Linear Equations

* **Simultaneous linear equations in 2 variables :-**

→ A pair of linear equations in 2 variables is said to form a system of simultaneous linear equations.

$x + 3y = 4 \rightarrow 1x + 3y - 4 = 0$ ✓

$x - 2y = 5 \rightarrow 1x - 2y - 5 = 0$

$2u + 5v + 1 = 0$

$1u - 2v + 9 = 0$

Linear Equations

* General / Standard form of simultaneous / pair of Linear Equations

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

coefficient of x

coefficient of y

constant term.

Here,

$a_1, b_1, c_1, a_2, b_2, c_2$ → real numbers

& they satisfy the condition

$$a_1^2 + b_1^2 \neq 0$$
$$a_2^2 + b_2^2 \neq 0$$

$$ax + by + c = 0$$
$$ap + bq + c = 0$$

Linear Equations

* Meaning of solution of pair of Linear Equations

- Mathematical Meaning
- Graphical Meaning.

Linear Equations

* **Mathematical Meaning**

→ The answer or the solution of 2 linear equations must satisfy both the equations.

→ $2x + 3y - 6 = 0$

→ $8x - 9y + 3 = 0$

Solution → $x = \alpha$, $y = \beta$

$2\alpha + 3\beta - 6 = 0$

$8\alpha - 9\beta + 3 = 0$

Linear Equations

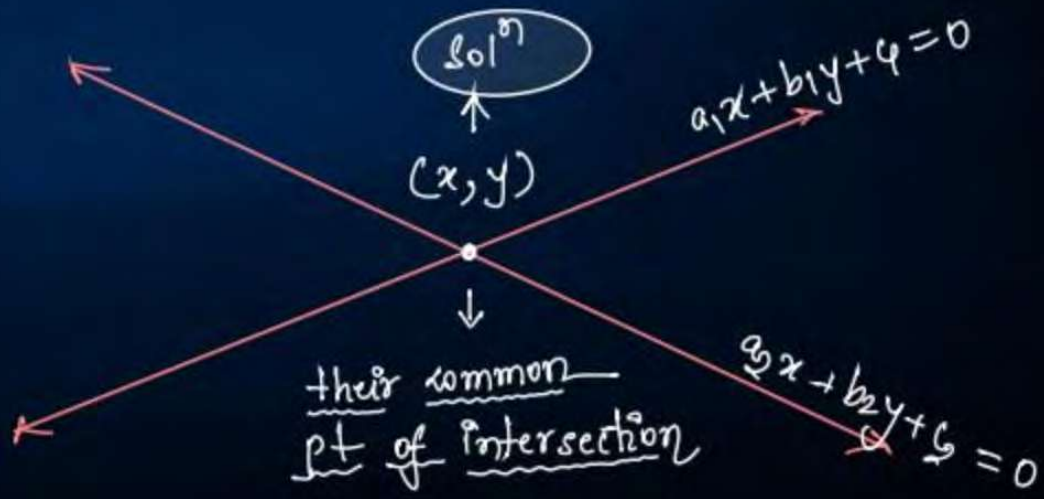
* **Graphical Meaning**

$x = \alpha$, $y = \beta$ → (α, β) ; (x, y)

linear equation in 2 variables ⇌ straight line ✓

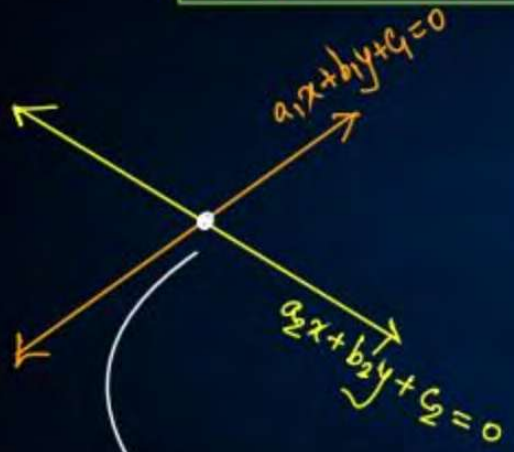
∴ Solution of 2 linear equation means (x, y) is a point which lies on both straight lines.

