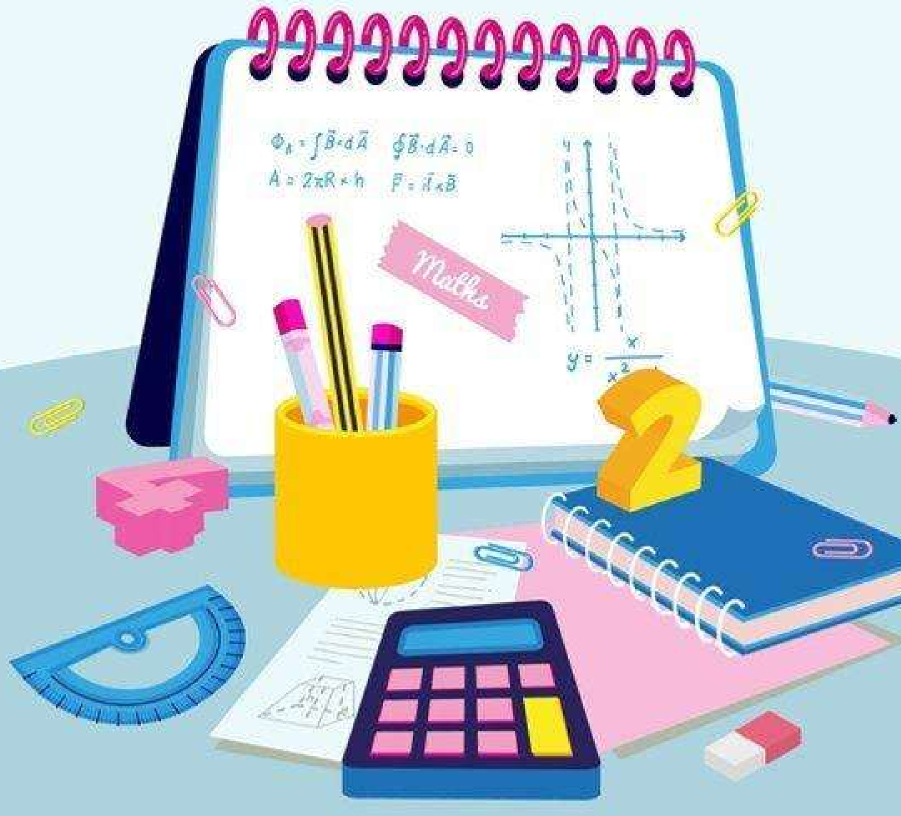

Matrices

Matrices

R

arrangement

Definition: Rectangular array of numbers.

$$\begin{array}{cc}
\begin{bmatrix} 15 & 6 \\ 10 & 2 \\ 13 & 5 \end{bmatrix} &
\begin{array}{l} \leftarrow \textbf{First row} \\ \leftarrow \textbf{Second row} \\ \leftarrow \textbf{Third row} \end{array} \\
\begin{array}{cc} \uparrow & \uparrow \\ \textbf{First Column} & \textbf{Second Column} \end{array} &
\end{array}$$

$3 \times 2 \rightarrow$ order of Matrices.

$$\begin{bmatrix} a & b \\ c & 3 \\ 2 & 3 \end{bmatrix}$$

3 rows and 2 columns

3×2

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

2×3

General Matrix

$$
\begin{array}{c@{}c}
 & \begin{array}{ccccc} 1 & 2 & \cdots & n \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ \vdots \\ m \end{array} &
\begin{bmatrix}
a_{11} & a_{12} & \cdots & a_{1n} \\
a_{21} & a_{22} & a_{23} & a_{2n} \\
a_{31} & a_{32} & \cdots & a_{3n} \\
\vdots & \vdots & \vdots & \vdots \\
a_{m1} & a_{m2} & \cdots & a_{mn}
\end{bmatrix}_{m \times n}
\end{array}
$$

a_{23}

2nd row 3rd Column

Order of Matrix

$$A = \underbrace{\begin{bmatrix} -2 & 5 \\ 0 & \sqrt{5} \\ 3 & 6 \end{bmatrix}}_{3 \times 2},\quad B = \underbrace{\begin{bmatrix} 2+i & 3 & -\frac{1}{2} \\ 3.5 & -1 & 2 \\ \sqrt{3} & 5 & \frac{5}{7} \end{bmatrix}}_{3 \times 3},\quad C = \underbrace{\begin{bmatrix} 1+x & x^3 & 3 \\ \cos x & \sin x + 2 & \tan x \end{bmatrix}}$$

$\underline{2 \times 3}$

$= 6$

$= 9$ no. of element

The number of matrices of order 3 x 3, whose entries are either 0 or 1 and the sum of all the entries is a prime number, is _.

$\boxed{0 \quad \text{OR} \quad 1}$

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$$\begin{bmatrix} \checkmark & 0 & \checkmark \\ 0 & \checkmark & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Sum of all 9 elements = prime

= 2, 3, 5, 7

Sum = 2	Sum = 3	Sum = 5	Sum = 7
1,1,0,0,0,0,0,0,0	1,1,1,0,0,0 0,0,0		
$\boxed{{}^9C_2 \times 1}$	${}^9C_3$	${}^9C_5$	${}^9C_7$

$${}^9C_2 + {}^9C_3 + {}^9C_5 + {}^9C_7$$

$$\Rightarrow \boxed{282} \checkmark$$

Let A be a 3 x 3 matrix having entries from. The set {-1, 0, 1}. The number of all such matrices A having sum of all the entries equal to 5. is

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$$\begin{bmatrix} \_ & \_ & \_ \\ \_ & \_ & \_ \\ \_ & \_ & \_ \end{bmatrix}$$

$\{-1, 0, 1\}$

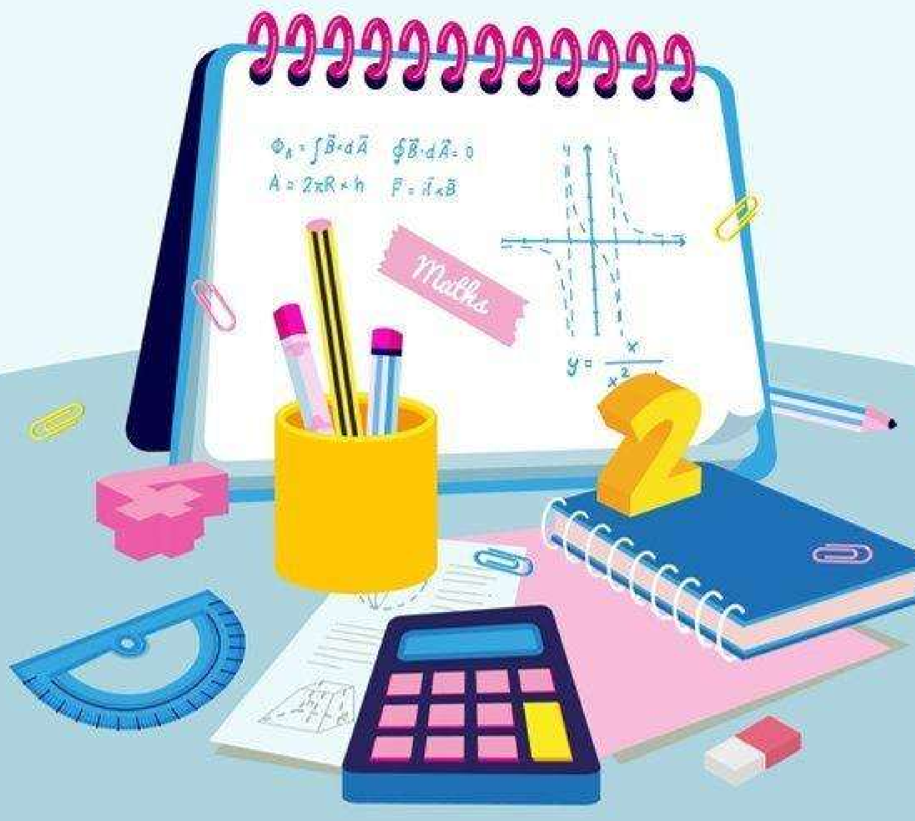
Sum = 5

C-1 $\quad 1, 1, 1, 1, 1, 0, 0, 0, 0 \Rightarrow \dfrac{9!}{5!\,4!}$

C-2 $\quad \boxed{1, 1, 1, 1, 1, 1}, -1, \boxed{0, 0} \Rightarrow \dfrac{9!}{6!\,2!}$

C-3 $\quad 1, 1, 1, 1, 1, 1, 1, -1, -1 \Rightarrow \dfrac{9!}{7!\,2!}$

414



Types of Matrices

Special Types of Matrices

Row Matrix

OR Row Vector

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}_{1 \times 4}$$

Column Matrix

OR Column Vector.

$$B = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}_{n \times 1}$$

Special Types of Matrices

Null Matrix or Zero Matrix

$$O = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{2\times 3}$$

Each element $= 0$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}_{2\times 2} \qquad \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}_{3\times 2}$$

$$\cancel{\begin{bmatrix} 0 & 0 \\ 0 & -1 \\ 0 & 0 \end{bmatrix}}$$

Special Types of Matrices

Horizontal Matrix

$\boxed{\text{row} < \text{Column}}$

$$\begin{bmatrix} 2 & 3 & -1 \\ 1 & 0 & 2 \end{bmatrix}_{2\times 3}$$

Vertical Matrix

$\boxed{\text{rows} > \text{Column}}$

$$\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}_{3\times 2}$$

Square Matrix

$\boxed{\text{rows} = \text{Column}}$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2} \quad \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3\times 3}$$

Special Types of Matrices

Square Matrix

$$A = \begin{bmatrix} 9 & 5 & 2 \\ 1 & 8 & 5 \\ 3 & 1 & 6 \end{bmatrix}_{3 \times 3}$$

$$Tr(A) = 9 + 8 + 6$$

Square Matrices

In a square matrix the pair of elements a_{ij} & a_{ji} are called **Conjugate Elements.**

a_{ij} a_{ji}

The elements $a_{11}, a_{22}, a_{33}, \ldots\ldots\ldots a_{nn}$ are called **Diagonal Elements.** The line along which the diagonal elements lie is called **"Principal or leading"** diagonal.

