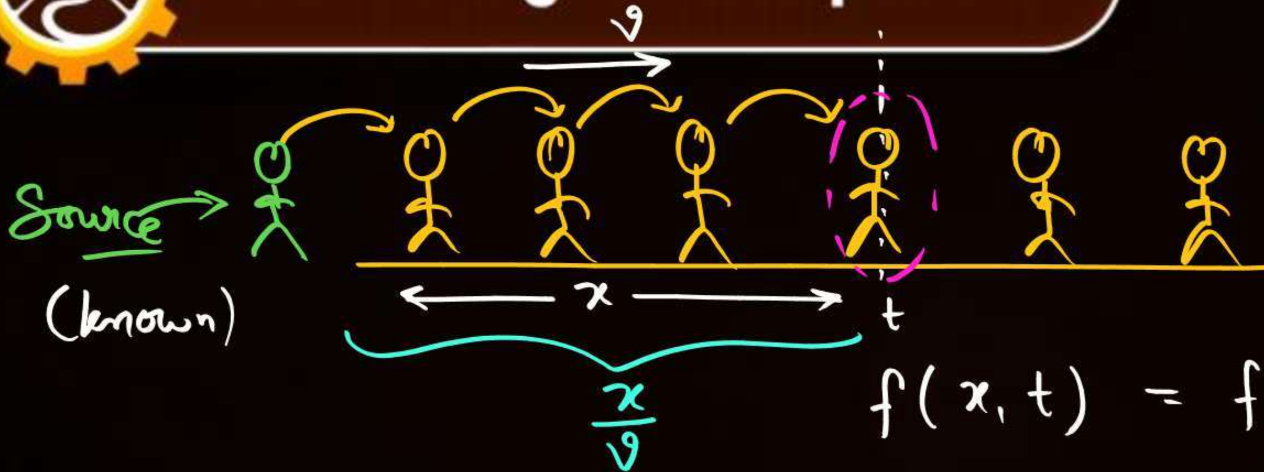
# Topics to be covered

1) Travelling Wave

2) Wave Interference

3) Standing Wave

# Travelling Wave Equation




$$f(x,t) = f\left(0,\ t-\frac{x}{v}\right)$$

Wave vel. → Medium only

Wave freq → Source only

Wave shape (waveform) → Source only

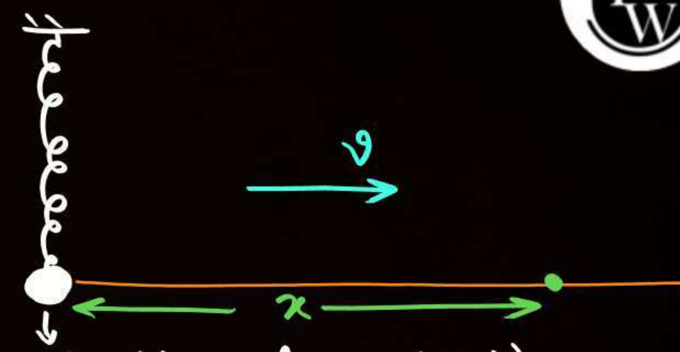
Eq. of travelling wave towards +ve $x$-axis →

$$y(x,t) = y\left(0, t - \frac{x}{v}\right) = f\left(t - \frac{x}{v}\right)$$

Eq. of travelling wave towards −ve axis →

$$y(x,t) = y\left(0, t + \frac{x}{v}\right) = f\left(t + \frac{x}{v}\right)$$

Sinosudal wave →




$$y(0,t) = A\sin(\omega t)$$

$$y(x,t) = y\left(0, t - \frac{x}{v}\right)$$

$$= A\sin\left[\omega\left(t - \frac{x}{v}\right)\right]$$

$$y = A\sin(\omega t - kx)$$

where, $k = \dfrac{\omega}{v}$

$$v = \frac{\omega}{k}$$

Wave propagation की direcn.

$$\frac{\omega}{v} = k$$

$y = A \sin(\omega t - kx)$ $\longrightarrow$ +ve $x$-axis

$y = A \sin(kx - \omega t)$ $\longrightarrow$ +ve $x$-axis.

$y = A \sin(\omega t + kx)$ $\longrightarrow$ -ve $x$-axis

$y = -A \sin(kx + \omega t)$ $\longrightarrow$ -ve $x$-axis.

QUESTION

Which of the following equations represents a traveling wave? [24 Feb, 2021 (S-II)]

(A) $y = A \sin x \cos \omega t$ ✗

(B) $y = A \sin (15x - 2t)$ ✓

(C) $y = Ae^{x} \cos (\omega t - \theta)$ ✗

(D) $y = Ae^{-x^2} (vt + \theta)$ ✗

QUESTION

If $y = (6x - 30t)^2$ then Find speed of waves.

$f\left(t - \frac{x}{v}\right)$

$f(vt - x)$

$\underset{\downarrow}{6}x - \underset{\downarrow}{30}t$

$k \quad \omega$

$v = \frac{\omega}{k} = \frac{30}{6} = 5\ m/s$ Ans

$\omega$ = Kendriya Vidhyalaya

$\omega = kv$

$y = A \sin(kx - \omega t)$

$k \to m^{-1}$, $\omega \to s^{-1}$

$v = \frac{\omega}{k}$

QUESTION

If $y = \frac{1}{(\boxed{4x+3t})^2+1}$ then find vel. of wave along with direction.

$\cancel{4x^2+3t}$

$\cancel{4x-3t^2}$

$\cancel{4x^2+3t^2}$

$\underset{k}{4}x + \underset{\omega}{3}t$

$v = \frac{\omega}{k} = \frac{3}{4}$ m/s

towards -ve x-axis

## QUESTION

The amplitude of wave disturbance propagating in the positive $x$-direction is given by $y = \frac{1}{1+(x)^2}$ at $t = 0$ and $y = \frac{1}{1+(x-2)^2}$ at $t = 1$s, where $x$ and $y$ are in metres. The shape of wave does not change during the propagation. The velocity of the wave will be 2 m/s.

[20 July, 2021 (S-I)]

$$y = \frac{1}{1+(x-2)^2} \quad \text{at } t = 1$$

$$\frac{1}{1+(\underset{k}{1}x - \underset{\omega}{2}\times 1)}$$

$$v = \frac{\omega}{k} = \frac{2}{1} = 2\ \text{m/s}.$$

# Wave Parameters

$$y = A\sin(kx - \omega t + \phi)$$

$A$ = Amplitude

$k$ = Angular wave number

$\omega$ = Angular frequency

$\phi$ = Phase const.

$kx - \omega t + \phi$ = Phase

wavelength

$$\lambda = \frac{2\pi}{k}$$

Time period

$$T = \frac{2\pi}{\omega}$$

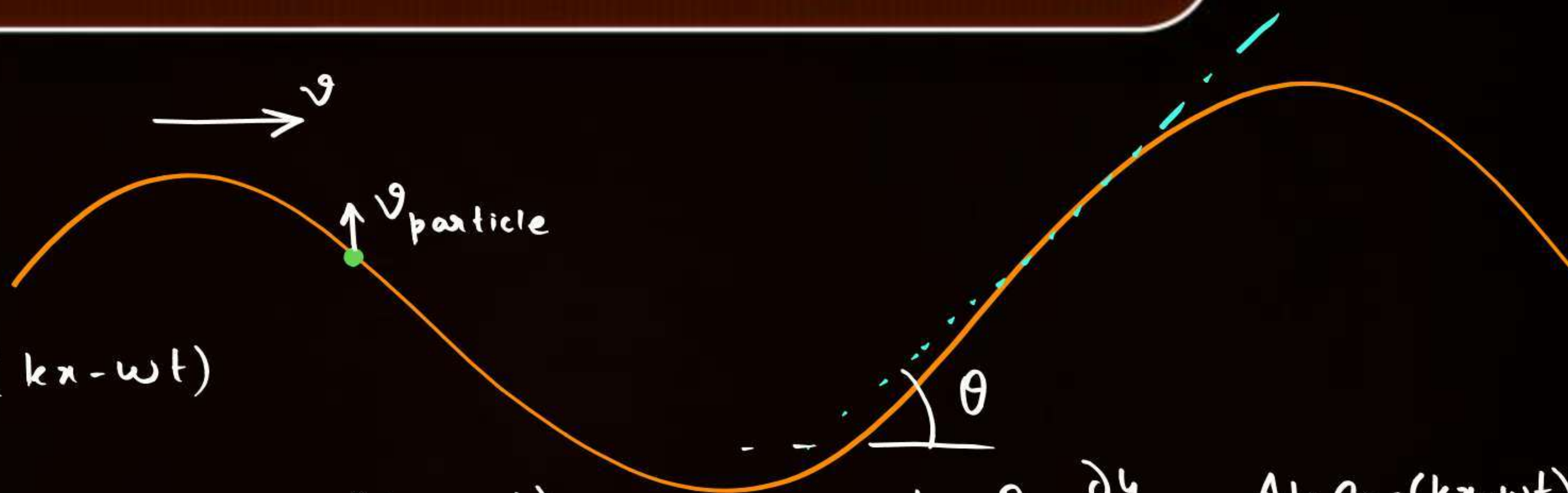
frequency

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

Wave vel.

$$v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f$$

# Wave Velocity, Particle Velocity and Slope




$$y = A\sin(kx - \omega t)$$

$$v_{particle} = \frac{\partial y}{\partial t} = -\underbrace{A\omega}_{v_{max.}}\cos(kx - \omega t)$$

$$\tan\theta = \frac{\partial y}{\partial x} = \underbrace{Ak}_{\text{max. slope}}\cos(kx - \omega t)$$

QUESTION

Marathon Q.

If $y = 4\sin(\underset{k}{\pi} x - \underset{\omega}{\frac{\pi}{2}} t)$ then find

1. Amplitude of wave ? $= 4$ m
2. Wavelength = ? $2$ m
3. Frequency = ? $\frac{1}{4}$ Hz
4. Wave speed = ? $\frac{1}{2}$ m/s
5. Speed of particle at $x = 1$ at $t = 3$s.

$= 0$ m/s.

2) $\lambda = \frac{2\pi}{k} = \frac{2\pi}{\pi} = 2m$

3) $f = \frac{\omega}{2\pi} = \frac{\cancel{\pi}/2}{2\cancel{\pi}} = \frac{1}{4}$ Hz

4) $v = \frac{\omega}{k} = \frac{\pi/2}{\pi} = \frac{1}{2}$ m/s

5) $v = \frac{\partial y}{\partial t} = 4\cos(\pi x - \frac{\pi}{2}t) \times -\frac{\pi}{2}$

$v = -2\pi\cos(\pi x - \frac{\pi}{2}t)$

$|v_{x=1, t=3}| = 2\pi\cos(\pi \times 1 - \frac{\pi}{2} \times 3)$

$= 2\pi\cos(-\frac{\pi}{2}) = 0$ m/s

## QUESTION

If $y = 4\sin\left(\pi x - \frac{\pi}{2}t\right)$ then find

6. Position of particle at $t = 1$ at $x = 2$.
7. Snapshot of wave at $t = 0.5$ s. → Photo → Video
8. Position of particle at $x = 0.5$m at any time.
9. Slope of tangent on wave at $x = 1$ at $t = 1$. $= 0$
10. Max. particle velocity. $\Rightarrow v_{max} = A\omega = 4 \times \frac{\pi}{2} = 2\pi \frac{m}{s}$
11. Phase diff. between two particles separated by 0.5 m.

11) $\Delta\phi = \frac{\cancel{2}\pi}{\cancel{2}} \times 0.5 = \frac{\pi}{2}$ Ans

6) $y_{x=2, t=1} = 4\sin\left(\pi \times 2 - \frac{\pi}{2} \times 1\right)$
$= 4\sin\left(\frac{3\pi}{2}\right)$
$= -4$ m

7) $y_{t=0.5} = 4\sin\left(\pi x - \frac{\pi}{2} \times \frac{1}{2}\right)$

$y_{t=0.5} = 4\sin\left(\pi x - \frac{\pi}{4}\right)$

8) $y_{x=0.5} = 4\sin\left(\pi \times \frac{1}{2} - \frac{\pi}{2}t\right)$
$= 4\sin\left(\frac{\pi}{2} - \frac{\pi}{2}t\right)$

$y_{x=0.5} = 4\cos\left(\frac{\pi}{2}t\right)$

$$y = 4\sin\left(\pi x - \frac{\pi}{2}t\right)$$

9)

$$\tan\theta = \frac{\partial y}{\partial x} = 4\cos\left(\underline{\pi x} - \frac{\pi}{2}t\right) \times \pi$$

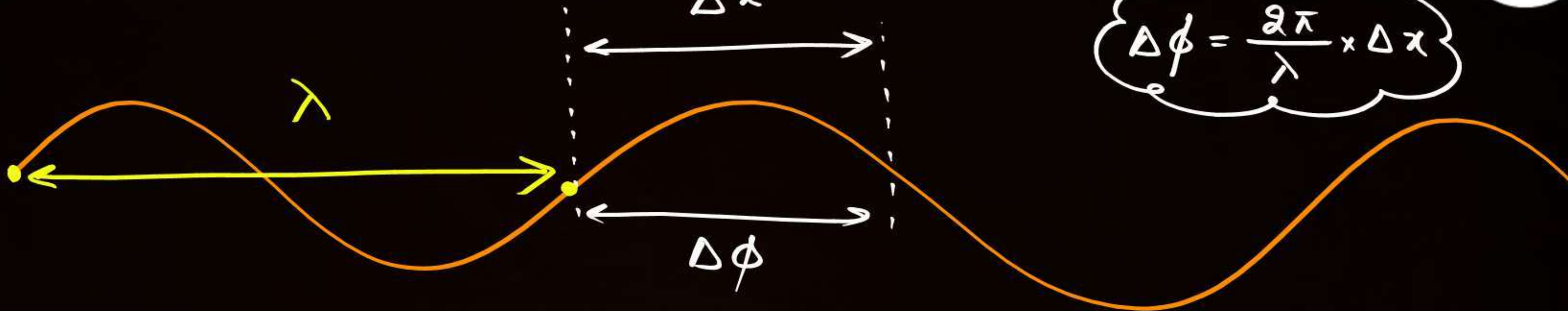
$$(\tan\theta)_{x=1,\, t=} = 4\pi\cos\left(\pi \times 1 - \frac{\pi}{2} \times 1\right)$$

$$= 0$$

Relation b/w Phase difference & Path difference

$\Delta x \rightarrow$ Path difference

$$\Delta\phi = \frac{2\pi}{\lambda} \times \Delta x$$




| Path Difference | | Phase difference |
|---|---|---|
| $\lambda$ | $\longrightarrow$ | $2\pi$ |
| $1$ | $\longrightarrow$ | $\frac{2\pi}{\lambda}$ |
| $\Delta x$ | $\longrightarrow$ | $\frac{2\pi}{\lambda}\Delta x = \Delta\phi$ |

