Unacademy Atoms

CROSS PRODUCT & RELATIONS

Cross Product (A x B)

Ordered Pair (x, y) ≠ (y, x)

$(x, y) \neq (y, x)$

$A = \{1, 2, 3\}$

$B = \{a, b\}$

$1 \in A$

$1 \in A \times B$

Number of elements

$n(A \times B) = n(A).n(B)$

$A \times B = \{(1, a), (1, b), (2, a), (2, b), (3, a), (3, b)\}$

$n(A \times B) = 6 = n(A) \cdot n(B)$

$n(B \times A) = n(B) \cdot n(A)$

$= 2 \times 3 = 6$

$B \times A = \{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3)\}$

$(a, 1) \in B \times A$ ✓

Q

A x B = {(1, a), (1, b), (2, a) (2, b), (3, a), (3, b)} True or False ?

A. $\boxed{1 \in A}$ True
B. 1 ∈ A x B False
C. (1, a) ∈ A x B True
D. { (1, a) } ⊂ A x B

{ (1, a) } ⊂ A x B True

How many subsets will the following sets have?

$A = \{1, 2\}$

$\{1, 2\}$

$\{1\}$

$\{2\}$

$\{\ \}$

2^n

$= 2^2$

$= 4$

$A = \{1, 2, 3\}$

$2^3 = 8$

$\{1, 2, 3\}$

$\{\ \}$ $\{1, 2\}$

$\{1\}$ $\{2, 3\}$

$\{2\}$ $\{1, 3\}$

$\{3\}$

Conclusion

Number of subsets of set A = 2^n

where n → number of element in set A

$n(A) = 10$

No. of subsets = 2^{10}

A x B = {(1, a), (1, b), (2, a) (2, b), (3, a), (3, b)}

How many subsets will A x B have ?

$n(A \times B) = 6$

No. of subsets of $A \times B = 2^6 = 64$

$\{(1, a), (1, b)\} \subset A \times B$

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Q

Let Z be the set of integers. If $A = \{x \in Z : 2^{(x+2)(x^2 - 5x + 6)} = 1\}$ and $B = \{x \in Z : -3 < 2x - 1 < 9\}$, then the number of subsets of the set A x B, is

A. 2^{15}
B. 2^{18}
C. 2^{12}
D. 2^{10}

JEE Main 2019

$$2^{(x+2)(x^2-5x+6)} = 1 = 2^0$$

$$(x+2)(x-2)(x-3) = 0$$

$$\boxed{x = -2, 2, 3}$$

$$\boxed{n(A) = 3}$$

$$-3 < 2x - 1 < 9$$

$$\frac{-2}{2} < \frac{2x}{2} < \frac{10}{2}$$

$$\boxed{-1 < x < 5}$$

$A = \{-2, 2, 3\}$

$B = \{0, 1, 2, 3, 4\}$

$n(A \times B) = 3 \times 5 = 15$

No. of subsets of A x B $\Rightarrow 2^{15}$

Relations

$\boxed{R : A \to B}$ $\boxed{aRb}$

→ A subset of **A x B**

If n(A) = m & n(B) = n then the number of Relations = 2^{mn}

$n(A) = m$

$n(B) = n$

$n(A \times B) = mn$

No. of Subsets of $A \times B = 2^{mn}$

No. of Relations $A \to B = 2^{mn}$

Visual Representation (Arrow Diagram)

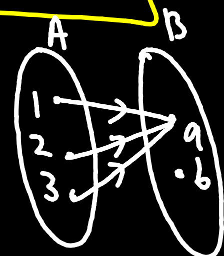
$A = \{1, 2, 3\}$

$B = \{a, b\}$

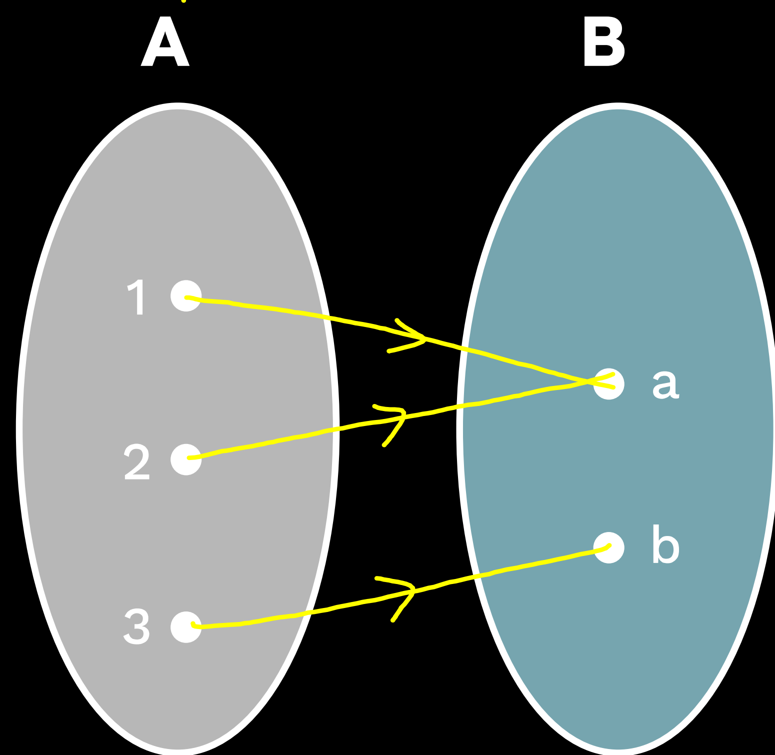
$A \times B = \{(1,a), (1,b), (2,a), (2,b), (3,a), (3,b)\}$

$\boxed{R = \{(1,a), (2,a), (3,b)\}}$

$R = \{(1,a)(2,a)(3,a)\}$



$\boxed{R: A \to B}$

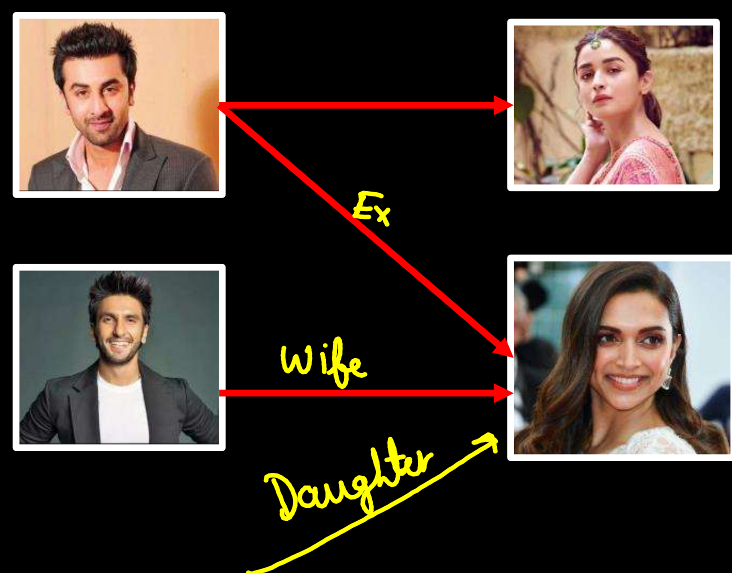


Relations in Real life

arrow

Kahi se bhi start

Kahi pe bhi Khatam



Q

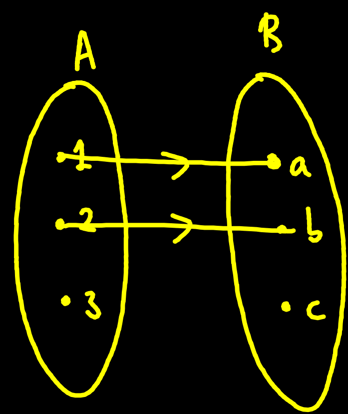
$A = \{1, 2, 3\}$

$B = \{a, b, c\}$

$R: A \rightarrow B$

$\boxed{n(A \times B) = 9}$

$R = \{(1, a), (2, b)\}$



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Identity Relations

Every element of A is related to itself only

$a, b \in N$

$R = \{(a,b) : a = b\}$

$R = \{(1,1), (2,2), (3,3), \ldots\}$

$R = \{(1,1), (1,2), (2,2)\} \longrightarrow$ No.

Representation of Relations

1. Roster form
2. Set builder form

TYPES OF RELATIONS

Types of Relations

$(a, b) \in R \quad (b, a) \in R$

$(a, b) \in R \quad (b, a) \notin R$

Reflexive

$(a, a) \in R$ for all $a \in A$

Symmetric

If $(a, b) \in R$ then $(b, a) \in R$

Transitive

If $(a, b) \in R$ and $(b, c) \in R$ then $(a, c) \in R$

Anti- Symmetric

If $(a, b) \in R$ and $(b, a) \in R \Rightarrow a = b$

Q

(2,1) (3,2) (3,1)

The relation $\boxed{R = \{\cancel{(1, 1)}, \cancel{(2, 2)}, \cancel{(3, 3)}, (1, 2), (2, 3), (1, 3)\}}$ on set A = {1, 2, 3} is -

A. Reflexive but not symmetric ✓

B. Reflexive but not transitive

C. Symmetric and transitive

D. Neither symmetric nor transitive

R on set A : $\boxed{R : A \to A}$

$n(A\times A) = n(A)\cdot n(A)$

$= 3\times 3$

$= 9$

$(3,2) \notin R$

$(1,2) \in R$

$(2,1) \notin R$

$(1,3) \in R$ $\quad (3,1) \notin R$

Yes

Ref

$(1,1) \in R$ ✓

$(2,2) \in R$ ✓

$(3,3) \in R$ ✓

$\boxed{\text{Yes}}$

Sym

$\cancel{(1,1) \in R}$

$\cancel{(2,2) \in R}$

$\cancel{(3,3) \in R}$

$(2,1) \notin R$

$(3,2) \notin R$

$(3,1) \notin R$

$\boxed{\text{No}}$

Trans.

Pairs / Jodi

(1,1) (1,2) $\quad$ (1,2)

(1,1) (1,3) $\quad$ (1,3)

(2,2) (2,3) $\quad$ (2,3)

(3,3) $\longrightarrow$ X

(1,2) (2,2) $\quad$ (1,2)

(2,3) (3,3) $\quad$ (2,3)

(1,3) (3,3) $\quad$ (1,3)

Shortcut (#NVStyle)

Sym / Trans.

$R = \{ (1,2), (2,3), (1,3) \}$

$(1,2) \in R$

$(2,1) \notin R$

Sym ✗

$(1,2)\ (2,3) = (1,3)$ ✓

$\cancel{(2,3)\ (2,?)}$ ✗

$\cancel{(1,3)\ (3,?)}$ ✗

Trans ✓

The relation R = {$\cancel{(1, 1)}$, $\cancel{(2, 2)}$, (1, 2), (2, 3)} on set A = {1, 2, 3} is -

A. Reflexive but not symmetric
B. Reflexive but not transitive
C. Symmetric and transitive
D. Neither symmetric nor transitive ✓

Ref ✗
$(3,3) \notin R$

$(2,1) \notin R$
Sym ✗

$(1,2)\,(2,3) = (1,3)$ ✗
Trans ✗

Unacademy Atoms

Q

Let R = {(~~3, 3~~), (~~6, 6~~), (~~9, 9~~), (~~12, 12~~) (6, 12) (3, 9) (3, 12), (3, 6)} be a relation on the set A = {3, 6, 9, 12}. The relation is -

A. An equivalence relation
B. Reflexive and symmetric only
C. Reflexive and transitive only ✓
D. Reflexive only

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Ref ✓

$(12, 6) \notin R$

Sym ✗

~~(6, 12)~~ → ✗

~~(3, 9)~~ → ✗

~~(3, 12)~~ → ✗

$(3, 6)\ (6, 12) = (3, 12)$ ✓

Y Y N

हाँ - ना bol

R = { (a, b): a is divisible by b}

R : a is divisible by b Ref ✓

YNY a is " " a (b Replace a)

Sym:- if a is divi by b then b is divi by a

(No)

Trans if a is divi by b and b is divi by c then a is divi by c

(Yes)

$16 \div 4$ and $4 \div 2$ $16 \div 2$ ✓

$R = \{(T_1, T_2) : T_1 \cong T_2\}$

Equivalence

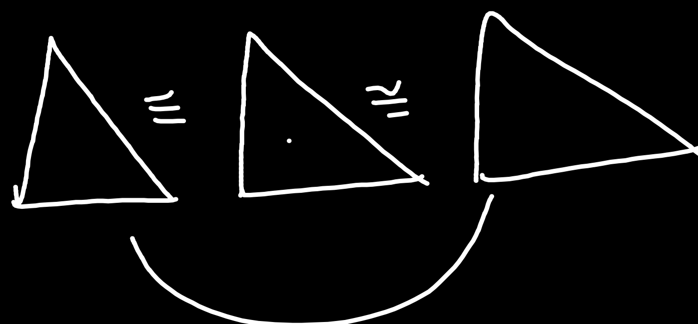
$T_1 \cong T_1$ Yes

Sym: if $T_1 \cong T_2$ then $T_2 \cong T_1$ Yes

Trans: if $T_1 \cong T_2$ and $T_2 \cong T_3$ then $T_1 \cong T_3$

Yes

Y Y Y



R = {(a, b) : a is brother of b}

Ref :- a is brother of a No

Sym: if $\underline{\underline{a}}$ " " " b then $\underline{\underline{b}}$ is ~~brother~~ sister of a

(Ladka) (Ladki)

(False)

Trans if a $\longrightarrow$ b and b $\longrightarrow$ c then a $\longrightarrow$ c

(Male) (Male) (M/F) (M) (M/F)

Yes

$\boxed{NNY}$

R = {(a, b) : a is cousin of b}

Ref : 'a' is Cousin of 'a' No

Sym: if a " " " b then b is cousin of a

Yes (True)

Trans:- if $a \to b$ and $b \to c$ then $a \to c$

(False) Chacha Ka ladka Me Me (Mama Ka ladka)

Unacademy Atoms

Q

For $\alpha \in N$, consider a relation R on N given by $R = \{(x, y) : 3x + \alpha y$ is a multiple of 7$\}$. The relation R is an equivalence relation if and only if :

$(3x + \alpha x)$ is multiple of 7

RV SV TV

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$\alpha = 14$ A. $\alpha = 14$

$\alpha = 4K$ B. α is a multiple of 4

$\alpha = 10K+4$ C. 4 is the remainder when α is divided by 10

$\alpha = 7K+4$ D. 4 is the remainder when α is divided by 7

$(3+\alpha)x \longrightarrow \times 7$

$(3+7K+4)x$

$(7K+7)x$

$7(K+1)x$

Let R_1 and R_2 be two relations defined on R by

$\boxed{aR_1b \Leftrightarrow ab \geq 0}$ and $\boxed{aR_2b \Leftrightarrow a \geq b}$, then

A. R_1 is an equivalence relation but not R_2

B. R_2 is an equivalence relation but not R_1

C. Both R_1 and R_2 are equivalence relations

✓D. Neither R_1 nor R_2 is an equivalence relation

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R_1 : $ab \geq 0$

Ref: $a^2 \geq 0$ (True)

$a = 2$

$b = 0$ Sym: if $ab \geq 0$ then $ba \geq 0$ (True)

$c = -3$ Trans: if $ab \geq 0$ and $bc \geq 0$ then $ac \geq 0$ (False)

$2 \times 0 \geq 0$ $\quad 0 \times (-3) \geq 0$ $\quad 2 \times (-3) \geq 0$

$-6 \geq 0$

Ref:- $a \geq a$ (True)

Sym: $a \geq b$ then $b \geq a$ (False)

$2 \geq 1$ $\quad$ $1 \geq 2$

(Nhi)

<u>Equi X</u>

Let $R_1 = \{(a, b) \in N \times N : \underline{|a - b| \leq 13}\}$ and

$R_2 = \{(a, b) \in N \times N : |a - b| \neq 13\}$. Then on N:

$|a-b| = |b-a|$

$|3-1| \quad |1-3|$

2

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A. Both R_1 and R_2 are equivalence relations

B. Neither R_1 nor R_2 is an equivalence relation

C. R_1 is an equivalence relation but R_2 is not

D. R_2 is an equivalence relation but R_1 is not

Ref: $|a-a| \leq 13$ (True)

Sym: if $|a-b| \leq 13$ then $|b-a| \leq 13$ (T)

Trans: if $|a-b| \leq 13$ and $|b-c| \leq 13$ then $|a-c| \leq 13$ (F)

$|1-3| \leq 13$ and $|3-16| \leq 13$ then $|1-16| \leq 13$ (F)

$|a-b| \neq 13$

Ref: $|a-a| \neq 13$ $\quad 0 \neq 13$ (T)

Sym: if $|a-b| \neq 13$ then $|b-a| \neq 13$ (T)

(Nhi) Trans:- if $|a-b| \neq 13$ and $|b-c| \neq 13$ then $|a-c| \neq 13$

$|1-3| \neq 13$ and $|3-14| \neq 13$ then $|1-14| \neq 13$ (false)

$\underline{2 \neq 13}$ $\quad \underline{11 \neq 13}$ $\quad \underline{13 \neq 13}$

Q

Let R be relation from the set {1, 2, 3 ....., 60} to itself such that $R = \{(a, b) : b = pq$, where $p, q \geq 3$ are prime numbers $\}$. Then, the number of elements in R is :

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A. 600

B. 660

C. 540

D. 720

(1, 9)
(2, 9)
(3, 9)
⋮
(60, 9)

(a, b)

60 × 11

660

b = p q

b	p	q
9	3	3
15	3	5
21	3	7
33	3	11
39	3	13
51	3	17
57	3	19

R : A → A

b	p	q
25	5	5
35	5	7
55	5	11
49	7	7

Q

Which of the following is not correct for relation R on the set of real numbers ?

$|x-x| \le 1$

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True A. $(x, y) \in R \Leftrightarrow 0 < |x| - |y| \le 1$ is neither transitive nor symmetric

False B. $(x, y) \in R \Leftrightarrow 0 < |x - y| \le 1$ is symmetric and transitive.

TRUE C. $(x, y) \in R \Leftrightarrow |x| - |y| \le 1$ is reflexive but not symmetric

TRUE D. $(x, y) \in R \Leftrightarrow |x - y| \le 1$ is reflexive and symmetric.

(A) $0 < |x| - |y| \le 1$ then $0 < |y| - |x| \le 1$

Trans if $0 < |x| - |y| \le 1$ and $0 < |y| - |z| \le 1$ then $0 < |x| - |z| \le 1$

$|2| - |1.5|$ $\quad$ $|1.5| - |0.5| \le 1$

if $0 < |x - y| \le 1$ then $0 < |y - x| \le 1$ (T)

if $0 < |x - y| \le 1$ and $0 < |y - z| \le 1$ then $\underbrace{0 < |x - z| \le 1}$

$\underbrace{0 < |2 - 1| \le 1}$ $\underbrace{0 < |1 - 0| \le 1}$ $\underbrace{0 < |2 - 0| \le 1}$

Trans — whi

---

if $|x| - |y| \le 1$ then $|y| - |x| \le 1 \longrightarrow$ (F/F)

$|1| - |3| \le 1$ $|3| - |1| \le 1$

$|x| - |x| \le 1$ (T) Ref ✓

$R: N \to N.$

Let N be the set of natural numbers and a relation R on N be defined by R = {(x,y) ∈ N x N : $x^3 - 3x^2y - xy^2 + 3y^3 = 0$}. Then the relation R is

$$x^2(x-3y) - y^2(x-3y) = 0$$

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A. Symmetric but neither reflexive nor transitive.

