Introduction To Inequalities

Inequalities vs Equations

$x + 3 > 2$ $\underline{x + 3 = 2}$

$<$

$\geq$

$\leq$

Unacademy Atoms

Types of Inequalities

- Strict inequality
- Slack Inequality

$$\begin{pmatrix} > \\ < \end{pmatrix} \qquad \begin{pmatrix} \geq \\ \leq \end{pmatrix}$$

Types of Intervals

$\cancel{[4, \infty]}$

Open Intervals	Close Intervals	Semi-open Semi-close Intervals
$3 < x < 5$	$3 \le x \le 5$	$3 < x \le 5$
$\boxed{x \in (3, 5)}$	$\boxed{x \in [3, 5]}$	$x \in (3, 5]$

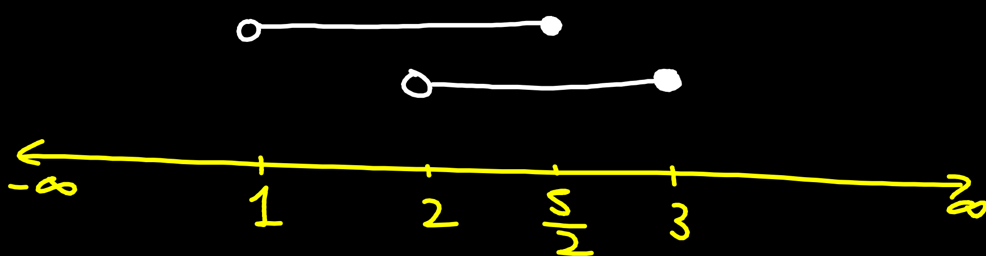
belongs to | element of

$x \in (-\infty, 2)$

$(-\infty, 2]$

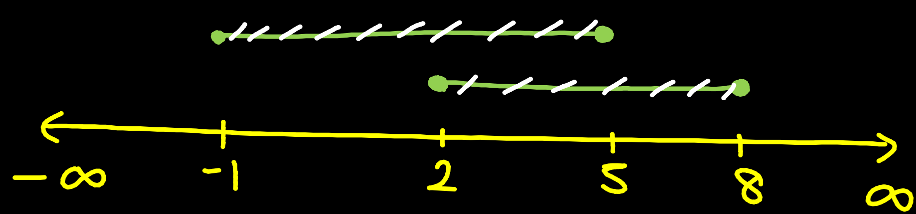
NOTE: " + ∞ " an A " - ∞ " is always open

$$(2, 3] \cup \left(1, \frac{5}{2}\right] = (1, 3]$$



Union of Intervals

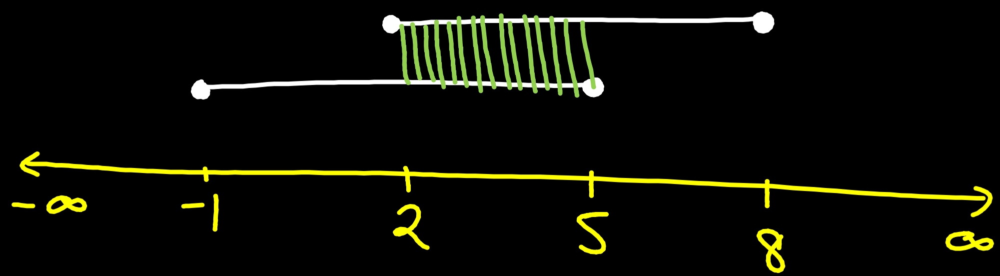
$[-1, 5] \cup [2, 8]$



Union :- $[-1, 8]$

Intersection of Intervals

$[-1, 5] \cap [2, 8] = [2, 5]$



Normal	Shortcut
$x^2 - 5x - 6$	$x^2 - 5x - 6$
$= x^2 - 6x + x - 6$	$(x-6)(x+1)$
$= x(x-6) + 1(x-6)$	
$= (x+1)(x-6)$	

$-5 = -6 + 1$

$1x^2 - 3x + 2 = (x-2)(x-1)$

$1x^2 - 5x + 4 = (x-1)(x-4)$

$1x^2 - x - 20 = (x-5)(x+4)$

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$\boxed{deg = 1}$

$3x - 2 = 5$

$\boxed{3x - 2 \geq 5 \atop \leq}$

Solving Linear Inequalities

Important points to Remember

1. Sign of inequality changes on multiplying OR dividing by a negative number
2. Never ever cross multiply a variable quantity
3. Cross multiply variable quantities if and only if we know its sign

$$(2) \qquad 2 > 1$$

$$-4 < -2$$

$$4 > 2$$

$$\frac{2}{x} < 1$$

$$\frac{2}{x} \times x < 1 \times x$$

$$\cancel{x}\left(\frac{2}{\cancel{x}}\right) = 1 \times x$$

$$\boxed{2 = x}$$

Q

$x \neq 0$	$x \neq 0$	$x \neq 0$
$\frac{2}{x^2} < 1$	$\frac{2}{\lvert x\rvert} < 1$	$x\left(\frac{2}{x}\right) < (1)x$
$\frac{2}{\cancel{x^2}} \cdot \cancel{x^2} < 1 \times x^2$	$\frac{2}{\lvert x\rvert} < 1$	$2 < x$
$2 < x^2$	$2 < \lvert x\rvert$	~~$\frac{2}{x} < 1 \Rightarrow 2 < x$~~
✓	✓	✗

Solving Linear Inequalities

Q-1 $3x - 14 > 22$

$$3x - \cancel{14} + \cancel{14} > 22 + 14$$

$$\frac{\cancel{3}x}{\cancel{3}} > \frac{36}{3}$$

$$\boxed{x > 12}$$

$$\boxed{x \in (12, \infty)}$$

Solving Linear Inequalities

$16 - 5x \geq 21$

$-5x \geq 21 - 16$

$\frac{-5x}{-5} \leq \frac{5}{-5}$

$\boxed{x \leq -1}$

$\boxed{x \in (-\infty, -1]}$

Solving Linear Inequalities

Solve the inequality $\frac{x}{4} < \frac{5x-2}{3} - \frac{7x-3}{5}$

$$\frac{x}{4} < \left(\frac{5x-2}{3} - \frac{7x-3}{5}\right)$$

$$\frac{x}{4} < \frac{25x-10-21x+9}{15}$$

$$15x < 4(4x-1)$$

$$15x < 16x-4$$

$$15x - 16x < -4$$

$$\frac{-x}{-1} > \frac{-4}{-1}$$

$$\boxed{x > 4} \qquad \boxed{x \in (4, \infty)}$$

Solving Linear Inequalities

Solve the inequality $\frac{1}{2}\left[\frac{3x}{5}+4\right] \geq \frac{1}{3}(x-6)$

$$3\left(\frac{3x}{5}+4\right) \geq 2(x-6)$$

$$\frac{9x}{5}+12 \geq 2x-12$$

$$\frac{9x}{5}-2x \geq -24$$

$$5\left(\frac{-x}{5} \geq -24\right)$$

$$(-1)\left(-x \geq -120\right)$$

$$\boxed{x \leq 120}$$

$$\boxed{x \in (-\infty, -120]}$$

Final Ans

$d = 2, 3, 4, \ldots$

Polynomial Inequalities

Polynomial Inequalities

Solve:- $x^2 - 3x < 0$ $\boxed{deg = 2}$

$x = 0$

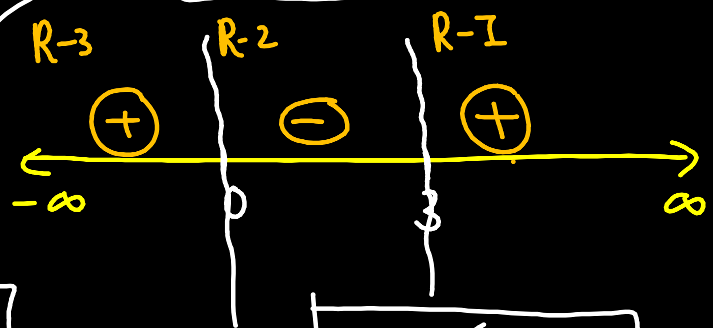
$x - 3 = 0$

$\boxed{\text{False}}$

$\boxed{0 < 0}$

$\boxed{0 < 0}$

$\underbrace{x}\,\underbrace{(x-3)} < 0$



$\boxed{x \in (0, 3)}$

$x(x-3)$

$4(4-3) = +$

$2(2-3) = -$

$(-1)(-1-3) = +$

Q

Solve the inequality -

$\boxed{x(x-2)(x+3) \ge 0}$

$\ge$ $\le$ •

$>$ $<$ ∘

S-5

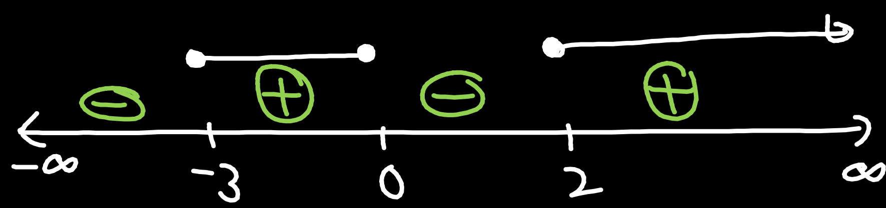
Even ⇒ same

odd ⇒ change

S-1 Factorize $1x(1x-2)(1x+3) \ge 0$

S-2 Coff. of $x = 1$ $\quad x^1(x-2)^1(x+3) \ge 0$

S-3



$\boxed{x \in [-3, 0] \cup [2, \infty)}$

Poly Ineq.

