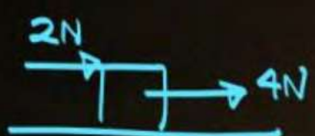
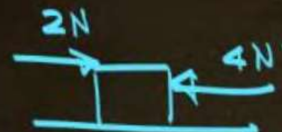
$\boxed{\text{Vectors}}$ $\longrightarrow$ Mag / direction.

PQ
(Direction)

(Addition / Subtraction)





$\boxed{F_{Total} = 6N}$ $\boxed{F_{Total} = 2N}$

(velocity, acc, displacement, force, Momentum)

$|\vec{F}|$ = length = Magnitude - - - direction.

$\vec{F} = (\text{Mag})(\text{direction})$

$\boxed{\vec{F} = |\vec{F}|\,\hat{F}}$ $\longrightarrow$ F Cap

direction of F

unit Vector along F.

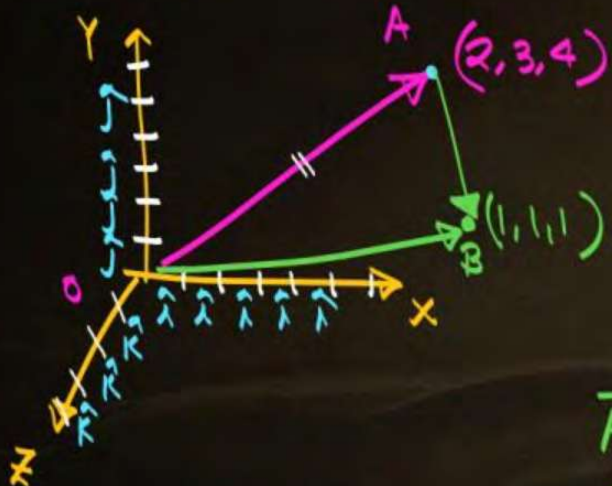
Ex:- $\vec{F} = 4N$ along x $|F| = 4N$ $\hat{F} = +x$

$\vec{F} = -4N$ along x $|F| = 4N$ $\hat{F} = -x$.

unit Vector (directions). Magnitude = 1.

(There are ∞ unit Vectors)

(3 Rectangular unit Vectors) $(\hat{i}, \hat{j}, \hat{k})$



$$\boxed{\vec{OA} = 2\hat{i} + 3\hat{j} + 4\hat{k}}$$ Position Vector of A wrt O.

$$|\vec{OA}| = \sqrt{2^2 + 3^2 + 4^2}$$

$$\hat{OA} = \frac{\vec{OA}}{|\vec{OA}|} = \frac{2\hat{i} + 3\hat{j} + 4\hat{k}}{\sqrt{29}}$$

$\vec{AB} = \vec{OB} - \vec{OA}$ (Displacement Vector).

$$\vec{AB} = -\hat{i} - 2\hat{j} - 3\hat{k}$$

$$|\vec{AB}| = \sqrt{(-1)^2 + (-2)^2 + (-3)^2} = \sqrt{14}$$

$$\hat{AB} = \frac{\vec{AB}}{|\vec{AB}|} = \frac{-\hat{i} - 2\hat{j} - 3\hat{k}}{\sqrt{14}}$$

$\text{Vector} = \vec{\text{Final}} - \vec{I}$

$= \vec{\text{Head}} - \vec{\text{Tail}}$

$\boxed{\text{Resolution}}$ + $\boxed{\text{Add + Sub}}$

Questions

2D Motion

Condition for particle to be always in 1D motion

Case 1.

u

$a = 0$ 1D

Case 2.

u, a ⟹ Speed up "accelerating"

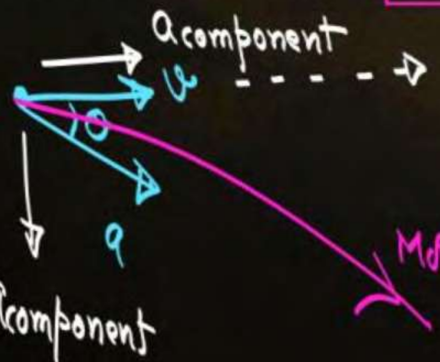
or

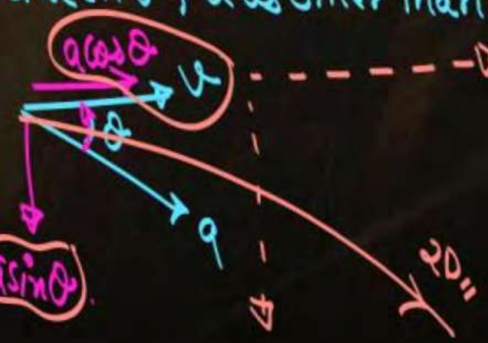
a ⟹ Retardation

(1D)

Condition for particle to be always in 2D motion

θ between v & a is other than 0 & π.

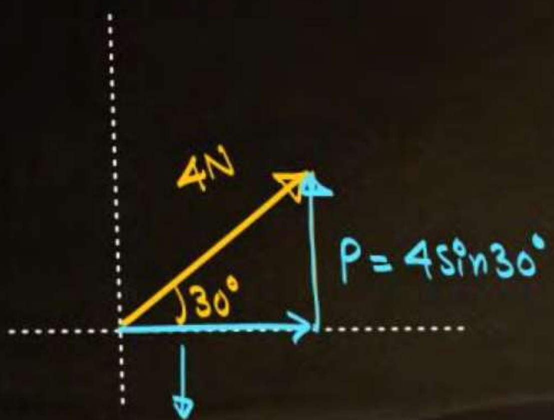




Resolution of Vectors

$\Downarrow$

"Todne"



Base = 4cos30

Vector = $4\cos 30\,\hat{i} + 4\sin 30\,\hat{j}$

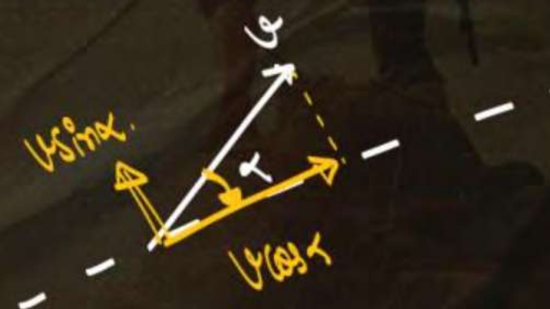
$$\boxed{\vec{V} = 2\sqrt{3}\,\hat{i} + 2\hat{j}}$$

Basic Maths

$\cos\theta = \dfrac{B}{H}$ $\qquad$ $\sin\theta = \dfrac{P}{H}$

$B = H\cos\theta$ $\qquad$ $P = H\sin\theta$

Vector Ko Todna hai



QUESTION 1

t → Vary x → Vary
Motion
y → Vary
Motion.

$y = \frac{x^2}{2}$ where $x = \frac{t^2}{2}$,
Velocity at $t = 2$ sec

$v_x = \frac{dx}{dt} = \frac{2t}{2} = t$

$y = \frac{x^2}{2} = \frac{1}{2}\frac{t^4}{4} = \frac{t^4}{8}$

$v_y = \frac{dy}{dt} = \frac{d}{dt}\left(\frac{t^4}{8}\right) = \frac{4t^3}{8} = \frac{t^3}{2}$

$\vec{V} = v_x\hat{i} + v_y\hat{j}$ at $t=2$

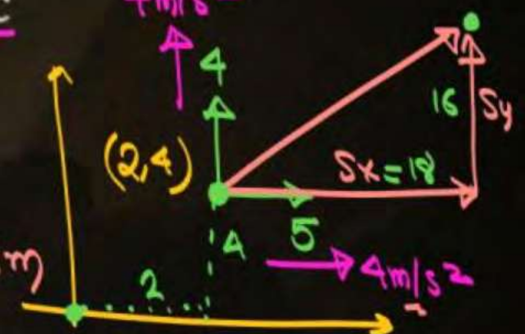
$\vec{V} = 2\hat{i} + 4\hat{j}$.

$|v| = \sqrt{2^2+4^2}$ $\hat{v} = \frac{2\hat{i}+4\hat{j}}{\sqrt{2^2+4^2}}$.

a = Const Kinematics Eqtn ✓.

QUESTION 2

initial position $(2\hat{\imath} + 4.0\hat{\jmath})$ at $t = 0$
initial velocity $(5.0\hat{\imath} + 4.0\hat{\jmath})$ ms^{-1}.
Constant acceleration $(4.0\hat{\imath} + 4.0\hat{\jmath})$
displacement in 2 sec



along x:-

$S_x = u_x t + \frac{1}{2} a_x t^2$

$= 5\times2 + \frac{1}{2}\cdot 4 \cdot 4 = 10+8 = 18m$

$S_y = u_y t + \frac{1}{2} a_y t^2$

$= 4\times2 + \frac{1}{2}\times4\times4$

$= 8+8 = 16m.$

$\boxed{\vec{S}_{net} = 18\hat{i} + 16\hat{j}}$ $a_x = 4$, $u_x = 5$; $a_y = 4$, $u_y = 4$.

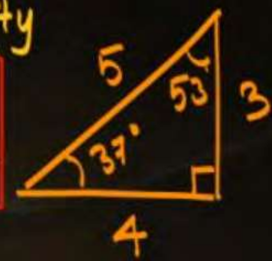
$\boxed{\text{Position}_{final} = 20\hat{i} + 20\hat{j}}$

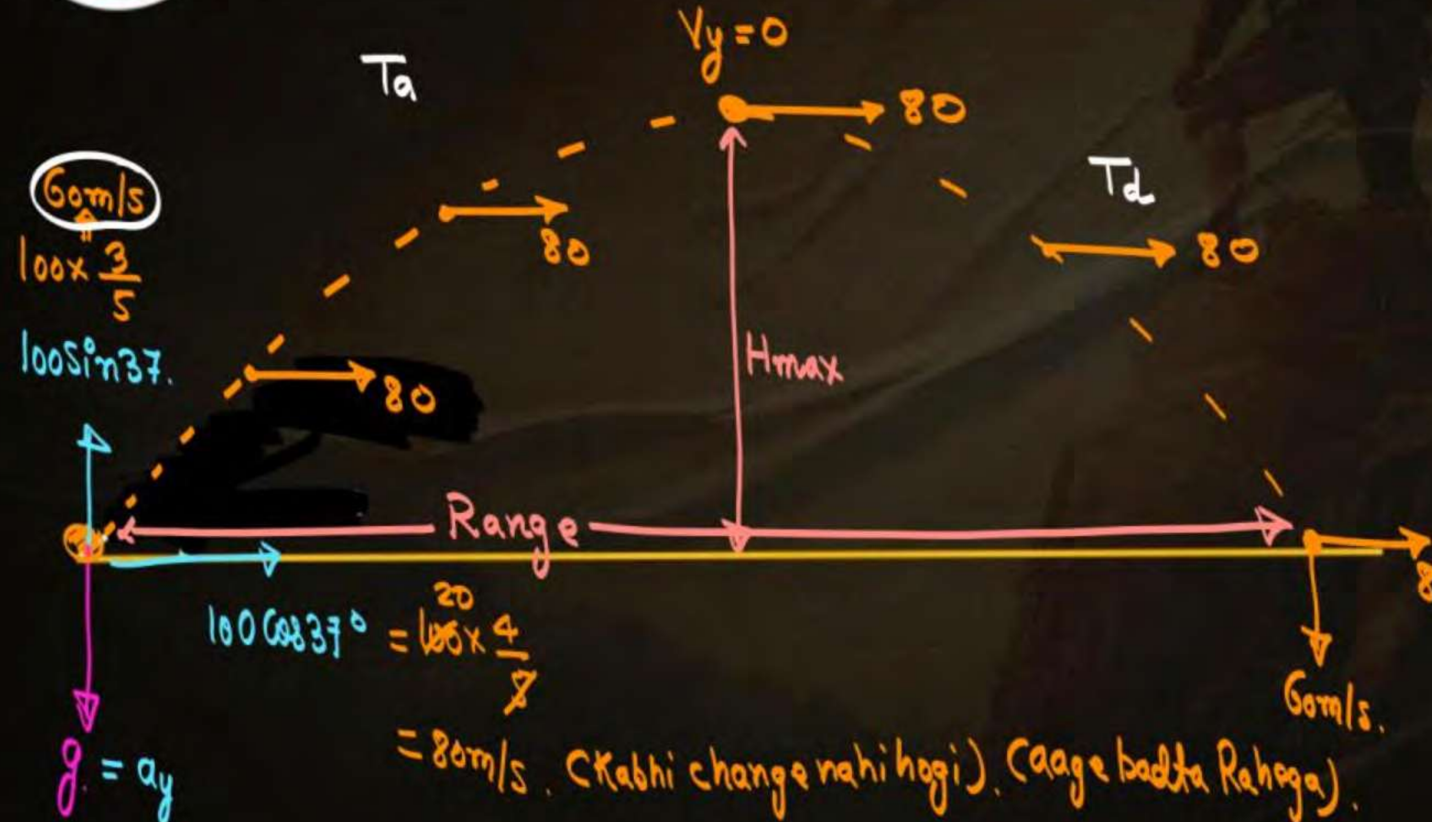
Ground to Ground Projection

Motion under Gravity

$u_x = 80 \quad u_y = 60$

$a_x = 0 \quad a_y = -10$





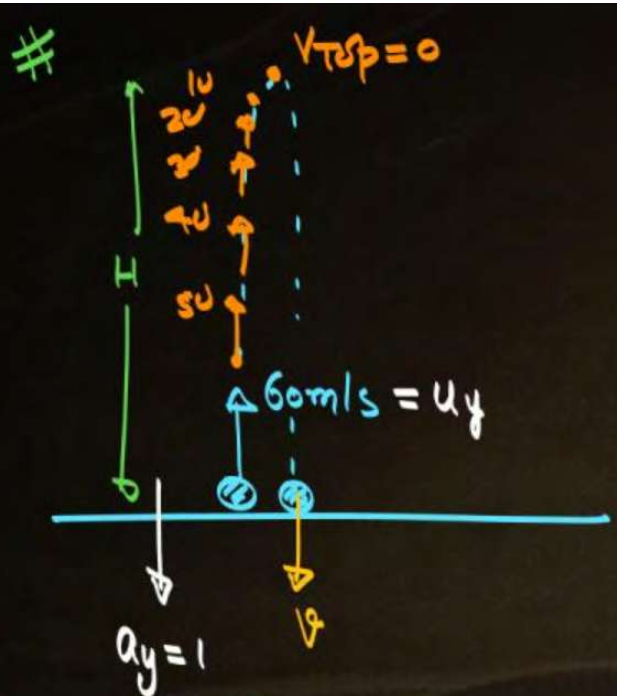
a) Time of flight.

$$T = \frac{2u_y}{|a_y|} = \frac{2\times 60}{10} = 12$$

b) $$H_{max} = \frac{u_y^2}{2a_y} = \frac{60\times 60}{20} = 180m$$

c) $$\text{Range} = S_x = u_x t + \cancel{\frac{1}{2} a_x t^2}^{\,0}$$

$$R = 80\times 12 = 960m. \quad = \frac{u_x\, 2u_y}{a_y}$$

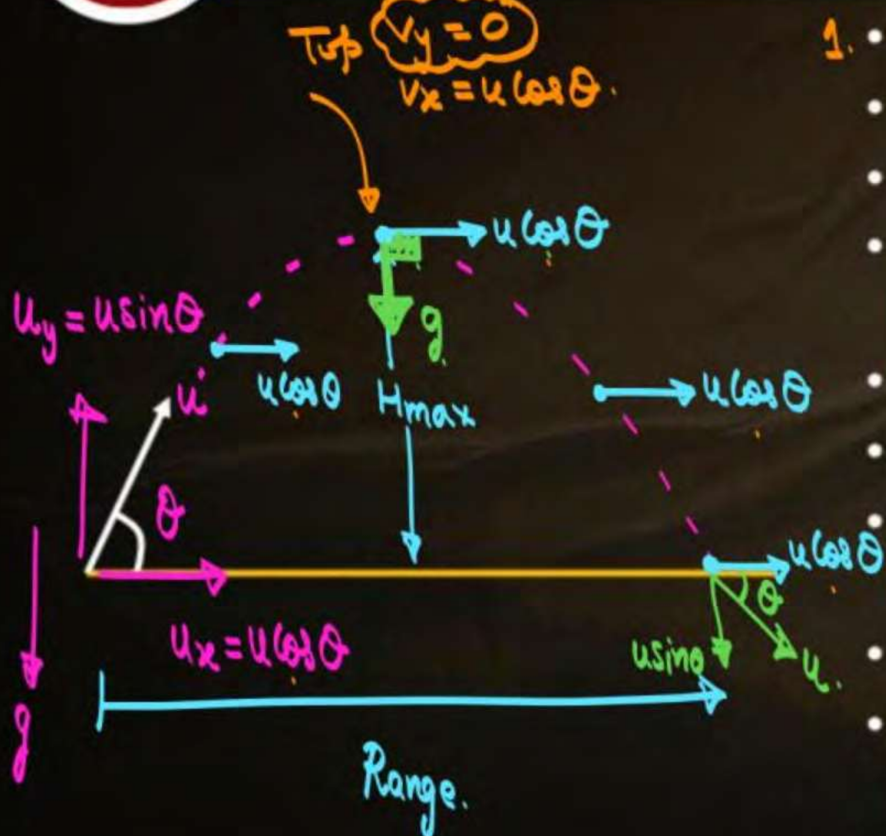


$$T = \frac{2u}{g} = \frac{2\times 60}{10} = 12 \text{ sec.}$$

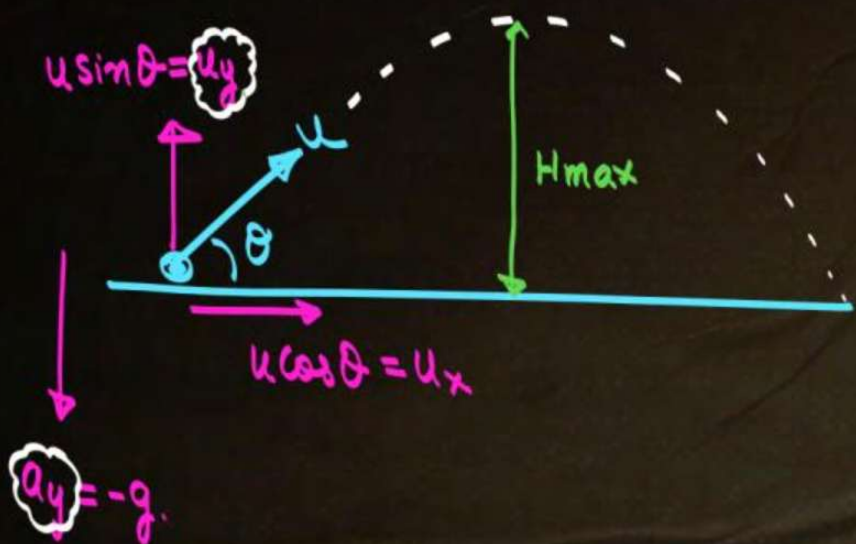
$$H_{max} = \frac{u^2}{2g} = \frac{60\times \cancel{60}^{3}}{\cancel{20}} = 180 m$$

$$\vec{v} = -60\hat{j}$$

Ground to Ground Projection



1. - There is no acceleration along x $a_x = 0$
- All acceleration is along y that is $-g$ $a_y = -g$
- Along x axis no change in velocity
- Time and vertical height is calculated along vertical direction
- Range is calculated along horizontal direction
- At top point of projectile velocity is minimum
- Vertical component of velocity at top point is zero
- Net velocity at top point is $u\cos\theta$.
- At top point net velocity and acceleration is perpendicular
- Speed at same horizontal level is same


u sinθ = u_y
u
θ
Hmax
u cosθ = u_x
a_y = -g.

Time of Flight

$$\boxed{T = \frac{2u_y}{a_y} = \frac{2u\sin\theta}{g}}$$

$$t_a = \frac{u\sin\theta}{g}$$

$$t_d = \frac{u\sin\theta}{g}$$

Maximum Height

$$\boxed{H_{max} = \frac{u_y^2}{2a_y} = \frac{u^2\sin^2\theta}{2g}}$$

Range of projectile

$$S_x = u_x t + \cancel{\frac{1}{2}a_x t^2}^{\,0}$$

$$S_x = \frac{u_x \cdot 2u_y}{a_y}$$

$$\boxed{R = \frac{2u_x u_y}{a_y}}$$

$$= \frac{2u\sin\theta\, u\cos\theta}{g} = \boxed{\frac{u^2\sin 2\theta}{g}}$$

❖ Maximum Range Condition

$$R = \frac{u^2 \sin 2\theta}{g} \rightarrow 1 \qquad \sin 2\theta = 1$$

$$\theta = 45^\circ$$

$\theta = ?\ R_{max}$

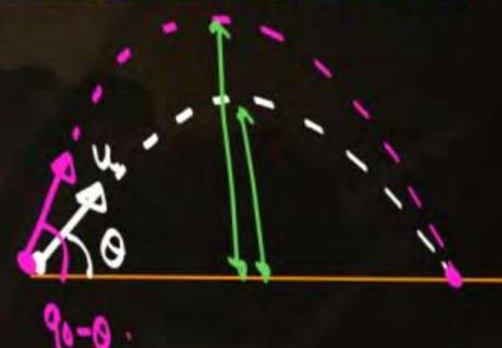
$$\boxed{R_{max} = \frac{u^2}{g}}$$

❖ Complimentary angles projection

Angles Sum = 90°

$\boxed{\theta \text{ or } 90-\theta}$

$\boxed{45+\theta \text{ or } 45-\theta}$



$$R_\theta = \frac{u^2\sin 2\theta}{g}$$

$$R_{90-\theta} = \frac{u^2 \sin 2(90-\theta)}{g} = \frac{u^2\sin 2\theta}{g}.$$

⊛ θ or $90-\theta$

$$\boxed{R_\theta = R_{90-\theta}}$$

\# $H_\theta = \dfrac{u^2 \sin^2\theta}{2g}$

$H_{90-\theta} = \dfrac{u^2 \sin^2(90-\theta)}{2g} = \dfrac{u^2 \cos^2\theta}{2g}.$

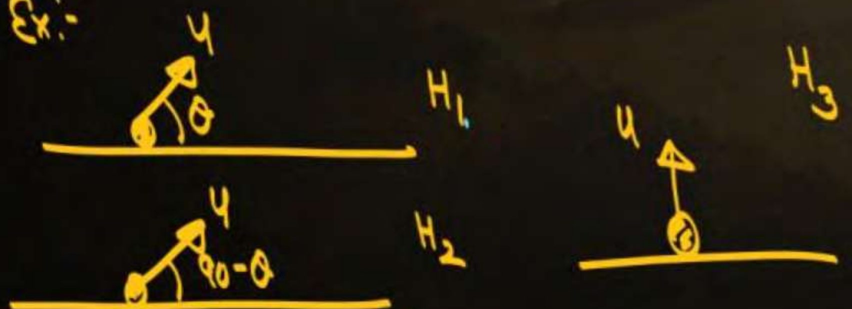
$$\boxed{\frac{H_\theta}{H_{90-\theta}} = \tan^2\theta}$$

\# $T_\theta = \dfrac{2u\sin\theta}{g}$

$T_{90-\theta} = \dfrac{2u\cos\theta}{g}.$

$$\boxed{\frac{T_\theta}{T_{90-\theta}} = \tan\theta}$$

Ex:-



Find Relation between H_1 H_2 H_3.

