Topics to be covered

1) Pressure and Thrust
2) Archimedes' Principle and Pascal's Law
3) Bernoulli's Theorem
4) Viscous Force and Stoke's Law
5) Surface Tension

Fluid Mechanics

Fluid Statics

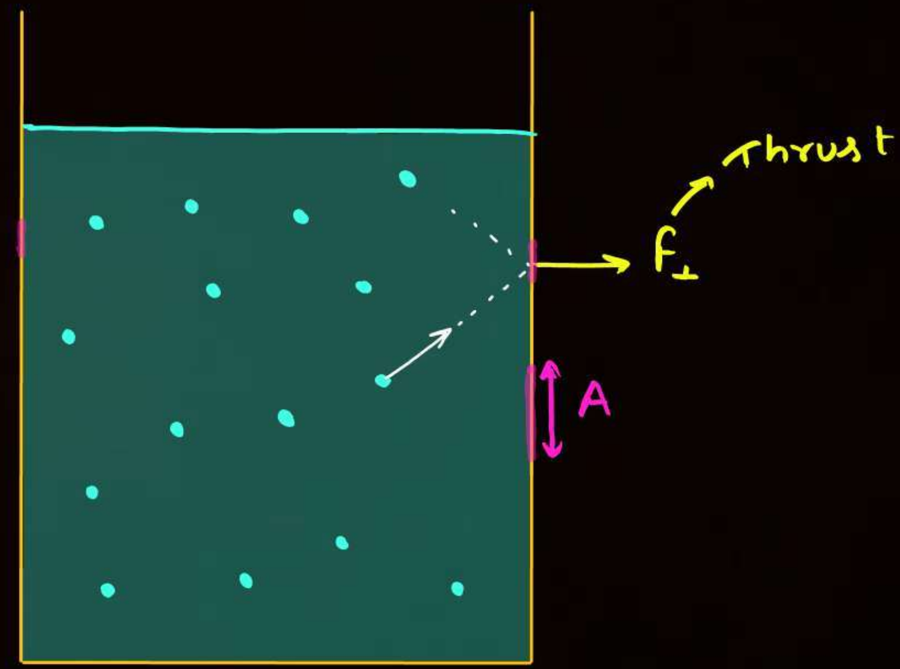
Fluid Dynamics.

Thrust

$\perp$ force exerted by liquid

$F_\perp \propto A$

$$P = \frac{\text{Thrust}}{\text{Area.}}$$



Pressure Inside Liquid

$$F_2 = F_1 + mg$$

$$P \cdot A = P_0 A + \rho V g$$

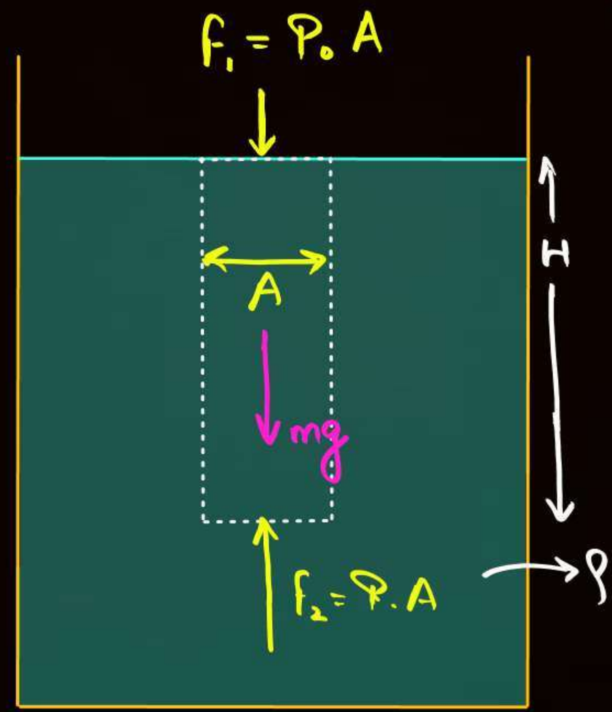
$$P\cancel{A} = P_0\cancel{A} + \rho \cancel{A} H g$$

$$\Rightarrow \boxed{P = P_0 + \underbrace{\rho g H}_{\text{Gauge pressure}}}$$

P : Absolute Pressure at H depth

P_0 : Atmospheric Pressure

$\rho g H$: Gauge pressure



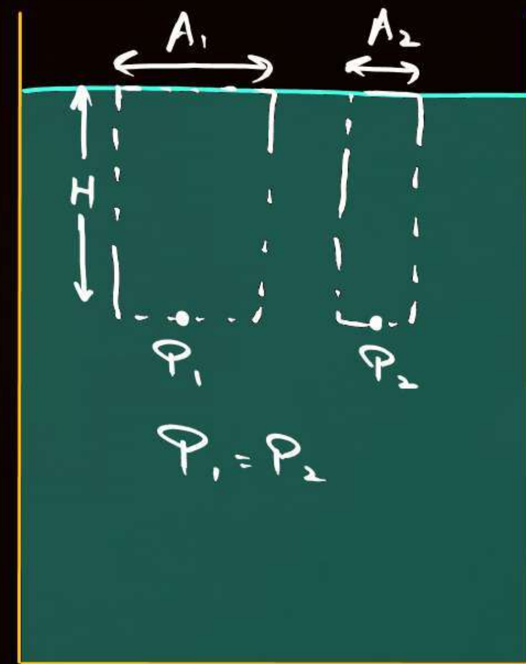
$$\text{Pressure} = \frac{\text{Thrust}}{\text{Area}} = \frac{F_\perp}{A}$$

- Scalar
- SI unit: N/m^2 = Pascal
- Other popular units —
  - ↳ Atmospheric pressure (Atm)
  - ↳ cm Hg.
  - ↳ Bar

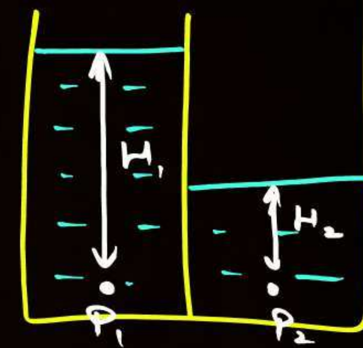
- P is independent of area of cross-section of liquid.
- At same depth, pressure inside a continuous uniform non-accelerating liquid is same

- $P_0 = 10^5\ N/m^2$
  or
  $10^5\ Pa.$

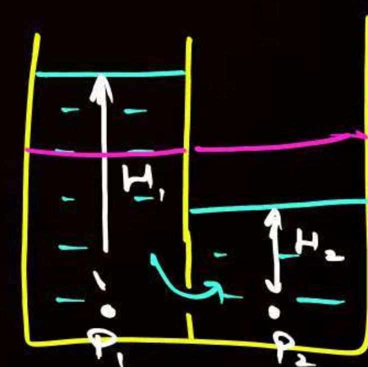
$P = 10^5\ Pa.$



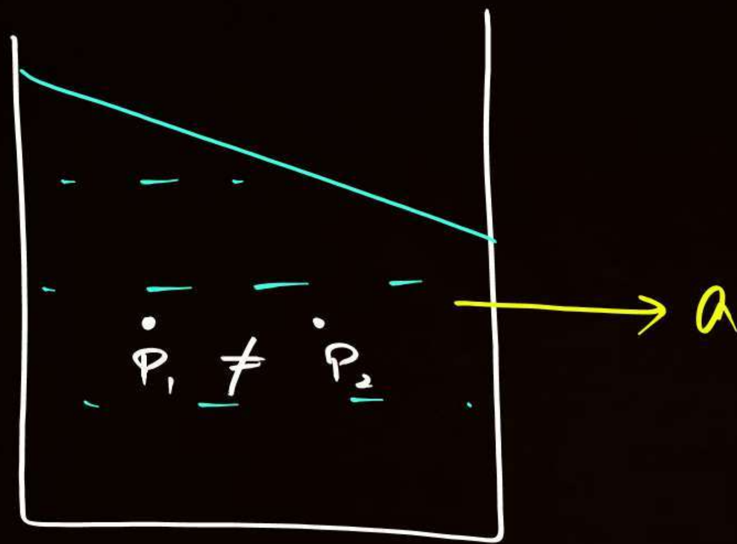
$P_1 = P_2$



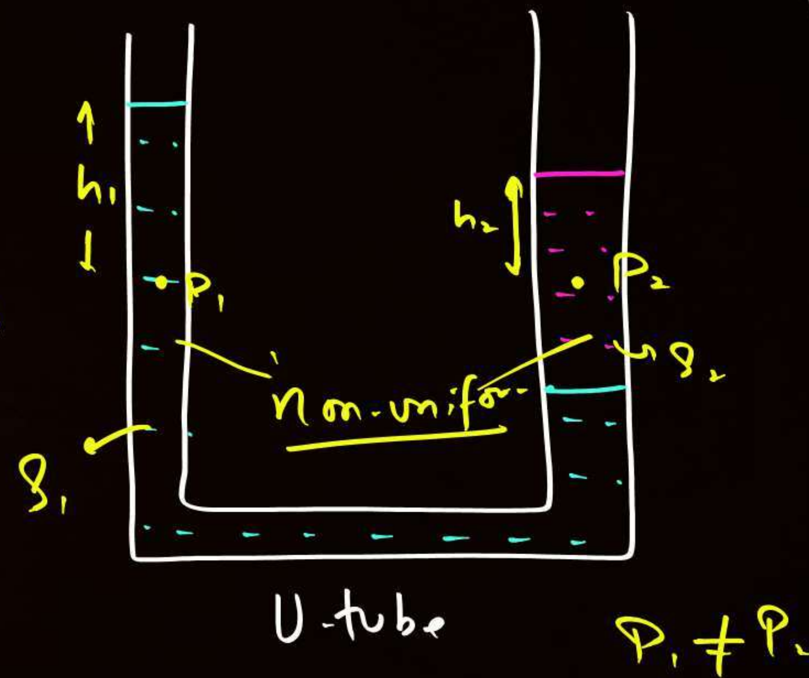
$P_1 \neq P_2$



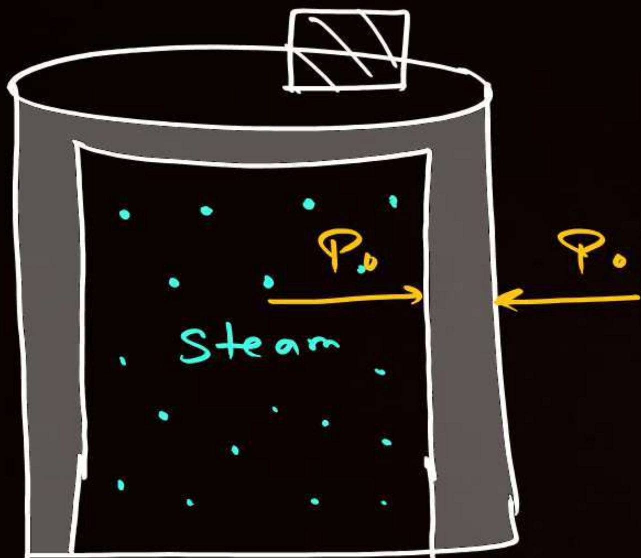
$P_1 \neq P_2$

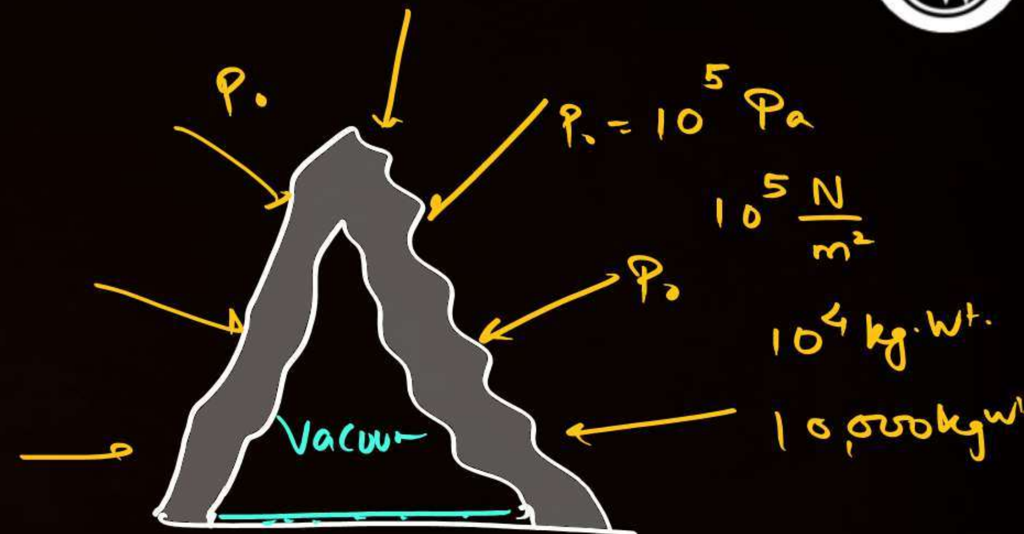


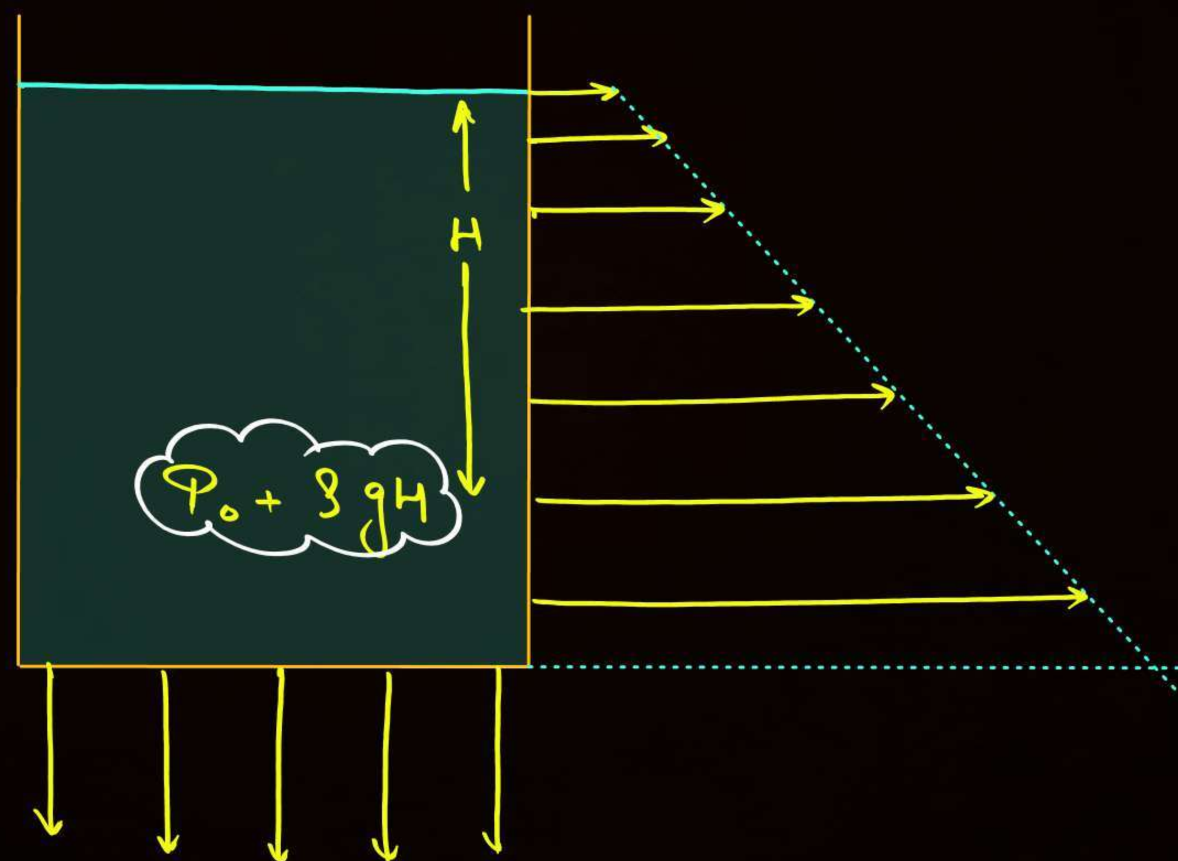
$P_1 \neq P_2$
a



h_1
h_2
P_1
P_2
ρ_1
ρ_2
non-uniform
U-tube
$P_1 \neq P_2$


P0
P0
Steam


P0
P0
Vacuum
P0 = 10^5 Pa
10^5 N/m^2
10^4 kg.wt.
10,000 kg wt



H
P₀ + ρgH

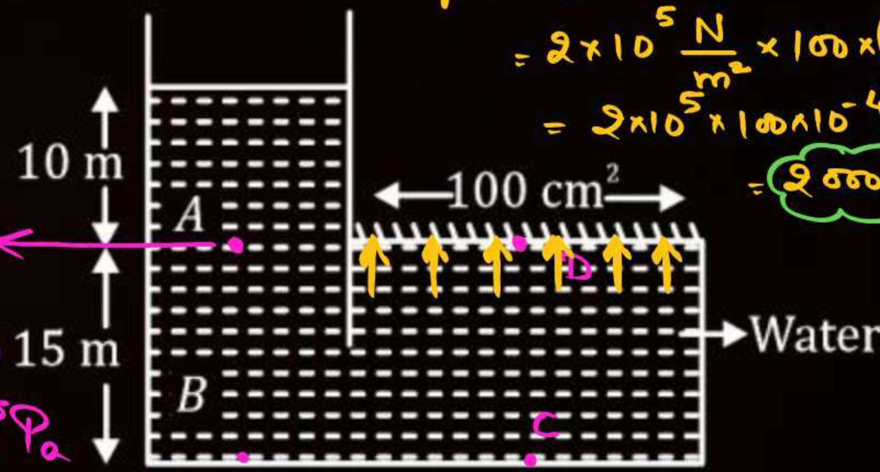
QUESTION

$1\ atm = 10^5\ Pa$

Find pressure at 'A', 'B', 'C' and 'D'. Also find the force exerted by the liquid on the roof.

$$\rho_{water} = 1000\ kg/m^3$$
$$= 1\ g/cm^3$$
$$= 1\ kg/litre$$

$$1\ litre = 1000\ cm^3$$



$P_D =$

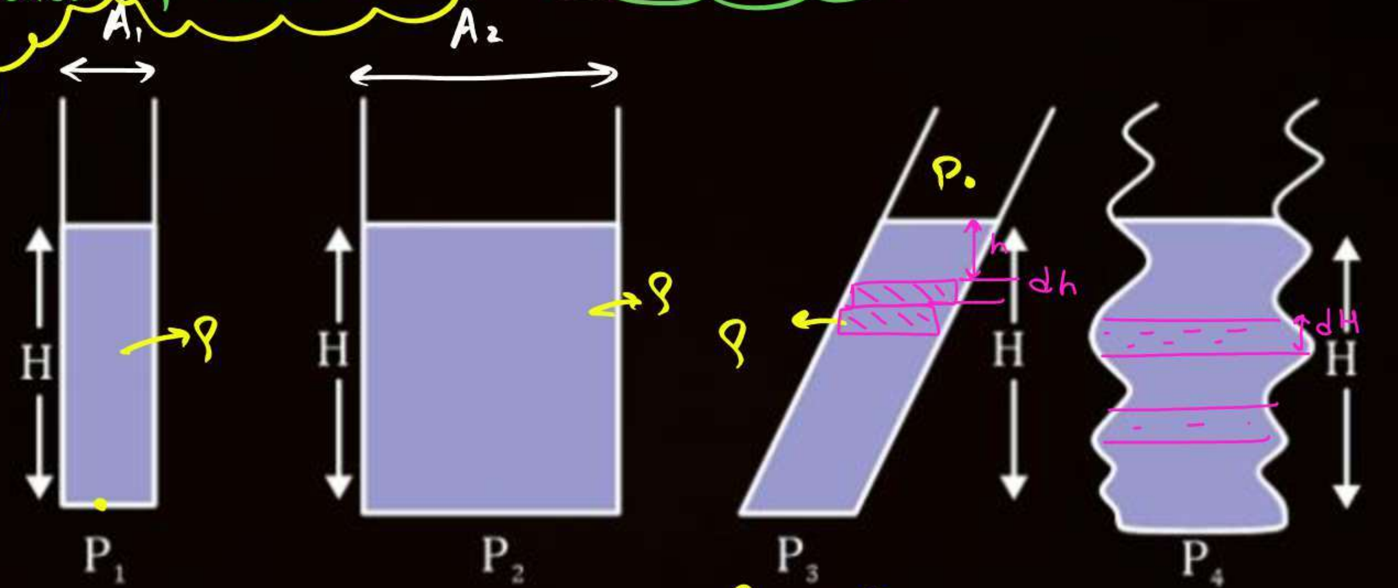
$$P_A = P_0 + \rho g H_1$$
$$10^5 + 1000\times10\times10$$
$$P_A = 10^5 + 10^5 = 2\times10^5\ Pa$$
$$\boxed{P_D = P_A = 2\times10^5\ Pa}$$

$$P_C = P_B = P_0 + \rho g H_2$$
$$= 10^5 + 10^3\times10\times25$$
$$= 10^5 + 2.5\times10^5 = \boxed{3.5\times10^5\ Pa}$$

$$F = P_D \cdot A$$
$$= 2\times10^5\ \frac{N}{m^2}\times100\times(10^{-2}\ m)^2$$
$$= 2\times10^5\times100\times10^{-4}\ N$$
$$= \boxed{2000\ N}$$

Pressure at depth is independent of shape & area of cross-section of container.

$P_1 = P_2 = P_3 = P_4$



$P_1 = P_0 + \rho g H$

$P_2 = P_0 + \rho g H$

$\int dP = \int \rho g \, dh$

$\Delta P = \rho g \int dh$

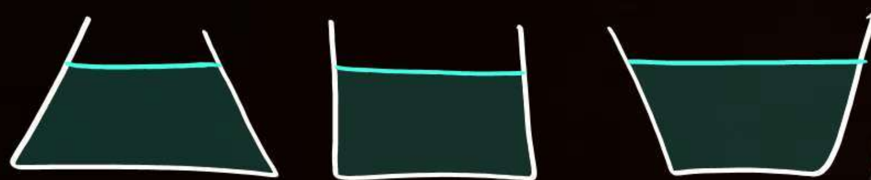
$P_3 - P_0 = \rho g H$

$P_3 = P_0 + \rho g H$

$P_4 = P_0 + \rho g H$

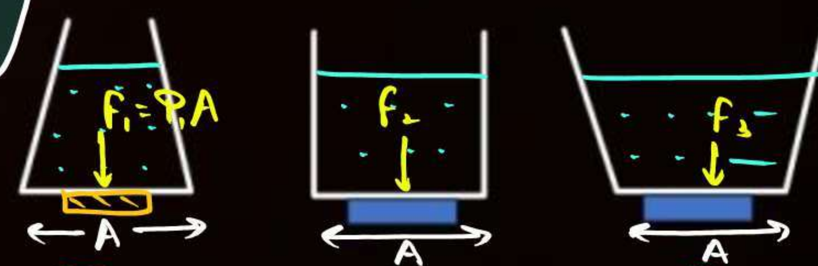
QUESTION

Find pressure exerted by liquids in all containers at the bottom of vessel. Find force on base of containers. Also compare reading of weighting machines on which these containers are placed.



$W_1 < W_2 < W_3$

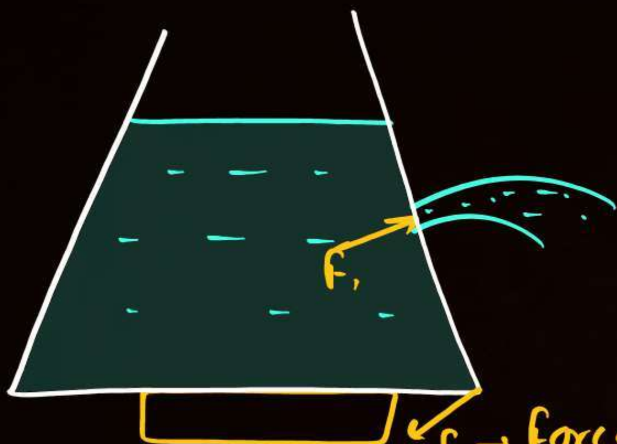
(1) (2) (3)



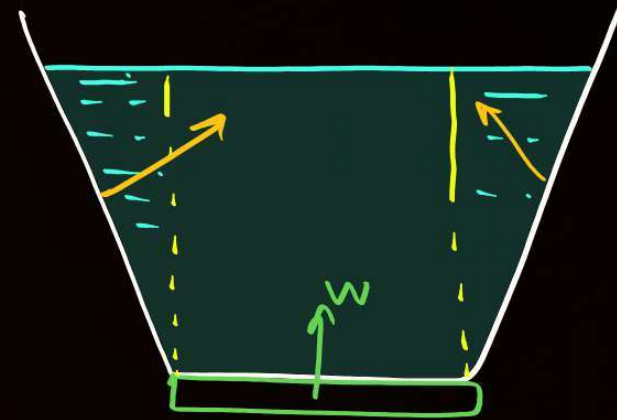
$P_1 = P_2 = P_3$

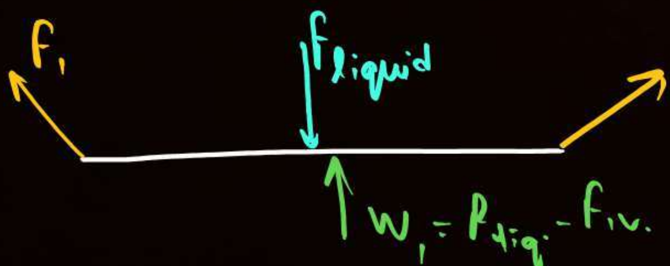
$F_1 = P_1A = F_2 = P_2A = F_3 = P_3A$

$W_1 \quad W_2 \quad W_3$



F_1 = force applied by base on side walls.





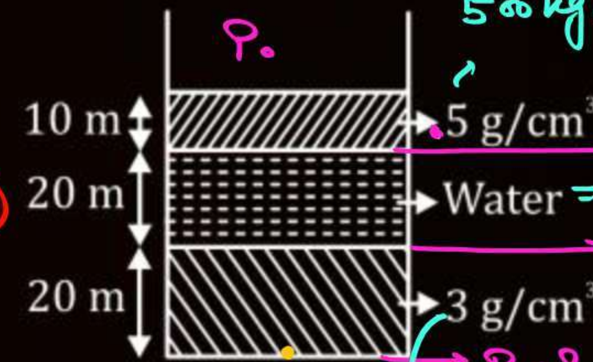
F_1 force applied by side walls on base

QUESTION

Find pressure at the bottom of the vessel.

$$0.5\,\frac{g}{cm^3} = 0.5\times\frac{10^{-3}\,kg}{(10^{-2}\,m)^3}$$
$$= 0.5\times\frac{10^{-3}}{10^{-6}}\,\frac{kg}{m^3}$$
$$= 0.5\times10^{3}\,kg/m^3.$$

$500\,kg/m^3$



Water $= 1\,g/cm^3 = 1000\,kg/m^3$

$\rho_1 g h_1 + P_0$

$P_0 + \rho_1 g h_1 + \rho_2 g h_2$

$P_0 + \rho_1 g h_1 + \rho_2 g h_2 + \rho_3 g h_3$

$3000\,kg/m^3$

$$P_{bottom} = P_0 + P_1 + P_2 + P_3$$

Common mistake

$$= 10^5 + 0.5\times10\times10 + 1\times10\times20 + 3\times10\times20$$

$$= 10^5 + 500\times10\times10 + 1000\times10\times20 + 3000\times10\times20$$

$$= 10^5 + .5\times10^5 + 2\times10^5 + 6\times10^5$$

$$= 9.5\times10^5\ Pa = 9.5\ atm.$$

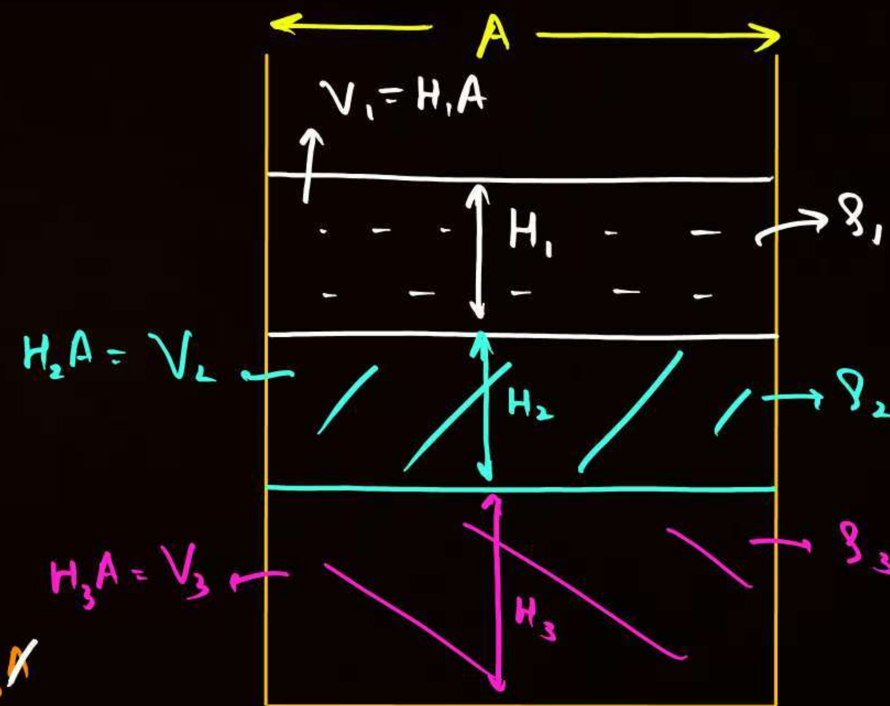
Density of Pure Liquid and Mixture

$$\rho = \frac{\text{mass}}{\text{vol.}}$$

$$\boxed{\rho_{mixture} = \frac{\text{Total mass}}{\text{Total vol.}}}$$

$$= \frac{\rho_1 V_1 + \rho_2 V_2 + \rho_3 V_3}{V_1 + V_2 + V_3}$$

$$= \frac{\rho_1 H_1 \cancel{A} + \rho_2 H_2 \cancel{A} + \rho_3 H_3 \cancel{A}}{H_1 \cancel{A} + H_2 \cancel{A} + H_3 \cancel{A}} = \frac{\rho_1 H_1 + \rho_2 H_2 + \rho_3 H_3}{H_1 + H_2 + H_3}$$



Specific Gravity

fancy name of Relative density.

$$\text{Rel. Density / Sp. gravity} = \frac{\text{Density of object}}{\text{Density of water} = 1\,g/cm^3}$$

Unitless

Dimensionless

QUESTION

Two litres of water is mixed with 3 litres of another liquid. If the density of liquid is $0.5\ \mathrm{g/cm^3}$. Find the equivalent density of mixture.

$$\rho_{eq} = \frac{\rho_1 V_1 + \rho_2 V_2}{V_1 + V_2}$$

$$= \frac{1 \times 2\,\cancel{\text{litre}} + 0.5 \times 3\,\cancel{\text{litre}}}{(2+3)\,\cancel{\text{litre}}}$$

$$= \frac{2 + 1.5}{5} = \frac{3.5}{5} = 0.7\ \mathrm{g/cm^3}$$

QUESTION

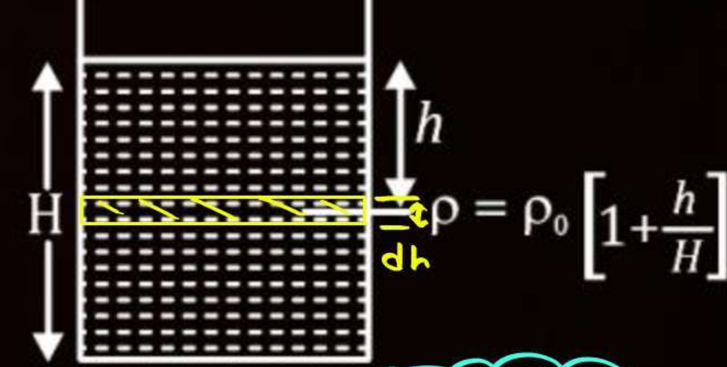
The density of a liquid varies with depth as $\rho = \rho_0\left[1+\frac{h}{H}\right]$ where H is the total depth of liquid. Find pressure at the base of containers.

$$\int dP = \int \rho g\, dh$$

$$\Delta P = \int_0^H \rho_0\left[1+\frac{h}{H}\right] g\, dh$$

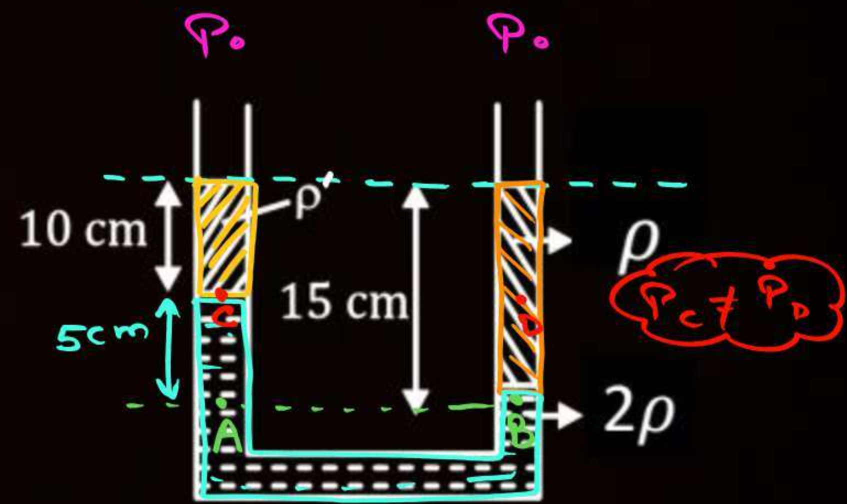
$$P - P_0 = \rho_0 g \int_0^H \left(1+\frac{h}{H}\right) dh$$

$$P - P_0 = \rho_0 g \left[h + \frac{h^2}{2H}\right]_0^H \Rightarrow P = P_0 + \rho_0 g\left[H + \frac{H^{\cancel{2}}}{2\cancel{H}}\right] = P_0 + \frac{3}{2}\rho_0 g H \quad \text{Ans}$$



QUESTION

Find the density of liquid in left arm (ρ') if the surface of liquids in both arms is at some horizontal level.



$$P_A = P_B$$

$$\cancel{P_0} + \rho' \cancel{g} \times 10 + 2\rho \cdot \cancel{g} \times 5 = \cancel{P_0} + \rho \cancel{g} \times 15$$

$$10\rho' = 15\rho - 10\rho$$

$$\Rightarrow 10\rho' = 5\rho$$

$$\rho' = \frac{1}{2}\rho$$ Ans.

QUESTION

In a U–tube as shown in a figure, water and oil are in the left side and right side of the tube respectively. The heights from the bottom for water and oil columns are 15 cm and 20 cm respectively. The density of the oil is [take $\rho_{water} = 1000\ kg/m^3$]

(A) $1200\ kg/m^3$

(B) $750\ kg/m^3$

(C) $1000\ kg/m^3$

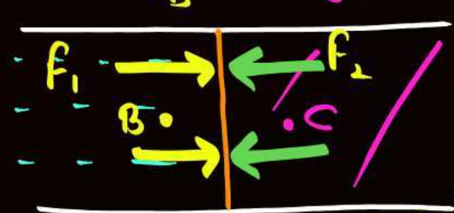
(D) $1333\ kg/m^3$

$P_A = P_B$

$P_C = P_D$

At Interface → $F_1 = F_2$

$P_B \cdot \cancel{A} = P_C \cdot \cancel{A}$

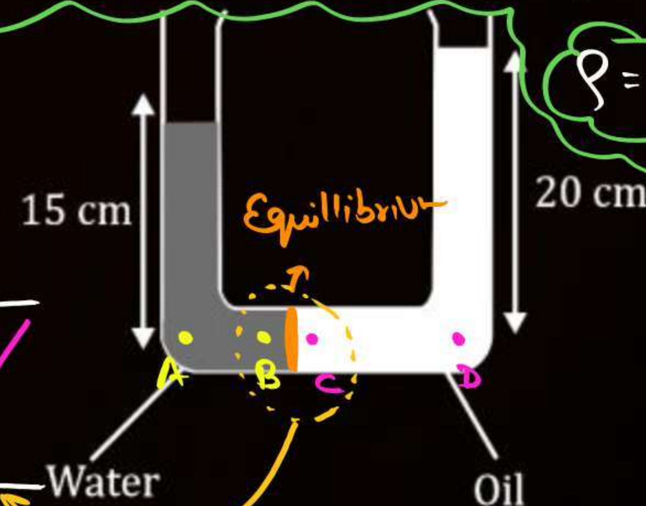


$P_B = P_C$

$P_B = P_C \Rightarrow \cancel{P_0} + \rho_1 \cancel{g} H_1 = \cancel{P_0} + \rho_2 \cancel{g} H_2$

$1000 \times 15 = \rho \times 20 \Rightarrow \rho = 1000 \times \frac{15}{20}$

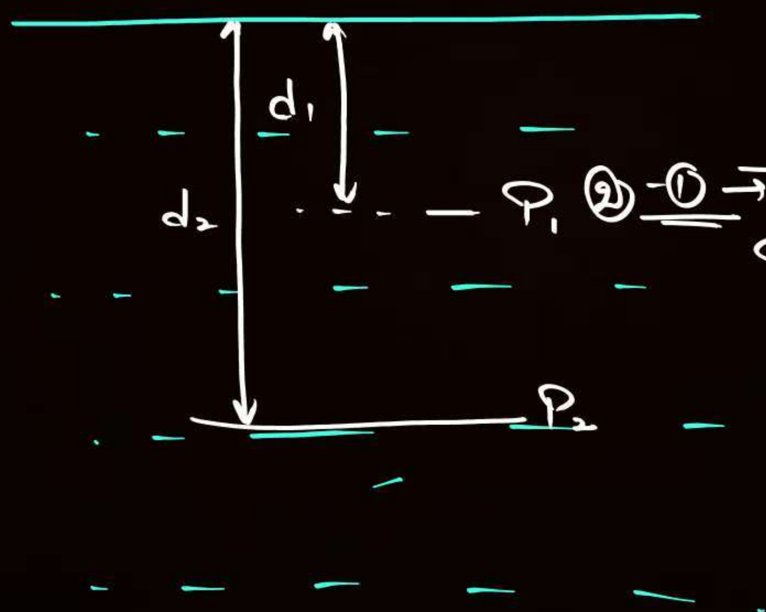
$\rho = 750\ kg/m^3$



QUESTION (JEE Mains – 8 April 2019 II)

A submarine experiences a pressure of 5.05×10^6 Pa at depth of d_1 in a sea. When it goes further to a depth of d_2, it experiences a pressure of 8.08×10^6 Pa. Then $d_2 - d_1$ is approximately (density of water = 10^3 kg/m^3 and acceleration due to gravity = 10ms^{-2}):

1. 300 m
2. 400 m
3. 600 m
4. 500 m



$P_1 = P_0 + \rho g d_1$ — (1)

$P_2 = P_0 + \rho g d_2$ — (2)

(2) − (1) →

$P_2 - P_1 = 0 + \rho g (d_2 - d_1)$

$3.03 \times 10^{6} = 10^{3} \times 10 (d_2 - d_1)$

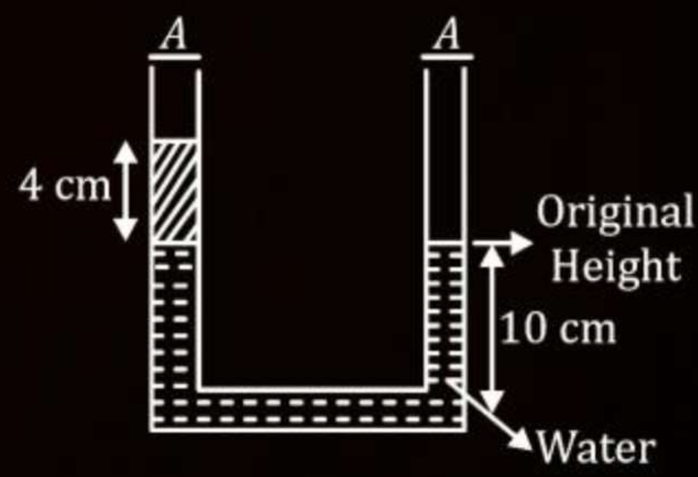
$\boxed{d_2 - d_1 = 303 \text{ m}}$ Ans.

