Topics to be covered

1) Work and Calculation of Work Done by Different Forces
2) Work Energy Theorem
3) Conservative and Non-Conservative Forces
4) Potential Energy
5) Conservation of Mechanical Energy
6) Vertical Circular Motion
7) Power

Important Points

1. Keep bottle of water with you.
2. Study on table chair.
3. Wear shoes and dress up before sitting for study.
4. Take regular breaks when you feel tired.
5. The objective is to

   UNDERSTAND THE LECTURE AND NOT TO MERELY COMPLETE IT.

↳ Center of Mass

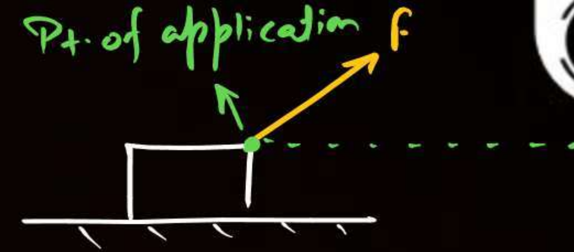
↳ Rotation

↳ Gravitation

↳ Electrostatic Pot. & Capacitance

Work

$$\text{Work} = \int \vec{F} \cdot d\vec{S}$$



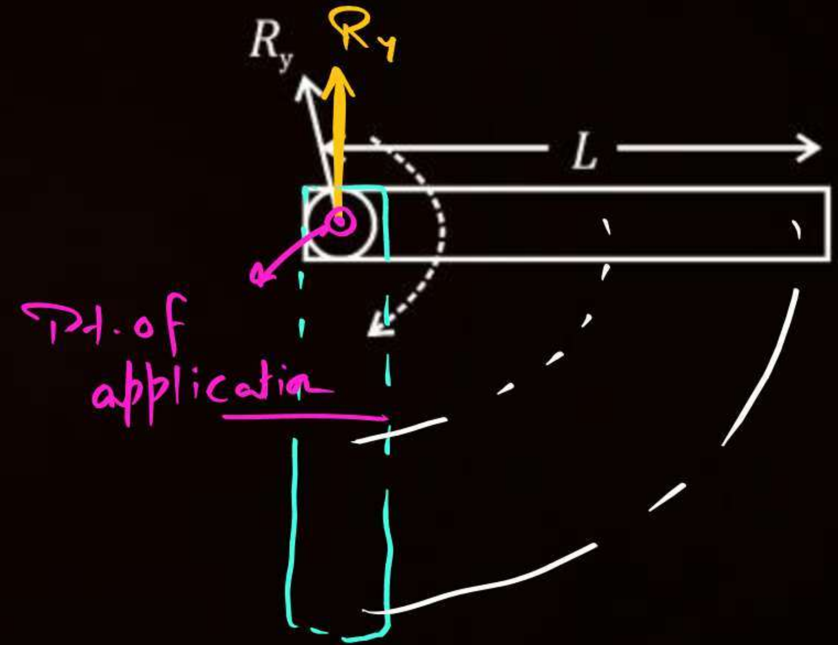
- Scalar
- SI unit : J
- $[W] = [ML^2T^{-2}]$

$d\vec{S}$ = Displacement of pt. of application of force.

QUESTION

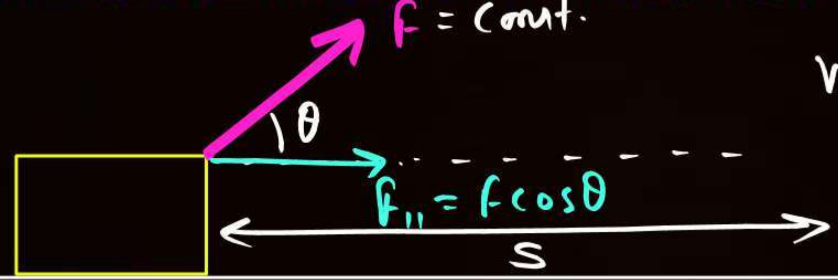
Find W.D. by R_y when rod becomes vertical.

$W_{R_y} = 0\ J$



Work Done by Different Forces

1. **Work done by constant force making angle θ with displacement:**



$$W = \int \vec{F} \cdot d\vec{S}$$

$$= \int F \, dS . \cos\theta$$

$$= F . \cos\theta . \int dS$$

$$W = F S \cos\theta$$

$$W = F_{\parallel} \cdot S$$

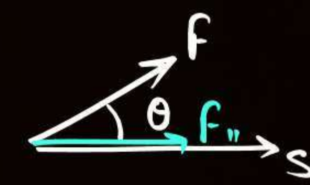
$$W = F \cdot S_{\parallel}$$

$$W = f \cdot s \cdot \cos\theta$$

(i) If $\theta = 0 \rightarrow W = f \cdot s \cdot \cos 0^\circ$

$W = f \cdot s$ — Max. work

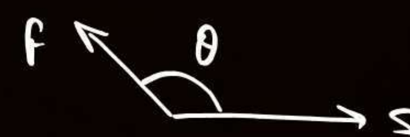
If $W > 0 \rightarrow 0^\circ \le \theta < 90^\circ$



(ii) $\theta = 90^\circ \longrightarrow W = f \cdot s \cdot \cos 90^\circ$

$W = 0$

If $W < 0 \rightarrow 90^\circ < \theta \le 180^\circ$



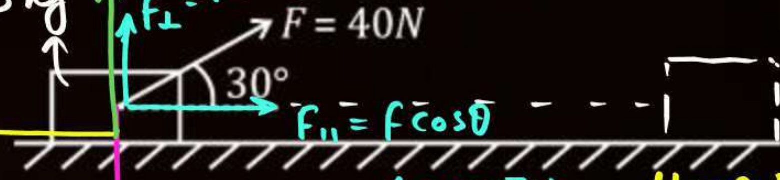
(iii) $\theta = 180^\circ \longrightarrow W = f \cdot s \cdot \cos 180^\circ$

$W = -f \cdot s$

QUESTION

fr. की औकात $\rightarrow fr \le \mu N$
$\le 0.5 \times 30 = 15N$

Find the work done by different forces when 5kg block displaces by 10m.



(A) $W_{mg} = 0J \; (mg \perp s)$

(B) $W_N = 0J \; (N \perp s)$ $\quad 15N = fr$

(C) $W_F = F.s.\cos\theta = F_{\parallel}.s = 20\sqrt{3} \times 10$
$= 200\sqrt{3}J$

(D) $W_{fr} = fr.s.\cos(180^\circ)$
$= 15 \times 10 \times (-1) = -150J.$

Common mistake.

$W_{fr} = -\mu mg s$
$= -0.5 \times 50 \times 10$
$= -250J$

Work Done by Different Forces

2. Work done by constant force vector:

$$F = F_x \hat{i} + F_y \hat{j}$$

$$W = \int \vec{F} \cdot d\vec{s}$$

$$W = \vec{F} \cdot \int d\vec{s}$$

$$W = \vec{F} \cdot \vec{s}$$

QUESTION

Const $\vec{F}$ Time pass करने

A force $\vec{F} = (2\hat{\imath} + 8\hat{\jmath} - \hat{k})$N displaces a 2kg block from $\underbrace{(0, 2, -4)}_{r_i}$m to $\underbrace{(4, 1, 2)}_{r_f}$m. What is the work done by force?

$$\vec{F} = 2\hat{i} + 8\hat{j} - \hat{k}$$

$$\vec{S} = \vec{r}_f - \vec{r}_i$$

$$= (4\hat{i} + \hat{j} + 2\hat{k}) - (0\hat{i} + 2\hat{j} - 4\hat{k})$$

$$\vec{S} = 4\hat{i} - 1\hat{j} + 6\hat{k}$$

$$W = \vec{F} \cdot \vec{S}$$

$$= (2\hat{i} + 8\hat{j} - \hat{k}) \cdot (4\hat{i} - \hat{j} + 6\hat{k})$$

$$= 2 \times 4 + 8 \times -1 + (-1) \times 6$$

$$= 8 - 8 - 6$$

$$= -6 \text{ J.}$$

QUESTION

A small particle moves to position $\overbrace{5\hat{\imath}-2\hat{\jmath}+\hat{k}}^{\vec{r}_f}$ from its initial position $\underbrace{2\hat{\imath}+3\hat{\jmath}-4\hat{k}}_{\vec{r}_i}$ under the action of force $\underbrace{5\hat{\imath}+2\hat{\jmath}+7\hat{k}N}_{\vec{F}}$. The value of work done will be 40 J.

[1 Feb, 2023 (Shift-I)]

$$\vec{F} = 5\hat{i}+2\hat{j}+7\hat{k}$$

$$\vec{S} = \vec{r}_f - \vec{r}_i$$

$$= (5\hat{i}-2\hat{j}+\hat{k}) - (2\hat{i}+3\hat{j}-4\hat{k})$$

$$\vec{S} = 3\hat{i}-5\hat{j}+5\hat{k}$$

$$W = \vec{F}\cdot\vec{S}$$

$$= (5\hat{i}+2\hat{j}+7\hat{k})\cdot(3\hat{i}-5\hat{j}+5\hat{k})$$

$$= 5\times3+2\times(-5)+7\times5$$

$$= 15-10+35$$

$$W = 40\,J$$ Ans.

Work Done by Different Forces

3. **Work done by variable force acting along/against displacement:**

$$W = \int \vec{F} \cdot d\vec{s}$$

$$W = \int F \cdot dx$$

QUESTION

Find work done by force $F = (3x^2 - x + 3)$N in displacing ~~displaying~~ a body from $x = 1$m to $x = 3$.

$$W = \int F \cdot dx$$

$$= \int_1^3 (3x^2 - x + 3)\, dx$$

$$= \left[\cancel{3} \times \frac{x^3}{\cancel{3}} - \frac{x^2}{2} + 3x\right]_1^3$$

$$= \left[3^3 - \frac{3^2}{2} + 3 \times 3\right] - \left[1^3 - \frac{1^2}{2} + 3 \times 1\right]$$

$$= 36 - \frac{9}{2} - 1 + \frac{1}{2} - 3 = 32 - 4 = 28\text{ J}$$

QUESTION

A force $\underbrace{F = (5 + 3y^2)}$ acts on a particle in the y direction, where F is newton and y is in meter. The work done by the force during a displacement from $\underbrace{y = 2\text{m}}$ to $\underbrace{y = 5\text{m}}$ is $\underline{132\,J}$.

[1 Feb, 2023 (Shift-II)]

$$f = 5 + 3y^2$$

$$W = \int_2^5 f \cdot dy = \int_2^5 (5 + 3y^2)\, dy = \left[5y + \cancel{3} \times \frac{y^3}{\cancel{3}}\right]_2^5$$

$$= (5 \times 5 + 5^3) - (5 \times 2 + 2^3)$$

$$= 25 + 125 - 10 - 8$$

$$= 132\,J \text{ Ans.}$$

Work Done by Different Forces

4. Work done by variable force vector:

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

$$d\vec{s} = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

$$W = \int \vec{F} \cdot d\vec{s}$$

$$W = \int (F_x\hat{i} + F_y\hat{j} + F_z\hat{k}) \cdot (dx\hat{i} + dy\hat{j} + dz\hat{k})$$

$$W = \int F_x dx + F_y dy + F_z dz$$

QUESTION

A force $F = (2x\hat{\imath} + 3y^2\hat{\jmath})$N displaces a body from $\underbrace{\text{origin}}_{r_i}$ to $\underbrace{(4, 6)}_{r_f}$. Find work done by the force.

$$\vec{F} = 2x\hat{i} + 3y^2\hat{j}$$

$$W = \int F_x dx + F_y dy$$

$$= \int_{(0,0)}^{(4,6)} 2x\,dx + 3y^2\,dy$$

$$= \left[\cancel{2}\cdot\frac{x^2}{\cancel{2}} + \cancel{3}\cdot\frac{y^3}{\cancel{3}}\right]_{(0,0)}^{(4,6)}$$

$$W = (4^2 + 6^3) - 0$$

$$= 16 + 36\times 6$$

$$= 16 + 216$$

$$= 232\text{ J}$$

Ans

QUESTION

$\quad\quad f_x \quad f_y$

Consider a force $\vec{F} = (-x)\hat{\imath} + (y)\hat{\jmath}$. The work done by this force in moving a particle from point A(1, 0) to B(0, 1) along the line segment is: (all quantities are in SI units)

[9 Jan. 2020 I]

(A) 2 J

(B) 1/2 J

(C) 1 J

(D) 3/2 J

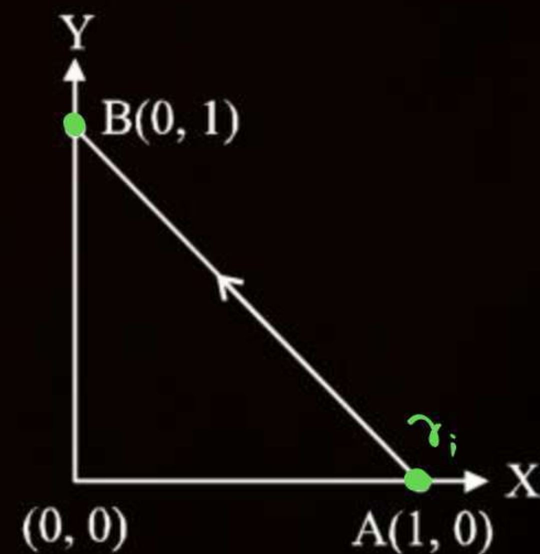
$$W.D = \int f_x\,dx + f_y\,dy$$

$$= \int -x\,dx + y\,dy$$

$$= \left[-\frac{x^2}{2} + \frac{y^2}{2}\right]_{(1,0)}^{(0,1)}$$

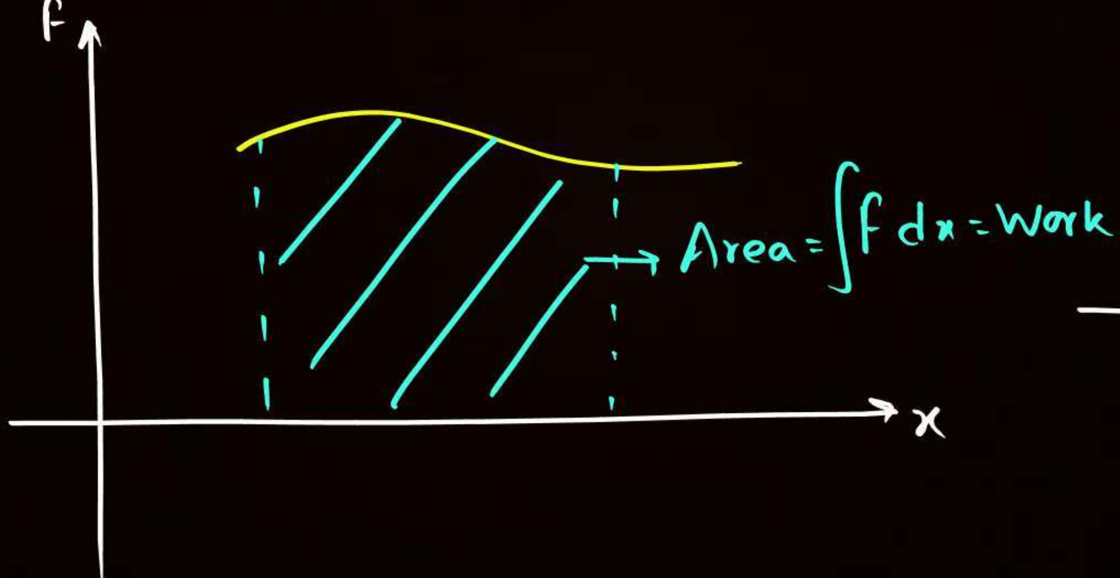
$$= \left(0 + \frac{1}{2}\right) - \left(-\frac{1}{2} + 0\right)$$

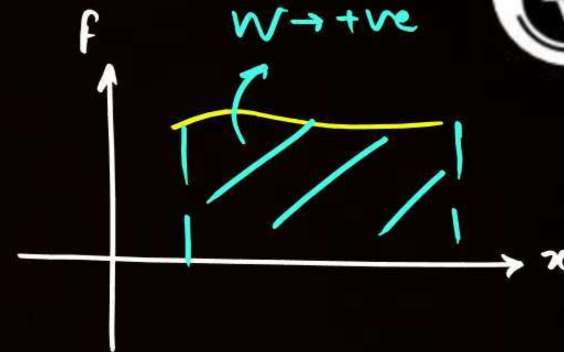
$$= 1\,J$$

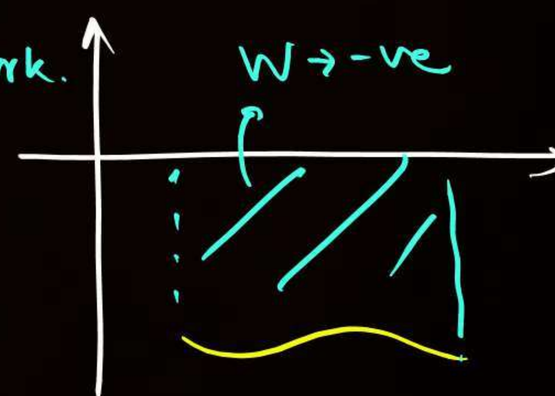


Work Done by Different Forces

5. Calculation of work from graph:





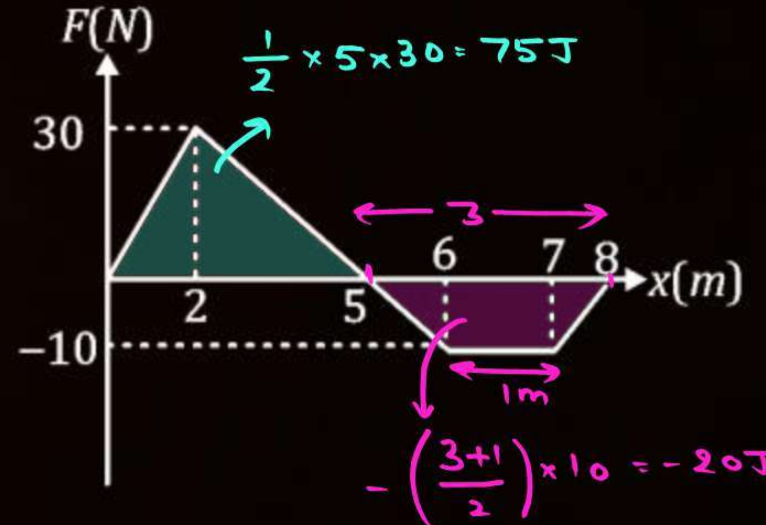


QUESTION

Find work done by force between $x = 0$ & $x = 8$.

$W = +75 - 20$

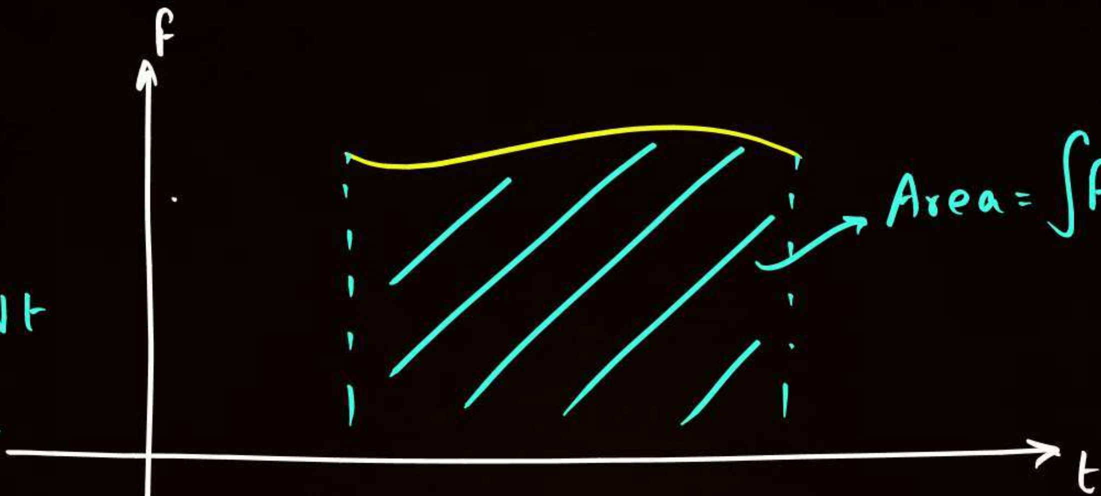
$W = 55\,J$ Ans



$$F = \frac{dP}{dt}$$

$$\int dP = \int F \cdot dt$$

$$\Delta P = \int F\, dt$$



$$\text{Area} = \int F \cdot dt = \underbrace{\Delta P}_{\text{change in momentum}} = \underbrace{J}_{\text{Impulse.}}$$

{ Area under F-t graph gives change in momentum }.

QUESTION

Arrange the four graphs in descending order of total work done; where W_1, W_2, W_3 and W_4 are the work done corresponding to figure A, B, C and D respectively.

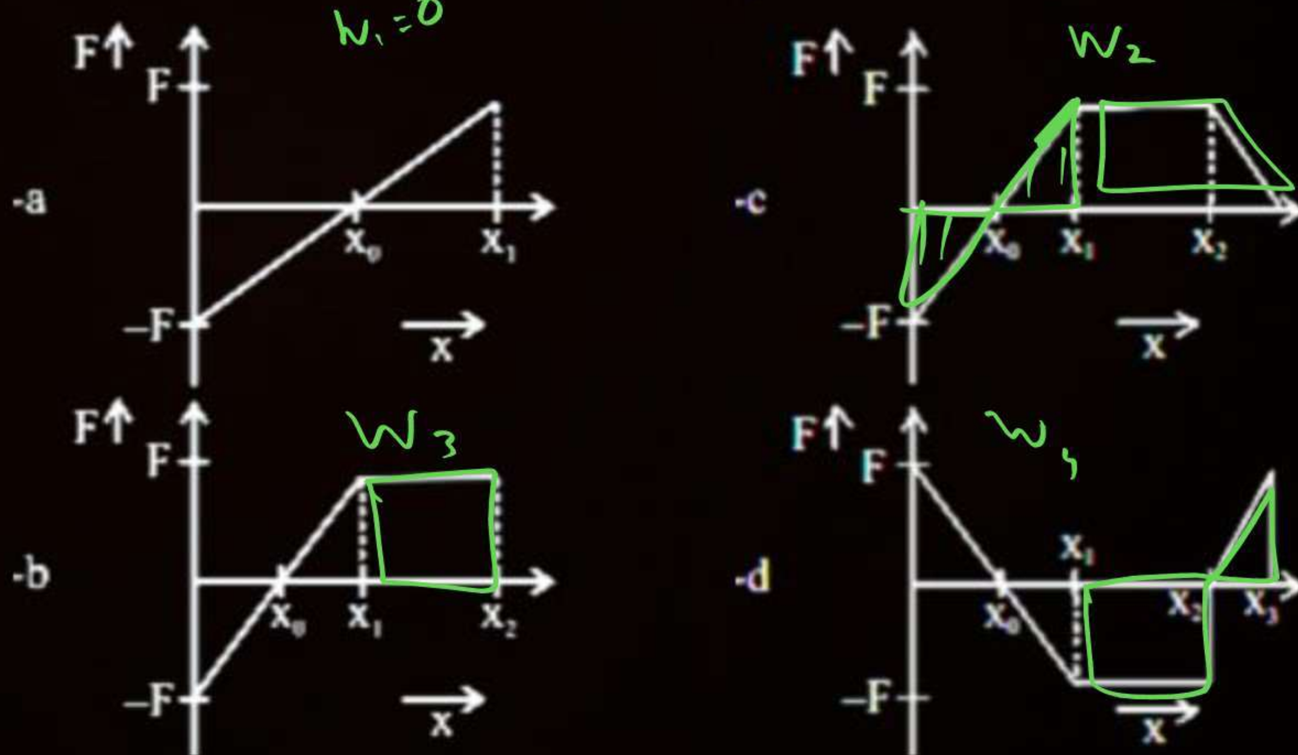
[26 June, 2022 (Shift-II)]

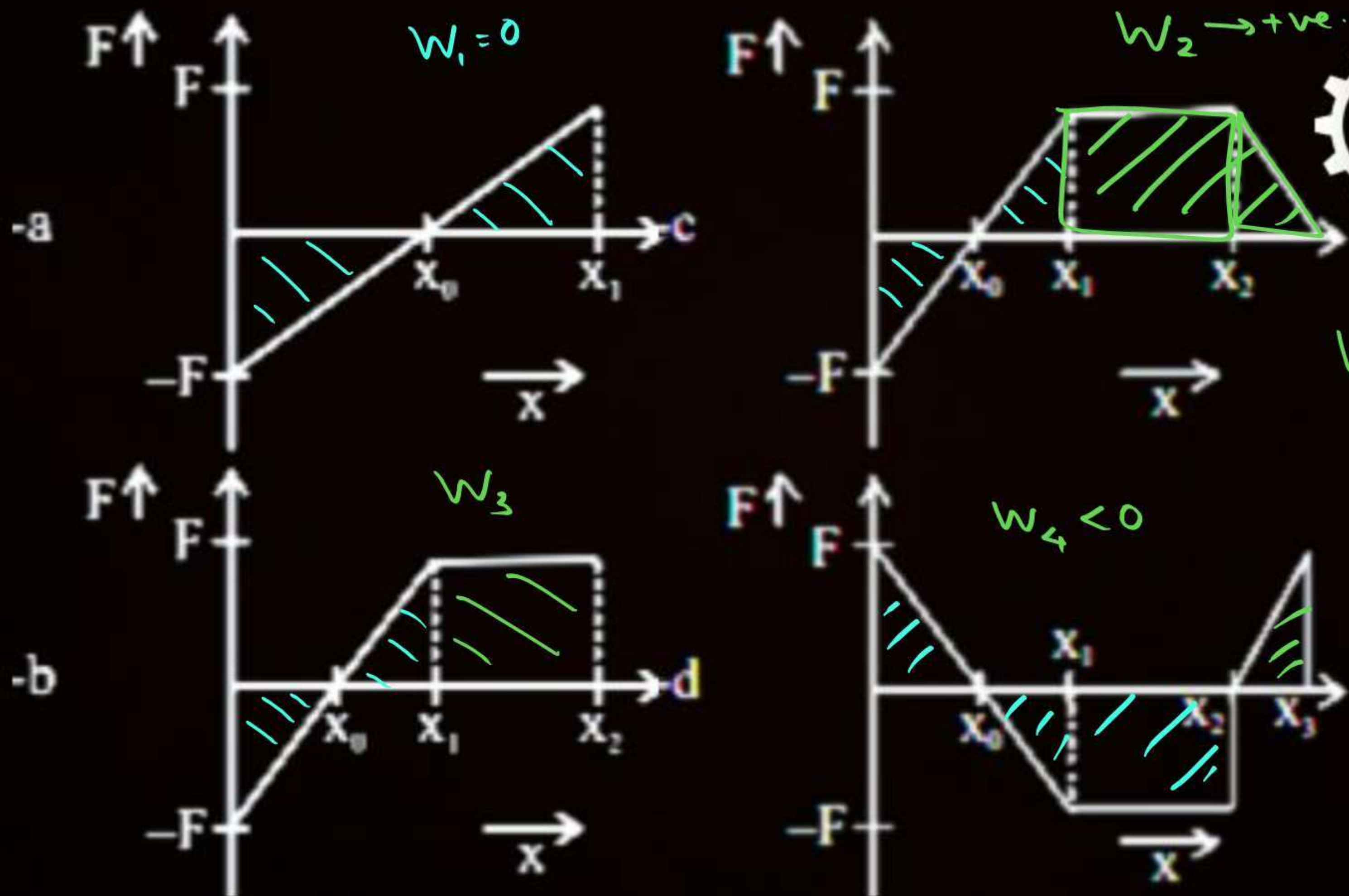
(A) $W_3 > W_2 > W_1 > W_4$

(B) $W_3 > W_2 > W_4 > W_1$

(C) $W_2 > W_3 > W_4 > W_1$

(D) $W_2 > W_3 > W_1 > W_4$



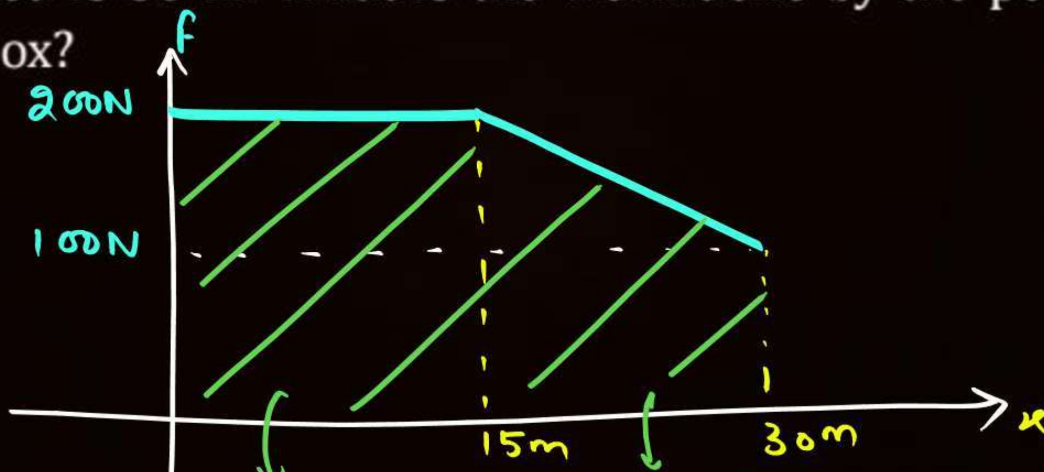


$W_2 > W_3 > W_1 > W_4$

QUESTION

A person pushes a box on a rough horizontal platform surface. He applies a force of 200 N over a distance of 15 m. Thereafter, he gets progressively tired and his applied force reduces linearly with distance to 100 N. The total distance through which the box has been moved is 30 m. What is the work done by the person during the total movement of the box?

