$y = \sin^{-1} x$

$f : [-\frac{\pi}{2}, \pi/2] \longrightarrow [-1, 1]$

$f(x) = \sin x$

$$[-\tfrac{\pi}{2}, \tfrac{\pi}{2}] \underset{f^{-1}}{\overset{f}{\rightleftarrows}} [-1, 1]$$

Range = C.D.

now $\sin x$ is $I \cap S$

$\Rightarrow$ Bijective $\Rightarrow$ Invertible

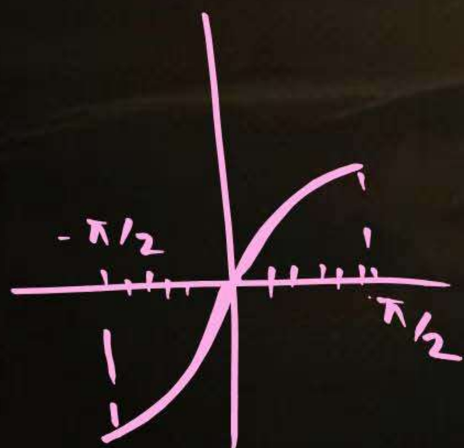
$f(x) = \sin x = y$

$x = \sin^{-1} y$

$x \leftrightarrow y$

$$\boxed{f^{-1}(x) = y = \sin^{-1} x}$$

$f^{-1} : [-1, 1] \longrightarrow [-\pi/2, \pi/2]$



$\sin\theta = 1/2$

$\theta = \pi/6, 5\pi/6, \ldots$

$\theta \to$ Infinite values

$\theta = \sin^{-1} 1/2 = \pi/6$ only

$\sin x = 1$

$x = \pi/2, 2\pi + \pi/2, \ldots$

But $\sin^{-1} 1 = \pi/2$

$\sin\theta = -1 \Rightarrow \theta = -\pi/2,\ 3\pi/2,\ 3\pi/2 + 2\pi, \ldots$

But $\sin^{-1}(-1) = 3\pi/2$ (WRONG)

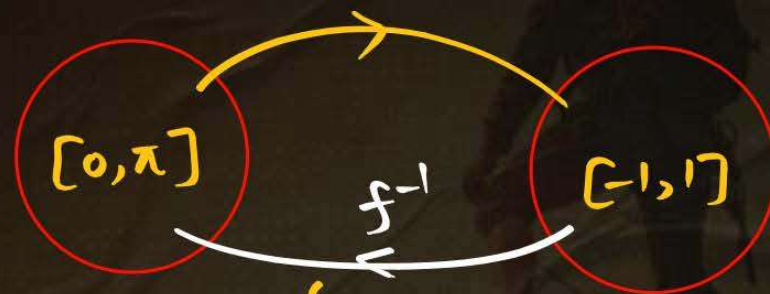
$\sin^{-1}(-1) = -\pi/2$ ✓

$\sin^{-1}(-\sqrt{3}/2) = -\pi/3$

$y = \cos^{-1} x$

$f : [0, \pi] \rightarrow [-1, 1]$

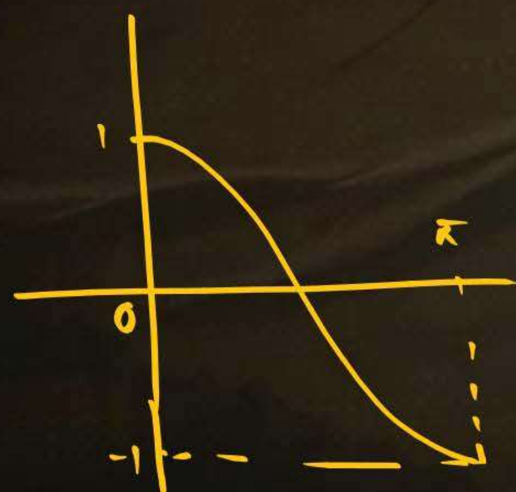
$f(x) = \cos x$



$f(x) = \cos x$

$f^{-1}(x) = \cos^{-1} x$

$f^{-1} : [-1, 1] \rightarrow [0, \pi]$



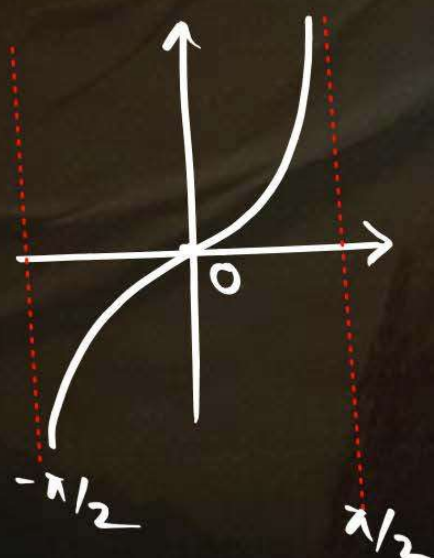
Ex: $\cos^{-1}\left(\frac{1}{2}\right) = \pi/3$

$\cos^{-1}\left(-\frac{1}{2}\right) = \pi - \pi/3 = 2\pi/3$

$\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = \pi - \pi/4 = 3\pi/4$

$y = \tan^{-1}x$

$$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \underset{f^{-1}}{\overset{f}{\rightleftarrows}} R$$



$$f^{-1} : R \rightarrow (-\pi/2, \pi/2)$$

Domain & Range of I.T.F.

$f(x) = \sec x$

$\sec \pi/2 = N \cdot D$

S.No.	f(x)	Domain	Range
(1)	$\sin^{-1} x$	$\underbrace{\lvert x\rvert \le 1}$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
(2)	$\cos^{-1} x$	$\lvert x\rvert \le 1$	$[0, \pi]$
(3)	$\tan^{-1} x$	$x \in R$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
(4)	$\cot^{-1} x$	$x \in R$	$(0, \pi)$
(5)	$\sec^{-1} x$	$\lvert x\rvert \ge 1$ $\boxed{x \geq 1 \text{ or } x \leq -1}$	$\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$ or $[0, \pi] - \{\pi/2\}$
(6)	$\operatorname{cosec}^{-1} x$	$\lvert x\rvert \ge 1$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

$f(x) = \operatorname{cosec} x$

$\operatorname{cosec} 0 = N \cdot D.$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION

[Ans. $\frac{3\pi}{2}$]

Find the value:

$$\underbrace{\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)}_{-\pi/4}+\underbrace{\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)}_{(\pi-\pi/4)}-\boxed{\tan^{-1}(-\sqrt{3})}+\boxed{\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)}$$

$\downarrow$ $-\pi/3$ $\qquad$ $\downarrow$ $\pi-\pi/3$

$$-\pi/4+\pi-\pi/4+\cancel{\pi/3}+\pi-\cancel{\pi/3}$$

$$2\pi-2\pi/4$$

$$2\pi-\pi/2$$

$$3\pi/2$$

$\cot\theta=\frac{1}{\sqrt{3}}$

$\theta=60^\circ$

$=\pi/3$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION [JEE Main 2023 Jan. (Shift-I)]

[Ans. C]

$\tan^{-1}\left(\frac{1+\sqrt{3}}{3+\sqrt{3}}\right) + \sec^{-1}\left(\sqrt{\frac{8+4\sqrt{3}}{6+3\sqrt{3}}}\right)$ is equal to

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{2}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{6}$

$\tan^{-1}\left[\frac{\cancel{1+\sqrt{3}}}{\sqrt{3}(\cancel{\sqrt{3}+1})}\right]$

$\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\downarrow$

$\pi/6 \; +$

$\sec^{-1}\sqrt{\frac{4\cancel{(2+\sqrt{3})}}{3\cancel{(2+\sqrt{3})}}}$

$\sec^{-1}\sqrt{4/3}$

$\sec^{-1}\frac{2}{\sqrt{3}}$

$\pi/6$

$\sec\theta = 2/\sqrt{3}$

$\Rightarrow \cos\theta = \sqrt{3}/2$

$2\pi/6 = \pi/3$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION

[Ans. A]

If $\boxed{\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = 3\pi}$, then the value of $\underbrace{x^{2024}}_{1} + \underbrace{y^{2024}}_{1} + \underbrace{z^{2024}}_{1} + \frac{1}{x^{2023} + y^{2023} + z^{2023}}$ is equal to

- (A) 8/3 ✓
- (B) 1
- (C) 0
- (D) 2

$\cos^{-1}x = \pi$ $\quad$ $\cos^{-1}y = \pi$ & $\cos^{-1}z = \pi$

$x = -1$ $\quad$ $y = -1$, $z = -1$

$$1+1+1+\frac{1}{-1-1-1}$$

$$3 - \frac{1}{3} = \frac{9-1}{3} = 8/3$$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION

If $\sum_{i=1}^{2024} cos^{-1} x_i = 0$ then find the value of $\sum_{i=1}^{2024} x_i$

$\Rightarrow \cos^{-1} x_i = 0 \quad \forall\, i \in [1, 2024]$

$x_i = \cos 0$

$x_i = 1$

$\sum_{i=1}^{2024} 1 = 2024$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION

Solution of equation $\tan(\underbrace{\cos^{-1}x}_{\phi}) = \underbrace{\sin\left(\cot^{-1}\frac{1}{2}\right)}_{2/\sqrt{5}}$ is

(A) $x = \frac{\sqrt{7}}{3}$

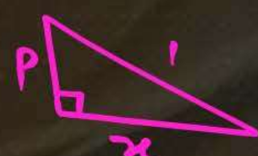
(B) $x = \frac{\sqrt{5}}{3}$ ✓

(C) $x = \frac{3\sqrt{5}}{2}$

(D) None of these

Let $\cos^{-1}x = \phi$

$\Rightarrow \cos\phi = \frac{x}{1}$

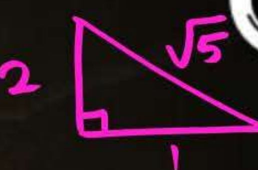


LHS: $\tan\phi$

$P = \sqrt{1-x^2}$

$\tan\phi = \frac{P}{x} = \frac{\sqrt{1-x^2}}{x}$

$\frac{\sqrt{1-x^2}}{x} = \frac{2}{\sqrt{5}} \Rightarrow \frac{1-x^2}{x^2} = 4/5$



Let $\cot^{-1} 1/2 = \theta$

$\Rightarrow \cot\theta = 1/2 = \frac{b}{p}$

RHS: $\sin\theta = \frac{P}{H} = 2/\sqrt{5}$

$5 - 5x^2 = 4x^2$

$5 = 9x^2 \Rightarrow x^2 = 5/9 \Rightarrow x = \sqrt{5}/3$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION

$\cos\left[\tan^{-1}\{\underbrace{\sin(\underbrace{\cot^{-1}x}_{\theta})}\}\right]$ is equal to-

(A) $\sqrt{\dfrac{x^2+2}{x^2+3}}$

(B) $\sqrt{\dfrac{x^2+2}{x^2+1}}$

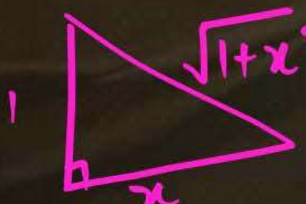
(C) ✓ $\sqrt{\dfrac{x^2+1}{x^2+2}}$

(D) None of these

Let $\cot^{-1}x = \theta$

$x = \cot\theta$

$\tan\theta = 1/x$

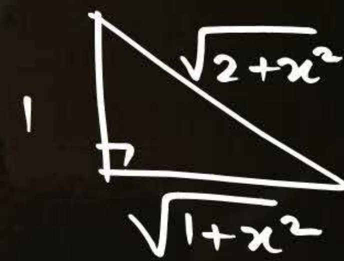


$\sin\theta = \dfrac{1}{\sqrt{1+x^2}}$

$\cos \underbrace{\tan^{-1}\left(\dfrac{1}{\sqrt{1+x^2}}\right)}_{\theta}$

Let $\tan^{-1}\dfrac{1}{\sqrt{1+x^2}} = \theta$

$\tan\theta = \dfrac{1}{\sqrt{1+x^2}}$



$\cos\theta = ?$

$\cos\theta = \dfrac{\sqrt{1+x^2}}{\sqrt{2+x^2}}$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION

[Ans. A,D]

$$\tan\left[\frac{1}{2}\underbrace{\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)}_{\theta}\right] =$$

(A) $\dfrac{3-\sqrt{5}}{2}$ ✓

(B) $\dfrac{3+\sqrt{5}}{2}$

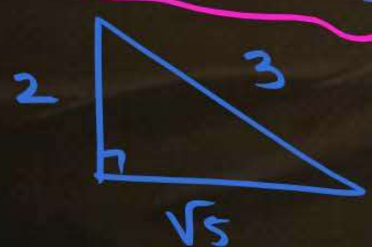
(C) $\dfrac{2}{3-\sqrt{5}}$

(D) $\dfrac{2}{3+\sqrt{5}}$ ✓

$\tan(\theta/2) = ?$

Let $\cos^{-1}\dfrac{\sqrt{5}}{3} = \theta$

$\boxed{\cos\theta = \sqrt{5}/3} = b/H$



$\tan\theta/2 = \dfrac{3-\sqrt{5}}{2}\cdot\dfrac{3+\sqrt{5}}{3+\sqrt{5}}$

$= \dfrac{4}{2(3+\sqrt{5})} = \dfrac{2}{3+\sqrt{5}}$

$\boxed{\tan^2\theta/2 = \dfrac{1-\cos\theta}{1+\cos\theta}}$

$\tan^2\theta/2 = \dfrac{1-\sqrt{5}/3}{1+\sqrt{5}/3}$

$\tan^2\theta/2 = \dfrac{3-\sqrt{5}}{3+\sqrt{5}}\cdot\dfrac{3-\sqrt{5}}{3-\sqrt{5}}$

$\dfrac{(3-\sqrt{5})^2}{9-5}$

$\tan^2\theta/2 = \left(\dfrac{3-\sqrt{5}}{2}\right)^2$

$\tan\theta/2 = \dfrac{3-\sqrt{5}}{2}$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION [Main June 26, 2022 (II)] [Ans. C]

If the inverse trigonometric functions take principal values, then

$\cos^{-1}\left(\frac{3}{10}\cos\left(\underbrace{\tan^{-1}\left(\frac{4}{3}\right)}_{\theta}\right)+\frac{2}{5}\sin\left(\underbrace{\tan^{-1}\left(\frac{4}{3}\right)}_{\theta}\right)\right)$ is equal to:

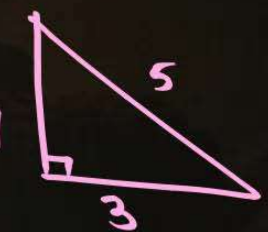
(A) 0

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{6}$

where $\tan\theta = 4/3$



$\cos^{-1}\left(\frac{3}{10}\cos\theta + \frac{2}{5}\sin\theta\right)$

$\cos^{-1}\left(\frac{3}{10}\times\frac{3}{5} + \frac{2}{5}\cdot\frac{4}{5}\right)$

$\cos^{-1}\left(\frac{9}{50} + \frac{16}{50}\right)$

$\cos^{-1}\left(\frac{25}{50}\right) = \cos^{-1}\frac{1}{2} = \pi/3$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION [Adv 2024]

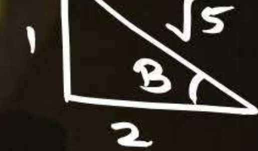
[Ans. B]

