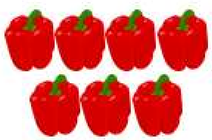
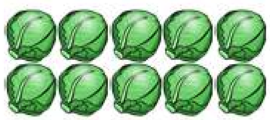
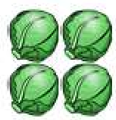
Permutation and Combination: It’s all about Counting

#Baspan



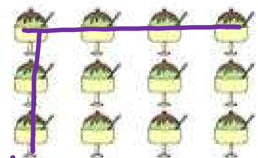
| 7 | + | 4 | = | 11 |





| | + | | = | |

1)

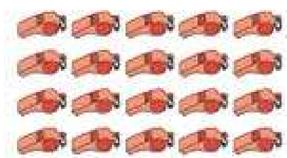


4 × 3 = 12

2)

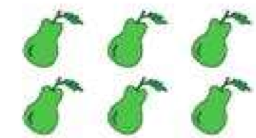
___ × ___ = ___

3)



___ × ___ = ___

4)



___ × ___ = ___

Factorial !

↳ $n!$

$3! = 3 \times 2 \times 1$

$4! = 4 \times 3 \times 2 \times 1$

$5! = 5 \times 4 \times 3 \times 2 \times 1$

__ __ __

A B C $3!$

A	B	C
A	C	B
B	A	C
B	C	A
C	A	B
C	B	A

$\rightarrow 3! = 6$

Factorial !

$0! = 1$

$1! = 1$

$2! = 2 \times 1 = 2$

$3! = 6$

$4! = 24$

$5! = 120$

Fundamental Principle of Counting

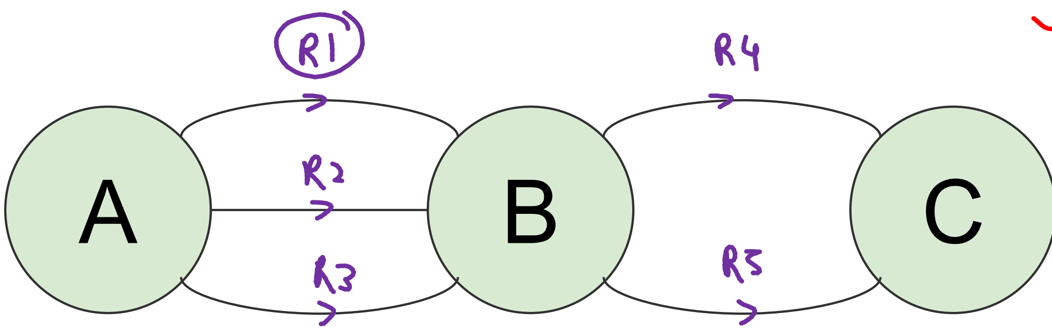
Fundamental Principle of Counting

AND $\Rightarrow$ $\times$

OR $\Rightarrow$ $+$

Fundamental Principle of Counting

City A and City B are connected by 3 different routes and City B and City C are connected by different routes. In how many ways can a person go from city A to city C via B?



✓ A – B **and** B – C

✗ A – B OR B – C

$3 \times 2 = \boxed{6}$

R1 R4
R1 R5
R2 R4
R2 R5
R3 R4
R3 R5

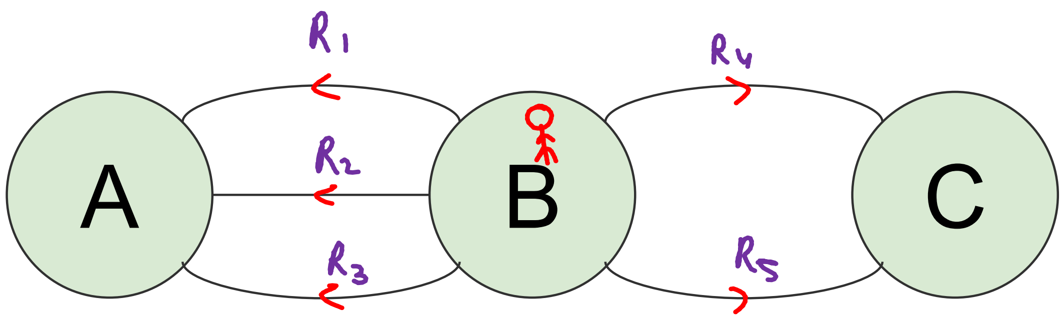
6 ways

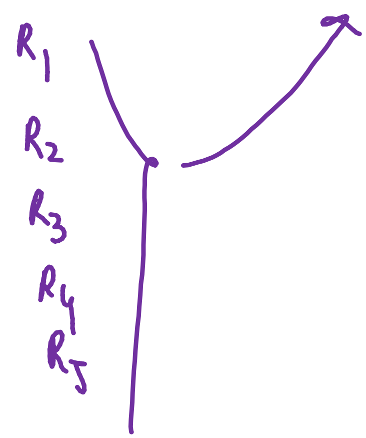
Fundamental Principle of Counting

In how many ways a person in city B can exit city B?

B — A OR B — C

$3 + 2 = 5$





Agar do Cases ⇒ Add

Case k andar ⇒ X

Concept of filling vacant spaces

Ya phir = OR

Example

How many 3 digit numbers can be formed by the digit 1, 2, 3, 4, 5

(i) without repetition

(ii) with repetition (Easy)

ii) 1|2|3|4|5 1|2|3|4|5

$5 \times 5 \times 5 \Rightarrow 125$

i) 1|2|3|4|5 1|2|4|5 1|4|5

3 2 4

5 4 3

$5 \times 4 \times 3 = 60$

How many **6 digits odd number** greater than 6,00,000 can be formed from the digits 5, 6, 7, 8, 9, 0 if repetition of digit is allowed ?

5, 6, 7, 8, 9, 0

6|7|8|9   5|6|7|8|9|0   5/7/9

_ _ _ _ _ _

4   6   6   6   6   3

$= 4 \times 6 \times 6 \times 6 \times 6 \times 3$

ii) Repe is Not allowed.

$\boxed{5, 6, 7, 8, 9, 0}$

C-1: 6 7 8 0 _ 5 (choices: 6/7/8/9, 7/8/9/0, 8/9/0, 0/9, 9)

$4 \times 4 \times 3 \times 2 \times 1 = 4 \times 4! = 96$

C-2: 6 _ _ _ _ 7 (choices: 6/8/9, 5/8/9/0)

$= 3 \times 4! = 72$

C-3: 7 _ _ _ _ 9 (choices: 6/7/8, 5/6/8/0)

$= 3 \times 4! = 72$

Add = 240

Rank of a word

Rank of Word

TAX A, T, X $\Rightarrow$ 3!

A T X

A X T

3rd $\boxed{T\ A\ X}$

T X A

X A T

X T A

6 words.

Shortcut - 1
Finding RANK
(Without repetition)

Find Rank of a Word “MATHS”

A. 32

B. 53

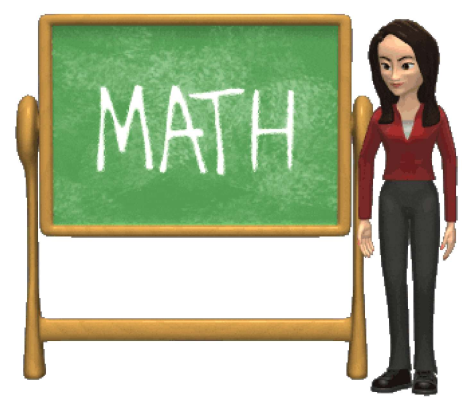
C. 54

D. 55

Without repetition

3	1	5	2	4
M	A	T	H	S
2	0	2	0	0
4!	3!	2!	1!	0!

48 + 0 + 4 + 0 + 0 + 1



Without repetition

Find Rank of a Word “MOTHER”

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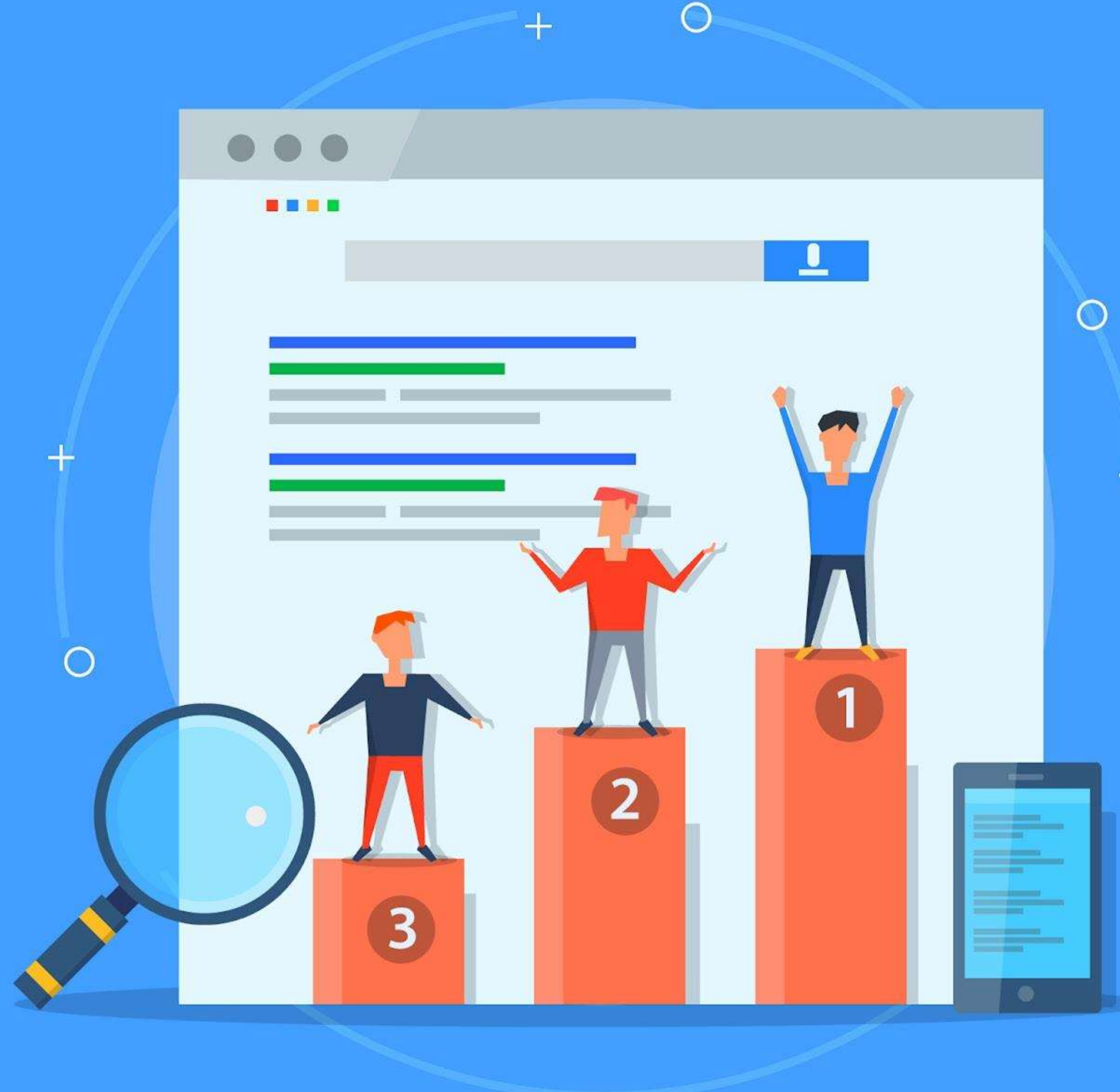
3	4	6	2	1	5
M	**O**	**T**	**H**	**E**	**R**
2	2	3	1	0	0
5!	4!	3!	2!	1!	0!

