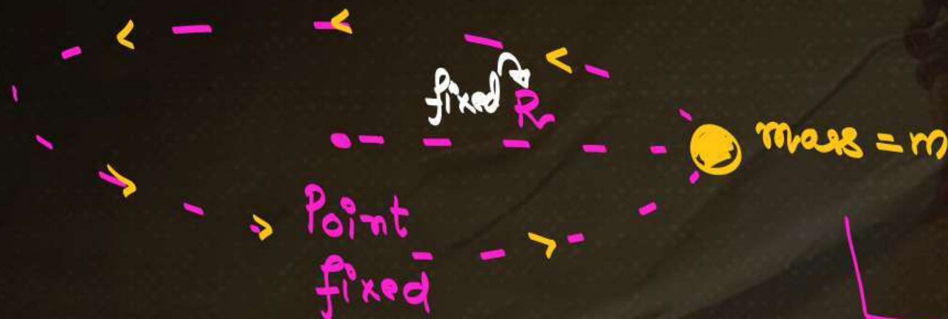
Circular Motion. "Plane".



Clockwise

Anticlockwise.

$x \rightarrow$ displacement $\Longrightarrow \theta$ "Angular displacement".

$v = \frac{dx}{dt} \Rightarrow$ velocity $\Longrightarrow$ "ω" "Angular velocity".

$a = \frac{dv}{dt} \Rightarrow$ acceleration $\Longrightarrow$ "α" "Angular acceleration.

Angular Displacement

$\Rightarrow$ θ = Angle Rotated by particle.

$\Rightarrow \dfrac{\text{Arc length}}{\text{Radius}}$

$\Rightarrow$ Radians $\Rightarrow [M^0 L^0 T^0]$.

$\Rightarrow$ $\boxed{180^\circ = \pi \text{ radians}}$

$\Rightarrow$ direction by RHTR.

Right hand $\rightarrow$ finger $\rightarrow$ dir of Motion

Thumb $\rightarrow$ θ dir.

on axis of Rotation.

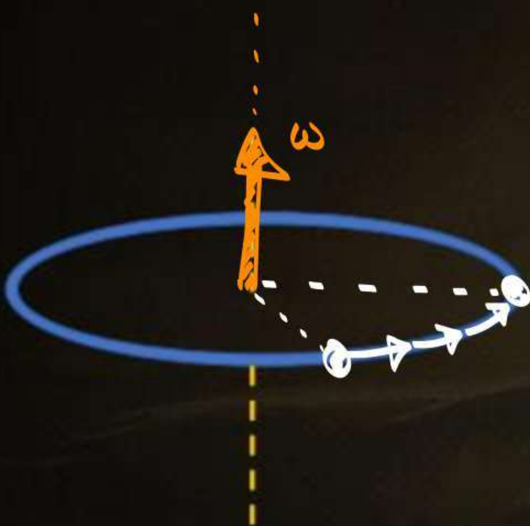
Angular Velocity

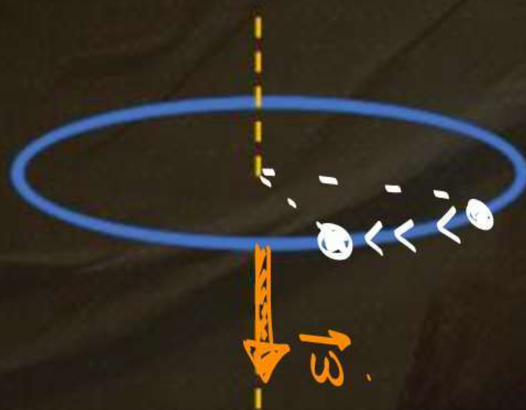
$\Longrightarrow \vec{\omega}$ = Angular Velocity

$\Rightarrow$ Rate of change of Angular displacement with time.

$$\omega = \frac{d\theta}{dt}$$

unit = $\frac{rad}{s}$ Dimension = $[T^{-1}]$





on axis of Rotation.

⊙ RHTR.

Right hand finger $\rightarrow$ dir of Motion.

Thumb $\rightarrow$ ω.

⊙ Instantaneous angular velocity.

$\theta = 2t^2$ (Circular Motion)

$$\omega = \frac{d\theta}{dt} = \frac{d}{dt}(2t^2) = 4t.$$

ω at any Instant. ω at $t = 1$ Sec.

$\omega_{t=1s} = 4$ rad/s.

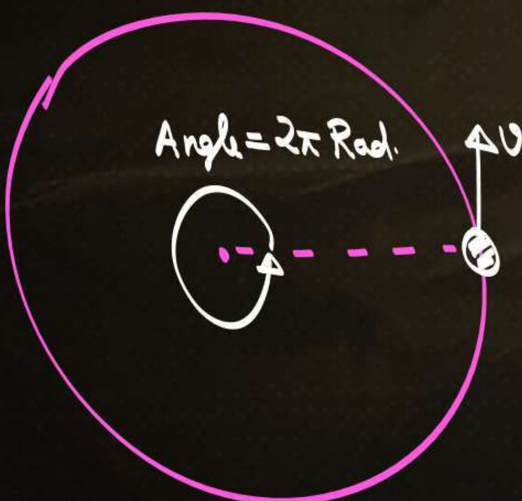
- Average angular velocity

$$\boxed{\omega_{av} = \frac{\text{Total Angle travelled}}{\text{Total time}} = \frac{\theta_2 - \theta_1}{t_2 - t_1}}$$

#

- If particle Cover 1 Rotation with Constant speed in time = T.

$$\omega = \frac{\text{Total Angle}}{\text{Total time}} = \frac{2\pi}{T}$$



$$\boxed{\begin{aligned} \omega &= \frac{2\pi}{T} \\ \omega &= 2\pi\nu \end{aligned}}$$

$\frac{1}{T}$ = frequency = ν ⟹

"No of Rotations per Second".

$\boxed{\text{unit} = \text{rps, rpm.}}$

120 rpm

$\text{rps} = \frac{\text{rpm}}{60}$

$\nu = \frac{120}{60}\text{ rps} = 2\text{ rps}$

$\omega = 2\pi\nu = 4\pi$ rad/s.

Angular Acceleration

$\Longrightarrow \alpha$ = Rate of change of angular Velocity wrt time.

$$\alpha = \frac{d\omega}{dt}$$

unit = $\frac{rad}{s^2}$ Dimension = $[T^{-2}]$

⊙ Instantaneous angular acceleration

$\theta = 2t^3 + 3t^2 + 4$

find α at $t = 1$ sec.

$\boxed{\omega = \frac{d\theta}{dt} = 6t^2 + 6t}$ → Speed up.

$\alpha = \frac{d\omega}{dt} = 12t + 6 \Rightarrow \alpha_{t=1} = 12 + 6 = 18 \frac{rad}{s^2}$.

Average Angular Acceleration.

$$\boxed{\alpha_{av} = \frac{\omega_f - \omega_i}{t_f - t_i}}$$

Kinematics

"Retardation" $\Longrightarrow$ Rukna.

$a = \oplus$ or $\ominus$ both ho Sakti hai.

v, a(−). Slow down

v, a(+) Slow down.

$\boxed{\text{v \& acc are anti parallel}}$.

① Similar in Circular Kinematics.

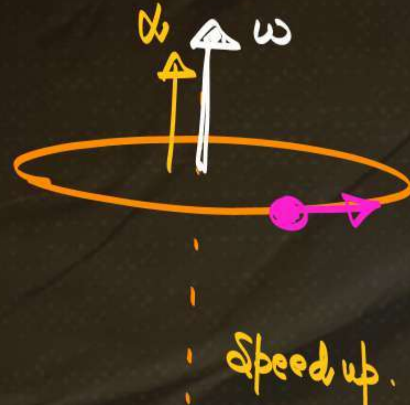
$\vec{\omega}$ & $\vec{\alpha}$ (axial Vectors) "Aise Vectors jo axis of Rotation par Lagte hai.

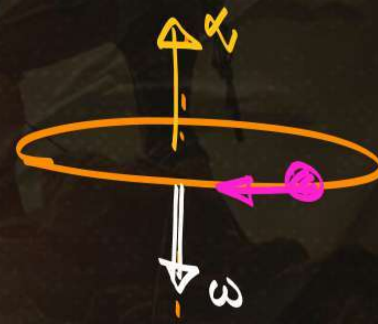
ω = Constant

Ex $\omega = 5 \frac{rad}{s}$

$$\boxed{\alpha = \frac{d\omega}{dt} = 0}$$

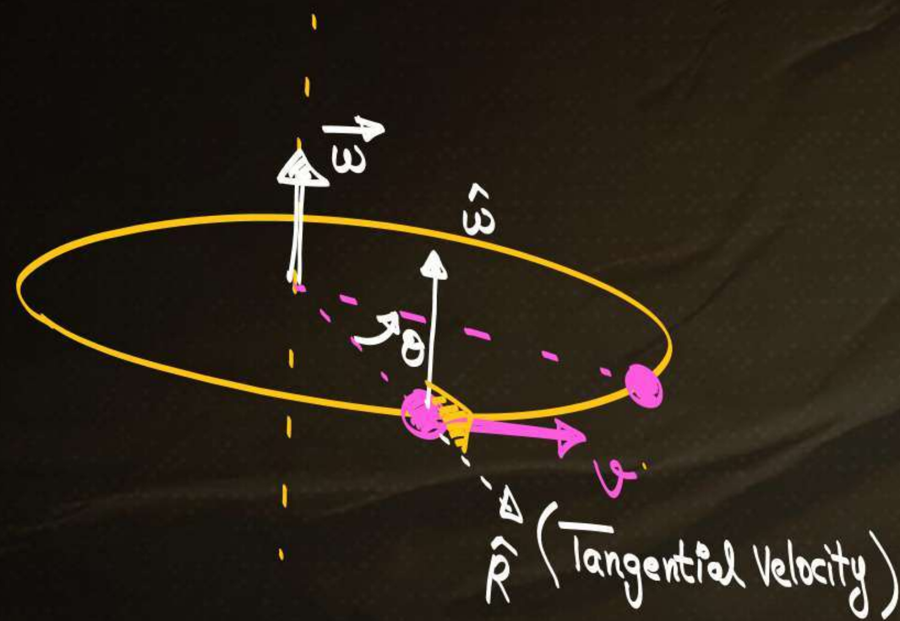
Neither Speed up Nor Slow down.





Important Relation.

Q)



There is a Relation between.

$v_{Tangential}$ & Angular velocity.

$$\boxed{\vec{v} = \vec{\omega} \times \vec{R}}$$

$$v = \omega R \sin 90$$

$$\boxed{v = R\omega}$$

$$\vec{\omega} = \hat{i} + \hat{j} - \hat{k}$$

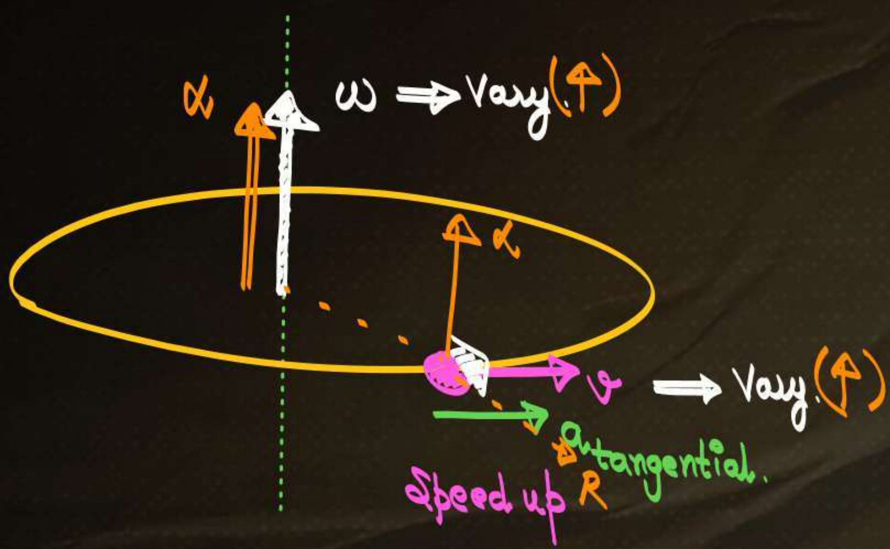
$$\vec{R} = 2\hat{i} - \hat{j} + 3\hat{k}$$

$$\vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 2 & -1 & 3 \end{vmatrix} = \hat{i}\,(3-(1)) - \hat{j}\,(3-(2)$$

$$\hat{k}\,(-1-2).$$

$$= \underline{\qquad}.$$

b) Relation between $\vec{a}_t$ & $\vec{\alpha}$.



⊙ $\vec{v} = \vec{\omega} \times \vec{R}$

⇓

Rate of change.

$$\frac{d}{dt}\vec{v}_T = \frac{d}{dt}(\bar{\omega} \times \bar{R})$$

$$\boxed{a_T = \vec{\alpha} \times \vec{R}}$$

$$a = \alpha R \sin 90$$

$$\boxed{a = R\alpha}$$

Kinematics of Circular Motion

- $\omega = \dfrac{d\theta}{dt}$

  $\omega_{av} = \dfrac{\theta_2 - \theta_1}{t_2 - t_1}$

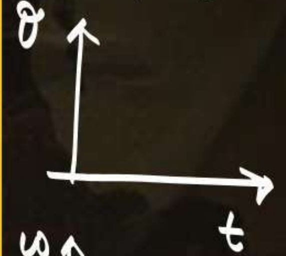
- $\alpha = \dfrac{d\omega}{dt} = \dfrac{d\omega}{dt} \times \dfrac{d\theta}{d\theta} = \omega \dfrac{d\omega}{d\theta}$

  $\alpha_{av} = \dfrac{\omega_2 - \omega_1}{t_2 - t_1}$

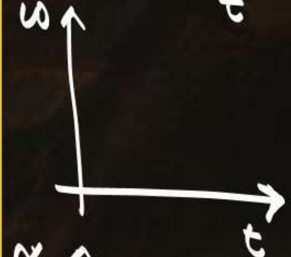
- $\alpha = \text{Constant.}$

$$\boxed{\begin{aligned} \omega_f &= \omega_0 + \alpha t \\ \theta &= \omega_0 t + \tfrac{1}{2}\alpha t^2 \\ \omega_f^2 - \omega_0^2 &= 2\alpha\theta \\ \theta_{n^{th}} &= \omega_0 + \tfrac{\alpha}{2}(2n-1) \end{aligned}}$$

Graphs.

