Today's Targets

1. Algebra
   (Quadratic, Sequence & Series, P & C, Binomial Theorem, Complex Number)

2. Coordinate Geometry
   (Straight Line, Circles, Parabola, Ellipse, Hyperbola)

3. Trigonometry
   (Trigonometric Ratio's, Trigonometric Equations, Solution of Triangle, H&D)

4. Mathematical Reasoning

5. Statistics

Quadratic Equation

Q. JEE Mains (2023)

Let $a \in R$ and let α, β be the roots of the equation $\boxed{x^2 + 60^{\frac{1}{4}}x + a = 0}$.

If $\boxed{\alpha^4 + \beta^4 = -30}$, then the product of all possible values of a is ....

β

α

$$(\alpha^2+\beta^2)^2 - 2\alpha^2\beta^2$$

$$\left((\alpha+\beta)^2 - 2\alpha\beta\right)^2 - 2\alpha^2\beta^2 = -30.$$

$$\left((-t)^2 - 2a\right)^2 - 2a^2 = -30.$$

$$t^4 + 4a^2 - 4at^2 - 2a^2 + 30 = 0$$

$$2a^2 - 4t^2(a) + 30 + t^4 = 0$$

$\alpha+\beta = -t.$

$\alpha\beta = a.$

$(60)^{\frac{1}{4}} = t$

Prod. of Root $= \dfrac{30+60}{2}$

$= 45.$

Q. JEE Mains (2023)

Root → 1

Let α be a root of the equation $\boxed{(a-c)x^2+(b-a)x+(c-b)=0}$ where a, b, c are distinct real numbers such that the matrix $\begin{bmatrix}\alpha^2 & \alpha & 1\\ 1 & 1 & 1\\ a & b & c\end{bmatrix}$ is singular.

Then the value of $\frac{(a-c)^2}{(b-a)(c-b)}+\frac{(b-a)^2}{(a-c)(c-b)}+\frac{(c-b)^2}{(a-c)(b-a)}$ is

A 6

B 3

C 9

D 12

$a-c=x$

$b-a=y$

$c-b=z$

$$\frac{x^2}{yz}+\frac{y^2}{xz}+\frac{z^2}{xy}=\left(\frac{x^3+y^3+z^3}{xyz}\right)=3.$$

\# $\boxed{0=x+y+z}$

$x^3+y^3+z^3-3xyz=0$

Q. JEE Mains (2023)

If the value of real number $a > 0$ for which $\boxed{x^2 - 5ax + 1 = 0}$ and $\boxed{x^2 - ax - 5 = 0}$ have a common real roots is $\frac{3}{\sqrt{2\beta}}$ then β is equal to ……13…… .

$$x^2 - 5ax + 1 = 0$$
$$x^2 - ax - 5 = 0$$
$$- \quad + \quad +$$
$$-4ax + 6 = 0$$
$$6 = 4ax$$
$$\# \quad \frac{3}{2a} = x \hookrightarrow \text{comm. R} \checkmark$$

$$x^2 = ax + 5$$
$$\left(\frac{3}{2a}\right)^2 = \cancel{a}\left(\frac{3}{2\cancel{a}}\right) + 5$$
$$\frac{9}{\cancel{4}^{2}a^2} = \frac{13}{\cancel{2}}$$
$$\frac{9}{26} = a^2 \Rightarrow a = \frac{3}{\sqrt{26}}$$
$$\hookrightarrow 2 \times 13$$

Q. JEE Mains (2023)

The equation $\boxed{e^{4x} + 8e^{3x} + 13e^{2x} - 8e^{x} + 1 = 0}$, $x \in R$ has:

A two solutions and both are negative

B no solution

C four solutions two of which are negative

D two solutions and only one of them is negative

$\div e^{2x}$

$\boxed{e^{2x} + 8e^{x} + 13 - \frac{8}{e^{x}} + \frac{1}{e^{2x}} = 0.}$

$e^{x} = t.$

$t^2 + 8t + 13 - \frac{8}{t} + \frac{1}{t^2} = 0.$

$\boxed{\left(t^2 + \frac{1}{t^2}\right) + 8\left(t - \frac{1}{t}\right) + 13 = 0.}$

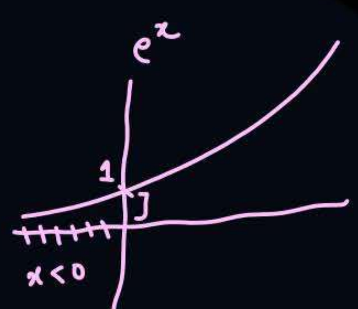
$\left(\left(t - \frac{1}{t}\right)^2 + 2\right) + 8\left(t - \frac{1}{t}\right) + 13 = 0$

$(\alpha^2 + 2) + 8\alpha + 13 = 0$

$\alpha^2 + 8\alpha + 15 = 0$

$(\alpha + 3)(\alpha + 5) = 0$

$\alpha = -3, -5$



$t - \frac{1}{t} = -3$, $t - \frac{1}{t} = -5$

$t^2 + 3t - 1 = 0$ $\quad$ $t^2 + 5t - 1 = 0.$

$t = \frac{-3 \pm \sqrt{9+4}}{2}$ $\quad$ $t = \frac{-5 \pm \sqrt{29}}{2}$

e^x

$e^x = t = \frac{-3 \pm \sqrt{13}}{2}$

$e^x = \frac{-3 + \sqrt{13}}{2} < 1$ $\quad$ $e^x = \frac{-5 + \sqrt{29}}{2} < 1.$

$\Downarrow$

$x < 0$ $\quad$ $\boxed{x<0}$

Q. JEE Mains (2022)

For a natural number n, let $a_n = 19^n - 12^n$. Then, the value of $\frac{31a_9 - a_{10}}{57a_8}$ is

Quad.

Newton's formula

$ax^2 + bx + c = 0$ → α, β

$S_n = \alpha^n \pm \beta^n$

$$\boxed{aS_n + bS_{n-1} + cS_{n-2} = 0}$$

$n \in N$.

$n \geq 2$

$$\boxed{x^2 - (19+12)x + 19\times 12 = 0}$$

$a_n = 19^n - 12^n$.

19, 12

$a_n - (31)a_{n-1} + 19\times 12\, a_{n-2} = 0$

$n = 10$

$a_{10} - 31a_9 + 19\times 12\, a_8 = 0$.

$$4 = \frac{31a_9 - a_{10}}{a_8(19\times 3 = 57)}$$

Q. JEE Mains (2019)

Let α and β the roots of the quadratic equation $\boxed{x^2 \sin\theta - x(\sin\theta\cos\theta + 1) + \cos\theta = 0}$ $= 0(0 < \theta < 45°)$, and $\alpha < \beta$.

Then $\sum_{n=0}^{\infty}\left(\alpha^n + \frac{(-1)^n}{\beta^n}\right)$ is equal to

(A) $\frac{1}{1+\cos\theta} - \frac{1}{1-\sin\theta}$

(B) $\frac{1}{1-\cos\theta} + \frac{1}{1+\sin\theta}$

(C) $\frac{1}{1-\cos\theta} - \frac{1}{1+\sin\theta}$

(D) $\frac{1}{1+\cos\theta} + \frac{1}{1-\sin\theta}$

$$\sum_{n=0}^{\infty}\alpha^n + \sum_{n=0}^{\infty}\frac{(-1)^n}{\beta^n}$$

$$\alpha^0 + \alpha^1 + \alpha^2 + \dots$$

$$\frac{1}{1-\alpha} = \frac{1}{1-\cos\theta}$$

$$(\sin\theta)x^2 - x\,s\theta\,c\theta - x + c\theta = 0$$

$$(x\,s\theta)\{x - c\theta\} - 1\{x - c\theta\} = 0.$$

$$\# \{x-\cos\theta\}\{x\sin\theta - 1\} = 0.$$

$$\boxed{x = \cos\theta,\ \operatorname{cosec}\theta}$$

$\cos\theta \to \alpha$, $\operatorname{cosec}\theta \to \beta$

$$1 - \frac{1}{\beta} + \frac{1}{\beta^2} - \frac{1}{\beta^3} + \dots \infty$$

$$\frac{1}{1-\left(-\frac{1}{\beta}\right)} = \frac{1}{1+\frac{1}{\beta}} = \frac{1}{1+\sin\theta}$$

Sequence & Series

Q. JEE Mains (2023)

The sum of the $\boxed{\text{common terms}}$ of the following three arithmetic progressions.

3, 7, 11, 15, ..., 399

2, 5, 8, 11, ..., 359 and

2, 7, 12, 17, ... 197 is equal to ....

3, 7, 11, 15, ... , 399 → $d = 4$

2, 5, 8, 11, ... , 359 → $d = 3$

$\Rightarrow$ C.T's :- $\boxed{11, 23, 35, 47 \ldots}$ → $d = 12$

$d = \text{LCM}(4, 3) = 12$

2, 7, 12, 17, 22, 27, 32, 37, 42, 47 → $d = 5$

CT's :- $\boxed{47, 107, 167}$

$d = 60$.

Sum = 321.

Q. JEE Mains (2023)

The 4^{th} term of GP is 500 and its common ratio is $\frac{1}{m}$, $m \in N$. Let S_n denote the sum of the first n terms of this GP. If $S_6 > S_5 + 1$ and $S_7 < S_6 + \frac{1}{2}$, then the number of possible values of m is ....12.... .

$T_4 = \boxed{ar^3 = 500}$

$r = \frac{1}{m}$

$\frac{a}{m^3} = 500$

$a = 500m^3$

$S_6 - S_5 > 1$

$\boxed{a_6 > 1}$

$\Downarrow$

$ar^5 > 1$

$\frac{a}{m^5} > 1$

$\frac{500\cancel{m^3}}{m^{\cancel{5}2}} > 1$

$\boxed{500 > m^2}$

$S_7 - S_6 < \frac{1}{2}$

$\Downarrow$

$\boxed{a_7 < \frac{1}{2}}$

$ar^6 < \frac{1}{2}$

$500\cancel{m^3} \cdot \frac{1}{m^{\cancel{6}3}} < \frac{1}{2}$

$1000 < m^3$

$\Downarrow$

$\boxed{10 < m}$

$\#\ \boxed{S_n - S_{n-1} = a_n}$

$\boxed{m = 11, 12, 13 \dots 22}$

12.

Q. JEE Mains (2023)

If $\frac{1^3+2^3+3^3+ \ldots\ldots \text{ upto n terms}}{\boxed{1\cdot3+2\cdot5+3\cdot7+ \ldots\ldots\text{upto n terms}}} = \frac{9}{5}$, then the value of n is

Ans: 05

Direct Ques.

QIBY!!

$S = 1.3 + 2\cdot5 + 3.7 + \ldots$

$T_n = n(2n+1) = 2n^2+n$

$S_n = \sum_{n=1}^{n}(2n^2+n)$

Q. JEE Mains (2023)

If $\boxed{a_n = \frac{-2}{4n^2-16n+15}}$, then $a_1 + a_2 + \cdots + a_{25}$ is equal to :

A $\frac{51}{144}$

B $\frac{49}{138}$

C $\frac{50}{141}$

D $\frac{52}{147}$

$$a_n = \frac{-2}{(2n-3)(2n-5)} \qquad -2 = (2n-5) - (2n-3)$$

$$a_n = \frac{1}{2n-3} - \frac{1}{2n-5}$$

$4n^2-16n+15$

$4n^2-10n-6n+15$

$2n(2n-5)-3(2n-5)$

$$a_1 = \cancel{\left(\frac{1}{-1}\right)} - \left(\frac{1}{-3}\right)$$

$$a_2 = \cancel{\frac{1}{1}} - \cancel{\left(\frac{1}{-1}\right)}$$

$$a_3 = \cancel{\frac{1}{3}} - \cancel{\left(\frac{1}{1}\right)}$$

$\vdots$

$$a_{25} = \frac{1}{47} - \cancel{\frac{1}{45}}$$

$$= \frac{1}{47} + \frac{1}{3}$$

$$= \frac{50}{141}$$

Q. JEE Mains (2023)

M.T. $\Rightarrow \left(\frac{n}{2}\right)^{th}, \left(\frac{n}{2}+1\right)^{th}$

Let $a_1 = 8, a_2, a_3, \dots a_n$ be an A.P. If the sum of its first four terms is 50 and the sum of its last four terms is 170 , then the product of its middle two terms is ....