B. Reflexive but neither symmetric nor transitive.

C. Reflexive and symmetric, but not transitive.

D. An equivalence relation.

$$\left\{(x,y) : \boxed{(x-3y)(x-y)(x+y) = 0}\right\}$$

$x = 9$
$y = 1$

Ref :- ✓ Sym ✗ $(3,1) \in R$

$(1,3) \notin R$

$(9,3)$ and $(3,1)$ $(9,1) \notin R$

$(9,3) \in R$ $\times$

$(3,1) \in R$

Summary

- Relations are subset of A x B
- Number of subsets of A x B = Number of relations defined from $A \longrightarrow B$
- $\boxed{\text{Number of subsets} = 2^n}$
- $\boxed{n(A \times B) = n(A).n(B)}$
- Number of relation which can be defined from $A \longrightarrow B$ is 2^{mn}
- Relations can be represented in roaster as well as set builder form
- Arrow starts from anywhere and ends anywhere (Relations are complicated)

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DOMAIN, CODOMAIN AND RANGE

Domain, Codomain and Range of Relations

A = {1, 3, 5, 7}; B = {2, 4, 6, 8}

$\boxed{R = \{(3, 2), (5, 4), (7, 2), (7, 4), (7, 6)\}}$

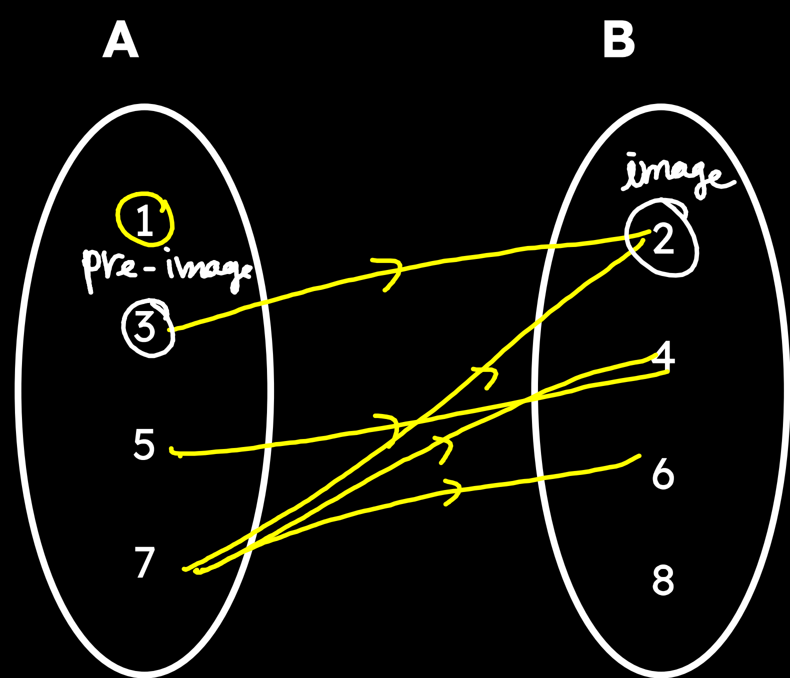
3 is pre-image of 2

2 is image of 3

Domain = {3, 5, 7}

(2nd set) Codomain = {2, 4, 6, 8}

Range = {2, 4, 6}



Domain, Codomain and Range of Relations

Domain of R : Collection of all elements of A which has a image in B

Range of R : Collection of all elements of B which has a pre-image in A

Q

If R = {(x, y)| x, y ∈ Z, $x^2 + y^2 \le 4$} is a relation in Z, then domain of R is -

x

A. {0, 1, 2}

B. {0, -1, -2}

C. {-2, -1, 0, 1, 2}

D. None of these

C

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$x^2 + y^2 \le 4$

x	y
0	0, ±1, ±2
±1	0, ±1
±2	0

$y^2 \le 4$

$(\pm 1)^2 + y^2 \le 4$

$(\pm 2)^2 + y^2 \le 4$

$(\pm 3)^2 + y^2 \le 4$

$y^2 \le -5$

Equivalence Class

Let R = { (1, 1), (2, 2), (3, 3), (1, 2), (2, 1) }

Equivalence class of 1 = [1] = { 1 , 2 }

Equivalence class of 2 = [2] = { 2 , 1 }

Equivalence class of 3 = [3] = { 3 }

26 Feb 2021 Shift 1

Q

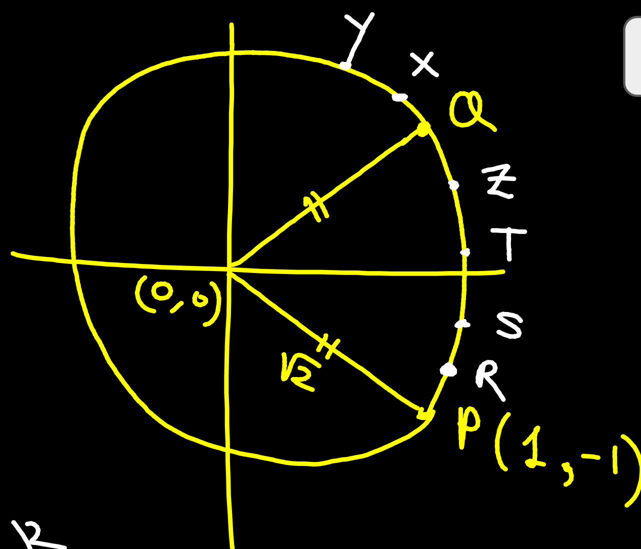
If R = {(P, Q) | P and Q are at the same distance from the origin } be a relation, then the equivalence class of (1, -1) is the set :

A. $S = \{ (x,y) \mid x^2 + y^2 = 1 \}$

B. $S = \{ (x,y) \mid x^2 + y^2 = 4\}$

C. $S = \{(x,y) \mid x^2 + y^2 = \sqrt{2} \}$

D. $S = \{ (x,y) \mid x^2 + y^2 = 2 \}$



$P \longrightarrow Q$

$(1,-1) \longrightarrow x^2+y^2=(\sqrt{2})^2$

INVERSE OF RELATION

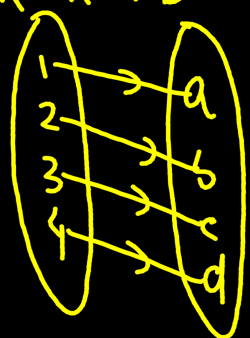
Inverse of a Relation

A = { 1, 2, 3, 4 }
B = { a, b, c, d }

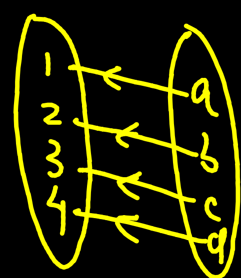
$R : A \rightarrow B$

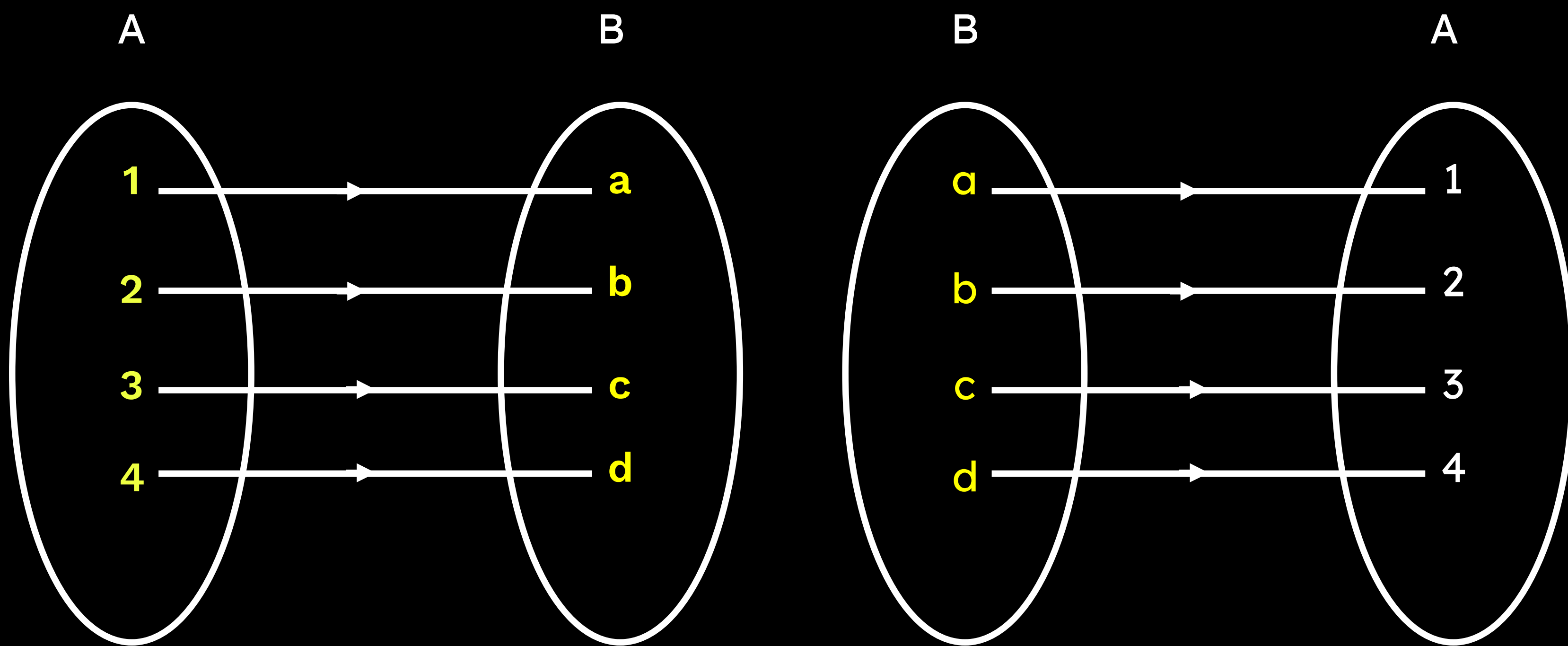
$R = \{(1, a), (2, b), (3, c), (4, d)\}$

$R : A \rightarrow B$



$R^{-1} : B \rightarrow A$




A
B
B
A
1
2
3
4
a
b
c
d
a
b
c
d
1
2
3
4

Domain of R = Range of R^{-1}

Range of R = domain of R^{-1}

Unacademy Atoms

Q

If $R = \{(x, y); x, y \in Z, x^2 + 3y^2 \leq 8\}$ is a relation on the set of integers Z, then the domain of R^{-1} is :

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A. {0, 1}

B. {-2, -1, 1, 2}

C. {-1, 0, 1}

D. {-2, -1, 0, 1, 2}

$x^2 \leq 8$

$x^2 + 3y^2 \leq 8$

x	y
$\pm 2, \pm 1, 0$	0
✓	1
✓	-1
X	2
X	-2

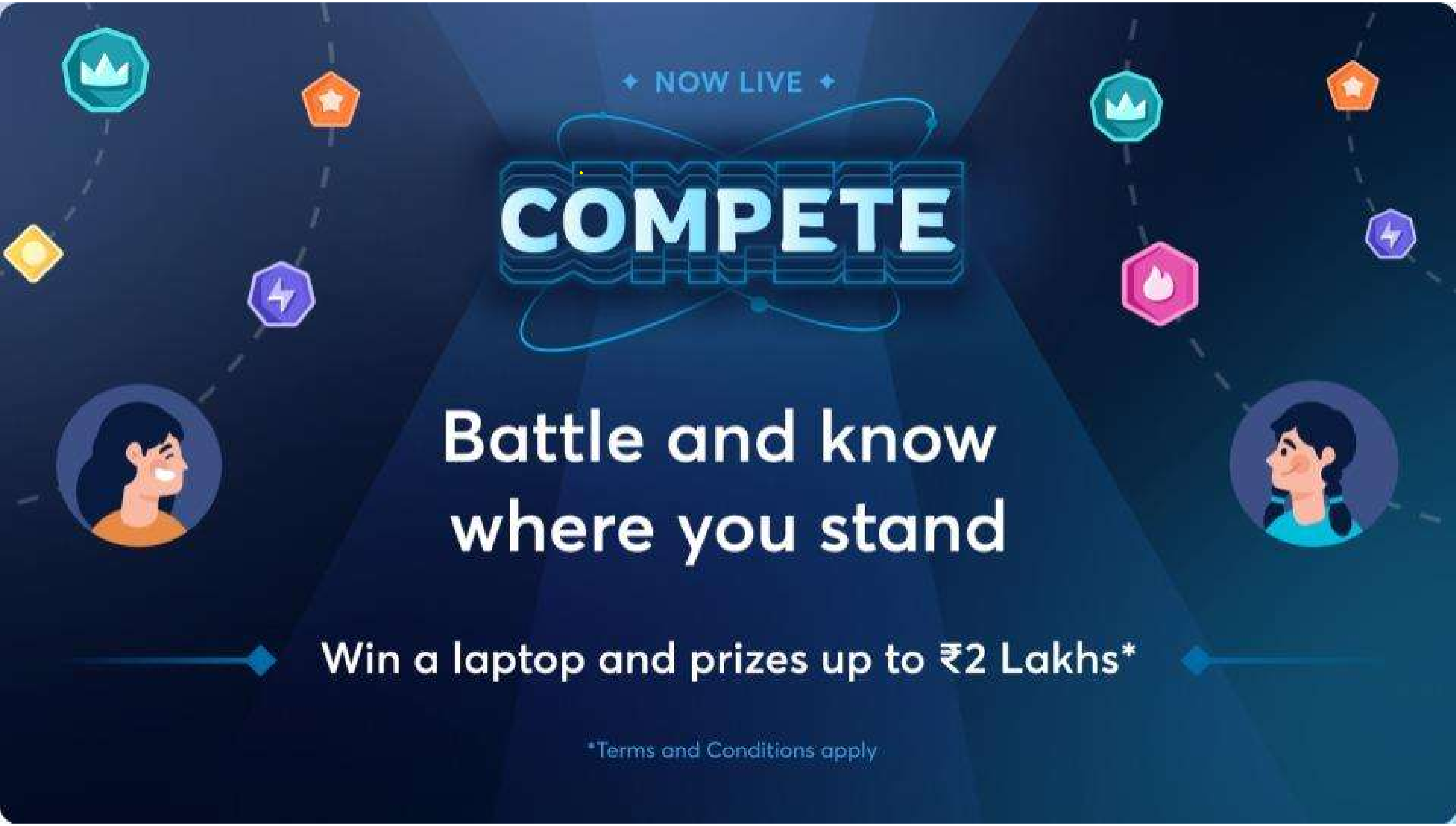
$x^2 + 12 \leq 8$

$x^2 \leq -4$

domain of R^{-1} = Range of R

(4)

(C)


NOW LIVE
COMPETE
Battle and know where you stand
Win a laptop and prizes up to ₹2 Lakhs*
*Terms and Conditions apply

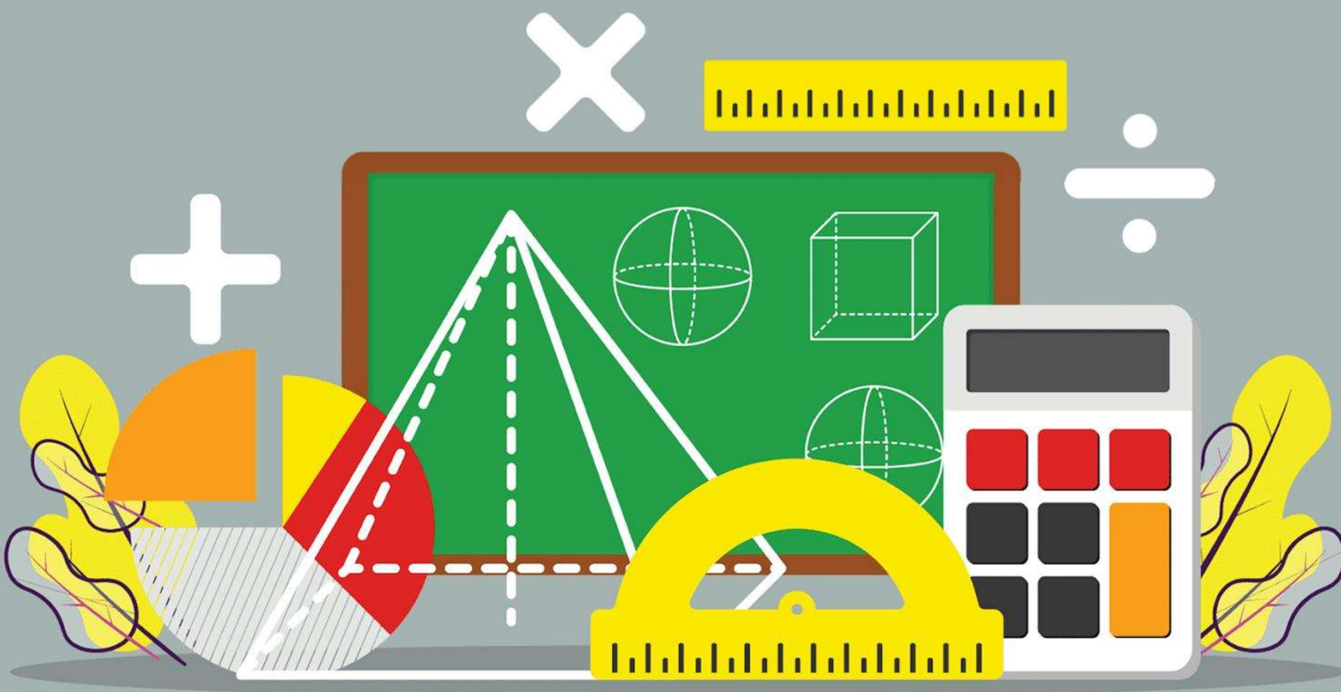
Functions

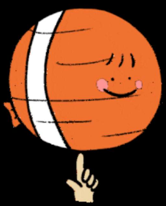
Functions

1. Modulus
2. Logarithmic
3. Exponentials
4. Greatest integer functions
5. Fractional part

Unacademy Atoms

MODULUS FUNCTIONS





Modulus Functions: Definition

↳ Absolute Value

$-3 \rightarrow |x| \rightarrow 3$

$5 \rightarrow |x| \rightarrow 5$

$0 \rightarrow |x| \rightarrow 0$

$-(-3)$

$|x|$

$|0| = 0$

$x>0 \rightarrow |x| \rightarrow x$

$0 \rightarrow |x| \rightarrow 0$

$x<0 \rightarrow |x| \rightarrow \boxed{-x}$

$$y = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Modulus Functions: Domain, Range, Graphs

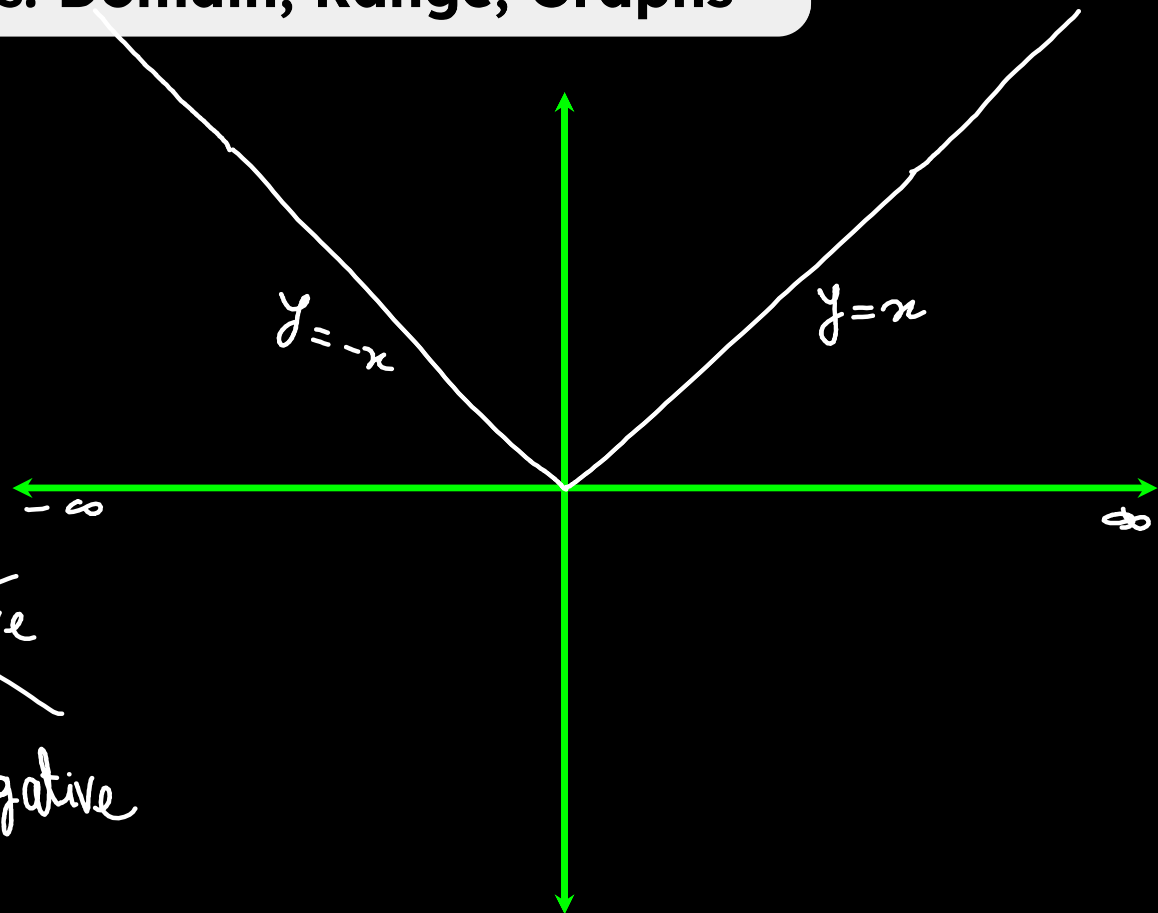
$f(x) = |x|$

Domain :- $x \in R$

Range : $y \in [0, \infty)$

~~positive~~

Non-negative



Opening Modulus Functions

$$|\square| = \begin{cases} +\square & \text{if } \square \geq 0 \\ -\square & \text{if } \square < 0 \end{cases}$$

Eg

$$|\boxed{x-1}| = \begin{cases} +(x-1) & \text{if } x-1 \geq 0 \\ -(x-1) & \text{if } x-1 < 0 \end{cases}$$

$x=1$ is c.p

$$|x-1| = \begin{cases} x-1 & \text{if } x \geq 1 \\ -x+1 & \text{if } x < 1 \end{cases}$$

$f(x) = |2x| - 3$

$2x = 0$

$\therefore \boxed{x=0}$ is c.p.