$240 + 48 + 18 + 2 + 0 + 0 + 1 = \underline{\underline{309}}$

Shortcut - 2

Finding RANK

(With repetition)



With repetition

Find Rank of a Word “India”

A. 46

B. 47

C. 55

D. 42

3 4 2 3 1

I	N	D	I	A

2 3 1 1 0

2!

4! 3! 2! 1! 0!

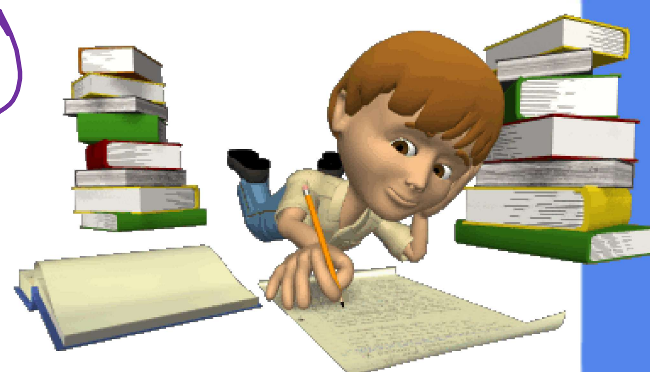
24 + 18 + 2 + 1 + 0 + 1

With repetition

Find Rank of a Word "IIT JEE" * (JEE Adv.)

2	2	4	3	1	1
I	I	T	J	E	E
$\frac{2}{2!\,2!}$	$\frac{2}{2!}$	$\frac{3}{2!}$	$\frac{2}{2!}$	$\frac{0}{2!}$	$\frac{0}{1!}$
$5!$	$4!$	$3!$	$2!$	$1!$	$0!$

$$60 + 24 + 9 + 2 + 0 + 0 + 1 = 96$$



The letters of the word COCHIN are permuted and all the permutations are arranged in an alphabetical order as in an English dictionary. The number of words that appear before the word COCHIN is

(a) 360 (b) 192 (c) 96 (d) 48

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1 5 1 2 3 4

C $\boxed{O\ C\ H\ I\ N}$

0 $\frac{4}{1!}$ 0 0 0 0

5! 4! 3! 2! 1! 0!

0 + 96 + 0 + 0 + 0 + 0

Basic Questions

If the digits are not allowed to repeat in any number formed by using the digits 0, 2, 4, 6, 8, then the number of all numbers greater than 10,000 is equal to

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Repe → x

0, 2, 4, 6, 8

2/4/6/8   0/4/6/8

2 0 _ _ _

4   4   3   2   1

$4 \times 4! = 96$

The total number of numbers, lying between 100 and 1000 that can be formed with the digits 1,2,3,4,5, if the repetition of digits is not allowed and numbers are divisible by either 3 or 5 is

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1|2|3|4

1, 2, 3, 4, 5 Repe → x

_ _ 5

4 3

$\boxed{\div 5} \Rightarrow 12$

$\boxed{\div 3} \Rightarrow 3! \times 4 = \boxed{24}$

1, 2, 3 ⇒ _ _ _ 3!

3 4 5 ⇒ _ _ 5 3!

1, 3, 5 ⇒ _ _ 5 3!

2, 3, 4 ⇒ _ _ _ 3!

$12 + 24 - 2 - 2$

$36 - 4$

32

The number of three-digit even numbers, formed by the digits 0, 1, 3, 4, 6, 7 if the repetition of digits is not allowed, is.

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0, 1, 3, 4, 6, 7

C-1 $\underset{5}{\_}\ \underset{4}{\_}\ \overset{0}{\_} = 20$

1|3|6|7

C-2 $\underset{4}{\overset{1}{\_}}\ \underset{4}{\_}\ \overset{4}{\_} = 16$

C-3 $\underset{4}{\_}\ \underset{4}{\_}\ \overset{6}{\_} = 16$

52

The number of 5 digit numbers which are divisible by 4, with digits from the set {1, 2, 3, 4, 5} and the repetition of digits is allowed, is __________.

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1|2|3|4|5

$\boxed{\div 4}$ last two digits $\div 4$

_ _ _ 1 2 = 125

5 5 5

_ _ _ 2 4 = 125

5 5 5

_ _ _ 3 2 = 125

5 5 5

_ _ _ 4 4 = 125

_ _ _ 5 2 = 125

$125 \times 5 = 5^4 = 625$

Number of zeros in n!

Finding the power of primes

Ex $100! = 100 \times 99 \times 98 \times 97 \times \ldots\ldots 1 = \ldots\ldots\ldots \underbrace{0000000}_{24 \text{ digits}}$

$= 2^2 \times 5^2 \times 3^2 \times 11 \times 2 \times 7^2 \ldots\ldots\ldots 1$

$= 2^{\boxed{97}} \; 3^{\boxed{?}} \; 5^{\boxed{24}}$

$\boxed{50+25+12+6+3+1}$

$\frac{100}{2} = [50]$ $\quad \frac{12.25}{2} = [6.125]$ $\quad \left[\frac{1.5m}{2}\right] = [0.5m]$

$\frac{50}{2} = [25]$ $\quad \frac{6.125}{2} = [3.5m]$

$\frac{25}{2} = [12.25]$ $\quad \frac{3.5m}{2} = [1.5m]$

Find number of Zeros in 100!

$$\frac{100}{5} = [20]$$

$$\frac{20}{5} = [4]$$

$$\frac{4}{5} = [0.\text{sm}]$$

$$24$$

Find number of Zeros in 1000!

$$\frac{1000}{5} = [200]$$

$$\frac{200}{5} = [40]$$

$$\frac{40}{5} = [8]$$

$$\frac{8}{5} = [1.5m]$$

$$\frac{1.5m}{5} = 0.5m$$

Add = 249

Find number of Zeros in 500!

$$\frac{500}{5} = [100]$$

$$\frac{100}{5} = [20]$$

$$\frac{20}{5} = [4]$$

$$\frac{4}{5} = [0.\text{sm}]$$

$$\underline{124}$$

Permutation vs Combination

Permutation vs Combination:

Arrangement

$$\boxed{{}^nP_r = \frac{n!}{(n-r)!}}$$

Selection

$$\boxed{{}^nC_r = \frac{n!}{r!\,(n-r)!}}$$

$$\boxed{{}^nP_r = {}^nC_r \times r!}$$

ABD ← A B C D E

A B C D E

⇓

Koi bhi 3

__ __ __

$$\boxed{{}^5P_3 = {}^5C_3 \times 3!}$$

${}^5C_3$

Permutation vs Combination :

(i) Out of A, B, C, D take 3 letter & form number plate of car. ${}^4P_3$

(ii) Out of four letters A, B, C, D take any 3 letters & form triangle. ${}^4C_3$

$\boxed{ABC}$ ←

$\boxed{CAB}$ ←

$\Delta ABC = \Delta BCA = \Delta CAB$

$4 \rightarrow 3$

Question based on Formula (^nC_r and ^nP_r)

If ${}^{1}P_1 + 2.{}^{2}P_2 + 3.{}^{3}P_3 + \ldots + 15.{}^{15}P_{15} = {}^{q}P_r - s$, $0 \le s \le 1$ then ${}^{q+s}C_{r-s}$ is equal to.

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$$S = 1! + 2(2!) + 3(3!) + \ldots + 15 \times 15!$$

$$= \sum_{r=1}^{15} r \times r!$$

$$= \sum (r+1-1)\, r!$$

$$= \sum (r+1)r! - r!$$

$$= \sum_{r=1}^{15} (r+1)! - r!$$

$$T_1 = \cancel{2!} - 1!$$

$$T_2 = \cancel{3!} - \cancel{2!}$$

$$\vdots$$

$$T_{15} = 16! - \cancel{15!}$$

$$S = 16! - 1$$

$$= {}^{16}P_{16} - 1 = {}^{q}P_r - s$$

$q = 16$

$r = 16$

$s = 1$

$${}^{17}C_{15} = \frac{17 \times 16}{2} = 136$$

$${}^{n}P_n = \frac{n!}{0!} = n!$$

The value of $(2.\,{}^{1}P_{0} - 3\,.\,{}^{2}P_{1} + 4.\,{}^{3}P_{2} - ..$ up to 51^{th} term$) + (1! - 2! + 3! - ...$ up to 51^{th} term$)$ is equal to :

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A. $1 - 51(51)!$

B. $1 + (51)!$

C. $1 + (52)!$

D. 1

$$\left[2(\cancel{1!}) - 3(\cancel{2!}) + 4(\cancel{3!}) \cdots + 52(51!)\right]$$

$$+$$

$$\left[1! - \cancel{2!} + \cancel{3!} \cdots + \cancel{51!}\right]$$

$$= \boxed{52! + 1}$$

Theorem - 1 :

Number of combination of n different things taken r at a time when p particular things are always included. = ${}^{n-p}C_{r-p}$

i.e. Find total number of ways of selecting 11 player out of 15 player when Mahendra singh Dhoni and Yuvraj Singh are always included

15 players $\longrightarrow$ 11 player.

