Unacademy Atoms

Trigonometric Equations

Examples of Trigonometric Equations

1 $\sin\theta = \frac{1}{2}$

✓ $\boxed{\sin\theta = \frac{1}{2}}$

2 $2\sin x - \cos x = 3$

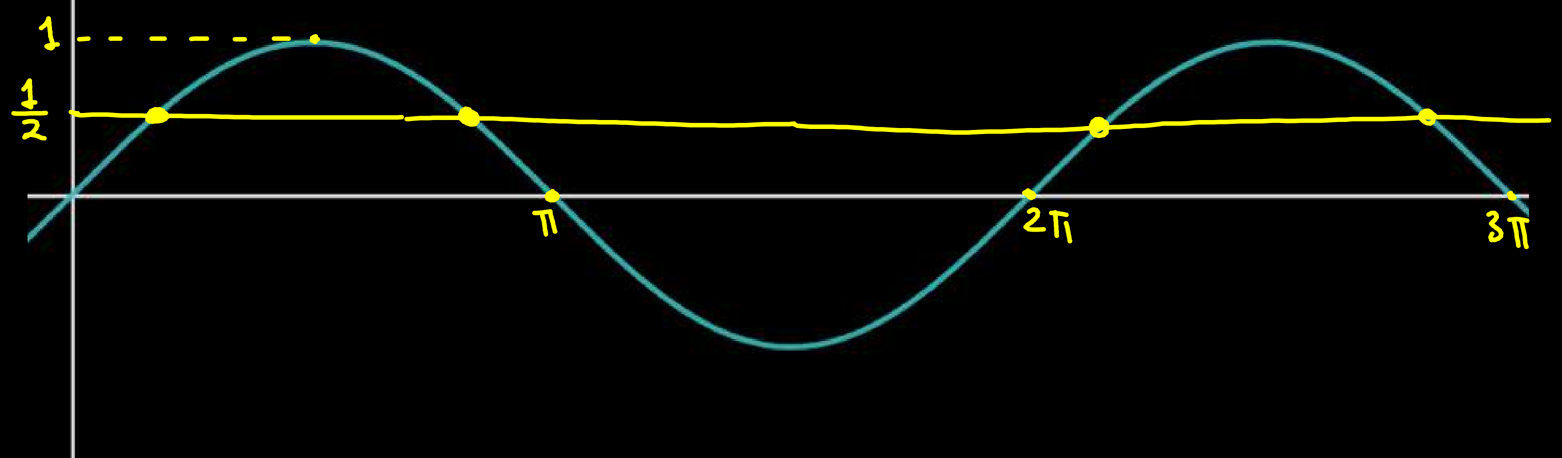
Solution of Trigonometric Equations

1

$$\sin\theta = \frac{1}{2}$$

$\boxed{\sin\theta = \boxed{\frac{1}{2}}}$

$y = \sin\theta$ $\quad$ $y = \frac{1}{2}$



Principal Solution

Principal Solution

$$\boxed{PS \in [0, 2\pi)}$$

$$\boxed{PS \in [0, 2\pi)}$$

Principal Solution - Shortcut Method

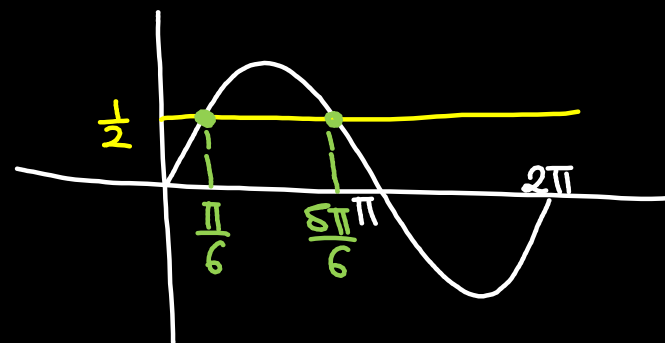
#NVSTYLE

1

$$\sin\theta = \frac{1}{2}$$

#NVStyle

S-1 $\sin\alpha = \frac{1}{2} \Rightarrow \boxed{\alpha = \frac{\pi}{6}}$



S-2

S	A
T	C

1st and 2nd

S-3

$\pi-\alpha$	α
$\pi+\alpha$	$2\pi-\alpha$

$\frac{\pi}{6}, \pi - \frac{\pi}{6} \Rightarrow \boxed{PS \Rightarrow \frac{\pi}{6}, \frac{5\pi}{6}}$

Principal Solution - Shortcut Method

#NVSTYLE

2

$$\sin x = -\frac{1}{2}$$

S-1 $\sin\alpha = \frac{1}{2}$ $\boxed{\alpha \Rightarrow \frac{\pi}{6}}$

S-2

S	A
T	C

III and IV

S-3

$\pi-\alpha$	α
$\pi+\alpha$	$2\pi-\alpha$

$PS = \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$

$$\boxed{PS = \frac{7\pi}{6}, \frac{11\pi}{6}}$$

Ans Verify

$\sin\left(\frac{7\pi}{6}\right)$

$= \sin\left(\pi + \frac{\pi}{6}\right)$

$= -\sin\frac{\pi}{6}$

$= \frac{-1}{2}$

Principal Solution - Shortcut Method

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3

$$\tan x = -\frac{1}{\sqrt{3}}$$

S-1 $\tan\alpha = \frac{1}{\sqrt{3}}$ $\boxed{\alpha = \frac{\pi}{6}}$

S-2

S	A
T	C

II and IV

S-3

$\pi-\alpha$	α
$\pi+\alpha$	$2\pi-\alpha$

$\pi - \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$

$\frac{5\pi}{6}, \frac{11\pi}{6}$

Principal Solution - Shortcut Method

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4

$$\operatorname{cosec} x = -2$$

$$\boxed{\sin x = \frac{-1}{2}}$$

S-1 $\sin\alpha = \frac{1}{2}$ $\quad \alpha = \frac{\pi}{6}$

S-2

S	A
T	C

S-3 PS $\Rightarrow \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$

General Solution

General Solution

$\sin\theta = \sin\alpha \qquad \theta = n\pi + (-1)^n \alpha \qquad n \in \mathbb{Z}$

$\cos\theta = \cos\alpha \qquad \theta = 2n\pi \pm \alpha$

$\tan\theta = \tan\alpha \qquad \theta = n\pi + \alpha$

General Solution

$$\left.\begin{aligned} \sin^2\theta &= \sin^2\alpha \\ \cos^2\theta &= \cos^2\alpha \\ \tan^2\theta &= \tan^2\alpha \end{aligned}\right\} \quad \boxed{\theta = n\pi \pm \alpha}$$

$n \in I$

General solutions

$\sin\theta = \sin\alpha$	$\Rightarrow$	$\theta = n\pi + (-1)^n \alpha$
$\cos\theta = \cos\alpha$	$\Rightarrow$	$\theta = 2n\pi \pm \alpha$
$\tan\theta = \tan\alpha$	$\Rightarrow$	$\theta = n\pi + \alpha$
$\sin^2\theta = \sin^2\alpha$	$\Rightarrow$	$\theta = n\pi \pm \alpha$
$\tan^2\theta = \tan^2\alpha$	$\Rightarrow$	$\theta = n\pi \pm \alpha$
$\cos^2\theta = \cos^2\alpha$	$\Rightarrow$	$\theta = n\pi \pm \alpha$

$$\sin^2\theta = \frac{1}{4}$$

$$\sin^2\theta = \sin^2\frac{\pi}{6}$$

$$\boxed{\theta = n\pi \pm \frac{\pi}{6}} \quad n \in \mathbb{Z}$$

General Solutions - Examples

1

$$\sin x = -\frac{\sqrt{3}}{2}$$

Method (G.S)

S-1 P.S i)
ii)
iii)

S-2 Use respective formula of G.S.

S-1 i) $\sin\alpha = \frac{\sqrt{3}}{2}$ $\boxed{\alpha = \frac{\pi}{3}}$

ii)

S	A
T	C

iii) PS: $\pi + \frac{\pi}{3}$, $2\pi - \frac{\pi}{3}$

PS: $\frac{4\pi}{3}$, $\frac{5\pi}{3}$

S-2 $\sin x = \sin\frac{4\pi}{3}$

$\boxed{x = n\pi + (-1)^n \frac{4\pi}{3}}$ $n \in \mathbb{Z}$

General Solutions - Examples

2

$$\sin^2 x = \frac{1}{4}$$

S-1 $\quad \sin^2 x = \left(\frac{1}{2}\right)^2$

$\sin^2 x = \sin^2 \frac{\pi}{6}$

S-2 $\quad \boxed{x = n\pi \pm \frac{\pi}{6}}$

General Solutions - Examples

3

$\cancel{\cot x = -1}$

cotx / secx / cosecx

★ $\tan x = -1$

S-1 i) $\tan\alpha = 1 \quad \alpha = \frac{\pi}{4}$

ii) S | A / T | C — II and IV

iii) PS: $\pi - \frac{\pi}{4}, \; 2\pi - \frac{\pi}{4}$

$\frac{3\pi}{4}, \; \frac{7\pi}{4}$

S-2 $\tan x = \tan\left(\frac{3\pi}{4}\right)$

GS: $\boxed{x = n\pi + \frac{3\pi}{4}}$ $\quad n \in \mathbb{Z}$

* PS ✓
* GS ✓
* No. of Sol^n

Number of Solution

Find Number of Solutions :