Mod k ander

$|\square|$

$\boxed{\square = 0}$

Opening Modulus Functions

$$|\underbrace{1-2x}| = \begin{cases} +(1-2x) & \text{if } \underline{1-2x \geq 0} \\ -(1-2x) & \text{if } 1-2x < 0 \end{cases}$$

$1-2x=0$

$x=\frac{1}{2}$

$$= \begin{cases} 1-2x & \text{if } x \leq 1/2 \\ -1+2x & \text{if } x > \frac{1}{2} \end{cases}$$

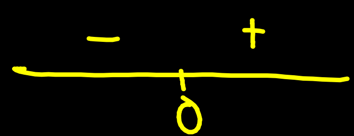
$\boxed{x=\frac{1}{2} \text{ is c.p.}}$

Opening Modulus Functions

Critical point

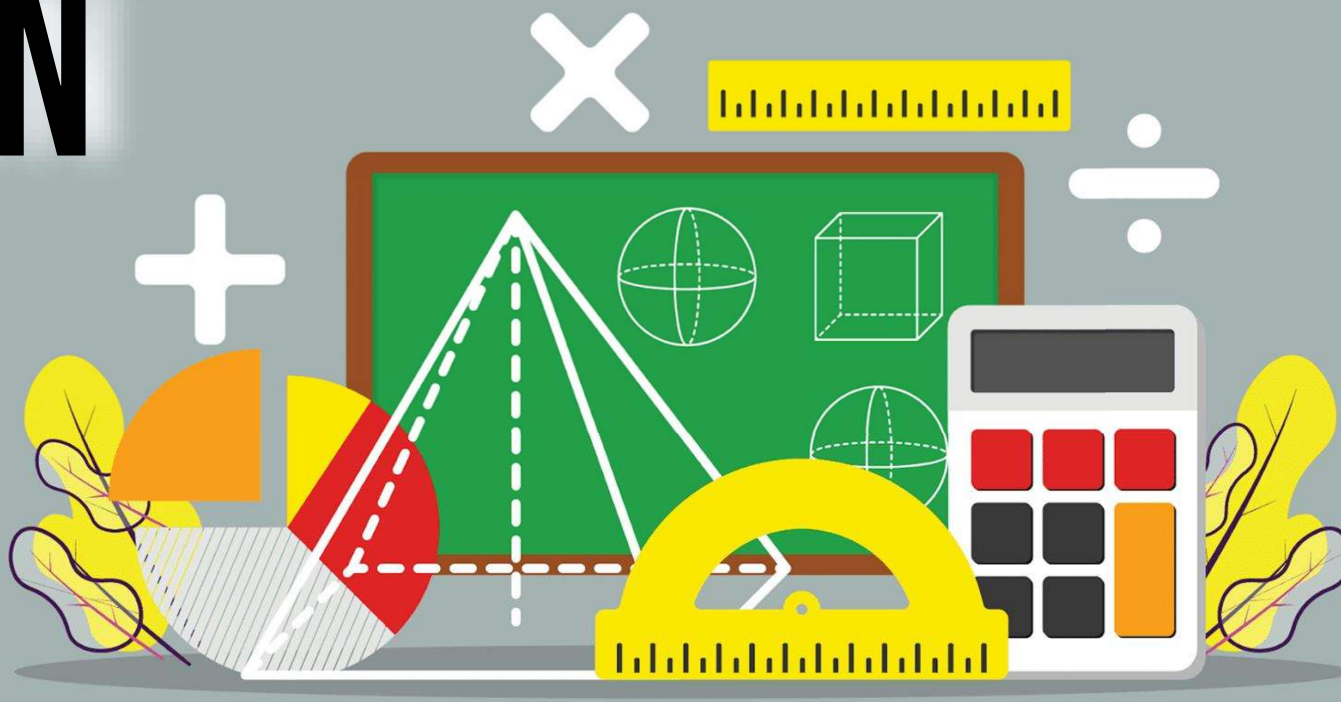
NOTE - On the right of critical point modulus opens with a plus sign and on the left it opens with a minus sign.

$$|\boxed{2x}| - 3 = \begin{cases} +(2x) - 3 & \text{if } \underline{2x \geq 0} \\ -(2x) - 3 & \text{if } 2x < 0 \end{cases}$$



$$= \begin{cases} 2x - 3 & \text{if } \underline{x \geq 0} \\ -2x - 3 & \text{if } x < 0 \end{cases}$$

GRAPH OF MODULUS FUNCTION



Graph of Modulus Functions

|x| + k   or |x| - k

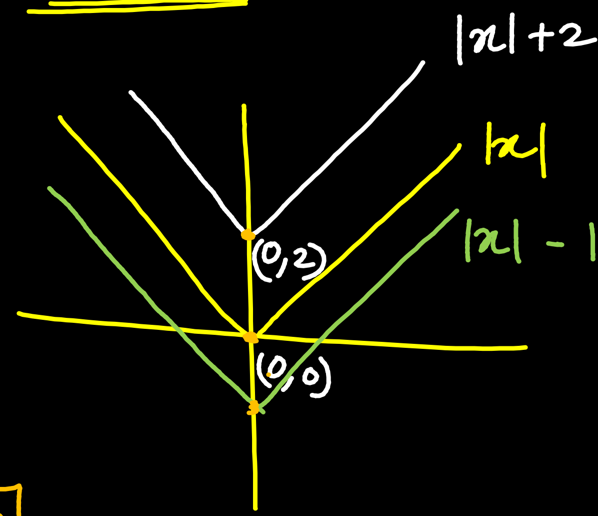
$|x| + k \longrightarrow$ uper 'k' units.

$|x| - k \longrightarrow$ neeche 'k' units.

$|x| + 2$

$|x| - 1$

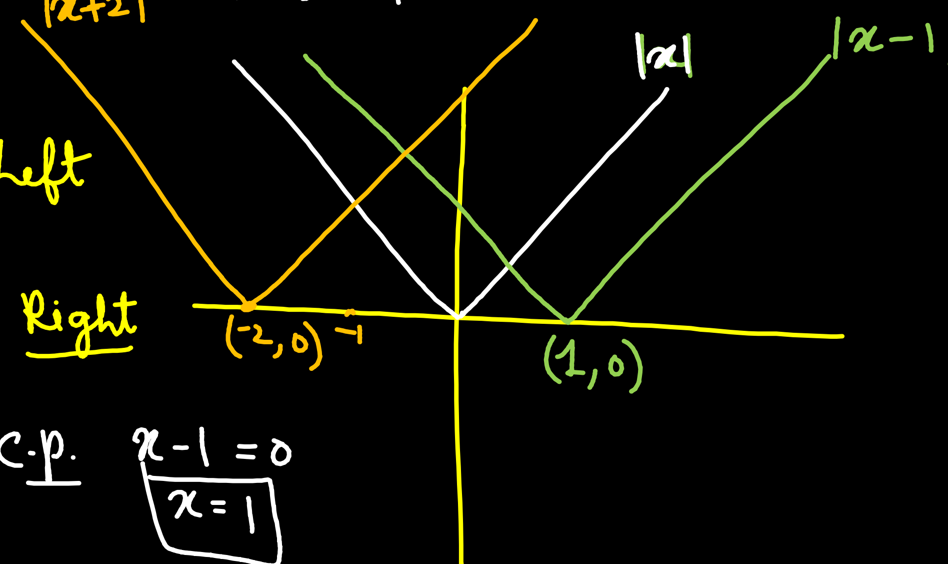
$\boxed{x = 0}$



Graph of Modulus Functions

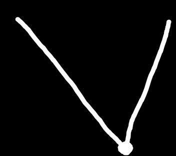
$|x + k|$ or $|x - k|$

1) $y = |x - 1|$ Right
2) $y = |x + 2|$ Left



$|x + k| \longrightarrow$ Shift to Left

$|x - k| \longrightarrow$ Shift to Right



$(1,0)$ $\boxed{y = |x-1|}$ c.p. $x - 1 = 0$, $\boxed{x = 1}$

$\boxed{y = |x+2|}$ $x + 2 = 0$, $x = -2$

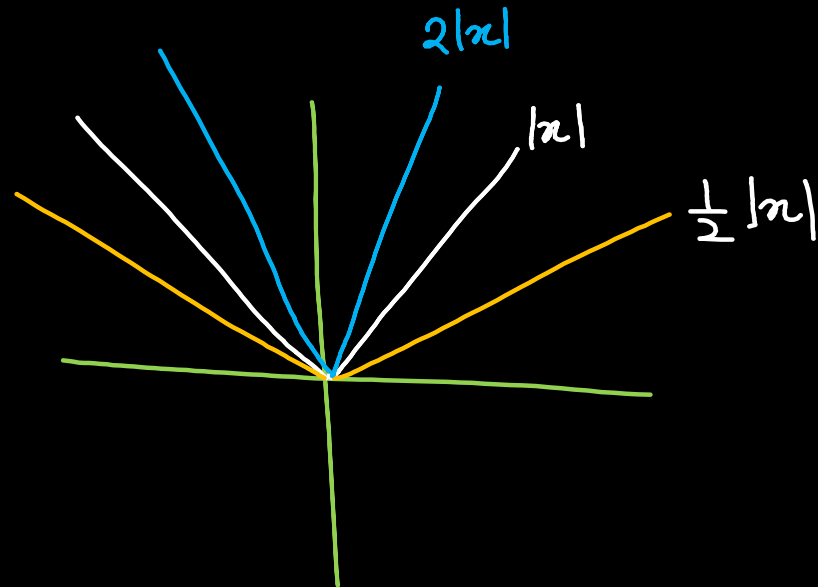
Graph of Modulus Functions

k|x| or |kx|

$k > 0$

$k > 1$ Contract

$k < 1$ Expand.

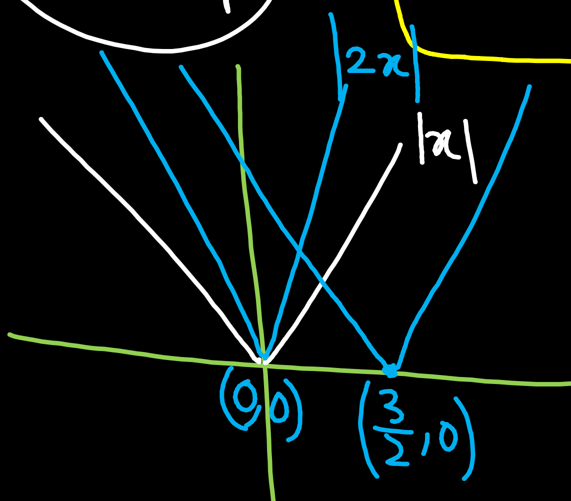


Define f(x) free of modulus and draw the graph f(x) = |2x - 3|

$$f(x) = |2x-3| = \begin{cases} +(2x-3) & \text{if } 2x-3 \geq 0 \\ -(2x-3) & \text{if } 2x-3 < 0 \end{cases}$$

$$\boxed{f(x) = \begin{cases} 2x-3 & \text{if } x \geq 3/2 \\ -2x+3 & \text{if } x < 3/2 \end{cases}}$$

$x = \frac{3}{2}$ is c.p.



$$|2x| = 2|x|$$

Q

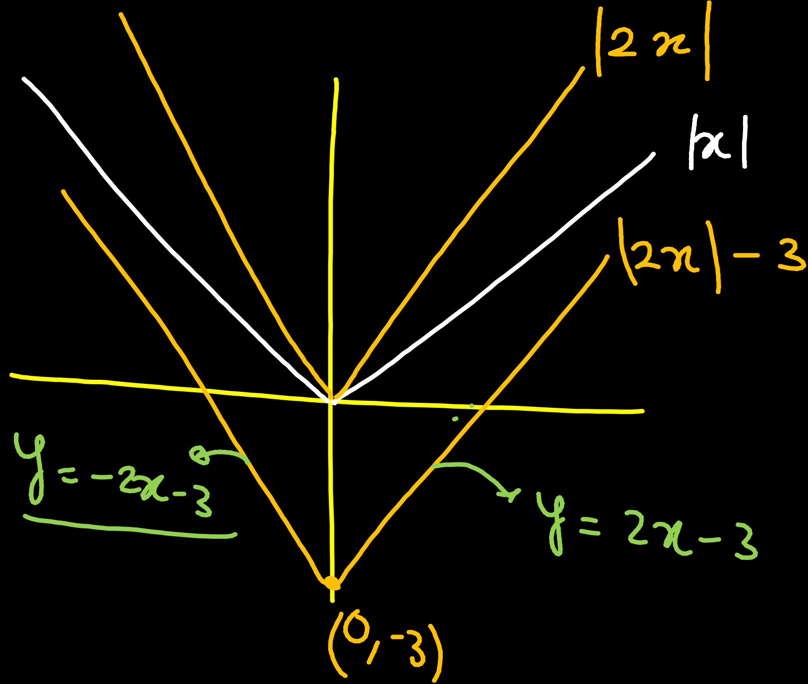
Define $f(x)$ free of modulus and draw the graph $f(x) = |2x| - 3$

$2x = 0$

$x = 0$

$y = |2x| - 3$

$(0, -3)$

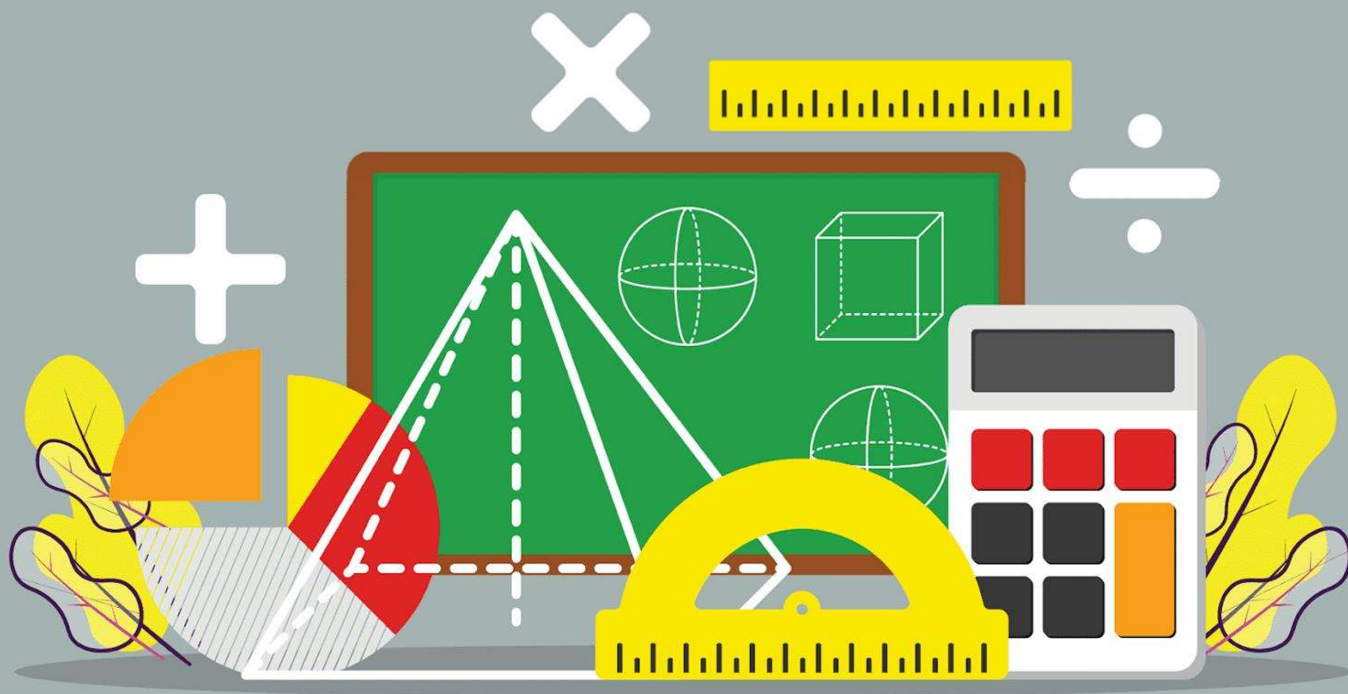


Vertex: $(0, -3) \equiv (CP, ?)$

CP : $x = 0$

Unacademy Atoms

DOUBLE MOD GRAPHS



Q

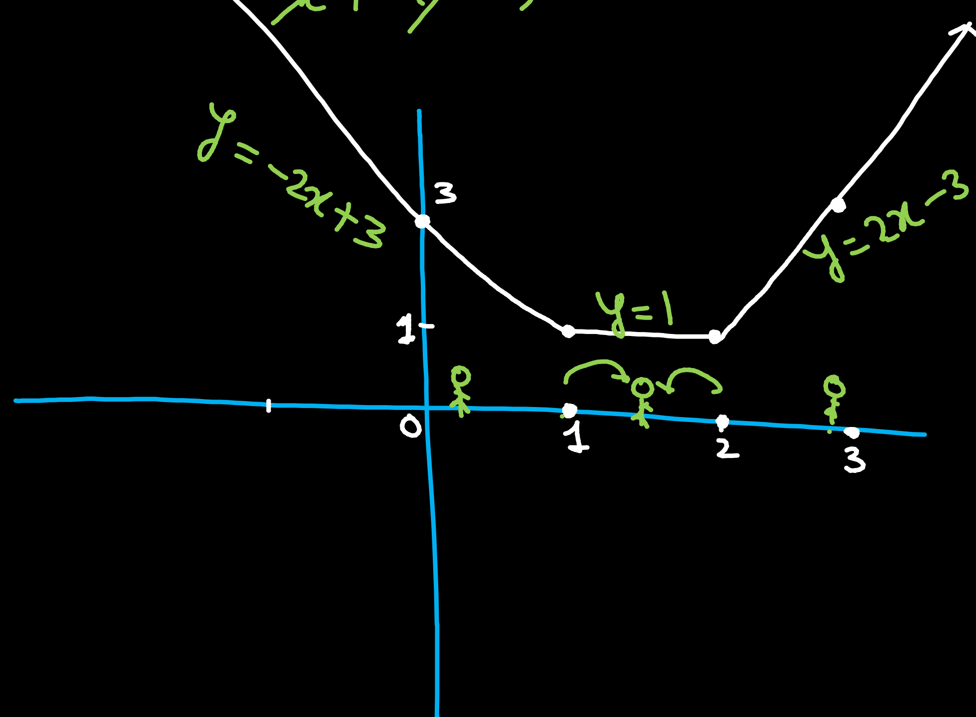
Shortcut

$-x+1 \quad -x+2$

$y = f(x) = |x - 1| + |x - 2|$

$y = x-1+x-2$

$\cancel{x}-1-(\cancel{x}-2)$



S-1 Find CPs.

