\sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cos^2\theta}$

#2 $\boxed{\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta}$

#1 $(\sin^2\theta)^2 + (\cos^2\theta)^2$

$$a^2+b^2 = (a+b)^2 - 2ab$$

$$= (\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta$$

$$= 1 - 2\sin^2\theta\cos^2\theta$$

#2 Proof :-

$$= \sin^6\theta + \cos^6\theta$$

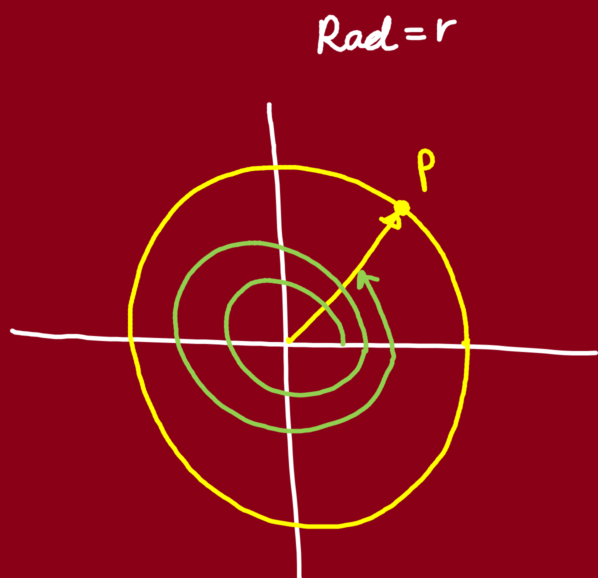
$$= (\sin^2\theta)^3 + (\cos^2\theta)^3$$

$$= (\sin^2\theta + \cos^2\theta)(\underline{\sin^4\theta + \cos^4\theta} - \sin^2\theta\cos^2\theta)$$

$$= (1)(1 - 2\sin^2\theta\cos^2\theta - \sin^2\theta\cos^2\theta)$$

$$= \underline{1 - 3\sin^2\theta\cos^2\theta}$$

CAST rule :-



Allied Angles

$\theta,\ 2\pi+\theta,\ 4\pi+\theta,\ \ldots$

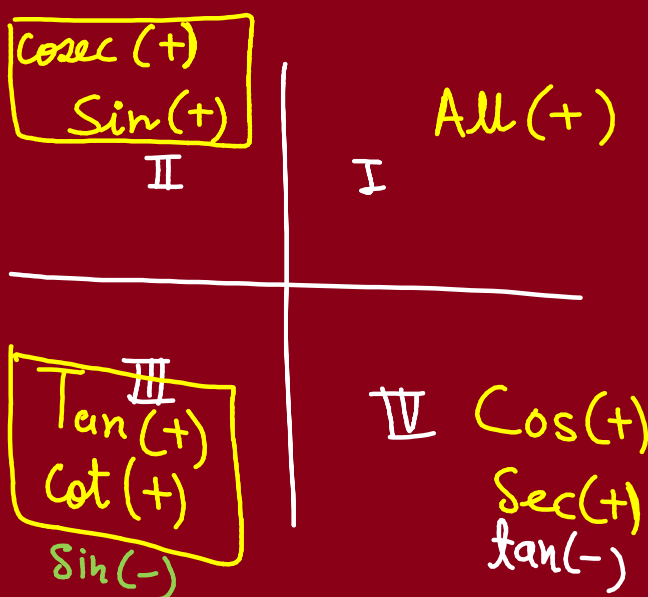
$\sin\theta = \sin(2\pi+\theta) = \sin(4\pi+\theta)$

	$0°$	$30°$	$45°$	$60°$	$90°$
$\sin\theta$	$\sqrt{\frac{0}{4}} = 0$	$\sqrt{\frac{1}{4}} = 1/2$	$\sqrt{\frac{2}{4}} = 1/\sqrt{2}$	$\sqrt{\frac{3}{4}}$ $\sqrt{3}/2$	$\sqrt{\frac{4}{4}}$ 1
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
$\tan\theta$					
$\cot\theta$					
$\sec\theta$					
$\mathrm{cosec}\,\theta$					

$\underline{\sin 120°}$

$\underline{\sin 180°}$

CAST / ASTC



$K \in \mathbb{Z}$

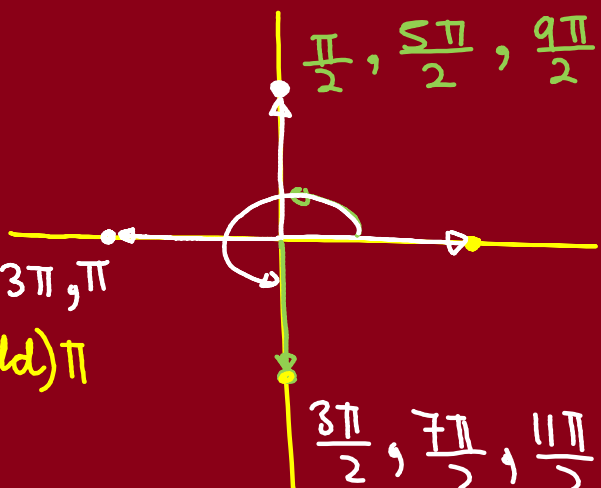
Rule 2

$$\sin(K\pi \pm \theta) = \sin\theta$$

$$\sin\left(\frac{K\pi}{2} \pm \theta\right) = \cos\theta$$

sin ⟷ cos
tan ⟷ cot
sec ⟷ cosec

$\frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \ldots \ldots \frac{(4k+1)\pi}{2}$



