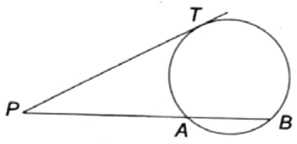
CIRCLE

1. THINGS TO REMEMBER

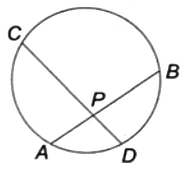
1. Equal chords of a circle are always be at a equal distance from centre and vice-versa.
2. Equal chord subtends equal angle at the centre and vice-versa.
3. Angles inscribed by a chord at the same segment are always equal.
4. Central angle is always be the twice of the inscribed angle.
5. Perpendicular dropped from centre to the chord always bisect the chord.
6. Let length of perpendicular dropped from centre to chord is p and radius is r then length of chord is $2\sqrt{r^2-p^2}$.

7. If PAB is a secant and PT is tangent, then



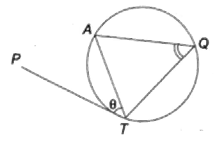
$PA \times PB = PT^2$

8. If two chords AB and CD intersect each other at point P, the



$PA \times PB = PC \times PD$

9. If PT is tangent and T is the point of contact, then if TA is chord such that $\angle ATP = \theta$, then



$\angle AQT = \angle ATP$

2. EQUATION OF CIRCLE IN VARIOUS FORMS

1. Centre Radius Form : Equation of circle with centre $(h,\ k)$ and radius r. $(x-h)^2+(y-k)^2=r^2$

2. Diameter form : Equation of circle with a diameter ends (x_1, y_1) and (x_2, y_2) are $(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$

3. General Form : $x^2+y^2+2gx+2fy+c=0$
Centre $\Rightarrow (-g,\ -f)$
Radius $\Rightarrow \sqrt{g^2+f^2-c}$

Above equation represent circle only if, $g^2+f^2-c>0$.
- If $g^2+f^2-c=0$, then equation represents a point sometimes called a degenerate circle.
- If $g^2+f^2-c<0$, then equation represents no locus. It is an imaginary equation.

3. EQUATION OF CIRCLE IN PARTICULAR CASES

1. Equation of circle with centre (h,k) & passing through origin.
$\Rightarrow (x-h)^2+(y-k)^2=h^2+k^2$
$\Rightarrow x^2+y^2-2hx-2ky=0$

2. Equation of circle with centre $(h,\ k)$ that torches x-axis
$(x-h)^2+(y-k)^2=k^2$

3. Equation circle with centre (h,k) that touches y-axis.
$(x-h)^2+(y-k)^2=h^2$

4. Equation of circle with radius r and which touches both the axes,
$(x\pm r)^2+(y\pm r)^2=r^2$.

4. PARAMETRIC EQUATION

1. If equation of circle is $x^2+y^2+2gx+2fy+c=0$, then its parametric form is

$$x+g=r\cos\theta;\ y+f=r\sin\theta;\ r=\sqrt{g^2+f^2-c};$$

$\theta=$ parameter

2. If equation of circle is $x^2+y^2=r^2$, then its parametric form is
$x=r\cos\theta,\ y=r\sin\theta,\ \theta=\text{parameter}.$

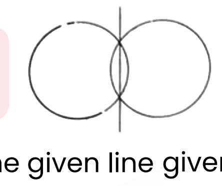
5. FAMILY OF CIRCLE

1. Family of circle passing through the point of intersections of circle $S=0$ and line $L=0$ is
$S+\lambda L=O$.

2. Equation of circle passing through two given points $(x_1, y_1)\ (x_2, y_2)$
$S+\lambda L=O$
Where $S=(x-x_1)(x-x_2)+(y-y_2)(y-y_2)$ and $L=0$ is line passing through these points.

3. Equation of circle passing through the intersection of two circles $S=0$ and $S'=0$ is

Note
$S-S'=0$ represents equation of common chord if circles intersects.



$S+\lambda(S-S')=0$

4. Equation of the circle touching the given line given point (x_1y_1) on it,
$(x-x_1)^2+(y-y_1)^2+\lambda(ax+by+c)=0$ $ax+by+c=0$ at the

5. Equation of circumcircle of triangle whose sides are defined by the lines $L_1=0,\ L_2=0$ and $L_3=0$

$$L_1L_2+\lambda L_2L_3+\mu L_3L_1=0$$

6. Equation of circle circumscribing the cyclic quadrilateral whose sides in order are given by lines $(L_1=0,\ L_2=0,\ L_3=0,\ L_4=0)$ $L_1L_3+\lambda L_2L_4=0$

6. INTERCEPT MADE BY CIRCLE ON COORDINATE AXES

Let the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ intersects x-axis at A and B and y-axis of C and D respectively, then :

1. Length of x-intercept, AB $= 2\sqrt{g^2 - c}$

2. Length of y-intercept, CD $= 2\sqrt{f^2 - c}$

Note
- Circle intersect x-axis at two distinct point only if $g^2 > c$. If $g^2 = c$, then circle touches x-axis. If $g^2 < c$, then circle neither intersect nor touches.
- If $f^2 > c$, then the circle intersect y-axis at two distinct points. If $f^2 = c$, then circle touches y-axis. If $f^2 < c$, neither touches nor intersect y-axis.
- Touches both axes simultaneously iff $g^2 = f^2 = c$

7. POSITION OF POINT WITH RESPECT TO CIRCLE

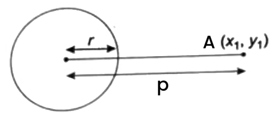
Let circle is given by $S(x, y) = x^2 + y^2 + 2gx + 2fy + c = 0$ and point is (x_1, y_1).