QUESTION

A wave travelling in the positive $x$-direction having displacement along y-direction as 1 m, wavelength $2\pi$ m and frequency of $\frac{1}{\pi}$ Hz is represented by

(A) $y = \sin(x - 2t)$

(B) $y = \sin(2\pi x - 2\pi t)$

(C) $y = \sin(10\pi x - 20\pi t)$

(D) $y = \sin(2\pi x + 2\pi t)$

$y = A\sin(kx - \omega t)$

$= 1\sin(1 \times x - 2t)$

Common mistake

$\lambda = \frac{2\pi}{k} \Rightarrow k = \frac{2\pi}{\lambda}$

$k = \frac{2\pi}{2\pi}$

$k = 1$

$f = \frac{\omega}{2\pi} \Rightarrow \omega = 2\pi f$

$= 2\pi \times \frac{1}{\pi}$

$f = 2$

QUESTION

$f = \frac{\omega}{2\pi} = \frac{2\pi}{2\pi} = 1\,Hz$

$\lambda = \frac{2\pi}{k} = \frac{2\pi}{10\pi} = \frac{1}{5}\,m = 0.2\,m.$

The wave described by

$$y = 0.25 \sin(\underbrace{10\pi}_{k} x - \underbrace{2\pi}_{\omega} t),$$

where, $x$ and $y$ are in metre and $t$ in second, is a wave travelling along the

(A) negative $x$-direction with frequency 1Hz

(B) positive $x$-direction with frequency $\pi$ Hz and wavelength $l = 0.2$m

(C) positive $x$-direction with frequency 1Hz and wavelength $l = 0.2$m

(D) negative $x$-direction with amplitude 0.25m and wavelength $l = 0.2$m

QUESTION

A transverse wave is represented by $\boxed{y = A\sin(\omega t - kx).}$ For what value of the wavelength is the wave velocity equal to the maximum particle velocity?

**A** $\pi A/2$

**B** $\pi A$

**C** $2\pi A$

**D** $A$

$$v = \frac{\omega}{k}$$

$$v_{max} = A\omega$$

$$\frac{\cancel{\omega}}{k} = A\cancel{\omega} \Rightarrow k = \frac{1}{A}$$

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{\left(\frac{1}{A}\right)} = 2\pi A$$

Ans

QUESTION

A transverse wave propagating along x-axis is represented by

$$y(x,t) = 8\,sin\left(\underbrace{0.5\pi x}_{k} - \underbrace{4\pi t}_{\omega} - \frac{\pi}{4}\right)$$

where, $x$ is in metre and $t$ is in second. The speed of the wave is

$$v = \frac{\omega}{k} = \frac{4\cancel{\pi}}{0.5\cancel{\pi}} = 8\,m/s$$

(A) $4\pi$ m/s

(B) $0.5\pi$ m/s

(C) $\pi/4$ m/s

(D) 8 m/s

QUESTION

A travelling harmonic wave is represented by the equation $y(x, t) = 10^{-3} \sin (\underbrace{50}_{\omega}t + \underbrace{2}_{k}x)$, where $x$ and $y$ are in meter and $t$ is in seconds. Which of the following is a correct statement about the wave?

**[JEE Mains 2019]**

(A) The wave is propagating along the negative $x$-axis with speed 25 $ms^{-1}$.

$v = \frac{\omega}{k}$

$= \frac{50}{2}$

$= 25\ m/s$

(B) The wave is propagating along the positive $x$-axis with speed 100 $ms^{-1}$.

(C) The wave is propagating along the positive $x$-axis with speed 25 $ms^{-1}$.

(D) The wave is propagating along the negative $x$-axis with speed 100 $ms^{-1}$.

QUESTION

A wave travelling in the positive $x$-direction having displacement along y-direction as 1 m, wavelength $2\pi$ m and frequency of $\frac{1}{\pi}$ Hz is represented by

(A) $y = \sin(x - 2t)$

Repeated

(B) $y = \sin(2\pi x - 2\pi t)$

(C) $y = \sin(10\pi x - 20\pi t)$

(D) $y = \sin(2\pi x + 2\pi t)$

QUESTION

The wave described by

$$y = 0.25 \sin(10\pi x - 2\pi t),$$

Repeat

where, $x$ and $y$ are in metre and $t$ in second, is a wave travelling along the

(A) negative $x$-direction with frequency 1Hz

(B) positive $x$-direction with frequency $\pi$ Hz and wavelength $l = 0.2$m

(C) positive $x$-direction with frequency 1Hz and wavelength $l = 0.2$m

(D) negative $x$-direction with amplitude 0.25m and wavelength $l = 0.2$m

QUESTION

The equation of waves is given by Y = $10^{-2} \sin 2\pi(160t - 0.5x + \pi/4)$ where $x$ and Y are in $m$ and $t$ in $s$. The speed of the wave is ______ km h$^{-1}$. [11 April, 2023 (S-I)]

$\omega$ $k$

$$v = \frac{\omega}{k} = \frac{160}{0.5} = 320 \frac{m}{s}$$

$$320 \times \frac{10^{-3}\ km}{1/3600\ hr}$$

$$3\cancel{2}0 \times 36\cancel{0}0 \times \cancel{10}^{-3} \frac{km}{hr}.$$

$$\frac{32 \times 36}{192\atop 96\times} \frac{km}{hr}$$

$$1152\ km/hr$$

QUESTION

A longitudinal wave is represented by $x = 10 \sin 2\pi(nt - x/\lambda) cm$. The maximum particle velocity will be four times the wave velocity if the determined value of wavelength is equal to:

[29 June, 2022 (S-I)]

(A) $2\pi$

(B) $5\pi$

(C) $\pi$

(D) $5\pi/2$

$$x = 10 \sin\left[2\pi\left(nt - \frac{x}{\lambda}\right)\right]$$

$$= 10 \sin\left(\underbrace{2\pi n}_{\omega} t - \underbrace{\frac{2\pi}{\lambda}}_{k} x\right)$$

$$v = \frac{\omega}{k} = \frac{\cancel{2\pi} n}{\frac{\cancel{2\pi}}{\lambda}} = n \times \lambda$$

$$v_{particle} = A\omega$$

$$= 10 \times 2\pi n$$

$$= 20\pi n$$

$$v_{particle} = 4 \times v$$

$$\cancel{20}^{5}\pi \cancel{n} = \cancel{4} \times \cancel{n} \lambda$$

$$\lambda = 5\pi$$

# Wave Velocity

$$y = A\sin(kx \pm \omega t)$$

$$\boxed{v_{wave} = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f}$$

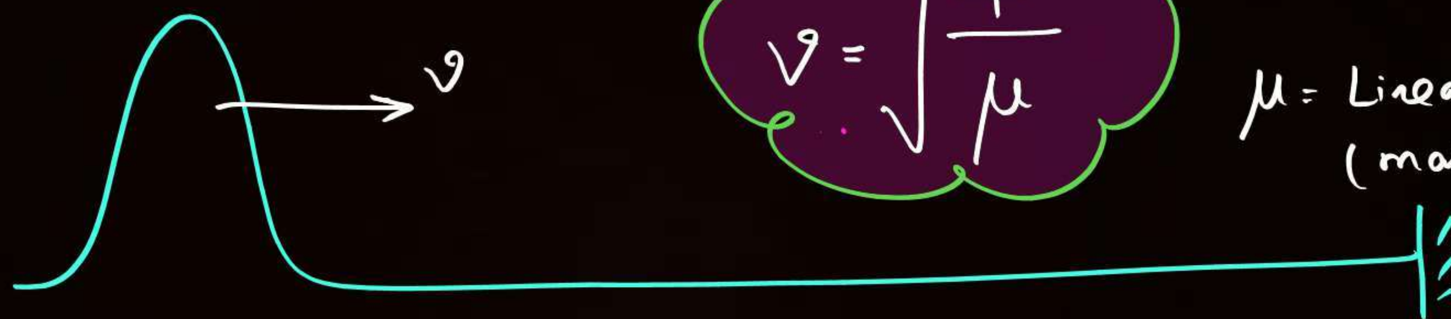
Depends on medium only.

# Transverse Wave Velocity in a string

$$v = \sqrt{\frac{T}{\mu}}$$

$T$ = Tension in string

$\mu$ = Linear mass density (mass per unit length)



QUESTION

A Steel wire with mass per unit length $\underbrace{7.0 \times 10^{-3}\ \text{kg}}_{\mu}$ is under tension of $\underbrace{70\ \text{N}}_{T}$. The speed of transverse waves in the wire will be:

**[01 Feb, 2023 (S-I)]**

(A) $200\pi$ m/s

(B) 100 m/s

(C) 10 m/s

(D) 50 m/s

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{\cancel{7}0 \cdot 10}{\cancel{7} \times 10^{-3}}}$$

$$= \sqrt{10^4}$$

$$= 100 \text{ m/s}$$

QUESTION

$\mu = 0.135 \times \frac{10^{-3}\,kg}{10^{-2}\,m} = 0.135 \times 10^{-1}\,\frac{kg}{m}$

The mass per unit length of a uniform wire is $\boxed{0.135\ g/cm.}$ A transverse wave of the form $\boxed{y = -0.21 \sin(x + 30t)}$ is produced in it, where x is in meter and t is in second. Then, the expected value of tension in the wire is $x \times 10^{-2}$ N. Value of $x$ is ______. (Round-off to the nearest integer)

**[26 Feb, 2021 (S-I)]**

$v = \frac{\omega}{k} = \frac{30}{1} = 30\ m/s$

$v = \sqrt{\frac{T}{\mu}} \Rightarrow 30 = \sqrt{\frac{T}{0.135 \times 10^{-1}}}$

$\Rightarrow 900 = \frac{T}{0.135 \times 10^{-1}}$

$T = 900 \times 0.135 \times 10^{-1}$

$= 12.15\ N$

$= 1215 \times 10^{-2}\ N$

Ans

**QUESTION** Adv. level

A uniform string of length 20 m is suspended from a rigid support. A short wave pulse is introduced at its lowest end. It starts moving up the string. The time taken to reach the supports is: [2016]

(A) $2\sqrt{2}$ s

(B) $\sqrt{2}$ s

(C) $2\pi\sqrt{2}$ s

(D) 2s

$$v = \sqrt{\frac{T}{\mu}}$$

$$T = m_x g$$

$$T = (\mu x) g$$

$$v = \sqrt{\frac{\cancel{\mu} x g}{\cancel{\mu}}}$$

$$v = \sqrt{gx}$$

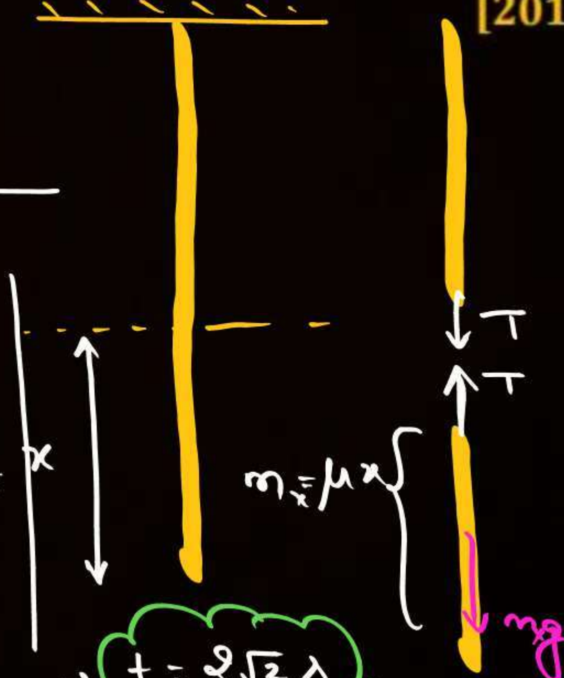
$$\Rightarrow \frac{dx}{dt} = \sqrt{xg}$$

$$\int_0^{20} \frac{dx}{\sqrt{x}} = \int_0^t \sqrt{g}\, dt$$

$$\left[\frac{x^{1/2}}{\left(\frac{1}{2}\right)}\right]_0^{20} = \sqrt{g}\, t$$

$$2\sqrt{20} = \sqrt{10}\, t \Rightarrow t = 2\sqrt{2}\ \text{s}$$

Ans.




$$v\uparrow = \sqrt{\frac{T}{\mu}}\uparrow$$

## QUESTION

A uniform thin rope of length 12 m and mass 6 kg hangs vertically from a rigid support and a block of mass 2 kg is attached to its free end. A transverse short wave-train of wavelength 6 cm is produced at the lower end of the rope. What is the wavelength of the wave-train (in cm) when it reaches the top of the rope? [Sep. 03, 2020 (I)]