[4 Sep. 2020 (II)]



$200 \times 15 = 3000$ J.

$\left[\frac{200+100}{2}\right] \times 15 = 150 \times 15 = 2250$ J

(A) 3280 J

(B) ~~2780 J~~

(C) 5690 J

(D) 5250 J

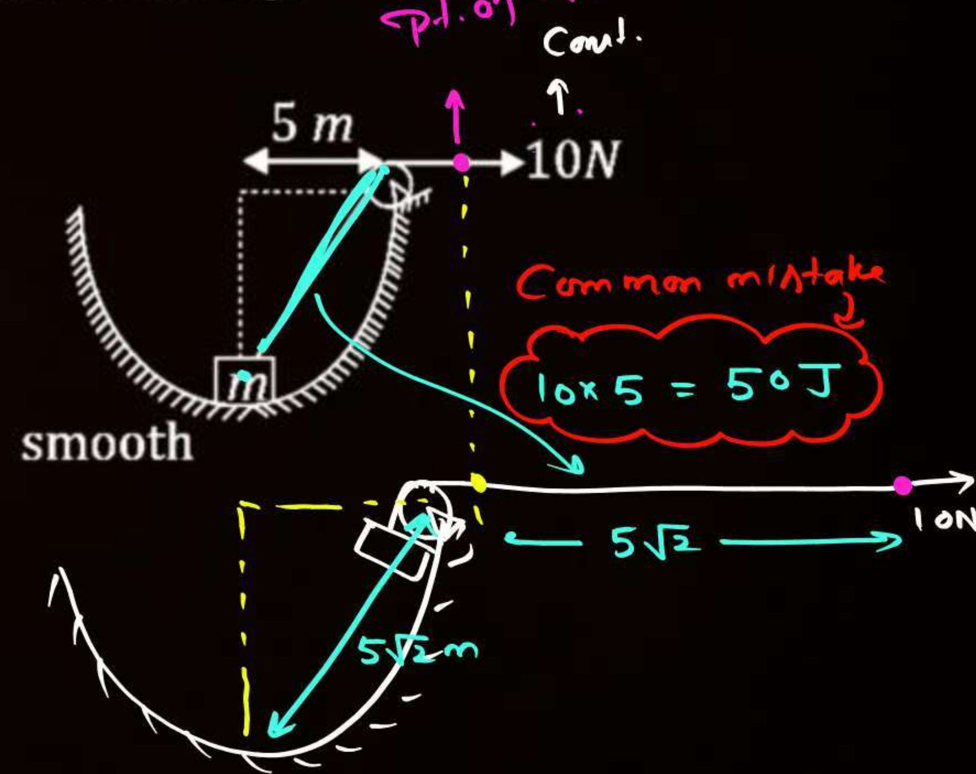
Find work done by 10N force when block reaches to the top.

a) 0 J   ~~b)~~ 50 J

c) $50\sqrt{2}$ J   d) None of them

$W = F \cdot s$

$= 10 \times 5\sqrt{2}$

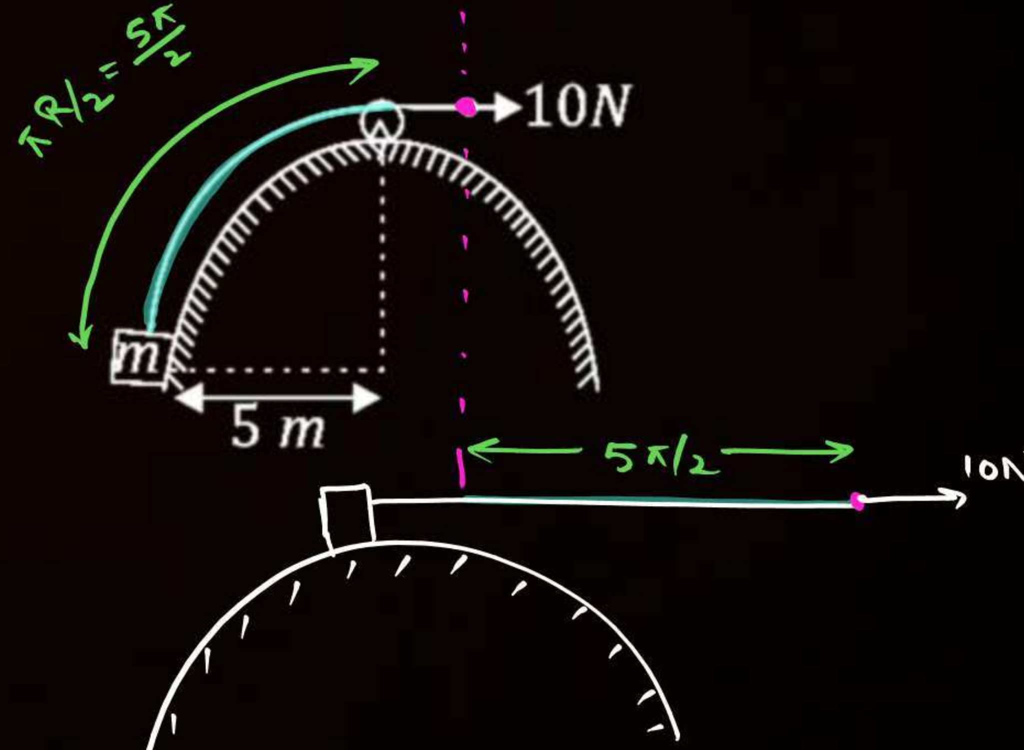


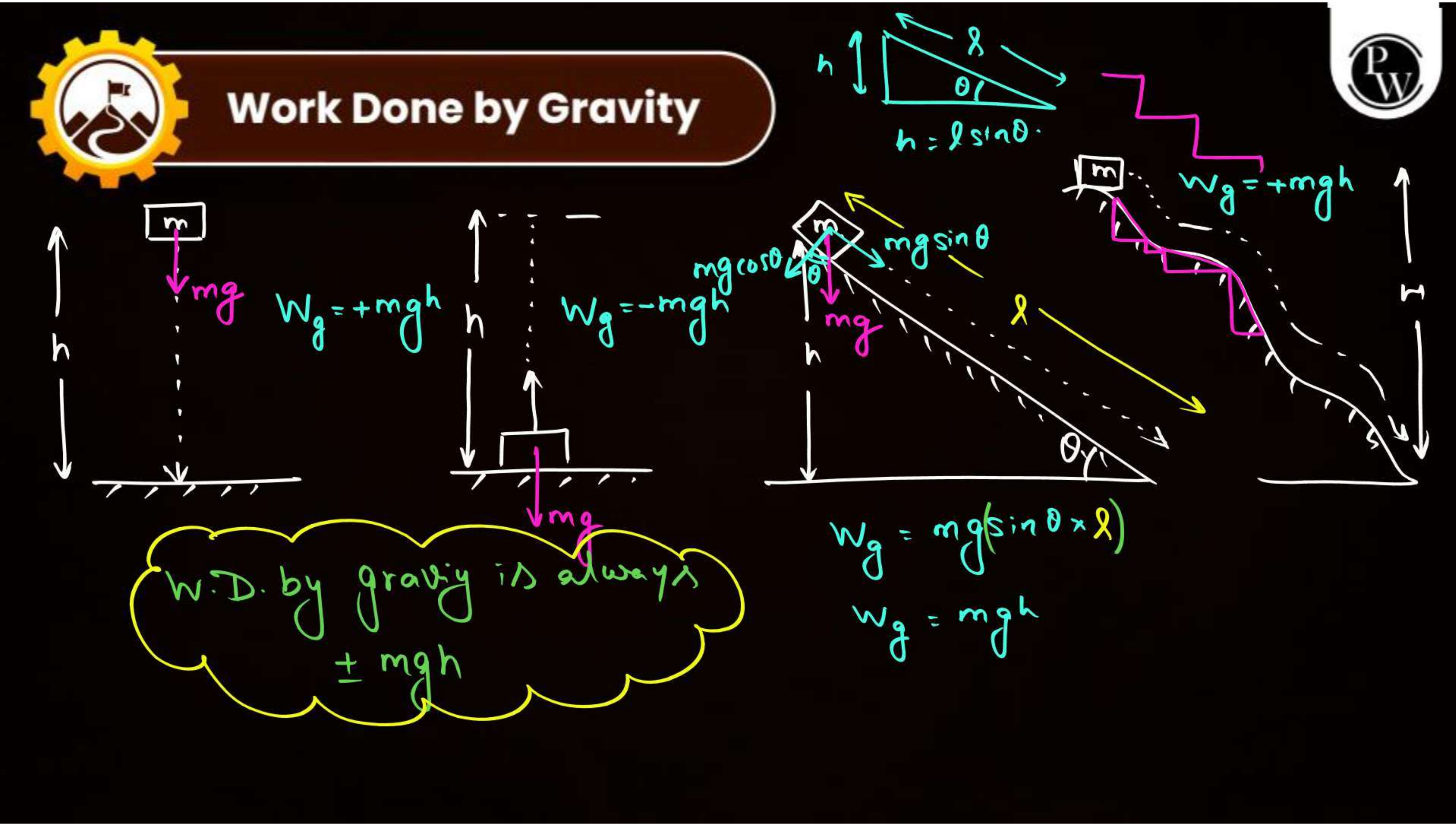
QUESTION

Find work done by 10N force when block reaches to the top.

$$W = 10 \times \frac{5\pi}{2}$$

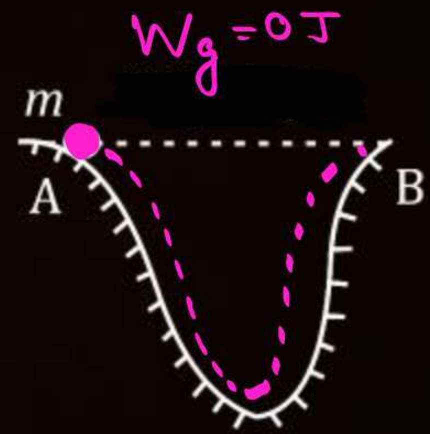
$$W = 25\pi \text{ J}$$

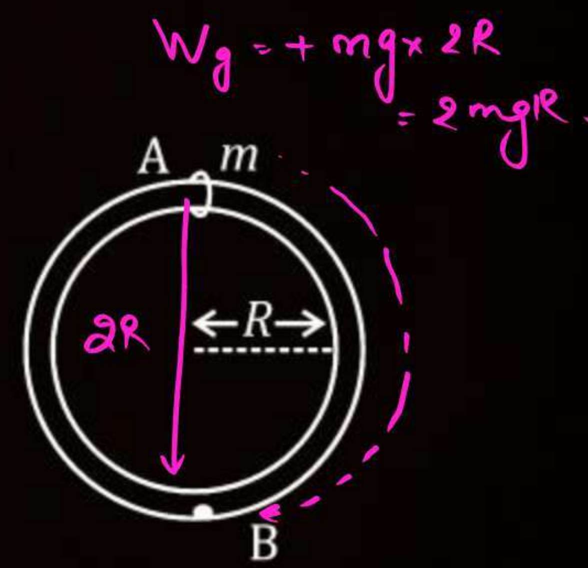



Work Done by Gravity
m
mg
h
$W_g = +mgh$
$W_g = -mgh$
$h = l \sin\theta$
l
θ
$mg\cos\theta$
$mg\sin\theta$
$W_g = mg(\sin\theta \times l)$
$W_g = mgh$
W.D. by gravity is always $\pm mgh$

QUESTION

Find work done by gravity when body reaches from point 'A' to 'B'





QUESTION

Find work done by gravity & constant force F when string makes 60° with vertical.

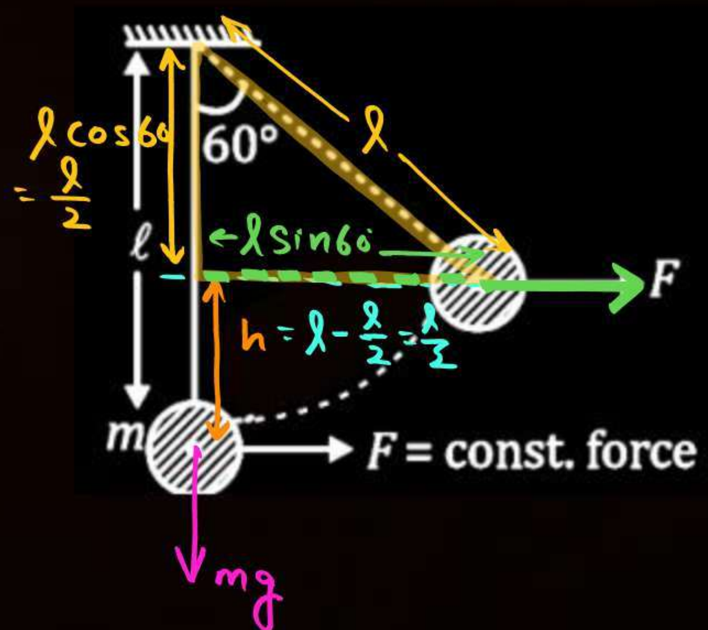
$$W_{mg} = -mgh$$

$$W_{mg} = -mg\frac{l}{2}$$

$$W_F = F \cdot S_{\parallel}$$

$$= F \cdot (l \sin 60^\circ)$$

$$= \frac{\sqrt{3}}{2} F \cdot l$$



QUESTION

(θ must be radians)

A variable force of constant magnitude acts tangentially on the pendulum as shown in figure. Find the work done by constant force F when the string makes angle θ with vertical.

$$\boxed{W_T = 0}$$

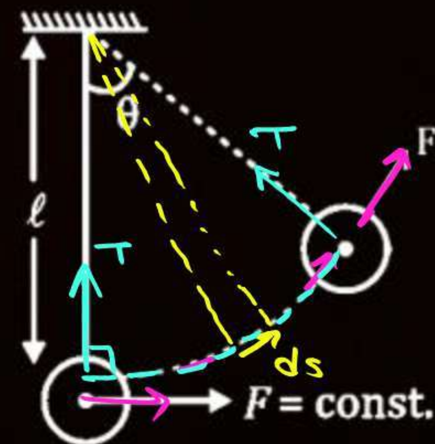
$$\int dW = \int F \cdot ds \cdot \cos 0^\circ$$

$$= F \int ds$$

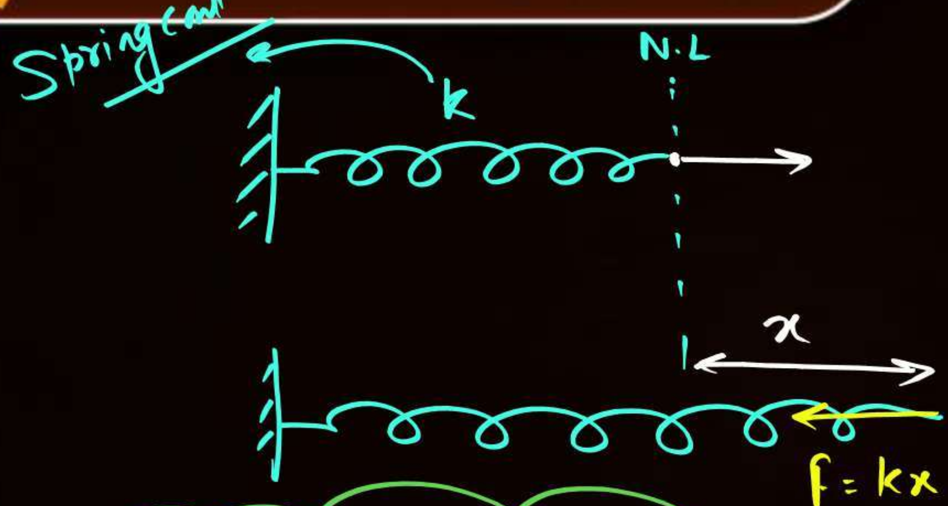
$$= F \cdot s$$

$$= F \cdot (R\theta)$$

$$W_F = F \cdot \ell\theta$$



Work Done by Spring



$$W = \int f \cdot dx \cos(180^\circ)$$
$$= \int kx \cdot dx \cos(180^\circ)$$
$$W = -\frac{1}{2} k x^2$$

$$\text{W.D. by Spring} = \pm \frac{1}{2} k x^2$$

x is the extension/compression in spring from natural length

QUESTION

Unit conversion

A block attached to a spring of spring constant $\underbrace{20\ \text{N/cm}}$ is released from rest as shown in figure. If the mass of block is $\underbrace{2\ \text{kg}}_{m}$. What is the work done by spring till block reaches its equilibrium positon?

magnitude

(A) ~~10 J~~

(B) 0.1 J ✓

(C) 1 J

(D) 0.2 J

Common mistake

$kx = mg$

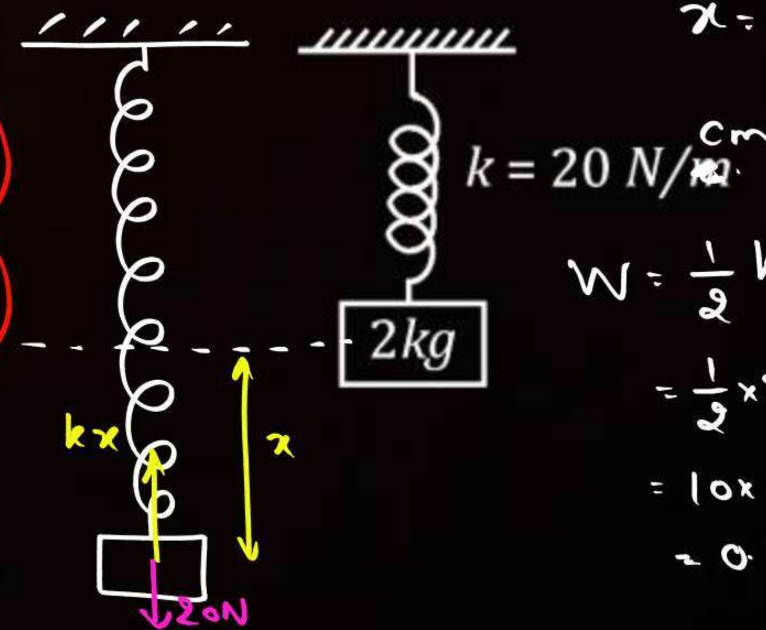
$\cancel{2}0 \times x = \cancel{2}0$

$x = 1\,\text{cm}$

$W_{spr.} = \frac{1}{2} k x^2$

$= \frac{1}{2} \times 20 \times 1^2$

$= -10\,\text{J}.$



$kx = mg$

$20 \frac{\cancel{N}}{cm} \times x = 20\cancel{N}$

$x = 1\,\text{cm}$

$W = \frac{1}{2} k x^2$

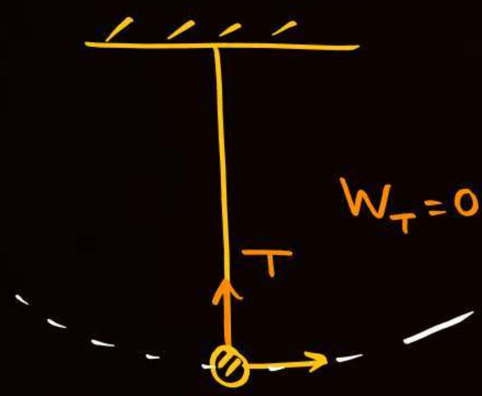
$= \frac{1}{2} \times \frac{20\,N}{\cancel{cm}} \times (1\,cm)^2$

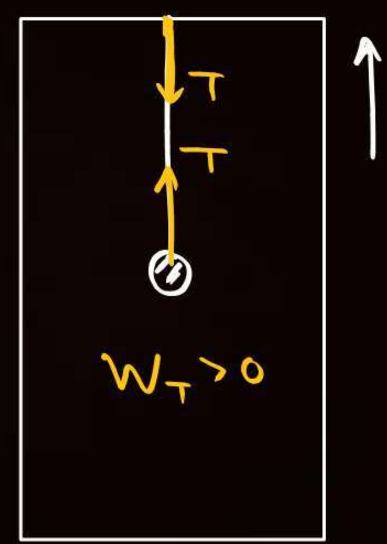
$= 10 \times 10^{-2}$

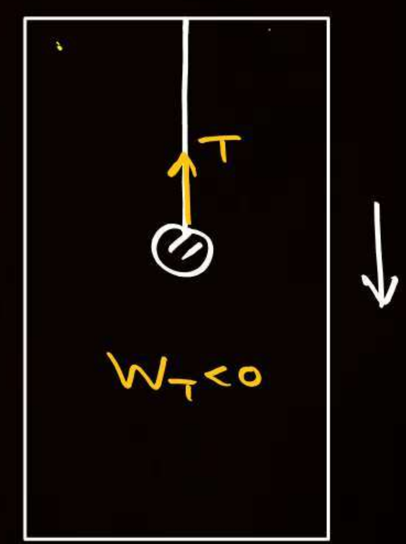
$= 0.1\,\text{J}.$

Work Done by Tension

- W.D. by tension may be +ve, -ve or zero.
- Total W.D. by string is always zero.







QUESTION

Find work done by tension in 2 sec. (Initially block is at rest.)

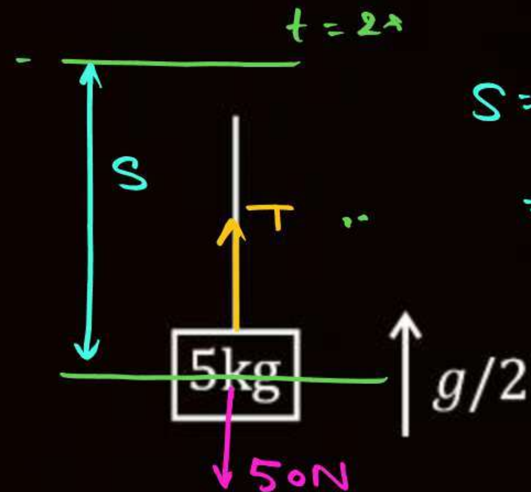
$F_{net} = ma$

$T - 50 = 5 \times 5$

$T = 75\,N \rightarrow$ const

$W = F \cdot S$

$= 75 \times 10 = 750\,J$ Ans

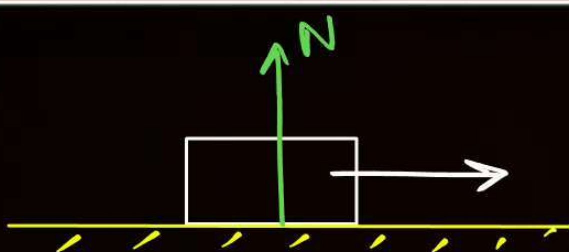


$S = ut + \frac{1}{2}at^2$

$= 0 + \frac{1}{\cancel{2}} \times 5 \times (2)^{\cancel{2}}$

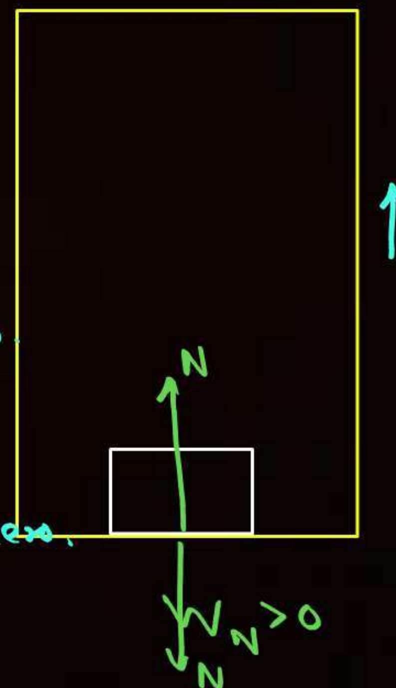
$= 10\,m$

Work Done by Normal Reaction

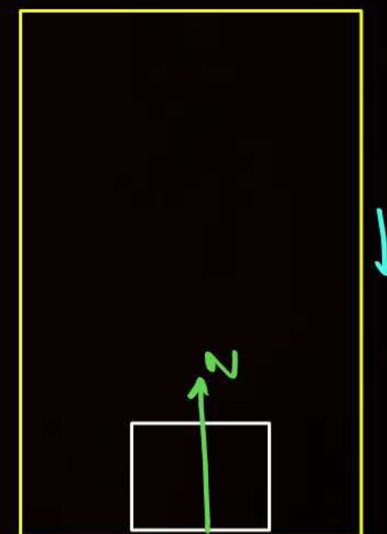


$W_N = 0$

- W.D. by N can be +ve, −ve or zero.
- Total W.D. by N. on both surfaces for perfectly rigid body is always zero.