$H_1 = \dfrac{u^2\sin^2\theta}{2g}$

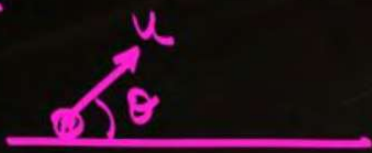
$H_2 = \dfrac{u^2\cos^2\theta}{2g}$

$H_3 = \dfrac{u^2}{2g}.$

$H_1 + H_2 = \dfrac{u^2}{2g}(\sin^2\theta + \cos^2\theta)$

$$\boxed{H_1 + H_2 = H_3}$$

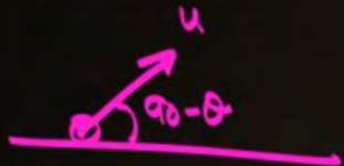
Ex:-



$T_1 = \frac{2u\sin\theta}{g}$

$T_3 = \frac{2u}{g}$

Find Relation between T_1 T_2 & T_3.



$T_2 = \frac{2u\cos\theta}{g}$

$\sin^2\theta + \cos^2\theta = 1$

$$\left(\frac{gT_1}{2u}\right)^2 + \left(\frac{gT_2}{2u}\right)^2 = 1$$

$$T_1^2 + T_2^2 = \left(\frac{2u}{g}\right)^2$$

$$\boxed{\sqrt{T_1^2 + T_2^2} = T_3}$$

Q. Time of flight = 3 sec.

Comp angle = 4 sec

Vertical proj =

$\sqrt{3^2 + 4^2} = T_V$

$T_V = 5$

Position of particle at any time *t*

$$\vec{S} = S_x\hat{i} + S_y\hat{j}$$

$$= \left(u_x t + \cancelto{0}{\tfrac{1}{2}a_x t^2}\right)\hat{i} + \left(u_y t + \tfrac{1}{2}a_y t^2\right)\hat{j}$$

$$\boxed{\vec{S} = (u\cos\theta\, t)\hat{i} + \left(u\sin\theta\, t - \tfrac{1}{2}g t^2\right)\hat{j}}$$

Velocity of particle at any time *t*

$$\vec{V} = v_x\hat{i} + v_y\hat{j}$$

$$= (u_x + a_x t)\hat{i} + (u_y + a_y t)\hat{j}$$

$$\boxed{\vec{V} = u\cos\theta\,\hat{i} + (u\sin\theta - g t)\hat{j}}$$

Angular Momentum [redacted]

$$\vec{L} = \vec{r} \times \vec{p} \Rightarrow \text{Determinant Method} = \begin{vmatrix} \hat{i} & -\hat{j} & \hat{k} \\ r_x & r_y & r_z \\ p_x & p_y & p_z \end{vmatrix} = \hat{i}(r_y p_z - p_y r_z) - \hat{j}(r_x p_z - p_x r_z) + \hat{k}(r_x p_y - p_x r_y).$$

$\vec{r}$ = Position vector

$\vec{p}$ = Momentum.

#

displacement & Velocity in 2sec.

$u_x = 80 \quad a_x = 0$

$u_y = 60 \quad a_y = -10.$

$m = 2kg.$

a) disp in $t = 2.$

$$S_x = u_x t + \cancel{\tfrac{1}{2} a_x t^2}^{\,0} = 80 \times 2 = 160m$$

$$S_y = u_y t + \tfrac{1}{2} a_y t^2 = 60 \times 2 - \tfrac{1}{2} \times 10 \times 4 = 120 - 20$$
$$= 100m.$$

$$\boxed{\vec{S} = 160\hat{i} + 100\hat{j}}$$

b) $\vec{V} = v_x\hat{i} + v_y\hat{j} = 80\hat{i} + 40\hat{j}.$

$v_x = u_x + a_x t = 80$

$v_y = u_y + a_y t = 60 - 10 \times 2 = 40.$

Find Angular Momentum about point of projection at $t=2$.

$m = 2\,kg$

at $t = 2$

$\vec{r} = 160\hat{i} + 100\hat{j}$

$\vec{v} = 80\hat{i} + 40\hat{j}$

$\vec{p} = m\vec{v} = 160\hat{i} + 80\hat{j}$.

$$\vec{L} = \vec{r} \times \vec{p} = \begin{vmatrix} \hat{i} & -\hat{j} & \hat{k} \\ 160 & 100 & 0 \\ 160 & 80 & 0 \end{vmatrix} = \hat{i}(0-0) - \hat{j}(0-0) + \hat{k}\underline{(160 \times 80 - 160 \times 100)}.$$

$\vec{L}$.

❖ Equation of Trajectory

$\Downarrow$

Raasta of particle.

$\Downarrow$

$\boxed{y = f(x)}$

Steps:

1. $X = F(t)$
2. $Y = F(t)$
3. Eliminate t

Example :

$x = t^2$, $y = t^4 + t^2$

Find equation of trajectory.

$x = t^2$

$y = f(x)$

$y = t^4 + t^2$

$y = (t^2)^2 + t^2$

$\boxed{y = x^2 + x}$ → Parabolic path.

#

$S_x = u_x t + \frac{1}{2} a_x t^2$

$\boxed{x = u\cos\theta\, t} \Rightarrow t = \frac{x}{u\cos\theta}$

$S_y = u_y t + \frac{1}{2} a_y t^2$

$\boxed{y = u\sin\theta\, t - \frac{1}{2} g t^2}$

$y = u\sin\theta\left(\frac{x}{u\cos\theta}\right) - \frac{1}{2} g \left(\frac{x}{u\cos\theta}\right)^2$

$\boxed{y = x\tan\theta - \frac{g x^2}{2u^2\cos^2\theta}} = \boxed{x\tan\theta\left[1 - \frac{x}{R}\right]}$

R = Range.

Horizontal Projectile

1) Motion under Gravity.

$u_x = 40 \qquad a_x = 0$

$\boxed{u_y = 0} \qquad a_y = -g.$

2) Kya ye 2D Motion?

❖ **Time of flight:**

$$\boxed{T = \sqrt{\frac{2H}{g}}}$$

$$S_y = u_y t + \frac{1}{2} a_y t^2$$

$$-H = -\frac{1}{2} g t^2$$

❖ **Range of projectile:**

$$R = S_x = u_x t + \cancel{\frac{1}{2} a_x t^2}$$

$$\boxed{R = u_x \sqrt{\frac{2H}{g}}}$$

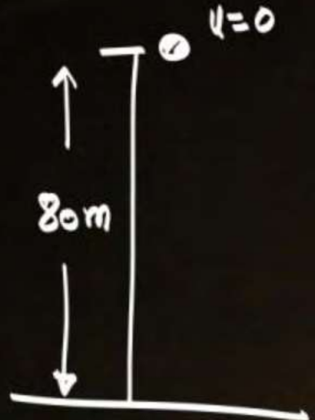
$$T = \sqrt{\frac{2 \times 80}{10}} = 4\,\text{sec}$$

$$R = S_x = u_x t$$

$$= 40 \times 4$$

$$= 160\,\text{m}$$

#



Time of flight $= \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times 80}{10}} = 4$ sec.

Velocity with which it hits ground =

$v^2 - u^2 = 2as$

$v^2 = 2\times g\times 80$

$v = 40$

$v = u + at$

$v = 0 - 10\times 4$

$= -40$ m/s.

Position at any time t

$$\vec{S} = S_x\hat{i} + S_y\hat{j}$$

$$= \left(u_x t + \frac{1}{2}a_x t^2\right)\hat{i} + \left(u_y t + \frac{1}{2}a_y t^2\right)\hat{j}$$

$$\boxed{\vec{S} = ut\,\hat{i} - \frac{1}{2}gt^2\,\hat{j}}$$

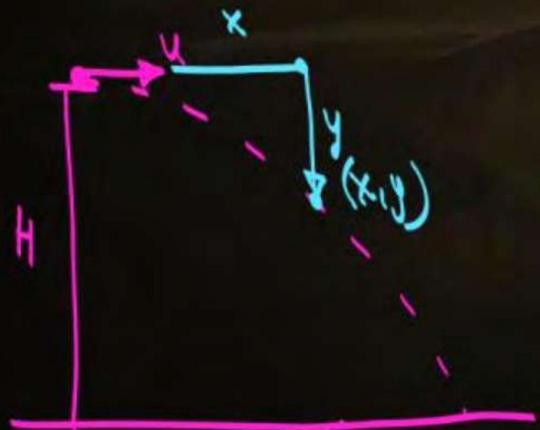
Velocity at any time t

$$\vec{V} = V_x\hat{i} + V_y\hat{j}$$

$$= (u_x + a_x t)\hat{i} + (u_y + a_y t)\hat{j}$$

$$\boxed{\vec{V} = u\,\hat{i} - gt\,\hat{j}}$$

Equation of trajectory



$$x = ut \qquad t = \frac{x}{u}$$

$$y = -\frac{1}{2}gt^2$$

$$\boxed{y = -\frac{1}{2}g\frac{x^2}{u^2}}$$

→ Parabolic.

QUESTION 3

4times.

For projectile.

Equation of trajectory $y = 2x - \frac{x^2}{2}$

- Find angle of projection $= 2x\left[1 - \frac{x^2}{2 \cdot 2x}\right] = 2x\left[1 - \frac{x}{4}\right]$
- Range of particle
- Initial velocity.

$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$ or $y = x\tan\theta\left[1 - \frac{x}{R}\right]$

Compare

$\frac{x}{4} = \frac{x}{R}$ $\quad$ $x\tan\theta = 2x$

$\tan\theta = 2$

$\frac{2u^2\sin\theta\cos\theta}{g} = \boxed{R = 4}$ $\quad$ $\boxed{\theta = \tan^{-1}(2)}$

Question 4

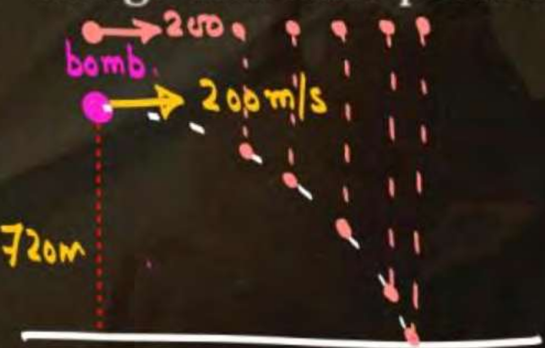
drop.

$\boxed{v_{person} = v_{bomb}}$

An airplane flying with 200 m/s at height 720m drops bomb.

Trajectory from ground and plane

Range and time period

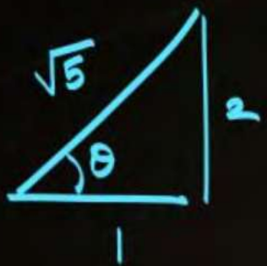


Parabolic. $\quad$ Straight line.

$T = \sqrt{\frac{2 \times 720}{10}} = 12$

$R = 200\,T = 200\sqrt{\frac{2 \times 720}{10}} = 2400.$

$\tan = \frac{2}{1} = \frac{P}{B}$



$$\text{Range} = 4 = \frac{u^2 \sin 2\theta}{g}$$

$\sin\theta = \frac{2}{\sqrt{5}}$

$\cos\theta = \frac{1}{\sqrt{5}}$

$$4 = \frac{u^2\, 2\sin\theta\cos\theta}{g}$$

$$4 = \frac{2u^2}{10} \times \frac{2}{\sqrt{5}} \times \frac{1}{\sqrt{5}}$$

$u = \_\_\_\_$.

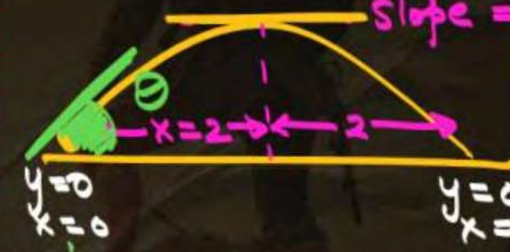
Method 2:- Angle of proj = slope at $x = 0$

$$y = 2x - \frac{x^2}{2} \qquad \left.\frac{dy}{dx}\right|_{x=0} = 2$$

$\tan\theta = 2$

$\theta = \tan^{-1} 2$

slope $= \frac{dy}{dx} = 0$



$\Rightarrow$ Range.

Range $y = 0 = 2x - \frac{x^2}{2}$

$$0 = 2x\left(1 - \frac{x}{4}\right)$$

$x = 0 \quad y = 0$

$x = 4 \quad y = 0$

hmax=? $\frac{dy}{dx} = 2 - x$ Range = 4

at hmax $\frac{dy}{dx} = 0$ $x = 2$

Hmax = ?
put $x = 2$ & find y.

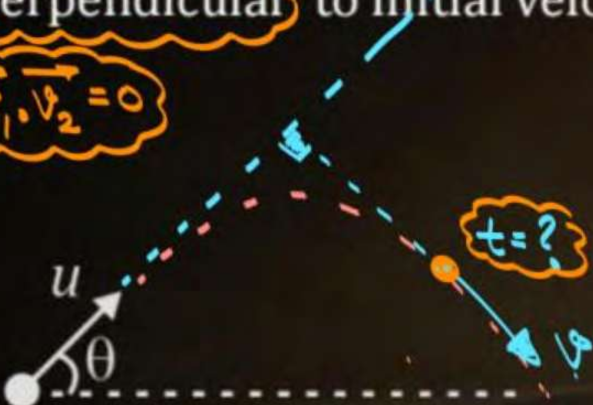
$y = 4 - 2$
$= 2 = H_{max}$

(Adv + JEE Mains)

Question 5

Time at which velocity is perpendicular to initial velocity

$\vec{v}_1 \cdot \vec{v}_2 = 0$



$t=0 \quad \vec{v}_i = u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}$

$t=t \quad \vec{v}_t = u\cos\theta\,\hat{i} + (u\sin\theta - gt)\,\hat{j}$

$\vec{v}_i \cdot \vec{v}_t = 0 = u^2\cos^2\theta + u^2\sin^2\theta - u\sin\theta(gt) = 0$

$u^2 = u\sin\theta\, g t \qquad \boxed{t = \frac{u}{g}\csc\theta}$

Question 6

#

$\vec{u} = \hat{\imath} + 2\hat{\jmath}, \vec{a} = -10\hat{\jmath}$

equation of trajectory $\rightarrow y = f(x)$

$S_x = u_x t + \frac{1}{2}a_x t^2$

$\boxed{x = 1 \times t}$

$S_y = u_y t + \frac{1}{2}a_y t^2$

$y = 2t - \frac{1}{2}10t^2$

$\boxed{y = 2t - 5t^2}$

$t = x$

$\boxed{y = 2x - 5x^2}$ Ans.

Question 7

JEE

Ratio of speed at projection to top is x angle of projection =

$$\frac{v_{Projection}}{v_{Top}} = x$$

$$\frac{\cancel{v}}{\cancel{v}\cos\theta} = x$$

$$\cos\theta = \frac{1}{x}$$

$$\boxed{\theta = \cos^{-1}\left(\frac{1}{x}\right)}$$

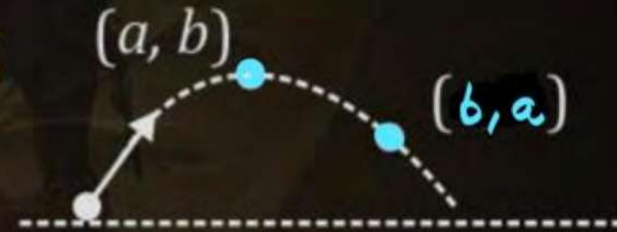
Question 8

(Level up).

Find the range of projectile.

Passes through (a,b) & (b,a)

Find Range.



$$y = x\tan\theta\left[1-\frac{x}{R}\right]$$

$$b = a\tan\theta\left[1-\frac{a}{R}\right]$$

$$a = b\tan\theta\left[1-\frac{b}{R}\right]$$

Ratio

$$\frac{b}{a} = \frac{a}{b}\left[\frac{R-a}{R-b}\right]$$

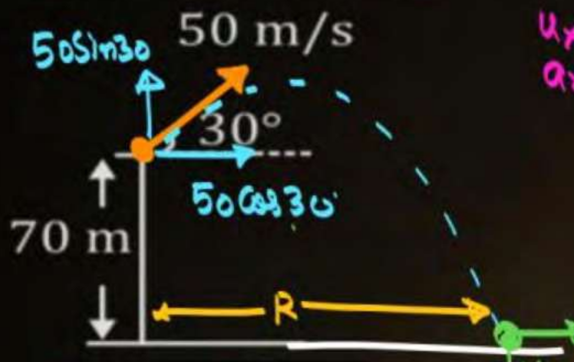
$$\frac{b^2}{a^2} = \frac{R-a}{R-b}$$

$R =$ ______.

Question 9

Time of flight
Range.

$u_x = 25\sqrt{3}$ $u_y = 25$
$a_x = 0$ $a_y = -10$.



a) Time of flight:

$\vec{S_y} = \vec{u_y}t + \frac{1}{2}\vec{a_y}t^2$

$-70 = +25t - \frac{1}{2}10t^2$

$-14 = 5t - t^2$

$t^2 - 5t - 14 = 0$

$t^2 - 7t + 2t - 14 = 0$

$t = 7$.

$S_x = u_x t + \cancel{\frac{1}{2}a_x t^2}$

$= 25\sqrt{3} \times 7$

$\boxed{R = 175\sqrt{3}}$

Velocity it hits the ground.

$\vec{V} = v_x\hat{i} + v_y\hat{j}$

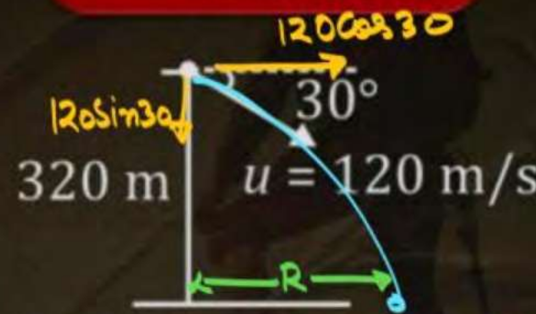
$= 25\sqrt{3}\hat{i} + (25 - 10\times 7)$

$= 25\sqrt{3}\hat{i} - 45\hat{j}$.

Question 10

(i) Time of flight
(ii) Range

$u_x = 60\sqrt{3}$ $u_y = -60$
$a_x = 0$ $a_y = -10$.



a) Time of flight.

$S_y = u_y t + \frac{1}{2}a_y t^2$

$-320 = -60t - \frac{1}{2}10t^2$

$-320 = -60t - 5t^2$

$t = 4$

$S_x = u_x t + \frac{1}{2}a_x t^2$

$R = 60\sqrt{3} \times 4$

$\boxed{R = 240\sqrt{3}}$

Question 11

JEE

If Range is same for two angle of projection. Relation between $h_1 h_2$ and R.

$\theta, 90-\theta$

$$h_1 = \frac{u^2 \sin^2\theta}{2g}$$

$$h_2 = \frac{u^2 \cos^2\theta}{2g}$$

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{2u^2 \sin\theta \cos\theta}{g}$$

$$R^2 = \frac{4u^4 \sin^2\theta \cos^2\theta}{g^2}$$

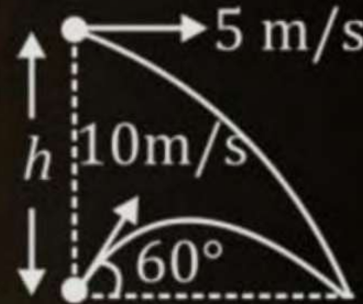
$$h_1 h_2 = \frac{u^4 \sin^2\theta \cos^2\theta}{(2g)^2} = \frac{4}{4\times 4}\underbrace{\frac{u^4 \sin^2\theta \cos^2\theta}{g^2}}_{R^2}$$

$$h_1 h_2 = \frac{R^2}{16} \qquad \boxed{R = 4\sqrt{h_1 h_2}}$$

Question 12

HW JEE

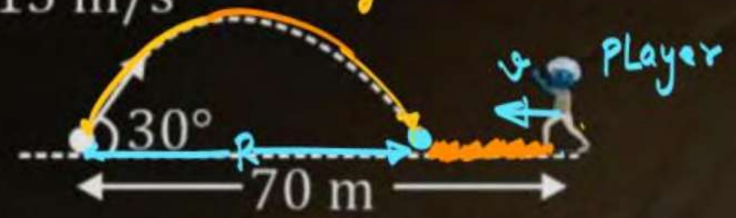
What is h? so that they meet at C simultaneously.