WAVY CURVE METHOD

Wavy Curve Method for Solving Inequalities

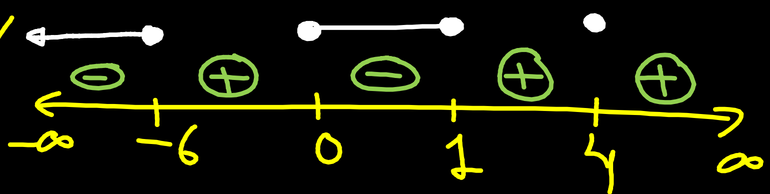
1. Factorize the expression into Linear Factors
2. Make Coefficient of x as 1
3. Plot roots on Number Line
4. Mark dot and circle
5. Put sign
   a. If power → Even then sign will remain same
   b. If power → Odd then sign will change
6. Take the required interval

Solve the inequality

$x^{1}(x-4)^{2}(x+6)^{3}(x-1)^{1} \le 0$

S-1 ✓

S-2 ✓



S-3

S-4 $(-\infty, -6] \cup [0, 1] \cup \{4\}$

S-5

Unacademy Atoms

Q

Solve for ‘x’ : $(x + 3)^1 (x - 7)^1 > 0$

A. $x \in (-3, 7)$

B. $x \in [-3, 7]$

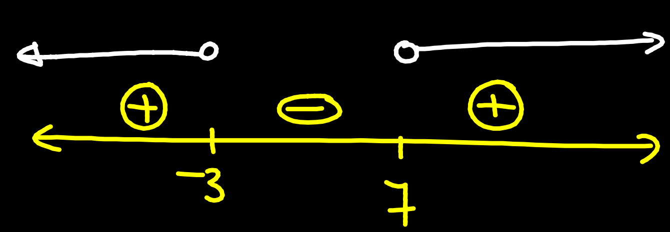
✓ C. $x \in (-\infty, -3) \cup (7, \infty)$

D. $x \in (-\infty, -3] \cup [7, \infty)$

S-1 ✓

S-2 ✓

S-3



$(-\infty, -3) \cup (7, \infty)$

Solve for ‘x’ : $\underbrace{(2 - x)}(x - 5) \geq 0$

A. $x \in (2, 5)$

B. $x \in [2, 5]$ ✓

C. $x \in (-\infty, 2) \cup (5, \infty)$

D. $x \in (-\infty, 5]$

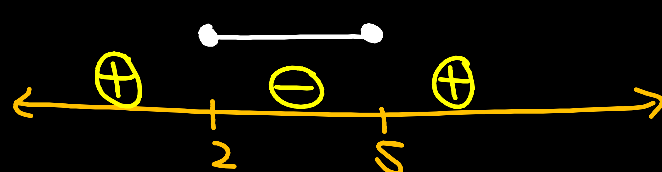
S-1 ✓

S-2 ✓

S-3

$$(x-2)(x-5) \leq \frac{0}{-1}$$

$$(x-2)^1(x-5)^1 \boxed{\leq 0}$$



Q

Solve for 'x' : $x(x - 5) \le 6$

A. $x \in (-1, 6)$

B. $x \in [-1, 6]$ ✓

C. $x \in (-\infty, -1) \cup (6, \infty)$

D. $x \in (-\infty, -1] \cup [6, \infty)$

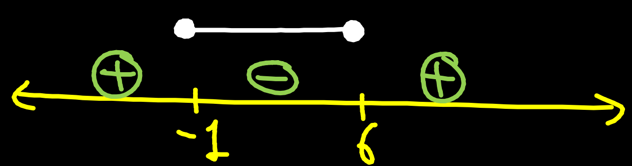
$$x^2 - 5x - 6 \le 0$$

$$(x-6)^1 (x+1)^1 \le 0$$

S-1 ✓

S-2 ✓

S-3



Unacademy Atoms

Q

Solve for 'x' : (3x + 2) (x - 1)^2 (x - 4) < 0

A. $x \in (-2/3, 4)$

B. $x \in [-2/3, 4]$

✓C. $x \in (-2/3, 1) \cup (1, 4)$

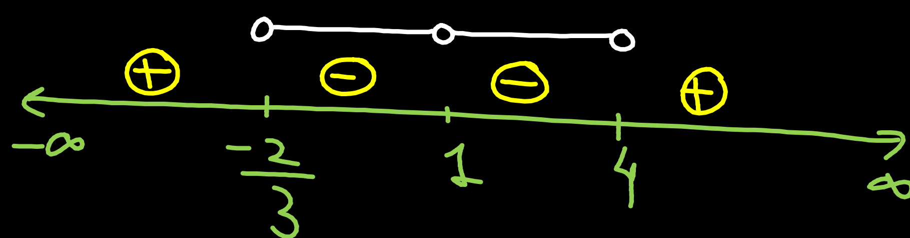
D. $x \in [-2/3, 1] \cup [1, 4]$

S-1 ✓

S-2 ✓

S-3

$$\left(x+\frac{2}{3}\right)^{1}(x-1)^{2}(x-4)^{1} < 0$$



$$\left(\frac{-2}{3}, 4\right) - \{1\}$$

Q

$(x - 1)^2 (x + 1)^3 (x - 4) < 0$

A. $x \in (-1, 4)$

B. $x \in [-1, 4]$

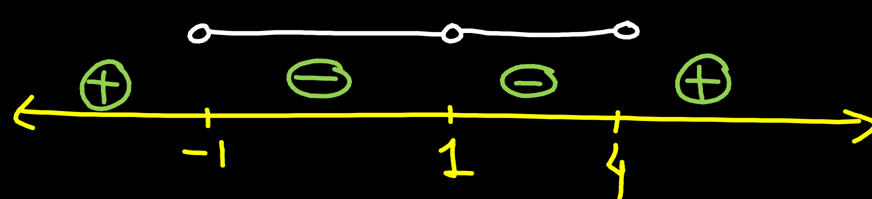
✓C. $x \in (-1, 1) \cup (1, 4)$

D. $x \in (-\infty, -1] \cup [4, \infty)$

S-1 ✓

S-2 ✓

S-3



Q

$\cancel{(x^2+1)}\,(x - 2)^2 (x - 3) < 0$, then x belongs to

$(x^2+1) = +$

A. $(-\infty, 2) \cup (2, 3)$
B. $(-\infty, 3)$
C. $(2, 3)$
D. None of these

$\cancel{(x-2)^2}\,(x-3) < 0$

S-1

$(x-2)^2 = \cancel{0}$ OR $+$

$x - 3 < 0$

$x < 3$

$x \neq 2$

$(-\infty, 3) - \{2\}$

$\frac{p}{q}$

RATIONAL INEQUALITY

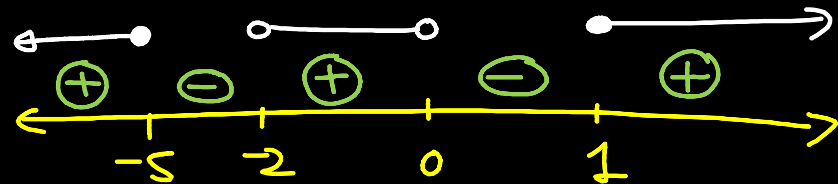
Q

Solve for 'x' : $\dfrac{(x-1)(x+5)}{\boxed{x(x+2)}} \geq 0$

S-1 ✓

S-2 ✓

S-3



$(-\infty, -5] \cup (-2, 0) \cup [1, \infty)$

Q

The complete solution set of the inequality $\dfrac{x^4-3x^3+2x^2}{x^2-x-30} \geq 0$ is:

A. $(-\infty, -5) \cup (1, 2) \cup (6, \infty) \cup \{0\}$

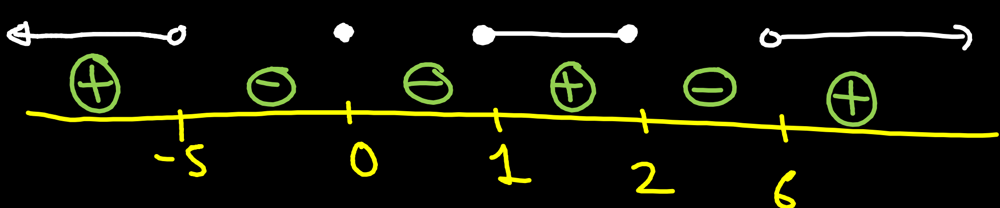
B. $(-\infty, -5) \cup [1, 2] \cup (6, \infty) \cup \{0\}$

C. $(-\infty, -5) \cup [1, 2] \cup (6, \infty)$

D. none of these

$$\frac{x^2(x^2-3x+2)}{(x^2-x-30)} \geq 0$$

$$\frac{x^2(x-1)^1(x-2)^1}{(x-6)^1(x+5)^1} \geq 0$$



Q

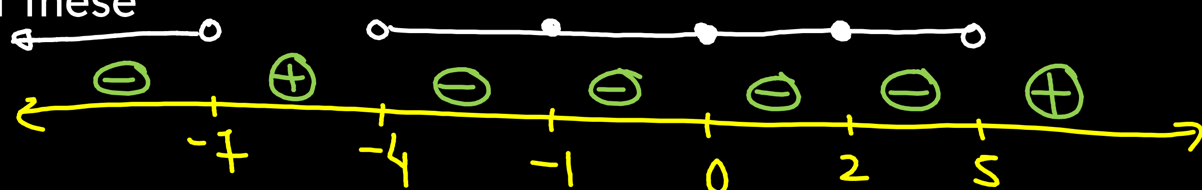
Solve for ‘x’ : $\dfrac{x^{2}(x+1)^{10}(x-2)^{4}}{(x+4)^{7}(x+7)^{5}(x-5)^{3}} \le 0$