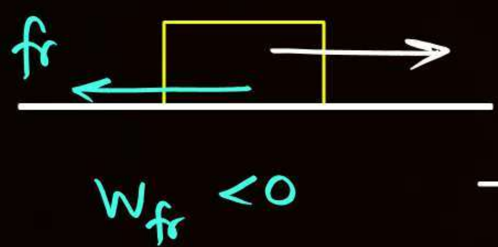


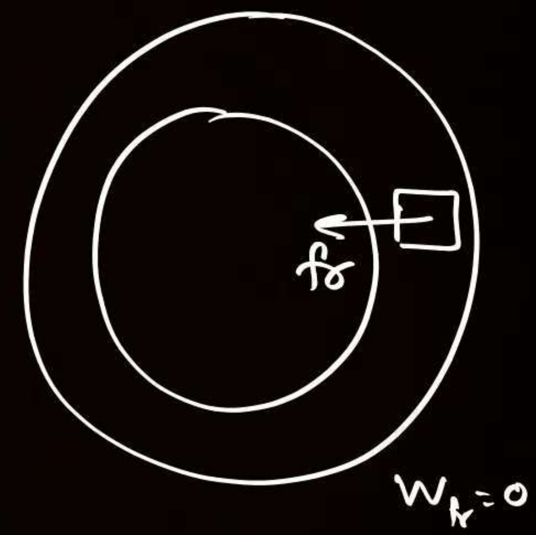
$W_N > 0$



$W_N < 0$

Work Done by Friction





- W.D. by friction may be +ve, −ve or zero.
- Total W.D. by static friction is always zero
- Total W.D. by kinetic friction is always −ve

10 min. Break.

DPP & Class notes will be available on PW app. in Manzil batch.

Kinetic Energy

$$K.E = \frac{1}{2} m v^2 = \frac{m v^2 \times m}{2m}$$

$$= \frac{m^2 v^2}{2m}$$

$$K.E = \frac{P^2}{2m} \rightarrow P = \sqrt{2m(K.E)}$$

Error / % change

$$y = x^n$$

$$\frac{\Delta y}{y} = \frac{n \cancel{x^{n-1}} \Delta x}{\cancel{x^n}}$$

Valid only if change is small (< 7%) →

$$\Rightarrow \boxed{\frac{\Delta y}{y} = n \frac{\Delta x}{x}}$$

$\frac{\Delta y}{y}$: fractional change in y

$\frac{\Delta x}{x}$: fractional change in x.

Ex. $y = x^3$

$$\frac{\Delta x}{x} \times 100 = 2\%$$

$$\frac{\Delta y}{y} \times 100 = 3 \times \left(\frac{\Delta x}{x} \times 100\right)$$

$$= 3 \times 2\%$$

$$= 6\%$$

QUESTION

If momentum of body is doubled, what will be its new Kinetic Energy

(A) $(k.\varepsilon_0)$

(B) $2(k.\varepsilon_0)$

(C) $1/2\ (k.\varepsilon_0)$

(D) $4(k.\varepsilon_0)$

$$4 \times k.\epsilon_0 = \frac{P_0^2}{2m} \quad \times 2$$

$$k.\epsilon = \frac{(2 \times P_0)^2}{2m}$$

QUESTION

The momentum of a body is increased by 2%. What will be the change in K.E of body?

(A) 2%

(B) 4%

(C) 1%

(D) 1/2%

$$K.E = \frac{p^2}{2m}$$

$$\% K.E = 2 \times (\% p)$$

$$\% K.E = 2 \times 2\%$$

$$= \underline{\underline{4\%}}$$

QUESTION

If momentum of a body is changed by $\underbrace{20\%}$ what will be the percentage change in kinetic energy?

बड़ा change

(A) 10 %

(B) 20 %

(C) 40 %

(D) 44 %

$$K.E = \frac{P^2}{2m}$$

Common mistake

$$\%\ K.E = 2 \times (\%\ P)$$
$$= 2 \times (20\%)$$
$$= 40\%$$

$$K.E_0 = \frac{P_0^2}{2m}$$

$$P = P_0 + \frac{20}{100} P_0$$
$$P = 1.2 P_0$$

$$K.E_{new} = \frac{P^2}{2m} = \frac{(1.2 P_0)^2}{2m}$$
$$= 1.44 \frac{P_0^2}{2m}$$

$$K.E_{new} = 1.44 \times K.E_0$$

Work Energy Theorem

Newton's II^{nd} Law $\longrightarrow$ $F_{net} = \frac{dP}{dt} = \frac{d(mv)}{dt}$

If $m = const$ — $F_{net} = ma$

a: $\frac{dv}{dt}$, $v\frac{dv}{dx}$

$$F_{net} = m \cdot \frac{v\,dv}{dx}$$

$$\underbrace{\int F_{net} \cdot dx} = \int_u^v m v\,dv$$

$$\text{Total work done} = \underbrace{\frac{mv^2}{2} - \frac{mu^2}{2}}_{\Delta K.E}$$

Total work done $= \Delta K.E$

QUESTION

What will be the speed of block when it comes down to the horizontal surface?

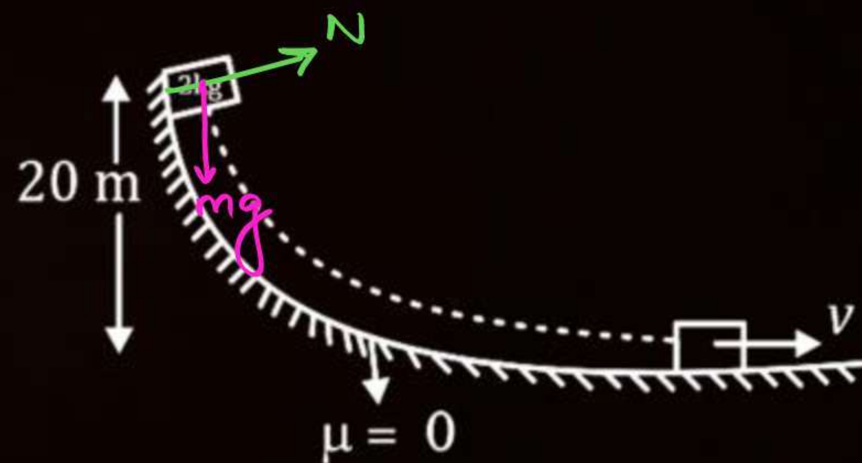
$$TWD = \Delta K.E$$

$$W_N + W_{mg} = K.E_f - K.E_i$$

$$0 + \cancel{m}g \times 20 = \frac{1}{2}\cancel{m}v^2 - 0$$

$$v = \sqrt{2 \times 10 \times 20}$$

$$v = 20\ m/s$$ Ans



Question

The spring is initially compressed by length $\ell/2$. A block of mass m is released from rest. Find the maximum height reached by block. (Block is not tied to spring)

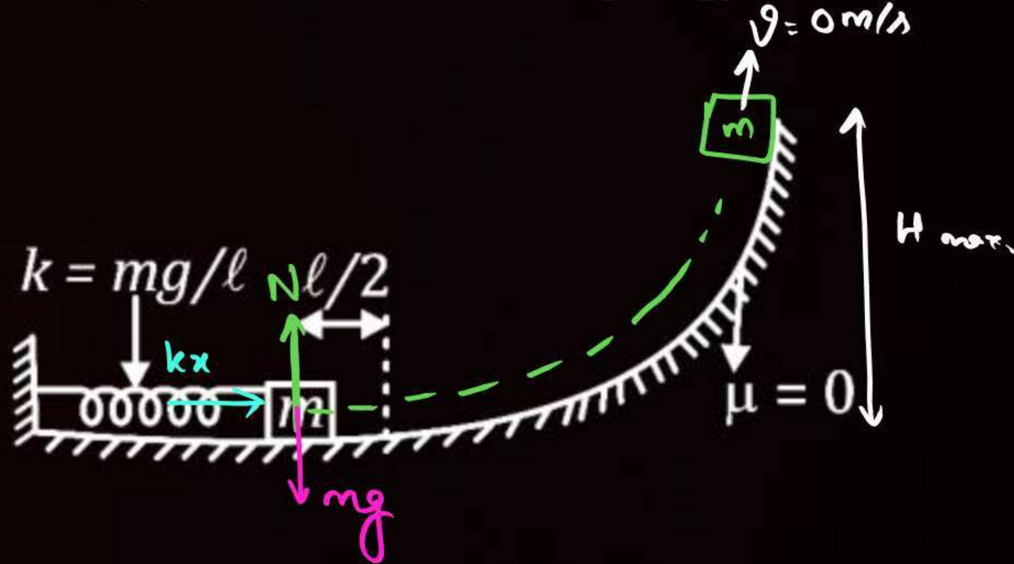
$$TWD = \Delta K.E$$

$$\Rightarrow W_{spr.} + W_N + W_{mg} = K.E_f - K.E_i$$

$$\Rightarrow +\frac{1}{2}kx^2 + 0 + (-mgH_{max}) = 0 - 0$$

$$\cancel{mg}H_{max} = \frac{1}{2} \times \frac{\cancel{mg}}{\ell} \times \left(\frac{\ell}{2}\right)^2$$

$$\boxed{H_{max} = \frac{\ell}{8}}$$ Ans



QUESTION

Find speed of 4kg block when it falls down by 24m.

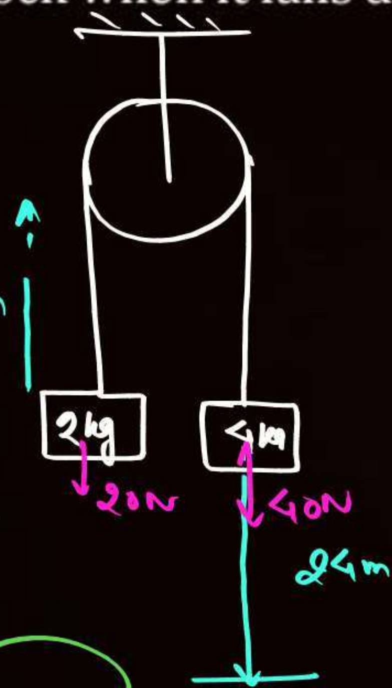
WPE

$TWD = \Delta KE$

$40 \times 24 - 20 \times 24 = \left[\frac{1}{2} \times 4 \times v^2 + \frac{1}{2} \times 2 \times v^2\right] - 0$

$20 \times \cancel{24}^{8} = \cancel{3}v^2$

$\boxed{v = \sqrt{160} \ m/s}$



NLM

$F_{net} = m_{total} \cdot a$

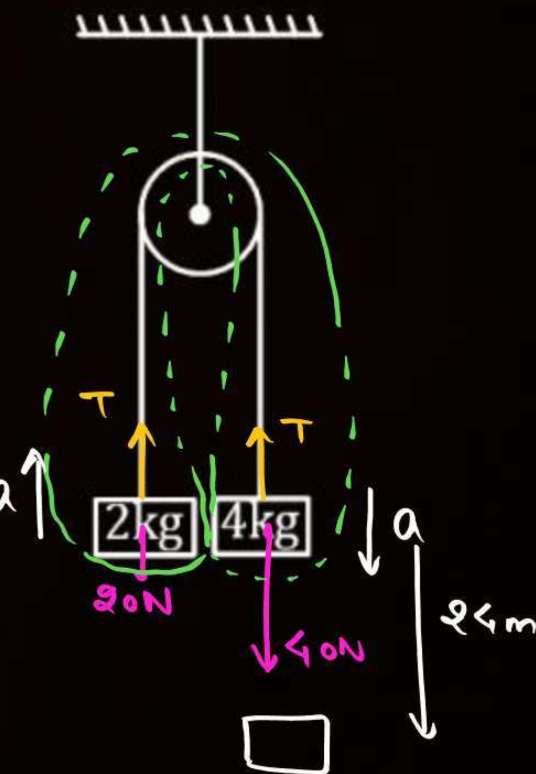
$40 - 20 = (4+2)\, a$

$a = \frac{20}{6} = \frac{10}{3} \ m/s^2$

$v^2 - u^2 = 2as$

$v^2 - 0 = 2 \times \frac{10}{\cancel{3}} \times \cancel{24}^{8}$

$v = \sqrt{160} \ m/s$



QUESTION

The speed of block when it falls down 40m is 20 m/s. Find the energy lost due to friction.

$$TWD = \Delta K.E$$

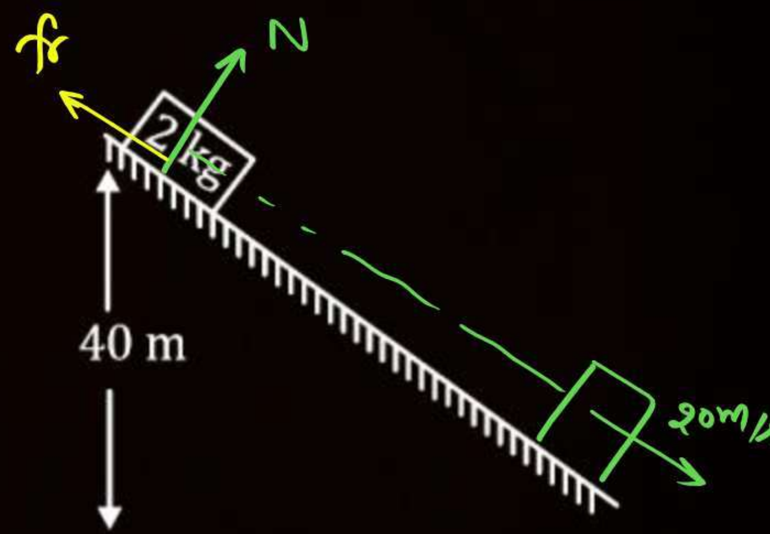
$$\Rightarrow W_{mg} + W_{fr} + W_N = K.E_f - K.E_i$$

$$\Rightarrow +mg \times 40 + W_{fr} + 0 = \frac{1}{2} \times m \times (20)^2 - 0$$

$$W_{fr} = \frac{1}{\cancel{2}} \times \cancel{2} \times 20^2 - 2 \times 10 \times 40$$

$$= 400 - 800$$

$$= -400\ J.$$



QUESTION

Find speed of bob when string becomes vertical.

$H = l - l\cos\theta$

$$TWD = \Delta K.E$$

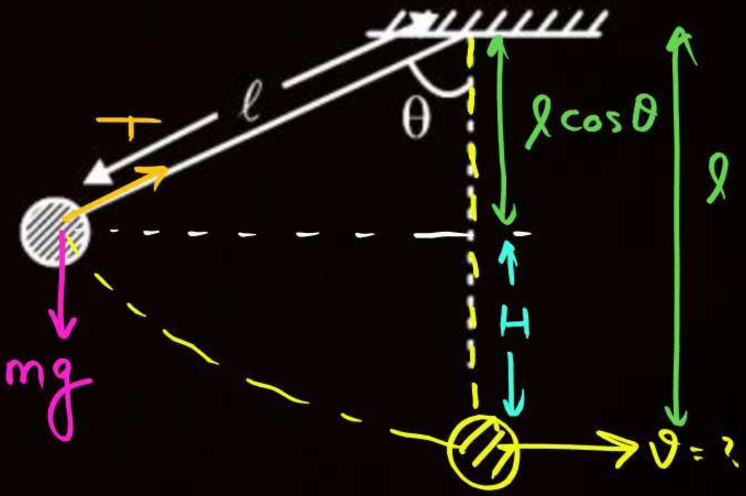
$$W_{mg} + W_T = \frac{1}{2}mv^2 - 0$$

$$\cancel{m}gH + 0 = \frac{1}{2}\cancel{m}v^2$$

$$v = \sqrt{2gH}$$

$$= \sqrt{2g(l - l\cos\theta)}$$

$$v = \sqrt{2gl(1-\cos\theta)}$$ Ans



QUESTION

- final speed is same, final vel. may be different.

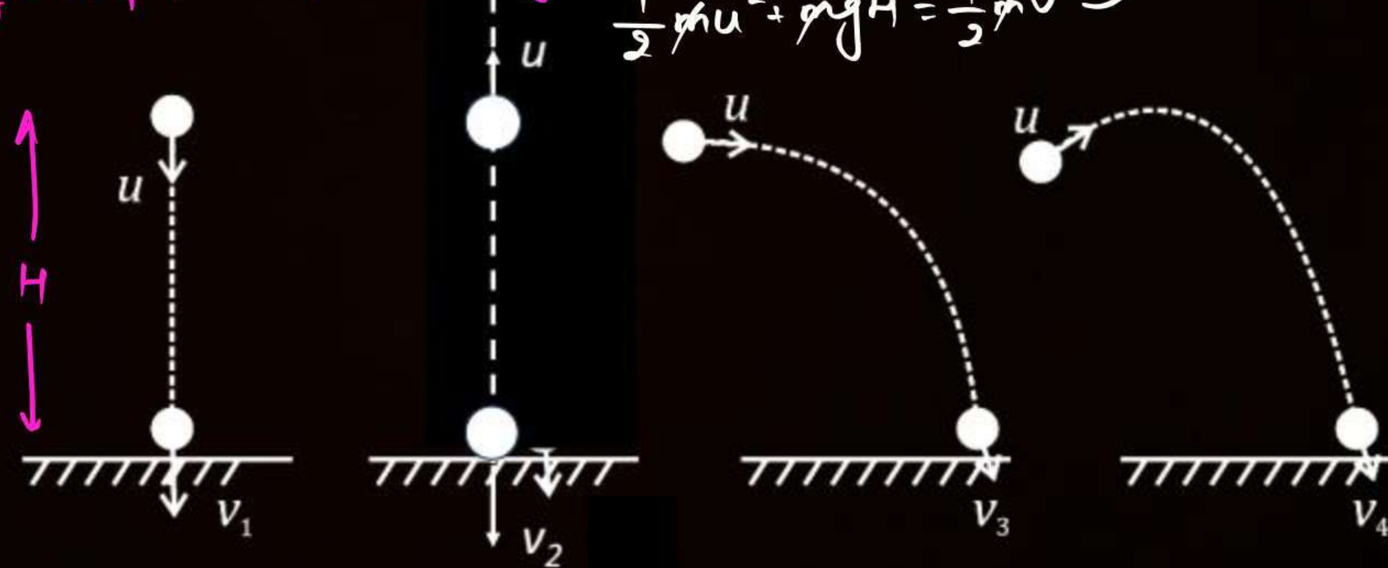
Compare final speeds v_1, v_2, v_3 and v_4. if each ball is thrown from same height.

$TWD = \Delta KE$

$mgH = K.E_f - K.E_i \Rightarrow K.E_f = \underbrace{K.E_i}_{} + \underbrace{mgH}_{}$

$v_1 = v_2 = v_3 = v_4 = \sqrt{u^2 + 2gH}$

$\frac{1}{2}\cancel{m}u^2 + \cancel{m}gH = \frac{1}{2}\cancel{m}v^2$



QUESTION

The spring is in natural length. A block attached to the spring is released. Find maximum extension in the spring.

Equillibrium

Common mistake → $mg = kx$

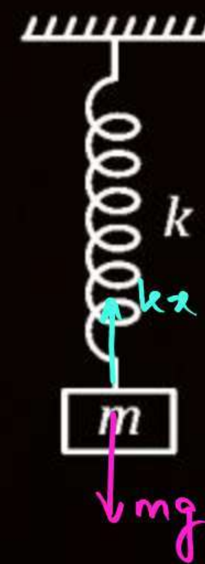
$x = mg/k$

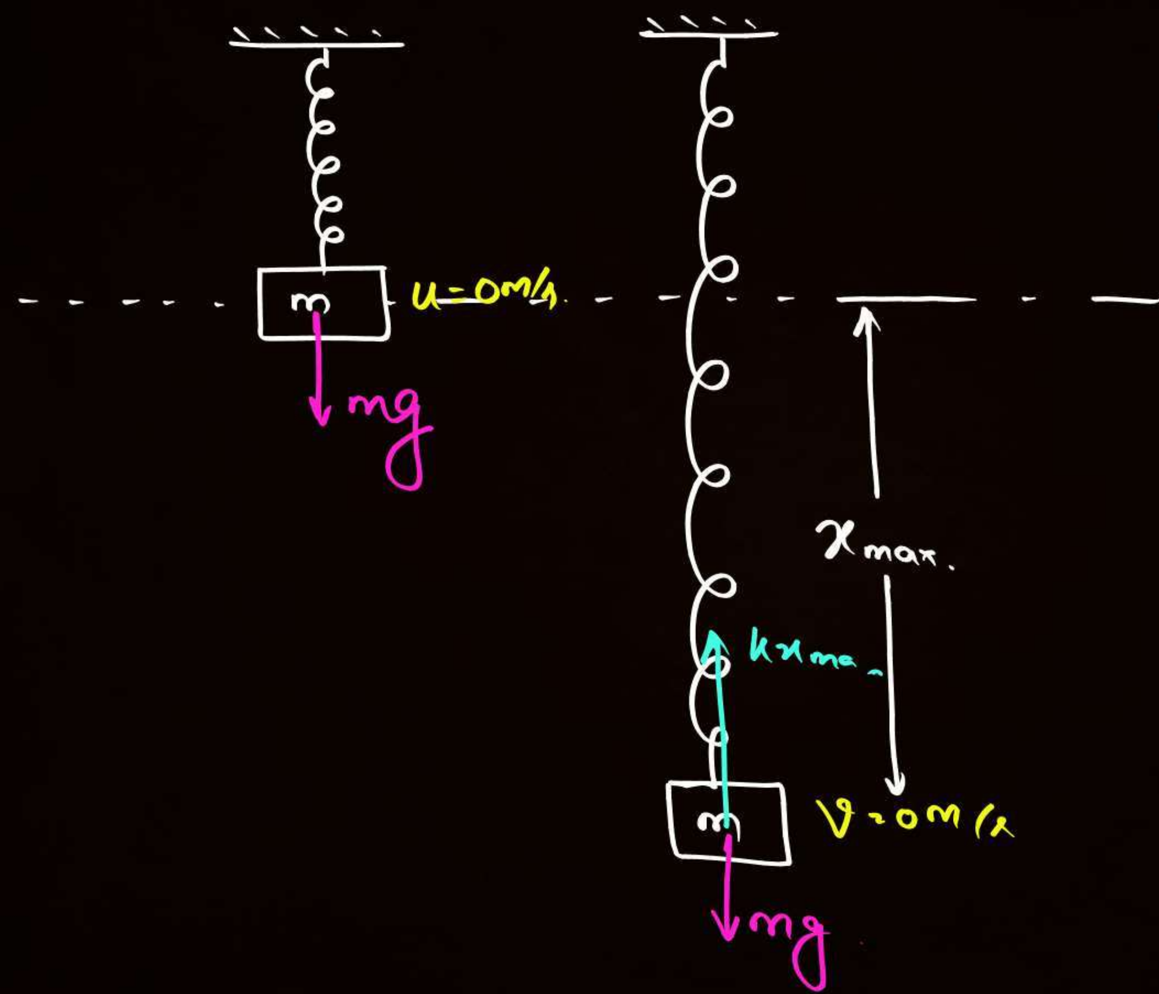
(A) mg/k

(B) $2mg/k$

(C) $4mg/k$

(D) None of these





$$TWD = \Delta K.E$$

$$W_{mg} + W_{spr.} = K.E_f - K.E_i$$

$$mg\,x_{max} - \frac{1}{2}k\,x_{max}^2 = 0 - 0$$

$$\frac{1}{2}k\,x_{max}^{\cancel{2}} = mg\,\cancel{x_{max}}$$

$$\boxed{x_{max} = \frac{2mg}{k}}$$

QUESTION

Find change in K.E. of body between t = 0 & t = 6s. Initially the body is at rest (mass = 2kg)   work done

$$K.E_f = \frac{P_f^2}{2m}$$

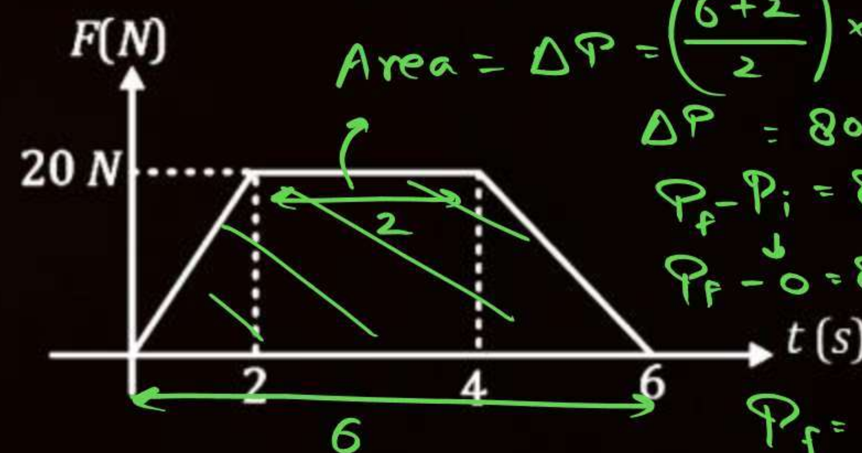
$$= \frac{(80)^2}{2 \times 2}$$

$$= \frac{6400}{4}$$

$$K.E_f = 1600\ J.$$

$$\Delta K.E = K.E_f - K.E_i = 1600 - 0$$

$$= \boxed{1600\ J}\ \text{Ans}$$



$$\text{Area} = \Delta P = \left(\frac{6+2}{2}\right) \times 20$$

$$\Delta P = 80$$

$$P_f - P_i = 80$$

$$P_f - 0 = 80$$

$$P_f = 80\ kg\,m/s.$$

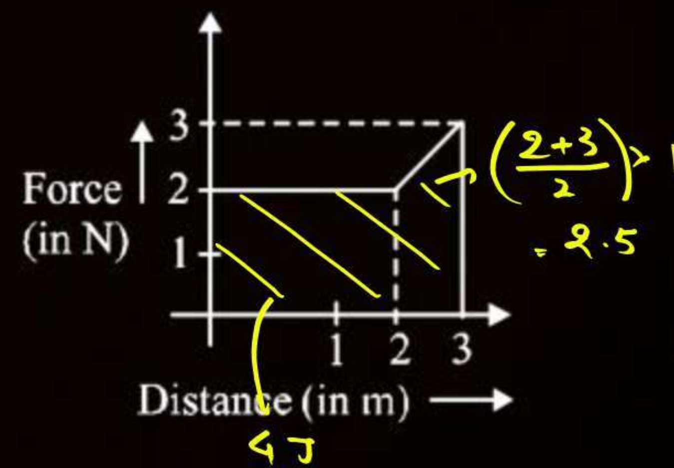
QUESTION

A particle moves in one dimension from rest under the influence of a force that varies with the distance travelled by the particle as shown in the figure. The kinetic energy of the particle after it has travelled 3 m is: [7 Jan. 2020 II]

W.D. = Area

Δk.E = 6.5

k.E = 6.5 J



(A) 4 J

(B) 2.5 J

(C) 6.5 J

(D) 5 J

QUESTION

H.W.