-2 -2

13 players $\longrightarrow$ 9 player.

MSD YS $\underbrace{\_\ \_\ \ldots\ \_}_{\text{9 player.}}$

${}^{13}C_9$

Theorem - 2 :

Number of combination of n different things taken r at a time when p particular things are always excluded. = $^{n-p}C_r$

15 players ⟶ 11 player

A B

−2

13 players ⟶ 11 players

$^{13}C_{11}$

Question based on Application of Formula(^nC_r and ^nP_r)

A test consists of 6 multiple choice questions, each having 4 alternative answers of which only one is correct. The number of ways, in which a candidate answers all six questions such that exactly four of the answers are correct, is ____.

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6 ques → 4 Correct + 2 Wrong.

___ ___ ___ ___ ___ ___

$$\boxed{{}^{6}C_{4} \times 1 \times 1 \times 1 \times 1 \times 3 \times 3} = 135$$

There are 15 players in a cricket team, out of which 6 are bowlers, 7 are batsmen and 2 are wicketkeepers. The number of ways, a team of 11 players be selected from them so as to include at least 4 bowlers, 5 batsmen and 1 wicketkeeper, is _

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6 Bowlers (4)	7 Batsmen (5)	2 W.K. (1)	
4	5	2	${}^6C_4\ {}^7C_5\ {}^2C_2$
4	6	1	${}^6C_4\ {}^7C_6\ {}^2C_1$
5	5	1	${}^6C_5\ {}^7C_5\ {}^2C_1$

Add :

A scientific committee is to formed from 6 indians and 8 foreigners, which includes at least 2 indians and double the number of foreigners as indians. Then the number of ways, the committee can be formed is :

A. 560

B. 1050

C. 1625

D. 575

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2 → double.

6 I	8 F	
2	4	$^6C_2\ ^8C_4$
3	6	$^6C_3\ ^8C_6$
4	8	$^6C_4\ ^8C_8$

Add :

A debate club consists of 6 girls and 4 boys. A team of 4 members is to be selected from this club including the selection of a captain (from among these 4 memoers) for the team. If the team has to include at most one boy, then the number of ways of selecting the team is

(a) 380 (b) 320 (c) 260 (d) 95

JEE Adv. 2016

max 1

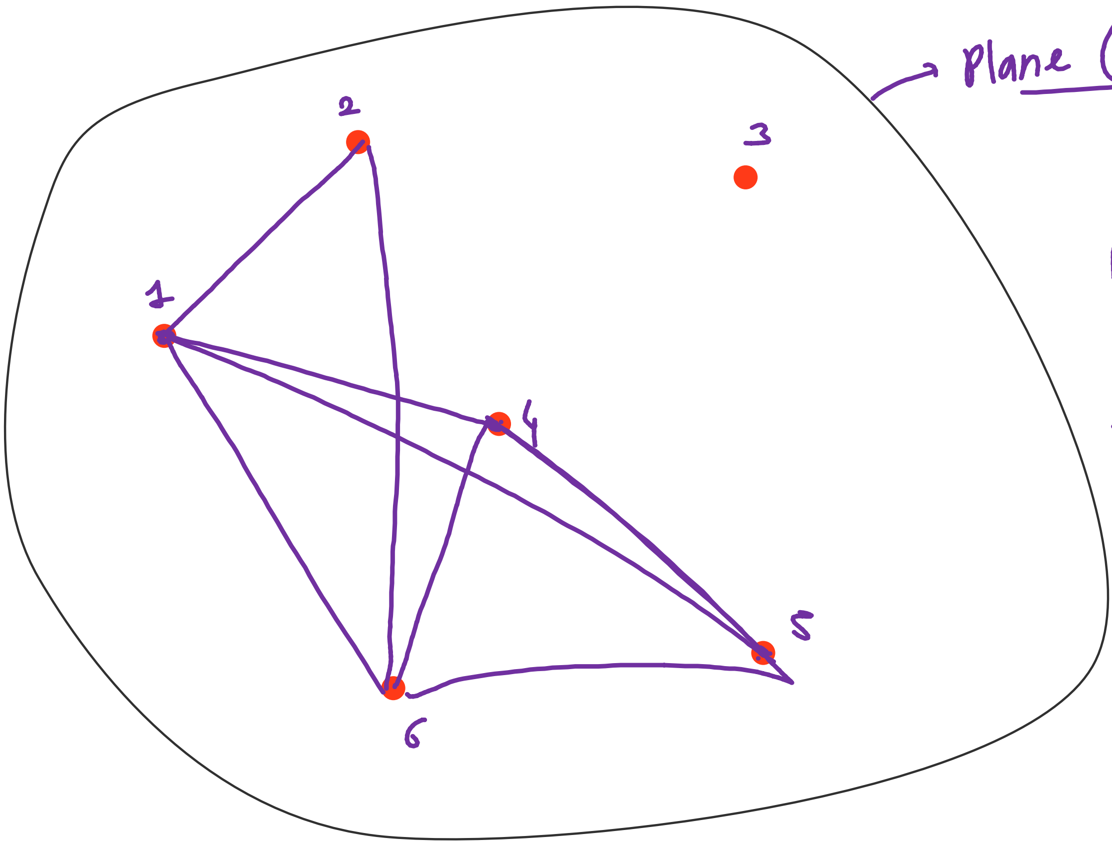
6 G	4 B
4	0
3	1

$${}^6C_4 \, {}^4C_0 \, {}^4C_1$$

$${}^6C_3 \, {}^4C_1 \, {}^4C_1$$

Number of Straight Lines and Triangles

Number of Straight Lines/ Triangles



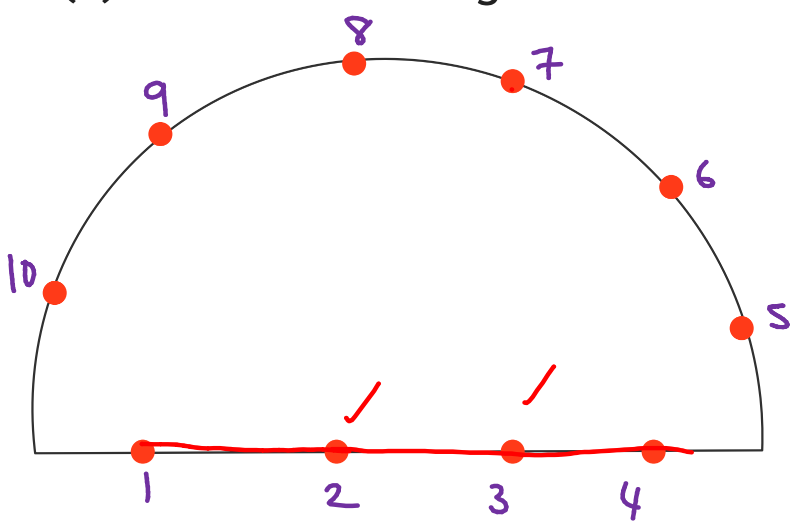
Plane ($n \rightarrow$ non-Collinear)

No. of SL $\Rightarrow {}^6C_2 \Rightarrow {}^nC_2$

No. of Δs $\Rightarrow {}^6C_3 \Rightarrow {}^nC_3$

There are 10 points in a plane of which 4 are collinear and rest are non - collinear. Find

(i) Number of straight lines
(ii) Number of triangles



#Method 1

C-1 2 N-C $\Rightarrow {}^6C_2 \Rightarrow 15$

C-2 1 NC and 1 C $\Rightarrow {}^6C_1\ {}^4C_1 = 24$

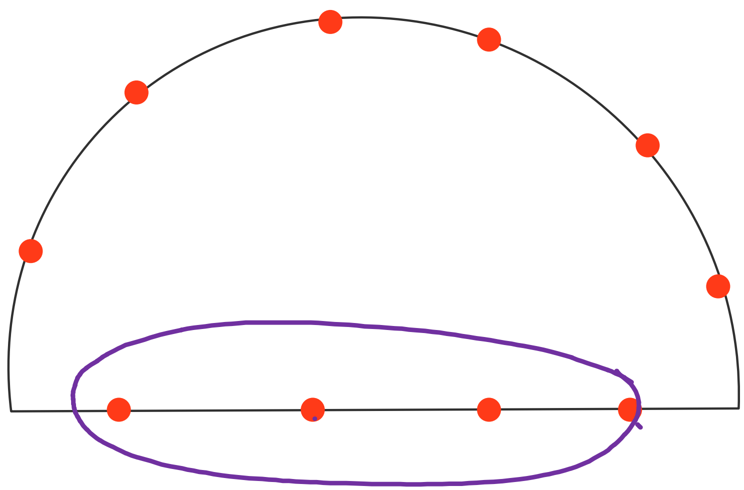
C-3 C - C $\Rightarrow 1$

40

There are 10 points in a plane of which 4 are collinear and rest are non - collinear. Find

(i) Number of straight lines
(ii) Number of triangles

#Method 2

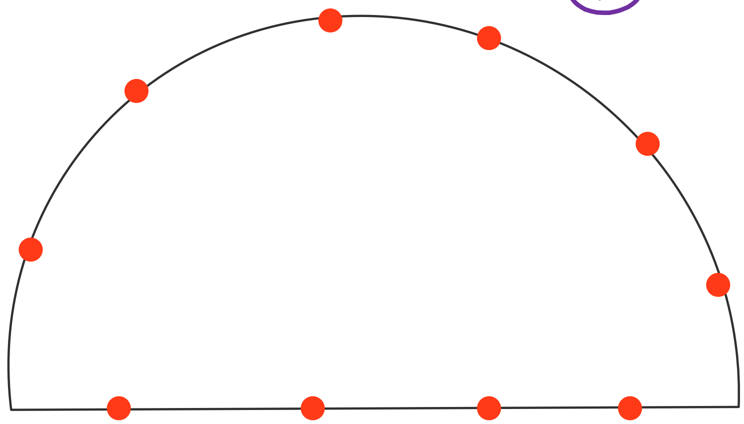


Total − Repeated

$^{10}C_2 - {^4C_2} + 1 \Rightarrow 40$

There are 10 points in a plane of which 4 are collinear and rest are non - collinear. Find