A. $x \in (-7, -4) \cup (5, \infty)$

B. $x \in (-\infty, -7) \cup (-4, 5)$

C. $x \in (-\infty, -4) \cup (5, \infty)$

D. none of these



Q

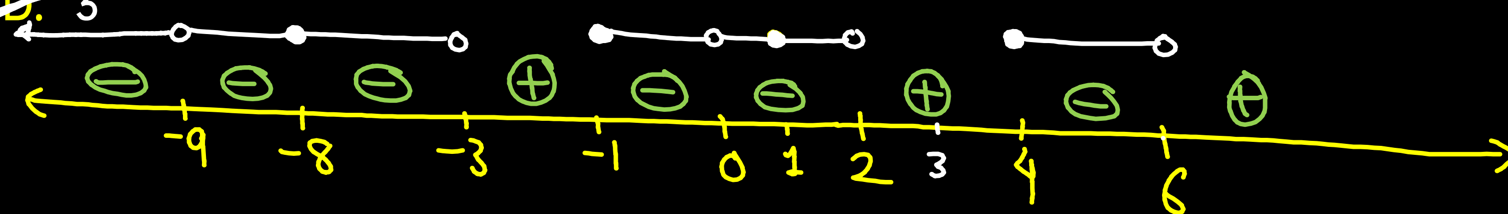
Number of positive integral values of x satisfying the inequality

$$\frac{(x-4)^{2013}.(x+8)^{2014}(x+1)}{x^{2016}(x-2)^{3}.(x+3)^{5}.(x-6)(x+9)^{2012}} \le 0 \text{ is}$$

A. 0
B. 1
C. 2
D. 3

$\boxed{x \Rightarrow 1, 4, 5}$

(D) 4



Important Concept

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If a > 0 and D < 0 then Q.E. → always positive

If a < 0 and D < 0 then Q.E. → always negative

$D = b^2 - 4ac$

$D = 1^2 - 4(1)(1)$

$D < 0$

$a < 0$
$D < 0$

$\boxed{-x^2 + x - 1}$ ⇒ always negative

$a = -1$

$a < 0$

$D = (+1)^2 - 4(-1)(-1)$

$D < 0$

$ax^2 + bx + c$ ⇒ Always Positive if $a > 0$ and $D < 0$

eg. $\boxed{x^2 + x + 1 \Rightarrow \text{Always Positive}}$

$ax^2 + bx + c$ ⇒ Always Negative if $a < 0$ and $D < 0$

Q

$$\frac{x^2-5x+12}{\boxed{x^2-4x+5}} > 3$$

A. $x \in (1/2\ ,\ 3)$ ✓

B. $x \in [-1,\ 3]$

C. $x \in (-\infty,\ 1/2)$

D. $x \in [1/2,\ 3]$

$1x^2-4x+5 \longrightarrow$ Always positive

$D = 16-4(1)(5)$

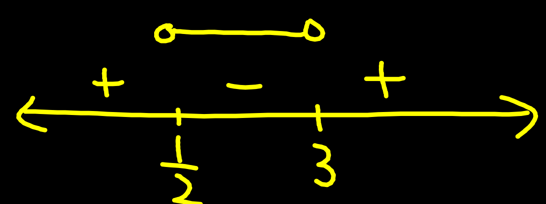
$\boxed{D<0 \quad a>0}$

$x^2-5x+12 > 3x^2-12x+15$

$0 > 2x^2-7x+3$

$2x^2-6x-x+3 < 0$

$(2x-1)(x-3) < 0$



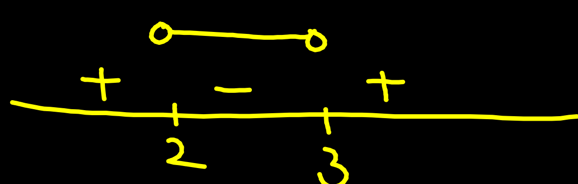
Q

$$\boxed{\frac{x^2-5x+6}{(x^2+x+1)} < 0}$$

$x^2+x+1 \Rightarrow$ Always positive

$$x^2-5x+6 < 0$$

$$(x-2)^1(x-3)^1 < 0$$



$$\boxed{x \in (2, 3)}$$

$$x^2 - 5x + 6$$

$$(x-2)(x-3)$$

Q

The solution of the inequality $\dfrac{(e^x-1)(2x-3)\boxed{\cancel{(x^2+x+2)}}}{(\cancel{\sin x-2})x(x+1)} \leq 0$

★★★ #NVTip

A. $\left[\dfrac{3}{2},\infty\right)$

✓ B. $(-\infty,-1)\cup\left[\dfrac{3}{2},\infty\right)$

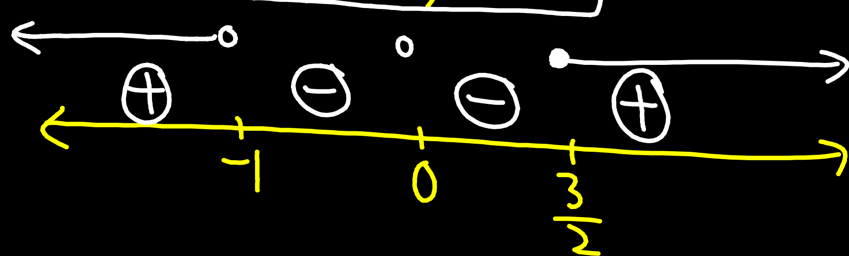
C. $(-1,0)\cup\left[\dfrac{3}{2},\infty\right)$

C. $R-\{0,-1\}$

$\boxed{e\sim 2.7}$

e^2-1

$$\boxed{\dfrac{(e^x-1)\left(x-\frac{3}{2}\right)}{x(x+1)}} \geq 0$$



$x=-\frac{1}{2}$

$D<0$

$a>0$

$\sin x \in [-1,1]$

$(\sin x-2) \Rightarrow$ Always Negative

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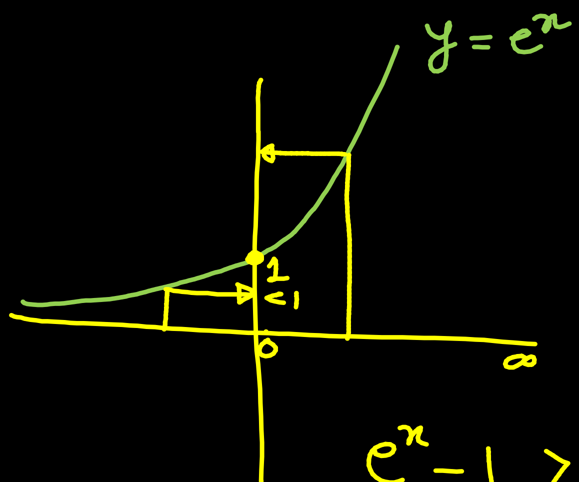


$e^x - 1 > 0$ if $x > 0$

$e^x - 1 < 0$ if $x < 0$

Q

Solve for ‘x’ : $$\frac{(x^2-3x+2)\cancel{(x^2+2x+2)}}{(x-3)\cancel{(-2x^2+x-1)}} \overset{\le}{\cancel{\ge}} 0$$

A. (-∞, ∞) - [1, 2]

B. (-∞, 1] U [2, 3)

C. (-∞, 2] U (3, ∞)

D. None of these

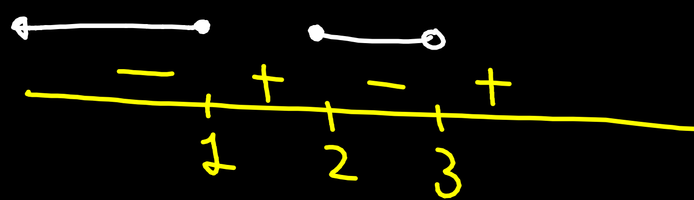
$x^2+2x+2 \rightarrow$ Always Positive

$a>0 \quad D<0$

$-2x^2+x-1 \Rightarrow$ Always Negative

$a<0$
$D<0$

$$\frac{(x-1)(x-2)}{(x-3)} \le 0$$



Unacademy Atoms

Q

Solve for 'x' : $\frac{x^3-2x^2+5x+2}{x^2+3x+2} \geq 1$

A. (-2, -1) U [2, ∞)

B. (-∞, -2) U (-1, 0) U [1, 2]

C. (-2, -1) U [0, 1] U [2, ∞)

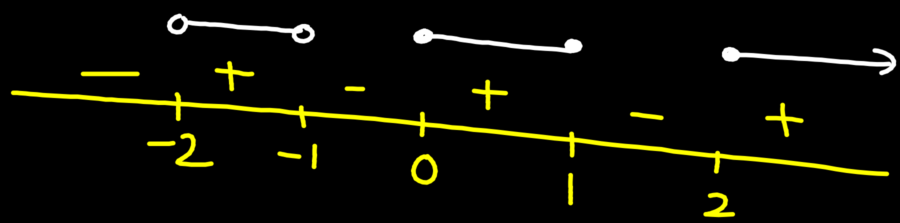
D. None of these

$$\frac{x^3-2x^2+5x+2}{x^2+3x+2} - 1 \geq 0$$

$$\frac{x^3-2x^2+5x+\cancel{2}-x^2-3x-\cancel{2}}{(x+1)(x+2)} \geq 0$$

$$\frac{x(x^2-3x+2)}{(x+1)(x+2)} \geq 0 \Leftarrow \frac{x^3-3x^2+2x}{(x+1)(x+2)} \geq 0$$

$$\frac{x^1(x-1)^1(x-2)^1}{(x+1)^1(x+2)^1} \geq 0$$



Domain & Range

How to find Domain?