$x = 1, 2$

S-2

x	y
0	3
1	1
2	1
3	3

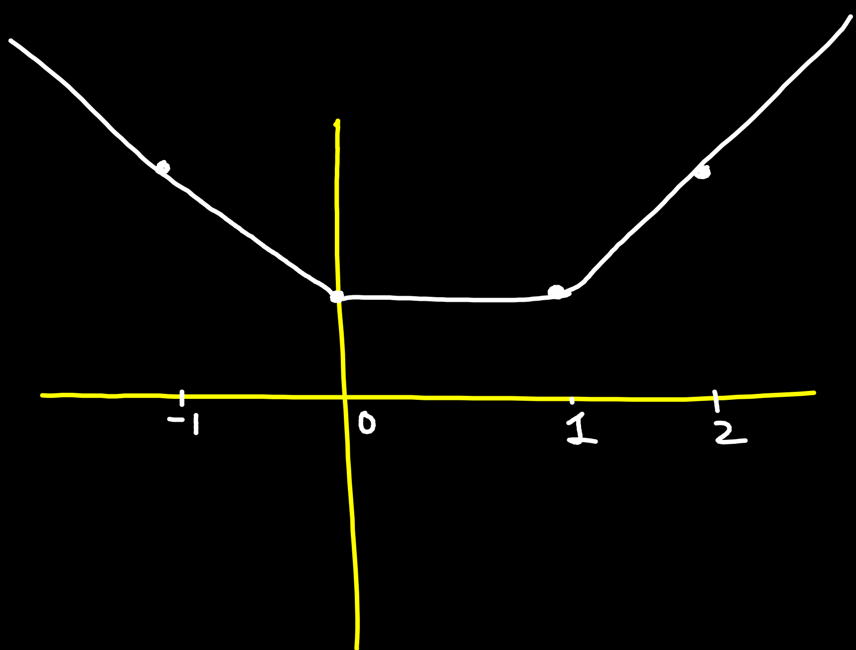
Unacademy Atoms

Q

$f(x) = |x| + |x - 1|$

Shortcut

Unacademy Atoms



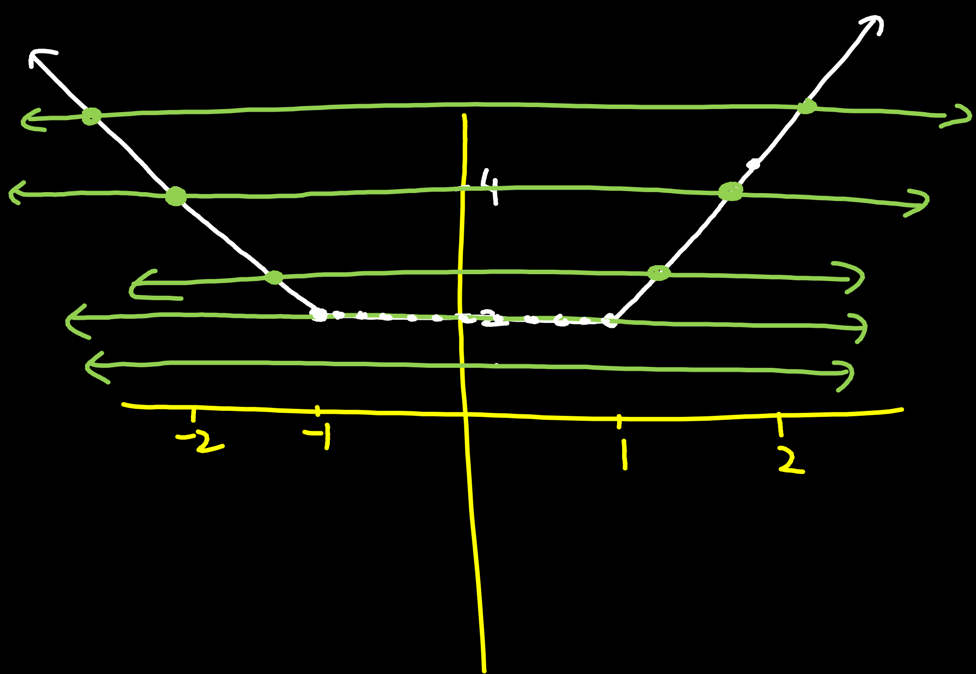
x	y
-1	3
0	1
1	1
2	3

Q

$f(x) = |x + 1| + |x - 1|$

Shortcut

Unacademy Atoms



$y = k$

LHS

$\overbrace{|x+1| + |x-1|}^{LHS} = \overbrace{k}$

find k —

i) if 4 Sol^n ϕ

ii) if 3 Sol^n ϕ

iii) if 2 Sol^n $k \in (2, \infty)$

iv) if 1 Sol^n ϕ

v) if 0 Sol^n $k < 2$

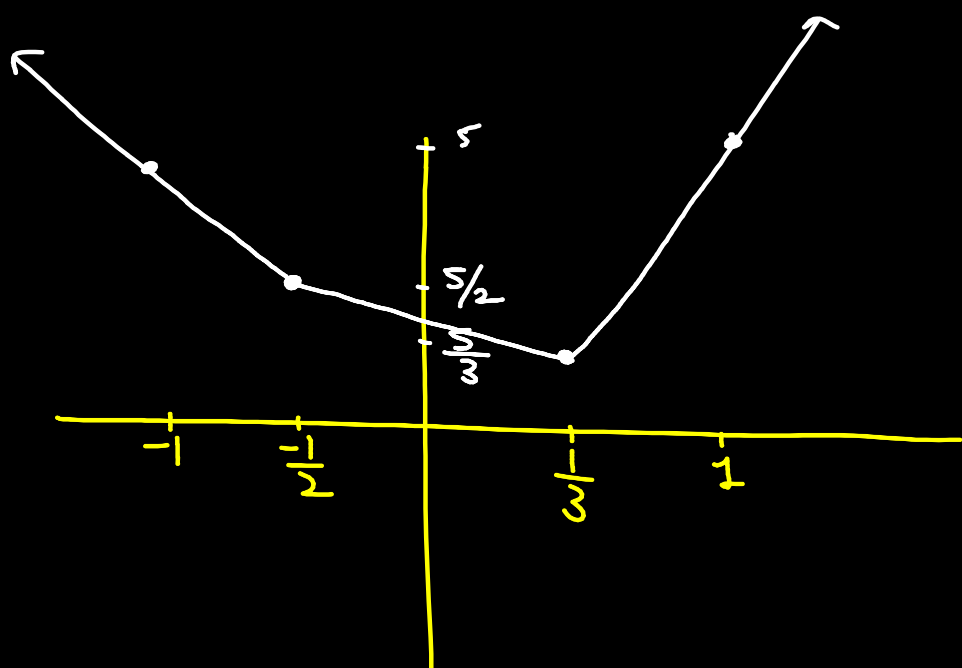
vi) if ∞ Sol^n $k = 2$

Shortcut

Q

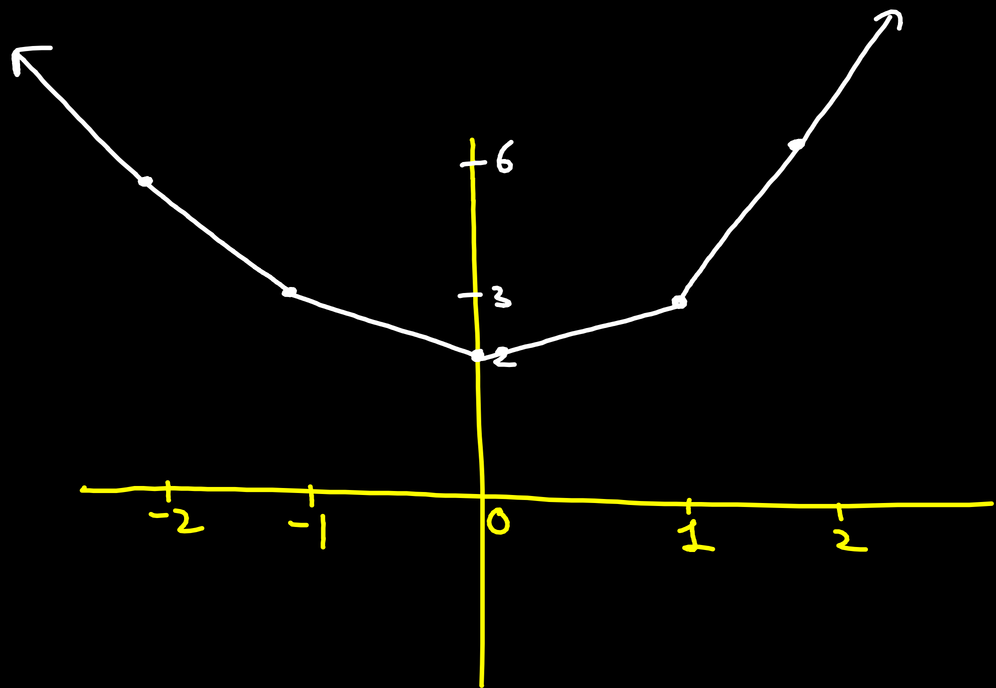
$f(x) = |2x + 1| + |3x - 1|$

$y = |2x+1| + |3x-1|$



x	y
-1	5
1	5
$-\frac{1}{2}$	$\frac{5}{2}$
$\frac{1}{3}$	$\frac{5}{3}$

Open modulus and draw graph of $f(x) = |x + 1| + |x| + |x - 1|$



x	y
0	2
1	3
-1	3
-2	6
2	6

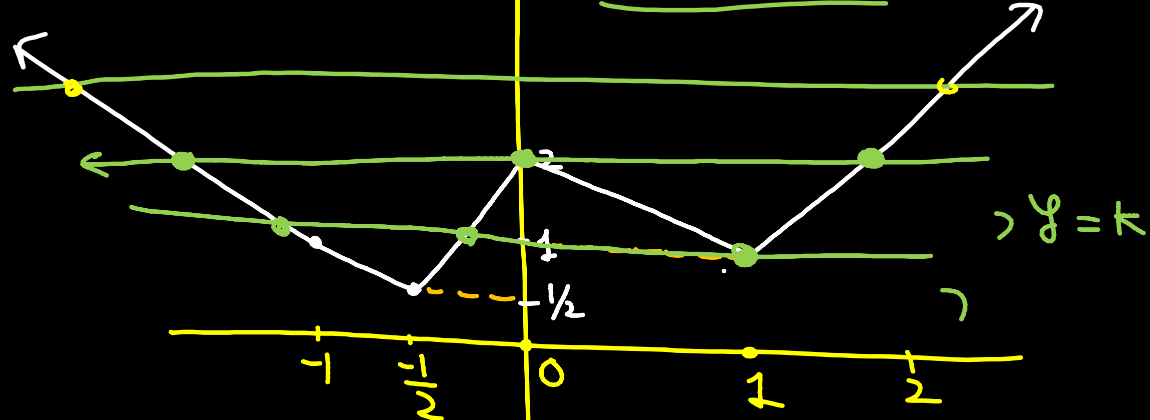
Find ‘k’ for which following equation has 4 solutions

$|x - 1| - 2|x| + |2x + 1| = k$

$y = |x-1| - 2|x| + |2x+1| \qquad y = k$

$$\boxed{\begin{gathered}3 \text{ Sol}^n \\ k \in \{2, 1\}\end{gathered}}$$

$\underline{k \in (1, 2)}$ 4 sol^n

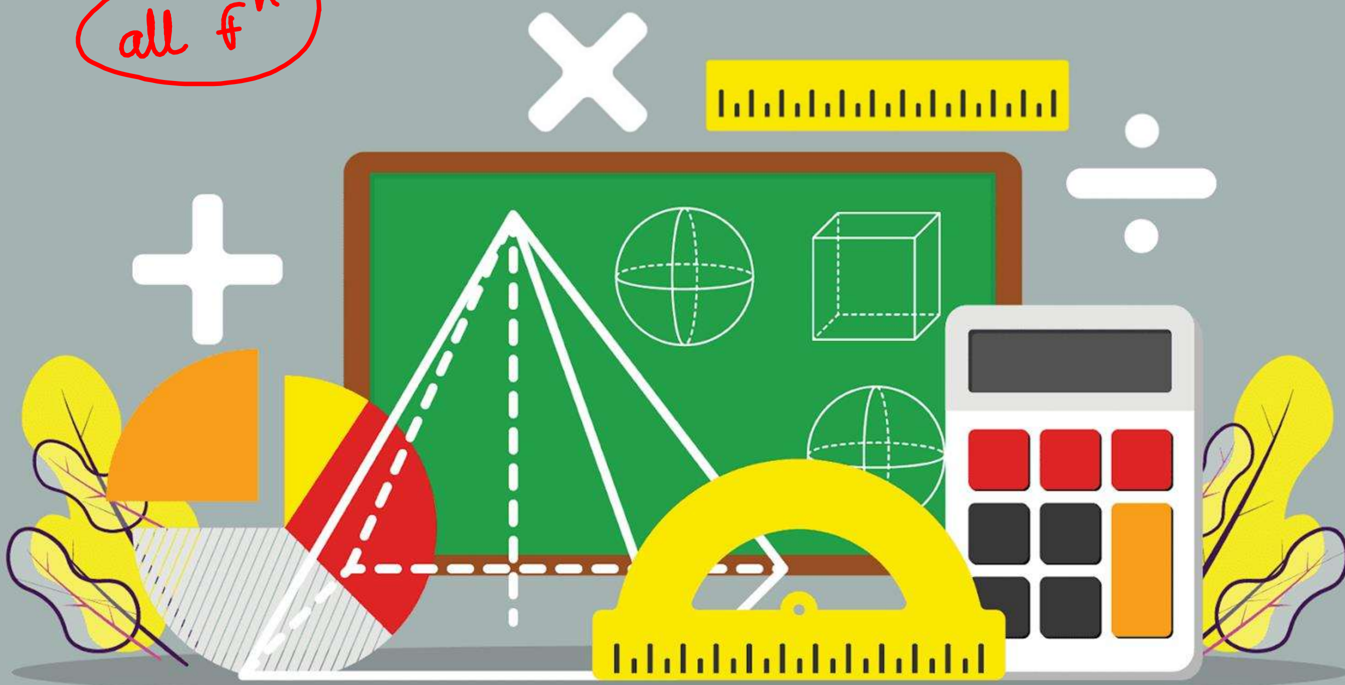


$\underline{k \in (\tfrac{1}{2}, 1) \cup (2, \infty)}$ 2 sol^n

x	y
0	2
1	$-2+3=1$
2	$1-4+5=2$
$\frac{-1}{2}$	$\frac{3}{2}-1=\frac{1}{2}$
-1	$2-2+1=1$

GRAPHICAL TRANSFORMATION

all f^n



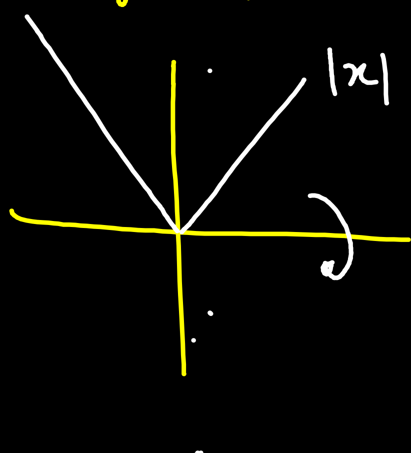
Graphical Transformation

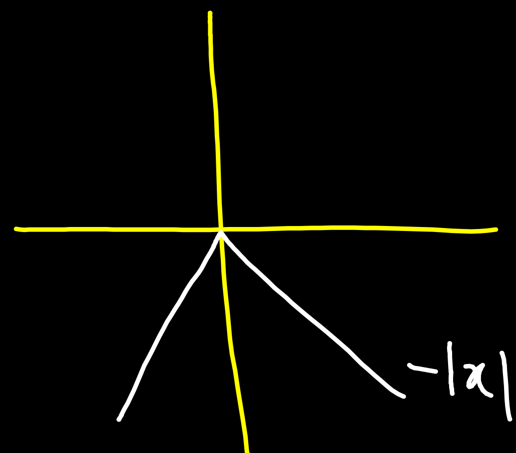
$y = f(x)$ $\xrightarrow{\text{Flip in x axis}}$ $y = -f(x)$

OR

$-y = f(x)$

$y = |x| \longrightarrow y = -|x|$

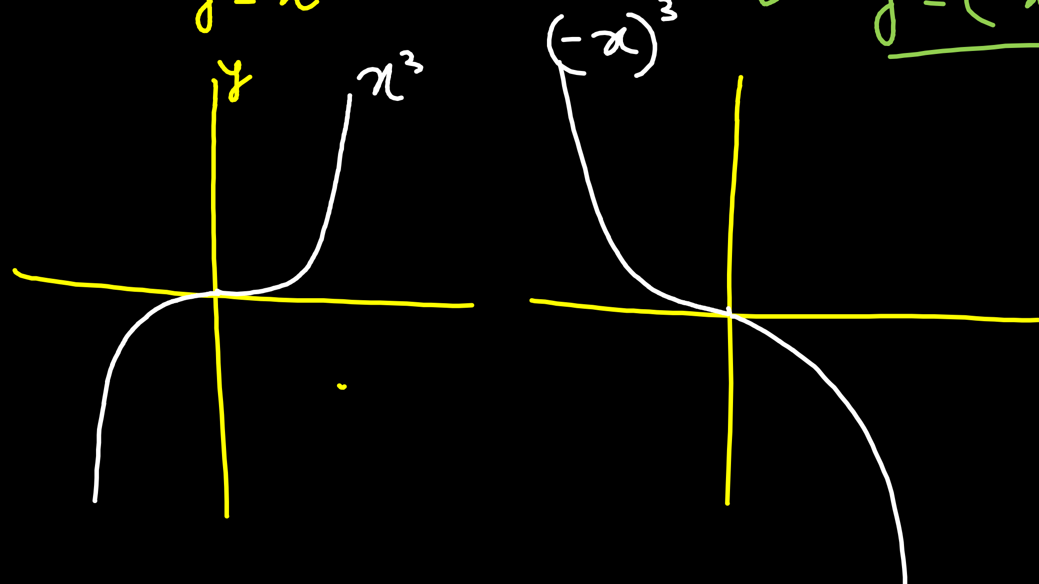




Graphical Transformation

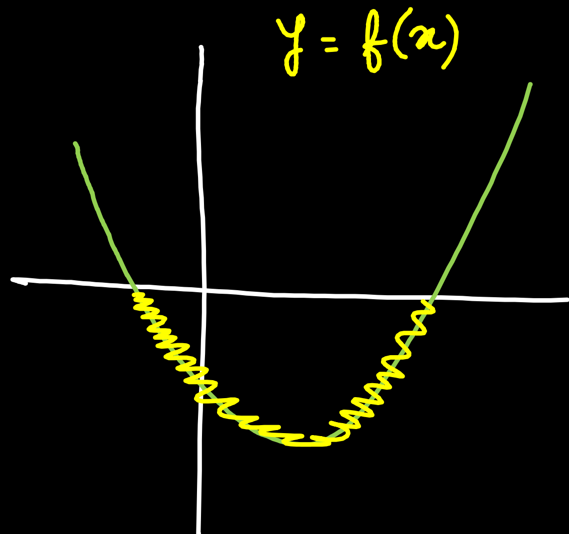
$y = f(x)$ $\xrightarrow{\text{Flip in y axis}}$ $y = f(-x)$

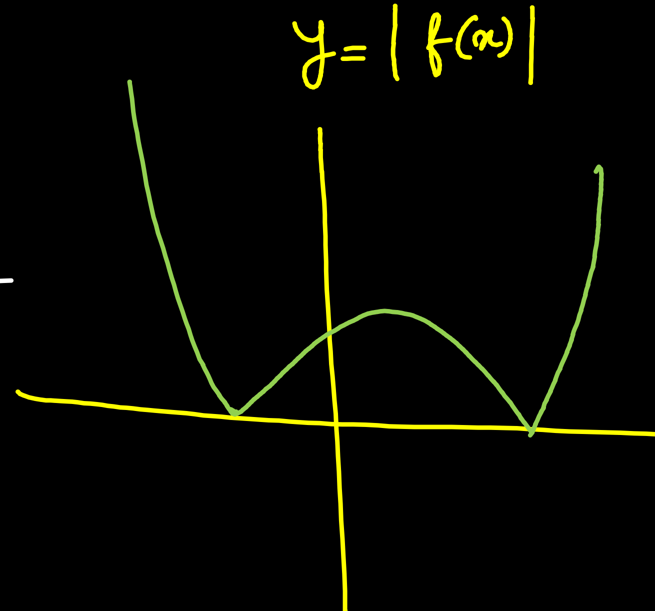
$y = x^3 \longrightarrow y = (-x)^3$ OR $y = -x^3$



Graphical Transformation

$y = f(x)$ $\xrightarrow{\text{neeche wala uper}}$ $y = |f(x)|$

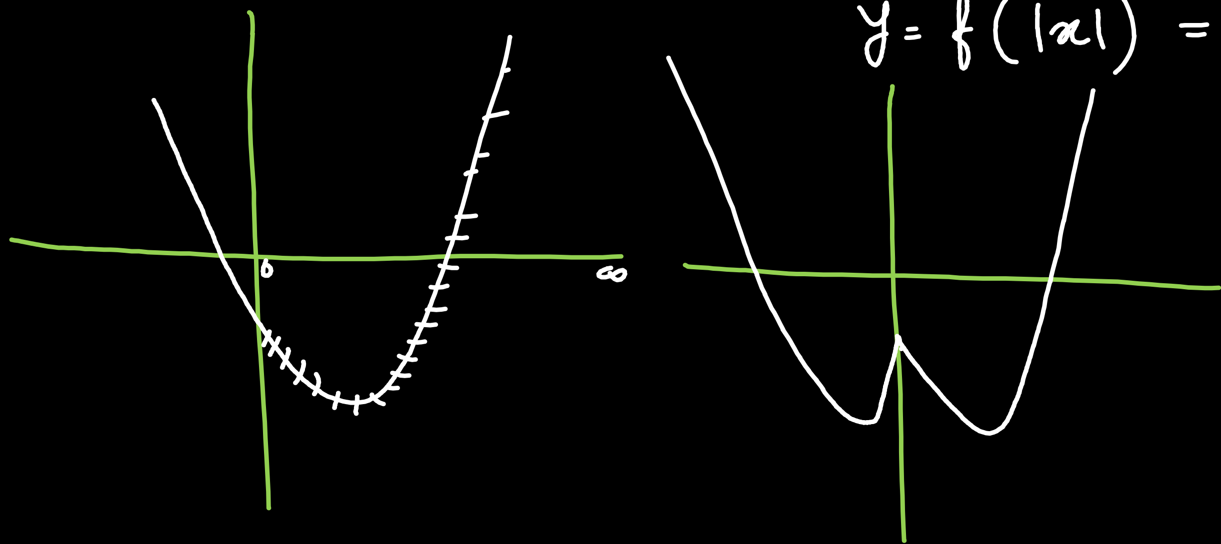




Graphical Transformation

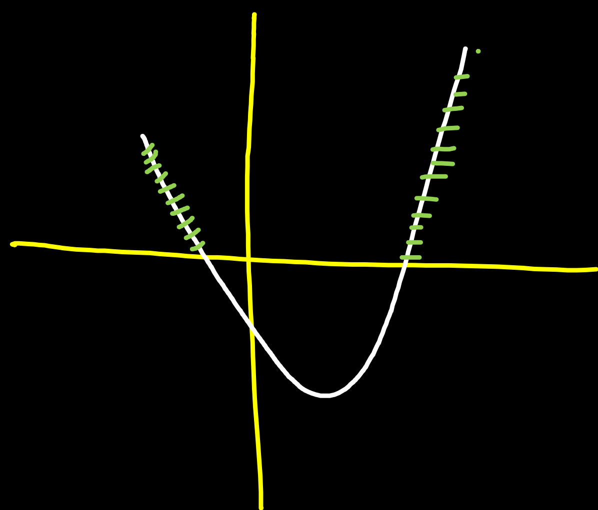
$y = f(x)$ $\xrightarrow[\text{Left side = Mirror}]{\text{Right side = Same}}$ $y = f(|x|)$

$$y = f(|x|) = \begin{cases} \boxed{f(x) \quad \text{if } x \geq 0} \\ f(-x) \quad \text{if } x < 0 \end{cases}$$

