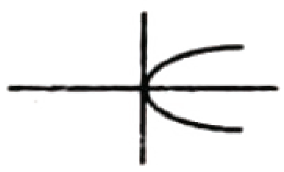
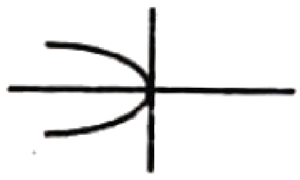
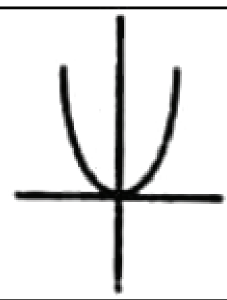
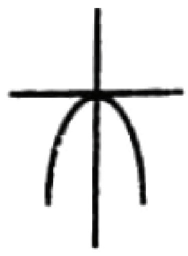
PARABOLA

01 STANDARD PARABOLAS

S.NO	CONCEPT	$y^2 = 4ax(a>0)$	$y^2 = -4ax(a>0)$	$x^2 = 4ay(a>0)$	$x^2 = -4ay(a>0)$
1.	GRAPH				
2.	FOCUS	$(a,0)$	$(-a,0)$	$(0,\ a)$	$(0,-a)$
3.	DIRECTRIX EQUATION	$x+a=0$	$x-a=0$	$y+a=0$	$y-a=0$
4.	VERTEX	$(0,0)$	$(0,0)$	$(0,0)$	$(0,0)$
5.	LATUS RECTUM (L.R.)	$4a$	$4a$	$4a$	$4a$
6.	END OF L.R.	$(a,\pm 2a)$	$(-a,\pm 2a)$	$(\pm 2a,a)$	$(\pm 2a,-a)$
7.	AXIS	$y=0$	$y=0$	$x=0$	$x=0$
8.	FOCAL DISTANCE OF POINT (X,Y)	$(x+a)$	$(a-x)$	$(y+a)$	$(a-y)$
9.	PARAMETRIC EQUATION	$x=at^2$, $Y=2at$	$x=at^2$, $Y=2at$	$x=2at,\ y=at^2$	$x=2at,\ y=-at^2$
10.	PARAMETRIC POINT	$(at^2,2at)$	$(-at^2,2at)$	$(2at,at^2)$	$(2at,-at^2)$

02 EQUAL OF PARABOLA IN VARIOUS CONDITIONS

1. Equation of parabola whose vertex is (α,β) and axis is parallel to x-axis is $(y-\beta)^2 = \pm 4a(x-\alpha)$ where 4 a is L.R

2. Equation of parabola whose axis is parallel to x-axis is $x = ay^2 + by + c$

3. Equation of parabola whose vertex is (α,β) and axis is parallel to y-axis is $(x-\alpha)^2 = \pm 4a(y-\beta)$, $4a$ is L.R

4. Equation of parabola whose axis is parallel to y-axis is $y = ax^2 + bx + c$

5. Equation of parabola whose axis is $ax+by+c=0$ and tangent at vertex is $bx-ay+d=0$

and whose latus rectum is 4A is $\left(\frac{ax+by+c}{\sqrt{a^2+b^2}}\right)^2 = \pm 4A\frac{(bx-ay+d)}{\sqrt{a^2+b^2}}$

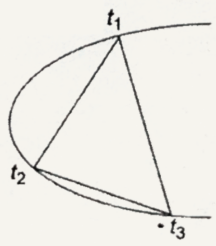
Equation of Chord Joining Points t_1 and t_2 on Parabola $y^2=4ax$ $y-2at_1 = \frac{2}{t_1+t_2}\left(x-at_1^2\right)$

NOTE

For the parabola $y^2 = 4ax$:

• Slope of the chord joining the points and $t_2 = \frac{2}{t_1+t_2}$

- The chord joining points t_1 and t_2 if intersects axis at point $(k,0)$, then $t_1t_2 = -\frac{k}{a}$. • Condition for the chord to be focal chord $t_1t_2 = -1$
- If one end of focal chord is $(at^2, 2at)$, then coordinate of its other end will be obtained by replacing t by $\frac{-1}{t}$, i.e., the coordinate of the other end will be $\left(\frac{a}{t^2}, \frac{2a}{t}\right)$. • Condition for the chord to subtend right angle at the vertex $t_1t_2 = -4$. • All the chord which subtends right angle at the vertex passes through fixed point $(4a,0)$. • Focal chord never subtend right angle at vertex or line subtending right angle at vertex never be a focal chord. • Let the variable chord is drawn from point $P(2a,0)$, meets parabola at Q and R then $\frac{1}{PQ^2} + \frac{1}{PR^2} = \frac{1}{4a^2}$ $(constant)$ • Area of triangle whose vertices are given by $A(t_1)$, $B(t_2)$ and $C(t_3)$ on parabola



$$\Delta = \frac{1}{2}\begin{vmatrix} at_1^2 & 2at_1 & 1 \\ at_2^2 & 2at_2 & 1 \\ at_3^2 & 2at_3 & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = a^2 \left|(t_1 - t_2)(t_2 - t_3)(t_3 - t_1)\right|$$

