THIS CHAPTER IS $\boxed{\text{SANTA'S GIFT}}$, YOU CAN EASILY SCORE 8 TO 16 MARKS.

CONCEPT HUMARA , CALCULATION (DIFFERENTIATION) TUMHARA.

TO GET THE 'BEST' FROM THIS CLASS

1. Keep a rough copy with you … Don't rush to write the notes …!
2. Listen to me carefully , have a smile !
3. Keep short notes copy with you & write what I request you to write.
4. Have Infinite Patience …. And enjoy the ride!!

TOPICS TO BE COVERED

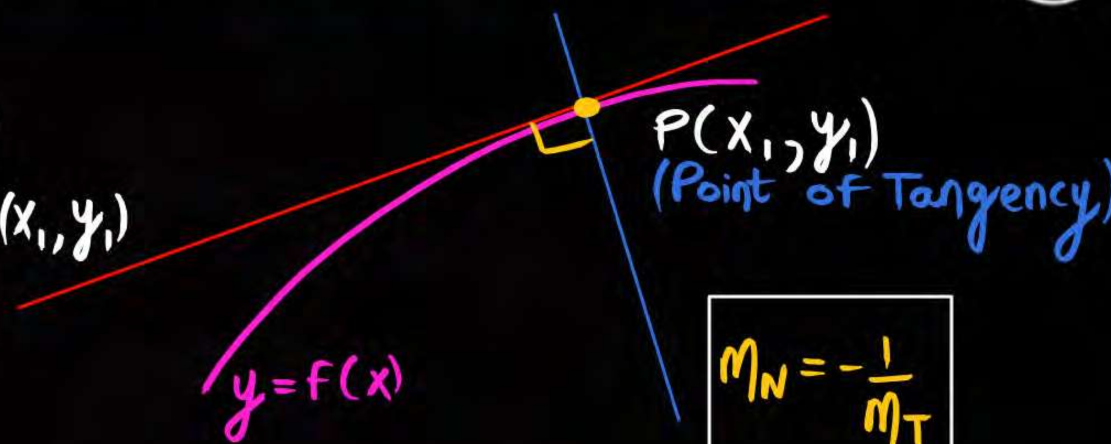
1. Tangent & Normal , Sub-Tangent & Sub-Normal ✓
2. Rate Measure
3. Monotonicity
4. Maxima & Minima
5. Graphs of Cubic

MEAN VALUE THEOREMS WILL BE TAUGHT IN CONTINUITY & DIFFERENTIABILITY.

TANGENTS AND NORMALS

SLOPE OF TANGENT AND NORMAL TO A CURVE AT A POINT LYING ON IT:

$$m_T = \left.\frac{dy}{dx}\right|_{(x_1, y_1)}$$



$$\boxed{m_N = -\frac{1}{m_T}}$$

EQUATION OF TANGENT AND NORMAL TO A CURVE AT A POINT LYING ON IT:

$$\frac{y - y_1}{x - x_1} = m_T$$

$$\frac{y - y_1}{x - x_1} = m_N$$

NOTE

$\boxed{(x_1, y_1)}$ *must be lying on the* $\boxed{\textit{Tangent}}$, $\boxed{\textit{Normal}}$ *as well as* $\boxed{\textit{on the curve.}}$

Q. If a curve with equation of the form $\boxed{y = ax^4 + bx^3 + cx + d}$ has zero $\boxed{\text{gradient}}$ at the point (0, 1) and also touches the x-axis at the point (–1, 0), then the values of x for which the curve has a negative gradient are :

(slope)

(A) $x > -1$

(B) $x > 1$

(C) $x < -1$

(D) $-1 \le x \le 1.$

(0,1) $\boxed{1 = d}$

(–1,0) $\boxed{0 = a - b - c + d}$ $d = 1$ $\boxed{b + c - a = 1}$ $c = 0$ $\boxed{b - a = 1}$

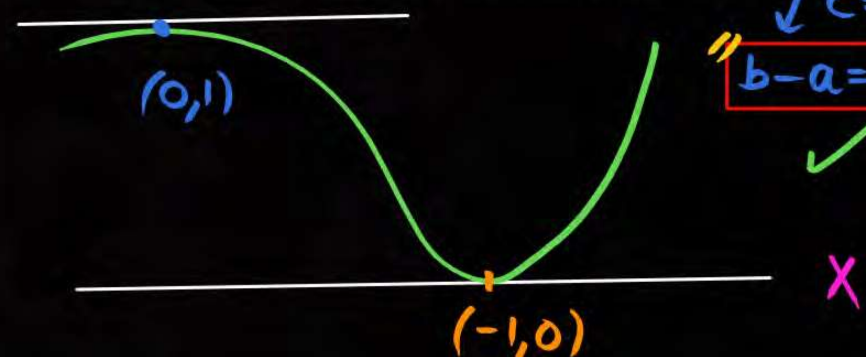
$$\boxed{\frac{dy}{dx} = 4ax^3 + 3bx^2 + c}$$

(0,1) $\boxed{0 = c}$

(–1,0) $\boxed{0 = -4a + 3b + c}$

$c = 0$ $\boxed{4a = 3b}$

$$\boxed{a = 3,\ b = 4}$$



$$\boxed{y = 3x^4 + 4x^3 + 1}$$

$$\frac{dy}{dx} = 12x^3 + 12x^2 < 0$$

$$\boxed{12x^2(x+1) < 0}$$

$$\boxed{x < -1}$$ Ans

Q. Let l be a line which is **normal** to the curve $\boxed{y = 2x^2 + x + 2}$ at a point P on the curve. If the point $Q(6, 4)$ lies on the **line l** and **O is origin**, then the **area of the triangle OPQ** is equal to ______.

(α, β)

$$\boxed{\beta = 2\alpha^2 + \alpha + 2}$$

[JEE Main-2022 (28 June - Shift 1)]

$$m_T = \frac{dy}{dx} = 4x + 1$$

$$\boxed{m_N = -\frac{1}{m_T} = -\frac{1}{4x+1}}$$

$$4\alpha^3 + 3\alpha^2 - 3\alpha - 4 = 0$$

$$\boxed{\alpha = 1}$$

N:

$$\frac{y - \beta}{x - \alpha} = -\frac{1}{4\alpha + 1}$$

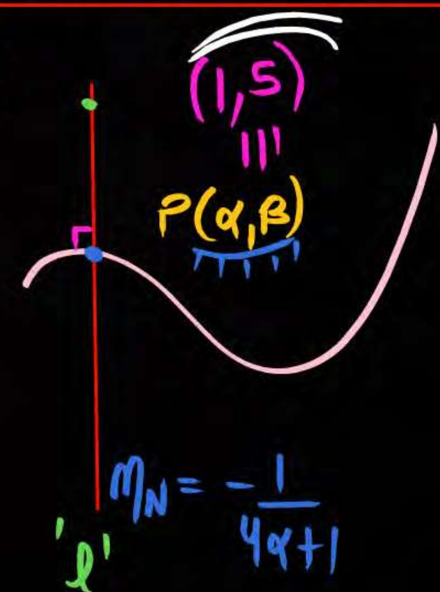
$(6, 4)$
$x = 6$, $y = 4$

$$\boxed{\frac{4 - \beta}{6 - \alpha} = -\frac{1}{4\alpha + 1}}$$

$$\beta = 2\alpha^2 + \alpha + 2$$

$$\frac{4 - (2\alpha^2 + \alpha + 2)}{6 - \alpha} = -\frac{1}{4\alpha + 1}$$

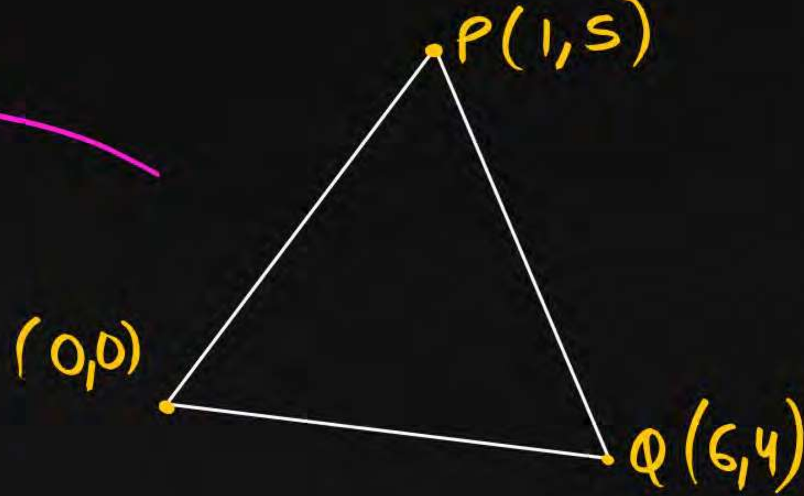
$(6,4)$ Q



area =

$$\frac{1}{2}\left|1(0-4)+0(4-5)+6(5-0)\right|$$

$= 13$ sq. units $\boxed{\text{Ans}}$



Q. If the tangent to the curve $y = x^3 - x^2 + x$ at the point (a, b) is also tangent to the curve $y = 5x^2 + 2x - 25$ at the point $(2, -1)$, then $|2a + 9b|$ is equal to ______.

$(a,b) \rightarrow b = a^3 - a^2 + a \xrightarrow{a=3} \boxed{b = 21}$

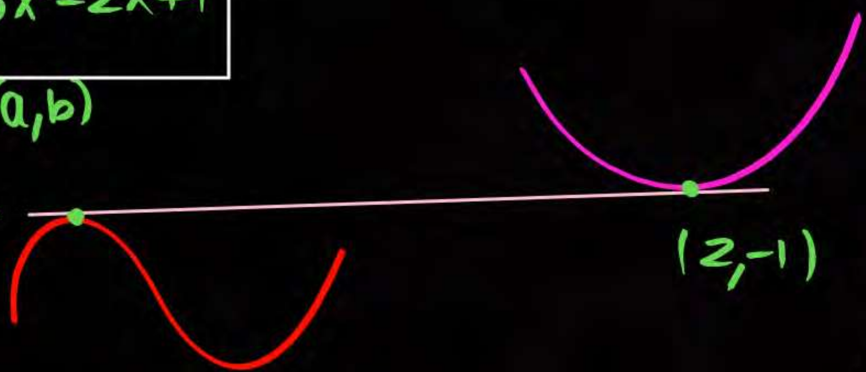
$\hookrightarrow (6 + 189) = 195$ Ans

[JEE Main-2022 (29 July - Shift 2)]

(BCS)

$$\boxed{\frac{dy}{dx} = 3x^2 - 2x + 1}$$

(a,b)



$y = x^3 - x^2 + x$

$m_T = 3a^2 - 2a + 1$

$$\boxed{\frac{y-b}{x-a} = 3a^2 - 2a + 1}$$

$$\frac{y - (a^3 - a^2 + a)}{x - a} = 3a^2 - 2a + 1$$

$y = 5x^2 + 2x - 25$

$\frac{dy}{dx} = 10x + 2 \xrightarrow{(2,-1)} m_T = 22$

$$\boxed{\frac{y-(-1)}{x-2} = 22} \rightarrow y + 1 = 22x - 44$$

$$\boxed{y - 22x + 45 = 0}$$

IDENTICAL LINES

$$\boxed{y - (3a^2 - 2a + 1)x + (2a^3 - a^2) = 0}$$

$$\boxed{\begin{aligned} &a_1x+b_1y+c_1=0\\ &a_2x+b_2y+c_2=0\\ &\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}\end{aligned}}$$

" Condⁿ for Identical lines "

$$\boxed{\frac{1}{1}=\frac{-22}{-(3a^2-2a+1)}=\frac{45}{2a^3-a^2}}$$

$a=3,-\frac{7}{3}$

$$\boxed{2a^3-a^2=45}$$

$\boxed{a=3\ \checkmark}$

$\cancel{a=-7/3}$

Q. If the angle made by the tangent at the point (x_0, y_0) on the curve $\boxed{x = 12(t + \sin t.\cos t)}$, $\boxed{y = 12(1 + \sin t)^2}$, $0 < t < \frac{\pi}{2}$, with the positive x-axis is $\frac{\pi}{3}$, then y_0 is :

$m = \tan\frac{\pi}{3}$ $x = f(t)$ $y = g(t)$ $\cos 2t = 2\cos^2 t - 1$

$\frac{dy}{dx} = ?$

[JEE Main-2022 (25 June - Shift 2)]

(A) $6(3 + 2\sqrt{2})$

(B) $3(7 + 4\sqrt{2})$

(C) 27 ✓

(D) 48

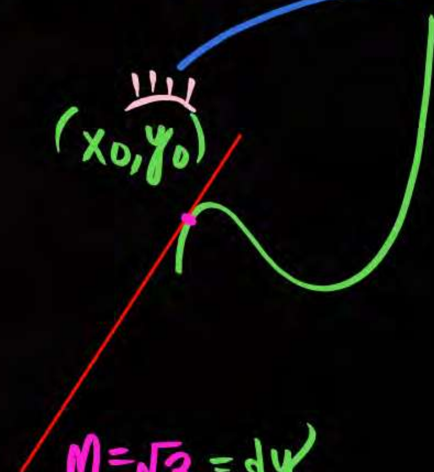
$\frac{dx}{dt} = 12(1 + \cos^2 t - \sin^2 t)$

$= 12(1 + \cos 2t)$

$\boxed{\frac{dx}{dt} = 24\cos^2 t}$

$\boxed{\frac{dy}{dt} = 24(1 + \sin t)\cos t}$

$\frac{dy}{dx} = \boxed{\frac{1 + \sin t}{\cos t} = \sqrt{3}}$



$m = \sqrt{3} = \frac{dy}{dx}$

$y_0 = 12\left(1 + \frac{1}{2}\right)^2$

$= 27$

Ans

$$\frac{1+\sin t}{\cos t}=\sqrt{3}$$

$$1+\sin t=\sqrt{3}\cos t$$

$$\boxed{1=\sqrt{3}\cos t-1\sin t}$$

Divide Both the sides by

$$\sqrt{(\sqrt{3})^2+(-1)^2}=2$$

$$\frac{1}{2}=\underbrace{\frac{\sqrt{3}}{2}}_{=\cos 30^\circ}\cos t-\underbrace{\frac{1}{2}}_{=\sin 30^\circ}\sin t$$

$$\boxed{\frac{1}{2}=\cos(t+30^\circ)}$$

$$\cos 60^\circ=\cos(t+30^\circ)$$

$$60^\circ=t+30^\circ$$

$$\boxed{t=30^\circ}$$

Q. Let M and N be the number of points on the curve $\boxed{y^5 - 9xy + 2x = 0}$, where the tangents to the curve are parallel to x-axis and y-axis, respectively. Then the value of $(M + N)$ equals 2 ____.