(A) 3

(B) 6

(C) 12

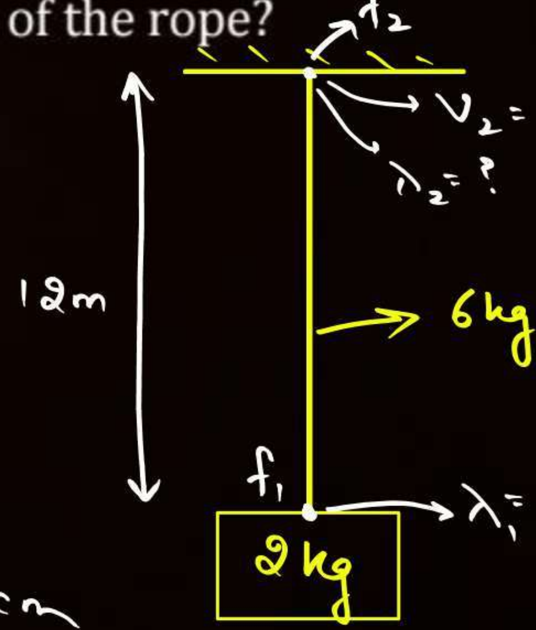
(D) 9

$$f_1 = f_2$$

$$\frac{v_1}{\lambda_1} = \frac{v_2}{\lambda_2}$$

$$\frac{\sqrt{\frac{2\cancel{0}}{\cancel{\mu}}}}{6} = \frac{\sqrt{\frac{8\cancel{0}}{\cancel{\mu}}}}{\lambda_2}$$

$$\lambda_2 = 6 \times \sqrt{\frac{8}{2}} = 12\,\text{cm}$$




$$v_2 = \sqrt{\frac{T_2}{\mu}} = \sqrt{\frac{20+60}{\mu}} = \sqrt{\frac{80}{\mu}}$$

$$\lambda_1 = 6\,\text{cm}, \quad v_1 = \sqrt{\frac{T_1}{\mu}} = \sqrt{\frac{20}{\mu}}$$

QUESTION

The percentage increase in the speed of transverse waves produced in a stretched string if the tension is increased by 4%, will be ......%, [25 Feb, 2021 (S-II)]

$$v = \sqrt{\frac{T}{\mu}}$$

$$\% v = \frac{1}{2} \times \% T$$

$$= \frac{1}{2} \times 4\%$$

$$= 2\%$$

$$y = x^n$$

$$\% y = n \times \% x$$

(for small % change)

# Longitudinal Wave

(Sound)

$$\Delta x = A \sin(kx - \omega t)$$

or $$S = A \sin(kx - \omega t)$$

$$v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f$$

# Longitudinal Wave Velocity

$v \propto \sqrt{\frac{\text{Elasticity}}{\text{Inertia}}}$

Solids →
(rod)

$$\boxed{v = \sqrt{\frac{Y}{\rho}}}$$

Y = Young's modulus of elasticity

$\rho$ = volumetric density

Liquid →

$$\boxed{v = \sqrt{\frac{B}{\rho}}}$$

B = Bulk modulus of elasticity.

Gases →

(Speed of sound in gas)

$$\gamma = \frac{C_p}{C_v} = 1 + \frac{2}{f}$$

Mono → $f = 3$

Diat. → $f = 5$

Poly → $f = 5$
(linear)

Poly → $f = 6$
(Non-linear)

$\gamma$ = Adiabatic exponent

P = Pressure of gas

$\rho$ = vol. density

$$v = \sqrt{\frac{B}{\rho}} = \underbrace{\sqrt{\frac{\gamma P}{\rho}}}_{\text{Newton's Law \& Laplace correction}} = \sqrt{\frac{\gamma RT}{M}}$$

→ Adiabatic process

T = Absolute temp.

M = Molar mass

{Depends only on T}

↓

For const temp., speed of sound does not depend on pressure & vol.

## Effect of temp. & Humidity.

$$v\uparrow = \sqrt{\frac{\gamma R T}{M\downarrow}}$$

(1) Temp $\longrightarrow$ $\boxed{v \propto \sqrt{T}}$

(2) Humidity $\longrightarrow$ Humidity $\uparrow \longrightarrow v\uparrow$

Atmosphere

$O_2$ $\downarrow$ 32g

$N_2$ $\downarrow$ 28g

$H_2O$ $\downarrow$ 18g

QUESTION

Diatomic, f = 5

Find ratio of speed of sound in $\underline{O_2}$ & $\underline{H_2}$ at same temperature.

$$\frac{v_{O_2} = \sqrt{\frac{\cancel{\gamma R T}}{M_{O_2}}}}{v_{H_2} = \sqrt{\frac{\cancel{\gamma R T}}{M_{H_2}}}} \Rightarrow \frac{v_{O_2}}{v_{H_2}} = \sqrt{\frac{M_{H_2}}{M_{O_2}}} = \sqrt{\frac{2}{32}}$$

$$= \sqrt{\frac{1}{16}}$$

$$= \frac{1}{4} \text{ Ans}$$

**QUESTION**

Two mono atomic ideal gases 1 and 2 of molecular masses $m_1$ and $m_2$ respectively are enclosed in separate containers kept at the same temperature. The ratio of the speed of sound in gas 1 to that in gas 2 is given by [2000S]

(A) $\sqrt{\dfrac{m_1}{m_2}}$

(B) $\sqrt{\dfrac{m_2}{m_1}}$

(C) $\dfrac{m_1}{m_2}$

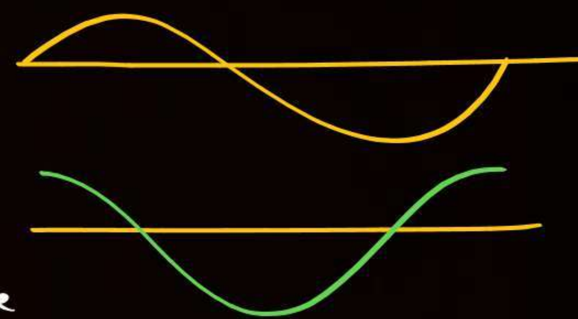
(D) $\dfrac{m_2}{m_1}$

$$\frac{v_1 = \sqrt{\dfrac{\gamma R T}{m_1}}}{v_2 = \sqrt{\dfrac{\gamma R T}{m_2}}}$$

# Pressure Wave

Displacement wave → $\Delta x = A \sin(\omega t - kx)$

Pressure wave → $\Delta P = P_0 \cos(\omega t - kx)$



Disp. amplitude

$$\boxed{P_0 = BAk}$$

Bulk modulus

Ang. wave no.

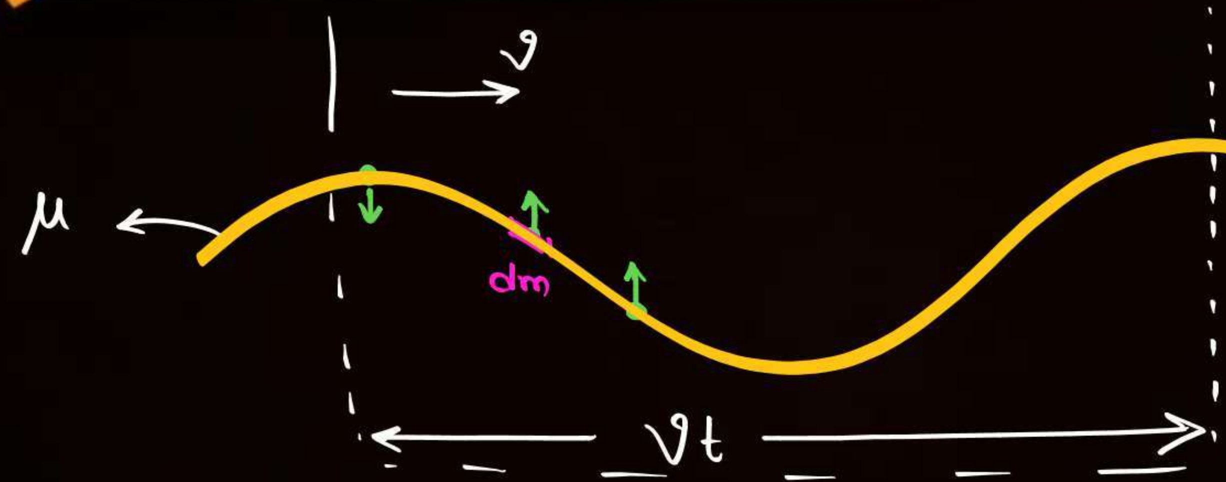
# Energy Transfer in String

Energy of small element $= \frac{1}{2} k A^2$

$$\int dE = \int \frac{1}{2}\, dm\, \omega^2 A^2$$

$$E = \frac{1}{2} m \omega^2 A^2$$

$$E = \frac{1}{2} (\mu v t)\, \omega^2 A^2$$




$$E = \frac{1}{2} \mu v \omega^2 A^2 t$$

$$m = \mu \times \lambda = \mu v t$$

$$\text{Power} = \frac{\text{Energy}}{\text{time}} = \frac{\frac{1}{2}\mu v \omega^2 A^2 \cancel{t}}{\cancel{t}} = \frac{1}{2}\mu v \omega^2 A^2$$

$$P \propto A^2$$

# Intensity

$$I = \frac{\text{Energy}}{\text{time} \times \text{Area}}$$

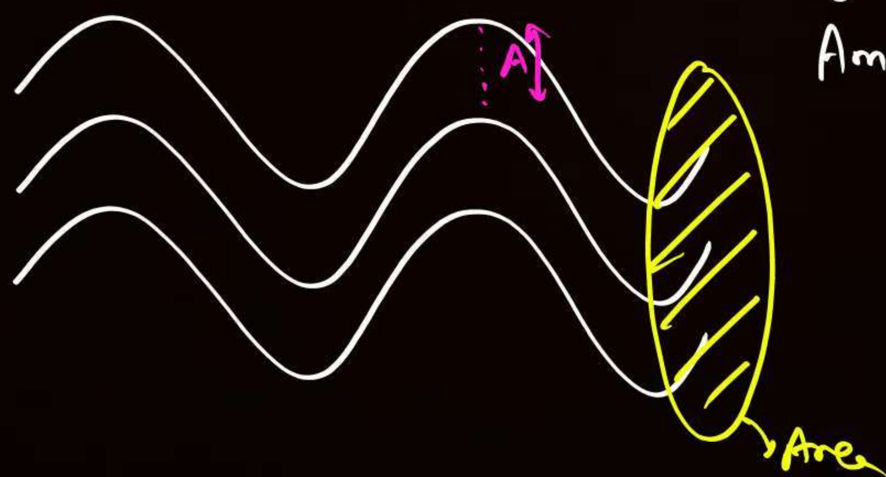
$$I = \frac{\text{Power}}{\text{Area}}$$

$$\boxed{I \propto A^2}$$

→ waves

→ wave optics

Amplitude of wave.



QUESTION

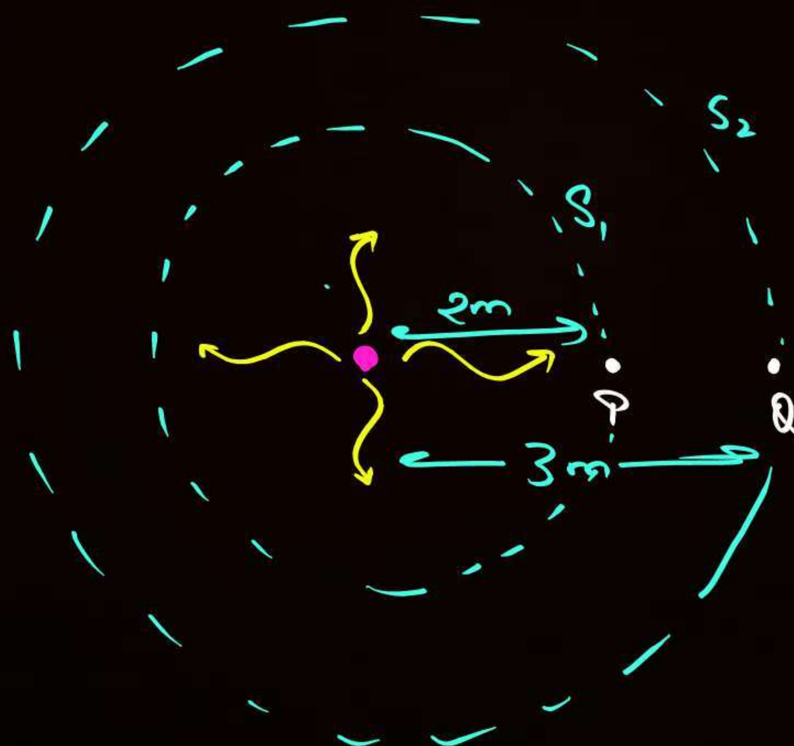
A point source emits sound equally in all directions in a non-absorbing medium. Two points P and Q are at distance of 2m and 3m respectively from the source. The ratio of the $\boxed{\text{intensities}}$ of the waves at P and Q is

(A) 9 : 4

(B) 2 : 3

(C) 3 : 2

(D) 4 : 9




$$E_{S_1} = E_{S_2}$$

$$\frac{I_{S_1} = \frac{\cancel{E_{S_1}}}{A_1}}{I_{S_2} = \frac{\cancel{E_{S_1}}}{A_2}} = \frac{A_2}{A_1}$$

$$= \frac{\cancel{4\pi} R_2^2}{\cancel{4\pi} R_1^2}$$

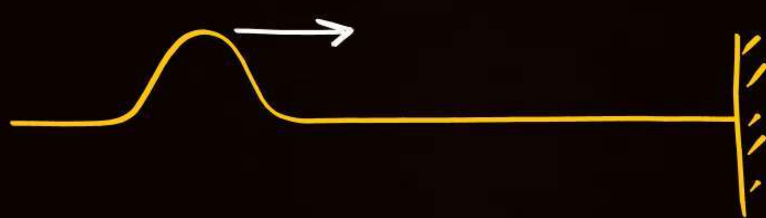
$$= \frac{3^2}{2^2} = \frac{9}{4}$$

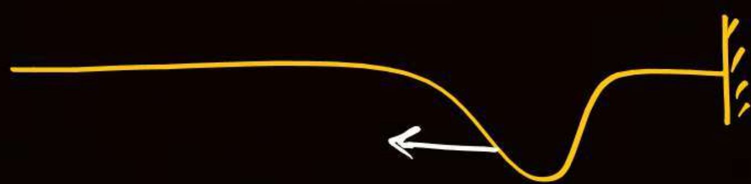
A.