(i) Number of straight lines
(ii) Number of triangles → 3



#Method 1

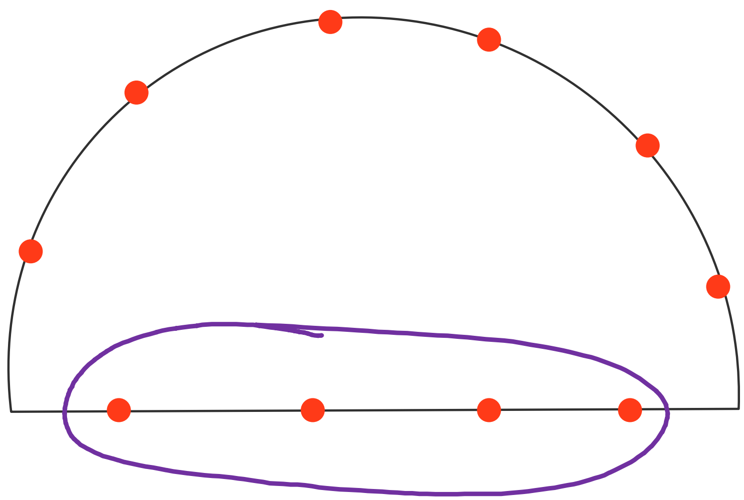
C-1 3 NC $\Rightarrow {}^6C_3$

C-2 2 NC and 1 C $\Rightarrow {}^6C_2\,{}^4C_1$

C-3 2 C and 1 N-C $\Rightarrow {}^4C_2\,{}^6C_1$

There are 10 points in a plane of which 4 are collinear and rest are non - collinear. Find

(i) Number of straight lines
(ii) Number of triangles



#Method 2

Total − (Repeated / Faltu)

$$\boxed{{}^{10}C_3 - {}^{4}C_3}$$

If the sides AB, BC and CA of a triangle ABC have 3,5 and 6 interior points respectively, then the total number of triangles that can be constructed using these points as vertices, is equal to

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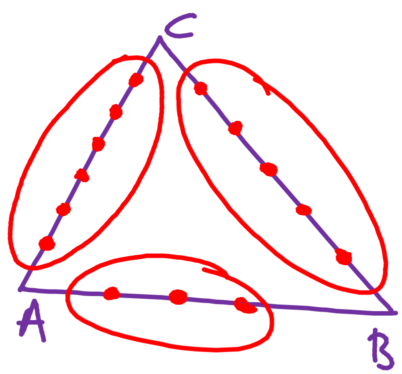
A. 354

B. 240

C. 333

D. 360

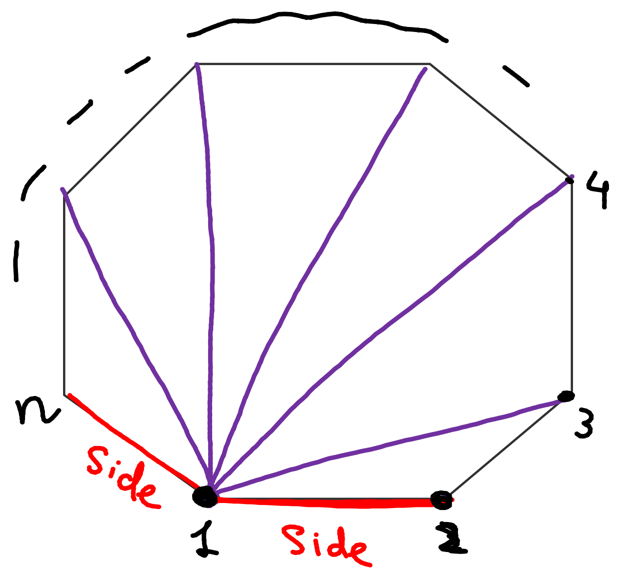


$${}^{14}C_3 - \left({}^{3}C_3 + {}^{5}C_3 + {}^{6}C_3\right)$$

$$= 333$$

Number of Diagonals in n-sided polygon

$$\boxed{{}^nC_2 - n}$$



$$\boxed{{}^nC_2 - n}$$

Let n > 2 be an integer. Suppose that there are n Metro stations in a city located along a circular path. Each pair of stations is connected by a straight track only. Further, each pair of nearest stations is connected by blue line, whereas all remaining pairs of stations are connected by red line. If the number of red lines is 99 times the number of blue lines, then the value of n is :

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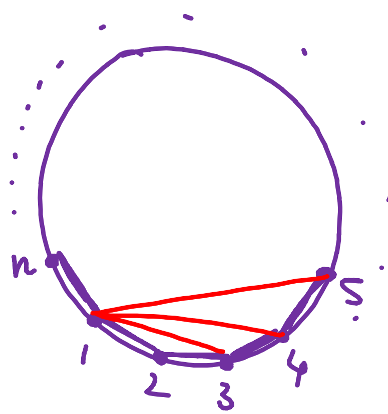
A. 201

B. 200

C. 101

D. 199



blue lines $\Rightarrow n$

Red lines $\Rightarrow {}^nC_2 - n$

$$\left({}^nC_2 - n\right) = 99(n)$$

$${}^nC_2 = 100n$$

$$\frac{\cancel{n}(n-1)}{2} = 100\cancel{n}$$

$$\boxed{n=201}$$

☆☆☆ ${}^nC_2 = \frac{n(n-1)}{2}$

Let $n \geq 2$ be an integer. Take n distinct points on a circle and join each pair of points by a line segment. Colour the line segment joining every pair of adjacent points by blue and the rest by red. If the number of red and blue line segments are equal, then the value of n is

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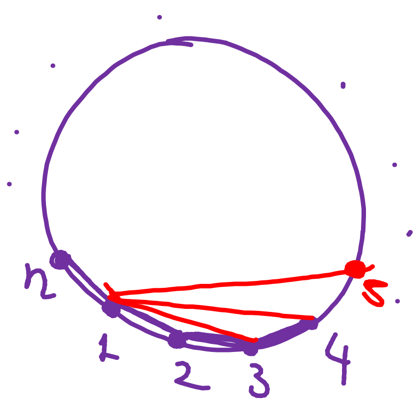

Red = Blue

$${}^nC_2 - n = n$$

$${}^nC_2 = 2n$$

$$\frac{\cancel{n}(n-1)}{2} = 2\cancel{n}$$

$$n = 5$$

Linear Permutation

4 Boys & 4 Girls are to be seated in a line find number of ways

(i) If B_1 and G_1 are always together

(ii) If B_1 and G_1 are never together

$B_1\ B_2\ B_3\ B_4\ \ G_1\ G_2\ G_3\ G_4 \Rightarrow 8!$

i) $\boxed{B_1 G_1}\ B_2\ B_3\ B_4\ G_2\ G_3\ G_4$

$\boxed{7! \times 2!}$

ii) $|B_2|B_3|B_4|G_2|G_3|G_4|$

$B_1 \quad G_1$

$\boxed{6!\ {}^7C_2 \times 2!}$

4 Boys & 4 Girls are to be seated in a line find number of ways

(iii) If "No two girls are together"

(iv) If not all the girls are together

iii) | B_1 | B_2 | B_3 | B_4 | $G_1 \quad G_2 \quad G_3 \quad G_4$

$$\boxed{4! \; {}^5C_4 \times 4!}$$

$B_1 \; B_2 \; B_3 \; B_4 \; \boxed{G_1 \; G_2 \; G_3 \; G_4}$

iv) Total − (all Girls are together)

$$\boxed{8! - 5! \times 4!}$$

4 Boys & 4 Girls are to be seated in a line find number of ways

(v) Boys and girls are alternate

(vi) If there are 4 married couples then number of ways in which they can be seated so that each couple is together.

(v) B G B G B G B G $\quad 4! \times 4!$

G B G B G B G B $\quad 4! \times 4!$

$4! \times 4! \times 2$

vi) $\boxed{B_1 G_1}$ $\boxed{B_2 G_2}$ $\boxed{B_3 G_3}$ $\boxed{B_4 G_4}$

$4! \times (2!)^4$

Two families with three members each and one family with four members are to be seated in a row. In how many ways can they be seated so that the same family members are not separated ?

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$A_1\ A_2\ A_3$ | $B_1\ B_2\ B_3$ | $C_1\ C_2\ C_3\ C_4$

A. 2! 3! 4!

B. $(3!)^3 . (4!)$

C. $(3!)^2 . (4!)$

D. $3! (4!)^3$

$3! \times 3! \times 3! \times 4!$

The number of six letter words(with or without meaning), formed using all the letters of the word 'VOWELS', so that all the consonants never come together, is.

JEE Main 2021

V O W E L S $\quad \boxed{V\ W\ L\ S}$ O E

Total − (All consonants are together)

$6! - (3! \times 4!)$

$\underline{576}$

Let n be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that all the girls stand consecutively in the queue. Let m be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that exactly four girls stand consecutively in the queue.