$\boxed{M=0}$

$\boxed{N=2}$

[JEE Main-2022 (27 July - Shift 1)]

D wrt 'x'

$$5y^4 \cdot \frac{dy}{dx} - 9\left(x \cdot \frac{dy}{dx} + y\right) + 2 = 0$$

$$\left(\frac{dy}{dx}\right)\left(5y^4 - 9x\right) = 9y - 2$$

$$\boxed{\frac{dy}{dx} = \frac{9y-2}{5y^4 - 9x}}$$

$$\frac{dy}{dx} = 0 \Rightarrow 9y - 2 = 0$$

$$\boxed{y = \frac{2}{9}}$$

$$y^5 - 9xy + 2x = 0$$

$$\left(\frac{2}{9}\right)^5 - \cancel{9}x \cdot \frac{2}{\cancel{9}} + 2x = 0$$

$$\boxed{\left(\frac{2}{9}\right)^5 = 0} \text{ False}$$

"No such point exists"

$D^R = 0$

$$\begin{cases} 5y^4 - 9x = 0 \\ \boxed{x = \dfrac{5y^4}{9}} \end{cases} \xrightarrow{\text{PUT}}$$

$$y^5 - 9xy + 2x = 0$$

$$y^5 - \cancel{9}y\left(\frac{5y^4}{\cancel{9}}\right) + 2y^4\frac{(5)}{9} = 0$$

$$y^5 - 5y^5 + \frac{10}{9}y^4 = 0$$

$$\boxed{\frac{10y^4}{9} = 4y^5}$$

$$0 = 4y^5 - \frac{10y^4}{9}$$

$$0 = \underline{\underline{y^4}}\left(4y - \frac{10}{9}\right)$$

✓ $\boxed{0 = x\ ,\ y = 0}$

✓ $\begin{cases} y = \dfrac{5}{18} \\ x = \dfrac{5}{9}\left(\dfrac{5}{18}\right)^4 \end{cases}$

Q. The slope of the tangent to the curve $(y - x^5)^2 = x(1 + x^2)^2$ at the point $(1, 3)$ is:

$\frac{dy}{dx} = ?$

....

[JEE (Advanced) – 2014 (Paper - 1)]

D WRT 'x'

$$2(y - x^5)\left(\frac{dy}{dx} - 5x^4\right) = x(2)(1+x^2)(2x) + (1+x^2)^2 \cdot 1$$

$$= (1+x^2)(5x^2+1)$$

$$\boxed{\frac{dy}{dx} = \frac{(1+x^2)(5x^2+1)}{2(y-x^5)} + 5x^4}$$

Ans

$x = 1$

$y = 3$

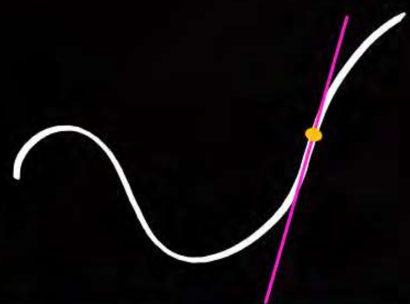
$\frac{dy}{dx} = \frac{12}{4} + 5$

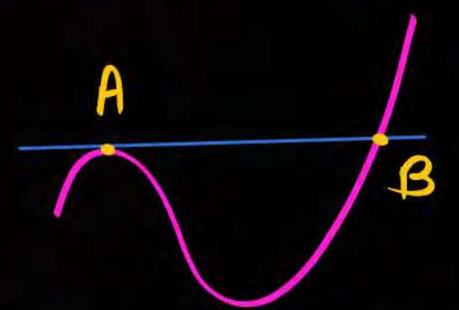
$= \boxed{8}$

Ans

GYAAN KA BHANDAAR

1. *Tangent can **cross the curve** at the **same point where it is Tangent**.*
2. *Tangent at **one point can intersect** the **curve again** at **another point**.*





3. **If a polynomial Curve in x & y passes through the (0,0) ,the equation of tangent at (0,0) can be directly written by equating the lowest degree term appearing in the curve's equation equal to zero.**

BKT

Q. *Write the tangent's equation passing through origin for :*

$$x^2 + y^2 - 3y + 3x = 0$$

Method 1

D WRT 'x'

$$2x + 2y\frac{dy}{dx} - 3\frac{dy}{dx} + 3 = 0$$

(0,0)

$$\boxed{\left(\frac{dy}{dx}\right) = \frac{-3-2x}{2y-3}}$$

$m_T = 1$

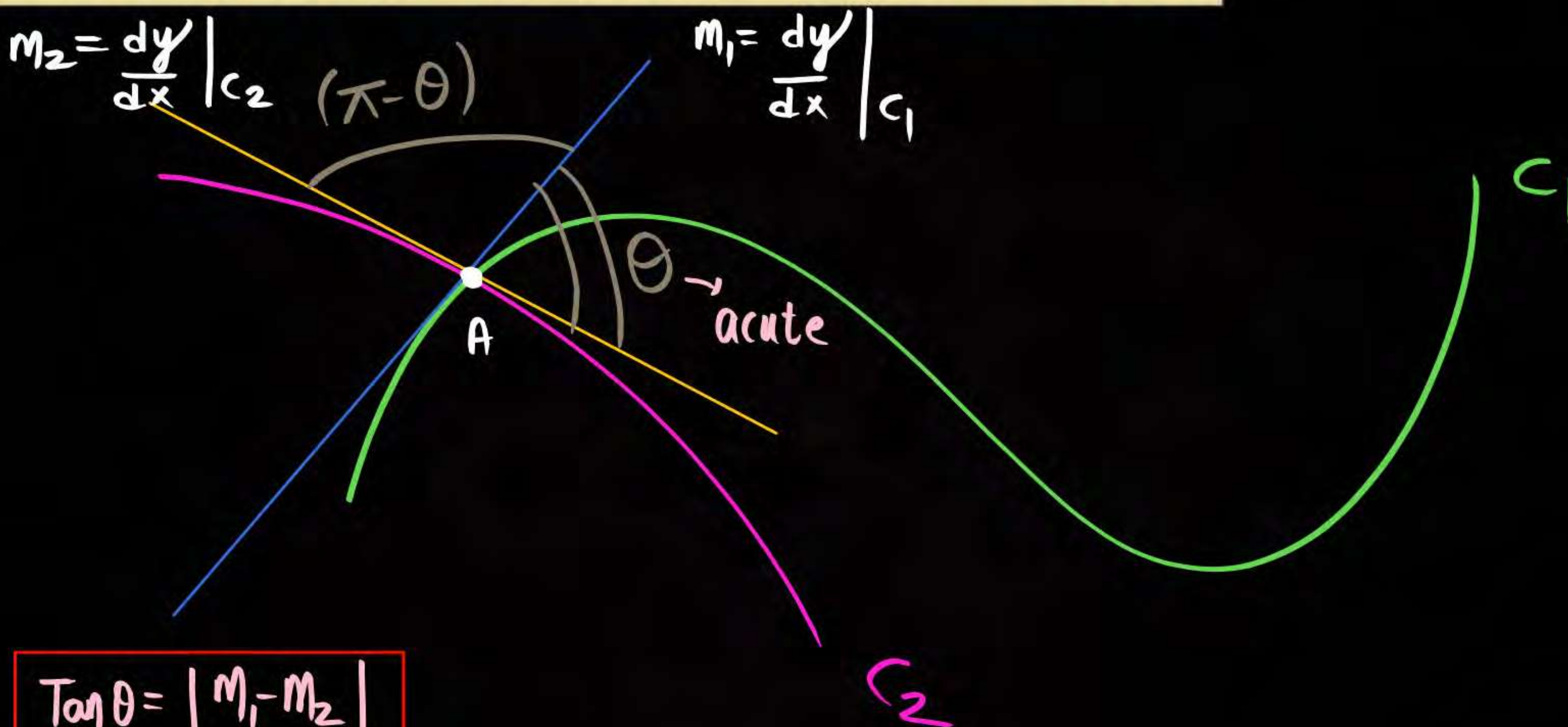
$$T: \frac{y-0}{x-0} = 1 \rightarrow \boxed{y = x} \text{ Ans}$$

Method 2 (Baba Jindagi)

$$-3y + 3x = 0$$

$$\boxed{y = x} \text{ Ans}$$

ANGLE OF INTERSECTION B/W TWO CURVES



$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

acute

"AOI" between 2 curves is same as the angle between the tangents at their POI.

Two curves can intersect at more than one point, therefore more than one AOI possible..

GYAAN

90°

"if and only if"

(ORTHOGONAL CURVES)

Two curves are orthogonal IFF:

$\left.\frac{dy}{dx}\right|_{c_1} \times \left.\frac{dy}{dx}\right|_{c_2} = (-1)$ at every point of intersection.

Q. Which of the following curves are orthogonal ?

Jab bhi Kabhi orthogonal check Karna ho, we never FIND POI.

always assume it.

(ORTHOGONAL)

(A) $y^2 = 4ax,\ y = e^{-x/2a}$

Let their POI be (α,β)

$\beta^2 = 4a\alpha$

$\beta = e^{-\alpha/2a}$

NOT ORTHO

(B) $y^2 = 4ax,\ x^2 = 4by.$

$2y\cdot\frac{dy}{dx} = 4a$

$\boxed{\frac{dy}{dx} = \frac{2a}{y}} \rightarrow \boxed{m_1 = \frac{2a}{\beta}}$ ✓

$\frac{dy}{dx} = \frac{e^{-x/2a}}{-2a} \rightarrow \boxed{m_2 = \frac{e^{-\alpha/2a}}{-2a} = \frac{-\beta}{2a}}$ ✓

$\boxed{m_1 m_2 = -1}$

(B) $\underline{y^2 = 4ax}$, $\underline{x^2 = 4by}$ → assume $(\alpha, \beta) \to$ POI

(α, β): $\boxed{\beta^2 = 4a\alpha}$

(α, β): $\boxed{\alpha^2 = 4b\beta}$

$\beta = \dfrac{\alpha^2}{4b}$

$\dfrac{\alpha^2}{\beta^2} = \dfrac{4b\beta}{4a\alpha}$

$\left(\dfrac{\alpha}{\beta}\right)^3 = \dfrac{b}{a}$

$\dfrac{a}{b} = \left(\dfrac{\beta}{\alpha}\right)^3$

$2y\dfrac{dy}{dx} = 4a \Rightarrow \boxed{\dfrac{dy}{dx} = \dfrac{2a}{y}}$

$\xrightarrow{(\alpha, \beta)}$ $\boxed{M_1 = \dfrac{2a}{\beta}}$ ✓

$2x = 4b\dfrac{dy}{dx}$

$\boxed{\dfrac{dy}{dx} = \dfrac{x}{2b}} \xrightarrow{(\alpha, \beta)} \boxed{M_2 = \dfrac{\alpha}{2b}}$ ✓

$M_1M_2 = \dfrac{a}{b} \cdot \left(\dfrac{\alpha}{\beta}\right) = \left(\dfrac{\beta}{\alpha}\right)^3 \left(\dfrac{\alpha}{\beta}\right) = \left(\dfrac{\beta}{\alpha}\right)^2 > 0$

$\neq -1$

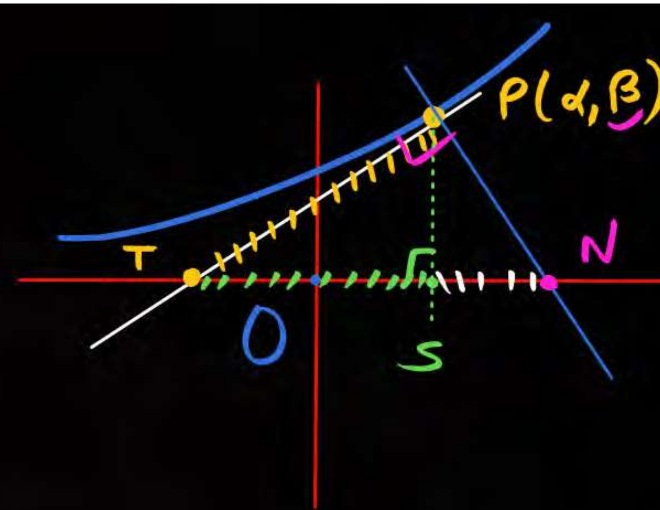
LENGTH OF TANGENT

$$PT = \left| \beta \cdot \sqrt{1 + \frac{1}{m^2}} \right|$$

$$m = \left.\frac{dy}{dx}\right|_{(\alpha, \beta)}$$

LENGTH OF NORMAL

$$PN = \left| \beta \cdot \sqrt{1 + m^2} \right|$$



TS → sub-Tangent

SN → sub-Normal

Q. *Find length of tangent for* $y = x^3 + 3x^2 + 4x - 1$ *at* $x = 0$.

$y = -1$

$\boxed{\alpha = 0, \beta = -1}$

$$\frac{dy}{dx} = 3x^2 + 6x + 4 \xrightarrow{x=0} m = 4$$

$$L_T = \left| (-1)\sqrt{1 + \frac{1}{16}} \right|$$

$$= \frac{\sqrt{17}}{4} \text{ units } \boxed{\text{Ans}}$$

LENGTH OF SUB-TANGENT

$$\left|\frac{\beta}{m}\right| = L_{ST}$$

LENGTH OF SUB-NORMAL

$$|\beta \cdot m| = L_{SN}$$

Q. If L_T, L_N, L_{ST} and L_{SN} denote the lengths of tangent, normal, sub-tangent and sub-normal respectively, of a curve $y = f(x)$ at a point $P(2009, 2010)$ on it, then:

$2009 = \alpha$, $2010 = \beta$

(A) $\dfrac{L_{ST}}{2010} = \dfrac{2010}{L_{SN}}$ $\quad L_{ST} \cdot L_{SN} = (2010)^2$

(B) $\left|\dfrac{L_T}{L_N}\sqrt{\dfrac{L_{SN}}{L_{ST}}}\right| = constant = 1$

(C) $1 - L_{ST}L_{SN} = \dfrac{2000}{2010}$ $\Rightarrow LHS = 1-(2010)^2 \neq \dfrac{2000}{2010}$

(D) $\left(\dfrac{L_T+L_N}{L_T-L_N}\right) = \dfrac{L_{ST}}{L_{SN}}$

$$\frac{\frac{L_T}{L_N}+1}{\frac{L_T}{L_N}-1} = \frac{\frac{1}{|m|}+1}{\frac{1}{|m|}-1} = \frac{1+|m|}{1-|m|} \neq \frac{1}{|m|^2}$$

$$\frac{L_{SN}}{L_{ST}} = |m|^2$$

$$\sqrt{\frac{L_{SN}}{L_{ST}}} = |m|$$

$$L_T = \left|2010\sqrt{1+\frac{1}{m^2}}\right|$$

$$L_N = \left|2010\sqrt{1+m^2}\right|$$

$$L_{ST} = \left|\frac{2010}{m}\right|$$

$$L_{SN} = |2010 \cdot m|$$

$$\left(\frac{L_T}{L_N}\right) = \frac{2010 \cdot \frac{\sqrt{m^2+1}}{|m|}}{2010\sqrt{m^2+1}} = \left(\frac{1}{|m|}\right)$$

$\boxed{\frac{dx}{dt} \longrightarrow \text{Rate of change of 'x' wrt 't'}}$

RATE MEASURE

$\boxed{\frac{dx}{dt} > 0} \longrightarrow$ 'X' increase

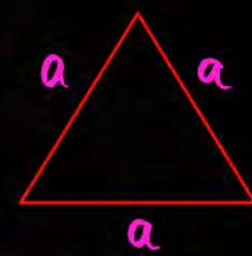
$\boxed{\frac{dx}{dt} < 0} \longrightarrow$ 'X' Decrease

Q. *If the sides of an **equilateral Δ** increase at a **rate of $\sqrt{3}$ cm/sec** and the **area increases at the rate of 12 cm²/sec**. Then the **sides** of the Δ are ?*

$\frac{da}{dt} = \sqrt{3}$

$\boxed{A = \frac{\sqrt{3}}{4} a^2} \rightarrow \boxed{\underbrace{\frac{dA}{dt}}_{12} = \frac{\sqrt{3}}{2} a \cdot \underbrace{\frac{da}{dt}}_{\sqrt{3}}} \rightarrow \frac{12 * 2}{3} = a$

$\boxed{a = 8\text{ cm}}$ Ans



Q. *If the area of the circle increases at a uniform rate. Prove that the rate of increase of the **perimeter varies inversely as its radius.***

$A = \pi R^2$

$\frac{dA}{dt} = \boxed{2\pi R \cdot \frac{dR}{dt} = K} \rightarrow \frac{dR}{dt} = \frac{K}{2\pi R}$

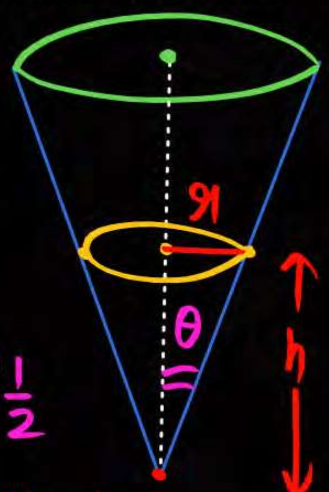
$P = 2\pi R \rightarrow \frac{dP}{dt} = 2\pi \cdot \frac{dR}{dt} = 2\pi \cdot \frac{K}{2\pi R} = \frac{K}{R}$

OTTE SIR

Q. *A water tank has the shape of a inverted right circular cone, whose semi-vertical angle is $\tan^{-1}\frac{1}{2}$. Water is poured into it at a constant rate of 5 cubic meter per minute. Then the rate (in m/min.), at which the level of water is rising at the instant when the depth of water in the tank is 10 m, is :*

....