# Wave Reflection and Refraction

Reflection →

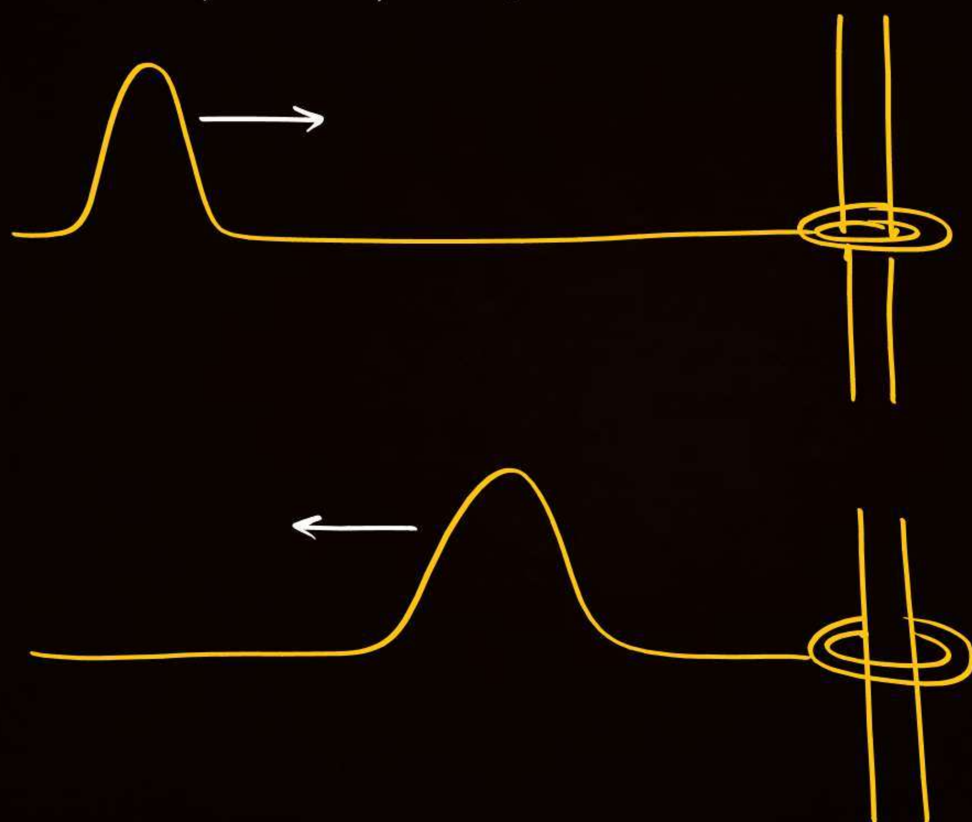
Case I :- Rigid end





Wave will get inverted after reflection from rigid end.

Case II :- Reflection from free end —



Wave does not get inverted upon reflection from free end.

Rarer $\longrightarrow$ Denser

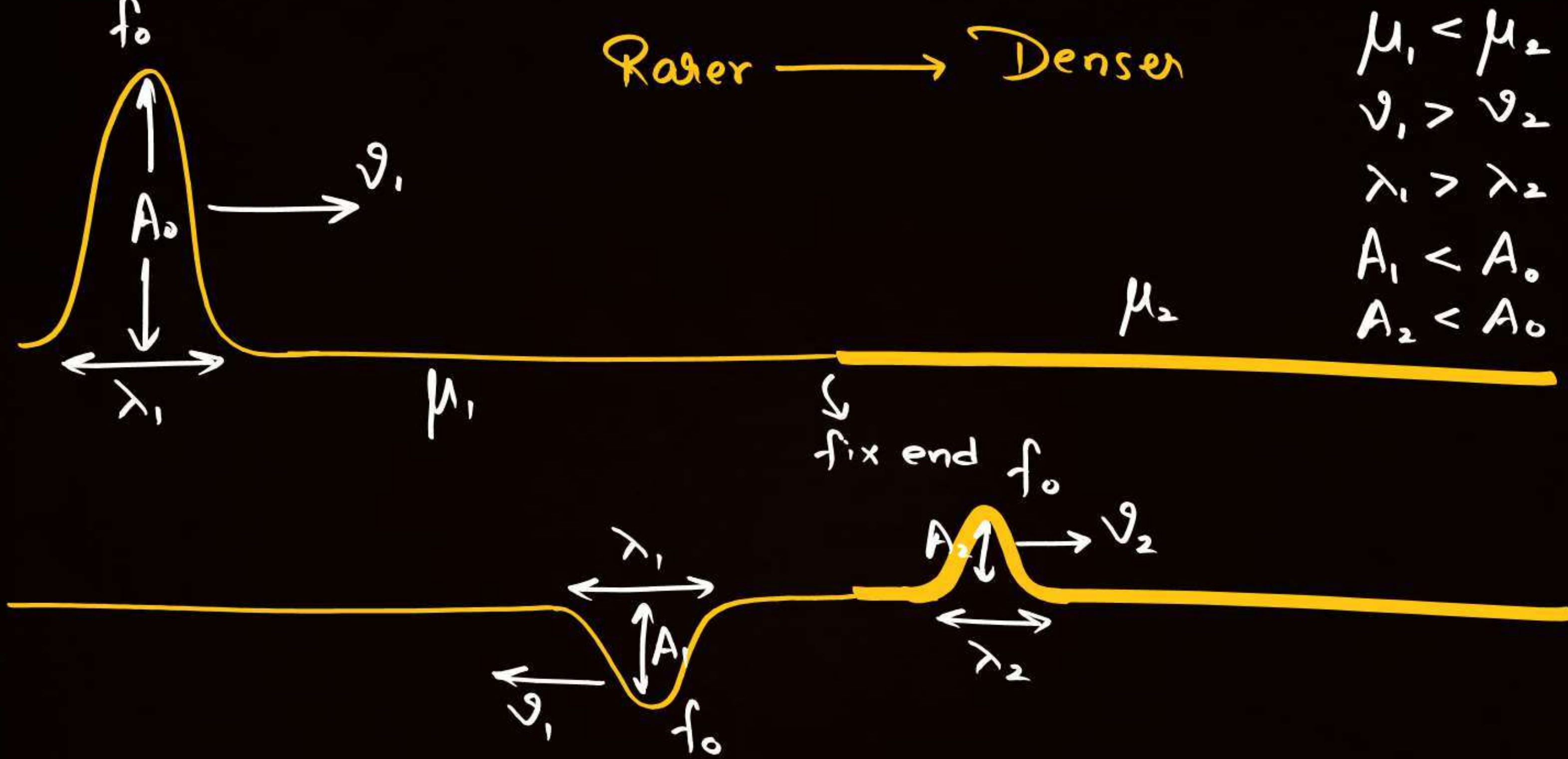
$\mu_1 < \mu_2$

$v_1 > v_2$

$\lambda_1 > \lambda_2$

$A_1 < A_0$

$A_2 < A_0$

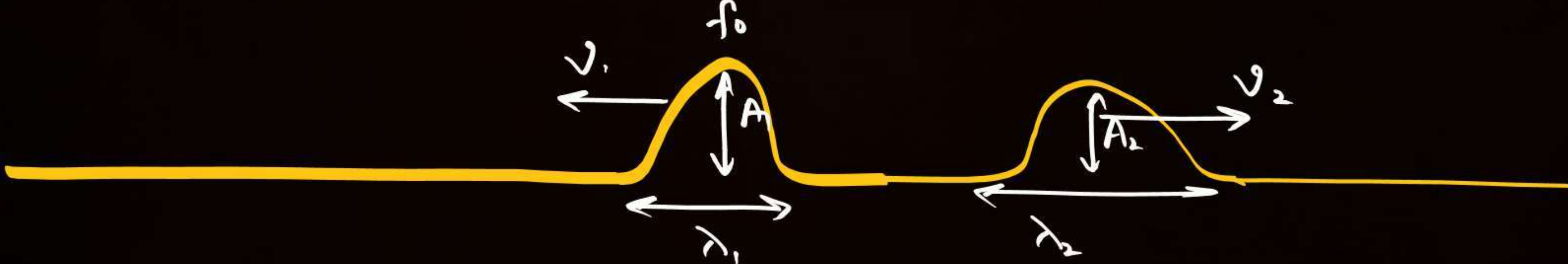


Denser $\longrightarrow$ Rarer

$\mu_1 > \mu_2$

$v_1 < v_2$

$\lambda_1 < \lambda_2$

QUESTION

Sound waves travel at 350 m/s through a warm air and at 3500 m/s through brass. The wavelength of a 700 Hz acoustic wave as it enters brass from warm air.

10x

A. increases by a factor 20

$$\underset{10\times}{\lambda} = \underset{10\times}{v} \times T$$

B. increases by a factor 10

C. decreases by a factor 20

D. decreases by a factor 10

# Wave Interference

(Sound)

① Waves of same frequency travelling in same direction

② Waves of same frequency and amplitude travelling in opposite direcn

(Standing Waves)

③ Waves of different frequencies travelling in same direcn

(Beats)

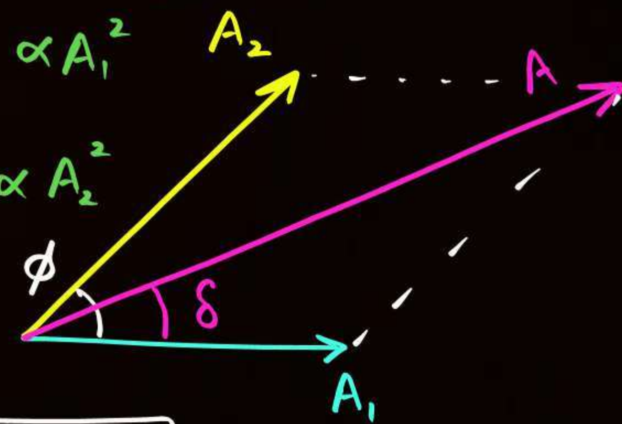
# Wave Interference

$\nu_1 = \nu_2$
$\omega_1 = \omega_2$
$\lambda_1 = \lambda_2 \Rightarrow k_1 = k_2$

**Case 1: Interference of waves of same frequency travelling in same direction:**

$y_1 = A_1 \sin(kx - \omega t)$ —— $I_1 \propto A_1^2$

$y_2 = A_2 \sin(kx - \omega t + \phi)$ —— $I_2 \propto A_2^2$




$y = y_1 + y_2 = A \sin(kx - \omega t + \delta)$

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$$

$$\boxed{\tan\delta = \frac{A_2 \sin\phi}{A_1 + A_2\cos\phi}}$$

$I \propto A^2$

$I \propto A_1^2 + A_2^2 + 2A_1A_2\cos\phi$

$$\boxed{I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi}$$

Special Cases

1. Constructive Interference ($\phi = 0$) $\rightarrow$

$y_1 = A_1 \sin(kx - \omega t)$ —— $I_1$

$y_2 = A_2 \sin(kx - \omega t)$ —— $I_2$

$y = y_1 + y_2 = (A_1 + A_2) \sin(kx - \omega t)$

$I \propto (A_1 + A_2)^2$

$I = (\sqrt{I_1} + \sqrt{I_2})^2$

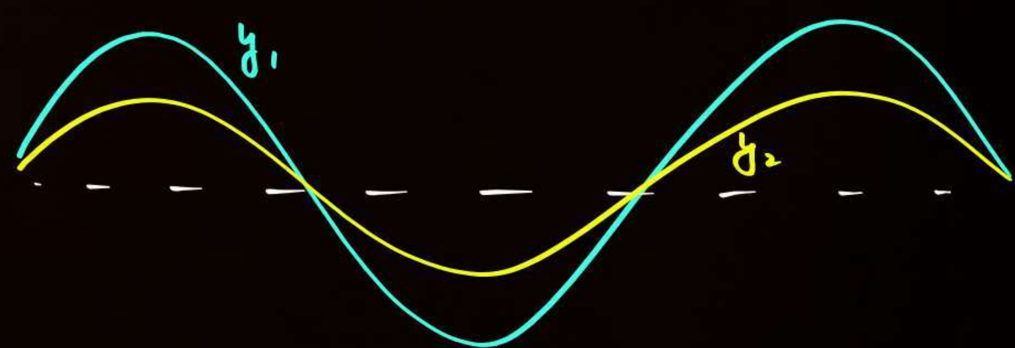
$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

$y_1 = A\sin(kx - \omega t)$ $\quad I$

$y_2 = A\sin(kx - \omega t)$ $\quad I$

$y = 2A\sin(kx - \omega t)$ $\quad 4I$

$\xrightarrow{A_2} \xrightarrow{A_1}$

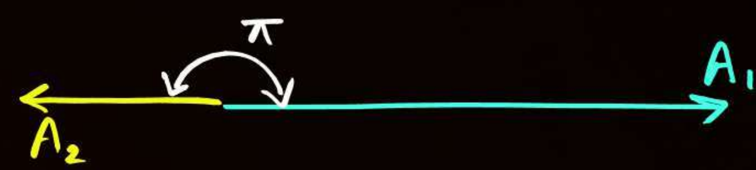


## (2) Destructive Interference ($\phi = \pi$) $\rightarrow$

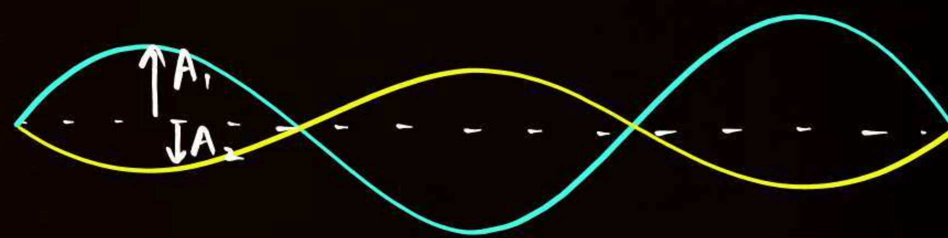
$$y_1 = A_1 \sin(kx - \omega t) \quad \text{———} \quad I_1$$

$$y_2 = A_2 \sin(kx - \omega t + \pi) \quad \text{———} \quad I_2$$

$$= -A_2 \sin(kx - \omega t)$$




$$y = y_1 + y_2 = (A_1 - A_2)\sin(kx - \omega t)$$




$$I \propto (A_1 - A_2)^2$$

$$I = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$$

$$I = I_1 + I_2 - 2\sqrt{I_1 I_2}$$

| | |
|---|---|
| $y_1 = A\sin(kx - \omega t)$ | $I$ |
| $y_2 = A\sin(kx - \omega t + \pi)$ | $I$ |
| $y = 0$ | $0$ |