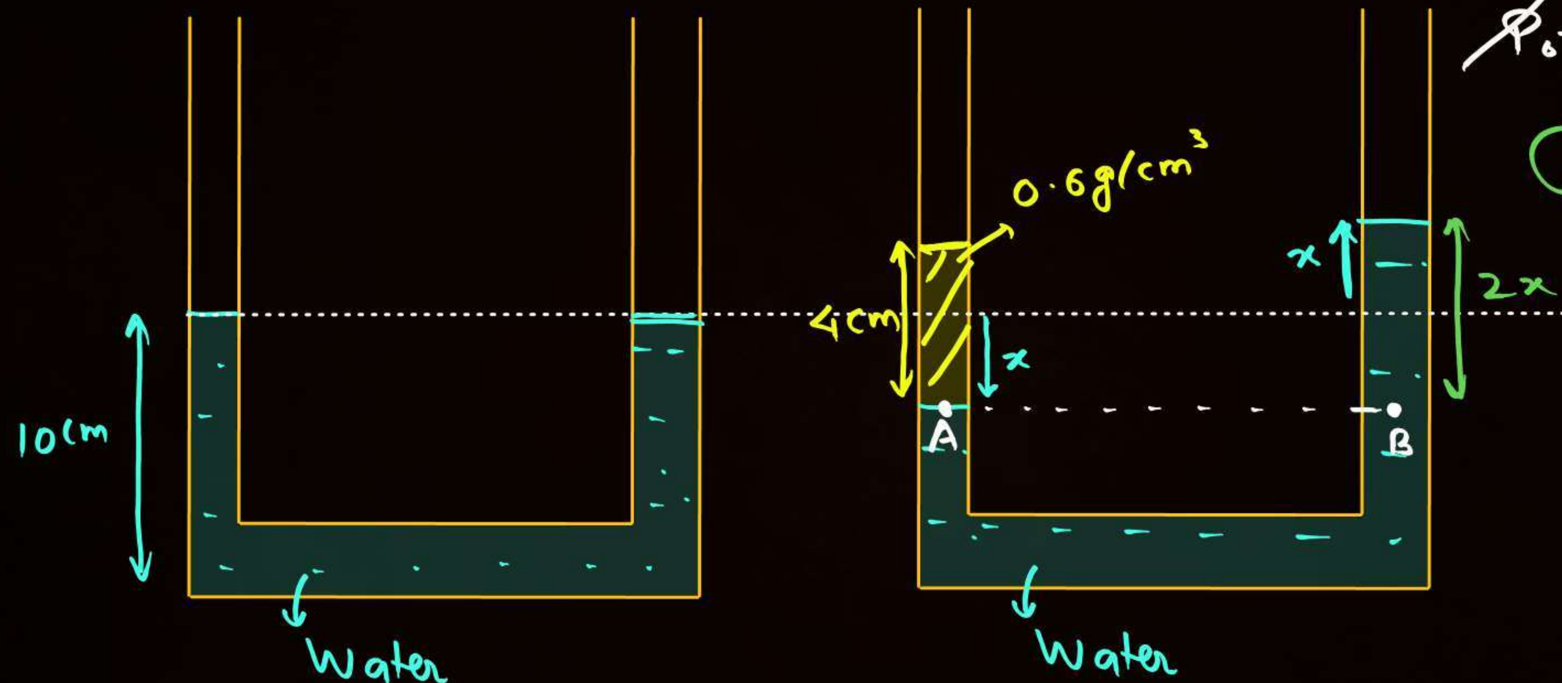
QUESTION

Adv. level.

A U-tube contains water in both arms upto height 10 cm. Oil having density of $0.6 g/cm^3$ is added to the left arm such that its height is 4 cm. Find the difference in water level in both arms. (Area of cross-section in same in both arms)

$2x = 2.4 \text{ cm}$





$$\cancel{P_0} + 0.6\,\cancel{g} \times 4 = \cancel{P_0} + 1 \times \cancel{g} \times 2x$$

$$2.4 = 2x$$

QUESTION (JEE Adv, 2020)

An open-ended U-tube of uniform cross-sectional area contains water (density 10^3 kg m^{-3}). Initially the water level stands at 0.29 m from the bottom in each arm. Kerosene oil (a water-immiscible liquid) of density 800 kg m^{-3} is added to the left arm until its length is 0.1 m, as shown in the schematic figure below. The ratio (h_1/h_2) of the heights of the liquid in the two arms is:

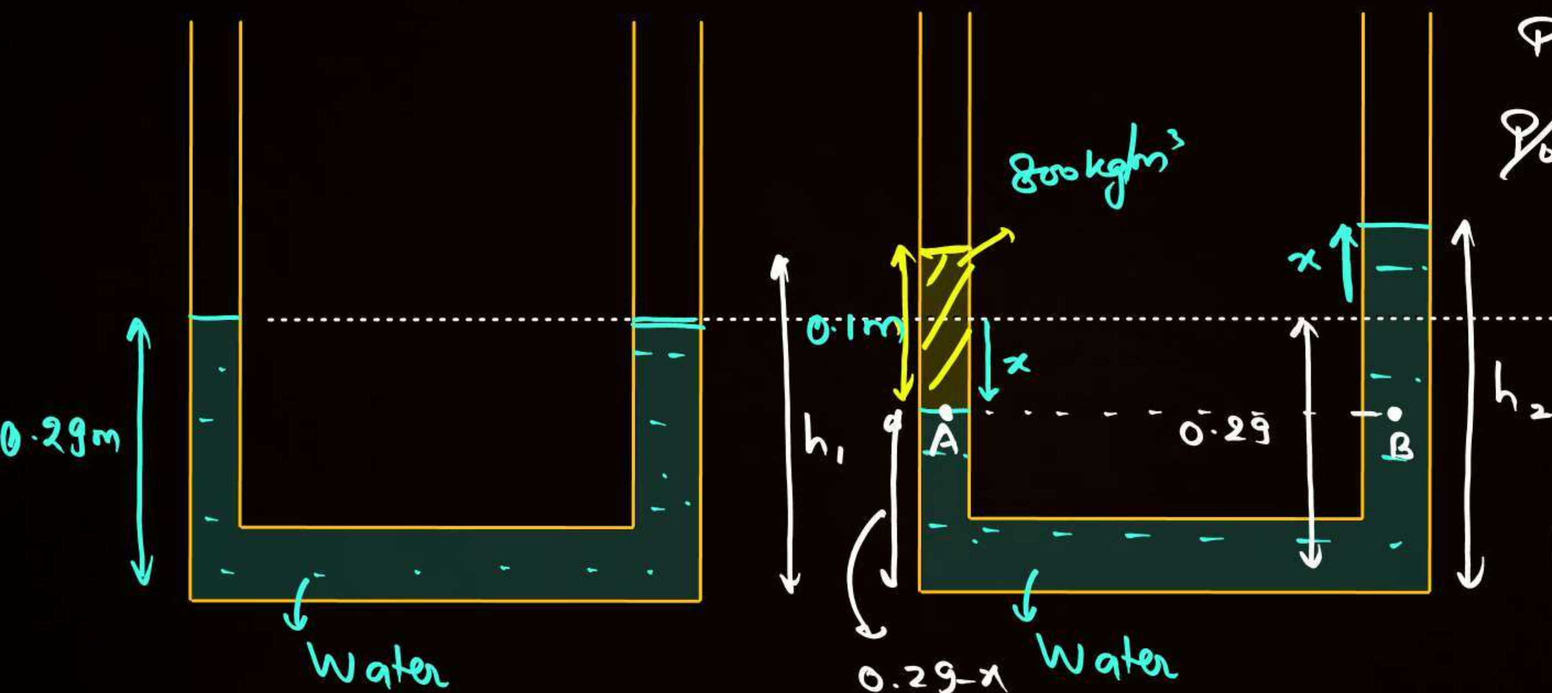
$$\frac{h_1}{h_2} = \frac{\cancel{0}.35}{\cancel{0}.33} = \frac{35}{33}.$$

(A) 15/14

(B) 35/33

(C) 7/6

(D) 5/4



$$P_A = P_B$$

$$\cancel{P_0} + 800 \times \cancel{g} \times 0.1 = \cancel{P_0} + 1000 \times \cancel{g} \times 2x$$

$$8\cancel{0} = 200\cancel{0}x$$

$$x = \frac{\cancel{8}}{2\cancel{00}} = \frac{4}{100} = 0.04$$

$$h_2 = 0.29 + 0.04$$

$$= 0.33$$

$$h_1 = (0.29 - x) + 0.1$$

$$= (0.29 - 0.04) + 0.1$$

$$h_1 = 0.35$$

QUESTION

If pressure at half the depth of a lake is equal to 2/3 pressure at the bottom of the lake then what is the depth of the lake

(A) 10 m

(B) 20 m

(C) 60 m

(D) 30 m

$$\frac{2\times10^5}{10^3\times10} = 20$$

$$P_A = P_0 + \rho g\left(\frac{H}{2}\right)$$

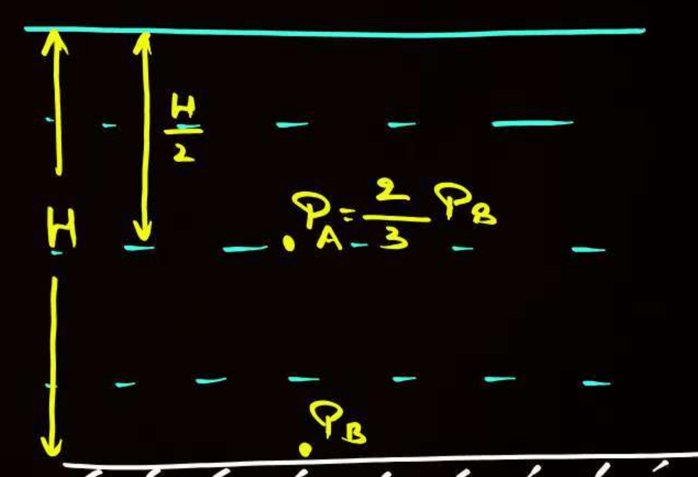
$$P_B = P_0 + \rho g H$$

$$P_A = \frac{2}{3} P_B$$

$$P_0 + \rho g\left(\frac{H}{2}\right) = \frac{2}{3}\left[P_0 + \rho g H\right]$$

$$P_0 + \frac{\rho g H}{2} = \frac{2}{3}P_0 + \frac{2}{3}\rho g H \Rightarrow \left[\frac{2}{3} - \frac{1}{2}\right]\rho g H = P_0 - \frac{2}{3}P_0$$

$$\left(\frac{4-3}{6}\right)\rho g H = \frac{P_0}{3} \Rightarrow H = \frac{P_0\times 6}{3\rho g} = \frac{2P_0}{\rho g}$$



QUESTION

A U-tube contains water and oil separated by mercury. The mercury column in the two arms are at the same level with 10 cm of water in one arm and 12.5 cm of oil in the other as shown. What is the relative density of oil?

(A) 0.8

(B) 1.0

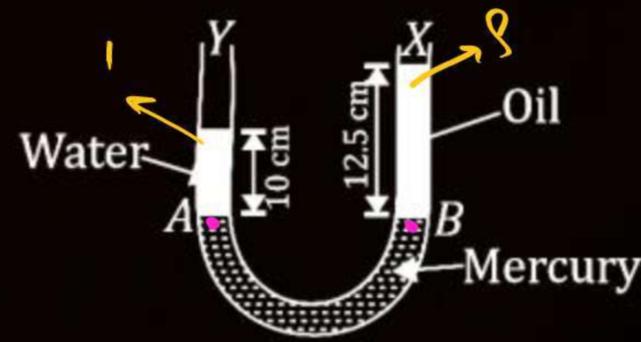
(C) 1.25

(D) 1.5

$$P_A = P_B$$

$$\cancel{P_0} + 1 \times \cancel{g} \times 10 = \cancel{P_0} + \rho \times \cancel{g} \times 12.5$$

$$\rho = \frac{10}{12.5} = \frac{4}{5} = 0.8$$



QUESTION (JEE Mains – 08 April, 2023 II)

Two cylindrical vessels of equal cross-sectional area 16 cm^2 contain water upto heights 100 cm and 150 cm respectively. The vessels are interconnected so that the water levels in them become equal. The work done by the force of gravity during the process, is [Take density of water = 10^3 kg/m^3 and g = 10 m/s^2]

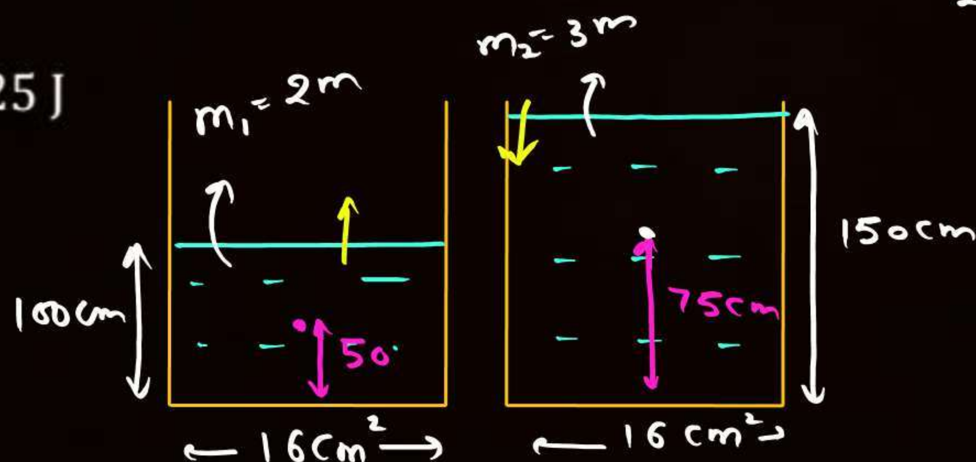
$$2m = \rho \cdot V \Rightarrow 2m = 10^3 \times 100 \times 16 \times 10^{-6}$$
$$2m = 1.6 \Rightarrow m = 0.8\ kg$$

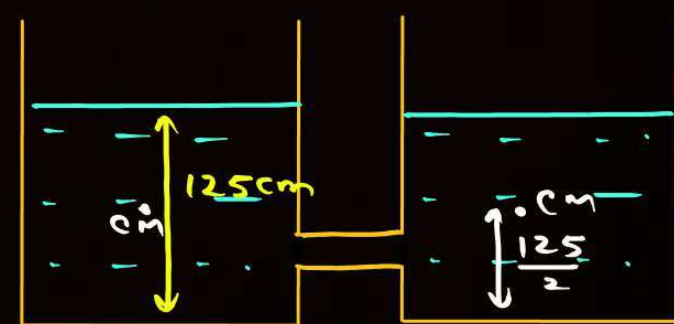
(A) 0.25 J

(B) 1 J

(C) 8 J

(D) 12 J





W.D. by grav. = Loss in grav. pot. Energy.

$$= U_i - U_f$$

$$= \left[m_1 g \times 50 + m_2 g \times 75\right] - \left[(m_1 + m_2) g \frac{125}{2}\right]$$

$$= \left[2m g \times 50 + 3m g \times 75\right] - \left[5m \times g \times \frac{125}{2}\right]$$

$$= 100mg + 225mg - \frac{625}{2}mg$$

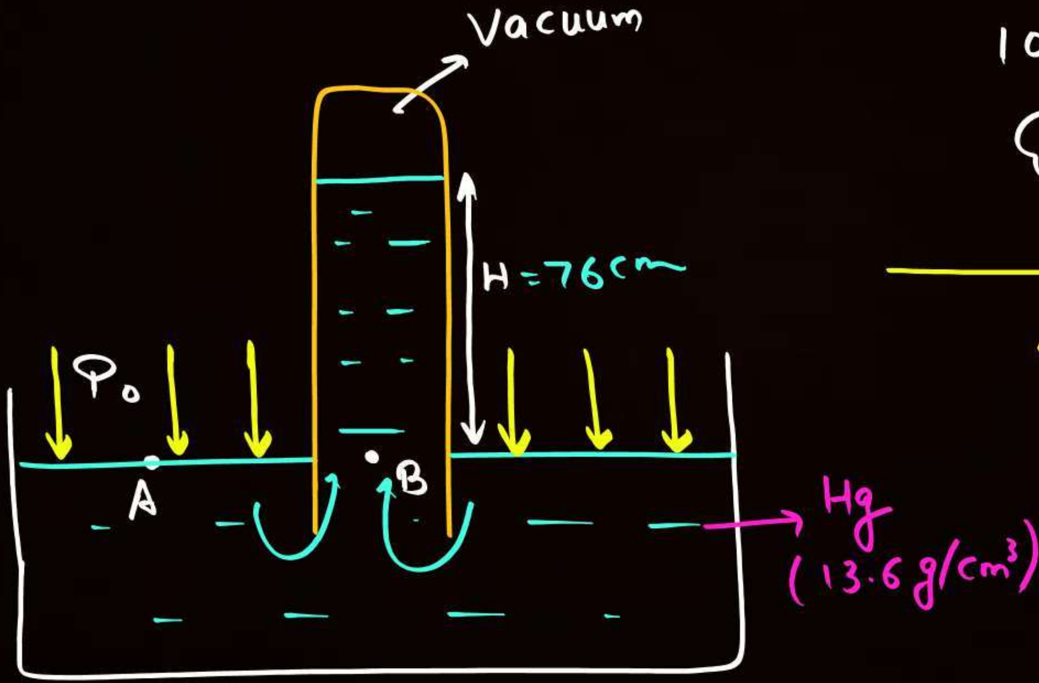
$$= \left[325 - \frac{625}{2}\right] mg$$

Measurement of Pressure

Barometer

Manometer.

Barometer



$$P_A = P_B$$

$$P_0 = \rho g H$$

$$10^5 = 10^3 \times 10 \times H$$

H = 10 m → Height of water rise

$$P_0 = \rho_{Hg} \cdot g H$$

H = 76 cm Hg.

QUESTION

Find l :

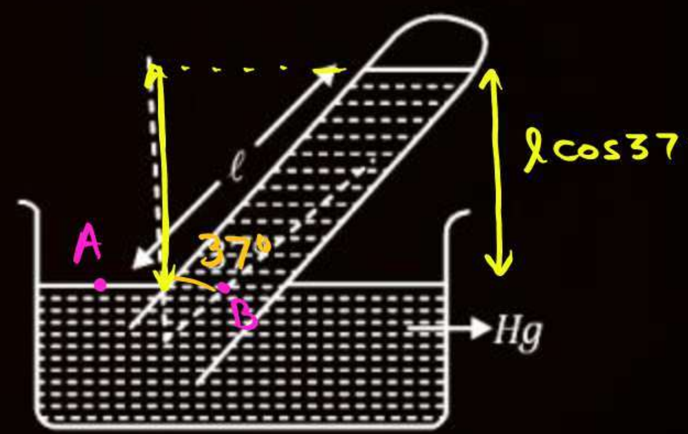
$$l \cos 37^\circ = 76 \text{ cm}$$

$$l \times \frac{4}{5} = 76 \text{ cm}$$

$$l = 76 \times \frac{5}{4}$$

$$= 19 \times 5$$

$$l = 95 \text{ cm}$$ Ans



QUESTION

Find normal reaction exerted by roof of tube.

$$P_A = P_B$$

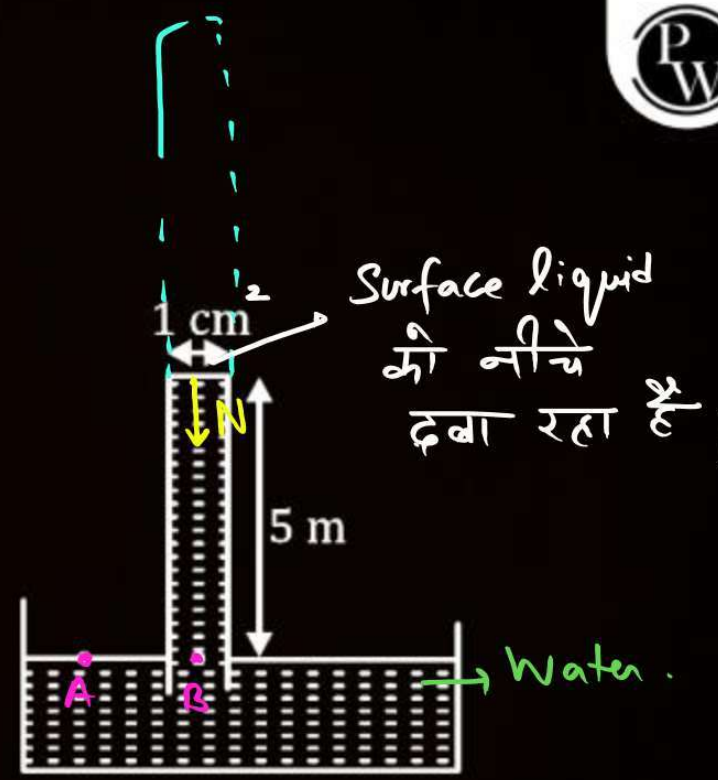
$$10^5 = \frac{N}{A} + \rho g H$$

$$10^5 = \frac{N}{(10^{-2})^2} + 10^3 \times 10 \times 5$$

$$\frac{N}{10^{-4}} = 10^5 - 0.5 \times 10^5$$

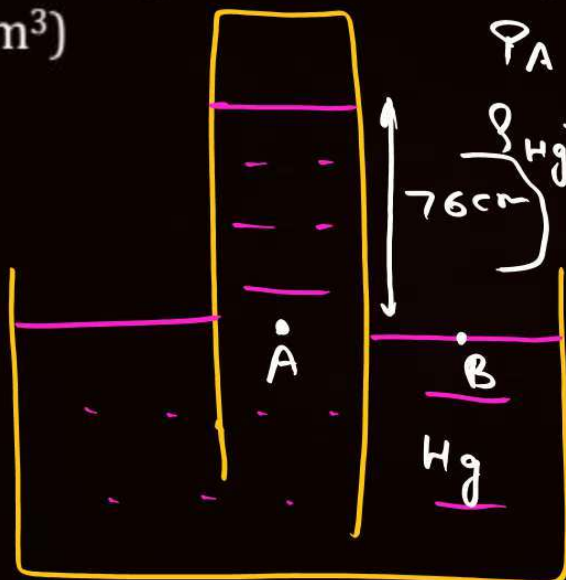
$$N = 0.5 \times 10^5 \times 10^{-4}$$

$$\boxed{N = 5\,N.}$$ Ans.



QUESTION

A barometer is constructed using a liquid (density = 760 kg/m^3). What would be the height of the liquid column, when a mercury barometer reads 76 cm? (Density of mercury = 13600 kg/m^3)



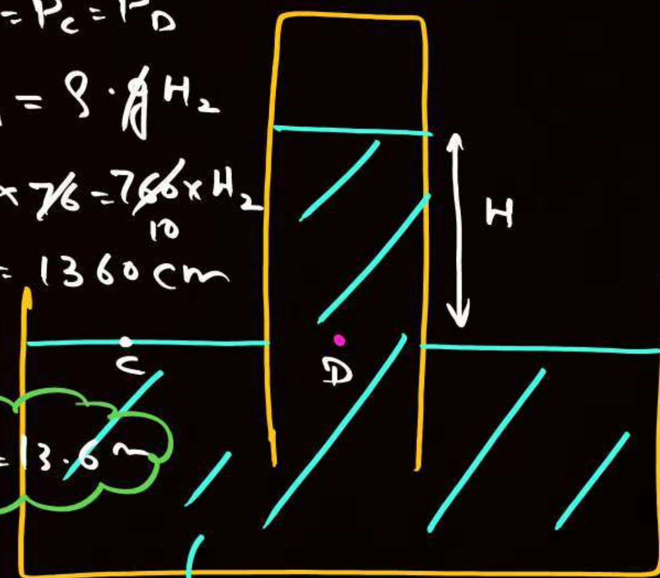
$P_A = P_B = P_C = P_D$

$\rho_{Hg} \cdot \cancel{g} \cdot H_1 = \rho \cdot \cancel{g} H_2$

$13600 \times \cancel{76} = \cancel{760}_{10} \times H_2$

$H_2 = 1360 \text{ cm}$

$H_2 = 13.6 \text{ m}$



$\rho = 760 \text{ kg/m}^3$

(A) 1.36 m

(B) 13.6 m

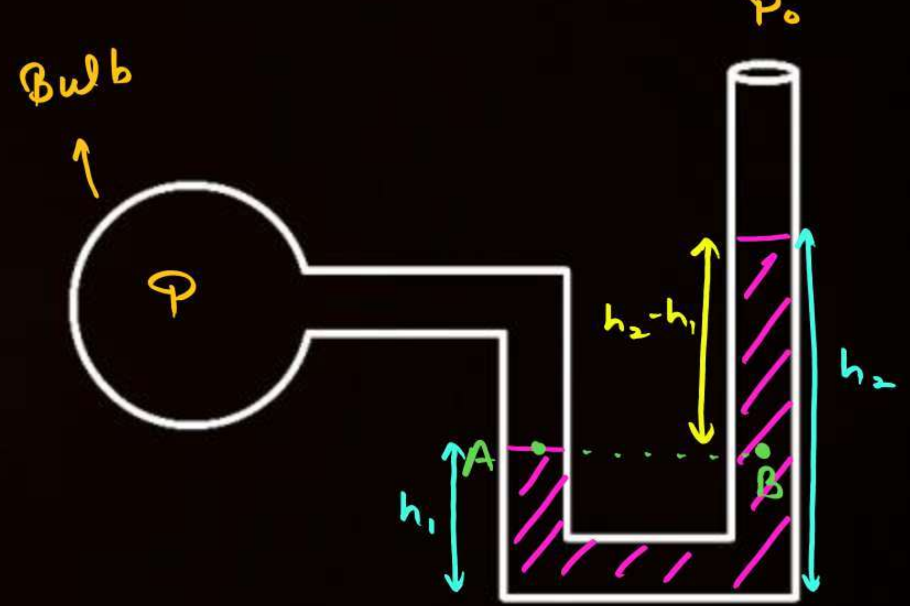
(C) 136 m

(D) 0.76 m

Manometer

$P_A = P_B$

$$\boxed{P = P_0 + \rho g (h_2 - h_1)}$$



QUESTION

Find pressure of gas inside the bulb. Given $P_0 = \underbrace{76 \text{ cm Hg}}_{38 \text{ cm Hg}} \simeq \underbrace{10^5}_{} \text{ Pa}$.

$38 \text{ cm Hg} \rightarrow \frac{10^5}{2} \text{ Pa}$

(A) 0.5×10^5 Pa

(B) 10^5 Pa

(C) 1.5×10^5 Pa

(D) None of these

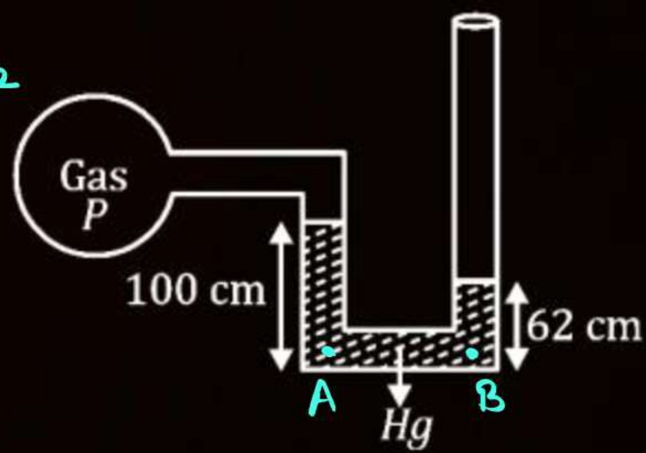
$$P_A = P_B$$

$$P + \rho g \times 100 = P_0 + \rho g \times 62$$

$$P = P_0 - \rho g \times 38$$

$$= 10^5 - \frac{10^5}{2}$$

$$= \frac{10^5}{2} \text{ Pa.}$$



QUESTION

Find:

$P = P_0 + \rho g \times 20$

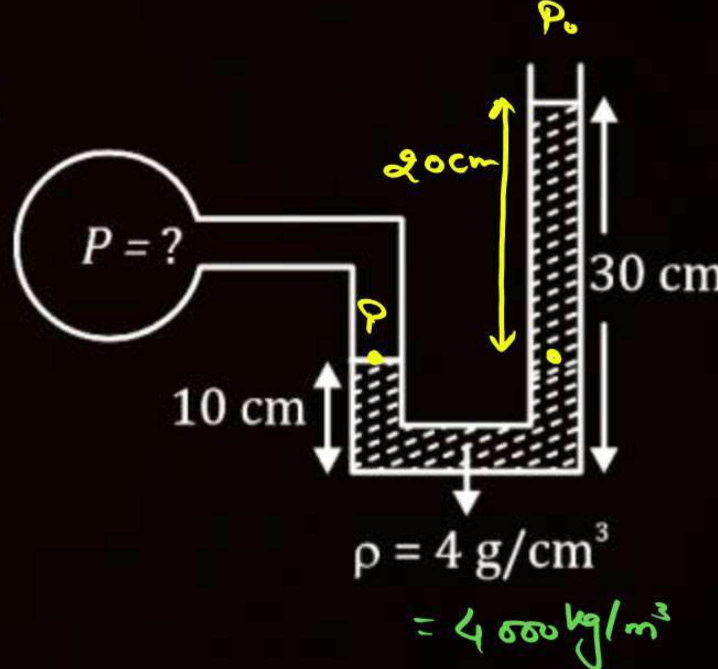
$= 10^5 + 400\cancel{0} \times 1\cancel{0} \times 20 \times 10^{\cancel{-2}}$

$= 10^5 + 8000$

$= 100 \times 10^3 + 8 \times 10^3$

$= 108 \times 10^3$ Pa.

$P = 1.08 \times 10^5$ Pa.