If the inverse trigonometric functions take principal values, then

$$\tan\left(\left(\underbrace{\sin^{-1}\left(\frac{3}{5}\right)}_{A}\right) - 2\underbrace{\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)}_{B}\right)$$ is equal to:

where $\sin A = \frac{3}{5} \Rightarrow \tan A = 3/4$

$\cos B = 2/\sqrt{5}$



(A) $\frac{7}{24}$

(B) $\frac{-7}{24}$ ✓

(C) $\frac{-5}{24}$

(D) $\frac{5}{24}$

$$\tan(A-2B) = \frac{\tan A - \tan 2B}{1+\tan A \tan 2B}$$

$$= \frac{3/4 - 4/3}{1 + \cancel{3}/\cancel{4} \times \cancel{4}/\cancel{3}}$$

$$= \frac{\frac{3}{4} - 4/3}{2} = \frac{9-16}{12\times 2} = -7/24$$

$\tan B = 1/2$

$$\tan 2B = \frac{2\tan B}{1-\tan^2 B}$$

$$= \frac{\cancel{2}\times 1/\cancel{2}}{1-\frac{1}{4}} = \frac{1}{3/4}$$

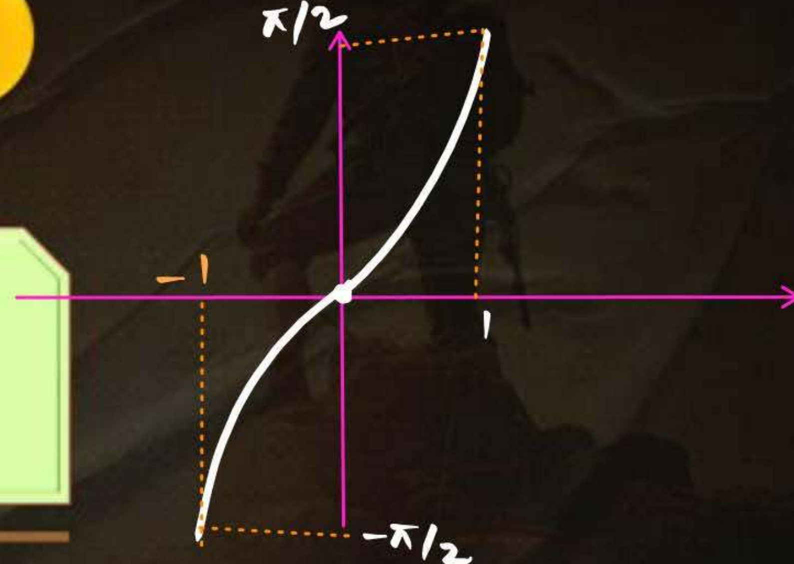
$= 4/3$

FOR NOTES & DPP CHECK DESCRIPTION

Graph of $y = \sin^{-1} x$

$f^{-1} : [-1, 1] \rightarrow [-\frac{\pi}{2}, \frac{\pi}{2}]$

$f^{-1}(x) = \sin^{-1}(x)$



$\sin^{-1}(x)$ is ↑

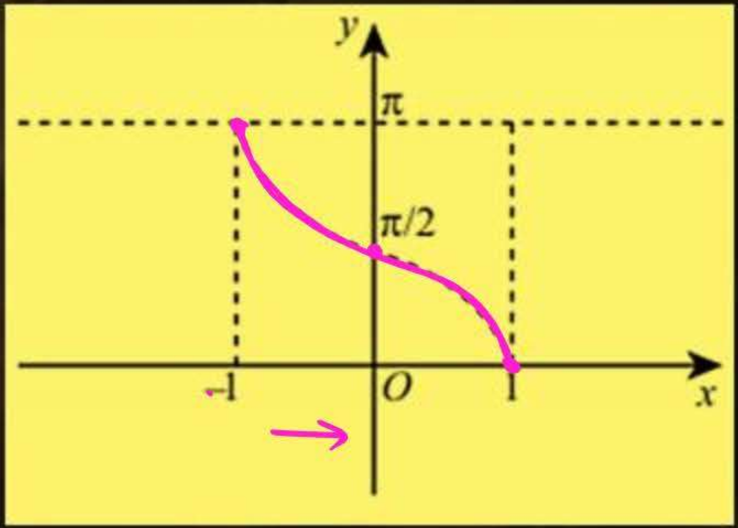
FOR NOTES & DPP CHECK DESCRIPTION

Graph of $y = \cos^{-1} x$

$f^{-1} : [-1, 1] \rightarrow [0, \pi]$ ✓

$f^{-1}(x) = \cos^{-1}(x)$

$\cos^{-1}(x)$ is ↓



$\cos^{-1} 0 = \pi/2$

$\cos^{-1} 1 = 0$

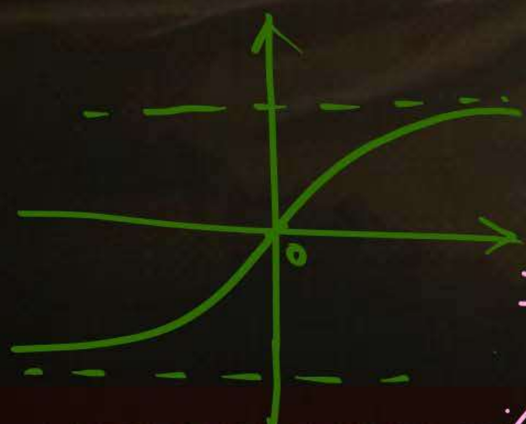
Graph of $y = \tan^{-1} x$

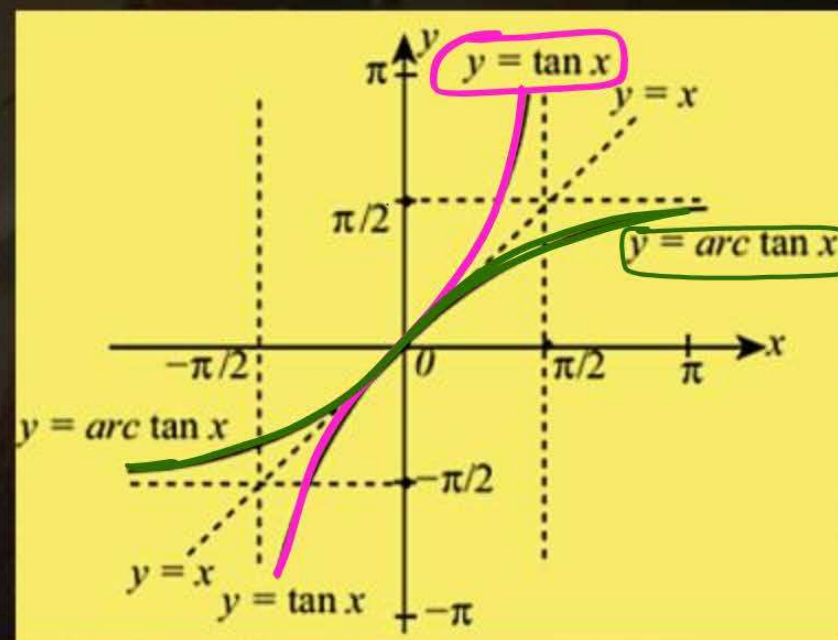
$\tan^{-1}(\infty) \rightarrow \pi/2$

$\tan^{-1}(-\infty) \rightarrow -\pi/2$

$f^{-1}: R \rightarrow (-\frac{\pi}{2}, \frac{\pi}{2})$

$f^{-1}(x) = \tan^{-1}(x)$



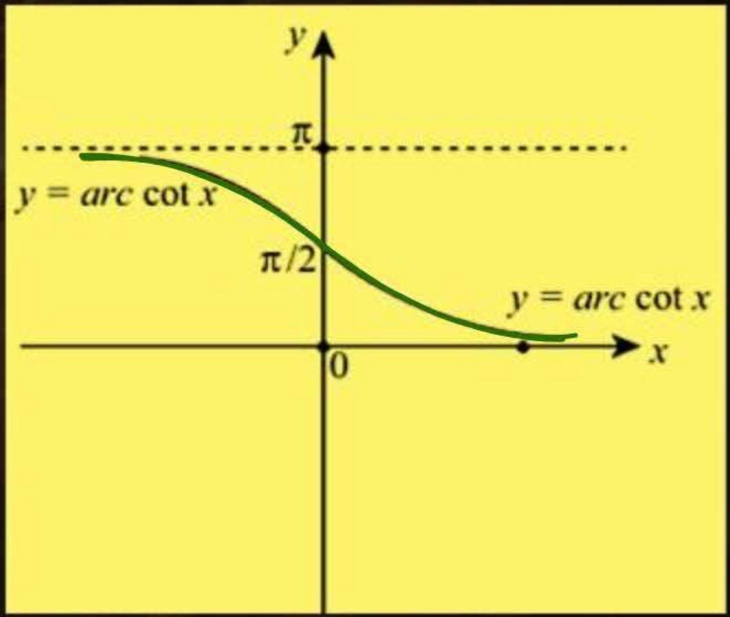


FOR NOTES & DPP CHECK DESCRIPTION

Graph of y = cot^{-1} x

$f^{-1}: R \to (0, \pi)$

$f^{-1}(x) = \cot^{-1}x$



$\cot^{-1}(\infty) \to 0$

$\cot^{-1}(-\infty) \to \pi$

FOR NOTES & DPP CHECK DESCRIPTION

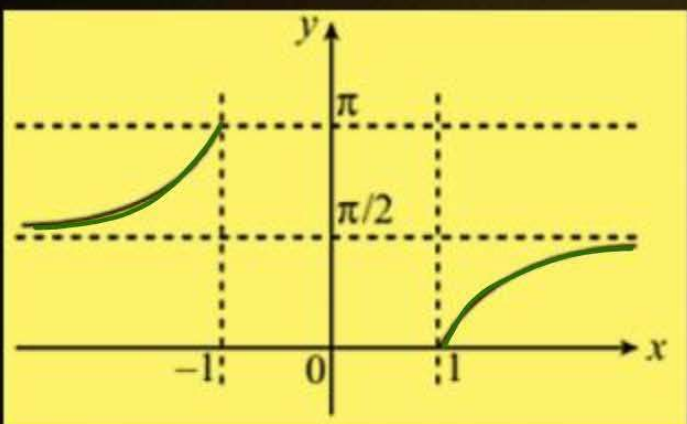
Graph of y = sec⁻¹ x & cosec⁻¹ x

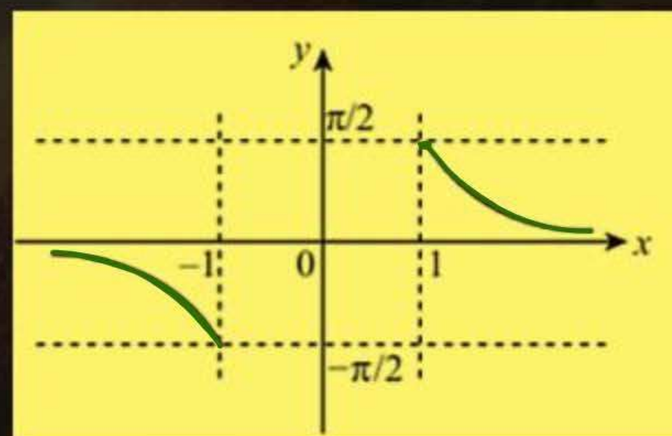
$f^{-1}: (-\infty, -1] \cup [1, \infty) \to [0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi]$

$f^{-1}(x) = \sec^{-1}x$

$f^{-1}: (-\infty, -1] \cup [1, \infty) \to [-\frac{\pi}{2}, 0) \cup (0, \frac{\pi}{2}]$

$f^{-1}(x) = \mathrm{cosec}^{-1}x$





Note

(1) $\underbrace{\cos^{-1}}x$, $\underbrace{\cot^{-1}x}$ & $\underbrace{\text{cosec}^{-1}}x$ are **decreasing** functions.

(2) $\underbrace{\sin^{-1}}x$, $\underbrace{\tan^{-1}x}$ and $\underbrace{\sec^{-1}x}$ are **increasing** functions.

(3) $\underbrace{\cos^{-1}x}$, $\underbrace{\cot^{-1}x}$ and $\underbrace{\sec^{-1}x}$ cant be **negative**.

— FOR NOTES & DPP CHECK DESCRIPTION —

QUESTION [Main July 28, 2022 (I)]

[Ans. B]

Considering only the principal values of the inverse trigonometric functions, the domain of the function $f(x) = \cos^{-1}\left(\frac{x^2 - 4x + 2}{x^2 + 3}\right)$ is:

(A) $\left(-\infty, \frac{1}{4}\right]$

(B) $\left[-\frac{1}{4}, \infty\right)$ ✓

(C) $\left(-\frac{1}{3}, \infty\right)$

(D) $\left(-\infty, \frac{1}{3}\right]$

$$-1 \le \frac{x^2-4x+2}{x^2+3} \le 1$$

x^2+3 +ve

$x \geq -1/4$

$-x^2-3 \le x^2-4x+2$

$\boxed{2x^2-4x+5 \geq 0}$

$D = 16-4\times2\times5$

$16-40 = -ve$. always true

$\Rightarrow$ always +ve

$\cancel{x^2}-4x+2 \le \cancel{x^2}+3$

$-4x \le 1$

$x \geq \frac{1}{-4}$

$\boxed{x \geq -1/4}$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION [Main July 22, 2021 (II)] **[Ans. A]**

If the domain of the function $f(x) = \dfrac{\cos^{-1}\sqrt{x^2 - x + 1}}{\sqrt{\sin^{-1}\left(\frac{2x-1}{2}\right)}}$ is the interval $(\alpha, \beta]$, then $\alpha + \beta$ is equal to:

$\alpha = 1/2,\ \beta = 1$

(A) 3/2 ✓

(B) 2

(C) 1/2

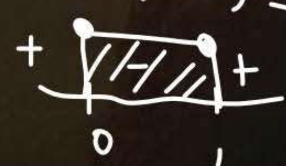
(D) 21

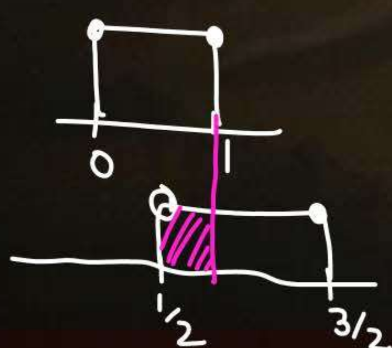
$\underbrace{-1 \le \sqrt{x^2 - x + 1}}_{\text{always true}}\ \underbrace{\le 1}_{\text{S.B.S}}$

$x^2 - x + \cancel{1} \le \cancel{1}$

$x(x-1) \le 0$

D_1:





Domain: $x \in (\frac{1}{2}, 1]$

$0 < \sin^{-1}\left(\frac{2x-1}{2}\right) \le \pi/2$

$\sin 0 < \frac{2x-1}{2} \le \sin \pi/2$

$0 < \frac{2x-1}{2} \le 1$

$2x - 1 > 0$

$x > 1/2$

$2x - 1 \le 2$

$x \le 3/2$

D_2: $x \in (\frac{1}{2}, \frac{3}{2}]$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION [Main June 29, 2022 (I)]

[Ans. D]

The domain of the function $\cos^{-1}\left(\frac{2\sin^{-1}\left(\frac{1}{4x^2-1}\right)}{\pi}\right)$ is:

(A) $R-\left\{-\frac{1}{2},\frac{1}{2}\right\}$

(B) $(-\infty,-1]\cup[1,\infty)\cup\{0\}$

(C) $\left(-\infty,\frac{-1}{2}\right)\cup\left(\frac{1}{2},\infty\right)\cup\{0\}$

(D) $\left(-\infty,\frac{-1}{\sqrt{2}}\right]\cup\left[\frac{1}{\sqrt{2}},\infty\right)\cup\{0\}$

$$-1 \le \frac{2\sin^{-1}\left(\frac{1}{4x^2-1}\right)}{\pi} \le 1$$

$$-\frac{\pi}{2} \le \sin^{-1}\left(\frac{1}{4x^2-1}\right) \le \frac{\pi}{2}$$

$$\boxed{-1 \le \frac{1}{4x^2-1} \le 1}$$

$$\frac{1}{4x^2-1} \le 1$$

$$\frac{1}{4x^2-1} - 1 \le 0$$

$$\frac{1-4x^2+1}{4x^2-1} \le 0$$

$$\frac{2-4x^2}{4x^2-1} \le 0$$

$$\frac{(x^2-1/2)}{4x^2-1} \ge 0$$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION [Main July 29, 2022 (II)]

[Ans. C]

The domain of the function $f(x) = \sin^{-1}\left(\frac{x^2 - 3x + 2}{x^2 + 2x + 7}\right)$ is:

(A) $[1, \infty)$

(B) $(-1, 2]$

(C) $[-1, \infty)$

(D) $(-\infty, 2]$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION [JEE Main 2023, (13 Apr Shift-II)]

[Ans. C]

The range of $f(x) = 4\sin^{-1}\left(\frac{x^2}{x^2+1}\right)$ is

(A) $[0, 2\pi]$

(B) $[0, \pi]$

(C) $[0, 2\pi)$

(D) $[0, \pi)$

$y = 4\sin^{-1} t$

$y_{max} \rightarrow 4\sin^{-1} 1 \rightarrow 4 \cdot \pi/2 = 2\pi$

$y_{min} \rightarrow 4\sin^{-1} 0 = 0$

$[0, 2\pi)$

$t = \frac{x^2+1-1}{x^2+1}$

$= \frac{\cancel{x^2+1}}{\cancel{x^2+1}} - \frac{1}{x^2+1}$

$t = 1 - \frac{1}{x^2+1}$

$t_{min} = 1-1 = 0$

t_{min} $\frac{1}{x^2+1} \rightarrow max$

t_{max} $\frac{1}{x^2+1} \rightarrow min$

$t \in [0, 1)$

t_{max} $x^2 \rightarrow \infty$

$t_{max} \rightarrow 1-0 \rightarrow 1$

FOR NOTES & DPP CHECK DESCRIPTION

PROPERTY- 1

1) $\boxed{\begin{array}{l}\sin^{-1}(-x) = -\sin^{-1}x \\ \tan^{-1}(-x) = -\tan^{-1}x \\ \operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}x\end{array}}$ $\forall x \in D$

$\boxed{\begin{array}{l}\cos^{-1}(-x) = \pi - \cos^{-1}x \\ \cot^{-1}(-x) = \pi - \cot^{-1}(x) \\ \sec^{-1}(-x) = \pi - \sec^{-1}(x)\end{array}}$

Ex $\tan^{-1}(-1/2) = -\underbrace{\tan^{-1}1/2}$

$\operatorname{cosec}^{-1}(-3) = -\underbrace{\operatorname{cosec}^{-1}3}$

$\cos^{-1}(-1/2) =$

FOR NOTES & DPP CHECK DESCRIPTION

PROPERTY-2

(i) $\boxed{\operatorname{cosec}^{-1} x = \sin^{-1}\frac{1}{x};}$ $\boxed{x \le -1 \text{ OR } x \ge 1}$