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$



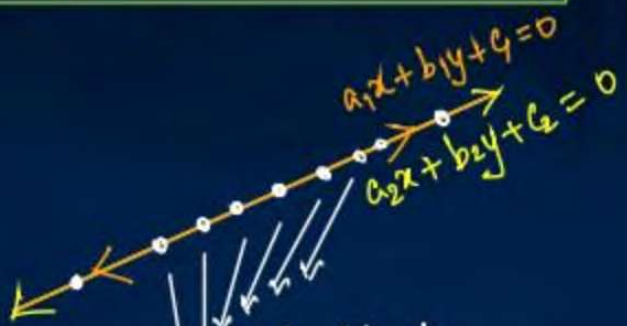
Linear Equations

* Graphical Representation of Straight lines



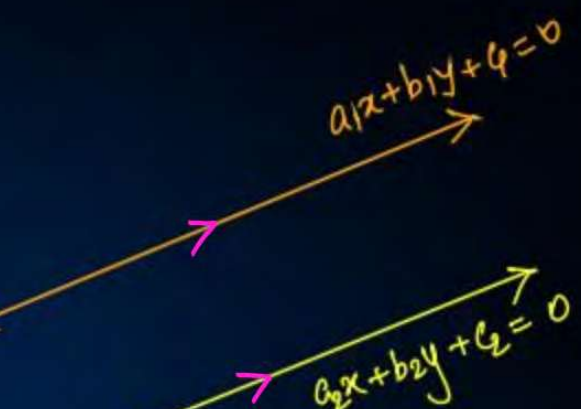
intersecting at one pt. (Unique Soln)

Case - I



coincident (∞ number of solutions)

Case - II



parallel (no soln)

Case - III

Linear Equations

* Conditions for solvability / consistency :—

a) Consistent soln / Solvable — if soln comes → unique soln

→ ∞ number of solns

b) Inconsistent soln / Non-consistent — if soln doesn't come → no soln

Linear Equations

* To check Solvability of equations without proper solving :-

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

Steps

i) First check for eqⁿ convert to standard form

ii) Then compare to find $a_1, b_1, c_1, a_2, \ldots$

iii) Then, put in condition.

a) Unique Soln — $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

b) ∞ Soln — $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

c) no Soln — $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

QUESTION

On comparing the ratios a_1/a_2, b_1/b_2 and c_1/c_2, find out whether the line representing the following pairs of linear equations intersect at a point, are parallel or coincident:

$5x - 4y + 8 = 0 \leftrightarrow a_1x + b_1y + c_1 = 0$

$7x + 6y - 9 = 0 \leftrightarrow a_2x + b_2y + c_2 = 0$

Soln

$a_1 = 5, b_1 = -4, c_1 = 8$

$a_2 = 7, b_2 = 6, c_2 = -9$

$\frac{a_1}{a_2} = \frac{5}{7}$

$\frac{b_1}{b_2} = \frac{-4}{6} = \frac{-2}{3}$

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \rightarrow$ Unique Soln

QUESTION

On comparing the ratios a_1/a_2, b_1/b_2 and c_1/c_2, find out whether the line representing the following pairs of linear equations intersect at a point, are parallel or coincident:

$9x + 3y + 12 = 0 \longleftrightarrow a_1x + b_1y + c_1 = 0$

$18x + 6y + 24 = 0 \longleftrightarrow a_2x + b_2y + c_2 = 0$

Soln

$a_1 = 9,\ b_1 = 3,\ c_1 = 12$

$a_2 = 18,\ b_2 = 6,\ c_2 = 24$

$\frac{a_1}{a_2} = \frac{9}{18} = \frac{1}{2}$

$\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$

$\frac{c_1}{c_2} = \frac{12}{24} = \frac{1}{2}$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \rightarrow \infty$

QUESTION

On comparing the ratios a_1/a_2, b_1/b_2 and c_1/c_2, find out whether the line representing the following pairs of linear equations intersect at a point, are parallel or coincident:

$6x - 3y + 10 = 0 \longleftrightarrow a_1x + b_1y + c_1 = 0$

$2x - 1y + 9 = 0 \longleftrightarrow a_2x + b_2y + c_2 = 0$

Soln

$a_1 = 6, \; b_1 = -3, \; c_1 = 10$

$a_2 = 2, \; b_2 = -1, \; c_2 = 9$

$\frac{a_1}{a_2} = \frac{3\cancel{6}}{\cancel{2}} = \frac{3}{1}$

$\frac{b_1}{b_2} = \frac{+3}{\cancel{-1}} = \frac{3}{-1}$

$\frac{c_1}{c_2} = \frac{10}{9}$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

no soln

QUESTION

On comparing the ratios a_1/a_2, b_1/b_2 and c_1/c_2, find out whether the following pair of linear equations are consistent, or inconsistent.