[JEE Main-2019/2022 (TWO SHIFTS)]



(A) $1/15\,\pi$

(B) $1/10\,\pi$

(C) $2/\pi$

(D) $1/5\,\pi$

$\theta = \tan^{-1}\frac{1}{2}$

$\tan\theta = \frac{r}{h} = \frac{1}{2}$

$\boxed{r = \frac{h}{2}}$

$V = \frac{1}{3}\pi r^2 h$

$\frac{dV}{dt} = 5$

$V = \frac{\pi}{3}\cdot\frac{h^2}{4}\cdot h = \frac{\pi}{12}h^3$

$\frac{dV}{dt} = \frac{\pi}{4}h^2\cdot\frac{dh}{dt} = 5$

$h = 10$

$\frac{\pi}{4}(10)^2\frac{dh}{dt} = 5$

$\boxed{\frac{dh}{dt} = \frac{1}{5\pi}\ m/min}$ ✓

Q. The surface area of a balloon of spherical shape being inflated, increases at a constant rate. If initially, the radius of balloon is 3 units and after 5 seconds, it becomes 7 units, then its radius after 9 seconds is:

$t = 9$

[JEE Main-2022 (24 June - Shift 1)]

(A) 9

(B) 10

(C) 11

(D) 12

$\boxed{S = 4\pi R^2}$

$K = \boxed{\frac{dS}{dt} = 8\pi R \cdot \frac{dR}{dt}}$

$\int K \cdot dt = \int 8\pi R \cdot dR$

$\boxed{K \cdot t = 4\pi R^2 + C}$

$t=0$, $R=3$: $0 = 36\pi + C \Rightarrow C = -36\pi$

$\boxed{Kt = 4\pi R^2 - 36\pi}$

$t=5$, $R=7$: $5K = 196\pi - 36\pi \Rightarrow \boxed{K = 32\pi}$

$(32\pi)t = 4\pi R^2 - 36\pi$

$t = 9$

$\boxed{R = 9}$ Ans

GYAAN KI BAAT

1. Polynomial functions, exponential functions, logarithmic functions are always Continuous & Derivable.

"Jo naa Tute"

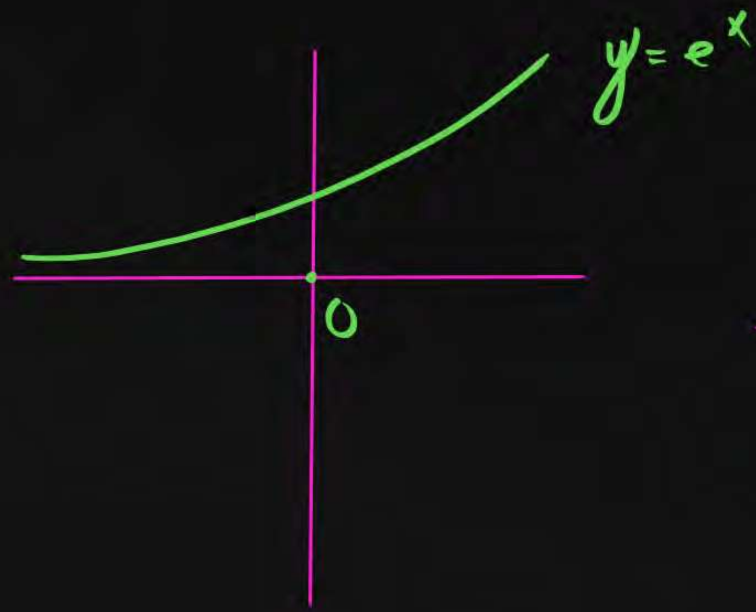
No sharp edge / Not DC / smooth curve.

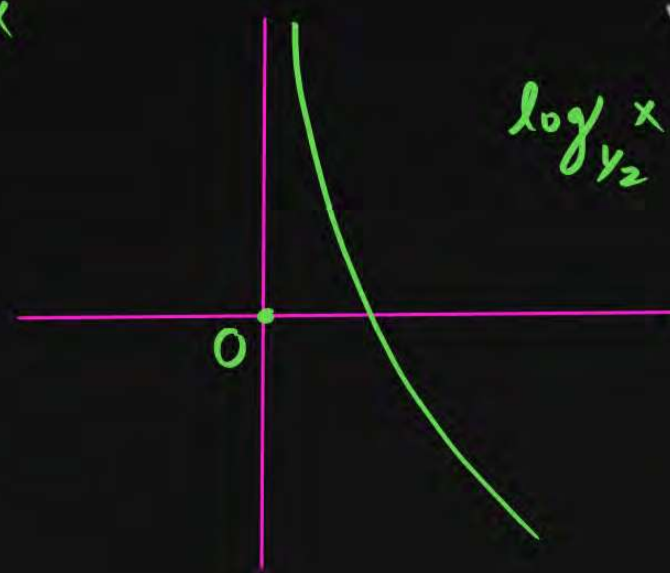
Only
Increasing
(Monotonically
Inc.)

MONOTONICITY

Only Dec.
(Monotonically
Dec.)

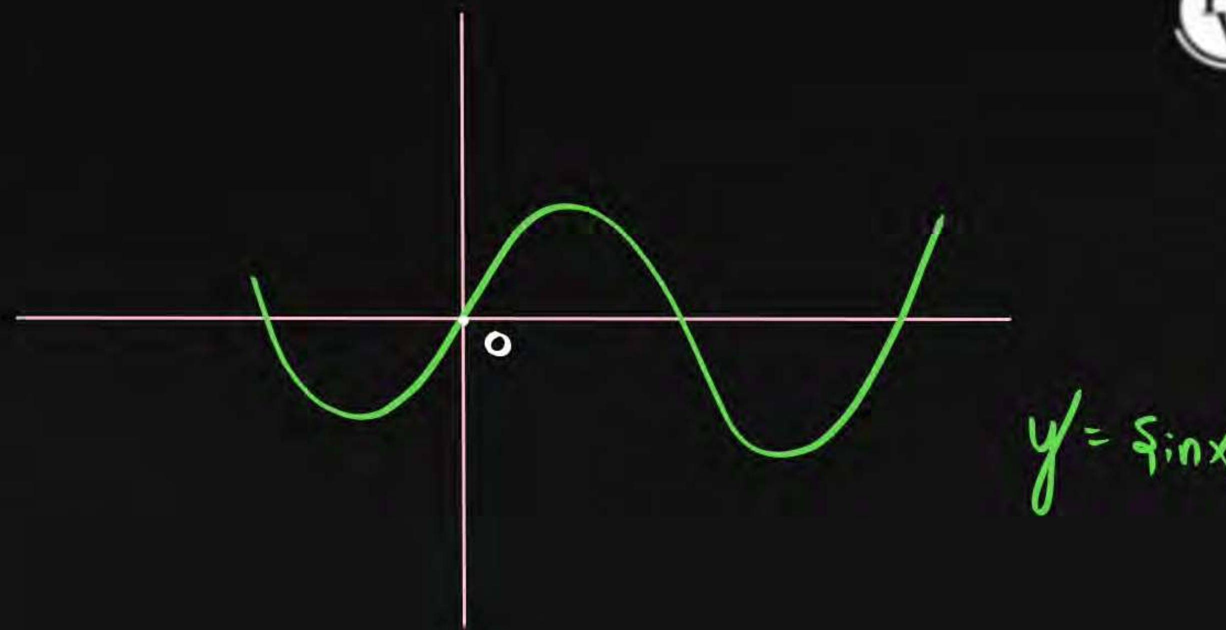
monotonic graphs





$\boxed{\text{Non-monotonic graphs}}$

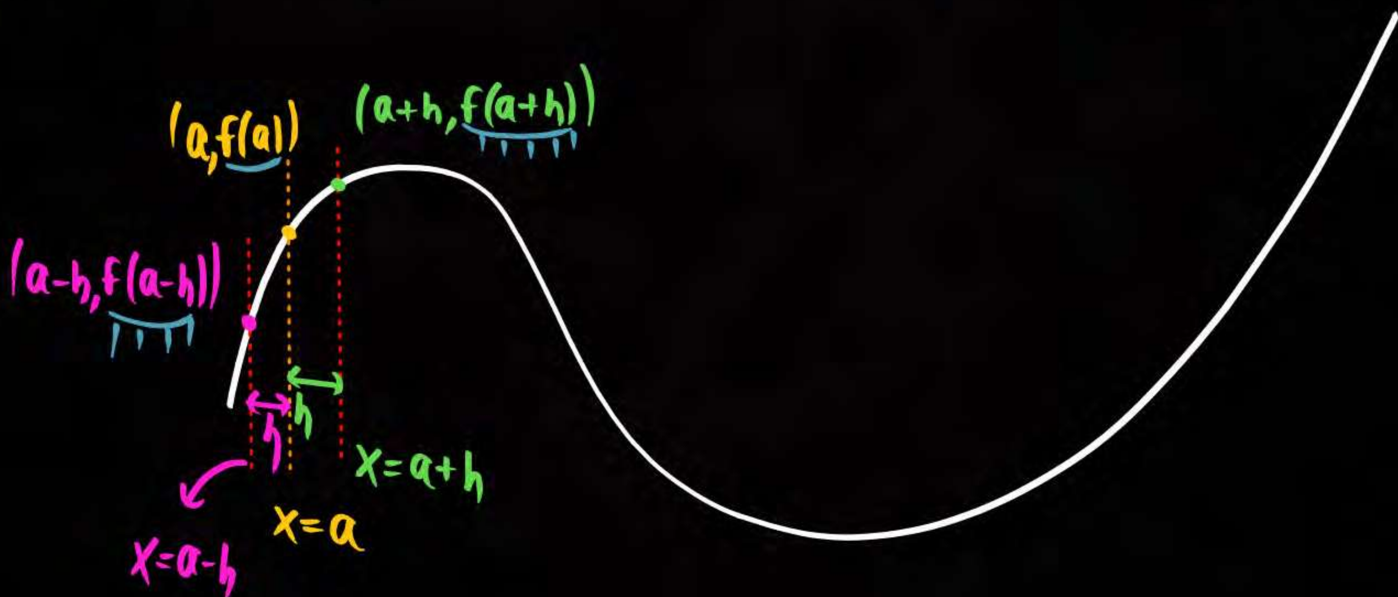
(Increasing as well as Decreasing)



MONOTONICITY OF A FUNCTION AT A POINT

A function $f(x)$ is said to be **monotonically increasing** at $x = a$, if $f(x)$ satisfies:

Here $\left.\begin{array}{l} f(a+h) > f(a) \\ f(a-h) < f(a) \end{array}\right\}$ ($\boxed{\textbf{Condition for Increasing}}$) , where $h \to 0^+$



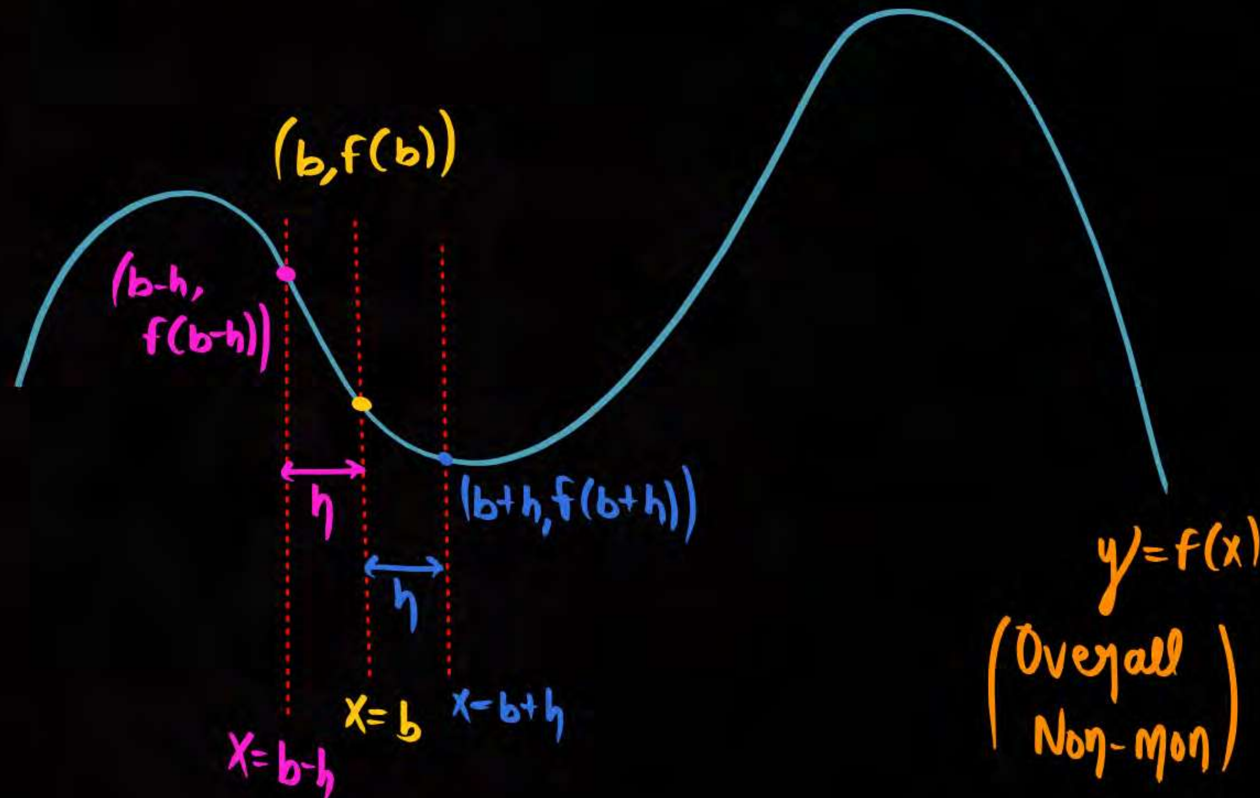
y = f(x)
(Overall Non-mon.)