Question 13

JEE

$T = \frac{2u\sin\theta}{g} = \frac{2\times 15\sin 30}{10} = \frac{3}{2}$



Speed of person to Catch ball

$R = \frac{u^2\sin(2\theta)}{g}$

$R = \frac{(15)^2\sin(2\times 30)}{10}$

$R = \frac{15^2\times\sin 60}{10}$

$\text{Speed} = \frac{D}{T}$

$\boxed{\text{Ans} = \frac{70-R}{3/2}}$

Question 14

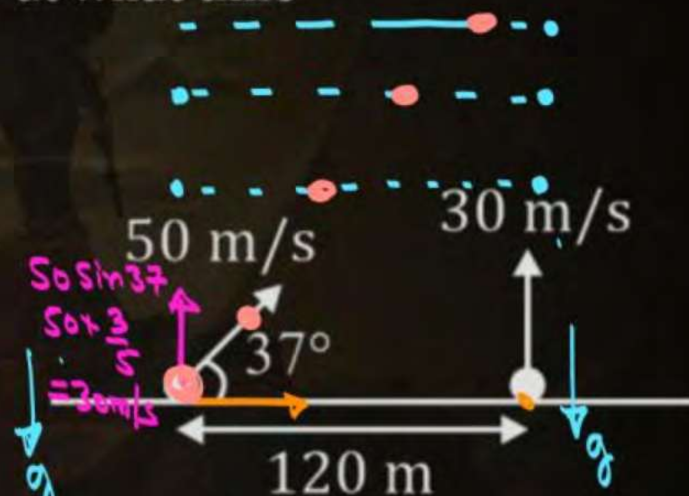
Will they meet? at what time

$t = \frac{D}{S}$

$= \frac{120}{40}$

$t = 3\,\text{sec}$

Along y axis both has Same u & a. at height all all time.



$50\sin 37$

$50\times\frac{3}{5}$

$= 30\,\text{m/s}$

$50\cos 37$

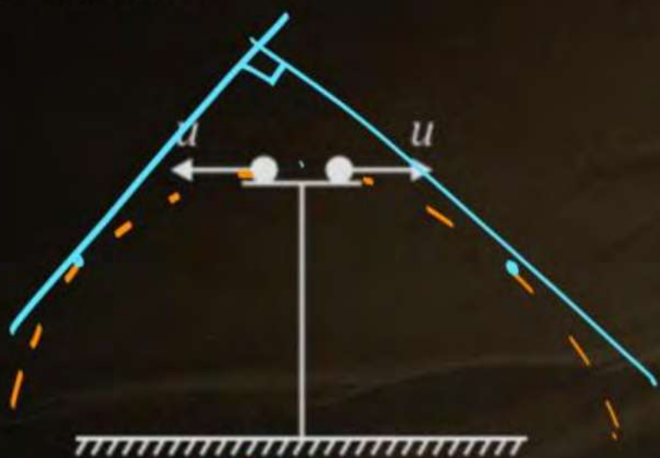
$\frac{50}{10}\times\frac{4}{5}$

40 m/s

Question 15

JEE Mains (HW)

Time at which velocity are per perpendicular

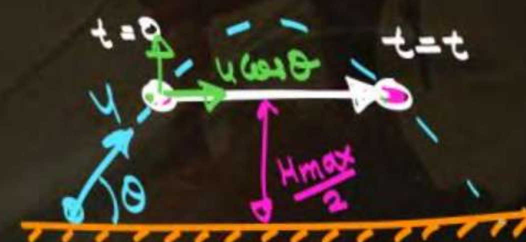


$\vec{v}_1 \cdot \vec{v}_2 = 0$

Question 16

HCV

Avg. velocity of projectile between instant it crosses Half of maximum height.



$$\frac{\text{Total disp}}{\text{Total time}} = \frac{(u\cos\theta)t}{t} = u\cos\theta$$

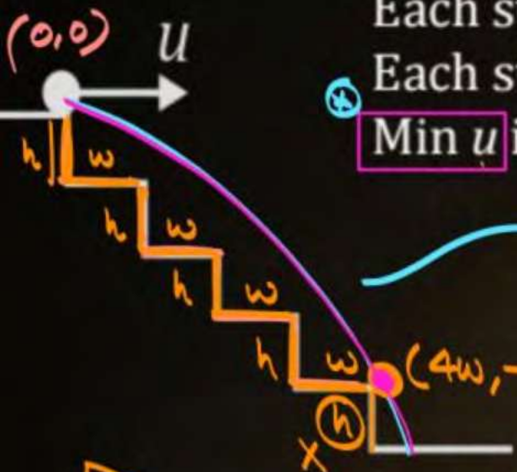
Question 17

JEE + HCV

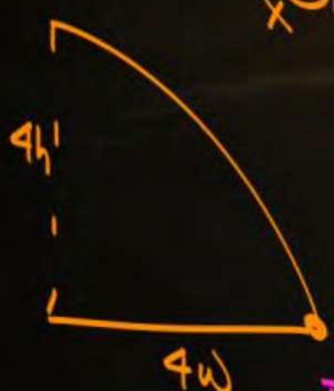
Each step height = h

Each step width = w

$\boxed{\text{Min } u}$ it hits $\boxed{\text{ground directly}}$



$$\boxed{y = -\frac{gx^2}{2u^2}}$$



$$-4h = -\frac{g(4w)^2}{2u^2}$$

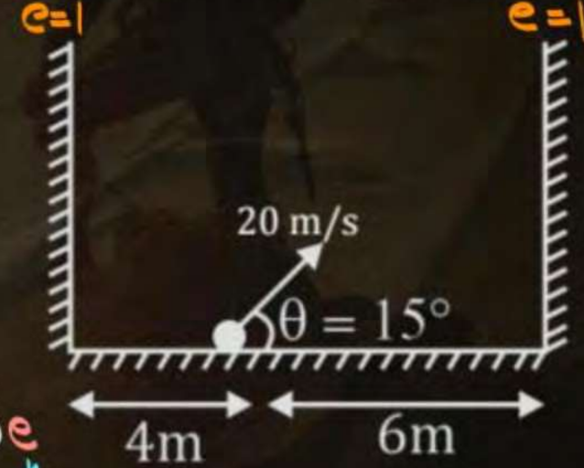
$u =$ ____

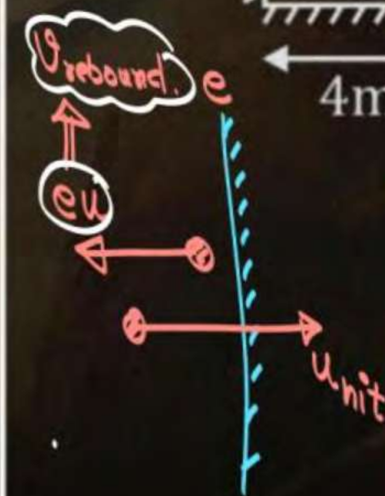
$T = \sqrt{\frac{2(4h)}{g}}$ $\quad R = 4w = u\sqrt{\frac{2(4h)}{g}}$

Question 18

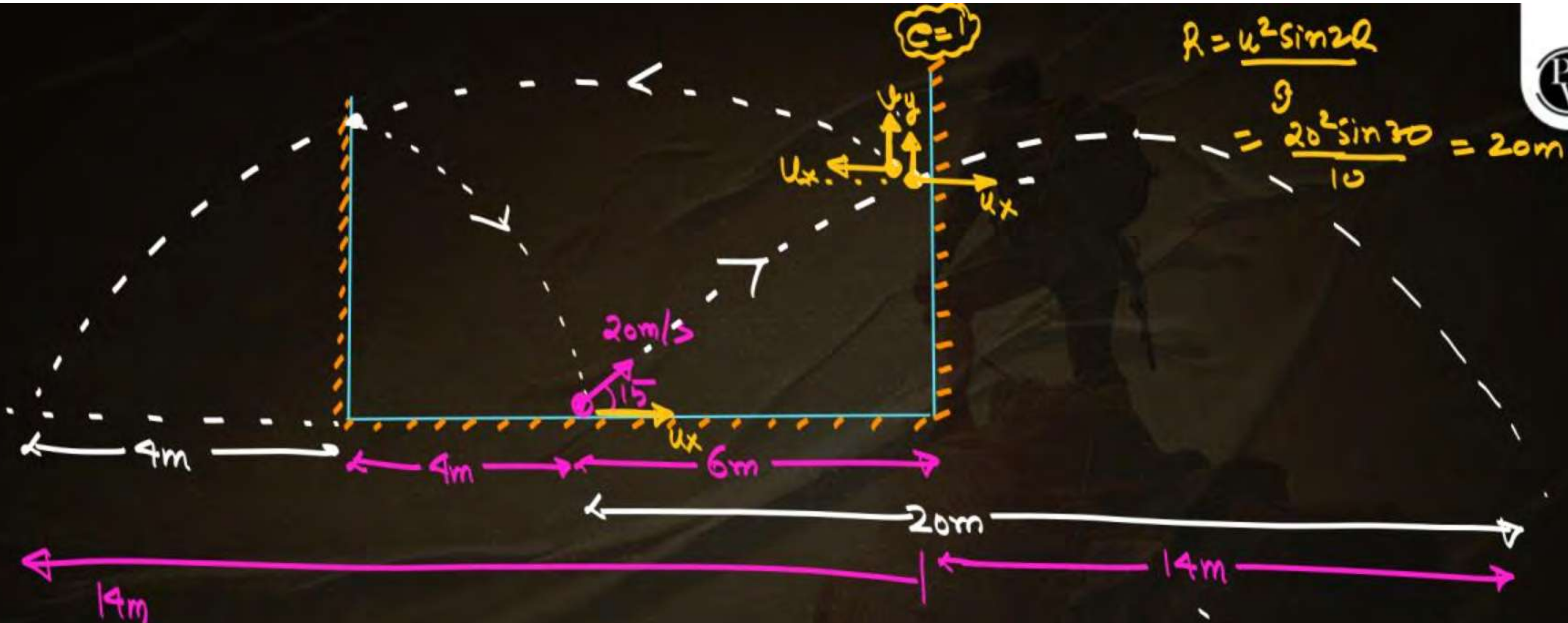
Collision (Adv)

$e = 1$ where ball will Land and time of flight



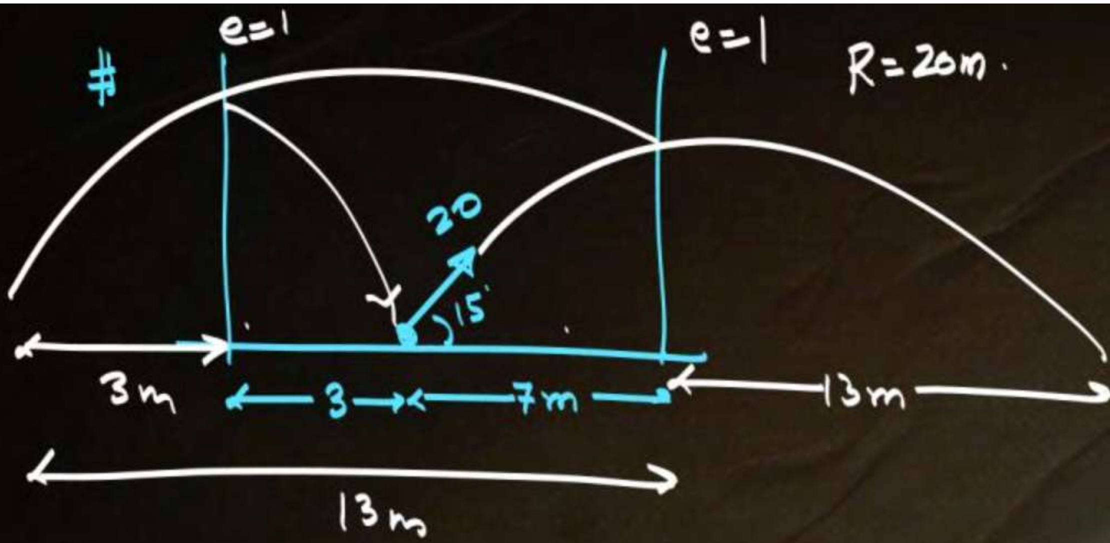


In this Que $e=1$



e = 1 both walls, distance between wall = $\frac{\text{Range}}{2}$, ball will Land at Same point.

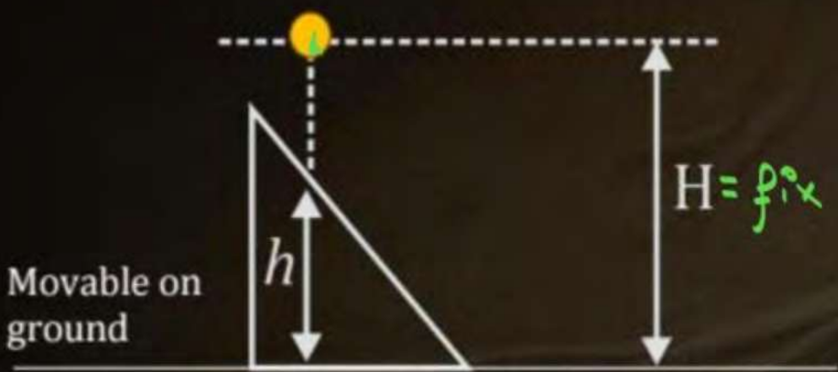
Time of flight $= \frac{2u\sin\theta}{g}$, $H_{max} = \frac{u^2\sin^2\theta}{2g}$.


#
e=1
e=1
R=20m.
20
15°
3m
3
7m
13m
13m

Question 19

(Advanced) (fluids).

What should be h/H so Range is maximum if velocity become horizontal



Question 20

JEE Mains x 3

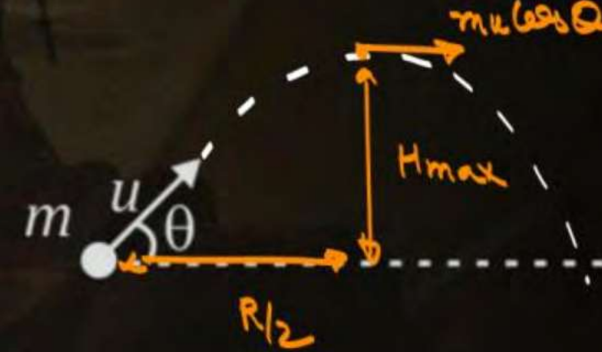
HW

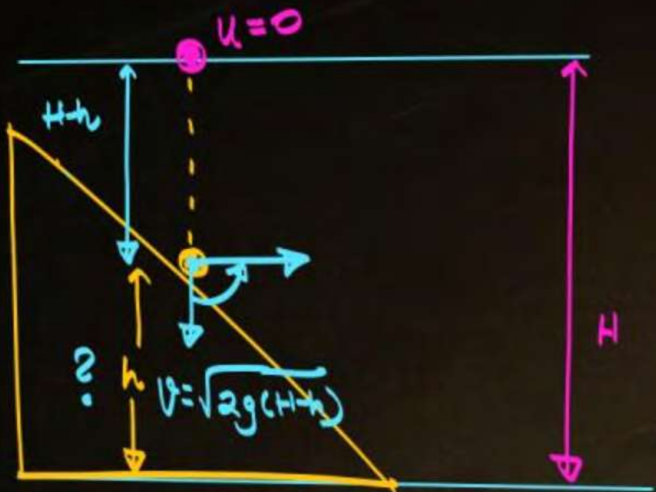
Angular momentum about O

- When at top
- When at maximum Range

$$\boxed{\begin{aligned}\vec{p} &= mu\cos\theta \\ \vec{r} &= \frac{R}{2}\hat{i} + H_{max}\hat{j}\end{aligned}}$$

$\vec{L} = \vec{r} \times \vec{p}$.





$$R^2 = 4(Hh - h^2)$$

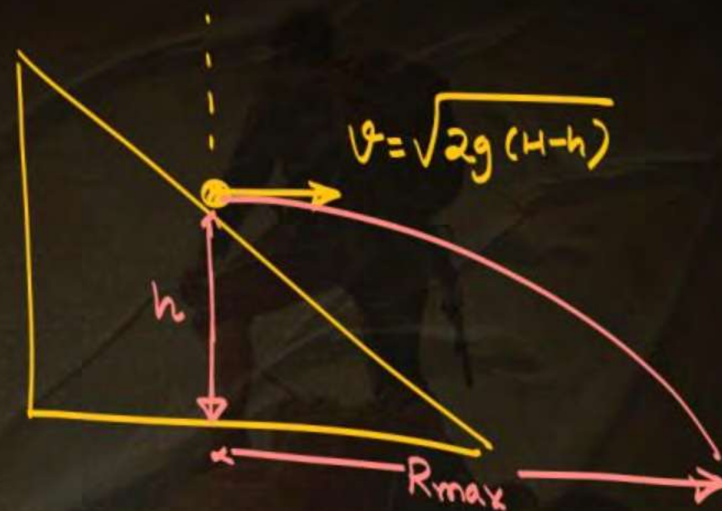
$$2R\frac{dR}{dh} = 4(H-2h)$$

$$\Downarrow$$

$$0$$

$$H = 2h$$

$$h = H/2$$



$$R = u_x t$$

$$R = \sqrt{2g(H-h)}\sqrt{\frac{2h}{g}}$$

QUESTION 21 (JEE Mains)

$R = \frac{u^2 \sin 2\theta}{g}$

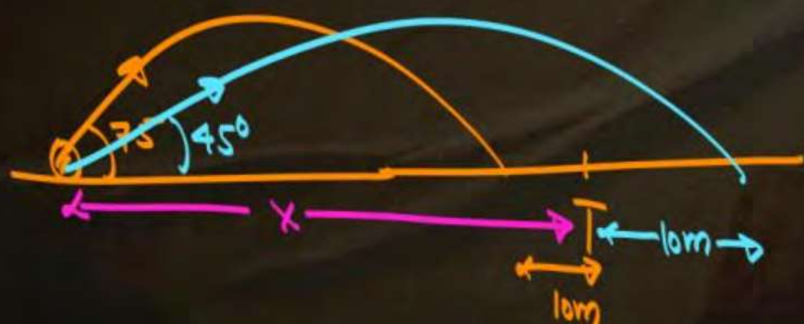
When the angle of projection is 75°, a ball falls 10 m shorter of the target. When the angle of projection is 45°, it falls 10 m ahead of the target. Both are projected from the same point with the same speed in the same direction, the distance of the target from the point of projection is

(A) 15 m

(B) 30 m

(C) 45 m

(D) 10 m



1) $\theta = 75°$

$x - 10 = \frac{u^2 \sin(150)}{g}$

$\theta = 45°$ $\quad x + 10 = \frac{u^2}{g}$

$\frac{x-10}{x+10} = \frac{1}{2}$

$2x - 20 = x + 10$

$x = 30$

QUESTION 22 (JEE Mains)

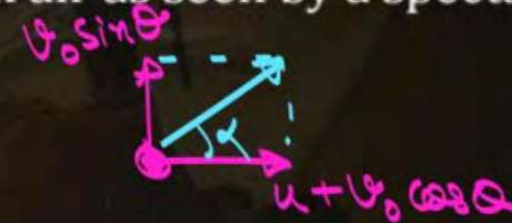
A cricket fielder can throw the cricket ball with a speed v_0. If he throws the ball while running with speed u at an angle θ to the horizontal, what is the effective angle to the horizontal at which the ball is projected in air as seen by a spectator?

(A) $\tan^{-1}\left[\frac{v_0\cos\theta}{u + v_0\sin\theta}\right]$

(B) $\tan^{-1}\left[\frac{v_0\sin\theta}{u + v_0\cos\theta}\right]$

(C) $\tan^{-1}\left[\frac{u}{v_0\cos\theta + v_0\sin\theta}\right]$

(D) $\tan^{-1}\left[\frac{v_0\sin\theta + v_0\cos\theta}{u}\right]$



$$\tan\alpha = \frac{v_0\sin\theta}{u+v_0\cos\theta}.$$

QUESTION 23

A train is moving along a straight line with a constant acceleration a. A boy standing in the train throws a ball forward with a speed of 10 m/s, at an angle of 60° to the horizontal. The boy has to move forward by 1.15 m inside the train to catch the ball back at the initial height. The acceleration of the train, in m/s^2, is:

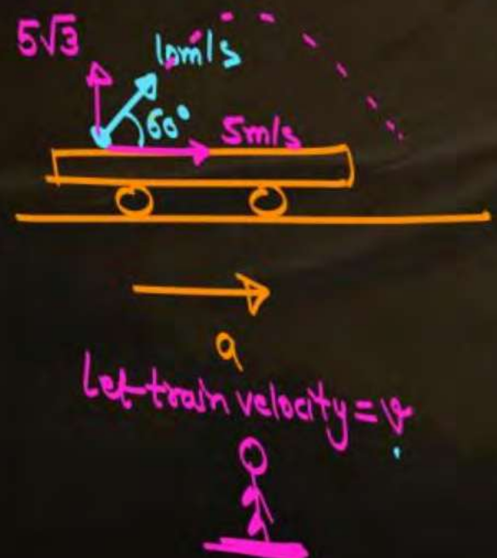
(IIT-JEE 2011)

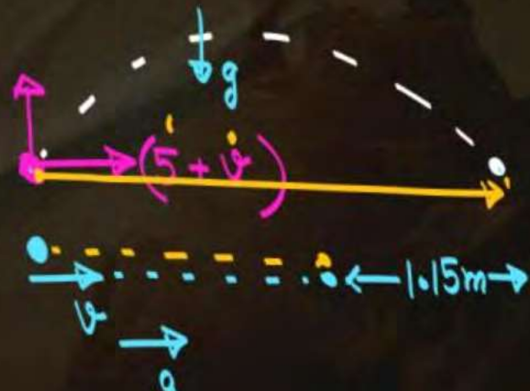
(A) $2\ m/s^2$

(B) $4\ m/s^2$

(C) $3\ m/s^2$

(D) $5\ m/s^2$





$S = vt + \frac{1}{2}at^2$

$= v\sqrt{3} + \frac{1}{2}a(\sqrt{3})^2$

$\boxed{S = v\sqrt{3} + 3a/2}$

$T = \frac{2u_y}{a_y} = \frac{2 \times 5\sqrt{3}}{10}$

$\boxed{T = \sqrt{3}}$

$d_{ball} = d_{person}$

$(5+v)\sqrt{3} = v\sqrt{3} + \frac{3a}{2} + 1.15$

$5\sqrt{3} + \cancel{v\sqrt{3}} = \cancel{v\sqrt{3}} + \frac{3a}{2} + 1.15$

$a =$ ____.

