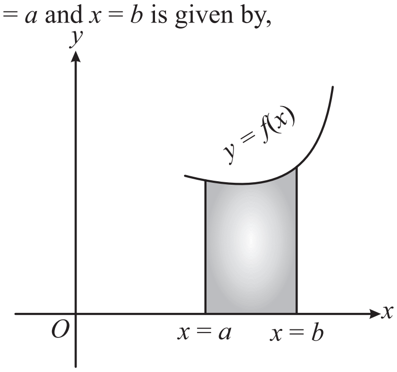
CHAPTER

10 Application of Integrals

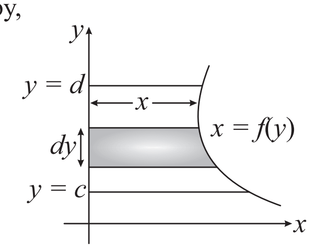
1. The area bounded by the curve $y = f(x)$, the x-axis and the ordinates $x = a$ and $x = b$ is given by,



$$A = \int_a^b f(x)dx = \int_a^b y\,dx$$

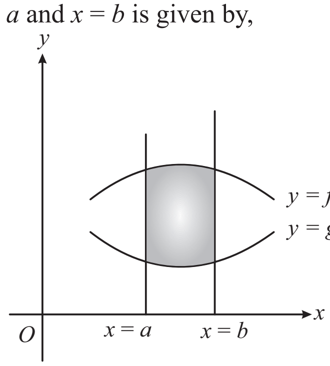
2. If the area is below the x-axis, then A is negative. The convention is to consider the magnitude only *i.e.* $A = \left|\int_a^b y\,dx\right|$ in this case.

3. The area bounded by the curve $x = f(y)$, y-axis and abscissa $y = c$, $y = d$ is given by,



$$\text{Area} = \int_c^d x dy = \int_c^d f(y)dy$$

4. Area between the curves $y = f(x)$ and $y = g(x)$ between the ordinates $x = a$ and $x = b$ is given by,



$$A = \int_a^b f(x)\,dx - \int_a^b g(x)dx$$

$$= \int_a^b [f(x) - g(x)]\,dx$$

5. Average value of a function $y = f(x)$ *w.r.t.* x over an interval $a \le x \le b$ is defined as:

$$y_{av} = \frac{1}{b-a}\int_a^b f(x)dx.$$

6. **Curve Tracing:**
The following outline procedure is to be applied in Sketching the graph of a function $y = f(x)$ which in turn will be extremely useful to quickly and correctly evaluate the area under the curves.
   (*a*) **Symmetry:** The symmetry of the curve is judged as follows:
      (i) If all the powers of y in the equation are even then the curve is symmetrical about the axis of x.
      (ii) If all the powers of x are even, the curve is symmetrical about the axis of y.
      (iii) If powers of x and y both are even, the curve is symmetrical about the axis of x as well as y.
      (iv) If the equation of the curve remains unchanged on interchanging x and y, then the curve is symmetrical about $y = x$.
      (v) If on interchanging the signs of x and y both the equation of the curve is unaltered then there is symmetry in opposite quadrants.
   (*b*) Find dy/dx and equate it to zero to find the points on the curve where you have horizontal tangents.
   (*c*) Find the points where the curve crosses the x-axis and also the y-axis.
   (*d*) Examine if possible the intervals when $f(x)$ is increasing or decreasing. Examine what happens to 'y' when $x \to \infty$ or $-\infty$.

7. **Useful Results:**
   (*a*) Whole area of the ellipse, $x^2/a^2 + y^2/b^2 = 1$ is $\pi\,ab$.
   (*b*) Area enclosed between the parabolas $y^2 = 4ax$ and $x^2 = 4by$ is $16ab/3$.
   (*c*) Area included between the parabola $y^2 = 4\,ax$ and the line $y = mx$ is $8\,a^2/3\,m^3$.