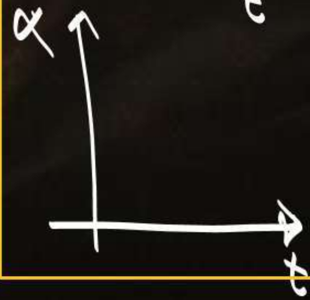


Slope = ω.



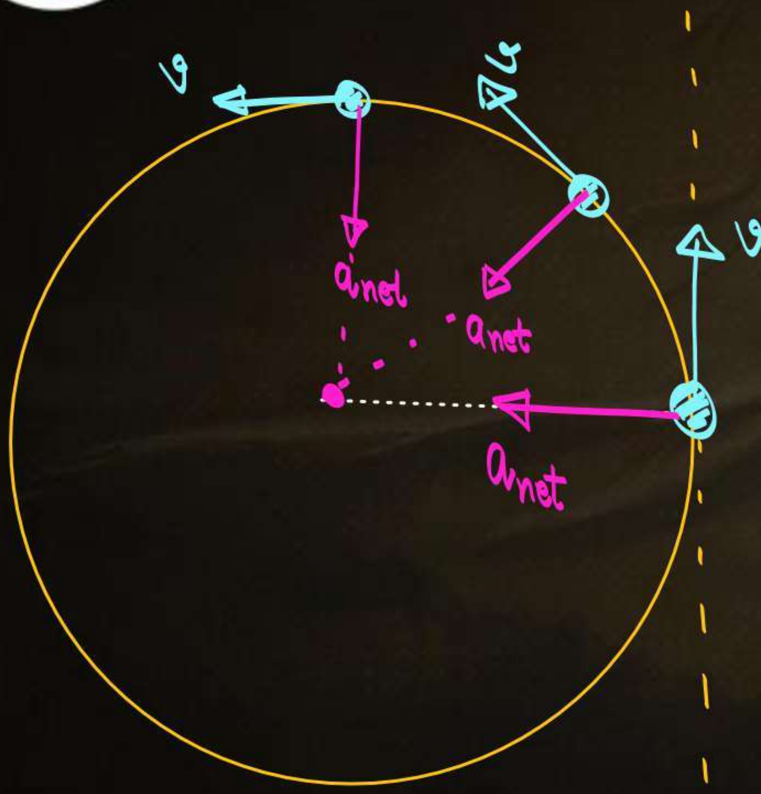
Slope = α

Area = $\theta_2 - \theta_1$



Area = $\omega_2 - \omega_1$.

Centripetal and Tangential Acceleration



Koi bhi particle agar CM mein hai toh ek acc towards Centre toh hogi (Must)

$\Rightarrow$ To acc towards Centre hai & direction change Kar rahi hai $\Rightarrow$ Centripetal Acceler

$$\boxed{a_c = a_{centripetal} = \frac{v^2}{R} = R\omega^2}$$

Towards Centre.

direction change Karane Ka Kaam.

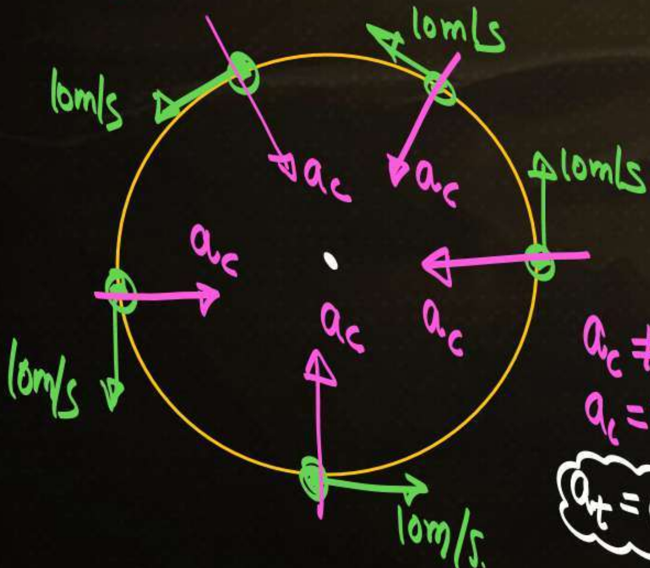
There are two type of CM.

UCM $\Rightarrow$ anet is towards Centre.

Uniform Circular Motion

ω = Constant Ex $\omega = 5\ \frac{rad}{s}$, $R = 2m$

$|v_{Tangential}|$ = Constant $v = R\omega = 10\ m/s$



UCM

$a_c \neq 0$

$a_c = R\omega^2 = \frac{v^2}{R}$

$a_t = 0$

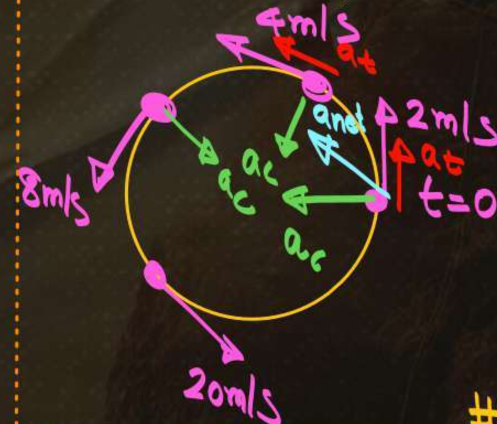
① $a_{net} = \sqrt{a_t^2 + a_c^2}$

Not toward Centre.

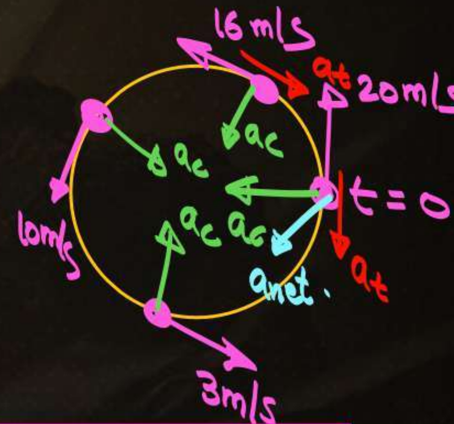
NUCM → ω = Vary
Ex:- $\omega = 2t^2$
OR
v = Vary.

(Non-uniform CM).

Speed up



Slow down.



$$\boxed{a_c \neq 0 = \frac{v^2}{R} = R\omega^2}$$

$v \rightarrow$ vary $a_c \rightarrow$ vary.

$$\boxed{a_{tangential} = R\alpha = \frac{d}{dt}|v|}$$

QUESTION 01

If θ depends on time t in following way

$\theta = 2t^2 + 3$ then

$t=0 \quad \theta = 2\cdot 0^2 + 3 = 3$

$t=3 \quad \theta = 2\cdot 9 + 3 = 21$

(a) Find out ω average upto 3 sec.

(b) ω at 3 sec (Instant)

a) $\omega_{avg} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{21-3}{3-0} = \frac{18}{3} = 6\ \frac{rad}{s}$.

b) $\omega = \frac{d\theta}{dt} = 4t$

at $t = 3$

$\omega = 4\times 3 = 12\ \frac{rad}{s}$.

QUESTION 02

$t=1$ $\quad \omega = 2(1)^2 + 1 = 3$ $\quad a_{net} = \sqrt{4^2 + 9^2}$

A particle moves in a circular path of radius 1 m with an angular speed

$\omega = 2t^2 + 1$ rad/sec

$t \rightarrow$ vary $\omega \rightarrow$ vary

$v \rightarrow$ vary

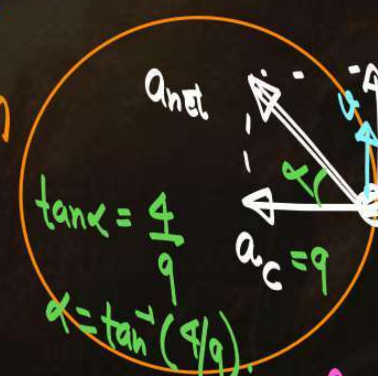
Find the angle between total acceleration and normal acceleration at $t = 1$ sec. NUCM.

Centripetal Acc.

Sol:-

$R = 1m$

Speed up.



$a_c = \frac{v^2}{R} = R\omega^2$

at $t=1$ $\quad a_c = R\omega^2$

$= 1\times(3)^2 = 9$

$\alpha = \frac{d\omega}{dt} = 4t$

at $t=1$ $\alpha = 4$

$a_t = R\alpha = 1\times 4 = 4\ m/s^2$.

QUESTION 03

If a particle moves in a circle describing equal angles in equal times, its "velocity" vector

$\omega = \frac{d\theta}{dt}$

$\boxed{\omega = \text{Constant}}$

$\boxed{|v| = \text{Constant}}$

UCM

$a_c = \checkmark$

$a_t = 0$

(A) ~~Remains constant~~

(B) ~~Changes in magnitude~~

(C) Change in direction ✓

(D) ~~Changes both in magnitude and direction~~

QUESTION 04 (Adv).

Match the matrix:

Column-I		Column-II	
(a)	UCM (r)(q)	(p)	$\left\|\frac{dv}{dt}\right\| = 0$ Magnitude of acc.
(b)	NUCM (r)(s) $a_t \neq 0$	(q)	$\frac{d\|v\|}{dt} = 0$ Mag of Velocity = Const → a_t
		(r)	$\left\|\frac{dv}{dt}\right\| \neq 0$
		(s)	$\frac{d\|v\|}{dt} \neq 0$

$a_{normal} = a_{centripetal} = a_{radial}$

QUESTION 05

The motion of a particle will be circular if:

UCM, NUCM

$a_c \neq 0$

$a_t = 0$ $\quad$ $a_t \neq 0$

(A) $a_r = 0$ but $a_t \neq 0$

(B) $a_r = 0$ and $a_t = 0$

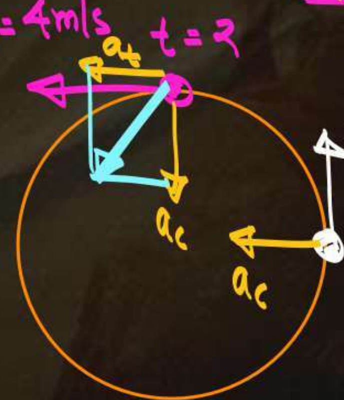
(C) $a_r \neq 0$ but $a_t = 0$ UCM

(D) $a_r \neq 0$ and $a_t \neq 0$ NUCM.

QUESTION 06

The speed of a particle moving in a circle of radius $r = 2$ m varies with time t as $v = t^2$ where, t is in second and v in ms^{-1}. The net acceleration at $t = 2$ s is **[2012]**

Speed up.



$v = t^2 \Rightarrow t \rightarrow$ vary $|v| \rightarrow$ vary.

NUCM.

$a_c = \dfrac{v^2}{R}$ $\quad$ $a_t = \dfrac{dv}{dt}$

$a_c = \dfrac{4\times4}{2} = 8\,m/s^2$ $\quad$ $= \dfrac{d}{dt}(t^2) = 2t$

at $t = 2$ $\quad$ $a_t = 4\,m/s^2$

$a_{net} = \sqrt{a_t^2 + a_c^2}$

$= \sqrt{8^2 + 4^2} = \sqrt{80}$.

QUESTION 07

K = Constant.

$\alpha = -k\sqrt{\omega}$, where ω is the angular velocity of body. Find the time after which body will come to rest if at $t = 0$, angular velocity of body was ω_0.

Sol $\alpha = -K\sqrt{\omega}$

$\Downarrow$

Vary as particle Stops

$t = 0 \quad \omega = \omega_0$

$t = t \quad \omega = 0.$

$$\frac{d\omega}{dt} = -K\omega^{1/2}$$

$$\int_{\omega_0}^{0} \frac{d\omega}{\omega^{1/2}} = \int_{t=0}^{T} -K\,dt \Rightarrow \left[2\sqrt{\omega}\right]_{\omega_0}^{0} = -K\left[t\right]_0^T$$

$$\boxed{T = \frac{2\sqrt{\omega_0}}{K}}$$

QUESTION 08 (PYQ)

What is the value of linear velocity, if $\vec{\omega} = 3\hat{i} - 4\hat{j} + \hat{k}$ and $\vec{r} = 5\hat{i} - 6\hat{j} + 6\hat{k}$

Given :- $\bar{v} = \bar{\omega} \times \bar{R}$

$$v = \begin{vmatrix} \hat{i} & -\hat{j} & K \\ 3 & -4 & 1 \\ 5 & -6 & 6. \end{vmatrix}$$

$= \_\_\_\_\_\_\_\_.$

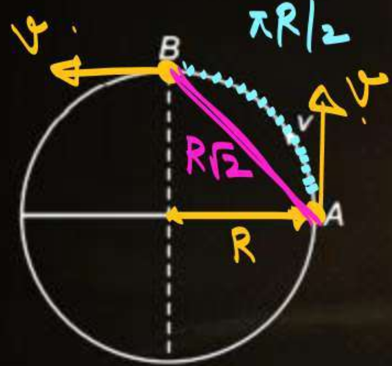
QUESTION 09

avg velocity = $\frac{\text{Total disp}}{\text{Total time}}$

$= \frac{R\sqrt{2}}{(\pi R/2v)}$

What is the average acceleration is going from A to B?

$|\text{avg acc}| = \frac{|\bar{v}_f - \bar{v}_i|}{\Delta T}$



$= \frac{v\sqrt{2}}{\left(\frac{\pi R/2}{v}\right)}$

$\bar{v}_f - \bar{v}_i =$

Time taken $= \frac{\frac{\pi R}{2}}{v}$

QUESTION 10

(PYQ)

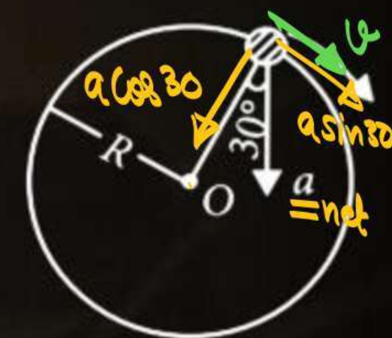
$a = 15\ \text{m s}^{-2}$ represents the total acceleration of a particle ,radius $R = 2.5$ m at a given instant of time. The speed of the particle is

$5/2$

UCM, NUCM

$\frac{v^2}{R} \Leftarrow a_c = a\cos 30$

$a_t = a\sin 30 \Rightarrow$ speed up.



$\frac{v^2}{R} = 15\cos 30$

$v = \sqrt{R \times 15 \times \frac{\sqrt{3}}{2}} = \sqrt{\frac{5}{2} \times 15 \times \frac{\sqrt{3}}{2}} =$ ___ .

QUESTION 11

(PYQ).