QUESTION 24

A ball is thrown from ground at an angle θ with horizontal and with an initial speed u_0. For the resulting projectile motion, the magnitude of $\boxed{\text{average velocity}}$ of the ball up to the point when it hits the ground for the first time is V_1

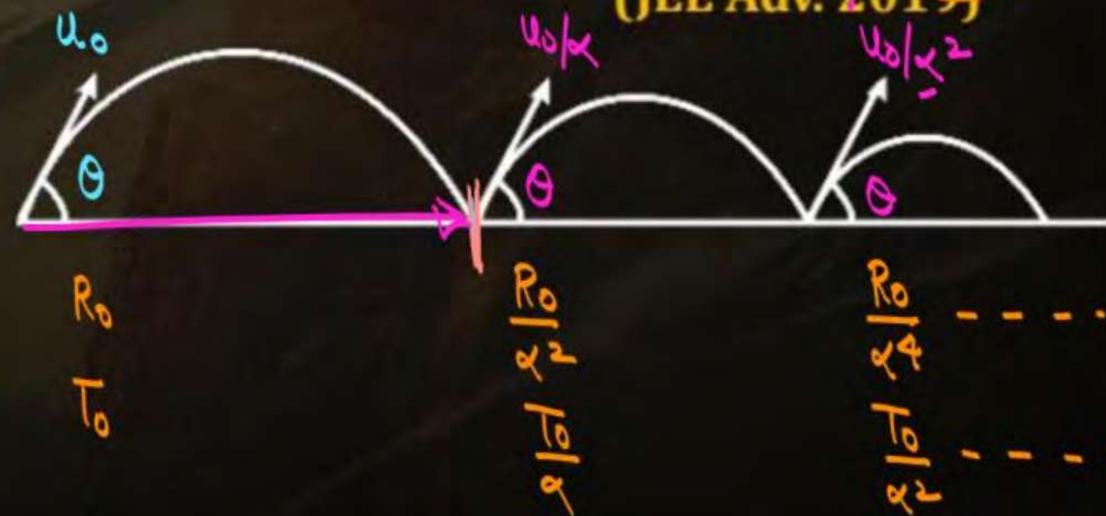
After hitting the ground, ball rebounds at the same angle θ but with a reduced speed of u_0/α. Its motion continues for a long time as shown in figure. If the magnitude of average velocity of the ball for entire duration of motion is $0.8V_1$, the value of α is

(JEE Adv. 2019)

$V_1 = \text{avg velocity} = \dfrac{R_0}{T_0}$

$R_0 = \dfrac{u_0^2 \sin 2\theta}{g}$

$T_0 = \dfrac{2u_0 \sin\theta}{g}$



$$\text{Avg velocity} = \frac{R_0 + R_1 + R_2 + R_3 \ldots}{T_0 + T_1 + T_2 + T_3 + \ldots}$$

given

$$0.8V_1 = \frac{R_0 + \frac{R_0}{\alpha^2} + \frac{R_0}{\alpha^4} + \frac{R_0}{\alpha^6} + \ldots}{T_0 + \frac{T_0}{\alpha} + \frac{T_0}{\alpha^2} + \frac{T_0}{\alpha^3} + \ldots}$$

$$0.8\frac{\cancel{R_0}}{\cancel{T_0}} = \frac{\cancel{R_0}\left[\frac{1}{1-\frac{1}{\alpha^2}}\right]}{\cancel{T_0}\left[\frac{1}{1-\frac{1}{\alpha}}\right]}$$

$$0.8 = \frac{1}{1+\frac{1}{\alpha}}$$

$$0.8 = \frac{\alpha}{\alpha+1}$$

$$0.8\alpha + 0.8 = \alpha$$

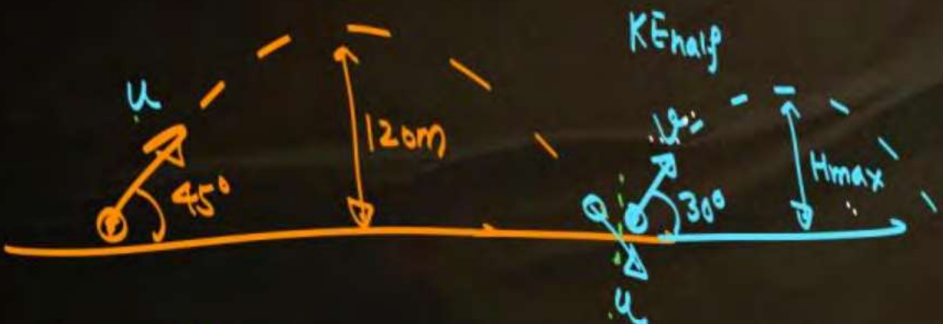
$$0.8 = 0.2\alpha$$

$$\alpha = 4$$

QUESTION 25

A ball is projected from the ground at an angle of 45° with the horizontal surface. It reaches a maximum height of 120m and returns to the ground. Upon hitting the ground for the first time, it loses half of its kinetic energy. Immediately after the bounce, the velocity of the ball makes an angle of 30° with the horizontal surface. The maximum height it reaches after the bounce in metres, is ________.

[JEE (Advanced) 2018]



$$H = \frac{u^2 \sin^2 45}{2g} =$$

$$\frac{u^2}{4g} = 120 \qquad \boxed{u^2 = 4800}$$

$$KE_f = \frac{1}{2} KE_i$$

$$\frac{1}{2} m v^2 = \frac{1}{2} \cdot \frac{1}{2} m u^2$$

$$v = \frac{u}{\sqrt{2}}$$

$$H_{max} = \frac{v^2 \sin^2 30}{2g}$$

$$= \frac{u^2}{8g}$$

$$= \frac{u^2}{2 \cdot 8g} = \frac{u^2}{160}$$

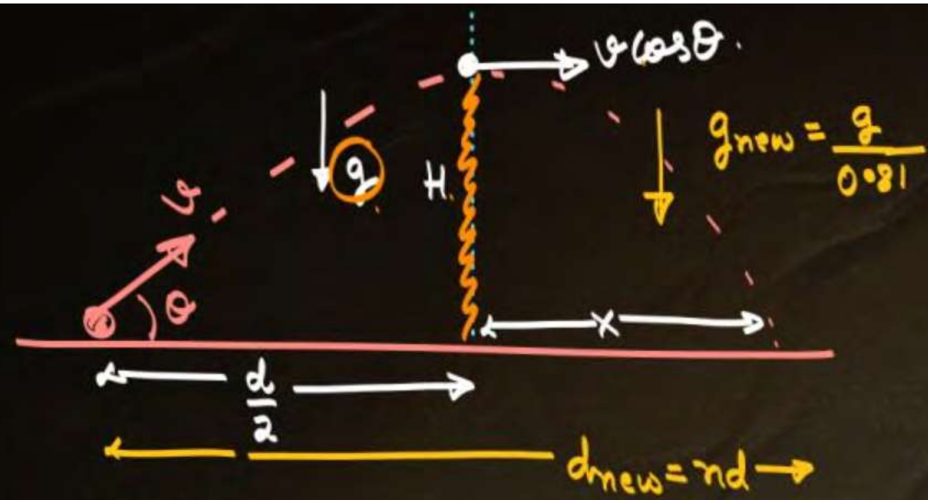
$$= \frac{4800}{160} = 30$$

QUESTION 26

A projectile is fired from horizonal ground with speed v and projection angle θ. When the acceleration due to gravity is g, the range of the projectile is d. If at the highest point in its trajectory, the projectile enters a different region where the effective acceleration due to gravity is $g' = \frac{g}{0.81}$, then the new range is $d' = nd$. The value of n is ______.

[JEE Advanced 2022]

$$\boxed{d = \frac{v^2 \sin 2\theta}{g}}$$



$$d_{new} = \frac{d}{2} + x$$

$$nd = \frac{d}{2} + \frac{0.9d}{2}$$

$$nd = \left(\frac{1.9}{2}\right)d$$

$$\boxed{n = 0.95}$$

$$H = \frac{v^2\sin^2\theta}{2g}$$

Horizontal proj.

$$x = v\cos\theta\sqrt{\frac{2(H)}{g_{eff}}}$$

$$= v\cos\theta\sqrt{\frac{\cancel{2}\times v^2\sin^2\theta\,(0.81)}{\cancel{2}g\cdot g}} = \frac{2}{2}v\cos\theta\times\frac{v}{g}\sin\theta\,(0.9) = \frac{0.9}{2}\,\frac{v^2\sin 2\theta}{g}$$

$$= \frac{0.9}{2}d.$$

QUESTION 27

HW

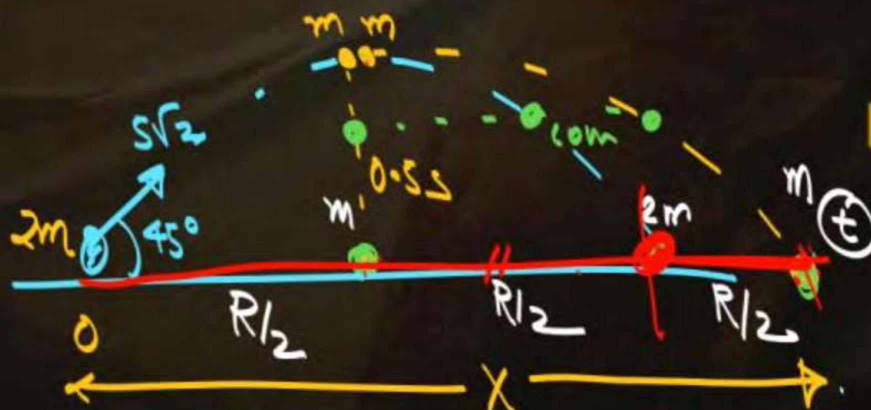
$\times$ 5

A projectile is thrown from a point O on the ground at an angle 45° from the vertical and with a speed $5\sqrt{2}$ m/s. The projectile at the highest point of its trajectory splits into two equal parts. One part falls vertically down to the ground, 0.5 s after the splitting. The other part, t seconds after the splitting, falls to the ground at a distance x metres from the point O. The acceleration due to gravity $g = 10\ \text{m/s}^2$.

The value of t is 0.5

The value of x is ______.

[Adv 2021]



$$x = \frac{3R}{2} = \frac{3}{2}\left(\frac{(5\sqrt{2})^2}{10}\right) = 7.5$$

QUESTION 28

HW

A ball is released from height $4h$ as shown in figure. The speed of ball when it leaves the surface is u. When the ball hits the ground, it hits with velocity v at angle θ with horizontal & rebounds. The coefficient of restitution of collision between ball and ground is $\frac{1}{\sqrt{3}}$. After hitting the ground, the ball rebounds and reaches maximum height of h.

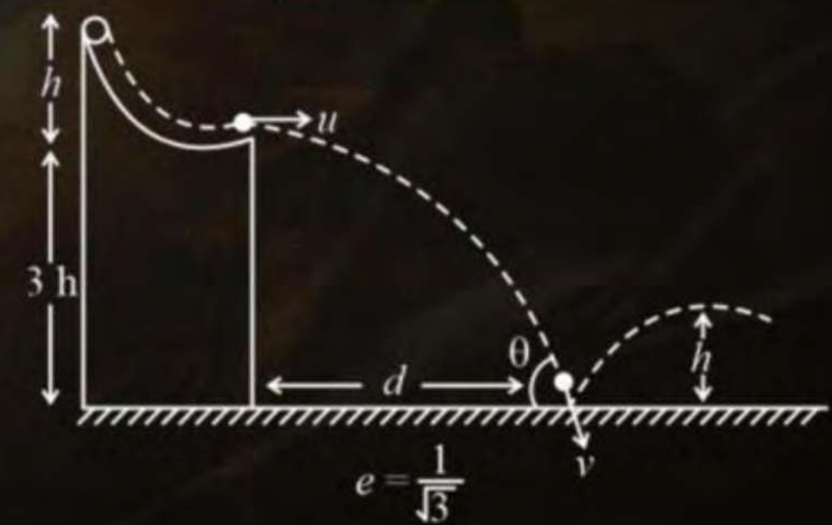
[Adv 2024]

(A) $\theta = 60°$

(B) $u = \sqrt{2gh}$

(C) $v = \sqrt{2gh}\,(\hat{\imath} - \hat{k})$

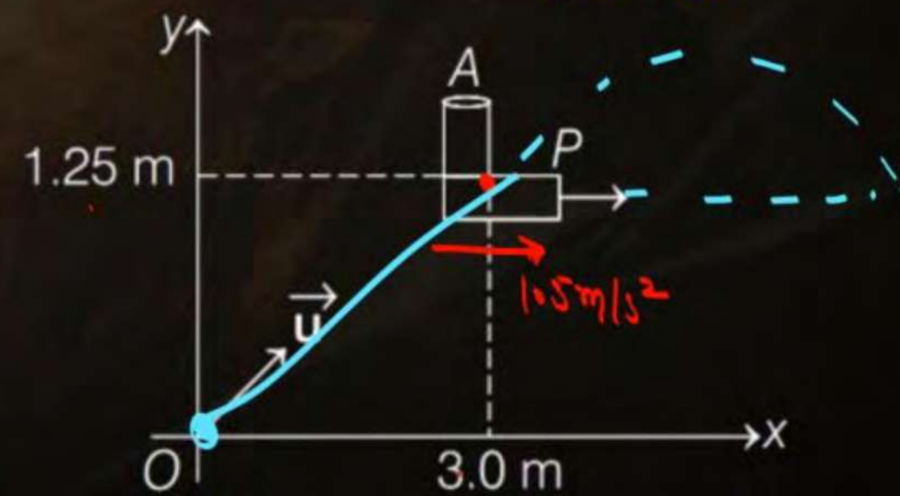
(D) $\frac{d}{h} = 2\sqrt{3}$

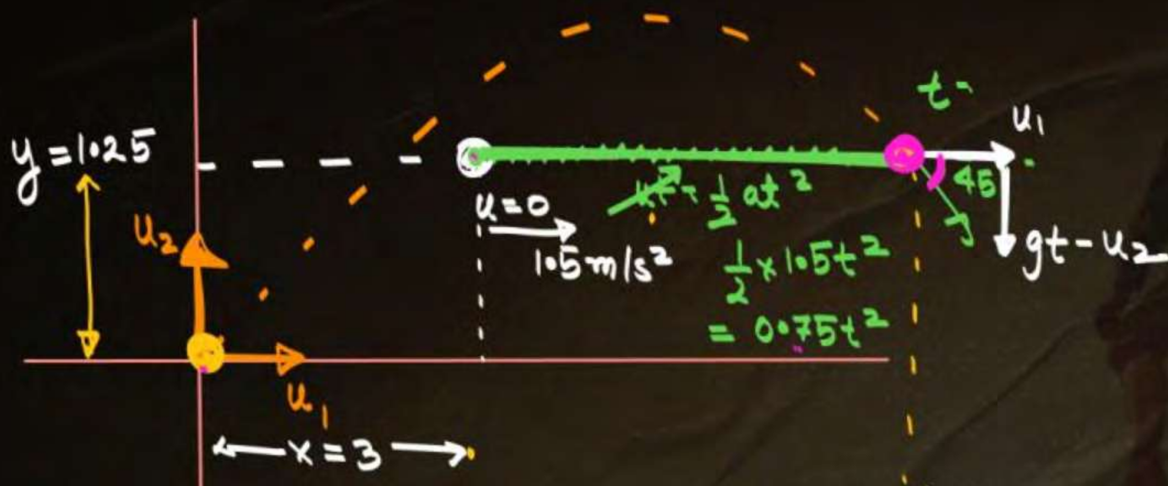


QUESTION 29

An object A is kept fixed at the point $x = 3$ m and $y = 1.25$ m on a plank P raised above the ground. At time $t = 0$, the plank starts moving along the $+x$-direction with an acceleration $1.5\ m/s^2$. At the same instant, a stone is projected from the origin with a velocity $\boldsymbol{u}$ as shown. A stationary person on the ground observes the stone hitting the object during its downward motion at an angle of 45° to the horizontal. All the motions are in x-y plane. Find $\boldsymbol{u}$ and the time after which the stone hits the object. (Take $g = 10\ m/s^2$).

[2000]





In time t

Projectile $x = 3 + 0.75t^2$

$$\boxed{u_1 t = 3 + 0.75t^2}$$

Projectile

$$S_y = u_y t + \frac{1}{2} a_y t^2$$

$$\boxed{1.25 = u_2 t - 5t^2}$$

$$\tan 45 = \frac{gt - u_2}{u_1}$$

$$\boxed{gt - u_2 = u_1}$$

$$v_y = u_y + a_y t$$

$$v_y = u_2 - gt$$

$$\tan(-45) = \frac{u_2 - gt}{u_1}$$

Incline Projectile

$u_x = u\cos\theta \qquad a_x = -g\sin\alpha$

$u_y = u\sin\theta \qquad a_y = -g\cos\alpha.$

for observer on incline.

along x
Const v ≠.

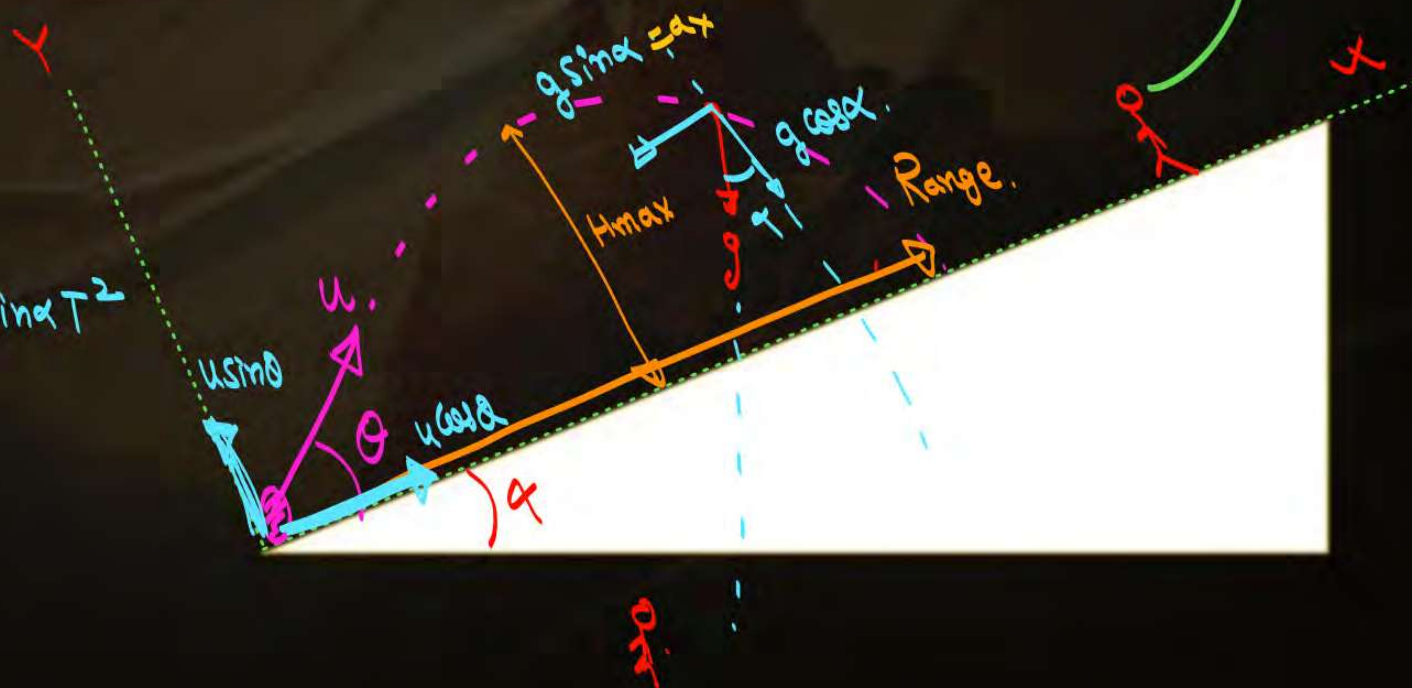
a) Time of flight $= T = \dfrac{2u_y}{a_y} = \boxed{\dfrac{2u\sin\theta}{g\cos\alpha}}$

b) $H_{max} = \dfrac{u_y^2}{2a_y} = \boxed{\dfrac{u^2\sin^2\theta}{2g\cos\alpha}}$

c) Range $= S_x = u_x t + \frac{1}{2}\boxed{a_x} t^2$

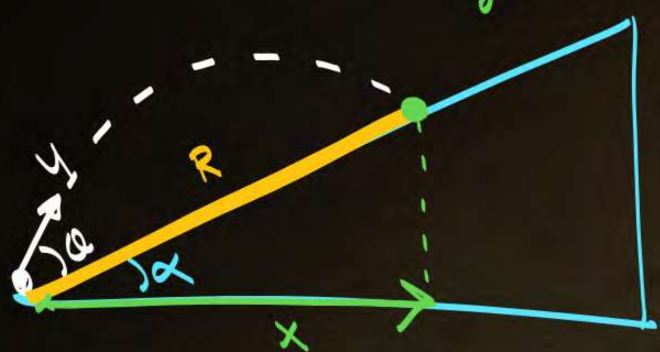
$= u\cos\theta(T) - \frac{1}{2} g\sin\alpha\, T^2$