$0, 2\pi, 4\pi \ldots \quad (\text{Even})\pi$

$\ldots 5\pi, 3\pi, \pi$

$(\text{Odd})\pi$

$\frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}, \ldots \ldots \frac{(4k+3)\pi}{2}$

$4 \,\overline{)\,2023}$

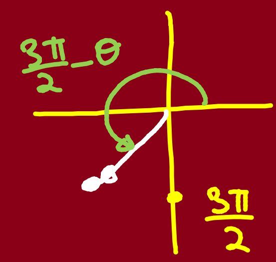
3

ACW(+)

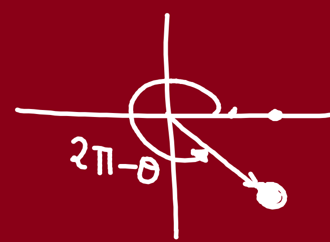
CW(−)

Examples.

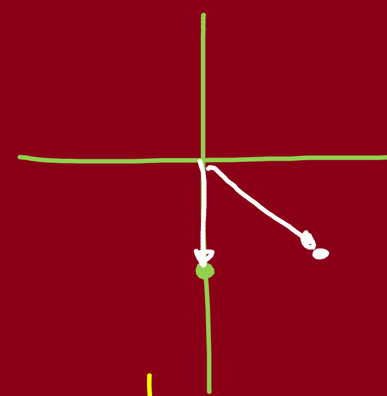
$$\# \quad \sin\left(\underbrace{\frac{3\pi}{2} - \theta}_{\text{III}}\right) = -\cos\theta$$



$$\# \quad \tan\left(\underbrace{2\pi - \theta}_{\text{IV}}\right) = -\tan\theta$$

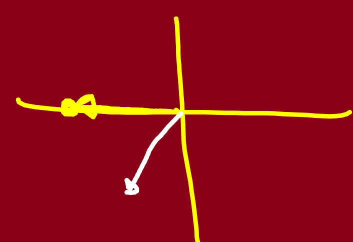


$\sec\left(\underbrace{\frac{11\pi}{2}+\theta}_{4th}\right) = +\operatorname{cosec}\theta$



$\sin\left(\underbrace{\frac{2023\pi}{2}+\phi}\right) = \boxed{-\cos\phi}$

$\cot\left(\underbrace{2021\pi + 2\theta}_{III}\right) = \boxed{+\cot(2\theta)}$



Negative Angles.