1. $\dfrac{N^r}{D^r} \Rightarrow D^r \neq 0$

$f(x) = \sqrt{x-1}$

2. $\sqrt{\square} \Rightarrow \boxed{\square \geq 0}$

$x - 1 \geq 0$

3. $\log_a \square \Rightarrow \boxed{\square > 0}$

$\boxed{x \geq 1}$

4. $\sin^{-1} \square$ OR $\cos^{-1} \square \Longrightarrow \boxed{-1 \leq \square \leq 1}$ $\boxed{\cap}$

5. $\sec^{-1} \square$ OR $\operatorname{cosec}^{-1} \square \Longrightarrow \boxed{\square \leq -1}$ OR $\boxed{\square \geq 1}$ $\boxed{\text{Union}}$

6. $\tan^{-1} \square \,/\, \cot^{-1} \square$ $\quad \square \in R$

Q

Domain of definition of the function $\boxed{f(x) = \frac{3}{4-x^2} + \log_{10}(x^3 - x)}$ is :

-0.5

JEE MAIN 2019

$f(3) = \frac{3}{4-3^2} + \log_{10}(3^3 - 3)$

~~A. (1, 2)~~

~~B. (-1, 0) ∪ (1, 2)~~

~~C.~~ (1, 2) ∪ (2, ∞)

~~D.~~ (-1, 0) ∪ (1, 2) ∪ (2, ∞)

-2 -1 0 1 2

$(-1, 0) \cup (1, 2) \cup (2, \infty)$

C-1 $\quad 4 - x^2 \neq 0$

and

$\boxed{x \neq \pm 2}$

$\boxed{x \in R - \{-2, 2\}}$

C-2 $\quad x^3 - x > 0$

$x(x-1)(x+1) > 0$

− + − +

-1 0 1

$\left(\frac{-1}{2}\right)^3 - \left(\frac{-1}{2}\right)$

$\Rightarrow \frac{-1}{8} + \frac{1}{2}$

$\Rightarrow +$

Q

#NVStyle

Unacademy Atoms

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The domain of the function

$$f(x)=\frac{\cos^{-1}\left(\frac{x^2-5x+6}{x^2-9}\right)}{\log_e(x^2-3x+2)}\text{ is}$$

A. $(-\infty,1)\cup(2,\infty)$

B. $(2,\infty)$

C. $\left[-\frac{1}{2},1\right)\cup(2,\infty)$

D. $\left[-\frac{1}{2},1\right)\cup(2,\infty)-\left\{\frac{3+\sqrt{5}}{2},\frac{3-\sqrt{5}}{2}\right\}$ $-\{3\}$

C-1 $\log_e(x^2-3x+2)\neq 0$

$$x^2-3x+2\neq 1$$

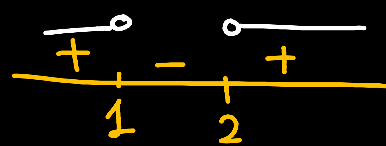
$$x^2-3x+1\neq 0$$

$$x\neq \frac{3\pm\sqrt{9-4}}{2}$$

$$\boxed{x\neq \frac{3\pm\sqrt{5}}{2}}$$

C-2 $x^2-3x+2>0$

$$(x-1)(x-2)>0$$



$$(-\infty,1)\cup(2,\infty)$$

C-3 $-1\leq \dfrac{x^2-5x+6}{x^2-9}\leq 1$

$$\frac{(x-2)\cancel{(x-3)}}{\cancel{(x-3)}(x+3)}$$

$$-1\leq \frac{x-2}{x+3}\leq 1$$

$$-1\leq \frac{x-2}{x+3} \qquad \frac{x-2}{x+3}\leq 1$$

C-4

$$x^2-9\neq 0$$

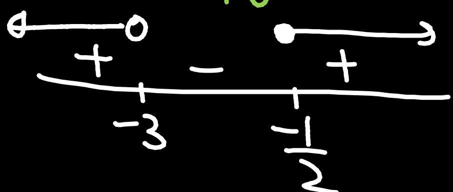
$$\boxed{x\neq \pm 3}$$

$$x\neq 3$$

$$-1 \le \frac{x-2}{x+3}$$

$$\frac{x-2}{x+3}+1 \ge 0$$

$$\frac{2x+1}{x+3} \ge 0$$

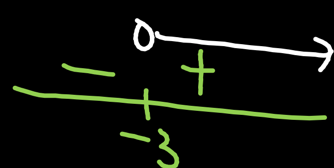


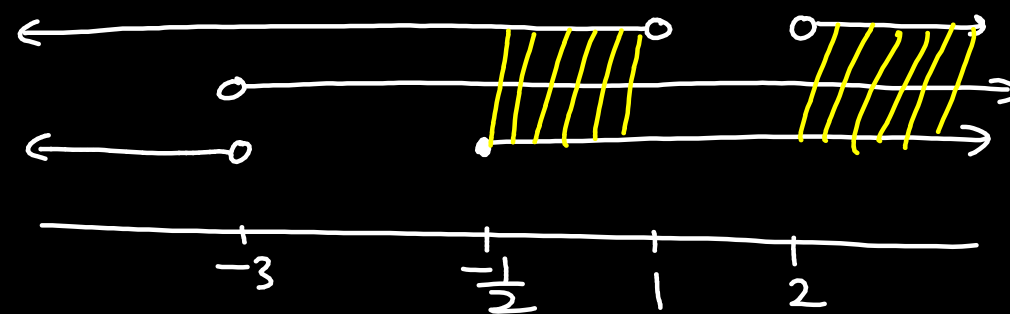
$$\frac{x-2}{x+3} \le 1$$

$$\frac{x-2}{x+3} - 1 \le 0$$

$$\frac{-5}{x+3} \le 0$$

$$\frac{1}{(x+3)} \ge 0$$





$$\left[-\frac{1}{2}, 1\right) \cup (2, \infty) - \left\{3, \frac{3 \pm \sqrt{5}}{2}\right\}$$

JEE MAIN 2022

Q

Considering only the principal values of the inverse trigonometric functions, the domain of the function $\boxed{f(x) = \underset{\text{Sin}}{\cancel{\cos}}^{-1}\left(\frac{x^2 - 4x + 2}{x^2 + 3}\right)}$ is :

A. $\left(-\infty, \frac{1}{4}\right]$

B. $\left[-\frac{1}{4}, \infty\right)$

C. $\left(-\frac{1}{3}, \infty\right)$

D. $\left(-\infty, \frac{1}{3}\right]$

$(-\sqrt{3})^2 + 3$

$\underline{x^2 + 3} = +$

C-1

$$-1 \le \frac{x^2-4x+2}{x^2+3} \le 1$$

$\left[-\frac{1}{4}, \infty\right)$

$-x^2-3 \le x^2-4x+2$

$0 \le 2x^2-4x+5$

$2x^2-4x+5 \ge 0$

$D = 16-4(2)(5)$

$D<0$

$a>0$

$\boxed{x \in R}$

$\cancel{x^2}-4x+2 \le \cancel{x^2}+3$

$-4x \le 1$

$\boxed{x \ge \frac{1}{-4}}$

$\cap$

Unacademy Atoms

Q The domain of the function $f(x)=\sin^{-1}\left(\dfrac{x^2-3x+2}{x^2+2x+7}\right)$ is.

JEE MAIN 2022

A. [1, ∞)
B. (-1, 2]
C. [-1, ∞)
D. (-∞. 2]

$$-1 \le \frac{x^2-3x+2}{\boxed{x^2+2x+7}} \le 1$$

x^2+2x+7 → Always Positive

$\boxed{\begin{matrix} a>0 \\ D<0 \end{matrix}}$

$$-x^2-2x-7 \le x^2-3x+2 \le x^2+2x+7$$

$a>0$
$D<0$

$$-x^2-2x-7 \le x^2-3x+2$$

$$0 \le 2x^2-x+9$$

Always Positive

$$\boxed{2x^2-x+9} \ge 0$$

$$\boxed{x \in R}$$

$$\cancel{x^2}-3x+2 \le \cancel{x^2}+2x+7$$

$$-5 \le 5x$$

$$\boxed{-1 \le x}$$

$$\cap$$

Unacademy Atoms

Q The domain of the function $f(x)=\sin^{-1}\left(\frac{3x^2+x-1}{\boxed{(x-1)^2}}\right)+\cos^{-1}\left(\frac{x-1}{x+1}\right)$ is #chalaki ≡ C

$\boxed{x \neq 1, -1}$

JEE MAIN 2021

A. $\left[0, \frac{1}{4}\right]$

B. $[-2,0] \cup \left[\frac{1}{4}, \frac{1}{2}\right]$

C. $\left[\frac{1}{4}, \frac{1}{2}\right] \cup \{0\}$

D. $\left[0, \frac{1}{2}\right]$

$\frac{3(0.01)+0.1-1}{0.81}$

$\frac{0.13-1}{0.81}$

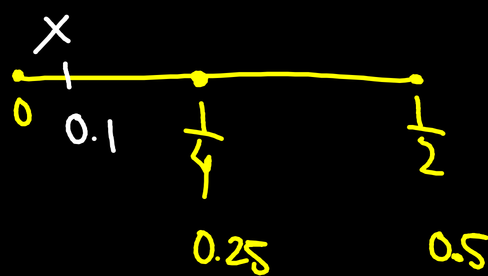
$-\frac{87}{81} \notin [-1,1]$

$\frac{0.1-1}{0.1+1}$

$\frac{-0.9}{1.1}$

$-\frac{9}{11}$

$[-1,1]$



$x \neq 1$

$$-1 \leq \frac{3x^2+x-1}{(x-1)^2} \leq 1 \qquad -1 \leq \frac{x-1}{x+1} \leq 1$$

$-x^2+2x-1 \leq 3x^2+x-1$	$3x^2+x-1 \leq x^2-2x+1$	$-1 \leq \frac{x-1}{x+1}$	$\frac{x-1}{x+1} \leq 1$
$\vdots$	$\vdots$	$\vdots$	$\vdots$

$\cap$

Ⓒ

Unacademy Atoms

Q

JEE MAIN 2022

$[-3] = -3$

$\boxed{\sqrt{5} = 2.2}$

The domain of the function

$1 - \cancel{5} + \cancel{5}$

$$f(x) = \sin^{-1}[2x^2 - 3] + \log_2\left(\log_{\frac{1}{2}}\left(x^2 - 5x + 5\right)\right),$$

$8 - 3$

0

where [t] is the greatest integer function, is :

$x = 2$ " "

$x = 0$ Not Satisfying

$x = 1$ " "

~~A.~~ $\left(-\sqrt{\frac{5}{2}}, \frac{5-\sqrt{5}}{2}\right)$

~~B.~~ $\left(\frac{5-\sqrt{5}}{2}, \frac{5+\sqrt{5}}{2}\right) = (1.4, 3.6)$

✓ C. $\left(1, \frac{5-\sqrt{5}}{2}\right)$

$(1, 1.4)$

~~D.~~ $\left[1, \frac{5+\sqrt{5}}{2}\right) = [1, 3.6)$

$$-1 \le [2x^2 - 3] \le 1$$

$$-1 \le [2x^2] - \cancel{3} \le 1$$

$$+3 \qquad +\cancel{3} \qquad +3$$

$$2 \le [2x^2] \le 4$$

$$[2x^2] \Rightarrow 2, 3, 4$$

$$2 \le 2x^2 < 5$$

$$\boxed{1 \le x^2 < \frac{5}{2}} \checkmark$$

$$\log_{\frac{1}{2}}(x^2 - 5x + 5) > 0$$

$$[\square] = 2, 3, 4$$

$$\boxed{2 \le \square < 5}$$

$$x^2 - 5x + 5 > 0$$

Q

Find Domain $f(x) = \sqrt{\log_{1/4}\left(\frac{5x-x^2}{4}\right)}$ is

A. [1, 4]
B. [1, 0]
C. [0, 5]
D. [5, 0]

H.W.