$=$



Method-2

$$T = \frac{2u\sin\theta}{g\cos\alpha}$$

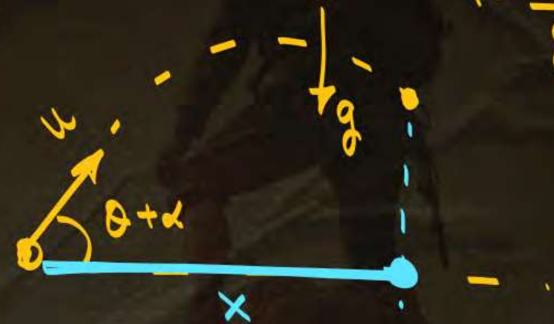


$$\cos\alpha = \frac{X}{R}$$

$$R = \frac{X}{\cos\alpha} = \boxed{\frac{2u^2\sin\theta\cos(\theta+\alpha)}{g\cos^2\alpha}}$$

ground frame

$$T = \frac{2u\sin\theta}{g\cos\alpha}$$



$\rightarrow a_x = 0$

$$S_x = X = u\cos(\theta+\alpha)\,T$$

$$= \frac{u\cos(\theta+\alpha)\,2u\sin\theta}{g\cos\alpha}$$

$$= \frac{2u^2\sin\theta\cos(\theta+\alpha)}{g\cos\alpha}$$

QUESTION 30

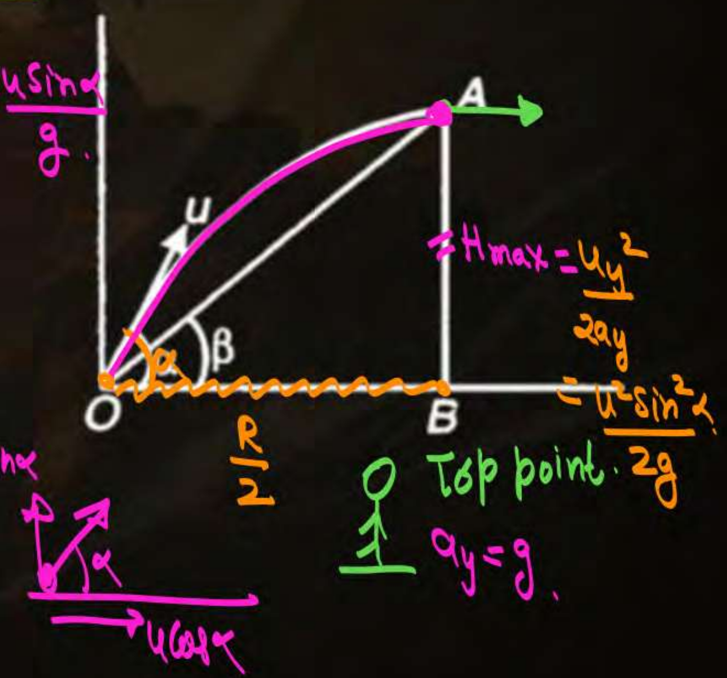
A particle is projected at an angle α with the horizontal from a point O on a plane which is inclined at an angle β to the horizontal and moves upwards in a vertical plane containing the line of greatest slope through the point. The particle is moving horizontally when it strikes the plane at a point A

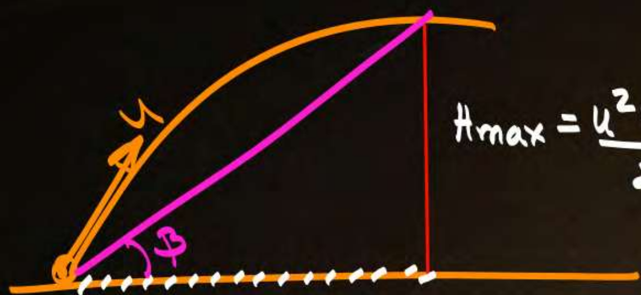
(A) The time taken to reach point A, $T = \dfrac{u \sin \alpha}{g}$. $t_a = \dfrac{u_y}{a_y} = \dfrac{u \sin\alpha}{g}$.

(B) The maximum height of the particle $= \dfrac{u^2 \sin^2 \alpha}{2g}$

(C) The horizontal distance $OB = \dfrac{u^2 \sin 2\alpha}{2g} = \dfrac{R}{2}$

(D) $\alpha = \tan^{-1}(2 \tan \beta)$





$$H_{max} = \frac{u^2 \sin^2\alpha}{2g}$$

$$\frac{u^2 \sin 2\alpha}{2g}$$

$$\tan\beta = \frac{P}{B} = \frac{\cancel{u^2} \sin^2\alpha}{\cancel{2g}} \times \frac{\cancel{2g}}{\cancel{u^2} \sin 2\alpha}$$

$$\tan\beta = \frac{\sin^{\cancel{2}}\alpha}{2 \cancel{\sin\alpha} \cos\alpha} \qquad \tan\beta = \frac{\tan\alpha}{2}$$

$$2\tan\beta = \tan\alpha$$

$$\boxed{\alpha = \tan^{-1}(2\tan\beta)}$$

QUESTION 31

A plane is inclined at an angle $\alpha = 30°$ with respect to the horizontal. A particle is projected with a speed $u = 2\ ms^{-1}$, from the base of the plane, making an angle $\theta = 15°$ with respect to the plane as shown in the figure. The distance from the base, at which the particle hits the plane is close to [Take, $g = 10\ ms^{-2}$]

[JEE Main 2019]

(A) 26 cm

(B) 20 cm

(C) 18 cm

(D) 14 cm

$$T = \frac{2u\sin\theta}{g\cos\alpha}$$

$$\boxed{T = \frac{2u\sin15}{g\cos30}}$$

$$S_x = u_x t + \frac{1}{2}a_x t^2$$

$$= 2\cos15(T) - \frac{1}{2}g\sin15(T)^2$$

$$T = \frac{2u \sin 15}{g \cos 30}$$

$$X_{ground} = u_x t = 2\cos 45 \left[\frac{2 \times 2 \sin 15}{g \cos 30}\right]$$

$$\cos 30 = \frac{x}{R}$$

$$R = \frac{x}{\cos 30} = \frac{\dfrac{2\cos 45 \times 4 \times \sin 15}{g \cos 30}}{\cos 30}$$

$$= \frac{8 \cos 45 \sin 15}{g \cos^2 30}$$

$$\sin(15) = \sin(45 - 30).$$

QUESTION 32

A particle is thrown from a point on an incline plane, with a velocity of 20 m/s at an angle of 60° with the horizontal, angle of incline plane is 30°. At what time the velocity of the projectile becomes parallel to the incline plane?

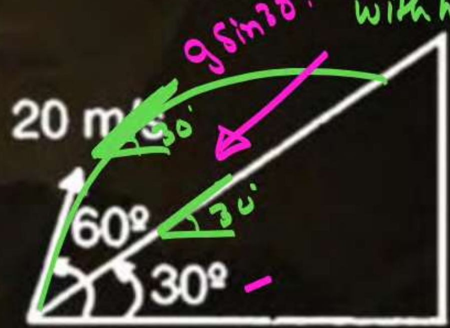
V_{net} makes 30° with horizontal.

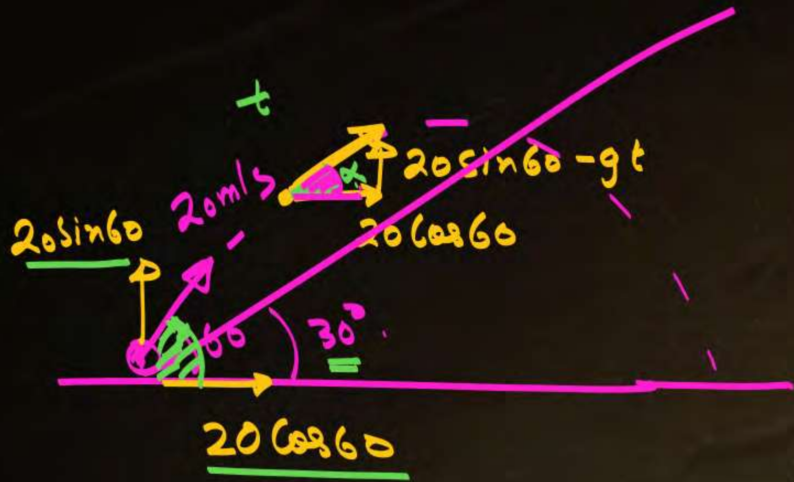
(A) $\frac{\sqrt{3}}{2}$ s

(B) $\frac{2}{\sqrt{3}}$ s

(C) $\sqrt{3}$ s

(D) $\frac{1}{\sqrt{3}}$ s





$$v_y = u_y + a_y t$$

$$v_y = 20\sin 60 - 10t$$

$$\boxed{\tan\alpha = \frac{20\sin 60 - 10t}{20\cos 60} = \tan 30}$$

$$\frac{10\sqrt{3} - 10t}{10} = \frac{1}{\sqrt{3}}$$

$$\sqrt{3} - t = \frac{1}{\sqrt{3}}$$

$$\sqrt{3} - \frac{1}{\sqrt{3}} = t$$

$$\frac{2}{\sqrt{3}} = t$$

QUESTION 33

PYQ

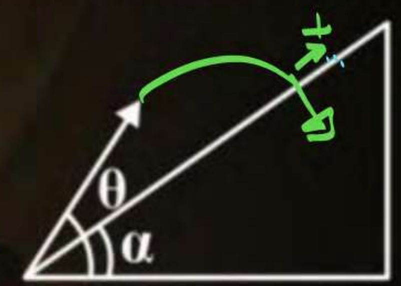
A projectile is fixed at an angle θ with the horizontal, (as shown in the figure), condition under which it lands perpendicular on an inclined plane of inclination α is

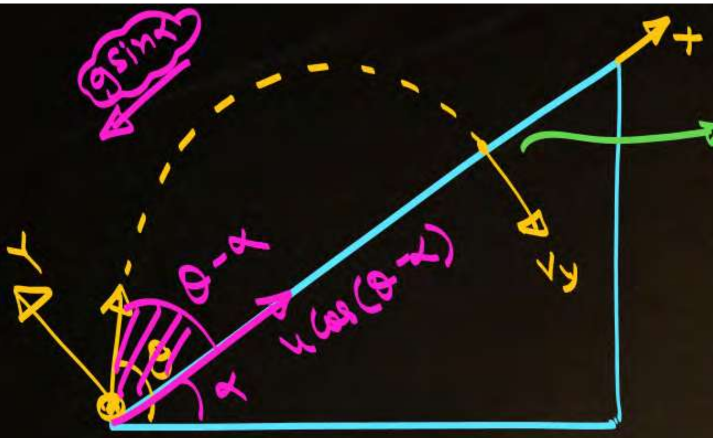
(A) $\sin\alpha = \cos(\theta - \alpha)$

(B) $\cos\alpha = \sin(\theta - \alpha)$

(C) $2\tan\alpha = \cot(\theta - \alpha)$

(D) $\cot(\theta - \alpha) = \sin\alpha$





$\perp$ hit Kare (No Velocity along x)

" $T = \dfrac{2u\sin(\theta-\alpha)}{g\cos\alpha}$ final $V_x = 0$.

$V_x = u_x + a_x t$

$0 = \cancel{u}\cos(\theta-\alpha) - \cancel{g}\sin\alpha\left(\dfrac{2\cancel{u}\sin(\theta-\alpha)}{\cancel{g}\cos\alpha}\right)$

$\cos(\theta-\alpha) = 2\tan\alpha\,\sin(\theta-\alpha)$

$\boxed{\cot(\theta-\alpha) = 2\tan\alpha}$ #

QUESTION 34

Find range of projectile which is projected perpendicular to the incline plane with velocity 20 m/s as shown in figure:

(A) 75 m

Ans

(B) 40 m

(C) 45 m

(D) 50 m

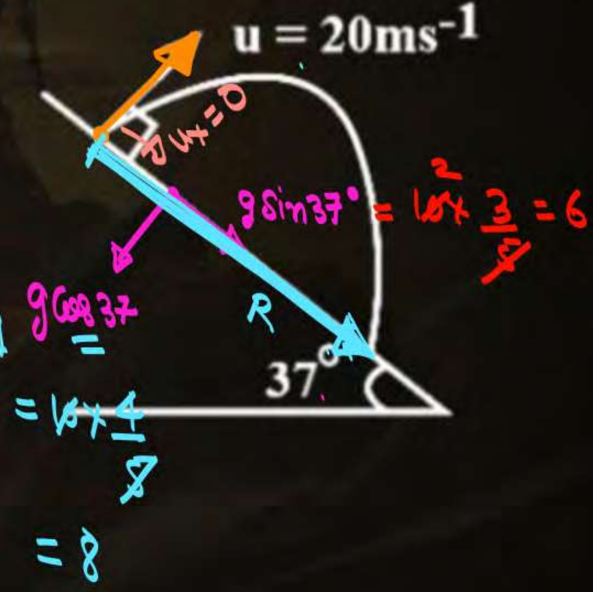
$T = \frac{2u_y}{a_y} = \frac{2 \times 20}{8} = 5$

$\boxed{T = 5}$

$S_x = \cancel{u_x t} + \frac{1}{2} a_x t^2$

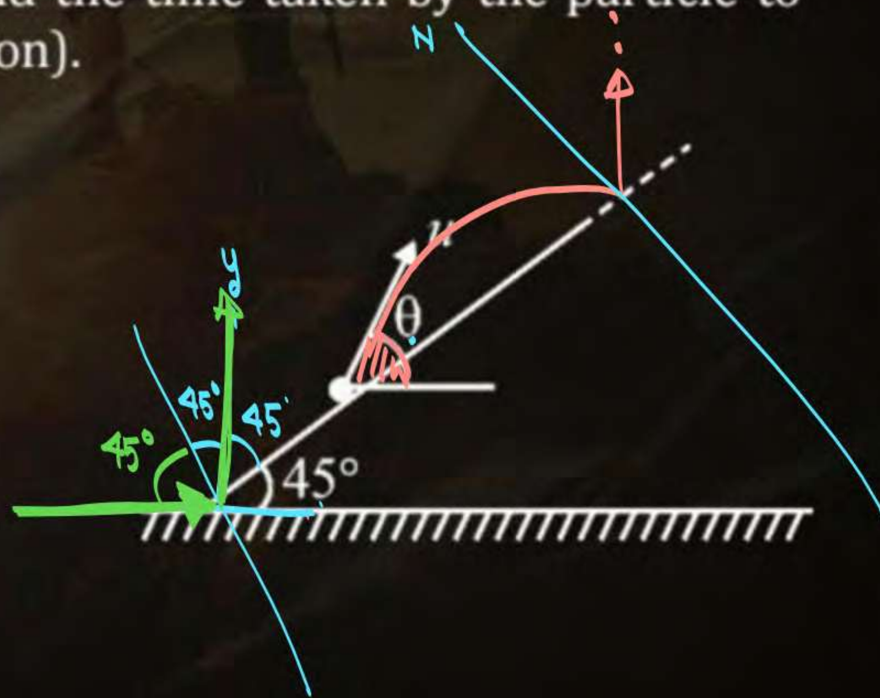
$= \frac{1}{2} \times 6 \times 5^2$

$= 75 m.$



QUESTION 35 (Level up)

A particle is projected from surface of the inclined plane with speed u and at an angle θ with the horizontal. After some time the particle collides elastically with the smooth fixed inclined plane for the first time and subsequently moves in vertical direction. Starting from projection, find the time taken by the particle to reach maximum height. (Neglect time of collision).



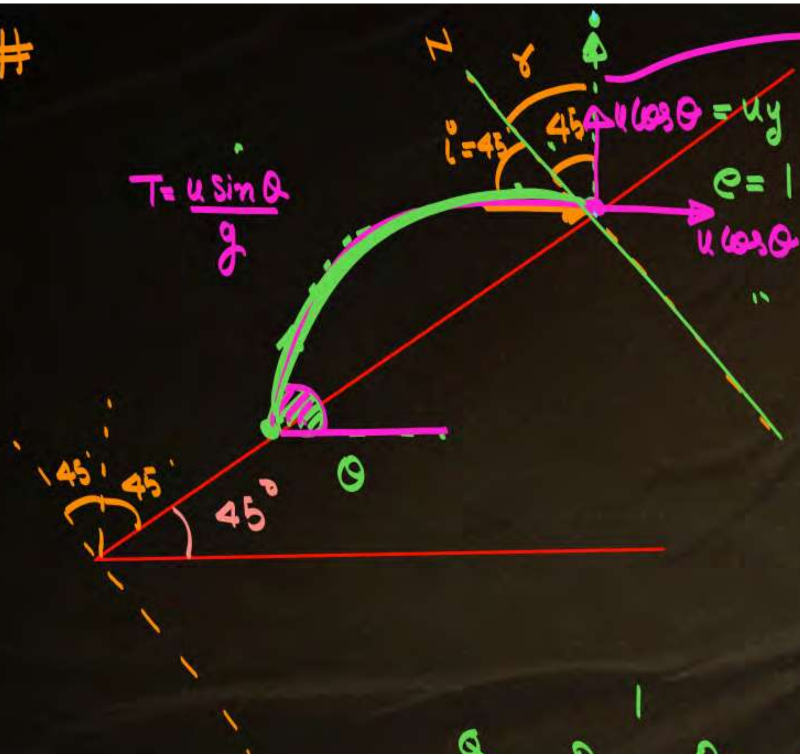
(A) $\frac{2u\cos\theta}{g}$

(B) $\frac{2u\sin\theta}{g}$

(C) $\frac{u(\sin\theta+\cos\theta)}{g}$ Ans

(D) $\frac{2u}{g}$

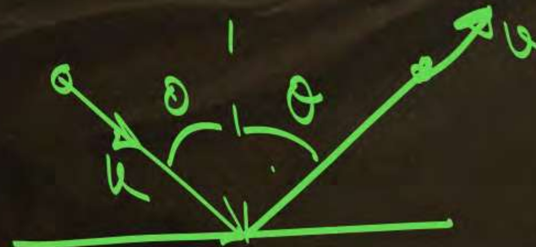
#



$$T_{ascent} = \frac{u_y}{a_y} = \frac{u\cos\theta}{g}.$$

$$T = \frac{u\sin\theta}{g} + \frac{u\cos\theta}{g}$$

$$\boxed{T = \frac{u}{g}(\sin\theta + \cos\theta)}$$



(12th & Droppers)

QUESTION 35

(Advanced)

$S_y = \cancel{u_y t}^{0} + \frac{1}{2} a_y t^2$

$\cancel{-}2h = \cancel{-}\frac{1}{2} \times 5 \times 4$

$2h = 10$

$h = 5$

If a particle is projected from on incline of angle 30^0 to another incline of 60^0 as shown such that while projection velocity is perpendicular to initial plane and while at landing it hits perpendicularly. If $u = 10\sqrt{3}$,find h, v, time of flight and distance at which it hits the other plane.

$v_x = u_x + a_x t$

$0 = 10\sqrt{3} - 5\sqrt{3}t$

$\frac{10\sqrt{3}}{5\sqrt{3}} = t = 2$

$a_x = -5\sqrt{3}$

$a_y = -5$

$u_x = 10\sqrt{3}$

$u_y = 0$

$\text{Range} = S_x = u_x t + \frac{1}{2} a_x t^2$

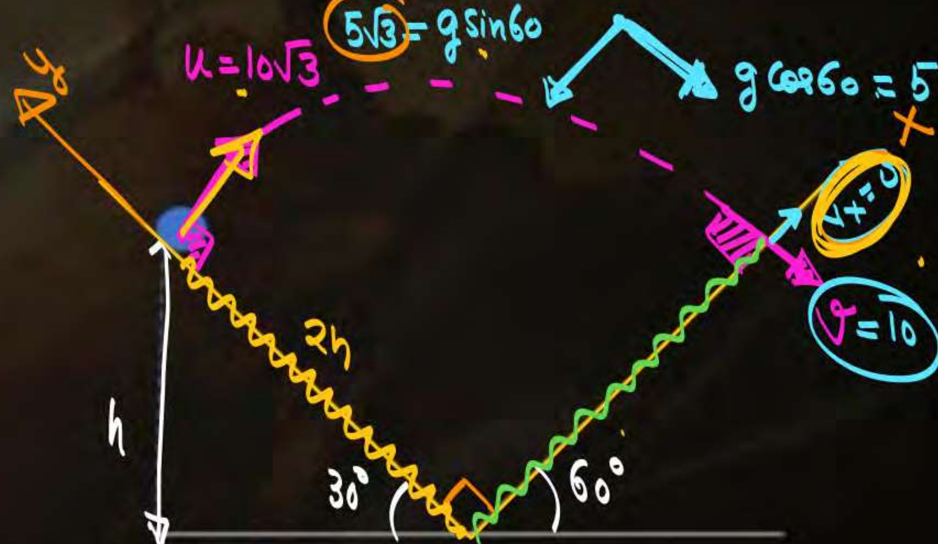
$= 10\sqrt{3} \times 2 - \frac{1}{2} 5\sqrt{3} \times 4$

$=$ ______

$v_y = u_y + a_y t$

$v_y = 0 - 5 \times 2$

$v_y = -10$



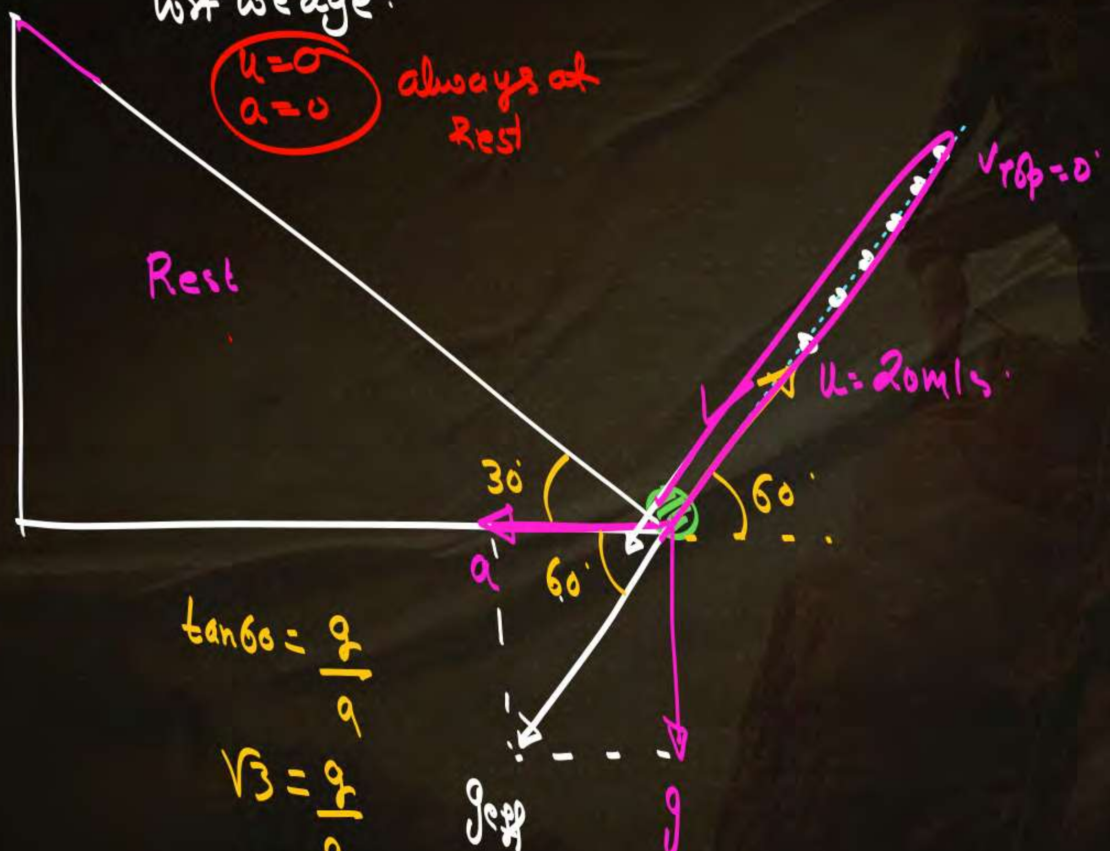
$\sin 30 = \frac{P}{H}$ $H = 2h$

QUESTION 36

Ball is projected at angle 60^0 as shown. At the same moment incline of angle 30^0 starts moving with acceleration. What should be a so that particle hits the plane perpendicularly.

wrt wedge.