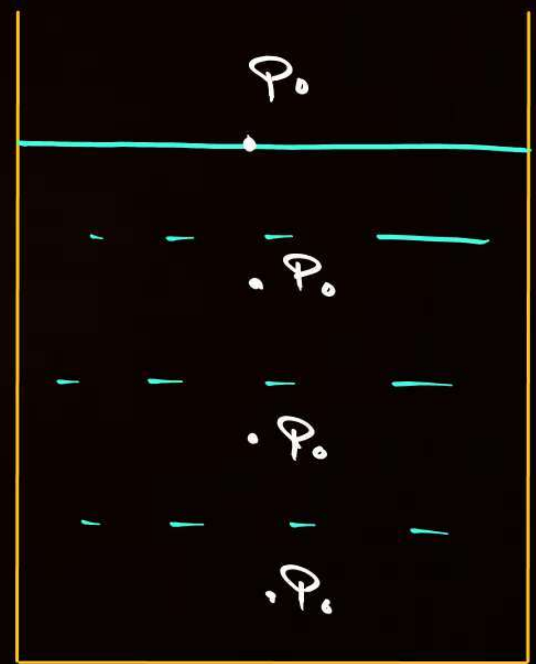


Pressure Inside Accelerating Liquid

Pressure in gravity free space:

$g = 0$

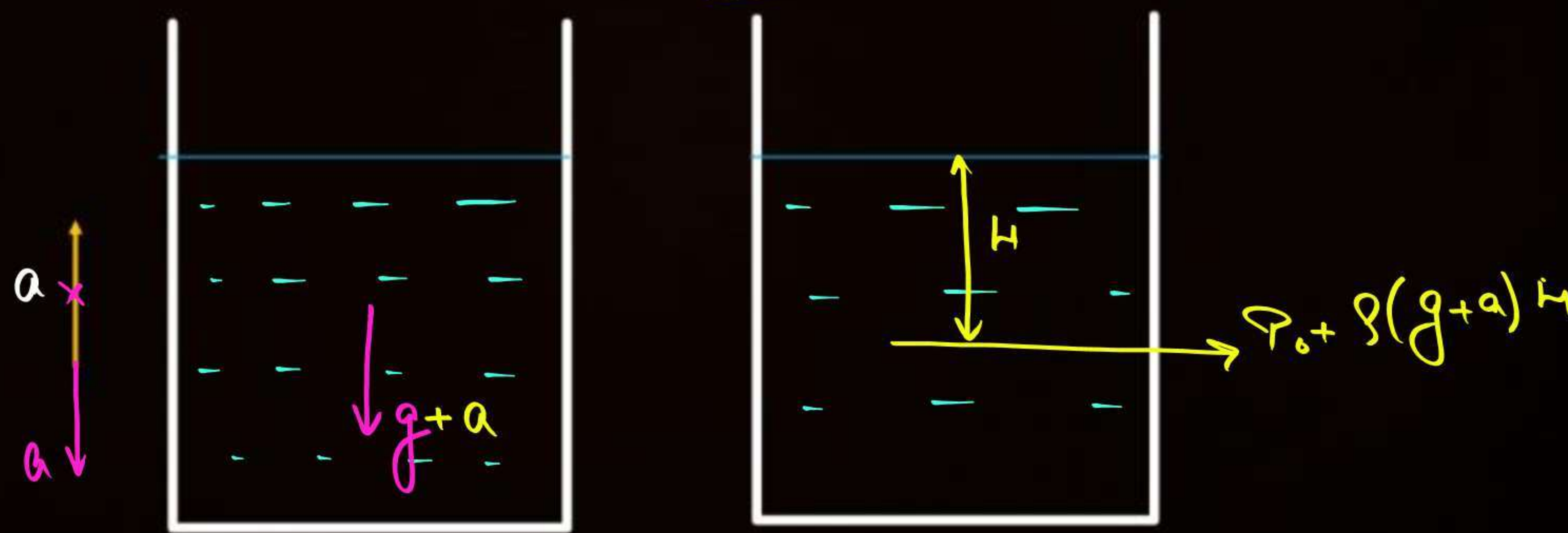
→ Pressure will remain same throughout the body of liquid



Pressure Inside Accelerating Liquid

Container Accelerating Upwards:

Brahmastra → Container को रोक दो |



Pressure Inside Accelerating Liquid

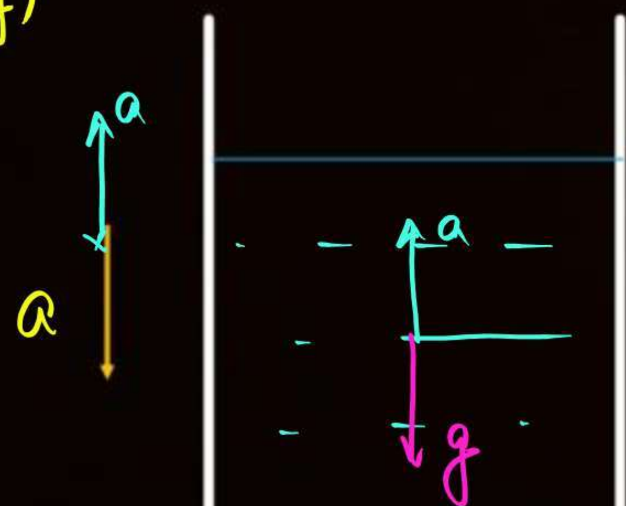
If $a > g \rightarrow$

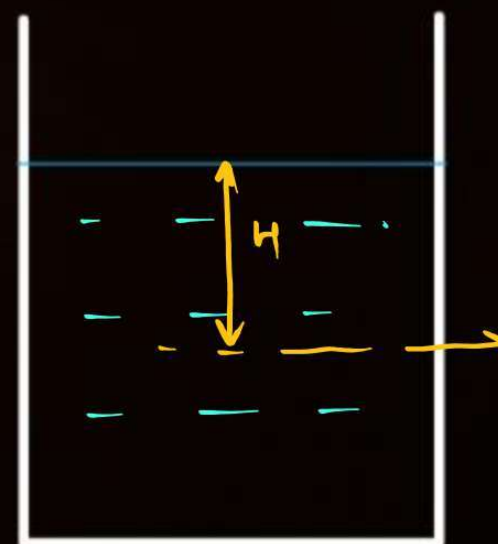
$\boxed{P = P_0}$ everywhere

{free fall}

Container Accelerating Downwards:

$(a < g)$



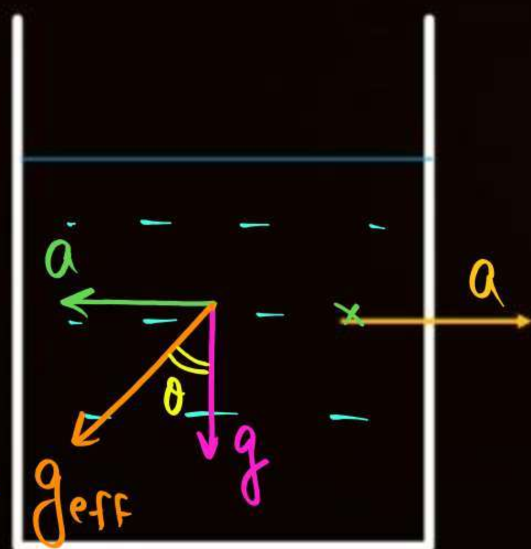


$\rightarrow P_0 + \rho(g-a)H$

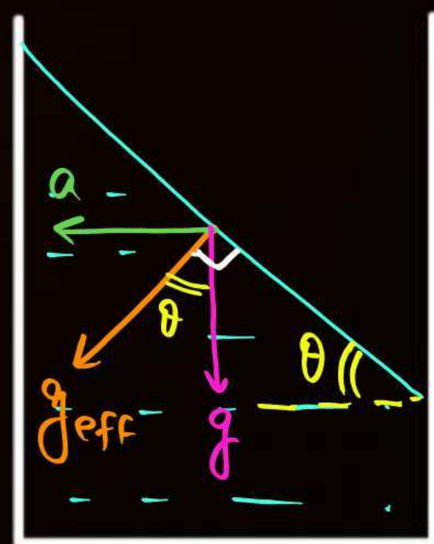
Pressure Inside Accelerating Liquid

Liquid का surface हमेशा $g_{eff.}$ के perpendicular होता है।

Container Accelerating Horizontally:



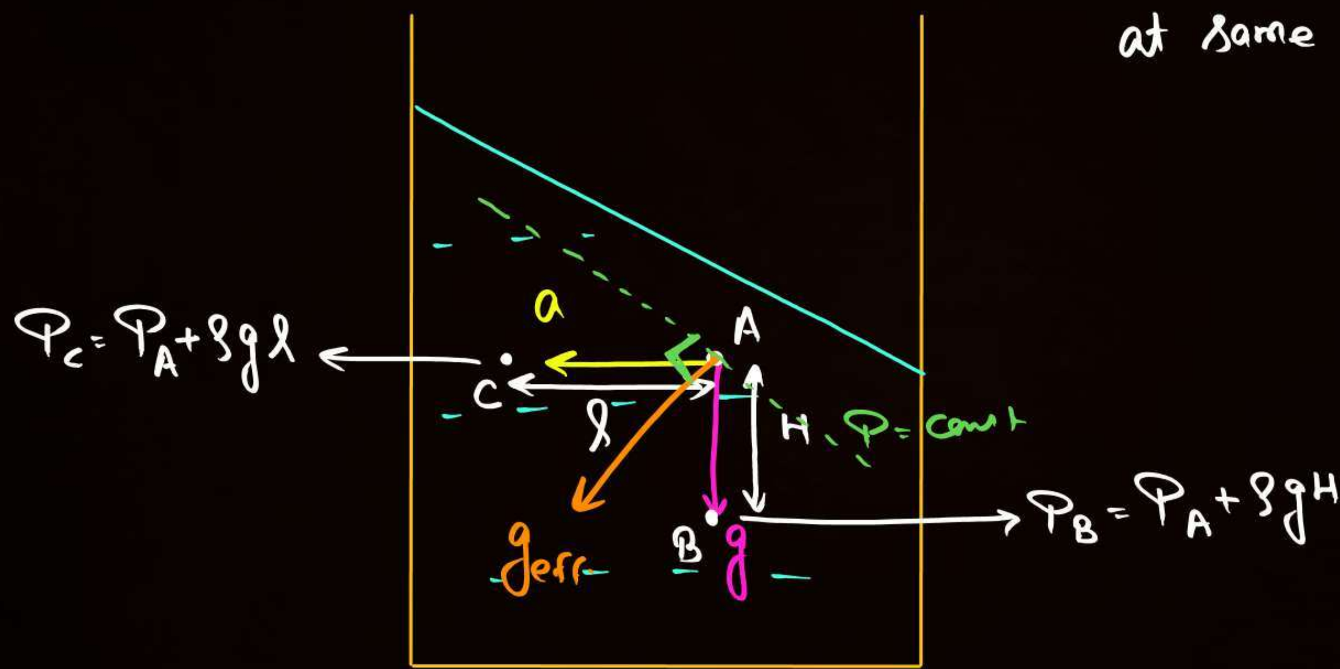
$$\sqrt{a^2+g^2} = g_{eff}$$



$$\tan\theta = \frac{a}{g}$$

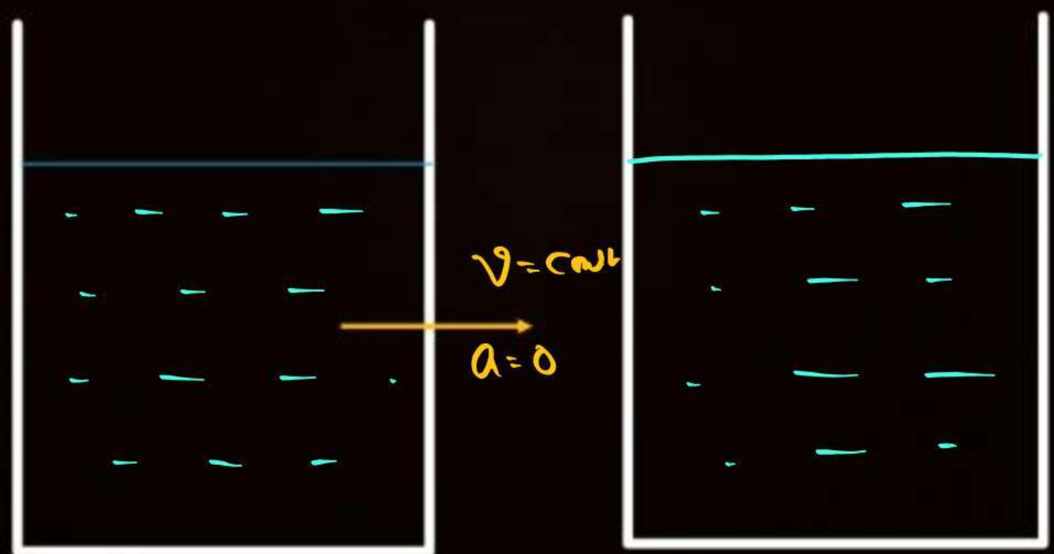
$$\theta = \tan^{-1}\left[\frac{a}{g}\right]$$

⇒ Pressure will not be const.
at same horizontal level.



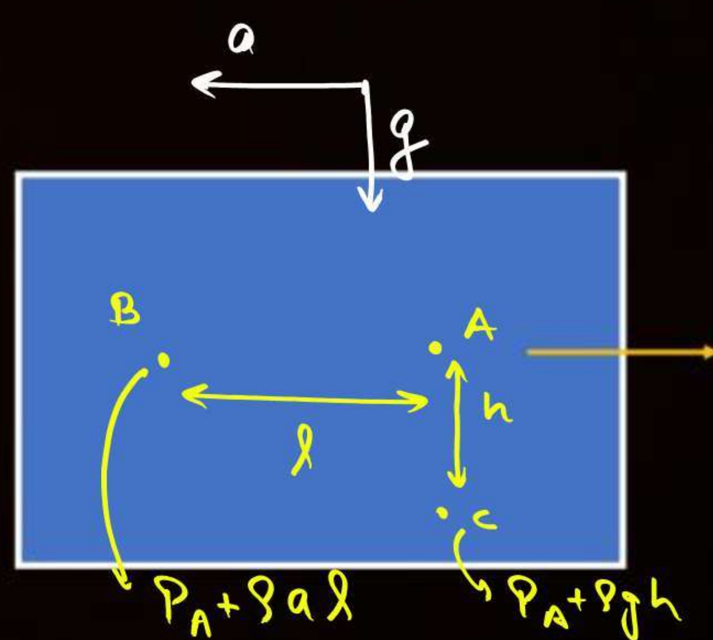
Pressure Inside Accelerating Liquid

Container Moving Horizontally at Constant Velocity:



Pressure Inside Accelerating Liquid

Completely Filled Closed Container

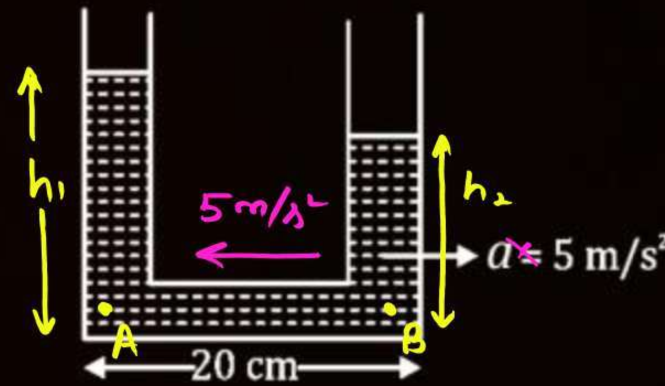


QUESTION

Find the difference in water level of two arms.

$$P_A = P_B + \rho a l$$

$$P_A - P_B = \rho a l$$



$$\Delta P = \rho a l$$

$$P_A = P_0 + \rho g h_1$$

$$P_B = P_0 + \rho g h_2$$

$$P_A - P_B = \rho g (h_1 - h_2)$$

$$\cancel{\rho} \times \cancel{5} \times \overset{10}{\cancel{20}} = \cancel{\rho}\, \overset{\cancel{2}}{\cancel{g}} \times (h_1 - h_2)$$

$$\boxed{h_1 - h_2 = 10\ cm}$$ Ans

QUESTION

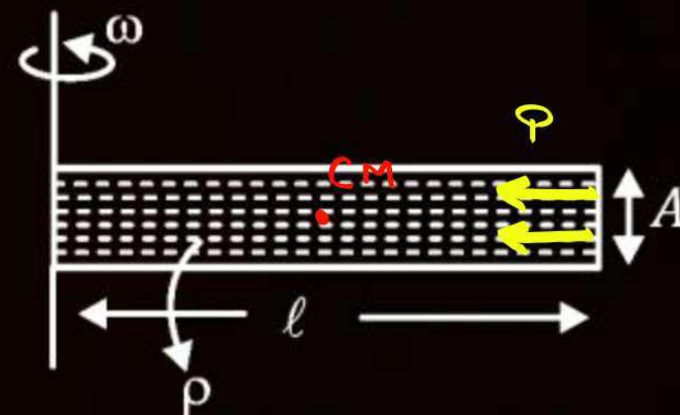
Find pressure of the outer end of tube.

$$F_{net} = m\omega^2 R$$

$$P \cdot \cancel{A} = [\rho \cdot \cancel{A}l]\omega^2 \frac{l}{2}$$

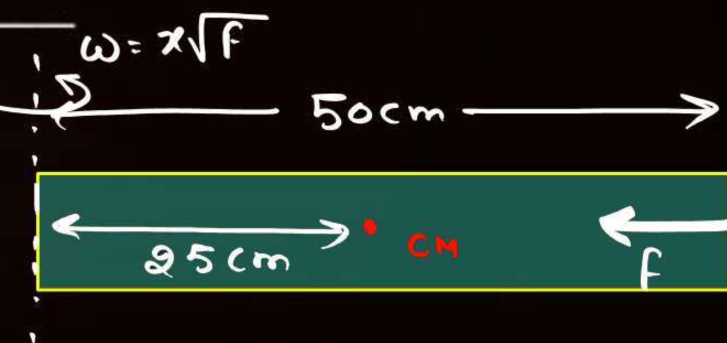
$$P = \frac{\rho \omega^2 l^2}{2}$$

Ans



QUESTION (JEE Mains – 29 July, 2022 (II))

A tube of length 50 cm is filled completely with an incompressible liquid of mass 250 g closed both ends. The tube is then rotated in horizontal plane about one of its ends with a uniform angular velocity $x\sqrt{F}$ rad s^{-1}. If F be the force exerted by the liquid at the other end then the value of x will be ____



$$F = m\omega^2 R$$

$$F = 250\times10^{-3}\times\omega^2\times(25\times10^{-2})$$

$$F = 25\times10^{-2}\times\omega^2\times25\times10^{-2}$$

$$\cancel{F} = 25^2\times10^{-4}\times x^2\cancel{F}$$

$$x^2 = \frac{1}{25^2\times10^{-4}} \Rightarrow x = \frac{1}{25\times10^{-2}} = \frac{100}{25} = 4 \text{ Ans}$$

QUESTION

Find difference in water level of two arms (Neglect thickness of tubes)

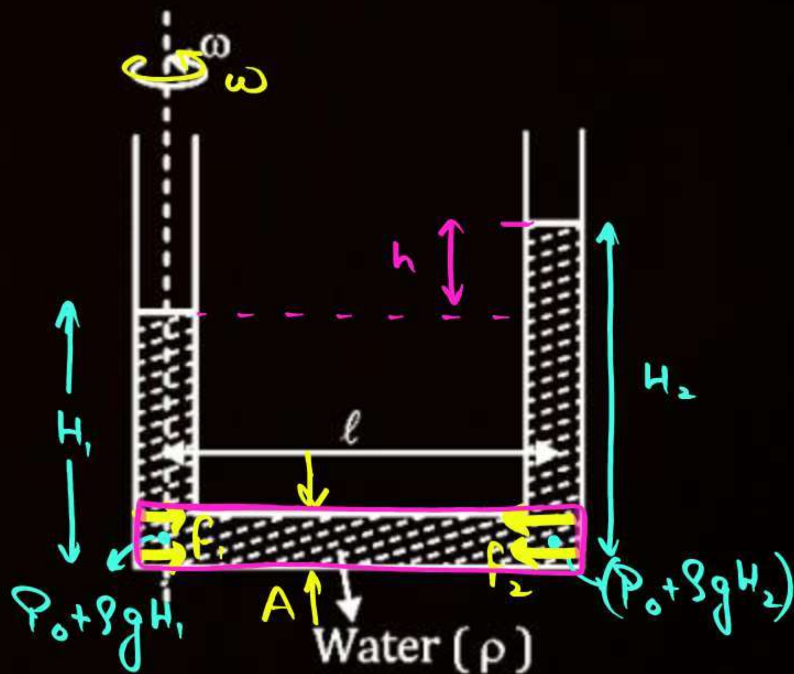
$$F_{net} = m\omega^2 R$$

$$[\cancel{P_0} + \rho g H_2]\cancel{A} - [\cancel{P_0} + \rho g H_1]\cancel{A} = [\rho \cdot l \cancel{A}]\omega^2 \frac{l}{2}$$

$$\cancel{\rho} g (H_2 - H_1) = \frac{\cancel{\rho}\,\omega^2 l^2}{2}$$

$$H_2 - H_1 = \frac{\omega^2 l^2}{2g}$$

Ans

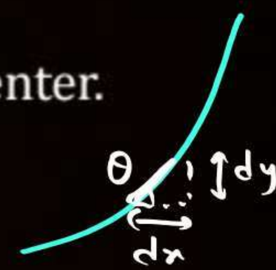


QUESTION

Adv. level

A cylinder of radius R containing water is rotated about its central axis as shown in figure.

1. angle made by the liquid surface with horizontal at distance r from center.
2. Difference in height of the liquid at the center of vessel and its sides.



1) $$\tan\theta = \frac{\cancel{m}\omega^2 r}{\cancel{m}g} \rightarrow \tan\theta = \frac{\omega^2 r}{g}$$

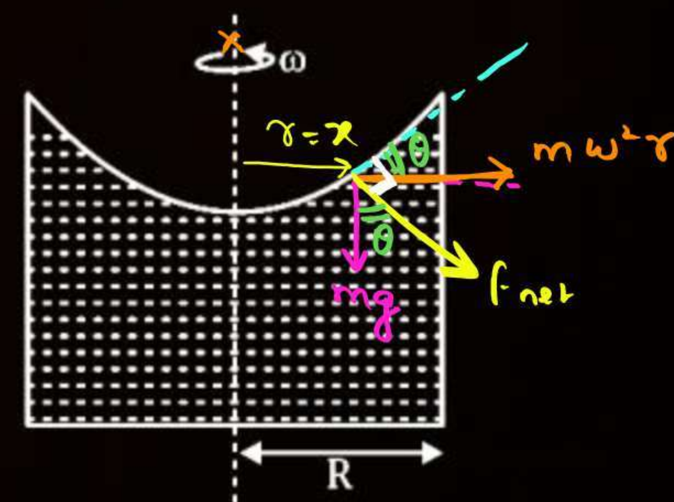
$$\theta = \tan^{-1}\left(\frac{\omega^2 r}{g}\right)$$

$$\frac{dy}{dx} = \frac{\omega^2 r}{g}$$

$$\int dy = \int \frac{\omega^2 r}{g}\, dx$$

$$\Delta y = \frac{\omega^2}{g} \cdot \frac{x^2}{2}$$

$$\Delta y = \frac{\omega^2 R^2}{2g}$$



QUESTION (JEE Adv, 2021)

A cylindrical tube, with its base as shown in the figure, is filled with water. It is moving down with a constant acceleration a along a fixed inclined plane with angle $\theta = 45°$. P_1 and P_2 are pressures at points 1 and 2, respectively, located at the base of the tube. Let $\beta = (P_1 - P_2)/(\rho g d)$, where ρ is density of water, d is the inner diameter of the tube and g is the acceleration due to gravity. Which of the following statement(s) is (are) correct?

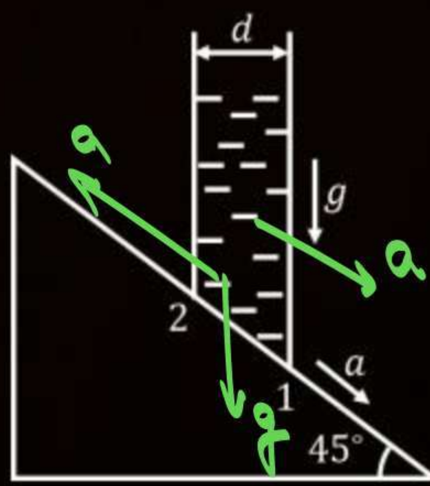
$$\beta = \frac{P_1 - P_2}{\rho g d}$$

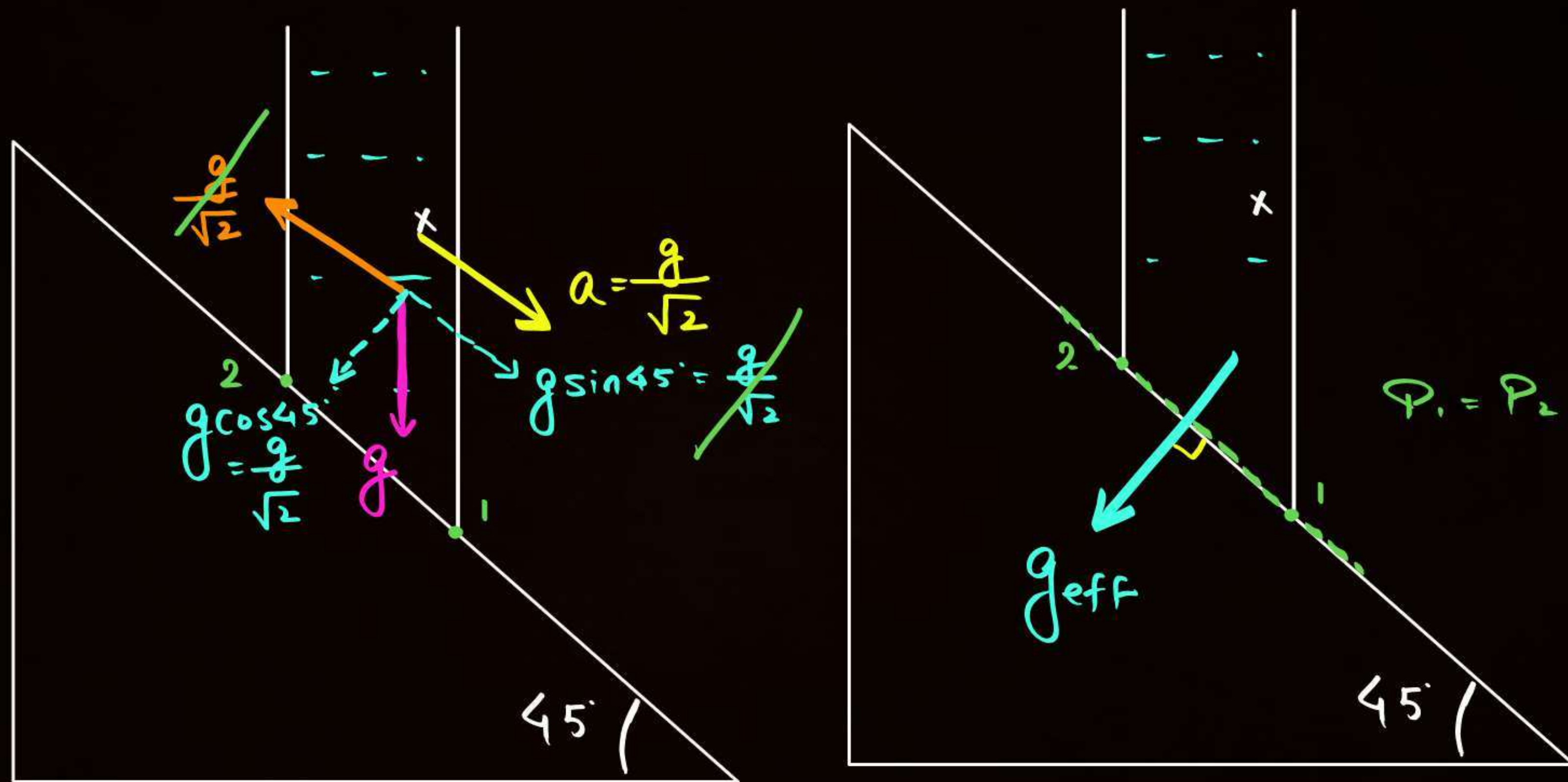
(A) $\beta = 0$ if $a = \frac{g}{\sqrt{2}}$

(B) $\beta > 0$ if $a = \frac{g}{\sqrt{2}}$

(C) $\beta = \frac{1}{\sqrt{2}}$ if $a = \frac{g}{2}$

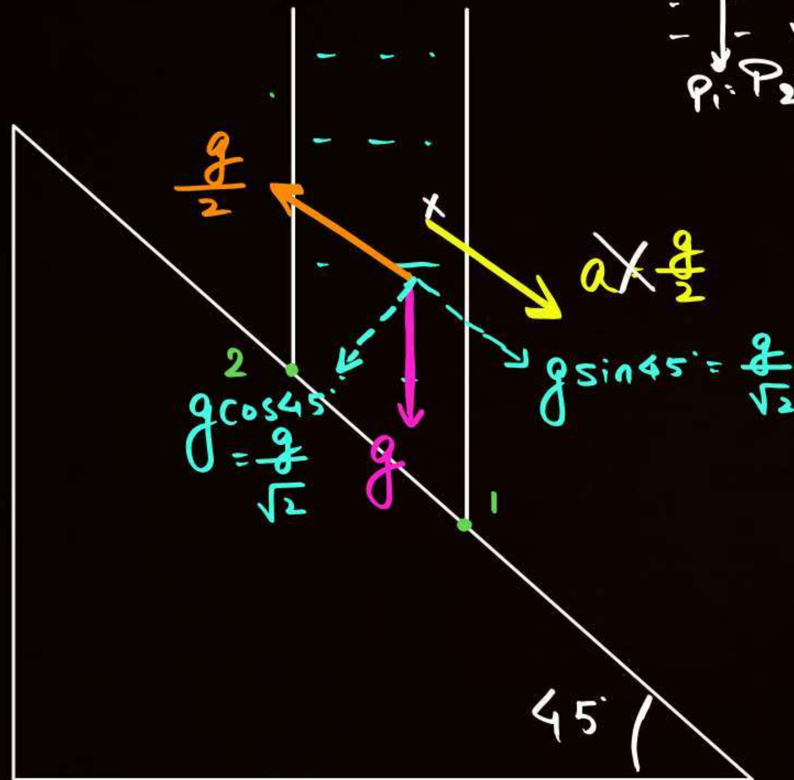
(D) $\beta = \frac{1}{\sqrt{2}} - 1$ if $a = \frac{g}{2}$

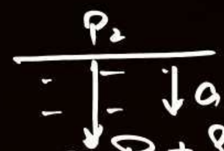




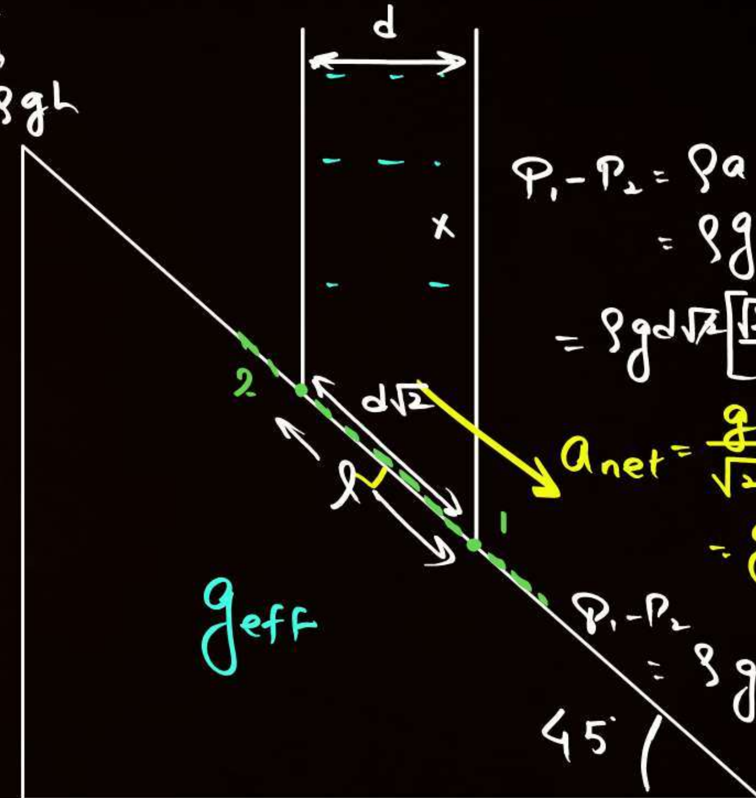
g/√2
x
a = g/√2
g sin45° = g/√2
2
g cos45° = g/√2
g
1
45°
x
2
P1 = P2
1
geff
45°

C, d





$P_1 = P_2 + \rho g h$



$P_1 - P_2 = \rho a l$

$= \rho g\left[\frac{1}{\sqrt{2}} - \frac{1}{2}\right] d\sqrt{2}$

$= \rho g d \cancel{\sqrt{2}}\left[\frac{\sqrt{2}-1}{2\cancel{\sqrt{2}}}\right]$

$a_{net} = \frac{g}{\sqrt{2}} - \frac{g}{2}$

$= g\left[\frac{1}{\sqrt{2}} - \frac{1}{2}\right]$

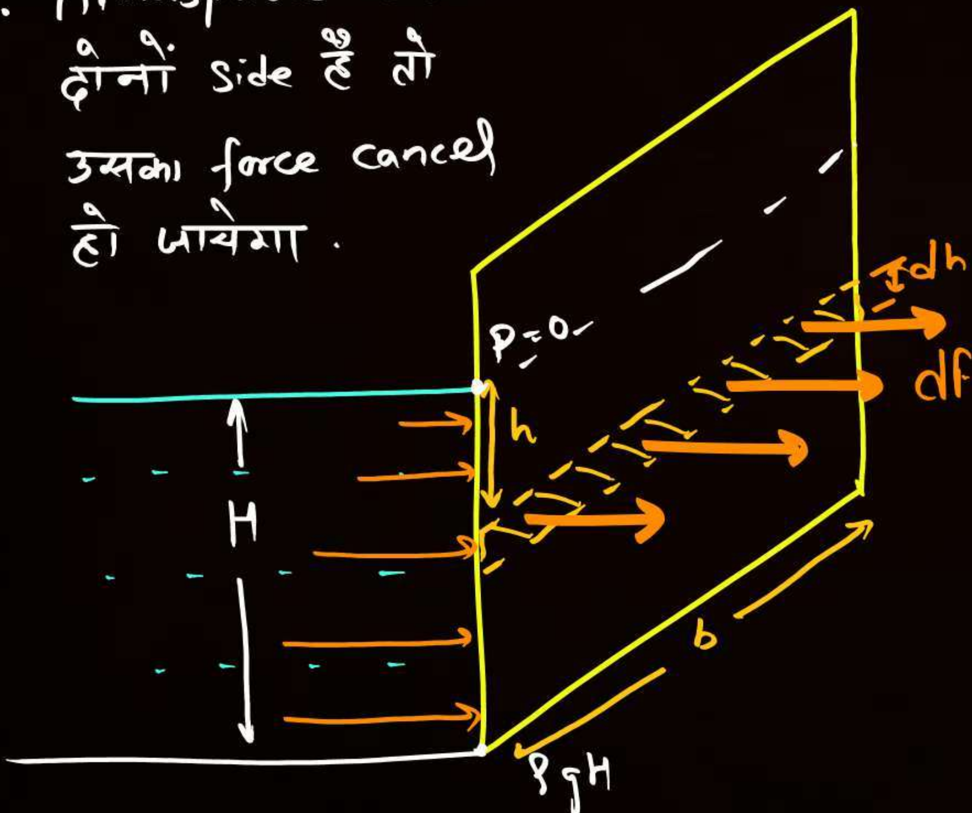
$P_1 - P_2 = \rho g d\left[1 - \frac{1}{\sqrt{2}}\right]$

$l \cos 45° = d$

$l = d\sqrt{2}.$

Force on Container Walls

$\because$ Atmosphere wall के दोनों side हैं तो उसका force cancel हो जायेगा.



$$\int dF = \int P \cdot dA$$

$$\Rightarrow F = \int_0^H \rho g h \cdot dh \cdot b$$

$$F = \rho g b \left[\frac{h^2}{2}\right]_0^H$$

$$F_{net} = \rho g \frac{bH^2}{2}$$

$$= \rho g \frac{H}{2} \cdot A$$

$$= P_{avg} \cdot A = \left[\frac{0+\rho g H}{2}\right] \cdot bH$$

$$F = P_{avg} \times A$$

$$= \left[\frac{0+\rho g H}{2}\right] \cdot Hb$$

$$= \frac{\rho g H^2 b}{2}$$

QUESTION (JEE Mains – 8 Jan 2020 II)

Two liquids of densities ρ_1 and ρ_1 ($\rho_2 = 2\rho_1$) are filled up behind a square wall of side 10m as shown in figure. Each liquid has a height of 5 m. The ratio of the forces due to these liquids exerted on upper part MN to that at the lower part NO is (Assume that the liquids are not mixing):

(A) 1/3

(B) 2/3

(C) 1/2

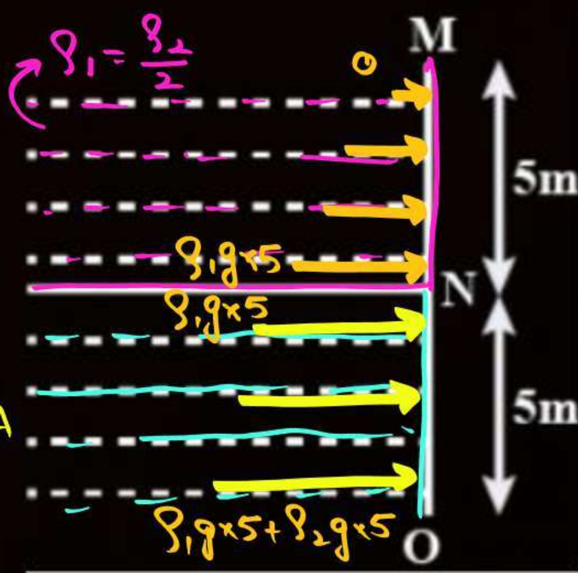
(D) 1/4

$$F_{MN} = P_{avg} \cdot A$$

$$F_{MN} = \left[\frac{0 + \rho_1 g \times 5}{2}\right] \cdot A$$

$$F_{NO} = P_{avg} \cdot A$$

$$F_{NO} = \left[\frac{\rho_1 g \times 5 + (\rho_1 g \times 5 + \rho_2 g \times 5)}{2}\right] A$$



$$\frac{f_{MN} = \dfrac{\rho_1 g \times 5 \times \cancel{A}}{\cancel{2}}}{f_{NO} = \left[\dfrac{\rho_1 g \times 10 + \rho_2 g \times 5}{\cancel{2}}\right]\cancel{A}}$$

$$\frac{f_{MN}}{f_{NO}} = \frac{5g\rho_1}{10g\rho_1 + 5g \times 2\rho_1}$$

$$= \frac{5\cancel{g}\cancel{\rho_1}}{20\cancel{g}\cancel{\rho_1}}$$

$$= \frac{1}{4}$$

QUESTION (JEE Mains – 2014) Adv. level

There is a circular tube in a vertical plane. Two liquids which do not mix and of densities d_1 and d_2 are filled in the tube. Each liquid subtends $90°$ angle at centre. Radius joining their interface makes an angle α with vertical. Ratio d_1/d_2 is:

(A) $\frac{1+\sin\alpha}{1-\sin\alpha}$

(B) $\frac{1+\sin\alpha}{1-\cos\alpha}$

(C) $\frac{1+\tan\alpha}{1-\tan\alpha}$

(D) $\frac{1+\sin\alpha}{1-\cos\alpha}$

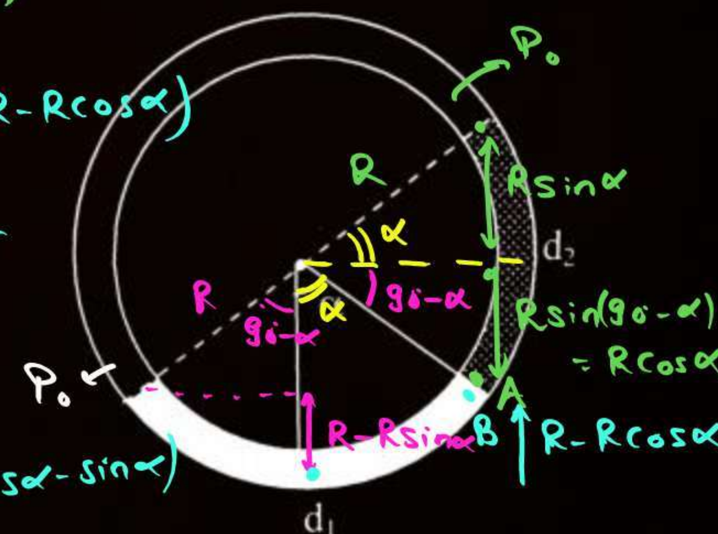
$$P_A = P_0 + d_2 g(R\sin\alpha + R\cos\alpha)$$

$$P_B = P_0 + d_1 g(R - R\sin\alpha) - d_1 g(R - R\cos\alpha)$$

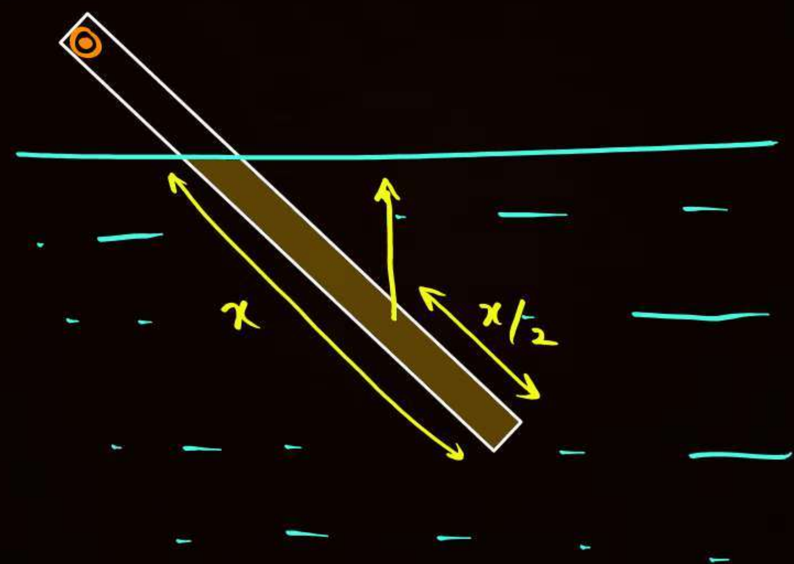
$$P_B = P_0 + (R\cos\alpha - R\sin\alpha) d_1 g$$

$$P_A = P_B$$

$$\cancel{P_0} + d_2 g R(\sin\alpha + \cos\alpha) = \cancel{P_0} + d_1 g R(\cos\alpha - \sin\alpha)$$



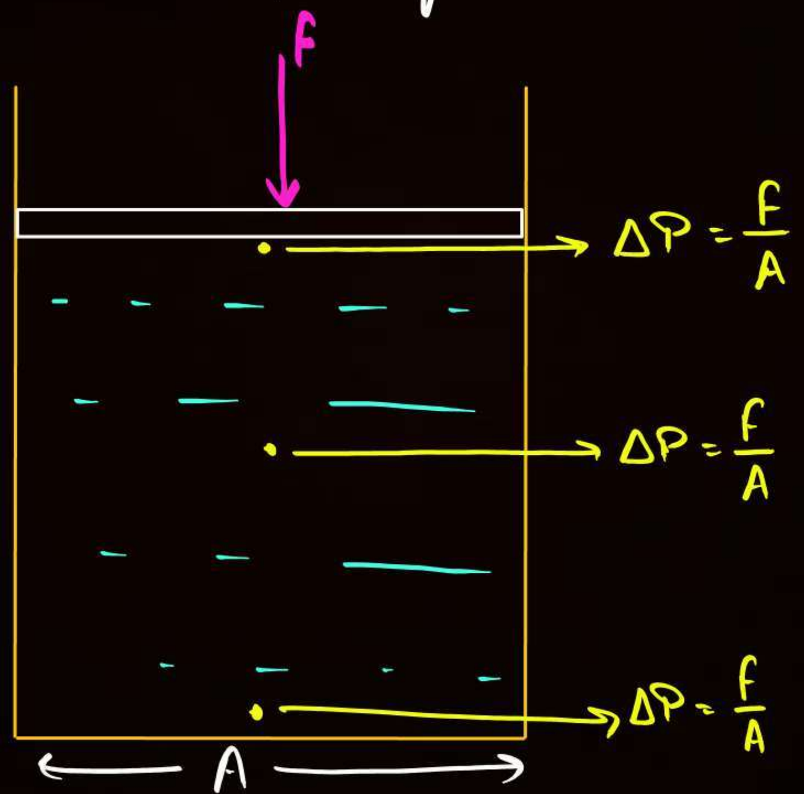
Point of Application

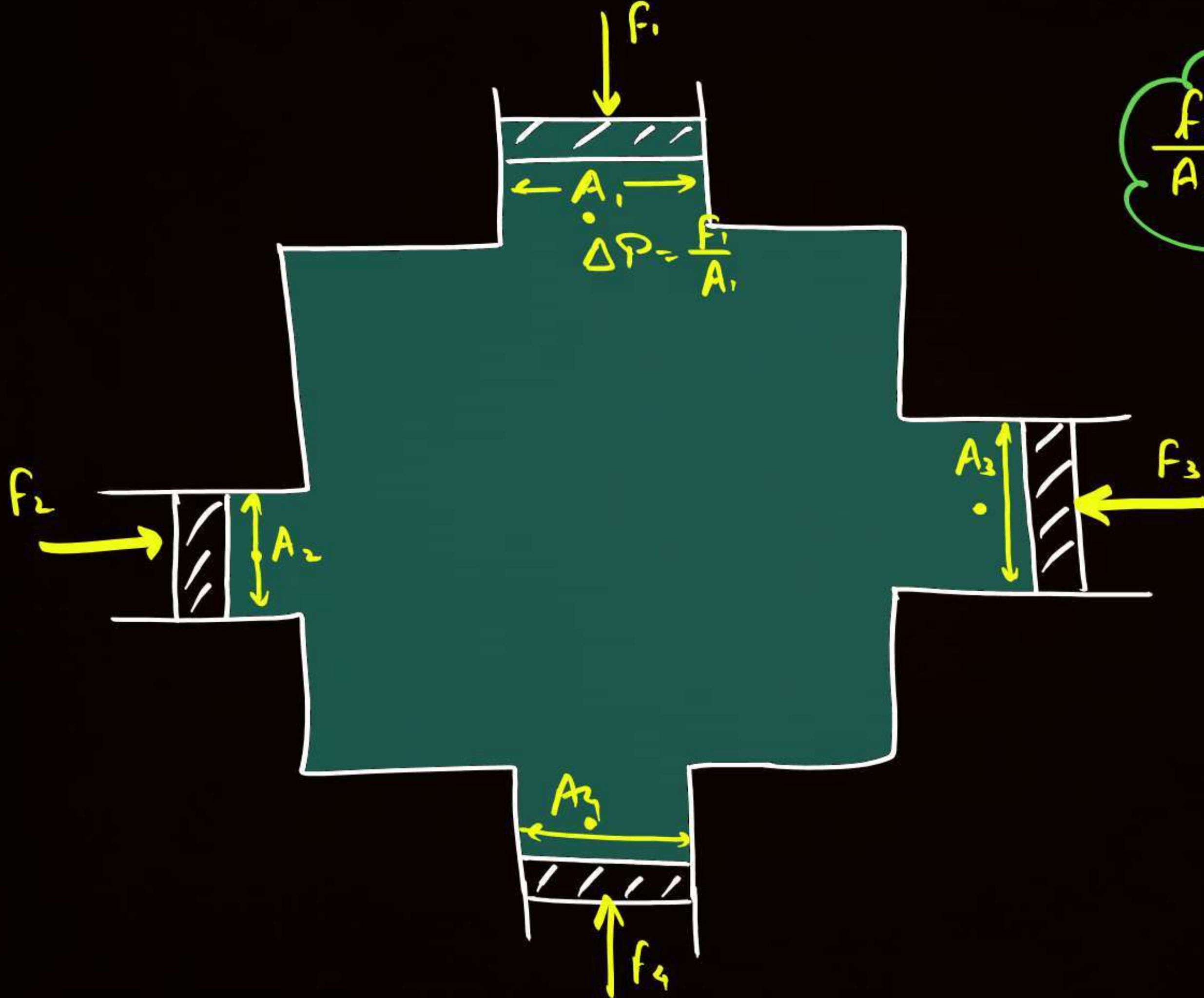


Pt. of application
↓
center of submerge portion.