$$\sin(-\theta) = -\sin\theta$$

$$\tan(-\theta) = -\tan\theta$$

$$\cot(-\theta) = -\cot\theta$$

$$\text{cosec}(-\theta) = -\text{cosec}\,\theta$$

$$\cos(-\theta) = \cos\theta$$

$$\sec(-\theta) = \sec\theta$$

Comp. Sci

GRAPH OF TRIG. FUNCTION

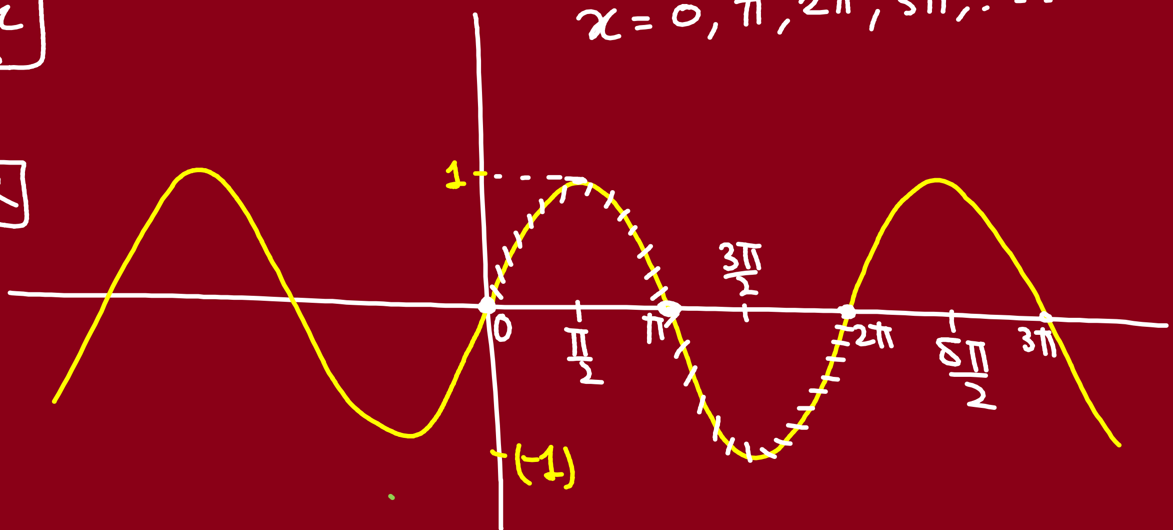
① $\boxed{y = \sin x}$

Domain: $\boxed{x \in R}$

Range: $[-1, 1]$

$\sin x = 0$

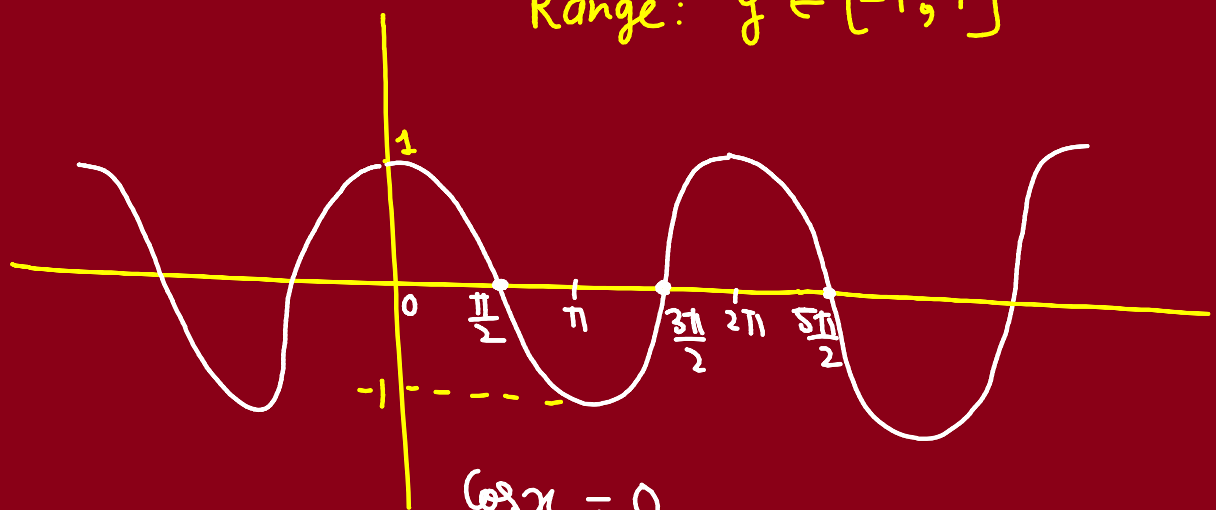
$x = 0, \pi, 2\pi, 3\pi, \ldots$



② $y = \cos x$

Domain: $x \in R$

Range: $y \in [-1, 1]$



$\cos x = 0$

$x = \frac{(2n+1)\pi}{2}$

Inc. f^n.

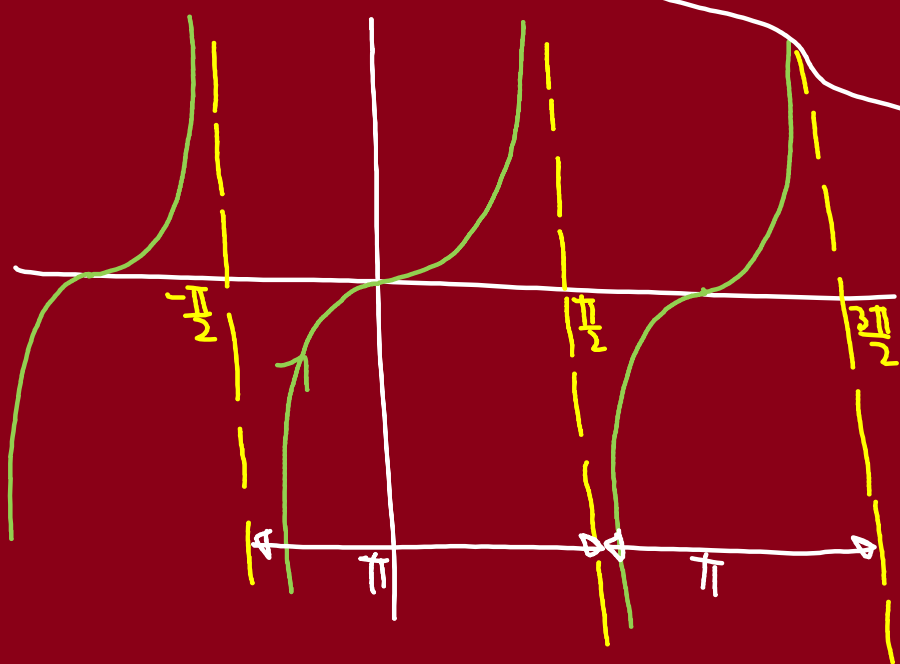
③ $\boxed{\tan x = y} = \dfrac{\sin x}{\cos x}$

