Polynomials

{ Number → real no.
Variable → ✓

$5x^5 + 6x^4 - 3x^2 - 2$

$8x^2 + 3x - 7x^0$

$2x + 3$

8

* How to check for Polynomial :- $f(x)$ ←

* General Representation of a Polynomial

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \cdots\cdots + a_1 x + a_0$$

→ coefficients (real nos.)

Representation way

→ variable of polynomial

Terms

variables (powers must be $0 / I^+$)

Polynomials

* Identify which of following is a Polynomial ?

i) $\frac{2}{5}\alpha^{2}+1\cdot\alpha^{1}$ ✓

ii) $\cancel{\frac{1}{(y^2-3y+8)}} \rightarrow (y^2-3y+8)^{-1}$

iii) $\cancel{2x^2-2\sqrt{x}+7}$ $\quad \sqrt{x} = x^{\frac{1}{2}}$

iv) $9x^0$ ✓

v) 0 ✓

Polynomials

* **Degree of a Polynomial** :- 3 → Cubic

→ Highest power of variable that is observed.

i) $f(x) = 3x^{1} + 5x^{0}$ → 1 → Linear

ii) $g(y) = 7y^{2} + 10y^{1} + 3y^{0}$ → 2 → Quadratic

iii) $h(t) = 9t^{0}$ → 0 → Constant

iv) $f(z) = 0$ → Not Defined → Zero

Polynomials

* Value of a Polynomial

→ If '$f(x)$' is a polynomial & 'α' be any real number, then $f(\alpha)$ is the value of the polynomial at $\boxed{x=\alpha}$.

$f(3) =$

Polynomials

$\text{Zeros} \rightarrow 2, -\frac{1}{2}$

* If $\boxed{f(x) = 2x^2 - 3x - 2}$, then find

i) $\boxed{f(1)} = -3$

* ii) $\boxed{f(2)} = 0$

iii) $\boxed{f(-2)} = 12$

* iv) $\boxed{f\left(-\frac{1}{2}\right)} = 0$

$$f(1) = 2\times(1)^2 - 3(1) - 2$$
$$= \cancel{2} - 3 - \cancel{2}$$
$$= -3$$

$$f(2) = 2\times(2)^2 - 3(2) - 2$$
$$= 8 - 6 - 2$$
$$= 0$$

$$f(-2) = 2(-2)^2 - 3(-2) - 2$$
$$= 8 + 6 - 2$$
$$= 12$$

$$f\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{2}\right)^2 - 3\left(-\frac{1}{2}\right) - 2$$
$$= \frac{1}{2} + \frac{3}{2} - 2$$
$$= 2 - 2 = 0$$

Polynomials

→ When $\boxed{f(\alpha) = 0}$, then

Zero of the polynomial $f(x)$.

Polynomials

* To find zero of a Polynomial

i) Linear Polynomial

zero → $\frac{5}{6}$

Eg :- Find zero of $(6x-5)$.

$$\boxed{6x-5=0}$$

$$\Rightarrow 6x = 5$$

$$\Rightarrow x = \frac{5}{6} \text{ Sol}^n$$

ii) Quadratic Polynomial

$4 \to 2\times2,\ 4\times1$

2 Methods

zeros → $2, -\frac{1}{2}$

Eg : Find zero of $(2x^2-3x-2)$

Method - I

$$2x^2-3x-2=0$$

$$-4x+1x$$

$$\Rightarrow 2x^2-4x+x-2=0$$

$$\Rightarrow 2x(x-2)+(x-2)=0$$

$$\Rightarrow (x-2)(2x+1)=0$$

$x-2=0$	$2x+1=0$
$\Rightarrow x=2$	$x=-\frac{1}{2}$

Polynomials

* To find zero of a Polynomial

ii) Quadratic Polynomial

$\boxed{ax^2+bx+c}$

$D = b^2-4ac$

$x \rightarrow \frac{-b+\sqrt{D}}{2a}$, $x \rightarrow \frac{-b-\sqrt{D}}{2a}$ $\longrightarrow \frac{-b\pm\sqrt{D}}{2a}$