Then the value of $\frac{m}{n}$ is $\rightarrow$ 5

JEE Adv. 2015

$$\frac{m}{n} = \frac{\cancel{5} \times \cancel{5!} \times \cancel{15}^{5} \times \cancel{2} \times \cancel{4!}}{\cancel{6}_{3} \times \cancel{5!} \times \cancel{5!}}$$

$= 5$

5B 5G

$G_1 G_2 G_3 G_4 G_5$ B_1 B_2 B_3 B_4 B_5

$$\boxed{n = 6! \times 5!}$$

$$\boxed{m = {}^5C_4\, 5!\, {}^6C_2 \times 2! \times 4!}$$

$G_1 G_2 G_3 G_4$ G_5

B_1 B_2 B_3 B_4 B_5

Words of length 10 are formed using the letters A, B, C, D, E, F, G, H, I, ~~J~~. Let x be the number of such words where no letter is repeated; and let y be the number of such words where exactly one letter is repeated twice and no other letter is repeated. Then, $\frac{y}{9x} =$

JEE Adv. 2017

A A B C D E F G H I

$$\boxed{\begin{aligned} x &= 10! \\ y &= {}^{10}C_1 \times {}^{9}C_8 \times \frac{10!}{2!} \end{aligned}}$$

$$\frac{y}{9x} = \frac{10 \times \cancel{9} \times \cancel{10!}}{\cancel{10!} \times \cancel{9} \times 2} = 5$$

Batting Order Questions

Batting Order Questions

Number of ways in which Indian team (11 players) can bat, if Yuvraj wants to bat before Dhoni and Jadeja wants to bat after Dhoni is

YS > MSD > J

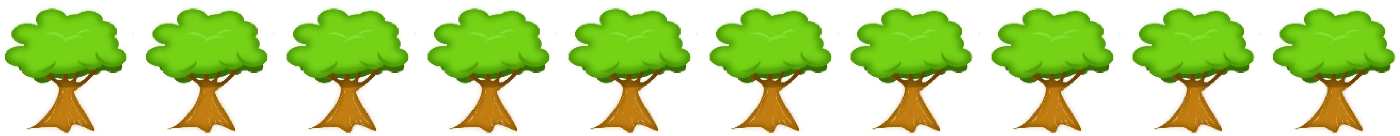
__ __ YS __ __ MSD __ __ J __ __

$$\boxed{{}^{11}C_3 \times 1 \times 8!}$$

Tick and Cross Method (No two consecutive selection)

Tick and Cross Method

There are 10 trees in a row. 4 trees are to be cut down. The number of ways that no two of the cut down trees are consecutive is



× ✓ × × ✓ × × ✓ × ✓

✓ × ✓ × ✓ × ✓ × × ×

Tick and Cross Method

$\underbrace{\checkmark\ \checkmark\ \checkmark\ \checkmark}_{4}\ \underbrace{\times\ \times\ \times\ \times\ \times\ \times}_{6}$

$\checkmark\ \checkmark\ \checkmark\ \checkmark$

$|\ \times\ |\ \times\ |\ \times\ |\ \times\ |\ \times\ |\ \times\ |\ \times\ |$

$$1 \times {}^{7}C_{4} \times 1 = 35$$

1. $\times\ \checkmark\ \times\ \times\ \checkmark\ \times\ \times\ \checkmark\ \times\ \checkmark$
2. $\checkmark\ \times\ \checkmark\ \times\ \checkmark\ \times\ \checkmark\ \times\ \times\ \times$

An engineer is required to visit a factory for exactly four days during the first 15 days of every month and it is mandatory that **no** two visits take place on consecutive days. Then the number of all possible ways in which such visits can be made by the engineer during 1-15 June 2021 is _____

JEE Adv. 2020

$\underbrace{\checkmark\ \_\_\ \_\_\ \checkmark\ \ldots\ \_\_}_{13\text{ days}} \longrightarrow 4\text{ day.}$

$\underbrace{\checkmark\checkmark\checkmark\checkmark}_{4}\ \underbrace{\times\times\times\times\times\ \times\times\times\times\times\times}_{11\text{ Cross}}$

$\times\times\times\times\times\ \times\times\times\times\times\ \times$

$\boxed{1 \times {}^{12}C_4 \times 1}$

495

Formation of Groups

OR

Teams

OR

Bundles.

Formation of Groups/Teams :

$P_1 \; P_2 \; P_3 \; P_4$

1 P $\qquad$ 3 P

$P_2 \qquad P_1 \; P_3 \; P_4$

$$^4C_1 = \frac{4!}{1!\,3!}$$

4

1 $\quad$ 3

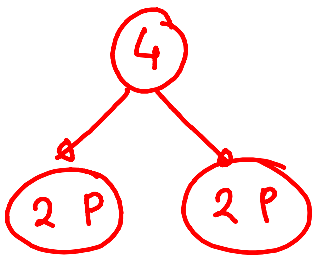
$$= \frac{4!}{1!\,3!}$$

8

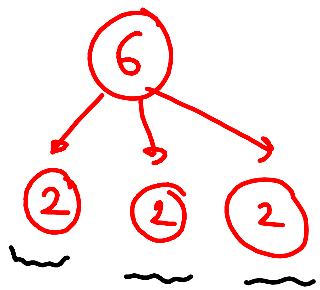
1 $\quad$ 2 $\quad$ 5

$$= \frac{8!}{1!\,2!\,5!}$$

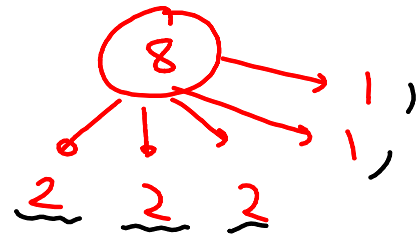
Formation of Groups/Teams :



$$\frac{4!}{2!\,2!\,2!}$$



$$\frac{6!}{2!\,2!\,2!\,3!}$$

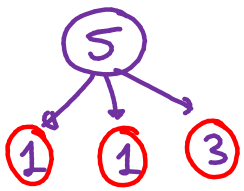


$$\frac{8!}{2!\,2!\,2!\,1!\,1!\,3!\,2!}$$

The total number of ways in which 5 balls of different colours can be distributed among 3 persons so that each person gets at least one ball is

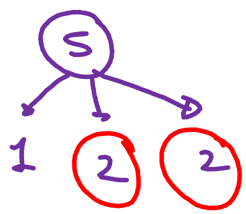
(a) 75 (b) 150 (c) 210 (d) 243

JEE Adv. 2012



$$\frac{5!}{1!\,1!\,3!\,2!} \times 3!$$

$$+$$



$$\frac{5!}{1!\,2!\,2!\,2!} \times 3!$$

150

A	B	C

B_1 B_2 B_3 B_4 B_5

$\boxed{B_1}$ $\boxed{B_2}$ $\boxed{B_3\ B_4\ B_5}$

In a hotel, four rooms are available. Six persons are to be accommodated in these four rooms in such a way that each of these rooms contains at least one person and at most two persons. Then the number of all possible ways in which this can be done is ____

JEE Adv. 2020

4 3 2 1

A B C D E F

1 1 2 2

A B CD EF

$$\boxed{\left(\frac{6!}{1!\,1!\,2!\,2!\quad 2!\,2!}\right)\times 4!}$$

Ans : 1080

Permutation of Alike Objects

same

identical.

Permutation of alike objects :

D	A	D	D	Y

$\Rightarrow \dfrac{5!}{3!}$

Find total number of word's formed by using all letters of the word "IITJEE".

I	I	T	J	E	E

$$\frac{6!}{2!\,2!}$$

Consider word ASSASSINATION, find number of ways of arranging the letters.

A > A > I > A > I > O

A, E, I, O, U

(i) Number of words using all.
(ii) If no two vowels are together.
(iii) vowels are in the same order.
(iv) Relative position of vowels and consonant remain same.

i) $\dfrac{13!}{3!\,4!\,2!\,2!}$

A, A, I, A, I, O

ii) | S | S | S | S | N | T | N |

$$\boxed{\left(\frac{7!}{4!\,2!}\right) {}^{8}C_{6} \times \frac{6!}{3!\,2!}}$$

A A I A I O

S S S S N T N

iii) $\Rightarrow {}^{13}C_6 \times 1 \times \dfrac{7!}{4!\,2!}$

A S S A S S I N A T I O N

vi)

$$\left(\frac{6!}{3!\,2!}\right)\left(\frac{7!}{4!\,2!}\right)$$

The number of words (with or without meaning) that can be formed from all the letters of the word "L~~E~~TT~~E~~R" in which vowels never come together is_____.

JEE Main 2020

L T T R (E E)

| L | T | T | R |

$$\left(\frac{4!}{2!}\right) {}^5C_2 \times 1 = 120$$

The number of seven digit integers with sum of the digits equal to 10 and formed by using the digits 1, 2 and 3 only is

JEE Main 2021

JEE Adv 2009

A. 77

B. 42

C. 35

D. 82

$x_1 + x_2 + \dots x_7 = 10$

C-1 1, 1, 1, 1, 1, 2, 3 $\Rightarrow \frac{7!}{5!} = 42$

C-2 $\underbrace{1, 1, 1, 1}$, $\underbrace{2, 2, 2}$ $\Rightarrow \frac{7!}{4!\,3!} = 35$

77

Permutation of Objects taken some at a time

Permutation of the objects taken some at a time :

How many 5 lettered words can be formed using the letters of the words. “INDEPENDENCE”.

12 → alpha

$\cancel{{}^{12}C_5}$

5 select → Arrange.

N D E P I

N N D E I

E E E N I

$N \to 3 \quad E \to 4 \quad D \to 2 \quad I \to 1 \quad P \to 1 \quad C \to 1$

NN EE DD

C-1 all 5 diff $\longrightarrow$ ${}^6C_5 \times 5!$

C-2 4 identical 1 Diff $\longrightarrow$ $1 \times {}^5C_1 \times \frac{5!}{4!}$

EEEE _

C-3 3 identical 2 diff $\longrightarrow$ $2 \times {}^5C_2 \times \frac{5!}{3!}$

N N N D P

C-4 3 identical 2 identical $\longrightarrow$ $2 \times 2 \times \frac{5!}{2!\,3!}$

N N N _ _

C-5 2 identical 3 diff $\longrightarrow$ $3 \times {}^5C_3 \times \frac{5!}{2!}$

N N _ _ _

C-6 2 identical 2 identical 1 diff $\longrightarrow$ ${}^3C_2 \times {}^4C_1 \times \frac{5!}{2! \times 2!}$

N N D D _

Find the number of words each consisting 5 letters from the letters of the word " M I S S S I S S I P P I"

D.I.Y.