$\boxed{\text{Power} = \vec{F} \cdot \vec{v}}$ $= P = Fv\cos\theta$

A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration (a) is varying with time t as $a = k^2rt^2$, where k is a constant. The power delivered to the particle by the force acting on it is given as

(A) Zero

(B) $mk^2r^2t^2$

(C) mk^2r^2t ✓

(D) mk^2rt

$\frac{v^2}{r} = a_c = k^2rt^2$

NUCM

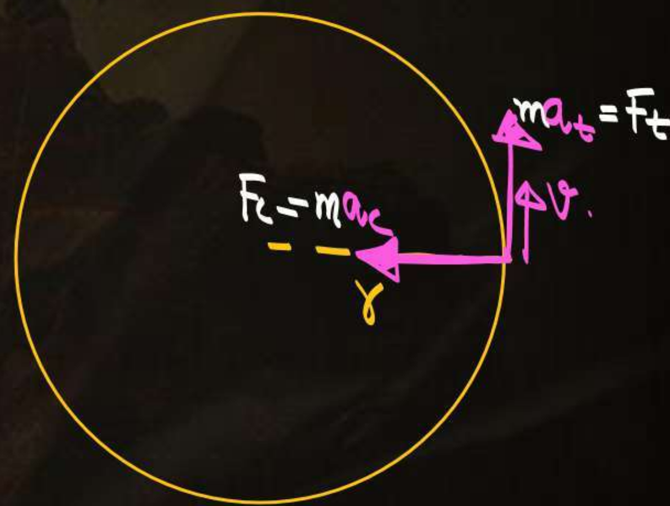
$v^2 = k^2r^2t^2$

$v = krt$

$\boxed{a_t = \frac{d}{dt}|v| = kr}$

F_c will not develop power.

$\theta = 90°$.



$\text{Power} = F_t \cdot v\cos 0$

$= ma_t \times krt$

$= m kr\, krt$

$= mk^2r^2t$.

QUESTION 12

(JEE)

$v = R\omega$

$A \rightarrow v_1 = R_1\omega$

$B \rightarrow v_2 = R_2\omega$

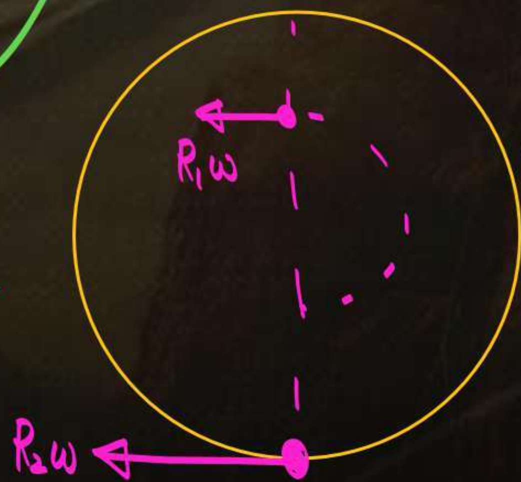
Two particles A and B are moving on two concentric circles of radii R_1 and R_2 with equal angular speed ω. At $t = 0$, their positions and direction of motion are shown in the figure. The relative velocity $\vec{v}_A - \vec{v}_B$ at $t = \dfrac{\pi}{2\omega}$ is given by

$\omega = \dfrac{2\pi}{T}$

$T = \dfrac{2\pi}{\omega}$

$\dfrac{T}{2} = \dfrac{\pi}{\omega}$

$t = \dfrac{1}{2}\left(\dfrac{T}{2}\right) = \dfrac{T}{4}$



(A) $\omega(R_1 + R_2)\hat{\imath}$

(B) $-\omega(R_1 + R_2)\hat{\imath}$

(C) $\omega(R_1 - R_2)\hat{\imath}$

(D) $\omega(R_2 - R_1)\hat{\imath}$

Ans

$\vec{V}_A - \vec{V}_B$

$\Rightarrow -R_1\omega\hat{\imath} - (-R_2\omega)\hat{\imath}$

$= (-R_1\omega + R_2\omega)\hat{\imath}$

$= (R_2 - R_1)\omega\hat{\imath}$

QUESTION 13

Centripetal.
Normal

For a particle moving along circular path, the $\boxed{\text{radial}}$ acceleration a_r is proportional to time t. If a_t is the tangential acceleration, then which of the following will be independent of time t?

$a_r \cdot a_t \propto t \cdot \frac{1}{\sqrt{t}}$

(A) a_t. ✗

(B) $a_r \cdot a_t$ ✗

(C) a_r / a_t

(D) $a_r (a_t)^2$ ✓

$a_c \propto t$

$a_c = Kt$ (NUCM)

$\frac{v^2}{R} = Kt$

$v = \sqrt{RKt}$

$a_t = \frac{d}{dt}|v| = \sqrt{Rk}\,\frac{1}{2\sqrt{t}}$

$a_r \cdot a_t^2 \propto \cancel{t} \cdot \frac{1}{\cancel{t}}$ Independent of t.

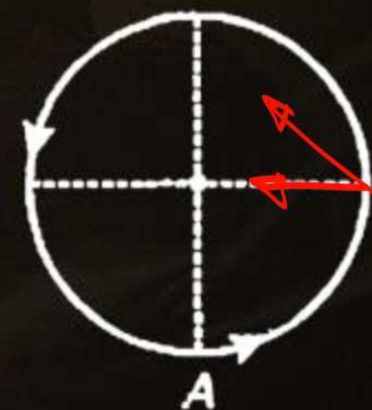
QUESTION 14

(Level up).

$|v| =$ Constant.
$a_t = 0$

UCM

A particle is describing uniform circular motion in the anti-clockwise sense such that its time period of revolution is T. At $t = 0$ the particle is observed to be at A. If θ_1 be the angle between acceleration at $t = \frac{T}{4}$ and average velocity in the time interval 0 to $\frac{T}{4}$ and θ_2 be the angle between acceleration at $t = \frac{T}{4}$ and the change in velocity in the time interval 0 to $\frac{T}{4}$, then



(A) $\theta_1 = 135°, \theta_2 = 45°$

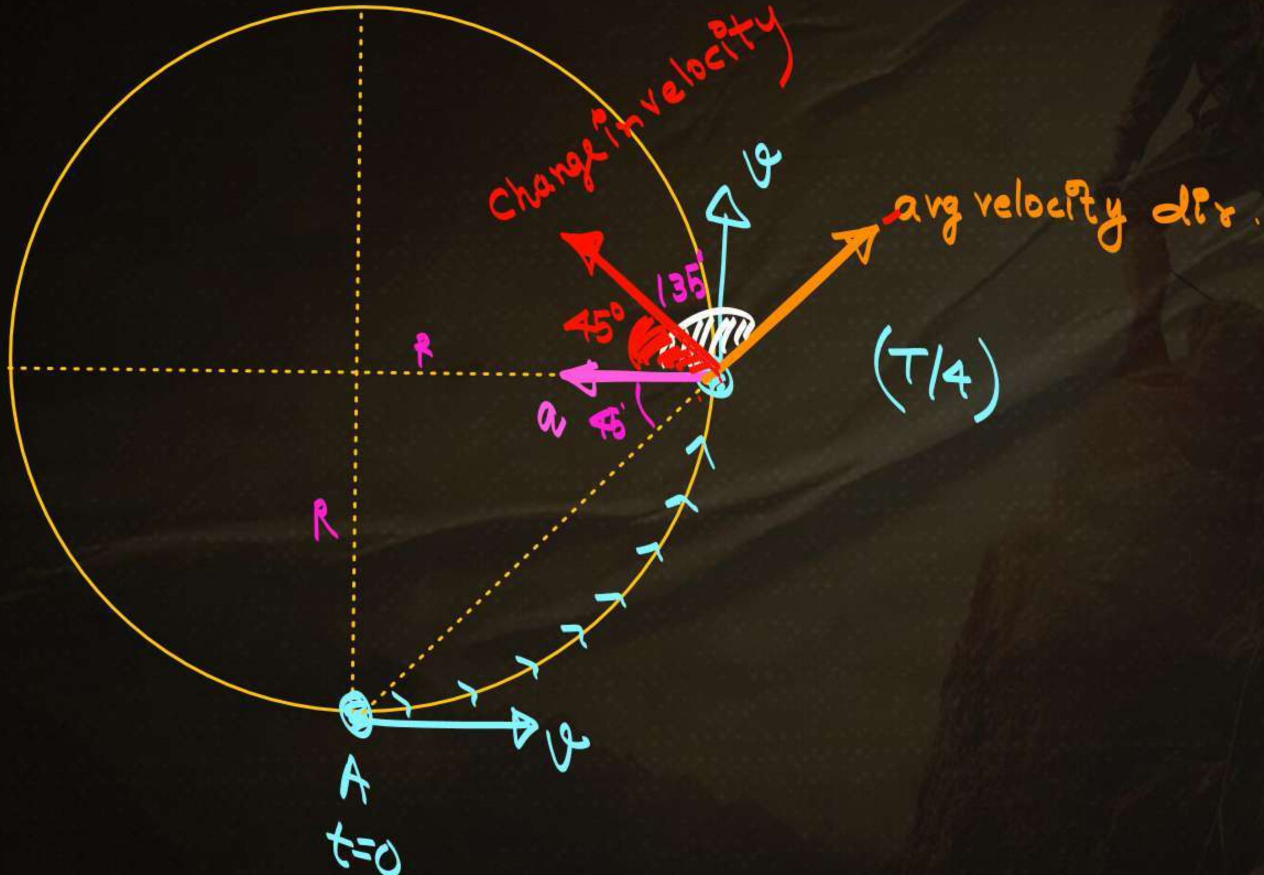
(B) $\theta_1 = 135°, \theta_2 = 135°$

(C) $\theta_1 = 45°, \theta_2 = 135°$

(D) $\theta_1 = 45°, \theta_2 = 45°$

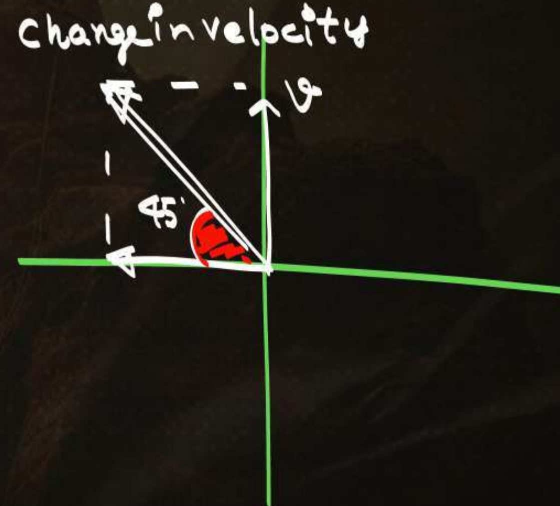
UCM $a_t = 0$

avg velocity $= \dfrac{\text{Total disp}}{\text{Total time}}$.



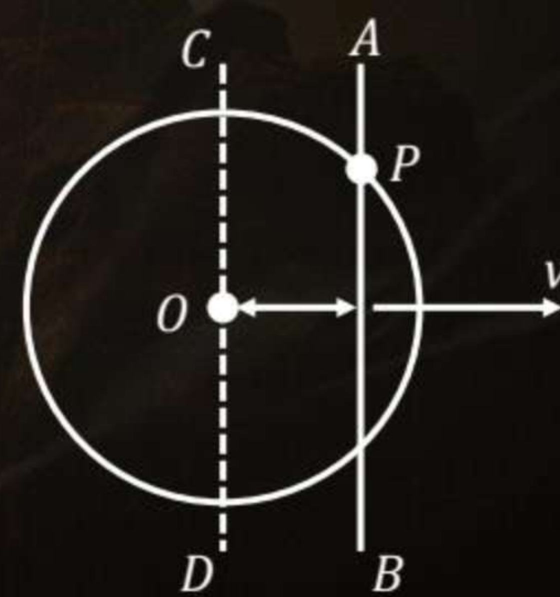
Change in velocity

$= \bar{v}_f - \bar{v}_i$



QUESTION 15

A rod AB is moving on a fixed circle of radius R with a constant velocity v as shown in figure. P is the point of intersection of rod and the circle. At an instant rod is at a distance $x = \frac{3R}{5}$ from centre of circle. The velocity of rod is normal to its length and rod always remain parallel to the diameter CD. Find the speed of point P at this instant.



Circular Dynamics

❖ **How to write force equation in Circular Motion.**

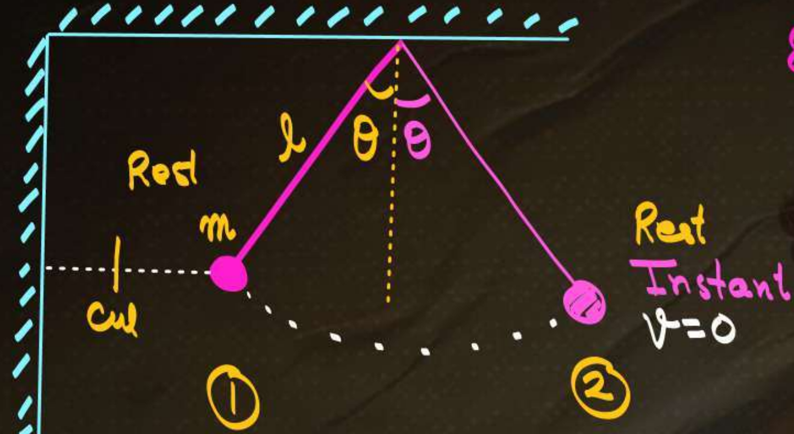
In CM

1. Draw FBD
2. Find Centre of Circle.
3. Resolve all forces along & $\perp$ to Radius.

along radius $F_{net\ towards\ Centre} = \frac{mv^2}{R} = mR\omega^2$

along Tangent $F_{net\ tangential} = ma_t = mR\alpha$.

Question.15.)



