Arithmetic Progressions (AP)

$7-5=2$

$5-7=-2$

* Observe the following sequences :-

2 2 2 2

5, 7, 9, 11, 13

2 2 2 2 2

6, 8, 10, 12, 14, 16

901th

10 10 10 10 10

10, 20, 30, 40, 50, 60 → 1000 terms

-3 -3 -3 -3

-3, -6, -9, -12, -15

-3 -3 -3 -3 -3

9, 6, 3, 0, -3, -6

For all these sequences, gap (difference) b/w successive terms is constant, indicating that the sequence is an A.P

Arithmetic Progressions (AP)

* To analyse an AP, only 2 things are needed :-

- a (first term) — $a = ?$
- d (common difference) — $d =$

$T_2 - T_1$

$T_3 - T_2$

$T_4 - T_3$

$\rightarrow$ (Agla – Pichla)

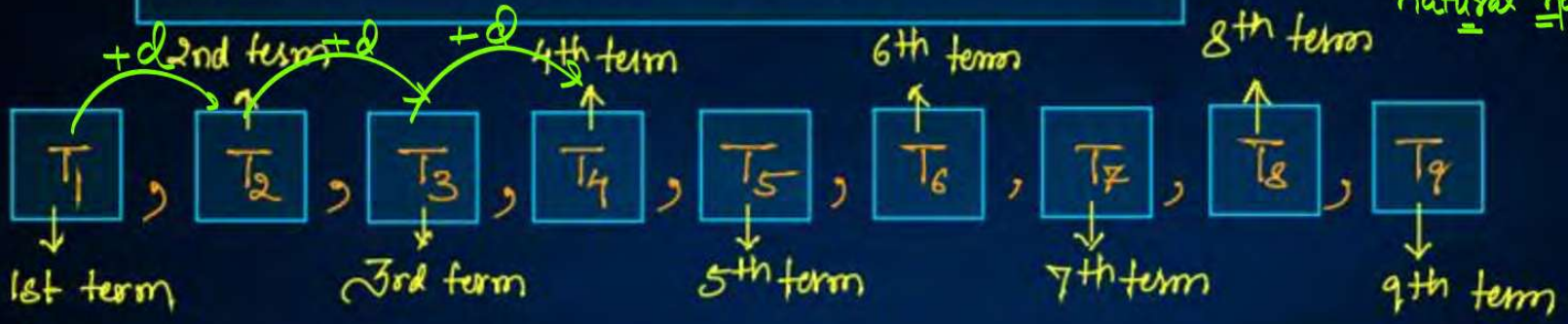
.
.
.

Arithmetic Progressions (AP)

$T_{900} = a + 899d$

$T_n = a + (n-1)d$, n → natural no.

* General Term / N^{th} term of an A.P



$T_1 = a$

$T_2 = T_1 + d \longrightarrow a+1d \longrightarrow a+(2-1)d$

$T_3 = T_2 + d \longrightarrow a+2d \longrightarrow a+(3-1)d$

$\therefore T_4 \rightarrow T_3 + d \longrightarrow a+3d$

$T_n = T_{n-1} + d \longrightarrow a+(n-1)d$

→ n^{th} term / General term of an AP.

QUESTION

For the AP: 3/2, 1/2, - 1/2, - 3/2, ..., write the first term a and the common difference d.

Soln

$\frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}, \ldots\ldots$

T_1 T_2

$a = \frac{3}{2}$

$d = T_2 - T_1$

$= \frac{1}{2} - \frac{3}{2}$

$= -\frac{2}{2} = -1$

$d = -1$

QUESTION

Which of the following list of numbers form an AP? If they form an AP, write the next two terms:

(i) 4, 10, 16, 22, 28, 34

T_1 T_2 T_3 T_4

Soln

$T_2 - T_1 = 10 - 4 = 6$

$T_3 - T_2 = 16 - 10 = 6$

$T_4 - T_3 = 22 - 16 = 6$

$d = 6$

Same

QUESTION

Which of the following list of numbers form an AP? If they form an AP, write the next two terms:

(ii) $-2, 2, -2, 2, -2, \ldots$ ⟶ A.P ✗

$T_1\ T_2\ T_3\ T_4\ T_5$

Soln

$T_2 - T_1 = 2 - (-2) = 4$

$T_3 - T_2 = -2 - 2 = -4$ ✗ Same

QUESTION

Which of the following list of numbers form an AP? If they form an AP, write the next two terms:

(iii) 1, 1, 1, 2, 2, 2, 3, 3, 3, ... → A.P ✗

T_1 T_2 T_3 T_4 T_5 T_6 T_7 T_8 T_9

$T_2 - T_1 = 1 - 1 = 0$

$T_3 - T_2 = 1 - 1 = 0$

$T_4 - T_3 = 2 - 1 = 1$

QUESTION

In which of the following situations, does the list of numbers involved make an arithmetic progression, and why?