$\cos x \neq 0$

Domain: $\boxed{x \neq \dfrac{(2n+1)\pi}{2}}$

Range : R

Period $= \pi$



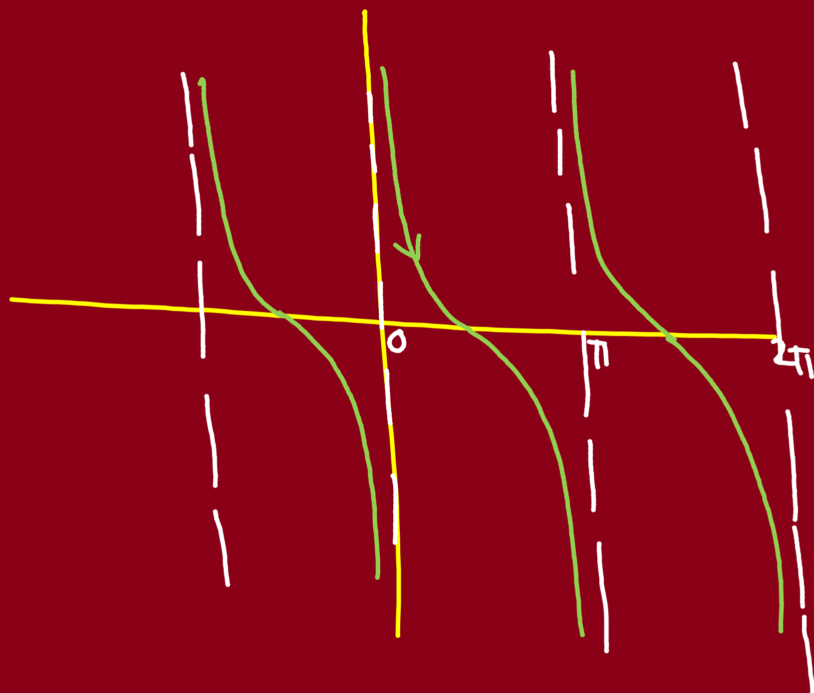
$\left.\begin{array}{l}\sin x\\ \cos x\\ \sec x\\ \operatorname{cosec} x\end{array}\right\}$ 2π