Unacademy Atoms

Logarithmic Inequality

Log Inequalities

Case I : base > 1

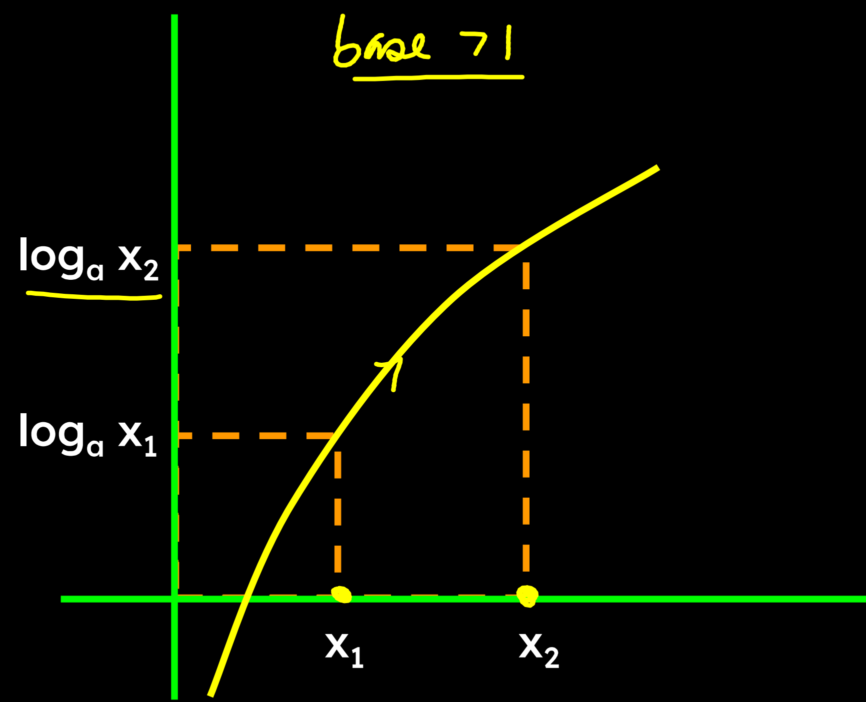
$$\boxed{y = \log_a x}$$

i) $x > 0$
ii) $a > 0$
iii) $a \neq 1$

$$x_1 < x_2$$

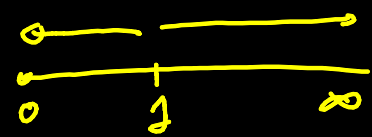
$$\log x_1 < \log x_2$$

base > 1

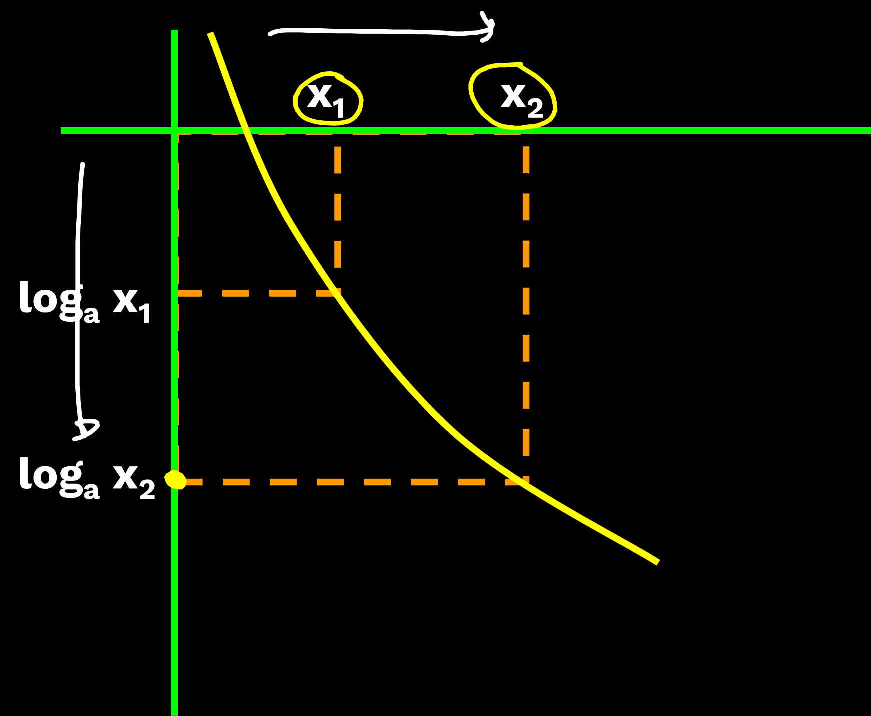


Log Inequalities

Case II : 0 < base < 1



$$\boxed{\begin{gathered} x_1 < x_2 \\ \log x_1 > \log x_2 \end{gathered}}$$



Concept

base > 1 ⇒ same
0 < base < 1 ⇒ change

Q

Solve for x : $\boxed{\log_3 x < 4}$

base = 3

$\log_3 x = 4$

$\boxed{x = 3^4}$

$x < 3^4$

$\boxed{x < 81}$

Domain

$\boxed{x > 0}$

$\cap$

$\cancel{(-\infty, 81)}$

-1, -2, -3 ....

Final $(0, 81)$

Unacademy Atoms

Log Inequalities

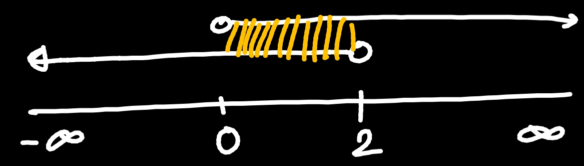
Step 1: Solve the inequality

Step 2: Find Domain

Step 3: Take intersection of answers of step 1 and step 2

Solve for x : $\log_{1/2} x > -1$

$$\log_{\frac{1}{2}} x > -1$$



Solve the inequality	Domain
base < 1	$\boxed{x > 0}$
$x < \left(\frac{1}{2}\right)^{-1}$	
$\boxed{x < 2}$	

$\boxed{x<2} \cap \boxed{x>0}$

$$\boxed{x \in (0, 2)}$$

Unacademy Atoms

Q

$$\log_{\frac{1}{3}}(5x-1) > 0$$

A. $x \in (1/5, 2/5)$

B. $x \in (1/5, \infty)$

C. $x \in (2/5, \infty)$

D. $x \in (0, 2/5)$

Solve	Domain
$5x-1 < \left(\frac{1}{3}\right)^0$	$5x-1 > 0$
$5x-1 < 1$	$\boxed{x > \frac{1}{5}}$
$5x < 2$	
$\boxed{x < \frac{2}{5}}$	

$f(x) = \sqrt{\text{Alia Bhatt}}$

$\boxed{\text{Alia Bhatt} \geq 0}$

$f(x) = \log_{\text{Kuch bhi}}(\text{Deepika Padukone})$

$\boxed{\text{Deepika Padukone} > 0}$

$$f(x) = \sin^{-1}\boxed{RK} \quad \text{OR} \quad \cos^{-1}\boxed{RK}$$

$$\boxed{-1 \leq RK \leq 1}$$

Solution set of the inequality, $2 - \log_2 (x^2 + 3x) \geq 0$ is.

~~A.~~ (-4, 1]

B. [-4, -3) U (0, 1]

~~C.~~ (-∞, -3) U (1, ∞)

~~D.~~ (-∞, -4) U [1, ∞)

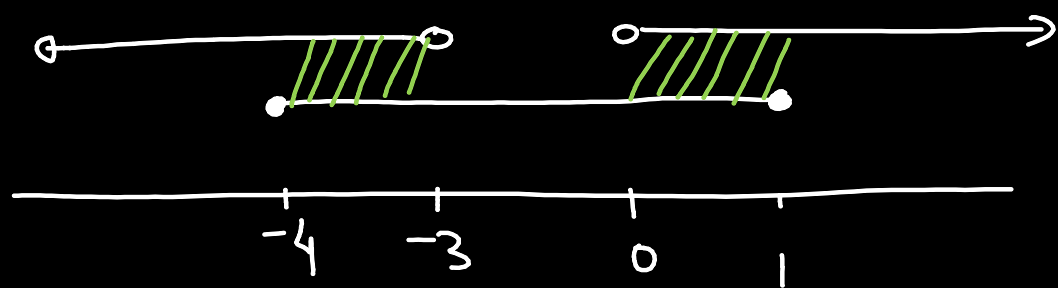
$$2 \geq \log_{2} (x^2 + 3x)$$

Solve	Domain
$4 \geq x^2 + 3x$	$x^2 + 3x > 0$
$0 \geq x^2 + 3x - 4$	$x(x+3) > 0$
$(x+4)(x-1) \leq 0$	+ − + ; −3, 0
+ − + ; −4, 1	

S-1 diagram.