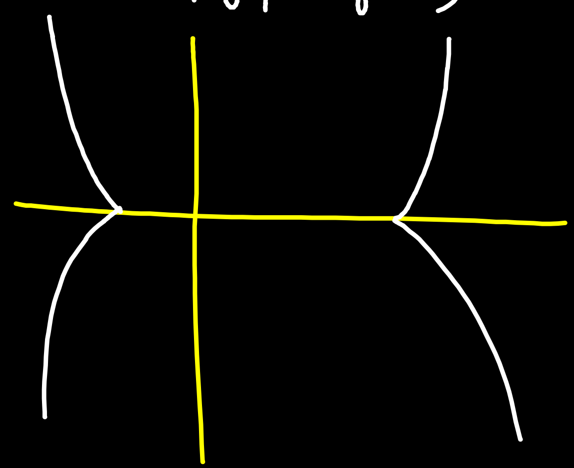


Graphical Transformation

$y = f(x)$ $\xrightarrow[\text{Neeche = Mirror}]{\text{Upar wala = Same}}$ $|y| = f(x)$

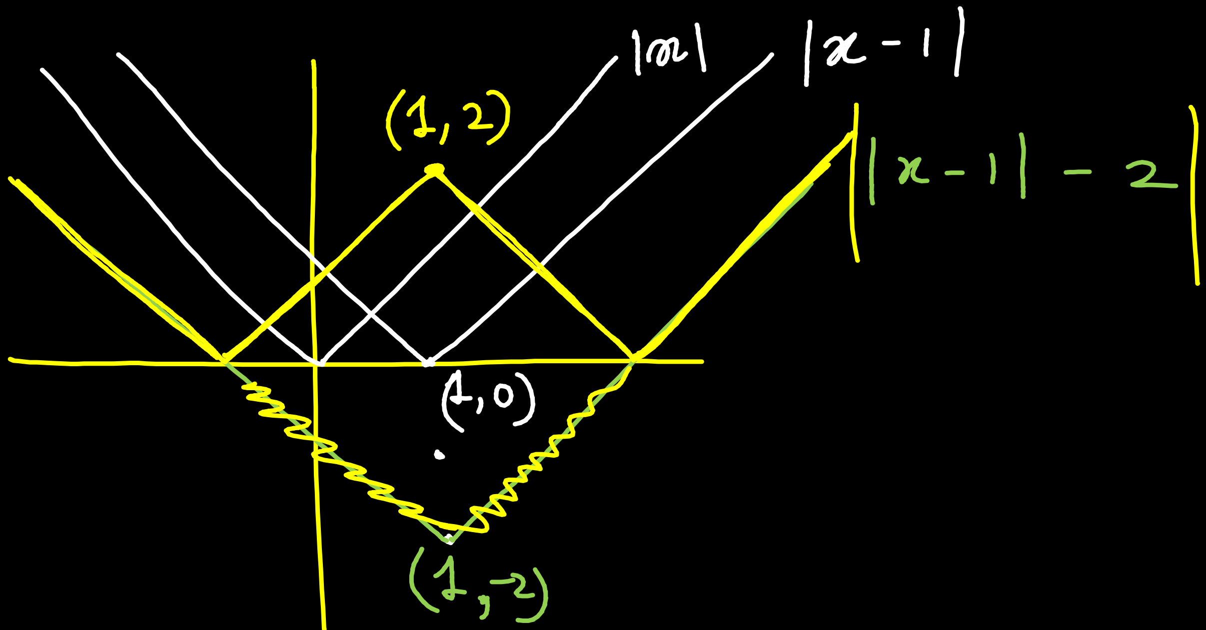


$|y| = f(x)$

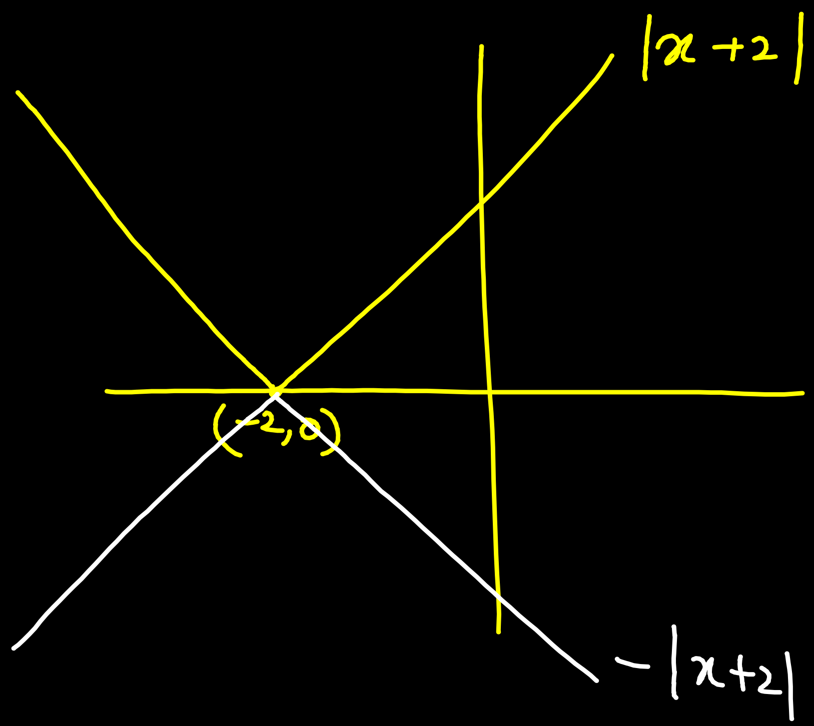


Q

Draw the graph of $y = ||x - 1| - 2|$



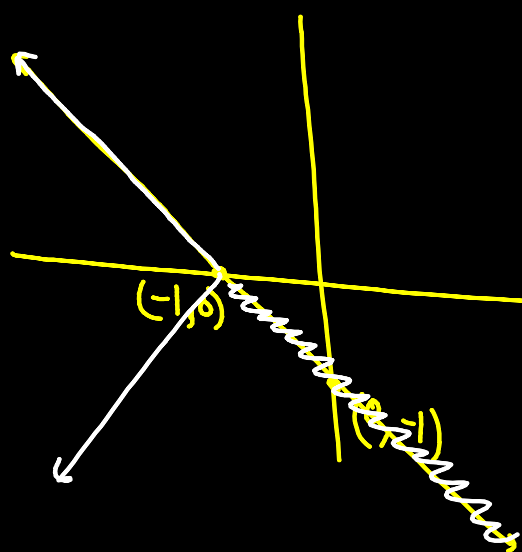
Draw the graph of $y = \underbrace{-|x + 2|}$

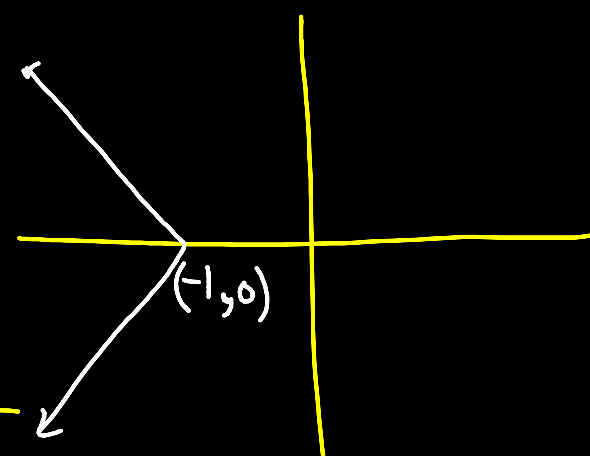


Draw the graph of $|y| + x = -1$

$$|y| = -1 - x$$

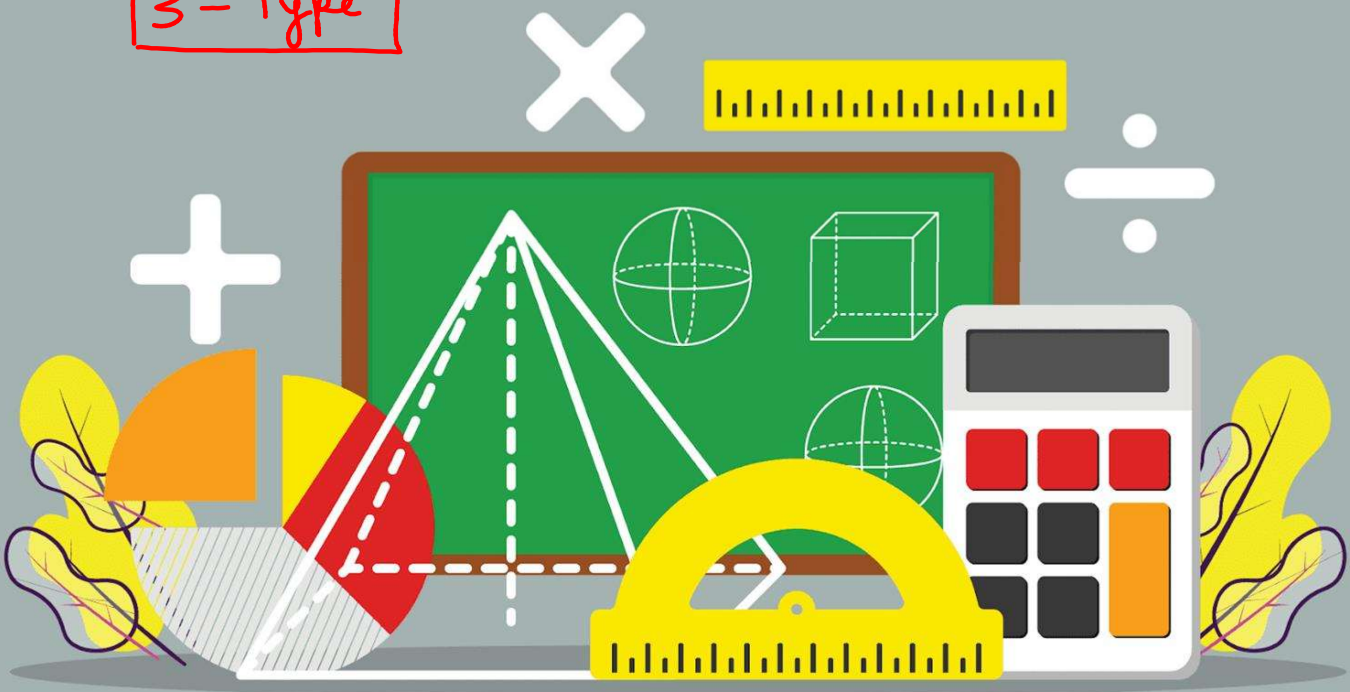
$$\boxed{|y| = -1 - x}$$





MODULUS EQUATIONS

3 - Type



Solving Modulus Equations

$|f(x)| = a \Rightarrow f(x) = \pm a$

Type-1

$|f(x)| = k$

$\boxed{f(x) = \pm k}$

Mod Hatao!!
aur $\pm$ lagao!!

Solving Modulus Equations (Method - 1)

Ex. $|x - 1| = 3$

$$|x-1| = 3$$

$$x-1 = \pm 3$$

$$x = 1 \pm 3$$

$$\boxed{x = -2, 4}$$

$$\left|\boxed{}\right| = k$$

$$\boxed{\boxed{} = \pm k}$$

Solving Modulus Equations (Method - 2)

Ex. $|x - 1| = 3$

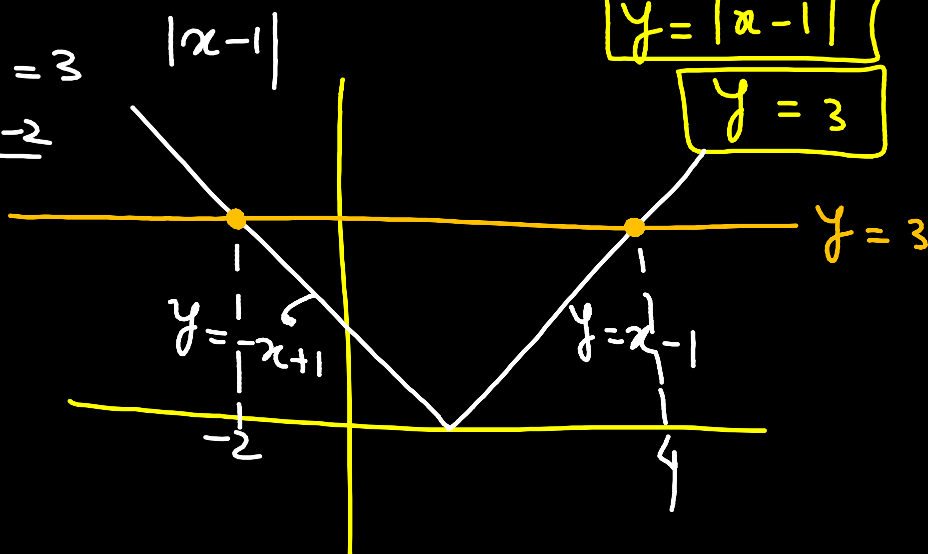
M-2

$y = |x - 1|$

$y = 3$

$-x + 1 = 3$

$\underline{x = -2}$



$x - 1 = 3$

$x = 4$

Solving Modulus Equations

Ex. $||x| - 1| = 7$

$$\left| |x| - 1 \right| = 7$$

$$|x| - 1 = \pm 7$$

$$|x| = 1 \pm 7$$

$|x| = 8$ | $|x| = -6$

$\boxed{x = \pm 8}$ | No Sol^n

No. of Sol^n = 2

Sol^n : 8, −8

Sum of Sol^n = 0

Unacademy Atoms

Q

Solve for x : $||x + 1| - 4| = 5$

$$|x+1| - 4 = \pm 5$$

$$|x+1| = 4 \pm 5$$

$$|x+1| = 9$$

$$x+1 = \pm 9$$

$$x = -1 \pm 9$$

$$\boxed{x = 8, -10}$$

$$\underbrace{|x+1|}_{} = \underbrace{-1}_{}$$

No Sol^n

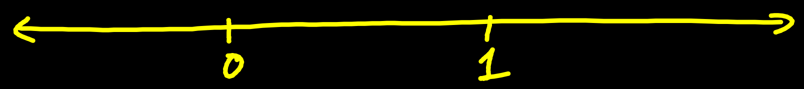
Q

Solve for x : $|x - 1| - |x| = 1$

M-1: Algebraic Method

Unacademy Atoms

Type-2



Final Ans : $(-\infty, 0]$

$x < 0$

$-(x-1) + x = 1$

$-\cancel{x} + 1 + \cancel{x} = 1$

$1 = 1$

always true

all Sol^n

$x \in (-\infty, 0)$

$0 \le x \le 1$

$-(x-1) - x = 1$

$-x + 1 - x = 1$

$\boxed{x = 0}$ ✓

$x > 1$

$(\cancel{x} - 1) - (\cancel{x}) = 1$

$-1 = 1$

Never true

No Sol^n

Q

Solve for x : |x - 1| - |x| = 1

M-2: Graphical Method

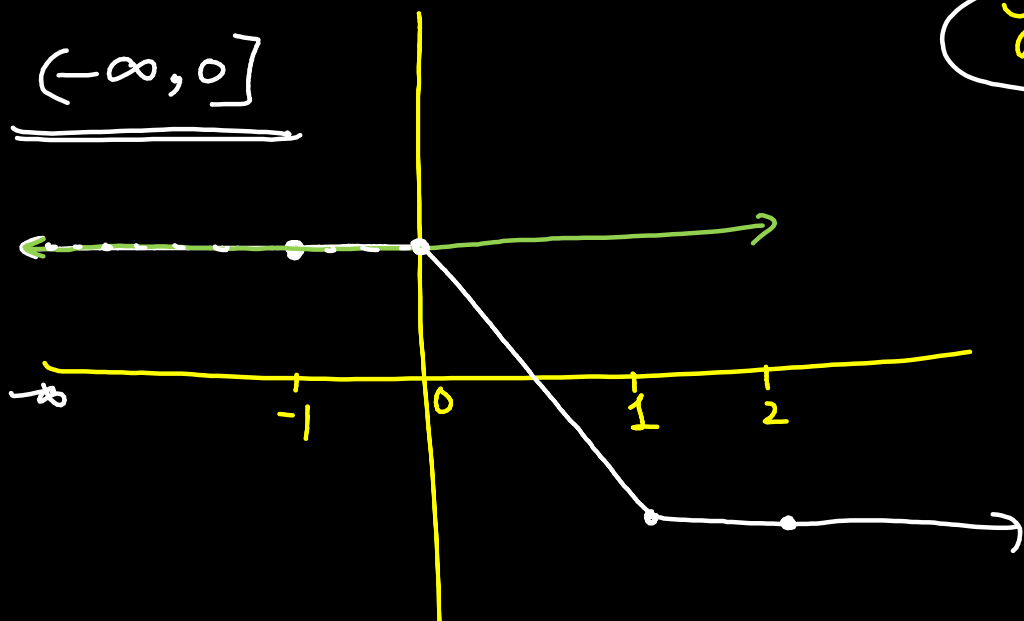
Unacademy Atoms

$y = |x-1| - |x|$

$y = 1$

∞ Sol^n

$(-\infty, 0]$



x	y
-1	1
0	1
1	-1
2	-1

$$|a| = |b|$$

$$a = b$$

$$\boxed{\begin{aligned} a &= -b \\ -a &= b \end{aligned}}$$

$$\cancel{-}a = \cancel{-}b$$

$$\boxed{\begin{aligned} a &= b \\ a &= -b \end{aligned}} \Rightarrow a = \pm b$$

Ty-1 $\quad |\square| = K \Rightarrow \square = \pm K$

Ty-2 $\quad \underbrace{|\square| \pm |\square| = K}_{\text{Graphical.}}$

Ty-3 $\quad |\square| = |\square| \Rightarrow \square = \pm \square$

Q

Solve the following linear equation $||x-1|-2| = |x-3|$

A. $x \in [1, \infty)$

B. $x \in [3, \infty)$

C. $x = 3$

D. $x = 1$

$\Rightarrow |x-1| - 2 = +(x-3)$

$\Rightarrow |x-1| = x-1$

$\Rightarrow \boxed{x \geq 1}$

"Union"