$\left.\begin{array}{l}\tan x\\ \cot x\end{array}\right\}$ π

④ $y = \cot x = \dfrac{\cos x}{\sin x}$ — dec

$\sin x \neq 0$

$\boxed{x \neq n\pi}$



Domain: $\mathbb{R} - \{n\pi\}$

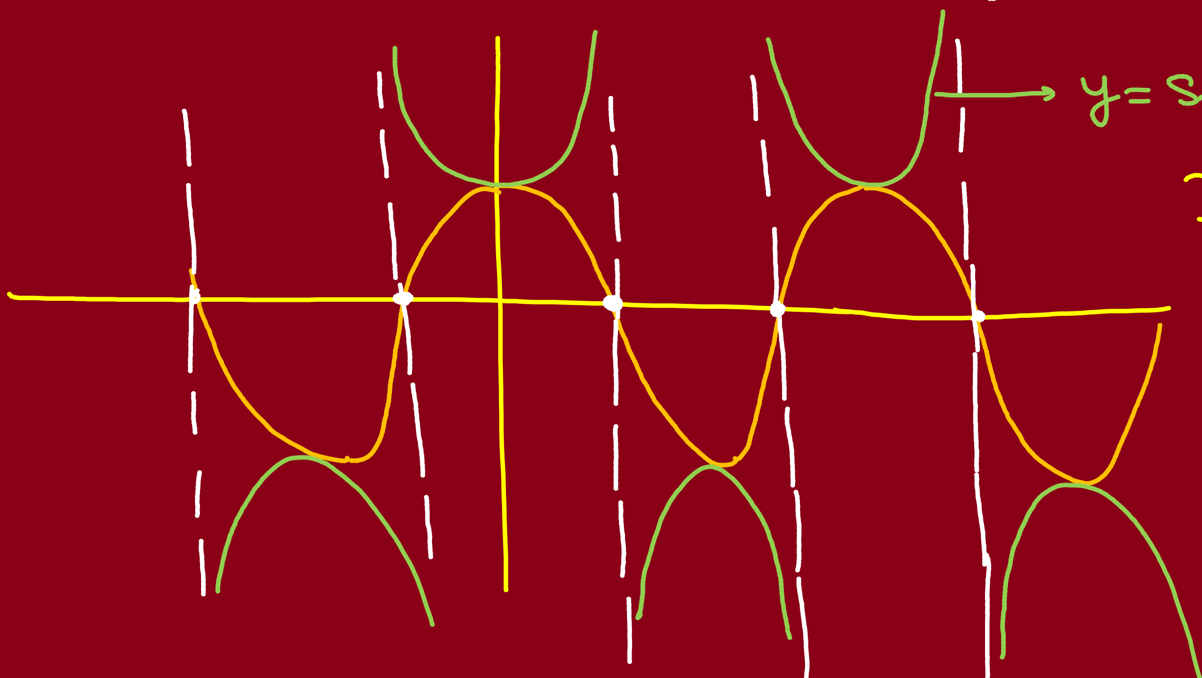
Range: $\mathbb{R}$

ULTI TOPI

$y = \sec x = \dfrac{1}{\cos x}$

$\cos x \neq 0$

$x \neq \dfrac{(2n+1)\pi}{2}$

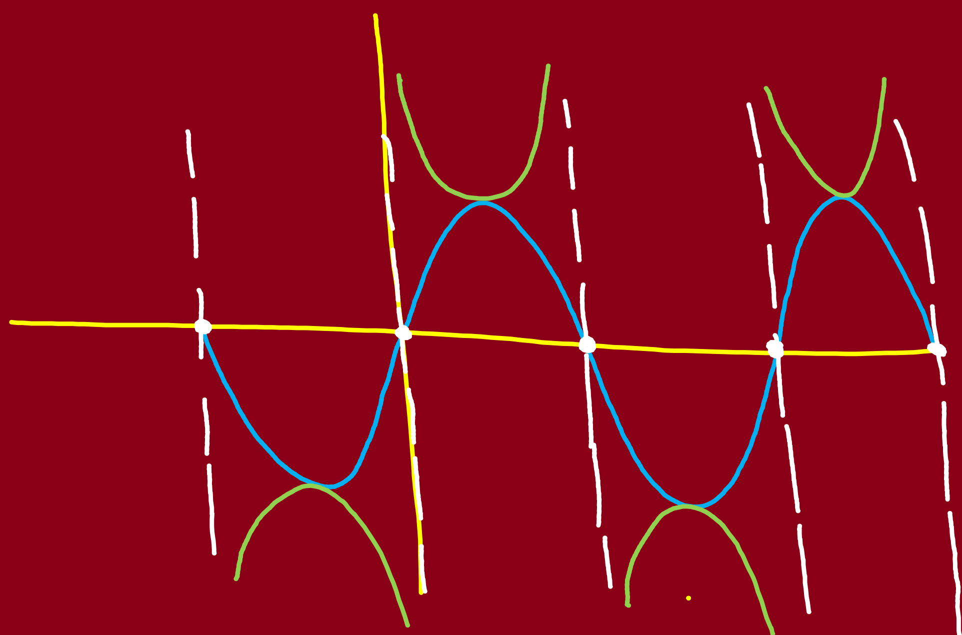


Domain: $R - \left\{ \dfrac{(2n+1)\pi}{2} \right\}$

Range: $(-\infty, -1] \cup [1, \infty)$

$y = \text{cosec}\,x = \dfrac{1}{\sin x}$ $\quad \sin x \neq 0$

$x \neq n\pi$



Domain : $R - \{n\pi\}$

Range : $(-\infty, -1] \cup [1, \infty)$

COMPOUND ANGLE

↳ Add 2 or more angles $(A+B)$

$\cos\theta = \sin(90-\theta)$

$\boxed{\sin(A+B) = \sin A \cos B + \cos A \sin B}$

$\sin(A-B) = \sin A \cos(B) - \cos A \sin(B)$

$\cos(A+B) = \sin(90 - A - B)$

$= \sin(90-A)\cos B - \cos(90-A)\sin B$

$= \cos A \cos B - \sin A \sin B$

$\cos(A-B) = \cos A \cos(B) + \sin A \sin(B)$

$\tan(A+B) = \dfrac{\sin(A+B)}{\cos(A+B)} = \dfrac{\dfrac{\sin A \cos B + \cos A \sin B}{\cos A \cos B}}{\dfrac{\cos A \cos B - \sin A \sin B}{\cos A \cos B}} = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

$B \to (-B)$

$$\boxed{\tan(A-B) = \frac{\tan A - \tan(B)}{1 + \tan A \tan(B)}}$$

$\cot(A+B) = \dfrac{\cos(A+B)}{\sin(A+B)}$

$$= \frac{\dfrac{\cos A \cos B - \sin A \sin B}{\sin A \sin B}}{\dfrac{\sin A \cos B + \cos A \sin B}{\sin A \sin B}}$$

$$= \frac{\cot A \cot B - 1}{\cot B + \cot A}$$

$\cot(A-B)$

$$= \frac{\cot(A)\cot(-B) - 1}{\cot(-B) + \cot A}$$

$$= \frac{\cot A \cot B + 1}{\cot B - \cot A}$$

//

$$\# \quad \tan\left(\frac{\pi}{4}+A\right) = \frac{\tan\frac{\pi}{4} + \tan A}{1 - \tan\frac{\pi}{4}\tan A}$$