$7^{th}, 8^{th}$

$a_1 = 8.$

$a_1 + a_2 + a_3 + a_4 = 50.$

$a_n + a_{n-1} + a_{n-2} + a_{n-3} = 170.$

add

$$\left(\underbrace{a_1+a_n}_{\alpha}\right) + \left(\underbrace{a_2+a_{n-1}}_{\alpha}\right) + \left(\underbrace{a_3+a_{n-2}}_{\alpha}\right) + \left(\underbrace{a_4+a_{n-3}}_{\alpha}\right) = 220.$$

$$\cancel{4}(a_1+a_n) = \cancel{220}\ 55$$

$$a_1 + a_n = 55$$

$$a + a+d + a+2d + a+3d = 50.$$

$$\Downarrow$$

$$4\underset{8}{a} + 6d = 50$$

$$6d = 18$$

$$d = 3.$$

ex:

$a_1 + a_{13} = a_8 + a_6 = 2a_7$

$a_7 = 8 + 6\times 3 = 26.$

$a_8 = 8 + 7\times 3 = 29$

$$a_n = 55 - \underset{8}{a_1}$$

$$8 + (n-1)d = 55 - 8$$

$$(n-1)3 = 47 - 8$$

$$3(n-1) = 39$$

$$n - 1 = 13$$

$$n = 14$$

54

Q. JEE Mains (2023)

Let $\sum_{n=0}^{\infty} \frac{n^3((2n)!) + (2n-1)(n!)}{(n!)((2n)!)} = ae + \frac{b}{e} + c$, where $a, b, c \in \mathbb{Z}$ and $e = \sum_{n=0}^{\infty} \frac{1}{n!}$ Then $a^2 - b + c$ is equal to ....

$5 \quad -1 \quad 0$

$25 - (-1) + 0$

$26.$

final ans $= 5e + \frac{e}{2} - \frac{1}{2e} - \frac{e}{2} - \frac{1}{2e}$

$= 5e - \frac{1}{e}$

$\frac{\cancel{2n}}{\cancel{2n} \cdot (2n-1)!} = \frac{1}{(2n-1)!}$

$\frac{n^3 \cdot \cancel{(2n)!}}{n! \, \cancel{(2n)!}} + \frac{2n \cdot \cancel{n!}}{\cancel{n!} \, (2n)!} - \frac{\cancel{n!}}{\cancel{n!} \, (2n)!}$

$\Downarrow$

$\frac{n^3}{n!} + \sum \frac{2n}{2n!} - \sum \frac{1}{(2n)!}$

$\frac{1}{(\text{evens are})!}$

$\sum \frac{1}{(n-3)!} + \frac{3}{(n-2)!} + \frac{1}{(n-1)!}$

$e \quad 3e \quad e$

$\sum \frac{1}{(2n-1)!} \to \frac{1}{2}\left(e - \frac{1}{e}\right)$

$\frac{1}{2}\left(e + \frac{1}{e}\right)$

$\frac{n^3}{n!} = \frac{n^{\cancel{3}\,2}}{\cancel{n} \cdot (n-1)!} = \frac{n^2 - 1 + 1}{(n-1)!} = \frac{\cancel{(n-1)}(n+1)}{(n-1)!} + \frac{1}{(n-1)!}$

$\cancel{(n-1)} \cdot (n-2)!$

$\frac{1}{(n-3)!} + \frac{3}{(n-2)!} = \frac{n-2}{(n-2)!} + \frac{3}{(n-2)!} = \frac{n+1}{(n-2)!}$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \ldots \infty$$

$x=1$

$$e = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots \infty$$

$$= \sum \frac{1}{\text{sare}!}$$

$$e^{-x} = 1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots \infty$$

$x=1$

$$e^{-1} = 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \ldots \infty$$

$$\frac{1}{e} = \sum \frac{\pm 1}{\text{sare}!}$$

add

$$e + \frac{1}{e} = 2\left(\frac{1}{0!} + \frac{1}{2!} + \ldots\right)$$

$$\frac{1}{2}\left(e + \frac{1}{e}\right) = \sum \frac{1}{(\text{even sare})!}$$

$$\frac{1}{2}\left(e - \frac{1}{e}\right) = \sum \frac{1}{(\text{oddsare})!}$$

Q. JEE Mains (2023)

Consider a sequence $a_1, a_2, \ldots\ldots\ldots, a_n$ given by $\boxed{a_n = a_{n-1} + 2^{n-1}}$, $a_1 = 1$ and another sequence given by $b_n = b_{n-1} + a_{n-1}$, $b_1 = 1$.

Also $P = \sum_{n=1}^{10} \frac{b_n}{2^n}$ and $Q = \sum_{n=1}^{10} \frac{n}{2^{n-1}}$, then $2^7(P - 2Q)$ is

$$a_n - a_{n-1} = 2^{n-1}$$

General

$a_1 \rightarrow \checkmark$

$a_2 \rightarrow \checkmark$

$a_3 \rightarrow \checkmark$

$a_4 \rightarrow \checkmark$

Pattern

$\downarrow$

\# G.T.

$$\cancel{a_2} - a_1 = 2^1$$
$$\cancel{a_3} - \cancel{a_2} = 2^2$$
$$\cancel{a_4} - \cancel{a_3} = 2^3$$
$$\vdots$$
$$a_n - \cancel{a_{n-1}} = 2^{n-1}$$

add

$$a_n - \underset{\hookrightarrow 1}{a_1} = 2^n - 2$$

$$\boxed{a_n = 2^n - 1}$$

$$b_n - b_{n-1} = 2^{n-1} - 1$$

$$\cancel{b_2} - b_1 = 2^1 - 1$$
$$\cancel{b_3} - \cancel{b_2} = 2^2 - 1$$
$$\cancel{b_4} - \cancel{b_3} = 2^3 - 1$$
$$\vdots$$
$$b_n - \cancel{b_{n-1}} = 2^{n-1} - 1$$

$$b_n - 1 = 2^n - 2 - (n-1)$$

$$b_n \cancel{-1} = 2^n - n \cancel{+1}$$

$$\boxed{b_n = 2^n - n}$$

Q. JEE Mains (2023)

Consider a sequence $a_1, a_2, \dots \dots \dots, a_n$ given by $\boxed{a_n = a_{n-1} + 2^{n-1}}$, $a_1 = 1$ and another sequence given by $b_n = b_{n-1} + a_{n-1}$, $b_1 = 1$.

Also $P = \sum_{n=1}^{10} \frac{b_n}{2^n}$ and $Q = \sum_{n=1}^{10} \frac{n}{2^{n-1}}$, then $2^7(P - 2Q)$ is

$$P - 2Q = \sum_{n=1}^{10}\left(\frac{b_n}{2^n} - \frac{4n}{2^n}\right) = \sum_{n=1}^{10}\left(\frac{2^n - 5n}{2^n}\right) = \sum_{n=1}^{10} 1 - 5\sum_{n=1}^{10}\frac{n}{2^n}$$

$b_n \to 2^n - n$

$$= 10 - 5(S)$$

$$= 10 - 5\left(\frac{2^{10}}{2^9} - \frac{6}{2^9}\right) = 10 - 5\left(2 - \frac{6}{2^9}\right)$$

$$\cancel{10} - \cancel{10} + \frac{5 \times 6}{2^9} = P - 2Q$$

$$\frac{5 \times \cancel{6}^{\,3}}{\cancel{2^2}} = 2^7(P - 2Q)$$

Ans

7.5

$S = \sum_{n=1}^{10} \frac{n}{2^n}$

$$S = \frac{1}{2^1} + \frac{2}{2^2} + \frac{3}{2^3} + \dots + \frac{9}{2^9} + \frac{10}{2^{10}}$$

$\times \frac{1}{2}$

$$\frac{S}{2} = 0 + \frac{1}{2^2} + \frac{2}{2^3} + \dots + \frac{9}{2^{10}} + \frac{10}{2^{11}}$$

$\ominus$

$$\frac{S}{2} = \boxed{\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \dots + \frac{1}{2^{10}}} - \frac{10}{2^{11}}$$

$$\frac{S}{2} = \cancel{\frac{1}{2}}\left(\frac{1-\left(\frac{1}{2}\right)^{10}}{\cancel{1-\frac{1}{2}}}\right) - \frac{10}{2^{11}} \quad \left(\frac{10}{2^{11}} = \frac{5}{2^{10}}\right) \Rightarrow \frac{S}{2} = \frac{2^{10}-1}{2^{10}} - \frac{5}{2^{10}}$$

$$\frac{S}{\cancel{2}} = \frac{2^{10}-6}{2^{\cancel{10}\,9}}$$

Permutations & Combinations

Q. JEE Mains (2023)

The number of 9 digit numbers, that can be formed using all the digits of the number 123412341 so that the even digits occupy only even places, is ....

\# 1, 1, 1, 2, 2, 3, 3, 4, 4

1	2	3	4	5	6	7	8	9
.	.	.	.	.	.	.	.	.

$$\# \; 60 = \left(\frac{4!}{2!\,2!}\right) \times \left(\frac{5!}{3!\,2!}\right) \Rightarrow \text{Ans.}$$

Q. **JEE Mains (2023)**

A boy needs to select five courses from 12 available courses, out of which 5 courses are language courses. If he can choose at most two language courses, then the number of ways he can choose five courses is

2 1 0

12 → 5 language

12 → 7 other

5 select

$$\boxed{\text{Ans} = \left({}^{5}C_2 \times {}^{7}C_3\right) + \left({}^{5}C_1 \cdot {}^{7}C_4\right) + {}^{7}C_5}$$

Q. JEE Mains (2023)

Let $S = \{1,2,3,5,7,10,11\}$. The number of nonempty subsets of S that have the sum of all elements a multiple of 3, is ……….

\# av. no.

$3n+0 \mid 3n \rightarrow \{3\}$

$3n+1 \rightarrow \{1, 7, 10\}$

$3n+2 \rightarrow \{2, 5, 11\}$

No. of elements in subsets :

1 element $\Rightarrow$ 1

2 element $\Rightarrow {}^3C_1 \times {}^3C_1 = 9$

3 element $\Rightarrow \left(1 \times {}^3C_1 \times {}^3C_1\right) + {}^3C_3 + {}^3C_3 = 11$

(0, 1, 2) (1, 1, 1) (2, 2, 2)

4 element $\Rightarrow {}^3C_2 \times {}^3C_2 + 1 \times 1 + 1 \times 1 = 11$

(1, 1, 2, 2) (1, 1, 1, 0) (2, 2, 2, 0)

\# 5 elements :- $\longrightarrow$ 9

2 ko drop

unka sum = 3 ka multiple

\# 6 elements $\rightarrow$ 1 $\{1, 2, 5, 7, 10, 11\}$

\# 7 elements $\rightarrow$ 1

final ans = 43.

Q. JEE Mains (2023)

If all the six digit numbers $\underbrace{x_1x_2x_3x_4x_5x_6}$ with $0 < x_1 < x_2 < x_3 < x_4 < x_5 < x_6$ are arranged in the increasing order, then the sum of the digits in the 72^{th} number is

?

digits → $\boxed{1, 2, 3 \ldots 9}$

ex: 123456
235678
145789
⋮

123456
123457
123458
⋮

$\boxed{}$ 72^{th}
Sum.

\# $1\boxed{\_\ \_\ \_\ \_\ \_} \Rightarrow {}^8C_5 = 56$

$2\ 3\boxed{\_\ \_\ \_\ \_} \Rightarrow {}^6C_4 = 15$ → 71

$\boxed{2\ 4\ 5\ 6\ 7\ 8} \leftarrow 72^{th}$

Sum = 32 Ans.

Q. JEE Mains (2023)

Let $S = \{1,2,3,5,7\}$. The rank of 35773 if all 5 digit number formed by the set S are arranged in a dictionary in ascending order and repetition of digits is allowed.

_ _ _ _ _

1 1 1 1 1
1 1 1 1 2
⋮

1 _ _ _ _ → 5^4
2 _ _ _ _ → 5^4
3 1 _ _ _ → 5^3
3 2 _ _ _ → 5^3
3 3 _ _ _ → 5^3
3 5 1 _ _ → 5^2
3 5 2 _ _ → 5^2
3 5 3 _ _ → 5^2

3 5 5 _ _ → 5^2
3 5 7 1 _ → 5
3 5 7 2 _ → 5
3 5 7 3 _ → 5
3 5 7 5 _ → 5
3 5 7 7 1 → 1
3 5 7 7 2 → 1
3 5 7 7 3 → 1

add

Q. JEE Mains (2023)

The letters of the word OUGHT are written in all possible ways and these words are arranged as in a dictionary, in a series. Then the serial number of the word TOUGH is :

A 89

B 84

C 86

D 79

Ans: A

$\boxed{G, H, \cancel{O}, \cancel{T}, U}$

G _ _ _ _ → 4!