1

$$\sin 4x = -\frac{1}{2} \; in \; x \in [0, 2\pi]$$

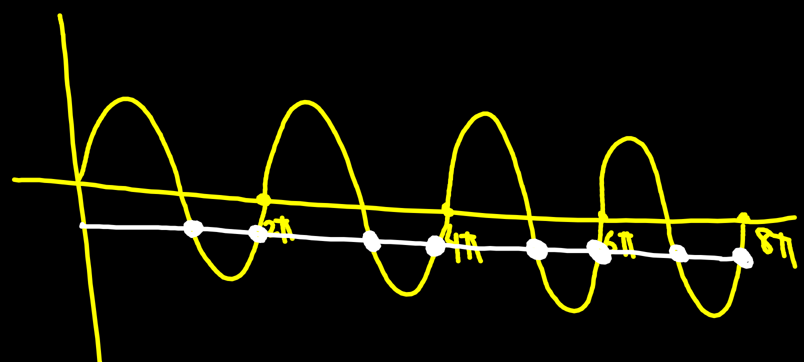
$\sin(\underline{4x}) = \frac{-1}{2}$ $\quad x \in [0, 2\pi]$

$\boxed{\text{No. of sol}^n = 8}$

$4x = \theta$

$\sin(4x) = \frac{-1}{2}$ $\quad 4x \in [0, 8\pi]$

$\sin(\theta) = \frac{-1}{2}$ $\quad \theta \in [0, 8\pi]$



Find Number of Solutions :

2

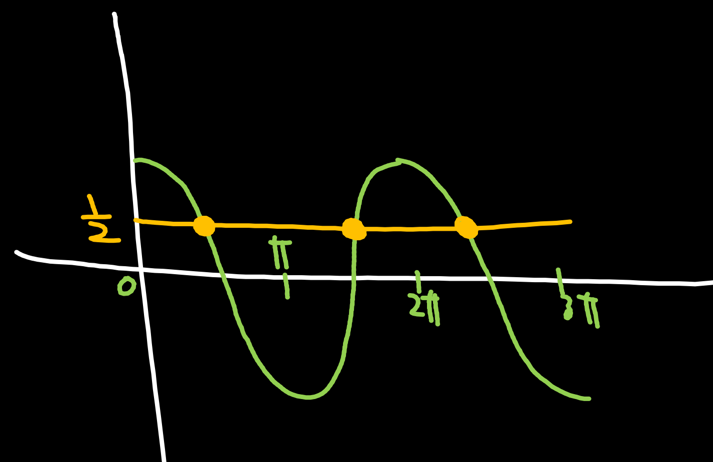
$$\sec 3x = 2 \; in \; x \in [0, \pi]$$

$\cos(3x) = \frac{1}{2}$ $\quad x \in [0, \pi]$

$\cos(3x) = \frac{1}{2}$ $\quad 3x \in [0, 3\pi]$

$\cos\theta = \frac{1}{2}$ $\quad \theta \in [0, 3\pi]$

$\boxed{\text{No. of sol}^n = 3}$



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The number of distinct solutions of the equation

$$\frac{5}{4}\cos^2 2x + \boxed{\cos^4 x + \sin^4 x} + \cos^6 x + \sin^6 x = 2$$

in the interval $[0, 2\pi]$ is

[JEE Adv. 2015]

$$\frac{5}{4}\cos^2 2x + \cancel{1} - 2\sin^2 x\cos^2 x + \cancel{1} - 3\sin^2 x\cos^2 x = \cancel{2}$$

$$\frac{5}{4}\cos^2 2x - \frac{5}{4}\left(4\sin^2 x\cos^2 x\right) = 0$$

$$\frac{5}{4}\left\{\cos^2 2x - \sin^2 2x\right\} = 0$$

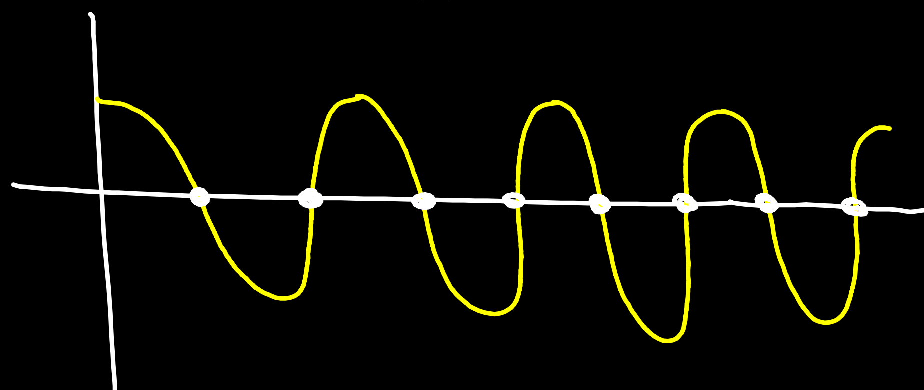
$\frac{5}{4}\cos 4x = 0$

$\Rightarrow \cos 4x = 0 \quad x \in [0, 2\pi]$

$\cos 4x = 0 \quad 4x \in [0, 8\pi]$

$\cos\theta = 0 \quad \theta \in [0, 8\pi]$

$\boxed{\text{No. of Sol}^n = 8}$



PS ✓

GS ✓

No. of Sol^n ✓

Type – 1

☆ Factorisation

Type 1: Factorization/ Quadratic Form

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$\sin x / \cos x \longrightarrow \boxed{Q.E}$

$\downarrow$

factorize

$\downarrow$

Roots — —

$\downarrow$

No. of Solⁿ | Sum of solⁿ

Solve $\boxed{(2 \sin x - \cos x)(1 + \cos x) = \sin^2 x}$ in $[0, 2\pi]$.

$$(2\sin x - \cos x)\cancel{(1+\cos x)} = (1-\cos x)\cancel{(1+\cos x)}$$

$$2\sin x - \cancel{\cos x} - 1 + \cancel{\cos x} = 0$$

CUTE

$$1+\cos x = 0$$

$$\boxed{\cos x = -1}$$

$$\boxed{x = \pi}$$

$$2\sin x = 1$$

$$\boxed{\sin x = \frac{1}{2}} \rightarrow [0, 2\pi]$$

$$\frac{\pi}{6}, \pi - \frac{\pi}{6}$$

$\longrightarrow$ 3 Sol^n : $\boxed{\pi, \frac{\pi}{6}, \frac{5\pi}{6}}$

$\boxed{\text{Sum of Sol}^n = 2\pi}$

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Q

If $\sqrt{3}\cos^2 x = (\sqrt{3}-1)\cos x + 1$, the number of solutions of the given equation when $x \in \left[0, \frac{\pi}{2}\right]$ is

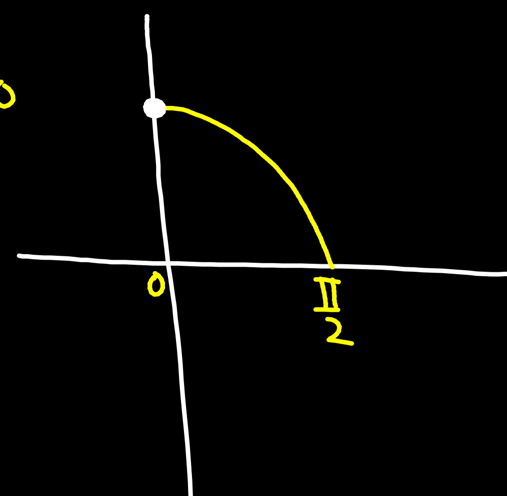
[JEE M 2021]

$$\sqrt{3}\cos^2 x - \sqrt{3}\cos x + \cos x - 1 = 0$$

$$\sqrt{3}\cos x\,(\cos x - 1) + 1\,(\cos x - 1) = 0$$

$$\therefore \cos x = 1 \quad \text{OR} \quad \cos x = \frac{-1}{\sqrt{3}}$$

$$\boxed{1 + 0} = 1$$



Q

[JEE M 2022]

Let $4+(-8)=-4$

$S=\{\theta\in[0,2\pi]: 8^{2\sin^2\theta}+8^{2\cos^2\theta}=16\}$. Then

$\overset{4}{n(S)}+\boxed{\sum_{\theta\in S}\left(\sec\left(\frac{\pi}{4}+2\theta\right)\operatorname{cosec}\left(\frac{\pi}{4}+2\theta\right)\right)}$ is equal to :

$\boxed{8^{2\sin^2\theta}}+8^{2-2\sin^2\theta}=16$ $\quad\boxed{8^{2\sin^2\theta}=t}$

A. 0

B. -2

C. -4

D. 12

$\Rightarrow t+\frac{64}{t}=16$

$\Rightarrow t^2-16t+64=0$

$\Rightarrow (t-8)^2=0$

$8^{2\sin^2\theta}=8^1$

$\therefore \boxed{\sin^2\theta=\frac{1}{2}}$

$1^{st}/2^{nd}$ $\quad$ $3^{rd}/4^{th}$

$$\underbrace{\sin\theta = \frac{+1}{\sqrt{2}}} \qquad \sin\theta = \frac{-1}{\sqrt{2}}$$

$$\frac{2}{\cos\pi} + \frac{2}{\cos 3\pi} + \frac{2}{\cos 5\pi} + \frac{2}{\cos 7\pi}$$

$$S = \left\{ \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \right\} \qquad \underline{n(S) = 4}$$

$$-2 \times 4 = -8$$

$$\sum_{\theta \in S} \frac{1 \times 2}{2\cos\left(\frac{\pi}{4}+2\theta\right)\sin\left(\frac{\pi}{4}+2\theta\right)} \Rightarrow \sum_{\theta \in S} \frac{2}{\sin\left(\frac{\pi}{2}+4\theta\right)}$$

$$\Rightarrow \sum_{\theta \in S} \left(\frac{2}{\cos 4\theta}\right)$$

Q

If the sum of solutions of the system of equations $2\sin^2\theta - \cos 2\theta = 0$ and $2\cos^2\theta + 3\sin\theta = 0$ in the interval $[0, 2\pi]$, then k is equal to__.