03 EQUATION OF TANGENT & NORMAL

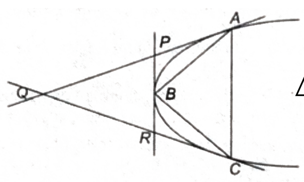
- Equation of Tangent at Point (x_1, y_1) on Parabola $y^2 = 4ax$ $T = 0 \Rightarrow yy_1 = 2a(x + x_1)$
- Equation of normal at 't', $tx + y = 2at + at^3$
- Equation of tangent at point t, $ty = x + at^2$
- Equation of normal with slope m, $y = mx - 2am - am^3$
- Point of normality, $(am^2, -2am)$
- Equation of tangent with slope m $y = mx + \frac{a}{m}$
- has point of contact $\frac{a}{m^2}, \frac{2a}{m}$

04 PROPERTIES OF TANGENT

- Condition for line with finite slope say $y = mx + c$ to be tangent to parabola is $c = \frac{a}{m}$. Hence $m \neq 0$, then, To be a chord $cm < a$. Neither chord nor tangent is $cm > a$. • Image of the focus in any tangent always lies on directrix. • At any point P if tangent is drawn at parabola meet the axis at T and normal to parabola meet axis on N then $ST = SN = SP$ where S is focus. i) Tangent at points t_1 and t_2 always intersect on $[at_1t_2, a(t_1 + t_2)]$.

ii) Area of triangle formed by tangents drawn on points $(t_1, t_2 and\ t_3)$ on parabola.

Let $A(t_1)$, $B(t_2)$ and $C(t_3)$ be the points on parabola an tangent at A, B and C forming ΔPQR.



$$\Delta PQR = \frac{1}{2}\Delta ABC$$

iii) Tangents if drawn by taking any point on its directrix always touches the parabola at the ends of a focal chord.

iv) If tangents are drawn from P touching the parabola at A and B. Then AB is called chord of contact of P with respect to parabola $P(h,k)$, then

- Length of $AB = \frac{\sqrt{S_1}\sqrt{k^2 + 4a}}{a}$; where $s_1 = k^2 - 4ah$
- $\Delta PAB = \frac{(s_1)^{\frac{3}{2}}}{2a}$

v) If tangents are drawn at any 3 points on the parabola, then orthocentre of triangle formed by them is always lie on directrix.

05 PROPERTIES OF NORMAL

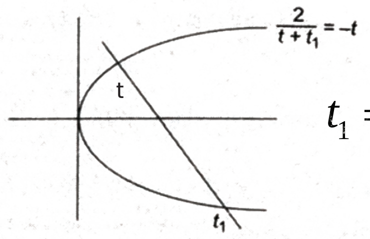
For the parabola $y^2 = 4ax$

i) Line $y = mx + c$ is a normal to the parabola if $c = -2am - am^3$ ii) From a point there can be maximum 3 normals to the parabola.

iii) If from point (h,k), three normals be drawn with slope $m_1 + m_2 + m_3 = 0$ m_1, m_2 and m_3, then and point of normalities corresponding to these are given by $(am_1^2 - 2a_1m_1)$, $(am_2^2 - 2am_2)$ and $(am_3^2 - 2am_3)$, called conormal points.

iv) If from a point 3 normals are possible, then sum of the ordinates of conormal points is always equal to zero.

v) If a normal is drawn at point t on parabola meets the parabola again at point t then t_1 is always equal to

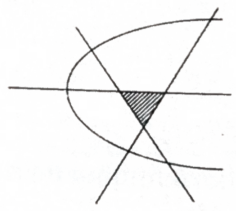


$$t_1 = -t - \frac{2}{t}$$

vi) If normals drawn at points t_1 and t_2 on parabola meets each other on parabola at point t_3 then $t_1 + t_2 + t_3 = 0$

vii) Point of intersection of normal drawn at t_1 and t_2 on parabola is given by $a\left(t_1^2 + t_2^2 + t_1 t_2 + 2\right), -a t_1 t_2 \left(t_1 + t_2\right)$

viii) Area of triangle formed by the normal drawn at points t_1, t_2 and t_3



$$\Delta = \frac{1}{2}\left|a^2\left(t_1 - t_2\right)\left(t_2 - t_3\right)\left(t_3 - t_1\right)\left(t_1 + t_2 + t_3\right)^2\right|$$

ix) From a point $(h,0)$ three normals are possibles on the parabola if $h > 2a$, i.e. if $h \le 2a$, then from $(h,0)$ there is only one normal, i.e. axis of parabola.

x) The length of subnormal at any point on parabola $y^2 = 4ax$ is constant known as semi latus rectum.