(ii) $\boxed{\sec^{-1} x = \cos^{-1}\frac{1}{x};}$ $\boxed{x \le -1 \text{ OR } x \ge 1}$

(iii) $\cot^{-1} x = \begin{cases} \boxed{\tan^{-1}\frac{1}{x};} & \boxed{x > 0} \\ \boxed{\pi + \tan^{-1}\frac{1}{x};} & x < 0 \end{cases}$

Ex →

$\operatorname{cosec}^{-1}(3/2) = \sin^{-1} 2/3$

$\sin^{-1} 1/2 = \operatorname{cosec}^{-1} 2$ ✓

$\boxed{\operatorname{cosec}^{-1}(-5/3)} = \boxed{\sin^{-1}(-3/5)}$

I ← $\boxed{\cot^{-1}(5/3)} = \boxed{\tan^{-1} 3/5}$ → I ✓

2nd Quad ← $\boxed{\cot^{-1}(-5/7)} = \boxed{\tan^{-1}(-7/5)}$ → 4th Quad → WRONG

$\sec^{-1}(-5/2) = \cos^{-1}(-2/5)$ ✓

↓ 2nd ↓ 2nd Q

FOR NOTES & DPP CHECK DESCRIPTION

$$\underbrace{\cot^{-1}(-5/7)}_{2^{nd}} = \underbrace{\pi + \tan^{-1}(-7/5)}_{2^{nd}\text{ Quad}}$$

$$\cot^{-1}(-3/2) = \tan^{-1}(-2/3) \quad \text{WRONG}$$

$$= \pi + \tan^{-1}(-2/3) \rightarrow \text{correct.}$$

PROPERTY-3

(i) $\boxed{\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2},}$ $\boxed{|x| \le 1}$

(ii) $\boxed{\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2},}$ $\boxed{x \in R}$

(iii) $\boxed{\operatorname{cosec}^{-1}x + \sec^{-1}x = \frac{\pi}{2},}$ $\boxed{|x| \ge 1}$ ✓

Ex → $\sin^{-1}\frac{1}{3} + \cos^{-1}\frac{1}{3} = \pi/2$

$\tan^{-1}\pi + \cot^{-1}\pi = \pi/2$

$\underbrace{\operatorname{cosec}^{-1}\frac{1}{2} + \sec^{-1}\frac{1}{2}}_{\text{(N.D)}} = \pi/2$ WRONG

$\operatorname{cosec}^{-1}3 + \sec^{-1}3 = \pi/2$

$\underbrace{\sin^{-1}x}_{\theta} + \underbrace{\cos^{-1}x}_{\phi} = \pi/2$ Proof:

$\sin\theta = x$ & $\cos\phi = x$

$\sin\theta = \cos\phi$

$\sin\theta = \sin(90° - \phi) \Rightarrow \theta = 90° - \phi$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION [JEE Main 2023 Apr. (Shift-II)]

[Ans. 0]

Let the inverse trigonometric functions take principal values. The number of real solutions of the equation $\boxed{2\sin^{-1}x + 3\cos^{-1}x = \frac{2\pi}{5}}$, is

$$\underbrace{2\sin^{-1}x + 2\cos^{-1}x} + \cos^{-1}x = 2\pi/5$$

$$2\left[\underbrace{\sin^{-1}x + \cos^{-1}x}\right] + \cos^{-1}x = 2\pi/5$$

$$\pi + \cos^{-1}x = 2\pi/5$$

$$\cos^{-1}(x) = -3\pi/5$$

$$x = \cos(-3\pi/5)$$ not possible

WRONG

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION

Find the range of the following functions

(1) $y = \sin^{-1}x + \cos^{-1}x + \tan^{-1}x$

$\underbrace{\sin^{-1}x}_{[-1,1]} \quad \underbrace{\cos^{-1}x}_{[-1,1]} \quad \underbrace{\tan^{-1}x}_{x \in R}$

Domain $x \in [-1, 1]$

(2) $y = (\sin^{-1}x)^2 + (\cos^{-1}x)^2$

(3) $y = (\sin^{-1}x)^3 + (\cos^{-1}x)^3$

1) $y = \pi/2 + \tan^{-1}x$

~~$y_{max} \to \pi/2 + \pi/2 \to \pi$~~

~~$y_{min} \to \pi/2 - \pi/2 \to 0$~~

WRONG.

$y_{max} = \pi/2 + \tan^{-1} 1 = \pi/2 + \pi/4 = \frac{3\pi}{4}$

$y_{min} = \pi/2 + \tan^{-1}(-1) = \pi/2 - \pi/4 = \pi/4$

$y \in [\pi/4, 3\pi/4]$

(2) Let $\boxed{\sin^{-1}x = \theta} \Rightarrow \cos^{-1}x = \pi/2 - \theta$

$\theta \in [-\pi/2, \pi/2]$

$y = \theta^2 + (\pi/2 - \theta)^2$

$y = \theta^2 + \pi^2/4 + \theta^2 - \pi\theta$

$\boxed{y = 2\theta^2 - \pi\theta + \pi^2/4} \to$ completing sq.

FOR NOTES & DPP CHECK DESCRIPTION

$$y = 2\left[\theta^2 - \frac{\pi\theta}{2} + \pi^2/8\right]$$

$$= 2\left[\left(\theta - \pi/4\right)^2 - \pi^2/16 + \pi^2/8\right]$$

$$2\left[\left(\theta - \pi/4\right)^2 + \pi^2/16\right]$$

$$\boxed{y = \frac{\pi^2}{8} + 2\left(\theta - \pi/4\right)^2}$$

$$y_{min} = \pi^2/8$$

$$y_{max} \begin{cases} \theta = \pi/2 \\ \theta = -\pi/2 \end{cases}$$

$$y_{max} = \frac{\pi^2}{8} + 2\left(-\pi/2 - \pi/4\right)^2$$

$$= \frac{\pi^2}{8} + \frac{2 \cdot 9\pi^2}{16} = \frac{10\pi^2}{8}$$

$$y \in \left[\frac{\pi^2}{8}, \frac{5\pi^2}{4}\right]$$

Short Cut

For y_{min} put $\sin^{-1}x = \cos^{-1}x = \pi/4$

For y_{max} Put Boundary pts of ITF.

$$y = (\sin^{-1}x)^2 + (\cos^{-1}x)^2$$

$$y_{min} = (\pi/4)^2 + (\pi/4)^2 = \frac{\pi^2}{16} + \frac{\pi^2}{16} = \frac{\pi^2}{8}$$

For y_{max}

$\sin^{-1}x$	$\cos^{-1}x$	
$\pi/2$	0	$\Rightarrow y = (\pi/2)^2 + 0^2$
$-\pi/2$	π	$\Rightarrow y = (-\pi/2)^2 + \pi^2$

$$y_{max} = \pi^2/4 + \pi^2$$
$$5\pi^2/4$$

$y = (\sin^{-1}x)^3 + (\cos^{-1}x)^3$

$y_{min} = (\pi/4)^3 + (\pi/4)^3 = \frac{2\pi^3}{64} = \frac{\pi^3}{32}$

For y_{max}

$\sin^{-1}x$	$\cos^{-1}x$	y
$\pi/2$	0	$(\pi/2)^3 + 0^3 = \pi^3/8$
$-\pi/2$	π	$(-\pi/2)^3 + \pi^3 = -\frac{\pi^3}{8} + \pi^3 = \frac{7\pi^3}{8}$

$y \in \left[\frac{\pi^3}{32}, \frac{7\pi^3}{8}\right]$

General method

$y = \pi/2\left[(\sin^{-1}x)^2 + (\cos^{-1}x)^2 - \sin^{-1}x\cos^{-1}x\right]$

QUESTION [JEE Main 2022 (24 June Shift 1)] [Ans. A]

The set of all values of k for which $\boxed{(\tan^{-1} x)^3 + (\cot^{-1} x)^3 = k\pi^3}$, $x \in R$, is the interval :

(A) $\left[\frac{1}{32}, \frac{7}{8}\right)$ ✓

(B) $\left(\frac{1}{24}, \frac{13}{16}\right)$

(C) $\left[\frac{1}{48}, \frac{13}{16}\right]$

(D) $\left[\frac{1}{32}, \frac{9}{8}\right)$

$y_{min} = (\pi/4)^3 + (\pi/4)^3 = \pi^3/32$

$\tan^{-1}x$	$\cot^{-1}x$	y
$\pi/2$	0	$\pi^3/8$
$-\pi/2$	π	$7\pi^3/8$

$y_{max} \to 7\pi^3/8$

$y \in \left[\frac{\cancel{\pi^3}}{32}, \frac{7\cancel{\pi^3}}{8}\right) = k\cancel{\pi^3}$

$\left[\frac{1}{32}, \frac{7}{8}\right) = k$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION

If the sum $\sum_{n=1}^{10} \sum_{m=1}^{10} \tan^{-1}\left(\frac{m}{n}\right) = k\pi$, find the value of $\boldsymbol{k}$.

FOR NOTES & DPP CHECK DESCRIPTION

PROPERTY-4

$\underbrace{\tan^{-1}x}_{A} + \underbrace{\tan^{-1}y}_{B} = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

$A + B =$

$$\boxed{\tan^{-1}x + \tan^{-1}y} = \begin{cases} \boxed{\tan^{-1}\dfrac{x+y}{1-xy}} \text{ where } \boxed{x>0, y>0} \text{ and } \boxed{xy<1} \\ \boxed{\pi + \tan^{-1}\dfrac{x+y}{1-xy}} \text{ where } \boxed{x>0, y>0} \text{ and } \boxed{xy>1}, \\ \boxed{\dfrac{\pi}{2}} \text{ where } x > 0, y>0 \text{ and } \boxed{xy=1} \end{cases}$$

If $xy = 1 \Rightarrow y = 1/x$

$\tan^{-1}x + \tan^{-1}\frac{1}{x}$

$\tan^{-1}x + \cot^{-1}x = \pi/2$

Note

1. $\left(\tan^{-1}1+\tan^{-1}2+\tan^{-1}3\right)=\pi$
2. $\left(\tan^{-1}1+\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}\right)=\frac{\pi}{2}$

1) $\underbrace{\tan^{-1}1}+\underbrace{\tan^{-1}2+\tan^{-1}3}$

$xy=6>1$

$\pi/4+\pi+\tan^{-1}\left(\frac{2+3}{1-2\times 3}\right)$

$\pi/4+\pi+\tan^{-1}\left(\frac{5}{1-6}\right)$

$\cancel{\pi/4}+\pi+\cancel{\tan^{-1}(-1)}$

π

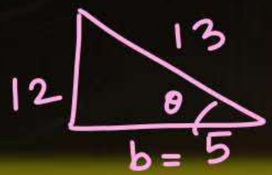
2) $\underbrace{\tan^{-1}1}+\tan^{-1}1/2+\tan^{-1}1/3$

$\pi/4+\tan^{-1}\left(\frac{1/2+1/3}{1-1/6}\right)$ $xy=\frac{1}{2}\cdot\frac{1}{3}=\frac{1}{6}<1$

$\pi/4+\tan^{-1}\left(\frac{\cancel{5/6}}{\cancel{5/6}}\right)$

$\pi/4+\tan^{-1}1=\pi/2$

QUESTION



$b=\sqrt{13^2-12^2}=5$

$\tan^{-1}(-x)=-\tan^{-1}x$

Find the value of $\sin^{-1}\frac{12}{13}+\cot^{-1}\frac{4}{3}+\tan^{-1}\frac{63}{16}=$

$$\underbrace{\tan^{-1}\left(\frac{12}{5}\right)+\tan^{-1}(3/4)}+\tan^{-1}\left(\frac{63}{16}\right)$$

$$xy=\frac{12}{5}\times\frac{3}{4}=\frac{36}{20}>1$$

$$\pi+\tan^{-1}\left(\frac{\frac{12}{5}+\frac{3}{4}}{1-\frac{12}{5}\times\frac{3}{4}}\right)$$

$$\pi+\tan^{-1}\left(\frac{48+15}{20-36}\right)$$

$$\pi+\tan^{-1}\left(\frac{63}{-16}\right)$$

$$=\pi-\cancel{\tan^{-1}\left(\frac{63}{16}\right)}+\cancel{\tan^{-1}\frac{63}{16}}$$

$$=\pi$$

QUESTION

Find the value of $\underbrace{\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{5}}_{}+\underbrace{\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{8}}_{}=$

HW

$xy = \frac{1}{3}\cdot\frac{1}{5} = \frac{1}{15}$

$\frac{1}{7}\times\frac{1}{8} < 1$

FOR NOTES & DPP CHECK DESCRIPTION

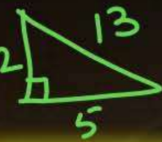
Properties of I.T.F. Property – 5

Property 5

(2) $\boxed{\tan^{-1} x - \tan^{-1} y = \tan^{-1} \frac{x-y}{1+xy}}$ where $\boxed{x > 0, y > 0}$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION [JEE Main – 2019 (April)]



[Ans. C]

The value of $\sin^{-1}\left(\frac{12}{13}\right) - \sin^{-1}\left(\frac{3}{5}\right)$ is equal to

(A) $\pi - \sin^{-1}\left(\frac{63}{65}\right)$

(B) $\pi - \cos^{-1}\left(\frac{33}{65}\right)$

(C) $\boxed{\frac{\pi}{2} - \sin^{-1}\left(\frac{56}{65}\right)}$ ✓

(D) $\frac{\pi}{2} - \cos^{-1}\left(\frac{9}{65}\right)$

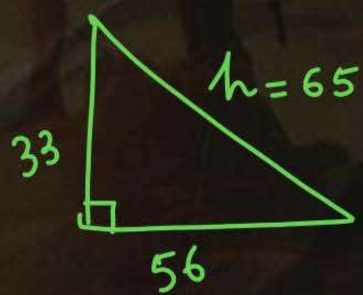
$$\tan^{-1}\left(\frac{12}{5}\right) - \tan^{-1}\left(3/4\right)$$

$$\tan^{-1}\left(\frac{\frac{12}{5} - 3/4}{1 + \frac{12}{5}\cdot 3/4}\right)$$

$$\tan^{-1}\left(\frac{48-15}{20+36}\right)$$

$$\tan^{-1}\left(\frac{33}{56}\right) = \sin^{-1}\frac{33}{65} = \cos^{-1}\frac{56}{65} = \pi/2 - \sin^{-1}\frac{56}{65}$$

$$h = \sqrt{33^2 + 56^2}$$



$$\sin^{-1}\frac{56}{65} + \cos^{-1}\frac{56}{65} = \pi/2$$

—— FOR NOTES & DPP CHECK DESCRIPTION ——

QUESTION [JEE Main 2024 Apr. (Shift-I)] **[Ans. 47]**

For $n \in N$, if $\cot^{-1}3 + \cot^{-1}4 + \cot^{-1}5 + \cot^{-1}n = \frac{\pi}{4}$, then n is equal to _____

$\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{n} = \tan^{-1}1 - \tan^{-1}\frac{1}{3}$

$= \tan^{-1}\left(\frac{1-\frac{1}{3}}{1+\frac{1}{3}}\right)$

$\tan^{-1}\frac{2/3}{4/3}$

$\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{n} = \tan^{-1}\frac{1}{2} - \tan^{-1}\frac{1}{4}$

$\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{n} = \tan^{-1}\left(\frac{\frac{1}{2}-\frac{1}{4}}{1+\frac{1}{8}}\right)$

$\tan^{-1}\frac{1/4}{9/8}$

$\tan^{-1}\frac{1}{n} = \tan^{-1}\frac{2}{9} - \tan^{-1}\frac{1}{5}$

$\tan^{-1}\frac{1}{n} = \tan^{-1}\left(\frac{\frac{2}{9}-\frac{1}{5}}{1+\frac{2}{9}\cdot\frac{1}{5}}\right)$

$\tan^{-1}\left(\frac{10-9}{45+2}\right)$

$\tan^{-1}\frac{1}{47} = \tan^{-1}\frac{1}{n}$

$\frac{1}{47} = \frac{1}{n}$

$n = 47$

FOR NOTES & DPP CHECK DESCRIPTION

Properties of I.T.F. Property–6

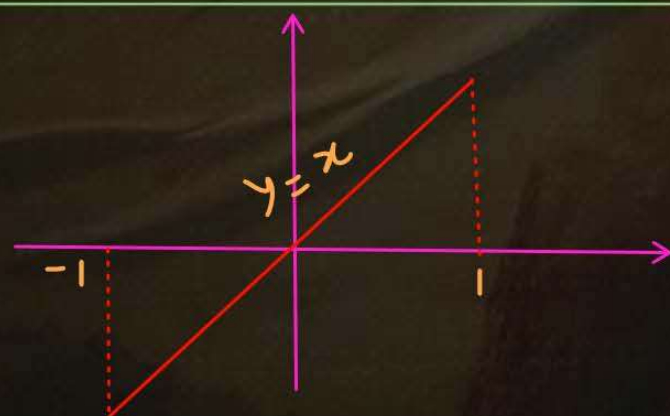
$f : A \to B$

$$\boxed{\begin{aligned} f f^{-1}(x) &= x \ \forall x \in B \\ f^{-1} f(x) &= x \ \forall x \in A \end{aligned}}$$ ✓