Sol. $S \to 4, I \to 4, P \to 2, M \to 1$

	Category	Selection	Arrangement
(1)	4 alike, 1 different	${}^{2}C_{1} \times {}^{3}C_{1} = 6$	$6 \times \frac{5!}{4!}$
(2)	3 alike, 2 alike	${}^{2}C_{1} \times {}^{2}C_{1} = 4$	$4 \times \frac{6!}{2!\,3!}$
(3)	3 alike, 2 different	${}^{2}C_{1} \times {}^{3}C_{1} = 6$	$6 \times \frac{6!}{3!}$
(4)	2 alike, 2 alike 1 different	${}^{3}C_{2} \times {}^{2}C_{1} = 6$	$6 \times \frac{5!}{2!\,2!}$
(5)	2 alike, 3 different	${}^{3}C_{1} \times {}^{3}C_{3} = 3$	$3 \times \frac{5!}{2!}$
	Total	Number of selection = 25	Number of arrangement = 1350

Then total number of words formed = 1350

The number of words, with or without meaning, that can be formed by taking 4 letters at a time from the letters of the word 'SYLLABUS' such that two letters are distinct and two letters are alike, is _.

JEE Main 2020

S Y L L A B U S

S → 2   L → 2   Y → 1   B → 1   A → 1   U → 1

2 identical 2 distinct $\Rightarrow$ $\boxed{2 \times {}^{5}C_{2} \times \frac{4!}{2!}}$

S S Y A

$\Rightarrow$ 240

Circular Permutation

Circular Permutation :

The number of circular permutation of n distinct objects is (n - 1)!

A B C

$(3-1)!$

A B C

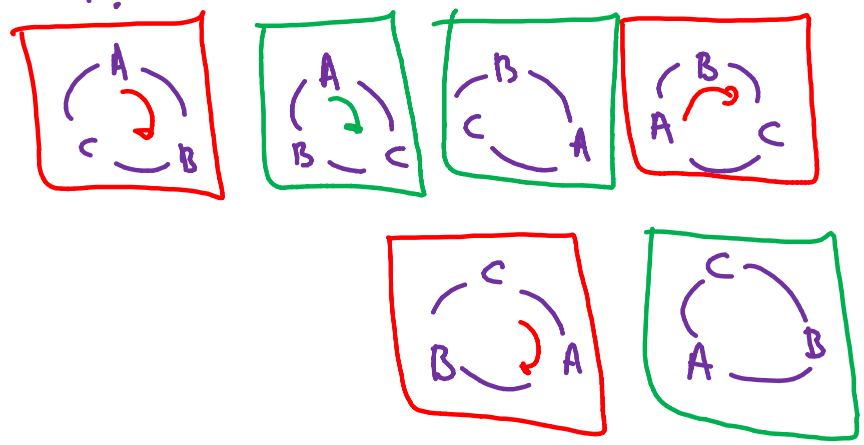
_ _ _

$3!$

A B C
A C B
B A C
B C A
C A B
C B A

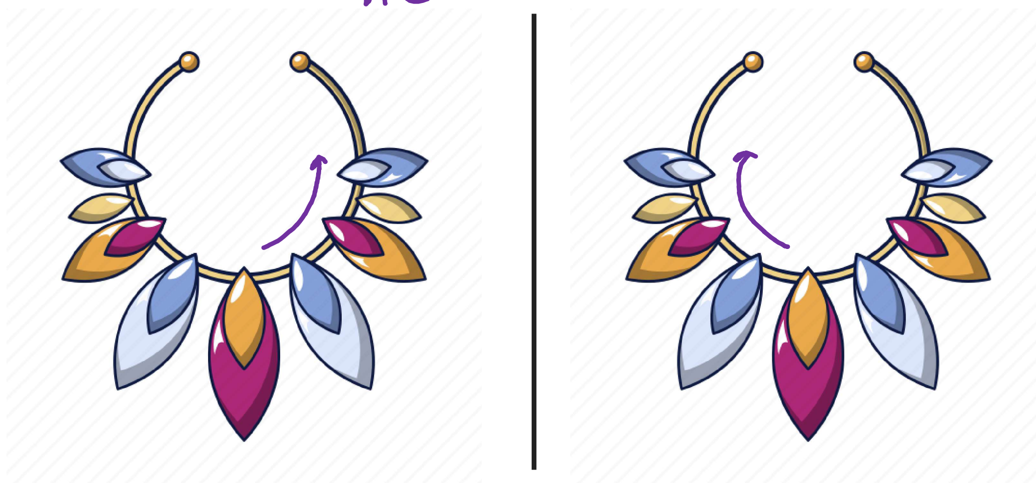
$$\frac{3!}{3} = \frac{n!}{n} = (n-1)!$$

$2!$



Circular Permutation :

ACW=CW



Necklace / Garland $\rightarrow \dfrac{(n-1)!}{2}$

The number of ways in which 5 boys and 3 girls can be seated on a round table if a particular boy B_1 and a particular girl G_1 never sit adjacent to each other, is :

~~A.~~ $5 \times 6!$

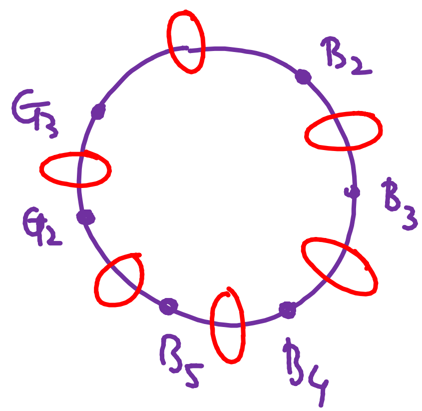
B. $6 \times 6!$

C. $7!$

D. $5 \times 7!$

JEE Main 2017

$B_1 \quad G_1$



$$\boxed{5! \; {}^6C_2 \times 2!}$$

$$\boxed{5! \times 6} \times 5 \; \cancel{\times 2!}$$

$$\underline{6! \times 5}$$

If seven women and seven men are to be seated around a circular table such that there is a man on either side of every woman, then the number of seating arrangements is

JEE Main 2012

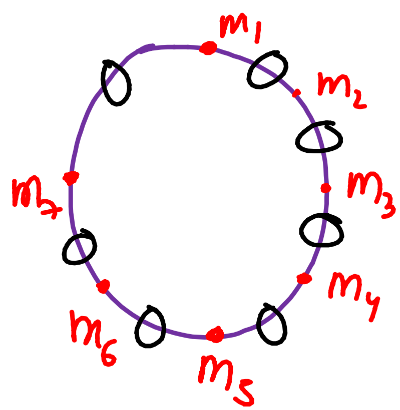
7 M 7 W

A. 7! 6!

B. $(6!)^2$

C. $(7!)^2$

D. 7!



$$\boxed{6! \ {}^7C_7 \times 7!}$$

$$= \underline{6! \times 7!}$$

Total number of Selections

Total number of selections/combination :

	n identical things	n different things
Number of ways of selecting zero things	1	${}^{n}C_0 = 1$
Number of ways of selecting one thing	1	${}^{n}C_1$
Number of ways of selecting two things	1	${}^{n}C_2$
.	.	.
.	.	.
.	.	.
Number of ways of selecting n things	1	${}^{n}C_n$
	$(n+1)$	2^n

Total number of selections/combination :

Number of ways of selecting zero or more objects from n distinct objects = 2^n

Number of ways of selecting one or more objects from n distinct objects = $2^n - 1$

Number of ways of selecting zero or more objects from n identical objects = $n+1$

Number of ways of selecting one or more objects from n identical objects = n

It is given that 4 Apples, 3 Mangoes, 2 Bananas, 2 Oranges, consider the following cases.

Case-I: Fruits of same species are alike and rests are different , then

(i) Find the number of ways, at least one fruit is selected.

(ii) Find the number of ways, at least one fruit of each kind is selected.

Case-II: Fruits of same species are different and rests are also different, then

(i) Find the number of ways, at least one fruit is selected.

(ii) Find the number of ways, at least one fruit of each kind is selected.

It is given that 4 Apples, 3 Mangoes, 2 Bananas, 2 Oranges, consider the following cases.

Case-I: Fruits of same species are alike and rests are different , then

(i) Find the number of ways, at least one fruit is selected.

(ii) Find the number of ways, at least one fruit of each kind is selected.

A A A A   M M M   O O   B B

i) Total − 1

$(4+1)(3+1)(2+1)(2+1) - 1$

A A A A   M M M   O O   B B

1 2 3 4   1 2 3

$(1+1+1+1)(1+1+1)$

ii) $(4)(3)(2)(2)$

A A A A   M M M   O O   B B

1 2 3 4   1 2 3   1 2   1 2

It is given that 4 Apples, 3 Mangoes, 2 Bananas, 2 Oranges, consider the following cases.

Case-II: Fruits of same species are different and rests are also different, then

(i) Find the number of ways, at least one fruit is selected.

(ii) Find the number of ways, at least one fruit of each kind is selected.

