CHAPTER 2

Vector and Calculus

Quadratic Equation

Roots of $ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Sum of roots $x_1 + x_2 = -\dfrac{b}{a}$

Product of roots $x_1 x_2 = \dfrac{c}{a}$

Binomial Approximation

If $x << 1$, then $(1 + x)^n \approx 1 + nx$ and $(1 - x)^n \approx 1 - nx$

Logarithm

$\log mn = \log m + \log n$

$\log m/n = \log m - \log n$

$\log m^n = n \log m$

$\log_e m = 2.303 \log_{10} m$

$\log 2 = 0.3010$

Componendo and Dividendo law

If $\dfrac{p}{q} = \dfrac{a}{b}$ then $\dfrac{p+q}{p-q} = \dfrac{a+b}{a-b}$

Arithmetic Progression-AP Formula

$a, a + d, a + 2d, a + 3d, \ldots, a + (n - 1)d$,

here d = common difference

Sum of n terms $S_n = \dfrac{n}{2}[2a + (n - 1)d]$

Note:

(i) $1 + 2 + 3 + 4 + 5 \ldots + n = \dfrac{n(n+1)}{2}$

(ii) $1^2 + 2^2 + 3^2 + \ldots + n^2 = \dfrac{n(n+1)(2n+1)}{6}$

Geometrical Progression-GP Formula

$a, ar, ar^2, \ldots$ here, r = common ratio

Sum of n terms $S_n = \dfrac{a(1-r^n)}{1-r}$

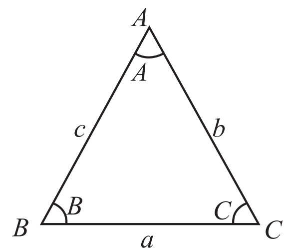
Sum of ∞ terms $S_\infty = \dfrac{a}{1-r}$

Sine law

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Cosine law

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}, \cos B = \frac{c^2 + a^2 - b^2}{2ca}, \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



Maxima and Minima of a Function y = f(x)

- For maximum value $\dfrac{dy}{dx} = 0$ & $\dfrac{d^2y}{dx^2} = -\text{ve}$
- For minimum value $\dfrac{dy}{dx} = 0$ & $\dfrac{d^2y}{dx^2} = +\text{ve}$

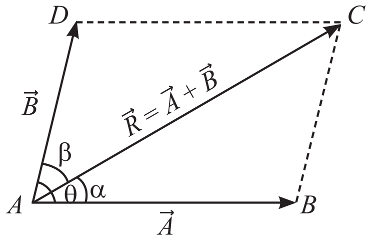
Average of a Varying Quantity

If $y = f(x)$ then $\langle y \rangle = \bar{y} = \dfrac{\int_{x_1}^{x_2} y\,dx}{\int_{x_1}^{x_2} dx} = \dfrac{\int_{x_1}^{x_2} y\,dx}{x_2 - x_1}$

- To convert an angle from degree to radian, we should multiply it by $\pi/180°$ and to convert an angle from radian to degree, we should multiply it by $180°/\pi$.
- By help of differentiation, if y is given, we can find dy/dx and by help of integration, if dy/dx is given, we can find y.
- The maximum and minimum values of function $A \cos\theta + B \sin\theta$ are $\sqrt{A^2 + B^2}$ and $-\sqrt{A^2 + B^2}$ respectively.

Parallelogram Law of Vector Addition

If two vectors are represented by two adjacent sides of a parallelogram which are directed away from their common point then their sum (i.e. resultant vector) is given by the diagonal of the parallellogram passing away through that common point.

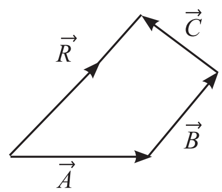


$$\overline{AB} + \overline{AD} = \overline{AC} = \overline{R} \text{ or } \vec{A} + \vec{B} = \vec{R} \Rightarrow R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta} \text{ and } \tan\beta = \frac{A\sin\theta}{B + A\cos\theta}$$

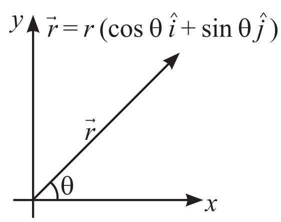
Addition of More than Two Vectors (Polygon Law)

If some vectors are represented by sides of a polygon in same order, then their resultant vector is represented by the closing side of polygon in the opposite order.