Eg: Find zero of $(2x^2-3x-2)$

Method-II $a=2$, $b=-3$, $c=-2$

$D = b^2-4ac$
$= (-3)^2-(4\times2\times-2)$
$= 9+16$
$\boxed{D=25}$

$\frac{-b+\sqrt{D}}{2a}$

$\frac{3+\sqrt{25}}{4}$

$\frac{3+5}{4} = \frac{8}{4} = 2$

$\frac{-b-\sqrt{D}}{2a}$

$\frac{3-\sqrt{25}}{4}$

$\frac{3-5}{4} = \frac{-\cancel{2}}{\cancel{4}_2} = -\frac{1}{2}$

Polynomials

* Some important graphs :-

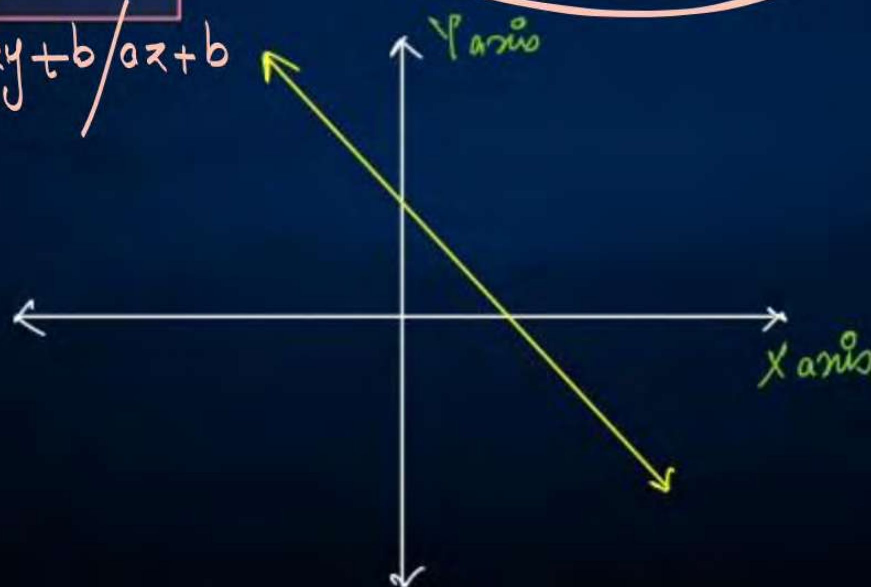
i) Linear Polynomial → Straight line

$ax+b$ / $ay+b$ / $az+b$

$2x+3$

$8x$

$\frac{7}{3}x-5$