$3x/2 + 5y/3 = 7$

$9x - 10y = 14$

Soln

$$\frac{3}{2}x + \frac{5}{3}y - 7 = 0$$

$$9x - 10y - 14 = 0$$

$$\frac{a_1}{a_2} = \frac{3/2}{9} = \frac{3}{2} \times \frac{1}{9_3} = \frac{1}{6}$$

$$\frac{b_1}{b_2} = \frac{5/3}{-10} = \frac{5}{3} \times \frac{1}{-10_2} = -\frac{1}{6}$$

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Unique Soln

↑

consistent

QUESTION

On comparing the ratios a_1/a_2, b_1/b_2 and c_1/c_2, find out whether the following pair of linear equations are consistent, or inconsistent.

$4x/3 + 2y = 8$

$2x + 3y = 12$

Soln

$\frac{4}{3}x + 2y - 8 = 0$

$2x + 3y - 12 = 0$

$\frac{a_1}{a_2} = \frac{\frac{4}{3}}{2} = \frac{2}{3} \times \frac{1}{2} = \frac{2}{3}$

$\frac{b_1}{b_2} = \frac{2}{3}$

$\frac{c_1}{c_2} = \frac{8}{12} = \frac{2}{3}$

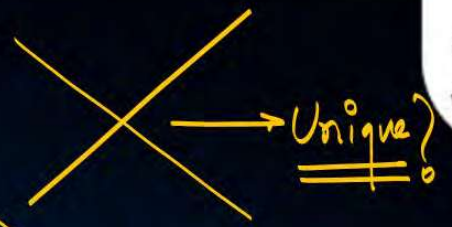
consistent

∞

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

QUESTION

Two rails are represented by the equations $x + 2y - 4 = 0$ and $2x + 4y - 12 = 0$. Will the rails cross each other?



→ No, they will be parallel to each other.

Soln

$1x + 2y - 4 = 0$

$2x + 4y - 12 = 0$

$\frac{a_1}{a_2} = \frac{1}{2}$

$\frac{b_1}{b_2} = \frac{x}{y_2} = \frac{1}{2}$

$\frac{c_1}{c_2} = \frac{-4}{-12} = \frac{1}{3}$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

no soln

QUESTION

$2x+6y+2024=0$

Given the linear equation $2x + 3y - 8 = 0$, write another linear equation in two variables such that the geometrical representation of the pair so formed is:

(i) intersecting lines

↓

Unique

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$2x + 3y - 8 = 0$$

$$\boxed{2}x + \boxed{6}y + \boxed{2024} = 0$$

QUESTION

$2x + 3y + 2025 = 0$

Given the linear equation $2x + 3y - 8 = 0$, write another linear equation in two variables such that the geometrical representation of the pair so formed is:

(ii) parallel lines

no soln

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$2x + 3y - 8 = 0$$

$$\boxed{2}x + \boxed{3}y + \boxed{2025} = 0$$

QUESTION

$\frac{1}{2}$

Given the linear equation $2x + 3y - 8 = 0$, write another linear equation in two variables such that the geometrical representation of the pair so formed is:

(iii) coincident lines

∞

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$2x + 3y - 8 = 0$

$\boxed{4}x + \boxed{6}y + \boxed{-16} = 0$

$4x + 6y - 16 = 0$

Linear Equations

* Mathematical Method to Solve linear equations :-

- Substitution Method
- Elimination Method

QUESTION

Solve the following pair of equations

$7x - 15y = 2$

$x + 2y = 3$

$x = \frac{49}{29}$, $y = \frac{19}{29}$

$\left(\frac{49}{29}, \frac{19}{29}\right)$

Subs

Soln

$7x - 15y = 2$

$x + 2y = 3$

$x = 3 - 2y$

$7(3 - 2y) - 15y = 2$

$\Rightarrow 21 - 14y - 15y = 2$

$\Rightarrow 21 - 29y = 2$

$29y = 19$

$y = \frac{19}{29}$

$x = 3 - 2\left(\frac{19}{29}\right)$

$= 3 - \frac{38}{29}$

$= \frac{87 - 38}{29}$

$x = \frac{49}{29}$

QUESTION

Elimⁿ

Solve the following pair of equations

$7x - 15y = 2$

$x + 2y = 3$

Solⁿ

$7x - 15y = 2$

$7(x + 2y = 3)$

$7x - 15y = 2$

$-\ 7x + 14y = 21$

$-29y = -19$

$y = \frac{19}{29}$

$x + 2y = 3$

$x + 2\left(\frac{19}{29}\right) = 3$

$x = 3 - \frac{38}{29} = \frac{87 - 38}{29} = \frac{49}{29}$, $x = \frac{49}{29}$