1. If $S(x_1, y_1) = 0$ point, lies on the circle.
2. $S_1 > 0$, point lies outside the circle
3. $S_1 < 0$, point lies inside the circle

8. DISTANCE BETWEEN A POINT AND CIRCLE

Let $S(x, y) = 0$ be the circle with radius r and $A(x_1, y_1)$ be the given point, then the distance between point A and circle S is equal to $|r - p|$

Where P is the distance between centre of circle and point P.



9. TANGENT TO A CIRCLE

- Equation of tangent at point (x_1, y_1) on $x^2 + y^2 = a^2$ is $xx_1 + yy_1 = a^2$
- Condition for line $y = mx + c$ to be tangent to circle

$$x^2 + y^2 = a^2,$$
$$c^2 = a^2(l + m^2)$$

And point at which it touches is $\left(-\frac{a^2 m}{c}, \frac{a^2}{c}\right)$

Note
- If line $lx + my + c = 0$, touches the fixed circle with center (h, k) and radius r, then to find center and radius we can compare the given relation with

$$\left|\frac{ly + mk + c}{\sqrt{l^2 + m^2}}\right| = r$$

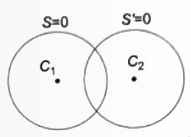
10. TWO CIRCLES CASES

Let $S = 0$ and $S' = 0$ be the circle with center C_1 and C_2 and radius r_1 and r_2 respectively. Let $C_1C_2 = d$.

INTERSECTING CIRCLE

$|r_1 - r_2| < d < r_1 + r_2$

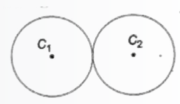
Number of common tangents = 2



EXTERNALLY TOUCHING CIRCLE

$d = r_1 + r_2$

Number of common tangents = 3



ONE CIRCLE TOUCHES INTERNALLY THE OTHER

$d = |r_1 - r_2|$

Number of common tangents = 1

DISJOINT CIRCLES

Have no point in common in their interior region and on the circles

$d > r_1 + r_2$

Number of common tangents = 4

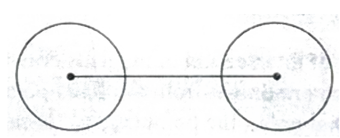
ONE CIRCLE CONTAINED IN OTHER

$d > |r_1 - r_2|$

Number of common tangents = 0

NOTE

- If the centre of both the circles lies on the same side of their common tangent, then such a tangent is said to the direct common tangent.



- Length of direct common tangent

$l = \sqrt{d^2 - (r_1 - r_2)^2}$

- Length of indirect common tangent

$l = \sqrt{d^2 - (r_1 + r_2)^2}$

Locus of centre of circle which is touched by two noncongruent circles externally is always a hyperbola.

11.

	CONCEPT	EQUATION
1.	Equation of pair of tangents drawn from external point (x_1, y_1) to the curve $S = 0$.	$SS_1 = T^2$
2.	Chord of contact : From point (x_1, y_1) two tangents are drawn to curve $S = 0$, touching it at point A and B, then chord joining AB is called chord of contact of (x_1, y_1) with respect to curve $S = 0$.	$T = 0$
3.	Equation of tangent at point (x_1, y_1) on the curve $S = 0$.	$T = 0$
4.	If (x_1, y_1) be the internal point, then equation of chord of curve $S = 0$ with mid-point (x_1, y_1).	$T = S_1$

12. POLE AND POLAR

Locus of point of intersection of tangents drawn at the extremities of chords which are drawn from the fixed point (x_1, y_1) is called polar and for that polar, the point (x_1, y_1) is called pole.

Equation of polar at point (x_1, y_1) w.r.t curve $S = 0$ is given by,

$$T = 0$$

1. If point $(x_1 y_1)$ i.e., pole is outside the curve, then chord of contact of it and polar to it has same equation.
2. If pole is lying on the curve, then tangent through it always coincide with polar of the pole.
3. If pole is inside the curve, then equation $T = 0$, represents only the polar.