Trace (A) = Sum of elements along principal diagonal. Notation **$t_r(A)$.**

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$t_r(A) =$ sum of diagonal element

Square Matrices

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -2 \end{bmatrix} = dia(1, 2, -2)$$

Triangular Matrix

Upper triangular

If $a_{ij} = 0 \; \forall \; i > j$

$$A = \begin{bmatrix} x & x & x \\ 0 & x & x \\ 0 & 0 & x \end{bmatrix}$$

P.D. ke neeche wale $= 0$

Lower triangular

If $a_{ij} = 0 \; \forall \; i < j$

$$A = \begin{bmatrix} x & 0 & 0 \\ x & x & 0 \\ x & x & x \end{bmatrix}$$

P.D ke uper wale $= 0$

0 / 0 nhi aaya

$$\begin{bmatrix} \cdot & 0 & 0 \\ 0 & \cdot & 0 \\ 0 & 0 & \cdot \end{bmatrix}$$

$$\begin{matrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{matrix}$$

Diagonal Matrix

$a_{ij} = 0$ if $i \neq j$

$$\begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix} = dia(d_1, d_2, d_3)$$

Scalar matrix

If $d_1 = d_2 = d_3 \; ..... = a$

$$\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$$

Identity / Unit Matrix

If $d_1 = d_2 = ... = 1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3$$

Types of Square Matrices

$$A = \begin{bmatrix} \alpha^2 & 6 & 8 \\ 3 & \beta^2 & 9 \\ 4 & 5 & \gamma^2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2\alpha & 3 & 5 \\ 2 & 2\beta & 6 \\ 1 & 4 & 2\gamma - 3 \end{bmatrix}$$

If $T_r(A) = T_r(B)$ then the value of $\left(\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}\right)$ is ③

A. 1

B. 2

C. 3

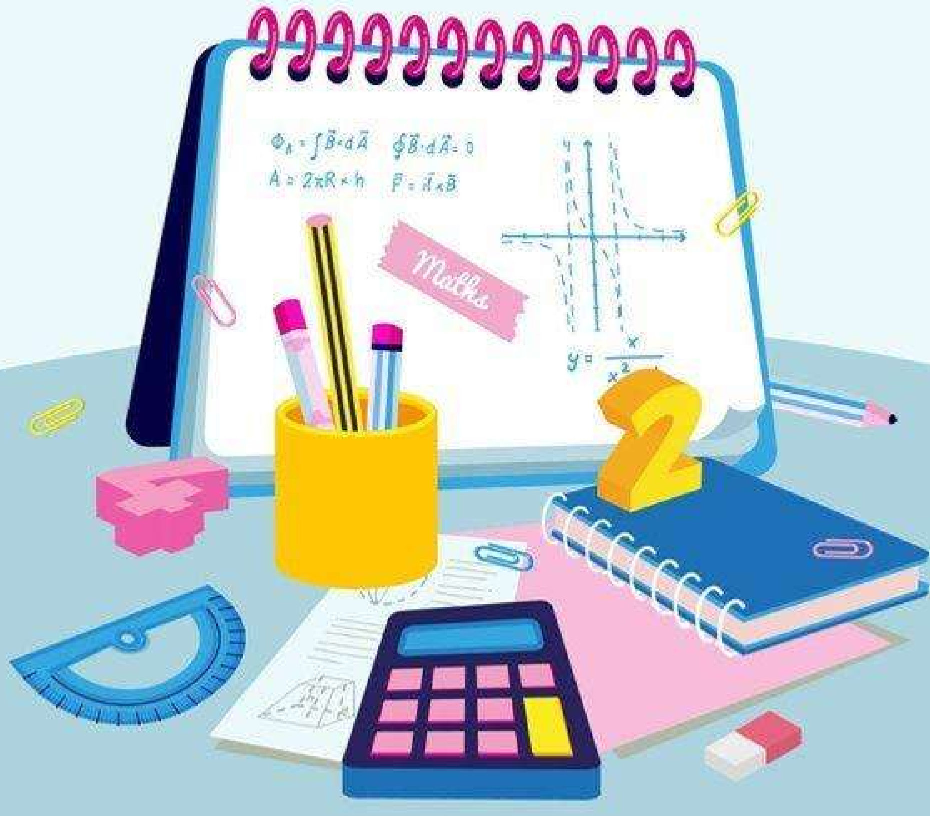
D. 4

$$\alpha^2 + \beta^2 + \gamma^2 = 2\alpha + 2\beta + 2\gamma - 1 - 1 - 1$$

$$\underbrace{\alpha^2 - 2\alpha + 1} + \underbrace{\beta^2 - 2\beta + 1} + \underbrace{\gamma^2 - 2\gamma + 1} = 0$$

$$(\alpha - 1)^2 + (\beta - 1)^2 + (\gamma - 1)^2 = 0$$

$$\boxed{\alpha = \beta = \gamma = 1}$$



Algebra of Matrices

Algebra of Matrices

- Adding/Subtracting
- Multiplication by Scalar
- Multiplying

$\frac{A}{B}$

$\Rightarrow K\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$A \times B$

$\frac{[\quad]}{[\quad]} = $ N.d.

Adding/Subtracting two Matrices

We can add/subtract two matrices only if they are of same order

$tr(A) = 5$

$tr(B) = 0$

$tr(A+B) = 5$

Given $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ *and* $B = \begin{bmatrix} -1 & 3 \\ -2 & 1 \end{bmatrix}$ **Find A + B**

$\underbrace{\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 2 & 3 \end{bmatrix}}_{\text{Nhi}}$

2x2 2x2

$$A + B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} -1 & 3 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 5 \\ 1 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} -1 & 3 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 5 & 3 \end{bmatrix}$$

Properties

Order should be same

$A + B = B + A$	(Commutative)
$(A + B) + C = A + (B + C)$	(Associative)
$t_r(A + B) = t_r(A) + t_r(B)$	(Square Matrix)
$t_r(A - B) = t_r(A) - t_r(B)$	(Square Matrix)
$t_r(kA) = k\, t_r(A)$	(Square Matrix)

$A = [\ \ ]$

$tr(12A)$

$= 12\, \underline{tr(A)}$

$= \checkmark$

$$tr(9A) = 9\ tr(A) = 9 \times 4$$

$$= 36$$

$$A = \begin{bmatrix} 3 & 5 & 3 \\ 4 & 2 & 9 \\ 7 & 3 & -1 \end{bmatrix}$$

Multiplication of a matrix by a scalar :

If $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$; $\quad kA = \begin{bmatrix} ka & kb & kc \\ kb & kc & ka \\ kc & ka & kb \end{bmatrix}$

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$tr(A) = 5$

$3A \Rightarrow 3\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

★ **k(A + B) = kA + kB**

$K(A+B) = KA + KB$

$3(A+B) = 3A + 3B$ ✓

$3A \Rightarrow \begin{bmatrix} 3 & 6 \\ 9 & 12 \end{bmatrix}$

$\times 3$

$tr(3A) = 15$

Additive Inverse :

$2 + (?) = 0$

If **A + B = O = B + A** (order of A = order of B)

Then A and B are **additive inverse of each other**

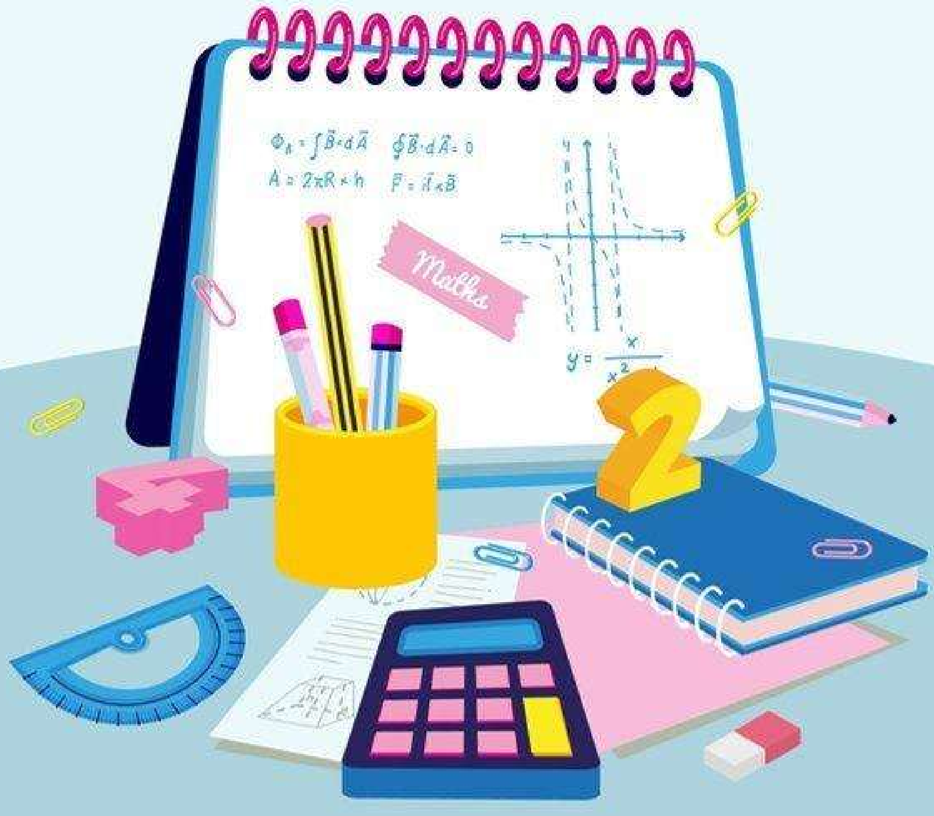
$(-1)\times \begin{bmatrix} \_ & \_ \\ \_ & \_ \end{bmatrix}$

$\begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} -2 & -3 \\ 1 & 0 \end{bmatrix}$

$\boxed{A + (?) = O}$

$-A$

$\boxed{A \longrightarrow -A}$



Main / Adv

Multiplication of Matrices

Multiplication of a matrices

AB exist if, A = m x n & B = n x p

* Kab?
* Kese?

$A_{m \times n} \, B_{a \times b}$

$\boxed{n = a} \rightarrow$ then Multi ✓

$\boxed{n \neq a} \rightarrow$ then Multi X

$A_{3\times2} \cdot B_{2\times4} = C_{3\times4}$

Multiplication of a matrices (Row by Column)

Find $\begin{bmatrix} 2 & 4 \\ 1 & 5 \end{bmatrix}_{2\times 2} \cdot \begin{bmatrix} 3 & 5 \\ 2 & 1 \end{bmatrix}_{2\times 2} = \begin{bmatrix} 14 & 14 \\ 13 & 10 \end{bmatrix}_{2\times 2}$

#NVSTYLE

Row x Column

$$\begin{array}{c|cc} & (3, 2) & (5, 1) \\ \hline (2, 4) & 6+8 & 10+4 \\ (1, 5) & 3+10 & 5+5 \end{array}$$

Multiplication of a matrices (Row by Column)

Find $\underset{2\times 2}{\begin{bmatrix} 2 & 4 \\ 1 & 5 \end{bmatrix}} \cdot \underset{2\times 2}{\begin{bmatrix} 3 & 5 \\ 2 & 1 \end{bmatrix}} = \underset{2\times 2}{\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}}$

Method - 2

$\times \quad a_{11} = 6 + 8 = 14$

$\times \quad a_{12} = 10 + 4 = 14$

$\checkmark \quad \boxed{a_{21} = 3 + 10 = 13}$

$\times \quad a_{22} = 5 + 5 = 10$

$\begin{bmatrix} 14 & 14 \\ 2 & 10 \end{bmatrix}$

Multiplication of a matrices (Row by Column)

#NVSTYLE

$$\underbrace{\begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix}}\cdot\underbrace{\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 2 \end{bmatrix}} = \begin{bmatrix} - & - & - \\ - & - & - \end{bmatrix}_{2\times 3}$$

$2\times 2 \quad 2\times 3$

$$\begin{array}{c|ccc} & (1, 2) & (2, 5) & (3, 2) \\ \hline (2, 1) & \left[\begin{matrix} 4 \\ \\ 13 \end{matrix}\right. & \begin{matrix} 9 \\ \\ 31 \end{matrix} & \left.\begin{matrix} 8 \\ \\ 19 \end{matrix}\right] \\ (3, 5) & & & \end{array}$$

Properties of Matrix Multiplication

In general, matrix multiplication is not Commutative
i.e. **AB ≠ BA** (in general).