Property 6

(i) $y = \sin(\underbrace{\sin^{-1}}\, x) = x \quad \forall x \in [-1, 1]$

(ii) $y = \cos(\cos^{-1} x) = x \quad \forall x \in [-1, 1]$



Ex →

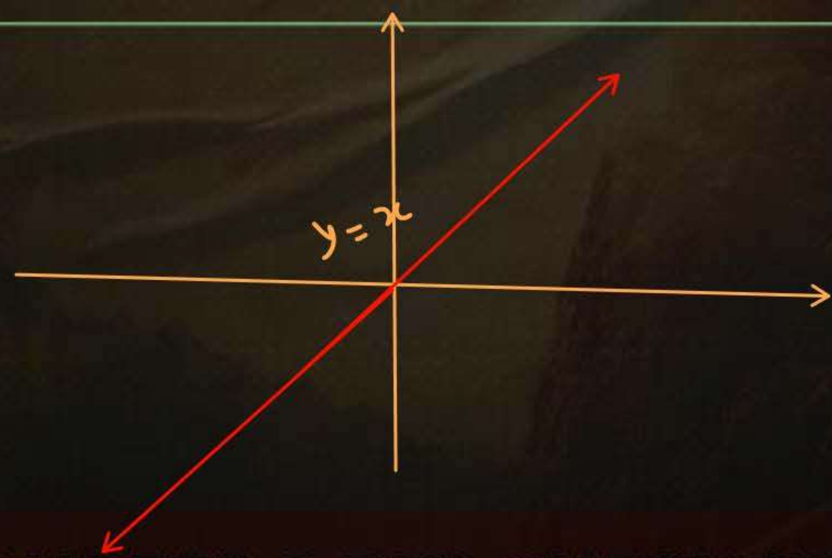
$\sin \sin^{-1} 1/5 = 1/5$

Properties of I.T.F. Property–6

Property 6

(iii) $y = \tan(\tan^{-1}x) = x \quad x \in R;$

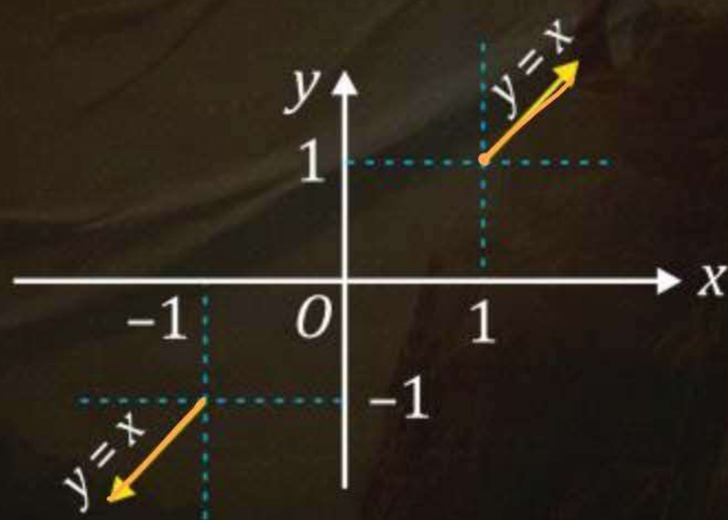
(iv) $y = \cot(\cot^{-1}x) = x, \quad x \in R$



Properties of I.T.F. Property–6

Property 6

$y = \sec(\sec^{-1} x) = x$, $y = \operatorname{cosec}(\operatorname{cosec}^{-1} x) = x$,
$|x| \geq 1, |y| \geq 1$



Properties of I.T.F. Property–7

$\pi = 3.14$

$\frac{\pi}{2} = 1.57$

Property 7

(i) $\boxed{y = \sin^{-1}(\sin x)} = x \;\forall\; x \in [-\pi/2, \pi/2]$

$x \in \underbrace{[-1.57, 1.57]}$

$y = \sin^{-1} \sin(200^\circ)$

$\sin^{-1} \sin(180 + 20^\circ)$

$\sin^{-1}(-\sin 20^\circ)$

$- \sin^{-1} \sin 20^\circ$

$= -20^\circ$

1) $\sin^{-1} \sin(\pi/8) = \pi/8$

2) $\sin^{-1} \sin(\pi/2) = \pi/2$

3) $\sin^{-1} \sin(5\pi/6) = \cancel{5\pi/6}$

$= \sin^{-1} \sin(\pi - \pi/6)$

$= \sin^{-1} \sin \pi/6 = \pi/6$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION

(i) $\sin^{-1}(\sin 1) = 1$

(ii) $\sin^{-1}(\sin 2) = \sin^{-1}\sin(\pi - 2) = \boxed{\pi - 2}$ ✓

(iii) $\sin^{-1}(\sin 3) = \sin^{-1}\sin(\pi - 3) = \pi - 3$

(iv) $\sin^{-1}(\sin 4) = (\pi - 4)$

(v) $\sin^{-1}(\sin 5) = \sin^{-1} - \sin(2\pi - 5) = -\sin^{-1}\sin(\underbrace{2\pi - 5}) = -[2\pi - 5] = 5 - 2\pi$

(vi) $\sin^{-1}(\sin 6) = \boxed{6 - 2\pi}$ ✓

(vii) $\sin^{-1}(\sin 8)$ $\cancel{8 - 2\pi}$ $\rightarrow \sin^{-1}\sin(3\pi - 8) = \boxed{3\pi - 8}$ ✓

(viii) $\sin^{-1}(\sin 10)$ $\rightarrow \boxed{3\pi - 10}$ ✓

$\pi - 2 \approx 3.14 - 2 = 1.14$

$\pi - 4 \approx 3.14 - 4 \approx -0.86$

$\pi - 5 \approx 3.14 - 5 \approx -1.86$

$2\pi - 5 \approx 2 \times 3.14 - 5 = 6.28 - 5 = 1.28$

$8 - 2\pi = 8 - 6.28$
$= 1.72$

$3\pi - 8 = 3 \times 3.14 - 8 = 9.42 - 8$

$\sin 5 = -\sin(2\pi - 5)$

$\sin 8 = \sin(3\pi - 8)$

$3\pi - 10 = 3 \times 3.14 - 10$
$= 9.42 - 10$
$= -0.58$

~~$\sin 10 = \sin(10 - 3\pi)$~~

$\cos(2\pi-\theta)=\cos\theta$

Properties of I.T.F. Property–7

Property 7

(ii) $y=\cos^{-1}(\cos x)=x \quad \forall\ x\in[0,\pi]$

$\underbrace{x\in[0,3.14]}$ $\cos^{-1}\cos 100^\circ = 100^\circ$

$\cos^{-1}\cos 5\pi/6 = 5\pi/6$

$\cos^{-1}\cos 7\pi/6 =$

$\cos^{-1}\cos\left(2\pi-5\pi/6\right)$

$\cos^{-1}\cos(5\pi/6) = 5\pi/6$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION

(i) $\cos^{-1}(\cos 1) = 1$
(ii) $\cos^{-1}(\cos 2) = 2$
(iii) $\cos^{-1}(\cos 3) = 3$
(iv) $\cos^{-1}(\cos 4) = 2\pi - 4$
(v) $\cos^{-1}(\cos 5) = 2\pi - 5$
(vi) $\cos^{-1}(\cos 6) = 2\pi - 6$
(vii) $\cos^{-1}(\cos 8) = 8 - 2\pi$
(viii) $\cos^{-1}(\cos 10) = 4\pi - 10$ ✓

$\cancel{\cos 8 = -\cos(3\pi - 8)}$

$\cancel{\cos 8 = \cos(8 - 3\pi)}$

$\cos 4 = \cos(2\pi - 4)$

$\cancel{\cos 4 = \cos(4 - \pi)}$

$2\pi - 4 = 6.28 - 4 = \underbrace{2.28}$ ✓

$\cancel{\cos 8 = \cos(3\pi - 8)}$

$\cos 8 = \cos(4\pi - 8)$

$4\pi - 8 = \underbrace{4.56}_{\text{outside}}$

$\cos 8 = \cos(2\pi - 8)$

$\cos 8 = \cos(8 - 2\pi)$

$4\pi - 10 = 2.56$ $\quad 2\pi - 8 = 6.28 - 8 = -1.72$

FOR NOTES & DPP CHECK DESCRIPTION

Properties of I.T.F. Property–7

Property 7

(iii) $y = \tan^{-1}(\tan x) = x \quad \forall x \in (-\pi/2, \pi/2)$

(iv) $y = \cot^{-1}(\cot x) = x \quad \forall x \in (0, \pi)$

(v) $y = \sec^{-1}(\sec x) = x \quad \forall x \in [0, \pi] - \{\pi/2\}$

(vi) $y = \operatorname{cosec}^{-1}(\operatorname{cosec} x) = x \quad \forall x \in [-\pi/2, \pi/2] - \{0\}$

QUESTION [Main June 27, 2022 (I)]

[Ans. A]

$\sin^{-1}\left(\sin\frac{2\pi}{3}\right)+\cos^{-1}\left(\cos\frac{7\pi}{6}\right)+\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$ is equal to:

(A) $\frac{11\pi}{12}$

(B) $\frac{17\pi}{12}$

(C) $\frac{31\pi}{12}$

(D) $-\frac{3\pi}{4}$

$\sin^{-1}\sin(\pi/3) + \cos^{-1}\cos(5\pi/6) + \tan^{-1}(-\tan\pi/4)$

$\pi/3 + 5\pi/6 - \pi/4$

$\frac{4\pi + 10\pi - 3\pi}{12} = \frac{11\pi}{12}$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION [JEE Mains 2019]

[Ans. D]

If $x = \sin^{-1}(\sin 10)$ and $y = \cos^{-1}(\cos 10)$, then $y - x$ is equal to –

$3\pi - 10$

$4\pi - 10$

(A) 7π

$(4\pi - 10) - (3\pi - 10)$

(B) 10

π

(C) 0

(D) π

QUESTION [JEE Main 2022 (25 June Shift 2)]

[Ans. B]

The value of $\tan^{-1}\left(\frac{\cos\left(\frac{15\pi}{4}\right)-1}{\sin\left(\frac{\pi}{4}\right)}\right)$ is equal to

(A) $-\frac{\pi}{4}$

(B) $-\frac{\pi}{8}$ ✓

(C) $-\frac{5\pi}{12}$

(D) $-\frac{4\pi}{9}$

$\cos 15\pi/4 =$

$\cos\left(4\pi - 15\pi/4\right)$

$\cos \pi/4$

$$\tan^{-1}\left(\frac{\cos\pi/4 - 1}{\sin\pi/4}\right)$$

$$-\tan^{-1}\left(\frac{1-\cos\pi/4}{\sin\pi/4}\right)$$

$$-\tan^{-1}\left(\frac{\cancel{2}\sin^2\pi/8}{\cancel{2}\,\cancel{\sin\pi/8}\cos\pi/8}\right)$$

$$-\tan^{-1}\tan(\pi/8)$$

$-\pi/8.$

$1-\cos 2\theta = 2\sin^2\theta$

$\boxed{\text{M-2}}$

$$\tan^{-1}\left(\frac{\left(\frac{1}{\sqrt{2}}-1\right)}{\frac{1}{\sqrt{2}}}\right)$$

$\tan^{-1}(1-\sqrt{2})$

$-\tan^{-1}(\sqrt{2}-1)$

$-\pi/8$

FOR NOTES & DPP CHECK DESCRIPTION

Problems on Telescoping Series

$$\boxed{T_r = a_{r+1} - a_r}$$

$T_1 =$

$T_2 =$

$$\boxed{\tan^{-1}\left(\frac{x-y}{\boxed{1+xy}}\right) = \tan^{-1}x - \tan^{-1}y}$$

$\boxed{x, y > 0}$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION

Evaluate $\sum_{p=1}^{\infty} \tan^{-1}\left(\frac{2p}{p^4+p^2+2}\right)$

$$T_p = \tan^{-1}\left(\frac{2p}{p^4+p^2+2}\right)$$

$$\tan^{-1}\left(\frac{2p}{1+\boxed{p^4+p^2+1}}\right)$$

$$\tan^{-1} \frac{2p}{1+(p^2+p+1)(p^2-p+1)}$$

$$\tan^{-1}\left[\frac{(p^2+p+1)-(p^2-p+1)}{1+(p^2+p+1)(p^2-p+1)}\right]$$

$$\tan^{-1}(p^2+p+1) - \tan^{-1}(p^2-p+1)$$

$$p^4+p^2+1 = (p^2+p+1)(p^2-p+1)$$

$$T_p = \tan^{-1}(p^2+p+1) - \tan^{-1}(p^2-p+1)$$

$$T_1 = \tan^{-1} 3 - \tan^{-1} 1$$

$$T_2 = \tan^{-1}(7) - \tan^{-1} 3$$

$$T_n = \tan^{-1}(n^2+n+1) - \sim$$

$$S_n = \tan^{-1}(n^2+n+1) - \tan^{-1} 1$$

$n \to \infty$

$$S_\infty = \pi/2 - \pi/4 = \pi/4$$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION [JEE Main 2023 (30 Jan.-II)]

[Ans. C]

HW

Let $a_1 = 1, a_2, a_3, a_4, \ldots..$ be consecutive natural numbers.

Then $\tan^{-1}\left(\frac{1}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{1}{1+a_2a_3}\right)+\ldots..+\tan^{-1}\left(\frac{1}{1+a_{2021}a_{2022}}\right)$ is equal to

(A) $\frac{\pi}{4}-\cot^{-1}(2022)$

(B) $\cot^{-1}(2022)-\frac{\pi}{4}$

(C) $\tan^{-1}(2022)-\frac{\pi}{4}$

(D) $\frac{\pi}{4}-\tan^{-1}(2022)$

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QUESTION [JEE Main 2022 (27 June Shift 2)]

[Ans. A]

The value of $cot\left(\underbrace{\sum_{n=1}^{50}\tan^{-1}\left(\frac{1}{1+n+n^2}\right)}\right)$ is

(A) $\frac{26}{25}$ ✓

(B) $\frac{25}{26}$

(C) $\frac{50}{51}$

(D) $\frac{52}{51}$

$T_n = \tan^{-1}\left(\frac{1}{1+n+n^2}\right)$

$\tan^{-1}\frac{1}{1+n(n+1)}$

$\tan^{-1}\left(\frac{(n+1)-n}{1+n(n+1)}\right)$

$\boxed{T_n = \tan^{-1}(n+1) - \tan^{-1}n}$

$T_n = \tan^{-1}(n+1) - \tan^{-1}n$

$T_1 = \tan^{-1}2 - \tan^{-1}1$

$T_2 = \tan^{-1}3 - \tan^{-1}2$

$\vdots$

$T_{50} = \tan^{-1}51 - \tan^{-1}50$

$S_{50} = \tan^{-1}51 - \tan^{-1}1$

$S_{50} = \tan^{-1}\frac{(51-1)}{1+51} = \tan^{-1}\left(\frac{50}{52}\right)$

$\tan^{-1}\left(\frac{25}{26}\right)$

$\cot^{-1}\left(\frac{26}{25}\right)$

$\cot\cot^{-1}\left(\frac{26}{25}\right) = 26/25$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION [JEE Main – 2019 (Jan.)]