A force acts on a 2 kg object so that its position is given as a function of time as $x = 3t^2 + 5$. What is the work done by this force in first 5 seconds? [9 Jan. 2019 II]

(A) 850 J

(B) 950 J

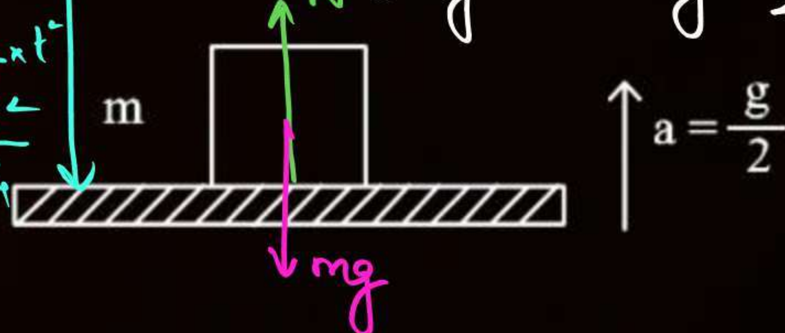
(C) 875 J

(D) 900 J

QUESTION

A block of mass m is kept on a platform which starts from rest with constant acceleration $g/2$ upward, as shown in fig. work done by normal reaction on block in time t is:

[10 Jan. 2019 I]



$S = \frac{1}{2}at^2$
$= \frac{1}{2}\frac{g}{2} \times t^2$
$= \frac{gt^2}{4}$

$N = mg + ma = mg + \frac{mg}{2} = \frac{3mg}{2}$

(A) $-\frac{mg^2t^2}{8}$

(B) $\frac{mg^2t^2}{8}$

(C) 0

(D) $\frac{3mg^2t^2}{8}$

$W_N = N \cdot S$
$= \frac{3mg}{2} \times \frac{gt^2}{4}$
$= \frac{3mg^2t^2}{8}$

QUESTION

$t=0,\ s=0$

A body of mass m starts moving from rest along x-axis so that its velocity varies as $v = a\sqrt{s}$ where a is a constant and s is the distance covered by the body. The total work done by all the forces acting on the body in the first t second after the start of the motion is:

[April 16, 2018]

(A) $\frac{1}{8}ma^4t^2$

(B) $4ma^4t^2$

(C) $8ma^4t^2$

(D) $\frac{1}{4}ma^4t^2$

$$v = \alpha\sqrt{s}$$

$$\frac{ds}{dt} = \alpha\sqrt{s}$$

$$\int\frac{ds}{\sqrt{s}} = \int\alpha\, dt$$

$$\frac{s^{1/2}}{1/2} = \alpha t \Rightarrow 2\sqrt{s} = \alpha t$$

$$s = \left(\frac{\alpha t}{2}\right)^2$$

$$W.D = \Delta KE$$

$$WD = KE$$

$$= \frac{1}{2}mv^2$$

$$= \frac{1}{2}m(\alpha\sqrt{s})^2$$

$$= \frac{1}{2}m\alpha^2 s$$

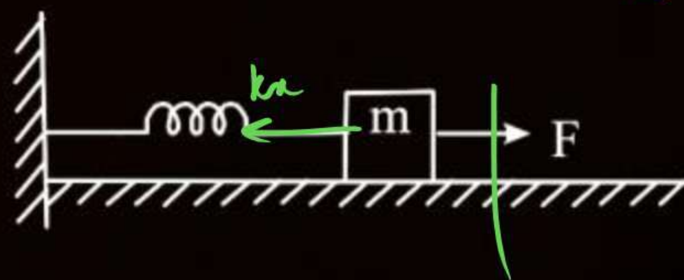
$$= \frac{1}{2}m\alpha^2\left(\frac{\alpha^2t^2}{4}\right)$$

$$\frac{1}{8}m\alpha^4t^2$$

QUESTION

H.W-2

A block of mass m, lying on a smooth horizontal surface, is attached to a spring (of negligible mass) of spring constant k. The other end of the spring is fixed, as shown in the figure. The block is initally at rest in its equilibrium position. If now the block is pulled with a constant force F, the maximum speed of the block is: **[9 Jan. 2019 I]**



(A) $\dfrac{2F}{\sqrt{mk}}$

(B) $\dfrac{F}{\pi\sqrt{mk}}$

(C) $\dfrac{\pi F}{\sqrt{mk}}$

(D) $\dfrac{F}{\sqrt{mk}}$ ✓

$$TWD = \Delta K.E$$

$$kx = F \Rightarrow x = \frac{F}{k}$$

$$F \cdot x - \frac{1}{2}kx^2 = \frac{1}{2}mv^2 - 0$$

$$F \cdot \frac{F}{k} - \frac{1}{2}k\left(\frac{F}{k}\right)^2 = \frac{1}{2}mv^2$$

$$\frac{F^2}{k} - \frac{F^2}{2k} = \frac{1}{2}mv^2$$

$$\frac{F^2}{\cancel{2}k} = \frac{1}{\cancel{2}}mv^2 \Rightarrow v = \frac{F}{\sqrt{mk}}.$$

QUESTION

Find the work done by force F when the pendulum is slowly raised till the string makes angle θ with vertical.

$$TWD = \Delta K.E$$

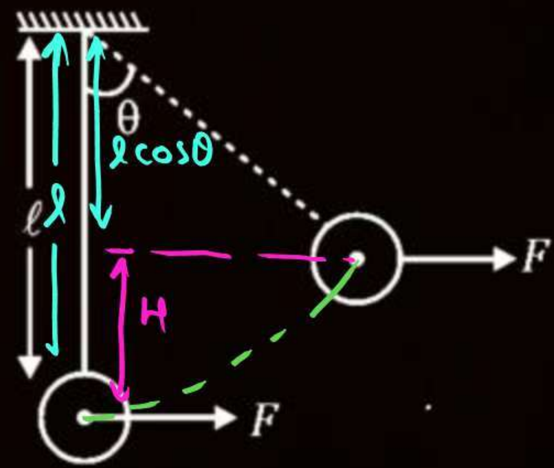
$$W_{mg} + W_F + W_T = K.E_f - K.E_i$$

$$-mgH + W_F + 0 = 0 - 0$$

$$W_F = mgH$$

$$= mg(\ell - \ell\cos\theta)$$

$$W_F = mg\ell(1-\cos\theta)$$ Ans.



QUESTION

HW

A body of mass 0.5kg travels on straight line path velocity $u = (3x^3 + 4)\text{m/s}$. The net work done by the force during its displacement from $x = 0$ to $x = 2\text{m}$ is:

[25 July, 2022 (Shift-I)]

(A) 64 J

(B) 60 J

(C) 120 J

(D) 128 J

Homework

Adv. level

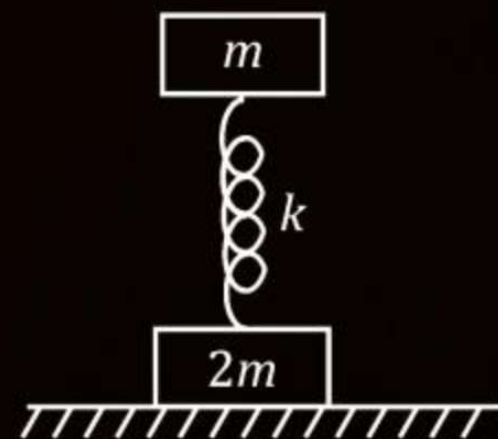
Find distance by which upper block of mass m should be compressed so that after releasing it can just lift the lower block of mass 2m off the ground.

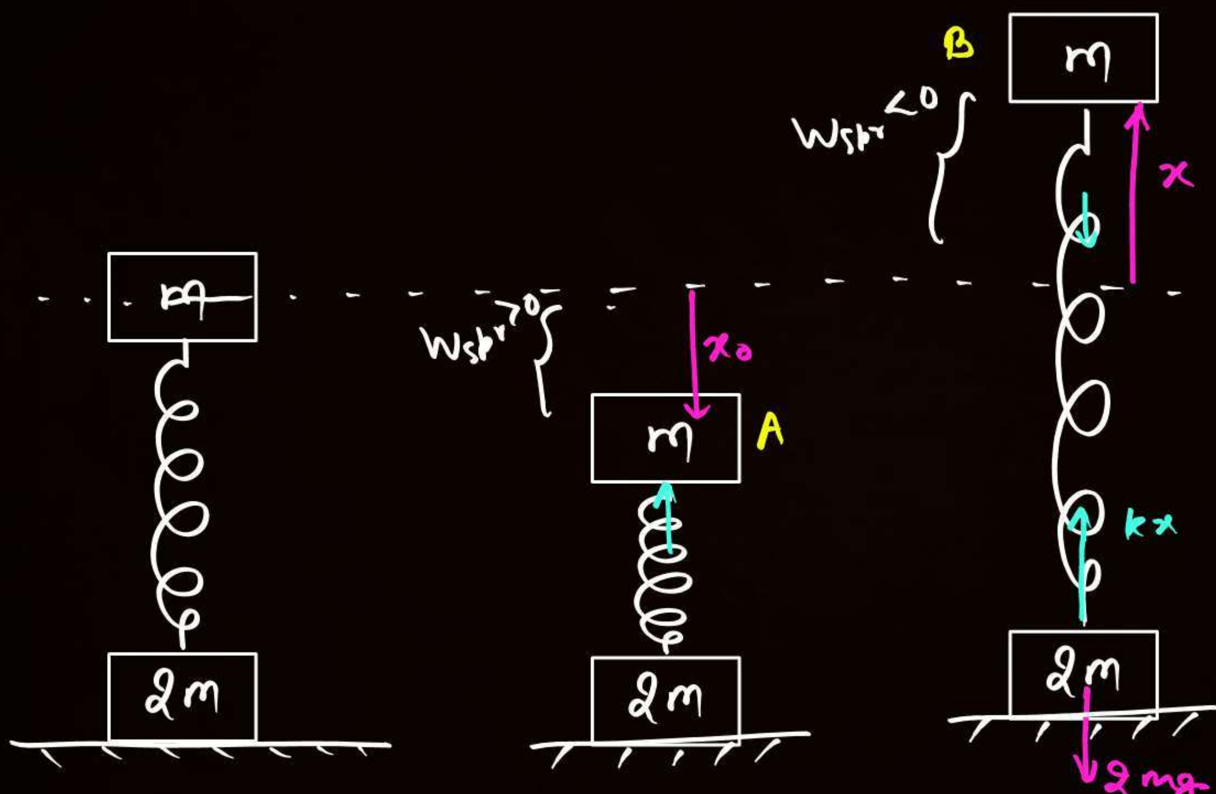
(A) mg/k

(B) $4mg/k$

(C) $2mg/k$

(D) None of these





WE theo. b/w A & B –

$$TWD = \Delta K.E$$

$$W_{mg} + W_{spr} = K.E_f - K.E_i$$

$$-mg(x_0 + x) + \left[\frac{1}{2}kx_0^2 - \frac{1}{2}kx^2\right] = 0 - 0$$

$$\Rightarrow -mg(x_0 + x) = \frac{1}{2}kx^2 - \frac{1}{2}kx_0^2$$

$$\Rightarrow -2mg\left(x_0 + \frac{2mg}{k}\right) = k\left(\frac{2mg}{k}\right)^2 - kx_0^2$$

$$\Rightarrow -2mgx_0 - \frac{4m^2g^2}{k} = \frac{4m^2g^2}{k} - kx_0^2$$

$$kx = 2mg \Rightarrow x = \frac{2mg}{k} =$$

$$kx_0^2 - 2mg\,x_0 - \frac{8m^2g^2}{k} = 0$$

$$x_0 = \frac{2mg \pm \sqrt{4m^2g^2 + 4\times\cancel{k}\times\frac{8m^2g^2}{\cancel{k}}}}{2k}$$

$$x_0 = \frac{2mg \pm \sqrt{36\,m^2g^2}}{2k}$$

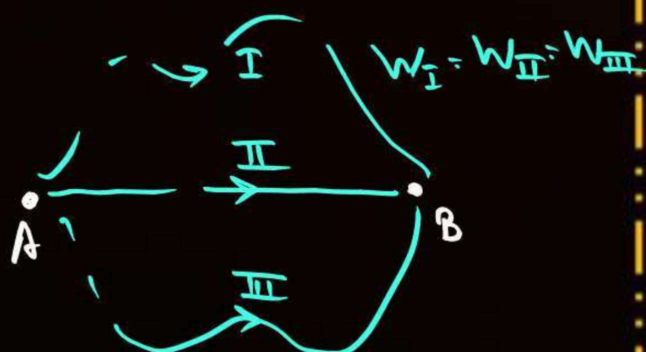
$$= \frac{2mg + 6mg}{2k} = 4mg/k.$$

Types of Forces Based on Work

Conservative Forces

- W.D. does not depend on path.

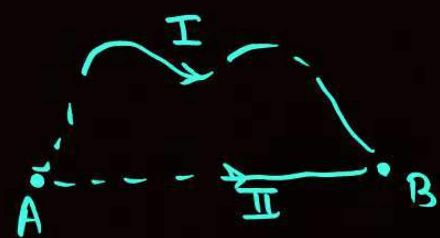
Ex. Gravitation, spring force.



Non-Conservative Forces

- W.D. by non-conservative forces depend on path

Ex. friction, air drag, viscous force



$W_I \neq W_{II}$

Types of Forces Based on Work

Conservative Forces

- $$W_{con.} = \int_{r_i}^{r_f} \vec{F} \cdot d\vec{S}$$

Integration can be done directly without needing the path information.

Non-Conservative Forces

- $$W_{non\text{-}con} = \int_{r_i}^{r_f} \vec{F} \cdot d\vec{S}$$

Above integral can not be calculated without the path information.

QUESTION

Conservative force

A force $\vec{F} = 3x^2\hat{\imath} + 2y\hat{\jmath}$ acts on a body. If the body is moved from origin to (1, 1). Find the work done by the force.

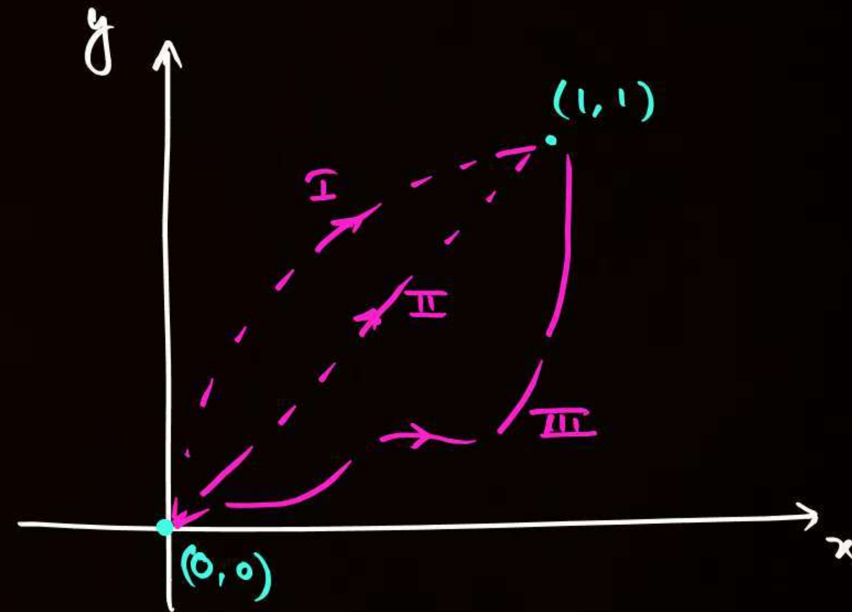
$$W = \int \vec{F} \cdot d\vec{s}$$

$$= \int_{(0,0)}^{(1,1)} F_x\,dx + F_y\,dy$$

$$= \int_{(0,0)}^{(1,1)} 3x^2\,dx + 2y\,dy$$

$$= \left[3 \times \frac{x^3}{3} + \frac{2y^2}{2}\right]_{(0,0)}^{(1,1)}$$

$$= \left[x^3 + y^2\right]_{(0,0)}^{(1,1)} = (1+1) - 0 = 2\,J$$



QUESTION

A force $\overbrace{\vec{F} = 3y\hat{\imath} + x^2\hat{\jmath}}^{\text{Non-conservative}}$ acts on a body. If the body is moved from origin to (1, 1) along path $y = x^2$ find the work done by the force.

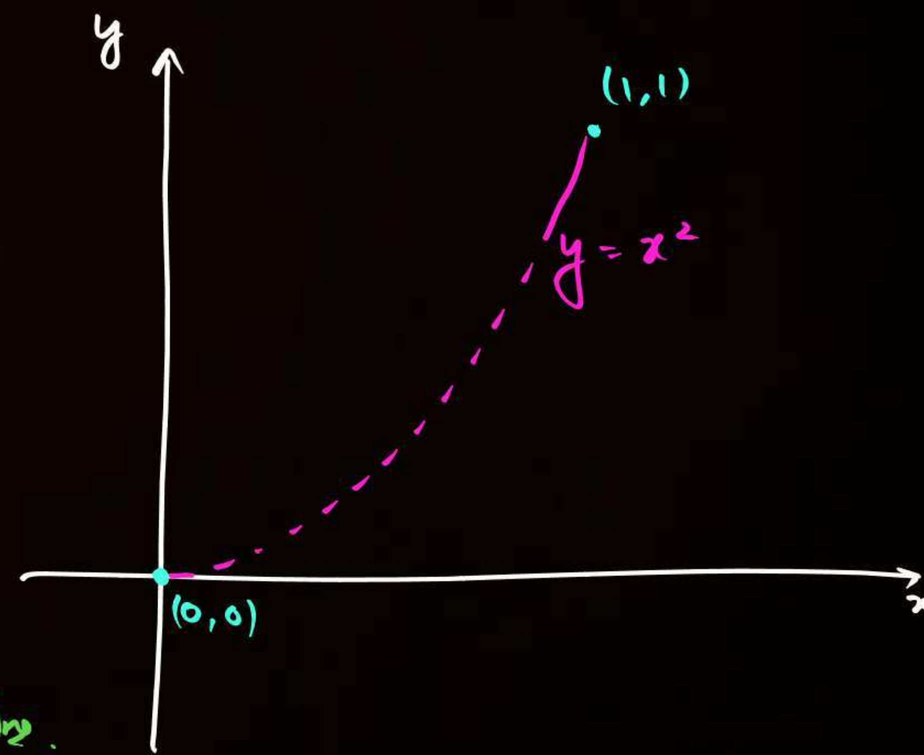
$$W = \int \vec{F} \cdot d\vec{s}$$

$$= \int_{(0,0)}^{(1,1)} 3y\,dx + x^2\,dy$$

$$= \int_{(0,0)}^{(1,1)} 3y\,dx + \int_{(0,0)}^{(1,1)} x^2\,dy$$

$$= \int 3x^2\,dx + \int y\,dy$$

$$= \left[x^3 + \frac{y^2}{2}\right]_{(0,0)}^{(1,1)} = \frac{3}{2}\text{ J} \quad \text{Ans.}$$



Types of Forces Based on Work

Conservative Forces

VS

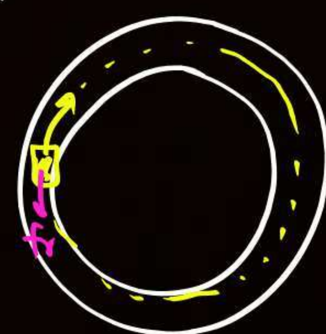
- If $S = 0$, $W_{cons.} = 0$

$$W = \int_{r_i}^{r_i} \vec{F} \cdot d\vec{s}$$

$$W = 0\,J$$

Non-Conservative Forces

- for non-con. forces, if $S = 0$, W.D. by non-con. force may or may not be zero.



Types of Forces Based on Work

ईमानदार force

Conservative Forces

- Concept of potential energy is defined only for conservative forces.

$$\boxed{\Delta U = -W_{cons.}}$$

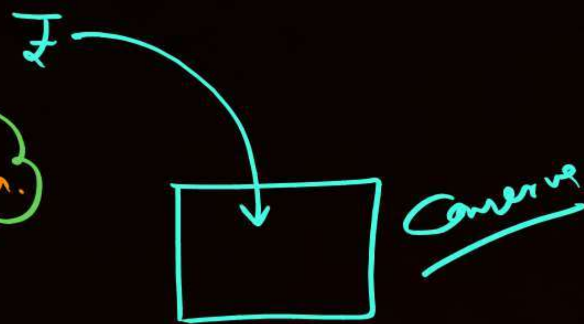
VS

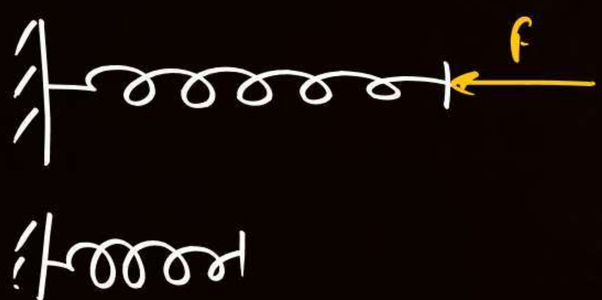
बेईमान force

Non-Conservative Forces

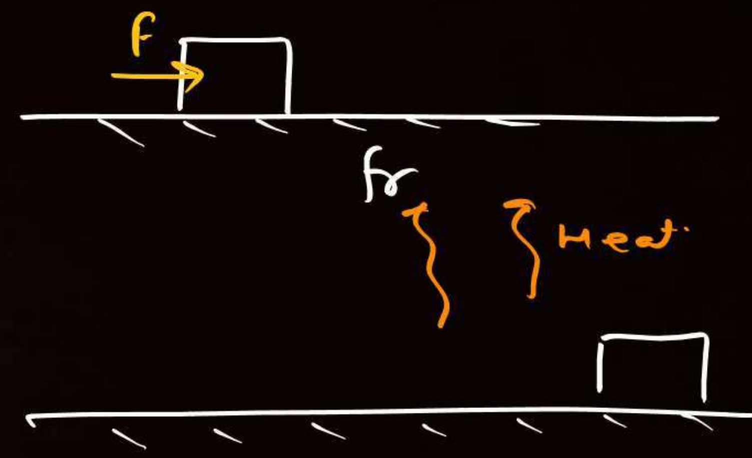
- For non-conservative forces, potential energy is not defined

$$\Delta U = -W_{con.}$$





$$U = \frac{1}{2} k x^2$$



Types of Forces Based on Work

Conservative Forces

- All constant forces are conservative

$$W = \int \vec{F} \cdot d\vec{s}$$

$$W = \vec{F} \cdot \vec{S}$$

VS

Non-Conservative Forces

- A variable force may or may not be non-conservative

QUESTION

Identify the conservative forces-

1. $\vec{F} = 4\hat{\imath} + 3\hat{\jmath} \longrightarrow$ Conservative
2. $\vec{F} = x\hat{\imath} + y\hat{\jmath} \longrightarrow$ Conservative
3. $\vec{F} = y^2\hat{\imath} + x\hat{\jmath} \longrightarrow$ Non-Conservative.
4. $\vec{F} = y\hat{\imath} + x\hat{\jmath} \longrightarrow$ Conservative
5. $\vec{F} = \underline{3x^2y^2}\hat{\imath} + \overline{2x^3y}\hat{\jmath} \rightarrow$ Conservative

5) $W = \int 3x^2y^2\,dx + 2x^3y\,dy$

$\int d(x^3 \cdot y^2)$

Partial diff.

2) $W = \int \vec{F} \cdot d\vec{s}$

$= \int x\,dx + y\,dy$

$= \frac{x^2}{2} + \frac{y^2}{2} + c$

3) $W = \int y^2\,dx + x\,dy$

4) $W = \int y\,dx + x\,dy$

$= \int d(xy)$

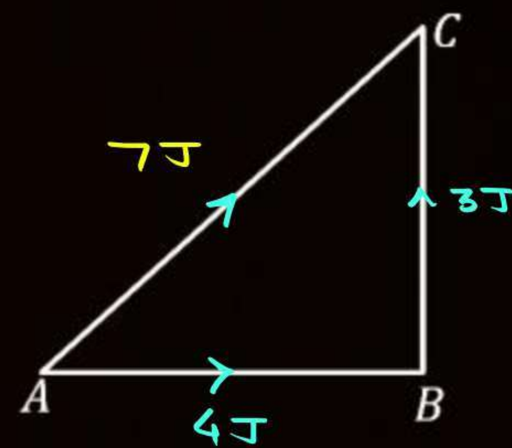
$W = xy$

QUESTION

If F is a conservative force and $W_{AB} = 4$ J, $W_{BC} = 3$ J then Find $W_{AC} = ?$ 7 J

scalar.

Common mistake → 5 J



QUESTION

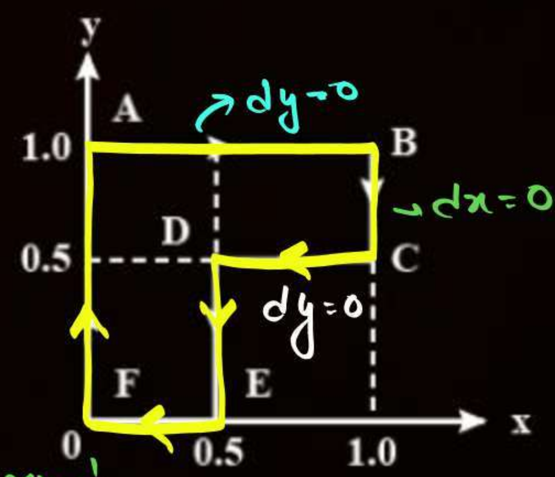
A particle is moved along a path AB-BC-CD-DE-EF-FA, as shown in figure, in presence of a force $\vec{F} = (\alpha y\hat{\imath} + 2\alpha x\hat{\jmath})N$ where x and y are in meter and a = –1 N/m^{-1}. The work done on the particle by the force $\vec{F}$ will be ____ Joule.