(i) The taxi fare after each km when the fare is Rs. 15 for the first km and Rs. 8 for each additional km.

1 km	2 km	3 km	4 km		
15	23	31	39		

$T_2 - T_1 = 23 - 15 = 8$

$T_3 - T_2 = 31 - 23 = 8$

$T_4 - T_3 = 39 - 31 = 8$

15, 23, 31, 39, ...
T_1 T_2 T_3 T_4

QUESTION

$\frac{3}{4}\left(\frac{9}{16}V\right)$ $\frac{3}{4}\left(\frac{3}{4}V\right)$ $\frac{3}{4}$ $\frac{1}{4}$

In which of the following situations, does the list of numbers involved make an arithmetic progression, and why?

(ii) The amount of air present in a cylinder when a vacuum pump removes 1/4 of the air remaining in the cylinder at a time.

$T_2 - T_1 = \frac{3}{4}V - V = -\frac{1}{4}V$

$T_3 - T_2 = \frac{9}{16}V - \frac{3}{4}V = -\frac{3V}{16}$

$V, \underset{T_2}{\frac{3}{4}V}, \underset{T_3}{\frac{9}{16}V}, \ldots\ldots\ldots$ (T_1)

QUESTION

Write first four terms of the AP, when the first term a and the common difference d are given as follows:

(i) a = 10, d = 10

→ 10, 20, 30, 40, ...

QUESTION

Write first four terms of the AP, when the first term a and the common difference d are given as follows:

(ii) $a = -2, d = 0$

$-2, -2, -2, -2, \ldots\ldots$

QUESTION

For the following APs, write the first term and the common difference:
(i) 1/3, 5/3, 9/3, 13/3, ...

T_1 T_2

$a = \frac{1}{3}$

$d = \boxed{T_2 - T_1} \rightarrow a_2 - a_1$

$= \frac{5}{3} - \frac{1}{3}$

$d = \frac{4}{3}$

T_n / a_n

$T_1, T_2, T_3, \ldots$

$a_1, a_2, a_3, a_4, \ldots$

QUESTION

For the following APs, write the first term and the common difference:
(ii) 0.6, 1.7, 2.8, 3.9,...

T_1 T_2

$a = 0.6$

$d = T_2 - T_1$

$= 1.7 - 0.6$

$d = 1.1$

QUESTION

Which of the following are APs? If they form an AP, find the common difference d and write three more terms.

(i) 2, 4, 8, 16,...

QUESTION

Which of the following are APs? If they form an AP, find the common difference d and write three more terms.

(ii) 2, 5/2, 3, 7/2, ...

$4, \frac{9}{2}, 5$

A.P

$d = \frac{1}{2}$

QUESTION

$(3+2\sqrt{2})-(3+\sqrt{2})$ $(3+3\sqrt{2})-(3+2\sqrt{2})$

$3+\sqrt{2}-3$ $3+2\sqrt{2}-3-\sqrt{2}$ $3+3\sqrt{2}-3-2\sqrt{2}$

$\sqrt{2}$ $\sqrt{2}$

Which of the following are APs? If they form an AP, find the common difference d and write three more terms.

(iv) $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}, \ldots$ $3+4\sqrt{2}$, $3+5\sqrt{2}$, $3+6\sqrt{2}$

$\sqrt{2}$ $\sqrt{2}$ $\sqrt{2}$

$d = \sqrt{2}$

QUESTION

$$\begin{array}{r} 0.22 \\ -0.20 \\ \hline 0.02 \end{array} \qquad \begin{array}{r} 0.222 \\ -0.220 \\ \hline 0.002 \end{array}$$

Which of the following are APs? If they form an AP, find the common difference d and write three more terms.

(v) 0.2,0.22, 0.222,0.2222,...

0.02 0.002

A.P ✗

QUESTION

Which of the following are APs? If they form an AP, find the common difference d and write three more terms.