H _ _ _ _ → 4!

O _ _ _ _ → 4!

T G _ _ _ → 3!

T H _ _ _ → 3!

T O G _ _ → 2!

T O H _ _ → 2!

T O U G H → 1

89

Dr. Sachin jee

Rx = Manzil 1.0

lec - P&C

Part -01

Rank → Topic

0 — 0 — X

Binomial Theorem

$\# \ {}^{13}C_0\,{}^{14}C_1 + {}^{13}C_1\,{}^{14}C_2 + {}^{13}C_2\,{}^{14}C_3 + \dots + {}^{13}C_{13}\,{}^{14}C_{14} = {}^{27}C_{13}$

${}^{14}C_{13} \quad {}^{14}C_{12} \quad {}^{14}C_{11} \quad {}^{14}C_0$

$= {}^{27}C_{14}$

Note :-

N^r – se gayab.

$$\# \ \boxed{r \cdot {}^{n}C_{r} = n \cdot {}^{n-1}C_{r-1}}$$

$$\# \ 3 \cdot {}^{15}C_{12} = 3 \cdot {}^{15}C_{3} = 15 \cdot {}^{14}C_{2}.$$

D^r se gayab.

$$\# \ \boxed{\frac{{}^{n}C_r}{r+1} = \frac{{}^{n+1}C_{r+1}}{n+1}}$$

\# Series :-

all B. Coeff. sum

$$\boxed{{}^{13}C_0 + {}^{13}C_1 + \dots + {}^{13}C_{13} = 2^{13}}$$

$$\sum_{r=0}^{n} {}^{n}C_r = 2^n.$$

$${}^{13}C_0 + {}^{13}C_2 + {}^{13}C_4 + \dots = {}^{13}C_1 + {}^{13}C_3 + {}^{13}C_5 + \dots = 2^{12}.$$

Q. JEE Mains (2023)

If $({}^{30}C_1)^2 + 2({}^{30}C_2)^2 + 3({}^{30}C_3)^2 + \cdots + 30({}^{30}C_{30})^2 = \frac{\alpha 60!}{(30!)^2}$, then α is equal to

A 30

B 60

C 15

D 10

$$\sum_{r=1}^{30} r\left({}^{30}C_r\right)^2 = r \cdot {}^{30}C_r \cdot {}^{30}C_r$$

$$\sum 30 \cdot {}^{29}C_{r-1}\, {}^{30}C_r$$

$$30\left(\sum_{r=1}^{30} {}^{29}C_{r-1}\, {}^{30}C_r\right)$$

$$30\left({}^{29}C_0\, {}^{30}C_1 + {}^{29}C_1\, {}^{30}C_2 + \cdots + {}^{29}C_{29}\, {}^{30}C_{30}\right) = \left({}^{59}C_{30}\right)30.$$

${}^{29}C_0 \to {}^{29}C_{29}$, ${}^{29}C_1 \to {}^{29}C_{28}$

$$\frac{\alpha \cdot 60!}{30!\,30!} = \left(\frac{59!}{30!\,29!}\right)30.$$

$$\alpha = \frac{30 \times 30}{60} = 15$$

Q. JEE Mains (2023)

Suppose $\sum_{r=0}^{2023} \underbrace{r^2\, {}^{2023}C_r}$ $= 2023 \times \alpha \times 2^{2022}$. Then the value of α is ....

$\#\ r \cdot \underbrace{r\, {}^{n}C_r}$

$\underline{\underline{r}} \cdot n \cdot {}^{n-1}C_{\underline{\underline{r-1}}}$

$n \cdot (\underline{\underline{r-1}} + 1)\, {}^{n-1}C_{\underline{\underline{r-1}}}$

$n\left(\underbrace{(\underline{\underline{r-1}})\, {}^{n-1}C_{\underline{\underline{r-1}}}} + {}^{n-1}C_{r-1}\right)$

$n\left((n-1)\, {}^{n-2}C_{r-2} + {}^{n-1}C_{r-1}\right)$

$2023\left(2022 \cdot \sum_{r=2}^{2023} {}^{2021}C_{r-2} + \sum_{r=1}^{2023} {}^{2022}C_{r-1}\right)$

$\underline{\text{all}}\ \underline{\text{BC's}}$

$2023\left(\underline{\underline{2}} \times 1011\ \underline{\underline{2}}^{2021} + \underline{\underline{2}}^{2022}\right)$.

$2023\ 2^{2022}\left(\underbrace{1011 + 1}_{\alpha}\right)$

$\sum r^3 \cdot {}^{n}C_r$

$\underline{\underline{\text{Try!}}}$

Q. JEE Mains (2023)

If a_r is the coefficient of x^{10-r} in the Binomial expansion of $(1+x)^{10}$, then $\sum_{r=1}^{10} r^3 \left(\frac{a_r}{a_{r-1}}\right)^2$ is equal to

A 4895

B 1210

C 5445

D 3025

Ans: B

$r^3 \cdot \left(\frac{{}^{10}C_r}{{}^{10}C_{r-1}}\right)^2$

$\downarrow$

$r^{3} \left(\frac{11-r}{r}\right)^2$

$\Sigma = \sum_{r=1}^{10} r(11-r)^2 \quad r \to 1+10-r$

$S = \sum_{r=1}^{10} (11-r) r^2$

$\Rightarrow {}^{10}C_r x^r$

${}^{10}C_{10-r} x^{10-r}$

$\# \boxed{{}^{10}C_r = a_r}$

$\frac{{}^{10}C_r}{{}^{10}C_{r-1}} = \frac{\cancel{10!}\,(r-1)!\,(11-r)!}{r!\,(10-r)!\,\cancel{10!}} = \frac{11-r}{r}$

Q. JEE Mains (2023)

The constant term in the expansion of $\left(2x + \frac{1}{x^7} + 3x^2\right)^5$ is ....

Multinomial

G.T. $= \dfrac{5!}{\alpha!\,\beta!\,\gamma!}(2x)^{\alpha}(x^{-7})^{\beta}(3x^2)^{\gamma}$

$\boxed{\alpha+\beta+\gamma=5}$

$= \left(\dfrac{5!}{\alpha!\,\beta!\,\gamma!}\,2^{\alpha}\cdot 3^{\gamma}\right)\ \underbrace{x^{\alpha}\,x^{-7\beta}\,x^{2\gamma}}$

$\hookrightarrow x^{\alpha-7\beta+2\gamma}$

$\boxed{\alpha-7\beta+2\gamma=0}$

or

$\alpha+2\gamma=7\beta$

α	β	γ
1	1	3

← O.P.C.

Ans. $\left(\dfrac{5!}{1!\,1!\,3!}\,2^1\cdot 3^3\right)$

Q. JEE Mains (2023)

100% Aayega.

The remainder when $(2023)^{2023}$ is divided by 35

Ans 7

$$(35\times 58-7)^{2023} = {}^{2023}C_0(35\times58)^{2023} - {}^{2023}C_1(35\times58)^{2022} + \dots - {}^{2023}C_{2023}7^{2023}$$

$$35\overline{)2023}\ 58$$
$$\begin{array}{r} 175 \\ \hline 273 \\ 280 \\ \hline -7 \end{array}$$

$-(7)^{2023} = -7^1\cdot 7^{2022} = -7\cdot(50-1)^{1011}$

$7^{2022} = (7^2)^{1011}$, $7^2 = 49$

$$= -7\left\{{}^{1011}C_0\,50^{1011} - {}^{1011}C_1\,50^{1010} + {}^{1011}C_2\,50^{1009}\dots + {}^{1011}C_{1010}(50) - {}^{1011}C_{1011}\right\}$$

${}^{1011}C_{1011} = 1$

$$= 35n + 7$$

Q. JEE Mains (2023)

The remainder on dividing 5^{99} by 11 is

$5n+4$

9

Pattern

$5^1 \longrightarrow 5 \div 11$ 5

$5^2 \longrightarrow 25 \div 11$ 3

$5^3 \longrightarrow 125 \div 11$ 4

$5^4 \longrightarrow 625 \div 11$ 9

$5^5 \longrightarrow 3125 \div 11$ 1

$(3124+1)$ 11✓

$5^6 \longrightarrow (\quad) \div 11$ 5

$5^7 \longrightarrow (\quad) \div 11$ 3

Ans: 9

DIBY!

Q. JEE Mains (2023)

Let the sixth term in binomial expansion of $\left(\sqrt{2^{\log_2(10-3^x)}} + \sqrt[5]{2^{(x-2)\log_2 3}}\right)^m$, in the increasing powers of $2^{(x-2)\log_2 3}$, be 21. If the binomial coefficients of the second, third and fourth terms in the expansion are respectively the first, third and fifth terms of an A.P., then the sum of the squares of all possible values of x is ……

$^mC_1 \quad {}^mC_2 \quad {}^mC_3$

I $\quad$ 3rd $\quad$ 5th $\to$ AP

$2\,{}^mC_2 = {}^mC_1 + {}^mC_3$

$\Downarrow$

solve m

$21 = T_6 = {}^mC_5 (\quad)^{m-5} (\quad)^5 \quad T_{r+1} = {}^mC_r$

$\downarrow$

x ✓

Chat

21BY! - 3

ans

Complex Numbers

Q. JEE Mains (2023)

The value of $\left(\dfrac{1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}}\right)^3$ is

(A) $\dfrac{-1}{2}(1-i\sqrt{3})$

(B) $\dfrac{1}{2}(1-i\sqrt{3})$

(C) $\dfrac{-1}{2}(\sqrt{3}-i)$

(D) $\dfrac{1}{2}(\sqrt{3}+i)$

$\theta = \frac{2\pi}{9}$

$\left(\dfrac{1+e^{i\left(\frac{\pi}{2}-\theta\right)}}{1+e^{i\left(\theta-\frac{\pi}{2}\right)}}\right)^3 = \left(\dfrac{1+e^{i\alpha}}{1+e^{i(-\alpha)}}\right)^3 = \left(\dfrac{\cancel{2\cos\frac{\alpha}{2}}\, e^{i\frac{\alpha}{2}}}{\cancel{2\cos\left(\frac{\alpha}{2}\right)}\, e^{i\left(-\frac{\alpha}{2}\right)}}\right)^3 = \left(e^{i\alpha}\right)^3$

$3\left(\frac{\pi}{2}-\theta\right) = \frac{3\pi}{2} - 3\theta$

$\cos\left(\frac{3\pi}{2}-3\theta\right) + i\sin\left(\frac{3\pi}{2}-3\theta\right) = e^{i3\alpha}$

$-\sin 3\theta - i\cos 3\theta$

$-i\left(\cos\frac{2\pi}{3} - i\sin\frac{2\pi}{3}\right)$

$-i\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$

$\frac{i}{2} - \frac{\sqrt{3}}{2}$

$\boxed{1+e^{i\theta} = 2\cos\frac{\theta}{2}\, e^{i\theta/2}}$

$\cos\theta - i\sin\theta = \cos(-\theta) + i\sin(-\theta)$
$= e^{i(-\theta)}$

$\sin\theta + i\cos\theta = \cos\left(\frac{\pi}{2}-\theta\right) + i\sin\left(\frac{\pi}{2}-\theta\right)$
$= e^{i\left(\frac{\pi}{2}-\theta\right)}$

$\sin\theta - i\cos\theta = -i(\cos\theta + i\sin\theta)$
$= e^{-\frac{\pi}{2}i} e^{i\theta} = e^{i\left(\theta-\frac{\pi}{2}\right)}$

Q. JEE Mains (2023)

Let $z_1 = 2 + 3i$ and $z_2 = 3 + 4i$. The set $S = \{z \in C: \boxed{|z - z_1|^2 - |z - z_2|^2 = |z_1 - z_2|^2}\}$ represents a

$z = x + iy$

$z - z_1 = (x-2) + i(y-3)$

$z - z_2 = (x-3) + i(y-4)$

$z_1 - z_2 = -1 - i$

(A) straight line with sum of its intercepts on the coordinate axes equals 14

$\frac{x}{7} + \frac{y}{7} = 1$

(B) hyperbola with the length of the transverse axis 7 ×

(C) straight line with the sum of its intercepts on the coordinate axes equals −18 ×

(D) hyperbola with eccentricity 2 ×

$\left((x-2)^2 + (y-3)^2\right) - \left((x-3)^2 + (y-4)^2\right) = 2$

$\cancel{x^2} + 4 - 4x + \cancel{y^2} + \cancel{9} - 6y - \cancel{x^2} - \cancel{9} + 6x - \cancel{y^2} - 16 + 8y = 2$

$2x + 2y = 14 \rightarrow \boxed{x + y = 7}$

Q. JEE Mains (2023)

Let $\alpha = 8 - 14i$, $A = \left\{z \in \mathbb{C} : \dfrac{\alpha z - \bar{\alpha}\bar{z}}{z^2 - (\bar{z})^2 - 112i} = 1\right\}$ and $B = \{z \in \mathbb{C} : |z + 3i| = 4\}$.