QUESTION

$x = 2$
$y = 3$

Solve the following pair of linear equations

$0.2x + 0.3y = 1.3$

$0.4x + 0.5y = 2.3$

$2(0.2x + 0.3y = 1.3) \longrightarrow 0.4x + 0.6y = 2.6$

$0.4x + 0.5y = 2.3 \longrightarrow (-)\ 0.4x + (-)\ 0.5y = (-)\ 2.3$

$0.1y = 0.3$

$0.2x + 0.3y = 1.3$

$0.2x + 0.9 = 1.3$

$0.2x = 0.4$

$\frac{0.2x}{0.2} = \frac{0.4}{0.2} \Rightarrow x = 2$

$\frac{0.1y}{0.1} = \frac{0.3}{0.1}$

$y = 3$

QUESTION

Solve the following pair of linear equations

$\sqrt{2}x + \sqrt{3}y = 0$

$\sqrt{3}x - \sqrt{8}y = 0$

$x = 0$
$y = 0$

$\sqrt{2}x = -\sqrt{3}y$

$x = -\frac{\sqrt{3}}{\sqrt{2}}y$

$x = 0$

$\sqrt{3}\left(-\frac{\sqrt{3}}{\sqrt{2}}y\right) - \sqrt{8}y = 0$

$-\frac{3}{\sqrt{2}}y - \sqrt{8}y = 0$

$y\left(-\frac{3}{\sqrt{2}} - \sqrt{8}\right) = 0$

$y = 0$

QUESTION

Solve the following pair of linear equations

$x/2 + 2y/3 = -1$

$x - y/3 = 3$

QUESTION

Solve $2x + 3y = 11$ and $2x - 4y = -24$ and hence find the value of 'm' for which $y = mx + 3$.

Sol^n

$$2x + 3y = 11$$
$$2x - 4y = -24$$

$(-)\ (+)\ (+)$

$$7y = 35$$
$$y = 5$$

$$2x + 3y = 11$$
$$2x + 15 = 11$$
$$2x = -4$$
$$x = -2$$

$x = -2$, $y = 5$

$$y = mx + 3$$
$$5 = -2m + 3$$
$$2m = -2$$
$$m = -1$$

QUESTION

Which of the following pairs of linear equations are consistent/inconsistent? If consistent, obtain the solution graphically:

$x + y = 5$

$2x + 2y = 10$

Soln

$1 \cdot x + 1 \cdot y - 5 = 0$

$2x + 2y - 10 = 0$

$(0,5)$ & $(5,0)$

$x+y=5$

x	0	5
y	5	0

$2x+2y=10$

x	0	5
y	5	0

$(0,5)$ $(5,0)$

$\frac{a_1}{a_2} = \frac{1}{2}$

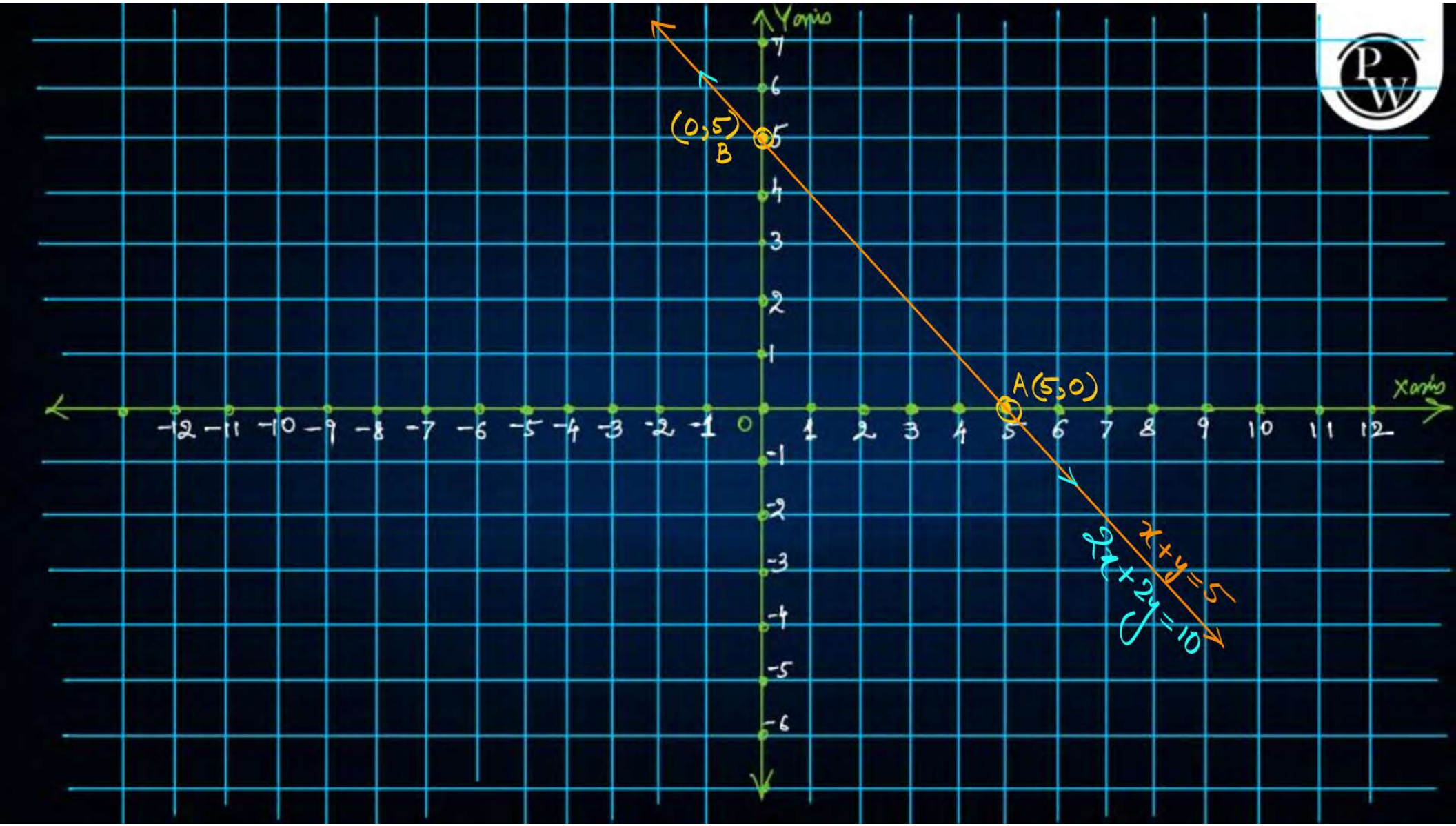
$\frac{b_1}{b_2} = \frac{1}{2}$

$\frac{c_1}{c_2} = \frac{1\cancel{0}}{1\cancel{0}_2} = \frac{1}{2}$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

consistent

∞ → coincident


Y axis
X axis
(0,5)
B
A(5,0)
x+y=5
2x+2y=10
-12 -11 -10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 11 12
7 6 5 4 3 2 1 -1 -2 -3 -4 -5 -6

QUESTION

Form the pair of linear equations in the following problems, and find their solutions graphically.