(vi) -1/2, -1/2, -1/2, -1/2, ..., $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}$

A.P ✓

$d = 0$

QUESTION

Which of the following are APs? If they form an AP, find the common difference d and

(vii) a, 2a, 3a, 4a, 5a, 6a, 7a

a a a

A.P

$d = a$

QUESTION

$a^2 - a = a(a-1)$ $\quad a^3 - a^2 = a^2(a-1)$

$0, 0, 0, 0$

Which of the following are APs? If they form an AP, find the common difference d and

(viii) $a, a^2, a^3, a^4, \ldots$

$a(a-1)$ $\quad a^2(a-1)$ $\quad \boxed{a \neq 0}$

A.P ✗

QUESTION

$\sqrt{6}-\sqrt{3}$

$\sqrt{2}\cdot\sqrt{3}-\sqrt{3}$

$\sqrt{3}(\sqrt{2}-1)$

$\sqrt{9}-\sqrt{6}$

$(\sqrt{3})^2-\sqrt{6}$

$(\sqrt{3})^2-\sqrt{3}\times\sqrt{2}$

$(\sqrt{5})^2=5$

Which of the following are APs? If they form an AP, find the common difference d and

(ix) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}$....

$\sqrt{3}(\sqrt{2}-1)$

$\sqrt{3}(\sqrt{3}-\sqrt{2})$

QUESTION

$a = 10$
$d = -3$

Choose the correct choice in the following and justify:

30th term of the AP: 10, 7, 4, ..., is

(A) 97 ✗
(B) 77 ✗
(C) -77
(D) -87

$T_{30} = a + 29d$
$= 10 + (29 \times -3)$
$= 10 - 87$
$= -77$

QUESTION

$a = 2$

$T_3 = 26$

In the following APs, find the missing terms in the boxes :

(i) 2, $\boxed{14}$, 26

T_1 T_2 T_3

$T_3 = 26$

$a + 2d = 26$

$2 + 2d = 26$

$2d = 24$

$d = 12$

$T_2 = a + d$

$= 2 + 12$

$= 14$

$x = \frac{2 + 26}{2}$

$= \frac{28}{2} = 14$

QUESTION

$a = -4$

$\boxed{T_6 = 6}$

In the following APs, find the missing terms in the boxes :

(iv) $-4, \boxed{-2}, \boxed{0}, \boxed{2}, \boxed{4}, 6$

$T_1 \quad T_2 \quad T_3 \quad T_4 \quad T_5 \quad T_6$

$T_6 = 6$

$a + 5d = 6$

$-4 + 5d = 6$

$5d = 10$ (2)

$d = 2$

QUESTION

$a = 3$
$d = 5$

Which term of the AP: 3, 8, 13, 18,...is 78?

$T_n \rightarrow T_{16}$

$T_n = 78$

$a + (n-1)d = 78$

$3 + 5(n-1) = 78$

$5(n-1) = 75$ (15)

$n - 1 = 15$

$n = 16$

QUESTION

$a = 7, d = 6$

Find the number of terms in each of the following APs:

(i) 7, 13, 19....., 205 $T_n \rightarrow T_{34}$ → 34 terms

$T_n = 205$

$a + (n-1)d = 205$

$7 + 6(n-1) = 205$

$6(n-1) = 198$ 33

$n - 1 = 33$

$n = 34$

QUESTION

its not a term of the given AP.

$a = 11$

$d = -3$

T_n

Check whether -150 is a term of the AP: 11, 8, 5, 2...

$$T_n = -150$$

$$\Rightarrow a + (n-1)d = -150$$

$$\Rightarrow 11 + (-3)(n-1) = -150$$

$$\Rightarrow 11 - 3(n-1) = -150$$

$$\Rightarrow -3(n-1) = -161$$

$$\Rightarrow n - 1 = \frac{161}{3}$$

$$\Rightarrow n = \frac{161}{3} + 1$$

QUESTION

T_{31}

Find the 31st term of an AP whose 11th term is 38 and the 16th term is 73.

$$210 - 32 = 178$$

$T_{11} = 38$ | $T_{16} = 73$

$a + 10d = 38$ | $a + 15d = 73$

$$a + 15d = 73$$
$$-\ a + 10d = 38$$
$$5d = 35$$
$$d = 7$$

$a + 10d = 38$

$a + 70 = 38$

$a = 38 - 70$

$a = -32$

$T_{31} = a + 30d$

$= -32 + (30 \times 7)$

$= -32 + 210$

$= 178$

QUESTION

If the 3rd and the 9th terms of an AP are 4 and −8 respectively, which term of this AP is zero?

