Arithmetic Progressions

Sequene

+3 +3 +3

(i) 4, 7, 10, 13, $\underline{16}$ → AP

×3 ×3 ×3 ×3

(ii) 2, 6, 18, 54, $\underline{162}$ → G.P

−3 −3 −3 −3

(iii) 18, 15, 12, 9, $\underline{6}$ → AP

(iv) 2, 7, 8, 18, 25, ___ X

+5 +1 +10 +7

Arithmetic Progressions

First term = a

AP:→ $\overset{+3}{\frown}$ 4, 7, 10, 13, 16 ..

$a_1, a_2, a_3, a_4, a_5 \ldots a_n$ → last term

First term

+ − 0

fixed number = d = Common-Difference = 3

$a_1 = 4 = a$

$a_2 = 7 = 4+3 = a+d$

$a_3 = 10 = 4+3+3 = 4+2(3) = a+2d$

$a_4 = 13 = 4+3+3+3 = 4+3(3) = a+3d$

$a_5 = a+4d$

$a_7 = a+6d$

$a_{100} = a+99d$

⋮

$$a_n = a+(n-1)d$$

a_n: last term; a: first term; n: no of term; d: common difference

AP:→ $a, a+d, a+2d, \ldots a+(n-1)d$

↳ General form of an A.P.

Common-Difference

$d = a_2 - a_1$

$d = a_3 - a_2$

$d = a_4 - a_3$

$$\boxed{d = a_n - a_{n-1}}$$

QUESTION

Write first four terms of the AP, when the first term a and the common difference d are given as follows:

$a = 4, d = -3$

AP:→ 4, 1, −2, −5 ...

AP:→ a, a+d, a+2d, a+3d..

$a = 4$

$a+d = 4+(-3) = 1$

$a+2d = 4+2(-3) = 4-6 = -2$

$a+3d = 4+3(-3) = 4-9 = -5$

QUESTION

For the given AP, write the first term and the common difference:
$-5, -1, 3, 7, \ldots$

$a = -5$

$d = a_2 - a_1 = -1 - (-5)$

$= -1 + 5$

$d = 4$

QUESTION

Which of the following are APs ? If they form an AP, find the common difference d and write three more terms.

(i) 2, 4, 8, 16, ...

(ii) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \ldots$

$\sqrt{2},\ 2\sqrt{2},\ 3\sqrt{2},\ 4\sqrt{2},\ 5\sqrt{2},\ 6\sqrt{2},\ 7\sqrt{2}$

(i) $d = a_2 - a_1 = 4 - 2 = 2$

$d = a_3 - a_2 = 8 - 4 = 4$

It is not an AP ✗

(ii) $d = a_2 - a_1 = \sqrt{8} - \sqrt{2}$

$= \sqrt{2\times2\times2} - \sqrt{2}$

$= 2\sqrt{2} - \sqrt{2}$

$= \sqrt{2}$

$d = a_3 - a_2$

$= \sqrt{18} - \sqrt{8}$

$= \sqrt{3\times3\times2} - 2\sqrt{2}$

$= 3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$

$d = a_4 - a_3$

$= \sqrt{32} - \sqrt{18}$

$= \sqrt{4\times4\times2} - 3\sqrt{2}$

$= 4\sqrt{2} - 3\sqrt{2}$

$= \sqrt{2}$

It is an AP.

QUESTION

Which term of the AP : 3, 15, 27, 39, . . . will be 132 more than its 54^{th} term?

$n = ?$

$a = 3$

$d = 15 - 3 = 12$

$$a_n = a_{54} + 132$$

$$\cancel{a} + (n-1)d = \cancel{a} + 53d + 132$$

$$(n-1)12 = 53(12) + 132$$

$$(n-1)12 = 636 + 132$$

$$(n-1)12 = 768$$

$$n - 1 = \frac{\cancel{768}}{\cancel{12}} = 64$$

$n - 1 = 64$

$n = 64 + 1$

$n = 65$

QUESTION

Find the 20th term from the last term of the AP : 3, 8, 13, . . ., 253.

$n = 20$

$a = 3$

$a_n = l = 253$

$d = 5$

$$\boxed{a_n = l - (n-1)d}$$

$a_n = 253 - (20-1)5$

$= 253 - (19)5$

$= 253 - 95$

$= 158$

Sum of first n terms of an AP

AP: → 2, 4, 6, 8, 10 . . . , 100

AP: → 2 + 4 + 6 + 8 + 10 + . . + 100

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_n = \frac{n}{2}\left[a + \underbrace{a + (n-1)d}\right]$$

$$S_n = \frac{n}{2}\left[a + a_n\right]$$

QUESTION

Show that $a_1, a_2, \ldots, a_n, \ldots$ form an AP where a_n is defined as below :
$a_n = 3 + 4n$
Also find the sum of the first 15 terms in each case

Note :→

a, b, c

$a + c = 2b$

$15 + 7 = 2(11)$

$22 = 22$

$a_n = 3 + 4n$

AP :→ $\overset{+4}{7, 11}, \overset{+4}{15} \ldots$

$a = 7$

$d = 11 - 7 = 4$

$n = 15$

$a_1 = 3 + 4(1) = 3 + 4 = 7$

$a_2 = 3 + 4(2) = 3 + 8 = 11$

$a_3 = 3 + 4(3) = 3 + 12 = 15$

$a-d, a, a+d$

$a-3d, a-d, a+d, a+3d$

$a-2d, a-d, a, a+d, a+2d$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{15}{2}[14 + 14(4)]$

$= \frac{15}{2}[14 + 56]$

$= \frac{15}{\cancel{2}} \times \cancel{70}\ 35$

$= 15 \times 35 \Rightarrow 525$