Pascal's Law

The pressure change in any part of liquid appears undiminished throughout the liquid





$$\frac{F_1}{A_1} = \frac{F_2}{A_2} = \frac{F_3}{A_3} = \frac{F_4}{A_4}$$

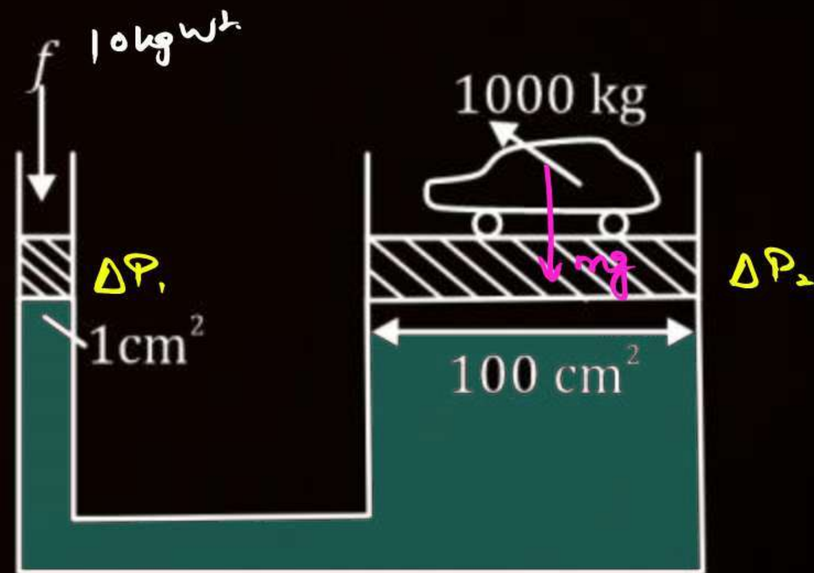
Application -

- Hydraulic Jach
- Hydraulic Lift
- Hydraulic bream.

QUESTION

$f = P \cdot A$

Find minimum force required to lift the car.



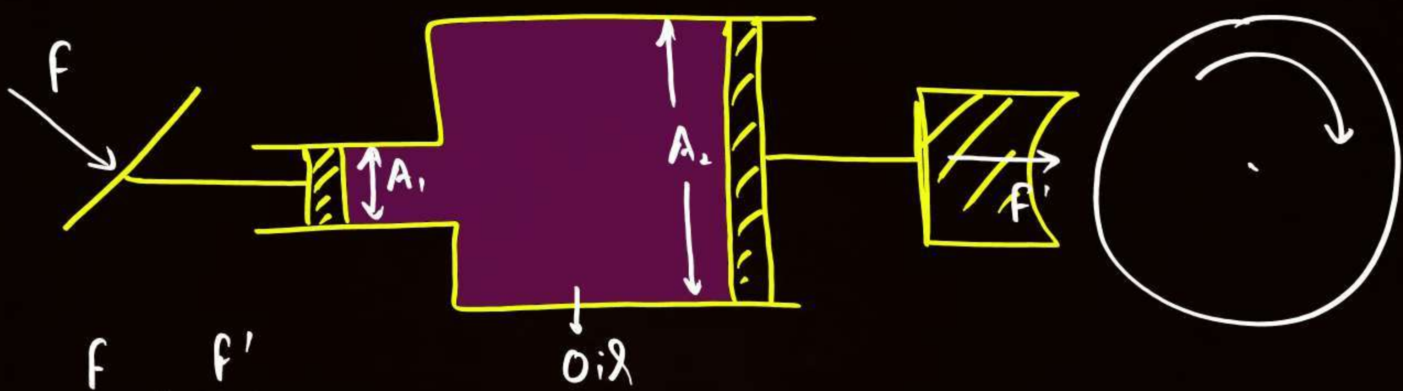
$$\frac{f}{1\,\cancel{cm^2}} = \frac{mg}{100\,\cancel{cm^2}}$$

$$f = \frac{mg}{100}$$

$$= \frac{10\cancel{00} \times 1\cancel{0}}{1\cancel{00}}$$

$$f = 100\ N$$

Hydraulic Breaks



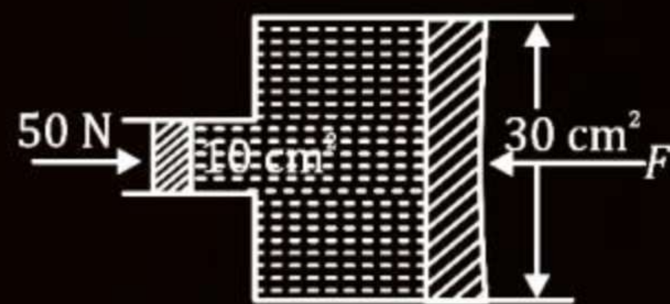
$$\frac{f}{A_1} = \frac{f'}{A_2}$$

$$f' = f \cdot \frac{A_2}{A_1}$$

QUESTION

Find force F needed to keep the piston is equilibrium.

$$\frac{50}{10} = \frac{F}{30} \Rightarrow F = 50 \times 3$$
$$= 150\,N.$$



QUESTION (JEE Mains – 08 April, 2023 (II))

A hydraulic automobile lift is designed to lift vehicles of mass 5000 kg. The area of cross section of the cylinder carrying the load is 250 cm^2. The maximum pressure the smaller piston would have to bear is: [Assume g = 100 m/s^2]:

(A) $200 \times 10^{+6}$ Pa

(B) $20 \times 10^{+6}$ Pa

(C) $2 \times 10^{+6}$ Pa ✓

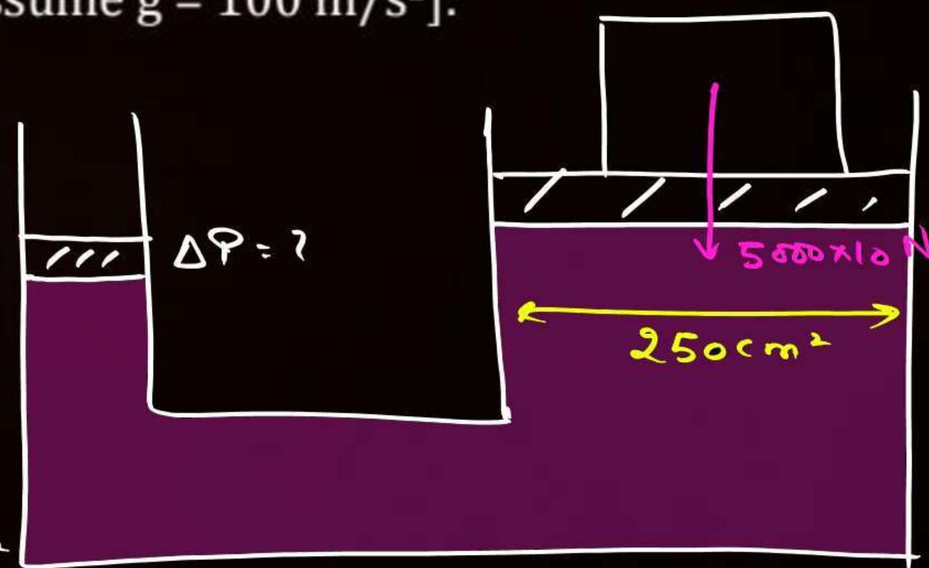
(D) $2 \times 10^{+5}$ Pa

$$\Delta P = \frac{mg}{A}$$

$$= \frac{5\cancel{000} \times 10}{\cancel{250} \times 10^{-4}}$$

$$= 200 \times 10^{4}$$

$$= 2 \times 10^{6}\ \text{Pa}$$



QUESTION (JEE Mains – 06 April, 2023 II)

Given below are two Statement: one is labelled as Assertion A and the other is labelled as Reason R.

Statement-1: When you squeeze one end of a tube to get toothpaste out from the other end, Pascal's Principle is observed.

Statement-2: A change in the pressure applied to an enclosed incompressible fluid is transmitted undiminished to every portion of the fluid and to the walls of its container.

In the light of the above statements, choose the most appropriate answer from the options given below:

(A) A is not correct but R is correct

(B) A is correct but R is not correct

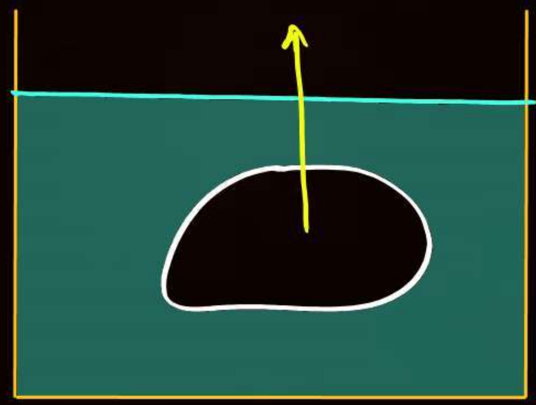
(C) Both A and R are correct and R is the correct explanation of A

(D) Both A and R are correct but R is NOT the correct explanation of A

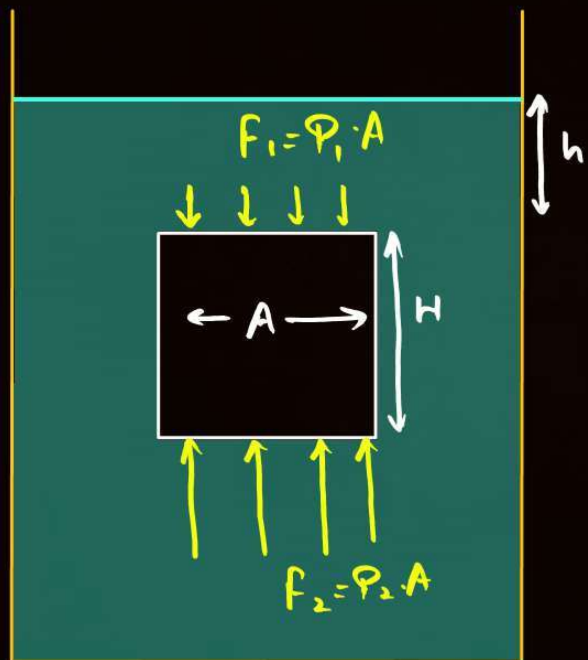
Archimedes' Principle

The liquid applies an upthrust on any object submerged inside it. This upthrust is equal to wt. of liquid displaced by the body.

U = Wt. of liquid displaced by body.



- U is independent of depth of body.

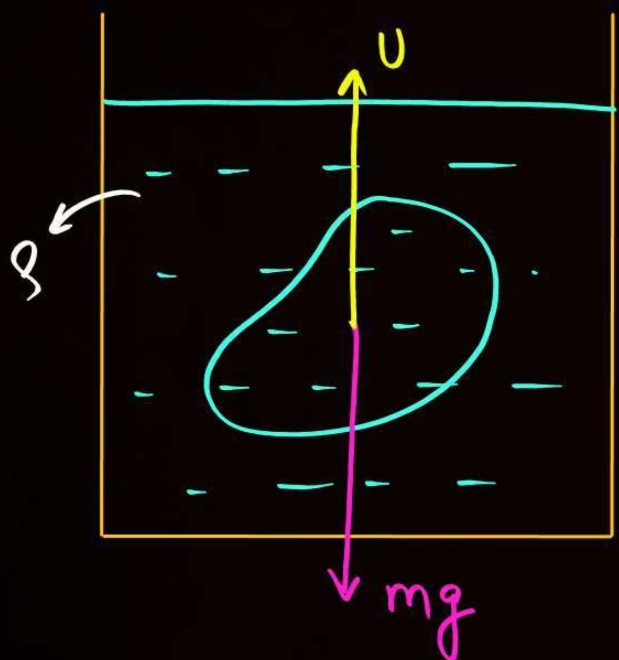


$$U = F_2 - F_1$$
$$= P_2 \cdot A - P_1 A$$
$$= [\cancel{P_0} + \rho g(H+h)]A - [\cancel{P_0} + \rho g h] \cdot A$$
$$= \rho g H \cdot A + \cancel{\rho g h \cdot A} - \cancel{\rho g h \cdot A}$$

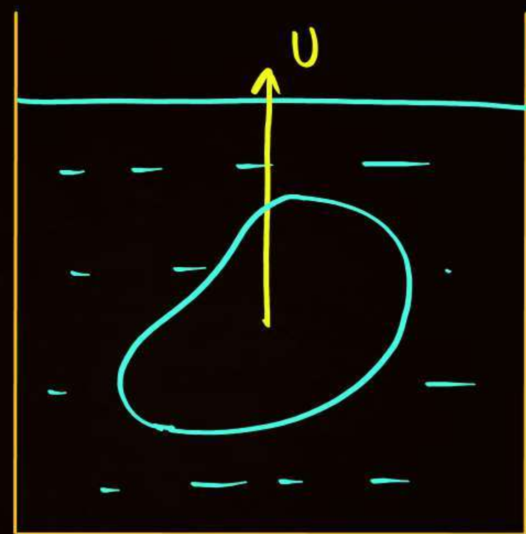
$$U = \rho g H \cdot A$$

$$U = \rho g V_{disp}$$
$$= (\rho V_{disp}) g$$
$$= \underset{\substack{\downarrow \\ \text{mass of liq. disp.}}}{m} g$$

$U = m\vec{g}$ wt. of liquid

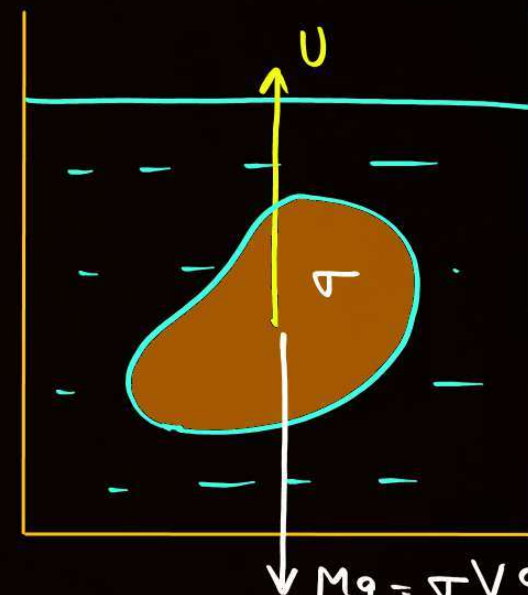


$U = mg$



$\boxed{U = mg = \rho V g}$

$\boxed{U = mg}$

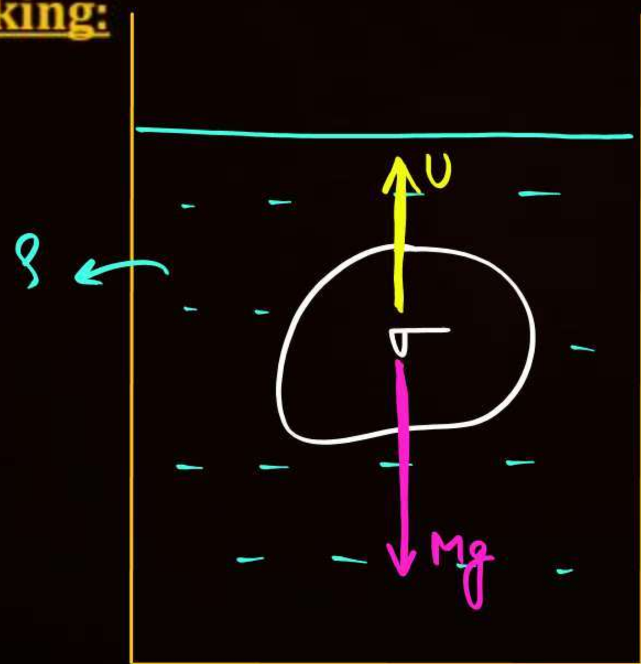


$Mg = \sigma V g$

wt. of body

Condition for Floating/Sinking of a Body

Condition for Sinking:



$$Mg \geq U$$

$$\sigma \cancel{V} \cancel{g} \geq \rho \cancel{V} \cancel{g}$$

$$\boxed{\sigma \geq \rho}$$

Effective density of obj

$$\sigma = \frac{M_{total}}{V_{disp.}}$$

Condition for Floating/Sinking of a Body

Condition for Floating:

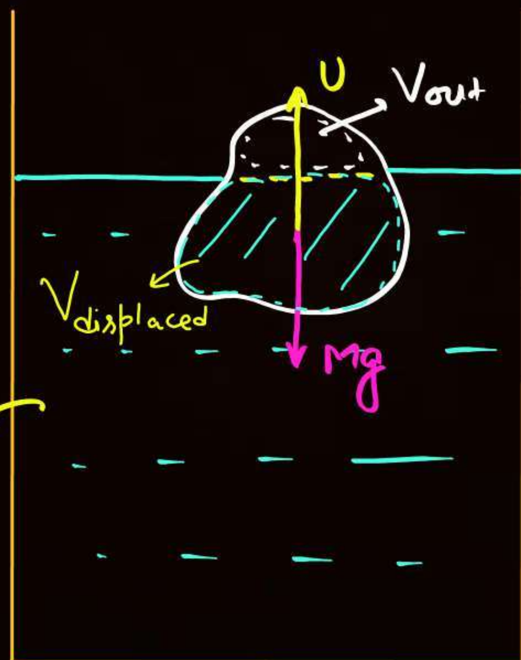
$\sigma < \rho$

$V_{out} = V_{total} - V_{in}$

$V_{out} = V_{total} - \frac{\sigma}{\rho} V_{total}$

$\boxed{V_{out} = V_{total}\left[1 - \frac{\sigma}{\rho}\right]}$

ρ



In equillibrium →

$U = Mg$

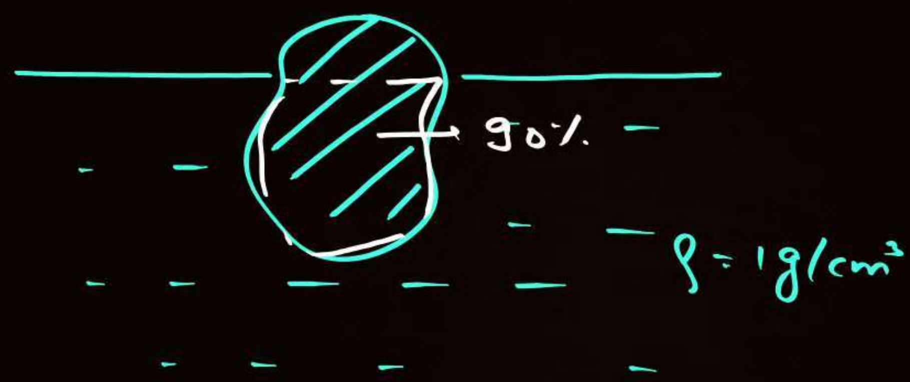
$\rho \cancel{g} V_{disp.} = \sigma \cancel{g} V_{total}$

$\boxed{\frac{V_{disp.}}{V_{total}} = \frac{\sigma}{\rho}}$

$V_{in} = V_{disp.}$ = Vol. of body inside the liquid

$\rho_{Ice} = 0.9\ g/cm^3$

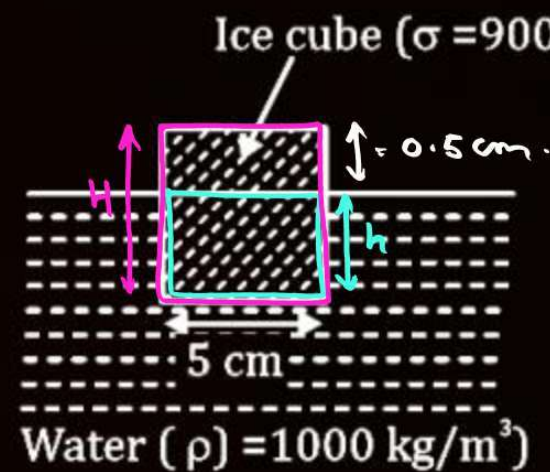
$$\frac{V_{in}}{V_{total}} = \frac{\sigma}{\rho} = \frac{0.9}{1}$$



QUESTION

Find-

1. Volume of ice out of water.
2. ~~Frictional~~ volume out of water. $\rightarrow \frac{1}{10}$
3. Height of cube visible from outside.



1) $V_{out} = 10\%$ of V_{total}

$$= 0.1 \times (5\,cm)^3$$

$$V_{out} = 12.5\ cm^3$$

$$\frac{V_{in}}{V_{total}} = \frac{\sigma}{\rho} = \frac{9}{10}$$

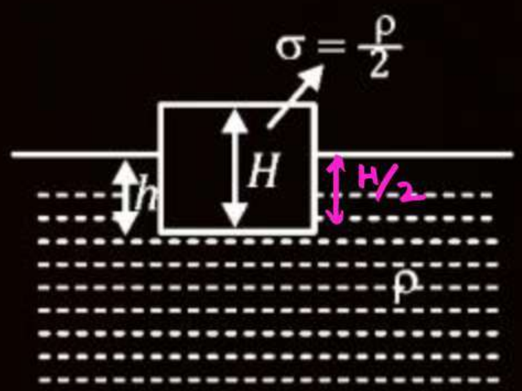
$$\frac{h.\cancel{A}}{H.\cancel{A}} = \frac{9}{10} \Rightarrow \frac{h}{H} = \frac{9}{10}$$

$$h = \frac{9}{10} \times 5 = 4.5\ cm.$$

3)

QUESTION

Find h/H.



QUESTION

Find σ/ρ for which sphere will be half immersed inside the liquid.

$U = Mg$

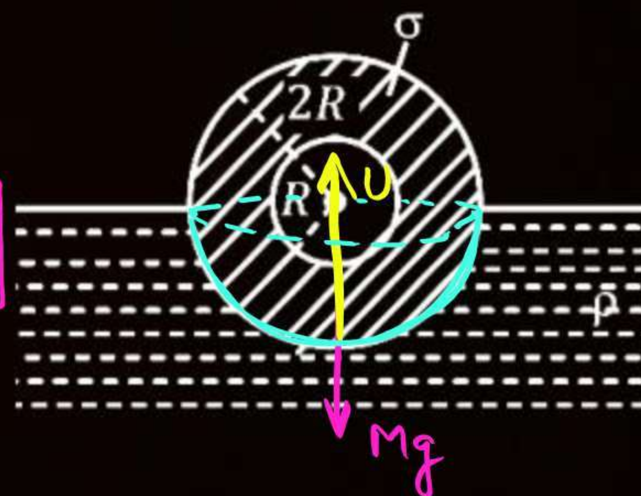
Vol. of material only $= V_{total} - V_{cavity}$

$\rho V_{disp.} \cancel{g} = \sigma V_{body} \cancel{g}$

$\rho \times \frac{2}{3}\pi(2R)^3 = \sigma \cdot \left[\frac{4}{3}\pi(2R)^3 - \frac{4}{3}\pi R^3\right]$

$\rho \times \cancel{8}^{4} \times \frac{\cancel{2}}{\cancel{3}}\cancel{\pi R^3} = \sigma \cdot 7 \times \frac{\cancel{4}^{2}}{\cancel{3}}\cancel{\pi R^3}$

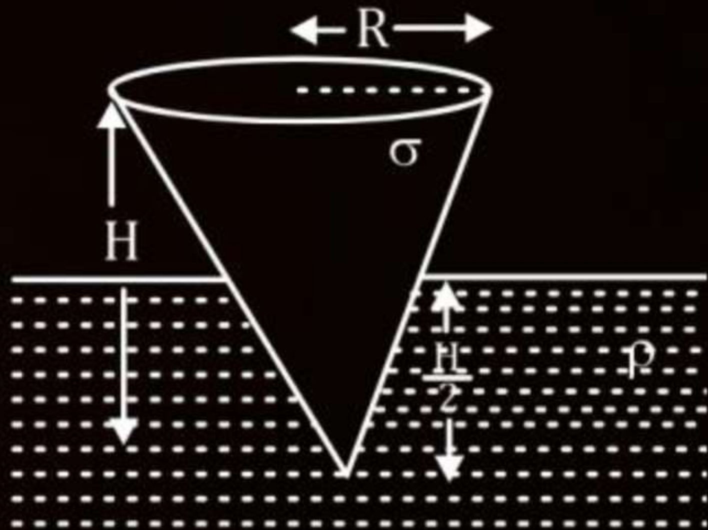
$4\rho = 7\sigma \Rightarrow \frac{\sigma}{\rho} = \frac{4}{7}$ Ans



QUESTION

H.W

Find σ/ρ for equilibrium of cone.



QUESTION (JEE Mains – 5 Sep. 2020 I)

A hollow spherical shell at outer radius R floats just submerged under the water surface. The inner radius of the shell is r. If the specific gravity of the shell material is 27/8, the value of r is:

(A) 8/9 R ✓

(B) 4/9 R ✗

(C) 2/3 R ✗

(D) 1/3 R ✗

$$U = Mg$$

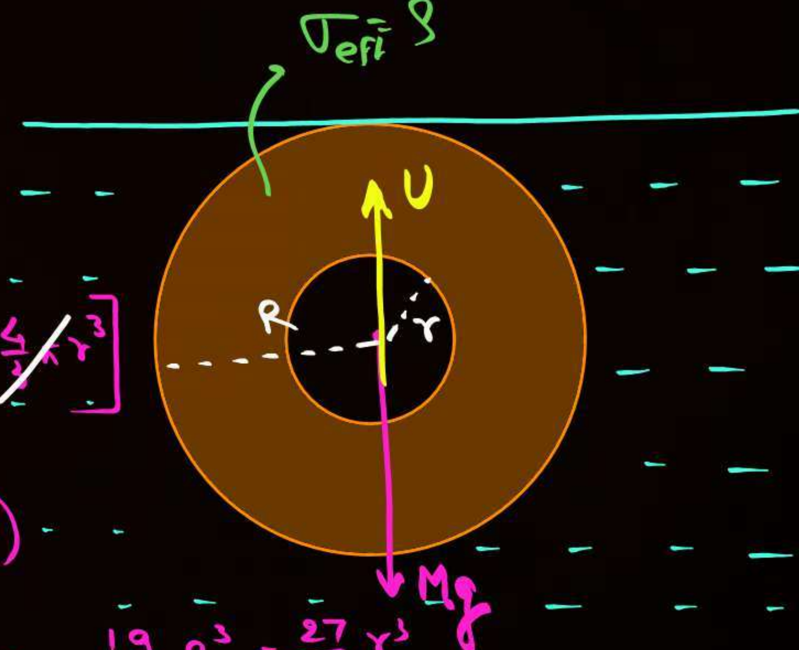
$$\rho V \cancel{g} = \sigma (V_1 - V_2) \cancel{g}$$

$$1 \times \frac{4}{3}\pi R^3 = \frac{27}{8}\left[\frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3\right]$$

$$R^3 = \frac{27}{8}(R^3 - r^3)$$

$$\left(\frac{27}{8} - 1\right)R^3 = \frac{27}{8} r^3 \Rightarrow \frac{19}{8} R^3 = \frac{27}{8} r^3$$

$$r = \frac{(19)^{1/3}}{3} R$$



QUESTION

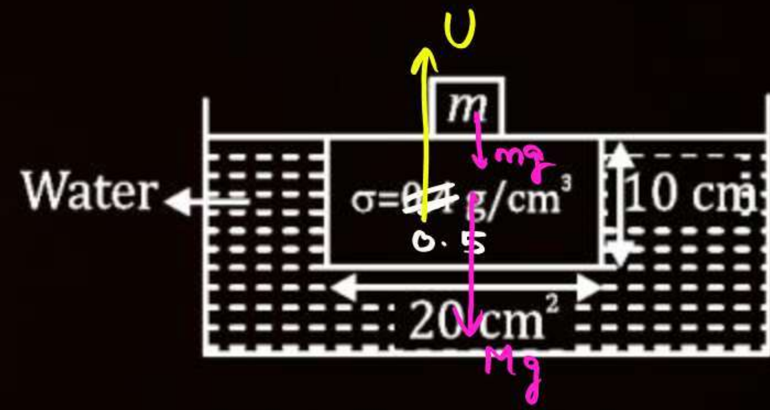
A block has base area 20 cm^2 & height 10 cm. The density of the block is 0.5 g/cm^3. Find mass m that should be kept on the block so that is just submerged into water.

$$U = Mg + mg$$

$$\rho \cancel{g} V = \sigma \cancel{g} V + m \cancel{g}$$

$$1 \frac{g}{\cancel{cm^3}} \times 200 \cancel{cm^3} = 0.5 \times 200 + m$$

$$\boxed{m = 100 \text{ grams}}$$



QUESTION (JEE Mains – 09 April, 2016 II)

$\sigma = \frac{4}{5}\rho$

A wooden block floating in a bucket of water has 4/5 of its volume submerged. When certain amount of an oil is poured into the bucket, it is found that the block is just under the oil surface with half of its volume under water and half in oil. The density of oil relative to that of water is:

(A) 0.5

(B) 0.7

(C) 0.6

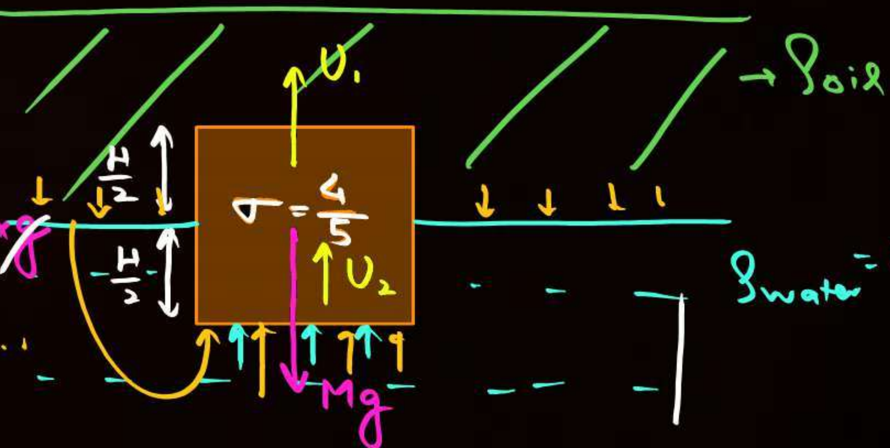
(D) 0.8

$$U_1 + U_2 = Mg$$

$$\rho_{oil}\cdot\frac{\cancel{V}}{2}\cancel{g} + 1\times\frac{\cancel{V}}{2}\times\cancel{g} = \frac{4}{5}\times\cancel{V}\times\cancel{g}$$

$$\frac{\rho_{oil}}{2} = \frac{4}{5} - \frac{1}{2}$$

$$= \frac{3}{10}\times 2 = \frac{3}{5}$$



QUESTION

Consider a solid sphere of radius R and mass density $\rho(r) = \rho_0\left(1 - \frac{r^2}{R^2}\right), 0 < r \le R$. The maximum density of a liquid in which it will float is

(A) $\rho_0/5$

(B) $2\rho_0/5$

(C) $2\rho_0/3$

(D) $\rho_0/3$

$$Mg = U$$

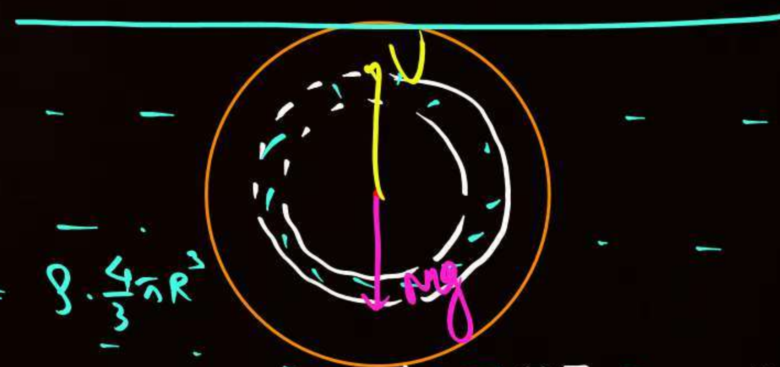
$$\int dM = \rho V$$

$$\int \rho\, dV$$

$$\int \rho_0\left(1 - \frac{r^2}{R^2}\right) \cdot 4\pi r^2 dr = \rho \cdot \frac{4}{3}\pi R^3$$

$$\rho_0 \cancel{4}\cancel{\pi} \int_0^R \left[1 - \frac{r^2}{R^2}\right] r^2 dr = \rho\, \frac{\cancel{4}}{3} \cancel{\pi} R^3 \Rightarrow \rho_0\left[\frac{R^3}{3} - \frac{R^5}{5R^2}\right] = \frac{\rho R^3}{3}$$

$$\rho_0\left[\frac{1}{3} - \frac{1}{5}\right] = \frac{\rho}{3} \Rightarrow \rho = \frac{2}{5}\rho_0$$



QUESTION

$\sigma = \frac{3}{4}\, g/cm^3$

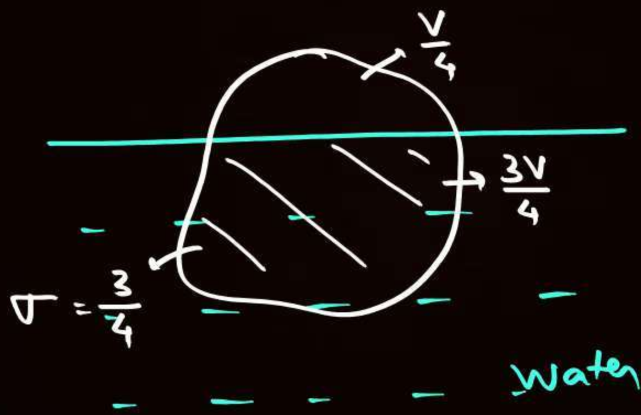
A body floats in water with one fourth of its volume outside water. The same body floats in a liquid of unknown density with one-tenth of its volume outside. Find the density of liquid.