[Ans. A]

The value of $\cot\left(\sum_{n=1}^{19}\cot^{-1}\left(1+\boxed{\sum_{p=1}^{n}2p}\right)\right)$ is :

(A) $\frac{21}{19}$ ✓

(B) $\frac{19}{21}$

(C) $\frac{22}{23}$

(D) $\frac{23}{22}$

$2\sum_{p=1}^{n} p$

$2[1+2+3+\dots+n]$

$\cancel{2}\,\frac{n(n+1)}{\cancel{2}} = n(n+1)$

$\sum_{n=1}^{19}\cot^{-1}(1+n(n+1))$

$T_n = \tan^{-1}\left(\frac{1}{1+n(n+1)}\right)$

$= \tan^{-1}\left(\frac{(n+1)-n}{1+n(n+1)}\right)$

$T_n = \tan^{-1}(n+1) - \tan^{-1}n$

$\boxed{T_n = \tan^{-1}(n+1) - \tan^{-1}n}$

$T_1 = \tan^{-1}2 - \tan^{-1}1$

$T_2 = \tan^{-1}3 - \tan^{-1}2$

$\vdots$

$T_{19} = \tan^{-1}20 - \tan^{-1}19$

$S_{19} = \tan^{-1}20 - \tan^{-1}1$

$= \tan^{-1}\frac{19}{21} = \cot^{-1}\frac{21}{19}$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION

$$\tan\left[\sum_{r=1}^{\infty}\tan^{-1}\left(\frac{4}{4r^2+3}\right)\right]=?$$

$$T_r = \tan^{-1}\left(\frac{4}{4r^2+3}\right)$$

$$\tan^{-1}\left(\frac{1}{r^2+3/4}\right)$$

$$\tan^{-1}\left(\frac{1}{1+\left(r^2-\frac{1}{4}\right)}\right) = \tan^{-1}\left[\frac{(r+\frac{1}{2})-(r-\frac{1}{2})}{1+(r-\frac{1}{2})(r+\frac{1}{2})}\right]$$

$$T_r = \tan^{-1}(r+1/2) - \tan^{-1}(r-1/2)$$

$$T_1 = \tan^{-1} 3/2 - \tan^{-1} 1/2$$
$$T_2 = \tan^{-1} 5/2 - \tan^{-1} 3/2$$
$$\vdots$$
$$T_n = \tan^{-1}(n+1/2) - \tan^{-1}(n-1/2)$$

$$S_n = \tan^{-1}(n+1/2) - \tan^{-1} 1/2$$

$$S_\infty = \pi/2 - \tan^{-1} 1/2$$

$$S_\infty = \cot^{-1} 1/2$$

$$= \tan^{-1} 2$$

$$\tan(\tan^{-1} 2) = 2$$

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BRAIN TEASER

$$\frac{(n+1)^2}{2}-\frac{(n-1)^2}{2}$$

If $\displaystyle\sum_{n=1}^{\infty} \tan^{-1} \frac{8n}{n^4-2n^2+5} = \pi - \cot^{-1} k$. Find k.

$$T_n = \tan^{-1}\left(\frac{8n}{\underbrace{n^4-2n^2+1}+4}\right)$$

$$\tan^{-1}\left(\frac{8n}{4+(n^2-1)^2}\right)$$

$$\tan^{-1}\left(\frac{2n}{1+\frac{(n^2-1)^2}{4}}\right)$$

$$\tan^{-1} \frac{2n}{1+\frac{(n-1)^2}{2}\cdot\frac{(n+1)^2}{2}}$$

$$\tan^{-1}\left(\frac{\frac{(n+1)^2}{2}-\frac{(n-1)^2}{2}}{1+\frac{(n+1)^2}{2}\frac{(n-1)^2}{2}}\right)$$

$$\boxed{T_n = \tan^{-1}\frac{(n+1)^2}{2} - \tan^{-1}\frac{(n-1)^2}{2}}$$

$$T_n = \tan^{-1}\frac{(n+1)^2}{2} - \tan^{-1}\frac{(n-1)^2}{2}$$

$$T_1 = \tan^{-1}\frac{2^2}{2} - \underbrace{\tan^{-1}\frac{0^2}{2}}$$

$$T_2 = \tan^{-1}\frac{3^2}{2} - \tan^{-1}\frac{1^2}{2}$$

$$T_3 = \tan^{-1}\frac{4^2}{2} - \tan^{-1}\frac{2^2}{2}$$

$$\frac{\pi}{2}+\frac{\pi}{2}-\tan^{-1}\frac{1}{2}$$

$$\boxed{\pi - \cot^{-1} 2} \qquad K=2$$

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QUESTION [JEE Main 2020 (September)]

[Ans. D]

If S is the sum of the first 10 terms of the series $\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{7}\right)+\tan^{-1}\left(\frac{1}{13}\right)+\tan^{-1}\left(\frac{1}{21}\right)+\cdots$ then $\tan(S)$ is equal to

(A) $-\frac{6}{5}$

(B) $\frac{5}{11}$

(C) $\frac{10}{11}$

(D) $\frac{5}{6}$

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Property 8

(a) $\boxed{\sin^{-1}x+\sin^{-1}y}=\begin{cases}\sin^{-1}\left[x\sqrt{1-y^2}+y\sqrt{1-x^2}\right] \\ \text{where } x>0, y>0 \ \& \boxed{\left(x^2+y^2\right)\le 1} \\ \pi-\sin^{-1}\left[x\sqrt{1-y^2}+y\sqrt{1-x^2}\right] \\ \text{where } x>0, y>0 \ \& \ \boxed{x^2+y^2>1}\end{cases}$

(b) $\boxed{\sin^{-1}x-\sin^{-1}y}=\sin^{-1}\left[x\sqrt{1-y^2}-y\sqrt{1-x^2}\right]$ where $x>0, y>0$

Property 9

(a) $\boxed{\cos^{-1} x + \cos^{-1} y} = \cos^{-1}\left[xy - \sqrt{1-x^2}\sqrt{1-y^2}\right]$

where $x > 0, y > 0$

(b) $\cos^{-1} x - \cos^{-1} y$

$$= \begin{cases} \cos^{-1}\left(xy + \sqrt{1-x^2}\sqrt{1-y^2}\right); & x < y, \quad x, y > 0 \\ -\cos^{-1}\left(xy + \sqrt{1-x^2}\sqrt{1-y^2}\right); & x > y, \quad x, y > 0 \end{cases}$$

QUESTION

Prove that $\cos^{-1}\frac{12}{13}+\sin^{-1}\frac{3}{5}=\sin^{-1}\frac{56}{65}$

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QUESTION

If $\underbrace{\cos^{-1} x + \cos^{-1} y} + \cos^{-1} z = \pi$, prove that $x^2 + y^2 + z^2 + 2xyz = 1$

$\cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right) = \pi - \cos^{-1} z$

$xy - \sqrt{1-x^2}\sqrt{1-y^2} = \cos(\pi - \cos^{-1} z)$

$= -\cos\cos^{-1} z$

$xy - \sqrt{1-x^2}\sqrt{1-y^2} = -z$

$(xy+z) = \sqrt{1-x^2}\sqrt{1-y^2}$

SBS

$x^2y^2 + z^2 + 2xyz = (1-x^2)(1-y^2)$

$\cancel{x^2y^2} + z^2 + 2xyz = 1 - x^2 - y^2 + \cancel{x^2y^2}$

$\boxed{x^2 + y^2 + z^2 + 2xyz = 1}$ ✓

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QUESTION

If $\boxed{\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \pi}$, prove that
$x^4 + y^4 + z^4 + 4x^2y^2z^2 = 2(x^2y^2 + y^2z^2 + z^2x^2)$

$$\sin^{-1}x + \sin^{-1}y = \pi - \sin^{-1}z$$

$$\sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right) = \pi - \sin^{-1}z$$

$$x\sqrt{1-y^2} + y\sqrt{1-x^2} = \sin(\pi - \sin^{-1}z)$$

$$= \sin\sin^{-1}z$$

$$\boxed{x\sqrt{1-y^2} + y\sqrt{1-x^2} = z}$$

$$x\sqrt{1-y^2} = z - y\sqrt{1-x^2}$$

S.B.S

$$x^2(1-y^2) = z^2 + y^2(1-x^2) - 2yz\sqrt{1-x^2}$$

$$x^2 - \cancel{x^2y^2} = z^2 + y^2 - \cancel{y^2x^2} - 2yz\sqrt{1-x^2}$$

$$2yz\sqrt{1-x^2} = y^2 + z^2 - x^2$$

SBS

$$4y^2z^2 - 4y^2z^2x^2 = y^4 + z^4 + x^4 + 2y^2z^2 - 2x^2z^2 - 2x^2y^2$$

$$\boxed{2y^2z^2 + 2x^2z^2 + 2x^2y^2 = x^4 + y^4 + z^4 + 4x^2y^2z^2}$$

QUESTION [JEE Main 2024 (31 Jan Shift 1)]

[Ans. A]

For $\alpha, \beta, \gamma \neq 0$. If $\boxed{\sin^{-1}\alpha + \sin^{-1}\beta + \sin^{-1}\gamma = \pi}$ and $\boxed{(\alpha+\beta+\gamma)(\alpha-\gamma+\beta) = 3\alpha\beta,}$ then γ equal to

(A) $\frac{\sqrt{3}}{2}$ ✓

(B) $\frac{1}{\sqrt{2}}$

(C) $\frac{\sqrt{3}-1}{2\sqrt{2}}$

(D) $\sqrt{3}$

$$\boxed{\alpha^4+\beta^4+\gamma^4} + 4\alpha^2\beta^2\gamma^2 = 2\alpha^2\beta^2 + 2\beta^2\gamma^2 + 2\gamma^2\alpha^2$$

$$(\alpha+\beta)^2 - \gamma^2 = 3\alpha\beta$$

$$\alpha^2+\beta^2+2\cancel{\alpha}\beta - \gamma^2 = 3\alpha\beta$$

$$\boxed{\alpha^2+\beta^2-\gamma^2 = \alpha\beta}$$

SBS

$$\underbrace{\boxed{\alpha^4+\beta^4+\gamma^4}} + 2\alpha^2\beta^2 - 2\beta^2\gamma^2 - 2\alpha^2\gamma^2 = \alpha^2\beta^2$$

$$2\alpha^2\beta^2 + \cancel{2\beta^2\gamma^2} + \cancel{2\gamma^2\alpha^2} - 4\alpha^2\beta^2\gamma^2 + 2\alpha^2\beta^2 - \cancel{2\beta^2\gamma^2} - \cancel{2\alpha^2\gamma^2} = \alpha^2\beta^2$$

$$3\alpha^2\beta^2 - 4\alpha^2\beta^2\gamma^2 = 0$$

$$\cancel{\alpha^2\beta^2}(3-4\gamma^2) = 0$$

$$3 = 4\gamma^2$$

$$\boxed{\gamma = \sqrt{3}/2}$$

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QUESTION [JEE Main 2024 (31 Jan Shift 1)]

[Ans. A]

For $\alpha, \beta, \gamma \neq 0$. If $\sin^{-1}\alpha + \sin^{-1}\beta + \sin^{-1}\gamma = \pi$ and $(\alpha + \beta + \gamma)(\alpha - \gamma + \beta) = 3\alpha\beta$, then γ equal to

(A) $\frac{\sqrt{3}}{2}$

(B) $\frac{1}{\sqrt{2}}$

(C) $\frac{\sqrt{3}-1}{2\sqrt{2}}$

(D) $\sqrt{3}$

$\sin^{-1}\alpha = A \Rightarrow \alpha = \sin A$

$\sin^{-1}\beta = B \Rightarrow \beta = \sin B$

$\sin^{-1}\gamma = C \Rightarrow \boxed{\gamma = \sin C}$

$A + B + C = 180^\circ$

$(\sin A + \sin B + \sin C)(\sin A + \sin B - \sin C) = 3\sin A \sin B$

$(\sin A + \sin B)^2 - \sin^2 C = 3\sin A \sin B$

HW

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Solving Inverse Trigonometric Equations

QUESTION

Solve the equation, $\tan^{-1}\frac{x+1}{x-1}+\tan^{-1}\frac{x-1}{x}=\tan^{-1}(-7)$

$$\tan^{-1}\left(\frac{\frac{x+1}{x-1}+\frac{x-1}{x}}{1-\frac{x+1}{x-1}\cdot\frac{x-1}{x}}\right)$$

$$\tan^{-1}\left(\frac{x(x+1)+(x-1)^2}{x(x-1)-(x^2-1)}\right)=\tan^{-1}(-7)$$

$$\tan^{-1}\left(\frac{x^2+x+x^2+1-2x}{-x+1}\right)=\tan^{-1}(-7)$$

$$\boxed{\frac{2x^2-x+1}{1-x}=-7}$$

$$2x^2-x+1=-7+7x$$

$$2x^2-8x+8=0$$

$$x^2-4x+4=0$$

$$(x-2)^2=0$$

$$\boxed{x=2}$$

LHS: $\tan^{-1}3+\tan^{-1}\frac{1}{2}=+$ ve angle

RHS $=\underbrace{\tan^{-1}(-7)}_{\text{Reject}}=-$ve angle

Reject $\Rightarrow$ no solⁿ

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QUESTION [JEE Main 2019 (January)] [Ans. C]

Considering only the principal value of inverse functions, the set

$$A = \left\{ x \geq 0 : \boxed{\tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}} \right\}$$

(A) contains two elements

(B) contains more than two elements

(C) is a singleton

(D) is an empty set

$\tan^{-1}\left(\frac{2x+3x}{1-2x\cdot 3x}\right) = \tan^{-1} 1$

$\frac{5x}{1-6x^2} = 1$

$5x = 1-6x^2$

$6x^2+5x-1=0$

$6x^2+6x-x-1=0$

$(6x-1)(x+1)=0$

$x = \frac{1}{6}$ or -1 ✗

$x = \frac{1}{6}$

Put

LHS:

$\tan^{-1} 2/6 + \tan^{-1} 3/6$

$\underbrace{\tan^{-1}\tfrac{1}{3}}_{+} + \underbrace{\tan^{-1}\tfrac{1}{2}}_{+}$

+ve

$x = \frac{1}{6}$ is accepted

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QUESTION [JEE Main 2024 (27 Jan Shift 2)]

[Ans. B]

Considering only the principal values of inverse trigonometric functions, the number of positive real values of x satisfying $\tan^{-1}(x) + \tan^{-1}(2x) = \frac{\pi}{4}$ is :

(A) More than 2

(B) 1 ✓

(C) 2

(D) 0

$\tan^{-1} x + \tan^{-1} 2x = \pi/4$

$\tan^{-1}\left(\frac{x+2x}{1-x.2x}\right) = \tan^{-1} 1$

$\boxed{\frac{3x}{1-2x^2} = 1}$

$3x = 1-2x^2$

$\boxed{2x^2+3x-1=0}$

$x = \frac{-3 \pm \sqrt{9-4\times2(-1)}}{2\times2} = \frac{-3\pm\sqrt{17}}{4} \rightarrow$ -ve Reject

$\boxed{x = \frac{\sqrt{17}-3}{4}} \rightarrow$ accept

Case → 2 $\quad x.2x > 1$

$\pi + \tan^{-1}\left(\frac{3x}{1-2x^2}\right) = \pi/4$

$\tan^{-1}\left(\frac{3x}{1-2x^2}\right) = -3\pi/4$

$\frac{3x}{1-2x^2} = \tan(-3\pi/4) \rightarrow 1$

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QUESTION

$$\underbrace{\sin^{-1} x}_{A} + \underbrace{\sin^{-1} 2x}_{B} = \frac{\pi}{3}$$

$\boxed{A+B=\pi/3}$ where $\sin A = x$ $\Rightarrow \cos A = \sqrt{1-x^2}$

$\sin B = 2x$ $\Rightarrow \cos B = \sqrt{1-4x^2}$

$$\cos(A+B) = \frac{1}{2}$$

$$\underbrace{\cos A}\,\underbrace{\cos B} - \underbrace{\sin A}_{x}\,\underbrace{\sin B}_{2x} = \frac{1}{2}$$

$$\sqrt{1-x^2}\sqrt{1-4x^2} - 2x^2 = \frac{1}{2}$$

$$\sqrt{1-x^2}\sqrt{1-4x^2} = \frac{1}{2} + 2x^2$$

SBS.

$$(1-x^2)(1-4x^2) = \frac{1}{4} + 4x^4 + 2x^2$$

$$1 - x^2 - 4x^2 + \cancel{4x^4} = \frac{1}{4} + \cancel{4x^4} + 2x^2$$

$$1 - 5x^2 = \frac{1}{4} + 2x^2$$

$$\frac{3}{4} = 7x^2$$

$$x^2 = \frac{3}{28}$$

$$x = +\sqrt{\frac{3}{28}}$$

QUESTION [JEE Main 2019 (Jan.)]