10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

Soln

No. of boys = x

No. of girls = y

$x + y = 10$

$y = 4 + x$

$x - y = -4$

$x + y = 10$

$x - y = -4$

$2x = 6$

$x = 3$

$y = 7$

$(3, 7)$

$x + y = 10$

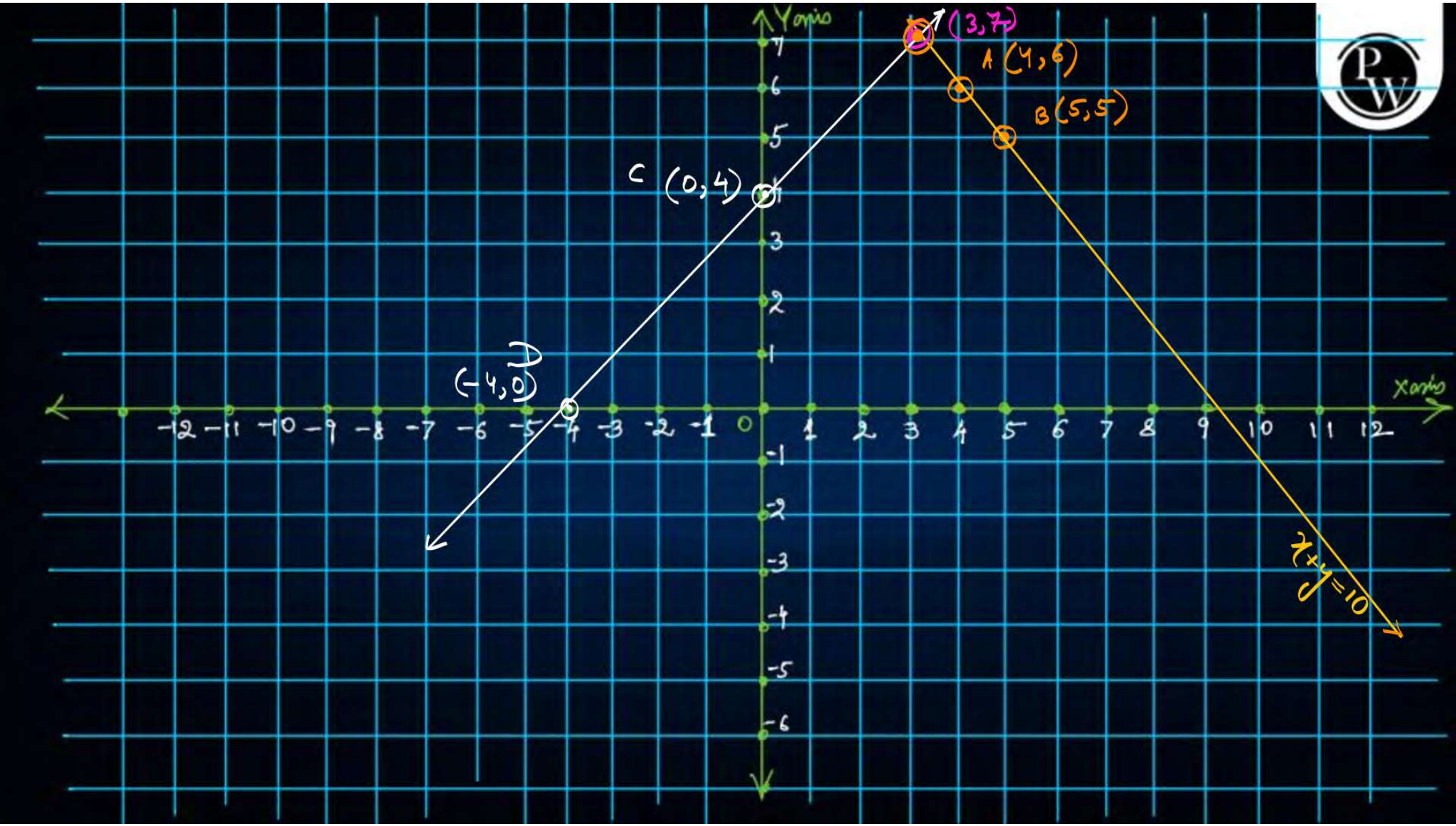
x	4	5
y	6	5

$(4, 6)$ & $(5, 5)$

$x - y = -4$

x	0	-4
y	4	0

$(0, 4)$ $(-4, 0)$


Yaxis
Xaxis
(3,7)
A (4,6)
B (5,5)
C (0,4)
D
(-4,0)
x+y=10

QUESTION

Draw the graphs of the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis, and shade the triangular region.

(2, 3)

Sol?

$x - y = -1$

$x - y + 1 = 0$

(0,1) & (−1,0)