$$\# \quad \tan\left(\frac{\pi}{4}+A\right) = \frac{1+\tan A}{1-\tan A}$$

$$\# \quad \tan\left(\frac{\pi}{4}-A\right) = \frac{1-\tan A}{1+\tan A}$$

$$(a+b)(a-b) = a^2 - b^2$$

$\boxed{\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B}$

$\boxed{\cos(A+B)\cos(A-B) = \boxed{\cos^2 A - \sin^2 B}}$

<u>Proof:-</u> $LHS = \sin(A+B).\sin(A-B)$

$$= (\sin A \cos B + \cos A \sin B)(\sin A \cos B - \cos A \sin B)$$

$$= \sin^2 A \cos^2 B - \cos^2 A \sin^2 B$$

$$= \sin^2 A(1 - \cancel{\sin^2 B}) - (1 - \cancel{\sin^2 A})\sin^2 B$$

$$= \underline{\sin^2 A - \sin^2 B}$$

TRANSFORMATION FORMULAS

(1) $\sin(A+B) + \sin(A-B) = 2\sin A \cos B$

$\frac{C+D}{2}$ $\qquad$ $\frac{C-D}{2}$

(2) $\sin(A+B) - \sin(A-B) = 2\cos A \sin B$

(3) $\cos(A+B) + \cos(A-B) = 2\cos A \cos B$

(4) $\cos(A+B) - \cos(A-B) = -2\sin A \sin B$

$$
\begin{array}{r}
A+B = C \\
A-B = D \\
\hline
2A = C+D
\end{array}
$$

$\therefore A = \frac{C+D}{2}$

$B = \frac{C-D}{2}$

1. $\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

2. $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

3. $\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

4. $\cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

$$\tan(\theta_1+\theta_2) = \frac{S_1}{1-S_2}$$

$$= \frac{\tan\theta_1 + \tan\theta_2}{1 - \tan\theta_1 \tan\theta_2}$$

$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

$$\tan(\theta_1+\theta_2+\ldots+\theta_n) = \frac{S_1 - S_3 + S_5 \cdots\cdots}{1 - S_2 + S_4 - S_6 \cdots\cdots}$$

$S_1 \Rightarrow \tan\theta_1 + \tan\theta_2 + \ldots + \tan\theta_n$

$S_2 \Rightarrow \tan\theta_1 \tan\theta_2 + \tan\theta_2 \tan\theta_3 + \ldots + \tan\theta_{n-1}\tan\theta_n$

$S_3 \Rightarrow \ldots$

$S_4 \Rightarrow \ldots$

$S_3 \quad S_4 \mid \cancel{S_5} \mid \cancel{S_6}$

$$\tan(\underline{\theta_1+\theta_2+\theta_3}) = \frac{S_1 - S_3}{1 - S_2}$$

$$= \frac{\tan\theta_1 + \tan\theta_2 + \tan\theta_3 - \tan\theta_1 \tan\theta_2 \tan\theta_3}{1 - (\tan\theta_1\tan\theta_2 + \tan\theta_2\tan\theta_3 + \tan\theta_3\tan\theta_1)}$$

Multiple and Sub-Multiple Angles:-

$\hookrightarrow 2A, 3A, \ldots$ $\hookrightarrow \frac{A}{2}, \frac{A}{4}, \ldots$

$c^2 - (1 - c^2)$

$\sin(2A) = \sin(A+A) = \sin A \cos A + \cos A \sin A = 2\sin A \cos A$

$\sin(2A) = 2\sin A \cos A$

$$= \frac{2\tan A}{1+\tan^2 A}$$

$\cos 2A = \cos^2 A - \sin^2 A$

$$\boxed{\cos 2A = 1 - 2\sin^2 A}$$

$$\boxed{\cos 2A = 2\cos^2 A - 1}$$

$$= \frac{1-\tan^2 A}{1+\tan^2 A}$$

$$\tan 2A = \frac{\sin 2A}{\cos 2A} = \frac{\dfrac{2\sin A\cos A}{\cos^2 A}}{\dfrac{\cos^2 A - \sin^2 A}{\cos^2 A}}$$

$$\boxed{\tan(2A) = \frac{2\tan A}{1-\tan^2 A}}$$

$$1+\cos 2A = 2\cos^2 A$$

$$\cos 2A = 1 - 2\sin^2 A$$

$$\boxed{2\sin^2 A = 1 - \cos 2A}$$

$$\boxed{\begin{aligned} 1-\cos 2\theta &= 2\sin^2\theta \\ 1+\cos 2\theta &= 2\cos^2\theta \end{aligned}}$$

sin = paap

sin = paap = $\ominus$

$1 - \cos 2\theta = 2\sin^2\theta$

$1 + \cos 2\theta = 2\cos^2\theta$

3 - 4 = $\ominus$

chote - bade

$\sin 3\theta = \underline{3}\sin^1\theta - \underline{4}\sin^3\theta$

$\cos 3\theta = \underline{\underline{4}}\cos^3\theta - \underline{\underline{3}}\cos\theta$

$15^\circ \quad , \quad 75^\circ$

$\to 45^\circ - 30^\circ$ $\quad \to 45^\circ + 30^\circ$

$\sin\theta = \cos(90-\theta)$

$\sin 15 = \cos 75$

$\sin 75 = \cos 15^\circ$

$\sin\left(\frac{\pi}{6}\right)$

$\cos\left(\frac{\pi}{3}\right)$

$$\sin 15^\circ = \sin(45^\circ - 30^\circ)$$

$$= \sin 45 \cos 30 - \cos 45 \sin 30$$

$$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$$

$$\boxed{15^\circ = \frac{\pi}{12} \qquad 75^\circ = \frac{5\pi}{12}}$$

$$\sin\left(\frac{\pi}{12}\right) = \sin 15^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

$$\boxed{\cos 15^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}}$$

$$\boxed{\tan 15 = \frac{\sqrt{3}-1}{\sqrt{3}+1}}$$

$$18^\circ, \quad 36^\circ, \quad 72^\circ, \quad 54^\circ$$

$$\frac{\pi}{10}, \quad \frac{\pi}{5}, \quad \frac{2\pi}{5}, \quad \frac{3\pi}{10}$$