OR Union

and $\cap$

$|x-1| - 2 = -(x-3)$

$|x-1| = 5-x$

$x-1 = 5-x$

$2x = 6$

$\boxed{x=3}$

$-(x-1) = 5-x$

$\cancel{-x}+1 = 5-\cancel{x}$

$1=5$

No soln

$|x| =$

Q

Number of solutions of the equations $|2x^2 + x - 1| = |x^2 + 4x + 1|$

A. 1 B. 2 C. 3 D. 4 ✓

$$2x^2 + x - 1 = x^2 + 4x + 1$$

$$\Rightarrow x^2 - 3x - 2 = 0$$

$$D = 9 - 4(-2)$$

$$D > 0$$

$$\boxed{2 \text{ Sol}^n}$$

$$2x^2 + x - \cancel{1} = -x^2 - 4x - \cancel{1}$$

$$3x^2 + 5x = 0$$

$$x(3x + 5) = 0$$

$$x = 0, -\frac{5}{3}$$

2 Solⁿ

Solutions of |4x + 3| + |3x - 4| = 12 are

A. $x = -\frac{7}{3}, \frac{3}{7}$

B. $x = -\frac{5}{2}, \frac{2}{5}$

C. $x = -\frac{11}{7}, \frac{13}{7}$

D. $x = -\frac{3}{7}, \frac{7}{5}$

Type-2 $|\square| + |\square| = k$

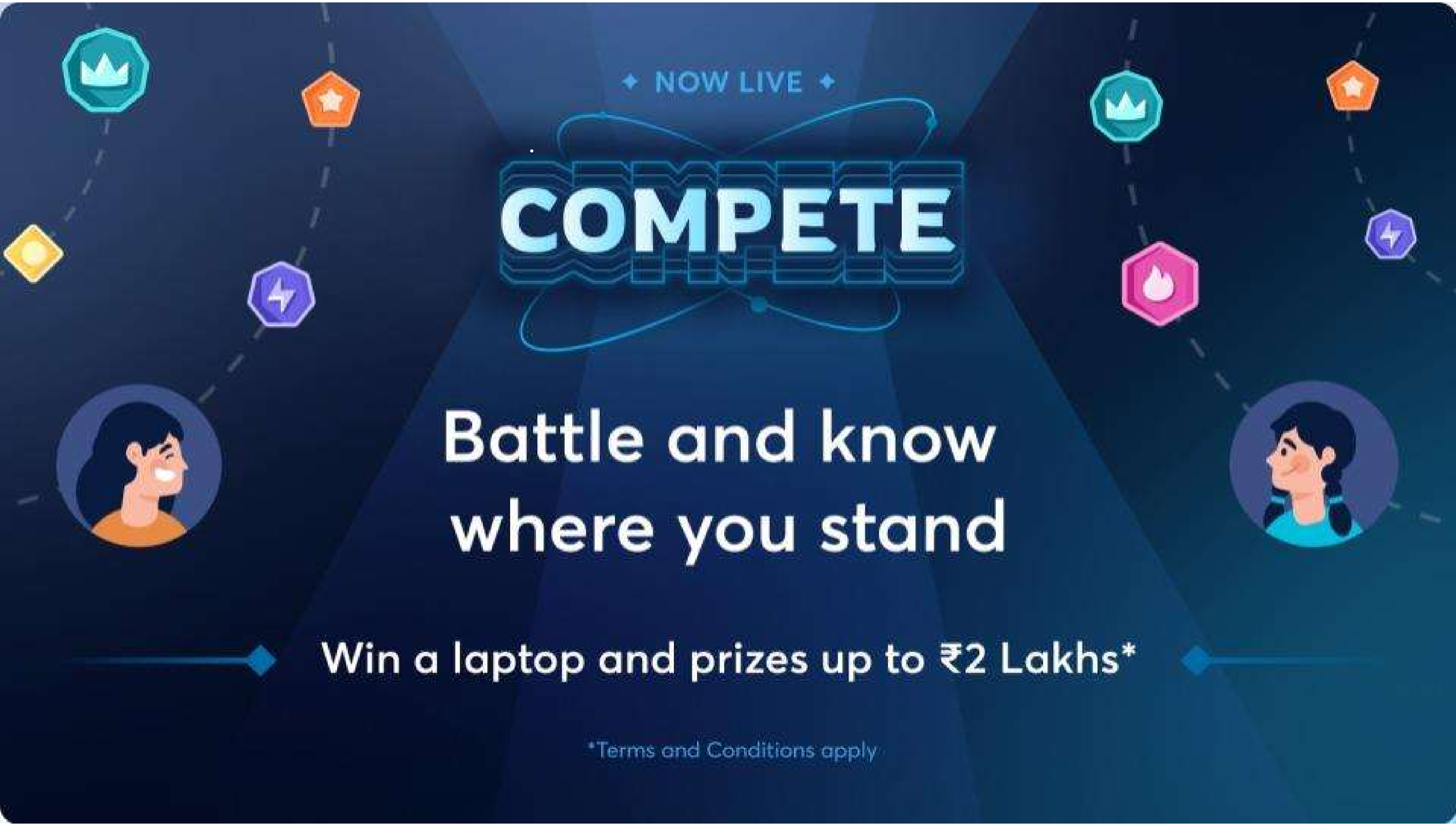
obj

$4x+3+3x-4 = 12$

$7x = 13$

$x = \frac{13}{7}$

[redacted]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Win a laptop and prizes up to ₹2 Lakhs*
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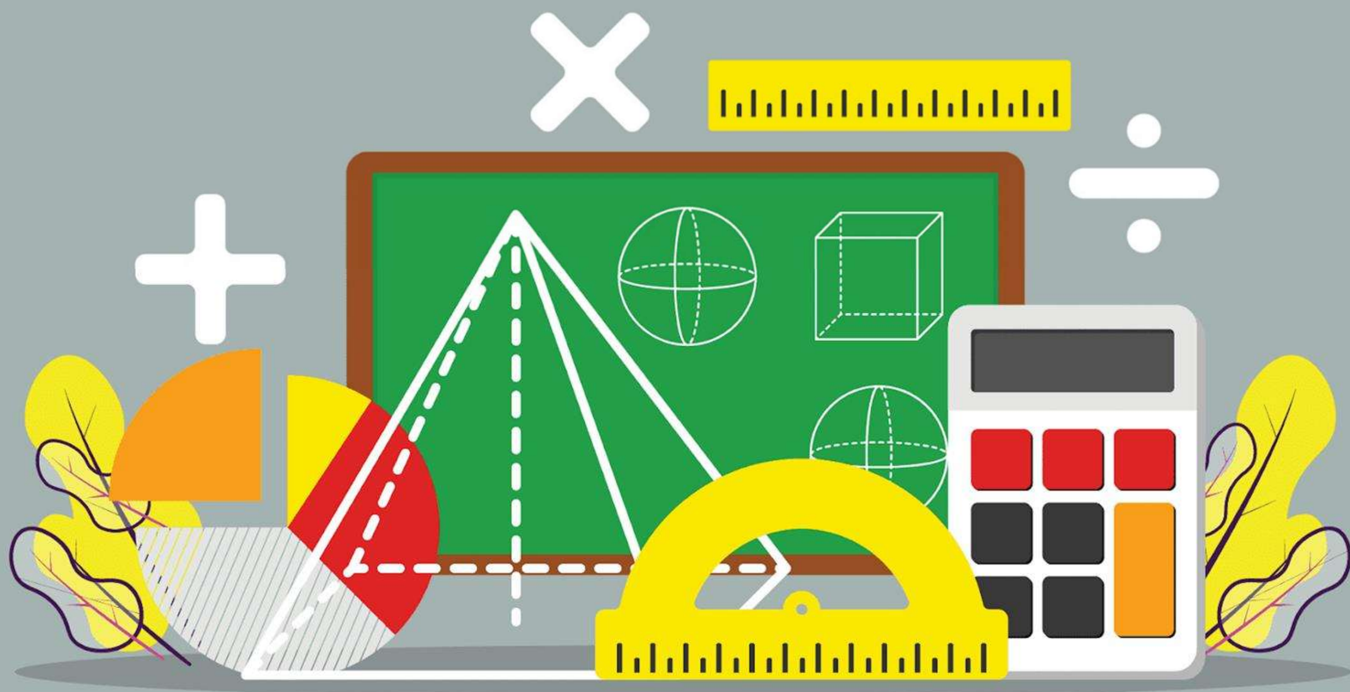
LOG FUNCTION



Log Functions : Definitions

power/ exponent/ index

$2^3 = 8$

base

a number.

Eg. $\log_{10} 100 = 2$

$\log_3 81 = 4$

$2^3 = 8 \longrightarrow \log_2 8 = 3$

Ans

base

Expo Style

Log Style

Find value of:

i. $\log_{81} 27 = ?$

$\log_{81} 27 = K$

$81^K = 27$

$(3^4)^K = 3^3$

$3^{4K} = 3^{3}$

$4K = 3$

$\therefore \boxed{K = 3/4}$

ii. $\log_{10} 100 = ?$

$= 2$

$10^2 = 100$

iii. $\log_{1/3} 9\sqrt{3} = K$

$\left(\frac{1}{3}\right)^K = 9\sqrt{3}$

$(3^{-1})^K = 3^2\, 3^{\frac{1}{2}}$

$3^{-K} = 3^{2+\frac{1}{2}}$

$\therefore \boxed{K = -\frac{5}{2}}$

$\underline{(a^m)^n = a^{mn}}$

Domain: Restriction on base and input

$\log_a x$ is defined if $a > 0$, $a \neq 1$ and $x > 0$

$$\boxed{y = \log_{a} x}$$

will be defined if

i) $\boxed{x > 0}$ Domain

ii) $a > 0$

iii) $a \neq 1$

$\log_2 (-5) = $ N.d.

$\log_0 22 = $ N.d.

$\log_{(-3)} 27 = $ N.d.

$\log_1 25 = $ N.d.

Frequently used base values

$\log_{10} N$

$\ln N = \log_e N$

If a = 10, then write log N rather than $\log_{10}$ N

If a = e, we write lnN rather than $\log_e$ N

$\log_{10} N$ $\quad$ $\log_e N = \underbrace{\ln N}_{\text{Natural Log}}$

$\log_{10} N$ $\quad$ e = Euler's No.

$\boxed{e \approx 2.71}$

$\boxed{\pi \approx 3.14}$

Find the value of

i. $\log_{(2-\sqrt{3})}(2+\sqrt{3}) = k$ $= (-1)$

ii. $\log_{(1/3)}(3\sqrt{3}) = k$

$$(2-\sqrt{3})^k = \frac{(2+\sqrt{3})(2-\sqrt{3})}{(2-\sqrt{3})}$$

$$= \frac{1}{(2-\sqrt{3})}$$

$$= (2-\sqrt{3})^{-1}$$

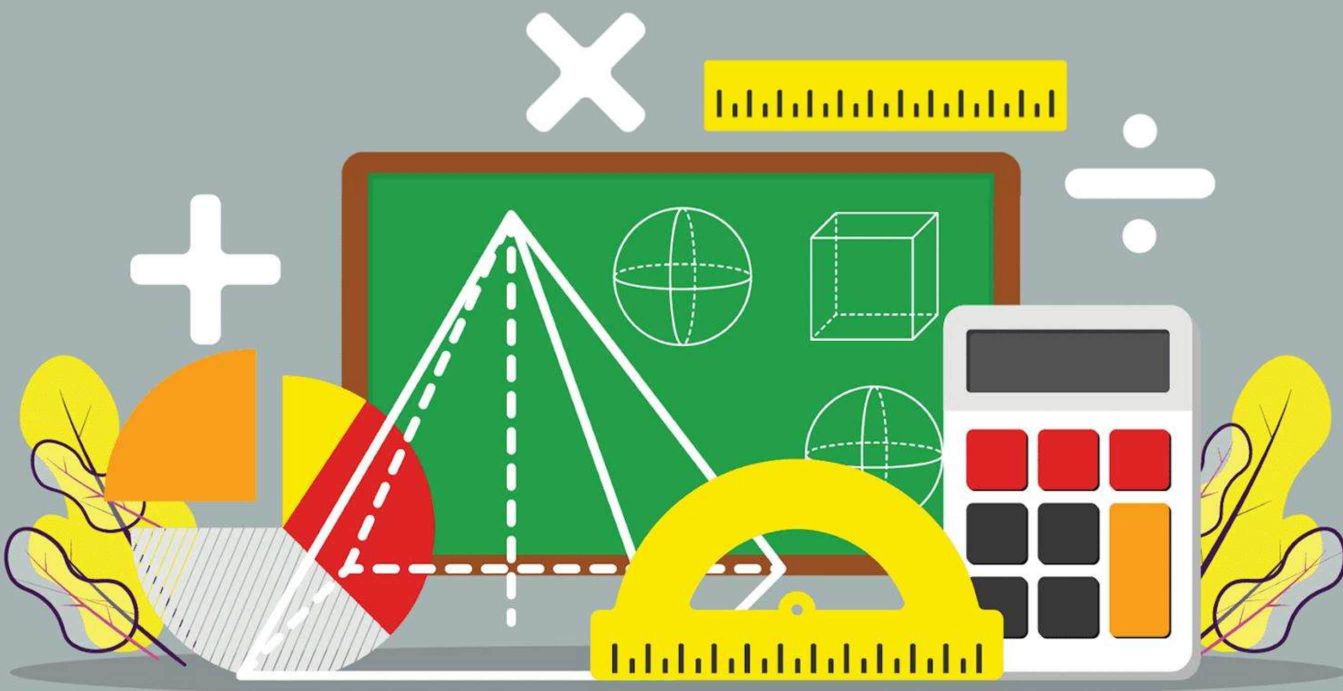
$$\left(\frac{1}{3}\right)^k = 3\sqrt{3}$$

$$3^{-k} = 3^1\ 3^{\frac{1}{2}}$$

$$3^{-k} = 3^{3/2}$$

$$k = \frac{-3}{2}$$

GRAPH OF LOG FUNCTION

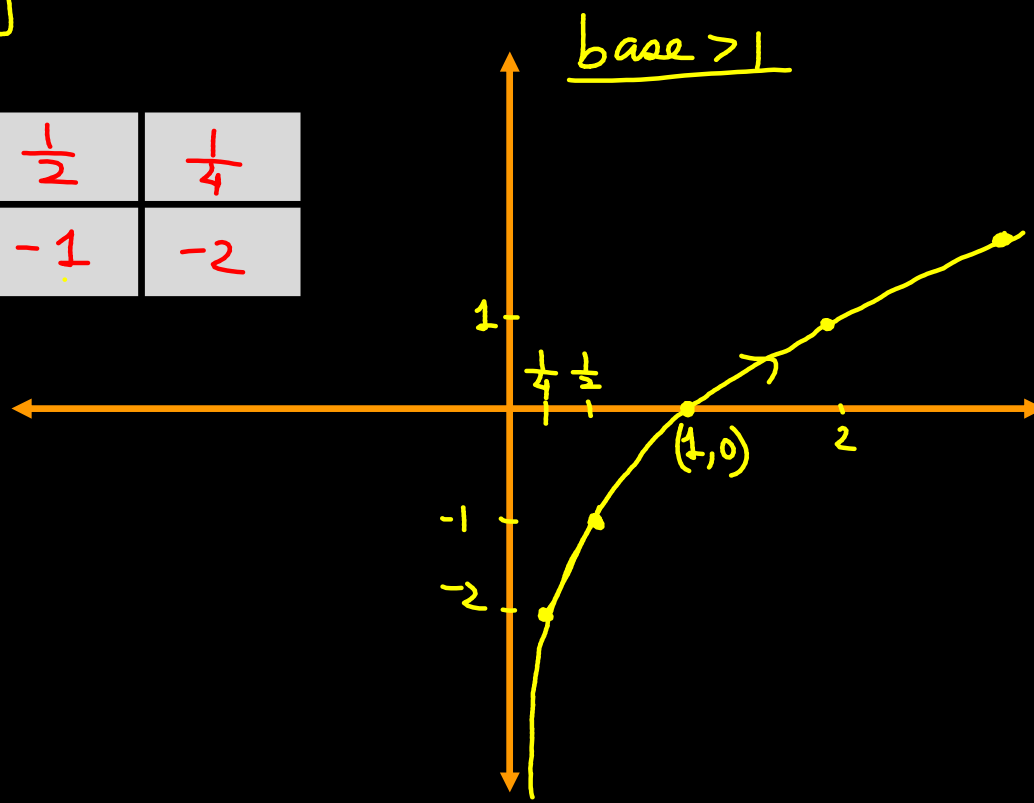


Graphs of Log Functions

$\boxed{base > 1}$

$y = \log_2 x \longleftrightarrow \boxed{2^y = x}$

x	1	2	4	$\frac{1}{2}$	$\frac{1}{4}$
y	0	1	2	-1	-2



Graphs of Log Functions

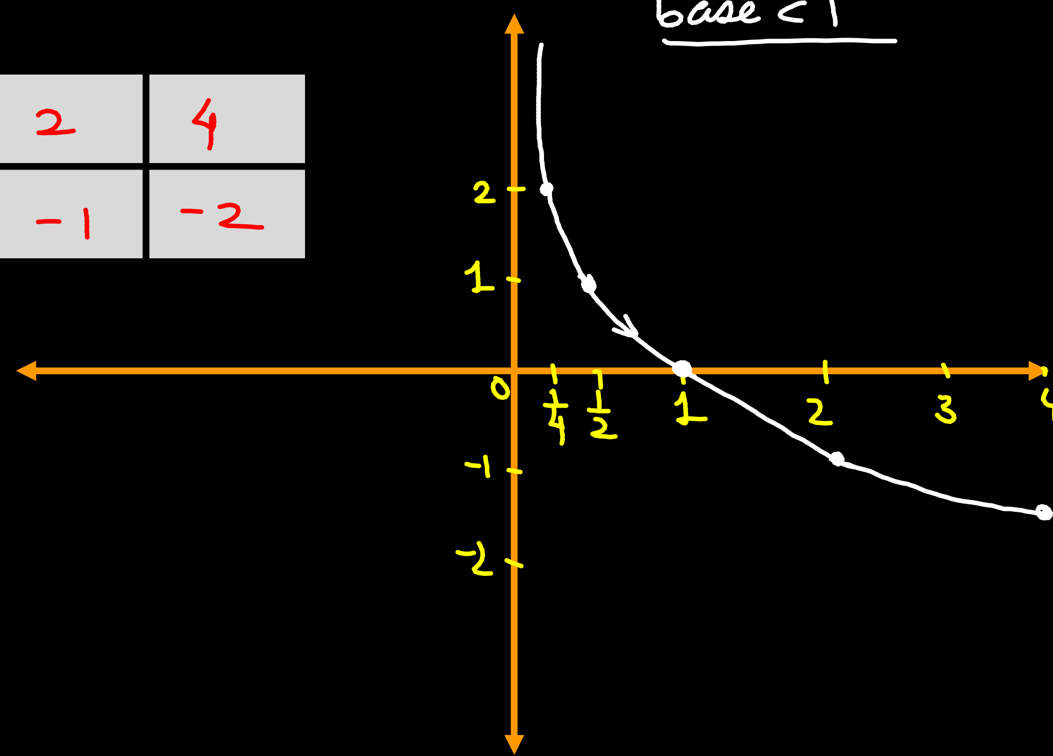
$\boxed{0 < base < 1}$

$y = \log_{1/2} x$

x	1	1/2	1/4	2	4
y	0	1	2	-1	-2

$\left(\frac{1}{2}\right)^{y} = x$

base < 1



Graphs of Log Functions

$y = \log_a x$

Graphs of Log Functions

$y = \log_2 (x + k)$ or $y = \log_2 (x - k)$

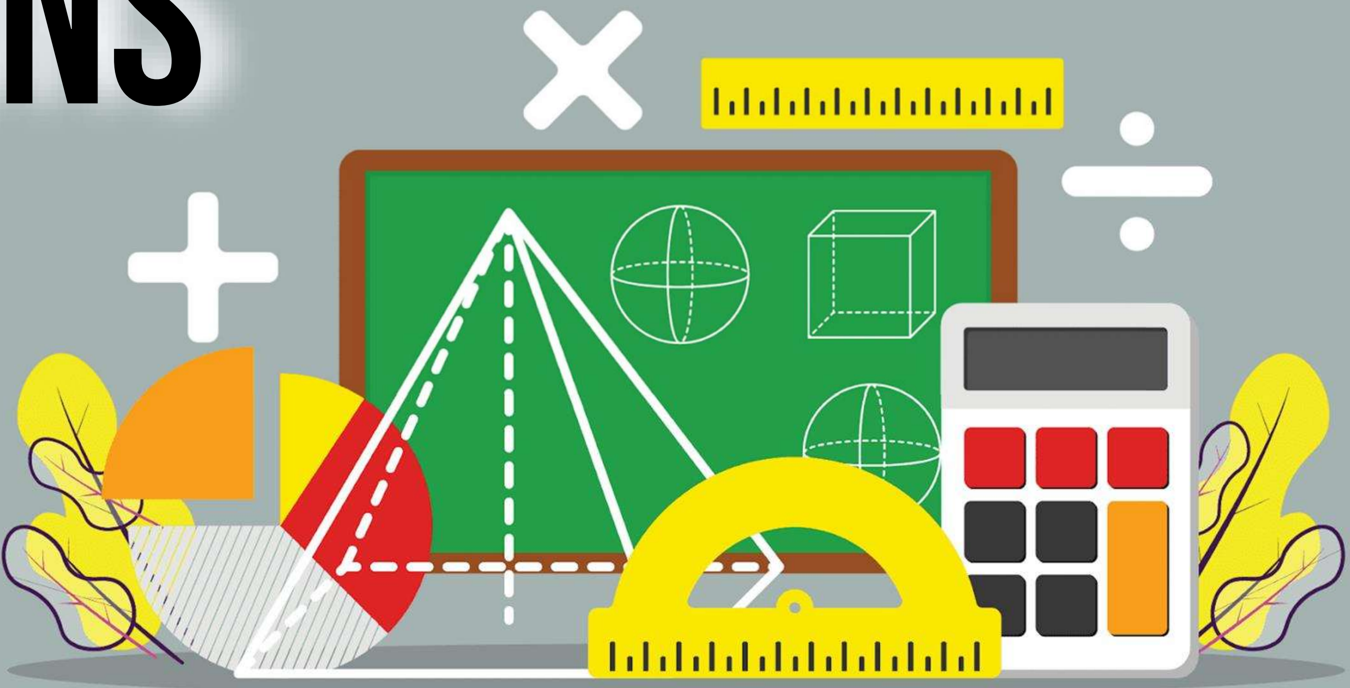
Graphs of Log Functions

$y = \log_2(x) + k$ or $y = \log_2(x) - k$

Graphs of Log Functions

$y = k \log_2 (x)$

PROPERTIES OF LOG FUNCTIONS



Properties of Log Functions

1. $\log_a 1 = 0$
2. $\log_a a = 1$
3. $\log_a (mn) = \log_a m + \log_a n$
4. $\log_a (m/n) = \log_a m - \log_a n$