x	0	−1
y	1	0

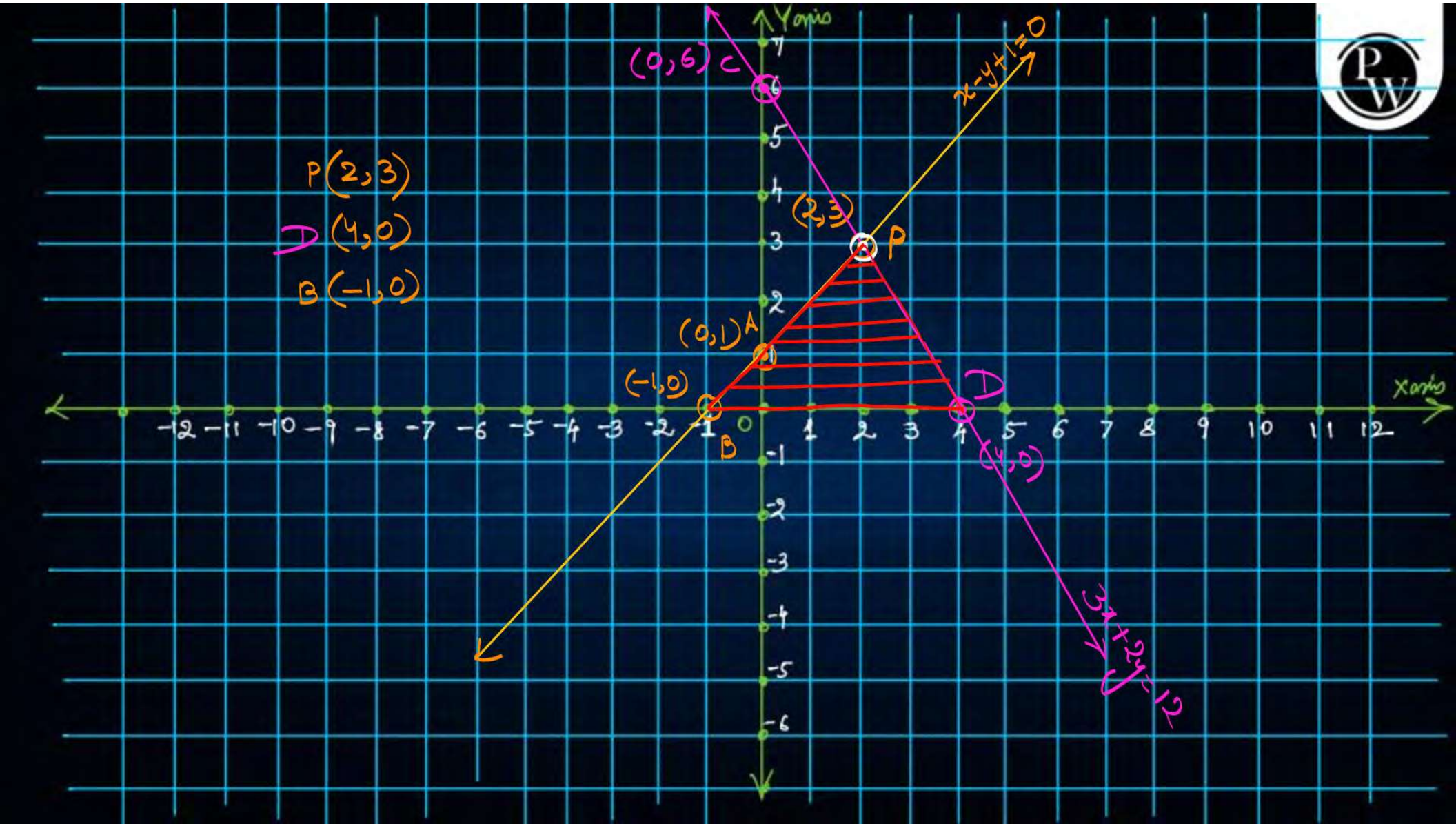
$3x + 2y - 12 = 0$

$3x + 2y = 12$

$y = \frac{12 - 3x}{2}$

x	0	4
y	6	0

(0,6) (4,0)


Yaxis
Xaxis
(0,6) C
x-y+1=0
P(2,3)
D (4,0)
B (-1,0)
(2,3)
P
(0,1) A
(-1,0)
B
A
(4,0)
3x+2y=12

Linear Equations

* Word Problems

→ Which 2 things are to be found form the 2 variables (x & y)

→ To solve 2 unknowns we need 2 equations.

→ Hence, directly / indirectly 2 infos must be there.

QUESTION

20m & 16m

Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.

Soln

length = x m → 20m

width = y m → 16m

$\frac{1}{2}$(Perimeter) = 36

$\Rightarrow \frac{1}{2}[2(x+y)] = 36$

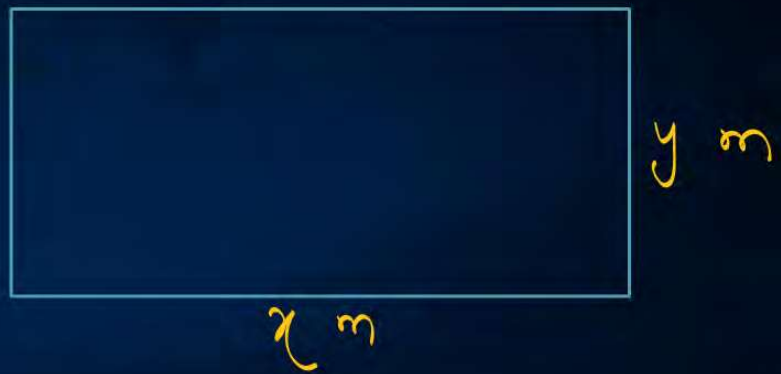
$\Rightarrow x+y=36$

$x = y + 4$

$x - y = 4$

$x = 20$ m

$y = 16$ m



QUESTION

Solve the following question

Aftab tells his daughter, "Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be."