$T_3 = 4$

$a + 2d = 4$

$T_9 = -8$

$a + 8d = -8$

$a + 8d = -8$

$a + 2d = 4$

$6d = -12$

$d = -2$

$a + 2d = 4$

$a + 2(-2) = 4$

$a - 4 = 4$

$a = 8$

$T_n = 0$

$a + (n-1)d = 0$

$8 + (-2)(n-1) = 0$

$8 - 2(n-1) = 0$

$2(n-1) = 8$

$n - 1 = 4$

$n = 5$

QUESTION

10 5

5

Two APs have the same common difference. The difference between their 100th terms is 100, what is the difference between their 1000th terms?

AP_1	AP_2
a, d	a', d
T_n	T_n'

$T_{100} - T'_{100} = 100$

$T_{1000} - T'_{1000} = 100$

$T_n - T_n'$

$= [a + (n-1)d] - [a' + (n-1)d]$

$= a + (n-1)d - a' - (n-1)d$

$= \boxed{a - a'}$

QUESTION

How many multiples of 4 lie between 10 and 250?

60 multiples

248 — 252

12, 16, 20, ..., 248

T_n

T_{60}

60 terms

$T_n = 248$

$a + (n-1)d = 248$

$12 + 4(n-1) = 248$

$4(n-1) = 236$

$n - 1 = 59$

$n = 60$

4) 10 (2
8
2

4) 8 (
|
0

4) 252 (62
24
12
12
0

248

QUESTION

For what value of n, are the nth terms of two APs: 63, 65, 67, and 3, 10, 17,... equal?

T_n {63, 65, 67, ... T_{13}

T_n' {3, 10, 17, ... T_{13}

$T_n = T_n'$

$\Rightarrow 63 + 2(n-1) = 3 + 7(n-1)$

$\Rightarrow 63 + 2n - 2 = 3 + 7n - 7$

$\Rightarrow 61 + 2n = 7n - 4$

$\Rightarrow 5n = 65$

13

$n = 13$

QUESTION

158

243 , 248

Find the 20th term from the last term of the AP: 3, 8, 13,...,253.

$a = 253$

$d = -5$

253, 248, 243, ..., 13, 8, 3

$$\begin{array}{r} 253 \\ -\ 95 \\ \hline 58 \end{array}$$

$T_{20} = a + 19d$

$= 253 + (19 \times -5)$

$= 253 - 95$

$= \boxed{158}$

QUESTION

Ramkali saved Rs. 5 in the first week of a year and then increased her weekly savings by Rs. 1.75. If in the nth week, her weekly savings become Rs. 20.75, find n. → 10