Then $\sum_{z \in A \cap B} (\operatorname{Re} z - \operatorname{Im} z)$ is equal to

$A \cap B$

$x + iy$

$(0, -3)$ $r = 4$

$(z + \bar{z})(z - \bar{z}) - 112i$

$(2x)(2iy) - 112i$

$i(4xy - 112)$

$4i(xy - 28)$

$N^r = (8 - 14i)(x + iy) - (8 + 14i)(x - iy)$

$= \cancel{8x} - 14xi + i8y + \cancel{14y} - \cancel{8x} + i8y - \cancel{14y} - 14ix$

$= i(-28x + 16y)$

$\dfrac{\cancel{4i}(-7x + 4y)}{\cancel{4i}(xy - 28)} = 1$

$-7x + 4y = xy - 28$

$7x - 4y + xy - 28 = 0$

$x(7 + y) - 4(y + 7) = 0$

$(x - 4)(7 + y) = 0$

A: $x = 4$ or $y = -7$ $\xleftarrow{\text{Solve}}$ B: $\boxed{x^2 + (y+3)^2 = 16}$

$x = 4 \to y = -3 \longrightarrow 4 - 3i$

$y = -7 \to x = 0 \longrightarrow 0 - 7i$

$4 - (-3)$

$+ \; 0 - (-7)$

$7 + 7 = 14$ ← Ans

Q. JEE Mains (2023)

The complex number $z = \frac{i-1}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}$ is equal to:

A $\sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$

B $\cos\frac{\pi}{12} - i\sin\frac{\pi}{12}$

C $\sqrt{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$

D $\sqrt{2}i\left(\cos\frac{5\pi}{12} - i\sin\frac{5\pi}{12}\right)$

$\Rightarrow e^{i(\pi/3)}$

$i - 1 = \underbrace{-1 + i}$

P.F.

QIBY1

4

Q. JEE Mains (2023)

If the center and radius of the circle $\left|\frac{z-2}{z-3}\right| = 2$ are respectively (α, β) and γ, then

$3(\alpha + \beta + \gamma)$ is equal to

A 11

B 9

C 10

D 12

$$|z-2|^2 = 2^2|z-3|^2$$

$$\boxed{(x-2)^2 + y^2 = 4\left((x-3)^2 + y^2\right)}$$

$$x^2 + 4 - 4x + y^2 = 4x^2 + 36 - 24x + 4y^2$$

$$3x^2 + 3y^2 - 20x + 32 = 0.$$

$$\boxed{x^2 + y^2 - \left(\frac{20}{3}\right)x + \frac{32}{3} = 0}$$

$\left|\frac{z - z_1}{z - z_2}\right| = k$

$k = 1$ ⇓ L.O.Z → z_1, z_2

$k \neq 1$ ⇓ Circle ⇓ eq^n? C&R

$z = x + iy$.

Q. JEE Mains (2022)

Let $A = \left\{z \in \mathbb{C} : \left|\frac{z+1}{z-1}\right| < 1\right\}$ and $B = \left\{z \in \mathbb{C} : \arg\left(\frac{z-1}{z+1}\right) = \frac{2\pi}{3}\right\}$. Then $A \cap B$ is:

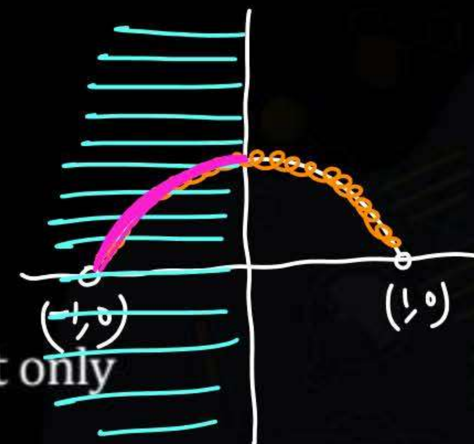
A a portion of a circle centred at $\left(0, \frac{1}{\sqrt{3}}\right)$ that lies in the second and third quad.

B ~~quadrants only~~ a portion of a circle centred at $\left(0, -\frac{1}{\sqrt{3}}\right)$ that lies in the second quadrant only

C an empty set

D a portion of a circle of radius $\frac{2}{\sqrt{3}}$ that lies in the third quadrant only

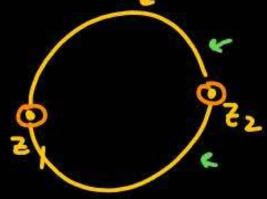
$\left|\frac{z+1}{z-1}\right| < 1 \Rightarrow |z+1| < |z-1|$



$\arg\left(\frac{z-z_1}{z-z_2}\right) = \theta$

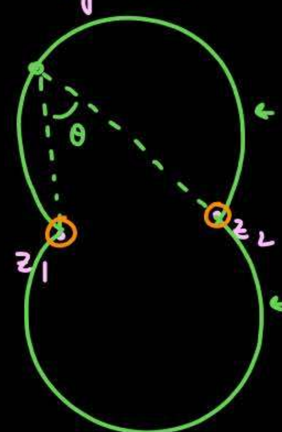
$\pm\frac{\pi}{2}$ → acute → obtuse.

$\pm\frac{\pi}{2}$

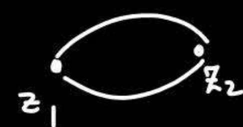


Semi circle

acute → Major arc.



obtuse → Minor arc

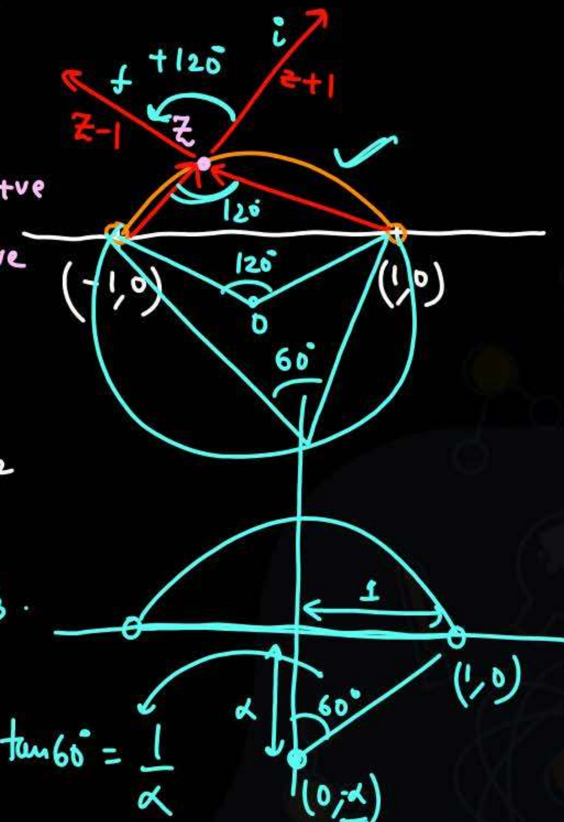


$\arg\left(\frac{\overset{f}{\boxed{z-1}}}{\underset{i}{\boxed{z+1}}}\right) = 120^\circ$

$i \to f$

$\theta \equiv +ve$

$\theta \equiv -ve$



$\alpha = \frac{1}{\sqrt{3}}$.

$\sqrt{3} = \tan 60^\circ = \frac{1}{\alpha}$

Q. JEE Mains (2022)

If $\boxed{z^2 + z + 1 = 0}$, $z \in \mathbb{C}$, then $\left|\sum_{n=1}^{15}\left(z^n + (-1)^n \cdot \frac{1}{z^n}\right)^2\right|$ is equal to 2

$z = \omega / \omega^2$

$\omega^{30} = 1$

$$\sum_{n=1}^{15} z^{2n} + \sum_{n=1}^{15} (-1)^{2n} \frac{1}{z^{2n}} + 2\sum_{n=1}^{15} (-1)^n = -2$$

$\cancel{(-1)} + \cancel{1} + \cancel{(-1)} + \cancel{(1)} \dots + (-1)$

$$\underbrace{z^2 + z^4 + \dots + z^{30}} + \frac{1}{z^2} + \frac{1}{z^4} + \frac{1}{z^6} + \dots + \frac{1}{z^{30}}$$

$$\underbrace{z^2\left(\frac{(z^2)^{15} - 1}{z^2 - 1}\right)}_{0} + \frac{1}{z^2}\left(\frac{1 - \left(\frac{1}{z^2}\right)^{15}}{1 - \frac{1}{z^2}}\right)$$

$$\left(\frac{z^{30} - 1}{\sim}\right) \to \underline{\underline{0}}$$

Q. JEE Mains (2021)

Let S_1, S_2 and S_3 be three sets defined as

$$S_1 = \{z \in \mathbb{C} : |z - 1| \le \sqrt{2}\}$$

$$S_2 = \{z \in \mathbb{C} : \mathrm{Re}((1 - i)z) \ge 1\}$$

$$S_3 = \{z \in \mathbb{C} : \mathrm{Im}(z) \le 1\}$$

Then the set $S_1 \cap S_2 \cap S_3$

Ans: C

2 IBY !!

A is a singleton

B has exactly two elements

C has infinitely many elements

D has exactly three elements

Coordinate Geometry

Q. JEE Mains (2023)

$y = -x$

$h = \frac{\sqrt{3}a}{2}$

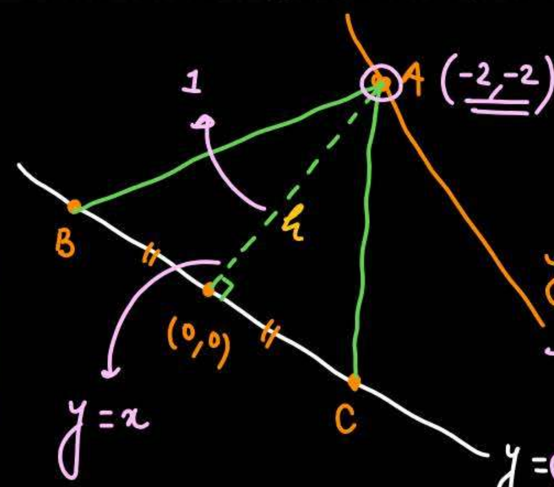
Let B and C be the two points on the line $y + x = 0$ such that B and C are symmetric with respect to the origin. Suppose A is a point on $y - 2x = 2$ such that $\triangle ABC$ is an equilateral triangle. Then, the area of the $\triangle ABC$ is