Pink thread:-

(Left Extreme) Tension in thread $= T_1$

(Right Extreme) Tension in thread $= T_2$

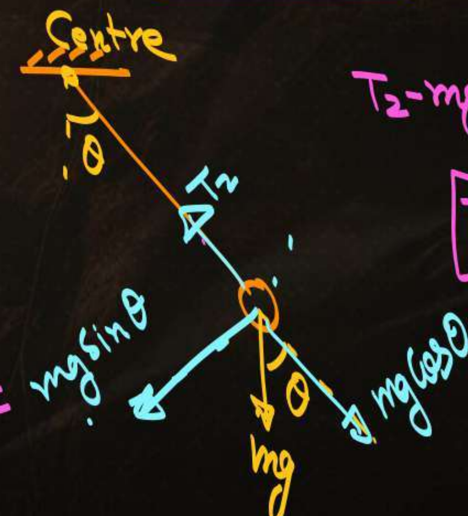
$$\frac{T_1}{T_2} = \frac{\cancel{mg}}{\cos\theta \times \cancel{mg}\cos\theta} = \frac{1}{\cos^2\theta} = \sec^2\theta$$

①

$T_1 \cos\theta = mg$

$$\boxed{T_1 = \frac{mg}{\cos\theta}}$$

Centre

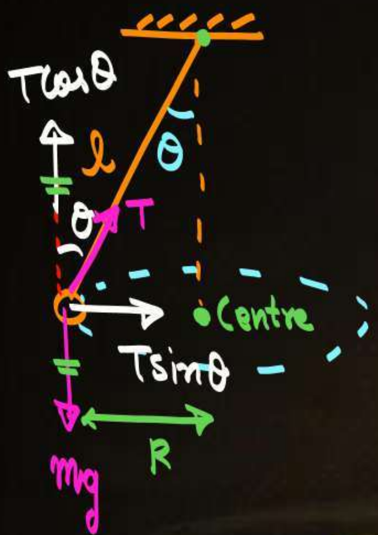


$f_t = mg\sin\theta$

$$T_2 - mg\cos\theta = \frac{m}{l}(0)^2$$

$$\boxed{T_2 = mg\cos\theta}$$

❖ Conical Pendulum



$$\boxed{R = l\sin\theta}$$

$$T\sin\theta = mR\omega^2$$
$$T\cos\theta = mg$$

$$\tan\theta = \frac{R\omega^2}{g}$$

$$\omega = \sqrt{\frac{g\tan\theta}{R}}$$

Time Period $= \omega = \frac{2\pi}{T}$

$$\frac{2\pi}{T} = \sqrt{\frac{g\tan\theta}{l\sin\theta}} = \sqrt{\frac{g\sin\theta}{l\sin\theta\cos\theta}}$$

$$\boxed{T = 2\pi\sqrt{\frac{l\cos\theta}{g}}}$$

❖ Bending of Cyclist

$$N\sin\theta = \frac{mv^2}{R}$$
$$N\cos\theta = mg$$

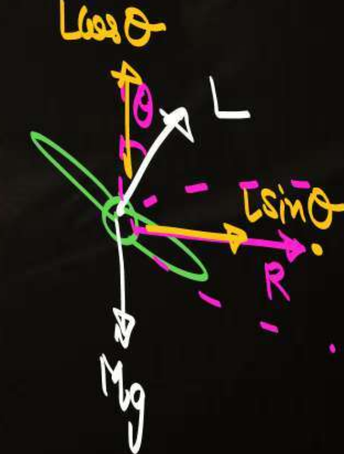
$$\tan\theta = \frac{v^2}{Rg}$$

$$\boxed{v = \sqrt{Rg\tan\theta}}$$

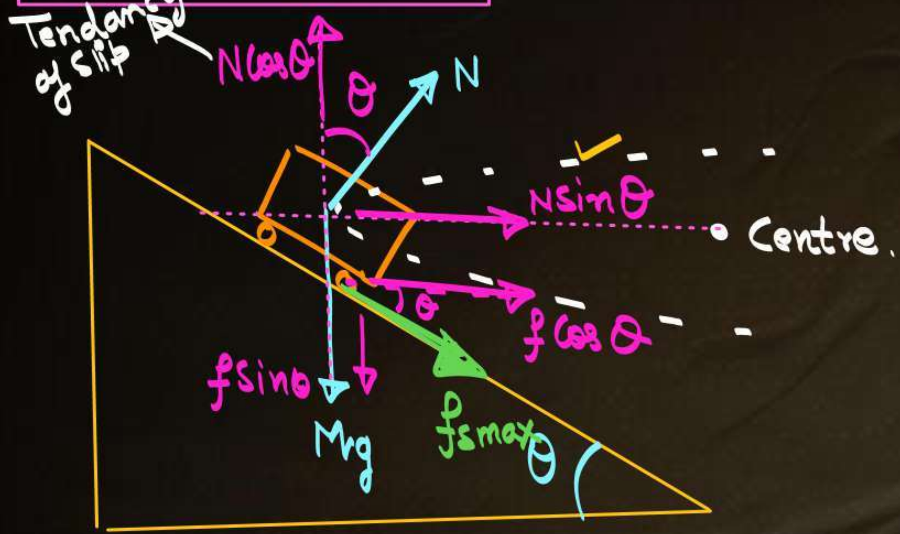
Banking of aeroplane / Train.

$$L\sin\theta = \frac{mv^2}{R}$$
$$L\cos\theta = mg$$

$$\boxed{\tan\theta = \frac{v^2}{Rg}}$$



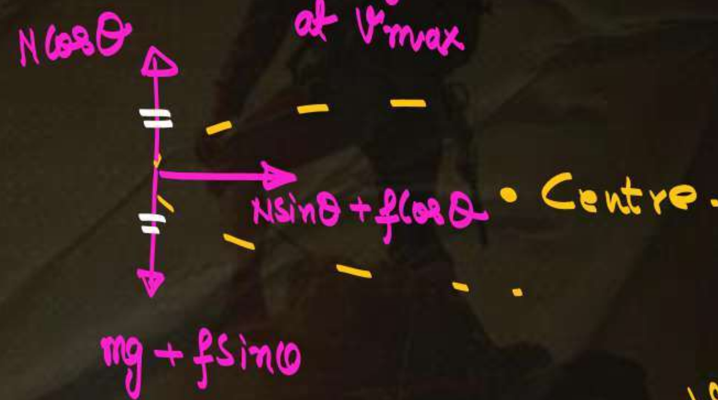
❖ Banking of Road



θ = Banking angle.

Finding $\boxed{v_{max}}$ for turn on banked Road.

Car Tendancy to slip = outward at v_{max}



$v \to$ Max

$f \to$ Max

$f = \mu_s N$

$$N\sin\theta + f\cos\theta = \frac{m v_{max}^2}{R}$$

$$N\cos\theta = mg + f\sin\theta$$

$$N(\sin\theta + \mu\cos\theta) = \frac{mu^2}{R}$$

$$N(\cos\theta - \mu\sin\theta) = mg$$

$$\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta} = \frac{v_{max}^2}{Rg}$$

$$\boxed{v_{max} \text{ for turn} = \sqrt{Rg\frac{(\sin\theta + \mu\cos\theta)}{(\cos\theta - \mu\sin\theta)}} = \sqrt{Rg\frac{(\tan\theta + \mu)}{(1 - \mu\tan\theta)}}}$$

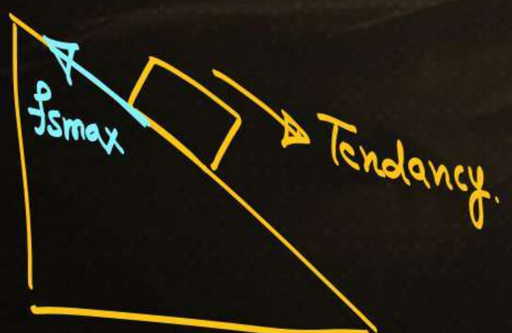
Rough Road but Levelled Road

$\theta = 0$

$$\boxed{v = \sqrt{\mu Rg}}$$

what should be v_{min} for banked Road.

$$\boxed{v_{min} = \sqrt{Rg\left(\frac{\tan\theta - \mu}{1 + \mu\tan\theta}\right)}}$$



$v_{min} \Rightarrow$

$\mu \rightarrow -\mu$

Replace

on banked Road $\mu = 0$ $\quad v = \sqrt{Rg\tan\theta}$

NIF $\Rightarrow$ observer accelerated.

❖ Centrifugal Force

$\Downarrow$

Pseudo force in CM.

Rest par maanega Earth ko — Observer (NIF)

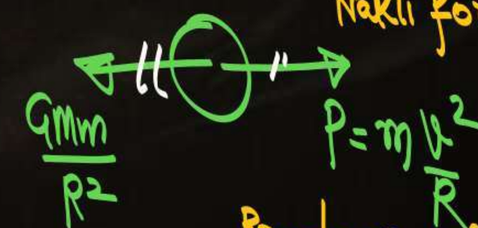
$$\boxed{\frac{GMm}{R^2} = \frac{mv^2}{R}}$$

$\frac{GMm}{R^2} = F_{centripetal}$ — Inertial frame.

$\frac{GMm}{R^2} = \frac{mv^2}{R}$.

Isliye CM ho Raha hai.

Nakli force



Pseudo Centrifugal.

QUESTION 16

Three identical particles are joined together by a thread as shown in figure. All the three particles are moving on a smooth horizontal plane about point O. If the speed of the outermost particle is v_0, then the ratio of tensions in the three sections of the string is: (Assume that the string remains straight)

(A) 3 : 5 : 7

(B) 3 : 4 : 5

(C) 7 : 11 : 6

(D) 3 : 5 : 6

$\omega = \frac{\text{Angle Rotated}}{\text{Time}}$

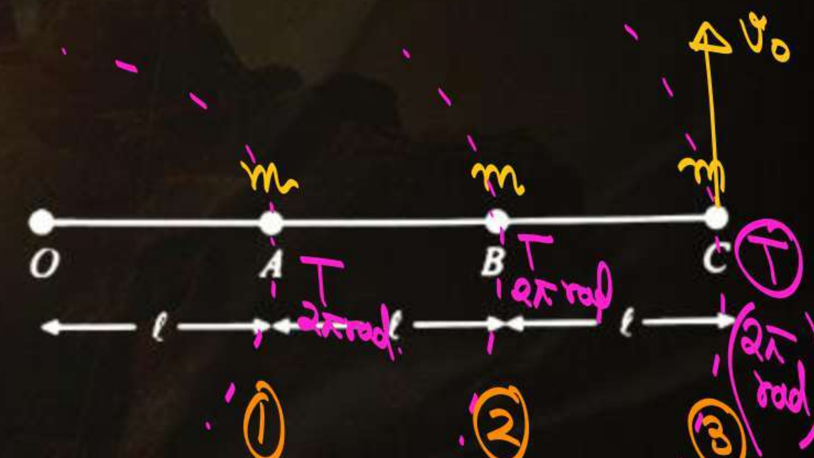
for all three particles

$\omega = \frac{2\pi}{T} \Rightarrow$ Same.

$v = r\omega$ $\quad v_3 = v_0 = 3l\omega$

$v_2 = 2l\omega$

$v_1 = l\omega$.



$$F_{net\ towards\ Centre} = mr\omega^2 = \frac{mv^2}{R}$$

$$T_1 - \cancel{T_2} = ml\omega^2$$

$$\cancel{T_2} - \cancel{T_3} = m(2l)\omega^2$$

$$\cancel{T_3} = m(3l)\omega^2$$

Add

$$T_1 = 6ml\omega^2$$

$$6ml\omega^2 - T_2 = ml\omega^2$$

$$\boxed{5ml\omega^2 = T_2}$$

$$T_1 = 6ml\omega^2$$

$$T_2 = 5ml\omega^2$$

$$T_3 = 3ml\omega^2$$

Ratio - 3:5:6.

QUESTION 17

Does particle has Same Speed at top in all Cases.

WET ⇒ $W_T = KE_f - KE_i$

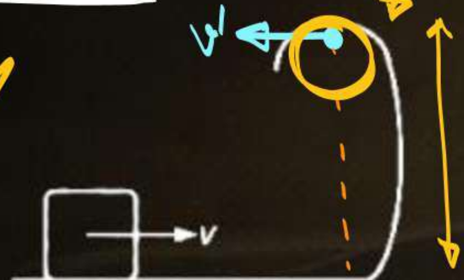
$-mgH = \frac{1}{2}mv'^2 - \frac{1}{2}mv^2$ v' = Same in all Cases.

A small block is shot into each of the four tracks as shown below. Each of the tracks rises to the same height. The speed with which the block enters the track is the same in all cases. At the highest point of the track, the normal reaction is maximum in

[2001] Adv.

Circular Motion

(A) Ans

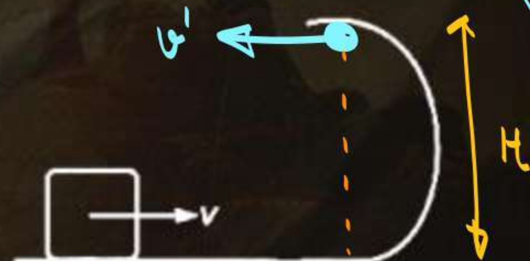


Least Radius.

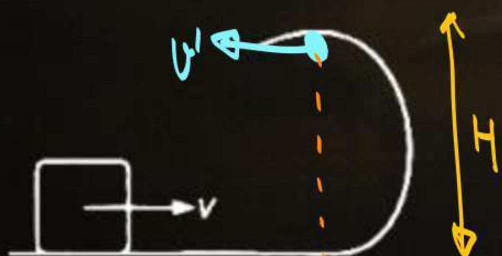
R↓

N↑.

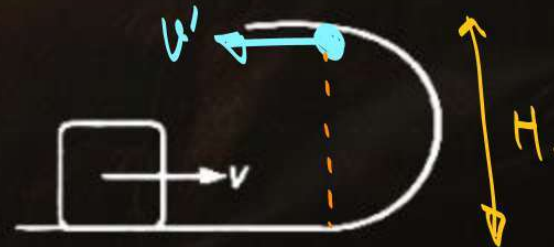
(B)

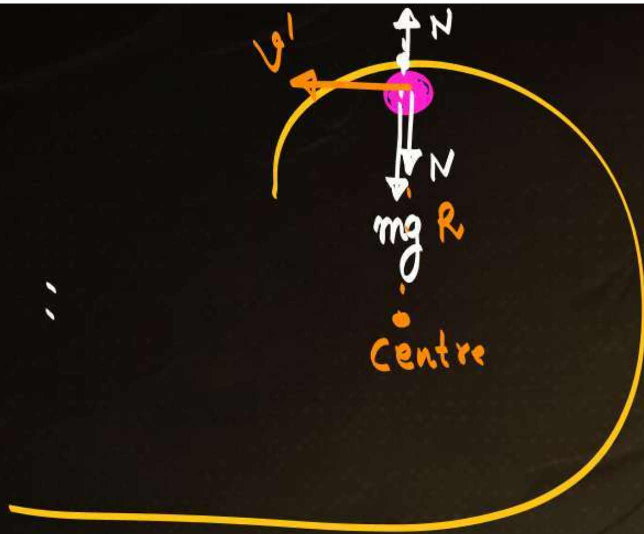


(C)



(D)





$$mg + N = \frac{mv'^2}{R}$$

$$\boxed{N = \frac{mv'^2}{R\downarrow} - mg}$$

Contact force → Normal

⊥ to surface (Push) away.