$a = 5$

$d = 1.75$

Weekly Savings

1st	2nd	3rd	4th	.....	T_n
5	6.75	8.5			20.75
					T_{10}

$T_n = 20.75$

$a + (n-1)d = 20.75$

$5 + 1.75(n-1) = 20.75$

$1.75(n-1) = 15.75$

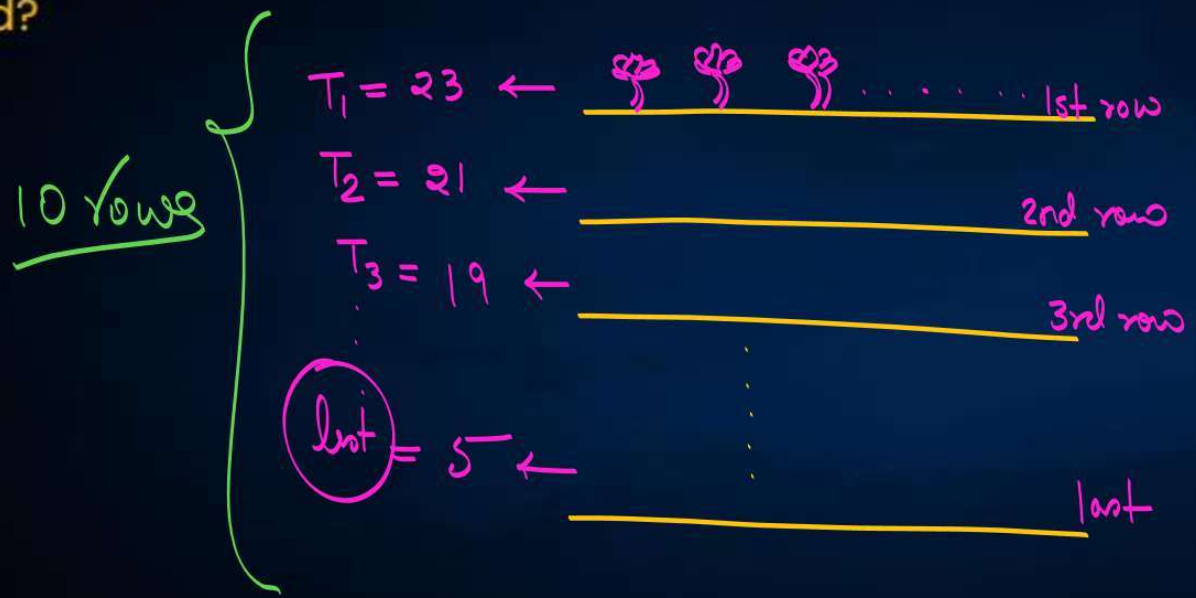
$n - 1 = \frac{15.75}{1.75} = 9$

$n - 1 = 9$

$n = 10$

QUESTION

In a flower bed, there are 23 rose plants in the first row, 21 in the second, 19 in the third, and so on. There are 5 rose plants in the last row. How many rows are there in the flower bed?



$a = 23$

$d = -2$

23, 21, 19, ... 5 (10 terms), T_n, T_{10}

$T_n = 5$

$a + (n-1)d = 5$

$23 + (-2)(n-1) = 5$

$23 - 2(n-1) = 5$

$\Rightarrow -2(n-1) = -18$

$n - 1 = 9$

$n = 10$

Arithmetic Progressions (AP)

Types of AP on the basis of number of terms

Finite A.P

- When AP has countable terms

i) 2, 4, 6, 8, 10, 12

ii) 1, 3, 5, ..., 13

Infinite A.P

- When number of terms in an AP is infinite.

i) 2, 4, 6, 8, ...

ii) ....., 1, 3, 5, 7, 9

Arithmetic Progressions (AP)

* Sum of 1st 'n' terms / Sum of 'n' terms of an AoP

S_n

$\frac{n}{2}\{2a + (n-1)d\}$

$\frac{n}{2}\{a + l\}$

To be used only when last term is somehow involved.

QUESTION

If the sum of the first 14 terms of an AP is 1050 and its first term is 10, find the 20th term.

$a = 10$

$S_{14} = 1050$

$\Rightarrow \frac{n}{2}[2a + (n-1)d] = 1050$

$\Rightarrow \frac{14}{2}[20 + 13d] = 1050$

$\Rightarrow 7(20 + 13d) = 1050$

$20 + 13d = 150$

$13d = 130$

$d = 10$

$T_{20} = a + 19d$

$= 10 + (19 \times 10)$

$= 10 + 190$

$\boxed{T_{20} = 200}$

QUESTION

$a = 2$
$d = 5$

Find the sum of the following AP:
2, 7, 12,..., to 10 terms.

$$S_{10} = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{5}{\cancel{10}_{\cancel{2}}}[4 + 5 \times 9]$$

$$= 5 \times 49$$

$$= 245$$

QUESTION

$a = 34$, $l = 10$, $d = -2$

Find the sum given below:

$34 + 32 + 30 + ... + 10$ → T_n, T_{13}

13 terms

$$S = \frac{n}{2}[a + l]$$

$$= \frac{13}{2} \times [34 + 10]$$

$$= \frac{13}{2} \times \cancel{44}^{22}$$

$$= 286$$

$T_n = 10$

$a + (n-1)d = 10$

$34 + (-2)(n-1) = 10$

$34 - 2(n-1) = 10$

$-2(n-1) = -24$

$n - 1 = 12$

$n = 13$

QUESTION

In an AP:
(i) given $a = 5$, $d = 3$, $a_n = 50$, find n and S_n.

16

$T_n = 50$

$a + (n-1)d = 50$

$5 + 3(n-1) = 50$

$3(n-1) = 45$

$n - 1 = 15$

$n = 16$

$S_{16} = \frac{n}{2}[2a + (n-1)d]$

$= \frac{16}{2}[10 + (15 \times 3)]$

$= 8 \times 55$

$= 440$

QUESTION

$a = 9$, $d = 8$ → 12 terms → 636

How many terms of the AP: 9, 17, 25,... must be taken to give a sum of 636?

n terms

$n = 20$, $n = 53$

5, 3, 1, −1, −3, ...