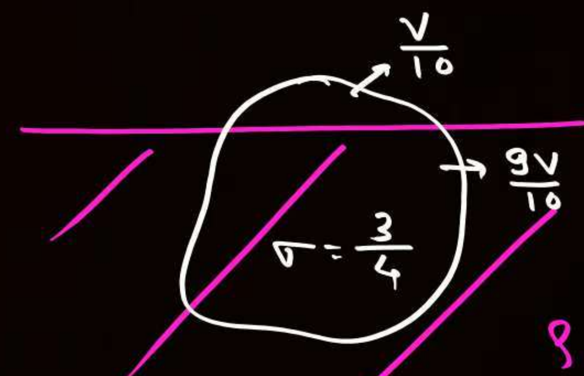
(A) $5/6\ g/cm^2$

(B) $6/5\ g/cm^2$

(C) $5/2\ g/cm^2$

(D) None of these





$$\frac{V_{in}}{V_{total}} = \frac{\sigma}{\rho} \Rightarrow \frac{9\cancel{V}/10}{\cancel{V}} = \frac{3/4}{\rho}$$

$$\rho = \frac{\cancel{3}}{\cancel{4}_2} \times \frac{\cancel{10}^5}{\cancel{9}_3} = \frac{5}{6}.$$

QUESTION

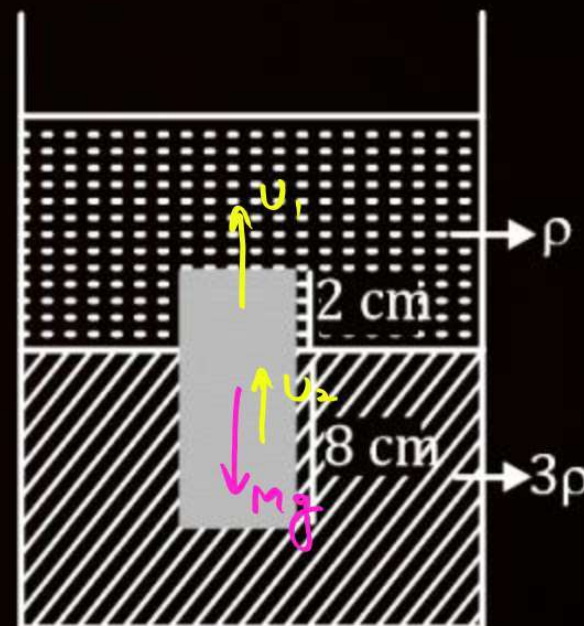
Find the density of block.

$$U_1 + U_2 = Mg$$

$$\rho \cancel{g} \times (2\cancel{A}) + 3\rho \cancel{g} (8 \times \cancel{A}) = \sigma \cancel{g} (10 \times \cancel{A})$$

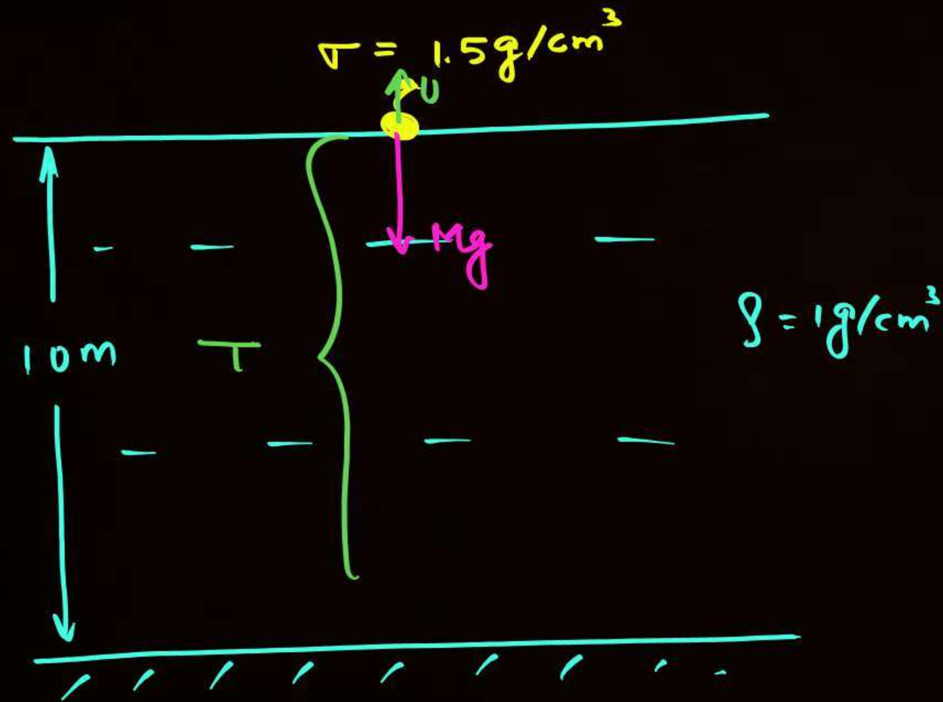
$$26\rho = 10\sigma$$

$$\boxed{\sigma = 2.6\rho}$$ Ans



QUESTION

A block of specific gravity 1.5 is dropped into a lake 10 m deep. Find time after which the block will hit the bottom of lake.



$$F_{net} = Mg - U$$

$$\sigma \cancel{V} \cdot a = \sigma g \cancel{V} - \rho g \cancel{V}$$

$$a = g\left[\frac{\sigma - \rho}{\sigma}\right]$$

$$a = g\left[1 - \frac{\rho}{\sigma}\right]$$

$$a = g\left[1 - \frac{1}{3/2}\right] = \frac{g}{3}$$

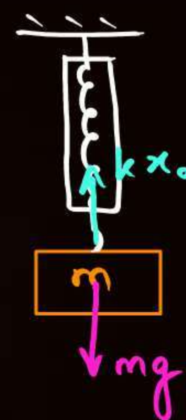
$$T = \sqrt{\frac{2H}{a}} = \sqrt{\frac{2\times 10}{10/3}} = \boxed{\sqrt{6}\ \text{s}}\ \text{Ans.}$$

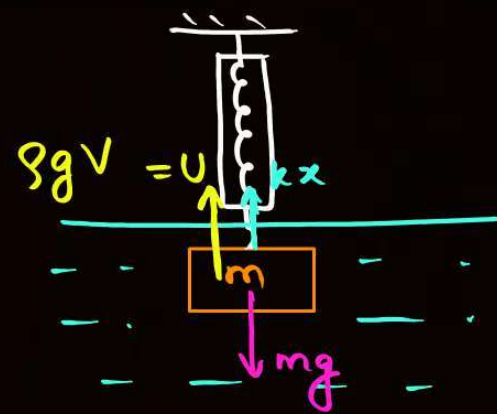
Application of Archimedes' Principle

1. To find weight of body inside liquid:

$$(\sigma > \rho)$$

$$W_{air} = mg = \sigma g V$$





$$kx + U = mg$$

$$W_{apparent} = mg - U$$

$$= mg\left[1 - \frac{U}{mg}\right]$$

$$= mg\left[1 - \frac{\rho \cancel{gV}}{\sigma \cancel{gV}}\right] \Rightarrow \boxed{W_{apparent} = mg\left[1 - \frac{\rho}{\sigma}\right]}$$

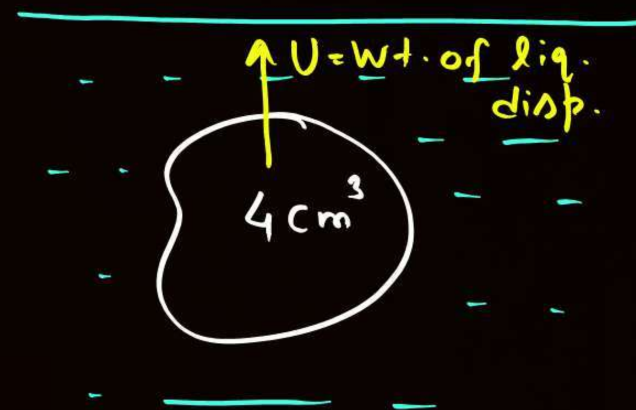
Loss in wt. $= U$

For water —

$\rho = 1 g/cm^3$

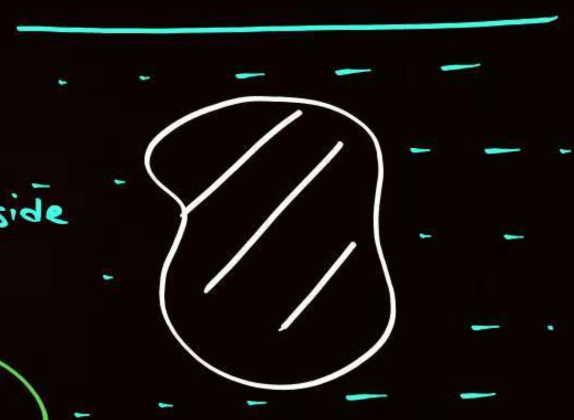
$1 cm^3 \longrightarrow 1 g$

$4 cm^3 \longrightarrow 4 g$ water disp.



Application of Archimedes' Principle

2. To find volume of irregular bodies:



W_{air}

W_{inside}

$U = W_{air} - W_{inside}$

$\rho g V = \text{Wt. loss}$

$$\boxed{V = \frac{\text{Wt. loss}}{\rho g}}$$

$W_{air} = 10 \text{ gram}$

$W_{inside} = 6 \text{ gm}$

$\text{Wt. loss} = 4 \text{ gram}$

$U = 4 \text{ gram.}$

$V_{disp.} = 4 \text{ cm}^3.$

QUESTION

A body of density σ is dropped onto the surface of a liquid of density ρ from height H. If ρ > σ, what will be the maximum depth upto which the body will sink?

(A) $\frac{\sigma H}{\delta}$

(B) $\frac{\delta H}{\sigma}$

(C) $\frac{\sigma H}{\delta - \sigma}$ ✓

(D) $\frac{\delta H}{\delta - \sigma}$

$$TWD = \Delta k.E$$

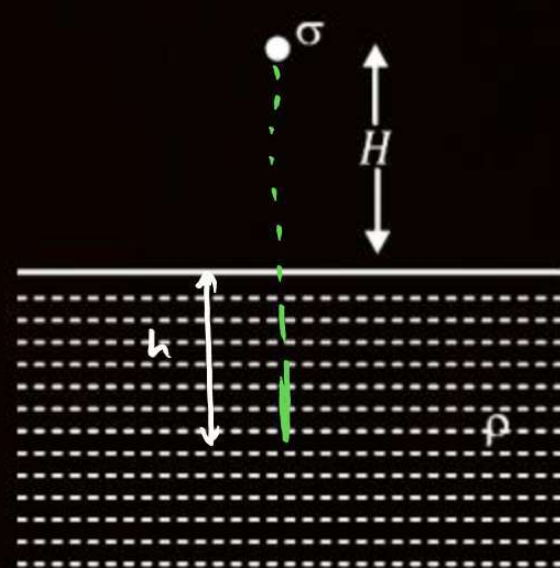
$$W_{mg} + W_U = k.E_f - k.E_i$$

$$+mg(H+h) - \rho g V.h = 0 - 0$$

$$\sigma \cancel{g}\cancel{V}(H+h) = \rho \cancel{g}\cancel{V} h$$

$$\sigma H = \rho h - \sigma h$$

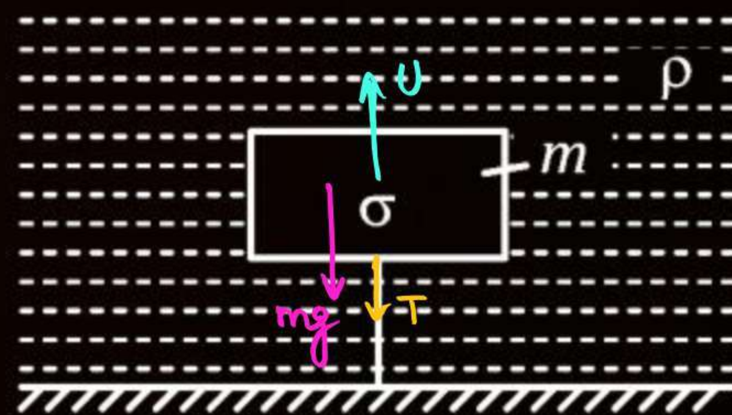
$$\boxed{h = \frac{\sigma H}{\rho - \sigma}}$$



QUESTION

A body of mass m and density σ $(\sigma < \delta)$ is tied to a string inside a liquid as shown in figure. Find the tension in string.

$\sigma V = m \Rightarrow V = \frac{m}{\sigma}$



(A) $mg\frac{\sigma}{\delta}$

(B) $mg\frac{\delta}{\sigma}$

(C) $mg\left[\frac{\delta}{\sigma}-1\right]$

(D) $mg\left[\frac{\sigma}{\delta}-1\right]$

$U = mg + T$

$T = U - mg$

$= \rho g V - mg$

$= \rho g \frac{m}{\sigma} - mg$

$= mg\left[\frac{\rho}{\sigma} - 1\right]$

QUESTION

H.W

The dimensions of base of a boat are 60 cm & 40 cm. If a 60 kg person sits in this boat, how much further will the boat sing in water?

QUESTION

H.W

Two non–mixing liquids of densities ρ and $n\rho$ ($n > 1$) are put in a container. The height of each liquid is h. A solid cylinder of length L and density d is put in this container. The cylinder floats with its axis vertical and length pL ($p < 1$) in the denser liquid. The density d is equal to

(A) $\{2 + (n + 1)p\}\rho$

(B) $\{2 + (n - 1)p\}\rho$

(C) $\{1 + (n - 1)p\}\rho$

(D) $\{1 + (n + 1)p\}\rho$

QUESTION

The approximate depth of an ocean is 2700 m. The compressibility of water is $45.4 \times 10^{-11}\ Pa^{-1}$ and density of water is $10^3\ kg/m^3$. What fractional compression of water will be obtained at the bottom of the ocean?

(A) 0.8×10^{-2}

(B) 1.0×10^{-2}

(C) 1.2×10^{-2} ✓

(D) 1.4×10^{-2}

$$k = \frac{1}{B} \Rightarrow B = \frac{1}{k}$$

$$\Delta P = -B \frac{\Delta V}{V}$$

$$\left|\frac{\Delta V}{V}\right| = \frac{\Delta P}{B} = \frac{\rho g H}{B} = \rho g H k$$

$$= 10^3 \times 10 \times 2700 \times 45.4 \times 10^{-11}$$

QUESTION

A block weights 200g in air, 150g in water and 100g in a liquid of unknown density. Find the density of the liquid.

$$\sigma = \frac{M}{V} = \frac{200}{50} = 4\,cm^3$$

$W_{air} = 200g$

$W_{water} = 150g$

$U = 50g.$

$V = 50cm^3$

$W_{liq} = 100g$

$$m\cancel{g}\left[1 - \frac{\rho}{\sigma}\right] = 100 \times \cancel{10}$$

$$\overset{2}{\cancel{200}} \times \left[1 - \frac{\rho}{4}\right] = 1\cancel{00}$$

$$\frac{\rho}{4} = \frac{1}{2} \rightarrow \boxed{\rho = 2g/cm^3}$$

QUESTION

A body weight $\underbrace{500g}$ in air & $\underbrace{400g}$ in water. Find the density of body & its volume.

Loss = 100 g

Vol. = 100 cm^3

$\sigma = \frac{500}{100} = 5 g/cm^3$.

Application of Archimedes' Principle

3. To find volume of cavity:

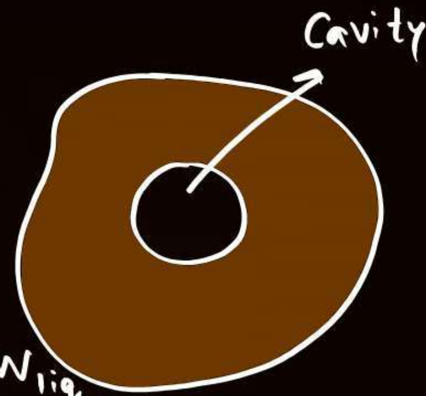
$(\sigma > \rho)$

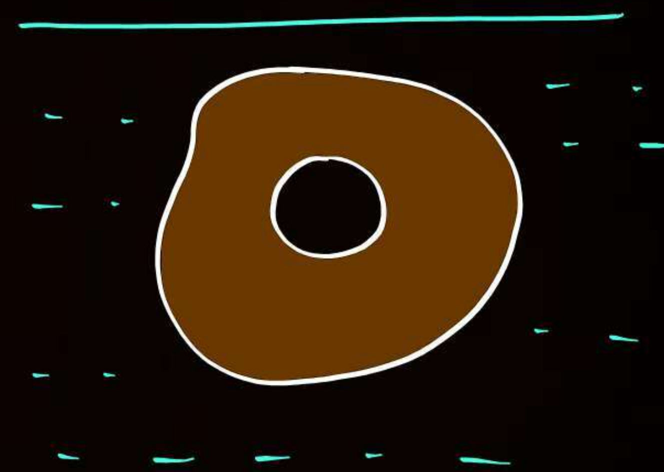
W_{air}, $W_{liq.}$ → Loss in wt., $V = W_{air} - W_{liq}$

$$V_{material} = \frac{W_{air}}{\sigma g}$$

$$\rho g V = W_{air} - W_{liq.}$$

$$V_{total} = \frac{W_{air} - W_{liq}}{\rho g}$$





$$V_{cavity} = V_{total} - V_{material}$$

$$= \frac{W_{air} - W_{liq}}{\rho g} - \frac{W_{air}}{\sigma g}$$

QUESTION

W_{air} $\quad$ W_{liq}

The weight of a gold ornament in air & water is 200g & 180g respectively. If the ornament has some air bubbles trapped inside, find the vol. of air bubbles (δ_{gold} = 20 g/cm^3)

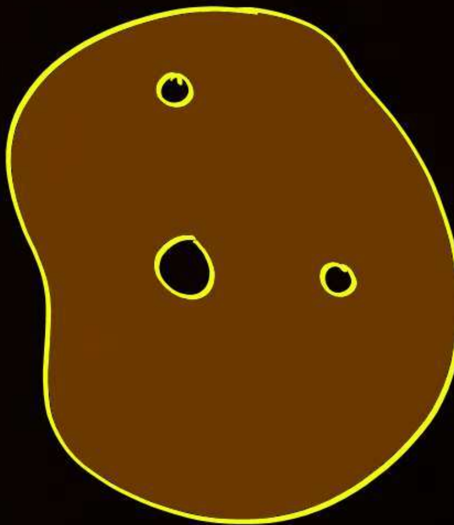
Wt. loss = 20g

Vol. = 20 cm^3

$V_{material} = \frac{200}{20} = 10\,cm^3$

$V_{cavity} = V_{total} - V_{material}$

$= 20 - 10 = 10\,cm^3$



QUESTION

$\rightarrow \sigma < \rho$

An object of mass m floats in liquid of density δ. If the density of the object is σ, find the apparent weight of object.

(A) $mg\frac{\sigma}{\delta}$

(B) $mg\left[1-\frac{\sigma}{\delta}\right]$

(C) $mg\left[1-\frac{\delta}{\sigma}\right]$ → Common mistake

(D) Zero

Application of Archimedes' Principle

4. Mixture of Metals (Alloys):

$$W_{air} = \sigma_1 V_1 + \sigma_2 V_2 \qquad V_1 + V_2 = V$$

$$W_{liq.} = W_{air} - \rho g V$$

QUESTION

An alloy is made up of copper ($\sigma_{cm} = 6\ g/cm^3$) & Aluminum ($\sigma_{Al} = 4\ g/cm^3$). The mass of alloy is 20g in air. When the alloy is submerged in water its apparent mass is 16g. Find the vol. of Cu and Al inside the alloy.

$\sigma_{Cu} = 6\ g/cm^3$

$\sigma_{Al} = 4\ g/cm^3$

$m_{air} = 20g$

$W_{liq.} = 16g.$

$U = 4g$

$V_{total} = 4\ cm^3$

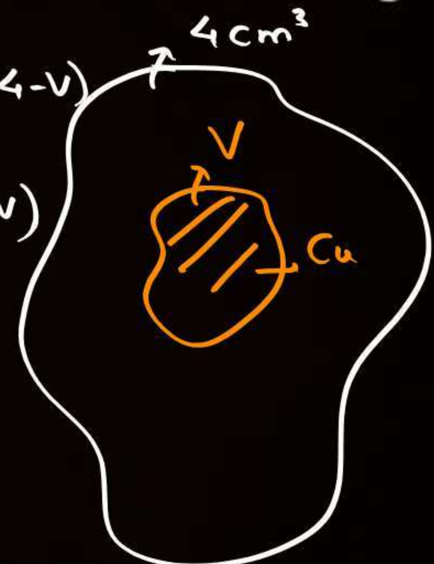
$W_{air} = \sigma_1 V + \sigma_2 (4-V)$

$20 = 6 \times V + 4(4-V)$

$20 = 2V + 16$

$\Rightarrow V_{Cu} = 2\ cm^3$

$V_{Al} = 2\ cm^3$



Variation in Level of Liquid

1. Dropping of Stones from a Boat:

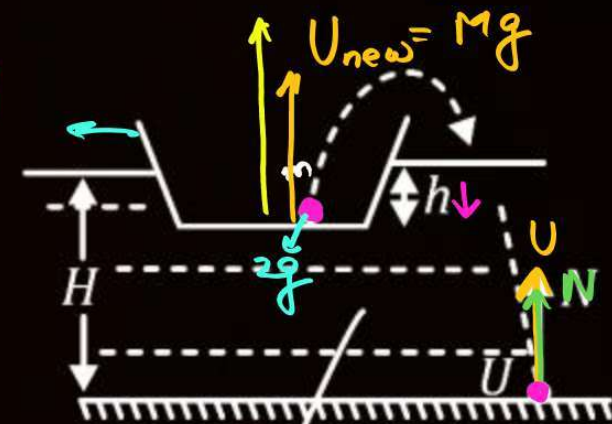
$U \downarrow \longrightarrow$ Vol. disp. $\downarrow$

$U_i = 12g$

$U_f = (10+1) = 11g.$

Initially $\rightarrow U = (M+m)g$

10g



h will decrease

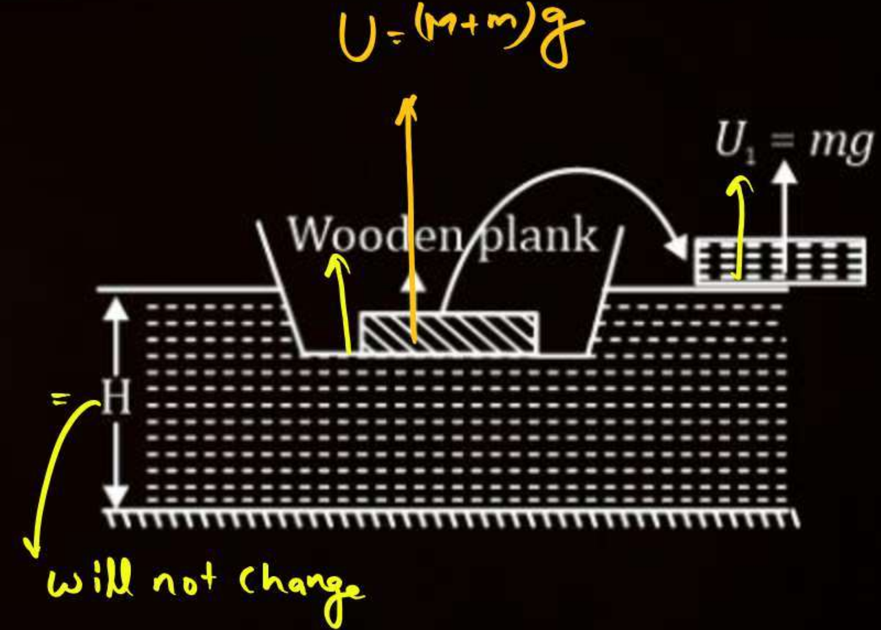
H will decrease

$U + N = mg$

Variation in Level of Liquid

2. Dropping of Wood from a Boat:

$U_i = U_f$



Variation in Level of Liquid

3. Melting of Ice:

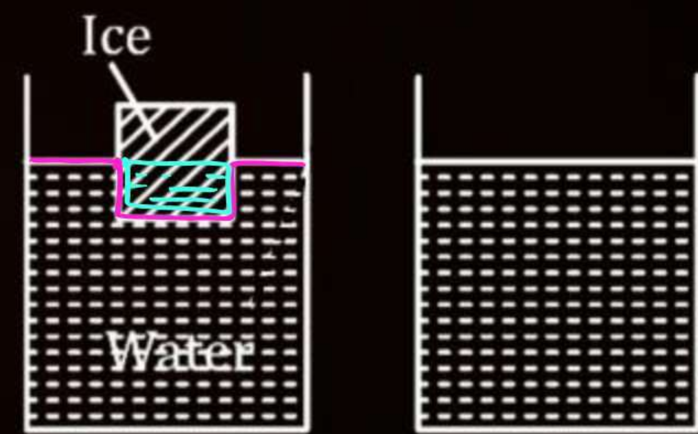
level will remain same.

Before → $U = mg = \sigma g V_{total}$

$\rho g V_{in} = \sigma g V_{total}$

After → $\cancel{\rho}\cancel{g} V_{in} = \cancel{\rho} g V.$

$V = V_{in}$



Before melting of ice

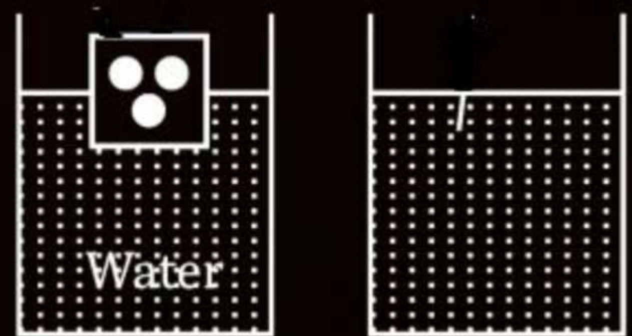
After melting of ice

Variation in Level of Liquid

4. Melting of Ice Having Air Bubbles:

U = same

V = same



Before melting of Ice

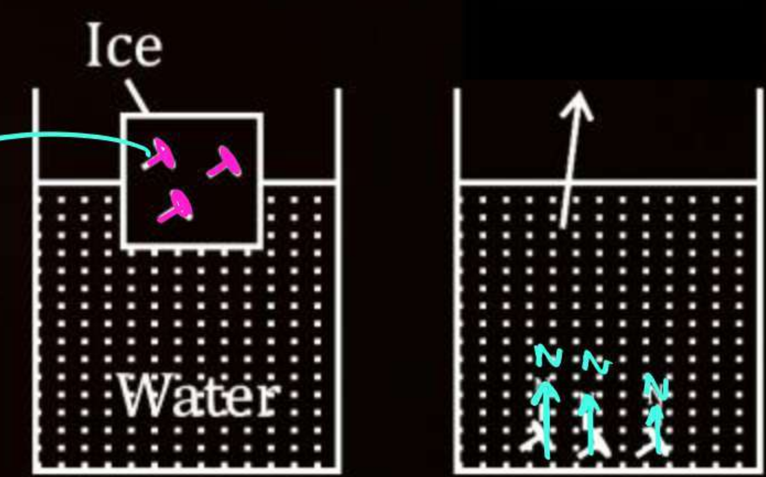
After melting of Ice

Variation in Level of Liquid

5. **Melting of Ice Having Iron Nails:**

V will decrease
↓
Level will also decrease.

Iron Nails



Before melting of Ice

After melting of Ice

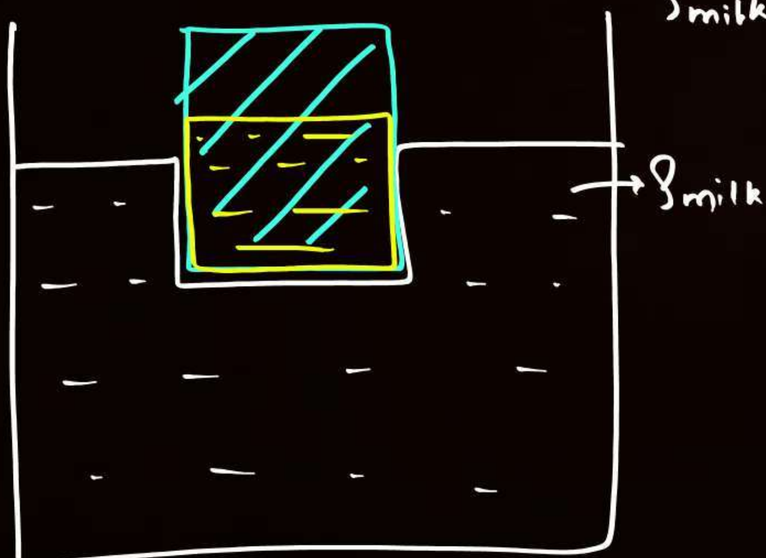
Variation in Level of Liquid

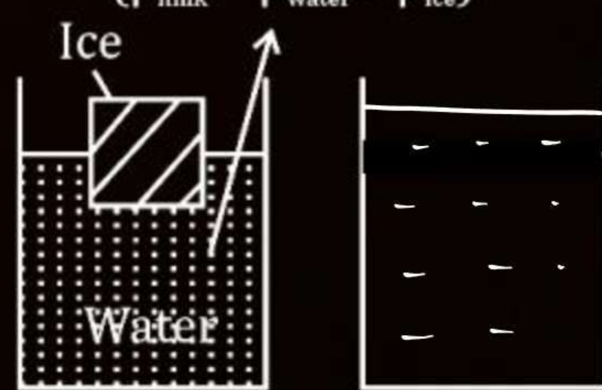
$\uparrow \rho_{milk} \downarrow V_{disp} = \downarrow \rho_{water} \cdot V_{water} \uparrow$

6. Melting of Ice in Higher Density Liquid:

$\rho_{milk} \cdot V_{disp.} = \sigma_{ice} \uparrow V_{total} \downarrow$ Milk

$(\rho_{milk} > \rho_{water} > \rho_{ice})$





Before melting of Ice

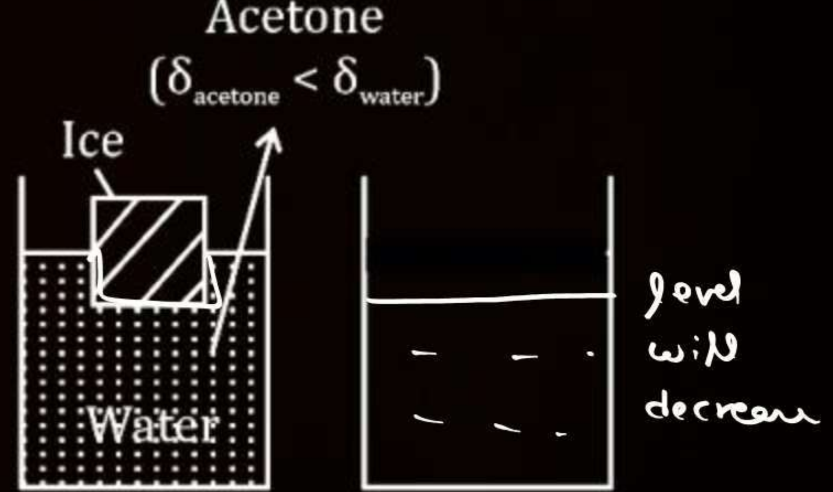
After melting of Ice

level will increase.

Variation in Level of Liquid

7. Melting of Ice in Lower Density Liquid:

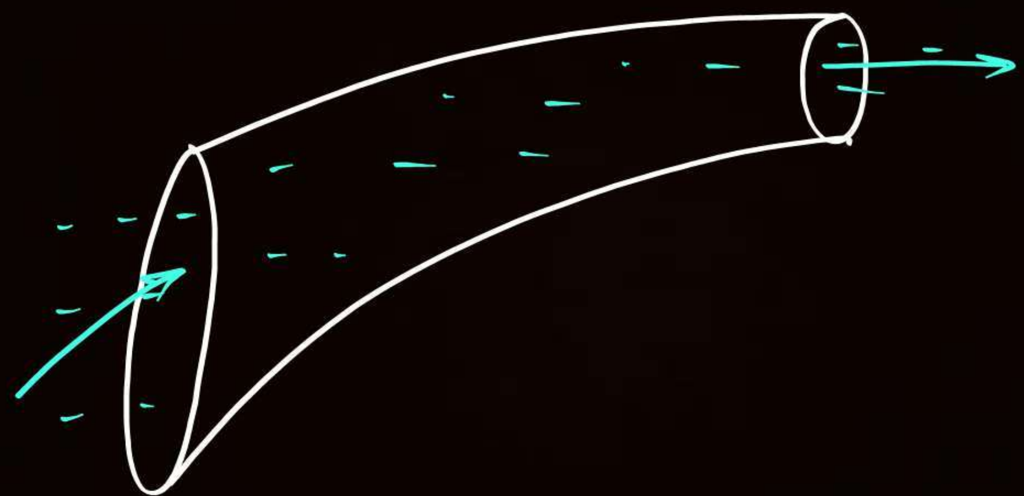
$\rho_1 V_1 = \rho_2 V_2$



Before melting of Ice

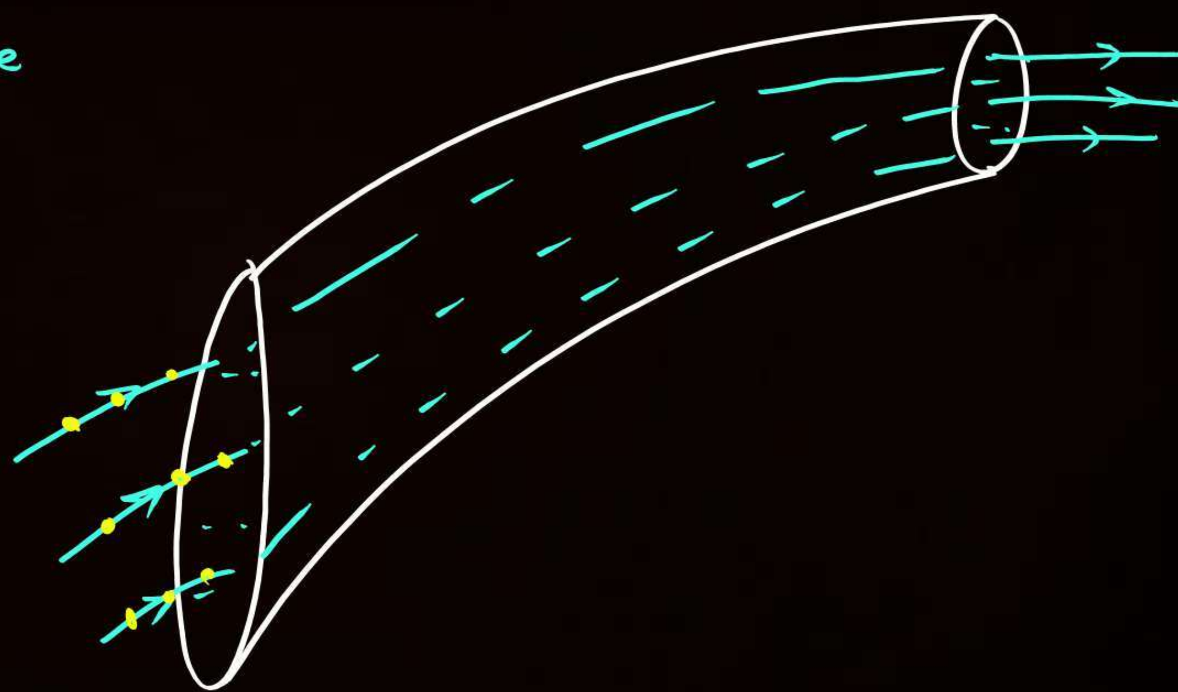
After melting of Ice

Fluid Dynamics



Assumptions for Ideal Liquid

- Incompressible
- Viscosity = 0
- Streamline
- No vortex are formed inside liquid

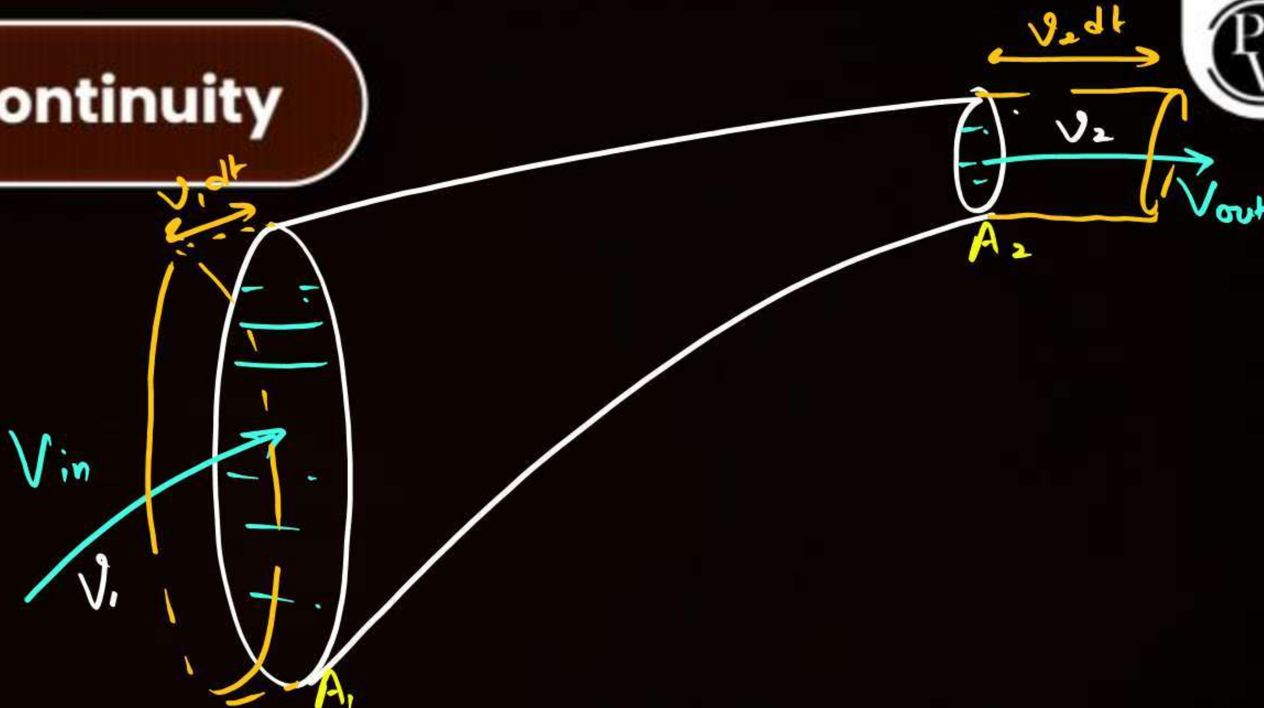


Equation of Continuity

$$V_{in} = V_{out}$$

$$A_1 v_1 \cancel{dt} = A_2 v_2 \cancel{dt}$$

$$A_1 v_1 = A_2 v_2$$



QUESTION

Find v.