It is said to be **monotonically decreasing** at $x = b$ if

$$f(b+h) < f(b)$$

and $f(b) < f(b-h)$

where $h \to 0^+$

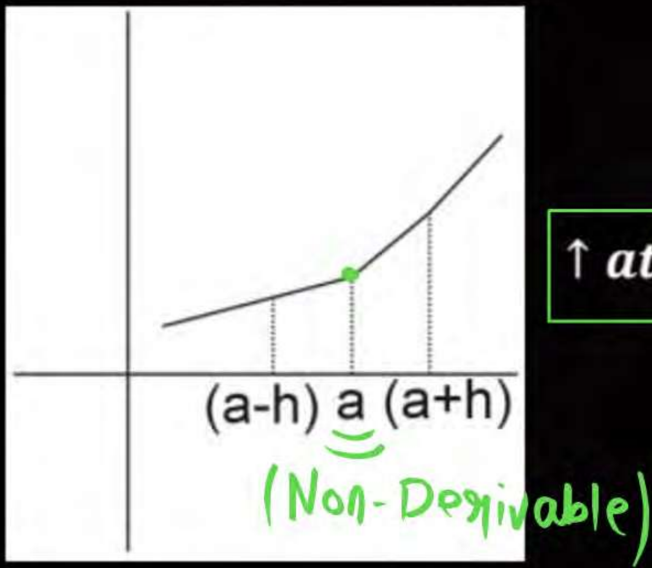


NOTE :

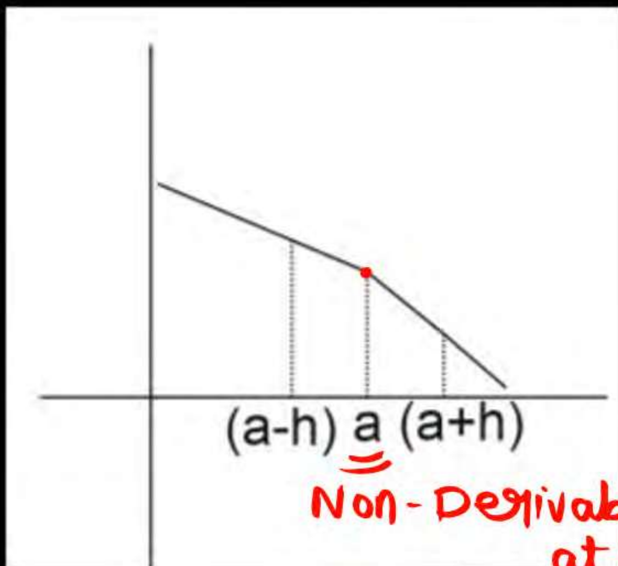
It must be noted that we can talk of monotonicity of $f(x)$ at $x = a$ only & only if $x = a$ lies in the domain of $f(x)$.

(without consideration of continuity or derivability of $f(x)$ at $x = a$.)

Example:

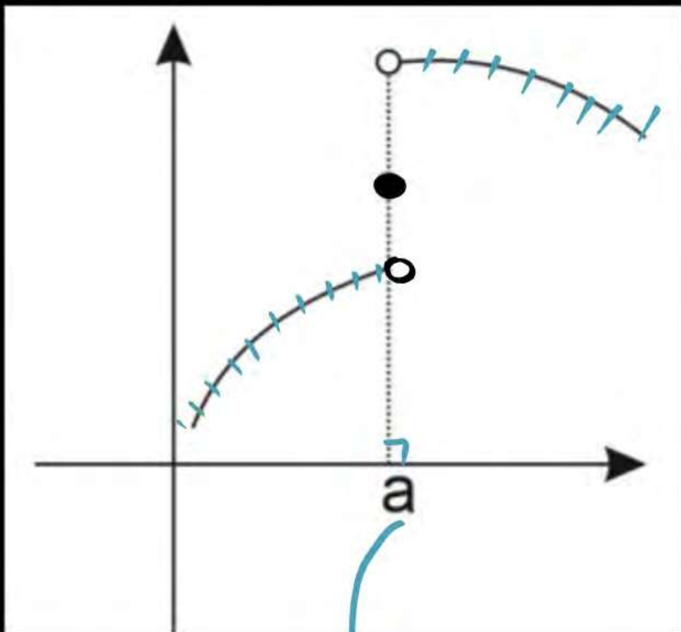


$\uparrow$ at $x = a$



Non - Derivable
at x = 0 . .

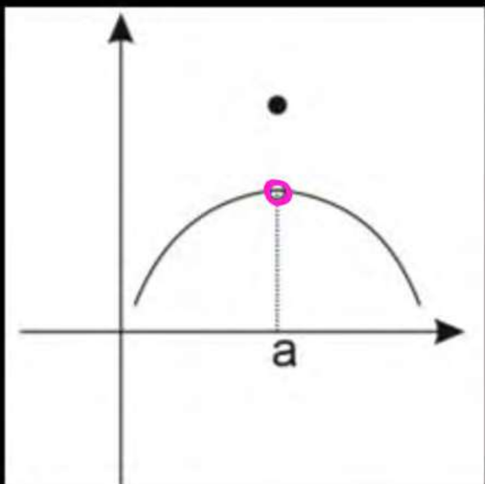
$$\boxed{\downarrow at\ x = a}$$



$at\ x = a \uparrow$

Discontinous at $x=a$

Non-Derivable at $x=a$



(Non-monotonic)

MONOTONICITY IN AN INTERVAL

1. If a function is $\boxed{\text{monotonically increasing}}$ at $\boxed{\text{each and every point in an interval}}$, then it is called monotonically ↑ in the interval.

Hence, $\underline{\underline{f'(x) > 0}}$ → (monotonically Increasing) → $y = f(x)$

2. For a monotonically decreasing function in some interval, $\underline{\underline{f'(x) < 0}}$

$y = f(x)$

***NOTE* (DHOMKA) :**

$\frac{dy}{dx}$ at some point may be equal to '0' but f(x) may still be increasing or decreasing at this point.

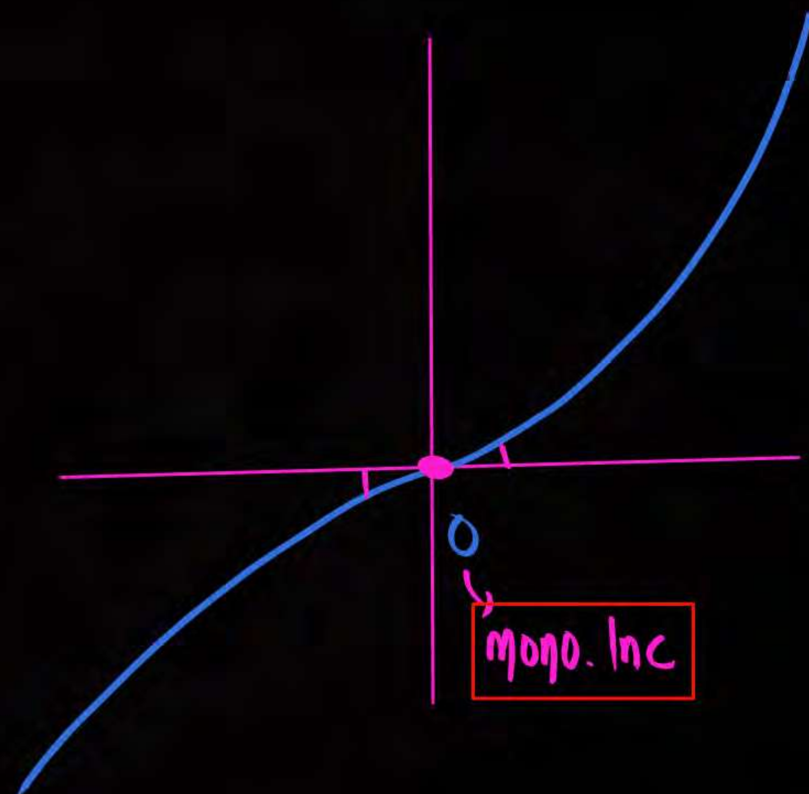
Consider

$f(x) = x^3$ which is increasing at $x = 0$, although

$\{ f'(x) = 0 \}$

As $\quad f(0 + h) > f(0)$

And $\quad f(0) > f(0 - h)$



$f(x) = x^3$

$\frac{dy}{dx} = f'(x) = 3x^2$

$x = 0$

$\frac{dy}{dx} = 0$

Monotonically Increasing $\xrightarrow{\text{Cond}^n}$ $f'(x) \geq 0$

Monotonically Decreasing $\xrightarrow{\text{Cond}^n}$ $f'(x) \leq 0$

BKG

Q. Find the values of 'a' for which $f(x) = ax - \sin x$ is monotonically increasing.

$\boxed{f'(x) \geq 0}$

$a - \cos x \geq 0 \rightarrow \boxed{a \geq \cos x}$

$a \geq (\cos x)_{max} \rightarrow \boxed{a \geq 1}$

$a \in [1, \infty)$ Ans

Q. If $f(x) = (a+2)x^3 - 3ax^2 + 9ax - 1$ is always decreasing $\forall\, x \in R$. Find $'a'$.

$f'(x) = \boxed{\underbrace{3(a+2)}_{A} x^2 - \underbrace{6a}_{=B} x + \underbrace{9a}_{C} \leq 0}$

$A < 0 \rightarrow 3(a+2) < 0 \rightarrow \boxed{a < -2}$

$D \leq 0$

$36a^2 - 4(9a)(3)(a+2) \leq 0$

$\rightarrow \boxed{a \in (-\infty, -3] \cup [0, \infty)}$

Ans $\rightarrow a \in (-\infty, -3]$

Q. Let λ^*, be the largest value of λ for which the function $f_\lambda(x) = 4\lambda x^3 - 36\lambda x^2 + 36x + 48$ is increasing for all $x \in R$. Then $f_{\lambda^*}(1) + f_{\lambda^*}(-1)$ is equal to:

[JEE Main-2022 (24 June - Shift 2)]

(A) 36

(B) 48

(C) 64

(D) 72

$f'_\lambda(x) = 12\lambda x^2 - 72\lambda x + 36$

$= 12(\lambda x^2 - 6\lambda x + 3) \geq 0$

$\boxed{\underset{A}{\lambda} x^2 + \underset{B}{(-6\lambda)} x + \underset{C}{3} \geq 0}$

$A > 0$, $D \leq 0$ | $\cap$

$\boxed{\lambda > 0}$

$36\lambda^2 - 12\lambda \leq 0$

$12\lambda(3\lambda - 1) \leq 0$

$\boxed{\lambda \in \left[0, \frac{1}{3}\right]}$

AFTER $\cap$:

$\boxed{\lambda \in \left(0, \frac{1}{3}\right]}$

$\lambda_{\text{largest}} = \boxed{\frac{1}{3} = \lambda^*}$

$f_{\lambda^*}(x) = \frac{4}{3}x^3 - 12x^2 + 36 + 48$

$x = 1$: $f_{\lambda^*}(1) = \frac{4}{3} - 12 + 36 + 48$

$\boxed{f_{\lambda^*}(-1) = -\frac{4}{3} - 12 - 36 + 48}$

Q. The maximum value of a, for which the function $f_a(x) = \tan^{-1} 2x - 3ax + 7$ is non-decreasing in $\left(-\frac{\pi}{6}, \frac{\pi}{6}\right)$, is $\bar{a}$, then $f_{\bar{a}}\left(\frac{\pi}{8}\right)$ is equal to

[JEE Main-2022 (26 July - Shift 2)]

(A) $8 - \frac{9\pi}{4(9+\pi^2)}$

(B) $8 - \frac{4\pi}{9(4+\pi^2)}$

(C) $8\left(\frac{1+\pi^2}{9+\pi^2}\right)$

(D) None of these

$\underbrace{\frac{\pi}{8}}_{=x}$

$$\boxed{f_a'(x) \geq 0}$$

$$\frac{2}{1+4x^2} - 3a \geq 0 \rightarrow \frac{2}{1+4x^2} \geq 3a \rightarrow \boxed{\frac{2}{3(1+4x^2)} \geq a}$$

$$\boxed{a \leq \frac{2}{3(1+4x^2)}}$$

$$a \leq \left(\frac{2}{3(1+4x^2)}\right)_{min}$$

$$\boxed{a \leq \frac{6}{\pi^2+9}}$$

$$\boxed{a_{max} = \bar{a} = \frac{6}{\pi^2+9}}$$

Ans = $\tan^{-1}\frac{\pi}{4} - 3\bar{a}x + 7$

$$\boxed{x \in \left(-\frac{\pi}{6}, \frac{\pi}{6}\right)}$$

$$x^2 \in \left(0, \frac{\pi^2}{36}\right)$$

$*4$

$$0 < 4x^2 < \frac{\pi^2}{9}$$

$+1$

$$1 < 4x^2+1 < \frac{\pi^2}{9}+1$$

$*3$

$$\boxed{3 < 3(4x^2+1) < \frac{\pi^2+9}{3}}$$

$\div 2$

$$\frac{3}{2} < \frac{3}{2}(4x^2+1) < \frac{\pi^2+9}{6}$$

$$R = \frac{2}{3} > \frac{2}{3(4x^2+1)} > \frac{6}{\pi^2+9}$$

$$R \in \left(\frac{6}{\pi^2+9}, \frac{2}{3}\right)$$

$x \in (-2, 4)$ (opp. signs)

$x^2 \in (0, 16)$

$x \in (-4, 2)$

$x^2 \in (0, 16)$

$x \in (2, 4)$ (same sign)

$x^2 \in (4, 16)$

$x \in (-4, -2)$

$x^2 \in (4, 16)$

$\boxed{\text{Baba ♡ ♡}}$

BKG

SIGN OF $f'(x)$

$\underset{a}{\overline{++++0++++}}$ (*INCREASING*) $\boxed{f(x)}$

$\underset{a}{\overline{----0----}}$ (*DECREASING*) $\boxed{f(x)}$

$\underset{a}{\overline{++++0----}}$ $\boxed{(\textit{NON-MONOTONIC})}$ maxima

f(x)

$\underset{a}{\overline{---\ 0+++}}$ $\boxed{(\textit{NON-MONOTONIC}).}$ minima

Q. *The function* $\boxed{f(x) = x \cdot e^{x(1-x)}}$, $x \in R$, *is*

....

