CHAPTER

7 Alternating Current

Instantaneous, RMS and Average Values

(i) Instantaneous value of alternating current
$I = I_0 \sin\omega t$ or $I = I_0 \cos\omega t$

(ii) Peak value of a.c. $= I_0$

(iii) Alternating emf. $= E = E_0 \sin\omega t$ or $E = E_0 \cos\omega t$

(iv) Mean or average value of a.c.

$$I_m \text{ or } I_a = \frac{2I_0}{\pi} = 0.637 I_0 \text{ for half cycle}$$
$$= 0 \text{ for full cycle.}$$

(v) R.m.s. value of ac $I_{rms} = I_0/\sqrt{2} = 0.707 I_0$.

Phase Difference

(i) If the emf leads the current by $\pi/2$, the reactance is called purely inductive.

(ii) If the emf lags behind the current by $\pi/2$, the reactance is called purely capacitive.
If the emf is in phase with the current, the circuit is called purely resistive.

Sign Convension

Sign for phase difference (ϕ) between I and E for series LCR circuit:

ϕ is positive, when $X_L > X_C$
ϕ is negative, when $X_L > X_C$
ϕ is zero, when $X_L = X_C$

Resonance

(i) The LCR circuit is said to be resonance when
$X_L = X_C$ i.e., when $\omega L = \dfrac{1}{\omega C}$ and $\omega = \omega_0 = \dfrac{1}{\sqrt{LC}}$ is called resonance frequency.

(ii) At series resonant frequency, $\omega_0 = \dfrac{1}{\sqrt{LC}}$, we have:

(*a*) $Z = R$ = minimum value of impedance.
(*b*) $I_0 = E_0/R$ = maximum value of peak current.
(*c*) $\phi = 0$ i.e., I and E are in phase with each other.
(*d*) V_L is equal and opposite to V_C.
(*e*) Potential drop across C and L together is zero.
(*f*) $E = V_R$

Energy Stored

(i) Energy stored in an inductor: $U = \dfrac{1}{2} L I_0^2$

(ii) Energy stored in a capacitor: $U = \dfrac{1}{2} CV^2 = \dfrac{1}{2}\dfrac{q_0^2}{C} = \dfrac{1}{2} q_0 V$

Power in AC Circuit

(i) The power in LCR circuit is given by
$P = EI = E_0 I_0 \sin\omega t \sin(\omega t - \phi)$.
Power in LCR circuit consists of two components

(*a*) Virtual power component $= \dfrac{1}{2} E_0 I_0 \cos(2\omega t + \phi)$.
It has frequency twice as that of A.C. Its value over the complete cycle is zero

(*b*) Real power component $= \dfrac{1}{2} E_0 I_0 \cos\phi$. It dissipates power and $\cos\phi$ is called power factor.

(ii) Inductive reactance: $X_L = \omega L$

Capacitive reactance: $X_C = \dfrac{1}{\omega C}$

Reactance: $X = X_L - X_C = \omega L - \dfrac{1}{\omega C}$

Impedance

(i) Impedance of LCR circuit: $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$= \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)}$$

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{IX_L - IX_C}{IR}$$

$$= \frac{X_L - X_C}{R} = \frac{\omega L - \dfrac{1}{\omega C}}{R}$$

Power, $P = E_{rms} \times i_{rms} \times \dfrac{R}{Z}$

(ii) Band width $= \omega_2 - \omega_1 = 2\Delta\omega$

(iii) Sharpness of resonance $= \dfrac{\omega_0}{2\Delta\omega} = \dfrac{\omega_0 L}{R}$

(iv) Q factor: $Q = \dfrac{\text{Voltage across } L \text{ or } C}{\text{Applied voltage}}$

$$Q = \frac{\omega_0 L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Transformer

(i) $\dfrac{E_s}{E_p} = \dfrac{N_s}{N_P} = k \text{ (say)}$ (transformer ratio)

(ii) For step up transformer, $k > 1$ and for step down transformer, $k < 1$

(iii) For step up transformer, $N_s > N_p$, therefore, $E_s > E_p$. And for the step down $N_s < N_p$ therefore, $E_s < E_p$.

(iv) The efficiency of the transformer is given by:

$$\eta = \frac{\text{Output power}}{\text{Input power}}$$

If I_p and I_s be the currents in the primary and secondary circuit, then

$$\eta = \frac{E_s I_s}{E_p I_p}$$

For ideal transformer $\eta = 1 = 100\%$. Therefore,

$E_s I_s = E_p I_p$ or $\dfrac{I_s}{I_p} = \dfrac{N_p}{N_s} = \dfrac{1}{k}$

Hence, for step up transformer, current in the secondary is less than that in the primary ($I_s < I_p$) and in a step down transformer, we have $I_s > I_p$.