S-2 $\cap$

$x = -5$ will not Satisfy



$[-4,-3) \cup (0,1]$

If $\log_{1/3}\left(\frac{3x-1}{x+2}\right)$ is less than unity then x must lie in the interval -

A. $(-\infty, -2) \cup (5/8, \infty)$

B. $(-2, 5/8)$

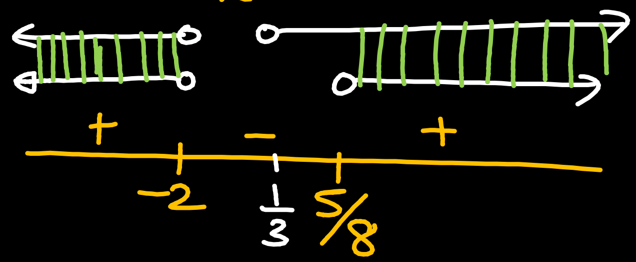
C. $(-\infty, -2) \cup (1/3, 5/8)$

D. $(-2, 1/3)$

$$\log_{\frac{1}{3}}\left(\frac{3x-1}{x+2}\right) < 1$$

Solve.	Domain
$\Rightarrow \frac{3x-1}{x+2} > \left(\frac{1}{3}\right)$	$\frac{3x-1}{x+2} > 0$
$\Rightarrow \frac{3x-1}{x+2} - \frac{1}{3} > 0$	+ − + ; −2, $\frac{1}{3}$
$\Rightarrow \frac{9x-3-x-2}{\cancel{3}(x+2)} > 0$	

$$\frac{8x-5}{x+2} > 0$$



$$\frac{5}{8} > \frac{1}{3}$$

$$(-\infty, -2) \cup \left(\frac{5}{8}, \infty\right)$$

$$\log_x (2x-1) > 1$$

Logarithmic Inequality

(with Variable Base)

Log Inequalities (with variable base)

$$\log_{\text{base}} \square \geq K$$

C-1 $\boxed{\text{base} > 1}$ OR C-2 $\boxed{0 < \text{base} < 1}$ and Domain

base > 1 same

0 < base < 1 change

$\cap$ C-I

C-I-Final

$\cap$ C-II

$$\big((\text{Case I}) \cup (\text{Case II})\big) \cap \text{Domain}$$

Q

$\log_x (x^3 - x^2 - 2x) < 3$

A. $x \in (-1, 0) \cup (2, \infty)$

B. $x \in (-\infty, -2) \cup (0, \infty)$

C. $x \in (0, \infty)$

D. $x \in (2, \infty)$

C-1 $x > 1$	C-2 $0 < x < 1$	Domain
$\cancel{x^3} - x^2 - 2x < \cancel{x^3}$	$\cancel{x^3} - x^2 - 2x > \cancel{x^3}$	$x^3 - x^2 - 2x > 0$
$-x(x+2) < 0$	$-x(x+2) > 0$	$x(x^2 - x - 2) > 0$
$x(x+2) > 0$	$x(x+2) < 0$	$x(x-2)(x+1) > 0$
$+\ \ -\ \ +$; $-2 \quad 0 \quad 1$	$+\ \ -\ \ +$; $-2 \quad 0 \quad 1$	$-\ \ +\ \ -\ \ +$; $-1 \quad 0 \quad 1 \quad 2$
$\boxed{x \in (1, \infty)}$	$\boxed{x \in \phi}$	

$$C\text{-}I \cup C\text{-}II \equiv (1, \infty)$$

$$(1, \infty) \cap \text{Domain} \equiv \underline{(2, \infty)}$$

Final Ans

Q

$$\log_{x}\left(2x-\frac{3}{4}\right) > 2$$

$x = 1 \rightarrow$ Nhi

$x = 2 \rightarrow$ Nhi

A. $x \in (3/8, 1/2) \cup (1, 3/2)$

~~C.~~ $x \in (1/2, \infty)$

~~B.~~ $x \in (3/8, 3/2)$

~~D.~~ $x \in (3/2, \infty)$

$$\log_2\left(4-\frac{3}{4}\right) > 2$$

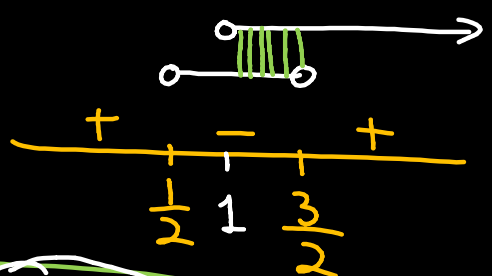
$$\frac{13}{4} > 4$$

$<$

C-I $x > 1$	C-II $0 < x < 1$	Domain
$4\left(2x-\frac{3}{4} > x^2\right)$	$2x - \frac{3}{4} < x^2$	$2x - \frac{3}{4} > 0$
$8x - 3 > 4x^2$		$x > \frac{3}{8}$
$4x^2 - 8x + 3 < 0$		
$4x^2 - 6x - 2x + 3 < 0$		
$(2x-1)(2x-3) < 0$		

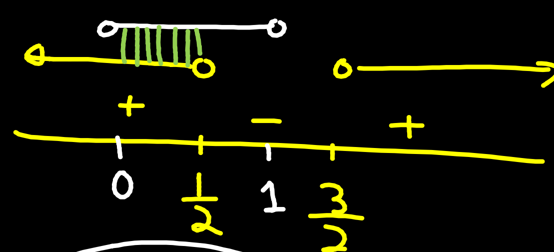
Unacademy Atoms

$(2x-1)(2x-3) < 0$



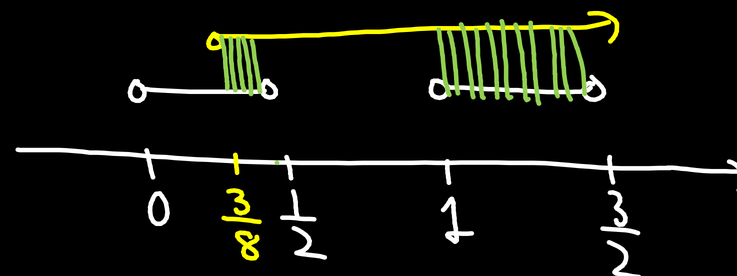
CI $\Rightarrow \left(1, \frac{3}{2}\right)$ $\cup$

$(2x-1)(2x-3) > 0$



$\left(0, \frac{1}{2}\right)$

A



$\left(0, \frac{1}{2}\right) \cup \left(1, \frac{3}{2}\right)$

Q

$\log_{2x}(x^2 - 5x + 6) < 1$

H.W.

$$\boxed{\sqrt{x^2+2x-3} > x+1}$$

Root
wali
Ineq

Irrational Inequality

Method to solve

$\sqrt{4} = +2$

$$\sqrt{f(x)} \geq g(x)$$

I. Find Domain $f(x) \geq 0$

II. a) If g(x) < 0 then all values of x will satisfy

b) If g(x) > 0 then solve $f(x) \geq g^2(x)$

$\sqrt{\text{Any}} = \oplus$

$$(II(a) \cup II(b)) \cap I$$

$\boxed{(C\text{I} \cup C\text{II}) \cap \text{Domain}}$

(0) $\sqrt{f(x)} \geq g(x)$

C-1 $g(x) \geq 0$	C-2 $g(x) < 0$	Domain
$\oplus \geq \oplus$	$\oplus \geq \ominus$	$f(x) \geq 0$
Squaring both Sides	All solⁿ	

Unacademy Atoms

Q

Complete set of values of x satisfying the inequality $x - 3 < \sqrt{x^2 + 4x - 5}$ is

A. $(-\infty, -5] \cup [1, \infty)$
B. $(-5, 3)$
C. $(3, 5)$
D. $(-5, 3)$

A

$$x - 3 < \sqrt{x^2 + 4x - 5}$$

C-I: $x - 3 \geq 0 \Rightarrow \boxed{x \geq 3}$

$(x-3)^2 < \left(\sqrt{x^2+4x-5}\right)^2$

$\cancel{x^2} - 6x + 9 < \cancel{x^2} + 4x - 5$

$14 < 10x$

C-II: $x - 3 < 0 \Rightarrow \boxed{x < 3}$

$x - 3 < \sqrt{x^2+4x-5}$

$\ominus < \oplus$ All

$\boxed{x \in (-\infty, 3)}$

Domain

$x^2 + 4x - 5 \geq 0$

$(x+5)(x-1) \geq 0$

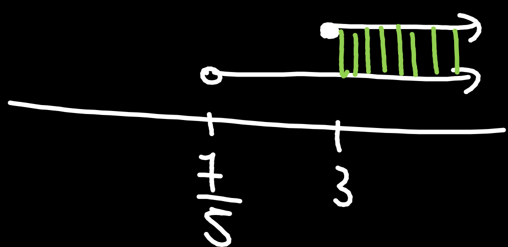
+ − +

−5 1

$C\text{-}I \cup C\text{-}II \quad (-\infty, \infty)$

$14 < 10x$

$\frac{7}{5} < x$



$C\text{-}I \equiv [3, \infty) \cup (-\infty, 3)$

Set of all real values of x satisfying the inequality

$\sqrt{x^2-5x-24} > x+2$

A. (-∞, -3]

B. (8, ∞)

C. (-∞, 8]

D. $\left(-\infty, \frac{-28}{9}\right]$

$x+2 \geq 0$

$\boxed{x \geq -2}$

$\cancel{x^2}-5x-24 >$
$\cancel{x^2}+4x+4$

$-28 > 9x$

$\boxed{\frac{-28}{9} > x}$

$-\frac{28}{9}$ -2

ϕ

$x+2 < 0$

$\boxed{x < -2}$

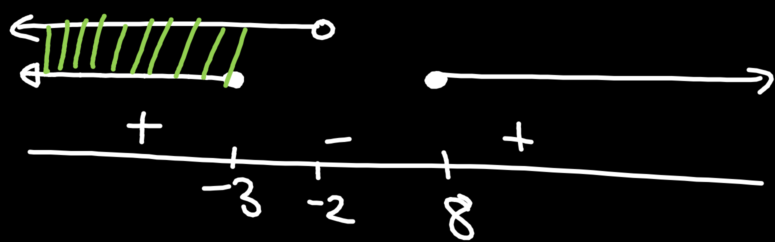
$\oplus > \ominus$

$x < -2$

$(-\infty, -2)$

$$x^2 - 5x - 24 \geq 0$$

$$(x-8)(x+3) \geq 0$$



$$(-\infty, -3]$$

Complete solution set of the inequality $\sqrt{x+18} < 2 - x$ is -

A. [-18, -2)
B. $(-\infty, -2) \cup (7, \infty)$
C. $(-18, 2) \cup (7, \infty)$
D. [-18, -2]

H.W.