$\sin 18^\circ = \cos 72^\circ$

$\cos 18^\circ = \sin 72^\circ$

$\sin 36 = \cos 54$

Proof:- $\Rightarrow \theta = 18^\circ$

$\Rightarrow 5\theta = 90^\circ$

$\Rightarrow 3\theta + 2\theta = 90^\circ$

$\Rightarrow 3\theta = 90 - 2\theta$

$\Rightarrow \cos(3\theta) = \cos(90 - 2\theta)$

$\cos 72^\circ$

$$\boxed{\begin{aligned} \sin 18^\circ &= \frac{\sqrt{5} - 1}{4} \\ \cos 36^\circ &= \frac{\sqrt{5} + 1}{4} \end{aligned}}$$

$$4\cos^3\theta - 3\cancel{\cos}\theta = 2\sin\theta\cancel{\cos}\theta$$

$$18^\circ \longrightarrow \mathrm{I}$$

$$4\underline{\underline{\cos^2\theta}} - 3 = 2\sin\theta$$

$$4(1-\sin^2\theta) - 3 = 2\sin\theta$$

$$4\sin^2\theta + 2\sin\theta - 1 = 0$$

$$\sin 18^\circ = \sin\theta = \frac{-2 \pm \sqrt{4+16}}{8} = \frac{-2 \pm 2\sqrt{5}}{8} \begin{cases} \dfrac{-1+\sqrt{5}}{4} \checkmark \\ \cancel{\dfrac{-1-\sqrt{5}}{4}} \times \end{cases}$$

$$\sin 18^\circ = \frac{\sqrt{5}-1}{4}$$

$$\cos 2\theta = 1 - 2\sin^2\theta$$

$$\theta = 18^\circ$$

$$\cos 36 = 1 - 2\sin^2 18^\circ$$

$$= 1 - 2\left(\frac{\sqrt{5}-1}{4}\right)^2$$

$$= \frac{\sqrt{5}+1}{4}$$

$\sin 36^\circ = ?$

$\cos 2\theta = 1 - 2\sin^2\theta$

$\theta = 36^\circ$

$\cos 72 = 1 - 2\sin^2 36$

$\frac{\sqrt{5}-1}{4} = 1 - 2\sin^2 36^\circ$

$2\sin^2 36^\circ = 1 - \frac{\sqrt{5}-1}{4}$

$\sin 36 = \sqrt{\frac{(5-\sqrt{5})(2)}{(8)(2)}}$

$$\boxed{\sin 36^\circ = \frac{\sqrt{10-2\sqrt{5}}}{4}}$$

$$\boxed{22.5^\circ = \frac{\pi}{8}} = \frac{45^\circ}{2}$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$\theta = \frac{\pi}{8}$$

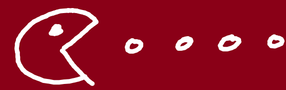
$$1 = \frac{2\tan\frac{\pi}{8}}{1-\tan^2\frac{\pi}{8}}$$

$$\boxed{1-\tan^2\frac{\pi}{8} = 2\tan\frac{\pi}{8}}$$

$$\tan\left(\frac{\pi}{8}\right) = \sqrt{2}-1$$

$$\cot\left(\frac{\pi}{8}\right) = \sqrt{2}+1$$

Cosine Series (Product)

pacman 

$$\boxed{\cos A \cdot \cos 2A \cos 4A \cos 8A \dots \cos\left(2^{n-1}A\right) = \frac{\sin\left(2^{n}A\right)}{2^{n}\sin A}}$$

Kab?

1. product
2. Angle → G.P. $(r=2)$
3. $n \rightarrow$ no. of terms

   $A \rightarrow$ First Angle | Smallest Angle

PROOF :-

$$\Rightarrow \frac{\boxed{2\sin A \cos A} \cos 2A \cos 4A \cos 8A \ldots \cos(2^{n-1}A)}{2\sin A}$$

$$\Rightarrow \frac{\boxed{2\sin 2A \cos 2A} \cos 4A \cos 8A \ldots \cos(2^{n-1}A)}{2^2 \sin A}$$

$2^{n-1}A \times 2$

$2^n A$

$\vdots$

$$\Rightarrow \frac{\sin(2^n A)}{2^n \sin A}$$

$\frac{\beta}{2} = $

Cosine Series (sum)

$$\cos\alpha + \cos(\alpha+\beta) + \cos(\alpha+2\beta) + \dots + \cos(\alpha+(n-1)\beta) = \frac{\sin\left(\frac{n\beta}{2}\right)}{\sin\left(\frac{\beta}{2}\right)} \cos\left(\alpha + \frac{(n-1)\beta}{2}\right)$$

Sine Series (sum)

$$\sin\alpha + \sin(\alpha+\beta) + \sin(\alpha+2\beta) + \dots + \sin(\alpha+(n-1)\beta) = \frac{\sin\left(\frac{n\beta}{2}\right)}{\sin\left(\frac{\beta}{2}\right)} \sin\left(\alpha + \frac{(n-1)\beta}{2}\right)$$

