CHAPTER

1 Units and Measurements

Fundamental Quantity	Derived Quantity
The physical quantities which do not depend on any other physical quantities for their measurements.	Those quantities which can be expressed in terms of fundamental/base quantities.
E.g., Mass, Length, Time Temperature, current, luminous Intensity & mole	E.g., velocity Acceleration, force etc.,

System of Units

(*a*) **FPS System:** Here length is measured in foot, mass in pounds and time in second.

(*b*) **CGS System:** In this system, L is measured in cm, M is measured in g and T is measured in sec.

(*c*) **MKS System:** In this system, L is measured in metre, M is measured in kg and T is measured in sec.

Principle of Homogeneity

According to this, the physical quantities having same dimension can be added or subtracted with each other and for a given equation, dimensions of both sides must be same.

For eg, in equation $F = A\sqrt{m} + \frac{B}{v} + C$,

all the three terms of R.H.S have same dimension as force on L.H.S.

Dimensions

The fundamental or base quantities along with their powers needed to express a physical quantity is called dimension.
E.g.: $[F] = [MLT^{-2}]$ is dimension of force.

Usage of Dimensional Analysis

(i) To check the correctness of a given formula.

(ii) To establish relation between quantities dimensionally.

(iii) To convert the value of a quantity from one system of units to other system.

Limitations of Dimensional Analysis

(i) It does not predict the numerical value or number associated with a physical quantity in a relation

eg, $v = \frac{u}{3} + \frac{1}{5}at$ & $v = u + at$

Both are dimensionally valid.

(ii) It does not derive any relations involving trigonometric, logarithmic or exponential functions

E.g. $P = P_0 e^{-bt^2}$ cannot be derived dimensionally.

(iii) It does not give any information about dimensionally constants or nature of a quantity (vector/scalar) associated with a relation.

Significant Figure or Digits

1. **Rules to find out the number of significant figures:**

I Rule: All the non-zero digits are significant E.g. 1984 has 4 SF.

II Rule: All the zeros between two non-zero digits are significant. E.g. 10806 has 5 SF.

III Rule: All the zeros to the left of first non-zero digit are not significant. E.g.00108 has 3 SF.

IV Rule: If the number is less than 1, zeros on the right of the decimal point but to the left of the first non-zero digit are not significant. E.g. 0.002308 has 4 SF.

V Rule: The trailing zeros (zeros to the right of the last non-zero digit) in a number with a decimal point are significant. E.g. 01.080 has 4 SF.

VI Rule: The trailing zeros in a number without a decimal point are not significant e.g. 010100 has 3 SF. But if the number comes from some actual measurement then the trailing zeros become significant. E.g. m = 100 kg has 3 SF.

VII Rule: When the number is expressed in exponential form, the exponential term does not affect the number of S.F. For example in $x = 12.3 = 1.23 \times 10^1 = .123 \times 10^2 = 0.0123 \times 10^3 = 123 \times 10^{-1}$, each term has 3 SF only.

2. **Rules for arithmetical operations with significant figures:**

I Rule: In addition or subtraction the number of decimal places in the result should be equal to the number of decimal places of that term in the operation which contain lesser number of decimal places. E.g. 12.587 – 12.5 = 0.087 = 0.1 ($\because$ second term contain lesser i.e. one decimal place)

II Rule: In multiplication or division, the number of SF in the product or quotient is same as the smallest number of SF in any of the factors. E.g. 5.0 × 0.125 = 0.625 = 0.62.

Rounding Off

Rules for rounding off the numbers:

I Rule: If the digit to be rounded off is more than 5, then the preceding digit is increased by one. e.g. $6.87 \approx 6.9$

II Rule: If the digit to be rounded off is less than 5, than the preceding digit is unaffected and is left unchanged. e.g. $3.94 \approx 3.9$

III Rule: If the digit to be rounded off is 5 then the preceding digit is increased by one if it is odd and is left unchanged if it is even. e.g. $14.35 \approx 14.4$ and $14.45 \approx 14.4$

Representation of Errors

1. Mean absolute error is defined as

$$\overline{\Delta a} = \frac{|\Delta a_1| + |\Delta a_2| + ... + |\Delta a_n|}{n} = \sum_{i=1}^{n} \frac{|\Delta a_i|}{n}$$

Final result of measurement may be written as:

$$a = a_m \pm \overline{\Delta a}$$

2. **Relative Error or Fractional Error:** It is given by

$$\frac{\overline{\Delta a}}{a_m} = \frac{\text{Mean absolute Error}}{\text{Mean value of measurement}}$$

3. **Percentage Error** $= \dfrac{\overline{\Delta a}}{a_m} \times 100\%$

Combination of Errors

(i) **In Sum:** If $Z = A + B$, then $\Delta Z = \Delta A + \Delta B$.

Maximum fractional error in this case is

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A+B} + \frac{\Delta B}{A+B}$$

(ii) **In Difference:** If $Z = A - B$, then maximum absolute error is $\Delta Z = \Delta A + \Delta B$ and maximum fractional error in this case

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A-B} + \frac{\Delta B}{A-B}$$

(iii) **In Product:** If $Z = AB$, then the maximum fractional error,

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

(iv) **In Division:** If $Z = A/B$, then maximum fractional error is

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

(v) **In Power:** If $Z = A^n$ then $\dfrac{\Delta Z}{Z} = n\dfrac{\Delta A}{A}$

In more general form if $Z = \dfrac{A^x B^y}{C^q}$

then the maximum fractional error in Z is

$$\frac{\Delta Z}{Z} = x\frac{\Delta A}{A} + y\frac{\Delta B}{B} + q\frac{\Delta C}{C}$$

To Find Smaller Measurements

Vernier Calliper

(i) Least count: Suppose movable Jaw is slided till the zero of vernier scale coincides with any of the mark of the main scale.

Let, n V.S.D = $(n - 1)$ MSD $\Rightarrow 1\text{VSD} = \left(\dfrac{n-1}{n}\right)$ M.S.D

$\therefore$ Vernier constant = 1 M.S.D – 1 V.S.D

$$= \left[1 - \frac{n-1}{n}\right]\text{MSD} = \frac{1}{n}\text{MSD}$$

(ii) Total reading = MSR + VSR

= MSR + n ×VC

where MSR = Main scale reading

VC = Vernier constant i.e. least count

n = n^{th} division of vernier scale coinciding with main scale.

Screw Gauge

This instrument works on the principle of micro-meter screw. It is used to measure very small (mm) measurements. It is provided with linear scale and a circular scale.

(i) Pitch of the screw gauge

$$= \frac{\text{Distance moved in } n\text{-rotation of cir-scale}}{\text{No. of full-rotation}}$$

(ii) L.C $= \dfrac{\text{Pitch}}{\text{Total number of division on the circular scale}}$

(iii) Total Reading (T.R) = L.S.R + C.S.R

L.S.R = Linear scale Reading = N where

C.S.R = Circular Scale Reading = $n \times$ L.C

If n^{th} division of circular scale coincides with the linear scale line, then

$\therefore$ Total reading = $N + n \times$ (L.C)