$T_1 + T_2 \rightarrow 8$

$T_1 + T_2 + T_3 + T_4 \rightarrow 8$

$S_n = 636$

$\Rightarrow \frac{n}{2}[2a + (n-1)d] = 636$

$\Rightarrow \frac{n}{2}[18 + (n-1) \times 8] = 636$

$\Rightarrow \frac{n}{2} \times 2[9 + 4(n-1)] = 636$

$\Rightarrow n(4n+5) = 636$

$\Rightarrow \boxed{4n^2 + 5n - 636 = 0}$

$4n^2 + 53n - 48n - 636 = 0$

$n(4n+53) - 12(4n+53) = 0$

$\Rightarrow (4n+53)(n-12) = 0$

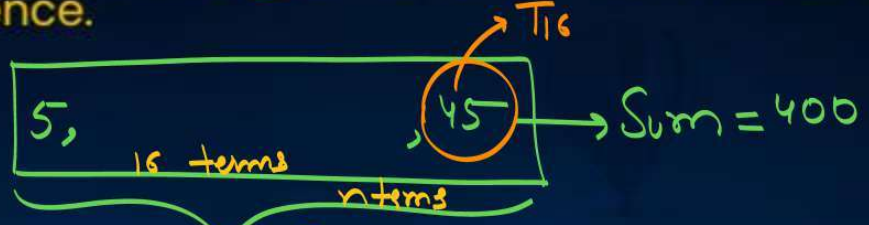
$n = -\frac{53}{4}$ | $n = 12$

2	636
2	318
3	159
	53

QUESTION

$a = 5$
$l = 45$

The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.



$S_n = 400$

$\frac{n}{2}[a+l] = 400$

$\Rightarrow \frac{n}{2}[5+45] = 400$

$\Rightarrow \frac{n}{2} \times 50 = 400$

$n = 16$

$T_{16} = 45$

$a + 15d = 45$

$\Rightarrow 5 + 15d = 45$

$15d = 40$

$d = \frac{8}{3}$

QUESTION

$S_7 = 49$ $S_{17} = 289$

If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

$S_7 = 49$

$\frac{7}{2}[2a+6d] = 49$

$\frac{7}{2} \times 2[a+3d] = 49$ 7

$a+3d = 7$

$a+6 = 7$

$a = 1$

$S_{17} = 289$

$\frac{17}{2}[2a+16d] = 289$

$\frac{17}{2} \times 2[a+8d] = 289$ 17

$a+8d = 17$

$a+8d = 17$

$a+3d = 7$ (−) (−) (−)

$5d = 10$

$d = 2$

$S_n = \frac{n}{2}[2a+(n-1)d]$

$= \frac{n}{2}[2+2(n-1)]$

$= \frac{n}{2} \times 2[1+n-1]$

$= n \times n$

$S_n = n^2$

$n = 1$

$T_n = a + (n-1)d$

$= a + nd - d$

$\boxed{T_n = nd + (a-d)}$

LP

$an + b$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2} \times 2a + \frac{n}{2}(n-1)d$

$= na + \frac{n^2}{2}d - \frac{n}{2}d$

$= \frac{n^2}{2}d + n\left(a - \frac{d}{2}\right)$

$\boxed{S_n = \frac{d}{2}n^2 + \left(a - \frac{d}{2}\right)n}$

QP

$an^2 + bn + c$

QUESTION

Show that $a_1, a_2, \ldots\ldots a_n$, form an AP where a_n is defined as below:

(i) $a_n = 3+4n$

(ii) $a_n = 9-5n$

Also find the sum of the first 15 terms in each case.

$T_n = 3+4n$ → 7, 11, 15, ...

$a = 7$, $d = 4$

$T_1 = 7$

$T_2 = 11$

$T_3 = 15$

$S_{15} = \frac{15}{2}[14 + 14 \times 4]$

$= \frac{15}{2} \times 14 \times [1+4]$

$= \frac{15}{2} \times 7 \times 14 \times 5$

$= 35 \times 15 \rightarrow 525$

QUESTION

$a = ?$ $T_3 \to ?$
$S_2 = ?$ $T_{10} \to ?$
$T_2 = ?$ $T_n \to ?$

If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (that is S_1)? What is the sum of first two terms? What is the second term? Similarly, find the 3rd, the 10th and the nth terms.