[JEE Adv. 2019]

$$\vec{F} = \alpha y\hat{i} + 2\alpha x\hat{j}$$

Non-cons

$$W_{AB} + W_{BC} + W_{CD} + W_{DE} + W_{EF} + W_{FA}$$



$W_{AB} \rightarrow \vec{F} = \alpha \times 1\hat{i} + 2\alpha x\hat{j}$

$$W_{AB} = \int \vec{F}\cdot d\vec{x} = \int \alpha\, dx = \alpha\Delta x = \alpha \times 1 = \alpha J$$

$W_{BC} \rightarrow \vec{F} = \alpha y\hat{i} + 2\alpha\hat{j}$

$$W_{AB} = \int \vec{F}\cdot d\vec{y} = \int 2\alpha\, dy = 2\alpha\,\Delta y = 2\alpha \times -\frac{1}{2} = -\alpha J$$

Potential Energy

$$\Delta U = -W_{cons.}$$

- Defined only for conservative forces

1D

$$\Delta U = -W_c$$

$$\Delta U = -\int f_c \, dx$$

$$f_c = -\frac{dU}{dx}$$

2D/3D

$$\Delta U = -W_c$$

$$\Delta U = -\int f_x dx + f_y dy + f_z dz$$

$$\vec{f} = -\left[\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j} + \frac{\partial U}{\partial z}\hat{k}\right]$$

Partial diff.

QUESTION

The potential energy of a body moving in a conservative force field is given by $U = 3x^2y$. Find the conservative force field.

$$U = 3x^2 y$$

$$\left\{ \frac{\partial U}{\partial x} \rightarrow \text{Treat } y \text{ \& } z \text{ as const} \right\}$$

$$\vec{F} = -\left[\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j}\right]$$

$$\vec{F} = -\left[3\times(2x)y\,\hat{i} + 3x^2\,\hat{j}\right]$$

QUESTION

The potential energy due to force $\underbrace{\vec{F} = x\hat{\imath} + y\hat{\jmath}}$ at origin is $\underbrace{1}_{U_{(0,0)}}$ J. Find the potential energy at (2, 4).

(0,0)

$$\Delta U = -W_c$$
$$= -\int x\,dx + y\,dy$$
$$U = -\left[\frac{x^2}{2} + \frac{y^2}{2}\right] + c$$

At (0,0), U = 1 J

$$1 = c$$

$$U = -\left[\frac{x^2}{2} + \frac{y^2}{2}\right] + 1$$

$$U_{(2,4)} = -\left[\frac{2^2}{2} + \frac{4^2}{2}\right] + 1$$
$$= -2 - 8 + 1$$
$$= -9\text{ J.}$$

QUESTION

A body is moving along x-axis under influence of force field given by $F = (x^2 - 2x)N$. Find potential energy of body at any x if the potential energy at origin is 0 J.

$$\Delta U = -W_c$$

$$\Delta U = -\int F\,dx$$

$$U_f - U_i = -\int_0^x (x^2 - 2x)\,dx$$

$$U_x - 0 = -\left[\frac{x^3}{3} - x^2\right]_0^x$$

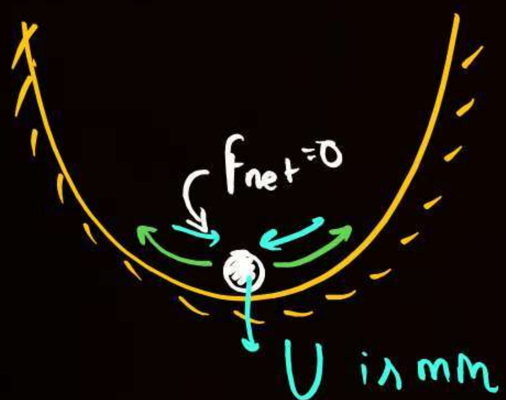
$$U_x = -\frac{x^3}{3} + x^2$$ Ans.

Equillibrium

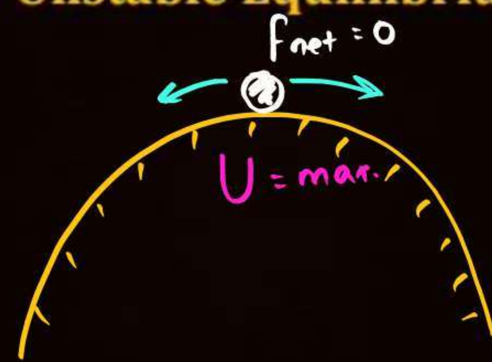
$F_{net} = 0$

Types of Equillibrium

Stable Equillibrium



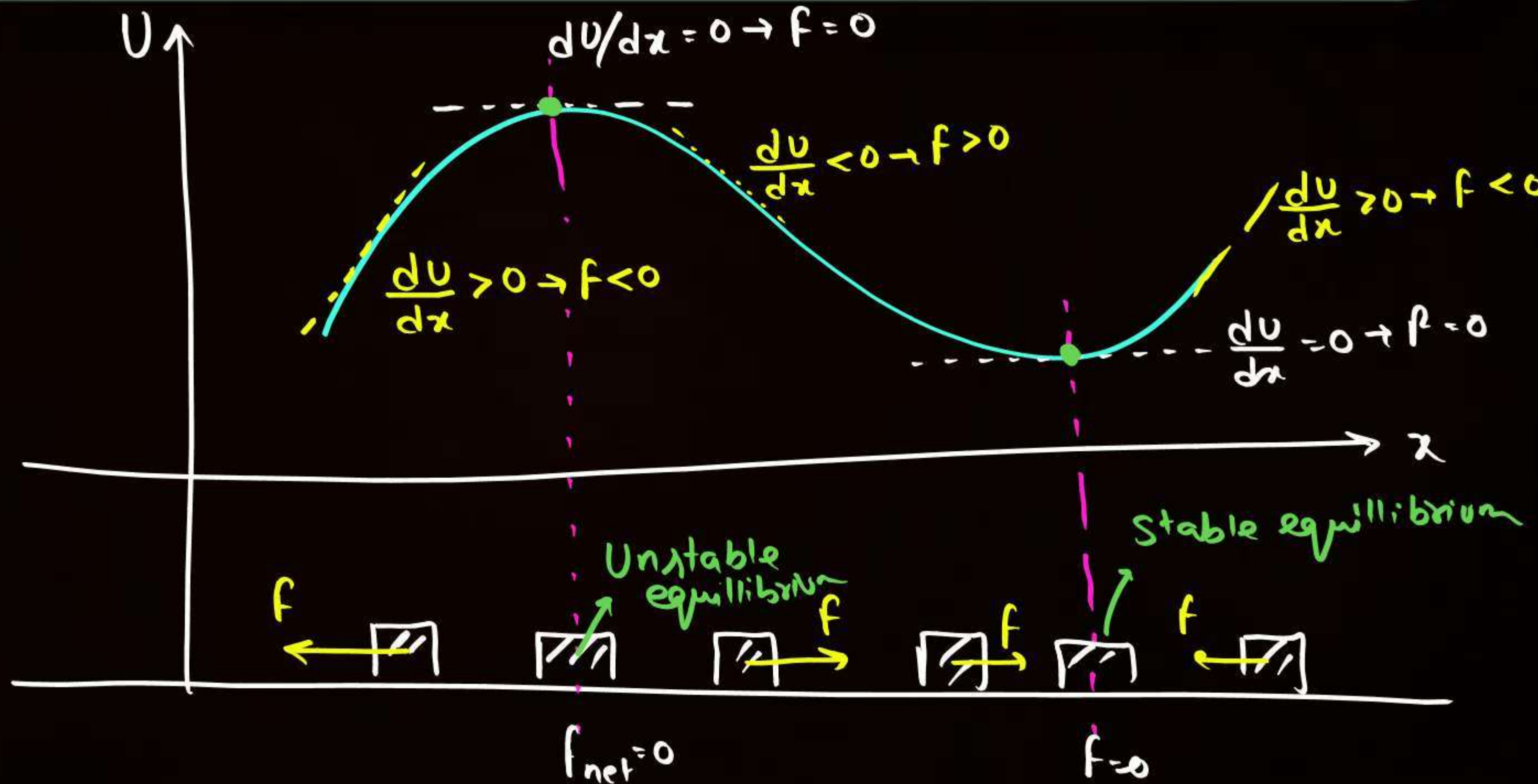
Unstable Equillibrium



Neutral Equillibrium

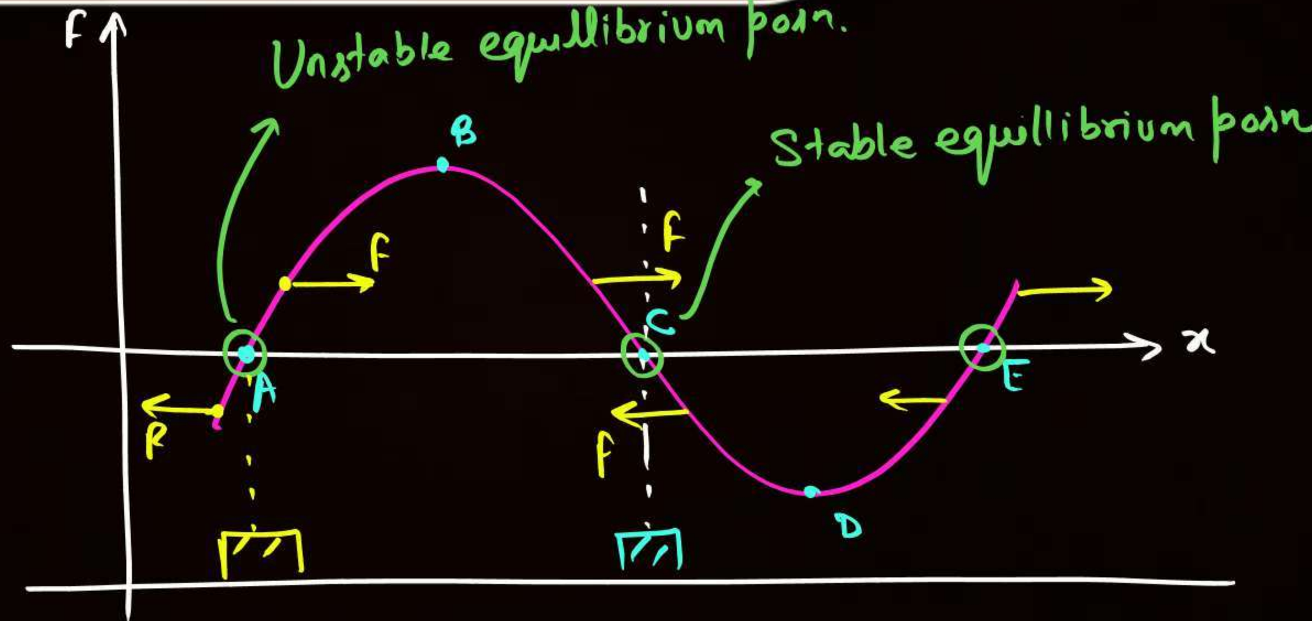
Graphical Relation b/w Conservative Force and Potential Energy

$$f = -\frac{dU}{dx}$$



Identification of Equillibrium Position from U-X Curve

Identification of Equillibrium Position from F-X Curve



QUESTION

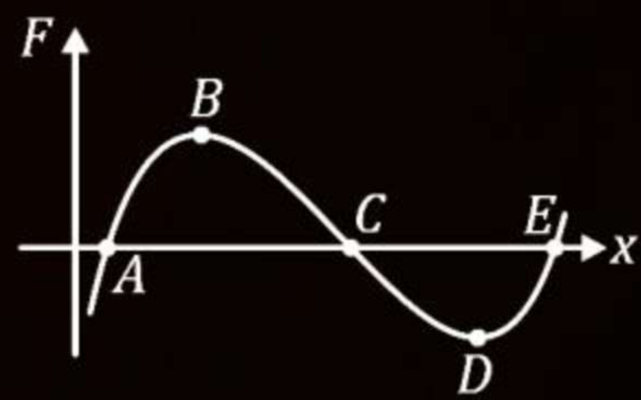
Equillibrium position.

A) B, D

B) A, C

C) A, C, E

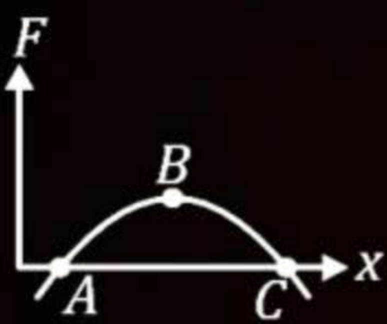
D) None of these

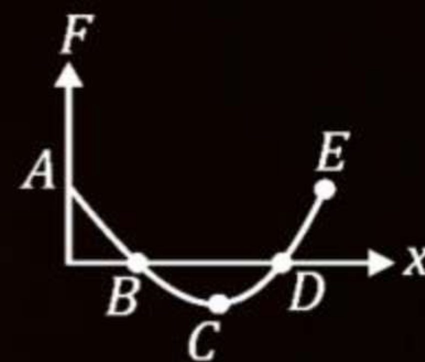


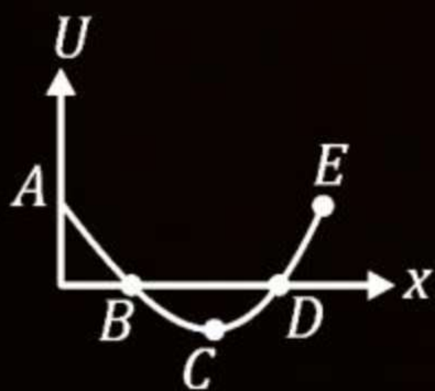
QUESTION

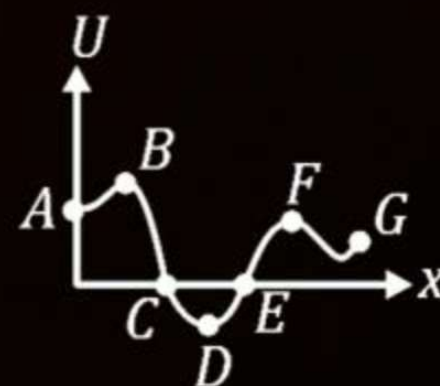
H.W

Identify the equilibrium positions and their types-

(A) 

(B) 

(C) 

(D) 

QUESTION

The potential energy of a body is given by $U = x^2 - 6x + 8$. Find position where the body will be in equilibrium.

$$U = x^2 - 6x + 8$$

$$F = -\frac{dU}{dx} = -2x + 6 = 0$$

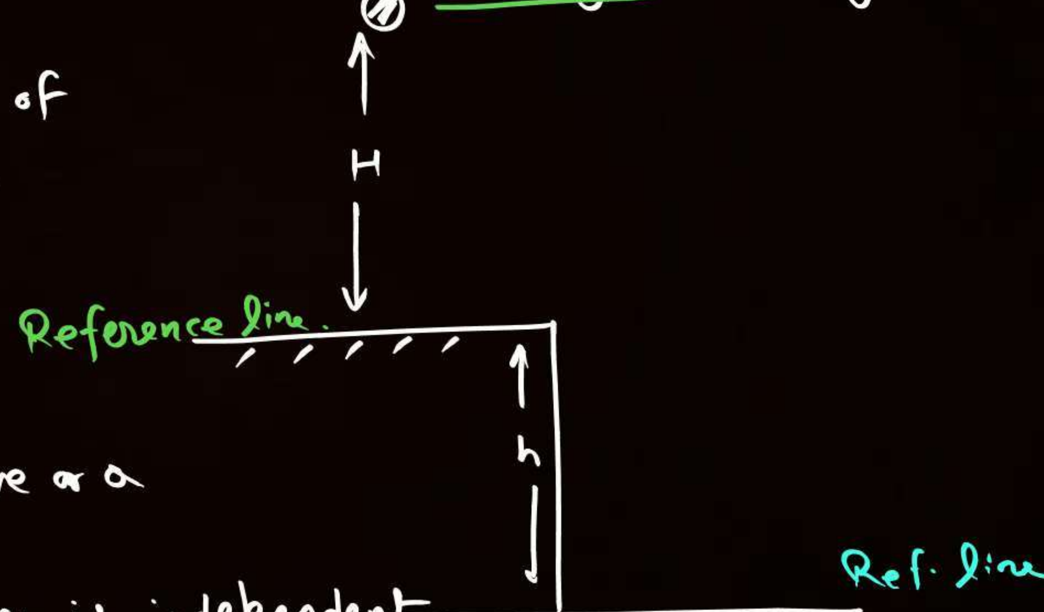
$$x = 3\,m$$

Gravitational Potential Energy

$$U = mgH$$

- Absolute grav. pot. energy of any pt is not defined
- Pot. energy depends on reference pt
- P.E can be +ve, -ve or 0
- Change in pot. energy is independent of reference pt.

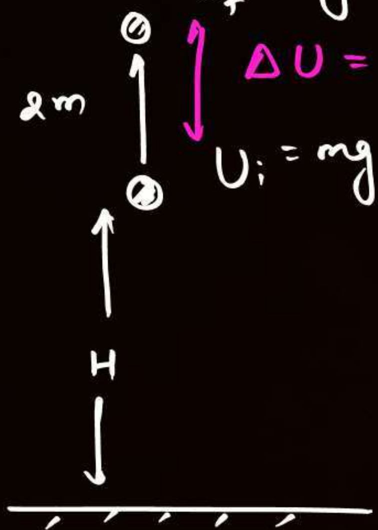
$U = +mgH$, $U = mg(H+h)$



$U_f = mg(H+2)$

$\Delta U = mg(H+2) - mgH = mg \times 2$.

$U_i = mgH$

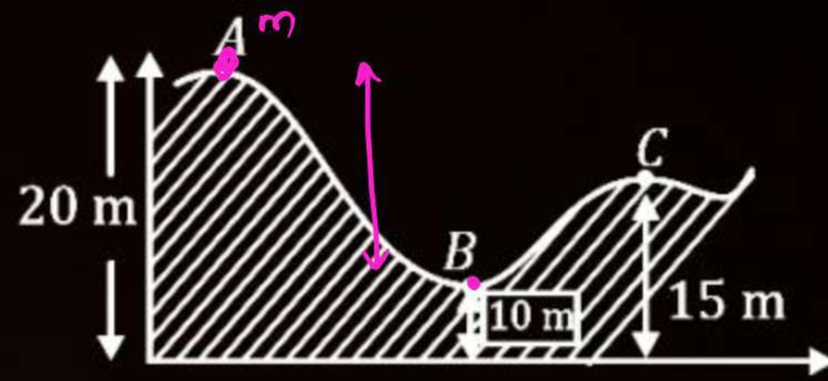


QUESTION

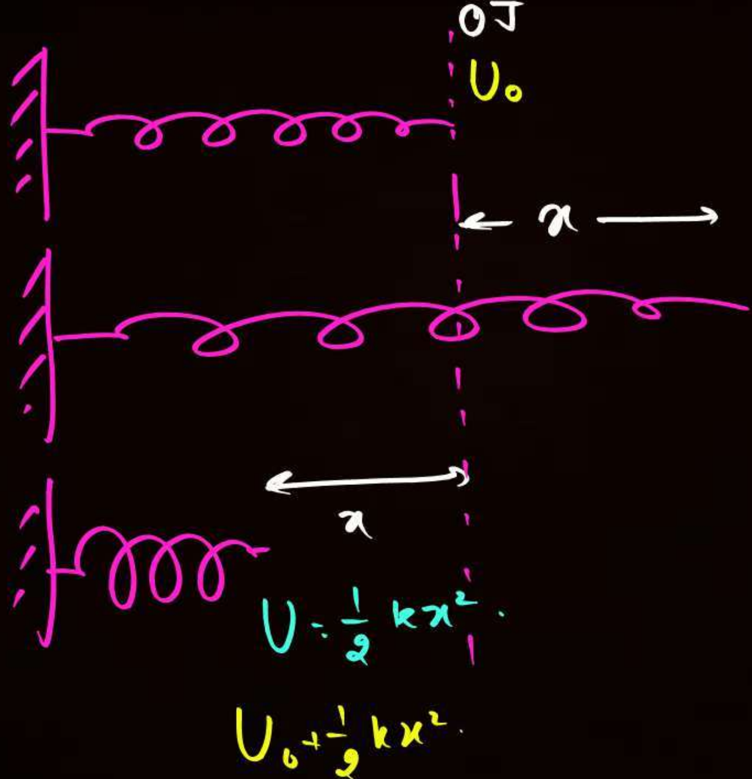
$U_A - U_B = ?$ $\quad mg \times (20-10) = 10mg$

$U_B - U_C = ?$

$mg \times 10 - mg \times 15 = -5mg$



Spring Potential Energy



$U = \frac{1}{2}kx^2 \;\|\; U_0 + \frac{1}{2}kx^2$

Always measure x from N. L. of spring

Law of Conservation of Mechanical Energy

If there are no non-cons. forces $\longrightarrow$

$$TWD = \Delta k.E \Rightarrow W_c + \underbrace{W_{NC}}_{0} = \Delta k.E$$

$$\Rightarrow W_c = \Delta k.E$$

$$\Rightarrow -\Delta U = \Delta k.E$$

Gain / Loss in U = Loss / Gain in $k.E$

$$\Rightarrow \Delta U + \Delta k.E = 0$$

$$\Rightarrow U_i + k.E_i = U_f + k.E_f$$

$$\Rightarrow M.E_i = M.E_f.$$

If $f_{non\text{-}con} \neq 0 \rightarrow$

$$W_c + W_{nc} = \Delta K.E$$

$$\Rightarrow -\Delta U + W_{n.c} = \Delta K.E$$

$$\Rightarrow W_{N.C} = \Delta K.E + \Delta U \longrightarrow$$

If $\Delta K.E = 0$

$$W_{N.C} = \Delta U$$

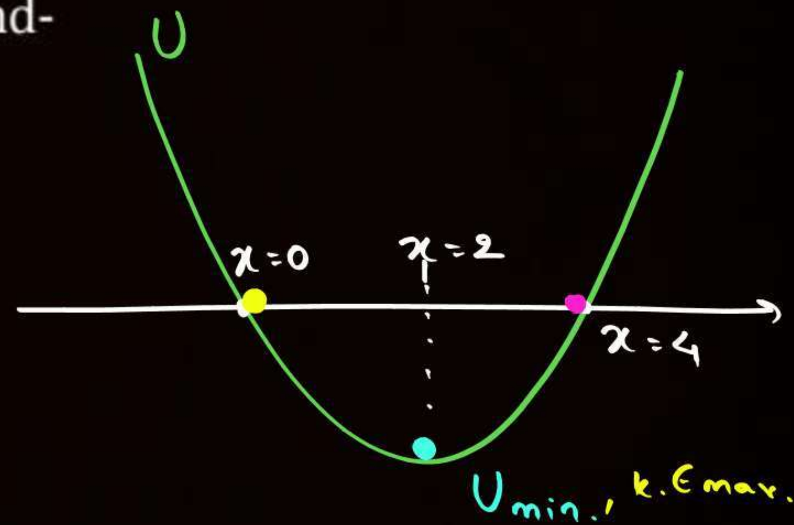
$$W_{N.C} = \Delta M.E$$

$$W_{ext} = \Delta M.E$$

QUESTION

A body of mass 2kg is released from rest from origin under the influence of a force field whose potential energy varies as $U = x^2 - 4x$. Find-

1. Maximum kinetic energy of body.
2. Maximum displacement along x-axis. $= 4\,m.$



At origin, $K.E = 0,\ U = 0,\ M.E = 0$

At $x = 2$, $U_{min.} = (2)^2 - 4 \times 2 = -4\,J$

$$U_{min.} + K.E_{max.} = M.E$$

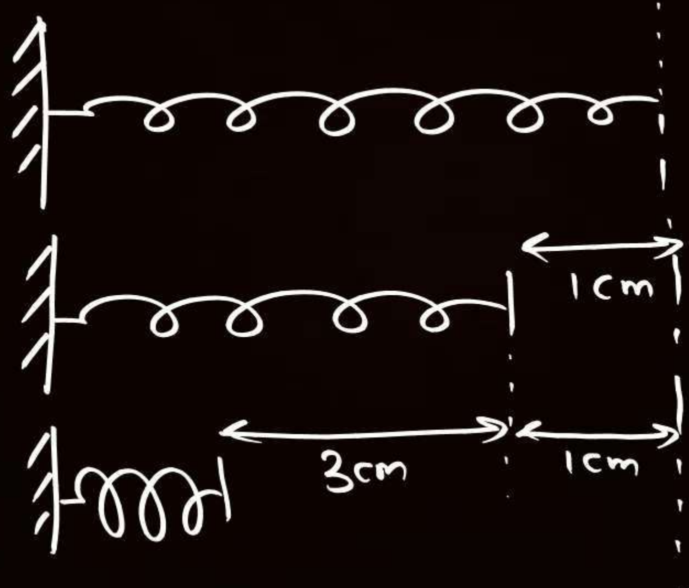
$$K.E_{max} = 0 - (-4)$$

$$= 4\,J.$$

QUESTION

A spring of spring constant k is initially compressed by 1 cm. The energy stored in the spring is U. If the spring is further compressed by 3 cm, the change in potential energy of spring will be-

$\Delta U = 16U - U = 15U.$



$U = \frac{1}{2} k (1)^2$

$U_f = \frac{1}{2} \times k \times (4)^2 = 16U$

(A) 16 U

(B) 3 U

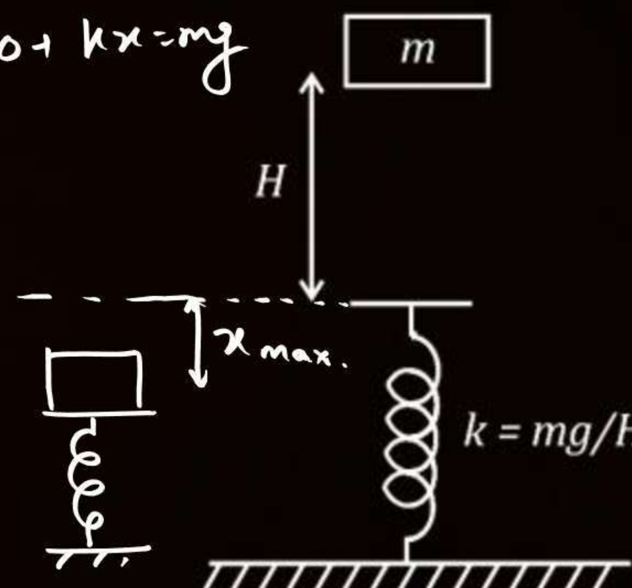
(C) 15 U

(D) 8 U common mistake

QUESTION

Find-

1. Maximum compression in string
2. Maximum speed of block $\longrightarrow$ When $F_{net} = 0 \rightarrow kx = mg$



1) $TWD = \Delta K.E$

$$mg(H + x_{max}) - \frac{1}{2} k x_{max}^2 = 0 - 0$$

$$\cancel{mg}(H + x_{max}) = \frac{1}{2} \times \frac{\cancel{mg}}{H} \times x_{max}^2$$

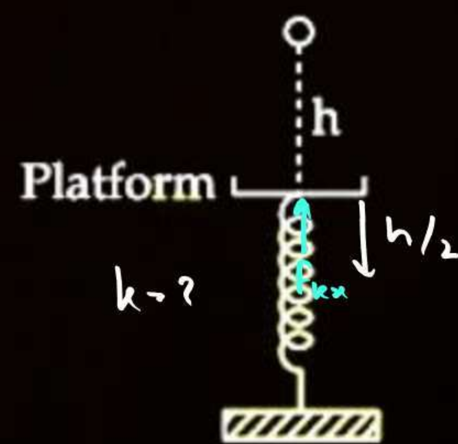
$$2H(H + x_{max.}) = x_{max}^2$$

QUESTION

H.W

A ball of mass 100g is dropped from a height $h = 10$ cm on a platform fixed at the top of a vertical spring (as shown in figure). The ball stays on the platform and the platform is depressed by a distance $h/2$. The spring constant is ______ Nm^{-1}.