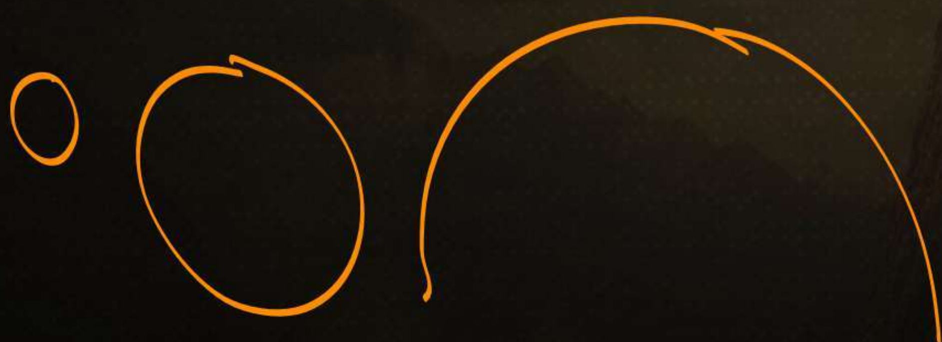
In all Cases $v' \rightarrow$ Same

Radius = ?

Radius ↓ N → Max.

More is the flatness of Surface R↑

Straight line → part of Circle whose $R \rightarrow \infty$



QUESTION 18 × 3 times.

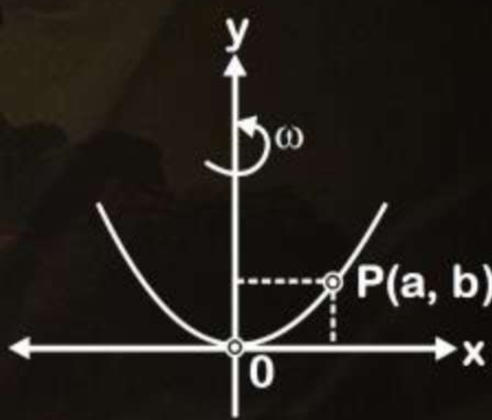
A bead of mass m stays at point $P(a, b)$ on a wire bent in the shape of a parabola $y = 4Cx^2$ and rotating with angular speed ω (see figure). The value of ω is (neglect friction) [JEE (Main)-2020]

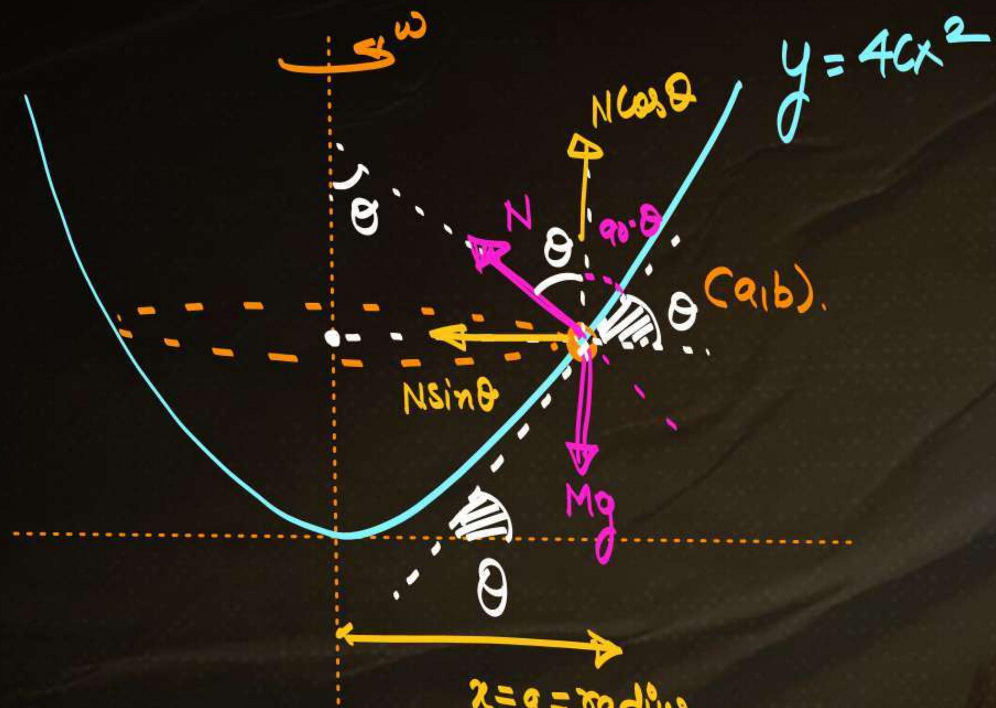
(A) $2\sqrt{2gC}$ Ans

(B) $2\sqrt{gC}$

(C) $\sqrt{\frac{2g}{C}}$

(D) $\sqrt{\frac{2gC}{ab}}$





$$y = 4cx^2$$

$$\tan\theta = \frac{dy}{dx} = 8cx$$

at $x = a$ $\quad \tan\theta = 8ca.$

$$N\sin\theta = mr\omega^2$$
$$N\cos\theta = mg$$

$$N\sin\theta = ma\omega^2$$
$$N\cos\theta = mg$$

$$\tan\theta = \frac{a\omega^2}{g}$$

$$\omega = \sqrt{\frac{g\tan\theta}{a}} = \sqrt{\frac{g\,8c\cancel{a}}{\cancel{a}}} = 2\sqrt{2gc}.$$

QUESTION 19

A modern grand-prix racing car of mass m is travelling on a flat track in a circular arc of radius R with a speed v. If the coefficient of static friction between the tyres and the track is μ_s, then the magnitude of $\boxed{\text{negative lift } F_L}$ acting downwards on the car is: (Assume forces on the four tyres are identical and g = acceleration due to gravity)

[JEE (Main)-2021]

(A) $m\left(\frac{v^2}{\mu_s R}+g\right)$

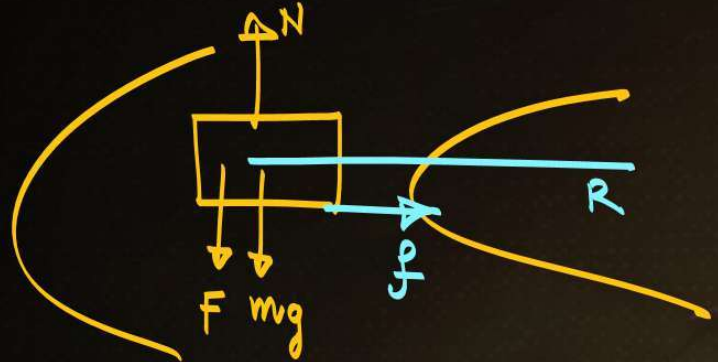
Ans

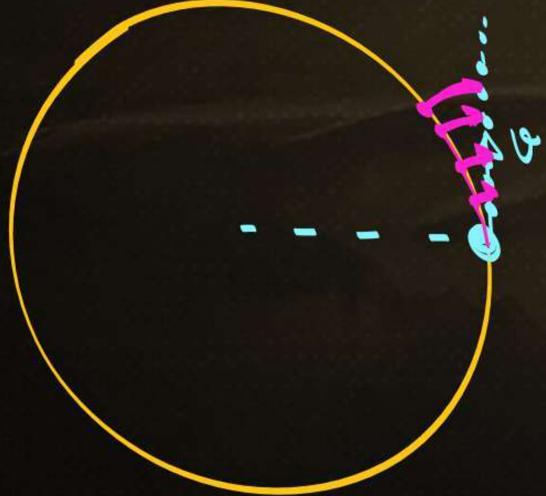
(B) $m\left(\frac{v^2}{\mu_s R}-g\right)$

(C) $-m\left(g+\frac{v^2}{\mu_s R}\right)$

(D) $m\left(g-\frac{v^2}{\mu_s R}\right)$

Tendency





$Y \rightarrow$ Equilibrium

$$N = Mg + F$$

$$f = \frac{mv^2}{R}$$

$$\mu N = \frac{mv^2}{R}$$

$$\mu(Mg + F) = \frac{mv^2}{R}$$

$$F = \frac{mv^2}{\mu R} - Mg$$

QUESTION 20

HCV

In figure shows a rod of length 20 cm pivoted near an end and which is made to rotate in a horizontal plane with a constant angular speed. A ball of mass m is suspended by a string also of length 20 cm from the other end of the rod. If the angle θ made by the string with the vertical is 30°, find the angular speed of the rotation. Take $g = 10\ m/s^2$.

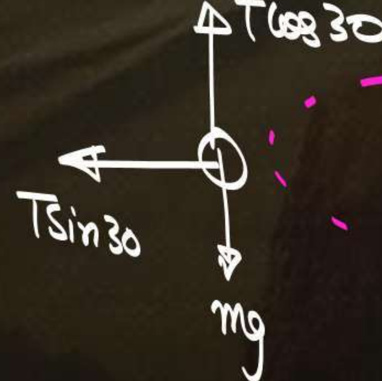
$T\cos 30 = mg$

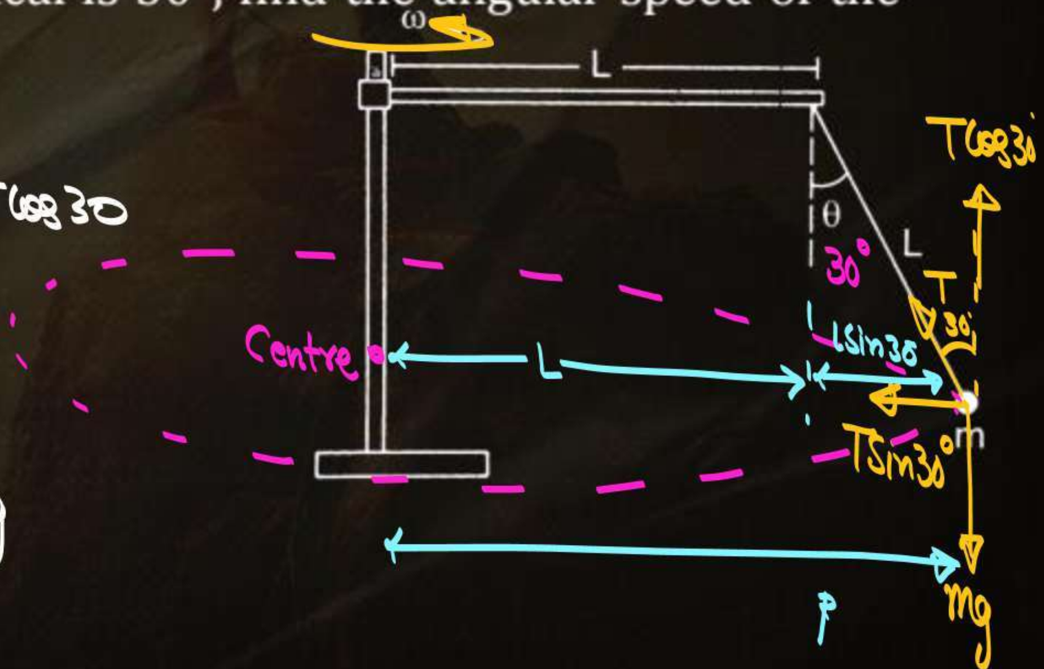
$T\sin 30 = mR\omega^2$

$T\sin 30 = m(l + l\sin 30)\omega^2$

$T\cos 30 = mg$

Ratio $\frac{1}{\sqrt{3}} = \frac{(3l/2)\omega^2}{g}$ $\omega =$ ___





QUESTION 21

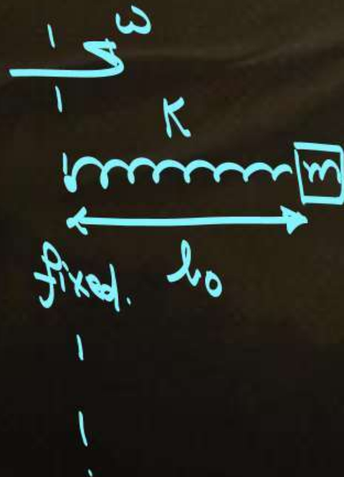
A particle of mass m is fixed to one end of light spring having force constant k and unstretched length l_0. The other end is fixed. The system is given an angular speed ω about the fixed end of the spring such that it rotates in a circle in gravity free space. Then the stretch in the spring is: **JEE Main 2020 - 8 Jan (Morning)**

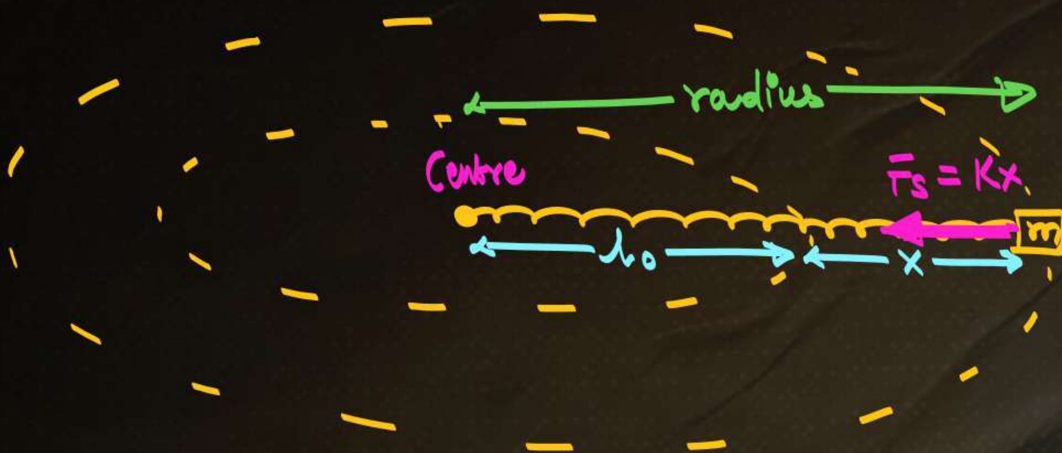
(A) $\dfrac{l_0 m\omega^2}{k - m\omega^2}$ Ans

(B) $\dfrac{l_0 m\omega^2}{k + m\omega^2}$

(C) $\dfrac{l_0 m\omega^2}{k - m\omega}$

(D) $\dfrac{l_0 m\omega^2}{k + m\omega}$





$F_s = -Kx$

$\downarrow$

Restoring Nature.