[Ans. A]

If $\cos^{-1}\left(\frac{2}{3x}\right) + \cos^{-1}\left(\frac{3}{4x}\right) = \frac{\pi}{2}, x > \frac{3}{4}$ then x is equal to:

(A) $\frac{\sqrt{145}}{12}$ ✓

(B) $\frac{\sqrt{145}}{10}$

(C) $\frac{\sqrt{146}}{12}$

(D) $\frac{\sqrt{145}}{11}$

$\cos^{-1}\left[\frac{2}{\cancel{3}x}\cdot\frac{\cancel{3}}{4x} - \sqrt{1-\frac{4}{9x^2}}\sqrt{1-\frac{9}{16x^2}}\right] = \pi/2$

$\frac{2}{4x^2} = \sqrt{1-\frac{4}{9x^2}}\sqrt{1-\frac{9}{16x^2}}$

$\left(\frac{1}{2x^2}\right)^2 = \left(1-\frac{4}{9x^2}\right)\left(1-\frac{9}{16x^2}\right)$

$\frac{1}{4\cancel{x^4}} = \frac{(9x^2-4)(16x^2-9)}{144\cancel{x^4}}$

$36 = (9x^2-4)(16x^2-9)$

$\cancel{36} = 144x^4 - 81x^2 - 64x^2 + \cancel{36}$

$144x^4 = 145x^2$

$144x^2 = 145$

$x = \frac{\sqrt{145}}{12}$

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QUESTION

Find the number of real solutions of the equation

$$\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$$

$$\tan^{-1}\boxed{\sqrt{x^2+x}} + \sin^{-1}\sqrt{1+x^2+x} = \pi/2$$

Domain

$$\boxed{x^2+x \geq 0}$$

$$\boxed{x^2+x=0} \rightarrow \text{Domain}$$

$$x(x+1)=0$$

$$\boxed{x=0,\ x=-1} \checkmark$$

LHS: $\tan^{-1}0 + \sin^{-1}1$

$0 + \pi/2 = \pi/2$

Ans: $x = 0, -1$

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QUESTION [JEE Main 2021 (March)]

$\tan(\theta-\pi)=\tan\theta$

[Ans. A]

The sum of possible values of **x** for $\tan^{-1}(x+1)+\underbrace{\cot^{-1}\left(\frac{1}{x-1}\right)}=\tan^{-1}\left(\frac{8}{31}\right)$ is:

(A) $-\frac{32}{4}$ ✓

(B) $-\frac{33}{4}$

(C) $-\frac{31}{4}$

(D) $-\frac{30}{4}$

Case → 1 $\boxed{x-1>0} \Rightarrow \boxed{x>1}$

$\tan^{-1}(x+1)+\tan^{-1}(x-1)=\tan^{-1}\frac{8}{31}$

$\tan^{-1}\left(\frac{x+\cancel{1}+x-\cancel{1}}{1-(x^2-1)}\right)=\tan^{-1}8/31$

$\tan^{-1}\left(\frac{2x}{2-x^2}\right)=\tan^{-1}8/31$

$\boxed{\frac{2x}{2-x^2}=8/31}$

$31x=8-4x^2$

$4x^2+31x-8=0$

$4x^2+32x-x-8=0$

$(4x-1)(x+8)=0$

$x=1/4,\ x=-8$

Reject

Case → 2

$\boxed{x-1<0}$ $\boxed{x<1}$ ✓

$\tan^{-1}(x+1)+\pi+\tan^{-1}(x-1)=\tan^{-1}\frac{8}{31}$

$\tan^{-1}(x+1)+\tan^{-1}(x-1)=\tan^{-1}\frac{8}{31}-\pi$

$\tan^{-1}\left(\frac{2x}{2-x^2}\right)=\tan^{-1}\frac{8}{31}-\pi$

$\frac{2x}{2-x^2}=\tan\left(\tan^{-1}8/31-\pi\right)$

$\tan\tan^{-1}8/31$

$\boxed{\frac{2x}{2-x^2}=8/31}$

$x=-8$ or $x=1/4$ ✗

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check $x = 1/4$

$$\tan^{-1}\left(\frac{1}{4}+1\right) + \cot^{-1}\left(\frac{1}{\frac{1}{4}-1}\right) = \underbrace{\tan^{-1}\frac{8}{31}}_{\text{acute}}$$

$$\underbrace{\tan^{-1}5/4}_{\text{1st Quad}} + \underbrace{\cot^{-1}(-4/3)}_{\text{2nd Quad}} =$$

1st Quad + 2nd Quad

Rejected.

$x = -8$ accepted

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Question [JEE MAIN 2022]

HW

[Ans. 130]

Let $x = \sin\left(2\underbrace{\tan^{-1}\alpha}_{\theta}\right)$ and $y = \sin\left(\frac{1}{2}\underbrace{\tan^{-1}}\frac{4}{3}\right)$.

If $S = [\alpha \in R : \underbrace{y^2 = 1 - x}]$, then $\sum_{\alpha \in S} 16\alpha^3$ is equal to______.

QUESTION [JEE Main 2024 (29 Jan Shift 2)]

[Ans. D]

Let $\boxed{x = \frac{m}{n}}$ (m, n are co-prime natural numbers) be a solution of the equation $\cos(2\underbrace{\sin^{-1}x}_{\theta}) = \frac{1}{9}$ and let $\alpha, \beta (\alpha > \beta)$ be the roots of the equation $mx^2 - nx - m + n = 0$. Then the point (α, β) lies on the line

(A) $3x + 2y = 2$

(B) $5x - 8y = -9$

(C) $3x - 2y = -2$

(D) $5x + 8y = 9$ ✓

$\cos 2\theta = 1/9$ $\quad \sin\theta = x$

$1 - 2\sin^2\theta = 1/9$

$1 - 2x^2 = 1/9$

$2x^2 = 1 - 1/9$

$2x^2 = 8/9$

$x^2 = 4/9$

$x = 2/3 = m/n$

$\boxed{\begin{matrix} m=2 \\ n=3 \end{matrix}}$

$2x^2 - 3x + 1 = 0$

$2x^2 - 2x - x + 1 = 0$

$(2x-1)(x-1) = 0$

$x = 1/2, 1$

$\alpha = 1, \beta = 1/2$

$(1, 1/2)$

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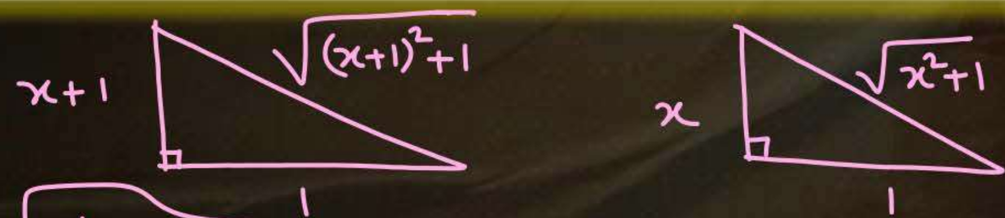
QUESTION [JEE Main 2023 (13 Apr Shift 1)]

[Ans. 4]

If $S = \left\{x \in \mathbb{R}: \sin^{-1}\left(\frac{x+1}{\sqrt{x^2+2x+2}}\right) - \underbrace{\sin^{-1}\left(\frac{x}{\sqrt{x^2+1}}\right)} = \frac{\pi}{4}\right\}$ then $\sum_{x \in S}\left(\sin\left(\left(x^2+x+5\right)\frac{\pi}{2}\right) - \cos\left(\left(x^2+x+5\right)\pi\right)\right)$ is equal to ....



$$\boxed{\tan^{-1}(x+1) - \tan^{-1}\frac{x}{1} = \pi/4}$$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION [JEE Main 2024 Apr. (Shift-II)]

[Ans. A]

Given that the inverse trigonometric function assumes principal values only. Let x, y be any two real numbers in $[-1,1]$ such that $\cos^{-1}x - \sin^{-1}y = \alpha, \frac{-\pi}{2} \leq \alpha \leq \pi$.

Then, the minimum value of $x^2 + y^2 + 2xy\sin\alpha$ is

(A) 0

(B) -1

(C) $\frac{1}{2}$

(D) $-\frac{1}{2}$

Solving Inverse Trigonometric In-Equations

1) $\boxed{\sin^{-1}x \geq \pi/3} \Rightarrow x \geq \sin \pi/3 \Rightarrow x \geq \sqrt{3}/2 \Rightarrow x \in [\sqrt{3}/2, 1]$

2) $\boxed{\cos^{-1}x \geq \pi/3} \Rightarrow x \leq \cos \pi/3 \Rightarrow x \leq 1/2 \Rightarrow x \in [-1, 1/2]$

3) $\boxed{1 < \cot^{-1}x < 3}$

$\cot 1 > x > \cot 3$

$\Rightarrow x \in (\cot 3, \cot 1)$

$0 < \cot^{-1}x < 5$

always true ✓

4) $\cos^{-1}x \geq 4 \rightarrow$ never true no sol^n

$\Rightarrow \boxed{x \leq \cos 4} \rightarrow$ WRONG

5) $\boxed{\sin^{-1}x \geq 2\pi/3} \Rightarrow x \geq \sin 2\pi/3 \Rightarrow x \geq \sqrt{3}/2$ WRONG

no sol^n

—— FOR NOTES & DPP CHECK DESCRIPTION ——

QUESTION [JEE Mains 2019]

[Ans. A]

All x satisfying the inequality $(\cot^{-1}x)^2 - 7(\cot^{-1}x) + 10 > 0$, lie in the interval.

$\cot^{-1}x = \theta$

(A) $(\cot 2, \infty)$

(B) $(\cot 5, \cot 4)$

(C) $(-\infty, \cot 5) \cup (\cot 4, \cot 2)$

(D) $(-\infty, \cot 2) \cup (\cot 5, \infty)$

$\theta^2 - 7\theta + 10 > 0$

$(\theta - 5)(\theta - 2) > 0$

$+ \quad 2 \quad - \quad 5 \quad +$

$\theta < 2$ or $\theta > 5$

$\cot^{-1}x < 2$ or $\cot^{-1}x > 5$

$\Rightarrow x > \cot 2$ never true.

$x \in (\cot 2, \infty)$

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Graph of $y = \sin^{-1}(\sin x)$

Periodic with Period 2π

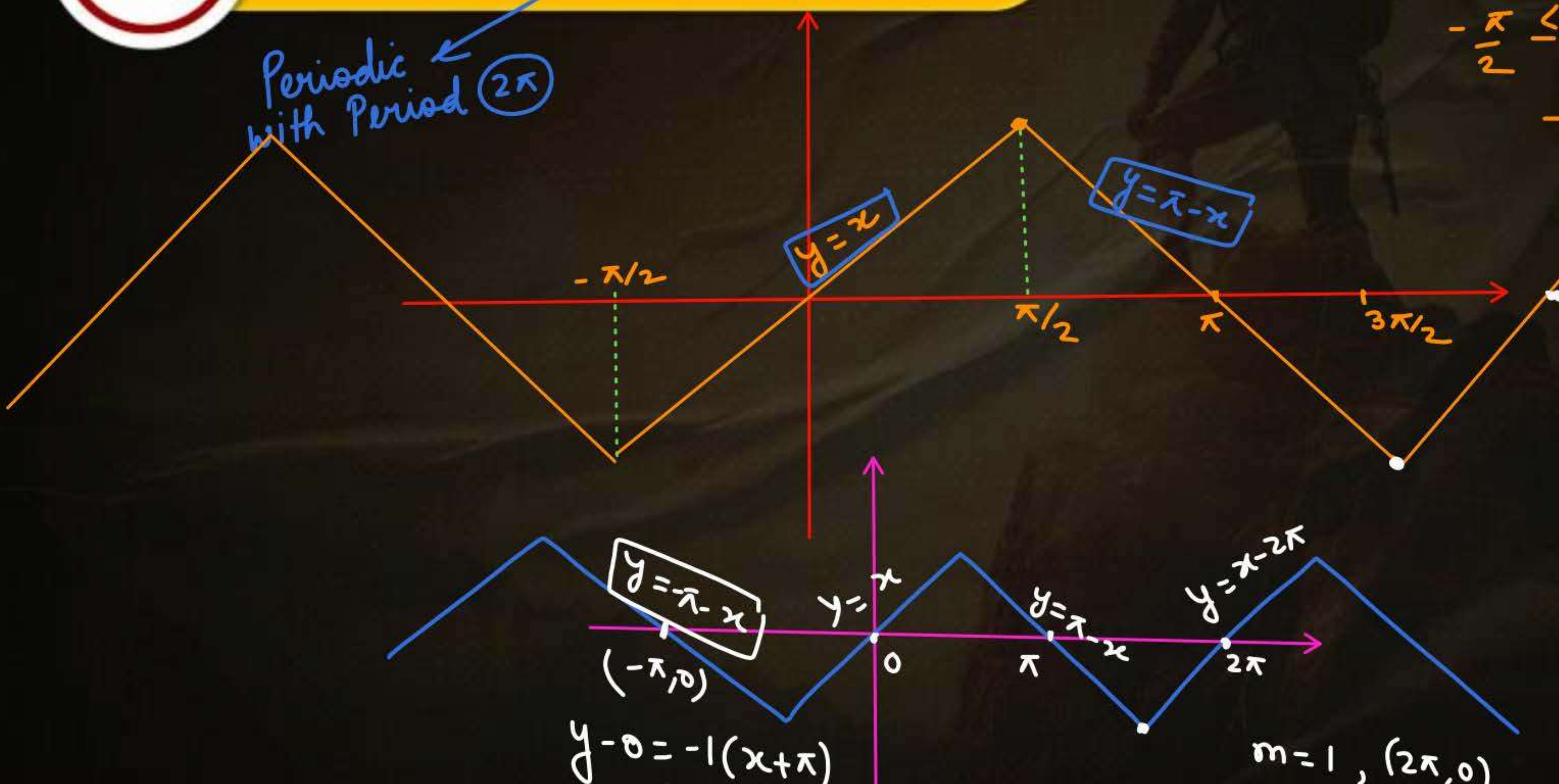
$y = \sin^{-1}\sin x = x \;\; \forall x \in [-\pi/2, \frac{\pi}{2}]$

$y = \sin^{-1}\sin(\pi - x) = \pi - x$ if

$-\frac{\pi}{2} \le \pi - x \le \pi/2$

$-3\pi/2 \le -x \le -\frac{\pi}{2}$

$\boxed{\frac{\pi}{2} \le x \le 3\pi/2}$



$y = \pi - x$ if

$x \in [\pi/2, 3\pi/2]$

$y - 0 = -1(x + \pi)$

$y = -\pi - x$

$m = 1, \; (2\pi, 0)$

$y - 0 = 1(x - 2\pi)$

$y = x - 2\pi$

Graph of $y = \cos^{-1}(\cos x)$ $= x$ if $x \in [0, \pi]$

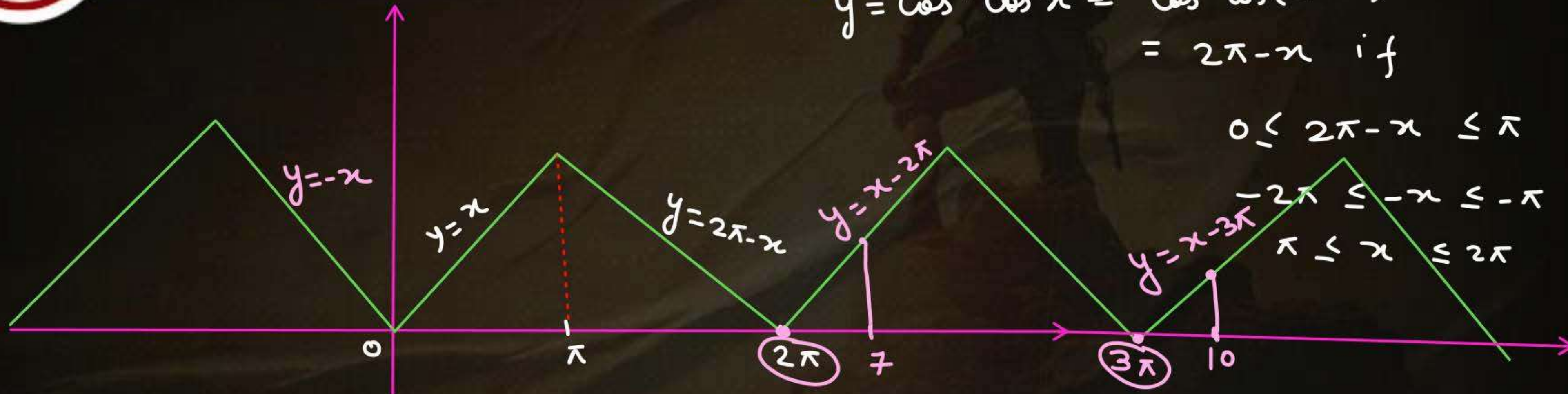
$y = \cos^{-1}\cos x = \cos^{-1}\cos(2\pi - x)$