[JEE Main-2022 (28 July - Shift 2)]

f(x)

(A) *Increasing in* $\left(-\frac{1}{2}, 1\right)$

(B) *decreasing in* $\left(\frac{1}{2}, 1\right)$

(C) *Increasing in* $\left(-1 - \frac{1}{2}\right)$

(D) *decreasing in* $\left(-\frac{1}{2}, \frac{1}{2}\right)$

$$f'(x) = x \cdot e^{x(1-x)}(1-2x) + e^{x(1-x)} \cdot 1$$

$$= \left(e^{x(1-x)}\right)\left(x - 2x^2 + 1\right)$$

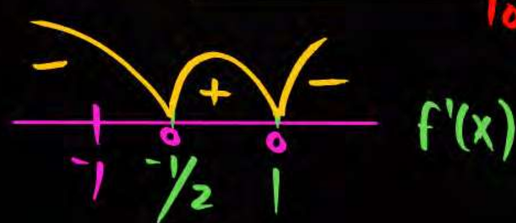
$$= -\left(e^{x(1-x)}\right)\left(2x^2 - x - 1\right)$$

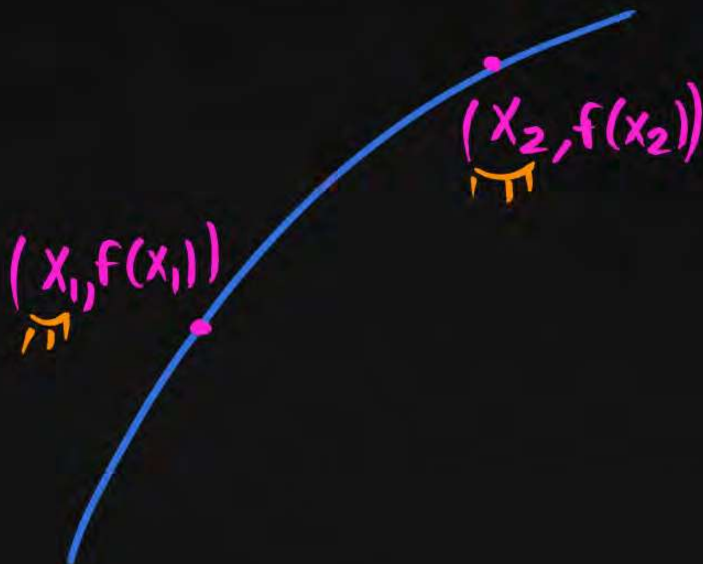
$$\boxed{f'(x) = -\left(e^{x(1-x)}\right)(2x+1)(x-1)}$$

POSITIVE

$f'(1) = 0$

$f'\left(-\frac{1}{2}\right) = 0$



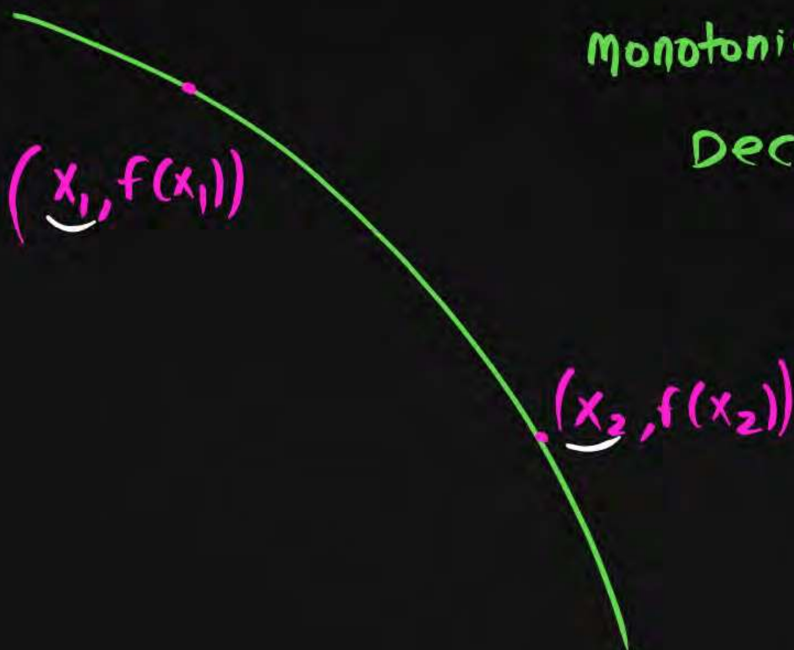


monotonically Increasing

$y = f(x)$

$\boxed{x_1 < x_2} \longrightarrow \boxed{f(x_1) < f(x_2)}$

Monotonically Dec. graph



$x_1 < x_2$

$f(x_1) > f(x_2)$

BKG

1. *Jab bhi **increasing function** lagaega **ya** hataega , sign **nahi** paltaega .*
2. *Jab bhi **decreasing function** lagaega **ya** hataega , sign jarur paltaega .*

Q. Let $f : R \to R$ and $g : R \to R$ be two functions defined by $f(x) = \ln(x^2 + 1) - e^{-x} + 1$ and $g(x) = \frac{1 - 2e^{2x}}{e^x}$. Then, for which of the following range of α the inequality $f\left(g\left(\frac{(\alpha-1)^2}{3}\right)\right) > f\left(g\left(\alpha - \frac{5}{3}\right)\right)$ holds?

[JEE Main-2022 (25 June - Shift 1)]

(A) $(2, 3)$

(B) $(-2, -1)$

(C) $(1, 2)$

(D) $(-1, 1)$

$\boxed{f'(x) > 0}$

$f(x)$ Inc.

$\boxed{g'(x) < 0}$

$g(x)$ Dec.

$$g\left(\frac{(\alpha-1)^2}{3}\right) > g\left(\alpha - \frac{5}{3}\right)$$

$$\boxed{\frac{(\alpha-1)^2}{3} < \alpha - \frac{5}{3}}$$

$\alpha \in (2, 3)$ Ans

ESTABLISHING INEQUALITIES

Format → $\boxed{f(x) \geq g(x) \quad \forall x \in (a,b)}$

To Prove :

WORKING RULE : $\boxed{f(x) - g(x)} \geq 0$

To Prove ← $H(x) \geq 0 \quad \forall x \in (a,b)$

always Do

using monotonicity.

Q. *Prove that*

2. $\sin x + \tan x \geqslant 3x \ \forall \ x \in [0, \frac{\pi}{2})$

$\boxed{2\sin x + \tan x - 3x \geq 0}$

To Prove:

$\boxed{f(x) \geq 0 \ \forall \ x \in [0, \frac{\pi}{2})}$

$f(x) = 2\sin x + \tan x - 3x$

$\boxed{f'(x) = \overbrace{2\cos x + \sec^2 x - 3}^{\geqslant 3}} \geqslant 0$

$\cos x, \cos x, \sec^2 x$

$AM \geqslant GM$

$$\frac{\cos x + \cos x + \sec^2 x}{3} \geqslant (1)^{1/3}$$

$\boxed{2\cos x + \sec^2 x \geqslant 3}$

$f(x) \rightarrow$ monotonically Increasing

'f' $\quad \boxed{0 \leq (x) < \frac{\pi}{2}^-}$

$f(0) \leq f(x) < f\left(\frac{\pi}{2}^-\right)$

$0 \leq f(x) < 2(1) + \infty - \frac{3\pi}{2}$

$\boxed{0 \leq f(x) < \infty}$

Hence Proved.

$$\tan\left(\frac{\pi}{2}^{-}\right) = +\infty$$

$$\tan\left(\frac{\pi}{2}^{+}\right) = -\infty$$

$$\tan\left(\frac{\pi}{2}\right) = ND$$

Q. Prove that $\boxed{\tan x > x}$ in $x \in (0, \pi/2)$.

$$\boxed{f(x) = \tan x - x}$$

To Prove: $\begin{cases} \tan x - x > 0 \\ f(x) > 0 \end{cases}$

$$f'(x) = \sec^2 x - 1$$
$$= \tan^2 x > 0$$

$$\boxed{f(x) \to \text{Inc.}}$$

$$0^+ < x < \frac{\pi}{2}^-$$

$$\boxed{f(0^+) < f(x) < f\left(\frac{\pi}{2}^-\right)}$$

$$0 < f(x) < \infty - \frac{\pi}{2}$$

$$\boxed{f(x) > 0}$$

Hence Proved

always simplify the Func^n and make RHS = 0

ISOLATING ROOT USING MONOTONICITY

Finding Root

Q.

Prove that the equation

$$\frac{x^3+1}{x^2+1} = 5 \text{ has no root in } [0,2]$$

OR,

$x^3+1 = 5x^2+5$

OR,

$x^3+1-5x^2-5=0$

$\boxed{\underbrace{x^3-5x^2-4}_{f(x)}=0}$ has no Root in $[0,2]$

$f(x) = x^3-5x^2-4$ doesnot intersect the x-axis in $[0,2]$

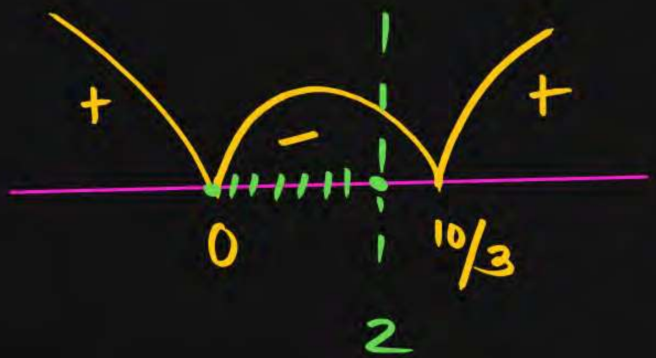
$$\boxed{f(x) = x^3 - 5x^2 - 4}$$

$f(0) = -4$

$f(2) = -16$

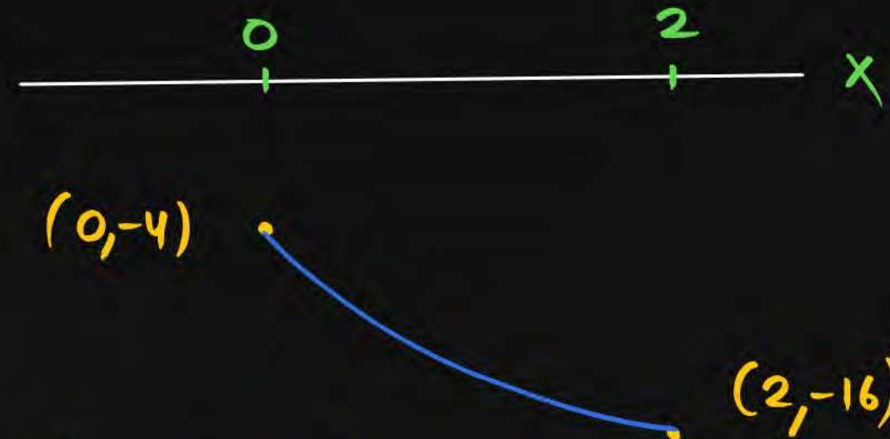
$f'(x) = 3x^2 - 10x$

$= x(3x - 10)$



sign scheme of $f'(x)$

$x \in [0,2]$; $f(x)$ is Decreasing



No Intersecⁿ pt with the x-axis in $[0,2]$

$\boxed{\text{No Root in } [0,2]}$ PROVED

Q. The Number of distinct real roots of the equation $x^7 - 7x - 2 = 0$ is

[JEE Main-2022 (24 June - Shift 2)]

(A) 5

(B) 7

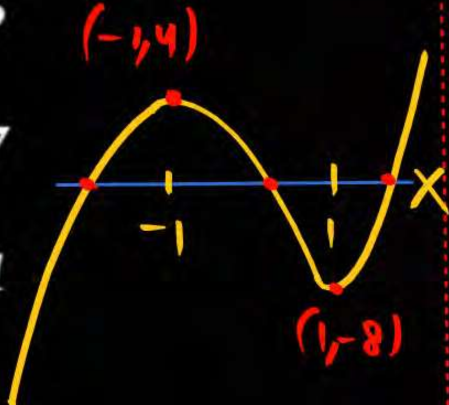
(C) 1

(D) 3 ✓

$F(1) = -8$

$f(\infty) = \infty$, $f(-1) = -1 + 7 - 2 = 4$

$f(-\infty) = -\infty$



$\boxed{f(x) = x^7 - 7x - 2}$

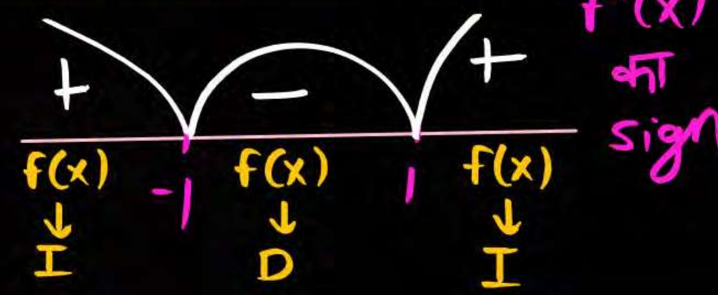
$f'(x) = 7x^6 - 7$

$= 7(x^6 - 1)$

$= 7((x^2)^3 - 1^3)$

$= 7(x^2 - 1)(x^4 + x^2 + 1)$

$= 7\underbrace{(x^4 + x^2 + 1)}_{\text{POSITIVE}}\underbrace{(x-1)(x+1)}_{\text{S.D.F.}}$



Q. The number of real solution of $x^7 + 5x^3 + 3x + 1 = 0$ is equal to ______.

$f(x) = x^7 + 5x^3 + 3x + 1$

" always Increasing "

[JEE Main-2022 (28 June - Shift 1)]

$\boxed{f'(x) = 7x^6 + 15x^2 + 3} \geq 3$

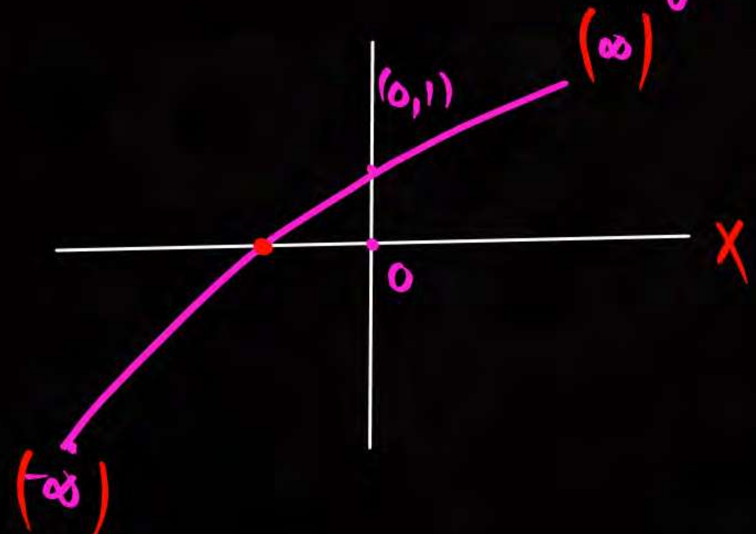
always Positive

(A) 0

(B) 1

(C) 3

(D) 5



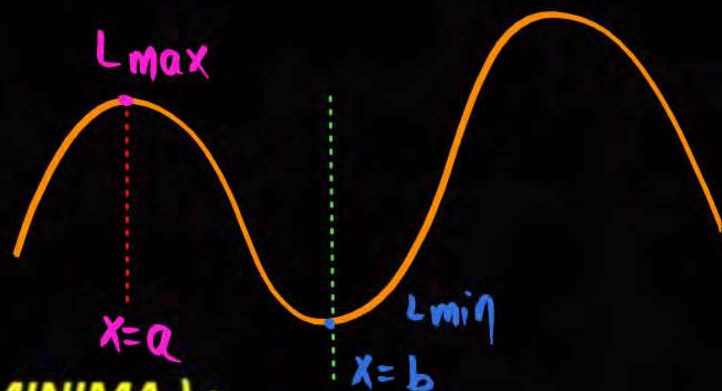
LOCAL MAXIMA AND LOCAL MINIMA

(FUNCTION IN SINGLE VARIABLE)

HOW LOCAL MAXIMA & LOCAL MINIMA ARE CLASSIFIED ?