Spring Elongate $F \rightarrow$ oppose

Compress $F \rightarrow$ oppose.

$$F_s = m(l+x)\omega^2$$

$$Kx = ml\omega^2 + mx\omega^2$$

$$x(K - m\omega^2) = ml\omega^2$$

$$\boxed{x = \frac{ml\omega^2}{K - m\omega^2}}$$

QUESTION 22

(HCV)

A car goes on a horizontal circular road of radius R, the speed increasing at a a_t - constant rate $\frac{dv}{dt}$. The friction coefficient between the road and the tyre is μ. Find the speed at which the car will skid.

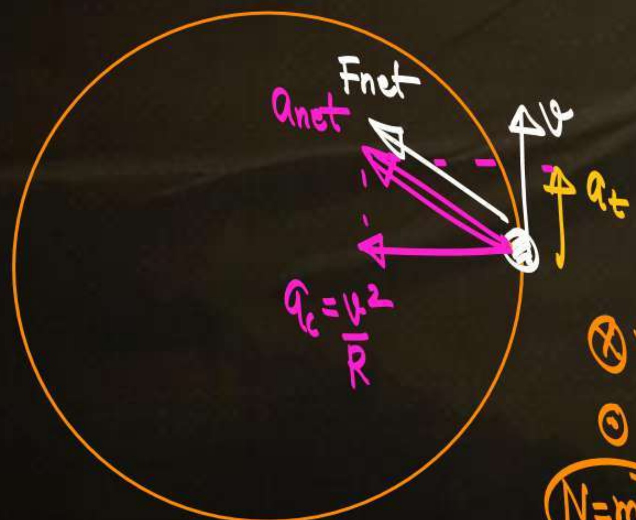
When car just slips

Top View

NUCM.

$f_{s\,max} = \mu N$.

$a_t = \frac{dv}{dt}$



$\boxed{f_{s\,max} = \mu mg}$

⊗ mg

⊙ N.

$N = mg$

$$a_{net} = \sqrt{a_t^2 + a_c^2} = \sqrt{\left(\frac{dv}{dt}\right)^2 + \left(\frac{v^2}{R}\right)^2}$$

$$a_{net} = \sqrt{\left(\frac{dv}{dt}\right)^2 + \frac{v^4}{R^2}}$$

$$F_{net} = ma_{net}$$

$$= m\sqrt{\left(\frac{dv}{dt}\right)^2 + \frac{v^4}{R^2}}$$

$$\text{frictional force} = F_{net} = \mu \cancel{m} g = \cancel{m}\sqrt{\left(\frac{dv}{dt}\right)^2 + \frac{v^4}{R^2}}$$

$$(\mu \cancel{m} g)^2 = \left(\frac{dv}{dt}\right)^2 + \frac{v^4}{R^2}$$

$$\boxed{v = \left[R^2\left[(\mu g)^2 - \left(\frac{dv}{dt}\right)^2\right]\right]^{1/4}}$$

QUESTION 23

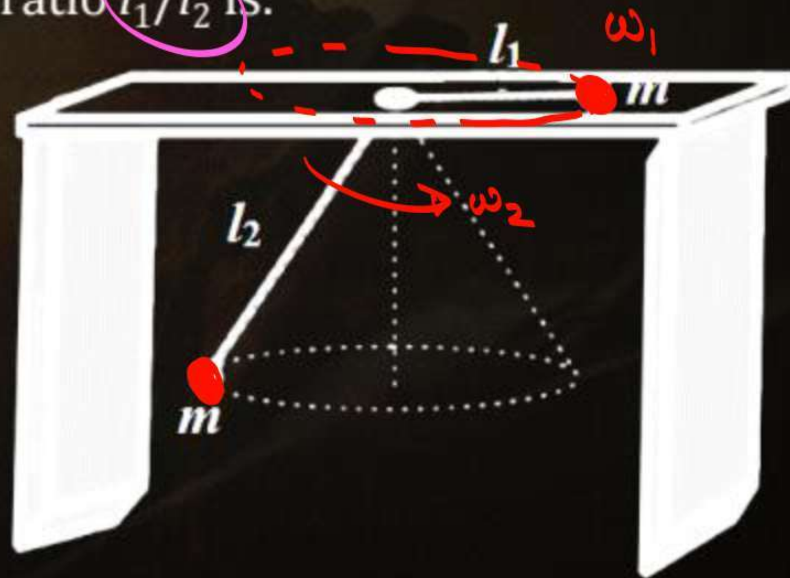
Two identical particles are attached at the ends of a light string which passes through a hole at the centre of a table. One of the particles is made to move in a circle on the table with angular velocity ω_1 and the other is made to move in a horizontal circle as a ~~contact~~ Concical pendulum with angular velocity ω_2. If l_1 and l_2 are the length of the string over and under the table, then in order that particle under the table neither moves down nor moves up the ratio l_1/l_2 is:

(A) $\frac{\omega_1}{\omega_2}$

(B) $\frac{\omega_2}{\omega_1}$

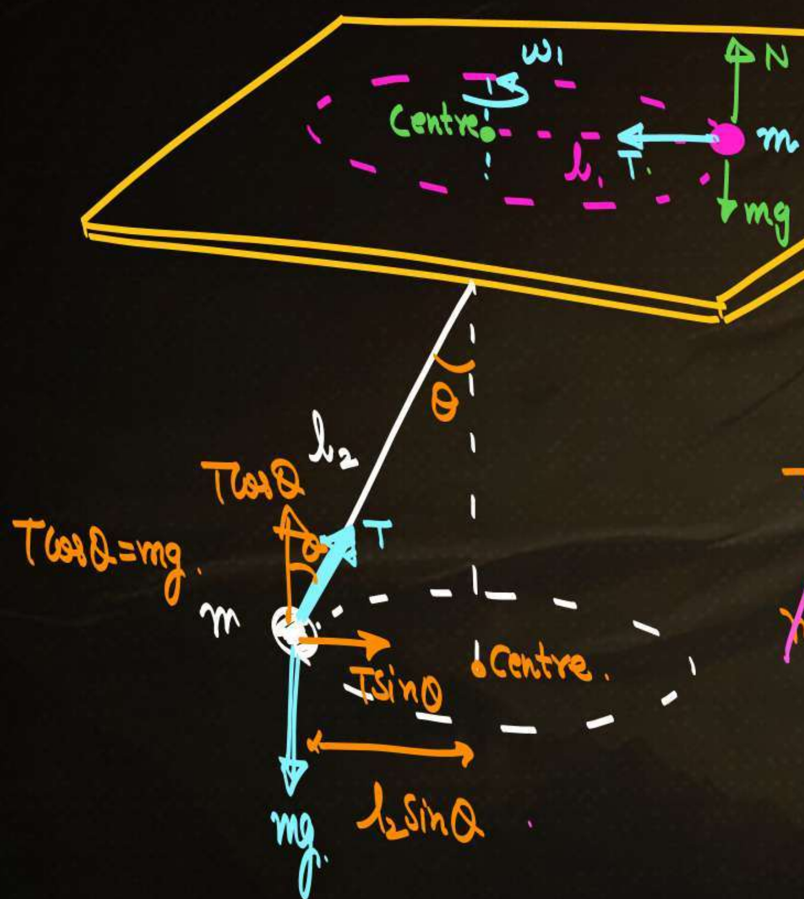
(C) $\frac{\omega_1^2}{\omega_2^2}$

Ans (D) $\frac{\omega_2^2}{\omega_1^2}$



$N = mg$

$$\boxed{T = m l_1 \omega_1^2}$$



$$T\sin\theta = m r \omega_2^2$$

$$\cancel{m} l_1 \omega_1^2 \cancel{\sin\theta} = \cancel{m}\, l_2 \cancel{\sin\theta}\, \omega_2^2$$

$$\frac{l_2}{l_2} = \frac{\omega_2^2}{\omega_1^2}$$

HW

QUESTION 24

$f = 0$

A smooth wire of length $2\pi r$ is bent into a circle and kept in a vertical plane. A bead can slide smoothly on the wire. When the circle is rotating with angular speed ω about the vertical diameter AB, as shown in figure, the bead is at rest with respect to the circular ring at position P as shown. Then the value of ω^2 is equal to

[JEE (Main)-2019]

(A) $\frac{(g\sqrt{3})}{r}$

(B) $\frac{2g}{r}$

(C) $\frac{2g}{(r\sqrt{3})}$

(D) $\frac{\sqrt{3g}}{2r}$

QUESTION 25

A uniform ring, having radius a and mass m is to be rotated in the horizontal plane about its own axis with constant angular velocity ω. What would be the tension in the ring and nature of force?

(A) $\frac{MR\omega^2}{2\pi}$ tensile

(B) $MR\omega^2$ tensile

(C) $\frac{MR\omega^2}{2}$ compressive

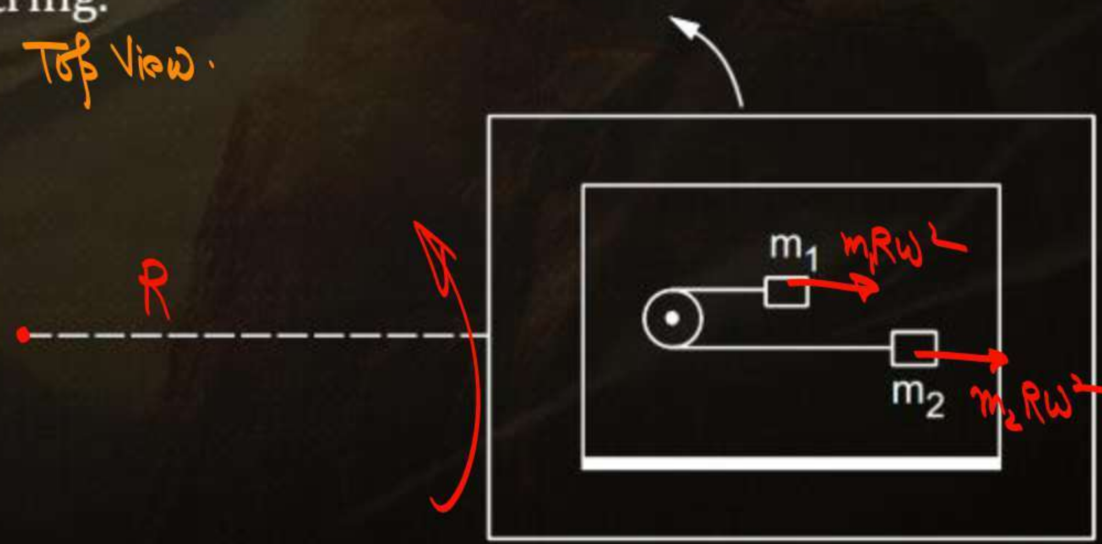
(D) $MR\omega^2$ compressive

QUESTION 26

HCV

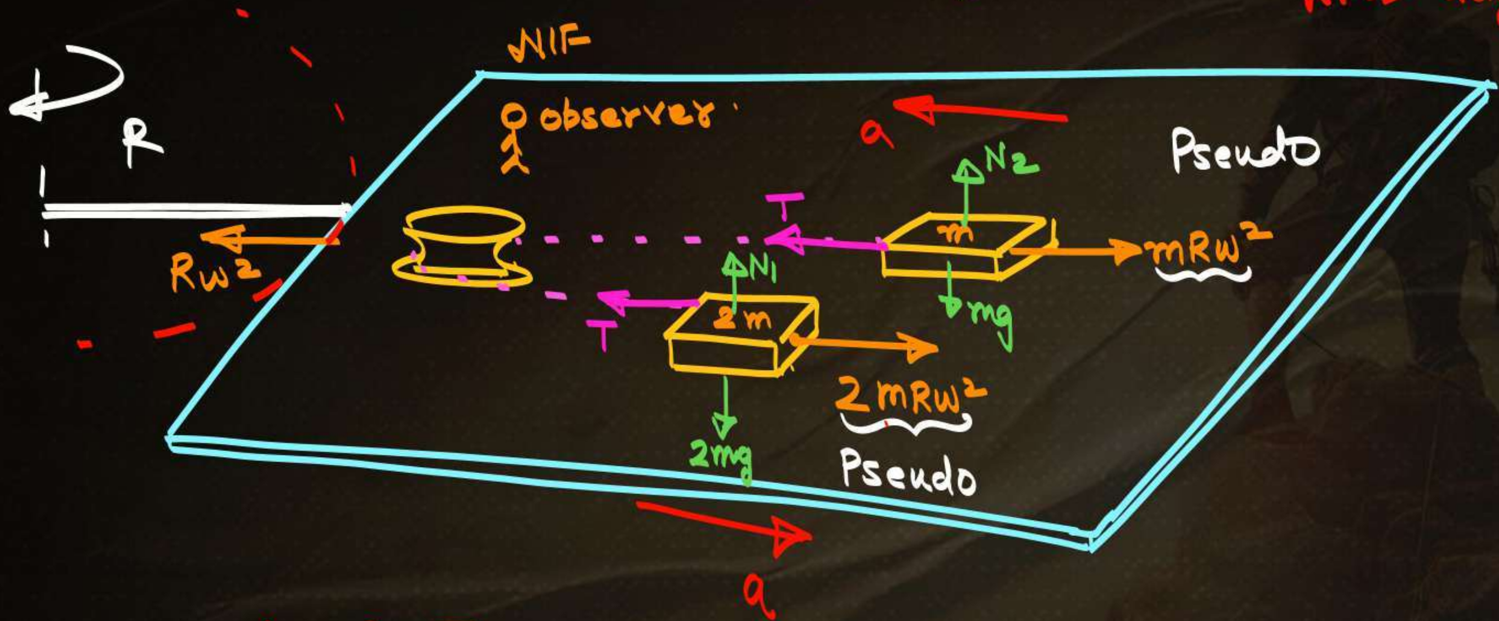
A table with smooth horizontal surface is placed in a cabin which moves in a circle of a large radius R. A smooth pulley of small radius is fastened to the table. Two masses m and $2m$ placed on the table are connected through a string going over the pulley. Initially the masses are held by a person with the strings along the outward radius and then the system is released from rest (with respect to the cabin). Find the magnitude of the initial acceleration of the masses as seen from the cabin and the tension in the string.

Top View.