[JEE M 2022]

$2\sin^2\theta = \cos 2\theta$

$2\sin^2\theta = 1 - 2\sin^2\theta$

$\sin^2\theta = \frac{1}{4}$

$\sin\theta = \frac{1}{2}$ or $\frac{-1}{2}$

$2\cos^2\theta + 3\sin\theta = 0$

$2(1-\sin^2\theta) + 3\sin\theta = 0$

$2\sin^2\theta - 3\sin\theta - 2 = 0$

$2\sin^2\theta - 4\sin\theta + \sin\theta - 2 = 0$

$(2\sin\theta + 1)(\sin\theta - 2) = 0$

$\sin\theta = \frac{-1}{2}$ or $\cancel{2}$

$\sin\theta = \frac{-1}{2} \quad [0, 2\pi]$

S-1 $\sin\alpha = \frac{1}{2} \quad \alpha = \frac{\pi}{6}$

S-2

S	A
T	C

3^{rd} and 4^{th}

sum $= 3\pi$

S-3 PS: $\pi + \frac{\pi}{6}$ and $2\pi - \frac{\pi}{6}$

$$\boxed{\frac{7\pi}{6}, \frac{11\pi}{6}}$$

Q

If $\underline{S} = \{\theta \in \underline{(0, 2\pi)} : \boxed{7\cos^2\theta - 3\sin^2\theta - 2\cos^2 2\theta = 2}\}$. Then, the sum of roots of all the equations $x^2 - 2(\tan^2\theta + \cot^2\theta)x + 6\sin^2\theta = 0$ $\theta \in S$, is_

[JEE M 2022]

$$\Rightarrow 7\left(\frac{1+\cos 2\theta}{2}\right) - 3\left(\frac{1-\cos 2\theta}{2}\right) - 2\cos^2 2\theta = 2$$

$$\Rightarrow \cancel{2} + 5\cos 2\theta - 2\cos^2 2\theta = \cancel{2}$$

$$\Rightarrow \cos 2\theta\,(5 - 2\cos 2\theta) = 0$$

$$\Rightarrow \boxed{\cos 2\theta = 0} \text{ OR } \cancel{\cos 2\theta = \frac{5}{2}}$$

$\cos 2\theta = 0$ $\quad \theta \in (0, 2\pi)$

$\cos(2\theta) = 0$ $\quad 2\theta \in (0, 4\pi)$

$2\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$

$\therefore \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$S = \left\{ \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \right\}$

Sum of Root $= \boxed{2(\tan^2\theta + \cot^2\theta)}$

$\Rightarrow 4 + 4 + 4 + 4$

$\Rightarrow 16$

Unacademy Atoms

Let

$$S = \left\{\theta \in [-\pi, \pi] - \left\{\pm\frac{\pi}{2}\right\} : \sin\theta\tan\theta + \tan\theta = \sin 2\theta\right\}.$$

If $\boxed{T = \sum_{\theta \in S} \cos 2\theta}$, then $\boxed{4+5}$ $T + n(S)$ is equal

[JEE M 2022]

A. $7 + \sqrt{3}$

B. 9

C. $8 + \sqrt{3}$

D. 10

$$\cancel{\tan}\theta(\sin\theta + 1) = \frac{2\cancel{\tan\theta}}{\boxed{1+\tan^2\theta}}$$

$\tan\theta = 0$

$$\sin\theta + 1 = 2\cos^2\theta$$

$$\sin\theta + 1 = 2(1 - \sin^2\theta)$$

$$2\sin^2\theta + \sin\theta - 1 = 0$$

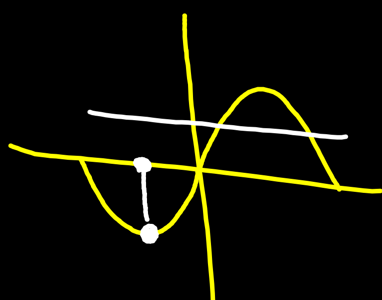
$$2\sin^2\theta + 2\sin\theta - \sin\theta - 1 = 0$$

$$(2\sin\theta - 1)(\sin\theta + 1) = 0$$

$$[-\pi, \pi] - \left\{\pm \frac{\pi}{2}\right\}$$

$\tan\theta = 0$	$\sin\theta = \frac{1}{2}$	$\sin\theta = -1$
$-\pi, 0, \pi$	$\frac{\pi}{6}, \frac{5\pi}{6}$	No Sol^n

$$S = \left\{-\pi, 0, \pi, \frac{\pi}{6}, \frac{5\pi}{6}\right\}$$



$$T = \sum_{\theta \in S} \cos 2\theta$$

$$= \cos(-2\pi) + \cos 0 + \cos 2\pi + \cos\frac{\pi}{3} + \cos\frac{5\pi}{3}$$

$$= 1 + 1 + 1 + \frac{1}{2} + \frac{1}{2}$$

$$= 4$$

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Q

The number of elements in the set $S = \{\theta \epsilon [-4\pi, 4\pi] : 3\cos^2 2\theta + 6\cos 2\theta - 10\cos^2\theta + 5 = 0\}$ is 32 ______.

[JEE M 2022]

$$3\cos^2 2\theta + 6\cos 2\theta - 5(2\cos^2\theta) + 5 = 0$$

$$3\cos^2 2\theta + 6\cos 2\theta - 5(1 + \cos 2\theta) + \cancel{5} = 0$$

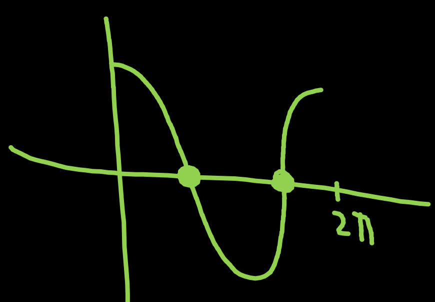
$$3\cos^2 2\theta + \cos 2\theta = 0$$

$$\cos 2\theta (3\cos 2\theta + 1) = 0$$

$\cos 2\theta = 0 \qquad \cos 2\theta = \frac{-1}{3}$

$\cos 2\theta = 0 \qquad \theta \in [-4\pi, 4\pi]$

$\cos x = 0 \qquad x \in [-8\pi, 8\pi]$



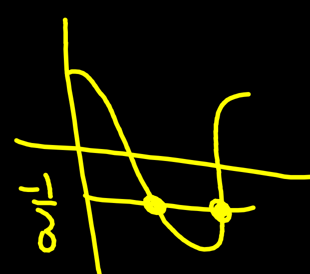
2×8

$= 16$

$\cos 2\theta = \frac{-1}{3} \qquad \theta \in [-4\pi, 4\pi]$

$\cos 2\theta = \frac{-1}{3} \qquad 2\theta \in [-8\pi, 8\pi]$

$\cos x = \frac{-1}{3} \qquad x \in [-8\pi, 8\pi]$



$2 \times 8 = 16$

$\star$ $\boxed{a \sin x + b \cos x = c}$

??

Type – 2

$\star$ $$\boxed{a \sin x + b \cos x = c}$$

Type 2

$$a \sin x + b \cos x = c$$

Method

$\Rightarrow$ divide by $\sqrt{a^2+b^2}$ both sides.

