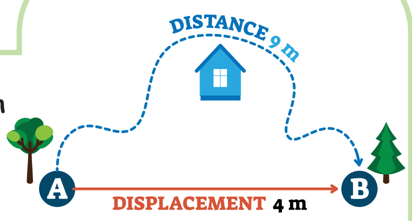
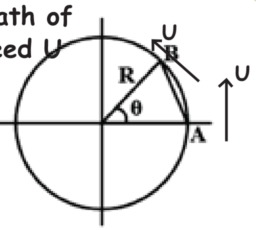
MOTION ALONG A STRAIGHT LINE

- Distance = Length of actual path
- Displacement = Length of shortest path
- Distance $\geq$ |displacement|



A particle moves from A to B in a circular path of radius R covering an angle θ with uniform speed U



- Distance $=\overset{\frown}{AB}=R\theta$
- Displacement $=AB=2R\sin\left(\frac{\theta}{2}\right)$
- Ratio of Displacement to Distance $=\dfrac{\sin\left(\frac{\theta}{2}\right)}{\frac{\theta}{2}}$
- Time $t=\dfrac{R\theta}{U}$
- Average Velocity $=\dfrac{2U\sin\left(\frac{\theta}{2}\right)}{\frac{\theta}{2}}$
- Average Acceleration $=\dfrac{U^2\sin\left(\frac{\theta}{2}\right)}{R\frac{\theta}{2}}$

For uniform motion

Displacement = velocity x time

Average speed = |average velocity|=|instantaneous velocity|

Time average speed

$$v_{av}=\frac{\text{Total distance covered}}{\text{Totaltime elapsed}}=\frac{s_1+s_2+s_3+\dots+s_n}{t_1+t_2+t_3+\dots+t_n}=\frac{v_1t_1+v_2t_2+v_3t_3+\dots\dots}{t_1+t_2+t_3+\dots\dots}$$

If $t_1=t_2=t_3=\dots\dots=t_n$

then

$$v_{av}=\frac{v_1+v_2+v_3+\dots\dots+v_n}{n}$$

for v_1 & v_2,

$V_{avg}=\dfrac{v_1+v_2}{2}$ (Arithmetic mean of speeds)

Distance average speed

$$v_{av}=\frac{\text{Total distance covered}}{\text{Totaltime elapsed}}=\frac{s_1+s_2+s_3+\dots\dots+s_n}{t_1+t_2+t_3+\dots\dots+t_n}=\frac{s_1+s_2+s_3+\dots\dots+s_n}{\frac{s_1}{v_1}+\frac{s_2}{v_2}+\frac{s_3}{v_3}+\dots\dots+\frac{s_n}{v_n}}$$

If $s_1=s_2=s_3=\dots\dots s_n$

then

$$v_{av}=\frac{n}{\frac{1}{v_1}+\frac{1}{v_2}+\frac{1}{v_3}+\dots\dots+\frac{1}{v_n}}$$

for v_1 & v_2, $V_{avg}=\dfrac{2V_1V_2}{V_1+V_2}$ (Harmonic mean of speeds)

Instantaneous Velocity $v=\dfrac{dx}{dt}$ $\quad \Delta x=\int v dt$

Instantaneous Acceleration $a=\dfrac{dv}{dt}$ $\quad \Delta v=\int a dt$

Case 1

$v=f(t)$ or $x=f(t)$

$a=\dfrac{dv}{dt}=\dfrac{d^2x}{dt^2}$

Case 2

$V=f(x)$

$a=V\dfrac{dV}{dx}$

Case 3

$t=f(x)$

then

$a=-(\text{double diff. of } t \text{ w.r.t. } x)\times V^3$

$$\text{Displacement}\ \underset{\text{Integration}}{\overset{\text{Differentiation}}{\rightleftarrows}}\ \text{Velocity}\ \underset{\text{Integration}}{\overset{\text{Differentiation}}{\rightleftarrows}}\ \text{Acceleration}$$

Motion with constant acceleration: Equations of motion

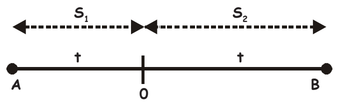
(i) $\boxed{v=u+at}$

(ii) $\boxed{S=ut+\frac{1}{2}at^2}$

- A Person travels from A to B covers unequal distances in equal interval of time with constant acceleration a then

initial velocity $U=\dfrac{3S_1-S_2}{2t}$

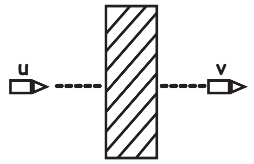
Acceleration $a=\dfrac{S_2-S_1}{t^2}$



(iii) $\boxed{v^2=u^2+2a.s}$

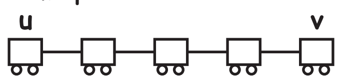
- The number of planks required to stop the bullet

$N=\dfrac{u^2}{u^2-v^2}$

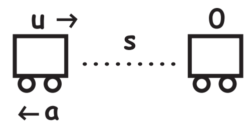


- The two ends of a train moving with constant acceleration pass a certain point with velocities u and v. The velocity with which the middle point of the train passes the same point is