QUESTION

If $A_1 : A_2 = 3 : 2$ then find the ratio of $I_{max}/I_{min}$. upon interference of two waves.

$$\frac{I_{max} \propto (A_1 + A_2)^2}{I_{min.} \propto (A_1 - A_2)^2} \Rightarrow \frac{I_{max}}{I_{min}} = \frac{(3A+2A)^2}{(3A-2A)^2} = \frac{25}{1}$$

Ans

QUESTION

$A_0$

There harmonic waves having equal frequency $\nu$ and same intensity $I_0$, have phase angles 0, $\frac{\pi}{4}$ and $-\frac{\pi}{4}$ respectively. When they are superimposed the intensity of the resultant wave is close to:

[9 Jan. 2020 I]

(A) $5.8\, I_0$

(B) $0.2\, I_0$

(C) $3\, I_0$

(D) $I_0$

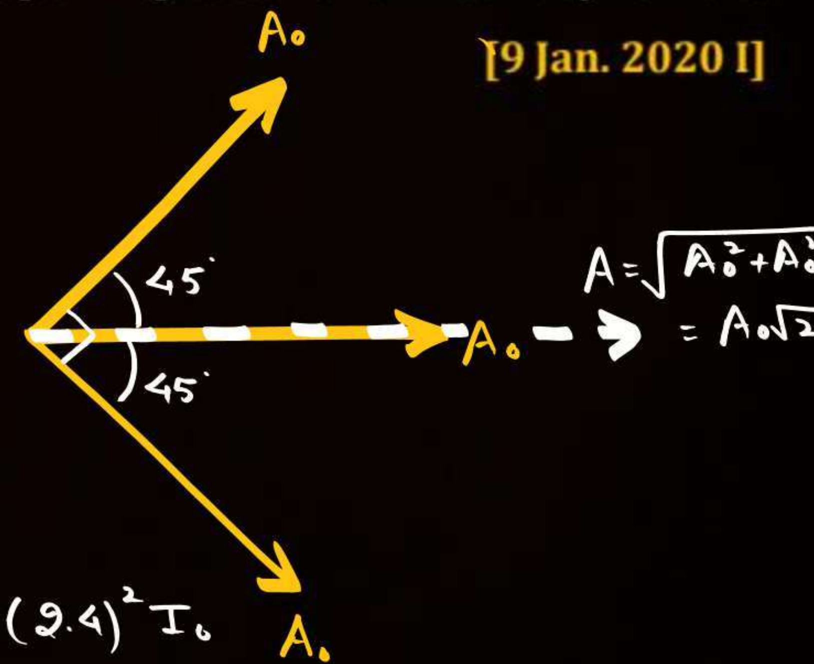
$A_{net} = A_0 + A_0\sqrt{2}$

$A_{net} = A_0(1+\sqrt{2})$

$I_{net} \propto A_{net}^2$

$I_{net} \propto [A_0(1+\sqrt{2})]^2$

$I_{net} = I_0(1+\sqrt{2})^2 \Rightarrow I = (2.4)^2 I_0$



## QUESTION

$y_1 = 5\sin(\underbrace{2\pi}_{k} x - 2\pi v t)$

$\lambda = \frac{2\pi}{2\pi} = 1$

The equations of two waves are given by:

$y_1 = 5\sin 2\pi(x - vt)$ cm

$y_2 = 3\sin 2\pi(x - vt + 1.5)$ cm. $= 3\sin(2\pi x - 2\pi v t + \underbrace{3\pi}_{\phi})$

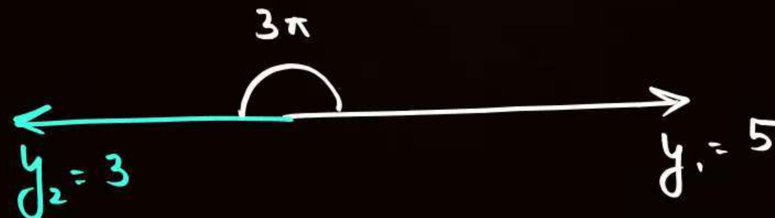
These waves are simultaneously passing through a string. The amplitude of the resulting wave is: [24 June, 2022 (S-I)]

(A) 2 cm

(B) 4 cm

(C) 5.8 cm

(D) 8 cm




$1 \longrightarrow 2\pi$

$1.5 \longrightarrow 3\pi$

## QUESTION

6 Ans

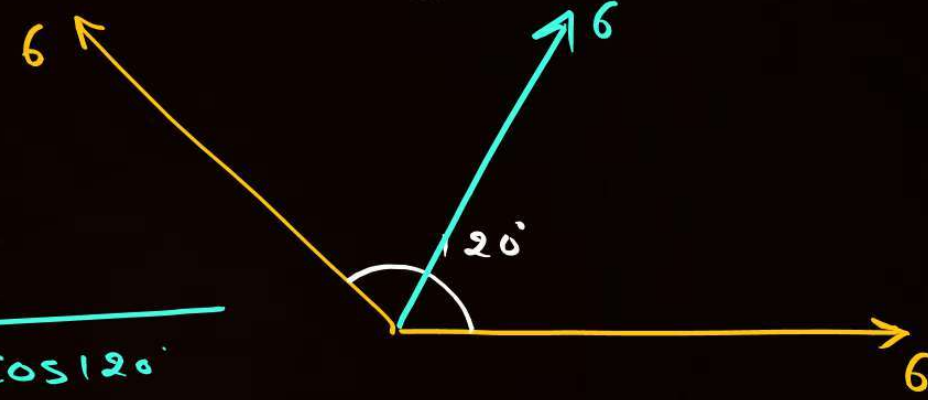
Amplitude of

Find the resultant wave upon interference of following two moves-

$y_1 = 6\sin(\pi x - \pi/2\, t)$

$y_2 = 6\sin(\pi x - \pi/2\, t + 2\pi/3)$

$120^\circ$




$$A_{net} = \sqrt{A^2 + A^2 + 2A^2\cos 120^\circ}$$

$$= \sqrt{2A^2 + 2A^2\left(-\tfrac{1}{2}\right)}$$

$$= A$$

## QUESTION

Two coherent sources produce waves of different intensities which interfere. After interference, the ratio of the maximum intensity to the minimum intensity is 16. The intensity of the waves are in the ratio

[9 Jan, 2019 (S-I)]

(A) 16 : 9

(B) 25 : 9

(C) 4 : 1

(D) 5 : 3

$$\frac{I_{max}}{I_{min}} = \frac{16}{1}$$

$$\frac{\left(\underset{A_1}{\sqrt{I_1}} + \underset{A_2}{\sqrt{I_2}}\right)^2}{\left(\sqrt{I_1} - \sqrt{I_2}\right)^2} = \frac{16}{1}$$

$$\Rightarrow \frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}} = 4$$

$$\Rightarrow \sqrt{I_1} + \sqrt{I_2} = 4\sqrt{I_1} - 4\sqrt{I_2}$$

$$\Rightarrow 3\sqrt{I_1} = 5\sqrt{I_2}$$

$$\Rightarrow \sqrt{\frac{I_1}{I_2}} = \frac{5}{3} \Rightarrow \frac{I_1}{I_2} = \frac{25}{9}$$

# Wave Interference

**Case 2: Interference of waves of same frequency and amplitude travelling in opposite direction:** (STANDING WAVE)

$$y_1 = A\sin(kx - \omega t)$$

$$y_2 = A\sin(kx + \omega t)$$

$$y = y_1 + y_2 = \underbrace{2A\sin(kx)}_{A_x}\cos(\omega t)$$ — Eq. of standing wave.

Posn. of nodes

$$\sin(kx) = 0$$

$$kx = n\pi$$

$$x = \frac{n\pi}{k}$$

Posn. of Anti-nodes

$$\sin(kx) = \pm 1$$

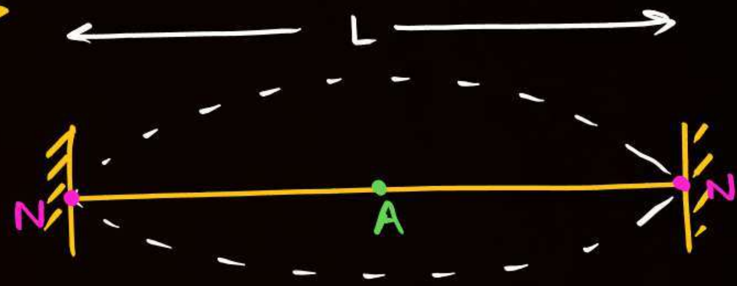
$$kx = (2n+1)\frac{\pi}{2}$$

$$x = (2n+1)\frac{\pi}{2k}.$$

$v = \sqrt{\frac{T}{\mu}}$

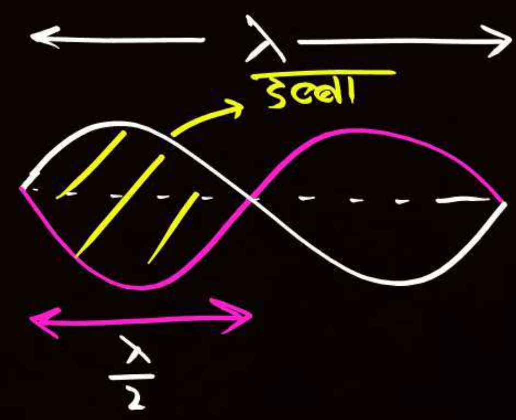
## Standing Wave in String fixed at both ends

(1) First Harmonic (fundamental Harmonic) →




$L = \frac{\lambda}{2} \Rightarrow \lambda = 2L$

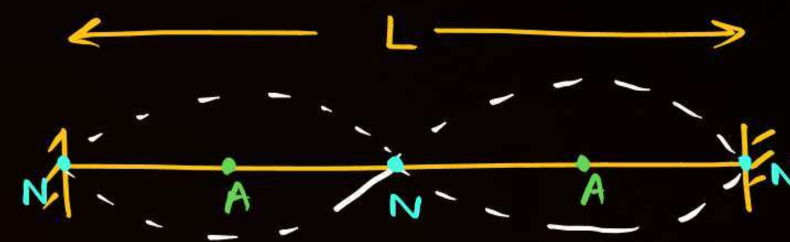
$$\boxed{f = \frac{v}{\lambda} = \frac{v}{2L}}$$ → fundamental frequency




Total no. of nodes = 2 Posn. of nodes = 0, L

Total no. of anti-nodes = 1 Posn. of anti-nodes = $\frac{L}{2}$

(2) $2^{nd}$ Harmonic ($1^{st}$ overtone) $\rightarrow$




$$L = 2\frac{\lambda}{2} \Rightarrow \lambda = L$$

$$f = \frac{v}{\lambda} = \frac{2v}{2L}$$

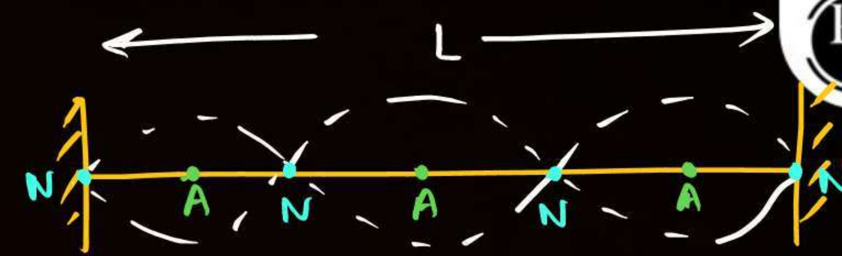
Total no. of Nodes = 3

Posn. of nodes = $0, \frac{L}{2}, L$

Total no. of Anti-nodes = 2

Posn. of Anti-nodes = $\frac{L}{4}, \frac{3L}{4}$

③ $3^{rd}$ Harmonic ($2^{nd}$ overtone) $\rightarrow$




$$L = \frac{3\lambda}{2} \Rightarrow \lambda = \frac{2L}{3}$$

$$f = \frac{v}{\lambda} \Rightarrow f = \frac{3v}{2L}$$

Total no. of nodes = 4

Posn. of nodes = $0, \frac{L}{3}, \frac{2L}{3}, \frac{3L}{3}$

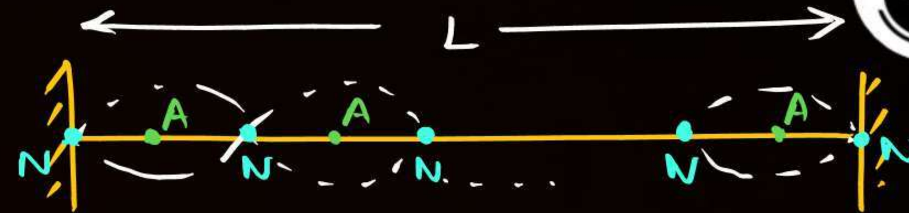
Total no. of anti-nodes = 3

Posn. of anti-nodes = $\frac{L}{6}, \frac{3L}{6}, \frac{5L}{6}$

(n) $n^{th}$ Harmonic ($(n-1)^{th}$ overtone) $\longrightarrow$

$$L = \frac{n\lambda}{2} \Rightarrow \lambda = \frac{2L}{n}$$

$$f = \frac{v}{\lambda} \Rightarrow \boxed{f = \frac{nv}{2L}}$$




Total no. of nodes = $n+1$

Total no. of anti-nodes = $n$

Posn. of nodes = $0, \frac{L}{n}, \frac{2L}{n}, \frac{3L}{n}, \ldots, \frac{nL}{n}$

Posn. of anti-nodes = $\frac{L}{2n}, \frac{3L}{2n}, \frac{5L}{2n} \ldots \frac{(2n-1)L}{2n}$

$v = \sqrt{\frac{\gamma P}{\rho}}$

## Standing wave in open organ pipe -

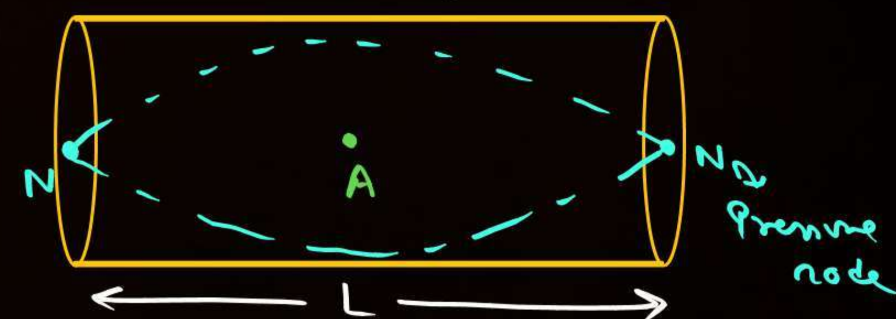
① First Harmonic ( Fundamental Harmonic) →