In fact if AB is defined it is possible that BA is not defined or may have different order.

$A_{2\times3} \cdot B_{3\times5}$ $\quad B_{3\times5} \cdot A_{2\times3} \Rightarrow \times$

$A_{2\times3} B_{3\times2}$ $\quad B_{3\times2} A_{2\times3}$

$(AB)_{2\times2} \neq (BA)_{3\times3}$

Properties of Matrix Multiplication

② If $\underline{A = O}$ or $\underline{B = O} \Rightarrow \underline{AB = O}$

A or B = O   AB = O

$AB = O \not\Rightarrow A = 0$ or $B = 0$

If $\underline{AB = O} \not\Rightarrow \underline{A = O \text{ or } B = O}$

E.g. $A = \begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 5 & 5 \\ 0 & 0 \end{bmatrix}$

$AB = \begin{bmatrix} \boxed{0 \quad -1} \\ \boxed{0 \quad 2} \end{bmatrix} \begin{bmatrix} \boxed{\begin{matrix} 5 \\ 0 \end{matrix}} & \boxed{\begin{matrix} 5 \\ 0 \end{matrix}} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$(x+1)(x-2) = 0$

$x+1=0 \quad x-2=0$

Properties of Matrix Multiplication

③ If $AB = AC \nRightarrow B = C$

$\cancel{A}B = \cancel{A}C \nRightarrow B = C$

But if $B = C \Rightarrow AB = AC$

$\boxed{AB \neq CA}$ ✓

$B = C$

$\Downarrow$

$AB = AC$ ✓

$BA = CA$ ✓

Properties of Matrix Multiplication

4 In case $\boxed{AB = BA} \Rightarrow$ A and B commute each other
if $\boxed{AB = -BA}$ then A and B anticommute each other.

E.g. $A = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ and $B = \begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix}$ **[AB = BA]**

(same order)
diagonal Matrix
$\downarrow$
$\boxed{AB = BA}$

$$AB = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix} = \begin{bmatrix} ac & 0 \\ 0 & bd \end{bmatrix}$$

$$BA = \begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix} \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} = \begin{bmatrix} ac & 0 \\ 0 & bd \end{bmatrix}$$

Properties of Matrix Multiplication

5. Multiplication of diagonal matrices of the same order will be commutative

#NVstye

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 12 \end{bmatrix} = dia(-1, 4, 12)$$

Properties of Matrix Multiplication

If A, B & C are comfortable for the product AB & BC, then **(A . B) . C = A . (B . C)**

$$(A\ B)\ C = A\ (B\ C)$$

$$A_{\underline{m}\times n}\ B_{n\times p}\ C_{p\times \underline{q}}$$

Distributivity :

$$A\,(B + C) = AB + AC$$

$$(A + B)\,C = AC + BC$$

$\cancel{CA}$ $\cancel{CB}$

Positive integral powers of a square matrix

$A^2A = (AA)A = A(AA) = A^3$

$I^m = I$ for all $m \in N$

$A^m . A^n = A^{m+n}$ and $(A^m)^n = A^{mn}$

$A^0 = I_n$, n being the order of A

$A^3 = AAA = A^2A = AA^2$

$\boxed{I^{any} = I}$ $\boxed{I^{2023} = I}$

$\boxed{A^5 . A^4 = A^9}$ $(A^2)^3 = A^6$

$\boxed{A^0 = I}$

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ such that $AB = B$ and $a + d = 2021$, then the value of $a\cancel{d}^{\,0} - bc$ is equal to 2020

#NVstyle

$d = 0$, $a = 2021$

$= -bc$

JEE Main 2021

$$\begin{bmatrix} 2021 & b \\ c & 0 \end{bmatrix}_{2\times 2} \begin{bmatrix} \alpha \\ \beta \end{bmatrix}_{2\times 1} = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

$$\begin{bmatrix} 2021\alpha + b\beta \\ c\alpha \end{bmatrix}_{2\times 1} = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}_{2\times 1}$$

$2021\alpha + b\beta = \alpha$

$\boxed{b\beta = -2020\alpha}$ — (1)

$\boxed{c\alpha = \beta}$ — (2)

$b(c\cancel{\alpha}) = -2020\cancel{\alpha}$

$\boxed{-bc = 2020}$

~~The trace of a square matrix is defined to be the sum of its diagonal entries~~. If A is a 2 x 2 matrix such that the trace of A is 3 and the trace of A^3 is – 18, then the value of the determinant of A is

$$A = \begin{bmatrix} 3 & a \\ b & 0 \end{bmatrix}_{2\times 2}$$

$tr(A) = 3$

$tr(A^3) = -18$

$$A^3 = \begin{bmatrix} 3 & a \\ b & 0 \end{bmatrix} \cdot \begin{bmatrix} 3 & a \\ b & 0 \end{bmatrix} \cdot \begin{bmatrix} 3 & a \\ b & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 9+ab & 3a \\ 3b & ab \end{bmatrix} \cdot \begin{bmatrix} 3 & a \\ b & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3(9+ab)+3ab & \_\_\_ \\ \_\_\_ & 3ab \end{bmatrix}$$

$3(9+ab) + 3ab + 3ab = -18$

$27 + 9ab = -18$

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$3 + ab = -2$

$\boxed{-ab = 5}$

$$\det(A) = \begin{vmatrix} 3 & a \\ b & 0 \end{vmatrix} = -ab = 5$$

Let M be a 3 x 3 matrix satisfying

$$M\begin{bmatrix}0\\1\\0\end{bmatrix}=\begin{bmatrix}-1\\2\\3\end{bmatrix},\quad M\begin{bmatrix}1\\-1\\0\end{bmatrix}=\begin{bmatrix}1\\1\\-1\end{bmatrix},\ \text{and}\ M\begin{bmatrix}1\\1\\1\end{bmatrix}=\begin{bmatrix}0\\0\\12\end{bmatrix}.$$

Then the sum of the diagonal entries of M is

(JEE adv 2011)

$tr(M) = a_1 + b_2 + c_3$

$= \underline{0} + \underline{2} + \underline{7}$

$= 9$

$$\begin{bmatrix}a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{bmatrix}\begin{bmatrix}0\\1\\0\end{bmatrix}=\begin{bmatrix}-1\\2\\3\end{bmatrix}$$

$$\begin{bmatrix}a_2\\b_2\\c_2\end{bmatrix}=\begin{bmatrix}-1\\2\\3\end{bmatrix}$$

$a_2 = -1,\ b_2 = 2$

$c_2 = 3$

$$\begin{bmatrix}a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{bmatrix}\begin{bmatrix}1\\-1\\0\end{bmatrix}=\begin{bmatrix}1\\1\\-1\end{bmatrix}$$

$$\begin{bmatrix}a_1-a_2\\b_1-b_2\\c_1-c_2\end{bmatrix}=\begin{bmatrix}1\\1\\-1\end{bmatrix}$$

$a_1 - a_2 = 1\ ;\ b_1 - b_2 = 1$

$c_1 - c_2 = -1$

$a_1 = 0$

$b_1 = 3$

$c_1 = 2$

$$\begin{bmatrix}a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix}=\begin{bmatrix}0\\0\\12\end{bmatrix}$$

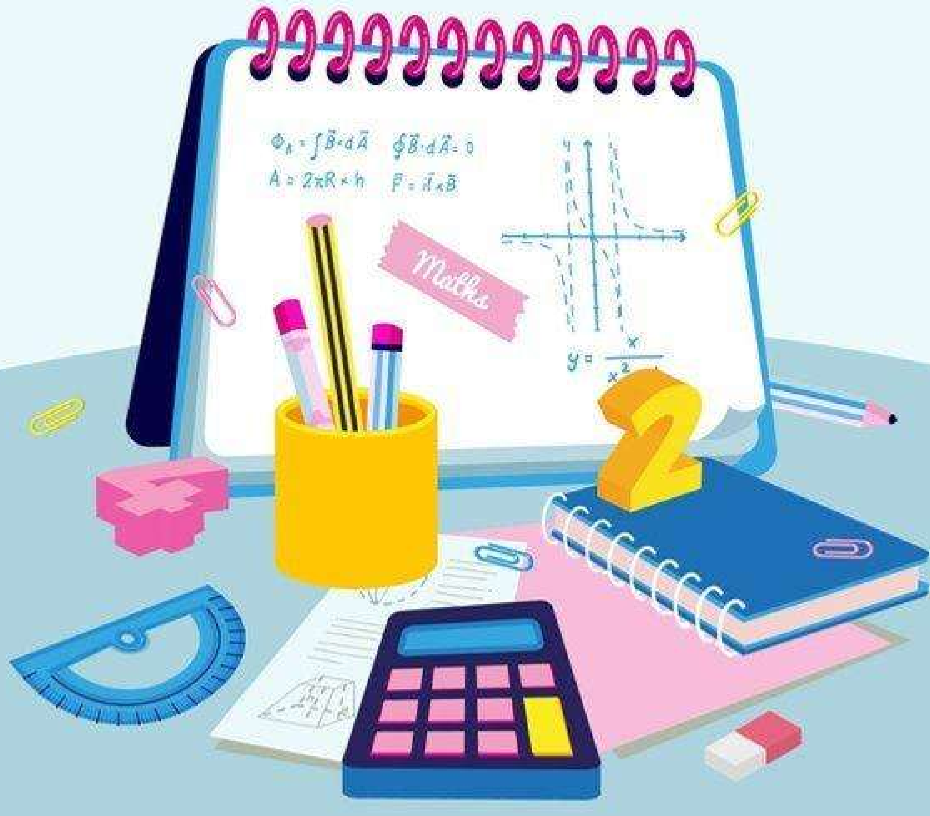
$a_1 + a_2 + a_3 = 0$ X

$b_1 + b_2 + b_3 = 0$ X

$c_1 + c_2 + c_3 = 12$

$2 + 3 + c_3 = 12$

$\boxed{c_3 = 7}$



Transpose of Matrices

Transpose of a Matrix

$$M \qquad\qquad M^T / M'$$

$$\begin{bmatrix} \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare \end{bmatrix} \qquad \begin{bmatrix} \quad & \quad & \quad \\ & & \\ & & \end{bmatrix}$$

Transpose of a Matrix

$$\begin{bmatrix} \boxed{1 \quad 2 \quad 3} \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}^T = \begin{bmatrix} \boxed{1 \quad 4 \quad 7} \\ \boxed{2 \quad 5 \quad 8} \\ \boxed{3 \quad 6 \quad 9} \end{bmatrix}^T = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Properties of Transpose