$\log_{a} 1 = 0$

Properties of Log Functions

5. $\log_{b^n} a^m = \frac{m}{n} \log_b a$

6. $\log_a b = \frac{\log_c b}{\log_c a}$

7. $\log_a b = \frac{1}{\log_b a}$

8. $a^{\log_a x} = x$

$a^{\log_a x} = x$

$\log_{2^4} 3^5$

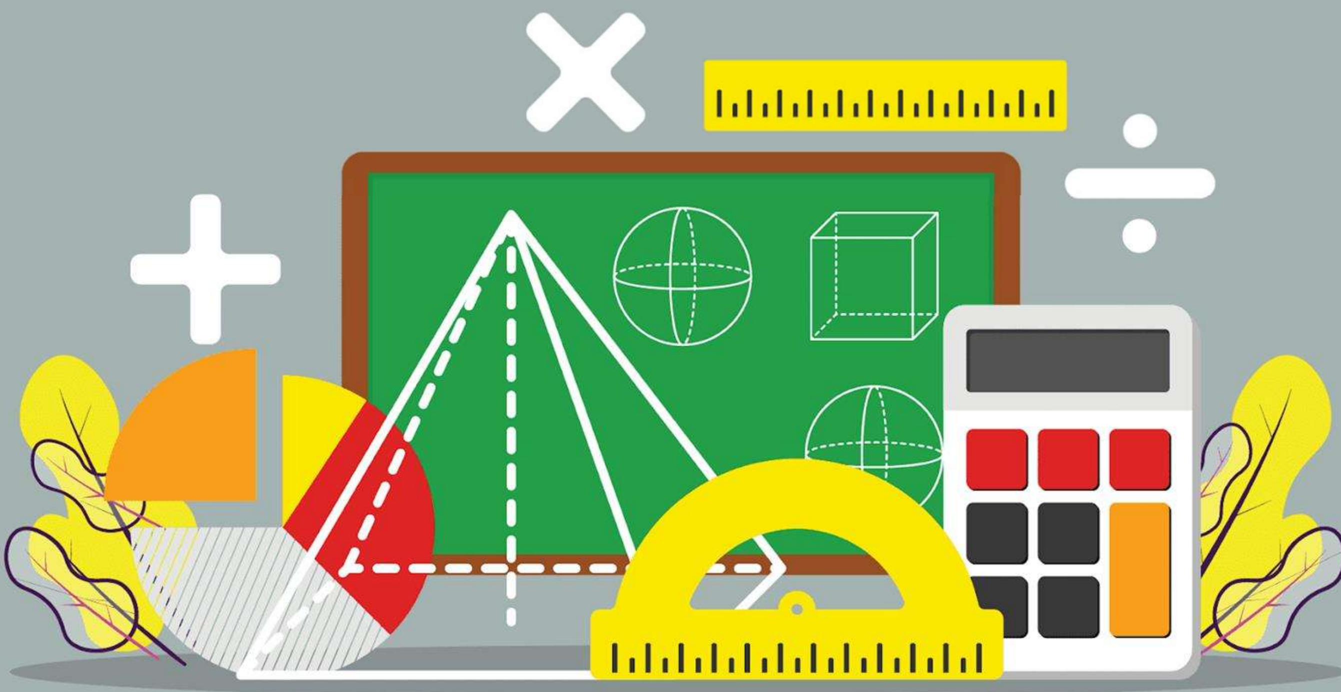
$= \frac{5}{4} \log_2 3$

$\boxed{7^{\log_7 18} = 18}$

$\log_a b = \frac{\log_{10} b}{\log_{10} a}$

$\boxed{\log_2 3 = \frac{1}{\log_3 2}}$

SOLVING LOG EQUATIONS



Solving Log Equations

$\log_2 x = 3$

$\Rightarrow \quad x = 2^3$

$\boxed{x = 8}$ ✓ $\quad \boxed{\text{Domain} : x > 0}$

Unacademy Atoms

Q

$(\log_2 3).(\log_3 4).(\log_4 5)\ ...\ \log_n(n+1) = 10$. Find n = ?

A. 512 **B.** 128 **C.** 1024 **D.** 1023

$$\cancel{\frac{\log 3}{\log 2}} \times \cancel{\frac{\log 4}{\log 3}} \times \cancel{\frac{\log 5}{\log 4}} \times \cdot \cdot \cdot \times \cancel{\frac{\log (n+1)}{\log n}} = 10$$

$$\frac{\log (n+1)}{\log 2} = 10$$

$$\Rightarrow \log_2^{\,n+1} = 10$$

$$\Rightarrow n+1 = 2^{10}$$

$$\Rightarrow \boxed{n = 2^{10} - 1}$$

Unacademy Atoms

Q

Solve $\frac{1}{\log_3 2}+\frac{2}{\log_9 4}-\frac{3}{\log_{27} 8}$

A. 0 **B.** 1 **C.** 2 **D.** 2

$\Rightarrow \log_2 3 + 2\log_4 9 - 3\log_8 27$

$\Rightarrow \log_2 3 + 2\frac{\cancel{2}}{\cancel{2}}\log_{2^2} 3^2 - 3\frac{\cancel{3}}{\cancel{3}}\log_{2^3} 3^3$

$\Rightarrow 1\log_2 3 + 2\log_2 3 - 3\log_2 3$

$\Rightarrow 0$

Unacademy Atoms

Q

Solve $x^2 + 7^{\log_7 x} - 2 = 0$

A. 0 **B.** 1 **C.** -2 **D.** -2, 1

IIT B

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = \cancel{-2}, 1$

$\boxed{7^{\log_7(-2)}}$

Find the value of x : $(x+1)^{\log_{10}(x+1)} = 100(x+1)$

$$\log_{10}(x+1)^{\log_{10}(x+1)} = \log_{10}\left(100(x+1)\right)$$

$$\left(\log_{10}(x+1)\right)^2 = \log_{10} 100 + \log_{10}(x+1)$$

$$t^2 = 2 + t$$

$$t^2 - t - 2 = 0$$

$$(t-2)(t+1) = 0$$

$$x = -0.9$$

$$x+1 = 0.1$$

$$\log_{10}(x+1) = 2 \text{ OR } -1$$

$$x+1 = 10^2 \text{ OR } 10^{-1}$$

$x+1 = 100$ | $x+1 = 0.1$

$x = 99$ ✓ | $\boxed{x = -0.9}$ ✓

Q

Find the value of x : $3^{\log_3^2 x} + x^{\log_3 x} = 162$

$(a^m)^n = a^{mn}$

$= 3^{(\log_3 x)(\log_3 x)}$

$= \left(3^{\log_3 x}\right)^{\log_3 x}$

$= x^{\log_3 x}$

$x^{\log_3 x} + x^{\log_3 x} = 162$

$x^{\log_3 x} = \frac{162}{2}$

$\log_3 x^{\boxed{\log_3 x}} = \log_3 81$

$(\log_3 x)^2 = 4$

$$\log_3 x = \pm 2$$

$$x = 3^2 \;/\; 3^{-2}$$

$$\boxed{x = 9 \;/\; \frac{1}{9}}$$

Q

The value of $\left((\log_2 9)^2\right)^{\frac{1}{\log_2(\log_2 9)}} \times (\sqrt{7})^{\frac{1}{\log_4 7}}$ is $\underline{\quad 8 \quad}$.

[JEE-Advanced 2018]

$\log_2 9 = t$

$(t^2)^{\frac{1}{\log_2 t}}$

$= t^{2 \times \log_t 2}$

$= t^{\log_t 2^2}$

$= 4$

$7^{\frac{1}{2} \times \log_7 4}$

$= 7^{\log_7 4^{1/2}}$

$= 2$

Q

If $3^x = 4^{x-1}$, then $x =$

(A) $\frac{2\log_3 2}{2\log_3 2 - 1}$ (B) $\frac{2}{2 - \log_2 3}$ (C) $\frac{1}{1 - \log_4 3}$ (D) $\frac{2\log_2 3}{2\log_2 3 - 1}$

$$3^x = 4^{x-1}$$

$$\log_3 3^x = \log_3 4^{(x-1)}$$

$$x = (x-1)\log_3 4$$

$$x = x\log_3 4 - \log_3 4$$

ABC

[JEE-Advanced 2013, 4, (-1)]

$$\log_3 4 = x(\log_3 4 - 1)$$

$$\therefore x = \frac{\log_3 4}{\log_3 4 - 1}$$

$$\boxed{x = \frac{2\boxed{\log_3 2}}{2\log_3 2 - 1}}$$

$$\frac{2}{2 - \frac{1}{\log_3 2}}$$

$$= \frac{2}{2 - \log_2 3}$$

$$= \frac{1}{1 - \log_{2^2} 3}$$

$$= \frac{1}{1 - \log_4 3}$$

Q

If $f(x)=\log_e\left(\frac{1-x}{1+x}\right), |x|<1$, then $f\left(\frac{2x}{1+x^2}\right)$ is equal to

[JEE-2009]

A. $2f(x)$

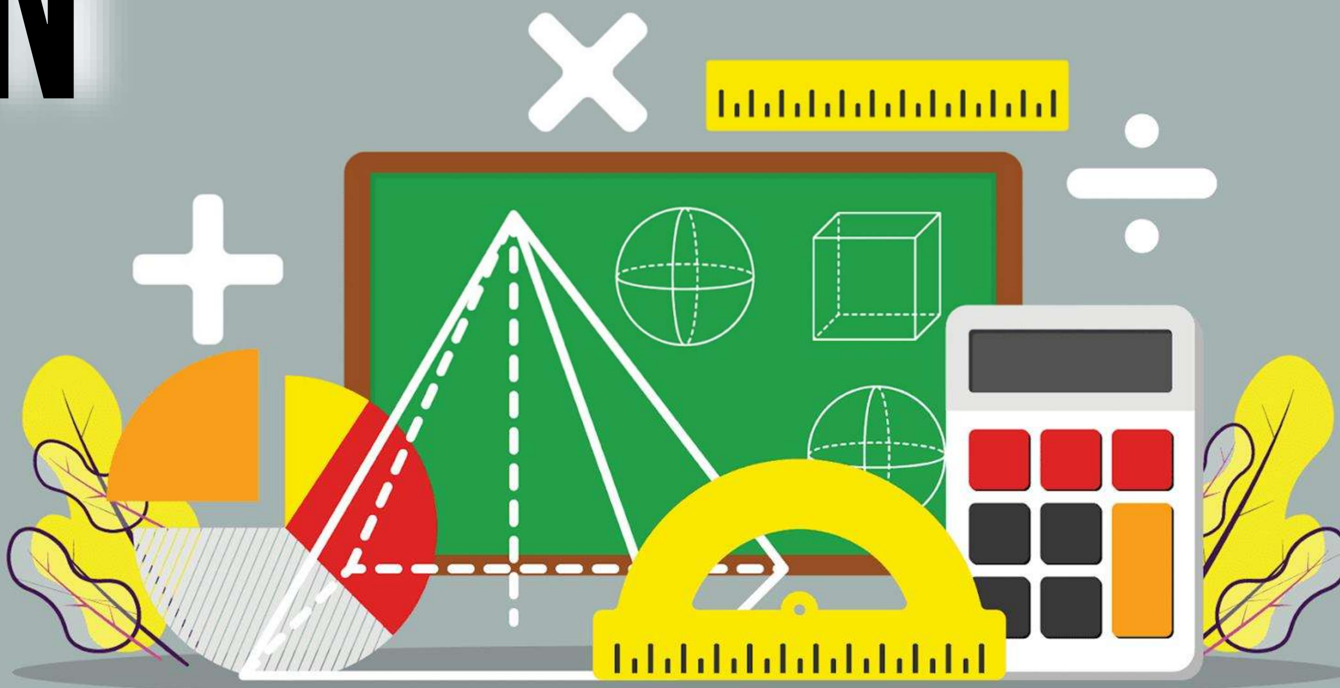
B. $2f(x^2)$

C. $(f(x))^2$

D. $-2f(x)$

H.W.

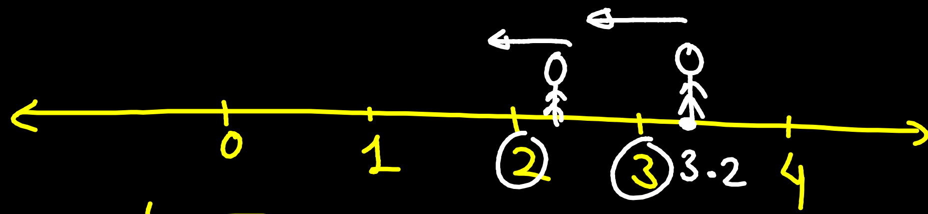
GREATEST INTEGER FUNCTION



Greatest Integer Function (G.I.F)

Definition: Greatest integer less than or equal to x.

$$f(x) = [x]$$

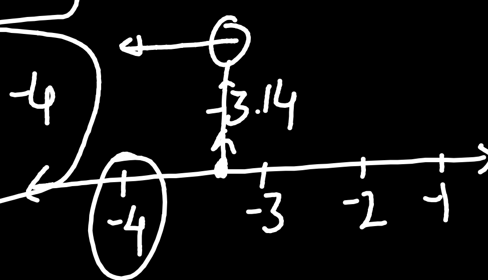


.... -2, -1, 0, 1, 2, 3

$[3.2] = 3$

$[2.1] = 2$

$[-3.14] = -4$



$$[-6.7] = -7$$

$$[2.339] = 2$$

$$[-1.927] = -2$$

$$[0] = 0$$

$$[7] = 7$$

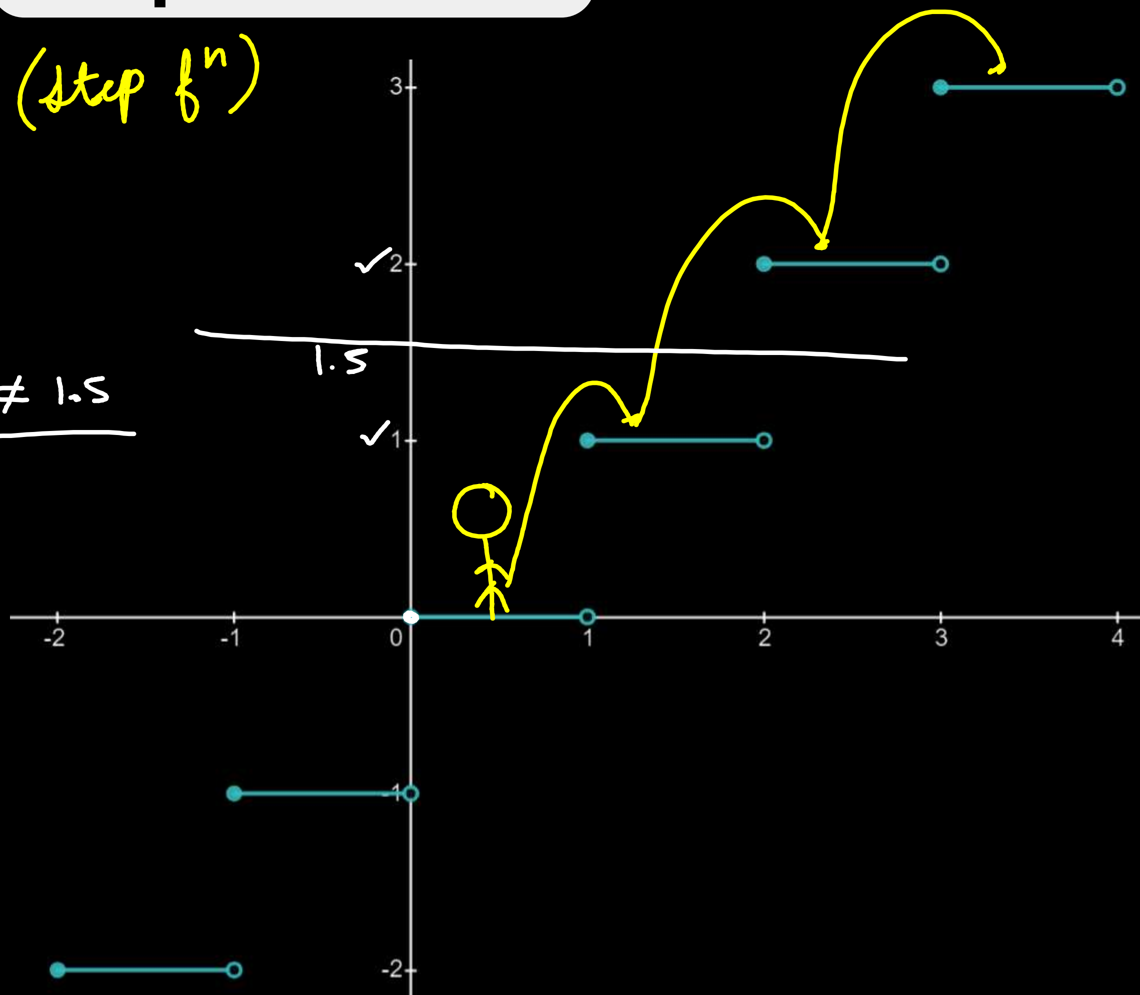
$$[52] = 52$$

$$[-47] = -47$$

Graph of G.I.F

Graph of G.I.F

(step f^n)



$[x] \neq 1.5$

Unacademy Atoms

Domain and Range of G.I.F.

$$\boxed{f(x) = [x]}$$

Domain: $\mathbb{R}$

Range: $\mathbb{Z}$

Properties of G.I.F

1. $x-1 < [x] \leq x$

$x-1 < [x] \leq x$

~~2. $[x+n] = [x]$~~ if n is integer

~~3. $[x+[x]] = [x] + [x]$~~

$$\boxed{[x+[x]] = [x]+[x]}$$

$$\boxed{[x+1] = [x]+1}$$

$$\boxed{x = 2.3}$$

$LHS = [2.3+1] = [3.3] = 3$

$RHS = [2.3]+1 = 3$

Properties of G.I.F

4) $$[x] + [-x] = \begin{cases} 0 & \text{if } x \in z \\ -1 & \text{if } x \notin z \end{cases}$$

$[2] + [-2]$

$= 2 + (-2)$

$= 0$

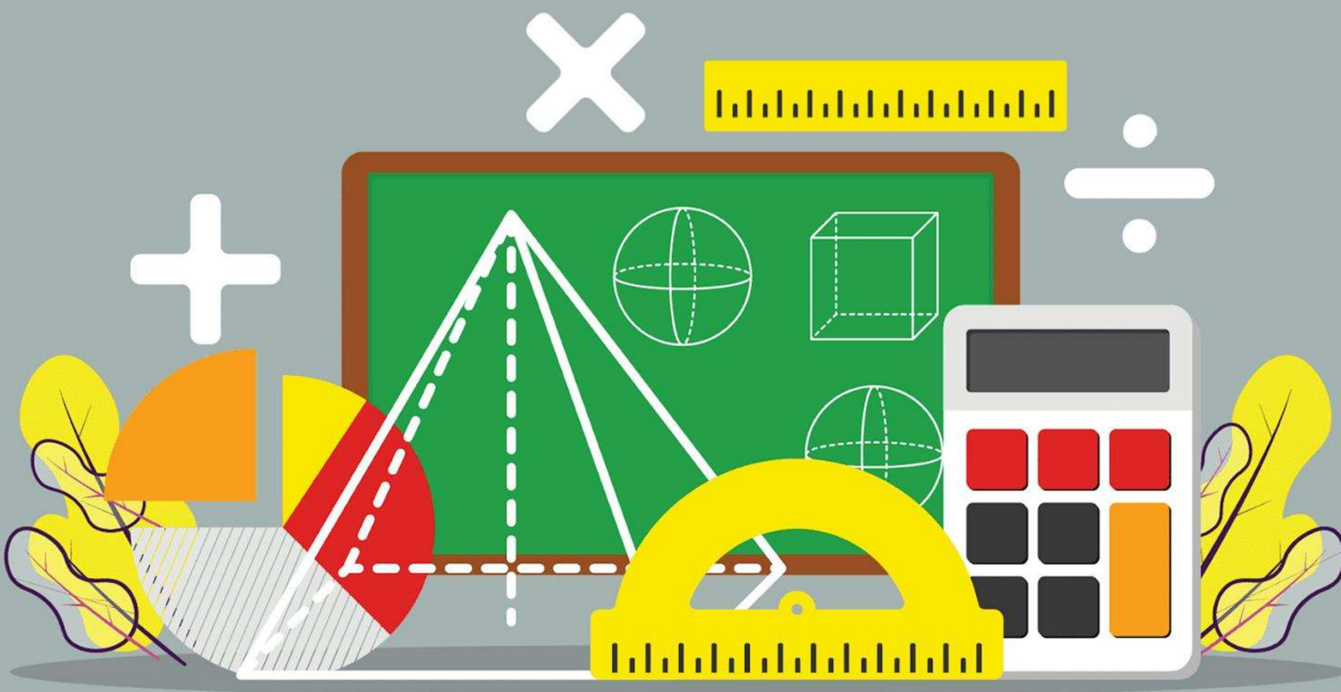
$[3] + [-3] = 0$

$[2.1] + [-2.1]$

$= 2 + (-3)$

$= -1$

FRACTIONAL PART FUNCTION



Fraction Part Function (F.P.F)