Q

Solution of the inequality, $x-3<\sqrt{x^2+4x-5}$ is -

A. $(-\infty,-5]\cup[1,\infty)$

B. $(-5,3]$

C. $(-\infty,-5]\cup\left(\frac{7}{5},\infty\right)$

D. $\left(\frac{7}{5},\infty\right)$

H.W.

Unacademy Atoms

1) Mod. Ineq.

2) Range

~~Oct End~~ ~~Jan~~ April

Modulus Inequalities

Solving Modulus Inequalities

$|\,\square\,| \;\; <,\; >,\; \leq,\; \geq \;\; K$

$|x-3| = 2$

$x-3 = \pm 2$

$x = 3 \pm 2$

$\boxed{x = 5, 1}$

Mod Eqn

Modulus Inequalities Type – 1

Solve the following inequalities :

$|x - 3| \geq 2$

$3 \geq 2$ ✓

$|\square| \geq 2$

$x-3 \leq -2$ OR $x-3 \geq 2$

$x \leq 1$ OR $x \geq 5$

$$\boxed{x \in (-\infty, 1] \cup [5, \infty)}$$

Concept

① $|\square| \geq K$

$\boxed{\square \leq -K}$ OR $\boxed{\square \geq K}$

(union)

② $|\square| < K$

$-K < \square < K$

$\boxed{\text{intersection}}$

Solve the following inequalities :

$$\left|\frac{3x}{x^2-4}\right| \le 1$$

$$-1 \le \boxed{\frac{3x}{x^2-4}} \le 1$$

And

$$-1 \le \frac{3x}{x^2-4}$$

$$0 \le \frac{3x}{x^2-4} + 1$$

$$0 \le \frac{3x+x^2-4}{x^2-4}$$

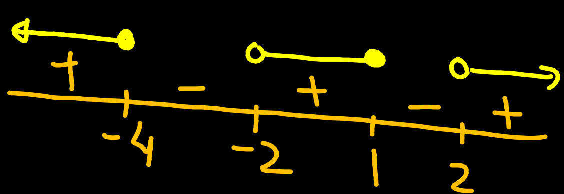
$$\frac{3x}{x^2-4} \le 1$$

$$\frac{3x}{x^2-4} - 1 \le 0$$

$$\frac{3x-x^2+4}{x^2-4} \le 0$$

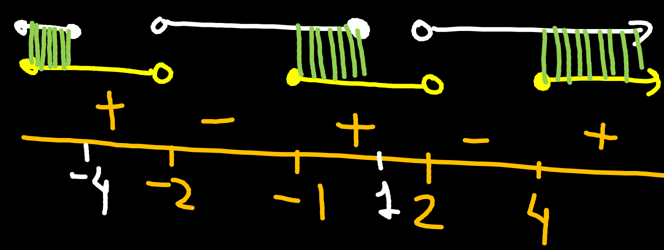
$$\frac{x^2+3x-4}{x^2-4} \geq 0$$

$$\frac{(x+4)(x-1)}{(x-2)(x+2)} \geq 0$$



$$\frac{x^2-3x-4}{x^2-4} \geq 0$$

$$\frac{(x-4)(x+1)}{(x-2)(x+2)} \geq 0$$



$$(-\infty,-4] \cup [-1,1] \cup [4,\infty)$$

① $|\square| < K$

② $|\square| > K$

Solve the following inequalities

$|x^2 - 4x + 4| \geq 1$

$|\square| \geq K$

$x^2 - 4x + 4 \leq -1$ OR $x^2 - 4x + 4 \geq 1$

Union

Modulus Inequalities Type – 2

Q

① $|\square| \le 0$

$\boxed{\square = 0}$ ***

Eg $\boxed{|x-3| \le 0}$

$x-3=0$

$\boxed{x=3}$ only Sol^n

$\boxed{x \in \{3\}}$

$|\square| \ge 0$

$\boxed{\square \in R}$

eg $|x-3| \ge 0$

$\boxed{x \in R}$

Solve the following inequalities :

$||x - 2| - 3| \leq 0$

$|x-2|-3=0$

$|x-2|=3$

$x-2=\pm 3$

$x=2\pm 3$

$\boxed{x=-1,5}$

$x \in \{-1,5\}$

Unacademy Atoms

Modulus Inequalities Type – 3

single mod

$\hookrightarrow$ Mod – open.

C-I
C-II

Solve the following inequalities :

$$\frac{|x+3|+x}{x+2} > 1$$

C–I

$x+3 \geq 0$

$\boxed{x \geq -3}$

$\frac{x+3+x}{x+2} - 1 > 0$

$\frac{2x+3-x-2}{x+2} > 0$

OR

C–II

$x+3 < 0$

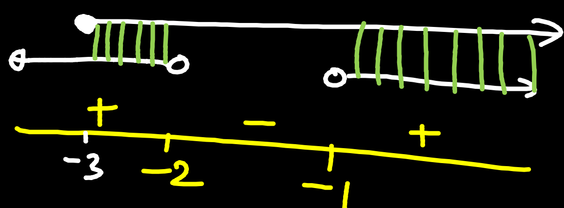
$\boxed{x < -3}$

$\frac{-\cancel{x}-3+\cancel{x}}{x+2} - 1 > 0$

$\frac{-3-x-2}{x+2} > 0$

C–I $\cup$ C–II

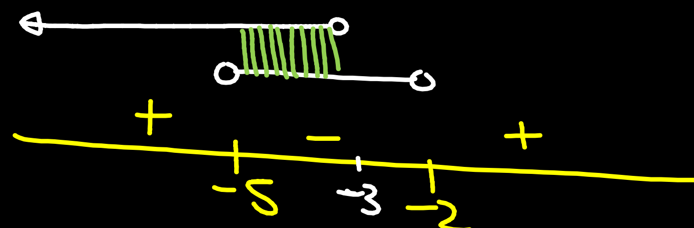
$$\frac{x+1}{x+2} > 0$$



$[-3,-2) \cup (-1,\infty)$

$$\frac{-x-5}{x+2} > 0$$

$$\frac{x+5}{x+2} < 0$$



union

$(-5,-3)$

$(-5,-2) \cup (-1,\infty)$ Final Ans

The set of all real numbers x for which $x^2 - \underbrace{|x + 2|} + x > 0$, is

[JEE 2002]

A. $(-\infty, -2) \cup (2, \infty)$

B. $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$ ✓

C. $(-\infty, -1) \cup (1, \infty)$

D. $(\sqrt{2}, \infty)$

C-I : $\boxed{x \geq -2}$

$$x^2 - (\cancel{x}+2) + \cancel{x} > 0$$

$$x^2 - 2 > 0$$

$$(x-\sqrt{2})(x+\sqrt{2}) > 0$$

C-II : $\boxed{x < -2}$

$$x^2 + (x+2) + x > 0$$

$$\boxed{x^2 + 2x + 2} > 0$$

$$(+) > 0$$

All

$$D = 2^2 - 4(2)$$

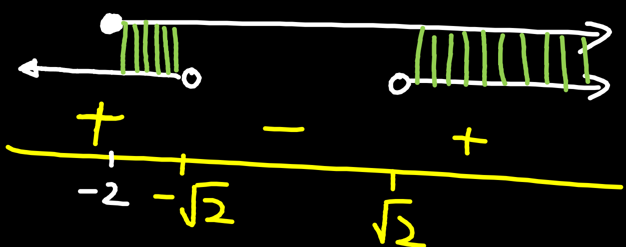
$$= 4 - 8$$

$$= (-)$$

D < 0

a > 0

$(x-\sqrt{2})(x+\sqrt{2}) > 0$



C-I Final Ans :- $[-2, -\sqrt{2}) \cup (\sqrt{2}, \infty)$

C II :-

$(-\infty, -2)$

Union

$(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$

Modulus Inequalities Type – 5

$|a|^2 > |b|^2$ $\qquad$ $|a| < |b|$

Square both side.

$a^2 > b^2$

$a^2 - b^2 > 0$

$\underline{(a-b)(a+b) > 0}$

Solve the following inequalities :

$|x + 3|^2 > |2x - 1|^2$

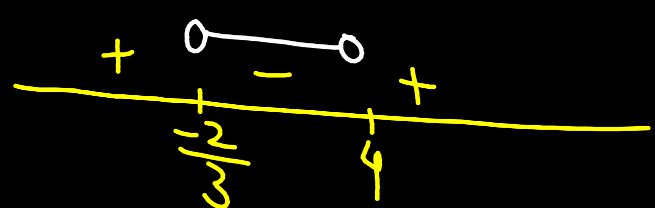
$$(x+3)^2 - (2x-1)^2 > 0$$

$$(x+3+2x-1)(x+3-2x+1) > 0$$

$$(3x+2)(-x+4) > 0$$

$$(3x+2)(x-4) < 0$$

$$\boxed{x \in \left(-\frac{2}{3}, 4\right)}$$



Solve the following inequalities :