$tr(kA) = k\, tr(A)$

$(kA)' = k A'$

1. $(A^T)^T = A$
2. $(A + B)^T = A^T + B^T$
3. $(A - B)^T = A^T - B^T$
4. $(kA)^T = k(A^T)$
5. $(AB)^T = B^T A^T$

$(A+B)' = A' + B'$

$$\left(\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}\right)'$$

$$= \begin{bmatrix} 2 & 2 \\ 2 & 6 \end{bmatrix}'$$

$$= \begin{bmatrix} 2 & 2 \\ 2 & 6 \end{bmatrix}$$

RHS

$$= \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 \\ 2 & 6 \end{bmatrix}$$

$(AB)' = B'A'$

$(ABC)' = C'B'A'$

$$(ABCD)' = D'C'B'A'$$

If P is a 3 x 3 matrix such that $\boxed{P^T = 2P + I}$, where P^T is the transpose of P and I is the **3 x 3 identity matrix**, then there exist a column matrix $\boxed{X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}$ such that

JEE Adv 2012

A. $PX = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

B. $PX = Y$

C. $PX = 2X$

$IX = X$

D. $PX = -X$ ✓

$P + I = 0$

$P\cancel{X} = -I\cancel{X}$

$\boxed{PX = -X}$

$(P^T)^T = (2P + I)^T$

$P = (2P)^T + I^T$

$P = 2(P^T) + I$

$P = 2(2P + I) + I$

$P = 4P + 3I$

$O = 3P + 3I$

If for the matrix $\mathbf{A} = \begin{bmatrix} 1 & -\alpha \\ \alpha & \beta \end{bmatrix}$, $\mathbf{AA}^{\mathsf{T}} = \mathbf{I}_2$, then the value of $\alpha^4 + \beta^4$ is :

JEE Main 2021

- **A.** 1
- **B.** 3
- **C.** 2
- **D.** 4

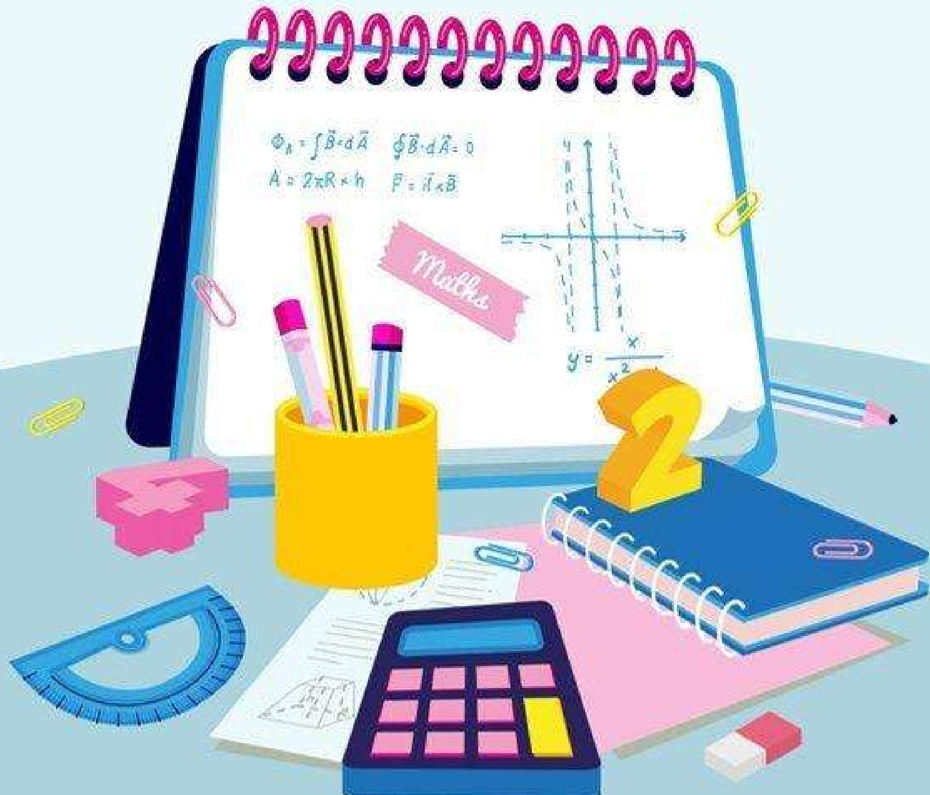
$$\begin{bmatrix} \boxed{1 \quad -\alpha} \\ \boxed{\alpha \quad \beta} \end{bmatrix} \cdot \begin{bmatrix} \boxed{\begin{matrix} 1 \\ -\alpha \end{matrix}} & \boxed{\begin{matrix} \alpha \\ \beta \end{matrix}} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \boxed{1+\alpha^2} & \alpha - \alpha\beta \\ \alpha - \alpha\beta & \cancel{\alpha^2} + \beta^2 \end{bmatrix} = \begin{bmatrix} \boxed{1} & 0 \\ 0 & 1 \end{bmatrix}$$

$$\alpha^2 = 0 \Rightarrow \alpha^4 = 0$$

$$\beta^2 = 1 \Rightarrow \beta^4 = 1$$

$$\alpha^4 + \beta^4 = 1$$



Symmetric and Skew-Symmetric Matrices

Symmetric and Skew-Symmetric Matrix

$\boxed{A^T = A}$

If $A^T = A$, then A is symmetric

$\boxed{A^T = -A}$

If $A^T = -A$, then A is skew symmetric

$\boxed{A^T = -A}$

$$\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}^T = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}^T = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \rightarrow$ Skew Sym

$$\begin{bmatrix} a & c \\ b & d \end{bmatrix} = \begin{bmatrix} -a & -b \\ -c & -d \end{bmatrix}$$

$c = -b$

$b = -c$

$a = -a$

$2a = 0$

$\boxed{a = 0}$

$d = -d$

$\boxed{d = 0}$

Visualize in #NVStyle

#NVstyle

(1) $\begin{bmatrix} \sqrt{3} & 2 & 3 \\ 2 & -1.5 & -1 \\ 3 & -1 & 1 \end{bmatrix} \longrightarrow$ Sym.

(2) $\begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix} \longrightarrow$ Skew Sym

Note: Diagonal elements of a Skew-symmetric = 0

Theorem 1

For any square matrix A with real number entries, **A + A’ is a symmetric matrix** and **A - A’ is a skew symmetric matrix.**

$A \to$ any. Sq. matrix

* $A + A^T \to$ Sym
* $A - A^T \to$ Skew Sym

$X = A + A^T$

To prove X is Sym.

$$\boxed{X^T = X}$$

Proof:- $X^T = (A + A^T)^T$

$= A^T + (A^T)^T$

$= A^T + A$

$= X$

$Y = A - A^T$

$Y^T = (A - A^T)^T$

$= A^T - (A^T)^T$

$= A^T - A$

$= -Y$

Theorem 2

Any square matrix can be expressed as the **sum of a symmetric and a skew symmetric matrix.**

$$\boxed{A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')}$$

$A \rightarrow$ any Sq. Matrix

$$\boxed{A = \text{Sym} + \text{Skew Sym}}$$

$$A = \frac{A + A^T + A - A^T}{2} = \underbrace{\frac{A + A^T}{2}}_{\text{Sym}} + \underbrace{\frac{A - A^T}{2}}_{\text{Skew Sym}}$$

Express the following matrices as **the sum of a symmetric and a skew symmetric matrix.**

$$\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} = \text{Sym} + \text{Skew Sym}$$

$$= \frac{A + A^T}{2} + \frac{A - A^T}{2}$$

$$= \frac{\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}}{2} + \frac{\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}}{2}$$

$$= \frac{\begin{bmatrix} 6 & 6 \\ 6 & -2 \end{bmatrix}}{2} + \frac{\begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}}{2}$$

$$\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

Let A be a symmetric matrix of order 2 with integer entries. If the sum of the diagonal elements of A^2 is 1, then the possible number of such matrices is :

$\boxed{a, b, c \in \mathbb{Z}}$

A. 6

B. 1

C. 4

D. 12

$$A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}_{2\times 2}$$

$$A^2 = \begin{bmatrix} \boxed{a \quad b} \\ \boxed{b \quad c} \end{bmatrix} \cdot \begin{bmatrix} \boxed{\begin{matrix} a \\ b \end{matrix}} & \boxed{\begin{matrix} b \\ c \end{matrix}} \end{bmatrix}$$

$$A^2 = \begin{bmatrix} a^2+b^2 & \underline{?} \\ \underline{?} & b^2+c^2 \end{bmatrix}$$

$$a^2 + 2b^2 + c^2 = 1$$

	a	b	c
1	1	0	0
2	-1	0	0
3	0	0	1
4	0	0	-1

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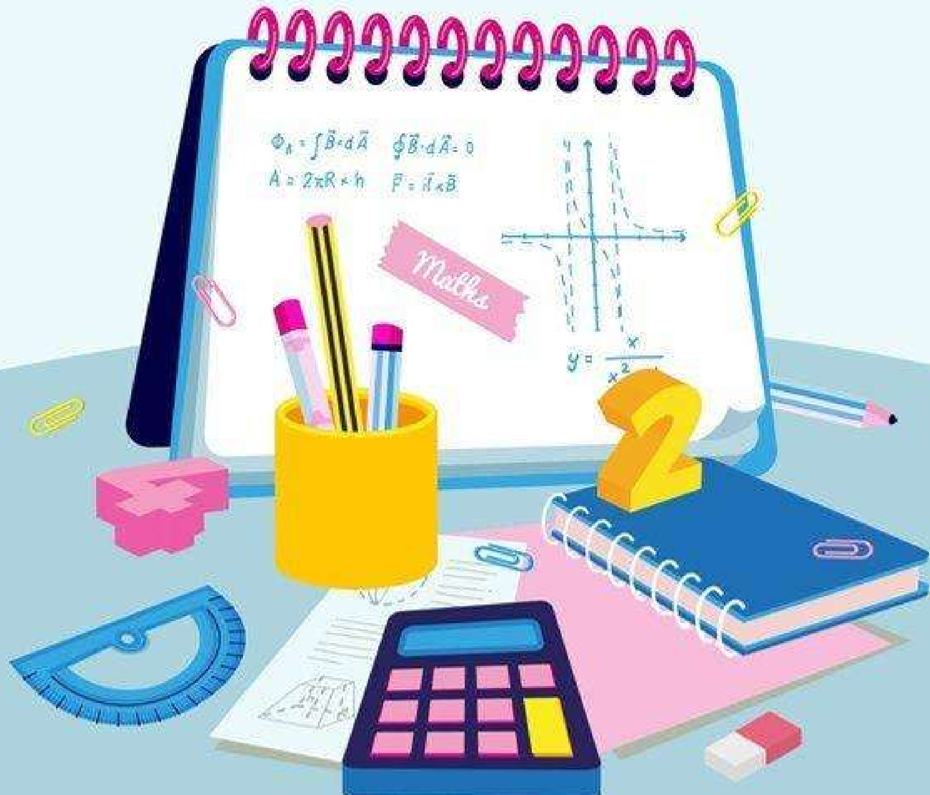

Properties of Symmetric and Skew-Symmetric

Properties of Symmetric & Skew-Symmetric

1. If A is a symmetric matrix, then **$-A$, kA, A^T, A^n, A^{-1}, B^TAB** are **also symmetric matrices**, where **$n \in N$, $k \in R$ and B is a square matrix of order that A.**

if $A \rightarrow$ Sym

$-A, kA, A^T, A^n, A^{-1}, B^TAB \rightarrow$ Sym

Given : $\boxed{A^T = A}$

To prove : $KA \rightarrow$ Sym

Proof: $X = KA$

$X^T = (KA)^T = KA^T = KA$

$= X$

Given: $A^T = A$

To prove:- $B^TAB \rightarrow$ Sym

$X = B^TAB$

$X^T = (B^TAB)^T = B^TA^TB = B^TAB = X$

$$\left(A^{2023}\right)^T$$

$$= \left(\underline{A}.A.A.A\ldots\ldots\underline{A}\right)^T$$

$$= \left(A^T\right)\left(A^T\right)\left(A^T\right)\ldots\left(A^T\right)$$

$$= \left(A^T\right)^{2023}$$

Properties of Symmetric & Skew-Symmetric

2. If A is a skew symmetric matrix, then

(a) A^{2n} is a **symmetric matrix** for **n ∈ N**,

(b) A^{2n+1} is a **skew-symmetric matrix** for **n ∈ N**,

(c) **kA** is also **skew-symmetric matrix**, where **k ∈ N**,

(d) B^TAB is also **skew-symmetric matrix** where B is a square matrix of order that of A.