$V_{Mid}=\sqrt{\dfrac{u^2+v^2}{2}}$



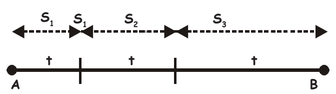
- Calculation of stopping distance $S=\dfrac{u^2}{2a}$



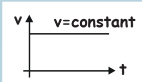
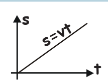
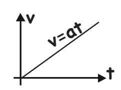
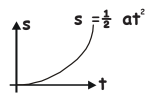
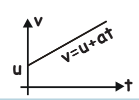
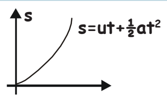
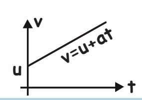
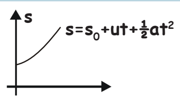
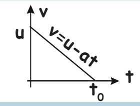
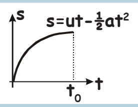
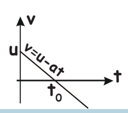
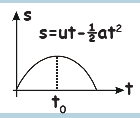
(iv) $\boxed{S_n=u+\frac{a}{2}(2n-1)}$

- Ratio of distance travelled in equal interval of time in a uniformly accelerated motion from rest

$S_1:S_2:S_3=1:3:5$



- for uniform accelerated motion $V_{avg}=\dfrac{u+v}{2}$

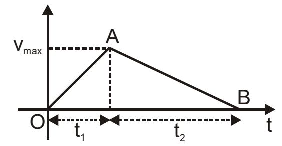
Different Cases	v-t graph	s-t graph
1. Uniform motion		
2. Uniformly accelerated motion with u =0 at t=0		
3. Uniformly accelerated with u ≠ 0 at t=0 & s=0 at t=0		
4. Uniformly accelerated motion with u≠0 and s=s₀ at t=0		
5. Uniformly retarded motion till velocity becomes zero		
6. Uniformly retarded then accelerated in opposite direction		

Important points about graphical analysis of motion

- Instantaneous velocity is the slope of position-time curve $\left[v=\dfrac{dx}{dt}\right]$
- Area of v-t curvegives displacement. $\left[\Delta x=\int v dt\right]$
- Slope of velocity-time curve = instantaneous acceleration $\left[a=\dfrac{dv}{dt}\right]$
- Area of a-t curve gives change in velocity. $\left[\Delta v=\int a dt\right]$

A car accelerates from rest at a constant rate α for some time, after which it decelerates at a constant rate β, to come to rest. If the total time elapsed is t, then

$V_{max}=\dfrac{\alpha\beta}{\alpha+\beta}t$ $\quad$ Total Distance $=\dfrac{1}{2}\left(\dfrac{\alpha\beta}{\alpha+\beta}\right)t^2$



MOTION UNDER GRAVITY

Sign Convention

(i) initial velocity

+ve = upward motion

-ve = downward motion

(ii) Acceleration

Always -ve

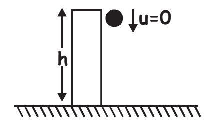
(iii) Displacement

+ve = final position is above initial position

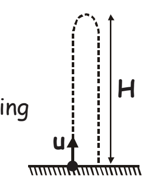
-ve = final position is below initial position

Zero = final position & initial position are at same level

- Object is dropped from top of a tower
  - (i) Ratio of displacement in equal interval of time $S_1:S_2:S_3\dots=1:3:5\dots$
  - (ii) Ratio of time of covering equal distance $t_1:(t_2-t_1):(t_3-t_2):\dots\dots:(t_n-t_{n-1})=1:(\sqrt{2}-\sqrt{1}):(\sqrt{3}-\sqrt{2}):\dots:(\sqrt{n}-\sqrt{n-1})$
  - (iii) Ratio of total distance covered at the end of time $t:2t:3t:\dots=1^2:2^2:3^2\dots$



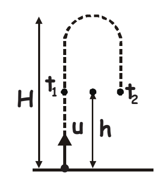
- If a body is thrown vertically up with a velocity u in the uniform gravitational field (neglecting air resistance) then
  - (i) Maximum height attained $H=\dfrac{u^2}{2g}$
  - (ii) Time of ascent = time of descent $\dfrac{u}{g}$
  - (iii) Total time of flight = $\dfrac{2u}{g}$
  - (iv) Velocity of fall at the point of projection = u (downwards)



- At any point on its path the body will have same speed for upward journey and downward journey. If a body thrown upwards crosses a point in time t_1 & t_2 respectively then

height of point $h=\frac{1}{2}gt_1t_2$ $\quad$ Maximum height $H=\dfrac{1}{8}g(t_1+t_2)^2$

Time of flight $=t_1+t_2=\dfrac{2u}{g}$



- A body is thrown upward, downward & horizontally with same speed takes time t_1, t_2 & t_3 respectively to reach the ground then

$t_3=\sqrt{t_1t_2}$ & height from where the particle was throw is $h=\frac{1}{2}gt_1t_2$