Find general solution of sin x + cos x = √2.

$$\sqrt{(1)^2+(1)^2} = \sqrt{2}$$

$$\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x = \frac{\sqrt{2}}{\sqrt{2}}$$

$$\sin\frac{\pi}{4}\sin x + \cos\frac{\pi}{4}\cos x = 1$$

$$2n\pi \pm \alpha$$

$$n\pi + (-1)^n\alpha$$

$$\# \quad \cos\left(x-\frac{\pi}{4}\right) = \cos 0$$

$$x - \frac{\pi}{4} = 2n\pi \pm 0$$

$$\therefore \boxed{x = 2n\pi + \frac{\pi}{4}} \quad n \in \mathbb{Z}$$

Unacademy Atoms

Q

Find general solution of √3 cos x + sin x = 2.

$$\sqrt{(\sqrt{3})^2 + 1^2} = 2$$

$$\boxed{\frac{\sqrt{3}}{2}} \cos x + \frac{1}{2} \sin x = \frac{2}{2}$$

$$\cos\frac{\pi}{6}\cos x + \sin\frac{\pi}{6}\sin x = 1$$

$$\cos\left(x - \frac{\pi}{6}\right) = \cos 0$$

$$x - \frac{\pi}{6} = 2n\pi$$

$$\therefore \boxed{x = 2n\pi + \frac{\pi}{6}}$$

Find the general solutions of equation sin x + cos x = 3/2

$$\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x = \frac{3}{2\sqrt{2}}$$

$$\Rightarrow \underbrace{\cos\left(x - \frac{\pi}{4}\right) = \frac{3}{2\sqrt{2}}}_{\text{No sol}^n}$$

$$\cos\left(x - \frac{\pi}{4}\right) = 1.5m$$

$$\frac{\sqrt{9}}{\sqrt{8}} = \sqrt{\frac{9}{8}}$$

$$\boxed{1.5m}$$

The number of integral values of 'k' for which the equation $\underbrace{3\sin x + 4\cos x} = k + 1$ has a solution, $k \epsilon R$ is

[JEE M 2021]

$\left[-\sqrt{a^2+b^2}, \sqrt{a^2+b^2}\right]$

$[-5, 5]$ $\quad k+1$

$-5 \le k+1 \le 5$

$\boxed{-6 \le k \le 4}$

$\underbrace{-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4}$

$\boxed{11 \text{ Values}}$

Concept : "sol^n Exist Karta hai"

$\underbrace{\sin x}_{[-1,1]} = 4$

No sol^n

$\underbrace{\sin x}_{[-1,1]} = \frac{1}{2}$

$\frac{\sqrt{3}}{2}$

$\frac{1}{3}$

$\frac{2}{3}$

$\sin x = k$

$\boxed{-1 \le k \le 1}$

Consider the following lists:

[JEE Adv. 2022]

List-I	List-II
(I) $\left\{x \in \left[-\frac{2\pi}{3}, \frac{2\pi}{3}\right] : \cos x + \sin x = 1\right\}$	(P) has two elements
(II) $\left\{x \in \left[-\frac{5\pi}{18}, \frac{5\pi}{18}\right] : \sqrt{3}\tan 3x = 1\right\}$	(Q) has three elements
(III) $\left\{x \in \left[-\frac{6\pi}{5}, \frac{6\pi}{5}\right] : 2\cos(2x) = \sqrt{3}\right\}$	(R) has four elements
(IV) $\left\{x \in \left[-\frac{7\pi}{4}, \frac{7\pi}{4}\right] : \sin x - \cos x = 1\right\}$	(S) has five elements
	(T) has six elements

$\tan\left(-\frac{5\pi}{6}\right)$
$= -\tan\frac{5\pi}{6}$
$= -\tan\left(\pi - \frac{\pi}{6}\right)$
$= \frac{1}{\sqrt{3}}$

$\tan 3x = \frac{1}{\sqrt{3}} \quad x \in \left[-\frac{5\pi}{18}, \frac{5\pi}{18}\right]$

$\tan(\theta) = \frac{1}{\sqrt{3}} \quad \theta \in \left[-\frac{5\pi}{6}, \frac{5\pi}{6}\right]$

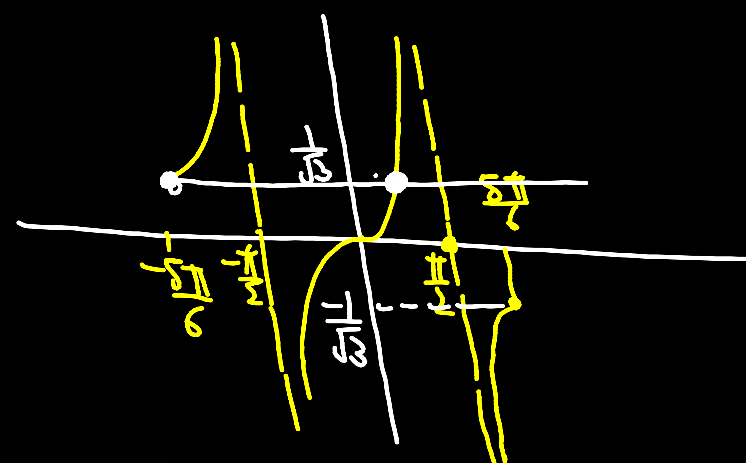
The correct option is:

(A) (I) → (P); (II) → (S); (III) → (P); (IV) → (S)

(B) (I) → (P); (II) → (P); (III) → (T); (IV) → (R)

(C) (I) → (Q); (II) → (P); (III) → (T); (IV) → (S)

(D) (I) → (Q); (II) → (S); (III) → (P); (IV) → (R)

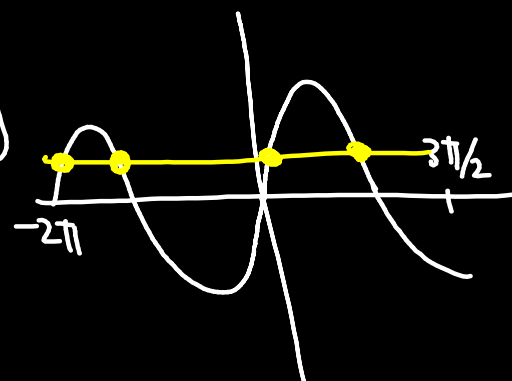


$-\frac{7\pi}{4}-\frac{\pi}{4}, \frac{7\pi}{4}-\frac{\pi}{4}$

$\frac{1}{\sqrt{2}}\sin x-\frac{1}{\sqrt{2}}\cos x=\frac{1}{\sqrt{2}}$ $\quad x\in\left[-\frac{7\pi}{4}, \frac{7\pi}{4}\right]$

$\sin\left(x-\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}$ $\quad \left(x-\frac{\pi}{4}\right)\in\left[-2\pi, \frac{3\pi}{2}\right]$

$\sin(\theta)=\frac{1}{\sqrt{2}}$ $\quad \theta\in\left[-2\pi, \frac{3\pi}{2}\right]$



① QE – factorise

② $a\sin x + b\cos x = c$

Type – 3

(Convert Sum to Product)

Type 3

Convert Sum to Product

If $0 \leq x < \frac{\pi}{2}$, then the number of values of x for which

$\boxed{\sin x - \sin 2x + \sin 3x = 0}$ is $x \in \left[0, \frac{\pi}{2}\right)$

[JEE M 2019]

A. 3
B. 1
C. 4
D. 2

$\underline{\sin x + \sin 3x} - \sin(2x) = 0$

$2 \sin(2x) \cos(x) - \sin 2x = 0$

$\sin(2x)\left(2\cos x - 1\right) = 0$

$\sin 2x = 0$ OR $\cos x = 1/2$

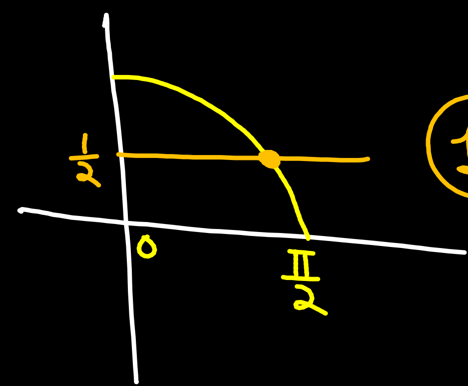
$\sin 2x = 0 \quad x \in [0, \frac{\pi}{2})$

$\sin 2x = 0 \quad 2x \in [0, \pi)$

$\sin \theta = 0 \quad \theta \in [0, \pi)$

$\boxed{\theta = 0}$

$\cos x = \frac{1}{2} \quad x \in [0, \frac{\pi}{2})$



1

$1 + 1 = \boxed{2}$

If the sum of values of x in $[0, 2\pi]$, for which $\sin x + \sin 2x + \sin 3x + \sin 4x = 0$, is equal to

[JEE M 2021]

A. 8π
B. 11π
C. 12π
D. 9π

$$2\sin\left(\frac{5x}{2}\right)\cos\left(\frac{3x}{2}\right) + 2\sin\left(\frac{5x}{2}\right)\cos\left(\frac{x}{2}\right) = 0$$

(D)

$$2\sin\left(\frac{5x}{2}\right)\left\{\cos\frac{3x}{2} + \cos\frac{x}{2}\right\} = 0$$

$$\frac{C-D}{2} = \frac{x-4x}{2} = \frac{-3x}{2}$$

$$2\sin\left(\frac{5x}{2}\right)\left\{2\cos x \cos\frac{x}{2}\right\} = 0$$

✓ ✓ ✓

$x \in [0, 2\pi]$

$\sin\frac{5x}{2} = 0$

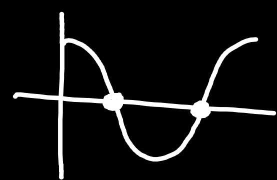
$\sin\left(\frac{5x}{2}\right) = 0 \qquad \frac{5}{2}x \in [0, 5\pi]$

$\frac{5x}{2} = 0, \pi, 2\pi, 3\pi, 4\pi, 5\pi$

$\therefore \boxed{x = 0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}, 2\pi}$

$x \in (0, 2\pi)$

$\cos x = 0$



$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}}$

$\cos\frac{x}{2} = 0 \qquad x \in [0, 2\pi]$

$\cos\left(\frac{x}{2}\right) = 0 \qquad \frac{x}{2} \in [0, \pi]$

$\frac{x}{\cancel{2}} = \frac{\pi}{\cancel{2}}$

$\therefore \boxed{x = \pi}$

$\boxed{\text{Sum of Sol}^n \Rightarrow 9\pi}$

The positive integer value of $n > 3$ satisfying the equation

$$\frac{1}{\sin\left(\frac{\pi}{n}\right)} = \frac{1}{\sin\left(\frac{2\pi}{n}\right)} + \frac{1}{\sin\left(\frac{3\pi}{n}\right)} \text{ is}$$