$\left(A^{2023}\right)^T$

$= \left(A^T\right)^{2023}$

Given:- $\boxed{A^T = -A}$

To prove:- $X = A^{2n+1} \rightarrow$ Skew Sym

Proof:- $X^T = \left(A^{2n+1}\right)^T = (-A)^{2n+1} = -A^{2n+1} = -X$

Properties of Symmetric & Skew-Symmetric

3. If A, B are two symmetric matrix, then

(a) **$A \pm B$, $AB + BA$** are also **symmetric matrix**,
(b) **$AB - BA$** is a **skew-symmetric matrix**,
(c) **AB** is a **symmetric matrix**, when **$AB = BA$**.

Given:- $A^T = A$ and $B^T = B$

To prove:- $AB - BA = X \longrightarrow$ Skew Sym.

Proof:- $X^T = (AB - BA)^T$

$= (AB)^T - (BA)^T$

$= B^T A^T - A^T B^T$

$= BA - AB = -X$

Properties of Symmetric & Skew-Symmetric

4. If A, B are two skew symmetric matrix, then

(a) $A \pm B$, $AB - BA$ are **skew-symmetric matrices**,
(b) $AB + BA$ is a **symmetric matrix**.

Let A and B be two symmetric matrices of order 3.

Statement-1: A(BA) and **(AB)A** are symmetric matrices.

Statement-2: AB is symmetric matrix if matrix multiplication of **A** with **B** is **commutative**.

JEE Adv. [2011]

A. Statement-1 is true, Statement-2 is true; Statement-2 is **not a** correct explanation for Statement-1.

B. Statement-1 is true, Statement-2 is false.

C. Statement-1 is false, Statement-2 is true.

D. Statement-1 is true, Statement-2 is true; is a correct explanation for Statement-1.

Given: $A^T = A$, $B^T = B$

$(ABA)^T$
$= A^T B^T A^T$
$= \underline{ABA}$

$(AB)^T$
$= B^T A^T$
$= BA$
$= AB$

Let **X and Y be two arbitrary, 3 x 3, non-zero, skew-symmetric matrices** and **Z be an arbitrary 3 x 3, non-zero, symmetric matrix.** Then which of the following matrices is (are) skew symmetric?

JEE Adv. [2015]

Given:- $X^T = -X$, $Y^T = -Y$, $Z^T = Z$

A. $Y^3Z^4 - Z^4Y^3$ (Sym)

B. $X^{44} + Y^{44}$ (Sym)

C. $X^4Z^3 - Z^3X^4$ (Skew-Sym)

D. $X^{23} + Y^{23}$ (Skew Sym)

$$C = X^4 Z^3 - Z^3 X^4$$

$$C^T = (X^4 Z^3)^T - (Z^3 X^4)^T$$

$$= (Z^T)^3 (X^T)^4 - (X^T)^4 (Z^T)^3$$

$$= Z^3 (-X)^4 - (-X)^4 (Z)^3$$

$$= -C$$

$$A = Y^3 Z^4 - Z^4 Y^3$$

$$A^T = (Y^3 Z^4 - Z^4 Y^3)^T$$

$$= (YYYZZZZ)^T - (ZZZZYYY)^T$$

$$= (Z^T)^4 (Y^T)^3 - (Y^T)^3 (Z^T)^4$$

$$= Z^4 (-Y)^3 - (-Y)^3 (Z)^4$$

$$= -Z^4 Y^3 + Y^3 Z^4$$

$$= A$$

$$B = X^{44} + Y^{44}$$

Let A and B be any two 3 x 3 symmetric and skew symmetric matrices respectively. Then which of the following is NOT true ?

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A. $A^4 - B^4$ is a symmetric matrix (true)

B. $AB - BA$ is a symmetric matrix (true)

C. $B^5 - A^5$ is a skew-symmetric matrix (False)

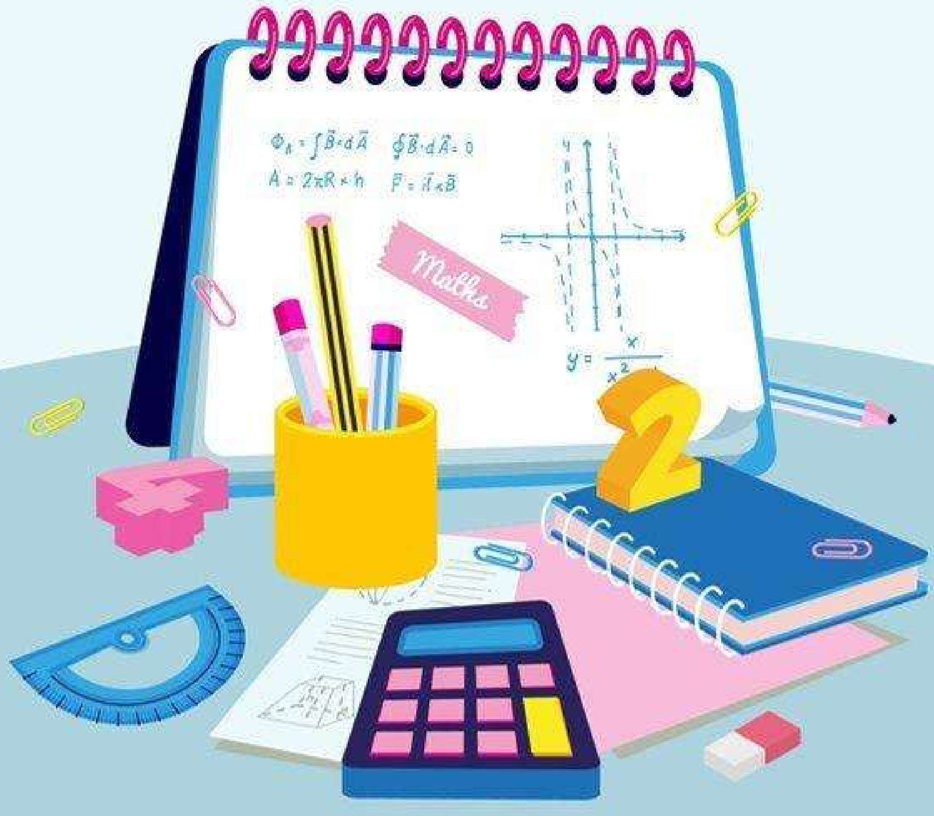
D. $AB + BA$ is a skew-symmetric matrix (true)

$A^T = A$
$B^T = -B$

$(AB - BA)'$
$(AB)' - (BA)'$
$B'A' - A'B'$
$(-B)(A) - (A)(-B)$
$-BA + AB$

$X = AB + BA$
$X^T = (AB)' + (BA)'$
$= (-B)(A) + (A)(-B)$
$= -BA - AB$
$= -X$

$X = B^5 - A^5$
$X^T = (B^5)^T - (A^5)^T$
$= (B^T)^5 - (A^T)^5$
$= (-B)^5 - (A)^5 = -(B^5 + A^5)$
$\neq X \quad \neq -X$



Types of Matrices

Type of Matrices :

Orthogonal Matrix	$A^TA = AA^T = I$
Idempotent Matrix ✓	$A^2 = A$ → $\boxed{A^3 = A^4 = A^n = A}$
Involutory Matrix	$A^2 = I_n$
Nilpotent Matrix	$A^k = O$
Periodic Matrix	$A^{k+1} = A$

$A^4 = 0$ $\boxed{A^5 = A^6 = \dots = 0}$

$\boxed{k \to \text{period}}$

Type of Matrices :

$A^2 = A$

$$A^3 = A \cdot A^2 = A \cdot A = A^2 = A$$

$$A^4 = A \cdot A^3 = A\,A = A^2 = A$$

$$A^{any} = A$$

Let $A = \begin{pmatrix} 2 & -2 \\ 1 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & 2 \\ -1 & 2 \end{pmatrix}$.

(1, 1)

Then the number of elements in the set $\{(n, m) : n, m \in \{1,2,\ldots,10\}$ and $nA^n + mB^m = I\}$ is_

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$$A^2 = \begin{pmatrix} 2 & -2 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 2 & -2 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 2 & -2 \\ 1 & -1 \end{pmatrix} = A$$

1

$$B^2 = \begin{pmatrix} -1 & 2 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} -1 & 2 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ -1 & 2 \end{pmatrix} = B$$

$$n(A) + m(B) = I$$

$$n\begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix} + m\begin{bmatrix} -1 & 2 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \boxed{2n-m} & -2n+2m \\ \boxed{n-m} & -n+2m \end{bmatrix} = \begin{bmatrix} \boxed{1} & 0 \\ \boxed{0} & 1 \end{bmatrix}$$

$$n = m \qquad 2n - m = 1$$

$$\boxed{n = m = 1}$$

Let $A = \underbrace{\begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix}}$ and $\boxed{B = A - I}$. If $\boxed{\omega = \frac{\sqrt{3}i - 1}{2}}$,

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then the number of elements in the set $\{n \in \{1, 2, \ldots, 100\} : A^n + (\omega B)^n = A + B\}$ is equal to ________.

$\boxed{\omega^{3n} = 1}$

\# $A^2 = A$

$\boxed{A^{\text{any}} = A}$

$A + (\omega B)^n = A + 1B$

$\boxed{\begin{array}{l} n = \text{multi of } 3 \\ n = \text{odd} \end{array}}$

$\{3, 9, 15, \ldots, 99\}$

17

$99 = 3 + (n-1)6$

$\boxed{n = 17}$

$B = A - I$

$B^{2} = (A - I)^2$

$= A^2 - 2A + I$

$= A - 2A + I$

$= -A + I$

$= -B$

$$\boxed{\begin{aligned} B^{\text{Even}} &= -B \\ B^{\text{odd}} &= B \end{aligned}}$$

$B^{3} = B \cdot B^2$

$= B(-B)$

$= -B^2$

$= -(-B)$

$= B$

<u>Concept</u>

$$\boxed{\begin{aligned} &(A - B)^2 \\ &= A^2 - AB - BA + B^2 \end{aligned}}$$

Let $A = \begin{pmatrix} 1+i & 1 \\ -i & 0 \end{pmatrix}$ where $i = \sqrt{-1}$.