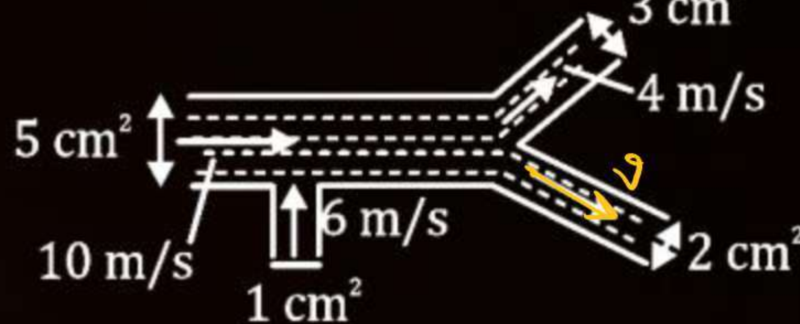
$$V_{in} = V_{out}$$

$$5 \times 10 + 1 \times 6 = 3 \times 4 + 2v$$

$$50 + 6 = 12 + 2v$$

$$2v = 56 - 12$$

$$v = \frac{44}{2} = 22\ m/s$$



QUESTION (JEE Mains – 8 April 2019 II)

The cylindrical tube of a spray pump has radius R, one end of which has n fine holes, each of radius r. If the speed of the liquid in the tube is v, the speed of the ejection of the liquid through the holes is

(A) $\dfrac{vR^2}{n^2r^2}$

(B) $\dfrac{vR^2}{nr^2}$ ✓

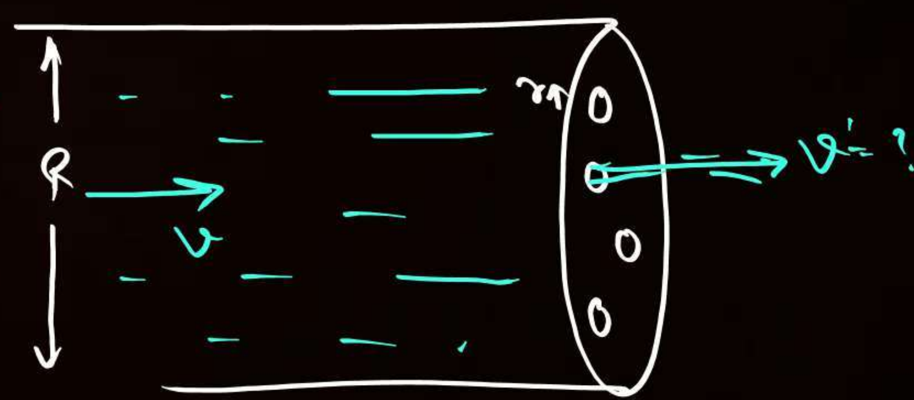
(C) $\dfrac{vR^2}{n^3r^2}$

(D) $\dfrac{v^2R}{nr}$

$$A_1 v_1 = A_2 v_2$$

$$\cancel{\pi} R^2 \times v = n \times \cancel{\pi} r^2 v'$$

$$v' = \frac{v R^2}{n r^2}$$



QUESTION (JEE Mains – 10 April, 2023 II)

Figure below shows a liquid being pushed out of the tube by a piston having area of cross section 2.0 cm^2. The area of cross section at the outlet is 10 mm^2. If the piston is pushed at a speed of 4 cm s^{-1}, the speed of outgoing fluid is _____ cm s^{-1}.

$$V_{in} = V_{out}$$

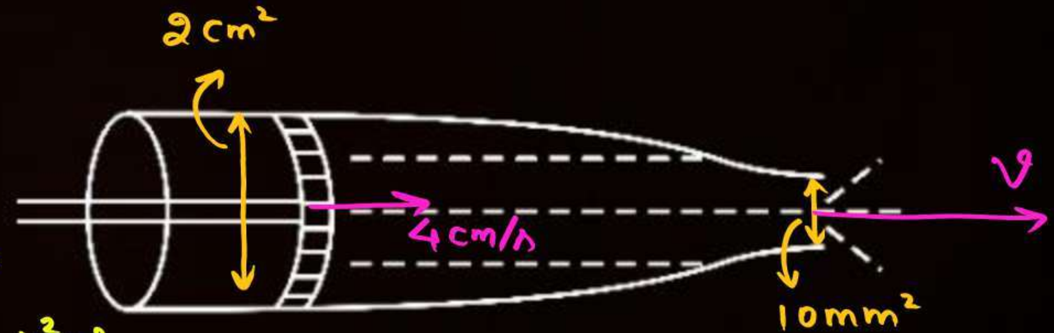
$$A_1 v_1 = A_2 v_2$$

$$2\ cm^2 \times 4\ \frac{cm}{s} = 10\ mm^2\ v_2$$

$$8\ \frac{cm^3}{s} = 10 \times (10^{-1}\ cm)^2\ v_2$$

$$8 = 10^{-1}\ v_2$$

$$v_2 = 80\ cm/s \text{ Ans}$$



QUESTION (JEE Mains – 01 Feb, 2023 II)

The surface of water in a water tank of cross section area 750 cm^2 on the top of a house is h m above the tap level. The speed of water coming out through the tap of cross section area 500 mm^2 is 30 cm/s. At that instant, dh/dt is $x \times 10^{-3}$ m/s. The value of x will be _____.

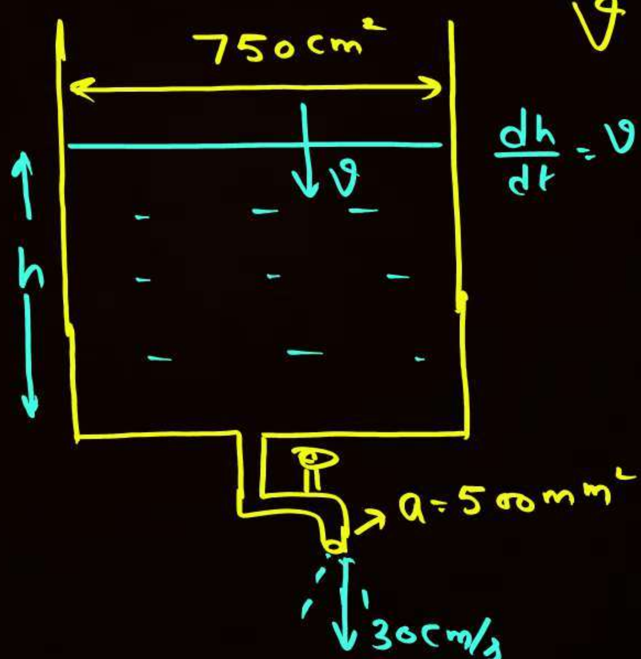
$$750 \times v = 500 \times (10^{-1})^2 \times 30$$

$$v = \frac{5 \times 3\cancel{0}}{75\cancel{0}}$$

$$= \frac{15}{75} \frac{cm}{s}$$

$$= 0.2 \frac{cm}{s}$$

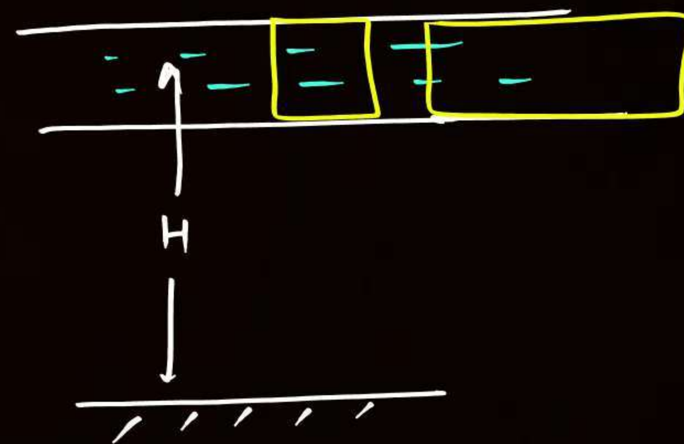
$$= 2 \times 10^{-3} \frac{m}{s}$$



Bernoulli's Theorem

$\longrightarrow$ Based on conservation of energy.

Gravitational Potential Energy:



$$\text{Pot. Energy density} = \frac{U}{V} = \frac{mgH}{V} = \rho g H$$

Kinetic Energy:

$$\frac{K.E}{V} = \frac{\frac{1}{2} m v^2}{V} = \frac{1}{2} \rho v^2$$

Bernoulli's Theorem

Pressure Energy:

Pressure Energy per unit vol. $= P$

Bernoulli's Theo. $\rightarrow$ Total Initial Energy = Total Final Energy.

$$\boxed{\rho g H_1 + \frac{1}{2}\rho v_1^2 + P_1 = \rho g H_2 + \frac{1}{2}\rho v_2^2 + P_2}$$

QUESTION

Find radius of water flow 15 m below the tap.

$$\cancel{P_0} + \rho g \times 15 + \frac{1}{2}\rho(10)^2 = \cancel{P_0} + 0 + \frac{1}{2}\rho v^2$$

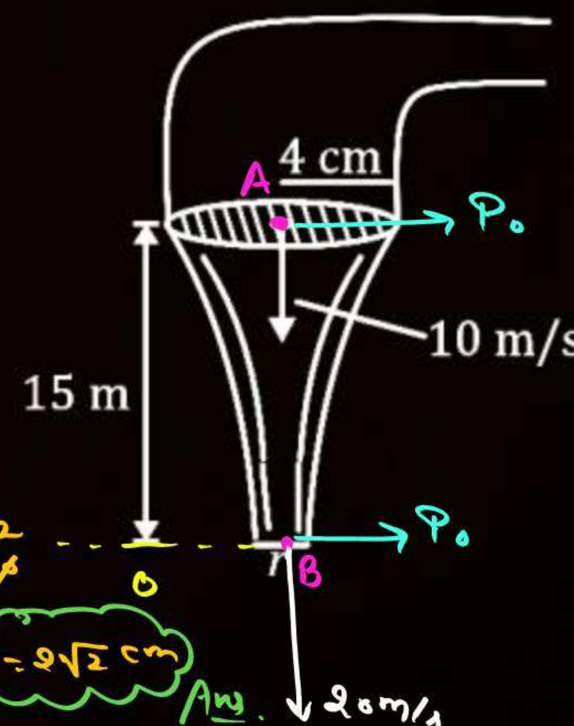
$$\cancel{10^3} \times 10 \times 15 + \frac{\cancel{10^3} \times 10^2}{2} = \frac{1}{2} \times \cancel{10^3} \times v^2$$

$$200 = \frac{v^2}{2}$$

$$v = 20\ m/s.$$

Eq of continuity b/w A & B — $\cancel{\pi} \times (4\ cm)^2 \times 1\cancel{0} = \cancel{\pi} r^2 \times 2\cancel{0}$

$r^2 = \frac{16}{2} = 8 \Rightarrow \boxed{r = 2\sqrt{2}\ cm}$ Ans.



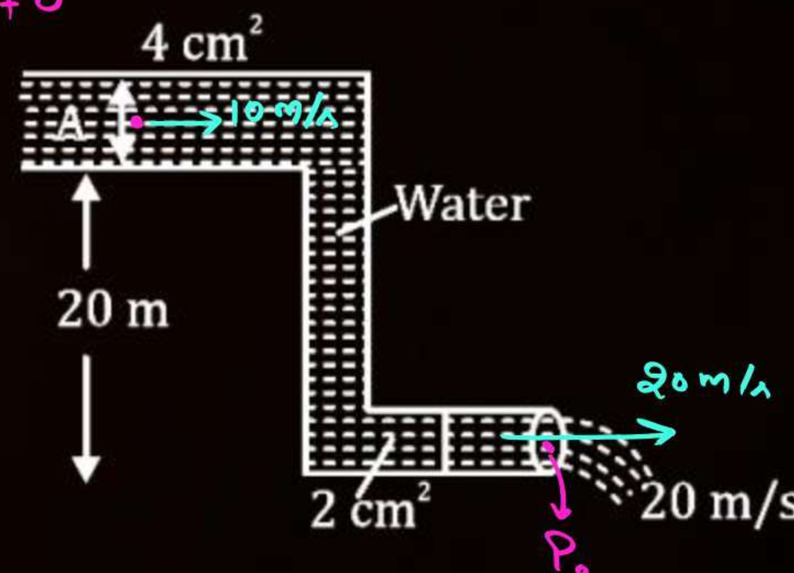
QUESTION

Find pressure at A.

$$P_A + \frac{1}{2}\rho\times(10)^2 + \rho g\times 20 = P_0 + \frac{1}{2}\rho\times(20)^2 + 0$$

$$P_A = 10^5 + \cancel{10^3\times 200} - \frac{10^5}{2} - \cancel{2\times 10^5}$$

$$P_A = \frac{10^5}{2}\ \text{Pa}\quad \text{Ans}$$



QUESTION

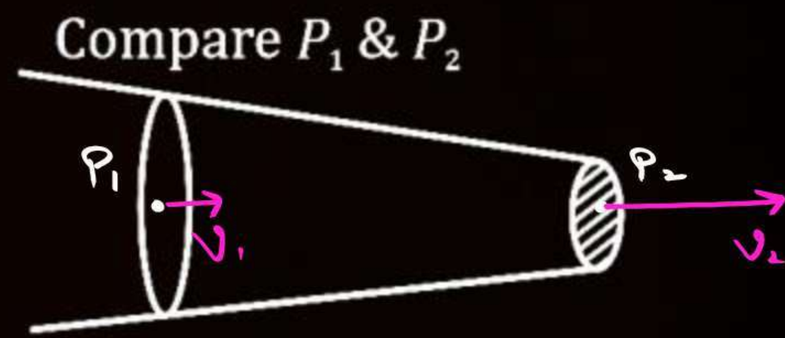
Compare P_1 & P_2.

(A) $P_1 = P_2$

(B) $P_1 > P_2$

(C) $P_1 < P_2$

(D) Can't Say.

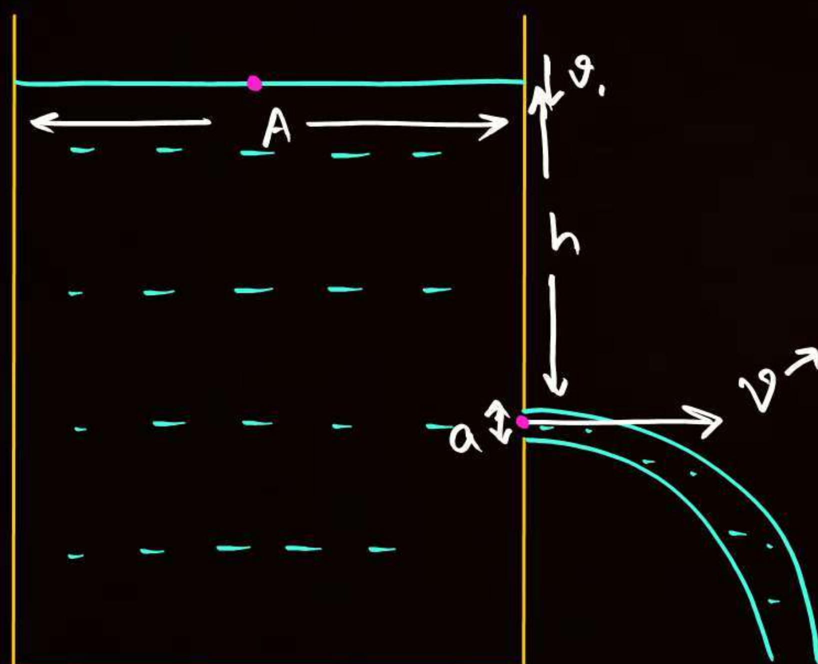


$v_2 > v_1$

$K.E_2 > K.E_1$

$P_2 < P_1$

Velocity of Efflux



$$A v_1 = a v$$

$$\cancel{P_0} + \cancel{\rho} g h + \frac{1}{2}\cancel{\rho} v_1^2 = \cancel{P_0} + 0 + \frac{1}{2}\cancel{\rho} v^2$$

$$\frac{v^2}{2} = \frac{v_1^2}{2} + gh$$

$$v^2 = v_1^2 + 2gh$$

$$v^2 = \left(\frac{av}{A}\right)^2 + 2gh$$

$$\left[1 - \left(\frac{a}{A}\right)^2\right] v^2 = 2gh$$

$$v = \sqrt{\frac{2gh}{\left[1 - \left(\frac{a}{A}\right)^2\right]}}$$

If $a << A$

$$v = \sqrt{2gh}$$

$v_1 \simeq 0$

QUESTION

Find h for which range of the path followed by ejected liquid is max. (Assume size of hole to be very small).

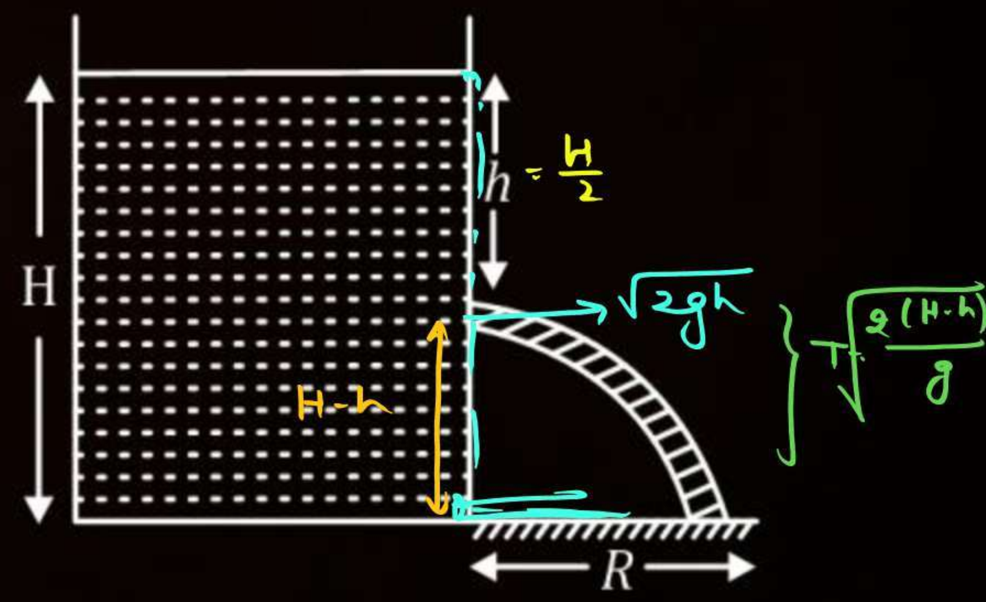
$$R = v \times T$$

$$= \sqrt{2gh} \times \sqrt{\frac{2(H-h)}{g}}$$

$$R = 2\sqrt{h(H-h)}$$

for max. R —

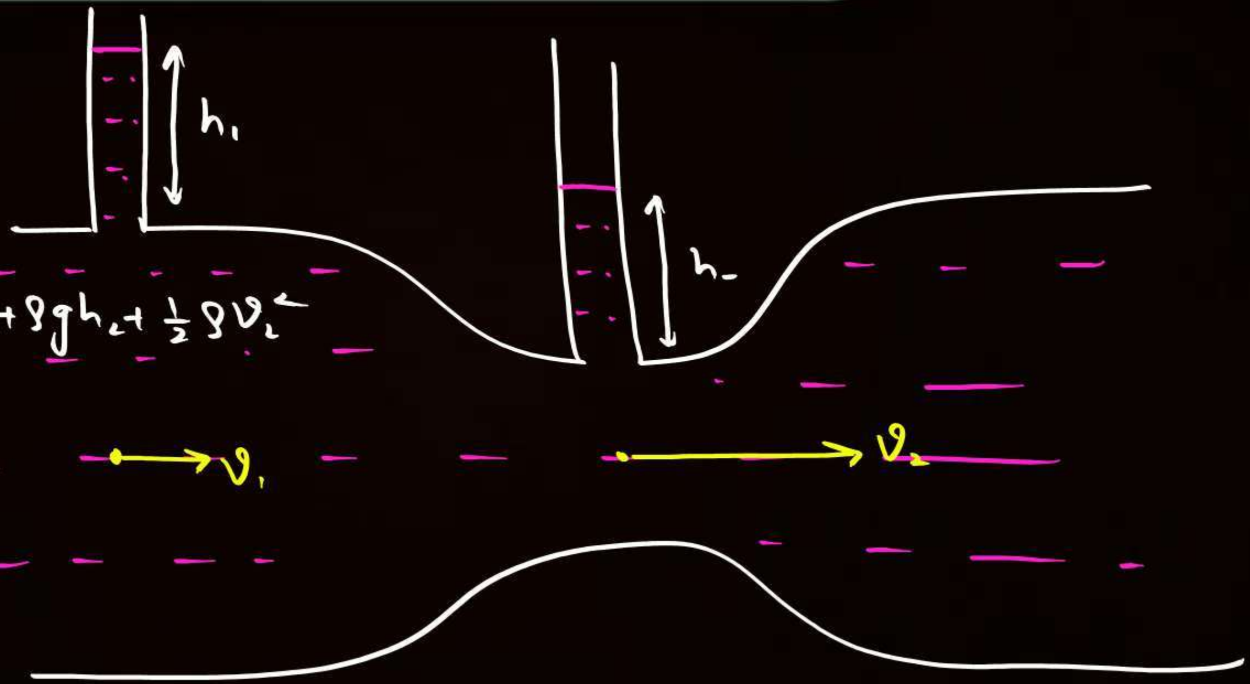
$$\frac{dR}{dh} = 0 \Rightarrow h = \frac{H}{2}$$ Ans.



Application of Bernoulli's Theorem

1. Venturi Meter:

$$P_0 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_0 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$



QUESTION (JEE Mains – 9 Jan. 2020 I)

Water flows in a horizontal tube (see figure). The pressure of water changes by 700 Nm^{-2} between A and B where the area of cross section are 40 cm^2 and 20 cm^2, respectively. Find the rate of flow of water through the tube. (density of water = 1000 kgm^{-3})

(A) 3020 cm^3/s

(B) 2720 cm^3/s

(C) 2420 cm^3/s

(D) 1810 cm^3/s

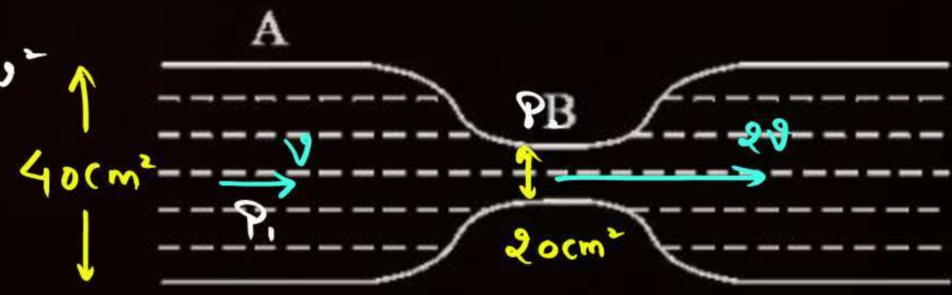
$$P_1 + \frac{1}{2}\rho v^2 = P_2 + \frac{1}{2}\rho(2v)^2$$

$$P_1 - P_2 = \frac{1}{2}\rho \times 3v^2$$

$$7\cancel{00} = \frac{1\cancel{000}}{\cancel{2}} \times 3v^2$$

$$7 = 15v^2$$

$$v = \sqrt{\frac{7}{15}}$$



Rate of water flow $= A\cdot v$

$$= 40 \times \sqrt{\frac{7}{15}} \times 10 \ \frac{cm^3}{s}.$$

QUESTION (JEE Mains – 19 Apr. 2014)

H.W

ΔP

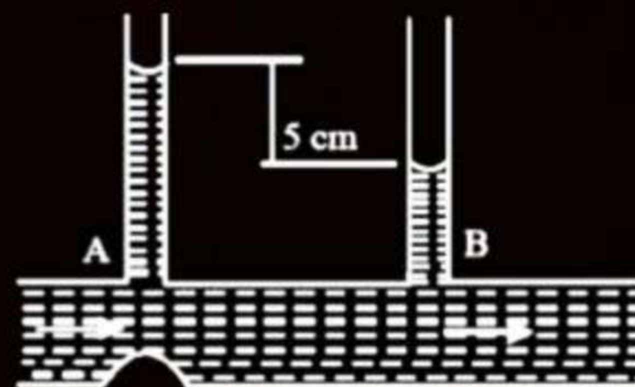
In the diagram shown, the difference in the two tubes of the manometer is 5 cm, the cross section of the tube at A and B is $6\ mm^2$ and $10\ mm^2$ respectively. The rate at which water flows through the tube is $(g = 10\ ms^{-2})$

(A) 7.5 cc/s

(B) 8.0 cc/s

(C) 10.0 cc/s

(D) 12.5 cc/s



QUESTION

A wind with speed 40 m/s blows parallel to the roof of a house. The area of the roof is 250 m^2. Assuming that the pressure inside the house is atmospheric pressure, the force exerted by the wind on the roof and the direction of the force will be ($\rho_{air} = 1.2\ kg/m^3$)

(A) 4.8×10^5 N, downwards

(B) 4.8×10^5 N, upwards

(C) 2.4×10^5 N, upwards

(D) 2.4×10^5 N, downwards

$$P_0 + 0 = P + \frac{1}{2} \times 1.2 \times 40^2$$

$$P_0 - P = 0.6 \times 1600$$

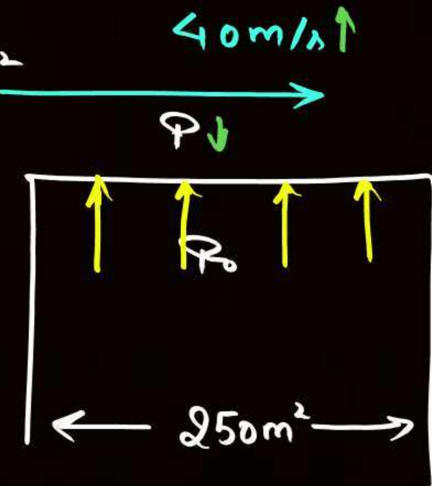
$$= 960\ Pa$$

$$F = (P_0 - P)A$$

$$= 960 \times 250$$

$$= 240000$$

$$= 2.4 \times 10^5\ N$$



QUESTION (JEE Mains – 6 Sep. 2020 II)

A fluid is flowing through a horizontal pipe of varying cross-section, with speed v ms^{-1} at a point where the pressure is P Pascal. At another point where pressure is $P/2$ Pascal its speed is V ms^{-1}. If the density of the fluid is ρ kg m^{-3} and the flow is streamline, then V is equal to:

(A) $\sqrt{\frac{P}{\rho}+v}$

(B) $\sqrt{\frac{2P}{\rho}+v^2}$

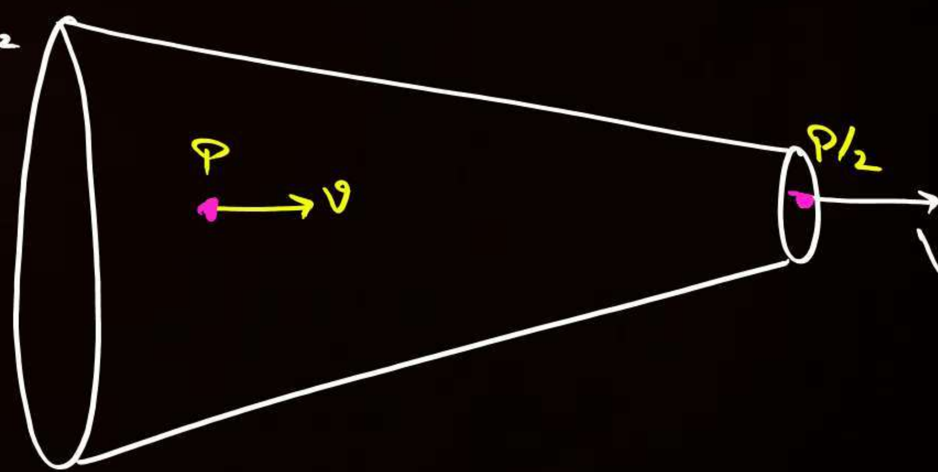
(C) $\sqrt{\frac{P}{2\rho}+v^2}$

(D) $\sqrt{\frac{P}{\rho}+v^2}$

$$P+\frac{1}{2}\rho v^2=\frac{P}{2}+\frac{1}{2}\rho V^2$$

$$\frac{P}{\cancel{2}}+\cancel{\frac{1}{2}}\rho v^2=\frac{1}{\cancel{2}}\rho V^2$$

$$V=\sqrt{\frac{P}{\rho}+v^2}$$



QUESTION (JEE Mains – 10 Jan. 2019 I)

Water flows into a large tank with flat bottom at the rate of $10^{-4}\ m^3\ s^{-(1)}$ Water is also leaking out of a hole of area $1\ cm^2$ at its bottom. If the height of the water in the tank remains steady, then this height is:

(A) 5.1 cm

(B) 7 cm

(C) 4 cm

(D) 9 cm

For const. H —

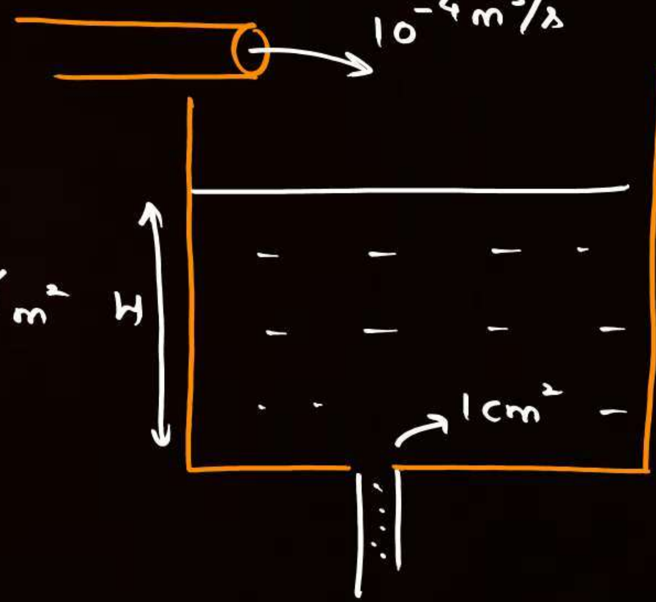
$$V_{in} = V_{out}$$

$$\cancel{10^{-4}} = \left(\sqrt{2gH}\right) \cdot \cancel{10^{-4}}\ m^2$$

$$\sqrt{2gH} = 1$$

$$H = \frac{1}{20}\ m$$

$$H = 5\ cm$$



QUESTION (JEE Mains –2014)

A cylindrical vessel of cross-section A contains water to a height h. There is a hole in the bottom of radius 'a'. The time in which it will be emptied is:

(A) $\dfrac{2A}{\pi a^2}\sqrt{\dfrac{h}{g}}$

(B) $\dfrac{\sqrt{2}A}{\pi a^2}\sqrt{\dfrac{h}{g}}$ ✓

(C) $\dfrac{2\sqrt{2}A}{\pi a^2}\sqrt{\dfrac{h}{g}}$

(D) $\dfrac{A}{\sqrt{2}\pi a^2}\sqrt{\dfrac{h}{g}}$

Rate of vol. —

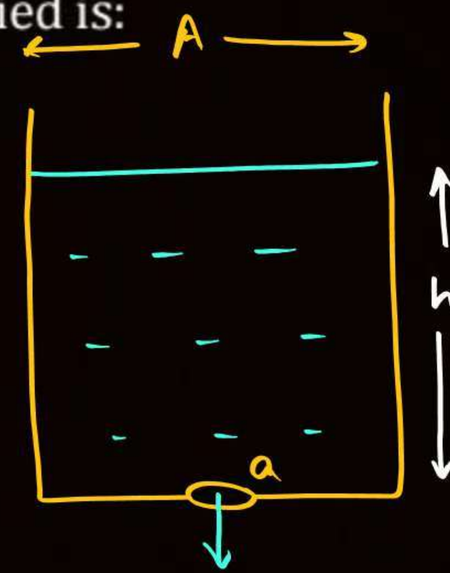
$$\frac{dV}{dt} = -\pi a^2 \cdot v$$

$$A \cdot \frac{dh}{dt} = -\pi a^2 \cdot \sqrt{2gh}$$

$$\int_h^0 \frac{dh}{\sqrt{h}} = -\int_0^t \frac{\pi a^2 \sqrt{2g}}{A}\, dt$$

$$\left[\frac{h^{1/2}}{1/2}\right]_h^0 = -\frac{\pi a^2}{A}\sqrt{2g}\, t$$

$$-2\sqrt{h} = -\frac{\pi a^2}{A}\sqrt{2g}\, t, \quad t = \frac{\sqrt{2}A}{\pi a^2}\sqrt{\frac{h}{g}}$$



$v = \sqrt{2gh}$

QUESTION (JEE Mains – 7 Jan. 2020 II)

An ideal fluid flows (laminar flow) through a pipe of nonuniform diameter. The maximum and minimum diameters of the pipes are 6.4 cm and 4.8 cm, respectively. The ratio of the minimum and the maximum velocities of fluid in this pipe is:

(A) 9/16

(B) $\sqrt{3}/2$

(C) 3/4

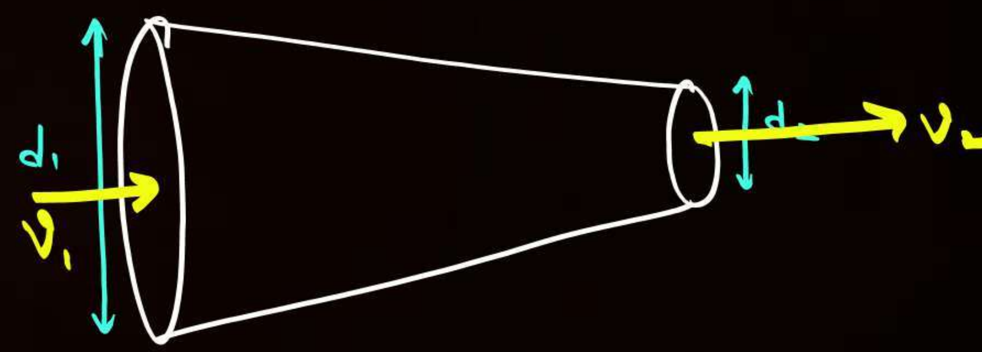
(D) 80/256

$$\cancel{\pi}\left(\frac{d_1}{\cancel{2}}\right)^2 v_1 = \cancel{\pi}\left(\frac{d_2}{\cancel{2}}\right)^2 \cdot v_2$$

$$\frac{v_1}{v_2} = \frac{d_2^2}{d_1^2}$$

$$= \left(\frac{\cancel{4.8}^{\,3}}{\cancel{6.4}_{\,4}}\right)^2$$

$$= \frac{9}{16}$$



QUESTION (JEE Mains – 10 April 2019 II)

HW

Water from a tap emerges vertically downwards with an initial speed of $1.0\ ms^{-1}$. The cross-sectional area of the tap is $10^{-4}\ m^2$. Assume that the pressure is constant throughout the stream of water and that the flow is streamlined. The cross-sectional area of the stream, 0.15 m below the tap would be : [Take $g = 10\ ms^{-2}$]

(A) $2 \times 10^{-5}\ m^2$

(B) $5 \times 10^{-5}\ m^2$

(C) $5 \times 10^{-4}\ m^2$

(D) $1 \times 10^{-5}\ m^2$

QUESTION (JEE Mains – 2012)

H.W.