$L = \frac{\lambda}{2} \Rightarrow \lambda = 2L$

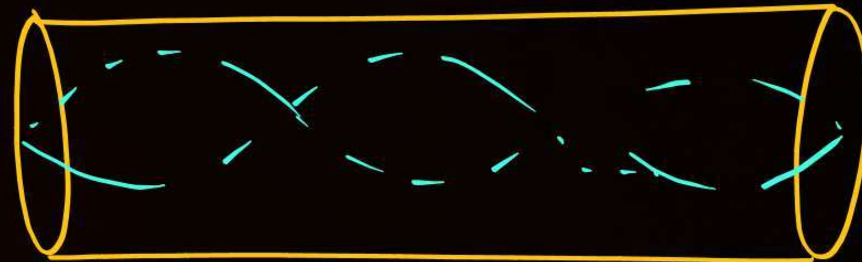
$$f = \frac{v}{\lambda} = \frac{v}{2L}$$

Disp. wave


Pressure wave



(n) $n^{th}$ Harmonic $[(n-1)^{th}$ overtone$]$ $\longrightarrow$



$$L = \frac{n\lambda}{2} \Rightarrow \lambda = \frac{2L}{n}$$

$$f_n = \frac{v}{\lambda} = \frac{nv}{2L}$$

$\vartheta = \sqrt{T/\mu}$

## Standing wave in string free at one end

(only odd Harmonics are obtained)

(1) first Harmonic (fundamental Harmonic) →


$$L = \frac{\lambda}{4} \Rightarrow \lambda = 4L$$

$$\boxed{f = \frac{\vartheta}{\lambda} = \frac{\vartheta}{4L}}$$

Total no. of nodes = 1

Posn. of node = 0

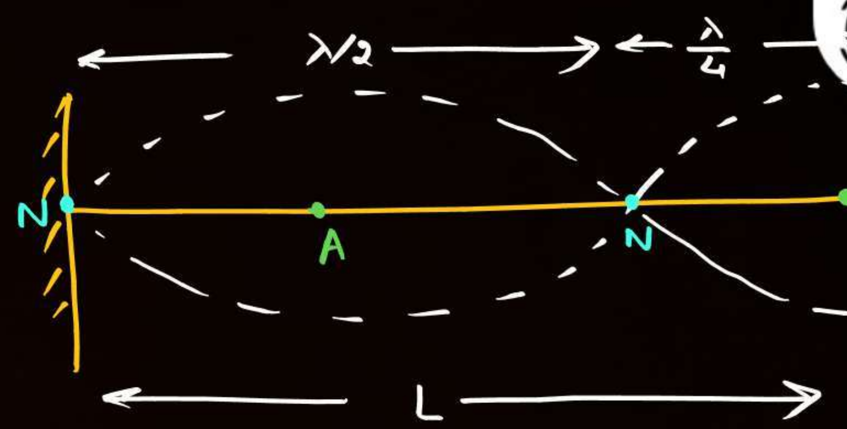
Total no. of anti-nodes = 1

Posn. of anti-nodes = L

② $\underline{\underline{3^{rd}}}$ Harmonic ( $1^{st}$ overtone) $\longrightarrow$

$$L = \frac{3\lambda}{4} \Rightarrow \lambda = \frac{4L}{3}$$

$$f = \frac{v}{\lambda} = \frac{3v}{4L}$$




Total no. of nodes = 2

Posn. of nodes = $0, \frac{2L}{3}$

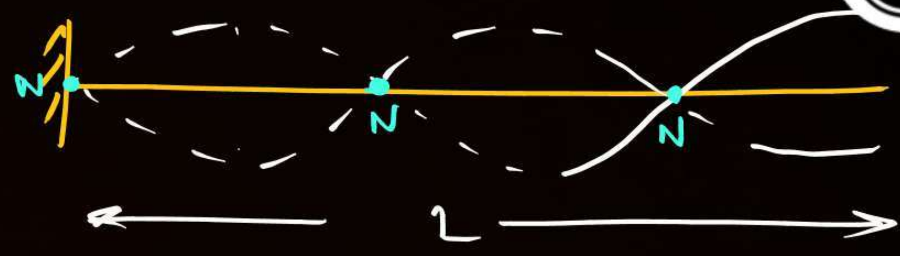
Total no. of anti-nodes = 2

Posn. of anti-nodes = $\frac{L}{3}, \frac{3L}{3}$

(3) 5th Harmonic (2nd overtone) $\longrightarrow$

$$L = \frac{5\lambda}{4} \Rightarrow \lambda = \frac{4L}{5}$$

$$f = \frac{v}{\lambda} = \frac{5v}{4L}$$




Total no. of nodes = 3

Posn. of nodes = 0, $\frac{2L}{5}$, $\frac{4L}{5}$

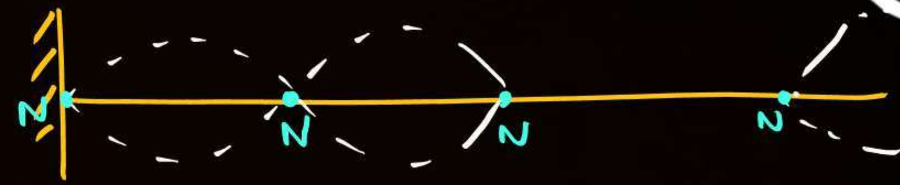
Total no. of anti-nodes = 3

Posn. of anti-nodes = $\frac{L}{5}$, $\frac{3L}{5}$, $\frac{5L}{5}$

$(2n+1)^{th}$ Harmonic ($n^{th}$ overtone) $\longrightarrow$

$n = 0, 1, 2 \ldots$

$L = (2n+1)\frac{\lambda}{4} \Rightarrow \lambda = \frac{4L}{2n+1}$




$$f = \frac{v}{\lambda} = \frac{(2n+1)v}{4L}$$

Total no. of nodes $= n+1$

Posn. of nodes $= 0, \frac{2L}{2n+1}, \frac{4L}{2n+1}, \ldots, \frac{2nL}{2n+1}$

Total no. of anti-nodes $: n+1$

Posn. of anti-nodes $= \frac{L}{2n+1}, \frac{3L}{2n+1}, \frac{5L}{2n+1}, \ldots, L$

$v = \sqrt{\frac{\gamma P}{\rho}}$

Standing wave in closed organ pipe →

(1) First Harmonic (fundamental Harmonic)


$$L = \frac{\lambda}{4} \Rightarrow \lambda = 4L$$

$$f = \frac{v}{\lambda} = \frac{v}{4L}$$

$(2n+1)^{th}$ Harmonic ($n^{th}$ overtone) ——

$$f = \frac{(2n+1)\,v}{4L}$$

QUESTION

Two travelling waves of equal amplitudes and frequencies move in opposite directions along a string. They interfere to produce a stationary wave whose equation is given by $y = \left(\underbrace{10\cos\pi x}_{A} \sin\frac{2\pi t}{T}\right)$. The amplitude of the particle at $x = 4/3$ cm will be ____ cm.

[24 June, 2022 (S-II)]

$$A_x = 10\cos(\pi x)$$

$$A_{x=\frac{4}{3}} = 10\cos\left(\pi \times \frac{4}{3}\right)$$

$$= 10\cos(240^\circ)$$

$$= |10\cos 60^\circ|$$

$$= |5|$$

## QUESTION

4 डब्बे

A string is clamped at both the ends and it is vibrating in its 4th harmonic. The equation of the stationary wave is $Y = 0.3 \sin(\underbrace{0.157}_{k}x) \cos(200t)$. The length of the string is: (All quantities are in SI units.) **[9 April, 2019 (S-I)]**

(A) 20 m

(B) 80 m

(C) 60 m

(D) 40 m

$$Y = 0.3 \sin(kx) \cos(\omega t)$$

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.157} = \frac{2 \times \cancel{3.14}^{\,2}}{\cancel{1.57} \times 10^{-1}}$$

$$\lambda = 40\ \text{m}$$

$$L = 4 \times \frac{\lambda}{2} = 2\lambda = 80\ \text{m}$$

QUESTION

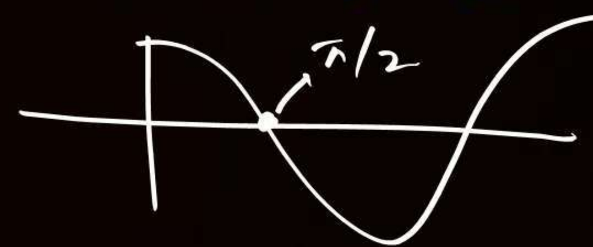
Two travelling waves produces a standing wave represented by equation, $y = 1.0$ mm $\cos(1.57\ \text{cm}^{-1})\ x \sin(78.5\ \text{s}^{-1})t$. The node closest to the origin in the region $x > 0$ will be at $x =$ ______ cm.

[26 Aug, 2021 (S-I)]

$$y = 1 \cos\left(\underbrace{\frac{\pi}{2} \times x}\right) \sin(\omega t)$$

$$\cancel{\frac{\pi}{2}} x = \cancel{\frac{\pi}{2}}$$

$$x = 1\text{cm}$$



QUESTION

The two nearest harmonics of a tube closed at one end and open at other end are 220 Hz and 260 Hz. What is the fundamental frequency of the system?

(A) 10 Hz

(B) 20 Hz

(C) 30 Hz

(D) 40 Hz

$$\left.\begin{matrix} 220\,Hz \\ 260\,Hz. \end{matrix}\right] 40\,Hz = \frac{v}{2L}$$

$$\frac{v}{4L},\ \frac{3v}{4L},\ \frac{5v}{4L},\ \frac{7v}{4L}$$

$$\frac{v}{2L} \quad \frac{v}{2L} \quad \frac{v}{2L}$$

$$f = \frac{v}{4L} = \frac{40}{2} = 20\,Hz$$

Ans

QUESTION

ONE OR MORE THAN ONE OPTION(S) MAY BE CORRRECT

If we study the vibration of a pipe open at both ends, which of the following statements is/are not true?

(A) Open end will be pressure antinode - false

(B) Odd harmonics of the fundamental frequency will be generated - false

(C) All harmonics of the fundamental frequency will be generated - True

(D) Pressure change will be maximum at both ends - false

QUESTION

The fundamental frequency in an open organ pipe is equal to the third harmonic of a closed organ pipe. If the length of the closed organ pipe is 20 cm, the length of the open organ pipe is

(A) 12.5 cm

(B) 8 cm

(C) 13.3 cm

(D) 16 cm

$$\frac{\cancel{v}}{2L_1} = \frac{3\cancel{v}}{4L_2}$$

$$\frac{1}{2 \times L_1} = \frac{3}{4 \times 20}$$

$$L_1 = \frac{\cancel{8}0^{4}0}{\cancel{2} \times 3}$$

$$L_1 = 13.3\ \text{cm}$$

## QUESTION

A wave of frequency $\boxed{100\ \text{Hz}}$ is sent along a string towards a fixed end. When this wave travels back after reflection, a node is formed at a distance of 10 cm from the fixed end of the string. The speeds of incident (and reflected) wave are

(A) 5 m/s

(B) 10 m/s

(C) 20 m/s ✓

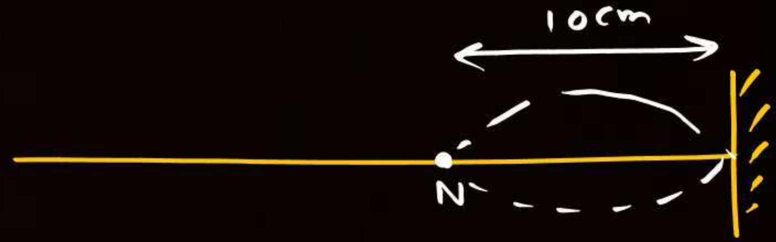
(D) 40 m/s

$$\frac{\lambda}{2} = 10\ \text{cm} \Rightarrow \lambda = 20\ \text{cm}$$

$$v = \lambda \times f$$

$$= 20 \times \cancel{10^{-2}} \times 1\cancel{00}$$

$$= 20\ \text{m/s}$$



**QUESTION**

Only odd Harmonics

A closed organ pipe (closed at one end) is excited to support the third overtone. It is found that air in the pipe has

$(2n+1)^{th}$ Harmonic

$2 \times 3 + 1 = 7^{th}$ Harmonic

(A) three noes and three antinodes

Common mistake

(B) three nodes and four antinodes

(C) four nodes and three antinodes

(D) four nodes and four antinodes

$1^{st}$ Harmonic

$3^{rd}$ Harmonic – $1^{st}$ O.T

$5^{th}$ – $2^{nd}$ O.T

$7^{th}$ Harmonic – $3^{rd}$ O.T

QUESTION

L $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $v$

An organ pipe 40 cm long is open at both ends. The speed of sound in air is 360 $ms^{-1}$. The frequency of the second harmonic is ____ Hz.

**[08 April, 2023 (S-II)]**

$$f_2 = \frac{\cancel{2} v}{\cancel{2} L} = \frac{36\cancel{0}}{4\cancel{0} \times 10^{-2}}$$

$$= 900 \text{ Hz.}$$

QUESTION

$\mu = \frac{m}{L} = \frac{2\times10^{-5}}{1} = 2\times10^{-5}$

A string of length $\boxed{1\ \text{m}}$ and mass $\boxed{2 \times 10^{-5}\ \text{kg}}$ is under tension T. When the string vibrates, two successive harmonics are found to occur at frequencies 750 Hz and 1000 Hz. The value of tension T is 5 Newton. [JEE Adv, 2023]

$\frac{v}{2L}$ $\left[\ \frac{v}{2L} \longrightarrow 1^{st}\right.$, $\frac{2v}{2L} \longrightarrow 2^{nd}$

$\frac{v}{2}$ $\left[\ \frac{2v}{2L}\right.$, $\frac{3v}{2L} \longrightarrow 3^{rd}$

$\left.\begin{matrix}750\text{Hz}\\1000\text{ Hz}\end{matrix}\right] 250 = \frac{v}{2L} \Rightarrow v = 500 \times 1$

$v = 500\ \frac{m}{s}$

$\sqrt{\frac{T}{\mu}} = 500$

$\frac{T}{2\times10^{-5}} = (500)^2 \Rightarrow T = 2\times10^{-5} \times 25\times10^{4}$

$= 5\cancel{0} \times \cancel{10^{-1}} = 5\text{ N}$ Ans

QUESTION

A guitar string of length 90 cm vibrates with a fundamental frequency of 120 Hz. The length of the string producing a fundamental frequency of 180 Hz will be _____ cm.