$= 2\pi - x$ if

$0 \le 2\pi - x \le \pi$

$-2\pi \le -x \le -\pi$

$\pi \le x \le 2\pi$



$m = 1$, Point $(2\pi, 0)$

$y - 0 = 1(x - 2\pi)$

$y = x - 2\pi$

$\cos^{-1}(\cos 7) = 7 - 2\pi$

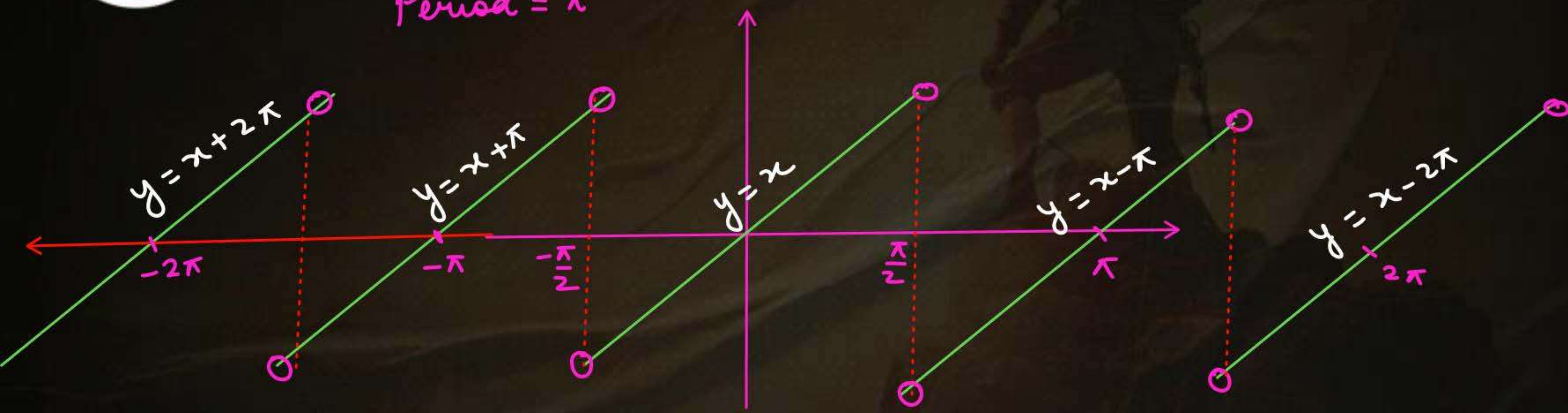
$\cos^{-1}(\cos 8) = 8 - 2\pi$

$\cos^{-1}\cos 10 = 10 - 3\pi$

FOR NOTES & DPP CHECK DESCRIPTION

Graph of $y = \tan^{-1}(\underbrace{\tan x}) = x$ if $x \in (-\pi/2, \pi/2)$

Period $= \pi$



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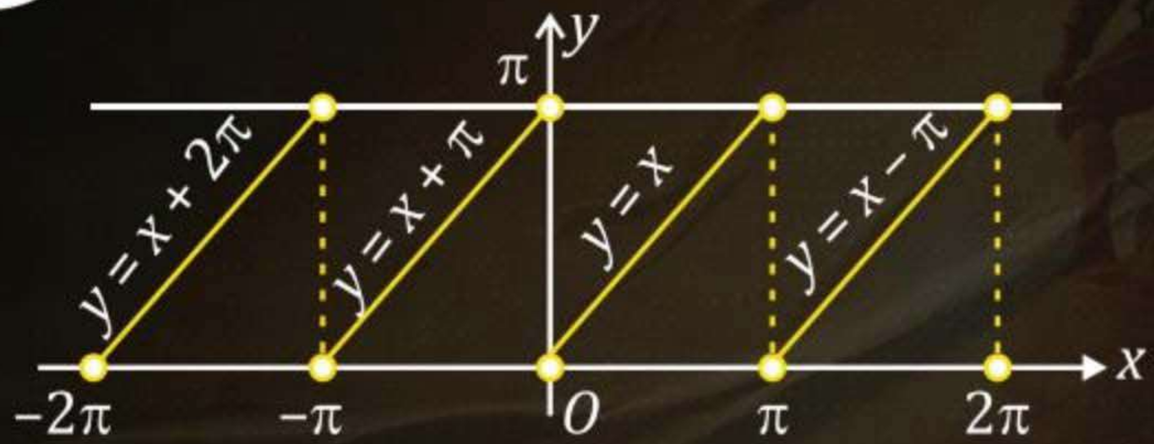
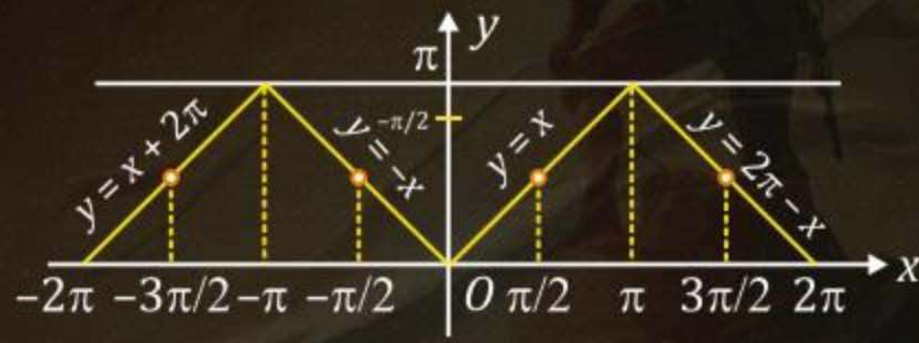
Graph of y = cot⁻¹(cot x)



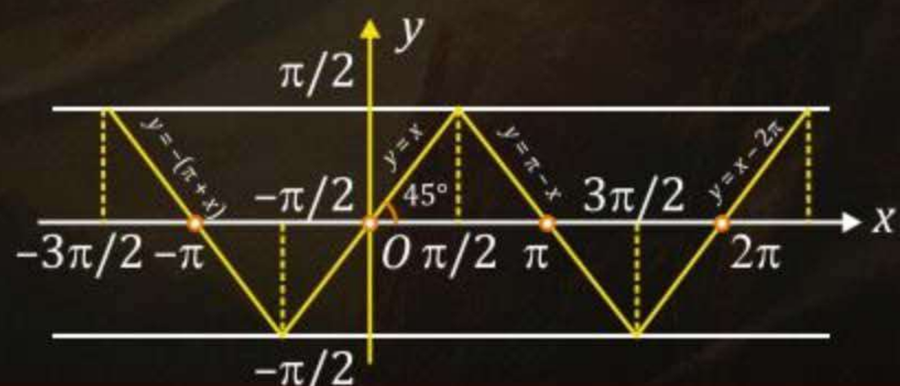
FOR NOTES & DPP CHECK DESCRIPTION

Graph of y = sec⁻¹(sec x) and y = cosec⁻¹(cosec x)

$y = \sec^{-1}(\sec x)$



$y = \text{cosec}^{-1}(\text{cosec } x)$

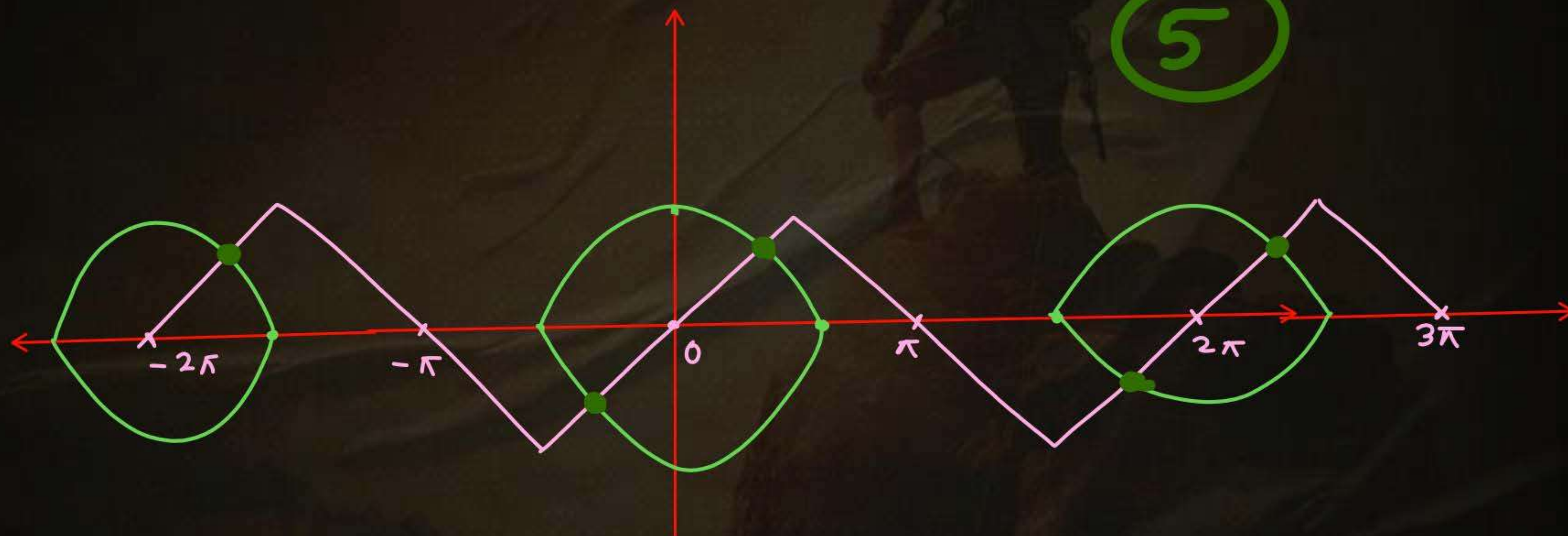


FOR NOTES & DPP CHECK DESCRIPTION

QUESTION

Find the number of solutions of $f(x, y)$ which satisfy $|y| = \cos x$ and $y = \sin^{-1}(\sin x)$, where $-2\pi \leq x \leq 3\pi$.

5



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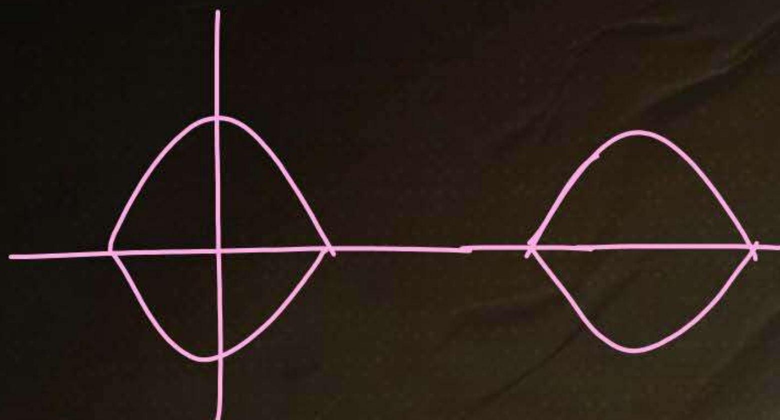

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION [JEE Adv. 2023]

$\sqrt{1+\cos 2x} = \sqrt{2\cos^2 x}$

$\sqrt{2}|\cos x|$

[Ans. 3]

Let $\tan^{-1}(x) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, for $x \in R$. Then the number of real solutions of the equation $\sqrt{1+\cos(2x)} = \sqrt{2}\tan^{-1}(\tan x)$ in the set $\left(-\frac{3\pi}{2}, -\frac{\pi}{2}\right) \cup \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ is equal to

$\cancel{\sqrt{2}}|\cos x| = \cancel{\sqrt{2}}\tan^{-1}\tan x$

$|\cos x| = \tan^{-1}\tan x$

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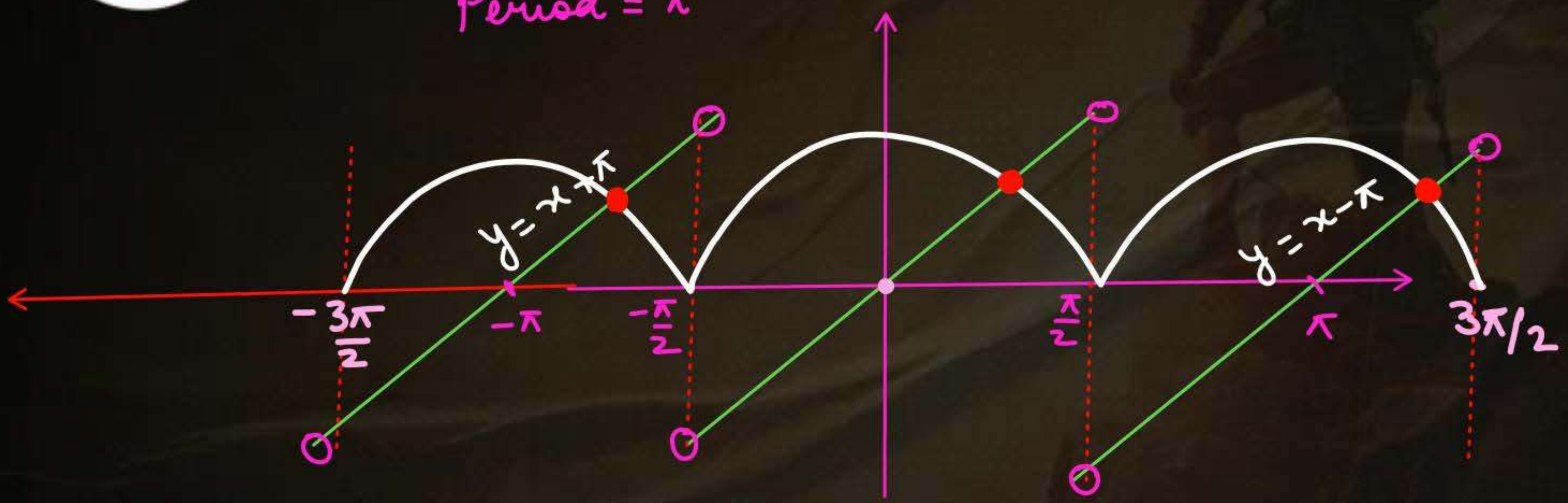
Graph of $y = \tan^{-1}(\underbrace{\tan x})$ $= x$ if $x \in (-\pi/2, \pi/2)$

Period $= \pi$



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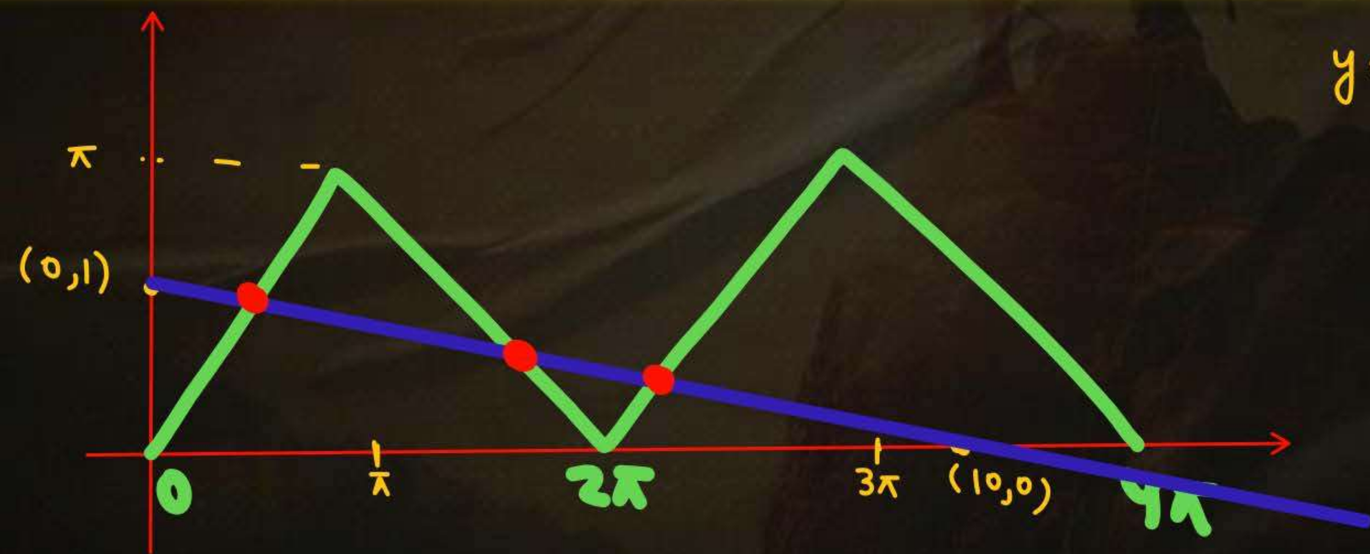
QUESTION [JEE Advanced 2014]

[Ans. 3]

Lef $f: [0, 4\pi] \to [0, \pi]$ be defined by $f(x) = \cos^{-1}(\cos x)$.

The number of points $x \in [0, 4\pi]$ satisfying the equation $f(x) = \frac{10 - x}{10}$ is



$y = \frac{10 - x}{10}$

$x = 0, y = 1$

$x = 10, y = 0$

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Double Angle Formulas in I.T.F

$$\sin^{-1}\frac{2x}{1+x^2} = \begin{cases} 2\tan^{-1}x & -1 \le x \le 1 \\ \pi - 2\tan^{-1}x & \text{if } x \ge 1 \\ -\pi - 2\tan^{-1}x & x \le -1 \end{cases}$$

$$\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$$

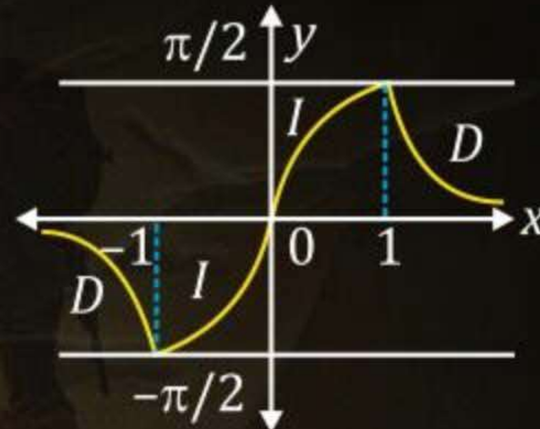
$$2\theta = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan\theta}\right)$$

$$\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

Double Angle Formulas in I.T.F

$$\sin^{-1}\frac{2x}{1+x^2} = \begin{cases} 2\tan^{-1}x & -1 \le x \le 1 \\ \pi - 2\tan^{-1}x & \text{if } x \ge 1 \\ -\pi - 2\tan^{-1}x & x \le -1 \end{cases}$$



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QUESTION [2023 (13 Apr Shift 2)]

[Ans. 2]

For $x \in (-1, 1]$, the number of solutions of the equation $\sin^{-1} x = 2\tan^{-1} x$ is equal to

$\sin^{-1} x = \sin^{-1} \frac{2x}{1+x^2}$

$x = \frac{2x}{1+x^2}$

$\cancel{x}(1+x^2) = 2\cancel{x}$

$\boxed{x=0}$ ✓ $\quad 1+x^2 = 2$

$x^2 = 1$

$x = \pm 1$

✓ $\boxed{x=1}$ $\quad x = -1$ is Rejected

$2\tan^{-1}(1/2) = \sin^{-1} \frac{\cancel{2} \times 1/\cancel{2}}{1+(1/2)^2}$

$\sin^{-1} \frac{1}{1+1/4}$

$= \sin^{-1}(4/5)$

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QUESTION [JEE Main 2022 (26 July Shift 1)]

[Ans. B]