06 OPTICAL PROPERTY OF PARABOLA

Let $y^2 = 4ax$ be a parabola then all the rays from $x \to \infty$ parallel the axis of parabola after reflecting from internal surface of parabolic mirror always passes through focus of parabola.

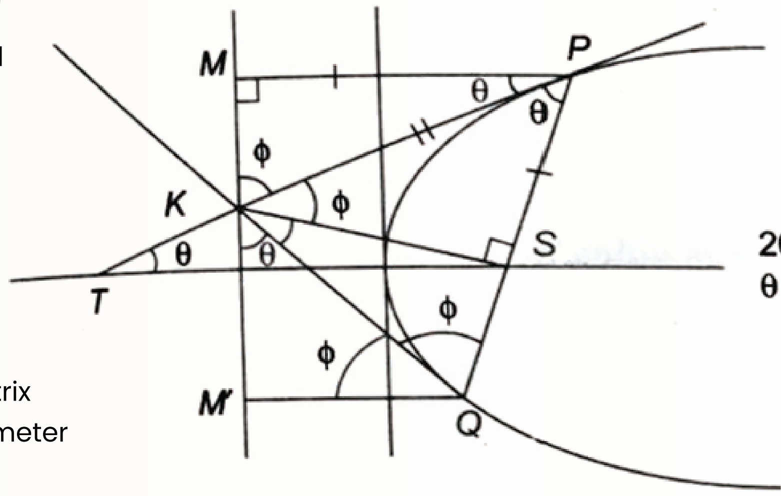
Features of Parabola

Let P be any point on the parabola and tangent at P meets the axis at T and directrix at K. Let tangent from K meets parabola at P and Q. Let S be the focus of parabola, PM is perpendicular to directrix and QM is also perpendicular to directrix.

- $\angle TPM = \angle TPS = \angle STP = \angle SKQ$
- Tangents at the ends of focal chord always meets each other on directrix at an angle of Therefore, mutually orthogonal tangents touches the parabola at the end of a focal chord. Since tangent form directrix are mutually perpendicular. Hence, directrix is called director circle of parabola.
- $\because \angle PKQ = 90°$

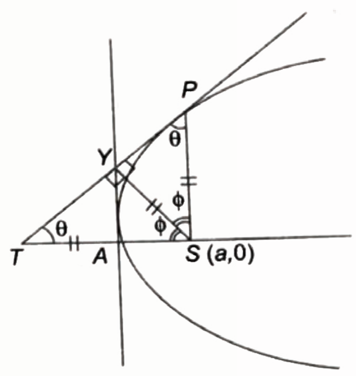
  $\therefore$ Taking PQ, *i.e.*, focal chord as a diameter if a circle being drawn that always touches the directrix at the point where tangent at point P & Q on the parabola meet the directrix.
- In the above figure $\Delta PMK \cong \Delta PSK$ and $\Delta SKQ \cong \Delta M'KQ$

  $\therefore \angle KSP = 90°$, *i.e.*, the intercept of tangent between point of contact and directrix always subtend right angle at the focus, therefore taking that intercept as a diameter if a circle being drawn that always passes through focus of parabola.



2θ + 2ϕ = 180°
θ + ϕ = 90°

- If tangent being drawn at point P on parabola meets tangent at vertex at Y.
- If A be the vertex. $\Rightarrow \frac{SY}{SP} = \frac{AS}{SY} \Rightarrow SY^2 = AS.SP$ Hence, AS, SY, SP are always in G.P.
- $\Delta SPY \cong \Delta STY$
- Perpendicular drawn from focus to the tangent always bisects the angle between axis and focal radii of point of contact.



07 CONGRUENT OR EQUAL PARABOLA

- Two parabolas are said to be equal or congruent if they have the same latus rectum
- Two equal parabolas with the same axis different vertex if sketch in the same irection never meets each other.

Length of focal chord, $l = \left|a\left(t_1 - t_2\right)^2\right|$ $l = \left|a\left(t_1 + \frac{1}{t_1}\right)^2\right|$

08 DIAMETER OF A PARABOLA

$\Rightarrow$ Lactus of mid-point of the set of parallel chords is called diameter let $y^2 = 4ax$ be the parabola and chords are drawn with slope m, then equation of corresponding diameter will be, $y = \frac{2a}{m}$

- Each and every line parallel to axis of parabola is its diameter.
- Tangents at the end of the chord always meet each other on the diameter corresponding to the chord.