[08 April, 2023 (S-II)]

$$f = \frac{v}{2L} \Rightarrow v = f \times 2L$$

$$= 12\cancel{0} \times 2 \times 9\cancel{0} \times \cancel{10^{-2}}$$

$$v = 24 \times 9 \ \frac{m}{s}$$

$$180 = \frac{24 \times 9}{2L}$$

$$L = \frac{24 \times 9}{180 \times 2} \ m$$

$$= \frac{24 \times \cancel{9}}{\cancel{180} \times 2} \times \cancel{100}^{5} \ cm = 60$$

QUESTION

For a certain organ pipe, the first three resonance frequencies are in the ratio of 1 : 3 : 5 respectively. If the frequency of fifth harmonic is 405 Hz and the speed of sound in air is 324 ms$^{-1}$ the length of the organ pipe is ____ m. [12 April, 2023 (S-I)]

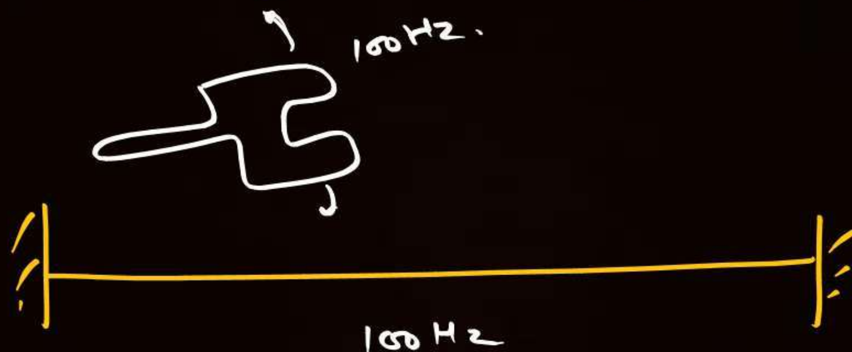
$$f_5 = \frac{5v}{4L}$$

$$405 = \frac{5 \times 324}{4 \times L}$$

$$L = \frac{\cancel{5} \times 3\cancel{2}4}{\cancel{4} \times \cancel{405}} = 1$$

# Resonance

$f_{ext} = f_{standing\ wave}.$



QUESTION

L

A string is stretched between fixed points separated by 75.0 cm. It is observed to have resonant frequencies of 420 Hz and 315 Hz. There are no other resonant frequencies between these two. The lowest resonant frequency for this strings is:

$$f = f_n - f_{n-1}$$

$$= 420 - 315$$

$$= 105\ Hz.$$

(A) 155 Hz

(B) 205 Hz

(C) 10.5 Hz

(D) 105 Hz

**QUESTION**

$\mu$

A wire of length $L$ and mass per unit length $\boxed{6.0 \times 10^{-3}\ \text{kgm}^{-1}}$ is put under tension of 540 N. $T$ Two consecutive frequencies that it resonates at are: 420 Hz and 490 Hz. Then $L$ in meters is:

[9 Jan. 2020 (II)]

(A) 2.1 m

(B) 1.1 m

(C) 8.1 m

(D) 5.1 m

$$f = 490 - 420 = 70\,\text{Hz}$$

$$\frac{v}{2L} = 70$$

$$\frac{1}{2L}\sqrt{\frac{T}{\mu}} = 70$$

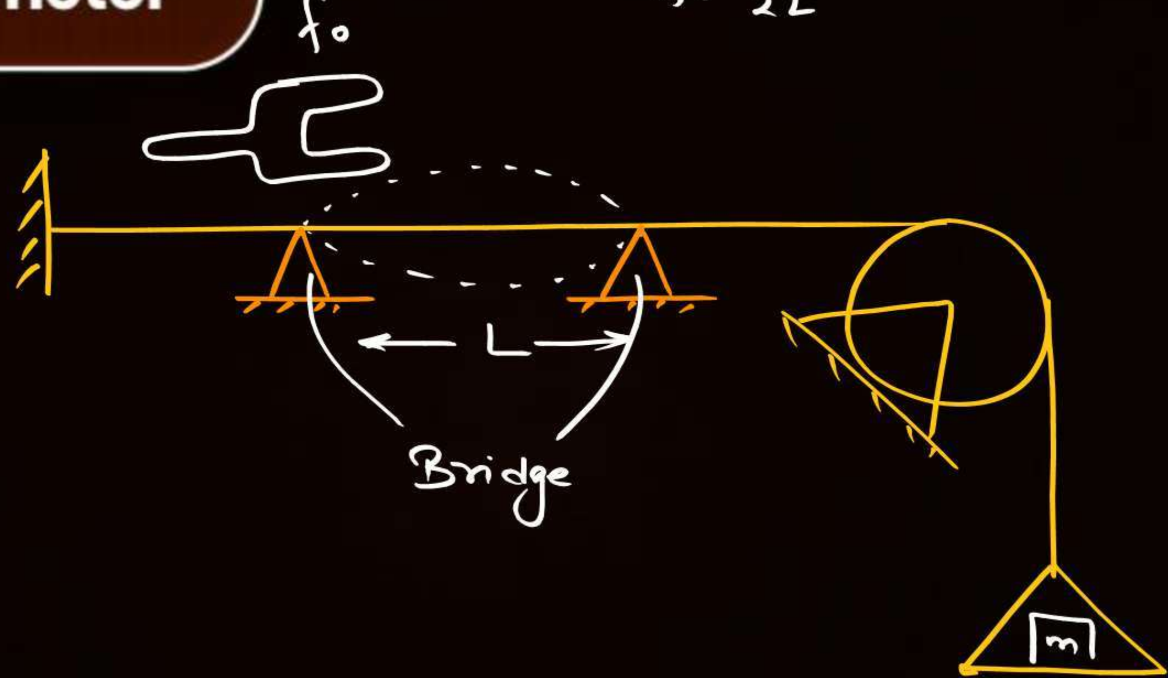
$$L = \frac{1}{140} \times \sqrt{\frac{5\cancel{4}0\ 90}{\cancel{6} \times 10^{-3}}}$$

$$\frac{\sqrt{9 \times 10^4}}{140}$$

$$L = \frac{3\cancel{0}0}{14\cancel{0}} = 2.1\,\text{m}$$

# Sonometer

$$f_0 = \frac{v}{2L}$$



QUESTION

The length of the string of a musical instrument is 90 cm and has a fundamental frequency of 120 Hz. Where should it pressed to produce fundamental frequency of 180 Hz?

A) 75 cm

B) 60 cm

C) 45 cm

D) 80 cm

$$f = \frac{v}{2L}$$

$$v = 2fL$$

$$f_1 L_1 = f_2 L_2$$

$$\cancel{120}^{2} \times \cancel{90}^{30} = \cancel{180}^{3} \times L_2$$

$$\boxed{L = 60\ cm}$$

QUESTION

If $n_1$, $n_2$ and $n_3$ are the fundamental frequencies of three segments into which a string is divided, then the original fundamental frequency $n$ of the string is given by

(A) $\frac{1}{n} = \frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3}$

(B) $\frac{1}{\sqrt{n}} = \frac{1}{\sqrt{n_1}} + \frac{1}{\sqrt{n_2}} + \frac{1}{\sqrt{n_3}}$

(C) $\sqrt{n} = \sqrt{n_1} + \sqrt{n_2} + \sqrt{n_3}$

(D) $n = n_1 + n_2 + n_3$

$n_1 = \frac{v}{2L_1} \Rightarrow L_1 = \frac{v}{2n_1}$, $n_2 = \frac{v}{2L_2}$ $\quad n_3 = \frac{v}{2L_3}$


$$L = L_1 + L_2 + L_3$$

$$\frac{\cancel{v}}{\cancel{2}n} = \frac{\cancel{v}}{\cancel{2}n_1} + \frac{\cancel{v}}{\cancel{2}n_2} + \frac{\cancel{v}}{\cancel{2}n_3}$$

$$\frac{1}{n} = \frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3}$$

QUESTION

A stretched string resonates with tuning fork of frequency $\boxed{512\text{ Hz}}$ when length of the string is 0.5 m. The length of the string require to vibrate resonantly with a tuning fork of frequency $\boxed{256\text{ Hz}}$ would be

(A) 0.25 m

(B) 0.5 m

(C) 1 m

(D) 2 m

$$f = \frac{v}{2L} \Rightarrow v = 2fL$$

$$\cancel{512}^{\,2} \times 0.5 = \cancel{256} \times L$$

$$L = 1\text{ m}$$

**QUESTION**

In an experiment with sonometer when a mass of 180 g is attached to the string, it vibrates with fundamental frequency of 30 Hz. When a mass m is attached, the string vibrates with fundamental frequency of 50 Hz. The value of m is 500 g.

**[13 April, 2023 (S-II)]**


$$f = \frac{v}{2L} = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

$$f = \frac{1}{2L}\sqrt{\frac{mg}{\mu}}$$

$$\frac{f_1}{f_2} = \sqrt{\frac{m_1}{m_2}} \Rightarrow \frac{3\cancel{0}}{5\cancel{0}} = \sqrt{\frac{180}{m}}$$

$$\frac{9}{25} = \frac{180}{m}$$

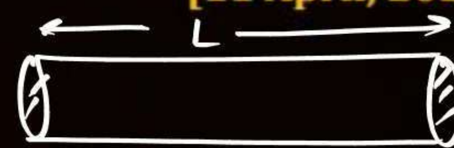
$$m = \frac{\cancel{180}^{20} \times 25}{\cancel{9}}$$

## QUESTION

$\rho$ $\quad$ $L$

A wire of density $8 \times 10^3$ kg/m$^3$ is stretched between two clamps $\boxed{0.5\text{m}}$ apart. The extension developed in the wire is $\boxed{3.2 \times 10^{-4}}$ $\Delta l$. If $Y = 8 \times 10^{10}$ N/m$^2$, the fundamental frequency of vibration in the wire will be ______ Hz.

**[11 April, 2023 (S-II)]**




$$\frac{F}{A} = Y\frac{\Delta l}{l}$$

$$F = \frac{AY\Delta l}{l}$$

$$v = \sqrt{\frac{T}{\mu}}$$

$$v = \sqrt{\frac{F}{\rho A}}$$

$$v = \sqrt{\frac{\cancel{A}Y\Delta l}{l \times \rho \cancel{A}}} = \sqrt{\frac{Y\Delta l}{\rho l}}$$

$$m = \mu \cancel{L} = \rho \times A \cancel{L}$$

$$\mu = \rho A$$

$$f = \frac{v}{2L}$$

$$f = \frac{v}{2L}$$

$$= \frac{1}{\cancel{2} \times \cancel{0}.5} \sqrt{\frac{\cancel{8} \times 10^{10} \times 3.2 \times 10^{-4}}{\cancel{8} \times 10^{3} \times 0.5}}$$

$$\sqrt{6.4 \times 10^{3}}$$

$$\sqrt{6400} = 80\,Hz$$

## QUESTION

$\frac{\rho_1}{\rho_2} = \sqrt{\frac{1}{4}}$

$m = \mu \cancel{l} = \rho A \cdot \cancel{l}$

$\mu = \rho A$

Two wires $W_1$ and $W_2$ have the same radius r and respective densities $\rho_1$ and $\rho_2$ such that $\boxed{\rho_2 = 4\rho_1}$. They are joined together at the point O, as shown in the figure. The combination is used as a sonometer wire and kept under tension T. The point O is midway between the two bridges. When a stationary waves is set up in the composite wire, the joint is found to be a node. The ratio of the number of antinodes formed in $W_1$ to $W_2$ is :

[Online April 8, 2017]

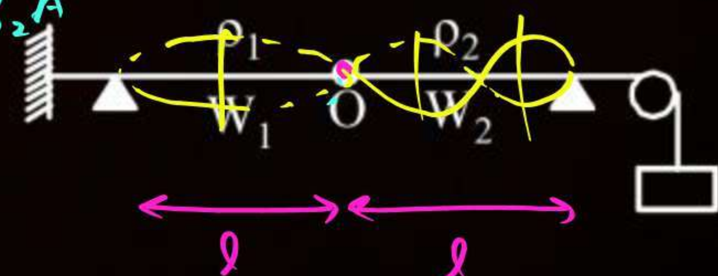
$f_1 = f_2$

$\frac{\mu_1}{\mu_2} = \frac{\rho_1 \cancel{A}}{\rho_2 \cancel{A}}$

$\frac{n_1 v_1}{\cancel{2L}} = \frac{n_2 v_2}{\cancel{2L}}$

$n_1\sqrt{\frac{\cancel{T}}{\mu_1}} = n_2\sqrt{\frac{\cancel{T}}{\mu_2}}$

$\frac{n_1}{n_2} = \sqrt{\frac{\mu_1}{\mu_2}} = \sqrt{\frac{\rho_1}{\rho_2}} = \boxed{\frac{1}{2}}$