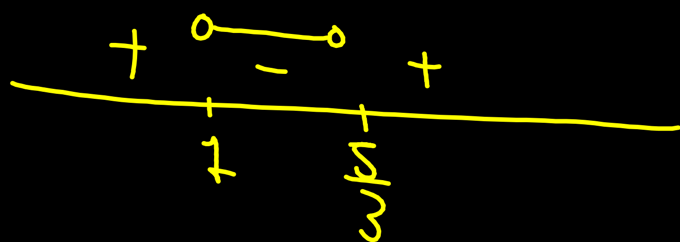
|x - 2| > |2x - 3|

$$(x-2)^2-(2x-3)^2>0$$

$$(x-2-2x+3)(x-2+2x-3)>0$$

$$(-x+1)(3x-5)>0$$

$$(x-1)(3x-5)<0$$



$\boxed{\left(1, \frac{5}{3}\right) : \text{Final Ans}}$

Q

The solution of the inequality $|x^2 - 2x - 3|^2 < |x^2 - x + 5|^2$ is -

A. $(-\infty, 5)$

B. $(-\infty, 2) \cup (3, 8) \cup (8, \infty)$

C. $(-8, \infty)$

D. $(3, 8)$

$$(x^2-2x-3+x^2-x+5)(\cancel{x^2}-2x-3-\cancel{x^2}+x-5)<0$$

$$D = 9-4(2)(2)$$

$$\boxed{\begin{matrix} D<0 \\ a>0 \end{matrix}}$$

$$(\cancel{2x^2-3x+2})(-x-8)<0$$

$$-(x+8)<0$$

$$x+8>0$$

$$\boxed{x>-8}$$

Unacademy Atoms

Modulus Inequalities Type – 6

Concept

$$\underbrace{|\cancel{a}+b|} = |\cancel{a}| + |b| \implies \boxed{a \cdot b \geq 0}$$

	a	b
✓	+2	+3
✗	-2	3
✗	2	-3
✓	-2	-3
✓	0	3
✓	-2	0

LHS = 5
RHS = 5
LHS = 1

a	b	a.b
+	+	+
−	−	+

Solve the following equations

$$|x^3 + (x^2 + x) + 1| = |x^3 + 1| + |x^2 + x|$$

$$|(x^3+1) + (x^2+x)| = |x^3+1| + |x^2+x|$$

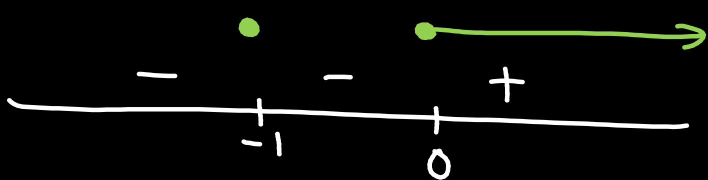
$$|a+b| = |a| + |b|$$

$$a \cdot b \geq 0$$

$$(x^3+1^3)(x^2+x) \geq 0$$

$$(x+1)(\quad)\,x(x+1) \geq 0$$

$$x(x+1)^{2} \geq 0$$



$$\boxed{x \in \{-1\} \cup [0, \infty)}$$

Modulus Inequalities Type – 7

$$|a-b| = |a| + |b| \implies \boxed{a \cdot b \le 0}$$

	a	b
✗	2	3
✓	-2	3
✓	2	-3
✗	-2	-3
✓	0	2

$1 \neq 5$

$2 = 2$

a	b	$a \cdot b$
$-$	$+$	$-$
$+$	$-$	$-$

Solve the following equations

$|x^2 - 4x + 3| + |x^2 - 6x + 8| = |2x - 5|$

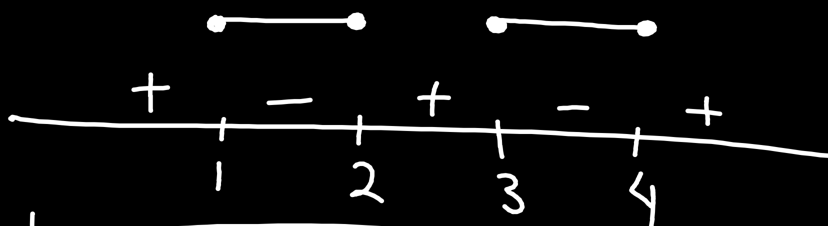
$$|\underline{x^2-4x+3}| + |\underline{x^2-6x+8}| = |(x^2-4x+3)-(x^2-6x+8)|$$

$$|a| + |b| = |a-b|$$

$$a \cdot b \leq 0$$

$$(x^2-4x+3)(x^2-6x+8) \leq 0$$

$$(x-1)(x-3)(x-2)(x-4) \leq 0$$



$$\boxed{x \in [1, 2] \cup [3, 4]}$$

over

Domain & Range

Methods to Find Range

1. $\boxed{f(x) = \dfrac{\text{Linear}}{\text{Linear}}}$

2. $f(x) = \dfrac{\text{Quadratic}}{\text{Quadratic}}$ (1 factor common)

3. $f(x) = \dfrac{\text{Quadratic}}{\text{Quadratic}}$ / $f(x) = \dfrac{\text{Linear}}{\text{Quadratic}}$ (No)

Method 1

$$f(x) = \frac{\text{Linear}}{\text{Linear}}$$

Method

Step - 1 Write $\boxed{x = f(y)}$ ✓

Step - 2 Find domain of f(y)

$$y = \boxed{f(x) = \frac{2x-3}{5x+7}}$$

find Range of f(x)

output

$R - \left\{\frac{2}{5}\right\}$

Find the range of following function $y = \dfrac{3x+2}{x-1}$ $\dfrac{L}{L}$

A. R

B. R - {3}

C. R - {3 ,1}

D. R - {1}

S-1 $y = \dfrac{3x+2}{x-1}$

$xy - y = 3x + 2$

$xy - 3x = y + 2$

$x(y-3) = y + 2$

$\boxed{x = \dfrac{y+2}{y-3}}$

S-2 $x = \dfrac{y+2}{y-3}$

Domain

condi 1

$y - 3 \neq 0$

$\boxed{y \neq 3}$

$\boxed{y \in R - \{3\}}$

$\frac{L}{L} \Rightarrow$ Range

#NVStyle

$$\underbrace{y = \frac{3x+2}{1x-1}}_{\text{Range}}$$

$R - \left\{\frac{3}{1}\right\}$

$\frac{-4}{2}$

$$y = \frac{-4x - 5^{2/3}}{2x + \sqrt{2023}}$$

Range: $R - \{-2\}$

Find the range of following function $y = \dfrac{x^2}{1+x^2}$

'x' wala

A. [0, 1)
B. [0, 1]
C. (0, 1)
D. None

$y = \dfrac{Q}{Q}$ (Coeff of $x = 0$)

↳ $\dfrac{L}{L}$ wala Method hi lagega

S-1 $y = \dfrac{x^2}{1+x^2}$ $\left(\dfrac{L}{L}\right)$

$y + x^2 y = x^2$

$\dfrac{y}{1-y} = x^2$

$y = x^2 - x^2 y$

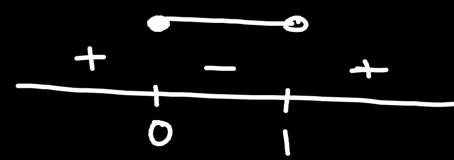
$y = x^2(1-y)$

$\dfrac{y}{1-y} = x^2$

$$x^2 = \frac{y}{1-y}$$

$$y \in [0, 1)$$

$$x = \pm\sqrt{\frac{y}{1-y}}$$



S-2 Domain

$1-y \neq 0$

$y \neq 1$

$$\frac{y}{1-y} \geq 0$$

$$\frac{y}{y-1} \leq 0$$

Method 2

Find the range of following function $y = \dfrac{x^2 - x}{x^2 + 2x} = \dfrac{\cancel{x}(x-1)}{\cancel{x}(x+2)}$

A. $R - \left\{1, -\dfrac{1}{2}\right\}$ ✓

B. R - {1}

C. R

D. $R - \left\{-\dfrac{1}{2}\right\}$

<u>Non-Zero quantity</u>

$y = \dfrac{1x - 1}{1x + 2}$

$x \neq 0$

$x \neq 0 \longrightarrow \boxed{y = \dfrac{x-1}{x+2}} \longrightarrow y \neq \dfrac{-1}{2}$

$R - \left\{1, \dfrac{-1}{2}\right\}$

CUTE DOUBT

$$\cancel{2} \times \frac{3}{\cancel{2}} = x \times 2$$

$$3 = 2x$$

$\sqrt{4} = 2$

$\sqrt{9} = 3$

$\sqrt{25} = 5$

Eqn.

$x^2 = 4 \longrightarrow x = \pm 2$

$x^2 - 4 = 0$

$(x-2)(x+2) = 0$

$x - 2 = 0 \quad x + 2 = 0$

$x = 2 \quad x = -2$

Method 3

$$f(x) = \frac{\text{Quadratic}}{\text{Quadratic}} \quad / \quad f(x) = \frac{\text{Linear}}{\text{Quadratic}}$$

Step - 1 Cross multiply and make Q.E. in 'x'

Step - 2 $D \geq 0$

Q

Find the range of following function $y = \frac{x+2}{x^2+3x+6}$ Real $\left(\frac{L}{Q} / \frac{Q}{Q}\right)$

A. $\left[-\frac{1}{5}, \frac{1}{3}\right]$ B. $\left(-\frac{1}{5}, \frac{1}{3}\right)$ C. $\left(-\frac{1}{5}, \infty\right)$ D. $\left(\frac{1}{3}, \infty\right)$

$$yx^2 + 3xy + 6y = x + 2$$

$$yx^2 + x(3y-1) + (6y-2) = 0 \quad (\text{QE in } 'x')$$

$x \rightarrow$ Real

$$D \geq 0$$

$$(3y-1)^2 - 4(y)(6y-2) \geq 0$$

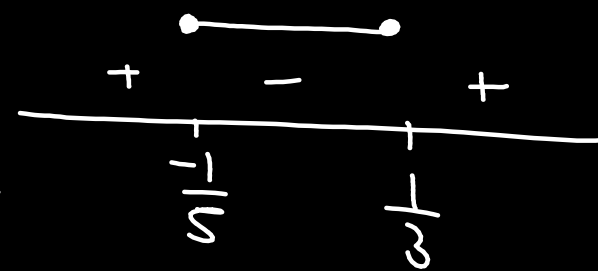
$$9y^2 - 6y + 1 - 24y^2 + 8y \geq 0$$

$$-15y^2 + 2y + 1 \geq 0$$

$$15y^2 - 2y - 1 \leq 0$$

$$15y^2 - 5y + 3y - 1 \leq 0$$

$$(5y+1)(3y-1) \leq 0$$



$$y \in \left[-\frac{1}{5}, \frac{1}{3}\right]$$