(Use $g = 10\ ms^{-2}$) [24 June, 2022 (Shift-I)]

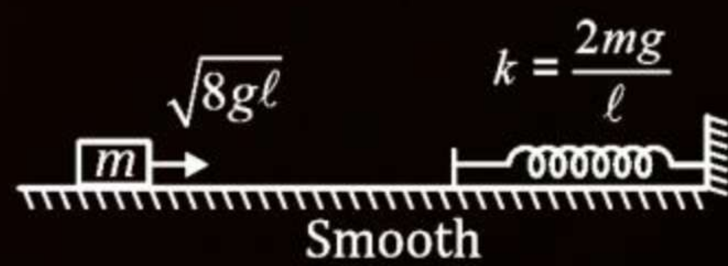


QUESTION

Find-

1. Maximum compression in the string.
2. Speed of block when spring compressed by ℓ.
3. What will be the maximum compression if the surface is rough with $\mu = 1/4$ and spring coefficient is $k = \frac{mg}{l}$?

Adv level



1)

$k = \frac{2mg}{l}$

Loss in K.E = Gain in spring U

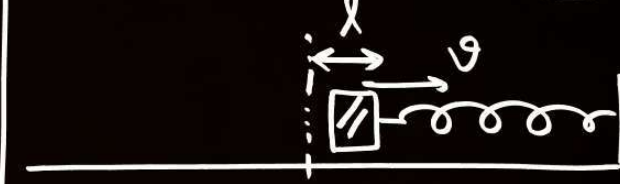
$$\frac{1}{2} m\left(\sqrt{8gl}\right)^2 = \frac{1}{2} k x^2$$

$$\frac{\overset{4}{\cancel{8}}\,m\cancel{g}l}{\cancel{2}} = \frac{1}{\cancel{2}} \times \frac{\cancel{2}\cancel{mg}}{l} x^2$$

$$x^2 = 4l^2$$

$$x = 2l.$$

2)



$$K.E_i = K.E + U$$

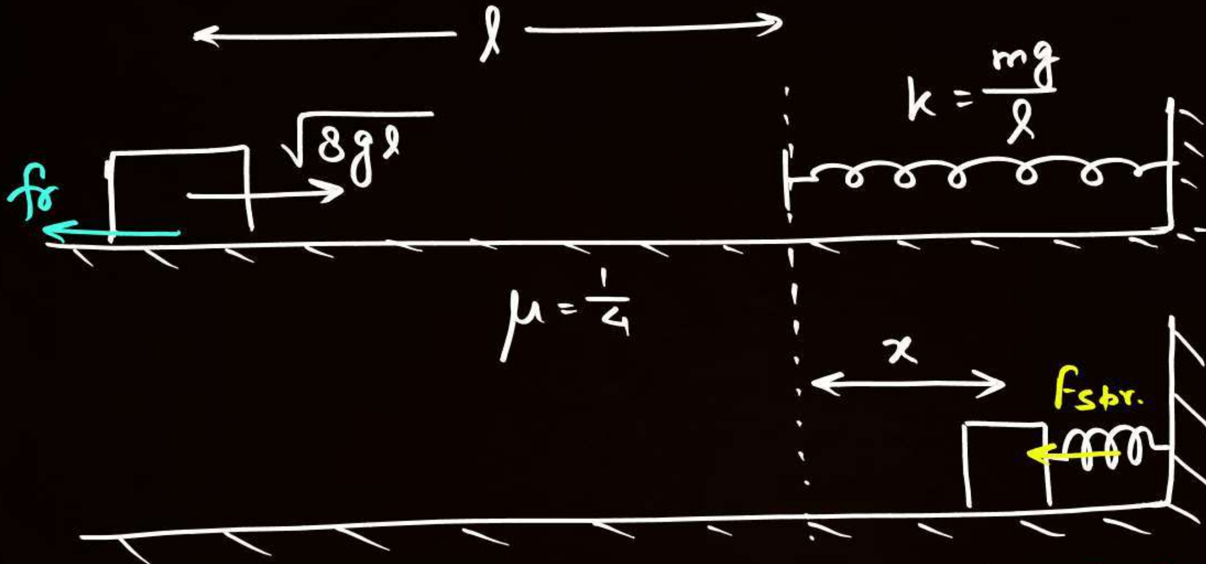
$$\frac{1}{2} m\left(\sqrt{8gl}\right)^2 = \frac{1}{2} m v^2 + \frac{1}{\cancel{2}}\left(\frac{\cancel{2}mg}{l}\right) l^2$$

$$4mgl = \frac{1}{2} m v^2 + mgl$$

$$\cancel{m} v^2 = 6\cancel{m}gl$$

$$v = \sqrt{6gl}$$

3)



$$TWD = \Delta K.E$$

$$-\mu mg \times (l+x) - \frac{1}{2} k x^2 = 0 - \frac{1}{2} m\left(\sqrt{8gl}\right)^2$$

$$\cancel{-}\frac{1}{4} \cancel{mg}(l+x) \cancel{-} \frac{1}{2} \frac{\cancel{mg}}{l} \cdot x^2 = \cancel{-}4\cancel{mg}l$$

$$\frac{l+x}{4} + \frac{x^2}{2l} = 4l$$

$$\frac{l^2 + xl + 2x^2}{4l} = 4l$$

$$2x^2 + xl - 15l^2 = 0$$

$$x = \frac{-l \pm \sqrt{l^2 + 4\times 2\times 15 l^2}}{4}$$

$$= \frac{-l + \sqrt{121\, l^2}}{4}$$

$$= \frac{-l + 11l}{4} = \frac{10l}{4}$$

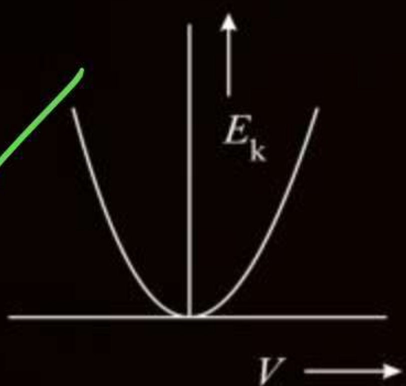
QUESTION

The graph between kinetic energy E_K and velocity V is-

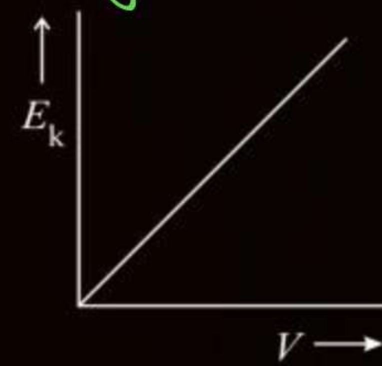
$\underline{k.E} = \frac{1}{2} m v^2$

$y = \frac{1}{2} m x^2 \quad y \propto x^2.$

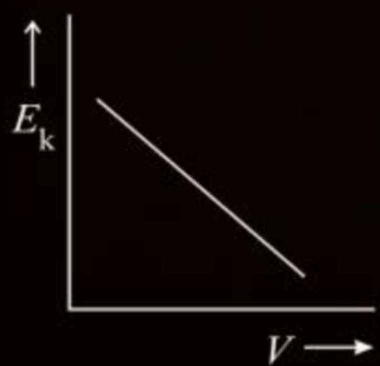
(A)



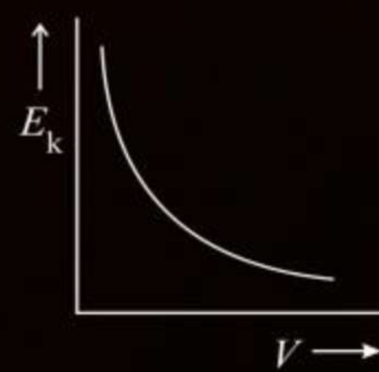
(B)



(C)



(D)



QUESTION

The graph between $\sqrt{E_k}$ and $1/p$ is (E_K = kinetic energy and p = momentum)-

y

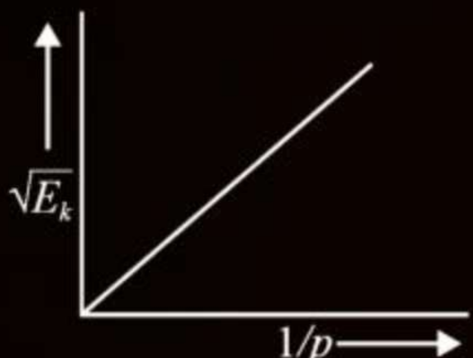
$$\sqrt{K.E} = \sqrt{\frac{p^2}{2m}}$$

$$y = \frac{p}{\sqrt{2m}}$$

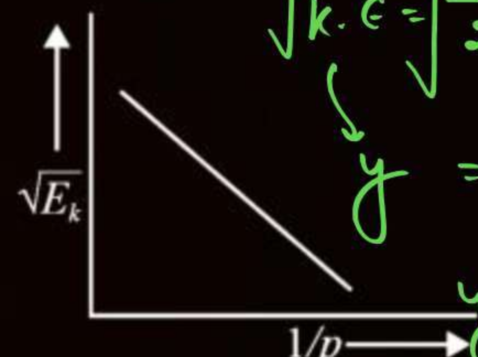
$$y = \frac{1}{\sqrt{2m}} \; \frac{1}{\underbrace{(1/p)}_{x}}$$

$$y \propto \frac{1}{x}$$

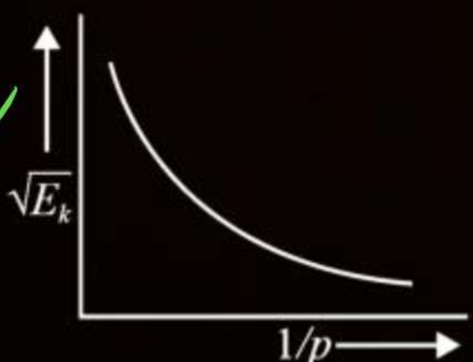
(A)



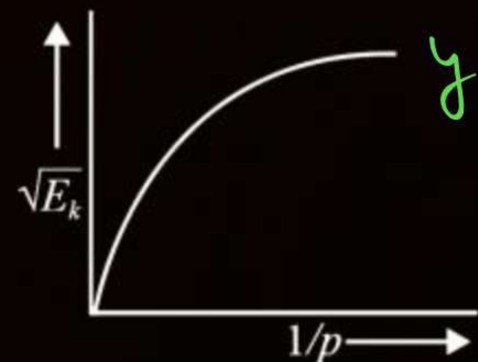
(B)



(C)



(D)

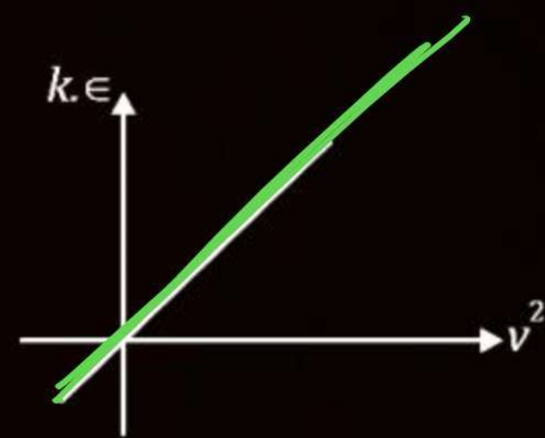


QUESTION

Plot graph between kinetic energy & V^2

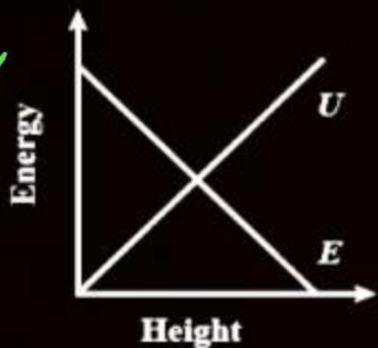
$$k.\epsilon = \frac{1}{2} m v^2$$

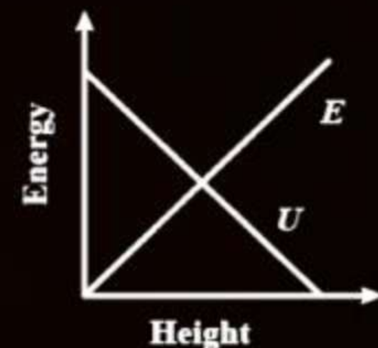
$$y = \frac{1}{2} m x$$

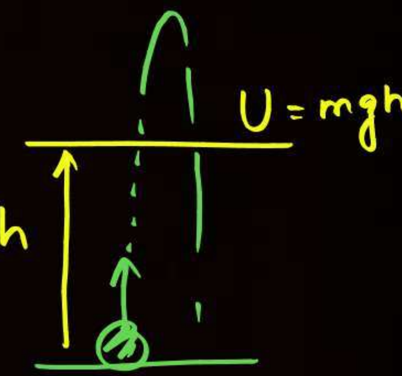


QUESTION

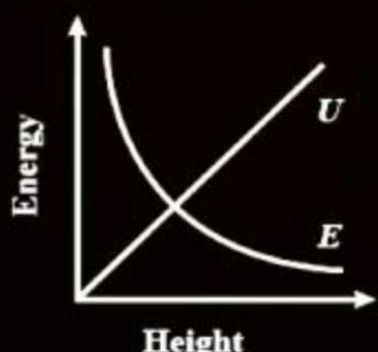
Which of the following graph is correct between kinetic energy (E) and potential energy (U) with height (h) from the ground of the particle?

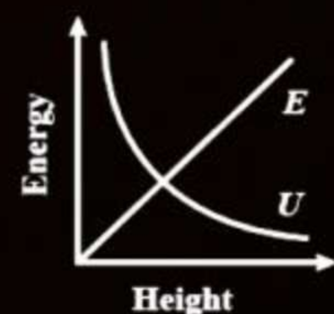
(A)


(B)




$\uparrow U + k.E \downarrow = \text{const}$

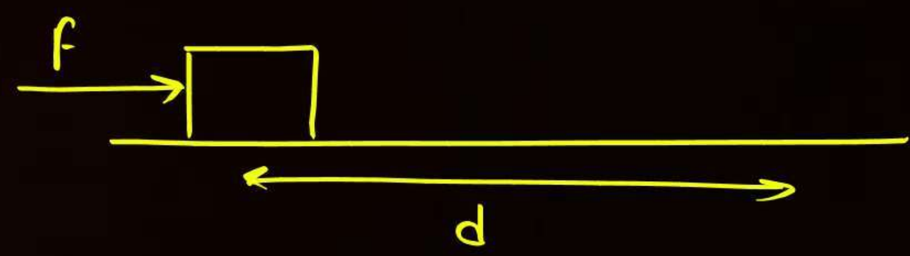
(C)


(D)


QUESTION

Kinetic energy acquired by a body of mass m initially kept at rest under the influence of a constant force F after travelling a certain distance d is-

1. Directly proportional to mass m.
2. Inversely proportional to mass m.
3. Directly proportional to $\sqrt{m}$.
4. Does not depend on m. ✓



$$T W D = \Delta K.E$$

$$F \cdot d = K.E - 0$$

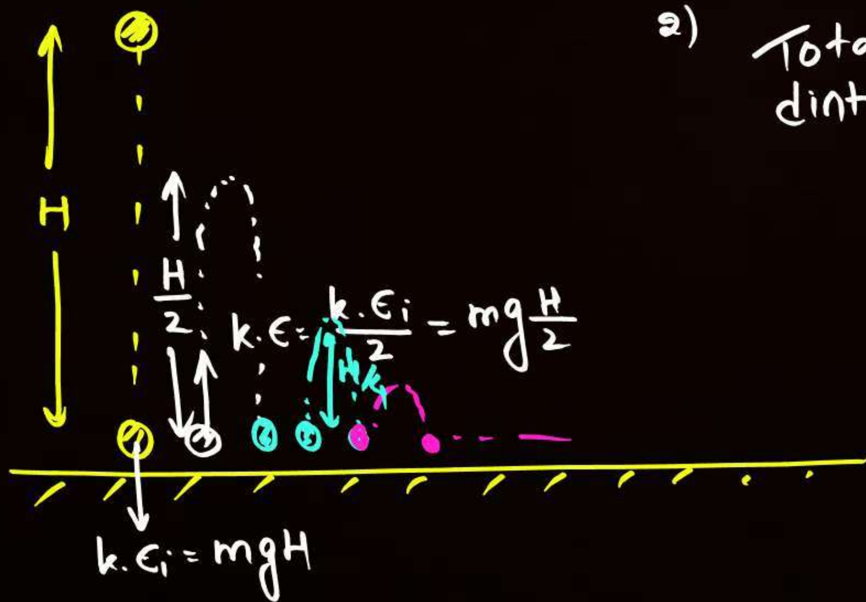
$$\boxed{K.E = F \cdot d}$$

$$\frac{1}{2} m v^2 = F \cdot d \quad v \propto \frac{1}{\sqrt{m}}.$$

QUESTION

A ball is dropped from a certain height H. If after striking the ground the ball loses 50% of its kinetic energy that is just before striking the surface, find-

1. Maximum height reached by the ball after 1st collision. $= \frac{H}{2}$
2. Total distance covered by ball before stopping.



2) $\text{Total dist} = H + 2\times\frac{H}{2} + 2\times\frac{H}{4} + 2\times\frac{H}{8} + \dots$

$= 2H + \frac{H}{2} + \frac{H}{4} + \frac{H}{8} + \dots$ Infinite GP.

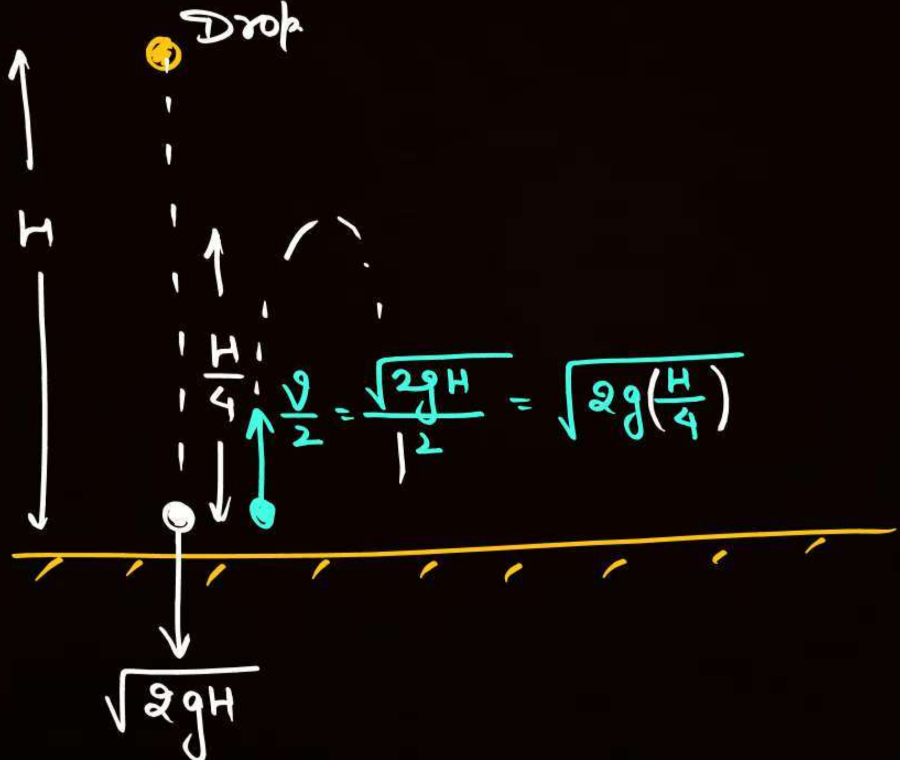
$= 2H + \frac{H}{2}\left[1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \dots\right]$

$= 2H + \frac{H}{2}\left[\frac{1}{1-\frac{1}{2}}\right] = 2H + \frac{H}{\cancel{2}}\times\cancel{2}$

$= 3H$ Ans.

QUESTION

A ball is dropped from height H. If the ball loses 50% of its speed after hitting the ground. How high will the ball rise after 1st collision? $\frac{H}{4}$.

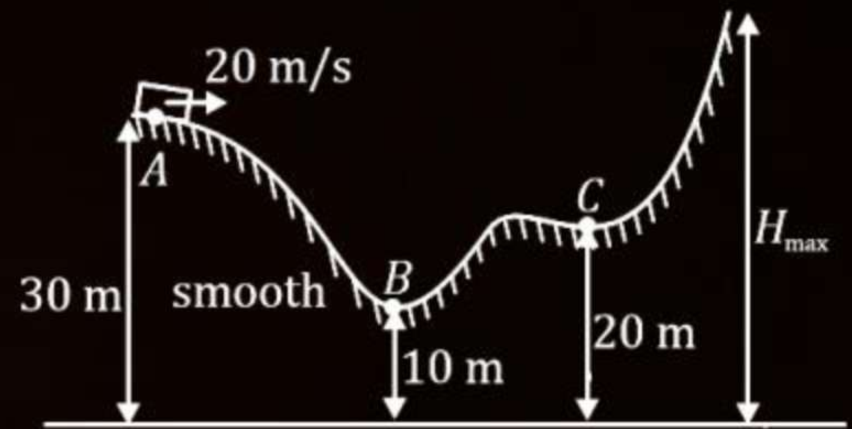


QUESTION

H.W

Find-

1. Speed of block at B
2. Speed of block at C
3. Max height upto which block can rise.



QUESTION

In the given figure, the block of mass m is dropped from the point 'A'. The expression for kinetic energy of block when it reaches point 'B' is: [29 June, 2022 (Shift-II)]

(A) $\frac{1}{2}mgy_0^2$

(B) $\frac{1}{2}mgy^2$

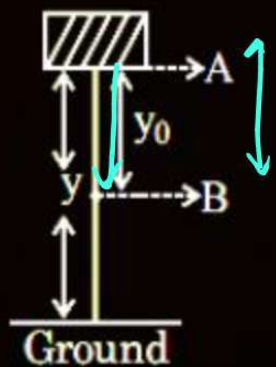
(C) $mg(y - y_0)$

(D) mgy_0

Gain in K.E = Loss in U

$K.E_f - K.E_i = mgy_0$

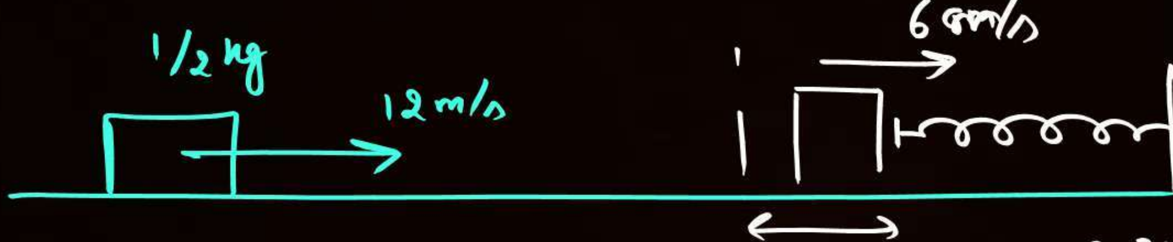
$K.E_f = mgy_0$



QUESTION

A 0.5 kg block moving at a speed of 12 ms^{-1} compresses a spring through a distance 30 cm when its speed is halved. The spring constant of the spring will be _____ Nm^{-1}.

[25 June, 2022 (Shift-I)]



$x = 30\text{ cm} = 0.3\text{ m}.$

$$\frac{1}{2}\times\frac{1}{2}\times(12)^2 = \frac{1}{2}\times\frac{1}{2}\times(6)^2 + \frac{1}{2}k\times(0.3)^2$$

$$\frac{\overset{36}{\cancel{144}}}{\cancel{4}} = \frac{\overset{9}{\cancel{36}}}{\cancel{4}} + \frac{0.09}{2}k$$

$$\frac{0.09}{2}k = 27 \Rightarrow k = \frac{\overset{6}{\cancel{54}}}{0.0\cancel{9}} = 600 \text{ Ans.}$$

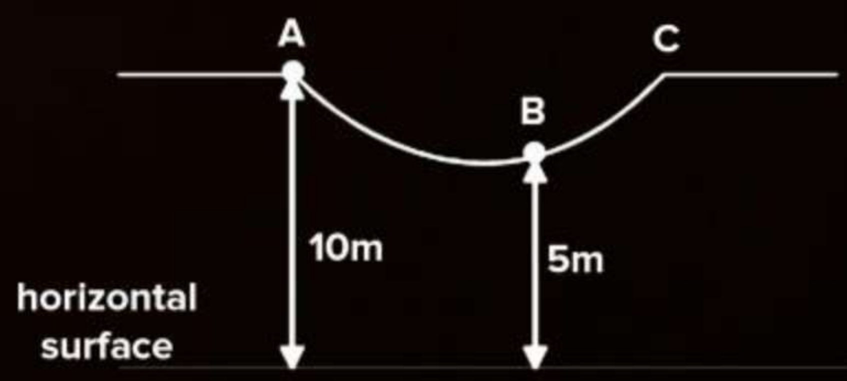
QUESTION

H.W

As shown in the figure, a particle of mass 10 kg is placed at a point A. When the particle is slightly displaced to its right, it starts moving and reaches the point B. The speed of the particle at B is x m/s. The value of 'x' to the nearest integer is

(Take $g = 10\ m/s^2$)

[18 March, 2021 (Shift-I)]

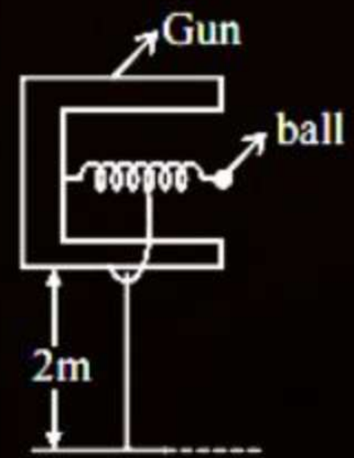


QUESTION

H.W

In a spring gun having spring constant 100 N/m a small ball 'B' of mass 100 g is put in its barrel (as shown in figure) by compressing the spring through 0.05m. There should be a box placed at a distance 'd' on the ground so that the ball falls in it. If the ball leaves the gun horizontally at a height of 2m above the ground. The value of d is ____m.