$S_n = 4n - n^2$

$S_1 = T_1 = a = 4 - 1$

$a = 3$

$S_2 = 4$

$T_1 + T_2 = 4$

$3 + T_2 = 4$

$T_2 = 1$

$S_2 = (4 \times 2) - 4$

$S_2 = 4$

$T_n = S_n - S_{n-1}$

$3, 1, -1, \ldots\ldots$

$a = 3$

$d = -2$

$T_n = a + (n-1)d$

$= 3 + (-2)(n-1)$

$= 3 - 2(n-1)$

$= 3 - 2n + 2$

$T_n = 5 - 2n$

QUESTION

$1, 2, 3, \ldots\ldots$

Find the sum of the first 40 positive integers divisible by 6.

$a = 6$

$d = 6$

$6, 12, 18, \ldots\ldots$ (40 terms)

$$S_{40} = \frac{20}{\cancel{40}} \left[12 + (39 \times 6)\right]$$

$$= 20 \times \left[12 + 234\right]$$

$$= 20 \times 246$$

$$= \boxed{4920}$$

QUESTION

1 Dec 3

A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: Rs. 200 for the first day, Rs. 250 for the second day, Rs. 300 for the third day, etc., the penalty for each succeeding day being Rs. 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days?

$a = 200$
$d = 50$

200, 250, 300, ...

Penalty

1st	2nd	3rd	4th		
200	250	300	350	⟶	30 terms

$$S_{30} = \frac{30}{2}[400 + 29 \times 50]$$

$$= \frac{30}{2} \times 50 \times [8 + 29]$$

$$= \frac{15}{\cancel{30}} \times 50 \times 37$$

$$= 15 \times 1850 \longrightarrow 27{,}750$$

QUESTION

In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of Class I will plant 1 tree, a section of Class II will plant 2 trees and so on till Class XII. There are three sections of each class. How many trees will be planted by the students?

$3, 6, 9, \ldots\ldots, 36$ (12 terms)

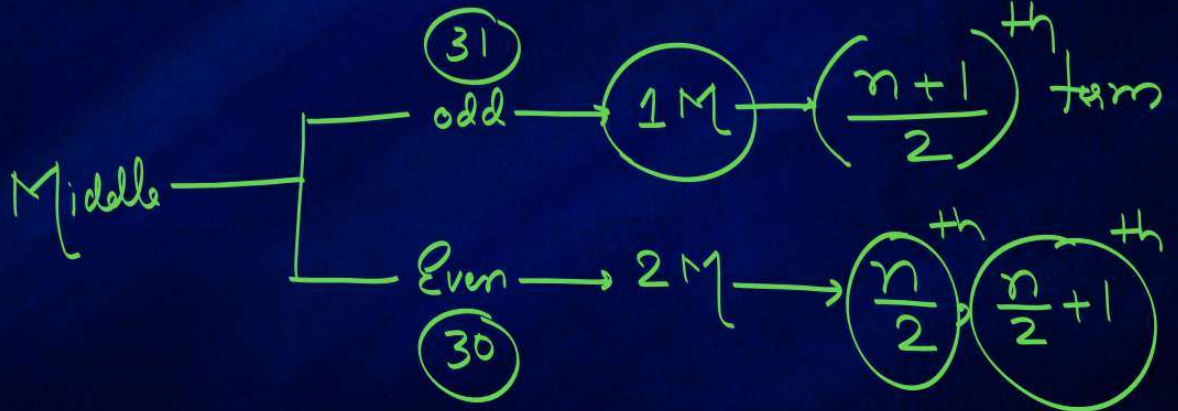
Class 1 – 12

Class 1	Class 2	Class 3	Class 4		Class 12
Trees → 1	2	3	4	. . . .	12
3	6	9	12		36

$S_{12} = \frac{n}{2}[3 + 36]$

$= \frac{12}{2} \times 39$

$= 234$ trees



$\underline{\quad a-d \quad}, \underline{\quad a \quad}, \underline{\quad a+d \quad}$

$\underline{\quad a-3d \quad}, \underline{\quad a-d \quad}, \underline{\quad a+d \quad}, \underline{\quad a+3d \quad}$