Definition: $x = [x] + \{x\}$

$$f(x) = \{x\}$$

$$x = [x] + \{x\}$$

$$3.2 = \underbrace{3}_{[x]} + \underbrace{0.2}_{\{x\}}$$

$$\{x\} = x - [x]$$

$1 - 0.1$

$$\{3.2\} = 3.2 - [3.2] = 0.2$$

$$\{-2.1\} = -2.1 - [-2.1]$$

$$= -2.1 - (-3)$$

$$= -2.1 + 3 = 0.9$$

$$\{3.2\} = \underline{0.2}$$

$$\{-3.2\} = 1 - 0.2 = 0.8$$

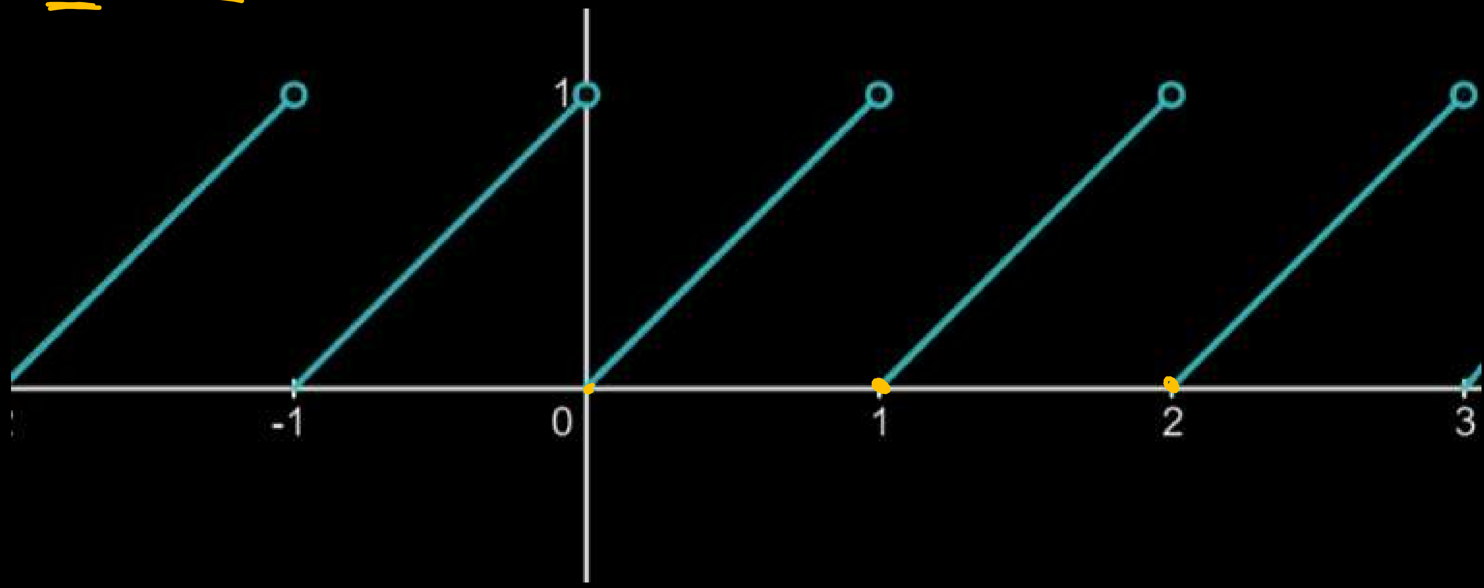
$$\{5.678\} = 0.678$$

$$\{-5.678\} = 1 - 0.678$$

Graph of Fractional Part Function

Graph of Fractional Part Function

$7.0 = 7 + 0$ $\{z\} = 0$ $[z] = z$



Domain and Range of {x}

$$\boxed{f(x) = \{x\}}$$

Domain: R

Range: $[0, 1)$

Properties of {x}

1. $0 \le \{x\} < 1$
2. $x = [x] + \{x\}$
3. $\{x + n\} = \{x\}$ if n is integer

$0 \le \{x\} < 1$

$\{1\} = 0$

$\{x + z\} = \{x\} + \cancel{\{z\}} = \{x\}$

$[x + z] = [x] + [z] = [x] + z$

Properties of {x}

$$\{x\} + \{-x\} = \begin{cases} 0 & if\ x \in Z \\ 1 & if\ x \notin Z \end{cases}$$

$x \in Z$

$\{2\} + \{-2\}$

$= 0 + 0$

$= 0$

$\{2.1\} + \{-2.1\}$

$= 0.\cancel{1} + 1 - 0.\cancel{1}$

$= 1$

Solve the equation

$\boxed{4[x] = x + \{x\}}$

S-1 $x = [x] + \{x\}$

Method

$4[x] = [x] + \{x\} + \{x\}$

$3[x] = 2\{x\}$

S-2 $\boxed{\{x\} = \frac{3[x]}{2}}$ — (1)

S-3 $0 \le \{x\} < 1$

$0 \le \frac{3[x]}{2} < 1$

$0 \le 3[x] < 2$

$0 \le [x] < \frac{2}{3}$

$[x] = 0$

S-4

$$[x] = 0$$

$$\{x\} = 0$$

$$x = [x] + \{x\}$$

$$= 0 + 0$$

$$\therefore \boxed{x = 0}$$

The number of solution(s) of the equation $[2x] - 3\{2x\} = 1$, is / are

(A) 1 (B) 2

(C) 3 (D) 0

S-1 ✗

S-2 $\boxed{\{2x\} = \dfrac{[2x]-1}{3}}$

S-3 $0 \le \{2x\} < 1$

$0 \le \dfrac{[2x]-1}{3} < 1$

$0 \le [2x]-1 < 3$

$\boxed{1 \le [2x] < 4}$

$[2x] = 1, 2, 3$

$\{2x\} = 0, \frac{1}{3}, \frac{2}{3}$

$2x = 1, \frac{7}{3}, \frac{11}{3}$

$\boxed{x = \frac{1}{2}, \frac{7}{6}, \frac{11}{6}}$

The number of solution(s) of the equation $[x] + \{-x\} = 2x$, is / are

(A) 1 (B) 2

(C) 3 (D) 0

H.W.

If [x] be the greatest integer less than or equal to x, then $\sum_{n=8}^{100}\left[\frac{(-1)^n n}{2}\right]$ is equal to.

[JEE-2021]

A. 0

B. 4

C. -2

D. 2

$$\sum_{n=8}^{100}\left[\frac{(-1)^n n}{2}\right] \qquad [-4.5] \qquad [-49.5]$$

$$\Rightarrow \left[\frac{(-1)^8 8}{2}\right] + \left[\frac{(-1)^9 9}{2}\right] + \left[\frac{(-1)^{10} 10}{2}\right] + \left[\frac{(-1)^{11} 11}{2}\right] + \dots + \left[\frac{(-1)^{99} 99}{2}\right] + \left[\frac{(-1)^{100} 100}{2}\right]$$

$$\Rightarrow 4 + \overbrace{(\cancel{-5}) + (\cancel{5}) + (\cancel{6}) + (\cancel{-6}) + \dots + (\cancel{-50}) + (\cancel{50})}^{92 \text{ terms}}$$

$$\Rightarrow 4$$

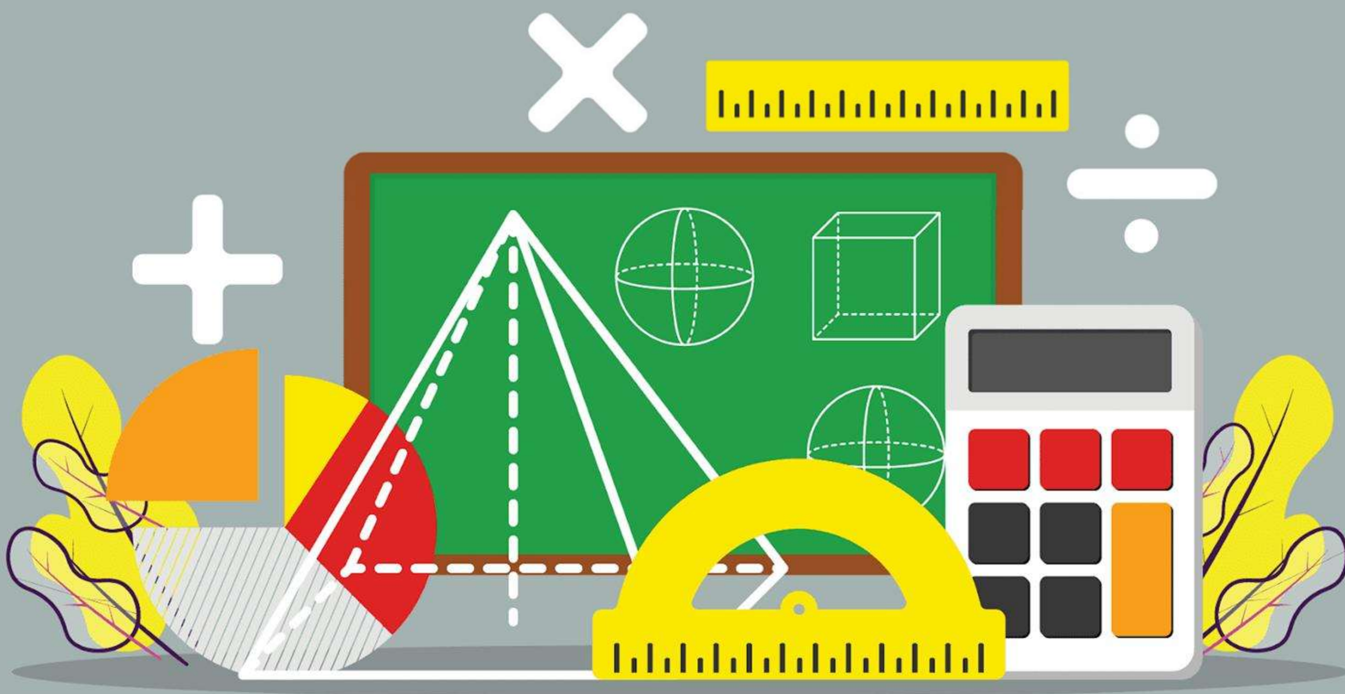
Q

Let $f(n) = \left[\frac{1}{2} + \frac{n}{100}\right]$, where [.] denotes the greatest integer function, then the value of $\sum_{n=1}^{151} f(n)$ is

(A) 101
(B) 102
(C) 104
(D) 103

H.W.

EXPONENTIAL FUNCTION



Exponential Functions

Unacademy Atoms

$$f(x) = a^x \text{ where } a > 0$$

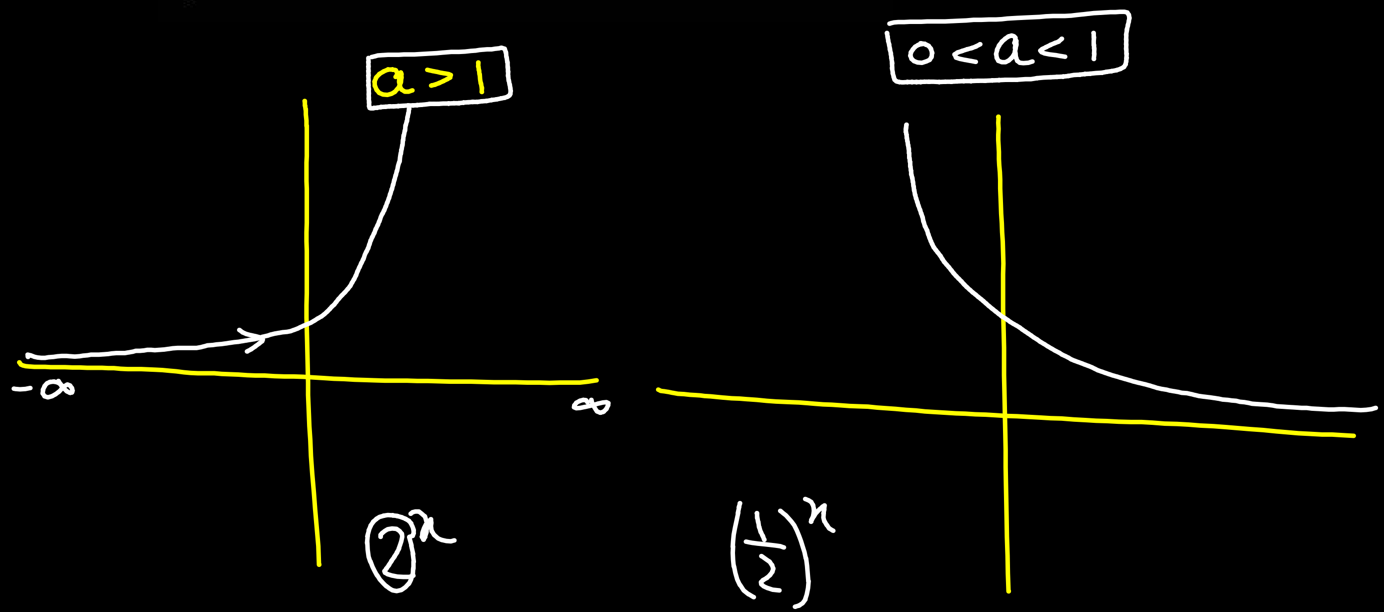
2^x , 3^x , 5^x $\left(\frac{1}{2}\right)^x$ e^x

$\cancel{(-2)^x}$

famous

Graph of Exponential Functions

$$f(x) = a^x \text{ where } a > 0$$



Exponential Functions

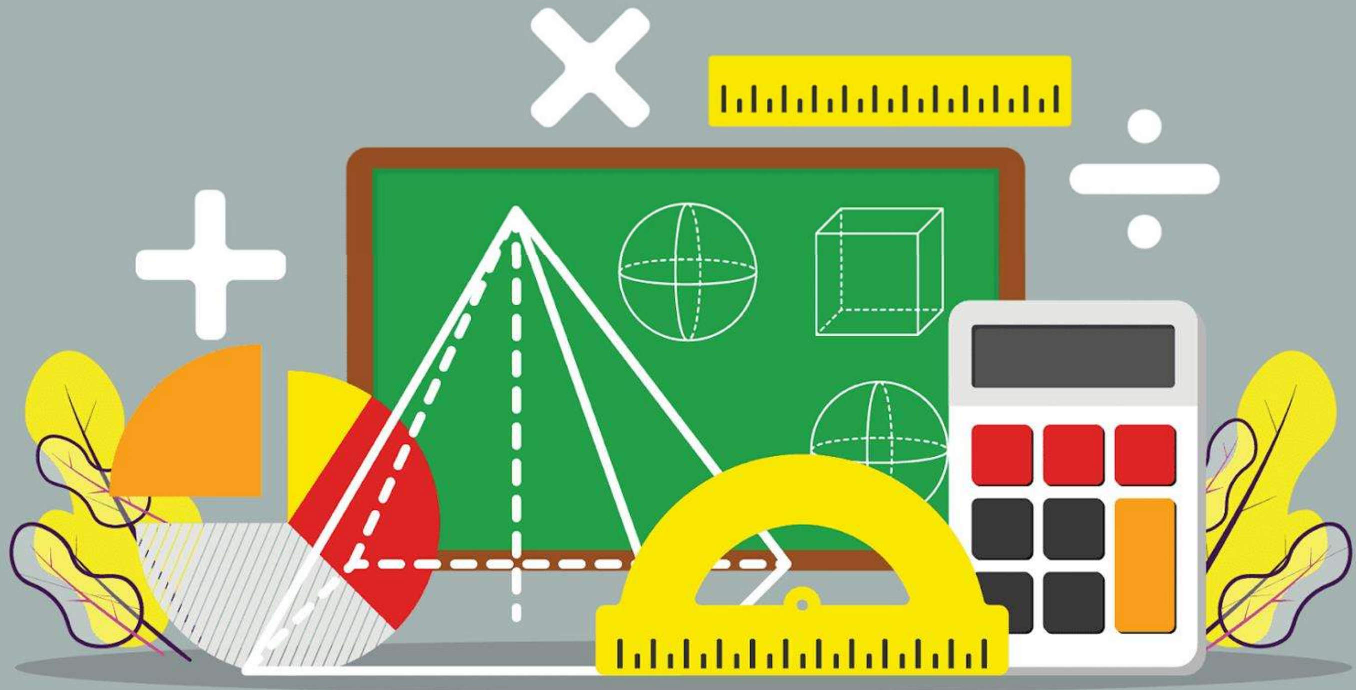
$$f(x) = a^x \ \ where\ a > 0$$

Domain: $(-\infty, \infty)$ OR R

Range: $(0, \infty)$ OR R^+

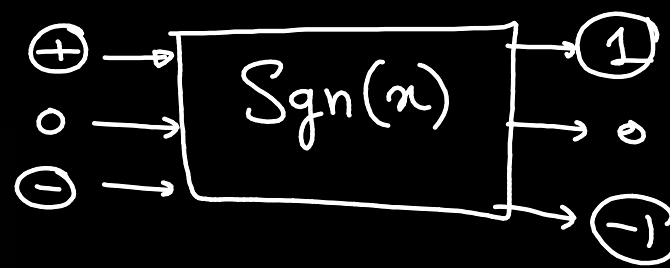
Unacademy Atoms

SIGNUM FUNCTION



Signum Function

$$f(x) = sgn(x) = \begin{cases} 1 & if\ x > 0 \\ 0 & if\ x = 0 \\ -1 & if\ x < 0 \end{cases}$$



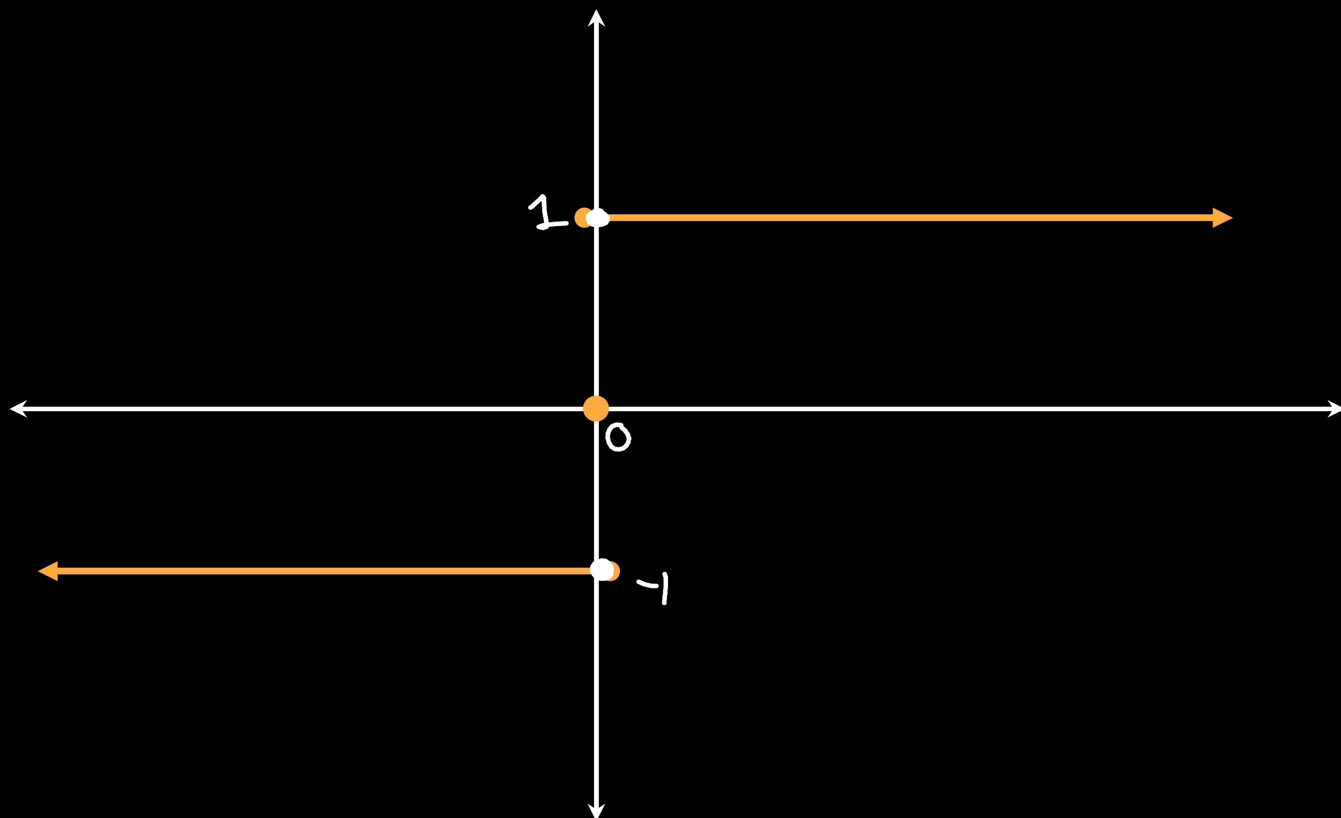
or

$$f(x) = sgn(x) = \begin{cases} \dfrac{|x|}{x} & if\ x \neq 0 \\ 0 & if\ x = 0 \end{cases}$$

$\frac{x}{x}$ $\quad x > 0$

$\frac{-x}{x}$ $\quad x < 0$

Graph of Signum Function



Q

Find the number of integers for which $sgn(x^2 - 2x - 8) = -1$

$x^2 - 2x - 8 < 0$

$(x-4)(x+2) < 0$

$x \in (-2, 4)$

$\boxed{-1, 0, 1, 2, 3}$

$\boxed{\text{Ans: } 5}$

Sgn $\square = 1 \Rightarrow \square > 0$

Sgn $\square = -1 \Rightarrow \square < 0$

Sgn $\square = 0 \Rightarrow \square = 0$

Q

$\text{sgn}(x^3 - 4x^2 + 3x) = 1$, $x \in Z$ and $x \in [-5, 10]$, then number of possible values of x is :

(A) 7 (B) 13

(C) 10 (D) 8

H.W.