$u=0$
$a=0$ always at Rest



$\tan 60 = \frac{g}{a}$

$\sqrt{3} = \frac{g}{a}$

$a = g/\sqrt{3}$

$g_{eff} = \sqrt{a^2+g^2}$

$T_{\text{to hit incline}} = \frac{2u_y}{a_y}$

$= \frac{2\times 20}{g_{eff}}$

QUESTION 37

Adv

$\boxed{T = t_1 + t_2}$

There are two parallel planes ,each inclined to the horizontal at an angle θ. A particle is projected from a point mid way between the foot of the two planes so that it grazes one of the planes and strikes the other at right angle . Find the angle of projection of the particle

surface ||

1. at Total time $V_x = 0$

$V_x = u_x + a_x t$

$0 = u\cos\alpha - g\sin\theta\, T$

$$\boxed{T = \frac{u\cos\alpha}{g\sin\theta}}$$

$$\boxed{t_1 = \frac{u_y}{a_y} = \frac{u\sin\alpha}{g\cos\theta}}$$

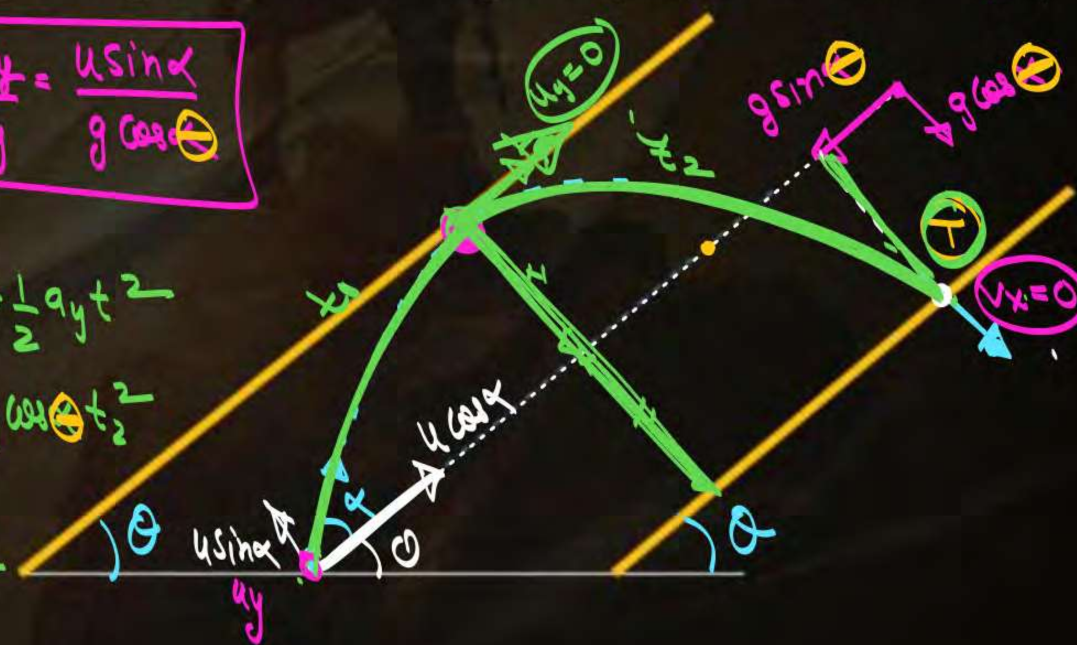
$S_y = u_y t + \frac{1}{2} a_y t^2$

$+2H = +\frac{1}{2} g\cos\theta\, t_2^2$

$\sqrt{\frac{4H}{g\cos\theta}} = t_2$

$S_y = u_y t + \frac{1}{2} a_y t^2$

$$\boxed{-H = u\sin\alpha\, T - \frac{1}{2} g\cos\theta\, T^2}$$



Concept of Relative Velocity

Observer → hamesha Rest par Maanega.

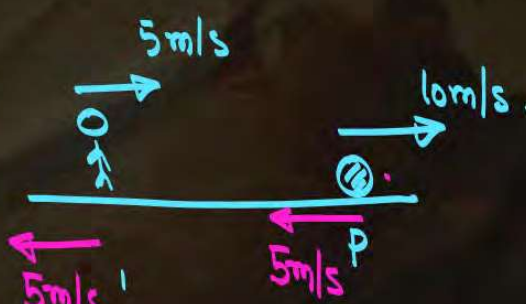
→ observer Ki velocity Ka ulta Kar Ke Sabko dedo.

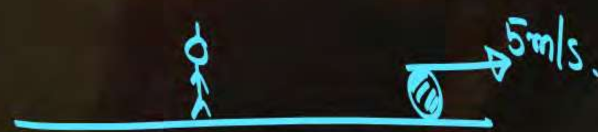
→ acc of observer ulta KarKe SabKo dedo.

Case.1

$\vec{V}_{PO} = 10\hat{i}$

Case.2

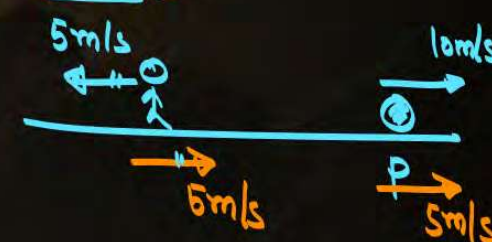




$v = 0$

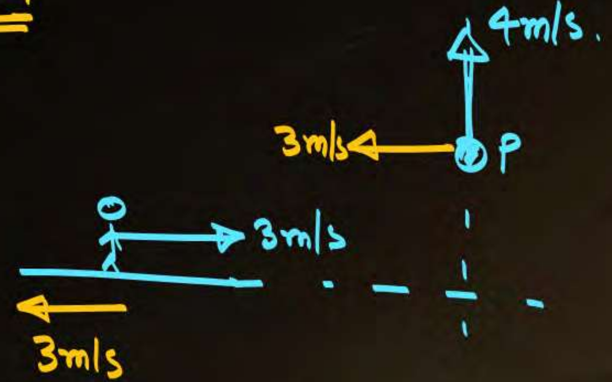
$\boxed{\vec{V}_{PO} = 5\hat{i}}$

Case. 3



$\boxed{\vec{V}_{PO} = 15\hat{i}}$

Case. 4

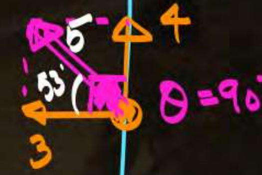


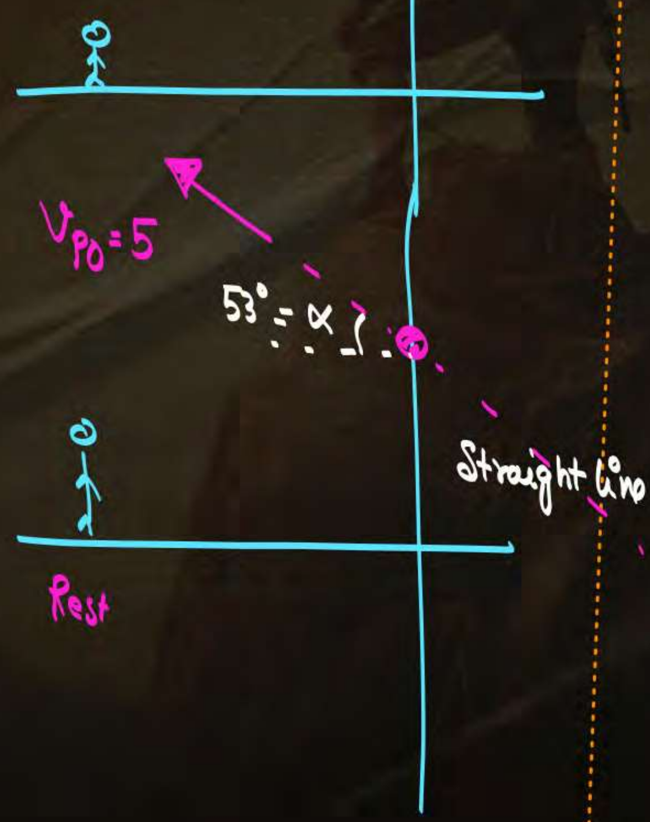
$$\boxed{\vec{V}_{PO} = \vec{V}_P - \vec{V}_O}$$

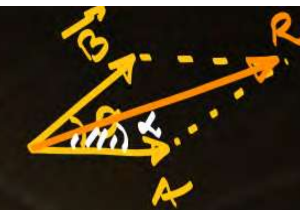
$$\boxed{\vec{V}_{PO} = \vec{V}_P + (-\vec{V}_O)}$$

$R = \sqrt{3^2 + 4^2}$

$R = 5.$







$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}.$

$\Downarrow$

Magnitude of Resultant

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}.$$

Case. 5   If observer is also accelerating.

$t=0$

5 ←

→ $5\,m/s$

O

→ $a = 1\,m/s^2$

5 ←

→ $10\,m/s$.

P

$t=0$ Rokne

$u=0$

O

→ 5 = $\boxed{\vec{u}_{PO} = 5\hat{i}}$

D

$1\,m/s^2$ ←

→ $a = 1\,m/s^2$

← $1\,m/s^2$

$V = \cancel{u} + \cancel{at}$

$V=0$

$u=0$

$a=0$

$a = 1\,m/s^2$

$\boxed{\begin{aligned}\vec{u}_{PO} &= 5\hat{i}\\ \vec{a}_{PO} &= -1\hat{i}\end{aligned}}$

always at Rest.

QUESTION 38

$\vec{V}_{AB}$ = velocity of A w.r.t B $= \vec{V}_A + (-\vec{V}_B)$.

observer

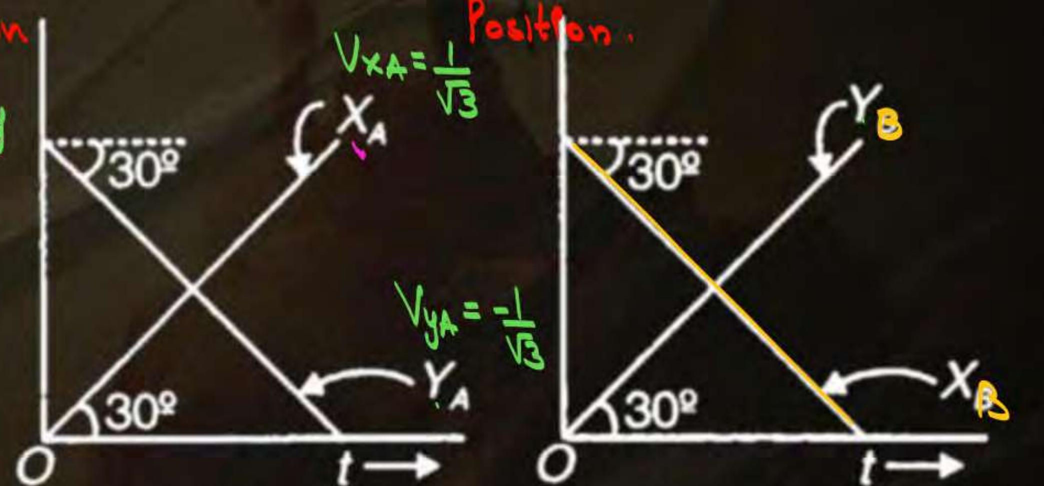
Slope

Displacement versus time plot for two particles A and B is shown below. X_A, X_B and Y_A, Y_B refer to x and y coordinate of particles A and B. velocity of particle A with respect to particle B is

Slope = velocity

$$\boxed{\vec{V}_A = \frac{1}{\sqrt{3}}\hat{i} - \frac{1}{\sqrt{3}}\hat{j}}$$

$$\boxed{\vec{V}_B = -\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j}}$$



$V_{xB} = -\frac{1}{\sqrt{3}}$ $\quad V_{yB} = +\frac{1}{\sqrt{3}}$

(A) $0\hat{i} + 0\hat{j}$

(B) Dependent of time t

(C) $\frac{2}{\sqrt{3}}\hat{i} - \frac{2}{\sqrt{3}}\hat{j}$ Ans

(D) $-\frac{2}{\sqrt{3}}\hat{i} + \frac{2}{\sqrt{3}}\hat{j}$

$\vec{V}_{AB} = \vec{V}_A + (-\vec{V}_B) = \frac{1}{\sqrt{3}}\hat{i} - \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{i} - \frac{1}{\sqrt{3}}\hat{j} = \frac{2}{\sqrt{3}}\hat{i} - \frac{2}{\sqrt{3}}\hat{j}$.

QUESTION 39

A man is moving with constant velocity 10 m/s in a direction $\boxed{60^\circ \text{ E of N}}$ and a dog is moving with constant velocity 10 m/s in direction $\boxed{30^\circ \text{ W of N}}$ on a horizontal field, then the path of the dog as observed by the man will be

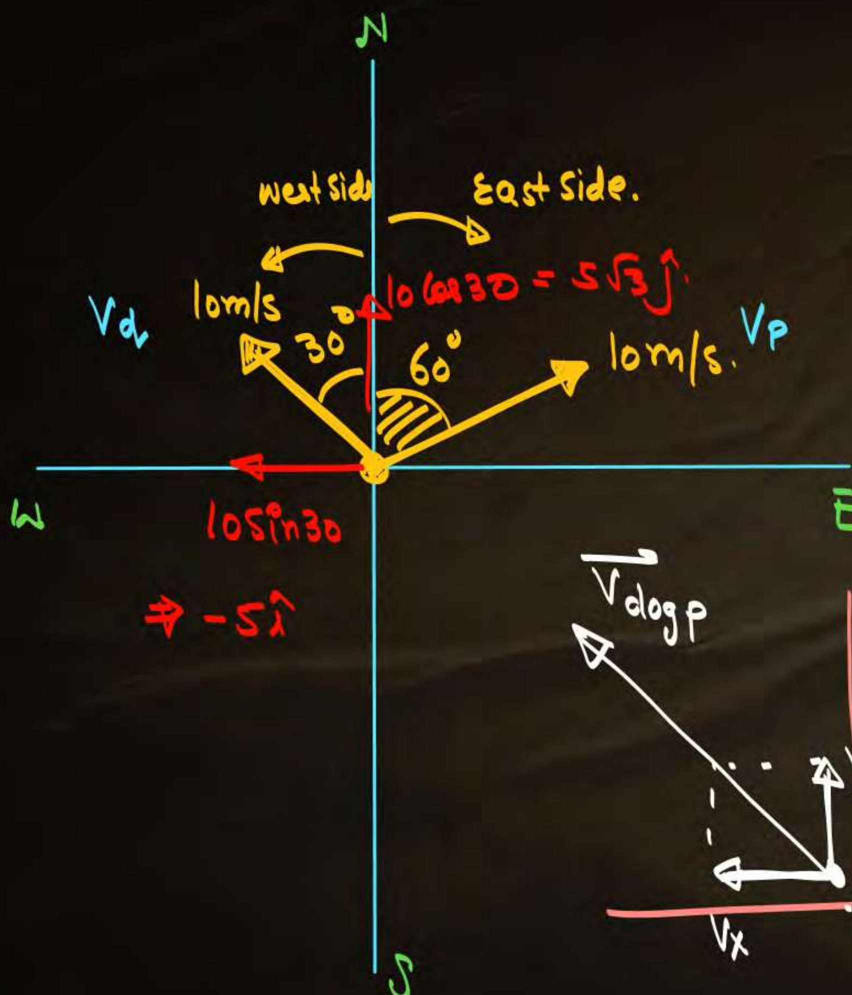
Ans

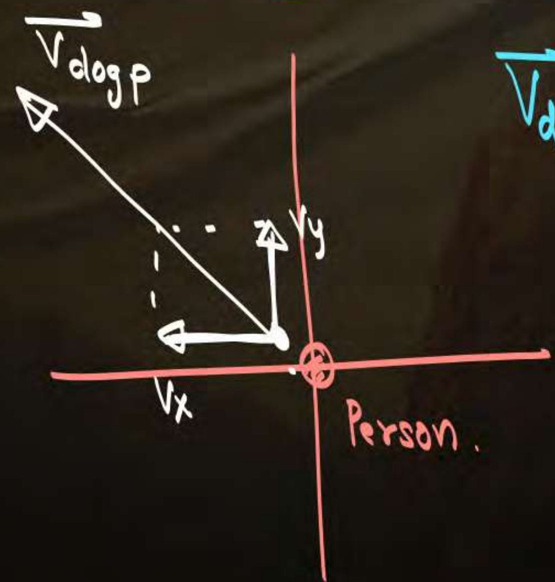
(A) Straight

(B) Parabolic

(C) Circular

(D) Dog will appear to be in rest





$$\vec{V_m} = 10\sin 60\,\hat{i} + 10\cos 60\,\hat{j}$$

$$\boxed{\vec{V_m} = 5\sqrt{3}\,\hat{i} + 5\hat{j}}$$

$$\vec{V_d} = -10\sin 30\,\hat{i} + 10\cos 30\,\hat{j}$$

$$\boxed{\vec{V_d} = -5\hat{i} + 5\sqrt{3}\,\hat{j}}$$

$$\vec{V_{dm}} = \vec{V_d} - \vec{V_m}$$

$$= -5\hat{i} + 5\sqrt{3}\,\hat{j} - 5\sqrt{3}\,\hat{i} - 5\hat{j}$$

$$= -(5+5\sqrt{3})\hat{i} + (5\sqrt{3}-5)\hat{j}.$$

QUESTION 40

Two cars A and B start moving from the same point with same speed $n = 5$ km/minute. Car A moves towards North and car B is moving towards East. What is the relative velocity of B with respect to A?

$\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$

$= 5\hat{i} - 5\hat{j}$

A observer

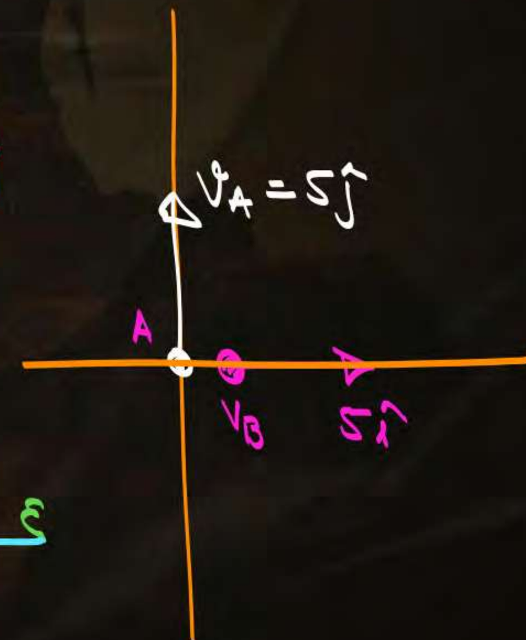
(A) $5\sqrt{2}$ km/min towards South-East

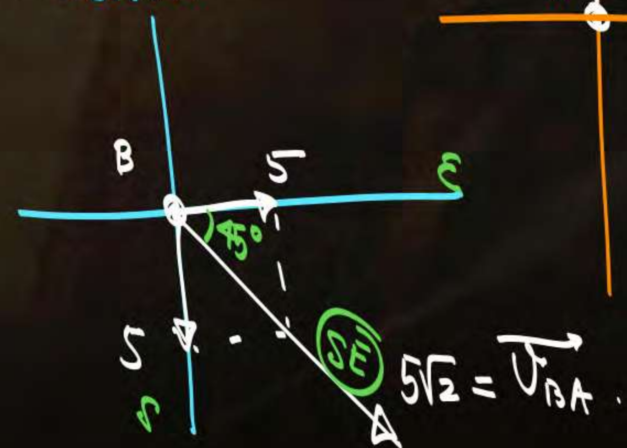
Ans

(B) $5\sqrt{2}$ km/min towards North-West

(C) $5\sqrt{2}$ km/min towards South-West

(D) $5\sqrt{2}$ km/min towards North-East





Concept of Catching or Overtaking

Case 1: when no particle is accelerating

time = ?