$\boxed{\text{Sum} \Rightarrow \text{Product}}$

[JEE Adv. 2011]

$$\frac{1}{\sin\left(\frac{\pi}{n}\right)} - \frac{1}{\sin\left(\frac{3\pi}{n}\right)} = \frac{1}{\sin\left(\frac{2\pi}{n}\right)}$$

$$\frac{\boxed{\sin\left(\frac{3\pi}{n}\right) - \sin\left(\frac{\pi}{n}\right)}}{\sin\left(\frac{\pi}{n}\right)\sin\left(\frac{3\pi}{n}\right)} = \frac{1}{\sin\left(\frac{2\pi}{n}\right)}$$

$$\frac{2\cos\left(\frac{2\pi}{n}\right)\cancel{\sin\left(\frac{\pi}{n}\right)}}{\cancel{\sin\left(\frac{\pi}{n}\right)}\sin\left(\frac{3\pi}{n}\right)} = \frac{1}{\sin\left(\frac{2\pi}{n}\right)}$$

$$\underbrace{2\cos\left(\frac{2\pi}{n}\right)\sin\left(\frac{2\pi}{n}\right)} = \sin\left(\frac{3\pi}{n}\right)$$

$$\underbrace{\sin\left(\frac{4\pi}{n}\right) = \sin\left(\frac{3\pi}{n}\right)}$$

Ans:-

$$\boxed{n = 7}$$

$$\sin\left(\frac{4\pi}{7}\right) = \sin\left(\frac{3\pi}{7}\right)$$

** $\boxed{\sin\theta = \sin(\pi - \theta)}$

Type – 4
(Convert Product to Sum)

Type 4

Convert Product into Sum

Unacademy Atoms

Number of solutions of the trigonometric equation in [0, π],

$\boxed{\sin 3\theta = 4 \sin\theta \cdot \sin 2\theta . \sin 4\theta}$ $\boxed{\sin(A-B)\cdot \sin(A+B) = \sin^2 A - \sin^2 B}$

A. 4

B. 6

C. 8 ✓

D. 10

$\Rightarrow \sin 3\theta = 4\sin\theta \, \sin(3\theta-\theta)\sin(3\theta+\theta)$

$\Rightarrow \sin 3\theta = 4\sin\theta\left(\sin^2 3\theta - \sin^2\theta\right)$

$\Rightarrow 3\sin\theta - \cancel{4\sin^3\theta} = 4\sin\theta\sin^2 3\theta - \cancel{4\sin^3\theta}$

$\Rightarrow 3\cancel{\sin\theta} = 4\cancel{\sin\theta}\sin^2 3\theta$

$\sin\theta = 0$ $\qquad 3 = 4\sin^2 3\theta$ $\qquad \theta \in [0, \pi]$

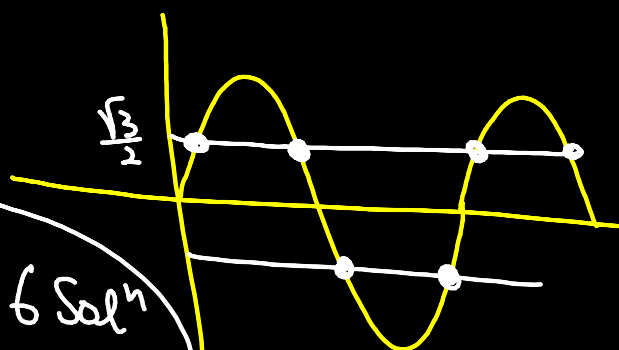
$\sin\theta = 0$

$\theta \in [0, \pi]$

$\boxed{\theta = 0, \pi}$

$\therefore \sin 3\theta = \pm \frac{\sqrt{3}}{2}$ $\qquad 3\theta \in [0, 3\pi]$

$\sin x = \pm \frac{\sqrt{3}}{2}$ $\qquad x \in [0, 3\pi]$



2 Sol^n + 6 Sol^n

* $\sin x \pm \cos x$ ✓
* $\sin x \cos x$ ✓

Type - 5

$\boxed{\sin x + \cos x = \sin x \cos x + 2}$

Method

Type – 5

$$f(\sin x \pm \cos x\,,\sin x\, cos x)$$

Type 5

(i) Equations of the form $P(\sin x \pm \cos x, \sin x.\cos x) = 0$, can be solved by the substituting $\boxed{\cos x \pm \sin x = t}$

(ii) Many equations can be solved by introducing a new variable

e.g. consider the equation $\sin^4 2x + \cos^4 2x = \sin 2x.\cos 2x$

$\sin x + \cos x = \sin x \cos x + 2$

Method $\sin x + \cos x = t$

$\Rightarrow (\sin x + \cos x)^2 = t^2$

$\Rightarrow 1 + 2\boxed{\sin x \cos x} = t^2$

$\Rightarrow \boxed{\sin x \cos x = \frac{t^2 - 1}{2}}$

Q.E

$$\boxed{t = \frac{t^2 - 1}{2} + 2}$$

Find general value of x satisfying the equation

$\sin^4 2x + \cos^4 2x = \sin 2x \cos 2x.$

$$2\left(1 - 2\sin^2 2x\cos^2 2x = \sin 2x\cos 2x\right)$$

$$\Rightarrow 2 - \sin^2 4x = \sin 4x$$

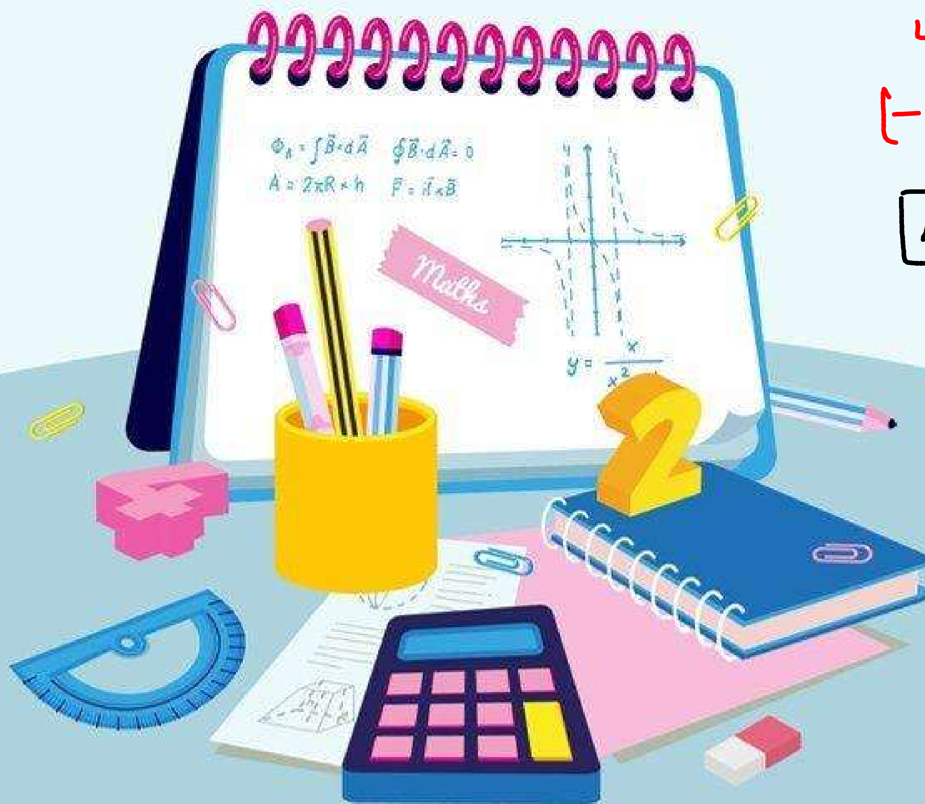
$$\Rightarrow \sin^2 4x + \sin 4x - 2 = 0$$

$$\Rightarrow (\sin 4x + 2)(\sin 4x - 1) = 0$$

$$\cancel{\sin 4x = -2} \qquad \sin 4x = 1$$

$$\sin 4x = \sin\frac{\pi}{2}$$

$$\boxed{x = \frac{n\pi}{4} + (-1)^n\frac{\pi}{8}}$$



$\underbrace{LHS}_{[-2,2]} = \underbrace{RHS}_{[2,8]}$

$\boxed{LHS=2 \quad RHS=2}$

Type - 6

Using Range of Functions

Type 6

Solving equations with the use of boundedness of the function.

Remember :-

$-1 \leq \sin x \leq 1$, $-1 \leq \cos x \leq 1$, $\tan x \in R$, $\cot x \in R$.

$|\operatorname{cosec} x| \geq 1$, $|\sec x| \geq 1$.

$$\left.\begin{matrix} \sec x \\ \operatorname{cosec} x \end{matrix}\right\} (-\infty, -1] \cup [1, \infty)$$

Most G.S.