$({}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4)$ $A_1 A_2 A_3 A_4$ $M_1 M_2 M_3$ $O_1 O_2$ $B_1 B_2$

1 2 3 4 1 2 3 1 2 1 2

i) Total − 1

$2^4 \times 2^3 \times 2^2 \times 2^2 - 1$

ii) $(2^4 - 1)(2^3 - 1)(2^2 - 1)(2^2 - 1)$

Number Theory

Total Number of Divisors :

$12 \rightarrow 1, 2, 3, 4, 6, 12$ $\qquad 12 = 2^{2} \times 3^{1}$

No. of divisors = 6 $\qquad (2+1)(1+1)$

$24 \rightarrow \boxed{1, 2, 3, 4, 6, 8, 12, 24}$ $\qquad 24 = 2^{3} \times 3^{1}$

No. of Divisors $\Rightarrow (3+1)(1+1)$

$\Rightarrow 8$

$2 \times 3 = 6$ $\qquad 12 \rightarrow 2^{2} \times 3^{1}$

$2 \times 3^{0} = 2$ $\qquad \rightarrow 2 \times 2 \times 3$

A A B

$\rightarrow (1+1+1)(1+1)$

A A B

1 2 0 1

Number of EVEN divisors

$12 \rightarrow 1, 2, 3, 4, 6, 12$

$12 = 2^2 \times 3^1$

Even $(\div 2) \Longrightarrow (2)(1+1) = 4$

$24 = 2^3 \times 3^1$ Even $\Rightarrow (3)(1+1) = 6$

Sum of Divisors

$12 \rightarrow 1+2+3+4+6+12$

$12 = 2^2 \times 3^1$

Sum of Divisors $\Rightarrow (2^0 + 2^1 + 2^2)(3^0 + 3^1)$

$1+3+2+6+4+12$

Number of ways of representing N as product of 2 numbers

$$No.of\ ways\ of\ representing\ N\ as\ product\ of\ two\ numbers$$

$$= \begin{cases} \boxed{\dfrac{Number\ of\ divisor}{2}} & if\ N \rightarrow not\ a\ perfect\ sq. \\ \dfrac{Number\ of\ divisor + 1}{2} & if\ N \rightarrow a\ perfect\ sq. \end{cases}$$

$$12 = 1 \times 12$$
$$= 2 \times 6$$
$$= 3 \times 4$$
3

No. of divisors = 6

25 = 1, 5, 25 3

$$25 = 1 \times 25$$
$$= 5 \times 5$$
2

Consider the number $N = 2^5 \times 3^4 \times 5^7 \times 7^2$

(i). Total number of divisor,

(ii). Number of proper divisor

i) $(5+1)(4+1)(7+1)(2+1) = \boxed{720}$

ii) proper = Total − 2

$= 720 - 2$

$= 718$

Consider the number N = $\cancel{2} \times 3^4 \times 5^{7} \times 7^2$

A A A A A A A — 7

1 2 3 4 5 6

(iii). Number of odd divisor

(iv). Number of even divisor $\div 2$

(v). Number of divisors divisible by 5. $\div 5$

(vi). Number of divisors divisible by 10. $\div 2$ and $\div 5$

(iv) $(5)(4+1)(7+1)(2+1) = 600$

iii) odd $= 720 - 600 = 120 \leftarrow (4+1)(7+1)(2+1)$

(v) $(5+1)(4+1)(7)(2+1) =$

vi) $(5)(4+1)(7)(2+1) =$

Consider the number $N = 2^5 \times 3^4 \times 5^7 \times 7^2$

(vii). Number of divisible by 2 but not by 4.

2 2 2 2 2

1

$(1)(4+1)(7+1)(2+1)$

Consider the number N = $2^5 \times 3^4 \times 5^7 \times 7^2$

22222

(viii). Sum of all divisors

(ix). Sum of even divisors (mini ek 2 ko select kro)

(x). Sum of odd divisors (ek bhi 2 ko select mat kro)

viii) $(2^0+2^1+\ldots+2^5)(3^0+3^1+\ldots+3^4)(5^0+\ldots+5^7)(7^0+7^1+7^2)$

(ix) $(2^1+2^2+\ldots 2^5)(\quad " \quad)(\quad " \quad)(\quad " \quad)$

(x) $(2^0)(\quad " \quad)(\quad " \quad)(\quad " \quad)$

Consider the number $N = 2^5 \times 3^4 \times 5^7 \times 7^2$ = ___ × ___

(xi). Number of ways in which N can be resolved as product of two divisor.

$$\frac{\text{No. of Divisors}}{2} = \frac{720}{2} = 360$$

A natural number has prime factorization given by $n = 2^x 3^y 5^z$, where y and z are such that $y + z = 5$ and $y^{-1} + z^{-1} = 5/6$, $y > z$. Then the number of odd divisors of n, including 1, is :

A. 11

B. 6x

C. 12

D. 6

JEE Main 2021

$y + z = 5$ — 1

$\frac{1}{y} + \frac{1}{3} = \frac{5}{6}$ — 2

$\frac{\cancel{y+z}}{yz} = \frac{\cancel{5}}{6}$

$\boxed{yz = 6}$ — 3

$\boxed{y = 3 \quad z = 2}$

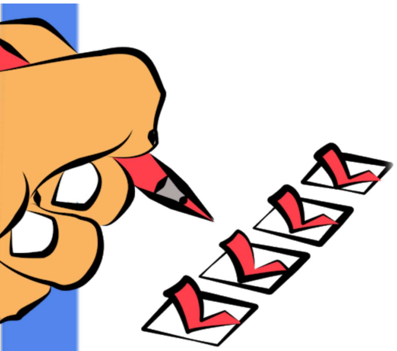
$\boxed{n = 2^x \, 3^3 \, 5^2}$

Odd divisor $= (1)(3+1)(2+1)$

$= 4 \times 3$

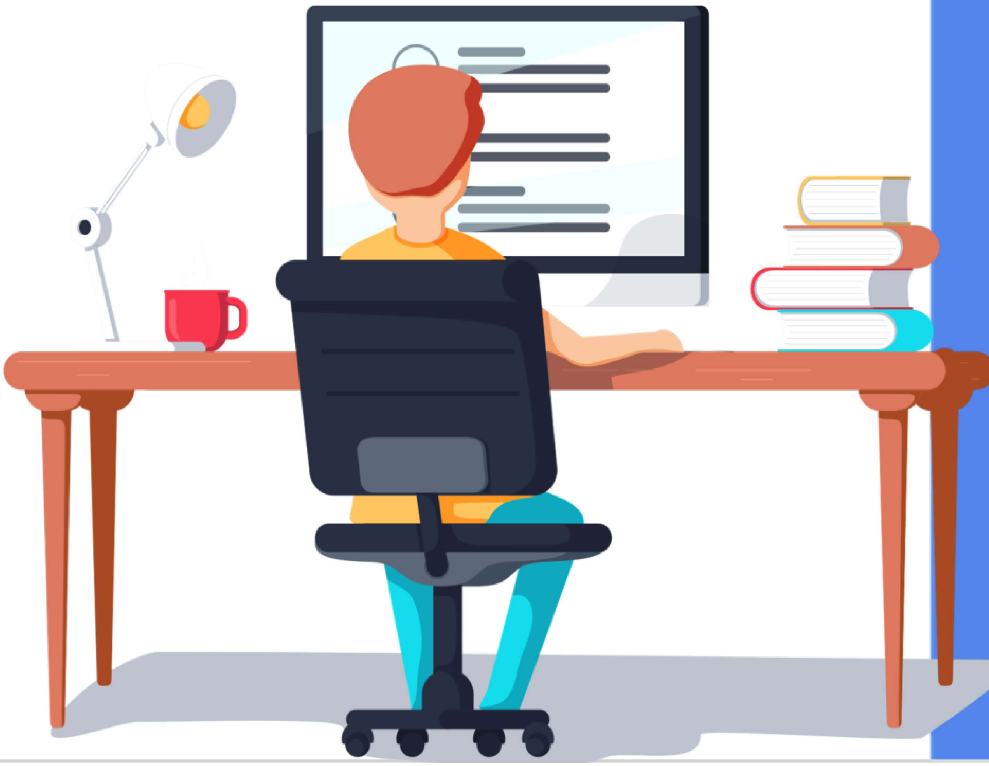
$= 12$

Summation of Numbers



Shortcut

Summation of Numbers



Find the sum of all 3 digit numbers that can be formed by using digits 1, 2, 3.

A. 1332

B. 1221

C. 1443

D. 1665

$$\begin{array}{r} {\scriptstyle 1\,1\,} \\ 123 \\ +\,132 \\ +\,213 \\ +\,231 \\ +\,312 \\ +\,321 \\ \hline 1332 \end{array}$$

$$111 \times (1+2+3) \times 2!$$

$$111 \times 6 \times 2$$

$$111 \times 12 = 1332$$

$$\begin{array}{r} 12 \\ 12 \\ 12 \\ \hline 1332 \end{array}$$

Find the sum of numbers greater than 10000 that can be formed by using digits 1, 2, 3, 4, 5 (without repetition).

A. 1111 x 15 x 24

B. 11111 x 15 x 24

C. 1111 x 15 x 120

D. 11111 x 15 x 120

11111 x 15 x 4!

11111 x 15 x 24

Sum of all the numbers greater than 10000 formed by the digits 1, 3, 5, 7, 9 if no digit being repeated.

$25 \times 24 = 5 \times 5 \times 2 \times 12$

$= 10 \times 60 = 600$

A. 6666600

B. 66666000

C. 6666000

D. 666660

$11111 \times 25 \times 4!$

$11111 \times 25 \times 24$

11111×600

6666600

$\begin{array}{r} 25 \\ \times 24 \\ \hline \end{array}$

Sum of all “3 digit numbers” that can be formed by using 1, 2, 3, 4 (without repetition).

A. 6660

B. 6666

C. 6600

D. 6000

1, 2, 3   1, 2, 4   1, 3, 4   2, 3, 4

$= 111 \times 6 \times 2! + 111 \times 7 \times 2! + 111 \times 8 \times 2! + 111 \times 9 \times 2!$

$= 111 \times 2! \times (6+7+8+9)$

$= 111 \times 2! \times 30$

$= \underline{6660}$

If 0 is included

0 (_ _)

Sum of all 3 digit numbers that can be formed by using 0, 1, 2.

A. 633

B. 666

C. 636

D. 663

0, 1, 2 — (1, 2)

$111 \times 3 \times 2! - 11 \times 3 \times 1!$

$666 - 33$

633

If digits are repeated

Sum of all 4 digit numbers that can be formed by using 1, 1, 3, 4.

A. 29997

B. 29977

C. 29797

D. 27779

1, 1, 3, 4

$1111 \times 9 \times \frac{3!}{2!}$

1111×27

$$\begin{array}{r} 27 \\ 27 \\ 27 \\ 27 \\ \hline 29997 \end{array}$$

If digits are repeated

Sum of all 4 digit numbers that can be formed by using 1, 1, 2, 2.