PARTICULARLY FOR A CIRCLE

$S(x, y) = x^2 + y^2 + 2gx + 2fy + c = 0$ and $P(x_1 y_1)$

$$S_1 = x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$$

$T = xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c$, then

1. Length of tangent drawn from external point $P(x_1 y_1)$ is $L = \sqrt{S_1}$

2. Square of length of tangent i.e., S_1 is called power of external point $P(x_1, y_1)$ with respect to circle.

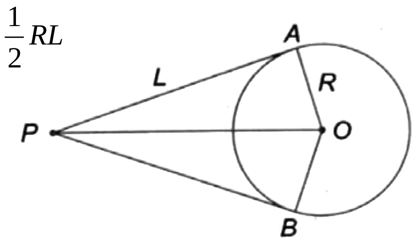
13. THINGS TO REMEMBER

If from $P(x_1, y_1)$ two tangents are drawn to circle with radius R and length of tangent from P to the circle be L

1. Length of chord of contact of point $P(x_1, y_1)$

$$AB = \frac{2RL}{\sqrt{R^2 + L^2}}$$

2. Area of $\Delta OAP = ar(\Delta OBP) = \frac{1}{2} RL$
3. Area of $\Delta OAB = \frac{R^3 L}{R^2 + L^2}$
4. Area of $\Delta PAB = \frac{RL^3}{R^2 + L^2}$



5. Area of quadrilateral PAOB = RL
6. If angle between two pair of tangent is given by θ, then,

$$\tan\left(\frac{\theta}{2}\right) = \frac{R}{L}$$

$$\Rightarrow \frac{\theta}{2} = \tan^{-1}\left(\frac{R}{L}\right)$$

$$\Rightarrow \theta = 2\tan^{-1}\left(\frac{R}{L}\right)$$

7. In above picture P, A, O and B are always be concylic points and circle which passes through them has OP always as its diameter.

14. DIRECTOR CIRCLE

- Locus of point of intersection of perpendicular tangents is called director circle.
- Equation of director circle of the circle $x^2 + y^2 = a^2$ is

$$x^2 + y^2 = 2a^2$$

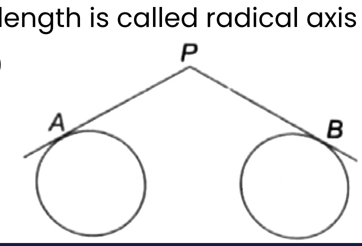
CIRCLE

- Director circle is always concentric with original circle.
- Ratio of radius of director circle o the radius of circle is $\sqrt{2} : 1$

RADICAL AXIS OF TWO CIRCLE

The locus of a point from where tangents drawn to the given circle are equal in length is called radical axis of that pair of circles.

$S - S' = 0$



NOTE

- If circles are intersecting circles, radical axis of the circle has same equation as equation of common chord.
- If circles are touching circles radical axis coincides with their common tangents at the common point of contact.
- Radical axis is always perpendicular to line joining the centre of the circles.

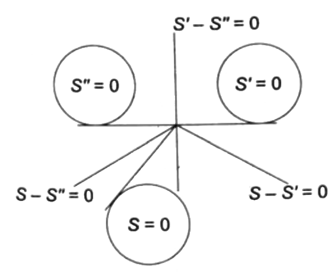
15. RADICAL CENTRE

The point in the plane from where tangents drawn to three given circle are equal in length is called radical centre of those three circles.

$S = 0,\ S' = 0$ and $S'' = 0$ be the circles in the standard form.

Radical centre lies on the line $S - S' = 0,\ S' - S'' = 0$ and $S - S'' = 0$,

i.e., radical centre always satisfy $S = S' = S''$



NOTE

If the point which satisfied of those three circles. $S = S' = S'$ is lie inside of any one of the circles, radical centres of the circles does not exist.

16. ANGLE OF INTERSECTION OF TWO CIRCLE

The angle between the tangents to the circle at the point of intersection of the curve is called angle of their intersection.

$$\cos\theta = \frac{r_1^2 + r_2^2 - d^2}{2r_1 r_2}$$

17. ORTHOGONAL CIRCLES

If angle of intersection of circles is $90°$, then circles are said to be orthogonal circle : Condition for orthogonality of two circles is

$$r_1^2 + r_2^2 = d^2$$

• If circles are given by $x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ and $x^2 + y^2 + 2gx_2 + 2f_2y + c_2 = 0$, then above condition imply.

$$g_1g_2 + f_1 + f_2 = \frac{c_1 + c_2}{2}$$

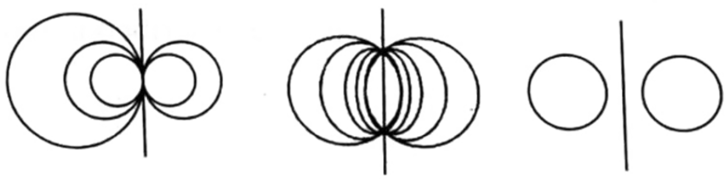
Note

1. Taking centre on radical axis of two circles and radius equal to length of tangent from that point on either of circle. If circle being drawn, then angle of intersection of that circle with each of the given two circles is $90°$.

2. Taking radical centre as a centre of circle and radius equal to the length of tangent from it to any of the three given circles if a circle being drawn, then that circle intersects all the three circles orthogonally.

18. COAXIAL FAMILY OF CIRCLES

The family of circles in which each pair of circle has same radical axis is called co-axial family of circles.



If two of the members of co-axial family are given by $S = 0$ and $S' = 0$, then co-axial family is described by

$$S + \lambda(S - S') = 0$$