~~Solve~~ for x : $\boxed{\cos x + \cos 2x + \cos 3x = 3}$

$[-1,1]$ $[-1,1]$ $[-1,1]$

$\cos x = 1$ $\cos 2x = 1$ $\cos 3x = 1$

$n \in \mathbb{Z}$

$\therefore \boxed{x = 2n\pi}$

$\{0, 2\pi, 4\pi, 6\pi, \ldots\}$

$\cancel{2}x = \cancel{2}n\pi$

$\therefore \boxed{x = n\pi}$

$\{0, \pi, 2\pi, 3\pi, 4\pi, \ldots\}$

$3x = 2n\pi$

$\therefore \boxed{x = \frac{2n\pi}{3}}$

$\{0, \frac{2\pi}{3}, \frac{4\pi}{3}, 2\pi, \frac{8\pi}{3}, \ldots\}$

$$\boxed{GS \in \{n\pi\} \cup \{\frac{2n\pi}{3}\}}$$

Q

All the pairs (x, y) that satisfy the inequality

$\boxed{\cancel{2}^{\sqrt{\sin^2 x - 2\sin x + 5}} \dfrac{1}{\underset{2}{\cancel{4}}^{\sin^2 y}} \leq 1}$ also satisfy the equation:

(1) $2|\sin x| = 3\sin y$ (2) $2\sin x = \sin y$

(3) $\sin x = 2\sin y$ (4) $\sin x = |\sin y|$ ✓

D

$$\sqrt{\sin^2 x - 2\sin x + 5} \leq 2\sin^2 y$$

$\underline{\sin x = -1}$

$$\underbrace{\sqrt{\cancel{(\sin x - 1)^2}} + 4}_{[2,\, 2\sqrt{2}]} \leq \underbrace{\boxed{2\sin^2 y}}_{[0,\, 2]}$$

$\boxed{\sin x = 1}$ $\sin^2 y = 1$

$\boxed{\sin y = \pm 1}$

$\sin x = |\sin y|$

[JEE M 2019]

The number of solutions of $\sin^7 x + \cos^7 x = 1$, $x \epsilon [0, 4\pi]$ is equal to

A. 11
B. 7
C. 5
D. 9

[JEE M 2021]

$\sin^7 x + \cos^7 x = 1$

$\sin x$	$\cos x$
1	0
0	1

$\boxed{x = \frac{\pi}{2}, \cancel{\frac{3\pi}{2}}, \frac{5\pi}{2}, \cancel{\frac{7\pi}{2}}}$

$x = 0, \cancel{\pi}, 2\pi, \cancel{3\pi}, 4\pi$

5 Sol^n

Unacademy Atoms

Q

For $x \in (0, \pi)$, the equation $\sin x + 2\sin 2x - \sin 3x = 3$ has

(a) infinitely many solutions
(b) three solutions
(c) one solution
(d) no solution

[JEE Adv. 2014]

$$-2\cos 2x \sin x + 2\sin 2x = 3$$

D

$$-2\cos 2x \sin x + 4\sin x \cos x = 3$$

$$\sin x\left(-2\cos 2x + 4\cos x\right) = 3$$

$$-2\left(2\cos^2 x - 1\right) + 4\cos x = \frac{3}{\sin x}$$

$$\underbrace{-1-4\cos^2 x+4\cos x}+3 = 3\,\mathrm{cosec}\,x$$

$$\boxed{3-(2\cos x-1)^2} = \boxed{3\,\mathrm{cosec}\,x}$$

$[-6, 3]$ $\qquad$ $[3, \infty)$

$\cos x = -1$

$\cos x = 1/2$

No Sol^n

$\boxed{\cos x = \frac{1}{2}}$

$\mathrm{cosec}\,x = 1$

$\boxed{\sin x = 1}$

(Root wale sawal)

Type – 7

$$f(x) = \sqrt{\varphi(x)}$$

Type 7

Solution of trigonometric equation of the form $f(x) = \sqrt{\varphi(x)}$

(i) $f(x) \geq 0$, $\varphi(x) \geq 0$

(ii) $f^2(x) = \varphi(x)$

* Jab bhi sq. both sides

$\Downarrow$

faltu Solⁿ

$\Downarrow$

Reject

Solve for x, $\boxed{\sin x = \sqrt{1 - \cos x}}$ in $x \in [0, 2\pi]$

$$\sin^2 x = 1 - \cos x$$

$$(1 - \cancel{\cos x})(1 + \cos x) = (1 - \cancel{\cos x})$$

$$1 + \cos x = 1$$

$$\boxed{\cos x = 1} \qquad \boxed{\cos x = 0}$$

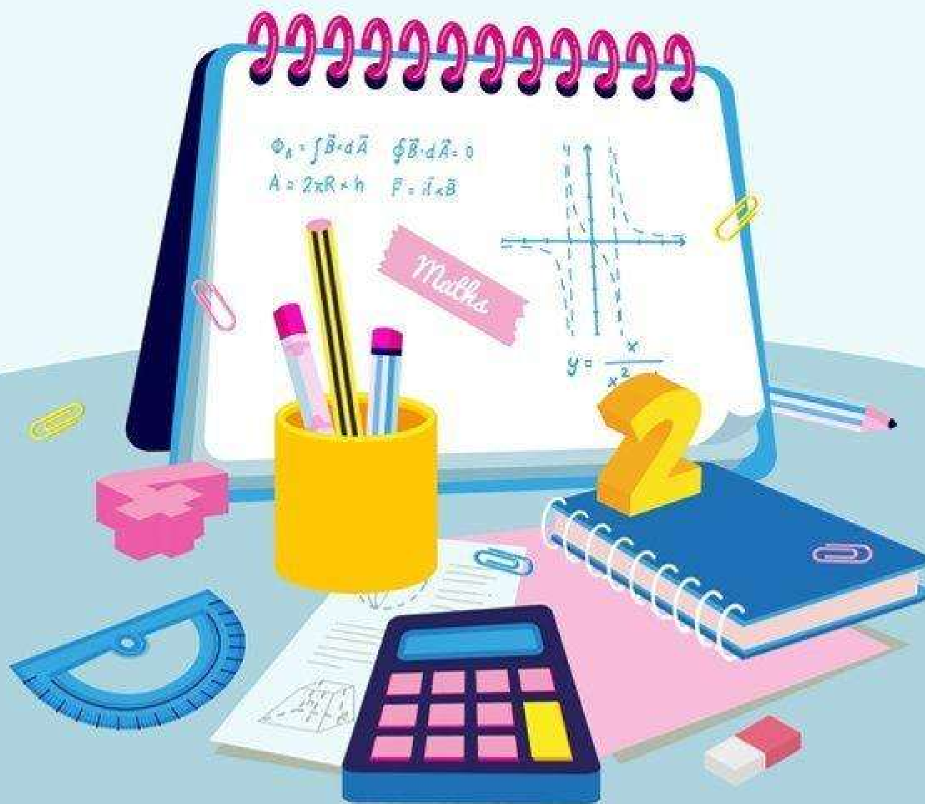
$$\boxed{x = 0, 2\pi} \qquad \boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}}$$

$$\sin x = \sqrt{1-\cos x}$$

$$\left\{\begin{array}{ll} x = 0 & 0 = \sqrt{1-1} \quad \checkmark \\ x = 2\pi & 0 = \sqrt{1-1} \quad \checkmark \\ x = \frac{\pi}{2} & 1 = \sqrt{1-0} \quad \checkmark \end{array}\right.$$

$$\cancel{\boxed{x = \frac{3\pi}{2}}} \qquad -1 = \sqrt{1-0} \quad \times$$

Unacademy Atoms



Type – 8
Log wale Questions

The number of distinct solutions of the equation,

$\log_{\frac{1}{2}} |\sin x| = 2 - \log_{\frac{1}{2}} |\cos x|$ in the interval $[0, 2\pi]$, is ____

[JEE M 2020]

$$\log_{\frac{1}{2}} |\sin x| + \log_{\frac{1}{2}} |\cos x| = 2$$

$$\log_{\frac{1}{2}} |\sin x| . |\cos x| = 2$$

$$\underbrace{|a||b| = |ab|}$$

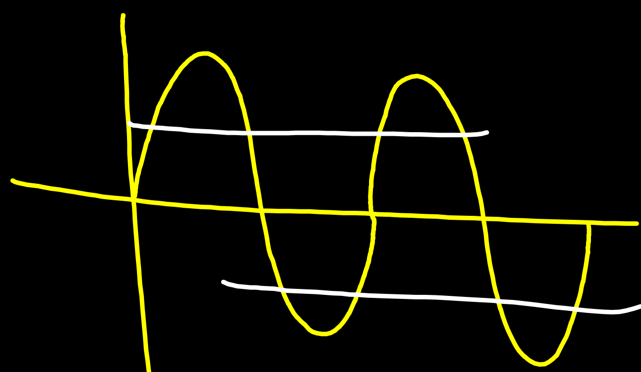
$$|2\sin x \cdot \cos x| = \frac{1}{4} \times 2$$

$$|\sin 2x| = \frac{1}{2}$$

$\sin 2x = \pm \frac{1}{2} \qquad x \in [0, 2\pi]$

$\sin(2x) = \pm \frac{1}{2} \qquad 2x \in [0, 4\pi]$

☆ $\sin(\theta) = \pm \frac{1}{2} \qquad \theta \in [0, 4\pi]$



8

If for $x \in \left(0, \frac{\pi}{2}\right)$, $\log_{10} \sin x + \log_{10} \cos x = -1$ and $\log_{10}(\sin x + \cos x) = \frac{1}{2}(\log_{10} n - 1)$, n > 0 then the value of n is

