CHAPTER

3 Current Electricity

Electric Current

I_{av}(average current) $= \dfrac{\Delta q}{\Delta t}$

I (instantaneous current) $= \lim\limits_{\Delta t \to 0} \dfrac{\Delta q}{\Delta t} = \dfrac{dq}{dt}$

SI unit : Ampere

Electric Current in a Conductor

$I = nqAv_d$

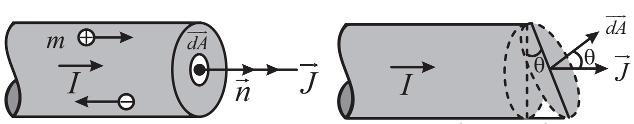
where I = current, n = number of charge carriers per unit volume, A = area of cross section, v_d = drift velocity.

$\vec{v}_d = -\dfrac{e\vec{E}}{m}\tau$

where e = charge of electron, m = mass of electron, $\vec{E}$ = electric field, τ = relaxation time.

Current Density(J) and Mobility (μ)

$\vec{J} = \dfrac{ne^2}{m}\tau\vec{E}$



$dA \cos\theta$

$I = \int \vec{J}.\vec{dA}$

$\mu = \dfrac{|v_d|}{E} = \dfrac{e\tau}{m}$

(**SI unit:** m^2/Vs)

Electrical Resistance (R) and Ohm's Law

$I = neAv_d = neA\left(\dfrac{eE}{m}\right)\tau$

$E = \dfrac{V}{l}$

So, $I = \left(\dfrac{ne^2\tau}{m}\right)\left(\dfrac{A}{l}\right)\times V = \left(\dfrac{A}{\rho l}\right)\times V = \dfrac{V}{R}$

$\Rightarrow V = IR$

ρ is called resistivity (it is also called specific resistance) and

$\rho = \dfrac{m}{ne^2\tau} = \dfrac{1}{\sigma}$, σ is called conductivity

SI Units: $R \to$ ohm (Ω), $\rho \to$ ohm–meter (Ω-m), $\sigma \to \Omega^{-1}m^{-1}$.

Dependence of Resistance on Temperature:

$R = R_0\,(1 + \alpha\ (T - T_0))$.

α = thermal coefficient of resistivity (**positive for conductors and negative for semi conductors and insulators**)

Electrical Power

$P = VI$

Energy $= \int P dt$

$P = I^2 R = VI = \dfrac{V^2}{R}$, Heat: $H = VIt = I^2\ Rt = \dfrac{V^2}{R}t$

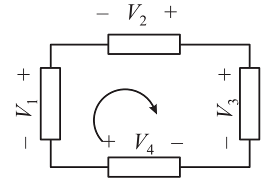
Kirchhoff's Laws

I. Law (Junction law or Nodal Analysis): This law is based on law of conservation of charge. It states that "The algebraic sum of the currents meeting at a junction is zero" or 'total currents entering a junction equals total current leaving the junction'.

$\Sigma I_{in} = \Sigma I_{out}$. It is also known as **KCL (Kirchhoff's current law)**.

II. Law (Loop analysis): The algebraic sum of all the voltages in closed circuit is zero.

$\Sigma\ IR + \Sigma\ EMF = 0$ in a closed loop . The closed loop can be traversed in any direction. While traversing a loop if higher potential point is entered, put a +ve sign in expression or if lower potential point is entered put a negative sign.



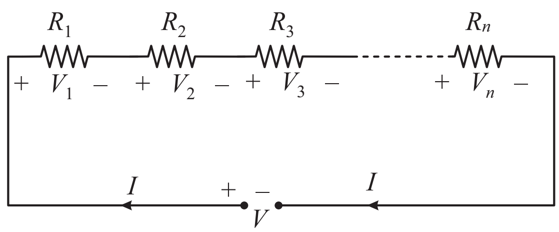
$-V_1 - V_2 + V_3 - V_4 = 0$. Boxes may contain resistor or battery or any other element (linear or non-linear).

It is also known as **KVL (Kirchhoff's voltage law).**

Combination of Resistances

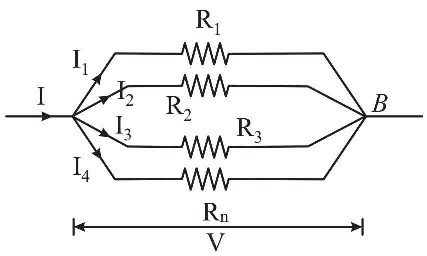
I. Resistances in Series:

$R = R_1 + R_2 + R_3 + ... + R_n$

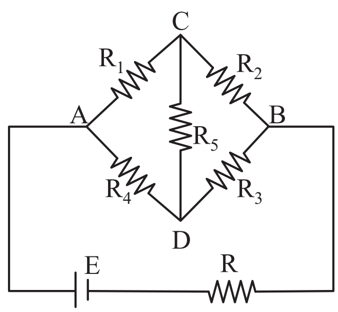


II. Resistances in Parallel:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ... + \frac{1}{R_n}$$



Wheatstone Network

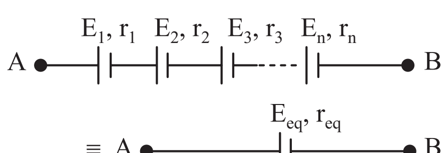


Current through the R_5 is zero (null point or balance point) if

$$\frac{R_1}{R_4} = \frac{R_2}{R_3}$$

Grouping of Cells

I. Cells in Series:



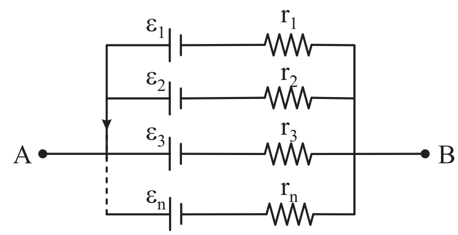
Equivalent EMF $E_{eq} = E_1 + E_2 + ... + E_n$ [write EMF's with polarity]

Equivalent internal resistance $r_{eq} = r_1 + r_2 + r_3 + r_4 + ... + r_n$

II. Cells in Parallel:

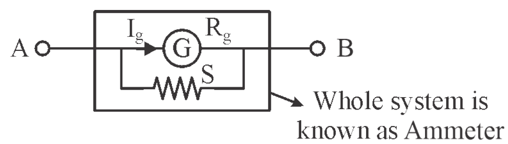
$$E_{eq} = \frac{\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} + ... + \frac{\varepsilon_n}{r_n}}{\frac{1}{r_1} + \frac{1}{r_2} + ... + \frac{1}{r_n}}$$ [Use emf with polarity]

$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + ... + \frac{1}{r_n}$$

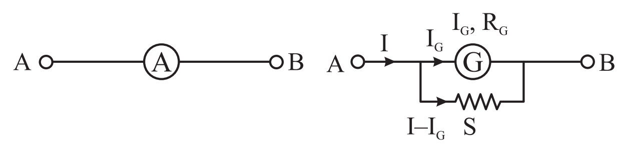


Ammeter

A shunt (small resistance) is connected in parallel with galvanometer to convert it into ammeter. An ideal ammeter has zero resistance



Ammeter is represented as follows :



If maximum value of current to be measured by ammeter is I then

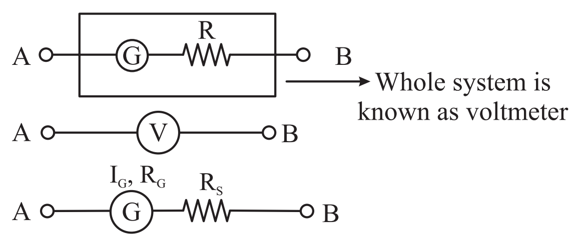
$I_G \cdot R_G = (I - I_G)\, S$

$$S = \frac{I_G \cdot R_G}{I - I_G} \Rightarrow S = \frac{I_G \times R_G}{I} \text{ (if } I >> I_G\text{).}$$

where, I = Maximum current that can be measured using the given ammeter.

Voltmeter

A high resistance is put in series with galvanometer. It is used to measure potential difference across a resistor in a circuit.



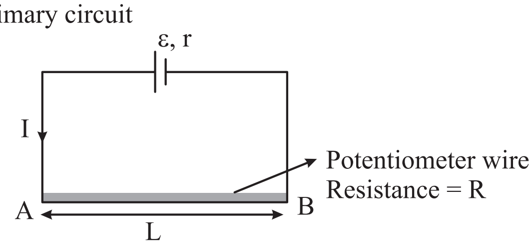
For maximum potential difference

$V = I_G \cdot R_S + I_G R_G$

If $R_G << R_S \Rightarrow R_S \approx \dfrac{V}{I_G}$

Potentiometer

Primary circuit



$$I = \frac{\varepsilon}{r+R}; \quad V_A - V_B = \frac{\varepsilon}{R+r} \cdot R$$

Potential gradient (x) $\rightarrow$ Potential difference per unit length of wire

$$x = \frac{V_A - V_B}{L} = \frac{\varepsilon}{R+r} \cdot \frac{R}{L}$$

Applications of potentiometer

(a) To find emf of unknown cell and compare emf of two cells.