LOCAL MAXIMA (RELATIVE MAXIMA) :

A Function is said to be a **Local maximum if at x=a** , f(a) is greater than any other value attained by f(x) in its **immediate neighbourhood.**



LOCAL MINIMA (RELATIVE MINIMA) :

A Function is said to be a **Local minimum if at x=b** , f(b) is lesser than any other value attained by f(x) in its **immediate neighbourhood.**

GYAAN BHARI BAATEIN

1. **MAXIMA & MINIMA** ARE ALSO CALLED AS **LOCAL MAXIMA & MINIMA** / **RELATIVE MAXIMA & MINIMA.**

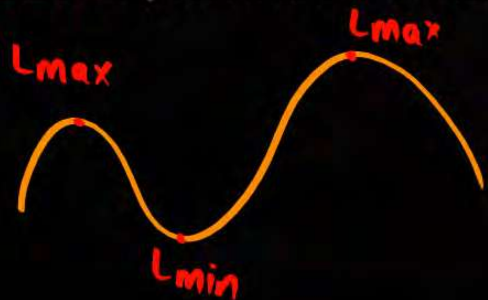
2. **TERM EXTREMUM** / **EXTREMAL** / **TURNING VALUE** IS USED FOR **BOTH** MAXIMA & MINIMA.

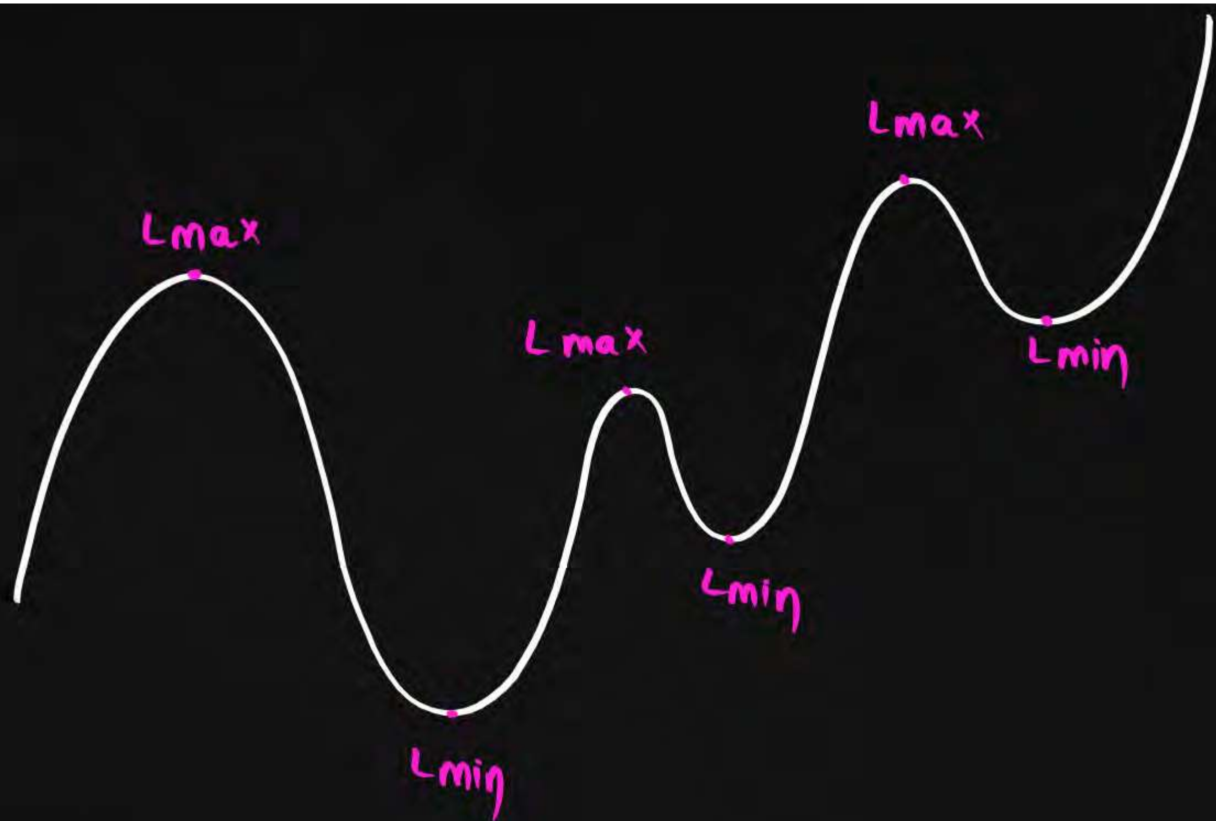
   Lmax, Lmin   Lmax, Lmin   Lmax, Lmin

3. *IF (a, f(a)) is Point of EXTREMA then :*

   x=a is extrema & f(a) is called VALUE OF EXTREMA.

4. *For Continous functions , maxima & minima occur alternatively.*




Lmax
Lmax
Lmax
Lmin
Lmin
Lmin

$$\boxed{L_{max} \begin{array}{c} > \\ = \\ < \end{array} L_{min}}$$

Stationary points

$$\frac{dy}{dx} = 0$$

⭐⭐⭐

CRITICAL POINTS

$$\frac{dy}{dx} = 0, \text{ DNE}$$

⭐⭐⭐

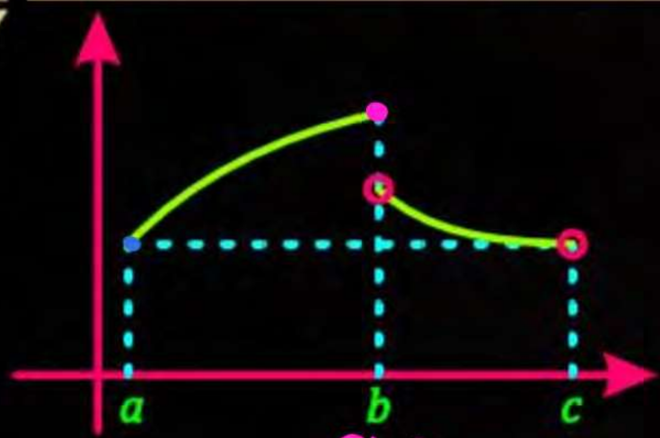
★★★★★

L_{min}, L_{max} के Liye us
point par $func^n$ का Derivable hona
ya continous hona Jaruri नहीं है

Bus Domain mai होना cahiye

SOME IMPORTANT GRAPHS

(1)

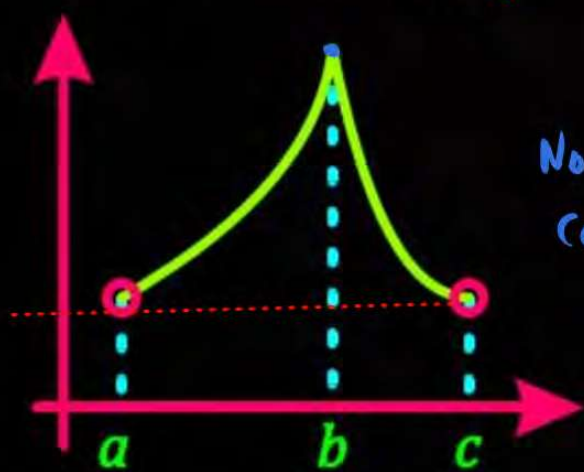


$x = a \rightarrow$ global minimum

$x = b \rightarrow$ global maximum/local maximum

(overall max)

(2)

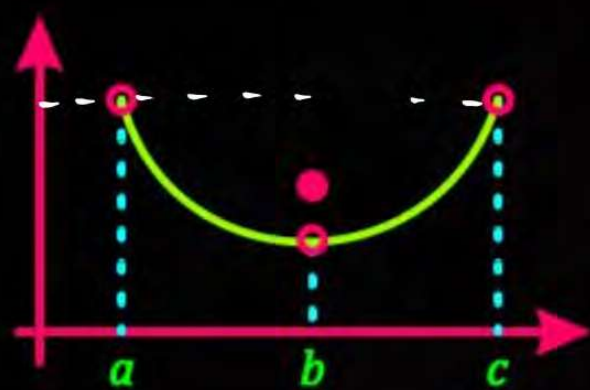


Non-Deri,
Continous

$x = b$ (Local maxima / Global maxima)

No Global minima

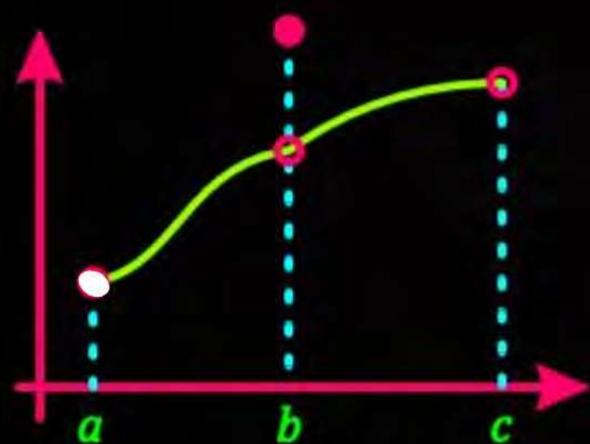
(3)



$x = b \rightarrow$ local maxima

No Global maxima / Global minima

(4)



DC,
Non-D

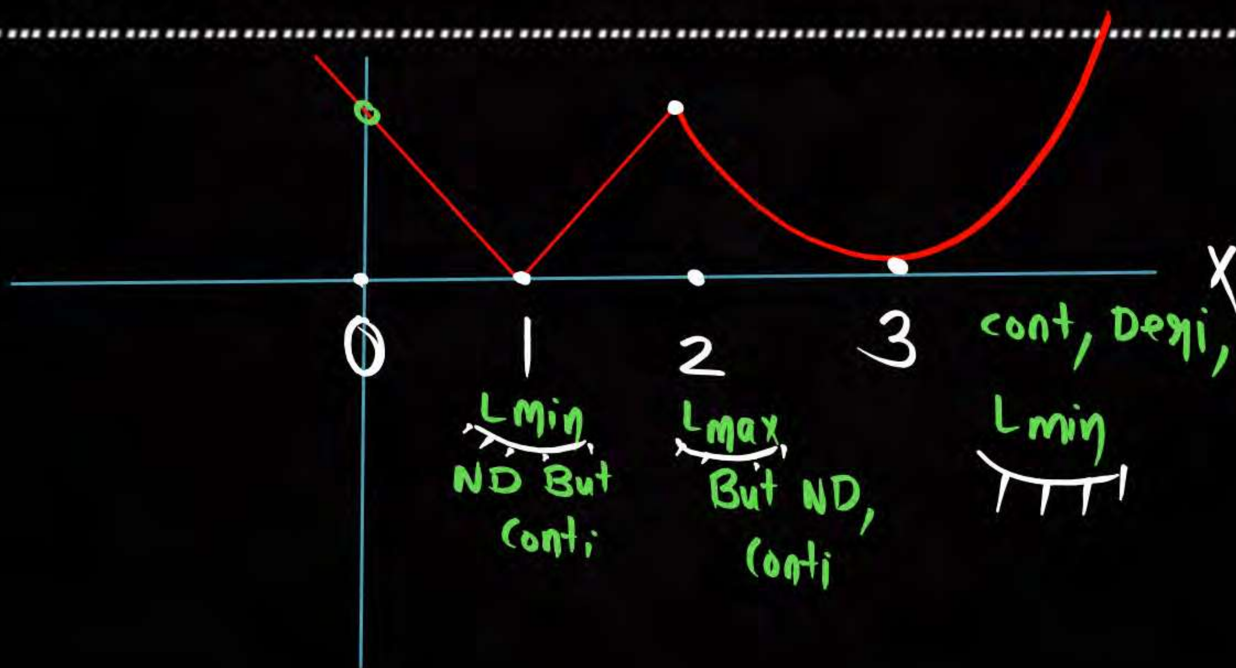
$x = b \rightarrow$ Local maxima /Global maxima

$x = a \rightarrow$ Global minima

DHYAAN SE DEKHO

Q. *Find minima & maxima graphically* :

$f(x) = |x-1|,\ 0 < x \le 2$
$\quad\;\; = (x-3)^2,\ x > 2$



FERMAT'S THEOREM

Necessary and sufficient condition for local maximum and minimum:-

If $f(x)$ has local maximum or minimum at $x = a$ and if $f'(a)$ exists, Then $f'(a) = 0$
(where 'a' is an internal point in the interval)

If funcⁿ is Derivable at the point of
Lmax OR Lmin ; $\frac{dy}{dx} = 0$

SIGN OF $f'(x)$

$$\frac{++++0++++}{a}$$ (INCREASING)

$$\frac{----0----}{a}$$ (DECREASING)

$$\frac{++++0----}{a}$$ (NON-MONOTONIC) Lmax

$$\frac{---0+++}{a}$$ (NON-MONOTONIC). Lmin

Q. Examine the **point of maxima & minima** in the following functions-

(i) $x.\ln x = f(x)$

(ii) $f(x) = x.e^{x-x^2}$.

C.C = Conti

D.D = Derivable

$$\boxed{f'(x) = (1+\ln x)}$$

$-$ 0 $+$

$\left(\frac{1}{e}\right)$

$f'(x)$ का sign

$\boxed{\text{Lmin}}$ Ans

(ii) $f'(x) = x.e^{x-x^2}.(1-2x)$

$\qquad + e^{x-x^2}.$

$= (e^{x-x^2})(x-2x^2+1)$

$= -(e^{x-x^2})(2x^2-x-1)$

$= -\underbrace{(e^{x-x^2})}_{\text{Pos.}}(2x+1)(x-1)$

$-$ 0 $+$ 0 $-$

$-\frac{1}{2}$ 1

$f'(x)$ का sign

Lmin   Lmax $\boxed{\text{Ans}}$

Q. Find the values of 'a' and 'b' for the $f(x) = a\ln x + bx^2 + x$ has Extrema at $x_1 = 1$ and $x_2 = 2$.

Lmax, Lmin

$$\boxed{f'(x) = \frac{a}{x} + 2bx + 1}$$

$x = 1$

$f'(1) = 0$

$$\boxed{a + 2b + 1 = 0}$$

$x = 2$

$f'(2) = 0$

$$\boxed{\frac{a}{2} + 4b + 1 = 0}$$

$a = -\frac{2}{3}$

$b = -\frac{1}{6}$

Ans

1^{st} Order Derivative Test

If $f(x)$ has local maximum or minimum at $x = a$, Then $f'(a) = 0$ / DNE & sign of derivative changes about x=a.

SIGN OF $f'(x)$

DNE

$++++0++++$ (INCREASING)

a

DNE

$----0----$ (DECREASING)

a

DNE

$++++0----$ (NON-MONOTONIC)

a

Lmax

DNE

$---0+++$ (NON-MONOTONIC).

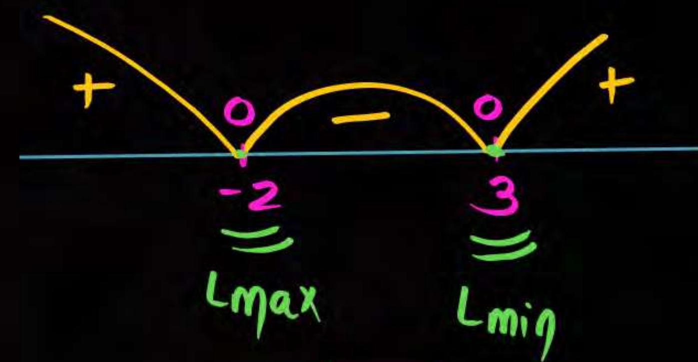
a

Lmin

Q. *Locate local maxima and local minima using first order derivative test for* $\boxed{f(x) = 2x^3 - 3x^2 - 36x + 53.}$ *(NCERT)*

"Derivable"

$$f'(x) = 6x^2 - 6x - 36$$
$$= 6(x^2 - x - 6)$$
$$= 6(x-3)(x+2)$$



sign of $f'(x)$

$\boxed{\text{Ans}}$

BKG

$\boxed{L_{max}, L_{min}}$ → of $f(x)$

✓ ① Find $f'(x)$.