Then, the number of elements in the set

$A^{4K+1} = A$

$\{n \in \{1, 2, \ldots, 100\} : A^{n} = A\}$ is

$A^4 = I \qquad A^8 = I$

$A^5 = A \cdot A^4 \qquad A^9 = A \cdot A^8$

$= A \cdot I \qquad = A \cdot I$

$= A \qquad = A$

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$$A^2 = \begin{pmatrix} 1+i & 1 \\ -i & 0 \end{pmatrix} \cdot \begin{pmatrix} 1+i & 1 \\ -i & 0 \end{pmatrix} = \begin{pmatrix} i & 1+i \\ 1-i & -i \end{pmatrix}$$

$$A^4 = A^2 \cdot A^2 = \begin{pmatrix} i & 1+i \\ 1-i & -i \end{pmatrix} \begin{pmatrix} i & 1+i \\ 1-i & -i \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

$4\cancel{k}+1$

$$\left\{ A^1, A^5, A^9, A^{13}, \ldots, A^{97} \right\}$$

$97 = 1 + (n-1)4$

$\boxed{n = 25}$

Let A = [a_{ij}] be a square matrix of order 3 such that $a_{ij} = 2^{j-i}$, for all i, j = 1, 2, 3. Then , the matrix $A^2 + A^3 + ... + A^{10}$ is equal to :

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A. $\left(\frac{3^{10}-3}{2}\right)A$

B. $\left(\frac{3^{10}-1}{2}\right)A$

C. $\left(\frac{3^{10}+1}{2}\right)A$

D. $\left(\frac{3^{10}+3}{2}\right)A$

$a_{ij} = 2^{j-i}$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 1 & 2 & 2^2 \\ \frac{1}{2} & 1 & 2 \\ \frac{1}{4} & \frac{1}{2} & 1 \end{bmatrix}$$

$\boxed{A^2 = 3A}$

$\Rightarrow 3A + 3^2A + 3^3A + ... + 3^9A$

$\Rightarrow \frac{3(3^9-1)}{3-1}A \Rightarrow \left(\frac{3^{10}-3}{2}\right)A$

$A^2 = 3A$

$A^3 = A \cdot A^2$

$= A \cdot (3A)$

$= 3A^2$

$= 3(3A)$

$= 3^2 A$

$A^4 = A \cdot A^3$

$= A(3^2 A)$

$= 3^2 A^2$

$= 3^2 (3A)$

$= 3^3 A$

$A^2 = 3^1 A$

$A^3 = 3^2 A$

$A^4 = 3^3 A$

Let $M=\begin{bmatrix}0 & -\alpha \\ \alpha & 0\end{bmatrix}$, where α is a non-zero real number an $N=\sum_{k=1}^{49} M^{2k}$. If $(I-M^2)N=-2I$, then the positive integral value of α is ____.

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H.W.

If $M = \begin{pmatrix} \frac{5}{2} & \frac{3}{2} \\ -\frac{3}{2} & -\frac{1}{2} \end{pmatrix}$, then which of the following matrices is equal to M^{2022} ?

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(A) $\begin{pmatrix} 3034 & 3033 \\ -3033 & -3032 \end{pmatrix}$

(B) $\begin{pmatrix} 3034 & -3033 \\ 3033 & -3032 \end{pmatrix}$

(C) $\begin{pmatrix} 3033 & 3032 \\ -3032 & -3031 \end{pmatrix}$

(D) $\begin{pmatrix} 3032 & 3031 \\ -3031 & -3030 \end{pmatrix}$

$$\boxed{\begin{aligned} &A^2 = 0 \\ &A^3 = A^4 = A^5 = \dots = 0 \end{aligned}}$$

$$M = \begin{pmatrix} \frac{5}{2} & \frac{3}{2} \\ -\frac{3}{2} & -\frac{1}{2} \end{pmatrix}$$

$$M = \begin{pmatrix} \frac{3}{2} & \frac{3}{2} \\ -\frac{3}{2} & -\frac{3}{2} \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$M = \frac{3}{2}\begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix} + I$$

$$\boxed{M = \frac{3}{2}A + I}$$

$$A = \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$A^3 = A \cdot A^2 = A \cdot 0 = 0$$

$$M^{2022} = \left(\frac{3}{2}A + I\right)^{2022}$$

$$= {}^{2022}C_0\, I + {}^{2022}C_1 \left(\frac{3}{2}A\right) + \cancelto{0}{{}^{2022}C_2 \left(\frac{3}{2}A\right)^2} + \cancelto{0}{\ldots} \cancelto{0}{\ldots}$$

$$= 1\, I + \overset{1011}{\cancel{2022}} \times \frac{3}{\cancel{2}} \times \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 3033 & 3033 \\ -3033 & -3033 \end{pmatrix}$$

$$= \begin{pmatrix} 3034 & 3033 \\ -3033 & -3032 \end{pmatrix}$$

Important Concept :

Gen Matrix

$$If\ A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, then$$

$$Tr(AA^T) = Tr(A^TA) = (\ )^2 + (\ )^2 + \ldots$$

$$\boxed{T_r(AA^T) = a_{11}^2 + a_{12}^2 + a_{13}^2 + a_{21}^2 + a_{22}^2 + a_{23}^2 + a_{31}^2 + a_{32}^2 + a_{33}^2}$$

$$AA^T = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \cdot \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$$

$$AA^T = \begin{bmatrix} a_{11}^2+a_{12}^2+a_{13}^2 & - & - \\ - & a_{21}^2+a_{22}^2+a_{23}^2 & - \\ - & - & a_{31}^2+a_{32}^2+a_{33}^2 \end{bmatrix}$$

How many **3 x 3 matrices M** with entries from **{0, 1, 2}** are there, for which the sum of the diagonal entries of **M^T^M** is 5 ?

JEE Adv. 2017

A. 126

B. 135

C. 162

D. 198

$$If\ M = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, then$$

$$5 = T_r(M^T M) = a_{11}^2 + a_{12}^2 + a_{13}^2 + a_{21}^2 + a_{22}^2 + a_{23}^2 + a_{31}^2 + a_{32}^2 + a_{33}^2$$

C-1 $\frac{9!}{5!4!}$ $\quad \boxed{1, 1, 1, 1, 1}, \boxed{0, 0, 0, 0}$

C-2 $\frac{9!}{7!}$ $\quad 2, 1, \boxed{0, 0, 0, 0, 0, 0, 0}$

198

Let S be the set containing all 3 x 3 matrices with entries from {-1 , 0, 1}.
The total number of matrices A ∈ S such that the sum of all the diagonal elements of $A^T A$ is 6 is_.

JEE Main 2022

$$If\ M = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, then$$

1/-1

$$6 = T_r(M^T M) = a_{11}^2 + a_{12}^2 + a_{13}^2 + a_{21}^2 + a_{22}^2 + a_{23}^2 + a_{31}^2 + a_{32}^2 + a_{33}^2$$

$$6 = (\pm 1)^2 + (\pm 1)^2 + (\pm 1)^2 + (\pm 1)^2 + (\pm 1)^2 + (\pm 1)^2 + 0 + 0 + 0$$

6 chairs ⇒ ±1 3 chair ⇒ 0

$$\boxed{{}^9C_6 \times 2^6 \times 1} = \boxed{5376}$$

Let M be any 3 x 3 matrix with entries from the set {0, 1, 2}. The maximum number of such matrices, for which the sum of diagonal elements of M^TM is seven is

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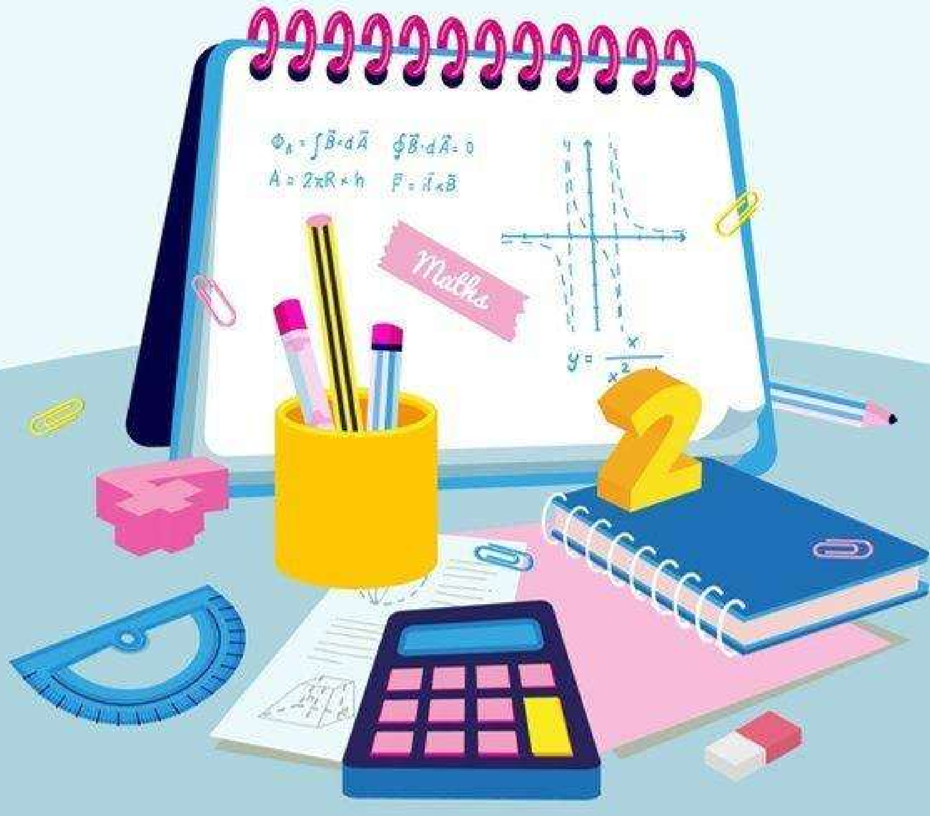
DIY



Determinant Basics

Minors

$$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$M_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$$

$$M_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$$

$$M_{31} = \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$$

Cofactor :

$$\boxed{C_{ij} = (-1)^{i+j} M_{ij}}$$

* $\boxed{C_{ij} = (-1)^{i+j} M_{ij}}$

$$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$D = \begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