A $3\sqrt{3}$

B $2\sqrt{3}$

C $\frac{8}{\sqrt{3}}$

D $\frac{10}{\sqrt{3}}$



$y - 2x = 2$

$y - x = 0$

$-x = 2$

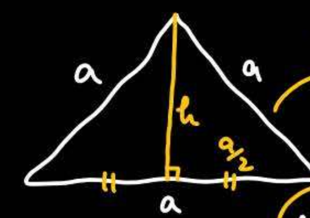
$x = -2$

$y = -x$

$\boxed{x + y = 0}$

$h = \left|\frac{4}{\sqrt{2}}\right|$

$h = 2\sqrt{2}$



$\frac{3a^2}{4} = h^2$

$a^2 = h^2 + \frac{a^2}{4}$

$\Delta = \frac{\sqrt{3}}{4}a^2$

$= \frac{\sqrt{3}}{4}\left(\frac{2h}{\sqrt{3}}\right)^2$

$= \frac{\sqrt{3}}{\cancel{4}} \times \frac{\cancel{4}h^2}{3}$

$\Delta = \frac{h^2}{\sqrt{3}}$

Q. JEE Mains (2023)

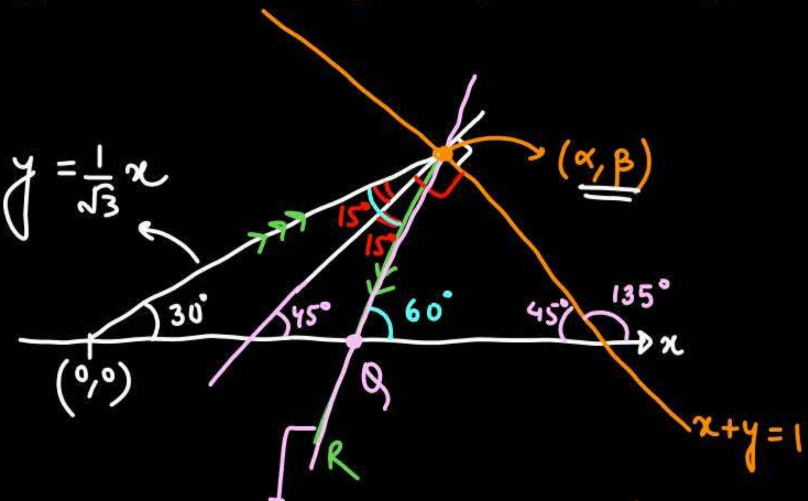
A light ray emits from the origin making an angle 30° with the positive x-axis. After getting reflected by the line $x + y = 1$, if this ray intersects x-axis at Q, then the abscissa of Q is

A $\dfrac{2}{(\sqrt{3}-1)}$

B $\dfrac{2}{3+\sqrt{3}}$

C $\dfrac{2}{3-\sqrt{3}}$

D $\dfrac{\sqrt{3}}{2(\sqrt{3}+1)}$



$m = \sqrt{3}$

$$\left.\begin{aligned} x + y &= 1 \\ y &= \frac{x}{\sqrt{3}} \end{aligned}\right\} \rightarrow x + \frac{x}{\sqrt{3}} = 1$$

$$x\left(\frac{\sqrt{3}+1}{\sqrt{3}}\right) = 1$$

$$x = \frac{\sqrt{3}}{\sqrt{3}+1} = \alpha$$

$$y = \frac{1}{\sqrt{3}+1} = \beta$$

$$y - \beta = \sqrt{3}(x - \alpha)$$

$$\Downarrow$$

$$y = 0$$

$$\boxed{\alpha - \frac{\beta}{\sqrt{3}} = x}$$

$$\rightarrow \frac{\sqrt{3}}{\sqrt{3}+1} - \frac{1}{\sqrt{3}}\left(\frac{1}{\sqrt{3}+1}\right)$$

$$\frac{1}{\sqrt{3}+1}\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) = \frac{2}{\sqrt{3}(\sqrt{3}+1)} = \frac{2}{3+\sqrt{3}}$$

C.G.

C, P, E, H

Tangent & Normal

Memorised

C.T.

E & H

Basic diag. ✓

Q. JEE Mains (2023)

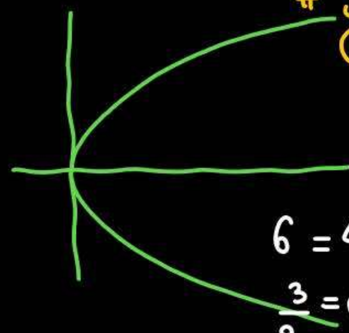
The equations of two sides of a variable triangle are $x = 0$ and $y = 3$, and its third side is a tangent to the parabola $\boxed{y^2 = 6x}$. The locus of its circumcentre is :

A $4y^2 - 18y - 3x - 18 = 0$

B $4y^2 + 18y + 3x + 18 = 0$

C $4y^2 - 18y + 3x + 18 = 0$

D $4y^2 - 18y - 3x + 18 = 0$

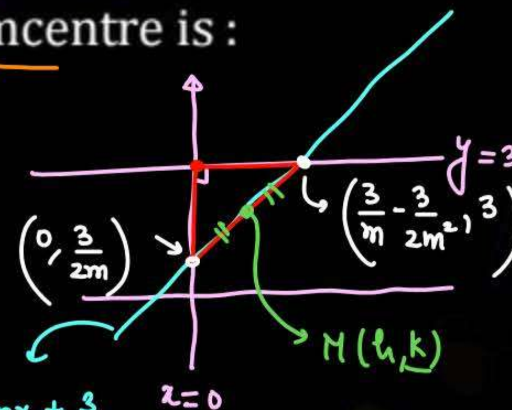


\# $y^2 = 6x$

$y = mx + \frac{a}{m}$

$6 = 4a$

$\frac{3}{2} = a$



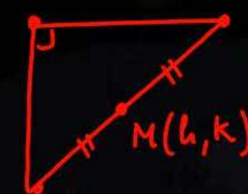
$y = mx + \frac{3}{2m}$

$$\boxed{2h = \frac{3}{m} - \frac{3}{2m^2}}$$

$$2k = \frac{3}{2m} + 3$$

$\Downarrow$

$$(2k-3) = \frac{3}{2m} \Rightarrow \boxed{\frac{1}{m} = \frac{2(2k-3)}{3}}$$



Q. JEE Mains (2023)

The combined equation of the two lines $ax + by + c = 0$ and $a'x + b'y + c' = 0$ can be written as $(ax + by + c)(a'x + b'y + c') = 0$. The equation of the angle bisectors of the lines represented by the equation $\boxed{2x^2 + xy - 3y^2 = 0}$ is

A $3x^2 + 5xy + 2y^2 = 0$

B $x^2 - y^2 + 10xy = 0$

C $3x^2 + xy - 2y^2 = 0$

D $x^2 - y^2 - 10xy = 0$

$a = 2,\ b = -3,\ h = \frac{1}{2}$

P.O.A.B

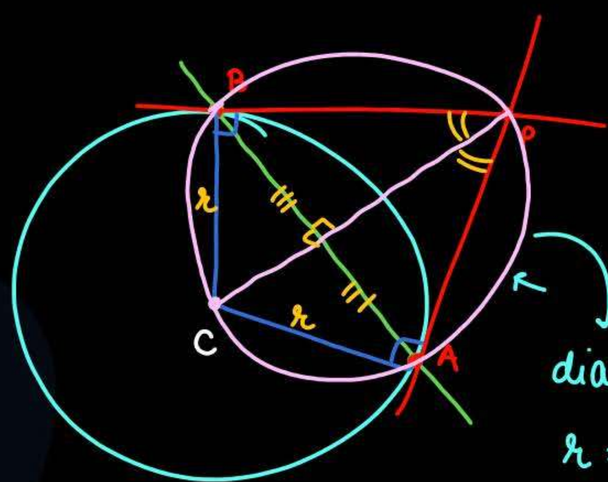
$ax^2 + 2hxy + by^2 = 0$

$\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$

$\frac{x^2 - y^2}{2 - (-3)} = \frac{xy}{\left(\frac{1}{2}\right)}$

$x^2 - y^2 = 10xy$

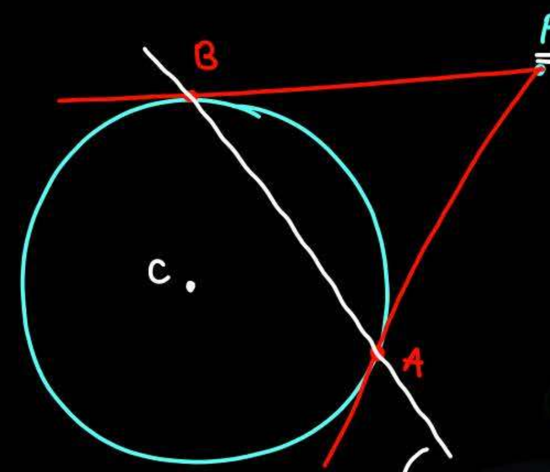
Most imp. diag. of Circles:



dia. → $\underline{\underline{PC}}$

$r = \frac{PC}{2}$ | centre m.p. of $\underline{P \& C}$

$\boxed{APBC \to \text{cyclic quad.}}$



$\underline{\underline{\text{c.o.c}}}$ w.r.to P

$\Downarrow$

$\boxed{T_1 = 0}$

Q. JEE Mains (2023)

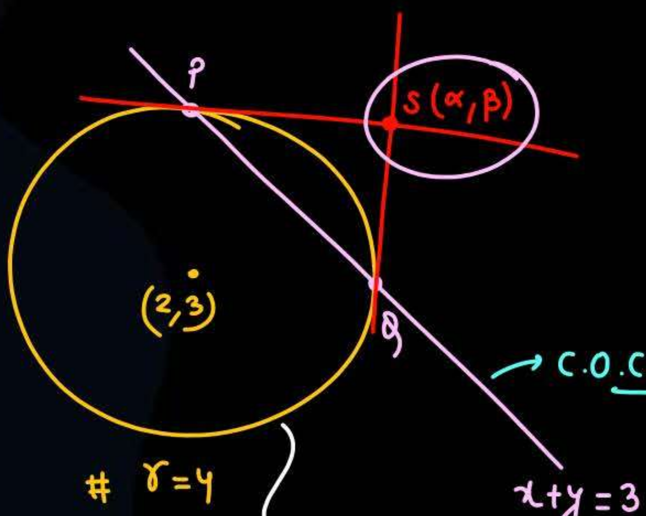
DIBY!

A circle with centre (2,3) and radius 4 intersects the line $\boxed{x + y = 3}$ at the points P and Q. If the tangents at P and Q intersect at the point $S(\alpha, \beta)$, then $4\alpha - 7\beta$ is equal to

$4(-6) - 7(-5)$

$-24 + 35$

11



$\# \ r = 4$

$(x-2)^2 + (y-3)^2 = 16$

$x^2 + y^2 + 13 - 4x - 6y = 16$

$\boxed{x^2 + y^2 - 4x - 6y - 3 = 0}$

C.O.C. w.r.to (S)

$T_1 = 0.$

PQ

$\alpha x + y\beta - \cancel{4}^{2}\left(\frac{x+\alpha}{\cancel{2}}\right) - \cancel{6}^{3}\left(\frac{y+\beta}{\cancel{2}}\right) - 3 = 0$

$(\alpha - 2)x + y(\beta - 3) - 2\alpha - 3\beta - 3 = 0$

$\boxed{(\alpha - 2)x + (\beta - 3)y = (2\alpha + 3\beta + 3)}$

$\boxed{\frac{\alpha - 2}{1} = \frac{\beta - 3}{1} = \frac{2\alpha + 3\beta + 3}{3}}$

$3\alpha - 6 = 2\alpha + 3\beta + 3$ | $\cancel{3\beta} - 9 = 2\alpha + \cancel{3\beta} + 3$

$\alpha - 3\beta = 9$ | $-12 = 2\alpha \rightarrow \alpha = -6$

$-3\beta = 15 \rightarrow \beta = -5$

Q. **JEE Mains (2023)**

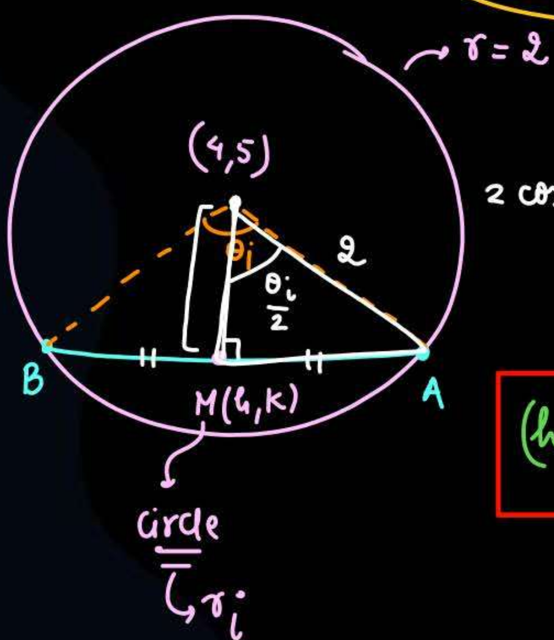
The locus of the mid points of the chords of the circle $C_1 : (x-4)^2 + (y-5)^2 = 4$ which subtend an angle θ_i at the centre of the circle C_1, is a circle of radius r_i. If $\theta_1 = \frac{\pi}{3}, \theta_3 = \frac{2\pi}{3}$ and $r_1^2 = r_2^2 + r_3^2$, then θ_2 is equal to