✓ ② Get those 'x' where $f'(x) = 0$, $\boxed{DNE}$

$D^R = 0$,

$\boxed{check}$ (f'(x) की Def^n change)

✓ ③ If sign changes abt these pts,
(of $f'(x)$)

then $f(x)$ has L_{max}, L_{min}

(+ −) (− +)

Q. If $f(x) = \int_0^x e^{t^2}(t-2)(t-3)dt$ for all $x \in (0, \infty)$, then

[IIT-JEE-2012 (ADVANCED)]

(A) f has a local maximum at $x = 2$

(B) f is decreasing on $(2, 3)$

(C) There exists some $c \in (0, \infty)$ such that $f''(c) = 0$

(D) f has a local minimum at $x = 3$

"Neend aa Jaegi"

"TREND" mai है।

CALCULATING NUMBER OF LOCAL MINIMA & MAXIMA

BKG

To calculate number of Local Maxima & Minima, following steps must be performed :

1. Simplify the function (i.e. Remove Modulus e.t.c)
2. Differentiate it .
3. Find those 'x' where $f'(x) = 0$ or ND (i.e. where $f'(x)$ is DC or denominator is zero)
4. Check if any change in sign of $f'(x)$ at those points.

Q. *The total number of local maxima and local minima of the function*

$f(x) = \begin{cases} (2+x)^3, & -3 < x \le -1 \\ x^{2/3}, & -1 < x < 2 \end{cases}$ *is :*

$\boxed{f'(x)=0}$ $\cancel{x=-2}$

$+++ \; +++$ at -2

[IIT-JEE-2008 (ADVANCED)]

(A) 0

(B) 1

(C) 2

(D) 3

$f'(x) = \begin{cases} 3(2+x)^2, & x \in (-3,-1] \\ \dfrac{2}{3x^{1/3}}, & x \in (-1,2) \end{cases}$

$D^R = 0$

$\boxed{x=0}$

$- - \; ND \; + +$ at 0 — $f'(x)$ का sign

Lmin

$f'(-1^-) = 3$, $f'(-1^+) = -\frac{2}{3}$

$+++ \; \boxed{ND} \; - -$ at $\boxed{-1}$ — $f'(x)$ का sign

Lmax

Q. Let $f(x) = |(x-1)(x^2-2x-3)| + x - 3$. If m and M are respectively the number of points of local minimum and local maximum in the interval $(0, 4)$, then $m + M$ is equal to 3.

2 1

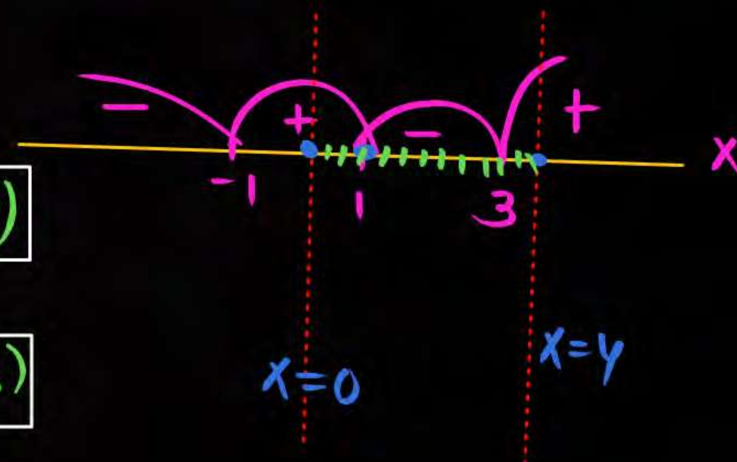
[JEE Main-2022]

$f(x) = |(x-1)(x-3)(x+1)| + (x-3)$

$x \in (0,1]$ → $x^2(x-3)$

$x \in (1,3]$ → $(x-3)(2-x^2)$

$x \in (3,4)$ → $x^2(x-3)$



$$f(x) = \begin{cases} x^3 - 3x^2 & ; x \in (0,1] \\ -x^3 + 3x^2 + 2x - 6 & ; x \in (1,3] \\ x^3 - 3x^2 & ; x \in (3,4) \end{cases}$$

$x = \cancel{0}, 2$

$$f'(x) = \begin{cases} 3x^2 - 6x, & x \in (0,1] \\ -3x^2 + 6x + 2; & x \in (1,3] \\ 3x^2 - 6x; & x \in (3,4) \end{cases}$$

$x = \cancel{0}, 2$

$f'(1^-) = -3$

$f'(1^+) = 5$

$\boxed{L_{min} \to x = 1}$ ✓

$-3x^2 + 6x + 2 = 0$

$x = 1 + \frac{\sqrt{15}}{3}, \cancel{1 - \frac{\sqrt{15}}{3}}$

L_{max}

$f'(3^-) = -27 + 18 + 2$
$= -7$

$f'(3^+) = 9$

$\boxed{x = 3 \to L_{min}}$ ✓

$\boxed{\text{BKG}}$

2nd Order Derivative Test

$$\boxed{\frac{dy}{dx} = 0}$$

$$\frac{d^2y}{dx^2} > 0 \longrightarrow \text{Lmin}$$

$$\frac{d^2y}{dx^2} < 0 \longrightarrow \text{Lmax}$$

GYAAN KA BHANDAAR

However, if $f'(c) = 0$ and $f''(c)$ is also zero. Then this test fails.

In this case, $f(x)$ can still have a point of maxima, a point of minima or point of inflection.

In such cases, we revert back to 1^{st} order derivative to ensure maxima and minima.

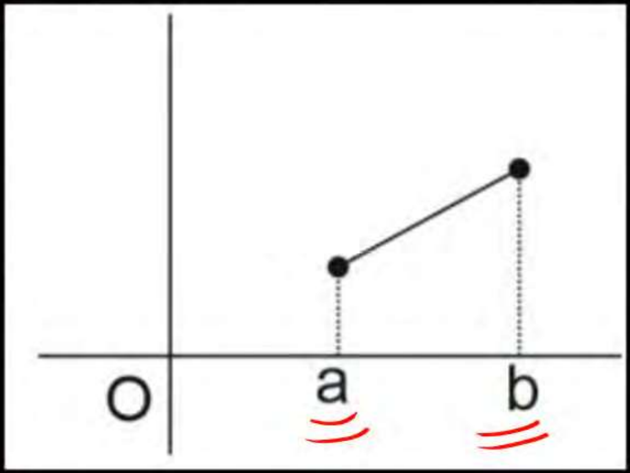
GLOBAL MAXIMA AND GLOBAL MINIMA

(GREATEST & LEAST VALUE OF A FUNCTION)

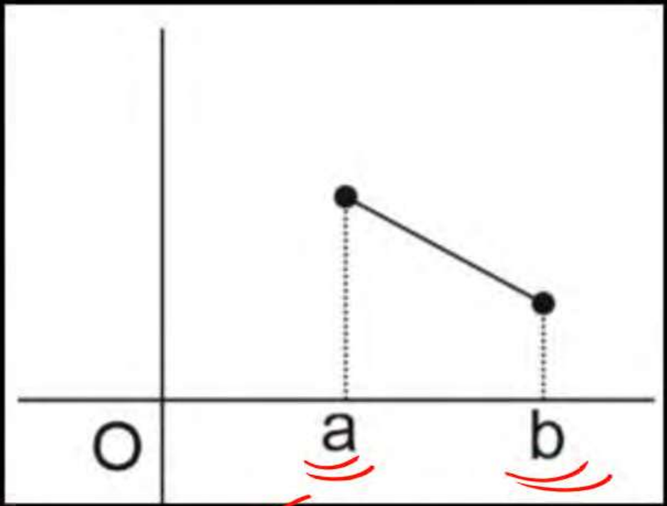
absolute max

absolute min

If a $\boxed{\text{continuous function } y = f(x)}$ is $\boxed{\text{strictly Increasing}}$ in $\boxed{\text{closed interval } [a, b]}$, then $f(a)$ is the least value and $f(b)$ is the greatest value of $f(x)$.



If $\boxed{f(x) \text{ is decreasing in } [a, b]}$ then f(b) is the least and f(a) is the greatest value of f(x).



Greatest

least Value

However if $\boxed{f(x) \text{ is non-monotonic in } [a, b]}$ and is continuous, Then the greatest and least value of $f(x)$ can occur at those points where $f'(x) = 0$ or $f'(x) = DNE$ or at the ends points i.e. at $x = a$ or $x = b$

⭐⭐⭐⭐⭐⭐⭐

$\boxed{\text{Compare}}$ Karna padhta $\overset{\checkmark}{\text{है}}$ |

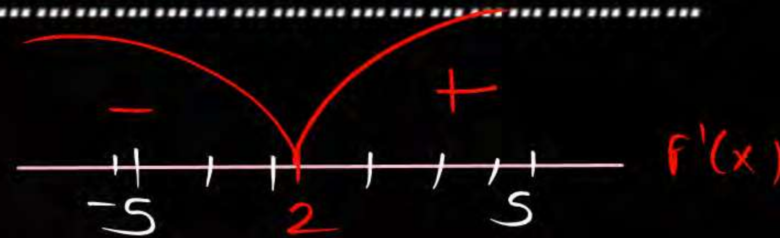
Q. If $f(x) = e^{x^2-4x+3}$ in $[-5,5]$.. Then find its greatest and least value.

$$f'(x) = \underbrace{\left(e^{x^2-4x+3}\right)}_{\text{POSITIVE}}(2x-4)$$

"Non-Monotonic"

$$f'(x) = 0$$

$$\boxed{x = 2}$$



#

$f(-5) = e^{48} \longrightarrow$ Global max

$f(5) = e^{8}$

$f(2) = e^{-1} \longrightarrow$ Global min

Ans

Q.

$y = \int_{5\pi/4}^{x} (3\sin t + 4\cos t)dt$ in $\left[\frac{5\pi}{4}, \frac{4\pi}{3}\right]$

....

Q. *The sum of absolute minimum and the absolute maximum values of the function $f(x) = |3x - x^2 + 2| - x$ in the interval [–1, 2] is :*

....

(JARUR DIKHEGA) /[JEE Main-2022]

(A) $\frac{\sqrt{17}+3}{2}$

(B) $\frac{\sqrt{17}+5}{2}$

(C) 5

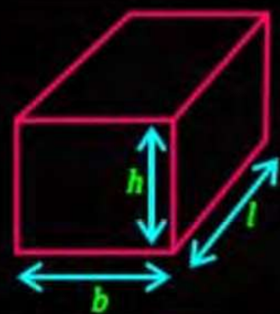
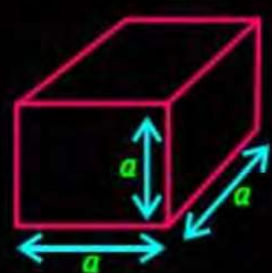
(D) $\frac{9-\sqrt{17}}{2}$

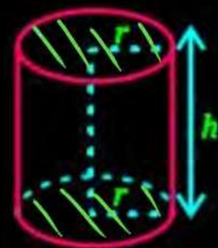
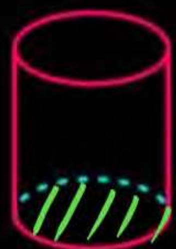
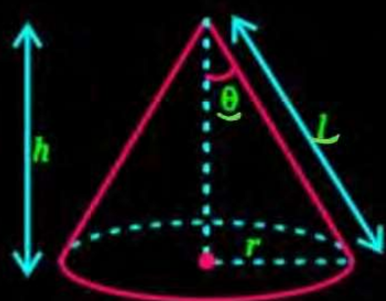
WORD/STORY PROBLEMS

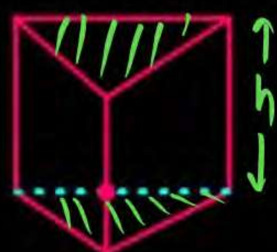
→ "JEE mains/Advanced के Favourite"

1. Draw diagram & introduce minimum number of variables.
2. Using geometric relations, make a function in single variable.
3. Check restriction of x.
4. Now differentiate & Analyse.

FLASH-PAD JINDABAD

SOLID	FIGURE	Curved Surface Area	Total Surface Area	VOLUME
Cuboid		Also known as lateral surface area = $2(lh + bh)$	$2(lb + bh + hl)$	$l.b.h$
Cube		Lateral surface area = $4a^2$	$4a^2 + 2a^2 = 6a^2$	a^3

		C.S.A	T.S.A.	Volume
Right circular cylinder (closed at top) ✓		Curved surface area = $2\pi rh$ ✓	$2\pi r\,(r+h)$ ✓ $2\pi rh + 2\pi r^2$	$\pi r^2 h$
Right circular cylinder (open at top)		Curved surface area = $2\pi rh$ ✓	$\pi r(2h+r)$ $2\pi rh + \pi r^2$	$\pi r^2 h$ ✓
Cone ✓		πrl ✓	$\pi r^2 + \pi rl$	$1/3\ \pi r^2 h$

		C.S.A	T.S.A.	✓
Sphere		$4\pi r^2$	$4\pi r^2$	$4/3\ \pi r^3$
Triangular Prism ✓		(Perimeter of base) × Height	(Lateral surface) + 2(area of base)	(area of base) × height
Triangular Pyramid ✓		½ (Perimeter of base) × slant height ✓	Lateral surface Area + (area of base) ✓	1/3 (base area) × height ✓

Q. A wire of length 22 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into an equilateral triangle. Then, the length of the side of the equilateral triangle, so that the combined area of the square and the equilateral triangle is minimum, is :

[JEE Main-2022 (29 June - Shift 1)]

(A) $\frac{22}{9+4\sqrt{3}}$

(B) $\frac{66}{9+4\sqrt{3}}$

(C) $\frac{22}{4+9\sqrt{3}}$

(D) $\frac{66}{9+4\sqrt{3}}$

$a = ?$

$\leftarrow x \rightarrow$ equilateral Δ $\quad$ square $\leftarrow 22-x \rightarrow$

a, a, a

$x = 3a$

$\boxed{a = \frac{x}{3}}$

b, b, b, b

$\boxed{4b = 22 - x}$

$b = \frac{22-x}{4}$

min. $\boxed{\text{area} = \frac{\sqrt{3}}{4}\left(\frac{x}{3}\right)^2 + \left(\frac{22-x}{4}\right)^2} = f(x)$

$$\boxed{f'(x)=0}$$

$$2\left(\frac{\sqrt{3}}{4}\right)\frac{x}{9}-2\left(\frac{22-x}{4}\right)\left(\frac{1}{4}\right)=0$$

$$\boxed{x=\frac{18*11}{4\sqrt{3}+9}=\frac{198}{4\sqrt{3}+9}}$$

$$a=\frac{x}{3}=\frac{66}{9+4\sqrt{3}}\ \boxed{\text{Ans}}$$

Q. Consider a **cuboid** of sides **2x, 4x and 5x** and a **closed hemisphere** of radius r. If the **sum of** their **surface area** is a **constant k**, then the ratio x : r, for which the sum of their volumes is **maximum**, is :

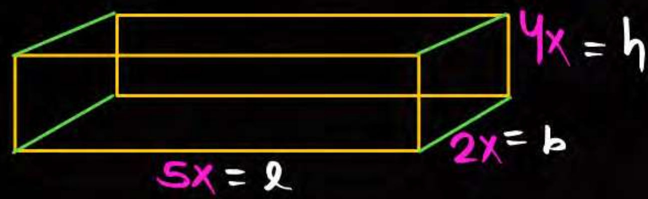
[JEE Main-2022]