A. 9900

B. 9000

C. 9999

D. 9990

1, 1, 2, 2

$$1111 \times 6 \times \frac{3!}{2!2!}$$

$$1111 \times 9$$

The sum of all the 4-digit distinct numbers that can be formed with the digits 1,2,2 and 3 is :

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A. 26664

B. 122664

C. 122234

D. 22264

1, 2, 2, 3

$1111 \times 8 \times \frac{3!}{2!}$

1111×24

$$\begin{array}{r} 24 \\ 24 \\ 24 \\ 24 \\ \hline 26664 \end{array}$$

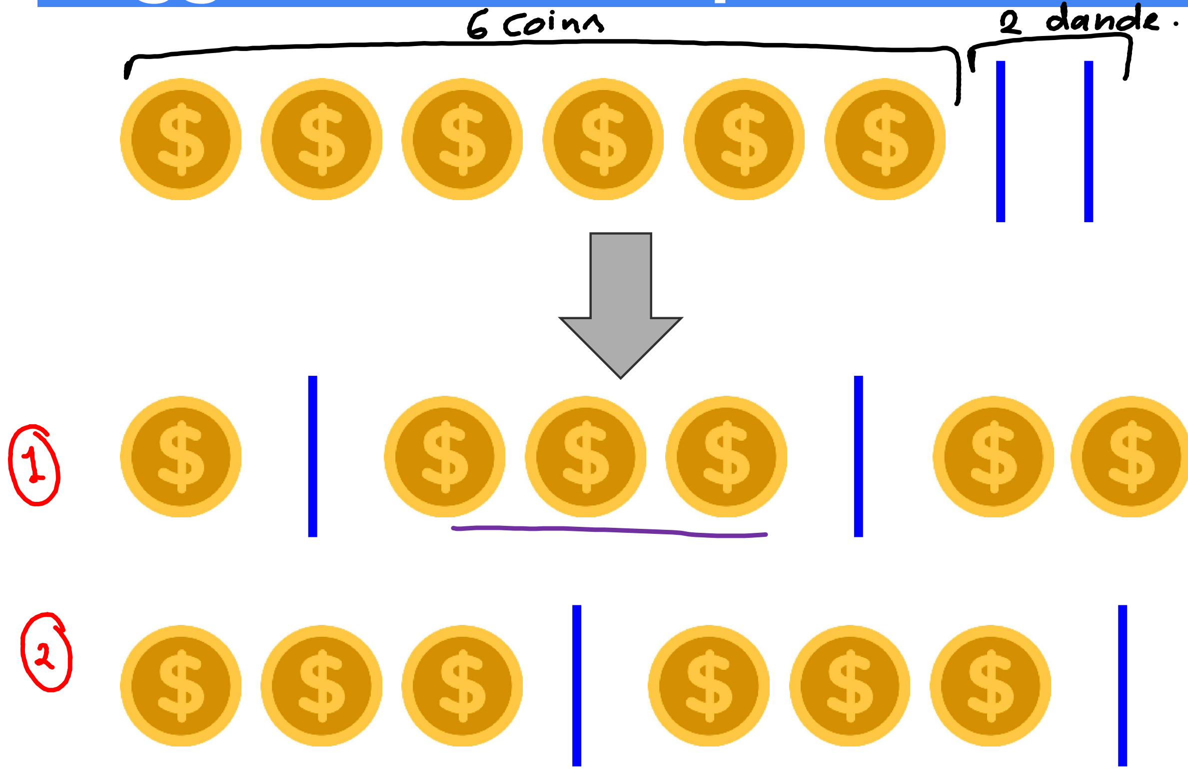
Distribution of Alike Objects

Beggar-Coin Principle/ Ball-stick Method

3 beggars.

	A	B	C
1	6	0	0
2	5	1	0
3	0	0	6
4	2	2	2
⋮	$\boxed{1}$	$\boxed{3}$	$\boxed{2}$
⋮	3	3	0
28			

Beggar-Coin Principle/ Ball-stick Method



$$\frac{8!}{6!\,2!} = {}^{8}C_{2} = 28$$

Find number of ways in which 30 mangos can be distributed among 5 persons.

$\underbrace{m\ m \ldots m}_{30}$ $\underbrace{|\ |\ |\ |}_{4 \text{ dande}}$.

$$\frac{(34)!}{30!\,4!}$$

Find total number of ways of distributing 7 identical computers to R|S|G so that each receive atleast one computer.

7 Comp.

R	S	G
0	0	0

$\cancel{0}\ \cancel{0}\ \cancel{0}\ \underbrace{0\ 0\ 0\ 0}$

$\underbrace{0\ 0\ 0\ 0}_{4}\ \underbrace{|\ \ |}_{2\ \text{dande}}$

$$\frac{6!}{4!\,2!}$$

Find number of natural solutions of equation x + y + z = 102, where x, y, z ∈ N.

$x + y + z = 102$

100	1	1
99	1	2
⋮		

102 coin
− 3

$\underbrace{99 \text{ coin}}\ \underbrace{|\ |}_{2 \text{ dande}}$

$$\frac{101!}{99!\,2!}$$

x	y	z
0	0	0

Number of non-negative integral solution of the inequality $x+y+z+t \leq 30$.

LHS $\leq$ RHS

$28 \leq 30$ ✓

$\underbrace{x+y+z+t} \leq 30$

$\boxed{\overset{22}{x}+\overset{2}{y}+\overset{2}{z}+\overset{2}{t}}+\overset{2}{\text{Gap}}=30$

30 coin | | | |

$\frac{34!}{30!\,4!} = {}^{34}C_4$

The total number of 3-digit numbers, whose sum of digits is 10, is _

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$x\ y\ z$

$x + y + z = 10$

$1 \le x \le 9 \quad 0 \le y \le 9 \quad 0 \le z \le 9$

$x' + y + z = 9$

9 0 0

9 coin ||

$$\boxed{\frac{11!}{9!\,2!} - 1} = 54$$

The total number of positive integral solutions (x, y, z) such that xyz = 24

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A. 36

B. 45

C. 24

D. 30

$xyz = 24$

(1)	(1)	(24)
(1)	(2)	(12)
(1)	(3)	(8)
(6)	(4)	(1)
(6)	(2)	(2)

x	y	z
2	2 2	–
3	–	–
2	2	2
3	–	–

2 2 2 | |

3 | |

$xyz = 2^3 \times 3^1$

$xyz = 2\ 2\ 2\ 3$

$\dfrac{5!}{3!\,2!}\ \dfrac{3!}{1!\,2!}$

$\boxed{30}$

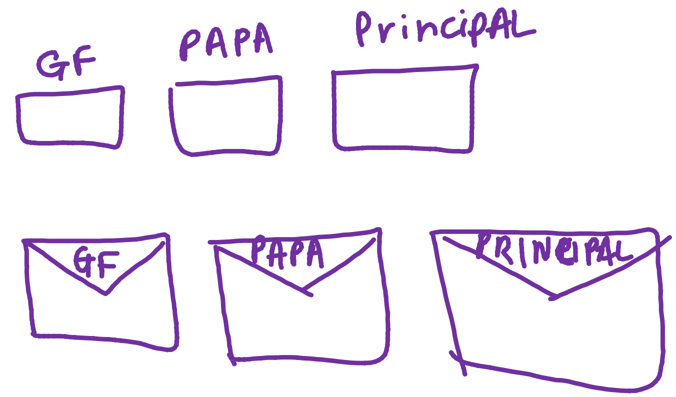
Derangement

Derangement :

If n things are arranged in a row, the number of ways they can be deranged so that r things occupy wrong places while (n - r) things occupy their original places, is

$$= {}^nC_r D_r$$

$$\text{Where } \boxed{D_r = r!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\ldots\ldots+(-1)^r\frac{1}{r!}\right)}$$



Remember

$$D_r = r!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\ldots\ldots+(-1)^r\frac{1}{r!}\right)$$

n Letter → r Wrong envelop.

${}^nC_r \times D_r$

5 Letter → 3 Letter Wrong
→ 2 Letter Right

${}^5C_3 \times D_3$

$D_0 = 1$

$D_1 = 0$

$D_2 = 1$

$D_3 = 2$

$D_4 = 9$

$D_5 = 44$

A person writes letters to five friends and addresses the corresponding envelopes. In how many ways can the letters be placed in the envelops so that

a. All letters are in the wrong envelopes.
b. Atleast three of them are in the wrong envelopes.

a) $^5C_5 \cdot D_5 = 44$

b) 3 Wrong + 4 Letter Wrong + 5 Letter Wrong.

$$^5C_3 D_3 + {^5C_4} D_4 + {^5C_5} D_5$$

$$(10)(2) + 5(9) + (1)(44)$$

$$20 + 45 + 44 = 109$$

Six cards and six envelopes are numbered 1, 2, 3, 4, 5, 6 and cards are to be placed in envelopes so that each envelope contains exactly one card and no card is placed in the envelope bearing the same number and moreover the card numbered 1 is always placed in envelope numbered 2. Then the number of ways it can be done is

(a) 264 (b) 265 (c) 53 (d) 67

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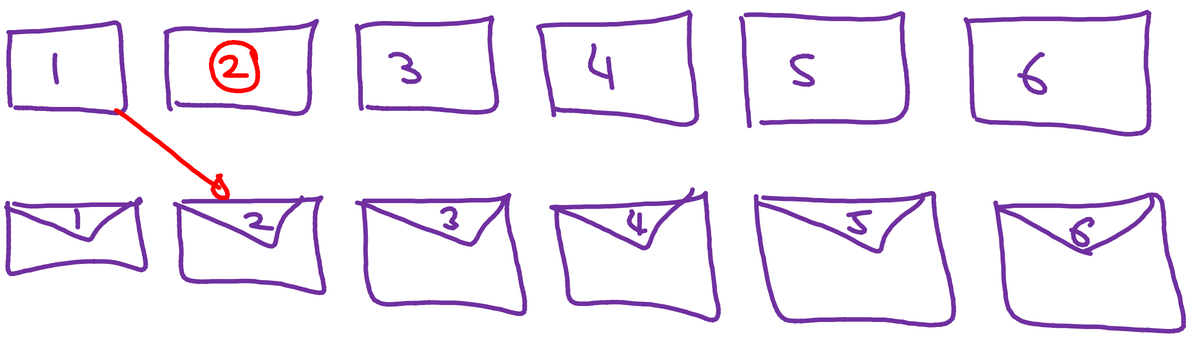

C-1 $\quad 2 \longrightarrow 1$

$$D_4 = 9$$

$$+$$

C-2 $\quad 2 \not\longrightarrow 1 \qquad D_5 = 44$

$$53$$