Polynomials

$\sqrt{-3}$ $\sqrt{\frac{-5}{2}}$

ay^2+by+c

ax^2+bx+c

ii) **Quadratic Polynomial**

$a \neq 0$ $\boxed{a}x^2+\underline{b}x+\underline{c}$

$\cup$ $a \to + \longrightarrow 2x^2-3x+4$

$\cup$ $a \to + \longrightarrow 1x^2+x+1$

$\cap$ $a \to - \longrightarrow -\frac{3}{2}x^2-5x+3$

$a>0$

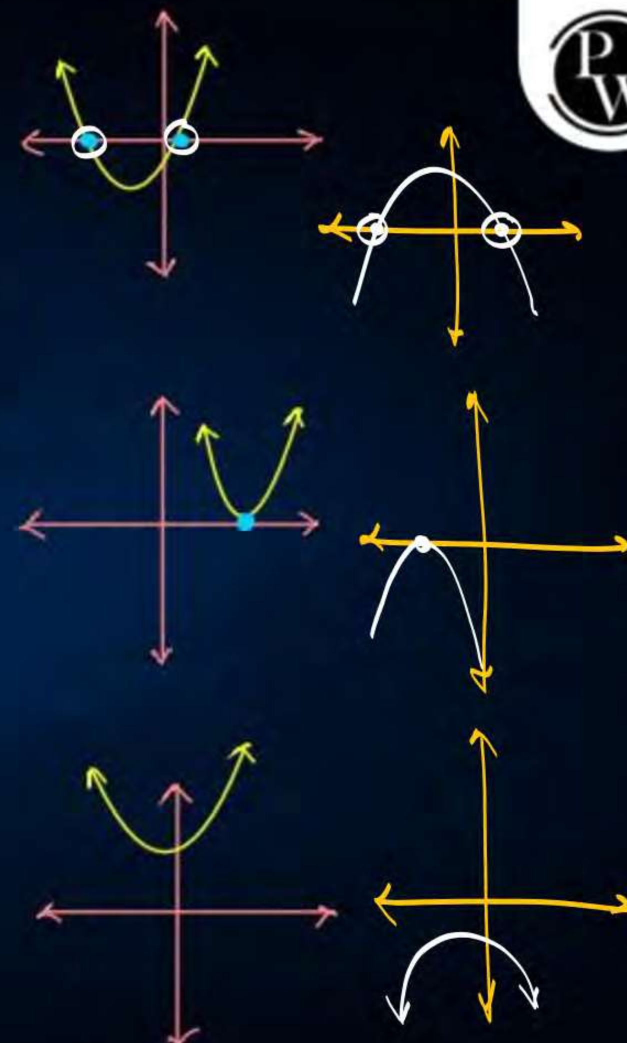
Parabola

$a<0$

Real & Distinct

Equal Real &

~~Non real~~



Polynomials

* Geometrical Meaning of zeros

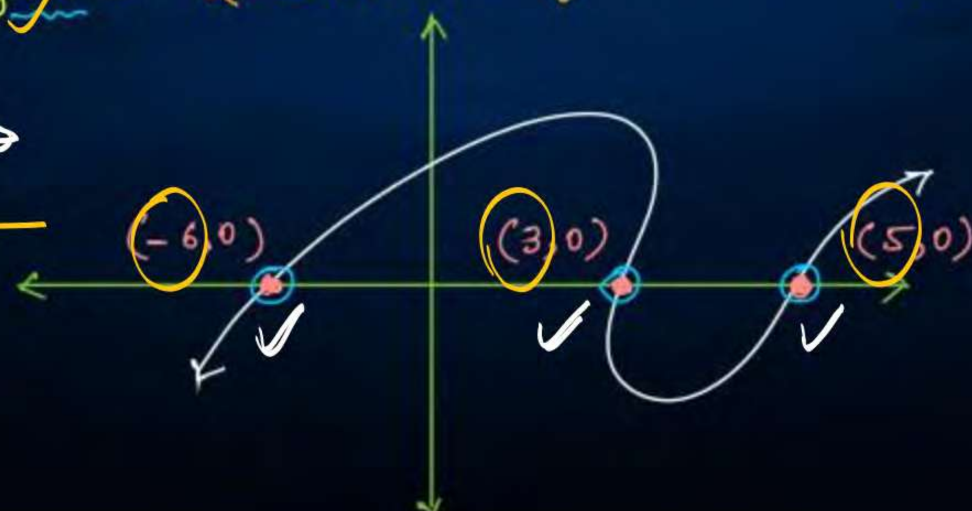
1 cut / 1 touch = 1 zero