General Vector in x-y Plane

$$\vec{r} = x\hat{i} + y\hat{j} = r(\cos\theta\hat{i} + \sin\theta\hat{j})$$



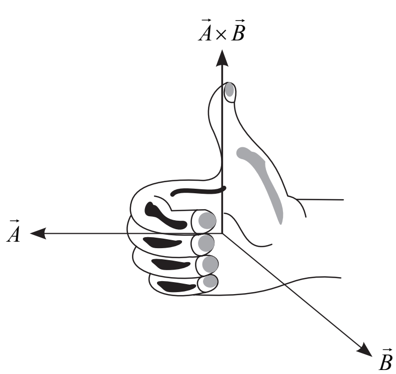
Scalar Product (Dot Product)

$$\vec{A}\cdot\vec{B} = AB\cos\theta \Rightarrow \boxed{\text{Angle between two vectors} = \cos^{-1}\left(\frac{\vec{A}\cdot\vec{B}}{AB}\right)}$$

e.g. work done = $\vec{F}\,.\,\vec{S}$ (where $\vec{F}$ is the Force vector and $\vec{S}$ is the displacement vector).

Cross Product (Vector Product)

$\vec{A}\times\vec{B} = AB\sin\theta\ \hat{}$ where $\hat{n}$ is a vector perpendicular to $\vec{A}$ and $\vec{B}$ or their plane and its direction given by right hand thumb rule.



- $\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$
- $\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$

$$= \hat{i}(A_yB_z - A_zB_y) - \hat{j}(A_xB_z - B_xA_z) + \hat{k}(A_xB_y - B_xA_y)$$

Area of Parallelogram

$\overrightarrow{\text{Area}} = \left(\left|\vec{A}\right|\left|\vec{B}\right|\sin\theta\right)\hat{n} = \vec{A}\times\vec{B}$ (where $\hat{n}$ is the unit vector normal to the plane containing $\vec{A}$ and $\vec{B}$)

Area of Triangle

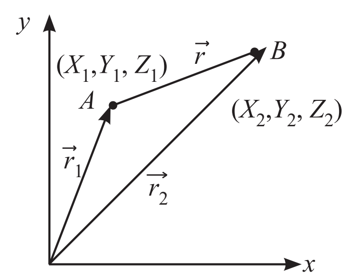
$$\text{Area} = \frac{|\vec{A}\times\vec{B}|}{2} = \frac{1}{2}AB\sin\theta$$

Differentiation of Vectors

- $\dfrac{d}{dt}(\vec{A}\cdot\vec{B}) = \dfrac{dA}{dt}\cdot\vec{B} + \vec{A}\cdot\dfrac{d\vec{B}}{dt}$
- $\dfrac{d}{dt}(\vec{A}\times\vec{B}) = \dfrac{d\vec{A}}{dt}\times\vec{B} + \vec{A}\times\dfrac{d\vec{B}}{dt}$

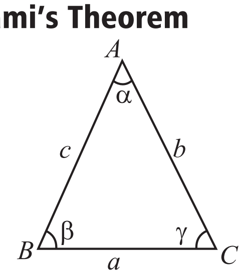
Displacement Vector

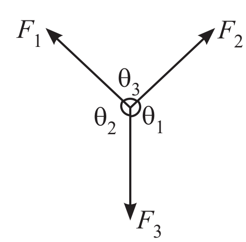
$$\vec{r} = \vec{r}_2 - \vec{r}_1 = (x_2\hat{i} + y_2\hat{j} + z_2\hat{k}) - (x_1\hat{i} + y_1\hat{j} + z_1\hat{k})$$
$$= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$



Magnitude $r = |\vec{r}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Lami's Theorem





$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \qquad \frac{F_1}{\sin\theta_1} = \frac{F_2}{\sin\theta_2} = \frac{F_3}{\sin\theta_3}$$

- A unit vector has no unit.
- Electric current is not a vector as it does not obey the law of vector addition.
- A scalar or a vector can never be divided by a vector.
- To a vector only a vector of same type can be added and the resultant is a vector of the same type.

CHAPTER

4 Projectile Motion

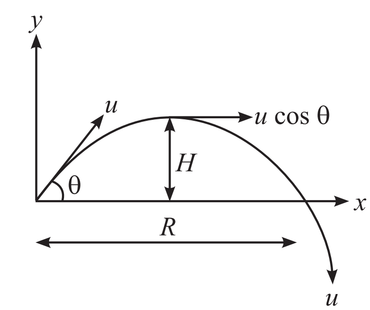
Projection Motion

Horizontal Motion of Projectile

$u \cos \theta = u_x$

$a_x = 0$

$x = u_x t = (u \cos \theta)t$



Vertical Motion of Projectile

$v_y = u_y - gt$ where $u_y = u \sin \theta$; $y = u_y t - \frac{1}{2}gt^2 = u \sin \theta t - \frac{1}{2}gt^2$

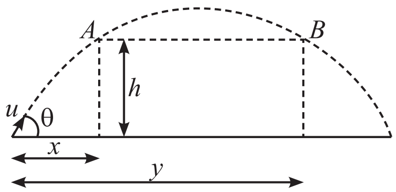
Net acceleration = $\vec{a} = a_x\hat{i} + a_y\hat{j} = -g\hat{j}$

At any Instant

$v_x = u \cos \theta, \quad v_y = u \sin \theta - gt$

Ground to Ground Projectile Motion

A body crosses two points at same height in time t_1 and t_2 the points are at distance x and y from starting point then



(*a*) $x + y = R$

(*b*) $t_1 + t_2 = T$

(*c*) $h = ½\, gt_1t_2$

(*d*) Average velocity from A to B is $u \cos \theta$

If a person can throw a ball to a maximum distance 'x' then the maximum height to which he can throw the ball will be (x/2).