Water is flowing through a horizontal tube having cross-sectional areas of its two ends being A and A' such that the ratio A/A' is 5. If the pressure difference of water between the two ends is 3×10^5 N m^{-2}, the velocity of water with which it enters the tube will be (neglect gravity effects)

(A) 5 m s^{-1}

(B) 10 m s^{-1}

(C) 25 m s^{-1}

(D) $50\sqrt{10} \text{ m s}^{-1}$

QUESTION (JEE Mains – 29 Jan, 2023 II)

A fully loaded aircraft has a mass of $\boxed{5.4 \times 10^5 \text{ kg}}$. Its total wing area is 500 m^2. It is in level flight with a speed of 1080 km/h. If the density of air is 1.2 kg m^{-3}, the fractional increase in the speed of the air on the upper surface of the wings relative to the lower surface in percentage will be ($g = 10$ m/s^2)

(A) 16

(B) 6

(C) 8

(D) 10

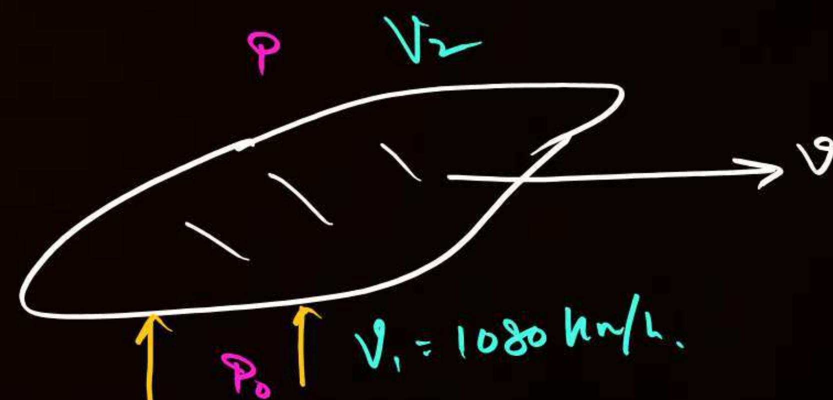
$$(P_0 - P)A = mg$$

$$P_0 - P = \frac{mg}{A}$$

$$\frac{1}{2}\rho v_1^2 + P_0 = \frac{1}{2}\rho v_2^2 + P$$

$$\frac{1}{2}\rho (v_2^2 - v_1^2) = P_0 - P$$

$$v_2^2 - v_1^2 = \frac{2mg}{A\rho}$$



QUESTION (JEE Mains – 13 April, 2023 I)

H.W

The figure shows a liquid of given density flowing steadily in horizontal tube of varying cross-section. Cross sectional areas at A is 1.5 cm^2 and B is 25 mm^2, if the speed of liquid at B is 60 cm/s then $(P_A - P_B)$ is:

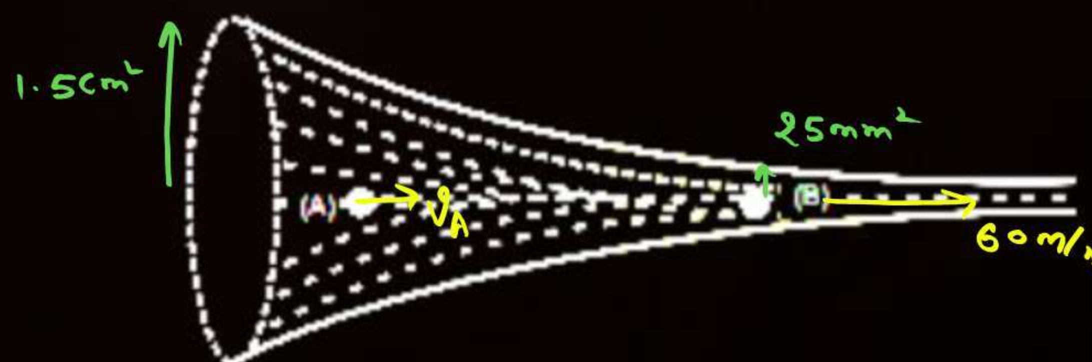
(Given P_A and P_B are liquid pressures at A and B points, Density $\rho = 1000 \text{ kg m}^{-3}$ A and B are on the axis of tube

(A) 175 Pa

(B) 27 Pa

(C) 135 Pa

(D) 36 Pa



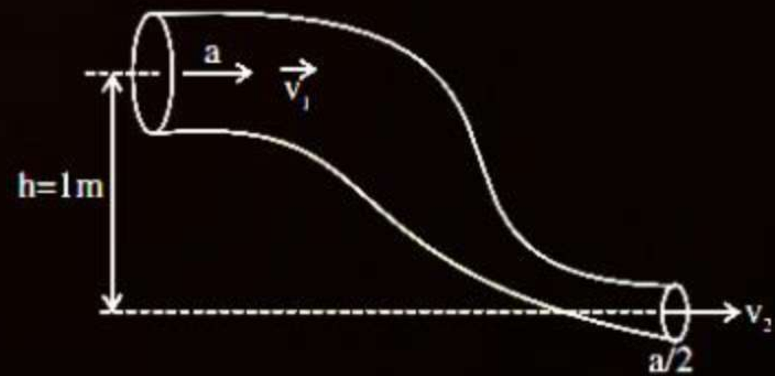
QUESTION (JEE Mains – 12 April, 2023 I)

H.W

Glycerine of density 1.25×10^3 kg m^{-3} is flowing through the conical section of pipe. The area of cross-section of the pipe at its ends is 10 cm^2 and 5 cm^2 and pressure drop across its length is 3 Nm^{-2}. The rate of flow of glycerine through the pipe is $x \times 10^{-5}$ m^3 s^{-1}. The value of x is

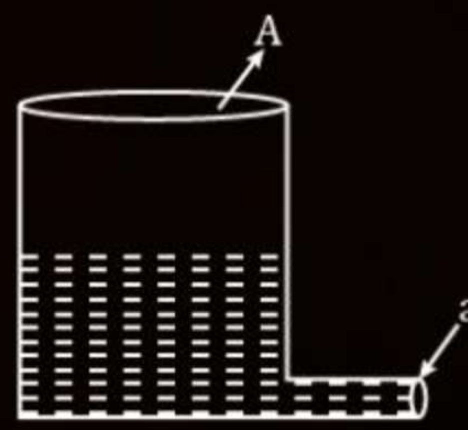
QUESTION (JEE Mains – 26 June, 2022 (I))

An ideal fluid of density 800 kgm^{-3}, flows smoothly through a bent pipe (as shown in figure) that tapers in cross-sectional area from a to $a/2$. The pressure difference between the wide and narrow sections of pipe is 4100 Pa. At wider section, the velocity of fluid is $\sqrt{x}/6$ ms^{-1} for $x =$ ____. (Given $g = 10$ ms^{-2})



QUESTION (JEE Mains – 27 July, 2021 (I))

A light cylindrical vessel is kept on a horizontal surface. Area of base is A. A hole of cross-sectional area 'a' is made just at its bottom side. The minimum coefficient of friction necessary to prevent sliding the vessel due to the impact force of the emerging liquid is (a << A):



(A) None of these

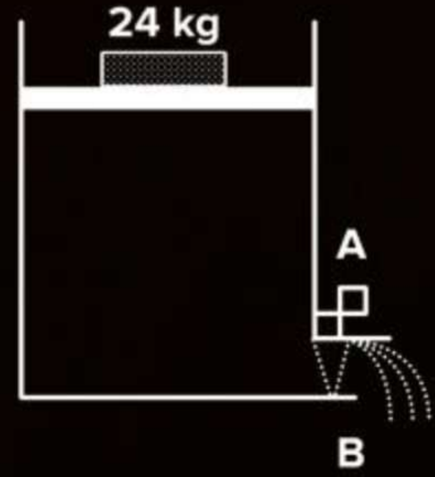
(B) $2a/\mathrm{A}$

(C) $\mathrm{A}/2a$

(D) a/A

QUESTION (JEE Mains – 18 March, 2021 (II))

Consider a water tank as shown in the figure. It's crosssectional area is $0.4\ m^{-2}$. The tank has an opening B near the bottom whose cross-sectional area is $1\ cm^2$. A load of 24 kg is applied on the water at the top when the height of the water level is 40 cm above the bottom, the velocity of water coming out the opening B is $v\ ms^{-1}$. The value of v, to the nearest integer, is _____ [Take value of g to be $10\ ms^{-2}$]



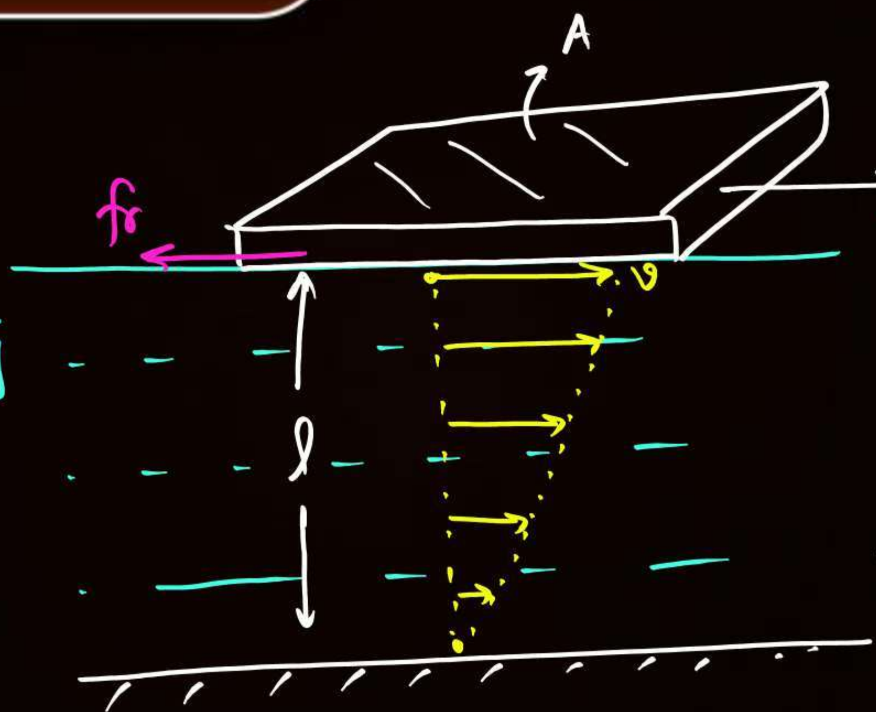
QUESTION (JEE Adv, 2020)

A train with cross-sectional area S_1 is moving with speed v_t inside a long tunnel of cross-sectional area $S_0 (S_0 = 4\ S_t)$. Assume that almost all the air (density ρ) in front of the train flows back between its sides and the walls of the tunnel. Also, the air flow with respect to the train is steady and laminar. Take the ambient pressure and that inside the train to be p_0. If the pressure in the region between the sides of the train and the tunnel walls is p, then $p_0 - p = \frac{7}{2N} \rho v_t^2$. The value of N is ______.

Viscous Force

$$\eta = \frac{fr \cdot l}{A \cdot v} = \frac{[M\cancel{L}T^{-2}][\cancel{L}]}{[\cancel{L^2}][LT^{-1}]}$$

$$[\eta] = [ML^{-1}T^{-1}]$$



vel gradient

$$fr \propto \frac{v}{l}$$

$$\propto A$$

$$\boxed{fr = \frac{\eta A v}{l}}$$

η = Coeff. of viscosity

CGS unit : Poise (P)

SI unit :- Decaporse

Frictional Force	Viscous Force
Independent of area of cross-section.	Viscous force $\propto$ Area of cross-section
f_{r_k} is const., it does not depend on magnitude of v_{rel}	Viscous force increases with increasing vel.
$f_r \le \mu N$	No upper limit.

QUESTION

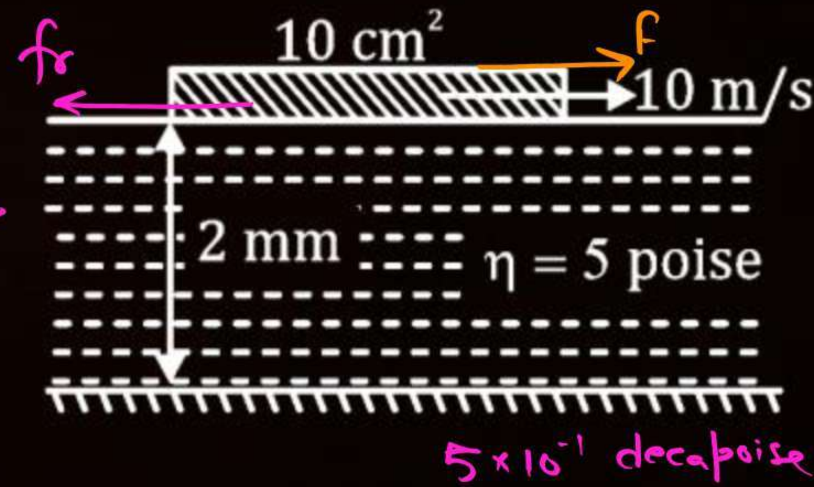
Find the force required to move the plate with constant vel. Of 10 m/s.

$$f = f_r$$

$$= \frac{\eta A v}{l}$$

$$= \frac{5 \times \cancel{10^{-1}} \times \cancel{10} \times \cancel{10^{-4}} \times \cancel{10}}{2 \times \cancel{10^{-3}}}$$

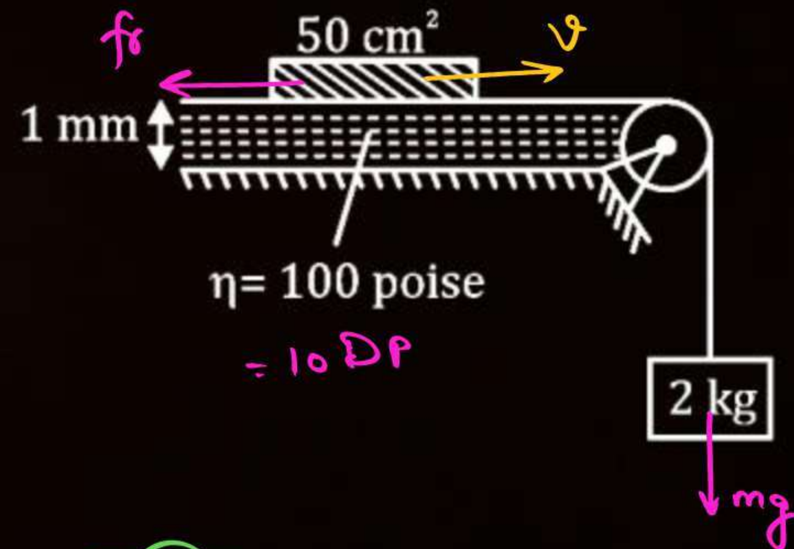
$$f = 2.5\,N$$ Ans.



QUESTION

Find max. vel of plate with which it can move over the liquid.

$\uparrow f_r = \frac{\eta A v}{l}$



At max. vel. —

$a = 0\ m/s^2$

$f_r = mg$

$\frac{\eta A v_{max}}{l} = 20$

$\frac{1\cancel{0}\times 5\cancel{0}\times \cancel{10^{-4}}\times v_{max}}{\cancel{10^{-3}}} = 2\cancel{0}$

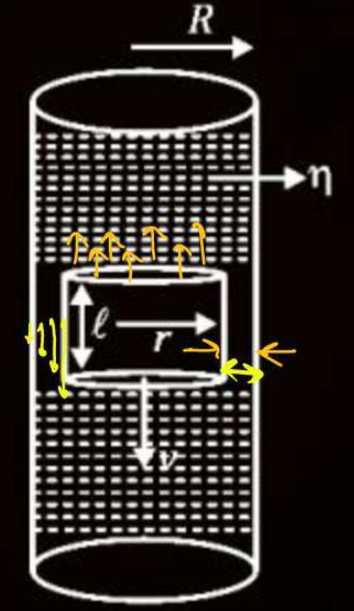
$v_{max} = \frac{2}{5}\ m/s = 0.4\ m/s$

QUESTION

Find viscous force acting on the cylindrical body moving inside the tube with vel. v as shown in figure.

$$f_r = \frac{\eta A v}{l}$$

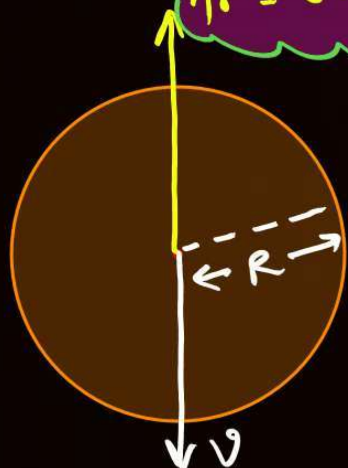
$$= \frac{\eta (2\pi r l) v}{(R-r)}$$



Stoke's Law

- Experimentally derived.
- Valid for spherical bodies only.

$$f_r = 6\pi\eta R v$$



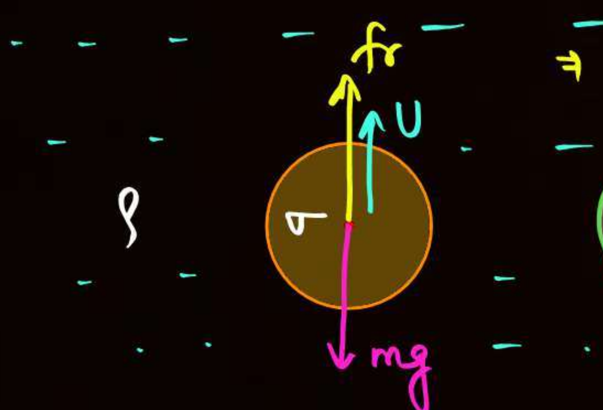
Terminal Velocity

At terminal vel., $F_{net} = 0 \Rightarrow f_r + U = mg$

$$\Rightarrow 6\pi\eta R v_{tr} = \sigma g V - \rho g V$$

$\uparrow f_r = 6\pi\eta R v \uparrow$

$$\Rightarrow 6\cancel{\pi}\eta \cancel{R} v_{tr} = (\sigma-\rho)g\times\frac{4}{3}\cancel{\pi}R^{\cancel{3}}$$



$$\boxed{v_{tr} = \frac{2}{9}\frac{gR^2}{\eta}(\sigma-\rho)}$$

mg

$v\uparrow$

QUESTION

$v = \text{const} \Rightarrow a = 0$

The velocity of a small ball of mass M and density d when dropped in a container filled with glycerine becomes constant after some time. If the density of glycerine is $d/2$, then the viscous force acting on the ball will be

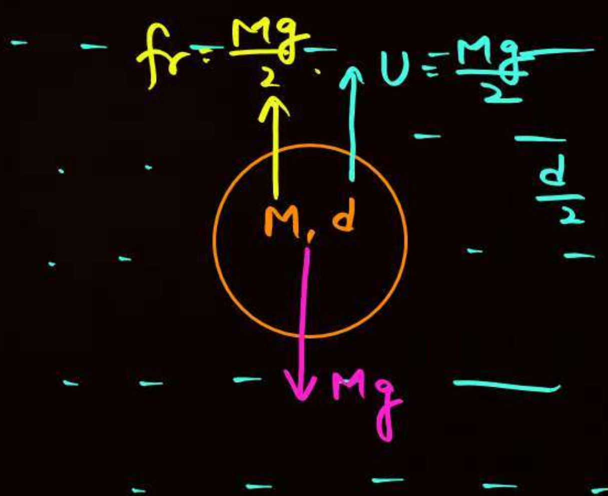
$U = \rho V g$

(A) $\frac{Mg}{2}$

(B) Mg

(C) $\frac{3}{2} Mg$

(D) $2\,Mg$



QUESTION (JEE Mains – 12 April 2019 II)

A solid sphere, of radius R acquires a terminal velocity v_1 when falling (due to gravity) through a viscous fluid having a coefficient of viscosity η. The sphere is broken into 27 identical solid spheres. If each of these spheres acquires a terminal velocity v_2 when falling through the same fluid, the ratio (v_1/v_2) equals:

(A) 9

(B) 1/27

(C) 1/9

(D) 27

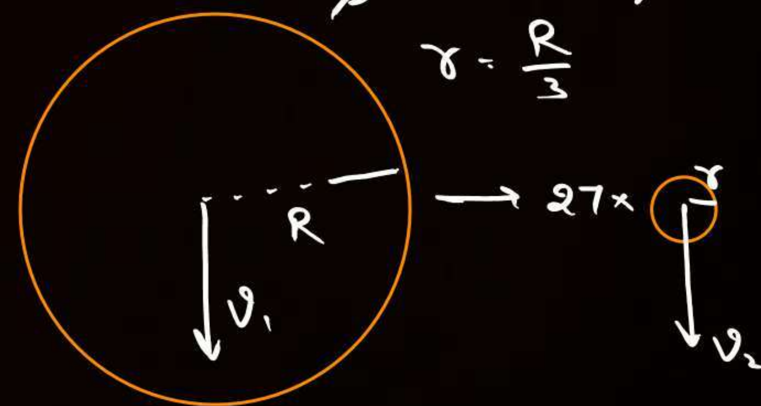
$$v_{tr} = \frac{2}{9}\frac{gR^2}{\eta}(\sigma - \rho)$$

$$\frac{\cancel{4}}{\cancel{3}}\cancel{\pi} R^3 = 27 \times \frac{\cancel{4}}{\cancel{3}}\cancel{\pi} r^3$$

$$r = \frac{R}{3}$$

$$v_1 = \frac{\cancel{2}}{\cancel{9}}\frac{\cancel{g}R^2}{\cancel{\eta}}\cancel{(\sigma-\rho)}$$

$$v_2 = \frac{\cancel{2}}{\cancel{9}}\frac{\cancel{g}(R/3)^2}{\cancel{\eta}}\cancel{(\sigma-\rho)}$$



$$\boxed{\frac{v_1}{v_2} = 9}$$

QUESTION

A small sphere of radius r falls from rest in a viscous liquid. As a result, heat is produced due to viscous force. The rate of production of heat when the sphere attains its terminal velocity, is proportional to

(A) r^5

(B) r^2

(C) r^3

(D) r^4

Power = Rate of prod. of Heat

$= f_r \cdot v$

$= 6\pi\eta R v \cdot v$

$\propto R \cdot v^2$

$\propto R \cdot R^4$

$\propto R^5$

$v_{tr} = \frac{2}{9}\frac{g R^2}{\eta}(\sigma - \rho)$

QUESTION

H.W

Eight equal drops of water are falling through air with a steady velocity of $10 cm^{-1}$. If the drops combine to form a single drop big in size, then the terminal velocity of this big drop is

(A) $80\ cms^{-1}$

(B) $30\ cms^{-1}$

(C) $10\ cms^{-1}$

(D) $40\ cms^{-1}$

QUESTION

Two small spherical metal balls, having equal masses, are made from materials of densities ρ_1 and ρ_2 ($\rho_1 = 8\rho_2$) and have radii of 1 mm and 2 mm, respectively. They are made to fall vertically (from rest) in viscous medium whose coefficient of viscosity equals η and whose density is $0.1\,\rho_2$. The ratio of their terminal velocities would be

(A) 79/72

(B) 19/36

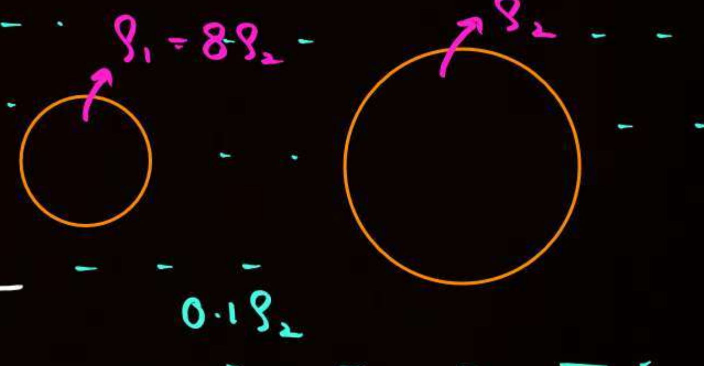
(C) 39/72

(D) 79/36

$$v_1 = \frac{2}{9}\frac{g R_1^2}{\eta}(\sigma - \rho)$$

$$\frac{v_1 = \frac{\cancel{2}}{\cancel{9}}\frac{\cancel{g}\,1^2}{\cancel{\eta}}(8\rho_2 - 0.1\rho_2)}{v_2 = \frac{\cancel{2}}{\cancel{9}}\frac{\cancel{g}\,2^2}{\cancel{\eta}}(\rho_2 - 0.1\rho_2)}$$

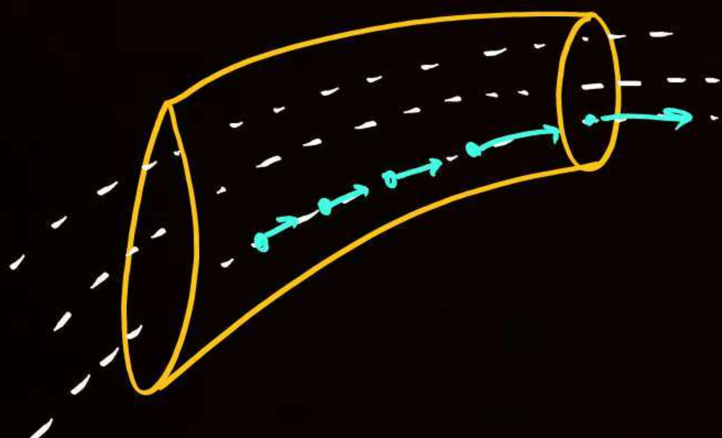
$$= \frac{7.9\cancel{\rho_2}}{4 \times 0.9\cancel{\rho_2}} = \frac{79}{36}$$

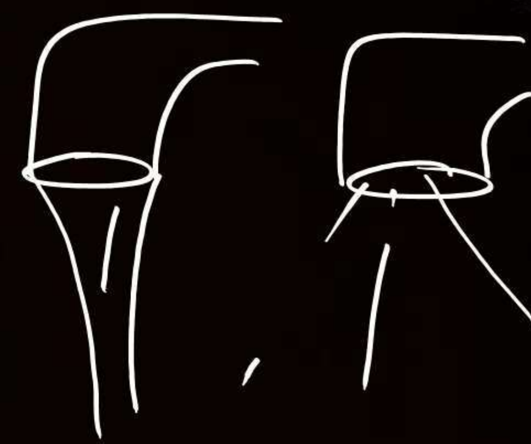


Types of Liquid Flow

Streamline flow

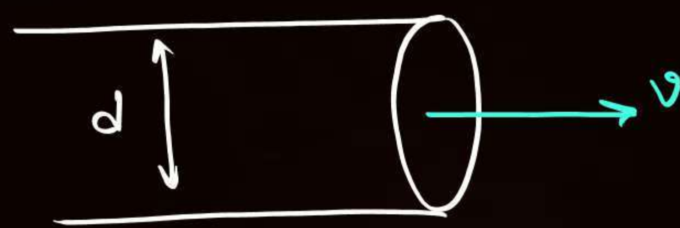
Turbulent flow





Reynold's Number

$$R = \frac{\rho v d}{\eta}$$



ρ = density

v = vel.

d = diameter

η = viscosity.

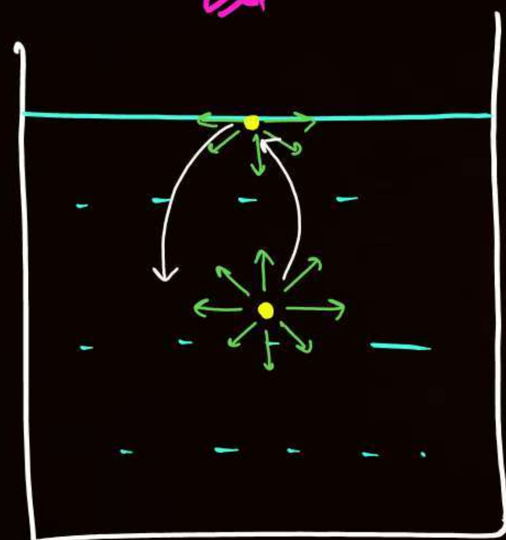
$R < 1000 \rightarrow$ Streamline

$1000 \leq R \leq 2000 \rightarrow$ Unstable

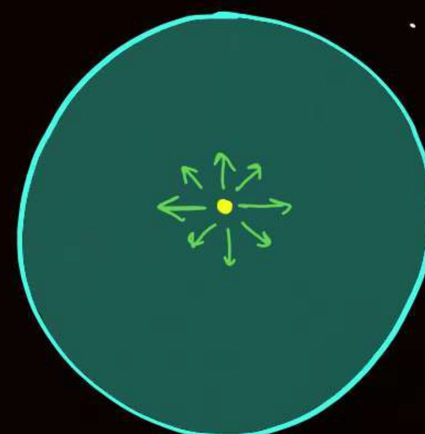
$R > 2000 \longrightarrow$ Turbulent

Surface Tension

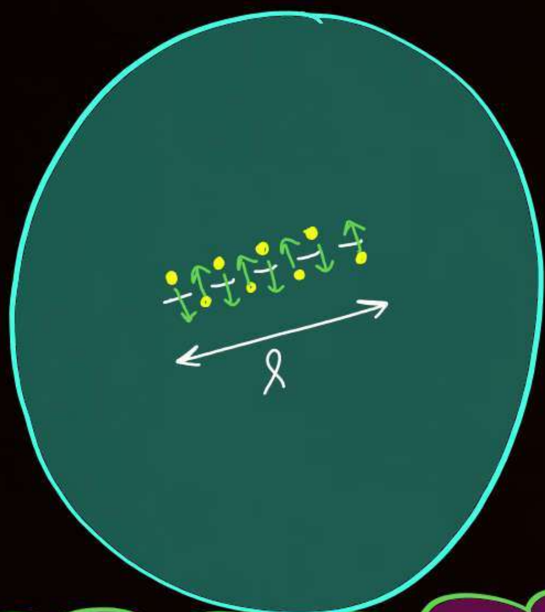
Surface → Interface b/w liquid & air.



Thickness of surface → 3-5 molecular diameter.



Trampoline



$$f \propto l$$

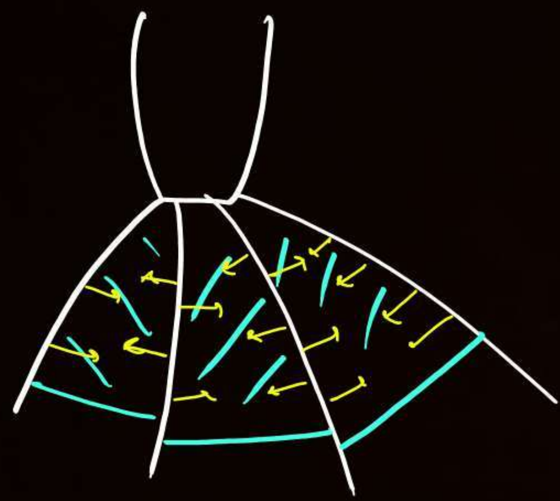
$$f = T \cdot l$$

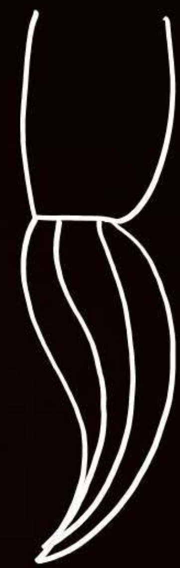
scalar ← $T = \dfrac{f}{l}$ → Surface की per unit length पर लगाने वाला force

Surface tension.

Unit: $\dfrac{N}{m}$

★★ किसी भी liquid का surface एक stretched rubber membrane की तरह behave करता है।





QUESTION

Find the force required to keep wire at rest.

Common mistake

(A) $T.\ell$

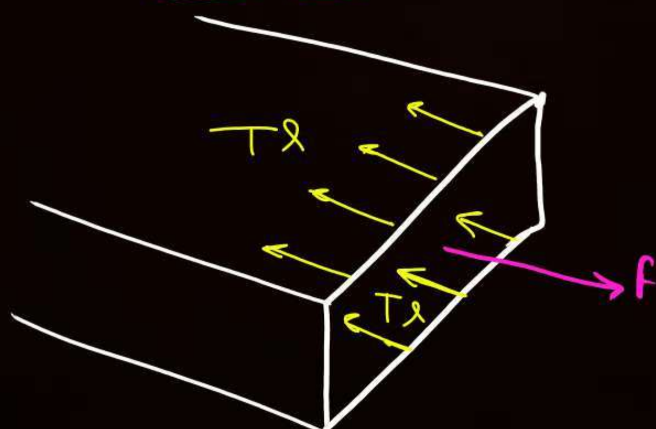
(B) $2\,T.\ell$

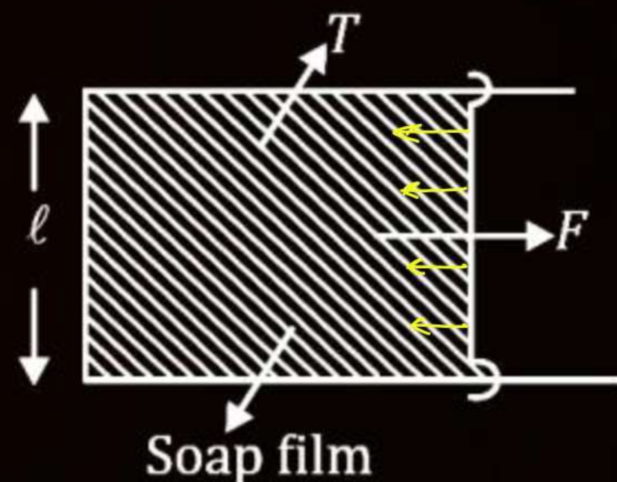
(C) $T\ell/2$

(D) 0

$F = T \cdot \ell$

$\boxed{F = 2 \times T\ell}$





QUESTION (JEE Mains – 2012)

A thin liquid film formed between a U-shaped wire and a light slider supports a weight of 1.5×10^{-2} N (see figure). The length of the slider is 30 cm and its weight is negligible. The surface tension of the liquid film is

(A) 0.0125 Nm^{-1}

(B) 0.1 Nm^{-1}

(C) 0.05 Nm^{-1}

(D) 0.025 Nm^{-1} ✓

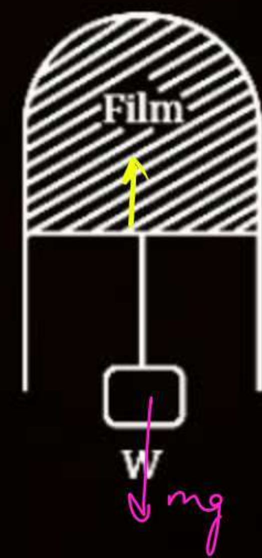
$$2T \cdot l = mg$$

$$T = \frac{mg}{2l}$$

$$= \frac{1.5 \times \cancel{10^{-2}}}{2 \times 30 \times \cancel{10^{-2}}}$$

$$= \frac{1.5}{60} = \frac{\cancel{15}}{\cancel{6}} \times 10^{-2}$$

$$= 2.5 \times 10^{-2} \text{ N}$$



QUESTION

Find the force required to lift a disc of negligible masses from the surface of a liquid having surface tension σ. Radius of the disc is R.

(A) $\sigma \times 2R$

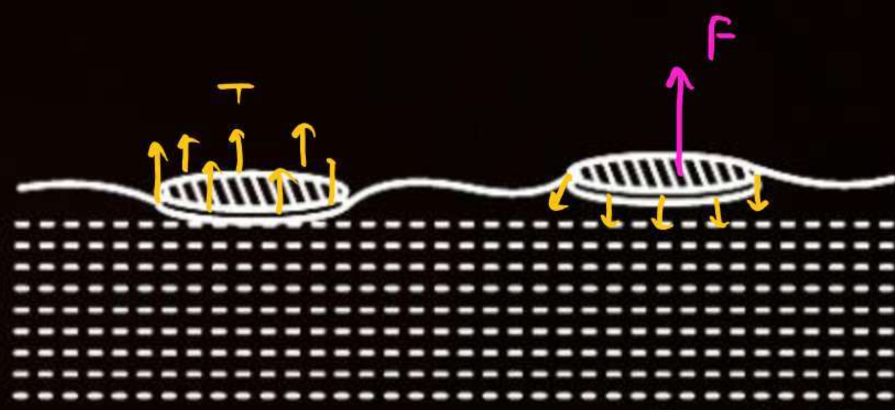
(B) $\sigma \times R$

(C) $\sigma \times 2\pi R$

(D) $\sigma \times 4\pi R$

$F = T \cdot 2\pi R.$

$= \sigma \cdot 2\pi R$



$F = 2\pi R$

QUESTION

Find the force required to lift a ring of negligible masses from the surface of a liquid having surface tension σ. Radius of the disc is R.