$C_{12} = (-1)^{1+2} M_{12} = -M_{12}$

$C_{11} = (-1)^{1+1} M_{11} = +M_{11}$

$C_{31} = M_{31}$

Determinant value of 3×3

$$\begin{vmatrix} 2 & -3 & 1 \\ 2 & 0 & -1 \\ 1 & 4 & 5 \end{vmatrix} = 2\begin{vmatrix} 0 & -1 \\ 4 & 5 \end{vmatrix} + 3\begin{vmatrix} 2 & -1 \\ 1 & 5 \end{vmatrix} + 1\begin{vmatrix} 2 & 0 \\ 1 & 4 \end{vmatrix}$$

$$= 2(4) + 3(11) + 8 = 49$$

$$\begin{vmatrix} 2 & -3 & 1 \\ 2 & 0 & -1 \\ 1 & 4 & 5 \end{vmatrix} = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$$

$$= (2)(4) + (-3)(-11) + (1)(8)$$

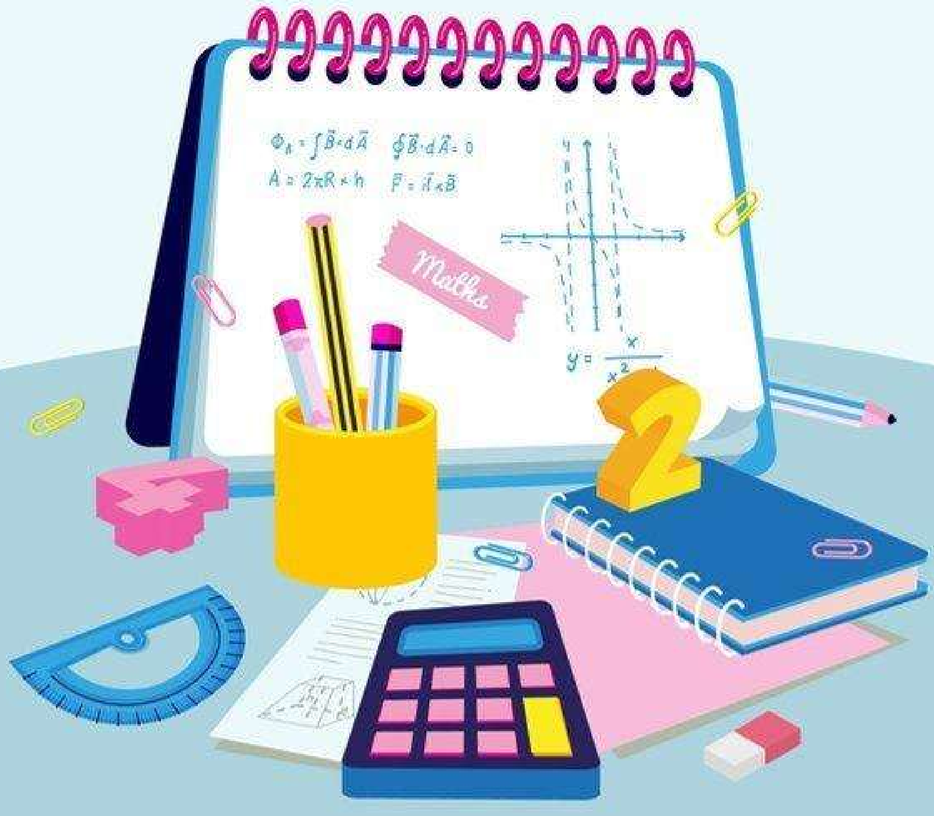
$$= 8 + 33 + 8$$

$$= 49$$

$C_{11} = M_{11} = 4$

$C_{12} = -M_{12} = -11$

$C_{13} = M_{13} = 8$



Adjoint of Matrix

Definition : Adjoint of A

adjoint = $(\text{cofactor})^T$

adj A = $(\text{Cofactor } A)^T$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$\text{Cofactor} = \begin{bmatrix} \boxed{C_{11} \quad C_{12} \quad C_{13}} \\ C_{21} \quad C_{22} \quad C_{23} \\ C_{31} \quad C_{32} \quad C_{33} \end{bmatrix}$$

$$\text{Adj } A = \begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix}$$

Adjoint of A

Find the adjoint of the matrix.

$$A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & -1 \end{bmatrix} \longrightarrow \text{Minor}(A) = \begin{bmatrix} 2 & -2 & 6 \\ 1 & -2 & 5 \\ -2 & 2 & -8 \end{bmatrix}$$

$$\text{Cof}(A) = \begin{bmatrix} \boxed{2 \quad 2 \quad 6} \\ -1 & -2 & -5 \\ -2 & -2 & -8 \end{bmatrix}$$

$$\text{adj}\, A = \begin{bmatrix} 2 & -1 & -2 \\ 2 & -2 & -2 \\ 6 & -5 & -8 \end{bmatrix}$$

#NVStyle

$$\begin{array}{c|ccccc}
3 & 1 & -1 & 3 & 1 \\
\hline
2 & -2 & 0 & 2 & -2 \\
1 & 2 & -1 & 1 & 2 \\
3 & 1 & -1 & 3 & 1 \\
2 & -2 & 0 & 2 & -2
\end{array}$$

$$\text{adj } A \Rightarrow \begin{bmatrix} 2 & -1 & -2 \\ 2 & -2 & -2 \\ 6 & -5 & -8 \end{bmatrix}$$

Properties of Adjoint

$\star\ \boxed{|kA| = k^n |A|}$

$\boxed{|\text{adj} A| = |A|^{n-1}}$

$\boxed{\text{NV10}}$ ← $A\,(\text{adj } A) = (\text{adj } A)\,A = |A|\, I_n$

$|\text{adj } A| = |A|^{n-1}$

$\text{adj}\,(AB) = (\text{adj } B).(\text{adj } A)$

$|\text{adj}(\text{adj } A)| = |A|^{(n-1)^2}$

$\text{adj}\,(kA) = k^{n-1}\,(\text{adj } A),\ (k \in R)$

$\star\ \text{adj}(A^{-1}) = (\text{adj } A)^{-1} = \dfrac{A}{|A|}$

$\text{adj}\,(\text{adj } A) = |A|^{n-2}\, A.$

$|\text{adj}(\text{adj}(\text{adj} A))| = |A|^{(n-1)^3}$

$n \rightarrow$ order of Matrix

1. $A \, \text{adj} A = |A| I$
2. $\text{adj}(AB) = (\text{adj} B)(\text{adj} A)$
3. $\text{adj}(KA) = K^{n-1} \, \text{adj} A$
4. $\text{adj} \, \text{adj} A = |A|^{n-2} A$
5. $|\text{adj} A| = |A|^{n-1}$
6. $|\text{adj} \, \text{adj} A| = |A|^{(n-1)^2}$
7. $\text{adj}(A^{-1}) = (\text{adj} A)^{-1} = \dfrac{A}{|A|}$
8. $|KA| = K^n |A|$
9. $|AB| = |A| \, |B|$

Let k be a positive real number and let

$$A = \begin{bmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 2k-1 & \sqrt{k} \\ 1-2k & 0 & 2\sqrt{k} \\ -\sqrt{k} & -2\sqrt{k} & 0 \end{bmatrix}$$

Skew Sym.

* property.

Skew Sym + odd order

$|A| = 0$

If det (adj A) + det(adj B) = 10^6. Then [k] is equal to

[**Note:** adj M denotes the adjoint of square matrix M and [k] denotes the largest integer less than or equal to k].

(JEE Adv. 2010)

$|\text{adj}\, A| + |\text{adj}\, B| = 10^6$

$|A|^2 + \cancel{|B|^2} = 10^6$

$|A|^2 = 10^6$

$|A| = 10^3$

$$A = \begin{bmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & -1 \end{bmatrix} = (2k+1)^3$$

$$(2k+1)^3 = 10^3$$

$$2k+1 = 10$$

$$\therefore [k] = [4.5] = 4$$

Let $S = \{\sqrt{n} : 1 \le n \le 50 \text{ and } \boxed{n \text{ is odd}}\}$ $= \{\sqrt{1}, \sqrt{3}, \sqrt{5}, \ldots, \sqrt{49}\}$

$a \in \{\sqrt{1}, \sqrt{3}, \sqrt{5}, \ldots, \sqrt{49}\}$

Let $a \in S$ and $A = \begin{bmatrix} 1 & 0 & a \\ -1 & 1 & 0 \\ -a & 0 & 1 \end{bmatrix}$ $\Rightarrow 1(1) + a(a) = 1 + a^2$

If $\boxed{\sum_{a \in S} \det(\operatorname{adj} A) = 100\lambda}$, then λ is equal to (JEE M 2022)

A. 218

B. ~~221~~

C. 663

D. 1717

$$\sum_{a \in S} |\operatorname{adj} A| = \sum_{a \in S} |A|^2$$

$$= \sum_{a \in S} (1 + a^2)^2$$

$$= 2^2 + 4^2 + 6^2 + \ldots + 50^2$$

$$= 2^2 \left(1^2 + 2^2 + 3^2 + \ldots + 25^2 \right)$$

$$= \frac{\cancel{4} \left(\cancel{25}\right) \left(\cancel{26}^{13}\right) \left(\cancel{51}^{17}\right)}{\cancel{6}} = \cancel{100}\,\lambda$$

$$\lambda = 13 \times 17$$

$$= \underline{221}$$

Let A be a 3 x 3 invertible matrix. If |adj (24A)| = |adj (3adj(2A))|, then $|A|^2$ = ? is equal to :

A. 6^6
B. 2^{12}
C. 2^6
D. 1

C

JEE Main 2022

$$|\text{adj}\,\square| = |\square|^{n-1}$$

$$|\text{adj}(24A)| = |\text{adj}\,\boxed{3\,\text{adj}(2A)}|$$

$$|24A| = |3\,\text{adj}(2A)|$$

$$\cancel{3}^3\, 8^3\, |A| = \cancel{3}^3\, |\text{adj}(2A)|$$

$$8^3\, |A| = |2A|\,|2A|$$

$$8^3\, |\cancel{A}| = 2^3\, |\cancel{A}|\, 2^3\, |A|$$

$$2^9 = 2^6\, |A|$$

$$\boxed{|A| = 8}$$

$$|A|^2 = 64$$

Let A and B be two 3 x 3 matrices such that $|AB| = |I|$ and $|A| = 1/8$ then $|adj(B\,adj(2A))|$ is equal to

A. 16
B. 32
C. 64
D. 128

$|A||B| = 1 \quad |A| = \frac{1}{8}$

$|B| = 8$ *JEE Main 2022*

$$\left| adj\left(\boxed{B\,adj(2A)}\right)\right|$$

$$\left(|AB| = |A||B|\right)$$

$$= |B\,adj(2A)|^2$$

$$= |B|^2\,|adj\boxed{2A}|^2$$

$$= |B|^2\left(|2A|^2\right)^2$$

$$= |B|^2\,|2A|^4$$

$$= |B|^2\,|2A|\,|2A|\,|2A|\,|2A|$$

$$= |B|^2\,2^{12}\,|A|^4$$

$$= 8^2 \times 2^{12} \times \left(\frac{1}{8}\right)^4$$

$$= \frac{2^{12}}{2^6} = 2^6$$

The positive value of the determinant of the matrix

A, whose $\left|Adj\left(Adj\left(A\right)\right)\right| = \left|\begin{pmatrix} 14 & 28 & -14 \\ -14 & 14 & 28 \\ 28 & -14 & 14 \end{pmatrix}\right|$,

JEE Main 2022

is ___________.