$\tan\left(2\tan^{-1}\frac{1}{5}+\sec^{-1}\frac{\sqrt{5}}{2}+2\tan^{-1}\frac{1}{8}\right)$ is equal to:

(A) 1

(B) 2 ✓

(C) $\frac{1}{4}$

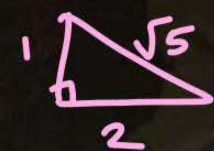
(D) $\frac{5}{4}$

$\tan\left[2\left(\underbrace{\tan^{-1}\tfrac{1}{5}+\tan^{-1}\tfrac{1}{8}}\right)+\sec^{-1}\left(\tfrac{\sqrt{5}}{2}\right)\right]$

$\tan\left[2\tan^{-1}\left(\frac{\frac{1}{5}+\frac{1}{8}}{1-\frac{1}{5}\cdot\frac{1}{8}}\right)+\tan^{-1}\frac{1}{2}\right]$

$\tan\left[2\tan^{-1}\left(\frac{13}{39}\right)+\tan^{-1}\frac{1}{2}\right]$

$\tan\left[2\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{2}\right]$



$\tan^{-1}\left(\frac{2\cdot\frac{1}{3}}{1-\frac{1}{9}}\right)+\tan^{-1}\frac{1}{2}$

$\tan^{-1}\left(\frac{2/3}{8/9}\right)+\tan^{-1}\frac{1}{2}$

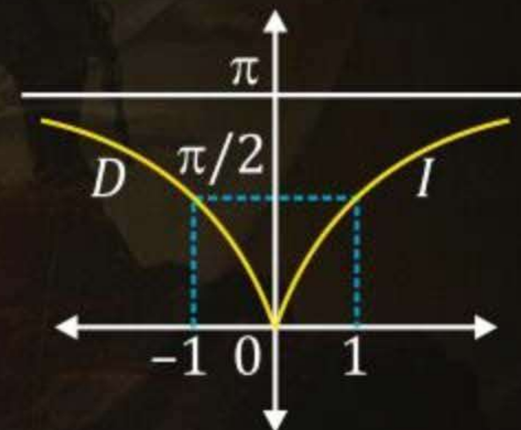
$\tan\left(\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{1}{2}\right)$

$\tan^{-1}\left(\frac{\frac{3}{4}+\frac{1}{2}}{1-\frac{3}{4}\cdot\frac{1}{2}}\right)=\frac{5/4}{5/8}$

$\tan\tan^{-1}2$

Double Angle Formulas in I.T.F

$$\cos^{-1}\frac{1-x^2}{1+x^2} = \begin{cases} 2\tan^{-1}x & x \geq 0 \\ -2\tan^{-1}x & x < 0 \end{cases}$$



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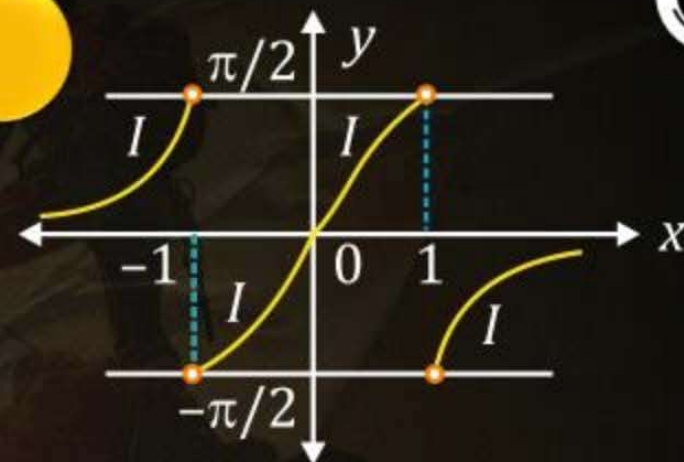
Double Angle Formulas in I.T.F

$$\tan^{-1}\frac{2x}{1-x^2} = \begin{cases} \pi + 2\tan^{-1}x & x < -1 \\ 2\tan^{-1}x & -1 < x < 1 \\ 2\tan^{-1}x - \pi & x > 1 \end{cases}$$



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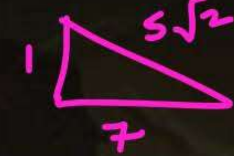
Double Angle Formulas in I.T.F



QUESTION

Prove that: $2\tan^{-1}\frac{1}{5}+\sec^{-1}\frac{5\sqrt{2}}{7}+2\tan^{-1}\frac{1}{8}=\frac{\pi}{4}$

$$2\left(\underbrace{\tan^{-1}\tfrac{1}{5}+\tan^{-1}\tfrac{1}{8}}\right)+\tan^{-1}\tfrac{1}{7}$$

Double Angle Formulas in I.T.F

QUESTION

Prove that: $\underbrace{4\tan^{-1}\frac{1}{5}} - \tan^{-1}\frac{1}{70} + \tan^{-1}\frac{1}{99} = \frac{\pi}{4}$

$2 \cdot \underbrace{2\tan^{-1}} 1/5$

$2\tan^{-1}\left[\frac{2 \cdot 1/5}{1 - 1/25}\right]$

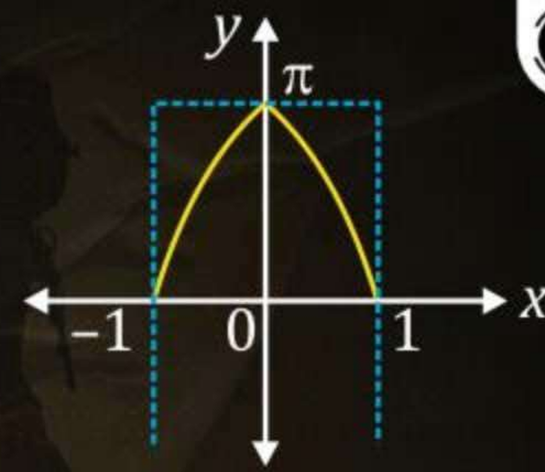
$2\tan^{-1}\left(\frac{2/5}{24/25}\right)$

$2\tan^{-1}\left(5/12\right)$

$\tan^{-1}\left(\frac{2 \cdot 5/12}{1-(5/12)^2}\right)$

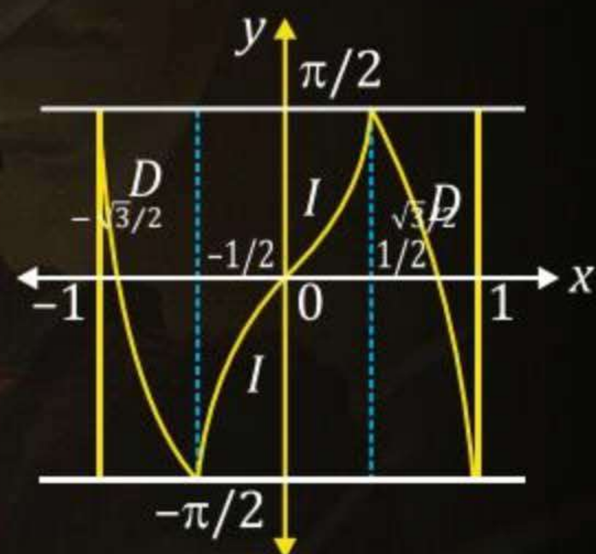
Double Angle Formulas in I.T.F

$$\cos^{-1}(2x^2-1)=\begin{cases}2\cos^{-1}x & \text{if } x\in[0,1]\\ 2\pi-2\cos^{-1}x & \text{if } x\in[-1,0]\end{cases}$$



Triple Angle Formulas in I.T.F

$$\sin^{-1}(3x-4x^3)=\begin{cases}-(\pi+3\sin^{-1}x) & \text{if } -1\le x\le -\frac{1}{2}\\ 3\sin^{-1}x & \text{if } -\frac{1}{2}\le x\le \frac{1}{2}\\ \pi-3\sin^{-1}x & \text{if } \frac{1}{2}\le x\le 1\end{cases}$$



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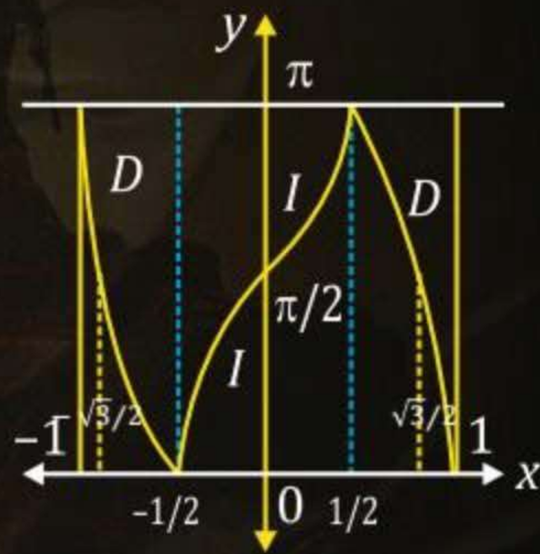
Triple Angle Formulas in I.T.F

$$\cos^{-1}(4x^3-3x)=\begin{cases}3\cos^{-1}x-2\pi & \text{if } -1\le x\le -\frac{1}{2}\\ 2\pi-3\cos^{-1}x & \text{if } -\frac{1}{2}\le x\le \frac{1}{2}\\ 3\cos^{-1}x & \text{if } \frac{1}{2}\le x\le 1\end{cases}$$



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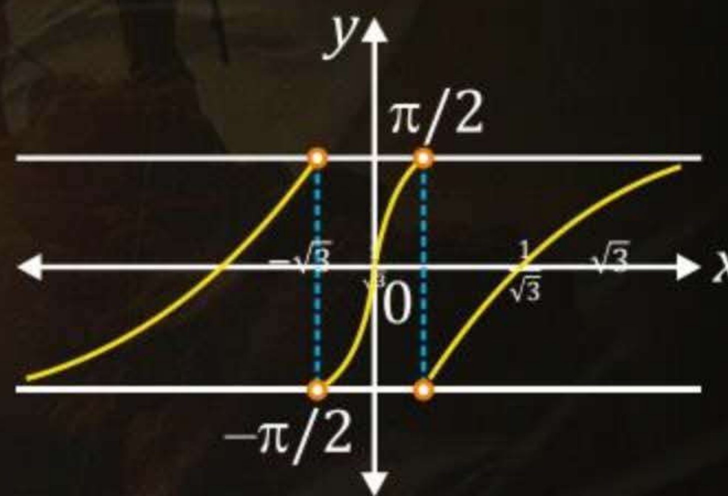
Triple Angle Formulas in I.T.F

$$\tan^{-1}\frac{3x-x^3}{1-3x^2}=\begin{cases} 3\tan^{-1}x & \text{if } -\frac{1}{\sqrt{3}}<x<\frac{1}{\sqrt{3}} \\ -\pi+3\tan^{-1}x & \text{if } x>\frac{1}{\sqrt{3}} \\ \pi+3\tan^{-1}x & \text{if } x<-\frac{1}{\sqrt{3}} \end{cases}$$



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QUESTION [JEE Main 2023 (25 Jan. -I)]

$\tan 15^\circ = 2-\sqrt{3}$

[Ans. 2]

If the sum of all the solutions of $\tan^{-1}\left(\frac{2x}{1-x^2}\right) + \boxed{\cot^{-1}\left(\frac{1-x^2}{2x}\right)} = \frac{\pi}{3}$, $\underbrace{-1 < x < 1}$, $x \neq 0$, is $\alpha - \frac{4}{\sqrt{3}}$, then α is equal to ?

Case→1 $\quad x \in (0,1)$

$\tan^{-1}\left(\frac{2x}{1-x^2}\right) + \tan^{-1}\left(\frac{2x}{1-x^2}\right) = \pi/3$

$2\tan^{-1}\underbrace{\left(\frac{2x}{1-x^2}\right)} = \pi/3$

$2\cdot 2\tan^{-1}x = \pi/3$

$\tan^{-1}x = \pi/12$

$x = \tan \pi/12$

$\boxed{x = 2-\sqrt{3}}$

Case→2 $\quad x \in (-1,0)$

$\tan^{-1}\left(\frac{2x}{1-x^2}\right) + \pi + \tan^{-1}\frac{2x}{1-x^2} = \pi/3$

$2\tan^{-1}\frac{2x}{1-x^2} = \frac{\pi}{3} - \pi$

$4\tan^{-1}x = -2\pi/3$

$\tan^{-1}x = -\pi/6$

$\boxed{x = -\frac{1}{\sqrt{3}}}$ ✓

Sum

$2-\sqrt{3}-\frac{1}{\sqrt{3}}$

$\frac{2\sqrt{3}-3-1}{\sqrt{3}} = \frac{2\sqrt{3}-4}{\sqrt{3}}$

$2-4/\sqrt{3}$

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QUESTION [JEE Adv. 2023]

[Ans. C]

For any $y \in \mathbb{R}$, let $\cot^{-1}(y) \in (0, \pi)$ and $\tan^{-1}(y) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.
Then the sum of all the solutions of the equation
$\tan^{-1}\left(\frac{6y}{9-y^2}\right) + \underbrace{\cot^{-1}\left(\frac{9-y^2}{6y}\right)} = \frac{2\pi}{3}$ for $0 < |y| < 3$, is equal to

(A) $2\sqrt{3} - 3$

(B) $3 - 2\sqrt{3}$

(C) $4\sqrt{3} - 6$ ✓

(D) $6 - 4\sqrt{3}$

$\tan^{-1}\left(\frac{6y/9}{1-y^2/9}\right)$

$\tan^{-1}\left(\frac{2y/3}{1-(y/3)^2}\right)$

$\boxed{y/3 = t}$

$\tan^{-1}\frac{2t}{1-t^2}$

$0 < \frac{|y|}{3} < 1 \quad 0 < |t| < 1$

$\tan^{-1}\frac{2t}{1-t^2} + \cot^{-1}\left(\frac{1-t^2}{2t}\right) = \frac{2\pi}{3}$

Case → 1 $\boxed{0 < t < 1}$ ←

$2\tan^{-1}\frac{2t}{1-t^2} = 2\pi/3$

$4\tan^{-1} t = 2\pi/3$

$\tan^{-1} t = \pi/6 \Rightarrow t = \frac{1}{\sqrt{3}} = y/3 \Rightarrow \boxed{y = \sqrt{3}}$ ✓

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Case → 2 $\quad -1 < t < 0$

$$\tan^{-1}\frac{2t}{1-t^2} + \pi + \tan^{-1}\frac{2t}{1-t^2} = 2\pi/3$$

$$2\tan^{-1}\frac{2t}{1-t^2} = \frac{2\pi}{3} - \pi$$

$$2\times 2\tan^{-1} t = -\pi/3$$

$$\tan^{-1} t = -\pi/12$$

$$t = \tan(-\pi/12)$$

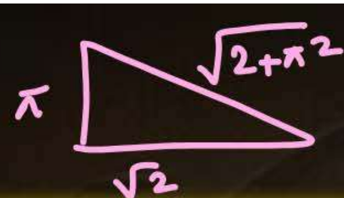
$$= -(2-\sqrt{3})$$

$$t = \sqrt{3}-2$$

$$y/3 = \sqrt{3}-2 \Rightarrow \boxed{y = 3\sqrt{3}-6}$$

Sum $= \boxed{4\sqrt{3}-6}$

QUESTION [JEE Adv. 2022]



[Ans. 2.35, 2.36]

Considering only the principal values of the inverse trigonometric functions, the value of $\frac{3}{2}\cos^{-1}\sqrt{\frac{2}{2+\pi^2}} + \underbrace{\frac{1}{4}\sin^{-1}\frac{2\sqrt{2}\pi}{2+\pi^2}} + \tan^{-1}\frac{\sqrt{2}}{\pi}$ is______.

$\frac{3}{2}\tan^{-1}\frac{\pi}{\sqrt{2}} + \cot^{-1}\frac{\pi}{\sqrt{2}} + \frac{1}{4}\sin^{-1}\frac{2\sqrt{2}\pi}{2+\pi^2}$

$\frac{1}{4}\sin^{-1}\left(\frac{2\sqrt{2}\pi/2}{1+\pi^2/2}\right)$

$\frac{1}{4}\sin^{-1}\left(\frac{2\,\pi/\sqrt{2}}{1+(\pi/\sqrt{2})^2}\right)$

$\sin^{-1}\frac{2t}{1+t^2} = \pi - 2\tan^{-1}t$, $t>1$

$t = \pi/\sqrt{2} > 1$

$\frac{3}{2}\tan^{-1}\frac{\pi}{\sqrt{2}} + \cot^{-1}\pi/\sqrt{2} + \frac{1}{4}\left(\pi - 2\tan^{-1}\pi/\sqrt{2}\right)$

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$$\underbrace{\frac{3}{2}\tan^{-1}\frac{\pi}{\sqrt{2}} + \cot^{-1}\frac{\pi}{\sqrt{2}} + \pi/4 - \frac{1}{2}\tan^{-1}}\frac{\pi}{\sqrt{2}}$$

$$\underbrace{\tan^{-1}\frac{\pi}{\sqrt{2}} + \cot^{-1}}\frac{\pi}{\sqrt{2}} + \pi/4$$

$$\pi/2 + \pi/4$$

$$3\pi/4$$

$$\frac{3 \times 22}{7 \times 4} \approx$$

Homework

"PYQs"

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