→ Number of zeros = Number of points where graph cuts / touches x-axis

→ Value of zero = x-coordinate of the above points

3 zeros

-6, 3 & 5



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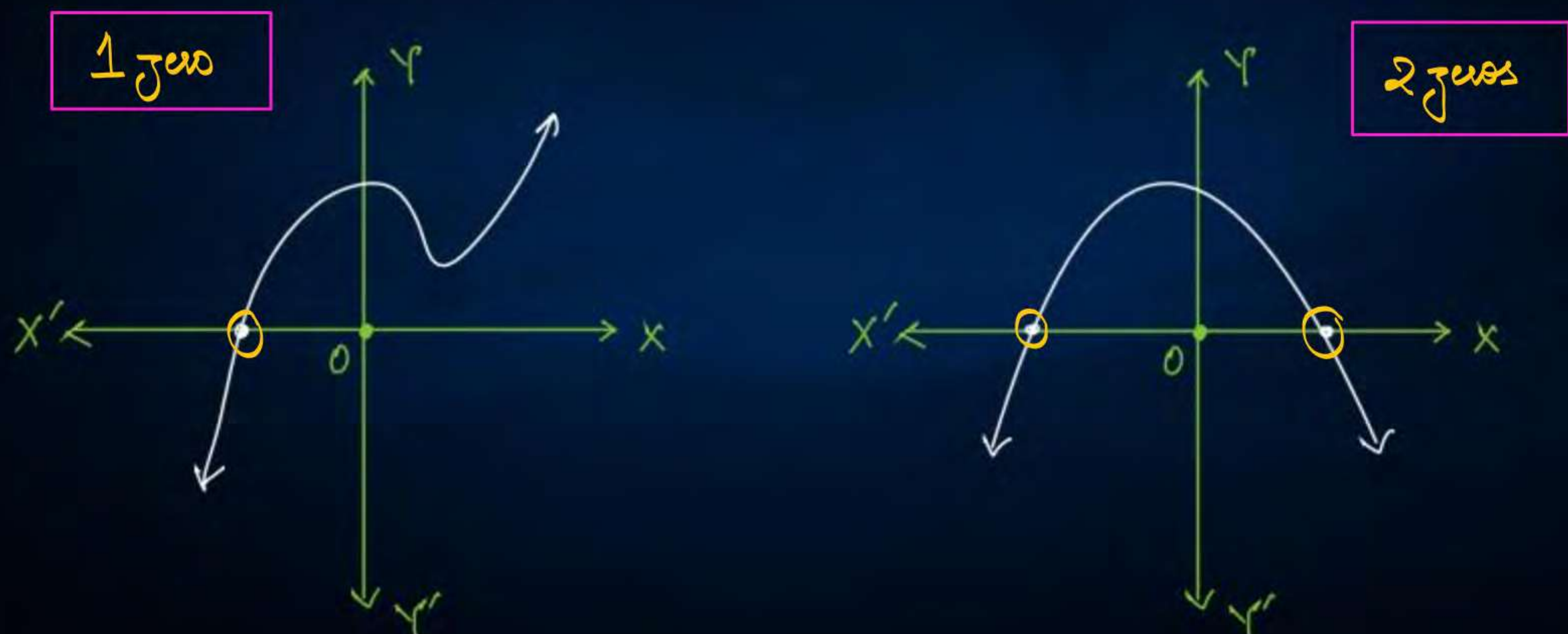
Look at the graphs in Fig. given below. Each is the graph of $y = p(x)$, where $p(x)$ is a polynomial. For each of the graphs, find the number of zeroes of $p(x)$.



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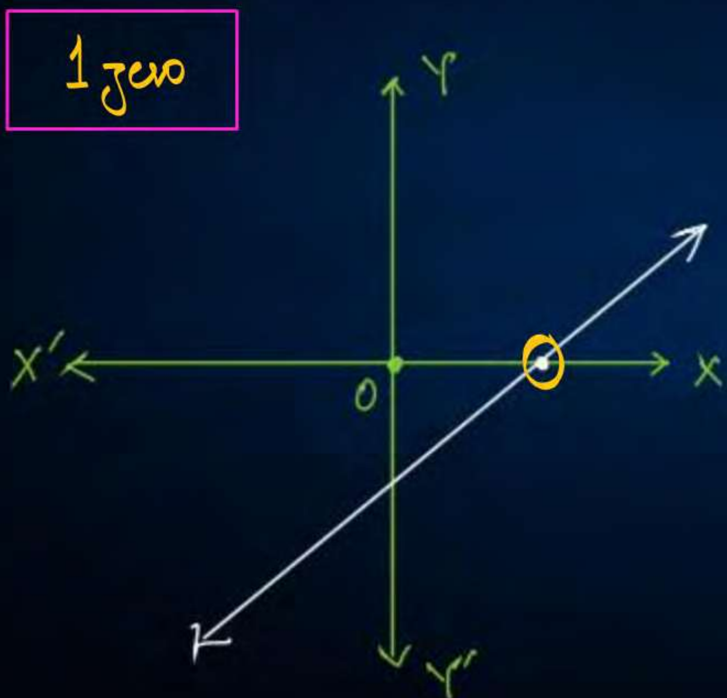