[20 July, 2021 (S-I)]



QUESTION

H.W

The ratio of spring constants of two springs is 3 : 2. If both of them are elongated upto same length. What will be the ratio of energy stored in them.

QUESTION

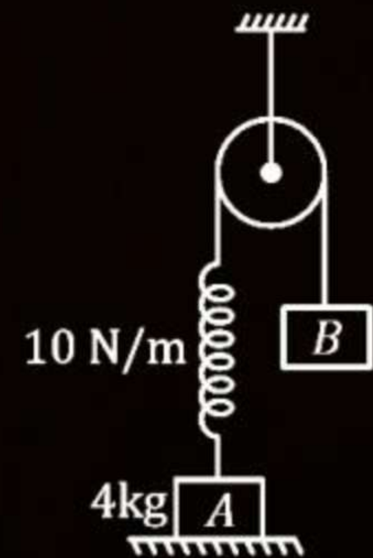
H.W.

The ratio of spring constants of two springs is 3 : 2. If both of them are elongated by same force. What is the ratio of energy stored in them in equillibrium condition?

QUESTION

H.W

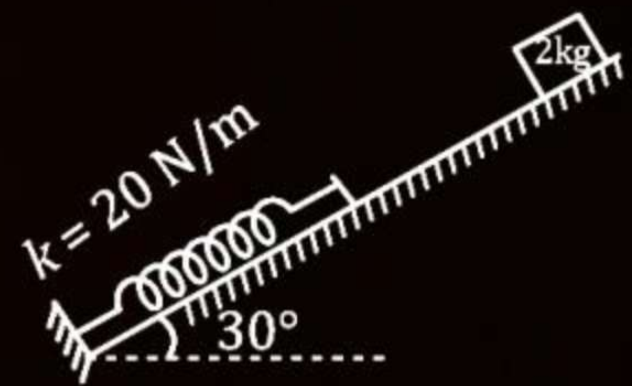
A block 'A' of mass 4 kg is connected to another block 'B' through a spring & pulley as shown in figure. Initially spring is unstretched. Find minimum value of mass of block 'B' which can just lift block 'A'.



QUESTION

H.W

A block released from rest over a smooth incline compresses the spring by 1 m. Find the distance covered by block before touching the spring.



QUESTION

A block is released into a hemispherical bowl of radius R as shown in figure. Find normal reaction when the block reaches bottommost point.

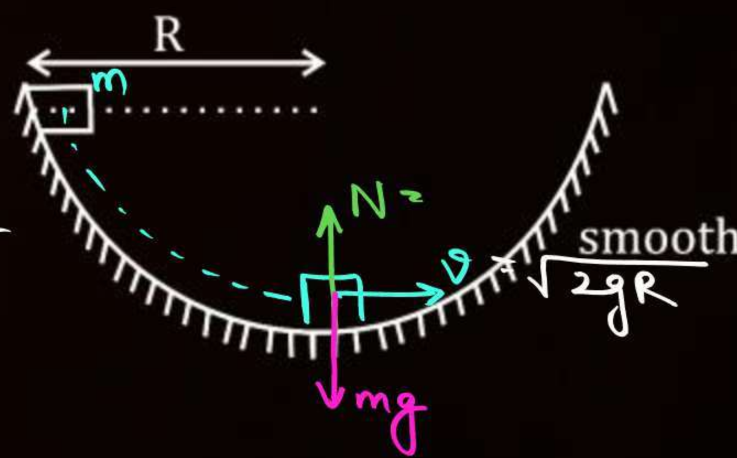
$$N \cancel{= mg}$$

$$N = mg + \frac{mv^2}{R}$$

$$= mg + \frac{m(\sqrt{2gR})^2}{R}$$

$$= mg + 2mg$$

$$= 3mg \text{ Ans}$$



QUESTION

$= \frac{2R}{3}$

A block is slightly displaced from top of a smooth hemispherical surface. At what height from the center will the block lose contact with ground?

When block is abt. to leave contact —

$$\cancel{m}g\cos\theta = \frac{\cancel{m}v^2}{R}$$

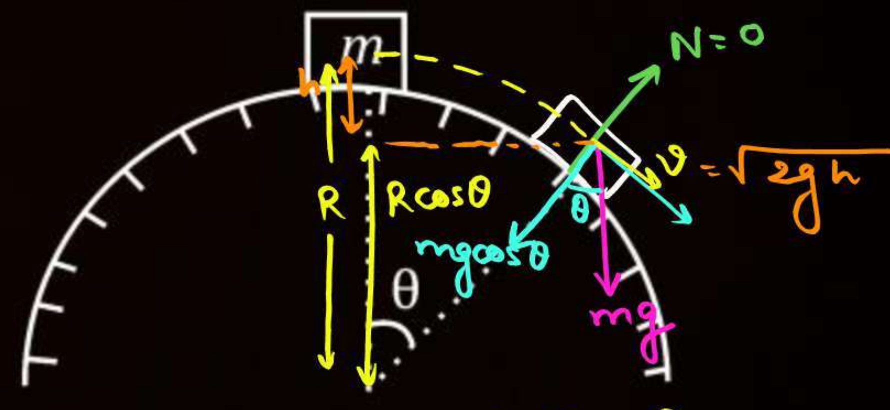
$$\cos\theta = \frac{v^2}{gR}$$

$$\cos\theta = \frac{2\cancel{g}\cancel{R}(1-\cos\theta)}{\cancel{g}\cancel{R}}$$

$$\cos\theta = 2 - 2\cos\theta$$

$$3\cos\theta = 2$$

$$\cos\theta = \frac{2}{3}$$

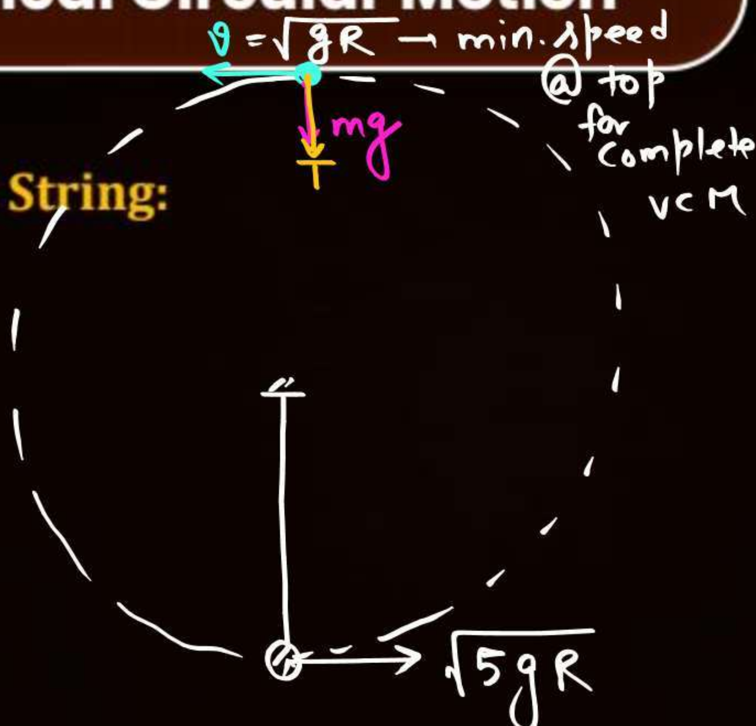


$h = R - R\cos\theta$

$v = \sqrt{2gR(1-\cos\theta)}$

Vertical Circular Motion

1. **Block Tied to String:**



At top –

$$\downarrow T + mg = \frac{mv^2}{R}\downarrow$$

When $T = 0$

$$\cancel{m}g = \frac{\cancel{m}v^2}{R}$$

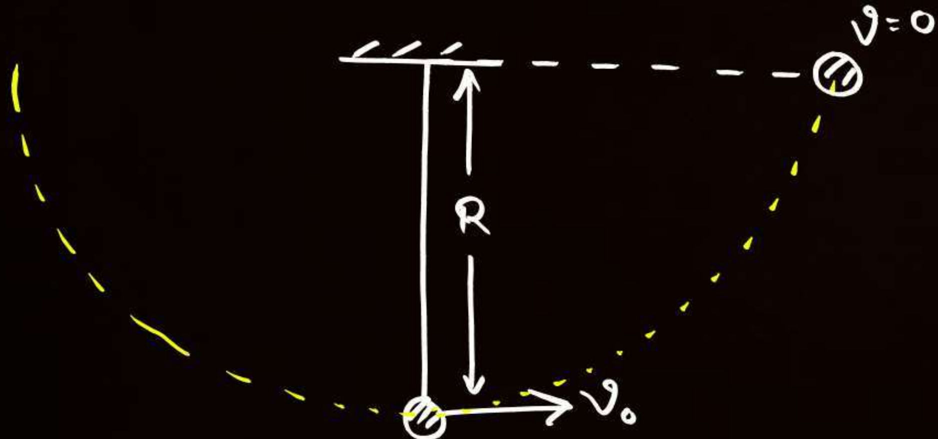
$$v = \sqrt{gR}$$

Case-II

Gain in U = Loss in k.E

$$mgR = \frac{1}{2} m v_0^2$$

$$v_0 = \sqrt{2gR}$$



$0 < v_0 \leq \sqrt{2gR} \rightarrow$ Oscillatory

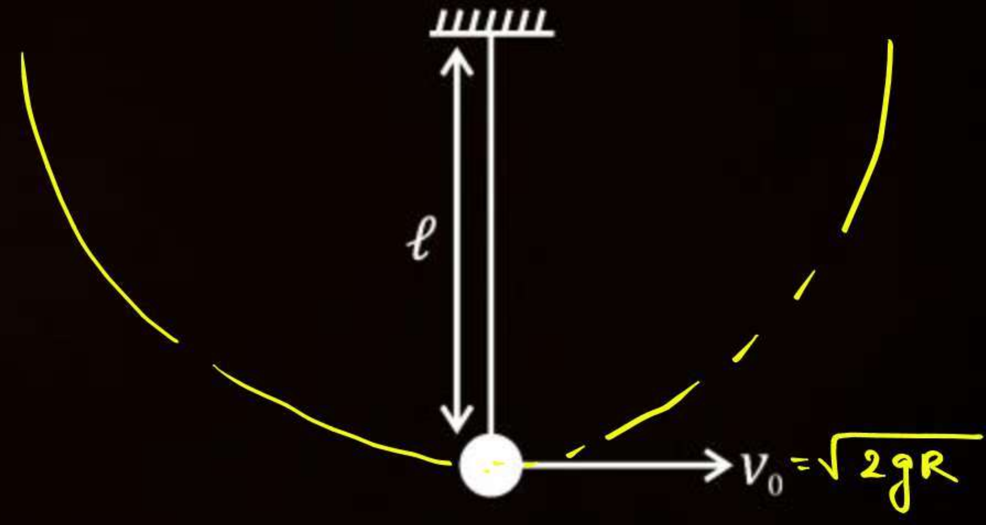
$\sqrt{2gR} < v_0 < \sqrt{5gR} \rightarrow$ String will slack at some angle θ.

$v_0 \geq \sqrt{5gR} \longrightarrow$ Complete VCM

QUESTION

Find min. speed at bottom so that string becomes horizontal.

$\vartheta = \sqrt{2gH}$



QUESTION

Find-

1. Max. angle θ string will make with vertical.
2. Tension at bottom.
3. T at θ_{max}.

1. Gain in U = Loss in k.E

$$mg(R - R\cos\theta_{max}) = \frac{1}{2} m \left(\sqrt{gR}\right)^2$$

$$\cancel{m}gR(1-\cos\theta) = \frac{\cancel{m}gR}{2}$$

$$1-\cos\theta = \frac{1}{2}$$

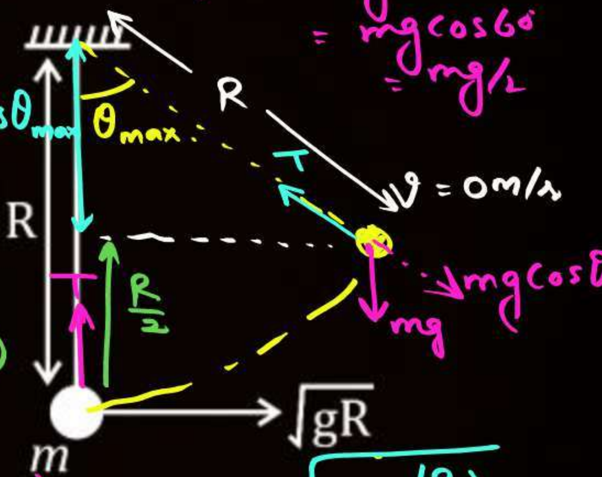
$$\cos\theta = \frac{1}{2}$$

$$\theta = 60^\circ$$

$$\frac{R}{2} = R\cos\theta_{max}$$

$$\cancel{R}\cos\theta_{max} = \frac{\cancel{R}}{2}$$

$$\theta_{max} = 60^\circ$$



$$= \sqrt{2g\left(\frac{R}{2}\right)}$$

$$\sqrt{2gh}$$

2) $T = mg + \frac{mv^2}{R}$

$$= mg + \frac{m\,g\cancel{R}}{\cancel{R}}$$

$$= 2mg$$

3) $T = mg\cos\theta$

$$= mg\cos 60^\circ$$

$$= mg/2$$

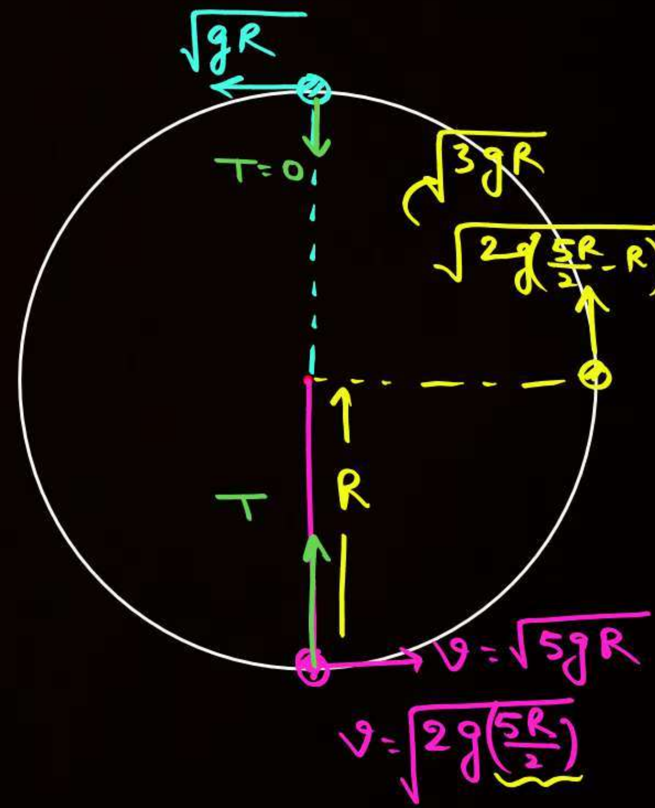
QUESTION

A small ball tied to a string of length ℓ is moving in vertical circle. The mass of ball is m. Find-

1. Minimum Speed at the top to perform complete VCM.
2. Minimum speed at the bottom to perform VCM.
3. Speed when string becomes horizontal.
4. Tension in the string at topmost & bottommost point.

$$T_{bottom} = mg + \frac{mv^2}{R} = mg + \frac{m(5gR)}{R}$$

$$T_{bottom} = 6mg.$$

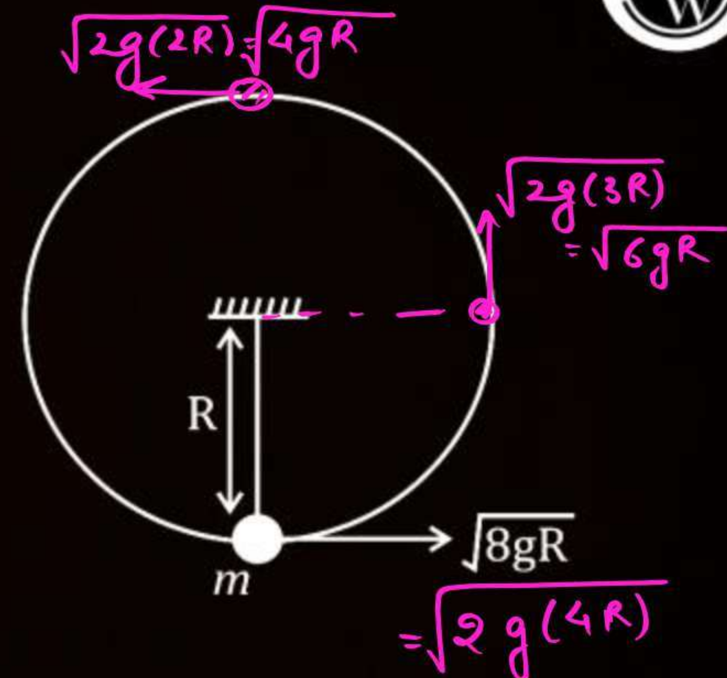


QUESTION

Find-

1. Velocity at topmost point $=\sqrt{4gR}$
2. Velocity when string becomes horizontal $=\sqrt{6gR}$
3. Tension in string when stone is at-
   (i) bottommost point
   (ii) topmost point
   (iii) when string becomes horizontal

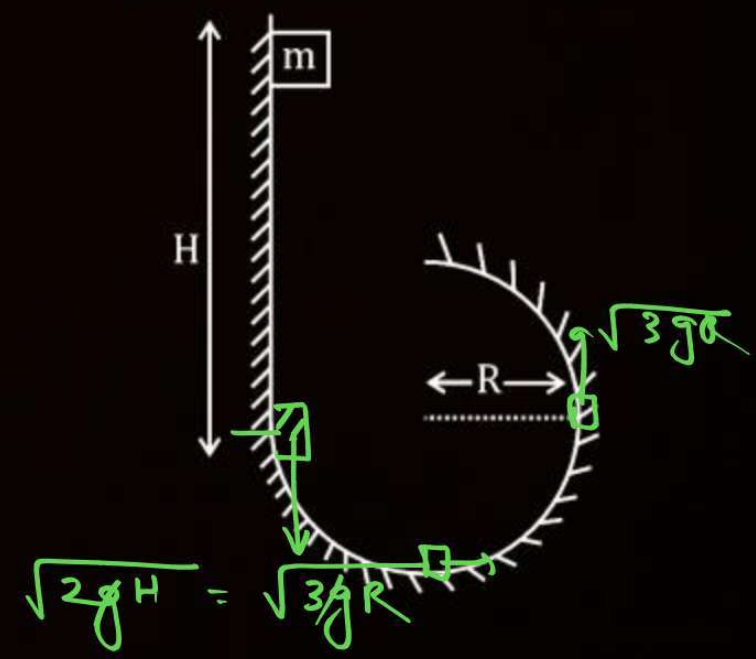
HW



QUESTION

Find min. height H from where block should be released so that it does not leave contact with surface anywhere.

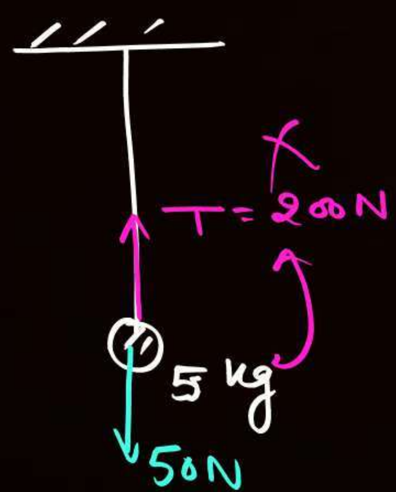
$$H = \frac{3R}{2}$$



QUESTION

String will break.

A block of mass 5 kg is tied to a string which can tolerate the max. tension of 200N will the block be able to perform VCM.



$$T_{bottom} = 6mg$$

$$= mg + m\frac{v^2}{R}$$

$$= mg + \frac{m\left(\sqrt{5gR}\right)^2}{R}$$

$$= 6mg$$

QUESTION

Find minimum speed at the bottom required to complete VCM. Also find tension at the point.

$v = \sqrt{5gR}$

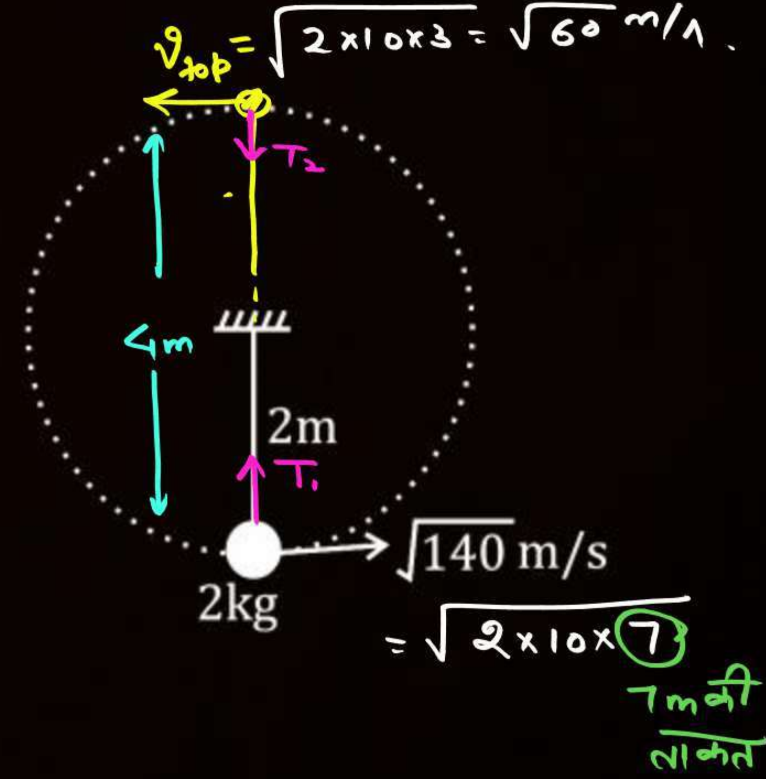
$= \sqrt{5 \times 10 \times 2}$

$= \sqrt{100}$

$= 10\ m/s.$

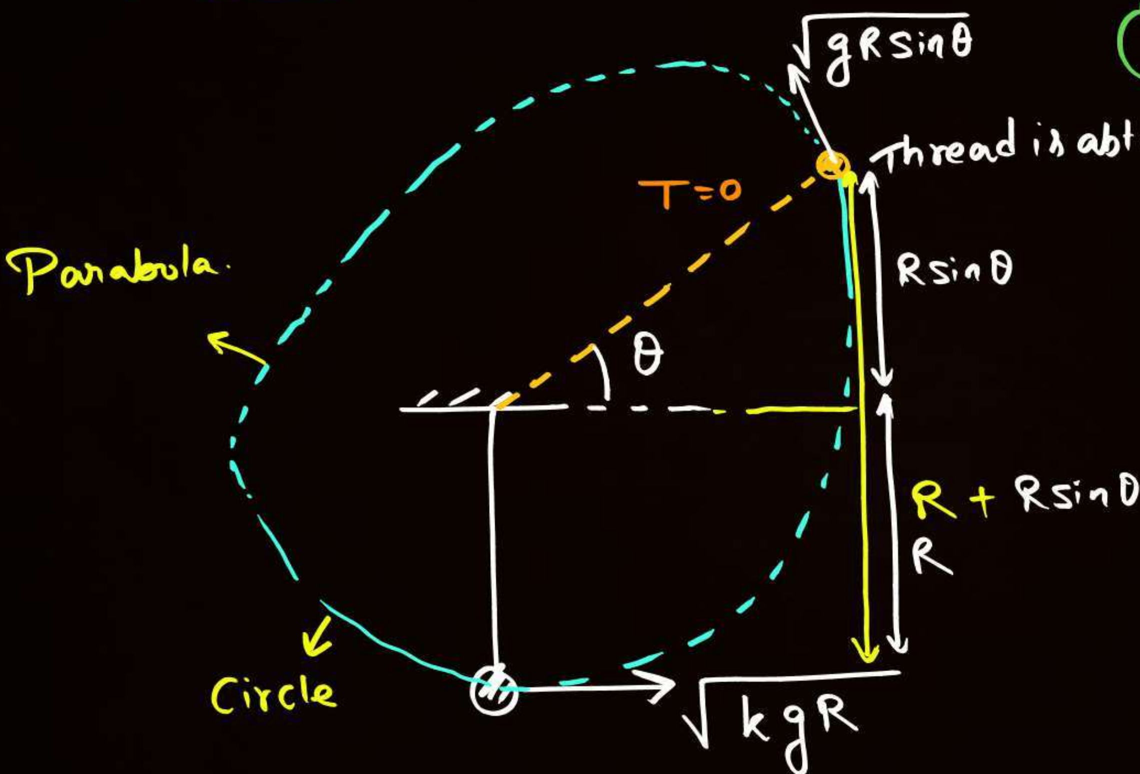
QUESTION

Find velocity & tension in string at bottommost & topmost point.