20m/s 5m/s

A 90m B $\longrightarrow$ observer.

$\xrightarrow{15}$ B

$\longleftarrow$ 90m $\longrightarrow$ Rest

$S = \frac{D}{t}$

$\bar{V}_{AB} = \bar{V}_A - \bar{V}_B$

$= 20\hat{i} - 5\hat{i}$

$= 15\hat{i}$

time to Catch $= \frac{90}{15} = 6$ sec.

Case 2: When one particle is accelerating

2m/s² 20m/s 5m/s

A 60m B

$a_{AB} = a_A - a_B$

$= +2 - 0 = +2.$

$\bar{V}_{AB} = \bar{V}_A - \bar{V}_B$

$= 20\hat{i} - 5\hat{i}$

$= 15\hat{i}$

2m/s²

$\xrightarrow{15}$

$\longleftarrow$ 60m $\longrightarrow$ Rest

$S = ut + \frac{1}{2}at^2$

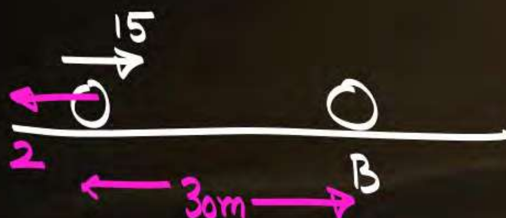
$60 = 15t + \frac{1}{\cancel{2}} \times \cancel{2} \times t^2$

$t^2 + 15t - 60 = 0$

$t = \_\_\_\_.$

Case 2: When one particle is accelerating

$$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$$
$$= 20\hat{i} - 5\hat{i}$$
$$= 15\hat{i}$$



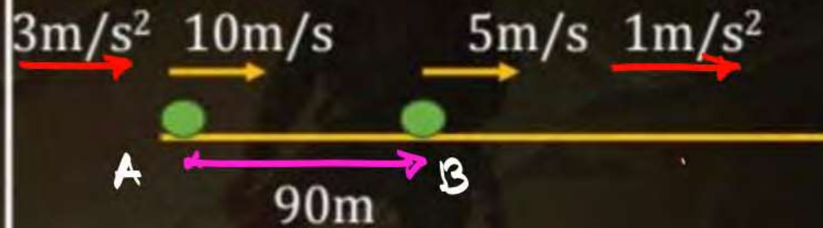
$$a_{AB} = \bar{a}_A - \bar{a}_B$$
$$= 0 - (2\hat{i})$$
$$= -2\hat{i}$$

$$s = ut + \frac{1}{2}at^2$$

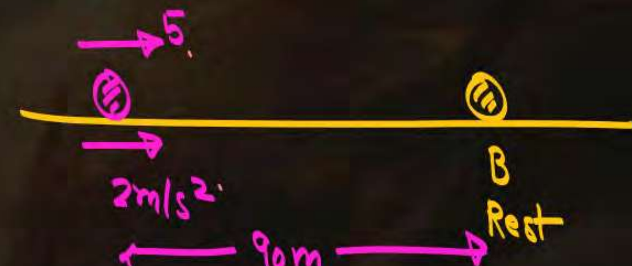
$$\boxed{30 = 15t - \frac{1}{2} \times 2 \times t^2}$$

$t =$ ___

Case 3: both particles are accelerating



$$\vec{V}_{AB} = \bar{V}_A - \bar{V}_B = 10\hat{i} - (5\hat{i}) = 5\hat{i}$$
$$\vec{a}_{AB} = \bar{a}_A - \bar{a}_B = 3\hat{i} - \hat{i} = 2\hat{i}$$



$$s = ut + \frac{1}{2}at^2$$

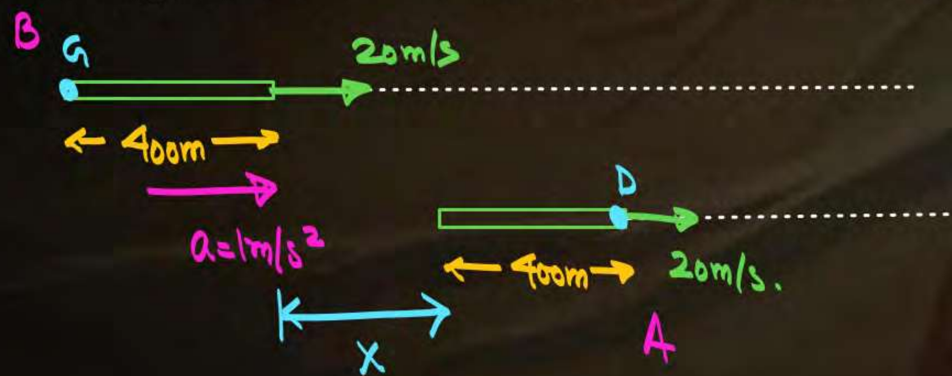
$$\boxed{90 = 5 \times t + \frac{1}{2} \times 2 \times t^2}$$

$t =$ ___

QUESTION 41

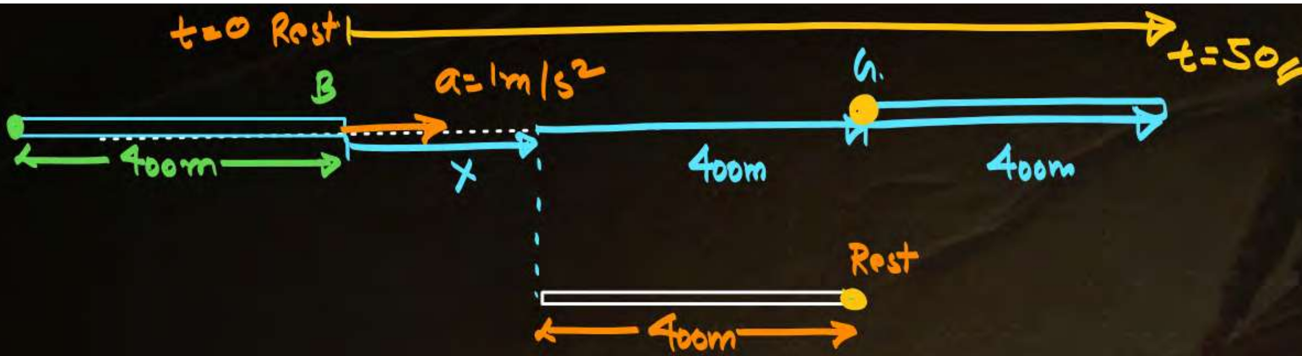
(NCERT) (overtaking of Extended objects)

Two trains A and B of length 400 m each are moving on two parallel tracks with a uniform speed of 20 m/s in the same direction, with A ahead of B. The driver of B decides to overtake A and accelerates by 1 m/s^2 If after 50 s, the guard of B just brushes past the driver of A, what was the original distance between them?



$\vec{V}_{BA} = \vec{V}_B - \vec{V}_A = 20 - 20 = 0.$

$\vec{a}_{BA} = \vec{a}_B - \vec{a}_A = 1\hat{i} - 0 = 1\hat{i}.$



distance by B to overtake A.

$d = 800 + x$

$u = 0$

$a = 1$

$t = 50$

$$S = ut + \frac{1}{2}at^2$$

$$800 + x = \frac{1}{2} \times 1 \times 50^2$$

$x = \_\_\_\_\_\_\_$.

QUESTION 42

An elevator is accelerating upward at a rate of 6 ft/sec^2 when a bolt from its ceiling falls to the floor of the lift (Distance = 9.5 feet). The time (in seconds) taken by the falling bolt to hit the floor is at time of fall

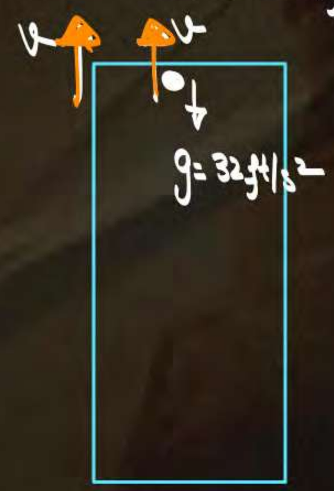
(A) $\sqrt{2}$

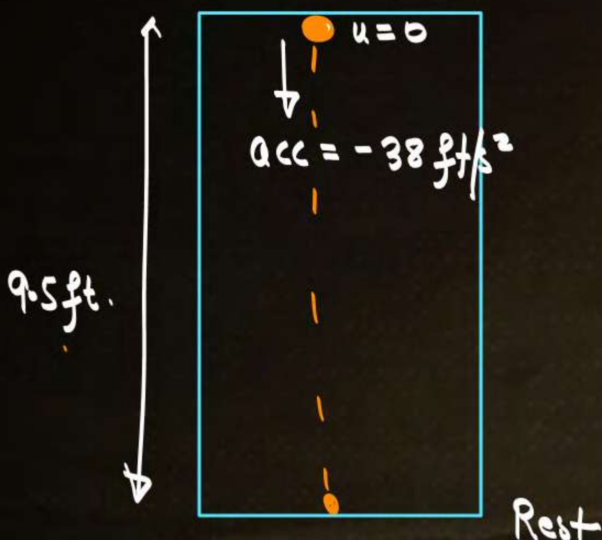
(B) $1/\sqrt{2}$

(C) $2\sqrt{2}$

(D) $1/2\sqrt{2}$

$$\vec{v}_{Be} = \vec{v}_B - \vec{v}_e$$
$$= v\hat{j} - v\hat{j} = 0$$
$$\vec{a}_{Be} = \vec{a}_B - \vec{a}_e$$
$$= -32 - 6$$
$$= -38\ ft/s^2$$





$$s = ut + \frac{1}{2}at^2$$

$$-9.5 = -\frac{1}{2} \times 38\, t^2$$

$$t = \_\_\_\_\_\_$$

QUESTION 43

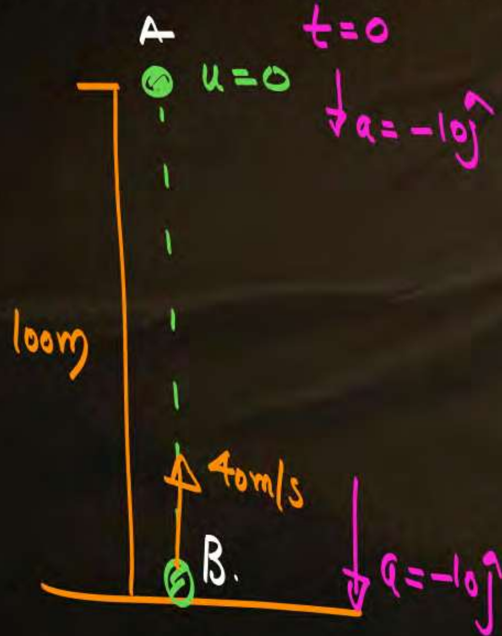
A ball is dropped from the top of a building 100 m high. At the same instant another ball is thrown upwards with a velocity of 40 ms^{-1} from the bottom of the building. The two balls will meet after

(A) 5 s

(B) 2.5 s

(C) 2 s

(D) 3 s



$$\vec{u}_{BA} = \vec{u}_B - \vec{u}_A = 40\hat{j} - 0 = 40\hat{j}$$

$$\vec{a}_{BA} = \vec{a}_B - \vec{a}_A = -10 - (-10) = 0.$$

$S = \frac{D}{t}$

$40 = \frac{100}{t}$

$t = \frac{10\cancel{0}}{4\cancel{0}} = 2.5$

QUESTION 44 (1997 JEE Adv + JEE Mains)

Two particles A and B are projected simultaneously from the two towers of height 10m and 20m respectively. Particle A is projected with an initial speed of $10\sqrt{2}$ m/s at an angle of 45° with horizontal, while particle B is projected horizontally with speed 10 m/s. If they collide in air, what is the distance d between the towers?

(A) 10 m

(B) 20 m

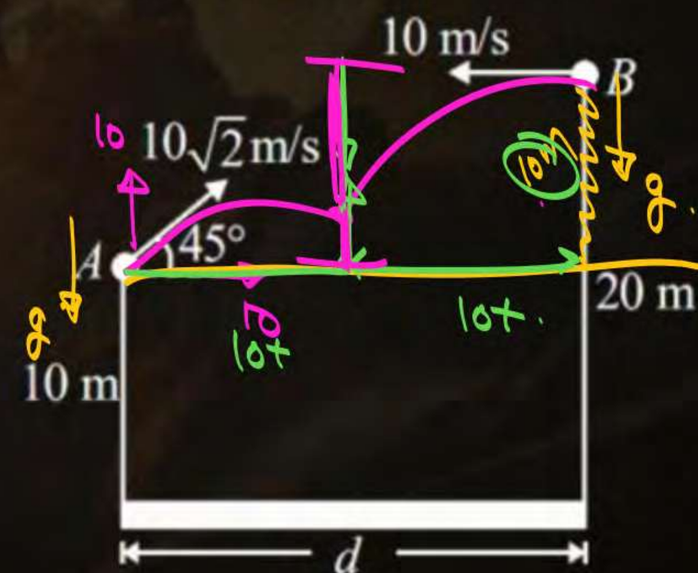
(C) 30 m

(D) 40 m

$\bar{a}_{AB} = \bar{a}_A - \bar{a}_B$

$= -10 - (-10) = 0.$

$20t = d.$



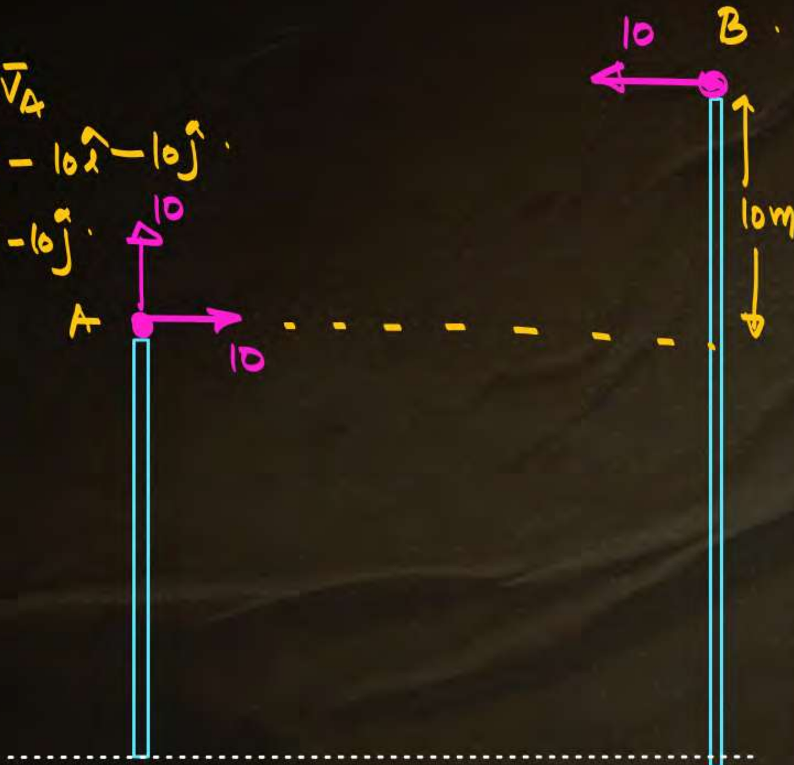
wrt A

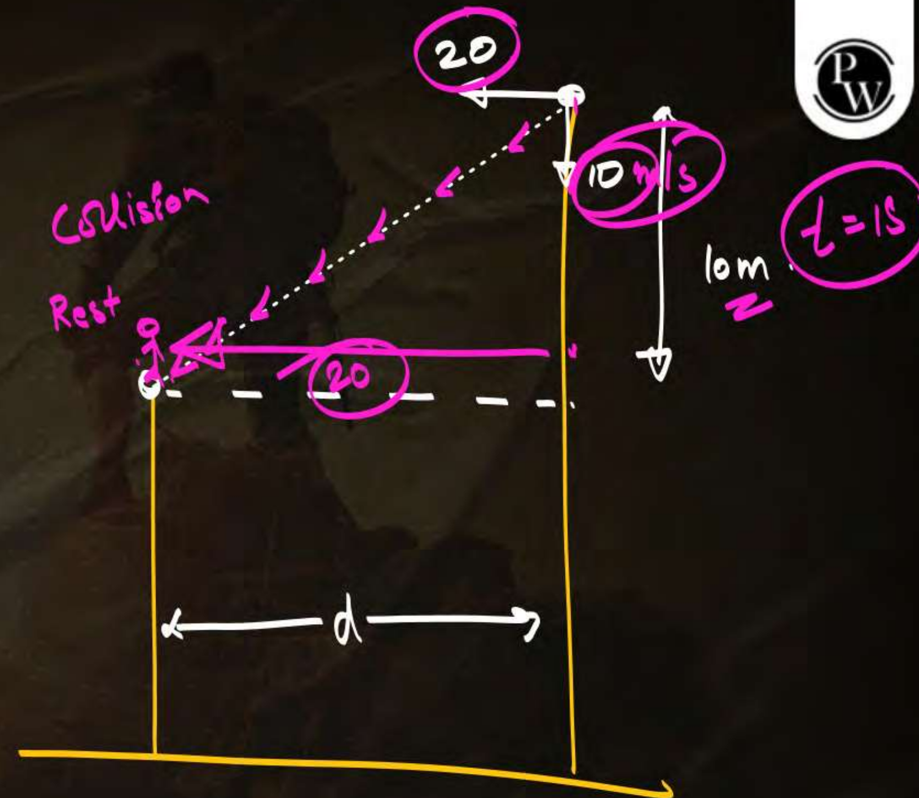
$\vec{a}_{BA} = 0$

$\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$

$= -10\hat{i} - 10\hat{i} - 10\hat{j}$

$= -20\hat{i} - 10\hat{j}$



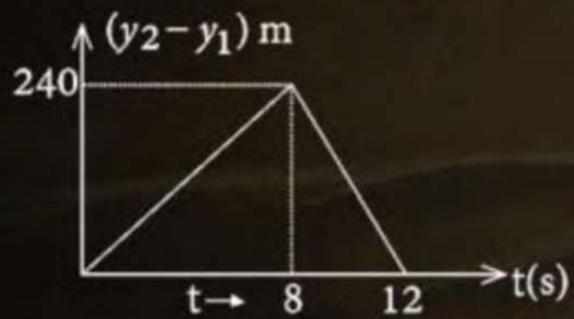


QUESTION 45

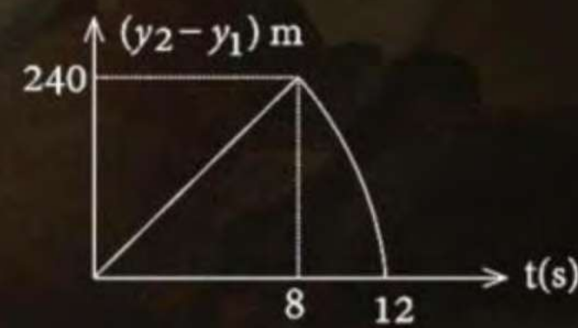
NCERT

Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed of 10 m/s and 40 m/s, respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first? (Assume stones do not rebound after hitting the ground and neglect air resistance, take $g = 10\ m/s^2$)

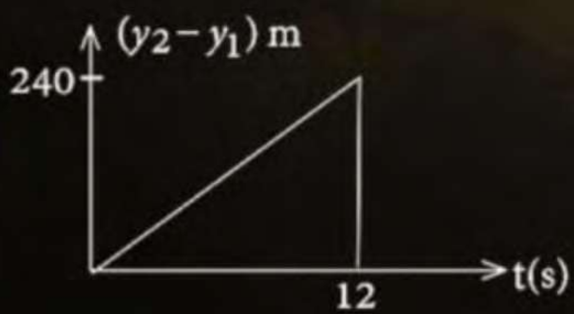
A



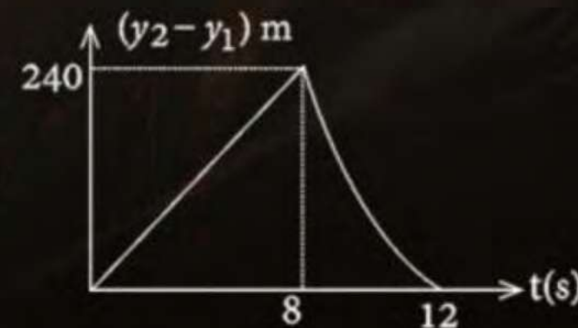
B



C



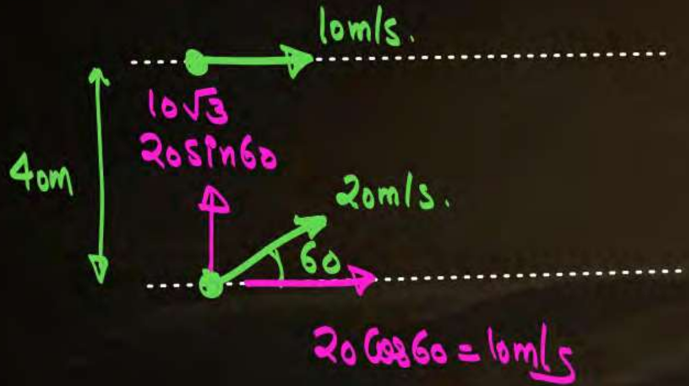
D



Concept of Collision

→ 2 particles → Same time par same position par hona.

a) Kya ye Collide Karenge.



Vapproach.

"Pass Lane Ki Koshish".