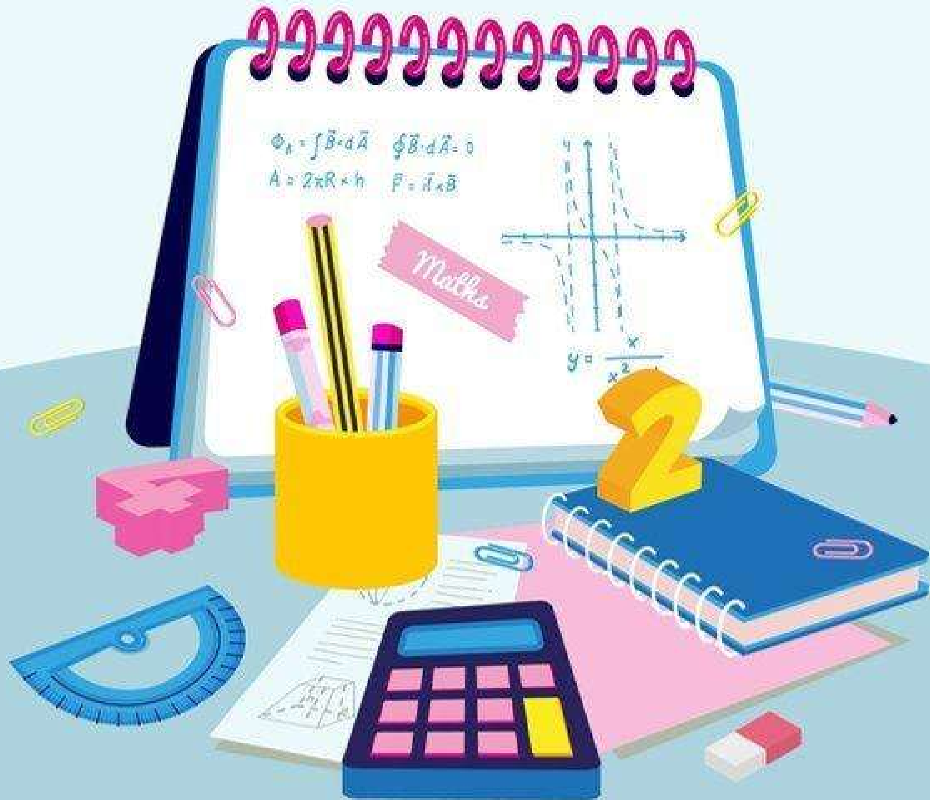
DIY

Let A be a matrix of order 3 x 3 and det (A) = 2, then det (det (A) adj (5 adj (A^3))) is equal to_.

A. 512×10^6
B. 256×10^6
C. 1024×10^6
D. 256×10^{11}

JEE Main 2022

DIY



Inverse of Matrix

Definition: Inverse of A

Reciprocal/ Multiplicative Inv.

$3 \to \frac{1}{3}$

A square matrix A said to be invertible (non singular) if there exists a matrix B such that **AB = I = BA**

B is called the inverse of A and is denoted by A^{-1}. Thus $A^{-1} = B \Leftrightarrow AB = I = BA$.

$A\left(\frac{adj A}{|A|}\right) = I$

$A^{-1} = \frac{adj A}{|A|}$

$(3)(?) = 1$

$(A)(?) = I$

$A\left(\frac{adj A}{|A|}\right) = I$

$A^{-1} = \frac{adj A}{|A|}$

Formula: Inverse of A

$$|A| = 0 \Rightarrow \text{Singular}$$

$$|A| \neq 0 \Rightarrow \text{Non-Singular.}$$

$$\boxed{A^{-1} = \frac{(\text{adj } A)}{|A|}}$$

Note: A^{-1} exists if A is non-singular.

$$|A| \neq 0$$

$|A| = 0 \quad A^{-1} = \text{D.N.E}$

$|A| \neq 0 \quad A^{-1} = \text{Exist}$

Shortcut: Inverse of 2 x 2

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$A^{-1} = \frac{\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}}{ad - bc}$$

$$\begin{bmatrix} 4 & 7 \\ 2 & 6 \end{bmatrix}^{-1} = \frac{\begin{bmatrix} 6 & -7 \\ -2 & 4 \end{bmatrix}}{24-14}$$

$$= \frac{1}{10}\begin{bmatrix} 6 & -7 \\ -2 & 4 \end{bmatrix}$$

Shortcut: Inverse of Diagonal matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{1} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$$

SC2

Shortcut: Inverse of 3x3 matrix

SC3

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 2 & -1 & 3 \\ 1 & 1 & 4 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj}\, A}{|A|}$$

$$\begin{array}{ccccc} 0 & 0 & 1 & 0 & 0 \\ 2 & -1 & 3 & 2 & -1 \\ 1 & 1 & 4 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 2 & -1 & 3 & 2 & -1 \end{array}$$

$$A^{-1} = \frac{\begin{bmatrix} -7 & 1 & 1 \\ -5 & -1 & 2 \\ 3 & 0 & 0 \end{bmatrix}}{3}$$

Shortcut: Inverse of 3x3 matrix

CRACK IN SECONDS!

30 SEC

Find inverse of

$$\begin{bmatrix} 0 & 0 & 1 \\ 2 & -1 & 3 \\ 1 & 1 & 4 \end{bmatrix}$$

Properties of Inverse

$A(?) = I$

$(A^k)^{-1} = (A^{-1})^k = A^{-k},\ k \in N$

$(A^T)^{-1} = (A^{-1})^T$

$(A^{-1})^{-1} = A$

$(A^T)^{-1} = (A^{-1})^T$

$|AA^{-1}| = |I|$

$|A|\,|A^{-1}| = 1$

$|A^{-1}| = \frac{1}{|A|}$

$AA^{-1} = A^{-1}A = I$

$|A^{-1}| = \frac{1}{|A|}$

$(AB)^{-1} = B^{-1}A^{-1}$

$(kA)^{-1} = \frac{1}{k}A^{-1}$

$$(AB)' = B'A'$$

$$\operatorname{adj}(AB) = (\operatorname{adj} B)(\operatorname{adj} A)$$

$$(AB)^{-1} = B^{-1}A^{-1}$$

$$(KA)^{-1} = \frac{1}{K}A^{-1}$$

For any 3×3 matrix M, let $|M|$ denote the determinant of M. Let I be the 3×3 identity matrix. Let E and F be two 3×3 matrices such that $(I - EF)$ is invertible. If $\boxed{G = (I - EF)^{-1}}$, then which of the following statements is (are) **TRUE** ?

A B C

(A) $|FE| = |I - FE||FGE|$

(B) $(I - FE)(I + FGE) = I$

(C) $EFG = GEF$

(D) $(I - FE)(I - FGE) = I$

JEE Adv. 2022

$GX = XG = I$

$G(I-EF) = (I-EF)G = I$

$G - GEF = \boxed{G - EFG = I}$

$EFG = G - I$

$|\cancel{F}||\cancel{E}| = |I-FE||\cancel{F}||G||\cancel{E}|$

$1 = |I-FE||G|$

$|G(I-FE)| = |I|$

$|G||I-FE| = 1$

$\cancel{X}G = \cancel{X}X^{-1}$

$XG = I$

$G\cancel{X} = X^{-1}\cancel{X}$

$GX = I$

(b) $(I - FE)(I + FGE)$

$$= I + FGE - FE - F\boxed{EFG}E$$

$$= I + FGE - FE - F(G - I)E$$

$$= I + \cancel{FGE} - \cancel{FE} - \cancel{FGE} + \cancel{FE}$$

$$= I$$

Let M be a 3 x 3 invertible matrix with real entries and let I denote the 3 x 3 identity matrix. If $M^{-1} = \text{adj}(\text{adj } M)$, then which of the following statements is/are ALWAYS TRUE?

~~A.~~ $M = I$ B. det M = 1   C. $M^2 = I$ D. $(\text{adj } M)^2 = I$ BCD

(JEE Adv. 2020)

$$|M^{-1}| = |\text{adj}(\text{adj } M)|$$

$$\frac{1}{|M|} = |M|^{(3-1)^2}$$

$$\frac{1}{|M|} = |M|^4$$

$$1 = |M|^5$$

$$\boxed{|M| = 1}$$

$$M^{-1} = \text{adj}(\text{adj } M)$$

$$\text{adj}(\text{adj } A) = |A|^{n-2} A$$

$$M^{-1} = \underbrace{|M|}\, M$$

$$M^{-1} = M \rightarrow M M^{-1} = M M$$

$$\boxed{I = M^2} \not\Rightarrow M = I$$

$$\frac{\text{adj } M}{|M|} = M$$

$$\text{adj } M = M$$

$$I = (\text{adj } M)^2$$

Let M and N be two 3 x 3 non-singular skew-symmetric matrices such that MN = NM. If P^T denotes the transpose of P, then $M^2N^2(M^TN)^{-1}(MN^{-1})^T$ is equal to

(JEE Adv 2011)

A. M^2

B. $-N^2$

C. $-M^2$

D. MN

Given : $M^T = -M$ and $N^T = -N$

$(MN) = (NM)$

To Find:- $MMNN(M^TN)^{-1}(MN^{-1})^T$

$\Rightarrow MM.N\boxed{NN^{-1}}(M^T)^{-1}(N^{-1})^T M^T$

$\Rightarrow MMN(M^T)^{-1}(N^T)^{-1} M^T$

$\Rightarrow MMN(N^TM^T)^{-1} M^T$

$\Rightarrow M(NM)(NM)^{-1} M^T = MM^T = M(-M) = -M^2$

$XX^{-1} = I$

$AA^{-1} = I$

Let $M = \begin{bmatrix} 0 & 1 & a \\ 1 & 2 & 3 \\ 3 & b & 1 \end{bmatrix}$ and $(\text{adj } M) = \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$ where a and b are real numbers. Which of the following options is/are correct ?

(JEE Adv. 2019)

A. $a + b = 3$

B. $\det(\text{adj } M^2) = 81$

C. $(\text{adj}M)^{-1} + \text{adj}M^{-1} = -M$

$\frac{M}{|M|} + \frac{M}{|M|}$

D. If $M \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, then $\alpha - \beta + \gamma = 3$

$$\begin{matrix} 0 & 1 & a & 0 & 1 \\ 1 & 2 & 3 & 1 & 2 \\ 3 & b & 1 & 3 & b \\ 0 & 1 & a & 0 & 1 \\ 1 & 2 & 3 & 1 & 2 \end{matrix}$$

$2 - 3b = -1$

$\therefore \boxed{b = 1}$

$-3a = -6$

$\therefore \boxed{a = 2}$

$$|M| = \begin{vmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{vmatrix} = -1(-8) + \underline{2(-5)} = -2$$

$$|\mathrm{adj}(M^2)| = |M^2|^2$$

$$= |M|^4$$

$$= (-2)^4$$

$$= 16$$

$$\neq 81$$

$$(\mathrm{adj}\, M)^{-1} = \mathrm{adj}(M^{-1}) = \frac{M}{|M|}$$

(c) $\cancel{2} \times \dfrac{M}{-\cancel{2}} = -M$

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} \beta + 2\gamma \\ \alpha + 2\beta + 3\gamma \\ 3\alpha + \beta + \gamma \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\underline{m^{-1} m} \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = m^{-1} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \frac{\operatorname{adj} m}{-2} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \frac{1}{-2} \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \frac{1}{-2} \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$