Brahmastra $\longrightarrow$

$$\sin\theta = \frac{(k-2)}{3}$$



$$2 < k < 5$$

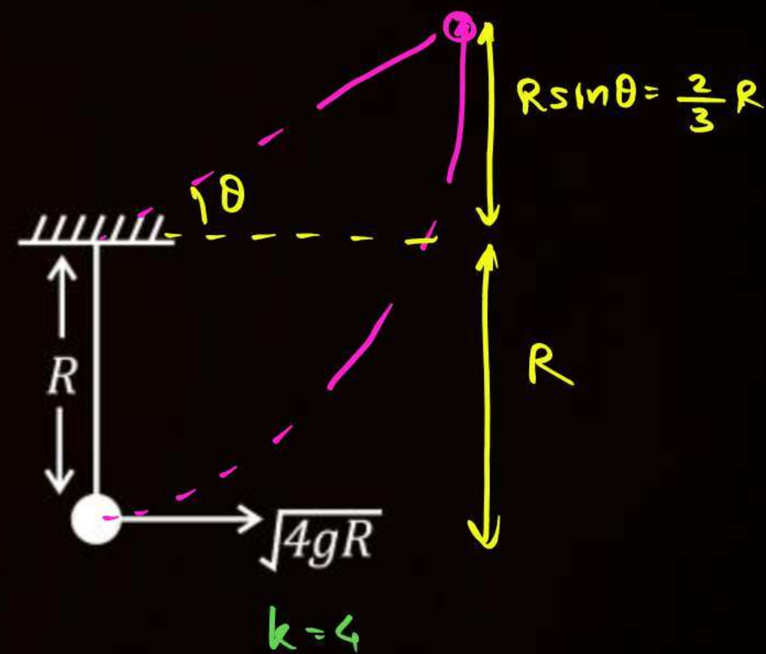
QUESTION

Find height at which thread will slack.

$$\sin\theta = \frac{k-2}{3}$$

$$= \frac{4-2}{3} = \frac{2}{3}$$

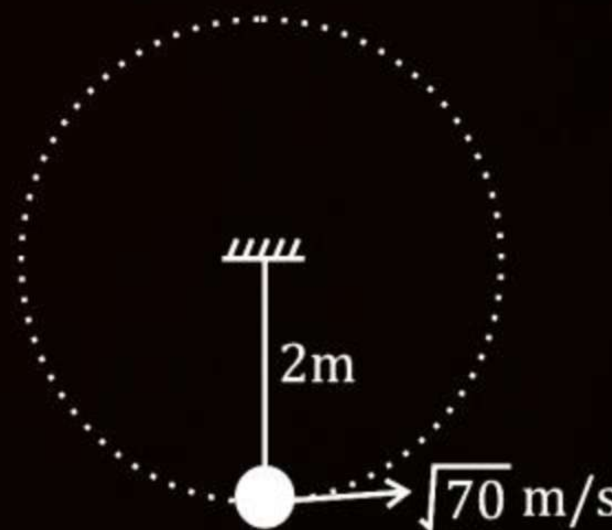
$$H = R + \frac{2R}{3} = \frac{5R}{3}.$$



QUESTION

HW

Find height from the lowest point where the string will start to slack. Also find speed at that point.



QUESTION

HW

A stone of mass m, tied to a string is being whirled in a vertical circle with a uniform speed. The tension in the string is [24 June, 2022 (Shift-II)]

(A) The same throughout the motion

(B) Minimum at the highest position of the circle path.

(C) Minimum at the lowest position of the circular path

(D) Minimum when the rope is the horizontal position.

QUESTION

H.W

A small bob tied at one end of a thin string of length 1 m is describing a vertical circle so that the maximum and minimum tension in the string are in the ratio 5 : 1. The velocity of the bob at the highest position is _____ m/s. (Take $g = 10\ m/s^2$)

[25 Feb, 2021 (Shift-I)]

QUESTION

H.W

A stone tied to a string of length L is whirled in a vertical circle with the other end of the string at the centre. At a acertain instant of time, the stone is at its lowest position and has a speed u. The magnitude of change in its velocity, as at reaches a position where the string is horizontal, is $\sqrt{x(u^2 - gL)}$. The value of x is

[27 June, 2022 (Shift-II)]

(A) 3

(B) 2

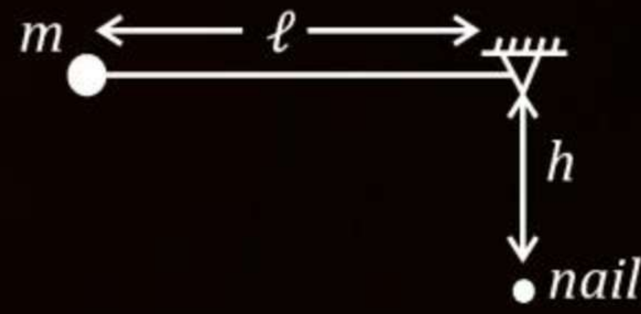
(C) 1

(D) 5

QUESTION

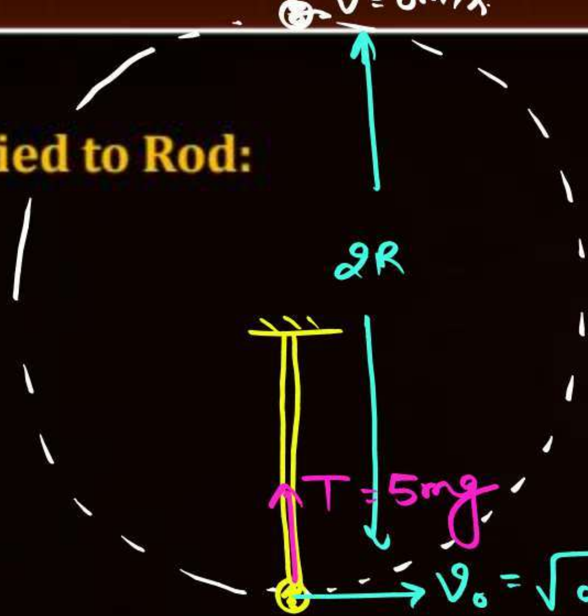
H.W

Find min. value of h for which body of mass m can perform complete VCM abt. the nail.



Vertical Circular Motion

2. Block Tied to Rod:

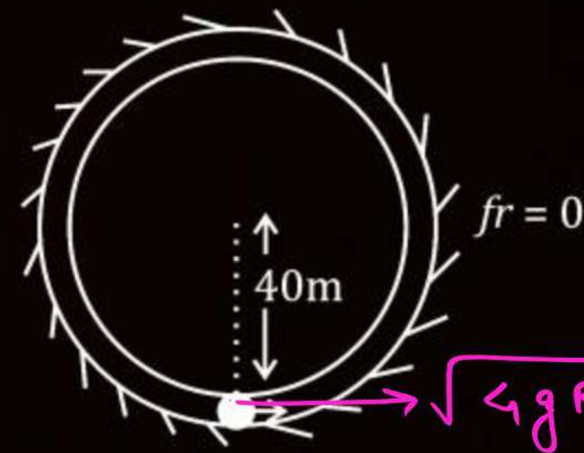


$v_0 = \sqrt{2g(2R)}$

$= \sqrt{4gR} \rightarrow$ min. speed at bottom

QUESTION

Find min. speed at bottom for complete VCM.



$\sqrt{4gR}$

$\sqrt{4\times10\times40}$

$= 40\,m/s$

QUESTION

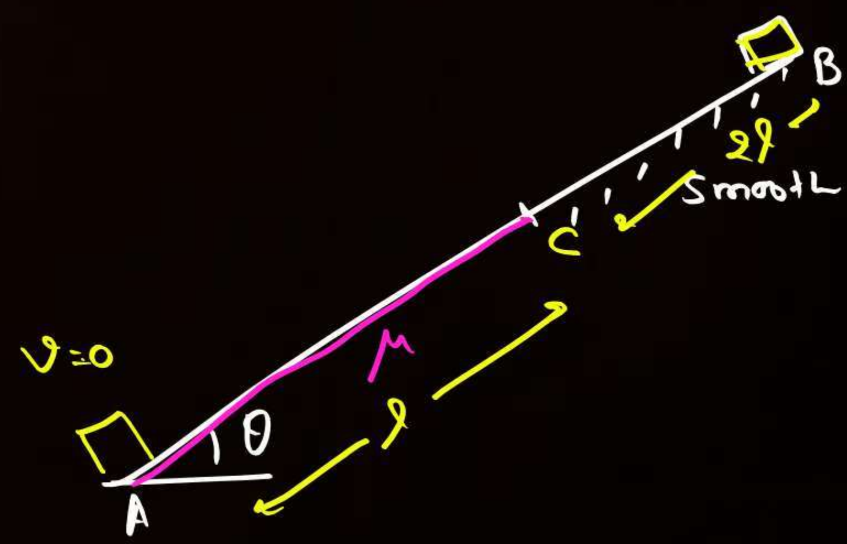
H.W

A small block starts slipping down from a point B on an inclined plane AB, which is making an angle θ with the horizontal section BC is smooth and the remaining section CA is rough with a coefficient of friction μ. It is found that the block comes to rest as it reaches the bottom (point A) of the inclined plane. If BC = 2AC, the coefficient of friction is given by μ = *k* tan θ. The value of *k* is ________. [NA 2 Sep. 2020 (I)]

$$TWD = \Delta K.E$$

$$W_{mg} + W_{fr} = 0 - 0$$

$$W_{mg} = -W_{fr}.$$



Homework

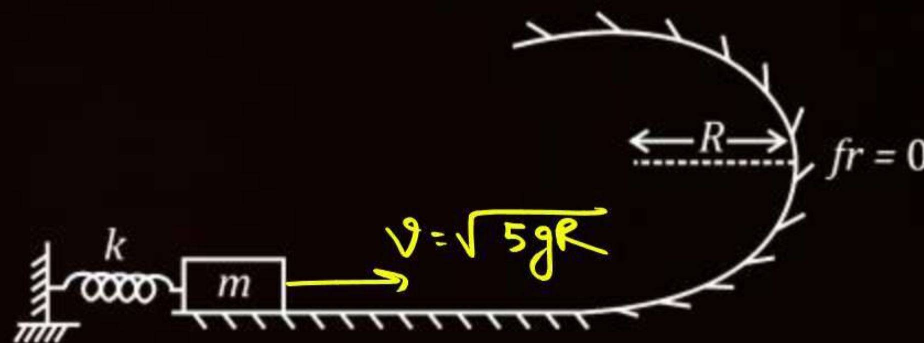
Find min. compression in spring so that block does not leave contact will surface anywhere.

Loss in U = Gain in k.E

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2$$

$$kx^2 = m\left(\sqrt{5gR}\right)^2$$

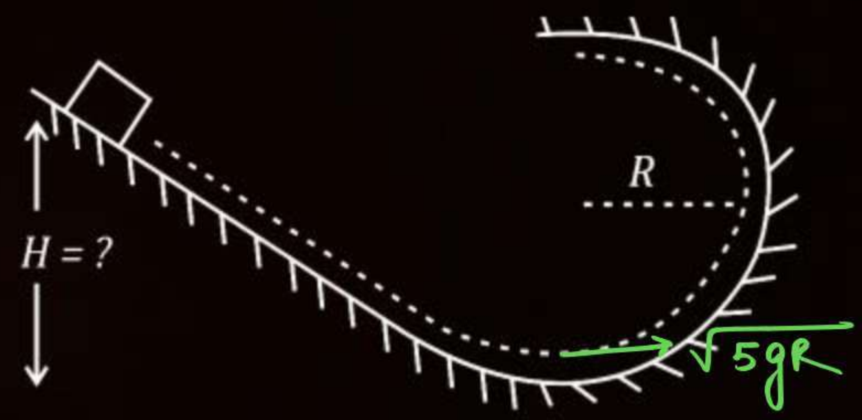
$$x = \sqrt{\frac{m}{k} \times 5gR}$$



QUESTION

H.W

Find:



Power

→ Rate of doing Work.

- Scalar
- SI Unit :- Watt = J/s.
- $[P] = [ML^2T^{-3}]$

Horsepower

$1\ HP = 746$ Watts.

Power

Average Power

$$P_{avg} = \frac{\text{Total Work}}{\text{Total time}}$$

$$P_{avg} = \frac{\Delta W}{\Delta t}$$

Instantaneous Power

$$P = \frac{dW}{dt}$$

$$P = \frac{\vec{F} \cdot d\vec{s}}{dt}$$

$$P = \vec{F} \cdot \vec{v}$$

QUESTION

Find-

1. Average power delivered by force in first 10 sec.
2. Power delivered by force at $t = 10s$.

$v = u + at$

$= 0 + 8 \times 10 = 80 m/s$

$a = \frac{F_{\parallel}}{m} = \frac{40}{5} = 8 m/s^2$

1) $P_{avg} = \frac{Work}{Time} = \frac{F_{\parallel} \cdot S}{t}$

$= \frac{40 \times 400}{10}$

$= 1600 W$

$S = \frac{1}{2} a t^2$

$= \frac{1}{2} \times 8 \times 10^2 = 400 m$

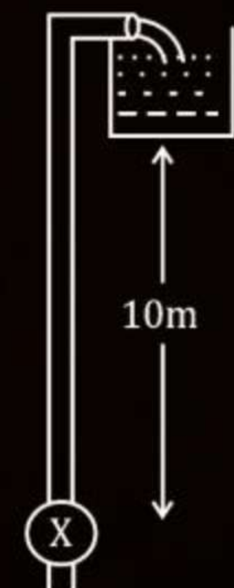
2) $P = \vec{F} \cdot \vec{v}$

$= F_{\parallel} \cdot v = 40 \times 80 = 3200 W$

QUESTION

A water pump can lift 60kg water to 10m height in one minute find power of the pump.

$$P = \frac{W}{T} = \frac{mgH}{T} = \frac{\cancel{60} \times 10 \times 10}{\cancel{60}} = 100\,W$$



QUESTION

Const. vector

Find average power delivered by force $F = \hat{\imath} + 2\hat{\jmath}$ if it produces a displacement of $(2\hat{\imath} - \hat{\jmath})$ in 5sec.

$$P_{avg} = \frac{W}{T} = \frac{\vec{F} \cdot \vec{S}}{T} = \frac{(\hat{i} + 2\hat{j}) \cdot (2\hat{i} - \hat{j})}{5}$$

$$= \frac{2 - 2}{5} = 0 \text{ watt.}$$

QUESTION

A person is pulling a thread with a force of 60N. Find power delivered by tension & gravity in first 10 sec (Initially block is at rest).

$$P_T = \frac{W_T}{t} = \frac{T \cdot S}{t} = \frac{60 \times 100}{10} = 600\,W$$

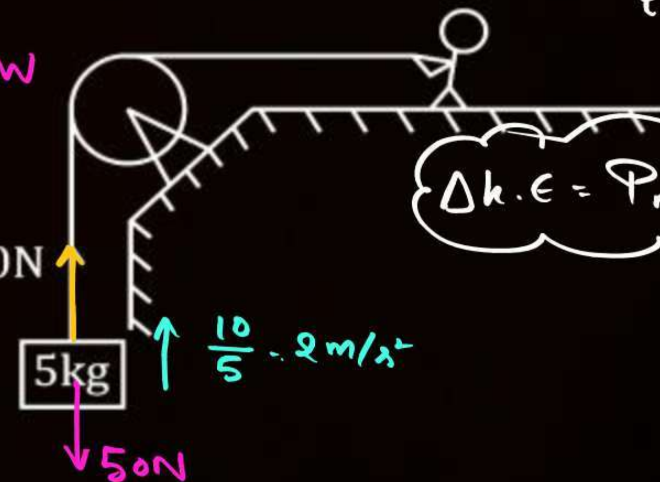
$$P_{mg} = \frac{W_{mg}}{t} = \frac{-mgH}{t} = \frac{-50 \times 100}{10} = -500\,W$$

$$S = \frac{1}{2} \times 2 \times 10^2 = 100\,m$$

$$P_{net} = 600 - 500 = 100\,W$$

$$P_{net} = \frac{W_{total}}{t} = \frac{\Delta K.E}{t}$$

$$\Delta K.E = P_{net} \times t$$



$$\frac{10}{5} = 2\,m/s^2$$

QUESTION

If total power delivered to a 5kg block is 100W. Find its kinetic energy after 10s. (Initially block is at rest).

$$100\,W = 100\,\frac{J}{s}$$

$$\Delta K.E = \text{Total energy delivered} = P\Delta t$$
$$= 1000\,J.$$

QUESTION

A car engine provides constant power of 2000W. Find acceleration of the car when its speed is 5 m/s. (Mass of car = 400kg). Neglect air resistance.

$$P = F \cdot v$$

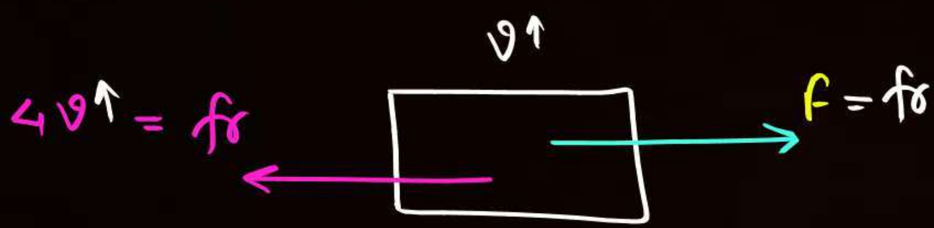
$$2000 = F \cdot 5$$

$$F = \frac{2000}{5} = 400\,N$$

$$a = \frac{F}{m} = \frac{400}{400} = 1\,m/s^2$$

QUESTION

Engine of a car provides constant power of 800W. If wind resistance after a retardation of 4v N on the car where v is the speed of car. Find top speed of the car.



$P = \downarrow F \cdot v \uparrow$

At v_{max} — $F = fr$

$800 = (4v) \cdot v$

$\cancel{4}v^2 = \cancel{800}\ 200$

$v = \sqrt{200}$

$v = 10\sqrt{2}$ m/s

QUESTION

H.W

A water pump fills 1000 litre water tank at height 30m from ground in 1 hr. Find electric power consumed by pump if it works at 50% efficiency.

QUESTION

Water falls from a height of 60 m at the rate of 15 kg/s to operate a turbine. The losses due to frictional force are 10% of the input energy. How much power is generated by the turbine? ($g = 10\ m/s^2$)

$$\frac{U}{t} = mgH = 15 \times 10 \times 60$$
$$= 9000\ \frac{J}{s}$$

$$P_{out} = 90\% \times P_{in}$$
$$= .9 \times 9000$$
$$= 8100\ W.$$

(A) 10.2 kW

(B) 8.1 kW

(C) 12.3 kW

(D) 7.0 kW

QUESTION

A body of mass 2 kg is driven by an engine delivering a constant power of 1 J/s. The body starts from rest and moves in a straight line. After 9 seconds, the body has moved a distance (in m) ______. 18m

[5 Sep. 2020 (II)]

$m = 2\,kg$

$P = 1\ J/s$

$T = 9\,s$

$\Delta K.E = \text{Energy delivered}$

$= 1 \times 9 = 9\,J$

$\frac{1}{2} \times 2 \times v^2 = 9$

$v = 3\,m/s$

$F \cdot v = 1$

$m a \cdot v = 1$

$2 \cdot a \cdot v = 1$

$a \cdot v = \frac{1}{2}$

$\frac{v\,dv}{dx} \cdot v = \frac{1}{2}$

$\int_0^v v^2\,dv = \int_0^x \frac{1}{2}\,dx$

$\frac{v^3}{3} = \frac{1}{2}x$

$\frac{(3)^{\cancel{3}}}{\cancel{3}} = \frac{x}{2}$

$x = 18\,m$

QUESTION

A 60 HP electric motor lifts an elevator having a maximum total load capacity of 2000 kg. If the frictional force on the elevator is 4000 N, the speed of the elevator at full load is close to: (1 HP = 746W, $g = 10\ \text{ms}^{-2}$) **[7 Jan. 2020 I]**

(A) $1.7\ \text{ms}^{-1}$

(B) $1.9\ \text{ms}^{-1}$

(C) $1.5\ \text{ms}^{-1}$

(D) $2.0\ \text{ms}^{-1}$

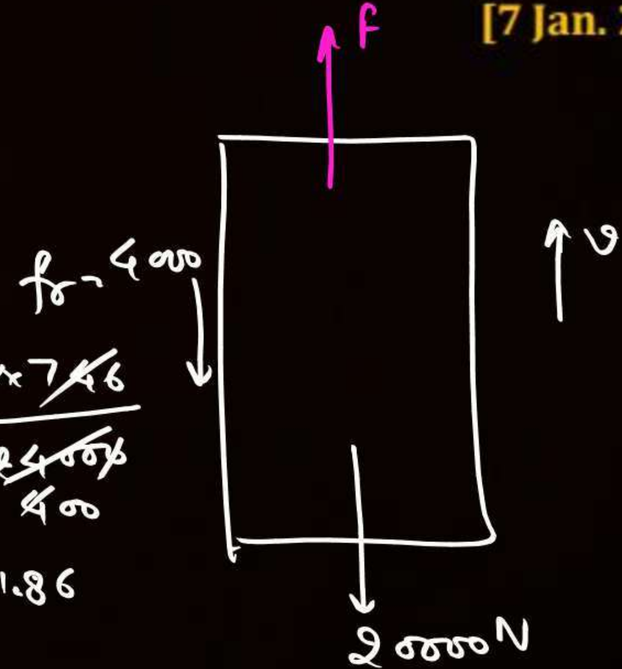
$$F = mg + f_r$$

$$= 20{,}000 + 4{,}000$$

$$F = 24{,}000\ \text{N}$$

$$P = F\cdot v \Rightarrow v = \frac{P}{F} = \frac{\cancel{60}\times 7\cancel{46}}{\cancel{24000}\ 400}$$

$$= 1.86$$



QUESTION

H.W

A particle of mass M is moving in a circle of fixed radius R in such a way that its centripetal acceleration at time t is given by $n^2 R t^2$ where n is a constant. The power delivered to the particle by the force acting on it, is: **[Online April 10, 2016]**

(A) $1/2\, M n^2 R^2 t^2$

(B) $M n^2 R^2 t$

(C) $M n R^2 t^2$

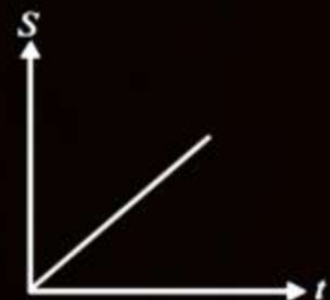
(D) $M n R^2 t$

QUESTION

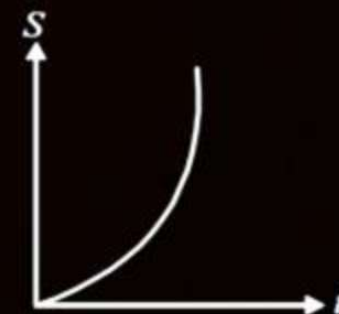
A particle is moving unidirectionally on a horizontal plane under the action of a constant power supplying energy source. The displacement (s) – time (t) graph that describes the motion of the particle is (graphs are drawn schematically and are not to scale):

[3 Sep. 2020 (II)]

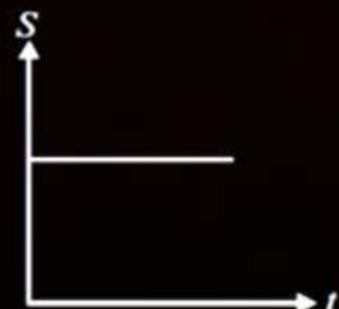
(A)



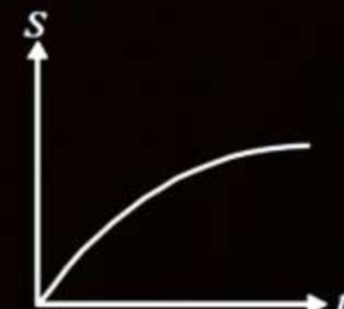
(B)



(C)



(D)



$P = \text{const.}$

$F.v = \text{const}$

$m.a.v = \text{const}$

$\frac{dv}{dt}.v = \frac{k}{m}$

$\int v\,dv = \int \frac{k}{m}\,dt$

$\frac{v^2}{2} = \frac{k}{m}t$

$v = \sqrt{\frac{2k}{m}t}$

$\frac{dx}{dt} = \sqrt{\frac{2k}{m}t}$

$\int dx \propto \int \sqrt{t}\,dt$

$x \propto \frac{t^{3/2}}{3/2}$

$x \propto t^{3/2}$

QUESTION

A particle of mass m is driven by a machine that delivers a constant power k watts. If the particle starts from rest, the force on the particle at time t is

(A) $\sqrt{\frac{mk}{2}}\,t^{-1/2}$

(B) $\sqrt{mk}\,t^{-1/2}$

(C) $\sqrt{2mk}\,t^{-1/2}$

(D) $\frac{1}{2}\sqrt{mk}\,t^{-1/2}$

QUESTION

H.W

A body of mass 2 kg is initially at rest. It starts moving unidirectionally under the influence of a source of constant power P. Its displacement in 4s is $1/3\ \alpha^2 \sqrt{P} m$. The value of α will be-

[30 Jan, 2023 (S-II)]

QUESTION

H.W

The ratio of power of two motors is $\frac{3\sqrt{x}}{\sqrt{x}+1}$, that are capable of raising 300 kg water in 5 minute and 50 kg water in 2 minutes respectively from a wall of 100m deep. The value of x will be

[13 April, 2023 (S-I)]

(A) 2

(B) 4

(C) 2.4

(D) 16

QUESTION

H.W

A body of mass 1 kg begins to move under the action of a time dependent force $\vec{F} = (t\hat{\imath} + 3t^2\hat{\jmath})N$. where $\hat{\imath}$ and $\hat{\jmath}$ are the unit vectors along x and y axis. The power developed by above force, at the time $t = 2s$. will be ____ W. [24 Jan, 2023 (S-II)]

QUESTION

H.W

If the maximum load carried by an elevator is 1400 kg (600 kg – Passenger + 800 kg – elevator), which is moving up with a uniform speed of 3 ms^{-1} and the frictional force acting on it is 2000 N, then the maximum power used by the motor is ____ kW.

($g = 10\ m/s^2$).

[10 April, 2023 (Shift-II)]

QUESTION

H.W

A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration (a) is varying with time t as $a = k^2rt^2$, where k is a constant. The power delivered to the particle by the force acting on it is given as

[28 June, 2022 (Shift-I)]

(A) Zero

(B) $mk^2r^2t^2$

(C) mk^2r^2t

(D) mk^2rt

QUESTION

H.W

Sand is being dropped from a stationary dropper at a rate of 0.5 kg s^{-1} on a conveyor belt moving with a velocity of 5 ms^{-1}. The power needed to keep the belt moving with the same velocity will be:

[27 July, 2022 (Shift-I)]

(A) 1.25 W

(B) 2.5 W

(C) 6.25 W

(D) 12.5 W

QUESTION

H.W

A body at rest is moved along a horizontal straight line by a machine delivering a constant power. The distance moved by the body in time 't' is proportional to:

[20 July, 2021 (Shift-II)]

A $t^{3/4}$

B $t^{3/2}$

C $t^{1/2}$

D $t^{1/4}$

Homework

H.W

→ DPP

→ Class Q.