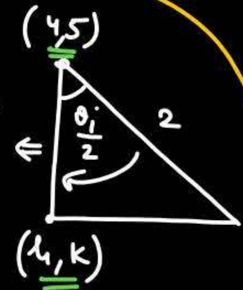
A $\frac{\pi}{4}$

B $\frac{3\pi}{4}$

C $\frac{\pi}{6}$

D $\frac{\pi}{2}$





$2\cos\frac{\theta_i}{2} \Leftarrow$

$$\boxed{(h-4)^2+(k-5)^2=\left(2\cos\frac{\theta_i}{2}\right)^2}$$

$r_i = 2\cos\frac{\theta_i}{2}$

$r_1 = 2\cos 30^\circ = 2\frac{\sqrt{3}}{2}$

$r_2 = ?$

$r_3 = 2\cos 60^\circ = 2\left(\frac{1}{2}\right)$

$\cancel{4}\left(\frac{3}{4}\right) = \cancel{4}\cos^2\frac{\theta_2}{2} + \cancel{4}\left(\frac{1}{4}\right)$

$\frac{1}{2} = \cos^2\frac{\theta_2}{2} \longrightarrow \cos\frac{\theta_2}{2} = \pm\frac{1}{\sqrt{2}}$

$\frac{\theta_2}{2} = \frac{\pi}{4}$

$\theta_2 = \pi/2$

Q. JEE Mains (2023)

If P(h, k) be point on the parabola $x = 4y^2$, which is nearest to the point Q(0,33), then the distance of P from the directrix of the parabola $y^2 = 4(x + y)$ is equal to :

(A) 2

(B) 4

(C) 8

(D) 6

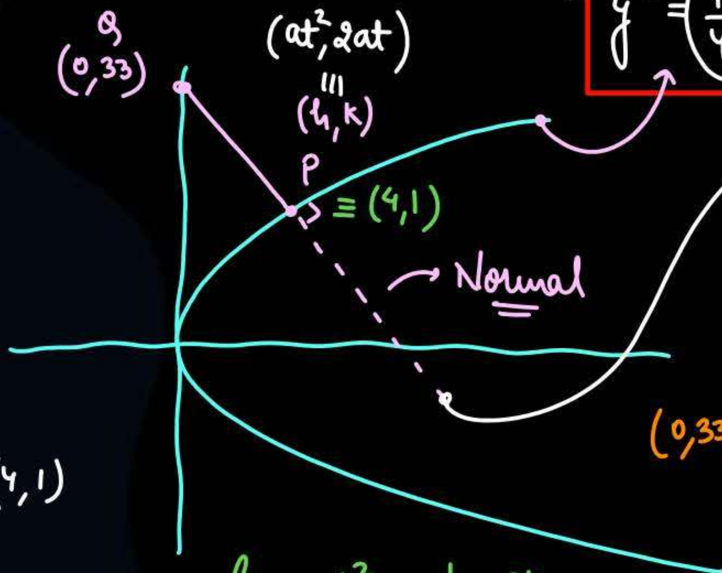
$\boxed{y^2 = \frac{1}{4}x}$

$4a = \frac{1}{4} \rightarrow a = \frac{1}{16}$

$y^2 - 4y + 4 = 4x + 4$

$(y-2)^2 = 4(x+1)$

$a = 1$



$y = -tx + 2at + at^3$

$y = -tx + 2\left(\frac{1}{16}\right)t + \frac{1}{16}t^3$

(0,33)

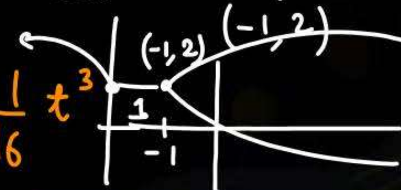
$33 = \frac{t}{8} + \frac{t^3}{16}$

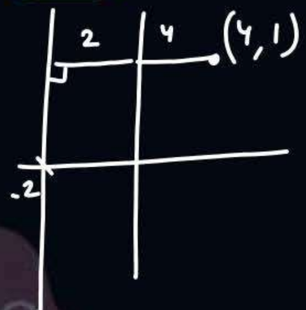
$33 = \frac{2t + t^3}{16}$

$\boxed{t^3 + 2t - 33 \times 16 = 0}$

$t(t^2+2)$

$t = 8$





$h = at^2 = \frac{1}{16} \times 64 = 4$

$k = 2at = 2\left(\frac{1}{16}\right)8 = 1$

Q. JEE Mains (2023)

$QR = \sqrt{6^2 + 6^2} = \sqrt{72}$

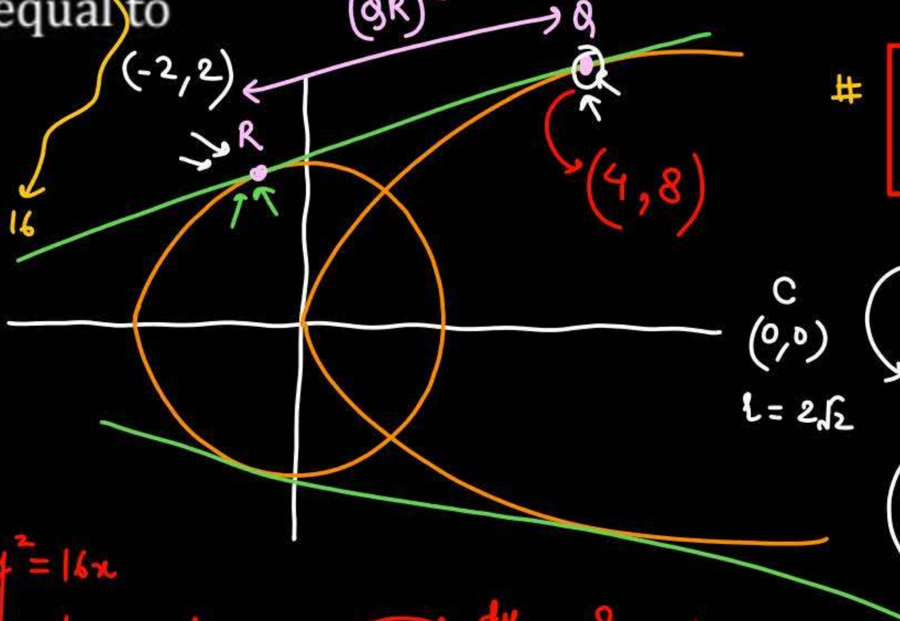
Let A be a point on the x-axis. Common tangents are drawn from A to the curves $x^2 + y^2 = 8$ and $y^2 = 16x$. If one of these tangents touches the two curves at Q and R, then $(QR)^2$ is equal to

(A) 64

(B) 76

(C) 81

(D) 72



$4a = 16$

$a = 4$

\# $\boxed{y = mx + \frac{4}{m}} \longrightarrow \boxed{y = x + 4}$ ($m = 1$)

$mx - y + \frac{4}{m} = 0$

C $(0,0)$

$r = 2\sqrt{2}$

$\left| \dfrac{4/m}{\sqrt{m^2+1}} \right| = 2\sqrt{2}$

$\dfrac{\cancel{16}^{\,2}}{m^2} = \cancel{8}(m^2+1)$

$2 = m^4 + m^2$

$m^4 + m^2 - 2 = 0 \rightarrow (m^2-1)(m^2+2) = 0$

$\boxed{m = \pm 1}$

$y^2 = 16x$

$\hookrightarrow 2y\dfrac{dy}{dx} = 16 \longrightarrow \dfrac{dy}{dx} = \dfrac{8}{y} = 1$

$y = 8$

$x^2 + (x+4)^2 = 8$

$2x^2 + 16 + 8x = 8$

$x^2 + 8 + 4x = 4 \rightarrow x^2 + 4x + 4 = 0 \rightarrow (x+2)^2 = 0$

Q. JEE Mains (2023)

If the maximum distance of normal to the ellipse $\frac{x^2}{4}+\frac{y^2}{b^2}=1$, $b \le 2$, from the origin is 1, then the eccentricity of the ellipse is:

A $\frac{1}{\sqrt{2}}$

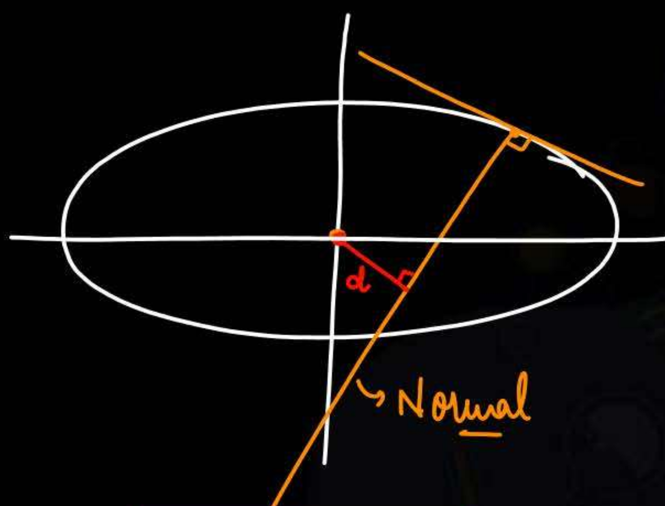
B $\frac{\sqrt{3}}{2}$

C $\frac{1}{2}$

D $\frac{\sqrt{3}}{4}$

$$\left.\begin{aligned} \# \ a-b &= 1 \\ a &= 2 \\ 2-b &= 1 \\ 1 &= b \end{aligned}\right\} \begin{aligned} e^2 &= 1-\frac{1}{4} \\ e^2 &= \frac{3}{4} \end{aligned}$$

Note :-



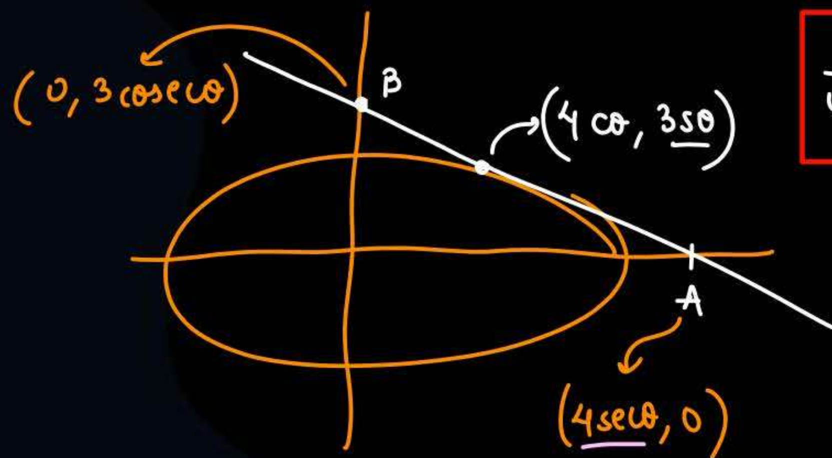
\# $\boxed{d_{max.} = a-b}$

Q. JEE Mains (2023)

Let a "tangent" to the Curve $9x^2 + 16y^2 = 144$ intersect the coordinate axes at the points A and B. Then, the minimum length of the line segment AB is ________.