In case I,

In figure, (1) is joint to (2) then balance length = l_1

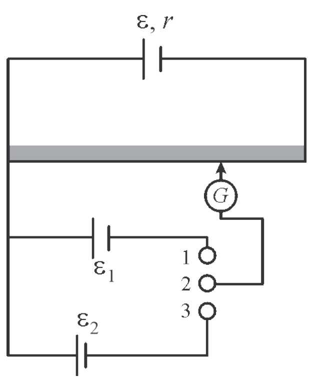
$$\varepsilon_1 = xl_1 \qquad \text{...(i)}$$

In case II,

In figure, (3) is joint to (2) then balance length = l_2

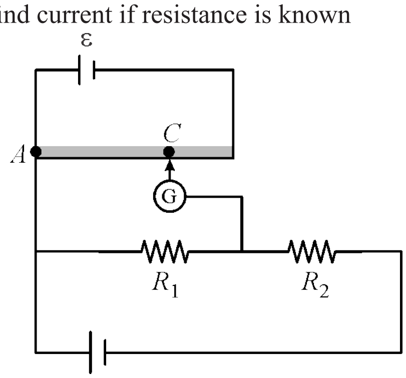
$$\varepsilon_2 = xl_2 \qquad \text{...(ii)}$$

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{l_1}{l_2}$$



If any one of ε_1 or ε_2 is known the other can be found. If x is known then both ε_1 and ε_2 can be found.

(b) To find current if resistance is known



$$V_A - V_C = xl_1$$

$$IR_1 = xl_1$$

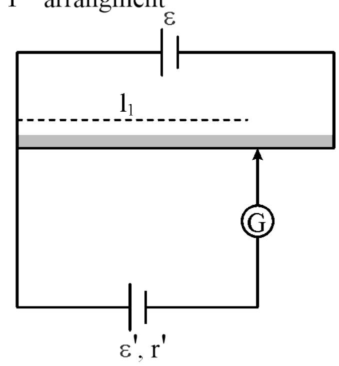
$$I = \frac{xl_1}{R_1}$$

Similarly, we can find the value of R_2 also.

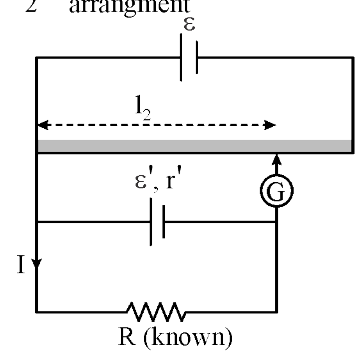
Potentiometer is ideal voltmeter because it does not draw any current from circuit at the balance point.

(c) To find the internal resistance of cell.

1st arrangment



2nd arrangment



By first arrangement $\quad \varepsilon' = xl_1 \quad$ ...(i)

By second arrangement $\quad IR = xl_2 \quad$ ...(ii)

$$I = \frac{xl_2}{R}, \text{ Also } I = \frac{\varepsilon'}{r'+R}$$

$$\therefore \quad \frac{\varepsilon'}{r'+R} = \frac{xl_2}{R} \Rightarrow \frac{xl_1}{r'+R} = \frac{xl_2}{R}$$

$$r' = \left[\frac{l_1 - l_2}{l_2}\right] R$$

Metre Bridge

It is used to measure unknown resistance.

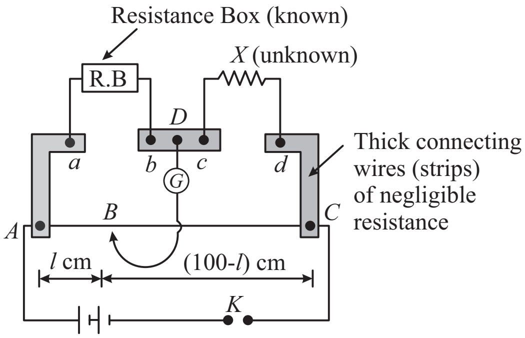
If $AB = l$ cm, then $BC = (100 - l)$ cm.

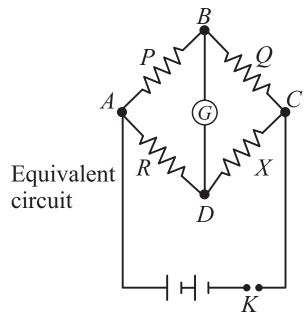
Resistance of the wire between A and B , $R \propto l$

[$\because$ Specific resistance ρ and cross-sectional area A are same for whole of the wire]

or $\quad R = \sigma l \qquad$ ...(i)

where σ is resistance per cm of wire.





If P is the resistance of wire between A and B, then

$$P \alpha l \quad \Rightarrow \quad P = \sigma (l)$$

Similarly, if Q is resistance of the wire between B and C, then

$$Q \propto 100 - l \quad \Rightarrow \quad Q = \sigma (100 - l) \qquad \text{...(ii)}$$

Dividing (i) by (ii),

$$\frac{P}{Q} = \frac{l}{100 - l}$$

Applying the condition for balanced Wheatstone bridge, we get $R\,Q = P\,X$

$$\therefore \quad X = R\,\frac{Q}{P} \qquad \text{or} \qquad X = \frac{100 - l}{l} R$$

Since R and l are known, therefore, the value of X can be calculated.