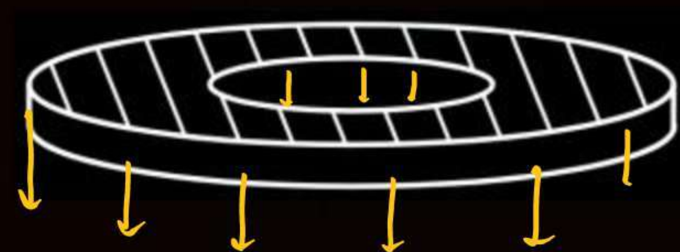
(A) $\sigma \times 2R$

(B) $\sigma \times R$

(C) $\sigma \times 2\pi R$

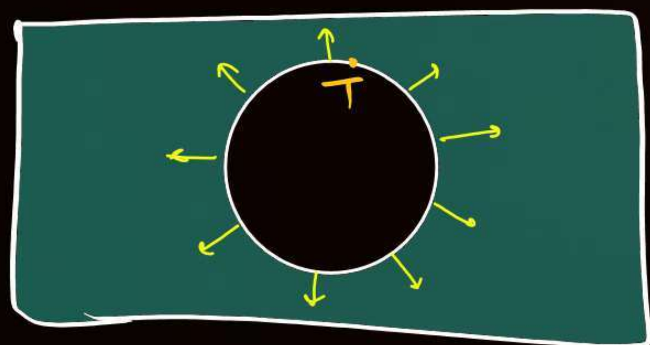
(D) $\sigma \times 4\pi R$

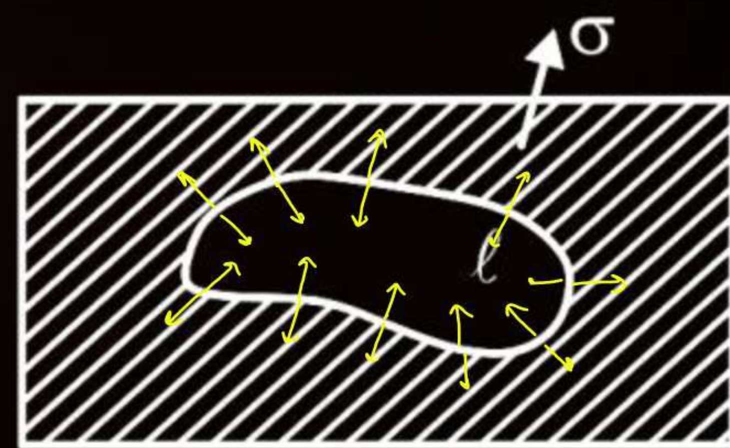
$F = \sigma \times 4\pi R$

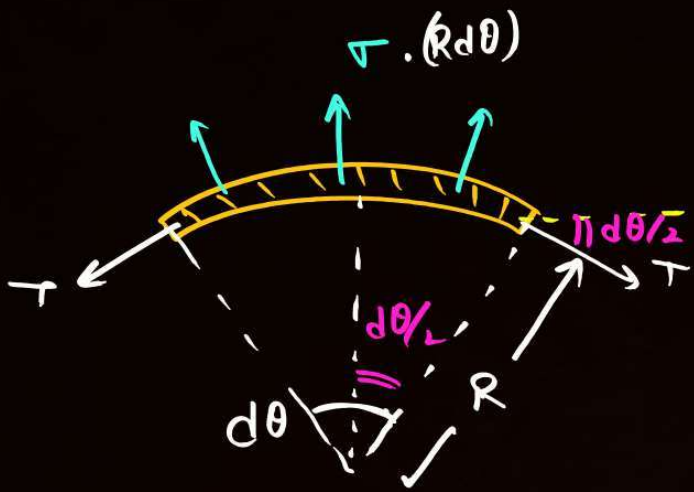


QUESTION

A thread loop is gently placed on a rectangular soap film as shown. What will happen when we pierce the inside region of soap film? Also find the tension in thread (surface tension : σ, length of loop : ℓ)





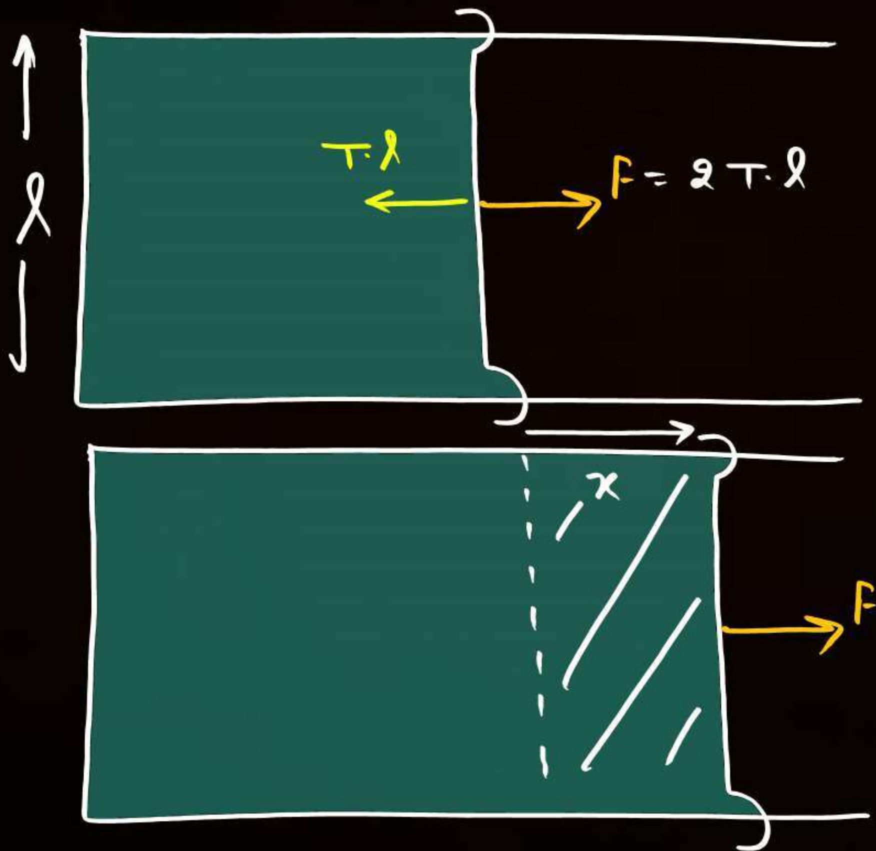


$$2T\sin\frac{d\theta}{2} = \sigma \cdot R\,d\theta$$

$$\Rightarrow \cancel{2}T\cdot\left(\frac{\cancel{d\theta}}{\cancel{2}}\right) = \sigma\cdot R\,\cancel{d\theta}$$

$$T = \sigma \cdot R$$

Energy Perspective of Surface Tension



$$\Delta U = -W_T = +W_f$$

$$= f \cdot x$$

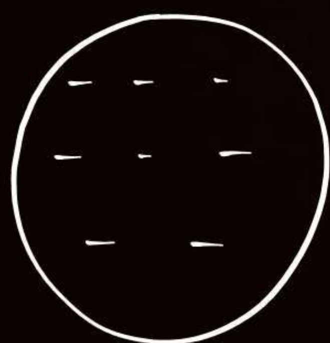
$$\Delta U = 2T \cdot l \cdot x$$

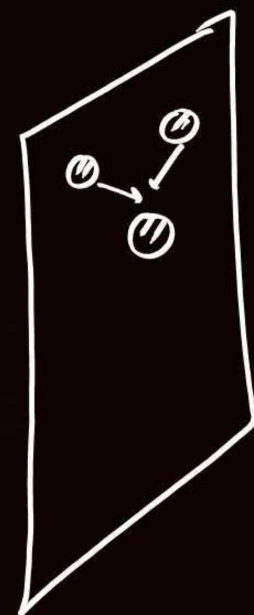
$$\Delta U = T \cdot (2lx)$$

$$\Delta U = T \cdot \Delta A$$

$$T = \frac{\Delta U}{\Delta A} = \frac{U-0}{A-0} \Rightarrow T = \frac{U}{A}$$

$$T = \frac{F}{l} = \frac{U}{A}$$



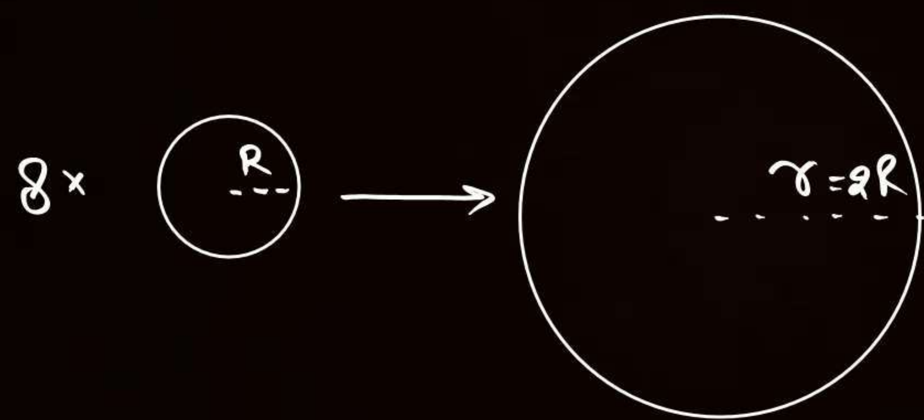


QUESTION

If 8 identical drops of radius R coalesce to form a bigger drop, find the energy released. (surface Tension = T)

$$8 \times \cancel{\tfrac{4}{3}}\pi R^3 = \cancel{\tfrac{4}{3}}\pi r^3$$

$$r = 2R$$



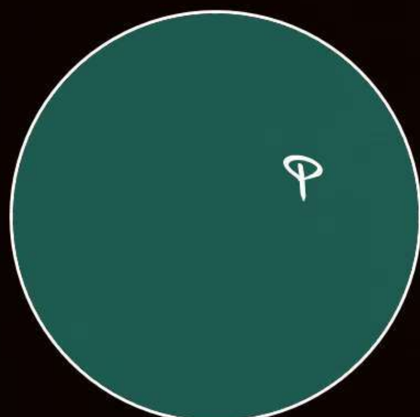
$$\text{Energy released} = T \times \text{loss in surface area}$$

$$= T\left[8 \times 4\pi R^2 - 4\pi(2R)^2\right]$$

$$= 16\pi R^2 T$$ Ans.

Excess Pressure Inside Drop

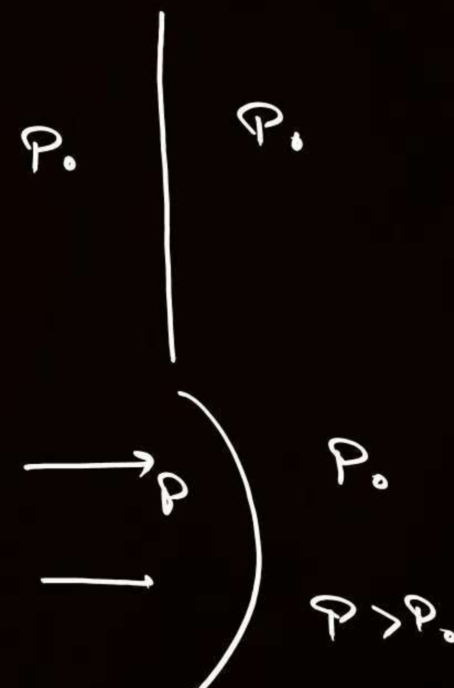
Pressure is greater in concave side.



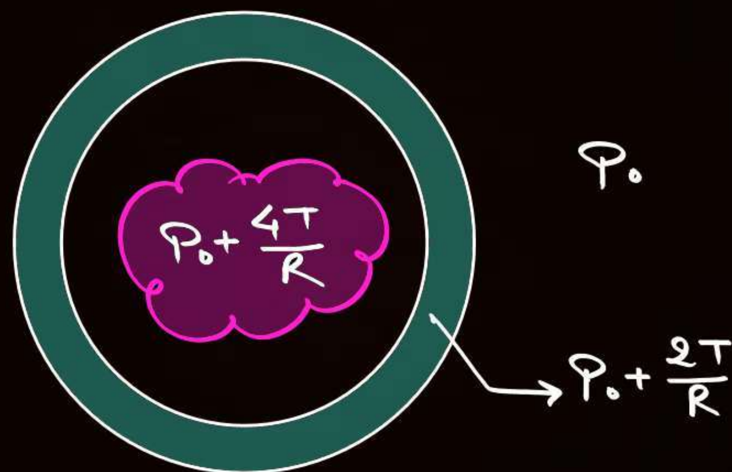
P_0

$P > P_0$

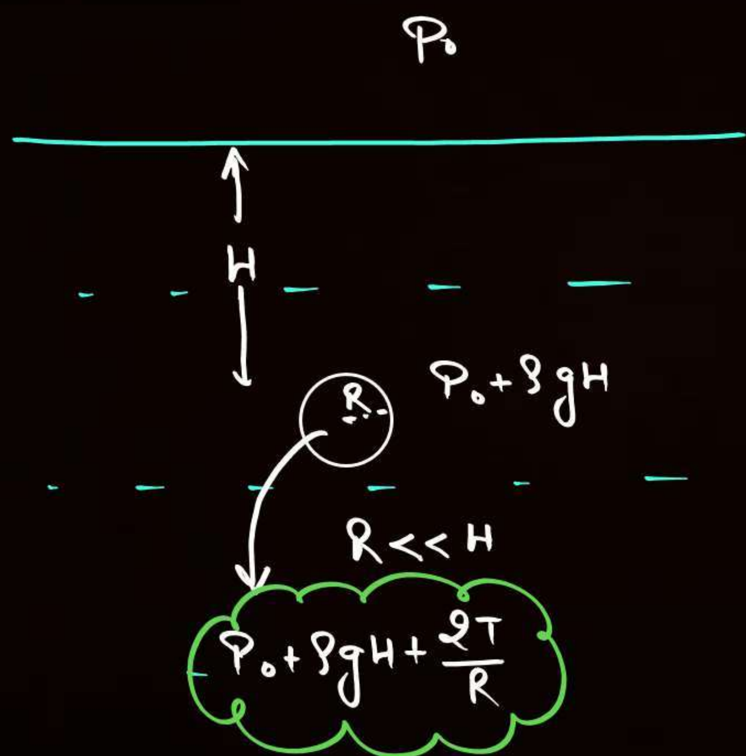
$P = P_0 + \frac{2T}{R}$



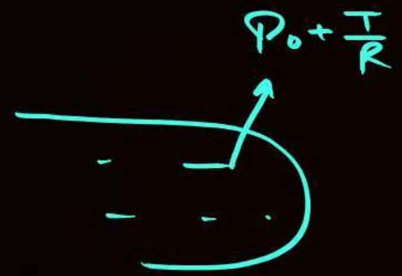
Excess Pressure Inside Soap Bubble

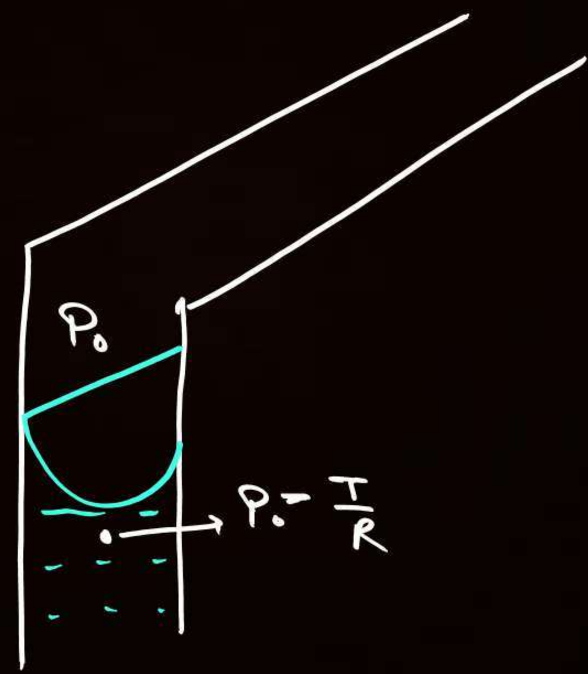


Excess Pressure Inside Air Bubble



Excess Pressure Inside Cylindrical Surface





QUESTION (JEE Mains – 03 Sep. 2020 I)

Pressure inside two soap bubbles are 1.01 and 1.02 atmosphere, respectively. The ratio of their volumes is :

Excess pressure, $\Delta P = \frac{4T}{R_1}$

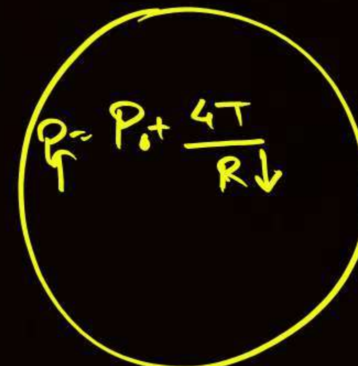
$$\frac{0.01\ atm = \frac{4T}{R_1}}{0.02\ atm = \frac{4T}{R_2}}$$

$$\frac{R_2}{R_1} = \frac{1}{2}$$

$$\frac{R_1}{R_2} = \frac{2}{1}$$

$$\frac{V_1}{V_2} = \frac{\frac{4}{3}\pi R_1^3}{\frac{4}{3}\pi R_2^3} = \frac{8}{1}$$ Ans

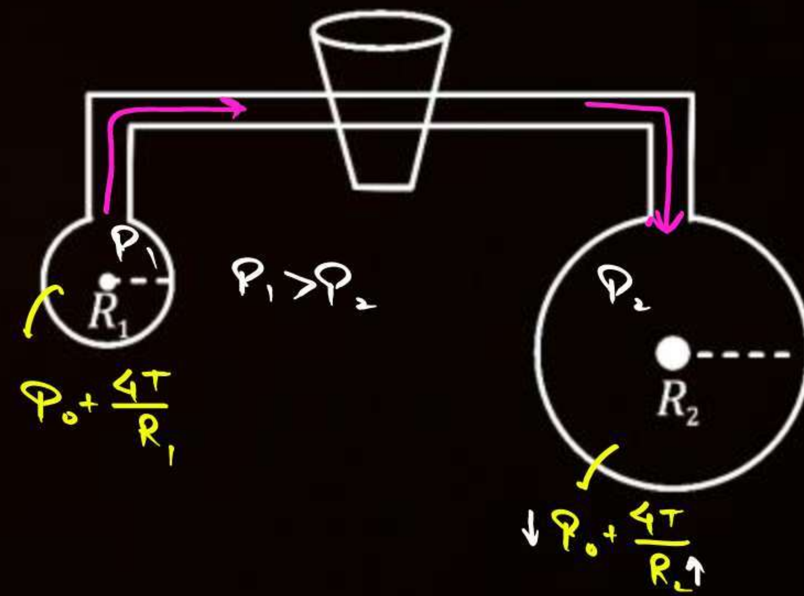
$P = P_0 + \frac{4T}{R}$



QUESTION

What will happened to the soap bubbles when the tap is opened?

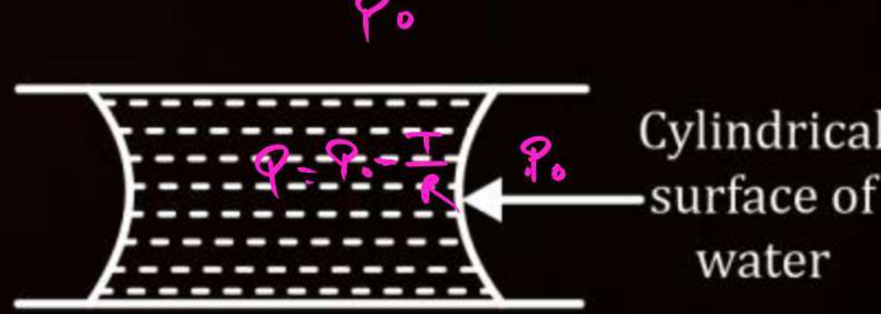
Smaller bubble will collapse



QUESTION (JEE Mains – 10 Apr. 2015)

If two glass plates have water between them and are separated by very small distance (see figure), it is very difficult to pull them apart. It is because the water in between forms cylindrical surface on the side that gives rise to lower pressure in the water in comparison to atmosphere. If the radius of the cylindrical surface is R and surface tension of water is T then the pressure in water between the plates is lower by:

$P < P_0$



(A) $2\,T/R$

(B) $4\,T/R$

(C) $T/4R$

(D) T/R ✓

QUESTION

Two spherical soap bubbles of radius R_1 & R_2 come close to each other as shown in figure find the radius R at the common interface b/w the bubbles.

(A) $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

(B) $\frac{1}{R} = \frac{1}{R_1} - \frac{1}{R_2}$

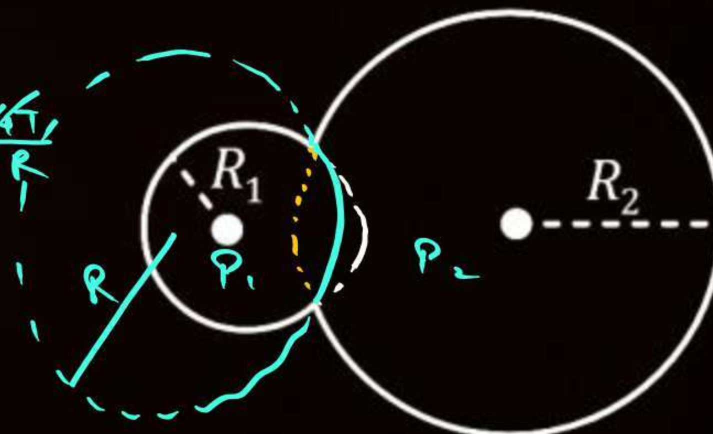
(C) $R = \frac{R_1 + R_2}{2}$

(D) $R = \sqrt{R_1 R_2}$

$P_1 - P_2 = \frac{4T}{R}$

$\left[\cancel{P_0} + \frac{4T}{R_1}\right] - \left[\cancel{P_0} + \frac{4T}{R_2}\right] = \frac{4T}{R}$

$\frac{1}{R} = \frac{1}{R_1} - \frac{1}{R_2}$



QUESTION (JEE Mains – 29 Jan, 2023 I)

H.W

Surface tension of a soap bubble is 2.0×10^{-2} Nm^{-1}. Work done to increase the radius of soap bubble from 3.5 cm to 7 cm will be : [Use $\pi = 22/4$]

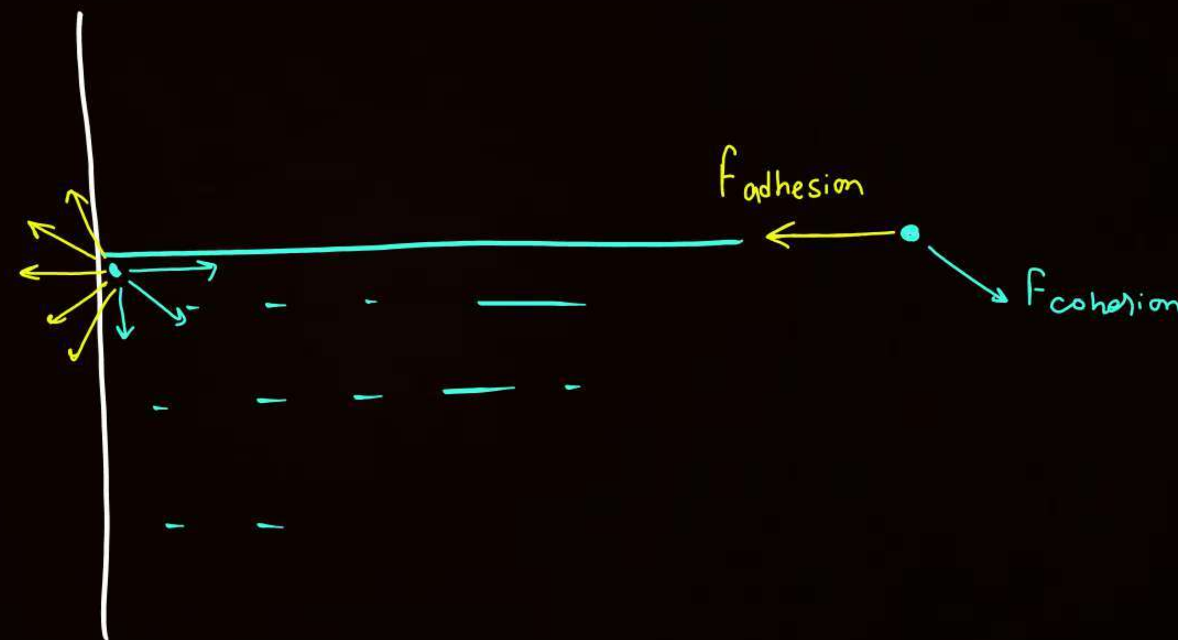
(A) 0.72×10^{-4} J

(B) 5.76×10^{-4} J

(C) 18.48×10^{-4} J

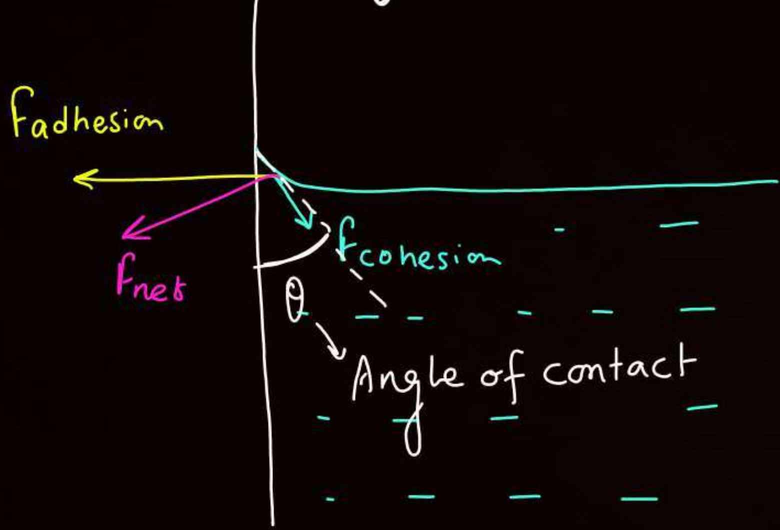
(D) 9.24×10^{-4} J

Cohesive and Adhesive Forces



$F_{adhesion} > F_{cohesion}$

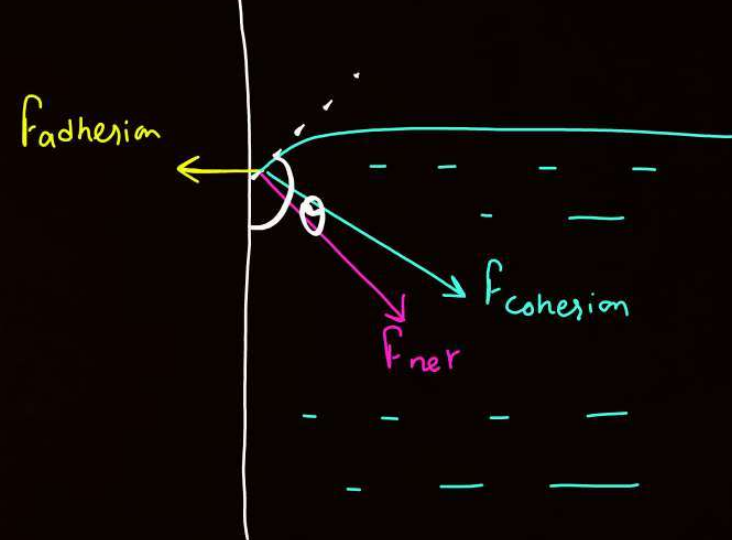
$\theta < 90^\circ$



Ex for glass-water = $\theta = 0^\circ$

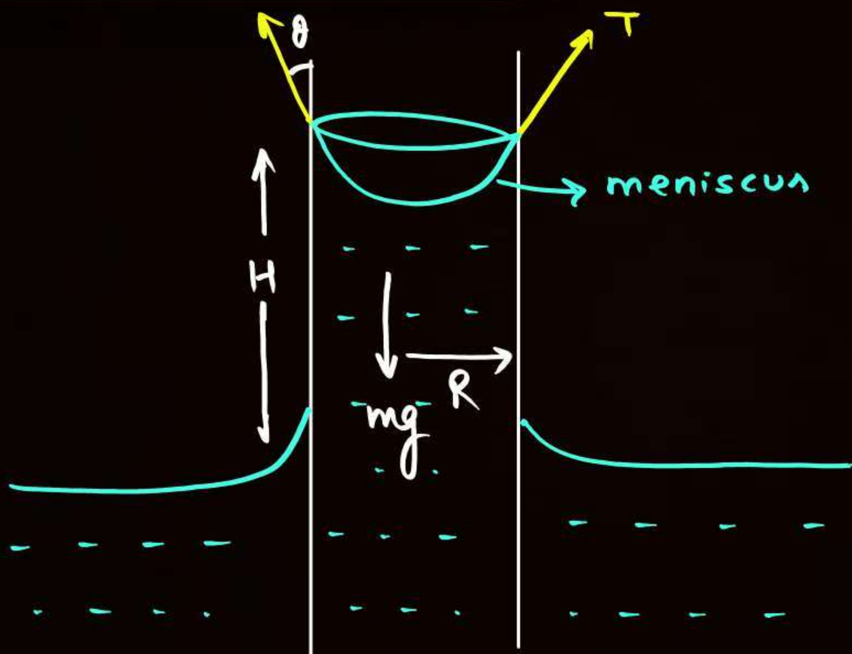
$F_{adhesion} < F_{cohesion}$

$\theta > 90^\circ$



Ex glass-Hg ($\theta = 135$)

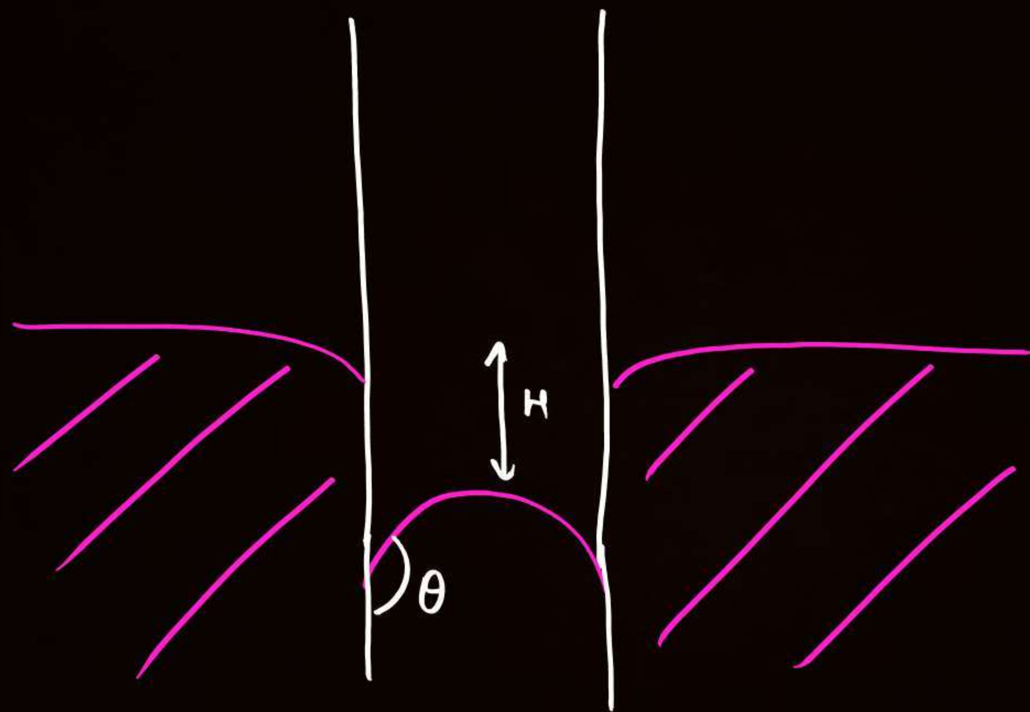
Capilliary Rise



$F_{up} = mg$

$T\cos\theta \cdot 2\cancel{\pi}\cancel{R} = \rho \cdot \cancel{\pi} R^{\cancel{2}} H g$

$H\uparrow = \dfrac{2T\cos\theta}{\rho g R\downarrow}$



$$H = \frac{2T\cos\theta}{\rho g R}$$

QUESTION

If the surface tension of water is 0.06 N/m then the capillary rise is a tube of diameter 1 mm is: ($\theta = 0°$)

(A) 1.22 cm

(B) 2.44 cm

(C) 3.12 cm

(D) 3.86 cm

$$H = \frac{2T\cos\theta}{\rho g R}$$

$$= \frac{2 \times 0.06 \times \cos 0°}{\cancel{10^3} \times \cancel{10} \times 0.5 \times \cancel{10^{-3}}}$$

$$= \frac{.12}{5}$$

$$= 24 \times 10^{-3}\ m$$

$$= 24\ mm = 2.4\ cm.$$

QUESTION (JEE Mains – 2013)

$h = \frac{2T\cos\theta}{\rho g R}$

This question has Statement-1 and Statement-2. Of the four choices given after the Statements, choose the one that best describes the two.

Statement-1: A capillary is dipped in a liquid and liquid rises to a height h in it. As the temperature of the liquid is raised, the height h increases (if the density of the liquid and the angle of contact remain the same).

Statement-2: Surface tension of a liquid decreases with the rise in its temperature.

(A) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation for Statement-1.

(B) Statement-1 is false, Statement-2 is true.

(C) Statement-1 is true, Statement-2 is false.

(D) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation for Statement-1.

QUESTION (JEE Mains – 10 Apr. 2019 I)

The ratio of surface tensions of mercury and water is given to be 7.5 while the ratio of their densities is 13.6. Their contact angles, with glass, are close to 135° and 0°, respectively. It is observed that mercury gets depressed by an amount h in a capillary tube of radius r_1, while water rises by the same amount h in a capillary tube of radius r_2. The ratio, (r_1/r_2), is then close to:

(A) 4/5

(B) 2/5

(C) 3/5

(D) 2/3

$$h = \frac{2T_1\cos\theta_1}{\rho_1 g R_1}$$

$$h = \frac{2T\cos\theta_2}{\rho_2 g R_2}$$

$$\frac{\cancel{2}T_1\cos\theta_1}{\rho_1 \cancel{g} R_1} = \frac{\cancel{2}T_2\cos\theta}{\rho_2 \cancel{g} R_2}$$

$$\frac{R_1}{R_2} = \frac{T_1}{T_2}\times\frac{\rho_2}{\rho_1}\times\frac{\cos 135^\circ}{\cos 0^\circ}$$

$$= 7.5\times 13.6\times\frac{1}{\sqrt{2}}$$

$$=$$

QUESTION

In a surface tension experiment with a capillary tube water rises upto 0.1m. If the same experiment is repeated in an artificial satellite, which is revolving around the earth ; water will rise in the capillary tube upto a height of:

(A) 0.1 m

(B) 0.2 m

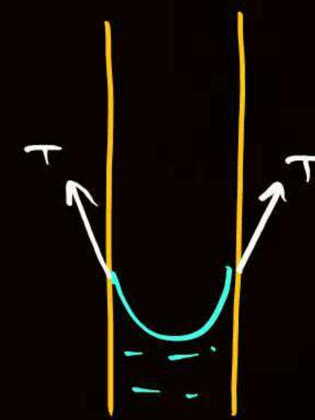
(C) 0.98 m

(D) Full length of tube

$g = 0$

$$H = \frac{2T\cos\theta}{\rho \underset{\downarrow 0}{g} R}$$

H = full tube length



QUESTION

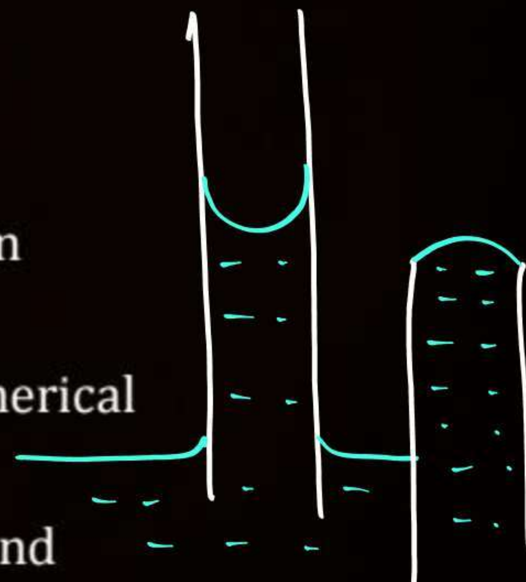
When a capillary tube is dipped in a liquid, the liquid rises to a height h in the tube. The free liquid surface inside the tube is hemispherical in shape. The tube is now pushed down so that the height of the tube outside the liquid is less than h:

(A) The liquid will ooze out of the tube slowly

(B) The liquid will come out of the tube like in a small fountain

(C) The free liquid surface inside the tube will not be hemispherical

(D) The liquid will fill the tube but not come out of its upper end



QUESTION

The lower end of a capillary tube touches a liquid whose angle of contact is $90°$, the liquid

(A) Rises into the tube

(B) Falls in the tube

(C) May rise of fall inside

(D) Neither rises nor falls inside the tube.

Question

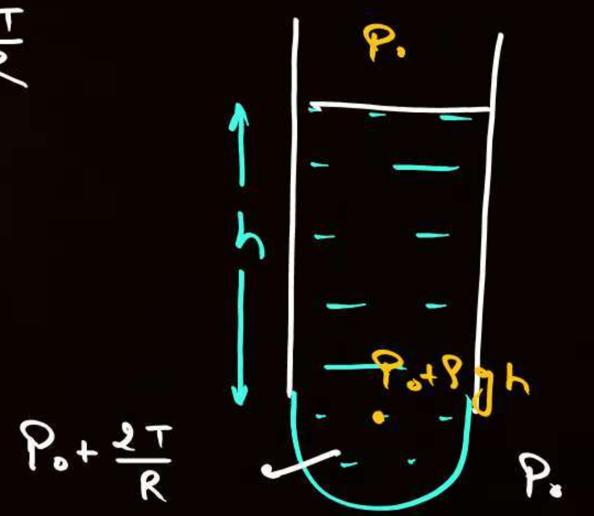
$R = 0.05\text{ mm}$

A vessel whose bottom has round holes with a diameter of $d = 0.1$ mm is filled with water. The maximum height of the water level h at which the water does not flow out, will be: (The water does not wet the bottom of the vessel). [S.T. of water = 70 dyn/cm]

1. $h = 20.0$ cm
2. $h = 25.0$ cm
3. $h = 26.0$ cm
4. $h = 28.0$ cm

$$\cancel{P_0} + \rho g h = \cancel{P_0} + \frac{2T}{R}$$

$$h = \frac{2T}{\rho g R}$$



QUESTION (JEE Mains – 02 Sep, 2020 I)

H W

A capillary tube made of glass of radius 0.15 mm is dipped vertically in a beaker filled with methylene iodide (surface tension = 0.05 Nm^{-1}, density = 667 kg m^{-3}) which rises to height h in the tube. It is observed that the two tangents drawn from liquid-glass interfaces (from opp. sides of the capillary) make an angle of 60° with one another. Then h is close to (g = 10 ms^{-2})

(A) 0.049 m

(B) 0.087 m

(C) 0.137 m

(D) 0.172 m

Homework

Class Q. + Module