A. 20
B. ~~12~~
C. 9
D. 16

[JEE M 2021]

$$\log_{10}(\sin x \cos x) = -1$$

$$\therefore \sin x \cos x = 10^{-1} = \frac{1}{10} \quad \text{--- (1)}$$

$$\log_{10}(\sin x + \cos x)^2 = \log_{10} n - \log_{10} 10$$

$$\cancel{\log_{10}}\left(1 + 2\sin x \cos x\right) = \cancel{\log_{10}}\left(\frac{n}{10}\right)$$

$$1 + 2 \times \frac{1}{10} = \frac{n}{10}$$

$$\therefore \boxed{n = 12}$$

Trig. + algebra

$\underbrace{\tan x}_{} = \underbrace{x - 10}_{}$

$y = \tan x$ $\quad$ $y = x - 10$

P.O.I

no. of sol^n

#

Type – 9

Graph wale Questions

The number of solutions of equation $x + 2\tan x = \frac{\pi}{2}$ in the interval $[0, 2\pi]$ is :

A. 3
B. 4
C. 2
D. 5

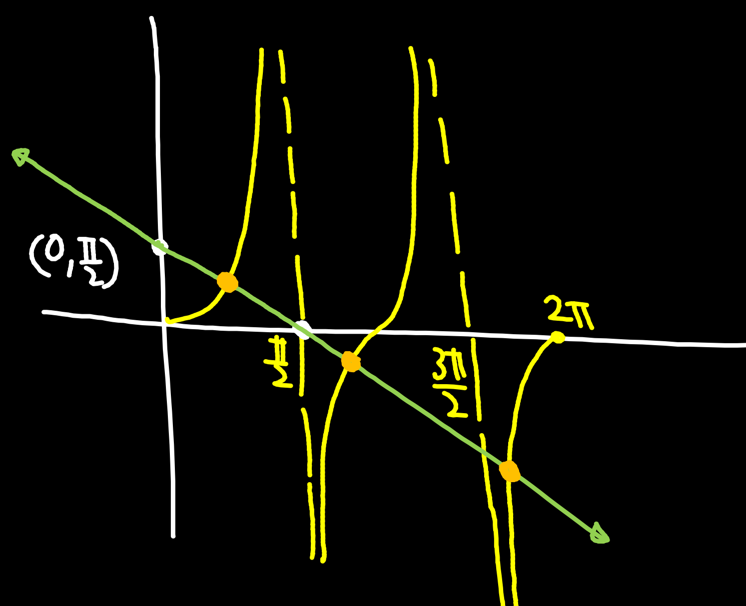
[JEE M 2021]

$$\underbrace{2\tan x} = \underbrace{\frac{\pi}{2} - x}$$

$y = 2\tan x$

$\underbrace{y = \frac{\pi}{2} - x}$

x	y
0	$\pi/2$
$\frac{\pi}{2}$	0



The number of solutions of $|\cos x| = \sin x$, such that $-4\pi \le x \le 4\pi$ is

[JEE M 2022]

A. 4

B. 6

C. 8

D. 12

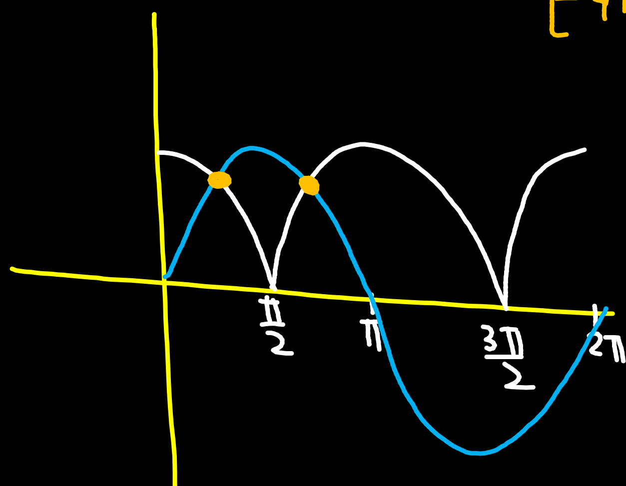
$[0, 2\pi] \longrightarrow 2$ sol^n

$[-4\pi, 4\pi] \rightarrow 2 \times 4$

$= 8$ sol^n

$y = |\cos x|$

$y = \sin x$



Q The number of solutions of the equation $2\theta - \cos^2\theta + \sqrt{2} = 0$ in R is equal to_. 1

[JEE M 2022]

$y = 2\theta + \sqrt{2}$

$(0, \sqrt{2})$ $\quad y = 2x + \sqrt{2}$

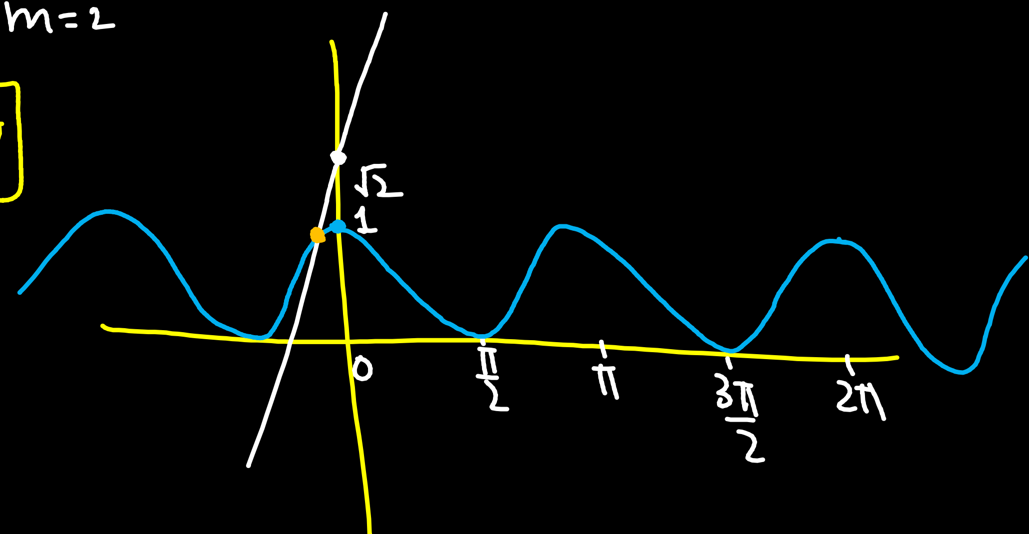
$m = 2$

$$\underbrace{2\theta + \sqrt{2}}_{} = \underbrace{\cos^2\theta}_{}$$

$\theta \in (-\infty, \infty)$

$\boxed{y = \cos^2\theta}$

θ	y
0	1
$\frac{\pi}{2}$	0
π	1
$\frac{3\pi}{2}$	0
2π	1



Q

The number of solutions of the equation sin x = $\cos^2 x$ in the interval (0,10) is_.

[JEE M 2022]