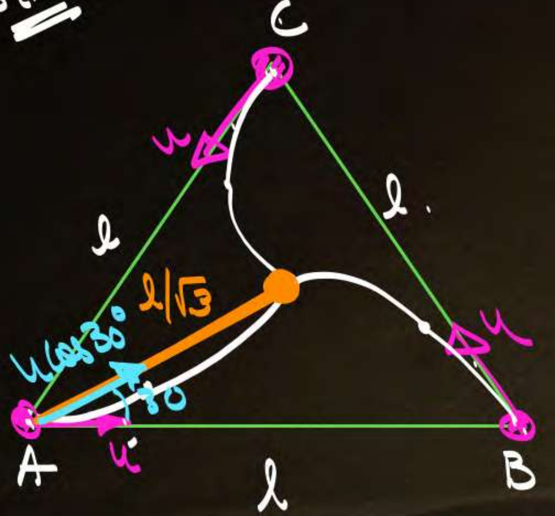
$v_{app} = 10\sqrt{3}$.

$$\text{time} = \frac{d_{app}}{v_{app}} = \frac{40}{10\sqrt{3}}$$

$$\boxed{t = \frac{4}{\sqrt{3}}\ \text{sec}}$$

HCV

हल Solved

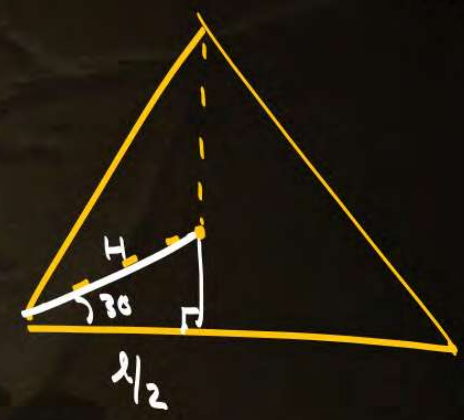


time of collision = $\frac{d_{app}}{v_{app}}$

$$t = \frac{\frac{l}{\sqrt{3}}}{u\cos 30}$$

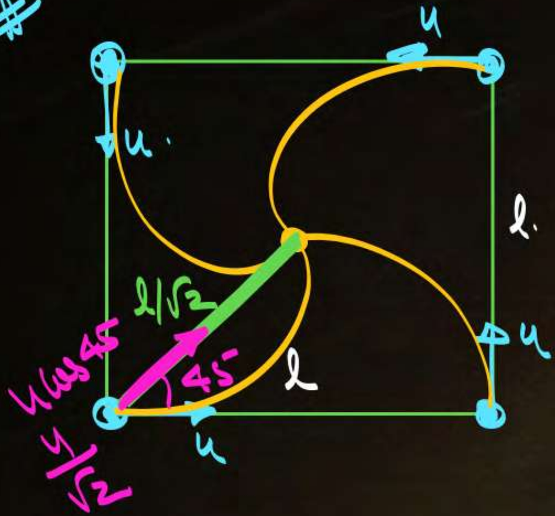
$$= \frac{l}{\sqrt{3}} \cdot \frac{2}{u\sqrt{3}}$$

$$\boxed{t = \frac{2l}{3u}}$$



$$\cos 30 = \frac{l}{2H}$$

$$H = \frac{l}{\cancel{2}\sqrt{3}} \times \cancel{2} = \frac{l}{\sqrt{3}}$$

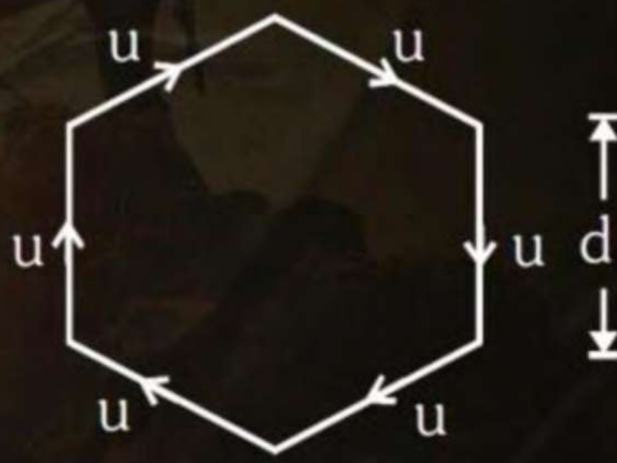


$$t_{collision} = \frac{d_{app}}{v_{app}} = \frac{l}{\sqrt{2}\,u}\sqrt{2} = \frac{l}{u}$$

QUESTION 46

HW

Six persons of same mass travel with same speed u along a regular hexagon of side 'd' such that each one always faces the other. After how what will they meet each other?



(A) $\frac{d}{u}$

(B) $\frac{2d}{3u}$

(C) $\frac{2d}{u}$

(D) $d\sqrt{3}u$

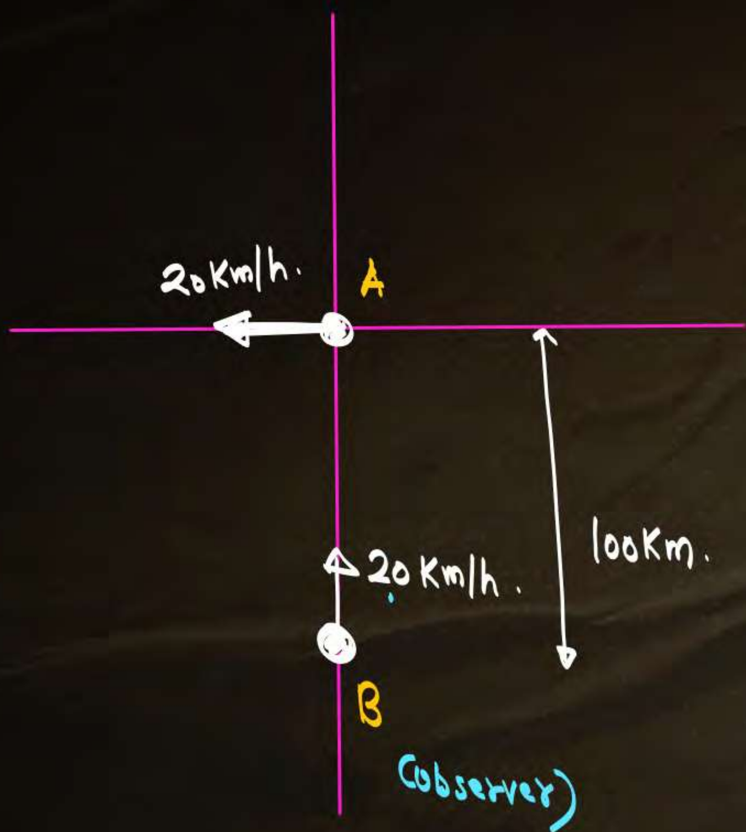
#

Concept of shortest distance

QUESTION 47

A ship A is moving towards the South with a speed 20 km per hour and another ship is moving towards the East with a speed of 20 km per hour. At a certain instant the Ship B is due south of ship A an is at a distance of 10 km from ship A find the shortest distance between the ships and the time after which they are closest to each other

Find Closest distance of approach



RAIN – MAN PROBLEM

QUESTION 48

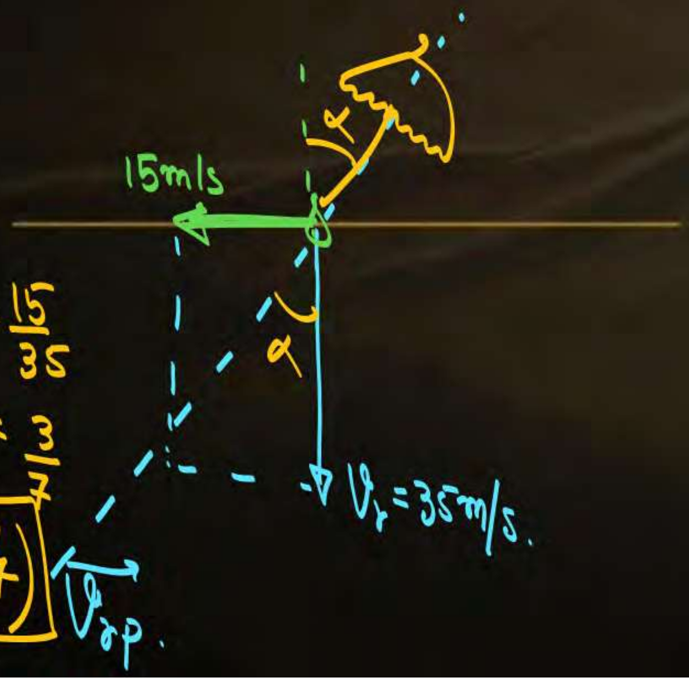
V_r =35m/s vertically downward

V_m = 15 m/s

What is the direction umbrella?

$\vec{V}_{rp} = \vec{V}_r - \vec{V}_p$

$= -35\hat{j} - 15\hat{i}$



$\tan\alpha = \frac{P}{B} = \frac{15}{35}$

$\tan\alpha = \frac{3}{7}$

$\boxed{\alpha = \tan^{-1}(3/7)}$

QUESTION 49

Rain falling at 20m/s at 30^0 with Vertical.

Velocity of person so that rain falls vertically on head.

$\vec{V}_p = -V_p\hat{i}$

$V_r = -10\hat{i} - 10\sqrt{3}\hat{j}$.

$\vec{V}_{rp} = \vec{V}_r - \vec{V}_p$

$= -10\hat{i} - 10\sqrt{3}\hat{j} - (-V_p\hat{i})$

$\vec{V}_{rp} = (V_p - 10)\hat{i} - 10\sqrt{3}\hat{j}$

$V_p - 10 = 0$

$V_p = 10$.

QUESTION 50

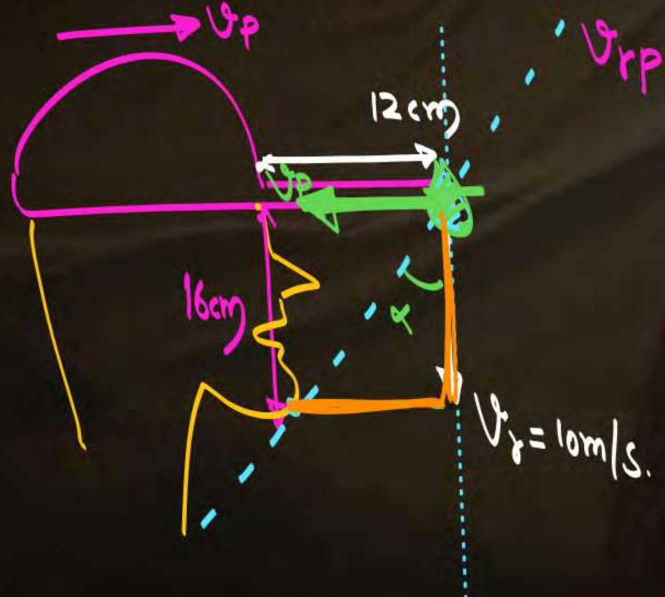
A man who is wearing a hat of extended length of 12 cm is running in rain falling vertically downwards with speed 10 m/s. The maximum speed with which man can run, so that rain drops do not fall on his face (the length of his face below the extended part of the hat is 16 cm) will be:

(A) 15/2

(B) 40/3

(C) 10

(D) 0



$$\tan\alpha = \frac{v_p}{\cancel{10}\,5} = \frac{\cancel{12}\,3}{\cancel{16}\,\cancel{4}\,2}$$

$$v_p = \frac{15}{2}$$

QUESTION 51

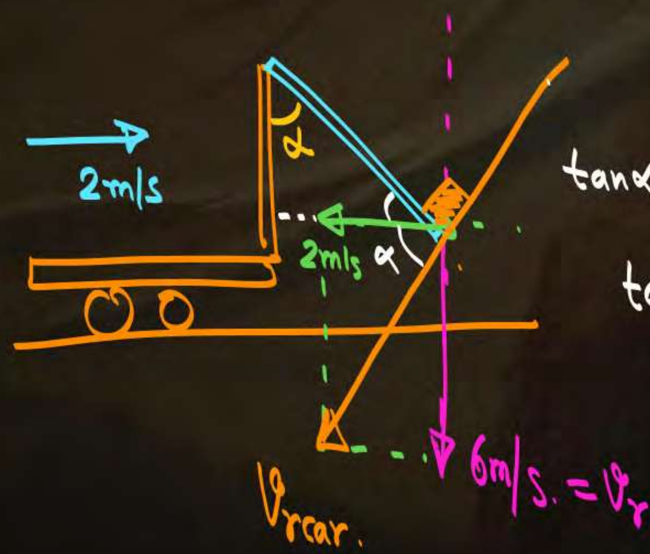
A glass wind screen whose inclination with the vertical can be changed is mounted on a car. The car moves horizontally with a speed of 2 m/s. At what angle α with the vertical should the wind screen be placed so that the rain drops falling vertically downwards with velocity 6 m/s strike the wind screen perpendicularly?

(A) $\tan^{-1}(3)$

(B) $\tan^{-1}(4)$

(C) $\tan^{-1}\left(\frac{1}{3}\right)$

(D) $\tan^{-1}\left(\frac{1}{4}\right)$



$\tan\alpha = \frac{6}{2}$

$\tan\alpha = 3$

$\boxed{\alpha = \tan^{-1}(3)}$

RIVER–BOAT (OR MAN) PROBLEM

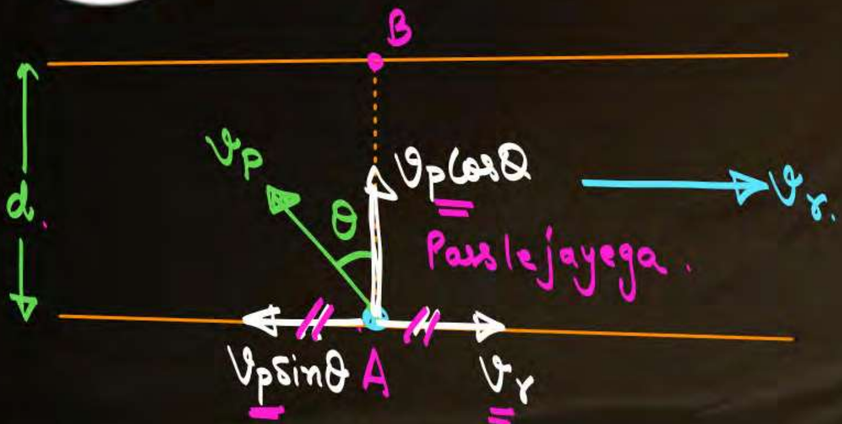
$$t = \frac{d}{v_p \cos\theta}$$

min

?

$\theta \to 0$

$$t_{min} = \frac{d}{v_p}$$



Min distance travel / Shortest path.

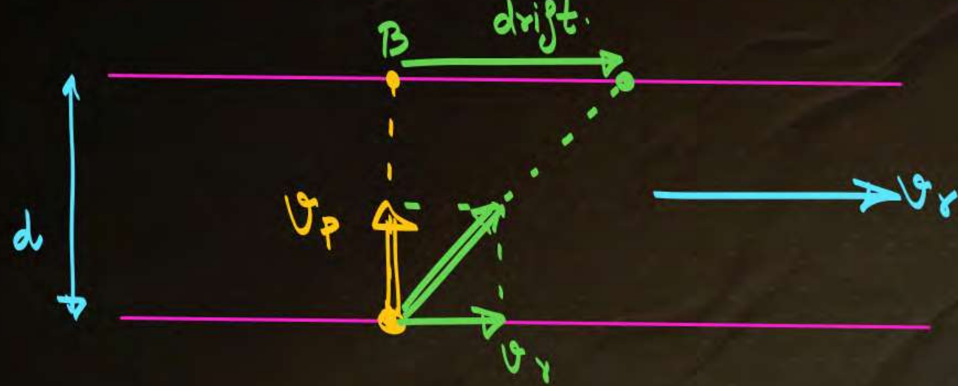
A ⟶ B

No net velocity along x

$$v_r = v_p \sin\theta \Rightarrow \sin\theta = \frac{v_r}{v_p} \qquad \theta = \sin^{-1}\left(\frac{v_r}{v_p}\right)$$

time to Reach B $t = \frac{d_{app}}{v_{app}} = \frac{d}{v_p\cos\theta} = \frac{d}{v_p\sqrt{1-\frac{v_r^2}{v_p^2}}} = \frac{d}{\sqrt{v_p^2 - v_r^2}}$

Case.2 Swim to Reach other Side in Min time → time min



$$S_x = u_x t = v_r \frac{d}{v_p}$$

$$\boxed{\text{drift} = d\left(\frac{v_r}{v_p}\right)}$$

$$\boxed{t_{min} = \frac{d_{app}}{v_{app}} = \frac{d}{v_p}}$$

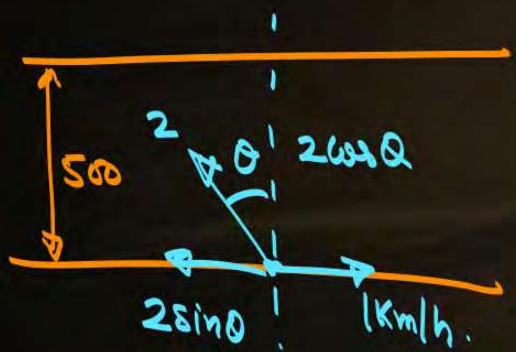
QUESTION 52 (PYQ)

Width of river 500m

Vp= 2km/h , Vr = 1Km/h.

Angle at which drift is minimum

If he swims for min time, drift?



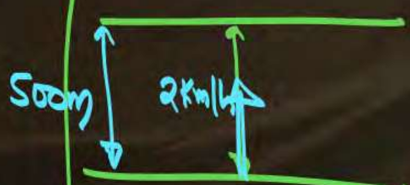
drift min

directly Reach B

$2\sin\theta = 1$

$\theta = 30^\circ$

tmin



$t = \frac{0.5\,km}{2}$

drift $= v_r t$

$= 1 \times 0.5/2$

QUESTION 53 (PYQ).

What should be angle so that swimmer can reach at B directly

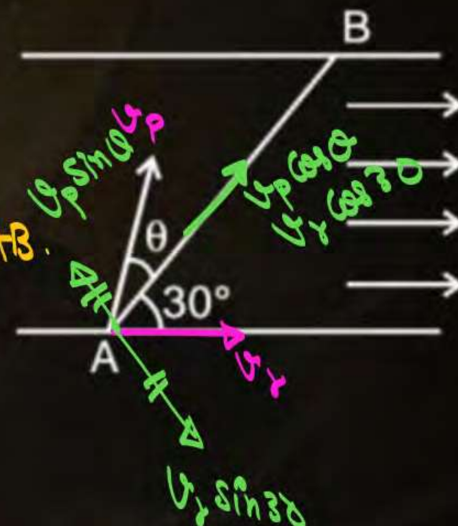
given $|v_r| = |v_p|$.

A ⟶ B

No velocity ⊥ to line AB.

$\cancel{v}_p \sin\theta = \cancel{v}_r \sin 30$

$\theta = 30^\circ$



QUESTION 54

HW

Statement-I: For an observer looking out through the window of a fast moving train, the nearby objects appear to move in the opposite direction to the train, while the distant objects appear to be stationary.

Statement-II: If the observer and the object are moving at velocities v_1 and v_2, respectively with reference to a laboratory frame, the velocity of the object with respect to the observer is $v_2 - v_1$.

(IIT-JEE 2008)

(A) Statement-I is true, Statement-II is true; Statement-II is the correct explanation for Statement-I

(B) Statement-I is the, Statement-II is true; Statement-II is not a correct explanation for Statement-I

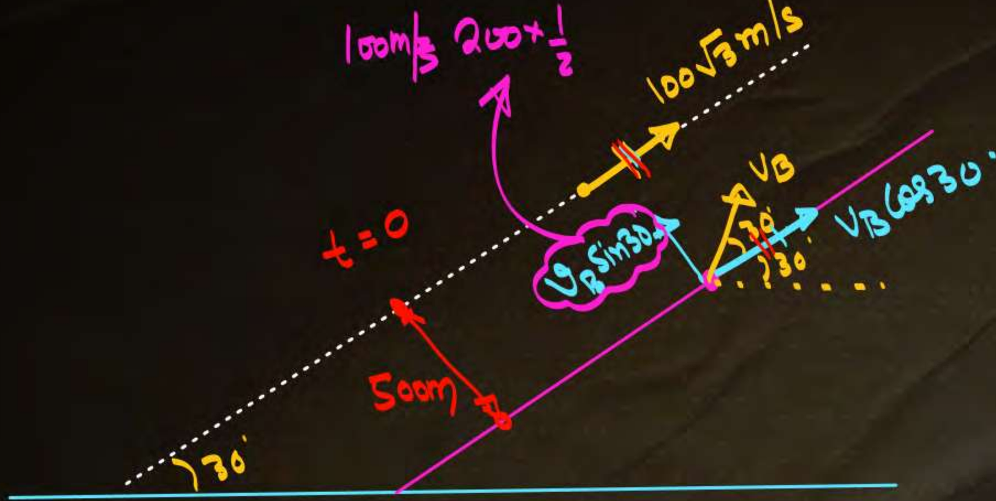
(C) Statement-I is true; Statement-II is false

(D) Statement-I is false; Statement-II is true

QUESTION 55

Airplanes A and B are flying with constant velocity in the same vertical plane at angles 30° and 60° with respect to the horizontal respectively as shown in the figure. The speed of A is $100\sqrt{3}\ \mathrm{ms}^{-1}$. At the time $t = 0$ s, an observer in A finds B at a distance of 500 m. This observer sees B moving with a constant velocity perpendicular to the line of motion of A. If at $t = t_0$, A just escapes being hit by B, t_0 in seconds is

(JEE Adv. 2014)



$$100\sqrt{3} = v_B \cos 30$$

$$100\sqrt{3} = v_B \frac{\sqrt{3}}{2}$$

$$200 = v_B$$

$$t_0 = t_{app} = \frac{d_{app}}{v_{app}} = \frac{500}{100} = 5.$$