Ans: 07

$\frac{x^2}{16} + \frac{y^2}{9} = 1$ $\Rightarrow \frac{4\cos\theta\, x}{16_4} + \frac{(3\sin\theta)\, y}{{}_3 9} = 1$



$$\boxed{\frac{x}{4\sec\theta} + \frac{y}{3\operatorname{cosec}\theta} = 1}$$

$4t = \frac{3}{t}$

$t^2 = \frac{3}{4}$

$\tan\theta = \pm\frac{\sqrt{3}}{2}$

$AB = \sqrt{16\sec^2\theta + 9\operatorname{cosec}^2\theta}$

$\sqrt{16(1+t^2) + 9\left(1+\frac{1}{t^2}\right)}$

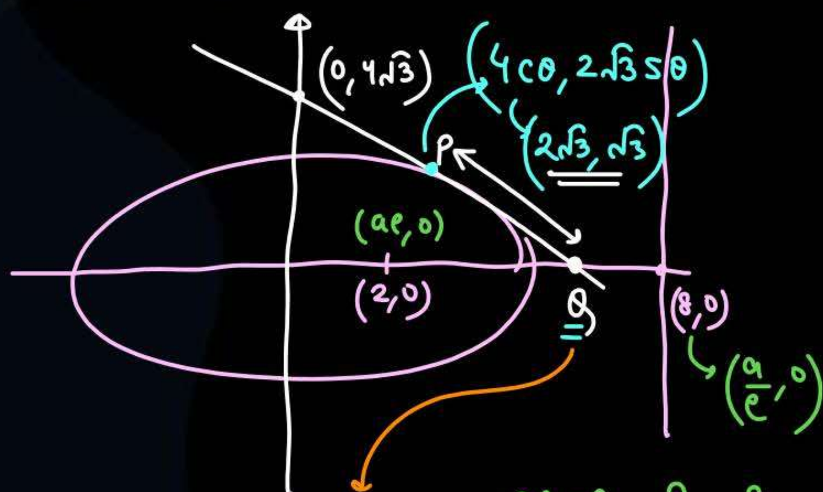
$\sqrt{16 + 16t^2 + 9 + \frac{9}{t^2}}$

$\sqrt{25 + (4t)^2 + \left(\frac{3}{t}\right)^2 - 24 + 24}$

$\sqrt{49 + \underbrace{\left(4t - \frac{3}{t}\right)^2}_{0}}$ $t = \tan\theta$

Q. JEE Mains (2023)

The line $\boxed{x = 8}$ is the directrix of the ellipse $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with the corresponding focus $(2, 0)$. If the tangent to E at the point P in the first quadrant passes through the point $(0, 4\sqrt{3})$ and intersects the x-axis at Q, then $(3PQ)^2$ is equal to ________.



$\left(\frac{8}{2\sqrt{3}}, 0\right) \equiv (4\sec\theta, 0)$

$ae = 2, \quad \frac{a}{e} = 8$

$a = \frac{2}{e}, \quad \frac{2}{e^2} = 8, \quad e^2 = \frac{1}{4}$

$e = \frac{1}{2}$

$a = 4 \qquad b^2 = 12$

$\frac{1}{4} = 1 - \frac{b^2}{16}$

$\frac{b^2}{16} = \frac{3}{4}$

$9PQ^2$

$\theta = 30^\circ$

$\sin\theta = \frac{1}{2}$

$\frac{4\sqrt{3}\sin\theta}{2\sqrt{3}} = 1$

$(0, 4\sqrt{3})$

$\boxed{\frac{x^2}{16} + \frac{y^2}{12} = 1}$

$T_P: \frac{x\cos\theta}{4} + \frac{y\sin\theta}{2\sqrt{3}} = 1$

$x = 4\sec\theta$

Q. JEE Mains (2023)

Let H be the hyperbola, whose foci are $(1 \pm \sqrt{2}, 0)$ and eccentricity is $\sqrt{2}$. Then the length of its latus rectum is ___2___.

$LR = \frac{2b^2}{a} = 2a. = 2.$

A 2

B 3

C $\frac{5}{2}$

D $\frac{3}{2}$

$\xleftrightarrow{2ae}$

$(1-\sqrt{2}, 0)$ $(-ae, 0)$ — $(1+\sqrt{2}, 0)$ $(ae, 0)$

$\cancel{2}ae = \cancel{2}\sqrt{2}$

$a = 1$

$e = \sqrt{2} \rightarrow \boxed{x^2 - y^2 = a^2}$

Trigonometry

Q. JEE Mains (2023)

If $\tan 15^\circ + \frac{1}{\tan 75^\circ} + \frac{1}{\tan 105^\circ} + \tan 195^\circ = 2a$ then the value of $\left(a + \frac{1}{a}\right)$ is :

(A) 4

(B) $4 - 2\sqrt{3}$

(C) 2

(D) $5 - \frac{3}{2}\sqrt{3}$

$\tan 15^\circ + \cancel{\tan 15^\circ} - \cancel{\tan 15^\circ} + \tan 15^\circ$

$\tan 15^\circ = 2 - \sqrt{3}$

$\cancel{2}(2-\sqrt{3}) = \cancel{2}a$

$2-\sqrt{3} + \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$

$\underbrace{2-\sqrt{3} + 2+\sqrt{3}}_{4}$

Q. JEE Mains (2023)

The set of all values of λ for which the equation $\cos^2 2x - 2\sin^4 x - 2\cos^2 x = \lambda$

$(s^2x)^2$

A $[-2,-1]$

B $\left[-2,-\frac{3}{2}\right]$

C $\left[-1,-\frac{1}{2}\right]$

D $\left[-\frac{3}{2},-1\right]$

$$(2\cos^2x-1)^2 - 2(1-\cos^2x)^2 - 2\cos^2x = \lambda$$

$$\cos^2x = \alpha$$

$$(2\alpha-1)^2 - 2(1-\alpha)^2 - 2\alpha = \lambda$$

$$4\alpha^2+1-\cancel{4\alpha}-2-2\alpha^2+\cancel{4\alpha}-2\alpha = \lambda$$

$$2\alpha^2-2\alpha-1 = \lambda$$

$$2[\alpha^2-\alpha]-1 = \lambda$$

$$2\left(\alpha^2-2\left(\tfrac{1}{2}\right)\alpha+\tfrac{1}{4}-\tfrac{1}{4}\right)-1 = \lambda$$

$$2\left(\left(\alpha-\tfrac{1}{2}\right)^2-\tfrac{1}{4}\right)-1 = \lambda$$

$$2\left(\alpha-\tfrac{1}{2}\right)^2-\frac{3}{2} = \lambda$$

$$\boxed{2\left(\cos^2x-\tfrac{1}{2}\right)^2-\frac{3}{2} = \lambda}$$

$$\lambda \in \left[-\frac{3}{2},-1\right]$$

Q. JEE Mains (2023)

Let $S = \{\theta \in [0, 2\pi) : \boxed{\tan(\pi\cos\theta) + \tan(\pi\sin\theta) = 0}\}$. Then $\sum_{\theta \in S} \sin^2\left(\theta + \frac{\pi}{4}\right)$ is equal to ______.

Ans: 02

$\tan(\pi\cos\theta) = \tan(-\pi\sin\theta)$

$\cancel{\pi}\cos\theta = n\cancel{\pi} - \cancel{\pi}\sin\theta$

$\tan\alpha = \tan\beta$

$\alpha = n\pi + \beta$

I

\# $\boxed{\cos\theta + \sin\theta = n}$ $\quad n \in I$

$[-\sqrt{2}, \sqrt{2}]$

$\Downarrow$

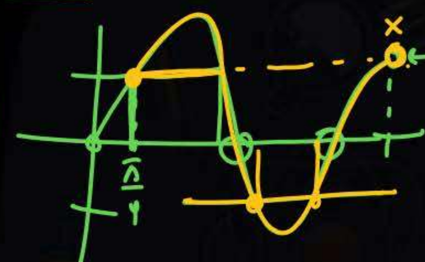
$-1, 0, 1$

$-1, 0, 1$. \# $\frac{\pi}{4} + \theta \in \left[\frac{\pi}{4}, 2\pi + \frac{\pi}{4}\right)$

$\sin\left(\frac{\pi}{4} + \theta\right) = \frac{-1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}$

2 times

2 time $\quad$ 2 time



$\frac{1}{2} + \frac{1}{2} + 0 + \frac{1}{2} + \frac{1}{2}$

$\Downarrow$

2

Q. JEE Mains (2022)

Let $S = \left\{\theta \in [0, 2\pi] : 8^{2\sin^2\theta} + 8^{2\cos^2\theta} = 16\right\}$.

Then $n(S) + \sum_{\theta \in S}\left(\sec\left(\frac{\pi}{4} + 2\theta\right)\operatorname{cosec}\left(\frac{\pi}{4} + 2\theta\right)\right)$ is equal to :

A 0

B −2

C −4

D 12

$n(S) \to 4$

$8^{2\sin^2\theta} = \alpha$ $\quad 8^{2(1-s^2\theta)} = 8^{2-2s^2\theta} = \frac{8^2}{8^{2\sin^2\theta}} = \frac{8^2}{\alpha}$

$\alpha + \frac{8^2}{\alpha} = 16$

$\alpha^2 - 16\alpha + 8^2 = 0$

$(\alpha - 8)^2 = 0$

$\alpha = 8$

$8^{2\sin^2\theta} = 8^1$

$\Downarrow$

$\sin^2\theta = \frac{1}{2}$

$\Downarrow$

$\sin\theta = \pm\frac{1}{\sqrt{2}}$

$\left(\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right)$

$\sec(\beta)\operatorname{cosec}(\beta)$

$\Downarrow$

$\frac{2 \cdot 1}{2\sin\beta\cos\beta} = \frac{2}{\sin 2\beta}$

$= \frac{2}{\sin\left(\frac{\pi}{2} + 4\theta\right)}$

$= \frac{2}{\cos 4\theta} = -2$

$\cos 4\theta \to \cos\pi / \cos 3\pi / \cos 5\pi / \cos 7\pi$

$4 + (-8) = -4$

Q. JEE Mains (2022)

Let a, b and c be the length of sides of a triangle ABC such that $\frac{a+b}{7}=\frac{b+c}{8}=\frac{c+a}{9}$. If r and R are the radius of incircle and radius of circumcircle of the triangle ABC, respectively, then the value of $\frac{R}{r}$ is equal to

(A) $\frac{5}{2}$

(B) 2

(C) $\frac{3}{2}$

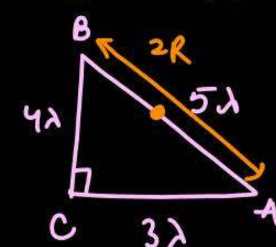
(D) 1

Solution of Triangle

$$\frac{a+b}{7}=\frac{b+c}{8}=\frac{c+a}{9}=\lambda$$

$$a+b=7\lambda$$
$$b+c=8\lambda$$
$$c+a=9\lambda$$
$$2(a+b+c)=24\lambda$$
$$a+b+c=12\lambda$$
$$c=5\lambda,\quad a=4\lambda,\quad b=3\lambda$$

$$c^2=a^2+b^2$$



$$2R=5\lambda$$
$$R=\frac{5\lambda}{2}$$
$$R=\frac{5(\lambda)}{2}$$

$$r=\frac{\frac{1}{2}(4\lambda)(3\lambda)}{6\lambda}=\frac{6\lambda^2}{6\lambda}$$
$$r=\lambda$$

$$\frac{R}{r}=\frac{5}{2}$$

Note :

$\boxed{\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} = 2R}$

$\boxed{\cos A = \dfrac{b^2+c^2-a^2}{2bc}}$

area of triangle

$\Delta = \dfrac{1}{2} ab\sin C = \dfrac{1}{2} bc\sin A = \dfrac{1}{2} ca\sin B$

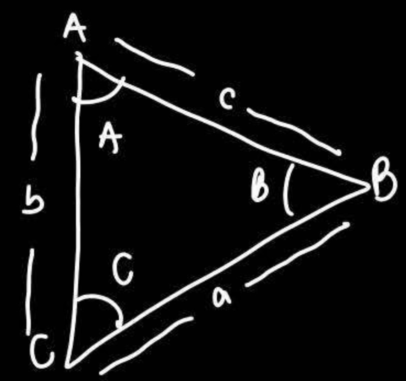
$\boxed{\Delta = \dfrac{abc}{4R}}$

circum-rad.