#1 Sum

#2 A.P

Where, $n \to$ no. of terms

$\alpha \to$ First Angle

$\beta \to$ Common diff

Cosec x Ki series

$$= \boxed{\cot\left(\frac{F.A.}{2}\right) - \cot(L.A.)}$$

$$\underbrace{\operatorname{cosec} x} + \underbrace{\operatorname{cosec} 2x} + \operatorname{cosec} \underline{4}x + \ldots + \operatorname{cosec}(2^n x) = \cot\left(\frac{x}{2}\right) - \cot(2^n x)$$

Proof: $T_1 = \operatorname{cosec} x = \dfrac{\sin\left(x - \frac{x}{2}\right)}{\sin x \sin\left(\frac{x}{2}\right)} = \dfrac{\sin x \cos\frac{x}{2} - \cos x \sin(x/2)}{\sin x \sin\left(\frac{x}{2}\right)}$

$$= \cot\left(\frac{x}{2}\right) - \cot x$$

$$T_1 = \cot\left(\frac{x}{2}\right) - \cancel{\cot(x)}$$

$$T_2 = \cancel{\cot(x)} - \cancel{\cot(2x)}$$

$$T_3 = \cancel{\cot(2x)} - \cancel{\cot(4x)}$$

$$\vdots$$

$$T_n = \cancel{\cot(2^{n-1}x)} - \cot(2^n x)$$

$$S = \cot\left(\frac{x}{2}\right) - \cot(2^n x)$$

$$\boxed{\#\text{Baarish}}$$

Max and mini Value of Trigo f^n :-

$f(x) = \boxed{}$

Range $\in [\alpha, \beta]$

Type 1 Use the Range of Known trig. f^n

$-1 \leq \sin x \leq 1$

$-1 \leq \cos x \leq 1$

$\left.\begin{matrix} \operatorname{cosec} x \\ \sec x \end{matrix}\right\} (-\infty, -1] \cup [1, \infty)$

$\left.\begin{matrix} \tan x \\ \cot x \end{matrix}\right\} R$

$$\left.\begin{array}{l}|\sin x|\\|\cos x|\end{array}\right\}\ [0,1]$$

$$\left.\begin{array}{l}|\tan x|\\|\cot x|\end{array}\right\}\ [0,\infty)$$

$$\left.\begin{array}{l}|\sec x|\\|\operatorname{cosec} x|\end{array}\right\}\ [1,\infty)$$

$\sin^{2n} x$

$$\left.\begin{array}{l}\sin^2 x\\\cos^2 x\end{array}\right\}\ [0,1]$$

$$\left.\begin{array}{l}\tan^2 x\\\cot^2 x\end{array}\right\}\ [0,\infty)$$

$$\left.\begin{array}{l}\sec^2 x\\\operatorname{cosec}^2 x\end{array}\right\}\ [1,\infty)$$

Ex: $y = 3 + \sin x \rightarrow [2, 4]$

$$-1 \le \sin x \le 1$$

$$\boxed{2 \le 3 + \sin x \le 4}$$

Type 2 $f(x) = a\sin x + b\cos x$

$$\boxed{-\sqrt{a^2+b^2} \leq a\sin x + b\cos x \leq \sqrt{a^2+b^2}}$$

$$-\sqrt{3^2+4^2} \leq 3\sin x + 4\cos x \leq \sqrt{3^2+4^2}$$

$$[-5, 5]$$

Type 3 Q.E in 'sinx' OR 'cosx'

$f(x) = \underbrace{\cos 2x} + 3\sin x$ Range of $f(x)$

$= 1 - 2\sin^2 x + 3\sin x$

$\dfrac{\frac{3}{2}}{2} = \left(\dfrac{3}{4}\right)$

$= -2\left(\sin^2 x - \frac{3}{2}\sin x - \frac{1}{2}\right)$

$= -2\left(\underbrace{\sin^2 x - \frac{3}{2}\sin x + \frac{9}{16}} \underbrace{- \frac{9}{16} - \frac{1}{2}}\right)$

$$f(x) = -2\left(\left(\sin x - \frac{3}{4}\right)^2 - \frac{17}{16}\right)$$

$$\boxed{f(x) = \frac{17}{8} - \boxed{\cancel{2\left(\sin x - \frac{3}{4}\right)^2}}}$$

Range of $f(x)$:-

$$\left[-4, \frac{17}{8}\right]$$

$\sin x = -1$

$\sin x = 3/4$

$$f_{min}(x) = \frac{17}{8} - 2\left(-1 - \frac{3}{4}\right)^2 = \frac{17}{8} - \frac{49}{8} = \frac{-32}{8} = -4$$

$$f_{max}(x) = \frac{17}{8} - 0 = \frac{17}{8}$$

Type 4

$$\boxed{\left(a^2\tan^2\theta + b^2\cot^2\theta\right)_{\text{mini}} = 2ab}$$

M-1 $\dfrac{a^2\tan^2\theta + b^2\cot^2\theta}{2} \geq \sqrt{a^2 b^2}$

M-2 $\left(a^2\tan^2\theta + b^2\cot^2\theta\right)_{\text{mini}} = \cancel{\left(a\tan\theta - b\cot\theta\right)^2}^{\to 0} + 2ab = 2ab$