$\boxed{\sqrt{5} \approx 2.2}$

$\boxed{\sqrt{3} \approx 1.7}$

M-1

$\sin x = \cos^2 x$

4

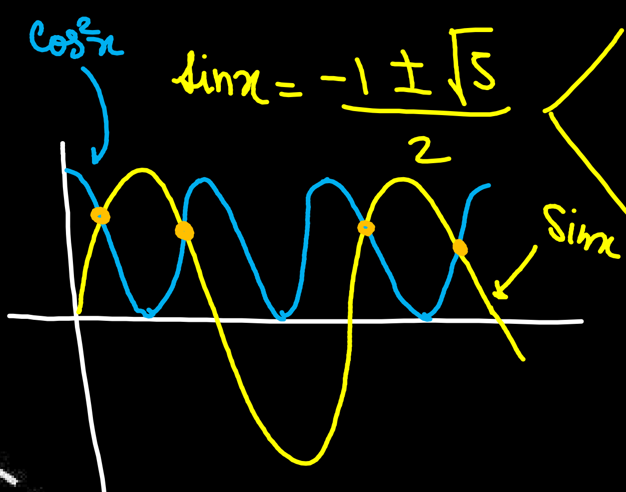
$\sin x = 1 - \sin^2 x$

$\sin^2 x + \sin x - 1 = 0$

$\sin x = \dfrac{-1 \pm \sqrt{5}}{2}$

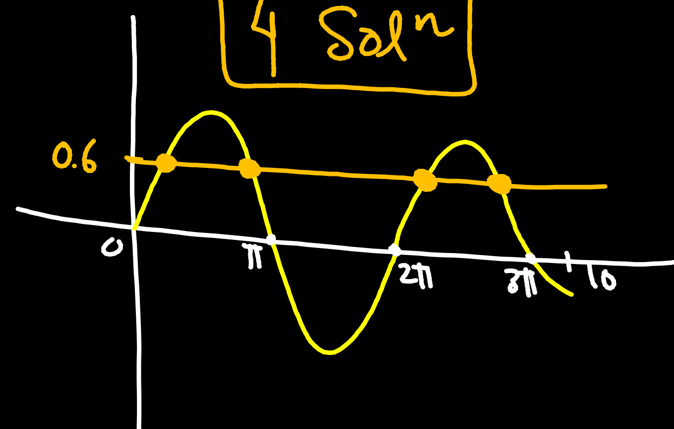
$\dfrac{-1+\sqrt{5}}{2} \approx 0.6$

$\cancel{\dfrac{-1-\sqrt{5}}{2} \approx -1.6}$



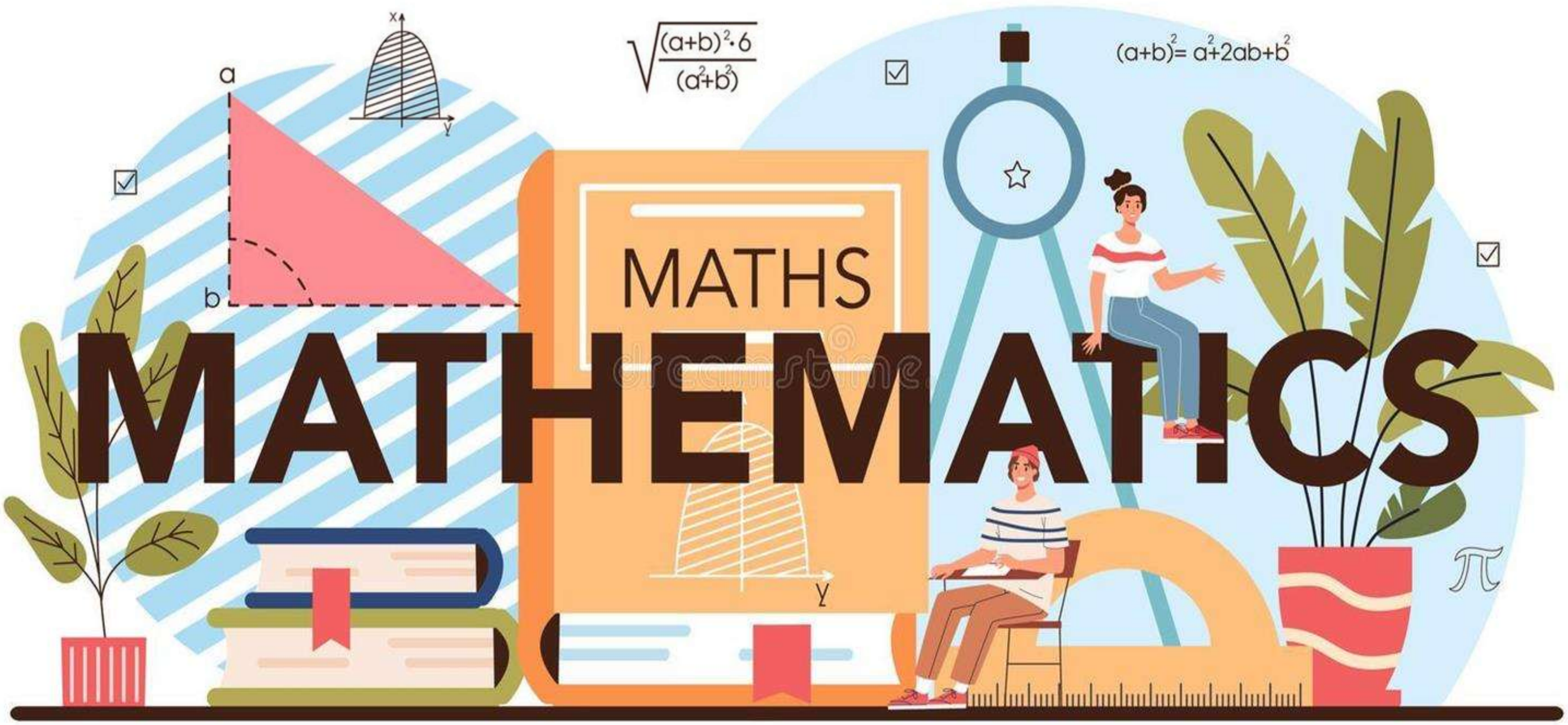
$\sin x = 0.6 \quad x \in (0,10)$

$\boxed{4 \text{ Sol}^n}$



Unacademy Atoms

PYQs



Unacademy Atoms

Q

The number of values of θ in the interval, $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ such that $\boxed{\theta \neq \frac{n\pi}{5}}$ for $n = 0, \pm 1, \pm 2$ and $\boxed{\tan\theta = \cot 5\theta}$ as well as $\boxed{\sin 2\theta = \cos 4\theta}$ is 3

[JEE Adv. 2010]

① $\tan\theta = \cot 5\theta$

$\tan\theta = \tan\left(\frac{\pi}{2} - 5\theta\right)$

$\theta = n\pi + \frac{\pi}{2} - 5\theta$

$6\theta = \frac{(2n+1)\pi}{2}$

$\therefore \boxed{\theta = \frac{(2n+1)\pi}{12}}$

$\theta = \frac{(2n+1)\pi}{12}$

$\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\theta \in \left(-\frac{6\pi}{12}, \frac{6\pi}{12}\right)$

$n=0$: $\frac{\pi}{12}$ ✓

$n=1$: $\frac{3\pi}{12}$ ✓

$n=2$: $\frac{5\pi}{12}$ ✓

$n=-1$: $-\frac{\pi}{12}$ ✓

$n=-2$: $-\frac{3\pi}{12}$ ✓

$n=-3$: $-\frac{5\pi}{12}$ ✓

$\theta \in (\quad\quad) \quad \left\{\frac{\pi}{12}, \frac{5\pi}{12}, \frac{-3\pi}{12}\right\}$

$\sin 2\theta = \cos 4\theta$

$\cos\left(\frac{\pi}{2} - 2\theta\right) = \cos 4\theta$

$\frac{\pi}{2} - 2\theta = 2n\pi \pm 4\theta$

$\frac{\pi}{2} - 2\theta = 2n\pi + 4\theta$

$\therefore \frac{\pi}{2} - 2n\pi = 6\theta$

$\therefore \boxed{\frac{(1-4n)\pi}{12} = \theta}$

$\frac{\pi}{2} - 2\theta = 2n\pi - 4\theta$

$2\theta = 2n\pi - \frac{\pi}{2}$

$\boxed{\theta = \frac{(4n-1)\pi}{4}}$

$n=0 \quad \frac{\pi}{12}$ ✓

$n=1 \quad \frac{-3\pi}{12}$ ✓

$n=2 \quad -\frac{7\pi}{12}$ X

$\vdots$

$n=-1 \quad \frac{5\pi}{12}$ ✓

$n=-2 \quad \frac{9\pi}{12}$ X

$\theta \in \left(\frac{-2\pi}{4}, \frac{2\pi}{4}\right)$

$n=0 \quad -\frac{\pi}{4}$ ✓

$n=1 \quad \frac{3\pi}{4}$ X

$\vdots$

$n=-1 \quad -\frac{5\pi}{4}$ X

Q

Let $S = \left\{x \in (-\pi, \pi) : x \neq 0, \pm\frac{\pi}{2}\right\}$. The sum of all distinct solutions of the equation $\sqrt{3}\sec x + \operatorname{cosec} x + 2(\tan x - \cot x) = 0$ in the set S is equal to

[JEE Adv. 2016]

(a) $-\frac{7\pi}{9}$

(b) $-\frac{2\pi}{9}$

(c) 0 c

(d) $\frac{5\pi}{9}$

$$\sqrt{3}\sec x + \operatorname{cosec} x + 2(\tan x - \cot x) = 0$$

$$\frac{\sqrt{3}}{\cos x} + \frac{1}{\sin x} + 2\left(\frac{\boxed{\sin^2 x - \cos^2 x}}{\sin x \cos x}\right) = 0$$

$$\frac{\sqrt{3}\sin x + \cos x}{\sin x \cos x} + \frac{2(-\cos 2x)}{\sin x \cos x} = 0$$

$$x \in (-\pi, \pi) \equiv \left(-\frac{9\pi}{9}, \frac{9\pi}{9}\right)$$

$\sin\frac{\pi}{3}$ $\quad$ $\cos\frac{\pi}{3}$

$$\frac{\sqrt{3}}{2}\sin x + \frac{\cos x}{2} = \cos 2x$$

$$\sin\frac{\pi}{3}\sin x + \cos\frac{\pi}{3}\cos x = \cos 2x$$

$$\cos\left(x - \frac{\pi}{3}\right) = \cos 2x$$

$$x - \frac{\pi}{3} = 2n\pi + 2x$$

$$-\frac{\pi}{3} - 2n\pi = x$$

$$\boxed{-\frac{(6n+1)\pi}{3} = x}$$

$$x - \frac{\pi}{3} = 2n\pi - 2x$$

$$3x = 2n\pi + \frac{\pi}{3}$$

$$\boxed{x = \frac{(6n+1)\pi}{9}}$$

$\boxed{n=0 \quad -\frac{\pi}{3}}$ ✓

$n=1 \quad -\frac{7\pi}{3}$ ✗

$n=-1 \quad \frac{5\pi}{3}$ ✗

$n=0 \quad \frac{\pi}{9}$ ✓

$n=1 \quad \frac{7\pi}{9}$ ✓

$n=2 \quad \frac{13\pi}{9}$ ✗

$n=-1 \quad -\frac{5\pi}{9}$ ✓

$n=-2 \quad -\frac{11\pi}{9}$ ✗

$$-\frac{\pi}{3} + \frac{\pi}{9} + \frac{7\pi}{9} - \frac{5\pi}{9} = \boxed{0}$$