Soln

$x - 7 = 7(y - 7)$

$\Rightarrow x - 7 = 7y - 49$

$\Rightarrow \boxed{x - 7y = -42}$

$x + 3 = 3(y + 3)$

$\Rightarrow x + 3 = 3y + 9$

$\Rightarrow \boxed{x - 3y = 6}$

$x = 6 + 3y$

$x = 42\text{yrs}$

$-4y = -48$

$y = 12\text{yrs}$

	Past (−7)	Present (yrs)	Future (+3)	
Aftab	$x - 7$	x (42yrs)	$x + 3$	45
Daughter	$y - 7$	y (12yrs)	$y + 3$	15
	35	5		

QUESTION

In a shop the cost of 2 pencils and 3 erasers is ₹9 and the cost of 4 pencils and 6 erasers is ₹18. Find the cost of each pencil and each eraser.

Sol^n

Cost of 1 pencil → x ₹

Cost of 1 eraser → y ₹

$2x + 3y = 9$

$x = \frac{9-3y}{2}$

$y = 2$ ₹, $x = 1.5$ ₹

$y = 1$ ₹, $x = 3$ ₹

→ Infinite ans are possible here.

$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

∞

$2x + 3y = 9$

$4x + 6y = 18$

QUESTION

B − S = +ve

The difference between two numbers is 26 and one number is three times the other. Find them.

$x > y$

Sol?

$N_1 \rightarrow x \uparrow \rightarrow 39$

$N_2 \rightarrow y \downarrow \rightarrow 13$

$x - y = 26$

$x = 3y$

$3y - y = 26$

$2y = 26$

$y = 13$

$x = 39$

QUESTION

The larger of two supplementary angles exceeds the smaller by 18 degrees. Find Them.

Sol^n

$x > y$

$\text{Angle}_1 \rightarrow x \rightarrow 99^\circ$

$\text{Angle}_2 \rightarrow y \rightarrow 81^\circ$

$x + y = 180$

$x - y = 18$

$x = 99$

$x = 99^\circ$

$y = 180 - 99$

$y = 81^\circ$

QUESTION

The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10 km the charge paid is Rs. 105 and for a journey of 15 km, the charge paid is Rs. 155. What are the fixed charges and the charge per km? How much does a person have to pay for travelling a distance of 25 km?

Sol^n

Fixed charge → ₹x (₹5)

Variable charge → ₹y/km (₹10/km)

$f + v = 105$

$x + 10y = 105$

$x + 15y = 155$

$5y = 50$

$y = 10$

$y = ₹10/km$

$x = 105 - 100$

$x = ₹5$

Total fare = $f + v$

$= 5 + (25 \times 10)$

$= ₹255$

QUESTION

A fraction becomes, 9/11 if 2 is added to both the numerator and the denominator. If, 3 is added to both the numerator and the denominator it becomes 5/6. Find the fraction.

Soln Fraction → Num → x, Den → y

$\frac{x}{y} \rightarrow \frac{7}{9}$

$$\frac{x+2}{y+2} = \frac{9}{11}$$

$$\Rightarrow 11x + 22 = 9y + 18$$

$$\Rightarrow \boxed{11x - 9y = -4}$$

$$\frac{x+3}{y+3} = \frac{5}{6}$$

$$\Rightarrow 6x + 18 = 5y + 15$$

$$\Rightarrow \boxed{6x - 5y = -3}$$

$6(11x - 9y = -4) \rightarrow 66x - 54y = -24$

$11(6x - 5y = -3) \rightarrow 66x - 55y = -33$

$(-)\ (+)\ (+)$

$y = 9$

$11x - (9 \times 9) = -4$

$11x = 81 - 4$

$11x = 77$

$x = 7$

QUESTION

$36 = 3\times10+6\times1$

The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.

Sol?

y	x
Tens	Unit

$No = (10y + x)$

$\text{In. No} = (10x + y)$

$x + y = 9$

$9(10y + x) = 2(10x + y)$

$90y + 9x = 20x + 2y$

$88y = 11x$

$x = 8y$

$y = 1$

$x = 8$

$No = 10y + x$

$= 18$

In No = 81

7 | 8

9

162