Velocity of Particle at Time *t*:

$$\vec{v} = v_x\hat{i} + v_y\hat{j} = u_x\hat{i} + (u_y - gt)\hat{j} = u\cos\theta\hat{i} + (u\sin\theta - gt)\hat{j}$$

If angle of velocity $\vec{v}$ with horizontal is α, then

$$\tan\alpha = \frac{v_y}{v_x} = \frac{u_y - gt}{u_x} = \frac{u\sin\theta - gt}{u\cos\theta} = \tan\theta - \frac{gt}{u\cos\theta}$$

At highest point: $v_y = 0$, $v_x = u \cos \theta$

Time of flight: $T = \frac{2u_y}{g} = \frac{2u\sin\theta}{g}$

Horizontal range: $R = (u \cos \theta)T$

$$= \frac{2u^2\sin\theta\cos\theta}{g} = \frac{u^2\sin 2\theta}{g} = \frac{2u_xu_y}{g}$$

It is same for θ and $(90° - \theta)$ and maximum for $\theta = 45°$.

Maximum height: $H = \frac{u_y^2}{2g} = \frac{u^2\sin^2\theta}{2g} = \frac{1}{8}gT^2$

$$\frac{H}{R} = \frac{1}{4}\tan\theta$$

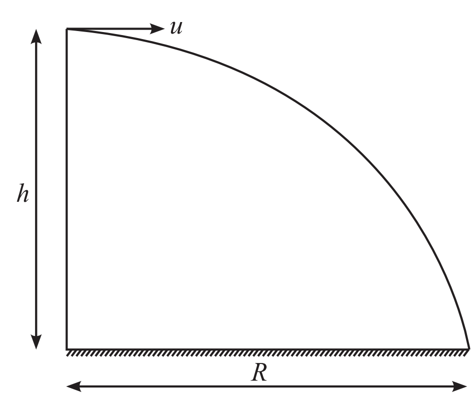
Equation of trajectory $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta} = x\tan\theta\left(1 - \frac{x}{R}\right)$

Horizontal Projection from a Height h

Time of flight $T = \sqrt{\frac{2h}{g}}$

Horizontal range $R = uT = u\sqrt{\frac{2h}{g}}$

Angle of velocity at any instant with horizontal $\theta = \tan^{-1}\left(\frac{gt}{u}\right)$



Projected from Some Height at Some Angle

Case-I: When projected at some angle θ with the horizontal towards upward direction.

time t_1 (time of flight) to strike the ground

$$\therefore \quad t_1 = \frac{T + \sqrt{T^2 + 8h/g}}{2}$$

$$\left(\text{where } T = \frac{2u\sin\theta}{g}\right)$$

$R = \Delta x = u_x t_1$

$v_x = u \cos\theta$

Case-II: When projected at angle θ with horizontal towards downward direction

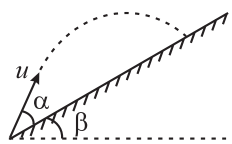
time t_2 to strike the ground then

$$\therefore \; t_2 = \frac{\sqrt{T^2 + 8h/g} - T}{2}$$

$R = u_x\, t_2$

$v_x = u \cos\theta$

Projection on an Inclined Plane



Case-I: Particle is projected up the incline

Time of flight (*T*):

$$T = \frac{2u\sin\alpha}{g\cos\beta} = \frac{2u_\perp}{g_\perp}$$

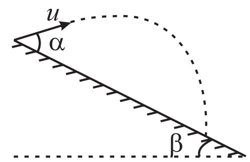
Maximum height from inclined plane (*H*):

$$H = \frac{u^2\sin^2\alpha}{2g\cos\beta} = \frac{u^2{}_\perp}{2g_\perp}$$

Range along the inclined plane (*R*):

$$\boxed{R = \frac{2u^2\sin\alpha\cos(\alpha+\beta)}{g\cos^2\beta}}$$

Case-II: Particle is projected down the incline



Time of flight (*T*):

$$T = \frac{2u\sin\alpha}{g\cos\beta} = \frac{2u_\perp}{g_\perp}$$

Maximum height (*H*):

$$H = \frac{u^2\sin^2\alpha}{2g\cos\beta} = \frac{u^2{}_\perp}{2g_\perp}$$

Range along the inclined plane (*R*):

$$\boxed{R = \frac{2u^2\sin\alpha\cos(\alpha-\beta)}{g\cos^2\beta}}$$