$N_1 = 2mg$ $N_2 = mg$

$R >>>$ length of string.



$$2mR\omega^2 - T = 2ma$$

$$T - mR\omega^2 = ma$$

$$\cancel{m}R\omega^2 = 3\cancel{m}a$$

$$\boxed{a = \frac{R\omega^2}{3}}$$

QUESTION 27

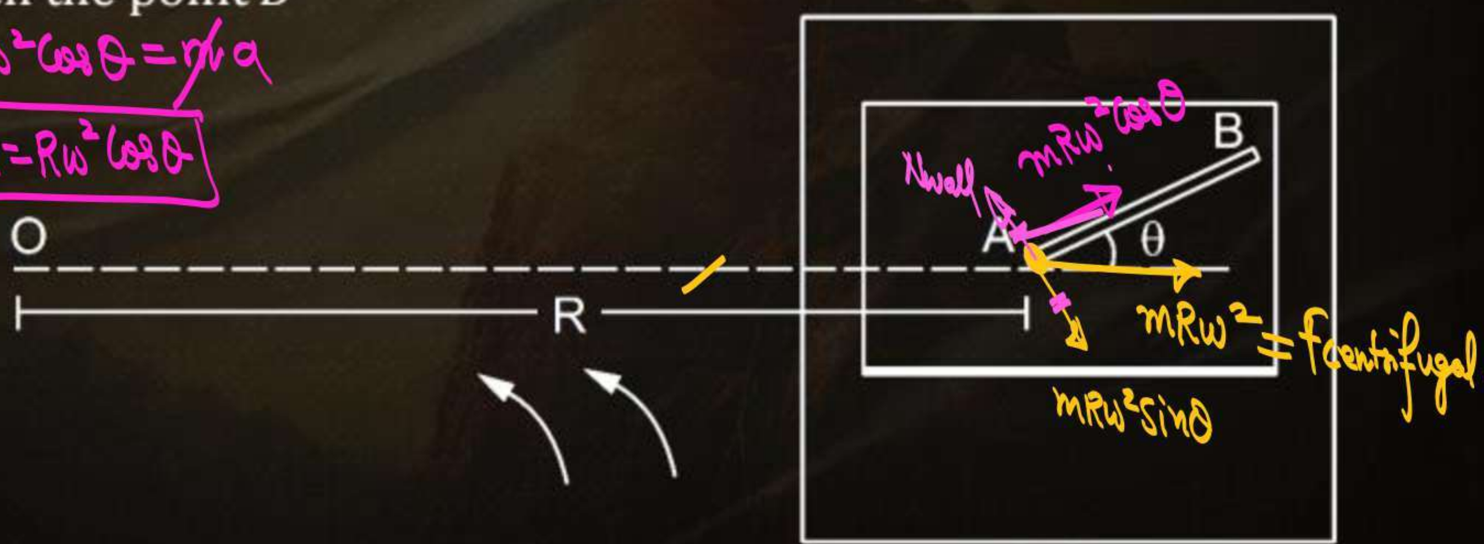
HCV

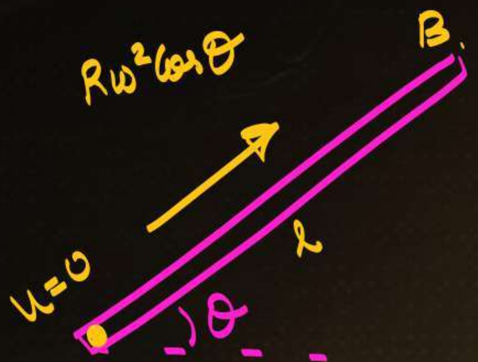
gadha.

A table with smooth horizontal surface is fixed in a cabin that rotates with a uniform angular velocity ω in a circular path of radius R. A smooth groove AB of length $L(<< R)$ is made on the surface of the table. The groove makes an angle θ with the radius OA of the circle in which the cabin rotates. A small particle is kept at the point A in the groove and is released to move along AB. Find the time taken by the particle to reach the point B

Force along groove $= \cancel{m}R\omega^2\cos\theta = \cancel{m}a$

$\boxed{a = R\omega^2\cos\theta}$





$a = R\omega^2\cos\theta$

Constant

$$s = \cancel{ut}^{\,0} + \frac{1}{2}at^2$$

$$l = \frac{1}{2}(R\omega^2\cos\theta)t^2$$

$$\sqrt{\frac{2l}{R\omega^2\cos\theta}} = t.$$

QUESTION 28

A ball of mass (m) 0.5 kg is attached to the end of a string having length (L)0.5 m. The ball is rotated on a horizontal circular path about vertical axis. The maximum tension that the string can bear is 324 N. The maximum possible value of angular "ω" velocity of ball (in radian/s) **[JEE 2011]**

$T_{max} = 324\,N.$

$T\sin\theta = mr\omega^2$

$T\cancel{\sin\theta} = mL\cancel{\sin\theta}\,\omega^2$

$T_{max} = mL\omega_{max}^2$

$\omega = \sqrt{\dfrac{324}{0.5\times0.5}} = \dfrac{18}{0.5} = 36$

QUESTION 29

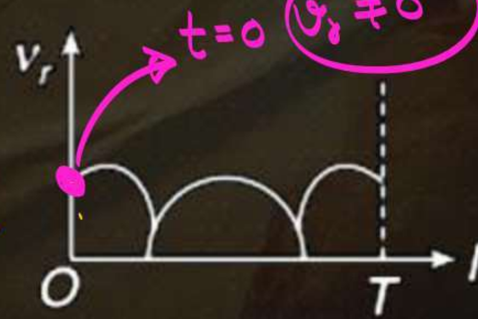
Two identical discs of same radius R are rotating about their axes in opposite directions with the same constant angular speed ω. The disc are in the same horizontal plane. At time $t = 0$, the points P and Q are facing each other as shown in the figure. The relative speed between the two points P and Q is v_r. as function of times best represented by

[IIT-JEE-2012]

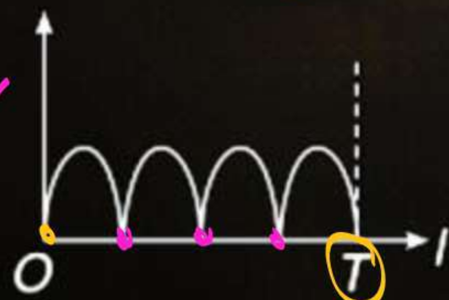
Ans

(A)

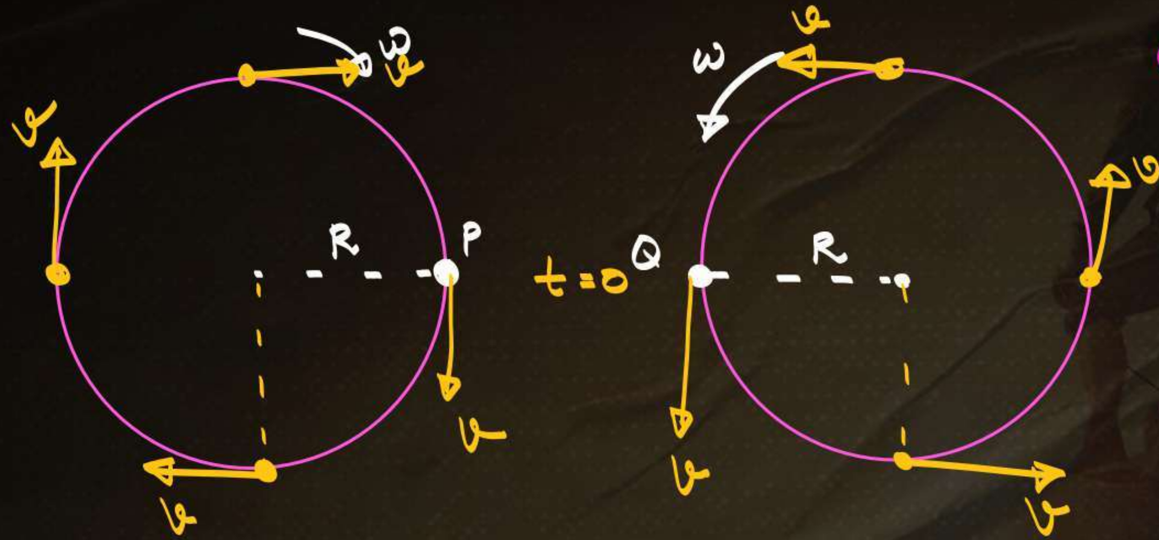
(B)



(C)



(D)



t	$\lvert v_r \rvert$
$t=0$	0
$t=T/4$	$2v$
$t=T/2$	0
$t=3T/4$	$2v$
$t=T$	0.

$t=0$

$v_r=0$

$$v_{rel} = \bar{v}_P - \bar{v}_Q$$

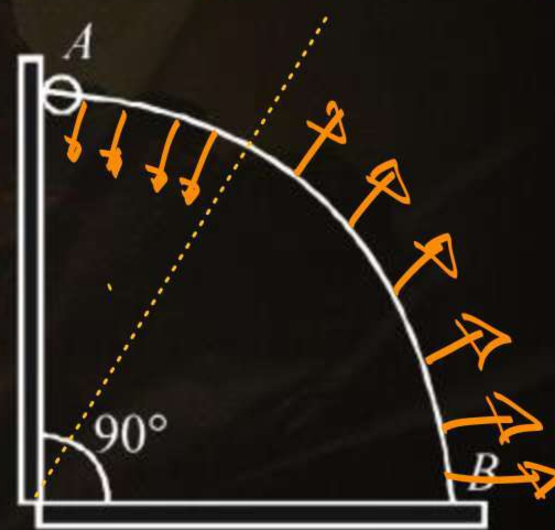
$$= -v-(+v)$$

$$= -2v.$$

Relative speed $= 2v$.

QUESTION 30

A wire, which passes through the hole is a small bead, is bent in the form of quarter of a circle. The wire is fixed vertically on ground as shown in the figure. The bead is released from near the top of the wire and it slides along the wire without friction. As the bead moves from A to B, the force it applies on the wire is **[JEE Adv 2014]**

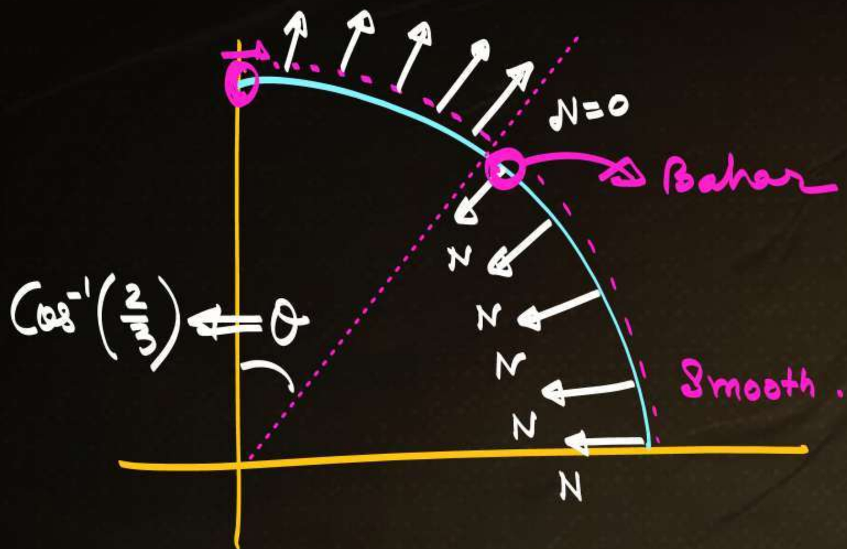


(A) always radially outwards

(B) always radially inwards

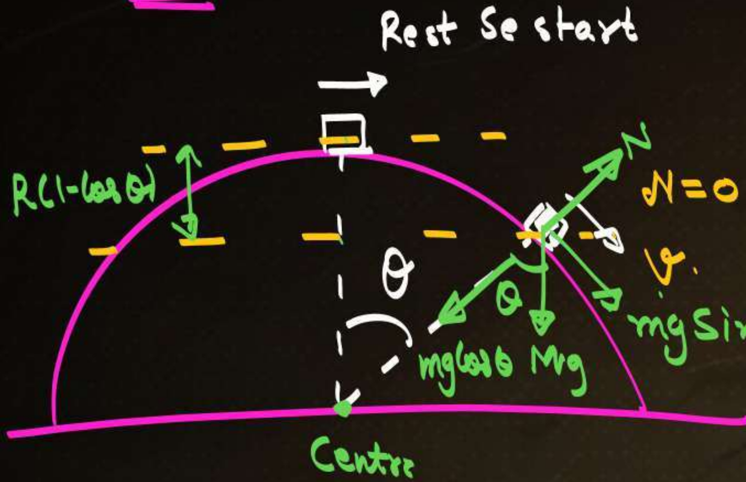
(C) radially outwards initially and radially inwards later

(D) radially inwards initially and radially outwards later.



$N \rightarrow$ Force on bead by wire.

VCM



$mg\sin\theta = F_t = ma_t$

Q) at what θ block leaves Contact.

$$mgR(1-\cos\theta) = \frac{1}{2}mv^2 - 0.$$

$v =$ ______

$$mg\cos\theta - N = \frac{mv^2}{R}$$

$N = 0$

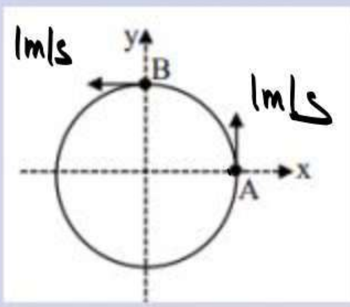
$$\boxed{\theta = \cos^{-1}\left(\frac{2}{3}\right)}$$

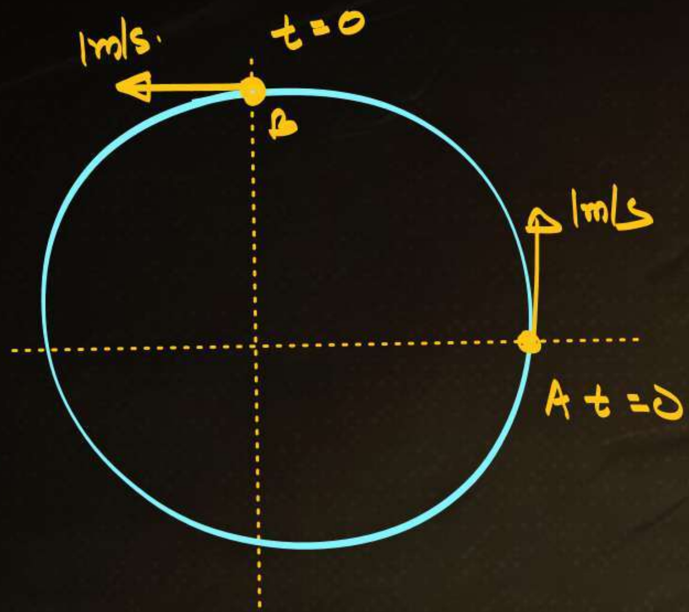
QUESTION 31

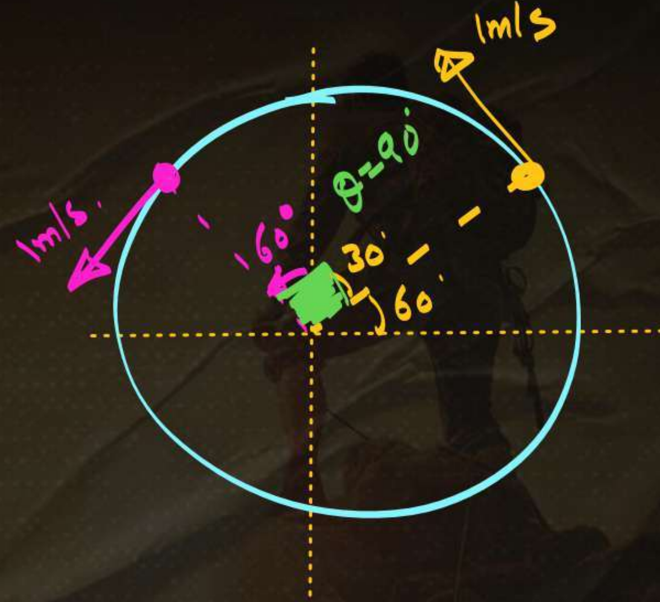
2022. (Advanced PYQ)

List-I describes four systems, each with two particles A and B in relative motion as shown in figure. List-II gives possible magnitudes of then relative velocities (in ms^{-1}) at time $t = \frac{\pi}{3}$ s.