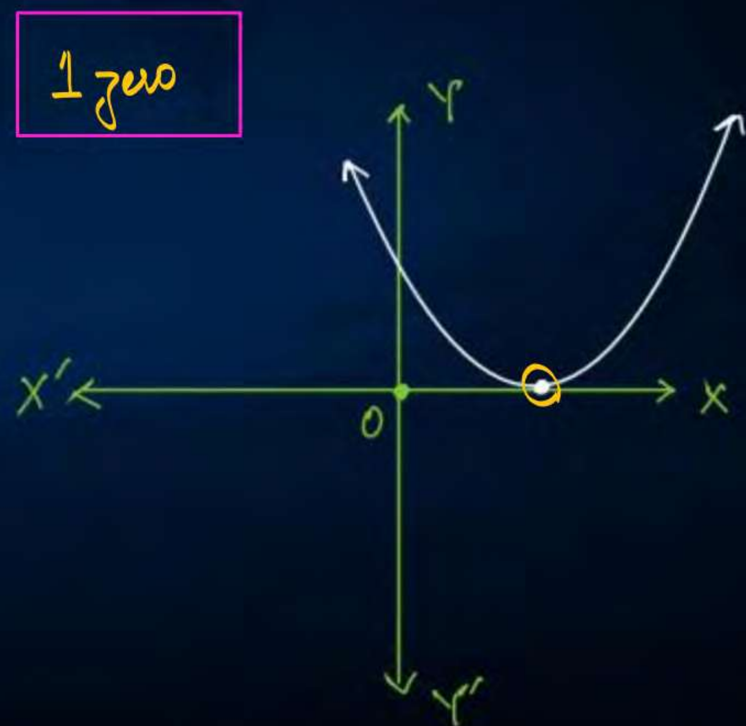

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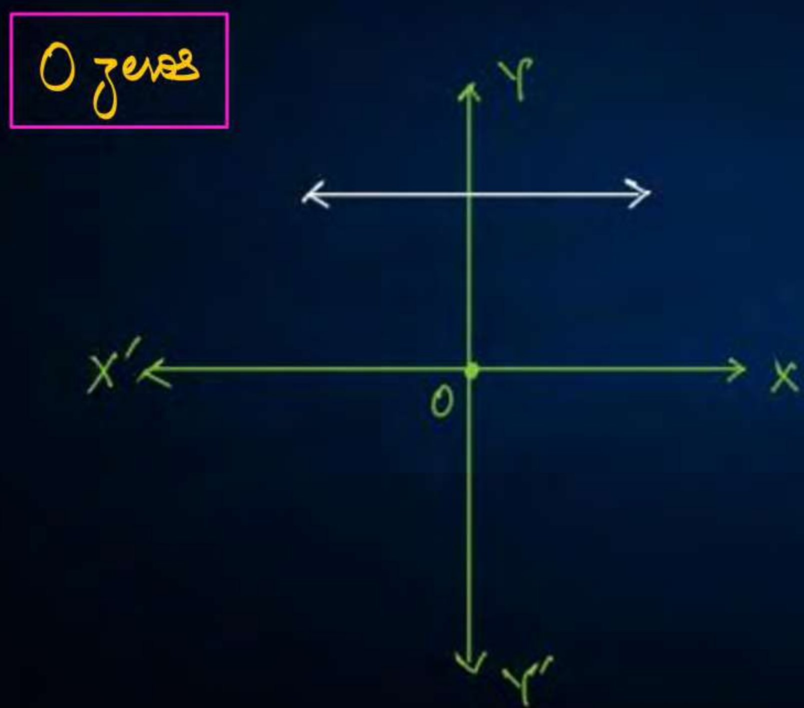

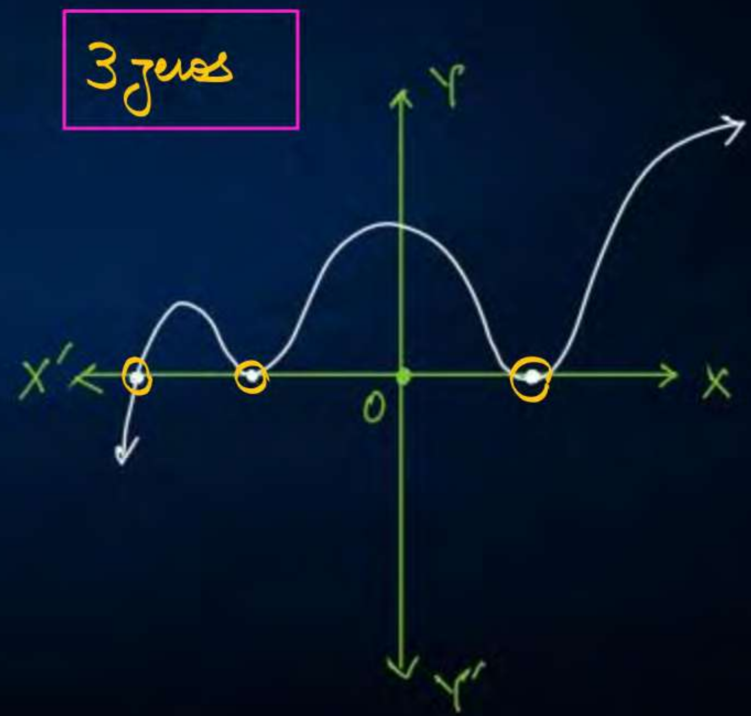

Polynomials

(Qudratic)