$\boxed{r = \dfrac{\Delta}{s}}$ # $s = \dfrac{a+b+c}{2}$

$\sin\dfrac{A}{2} = \sqrt{\dfrac{(s-b)(s-c)}{bc}}$ $\cos A/2 = \sqrt{\dfrac{s(s-a)}{bc}}$

$\tan A/2 = \dfrac{\Delta}{s(s-a)}$



Q. JEE Mains (2022)

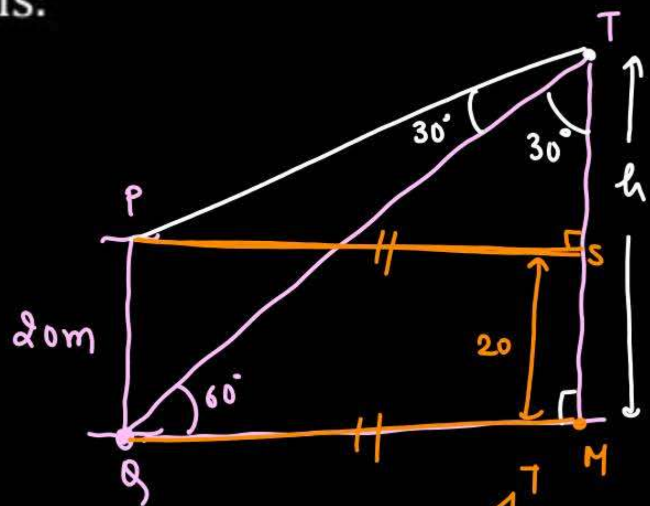
From the base of a pole of height 20 meter, the angle of elevation of the top of a tower is $60°$. The pole subtends an angle $30°$ at the top of the tower. Then the height of the tower is:

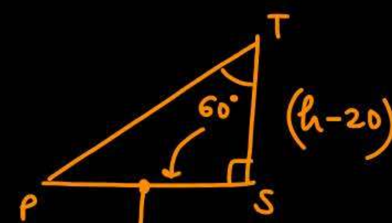
A $15\sqrt{3}$

B $20\sqrt{3}$

C $20 + 10\sqrt{3}$

D 30

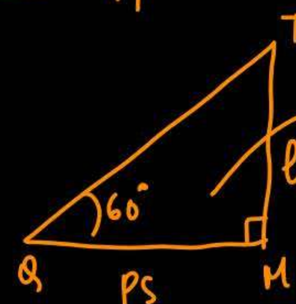




PS $= \frac{h}{\sqrt{3}}$

$$\tan 60° = \frac{h}{\sqrt{3}(h-20)}$$

$$\sqrt{3}$$



$$\sqrt{3} = \frac{h}{PS}$$

$$PS = \frac{h}{\sqrt{3}}$$

$$3(h-20) = h$$

$$3h - 60 = h$$

$$2h = 60$$

$$h = 30.$$

Mathematical Reasoning

JEE Mains (2023)

Q. The statement $B \Rightarrow ((\sim A) \vee B)$ is equivalent to

(A) $B \Rightarrow (A \Rightarrow B)$

(B) $A \Rightarrow (A \Leftrightarrow B)$

(C) $A \Rightarrow ((\sim A) \Rightarrow B)$

(D) $B \Rightarrow ((\sim A) \Rightarrow B)$

A	B	$\sim A$	$\sim A \vee B$	$B \to (\sim A \vee B)$	$A \to B$	$B \to (A \to B)$
T	T	F	T	T	T	T
T	F	F	F	T	F	T
F	T	T	T	T	T	T
F	F	T	T	T	T	T

NOTE :-

(1) $\boxed{p \to q \equiv (\sim p) \vee q}$

(2)
$$\sim (p \vee q) = \sim p \wedge \sim q$$
$$\sim (p \wedge q) = \sim p \vee \sim q$$

(3) If then :-

$\boxed{p \to q}$

Converse : $q \to p$

Inverse : $(\sim p) \to (\sim q)$

**Contrapositive : $(\sim q \to \sim p)$

(4) $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$

Q. JEE Mains (2023)

Let p and q be two statements. Then $\sim (p \wedge (p \Rightarrow \sim q))$ is equivalent to

A $p \vee (p \wedge (\sim q))$

B $p \vee ((\sim p) \wedge q)$

C $(\sim p) \vee q$

D $p \vee (p \wedge q)$

$$(p \to \sim q)$$
$$\Downarrow$$
$$\sim (p \wedge (\sim p \vee \sim q))$$

$$\sim(\sim p \,\underline{\underline{\vee}}\, \sim q)$$
$$(p \wedge q)$$

$$\boxed{\sim p \vee (p \wedge q)}$$

$$(\sim p \vee p) \wedge (\sim p \vee q) \equiv (\sim p \vee q)$$

$\Downarrow$

Complete

T

Q. JEE Mains (2023)

If p, q and r are three propositions, then which of the following combination of truth values of p, q and r makes the logical expression $\{(p \vee q) \wedge ((\sim p) \vee r)\} \rightarrow ((\sim q) \vee r)$ false ?

A $p = T, q = F, r = T$

B $p = T, q = T, r = F$

C $p = F, q = T, r = F$

D $p = T, q = F, r = F$

$$\boxed{\underbrace{(p \vee q) \wedge ((\sim p) \vee r)}_{T} \longrightarrow \underbrace{((\sim q) \vee r)}_{f}}$$

f

Q. JEE Mains (2023)

Consider the following statements:

P : I have fever

Q : I will not take medicine

R : I will take rest

The statement "If $\underbrace{\text{I have fever}}_{P}$, then $\underbrace{\text{I will take medicine}}_{\sim Q}$ and $\underbrace{\text{I will take rest}}_{R}$" is equivalent to:

A $((\sim P) \vee \sim Q) \wedge ((\sim P) \vee R)$

B $((\sim P) \vee \sim Q) \wedge ((\sim P) \vee \sim R)$

C $(P \vee Q) \wedge ((\sim P) \vee R)$

D $(P \vee \sim Q) \wedge (P \vee \sim R)$

\# $P \longrightarrow (\sim Q \wedge R)$

$\equiv (\sim P) \vee (\sim Q \wedge R)$

$(\sim P \vee \sim Q) \wedge (\sim P \vee R)$

$A \to B$

$\equiv (\sim A) \vee (B)$

Q. JEE Mains (2023)

Let $\Delta, \nabla \in \{\wedge, \vee\}$ be such that $(p \to q)\Delta(p \nabla q)$ is a tautology. Then

A $\Delta = \wedge, \nabla = \vee$ ×

B $\Delta = \vee, \nabla = \wedge$ ×

C $\Delta = \vee, \nabla = \vee$ ✓

D $\Delta = \wedge, \nabla = \wedge$

$(p \to q) \_ (p \_ q)$

(A) $(p \to q) \wedge (p \vee q)$ ×

$p \to T,\ q \to F$

(B) $\boxed{(p \to q) \vee (p \wedge q)}$

$F \vee F \equiv F$

(C) $\boxed{(p \to q) \vee (p \vee q)}$ → false

$(T \to F) \vee (T \vee F)$

$F \vee T$

T

Implies

$\boxed{p \to q}$

* $T \to F \equiv F$

otherwise always T.

p	q	$p \to q$	$q \to p$	$p \leftrightarrow q$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

double implies :-

$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

$$\underbrace{p \leftrightarrow q}_{\Downarrow} \equiv T$$

both same

$$\underbrace{p \leftrightarrow q} \equiv F$$

both opp.

Statistics

Q. JEE Mains (2023)

Let the six numbers $a_1, a_2, a_3, a_4, a_5, a_6$ be in $\boxed{\text{A.P.}}$ and $a_1 + a_3 = 10$. If the mean of these six numbers is $\frac{19}{2}$ and their variance is σ^2, then $8\sigma^2$ is equal to

A 220

B 210

C 200

D 105

Ans: B

$$\frac{a_1+a_2+\dots+a_6}{\cancel{6}\,3} = \frac{19}{\cancel{2}}$$

$$a_1+a_2+\dots+a_6 = 57$$

$$3(a_1+a_6) = 57$$

$$a_1+a_6 = 19 \rightarrow a+a+5d=19$$

$$a+a+2d=10$$

$$2a+5d=19$$

$$2a+2d=10$$

$$3d=9$$

$$d=3$$

Note :- Observ. $\rightarrow$ A.P.

$$\boxed{a,\ a+d,\ a+2d,\ \dots,\ a+(n-1)d}$$

$$\boxed{\sigma^2 = \frac{d^2(n^2-1)}{12}}$$

$$= \frac{d^2(6^2-1)}{12}$$

$$8\sigma^2 = \frac{\cancel{9}^{\,3}\times 35}{\cancel{12}\,\cancel{4}} \times \cancel{8}^{\,2} = 210$$

Q. JEE Mains (2023)

Let $X = \{11, 12, 13, \ldots, 40, 41\}$ (31 Terms.) and $Y = \{61, 62, 63, \ldots\ldots, 90, 91\}$ ($n =$ 31 Terms) be the two sets of observations. If $\bar{x}$ and $\bar{y}$ are their respective means and σ^2 is the variance of all the observations in $X \cup Y$, then $|\bar{x} + \bar{y} - \sigma^2|$ is equal to $|102 - 705|$

Ans: 603

$$\bar{x} = \frac{11 + 12 + \ldots + 41}{31}$$

$$= \frac{\frac{\cancel{31}}{2}(11 + 41)}{\cancel{31}}$$

$$= \frac{52}{2}$$

$$\bar{x} = 26$$

$$\bar{y} = \frac{\frac{\cancel{31}}{2}(61 + 91)}{\cancel{31}}$$

$$\bar{y} = 76.$$

$$\sigma_x^2 = \frac{d^2(n^2 - 1)}{12}$$

$$\sigma_x^2 = \frac{1 \times (31^2 - 1)}{12} = \sigma_y^2$$

$$31(\alpha + (\bar{x})^2) \Leftarrow \frac{\sum x_i^2}{31} - (\bar{x})^2 = \alpha = 80$$

$$\sigma^2 = \left(\frac{\sum x_i^2}{n}\right) - (\bar{x})^2$$

$$\sigma^2 = \left(\frac{\sum x_i^2 + \sum x_i^2}{2n}\right) - \left(\frac{\bar{x} + \bar{y}}{2}\right)^2$$

26, 76, $n \to 31$

$$\frac{\cancel{31}(\alpha + (\bar{x})^2) + \cancel{31}(\alpha + (\bar{y})^2)}{2 \times \cancel{31}} - \left(\frac{\bar{x} + \bar{y}}{2}\right)^2$$

$$\sigma^2 = \alpha + \frac{(\bar{x})^2}{2} + \frac{(\bar{y})^2}{2} - \left(\frac{\bar{x} + \bar{y}}{2}\right)^2$$

$$\sigma^2 = \alpha + \frac{(\bar{x} - \bar{y})^2}{4}$$

$$\sigma^2 = 80 + \frac{\cancel{2500}^{\,625}}{\cancel{4}}$$

$$\sigma^2 = 705$$

Q. JEE Mains (2023)

The mean and variance of the marks obtained by the students in a test are 10 and 4 respectively. Later, the marks of one of the students is increased from 8 to 12. If the new mean of the marks is 10.2. then their new variance is equal to :

Ans: C

A 4.04

B 4.08

C 3.96

D 3.92

$\bar{x} = 10 \rightarrow \dfrac{\boxed{x_1 + x_2 + \dots + x_{n-1}} + 8}{n} = 10 \Rightarrow x_1 + \dots + x_{n-1} = 10n - 8$

$\sigma^2 = 4 \rightarrow \left(\dfrac{x_1^2 + x_2^2 + \dots + x_{n-1}^2 + 8^2}{n}\right) - (10)^2 = 4 \Rightarrow \underbrace{x_1^2 + \dots + x_{n-1}^2} = 104 \times 20 - 8^2$

$\dfrac{x_1 + x_2 + \dots + x_{n-1} + 12}{n} = 10.2$

$10n - 8 + 12 = 10.2n$

$4 = 0.2n$

$\# \; 20 = n$

$\dfrac{\underbrace{x_1^2 + \dots + x_{n-1}^2} + 12^2}{n} - (10.2)^2 = ?$

$\dfrac{104 \times 20 - 64 + 12^2}{20} - (10.2)^2 = \underline{\underline{3.96}}$

Q. JEE Mains (2022)

The mean of the numbers a, b, 8, 5, 10 is 6 and their variance is 6.8. If M is the mean deviation of the numbers about the mean, then 25M is equal to:

(A) 60

(B) 55

(C) 50

(D) 45

Ans: A

$$\frac{a+b+8+5+10}{5} = 6$$

$$a+b = 30-23$$

$$\boxed{a+b=7}$$

$$\sigma^2 = 6.8$$

$$\frac{a^2+b^2+8^2+5^2+10^2}{5} - (6)^2 = 6.8$$

$$a^2+b^2+64+25+100 = (36+6.8)\,5$$

$$a^2+b^2 = 214-189$$

$$a^2+b^2 = 25$$

$$\left\{\begin{matrix} a=4 \\ b=3 \end{matrix}\right\}$$

$$M = \frac{|a-6| + |b-6| + |8-6| + |5-6| + |10-6|}{5}$$

$$\frac{2+3+2+1+4}{5}$$

$$25M = \frac{12}{\cancel{5}} \times \cancel{25}^{5} = 60$$