$\bar{V}_A - \bar{V}_B$

	List-I		List-II
I.	A and B are moving on a horizontal circle of radius 1 m with uniform angular speed $\omega = 1 rads^{-1}$. The initial angular positions of A and B at time $t = 0$ are $\theta = 0$ and $\theta = \frac{\pi}{2}$ respectively.  $v = r\omega = 1 \times 1 = 1 m/s$.	P.	$\frac{\sqrt{3}+1}{2}$





$\omega = 1$

$t = \pi/3$

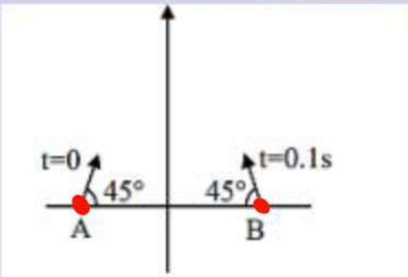
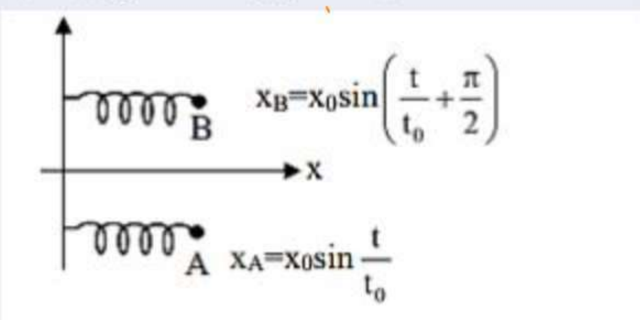
angle Rotated. $\theta = \omega t + \cancel{\frac{1}{2}\alpha t^2}$ $\alpha = 0$

$= 1 \times \frac{\pi}{3}$

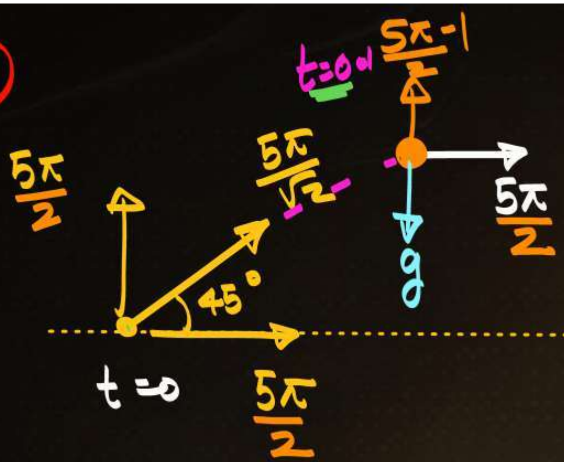
$= 60°$

$v_{rel} = \sqrt{1^2 + 1^2 - 2\cdot 1\cdot 1 \cos 90}$

$= \sqrt{2}$

	List-I		List-II
II.	Projectiles A and B are fired (in the same vertical plane) at $t = 0$ and $t = 0.1$ s respectively, with the same speed $v = \frac{5\pi}{\sqrt{2}}\,\text{ms}^{-1}$ and at $45°$ from the horizontal plane. The initial separation between A and B is large enough so that they do not collide, $(g = 10\ \text{ms}^{-2})$.  v_{rel} at $t = \frac{\pi}{3}$.	Q.	$\frac{\left(\sqrt{3}-1\right)}{\sqrt{2}}$
III.	Two harmonic oscillators A and B moving in the x direction according to $x_A = x_0 \sin\frac{t}{t_0}$ and $x_B = x_0 \sin\left(\frac{t}{t_0} + \frac{\pi}{2}\right)$ respectively, starting from $t = 0$. Take $x_0 = 1$ m, $t_0 = 1$ s.  	R.	$\sqrt{10}$

(11)



t = 0.1 sec.

$$\vec{V}_A = \frac{5\pi}{2}\hat{i} + \left(\frac{5\pi}{2} - 1\right)\hat{j}$$

$$\vec{V}_B = -\frac{5\pi}{2}\hat{i} + \frac{5\pi}{2}\hat{j}.$$

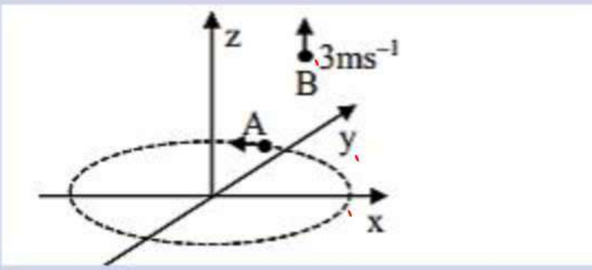
$$\vec{V}_A - \vec{V}_B = 5\pi\hat{i} - \hat{j}.$$

$$|V_A - V_B| = \sqrt{25\pi^2 + 1}.$$

$$V_y = u_y + a_y t$$
$$= \frac{5\pi}{2} - 10 \times 0.1$$
$$= \frac{5\pi}{2} - 1$$

$a_{rel} = 0$

Vel_{rel} appas mein Same after any time. $t = \pi/3$ nikalo ya $t = 0.1$ sec par nikalo baat ek hi hai.

List-I		List-II	
IV.	Particle A is rotating in a horizontal circular path of radius 1 m on the xy plane, with constant angular speed $\omega = 1\ \text{rads}^{-1}$. Particle B is moving up at a constant speed $3\ \text{ms}^{-1}$ in the vertical direction as shown in the figure. (Ignore gravity.) 	S.	$\sqrt{2}$
		T.	$\sqrt{25\pi^2 + 1}$

$v = r\omega$

$v = 1$

$v_B = \hat{z}$

$\perp$

$v_A = \hat{i}$ or $\hat{j}$ or $\hat{i} + \hat{j}$

$v_{rel} = \sqrt{3^2 + 1^2} = \sqrt{10}$

Which one of the following options is correct?

(A) I → R, II → T, III → P, IV → S

(B) I → S, II → P, III → Q, IV → R

(C) I → S, II → T, III → P, IV → R

(D) I → T, II → P, III → R, IV → S

I — S

Ans

QUESTION 32

(HW)

A long horizontal rod has a bead which can slide along its length and is initially placed at a distance L from one end A of the rod. The rod is set in angular motion about A with a constant angular acceleration α. If the coefficient of friction between the rod and bead is μ, and gravity is neglected, then the time after which the bead starts slipping is

(2000, 2M)

(A) $\sqrt{\dfrac{\mu}{\alpha}}$

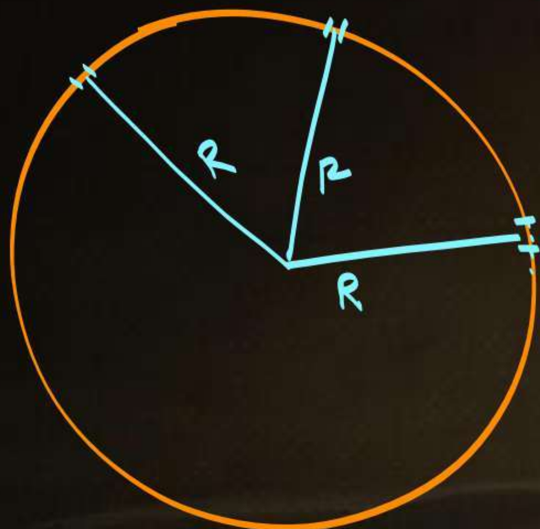
(B) $\dfrac{\mu}{\sqrt{\alpha}}$

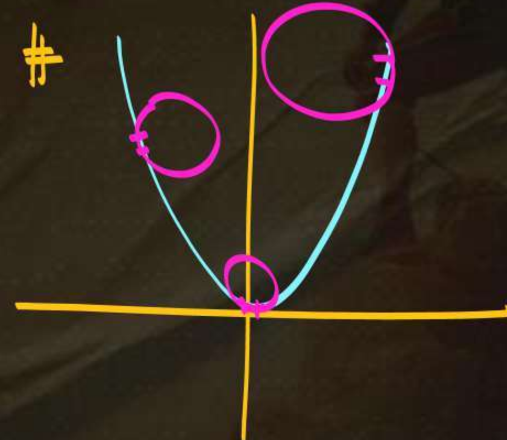
(C) $\dfrac{1}{\sqrt{\mu\alpha}}$

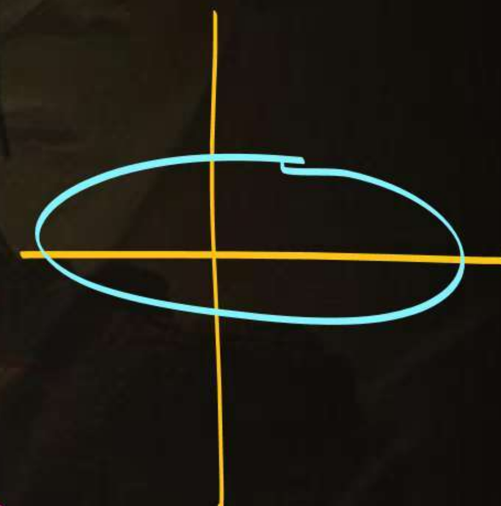
(D) infinitesimal

❖ Radius of curvature

Circle is a Geometrical Shape which has Same Radius at all points.







for other Curves $y = f(x)$

Radius will be different at different Points.

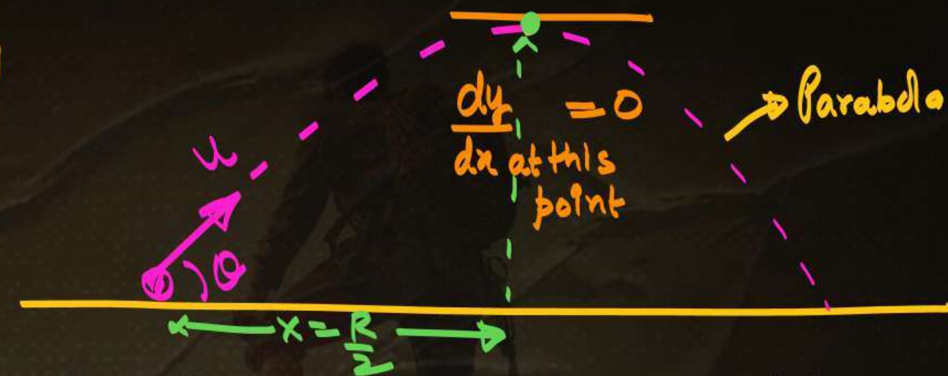
Koi bhi mathematical Eqtn dedu.

$y = f(x)$

$$\boxed{R = \frac{\left[1 + \left(\underbrace{\frac{dy}{dx}}_{\text{slope}}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}}$$

$$R_{\text{at top}} = \frac{\left[1 + (0)^2\right]^{3/2}}{\left|\frac{-g}{u^2\cos^2\theta}\right|} = \frac{u^2\cos^2\theta}{g}.$$

#



Find Radius of Curvature at top point.

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

Slope $\Rightarrow$ $$\frac{dy}{dx} = \tan\theta - \frac{gx}{u^2\cos^2\theta}$$ Put $x = \frac{R}{2}$.

$$\frac{d^2y}{dx^2} = 0 - \frac{g}{u^2\cos^2\theta}.$$

#) Straight line $R \to \infty$

$$y = mx + c$$

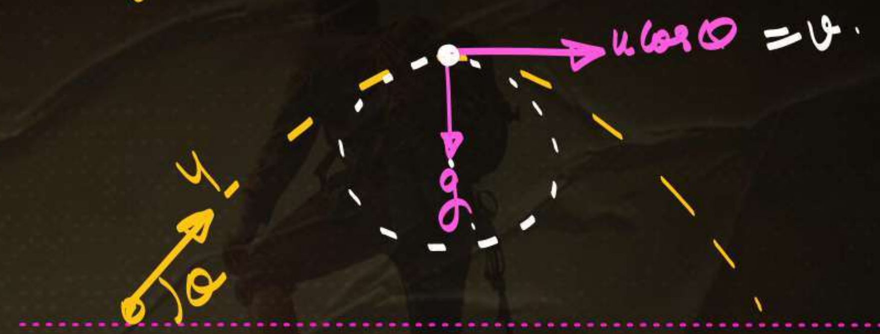
$$\frac{dy}{dx} = m$$

$$\frac{d^2y}{dx^2} = 0$$

$$R = \frac{\left[1 + m^2\right]^{3/2}}{0}$$

$$R \to \infty$$

Method-2 Physics.



$$g \Rightarrow a_{centripetal} = \frac{v^2}{R}$$

$$g = \frac{u^2\cos^2\theta}{R}$$

$$\boxed{R = \frac{u^2\cos^2\theta}{g}}$$

#

Find R of Curvature when particle makes angle $\theta/2$ with horizontal.

$$\frac{dy}{dx} = \tan(\theta/2)$$

$$\left|\frac{d^2y}{dx^2}\right| = \frac{g}{u^2\cos^2\theta}$$

$$R = \frac{\left[1+\tan^2\theta/2\right]^{3/2}}{\frac{g}{u^2\cos^2\theta}} =$$

QUESTION 33 (Irodov)

A particle moves at uniform speed on a parabiotic trajectory $y = ax^2$ at uniform speed v. Find the acceleration of particle when it passes point $x = 0$ and point (1, a).