(A) 2 : 5

(B) 19 : 45

(C) 3 : 8

(D) 19 : 15



$$TSA = 2(lb + bh + lh)$$
$$= 2(8x^2 + 10x^2 + 20x^2)$$
$$= 76x^2$$

$$T.S.A = 2\pi R^2 + \pi R^2$$
$$= 3\pi R^2$$

$$\text{Sum} = \boxed{K = 3\pi R^2 + 76x^2}$$

$$V = (2x)(4x)(5x) + \frac{2}{3}\pi R^3$$

$$K - 76x^2 = 3\pi R^2$$

$$\boxed{V = 40x^3 + \frac{2}{3}\pi R^3}$$

PUT

$$\boxed{\left(\frac{K-76x^2}{3\pi}\right)^{1/2} = R}$$

$$V = 40x^3 + \frac{2}{3}\pi\left(\frac{K-76x^2}{3\pi}\right)^{3/2}$$

$$\frac{dV}{dx} = 0$$

$$\frac{dV}{dx} = 120x^2 + \frac{2\pi}{3}\cdot\frac{3}{2}\underbrace{\left(\frac{K-76x^2}{3\pi}\right)^{1/2}}_{=R}\left(\frac{-76\times 2x}{3\pi}\right) = 0$$

$$120x + R\left(\frac{-152}{3}\right) = 0 \longrightarrow 120x = \frac{152R}{3} \longrightarrow \frac{x}{R} = \frac{152}{360} = \frac{38}{90} = \frac{19}{45}$$

Ans

Q. A cylindrical container is to be made from certain solid material with the following constraints : It has a fixed inner volume of V mm^3, has a 2 mm thick solid wall and is open at the top. The bottom of the container is a solid circular disc of thickness 2 mm and is of radius equal to the outer radius of the container. If the volume of the material used to make the container is minimum when the inner radius of the container is 10 mm, then the value of $\frac{V}{250\pi}$ is : 4

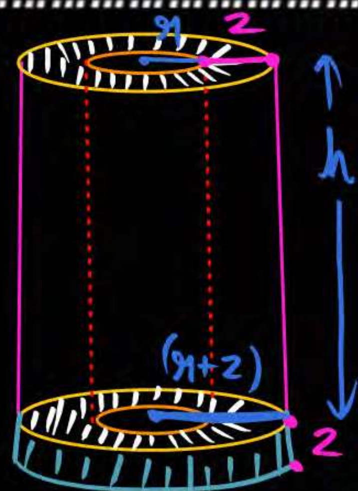
$(r = 10)$

[JEE (Advanced)-2015 (Paper-1)]

(Bhokaal ♡♡)

$V = \pi r^2 h$

$h = \frac{V}{\pi r^2}$



$$\boxed{V_{material} = V_{outer} - V + V_{DISC}}$$

$$\boxed{V_m = \pi (r+2)^2 h - V + \pi (r+2)^2 (2)}$$

$$V_m = f(r) = \pi (r+2)^2 \left(\frac{V}{\pi r^2} + 2\right) - V$$

$$\boxed{f'(10)=0}$$

$$(r+2)^2\left(-2Vr^{-3}\right)+\left(\frac{V}{r^2}+2\pi\right)2(r+2)-0\Big|_{r=10}=0$$

$$(144)\left(-\frac{2V}{1000}\right)+24\left(\frac{V}{100}+2\pi\right)=0$$

$$\boxed{V=1000\pi}$$

$$\frac{V}{250\pi}=\frac{1000\pi}{250\pi}=4$$

POINT OF INFLECTION

A Point where the graph of the function is Continous and has a Tangent line (may be vertical) and where the concavity changes is called as point of inflection.

METHOD TO FIND POI :

(i) At the $\boxed{\text{POI}}$ either $\frac{d^2y}{dx^2} = 0$ or fails the exist.

(ii) Now the point will be called as POI if the satisfies all above condition.

NOTE :

(i) At the POI, the curve crosses its tangent at that point.

(ii) A function can't have POI and EXTREMA at the same point.

(iii) If $\frac{d^2y}{dx^2} > 0$.. Then y is concave UP and if $\frac{d^2y}{dx^2} < 0$, then y is concave down.

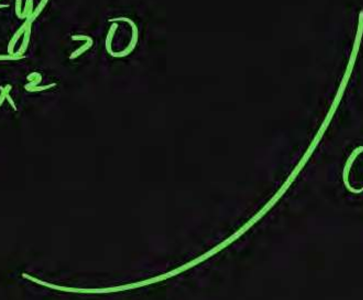
Baba Ka Gyaan

How to Find POI?

① $\frac{d^2y}{dx^2} = 0$, DONE $\longrightarrow$ $\boxed{x = ?}$

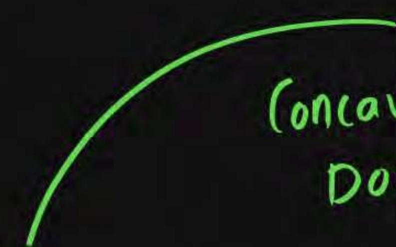
② At those 'x', check $\left[\begin{array}{l} \text{continous} ✓ \\ \text{Tangent} ✓ \\ \text{Concavity changes} ✓ \end{array}\right]$

$\frac{d^2y}{dx^2} > 0$



Concave up

$\frac{d^2y}{dx^2} < 0$



Concave Down

Q. (i) $f(x) = x^3$

Check Point of Inflecⁿ.

$$\boxed{\frac{d^2y}{dx^2} = 6x}$$

$x = 0$ → Continous ✓

→ Tangent line ✓

$-$ | $+$ $f''(x)$ ✓

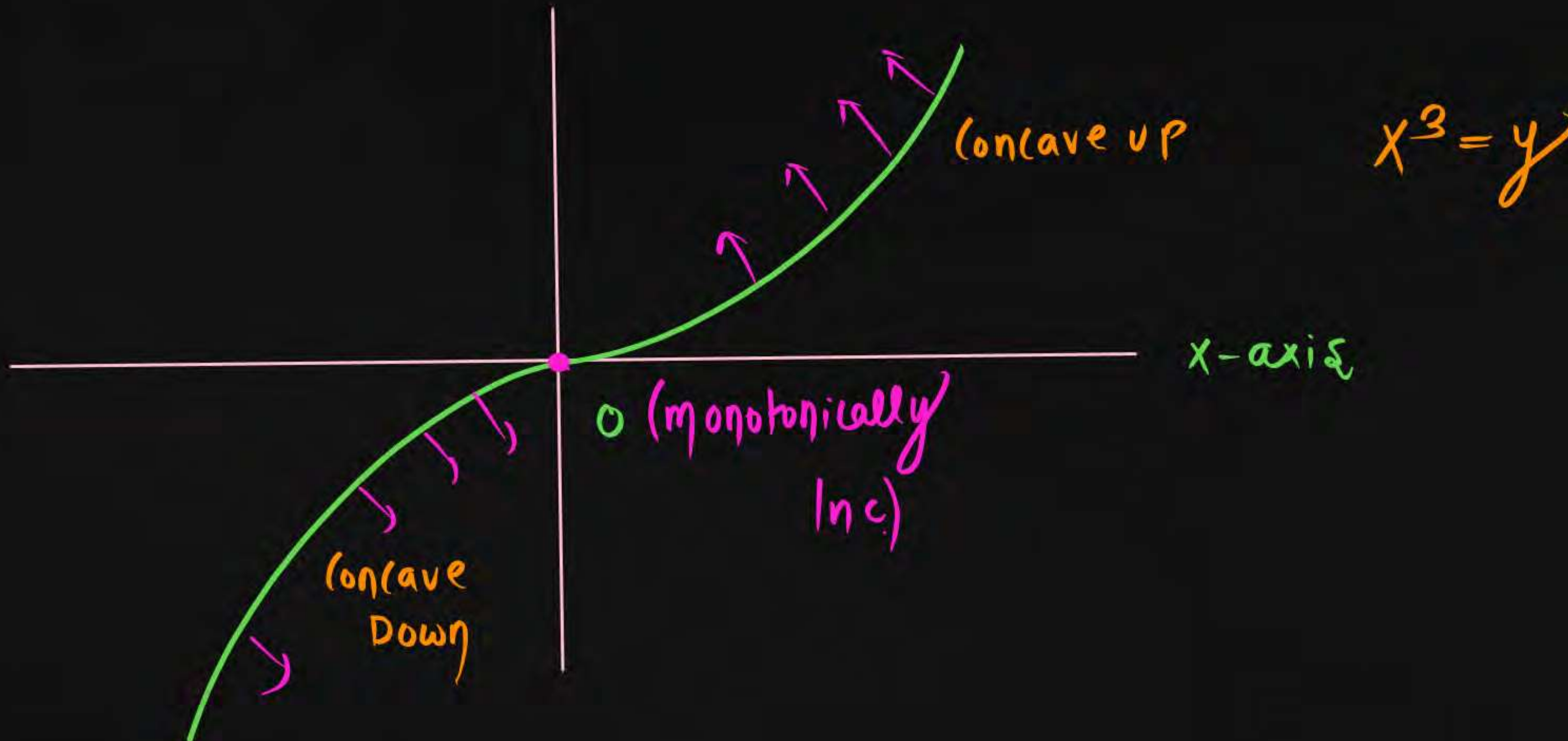
$x = 0$

$f(x)$ Concave Down

$f(x)$ Concave up

Yes, $x = 0$

Ans


Concave up
$x^3 = y$
x-axis
O (monotonically
Inc)
Concave
Down

Q. (ii) $\boxed{y = x^4}$ check P.O.I.

$\frac{d^2y}{dx^2} = 12x^2 \longrightarrow \frac{d^2y}{dx^2} = 0 \longrightarrow \boxed{x = 0}$

→ Conti. ✓

→ Tangent ✓

+ + + 0 + + + Sign of $f''(x)$

0

Concavity change ✗

$\boxed{\text{No POI}}$ Ans

Q. Let $f(x) = 3^{\left((x^2-2)^3+4\right)}$, $x \in R$. Then which of the following statements are true?

$P : x = 0$ is a point of local minima of f

$Q : x = \sqrt{2}$ is a point of inflection of f

$R : f'$ is increasing for $x > \sqrt{2}$

[JEE Main-2022 (29 July - Shift 1)]

A. Only P and Q

B. Only P and R

C. Only Q and R

D. All P, Q and R

GRAPHS OF CUBIC POLYNOMIALS

Condition for a cubic to have three distinct Real Roots

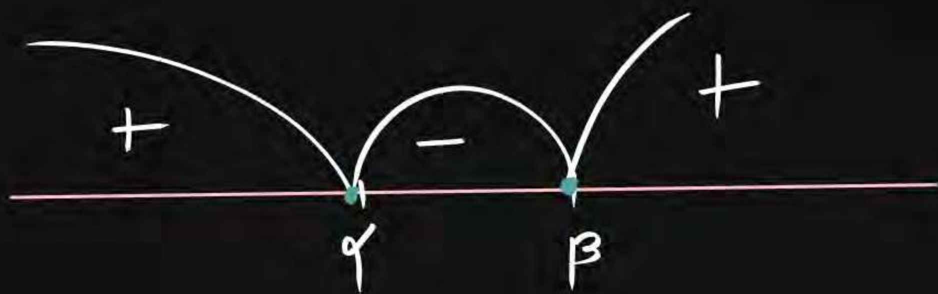
$\boxed{f(\alpha)\cdot f(\beta)<0}$

$\boxed{y = \underset{=}{a}x^3+bx^2+cx+d}$

$a>0$, $\frac{dy}{dx} = 3ax^2+2bx+c \rightarrow D>0 \rightarrow \alpha, \beta \rightarrow$ Roots of $f'(x)$

$$\boxed{3a\,(x-\alpha)(x-\beta) = f'(x)}$$

$\alpha < \beta$



Sign of $f'(x)$

BKG

any cubic $f(x) = ax^3 + bx^2 + cx + d$ has 3 Real, Distinct Roots...

Cond^n: $\boxed{f(\alpha).f(\beta) < 0}$

$\alpha, \beta \rightarrow$ Roots

$f'(x) = 0$

Q. If the equation $x^3 - 3x + [a] = 0$ has all three roots real and distinct, find the range of a.

([.] denotes GIF)

$f(x) = x^3 - 3x + [a]$

$f'(x) = 3x^2 - 3 = 0$

$\alpha = -1$

$\beta = 1$

$f(\alpha) \cdot f(\beta) < 0$

$\boxed{f(-1) \cdot f(1) < 0}$

$([a]+2)([a]-2) < 0$

$-2 < [a] < 2$

$[a] = -1, 0, 1$

$a \in (-1, 2)$ Ans

Min^m and Max^m Distance b/w 2 curves

- Always occurs along their common Normal (Parallel Tangents)

Q → A straight line L passes through (3,0) and (0,4). The point A lies on $y = 2x - x^2$. Find 'A' for which the distance b/w the parabola and st. line is least.

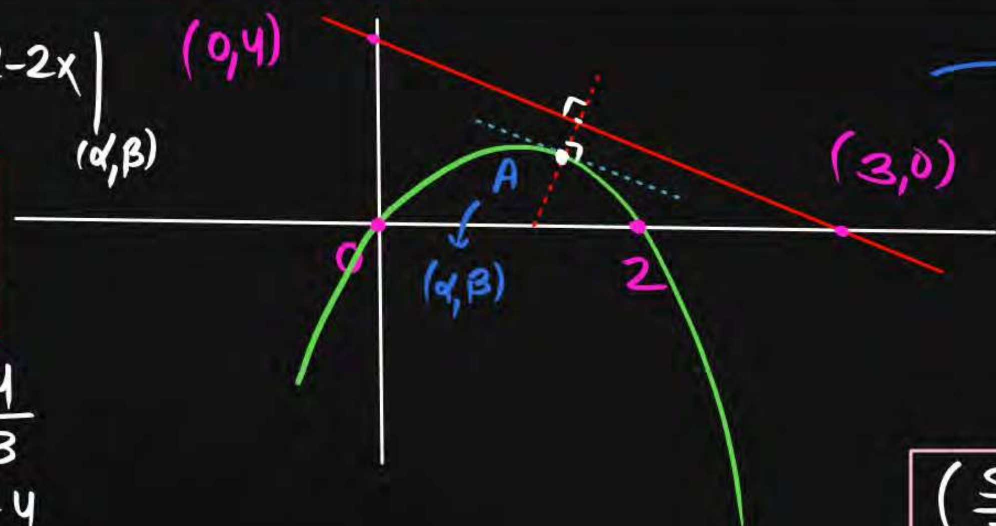
sol^n → $\left.\dfrac{dy}{dx} = 2-2x\right|_{(\alpha,\beta)}$

$\boxed{m_{T_A} = m_L}$

$2-2\alpha = -\dfrac{4}{3}$

$6-6\alpha = -4$

$\boxed{\alpha = \dfrac{5}{3}}$



→ $\dfrac{x}{3} + \dfrac{y}{4} = 1$

$\beta = 2\alpha - \alpha^2$
$= \dfrac{10}{3} - \dfrac{25}{9} = \dfrac{5}{9}$

$\boxed{\left(\dfrac{5}{3}, \dfrac{5}{9}\right) \text{ Ans}}$