* Relationship b/w zeros & coefficients of a Polynomial :-

$$\boxed{ax^2+bx+c} \xrightarrow{2\text{ zeros}} \begin{cases} \text{'}\alpha\text{'} \\ \text{'}\beta\text{'} \end{cases}$$

Sum: $\alpha+\beta = -\frac{b}{a}$

Product: $\alpha \cdot \beta = \frac{c}{a}$

QUESTION

$10 \to 10\times 1$, 2×5

$\xrightarrow{\text{2 zeroes}} -5 \; \& \; -2$

Find the zeroes of the quadratic polynomial $x^2 + 7x + 10$, and verify the relationship between the zeroes and the coefficients.

ax^2+bx+c

$\boxed{\text{Sol}^n}$

$$x^2+7x+10=0$$
$$\Rightarrow x^2+5x+2x+10=0$$
$$\Rightarrow x(x+5)+2(x+5)=0$$
$$\Rightarrow (x+5)(x+2)=0$$

$x+5=0$ | $x+2=0$

$x=-5$ | $x=-2$

$a=1,\ b=7,\ c=10$

$$S=(-5)+(-2)$$
$$=-5-2$$
$$\boxed{S=-7}$$

$$-\frac{b}{a} \Rightarrow \frac{-7}{1} \Rightarrow -7$$

$$P=(-5)\times(-2)$$
$$\boxed{P=10}$$

$$\frac{c}{a}=\frac{10}{1}=10$$

QUESTION

ax^2+bx+c $\quad \frac{3}{2}, -\frac{1}{3}$ $\quad a=6, b=-7, c=-3$

Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

$6x^2 - 3 - 7x \rightarrow 6x^2-7x-3 \xrightarrow{2\text{ zeroes}} \frac{3}{2}, -\frac{1}{3}$

Sol^n

$$\boxed{6x^2-7x-3=0}$$

$$\Rightarrow 6x^2-9x+2x-3=0$$

$$\Rightarrow 3x(2x-3)+(2x-3)=0$$

$$\Rightarrow (2x-3)(3x+1)=0$$

$2x-3=0$, $x=\frac{3}{2}$ | $3x+1=0$, $x=-\frac{1}{3}$

$$S=\frac{3}{2}+\left(-\frac{1}{3}\right)=\frac{3}{2}-\frac{1}{3}=\frac{9-2}{6}=\frac{7}{6}$$

$$-\frac{b}{a}=\frac{7}{6}$$

$$P=\frac{\cancel{3}}{2}\times -\frac{1}{\cancel{3}}$$

$$P=-\frac{1}{2}$$

$$\frac{c}{a}=-\frac{\cancel{3}}{\cancel{6}_2}=-\frac{1}{2}$$

Polynomials

$S = \alpha + \beta$

$P = \alpha \cdot \beta$

S, P

* If zeros of a Quadratic Polynomial are $\longrightarrow$ $\boxed{\alpha, \beta}$

→ Then reqd. Quadratic Polynomial is $\boxed{k\left[x^2 - Sx + P\right]}$

k → non zero real number

S → Sum

P → Product

QUESTION

$S = -3$
$P = 2$

Find a quadratic polynomial, the sum and product of whose zeroes are -3 and 2, respectively.

Sol^n

$k[x^2 - Sx + P]$

$k[x^2 - (-3)x + 2]$

$\boxed{k[x^2 + 3x + 2]}$, k is any real no except 0.

$k = 1$, $x^2 + 3x + 2$

QUESTION

$S = 0$
$P = \sqrt{5}$

Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

0, √5

Sol^n

$k[x^2 - Sx + P]$

$k[x^2 - 0x + \sqrt{5}]$

$\boxed{k[x^2 + \sqrt{5}]}$, where k is any real no except '0'.

$k = 1$ $x^2 + \sqrt{5}$