(A) 1 : 1

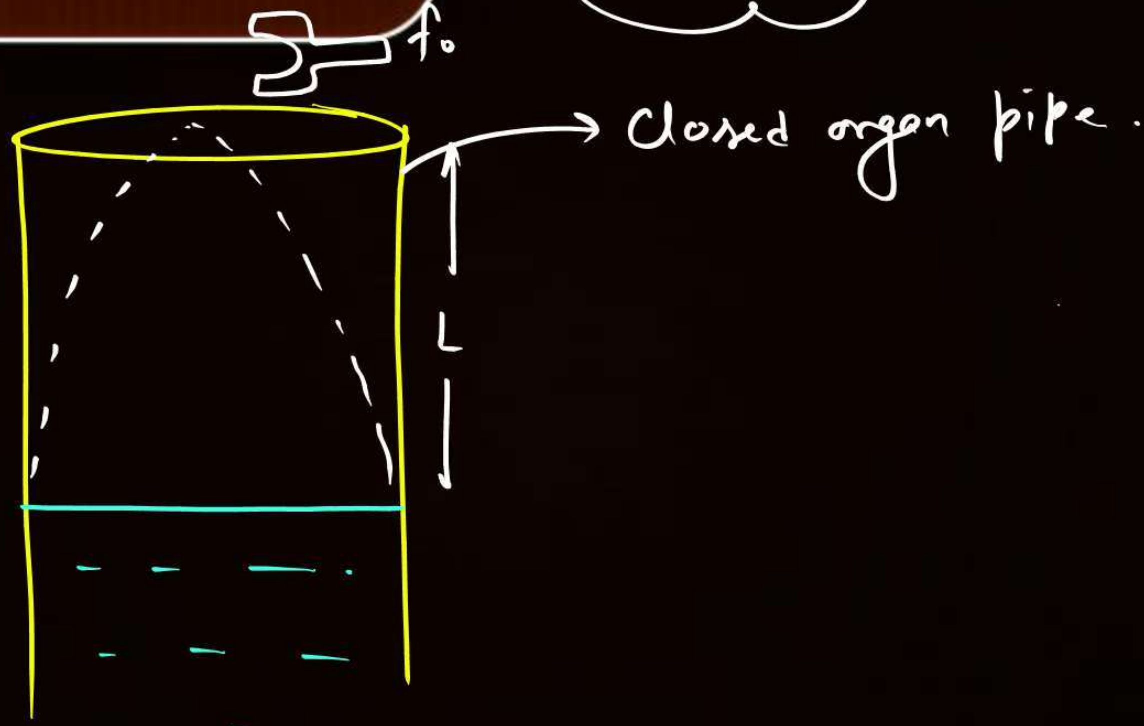
(B) 1 : 2

(C) 1 : 3

(D) 4 : 1

# Resonance Tube

$$f_0 = \frac{(2n+1)v}{4L}$$



QUESTION

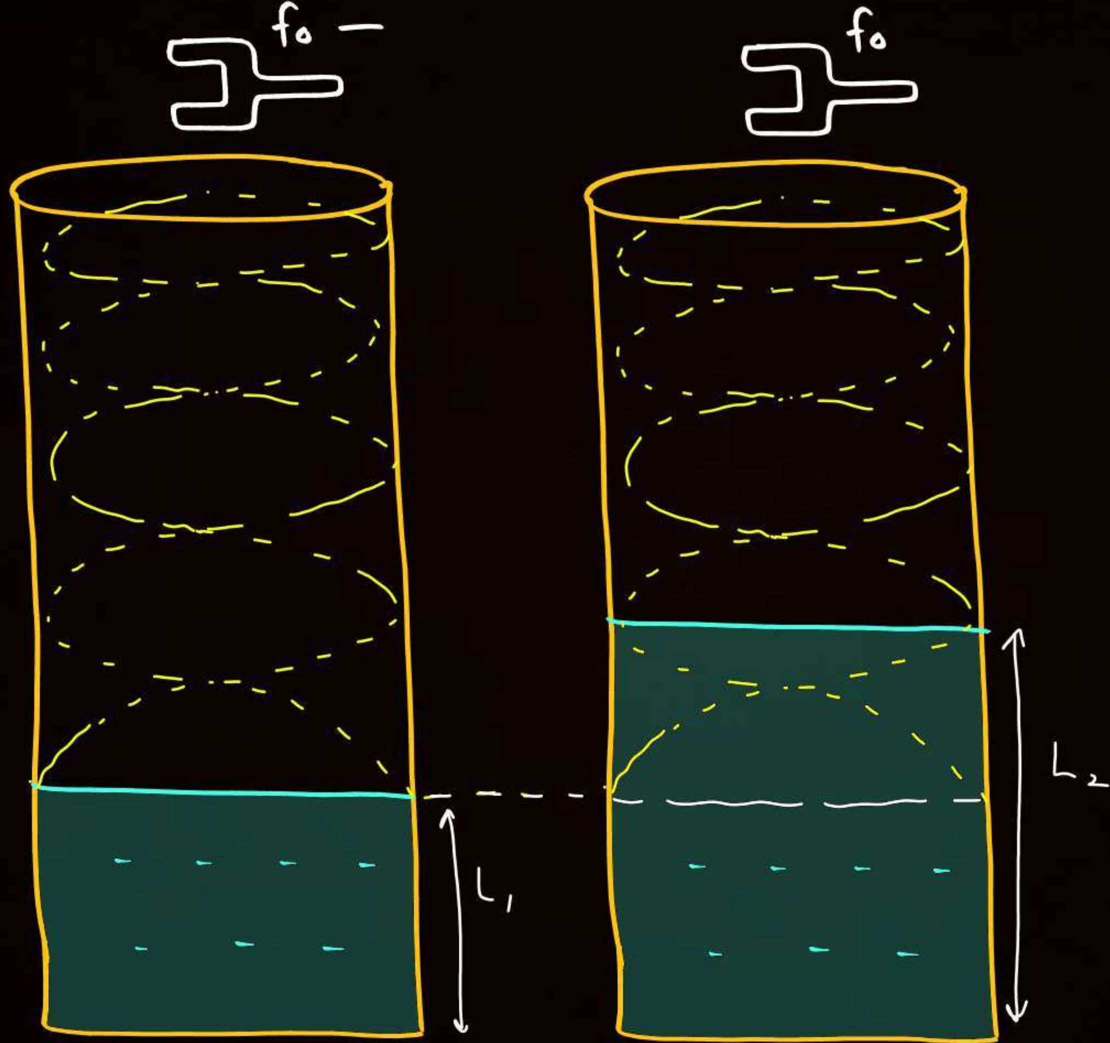
In a resonance tube experiment when the tube is filled with water up to a height of 17.0 cm from bottom, it resonates with a given tuning fork. When the water level is raised the next resonance with the same tuning fork occurs at a height of 24.5 cm. If the velocity of sound in air is 330 m/s, the tuning fork frequency is: **[Sep. 05, 2020 (I)]**

(A) 2200 Hz

(B) 550 Hz

(C) 1100 Hz

(D) 3300 Hz




$$|L_2 - L_1| = \frac{\lambda}{2}$$

$$24.5 - 17 = \frac{\lambda}{2}$$

$$\lambda = 7.5 \times 2 = 15\,cm$$

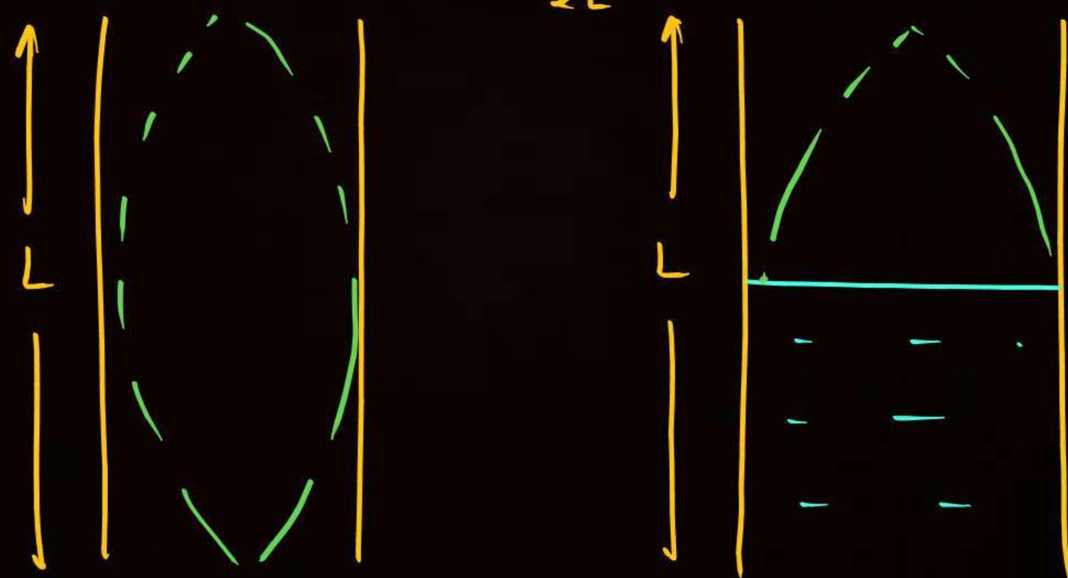
$$f = \frac{v}{\lambda} = \frac{\overset{22}{\cancel{330}}}{\cancel{15} \times 10^{-2}}$$

$$= 2200\,Hz$$

**QUESTION**

A cylindrical resonance tube open a both ends, has a fundamental frequency $f$, in air. If half of the length is dipped vertically in water, the fundamental frequency of the air column will be

$f$ $\quad f = \frac{v}{2L}$




$$f = \frac{v}{4(L/2)}$$

$$f = \frac{v}{2L}$$

(A) $2f$

(B) $3f/2$

(C) $f$

(D) $f/2$

QUESTION

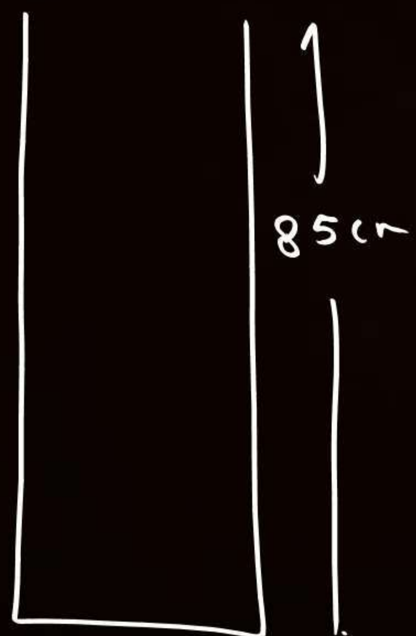
The number of possible natural oscillations of air column in a pipe closed at one end of length $\boxed{85\text{ cm}}$ whose frequencies lie below $\boxed{1250\text{ Hz are}}$ (velocity of sound = 340 $m^{-1}$)

(A) 4

(B) 5

(C) 7

(D) 6




$$f = \frac{v}{4L} = \frac{340}{4\times 85\times 10^{-2}}$$

$$= \frac{340\times 100}{340} = 100\text{ Hz}$$

$$f_1, f_3, f_5, f_7, f_9, f_{11}, \cancel{f_{13}}$$

QUESTION

A tuning fork with frequency $\boxed{800 \text{ Hz}}$ produces resonance in a resonance column tube with upper end open and lower end closed by water surface. Successive resonance are observed at length 9.75 cm, 31.25 cm and 52.75 cm. The speed of sound in air is

(A) 500 m/s

(B) 156 m/s

(C) 344 m/s

(D) 172 m/s

9.75 cm
31.25 cm
52.75 cm

$\frac{\lambda}{2} = 31.25 - 9.75$

$\frac{\lambda}{2} = 21.5 \text{ cm}$

$\lambda = 43 \text{ cm}$

$v = \lambda \times f$

$= 43 \times \cancel{10^{-2}} \times \cancel{800}$

$= 344 \text{ m/s}$

# Beats

Interference of waves having some frequency diff.

$f_1$

$f_2$

$$\boxed{f_{beat} = |f_1 - f_2|}$$

$f_1 = 200\,Hz$
$f_2 = 204\,Hz$ $\rightarrow f_{beat} = 4\,Hz$

QUESTION

Two sound waves with wavelengths $\boxed{5\ \text{m}}$ and $\boxed{5.5\ \text{m}}$ respectively, each propagate in a gas with velocity $\boxed{330\ \text{m/s}}$. We expect the following number of beat per second

(A) 12

(B) zero

(C) 1

(D) 6

$$f_1 = \frac{v}{\lambda} = \frac{330}{5} = 66\ \text{Hz}$$

$$f_2 = \frac{v}{\lambda} = \frac{\cancel{3300}}{\cancel{5.5}} = 60\ \text{Hz}$$

$$f_{beat} = |f_2 - f_1| = 6\ \text{Hz}.$$

## QUESTION

Two vibrating tuning forks produce progressive waves given by

$y_1 = 4 \sin \underbrace{500\pi}t$ and

$y_2 = 2 \sin 506\pi t$. Number of beat produced per minute is

(A) 360

(B) 180

(C) 3

(D) 60

$f_1 = \frac{\omega_1}{2\pi} = \frac{500\cancel{\pi}}{2\cancel{\pi}} = 250\,Hz$

$f_2 = \frac{506\pi}{2\pi} = 253\,Hz$

$f_{beat} = 3\,Hz.$

no. of beats $= f_{beat} \times t$

$= 3 \times 60$

$= 180$

QUESTION

Two sources of sound placed closed to each other, are emitting progressive wave given by $y_1 = 4 \sin \underbrace{600\pi t}$ and $y_2 = 5 \sin 608\pi t$. An observer located near these two sources of sound will hear $f_1 = \frac{600\pi}{2\pi} = 300\text{Hz}$ $f_2 = \frac{608\pi}{2\pi} = 304\text{Hz}$. $\mu\uparrow$ $\mu\downarrow$

$$\frac{I_{max}}{I_{min}} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2} = \frac{(4+5)^2}{(4-5)^2} = 81 : 1$$

(A) 4 beat/s with intensity ratio 25 : 16 between waxing and waning

(B) 8 beat/s with intensity ratio 25 : 16 between waxing and waning

(C) 8 beat/s with intensity ratio 81 : 1 between waxing and waning

(D) 4 beat/s with intensity ratio 81 : 1 between waxing and waning

QUESTION

Two waves of wavelength $\boxed{50\text{ cm}}$ and $\boxed{51\text{ cm}}$ produce $\boxed{12\text{ beat/s.}}$ The speed of sound is

(A) 306 m/s

(B) 331 m/s

(C) 340 m/s

(D) 360 m/s

$$f_1 = \frac{v}{\lambda_1} \qquad f_2 = \frac{v}{\lambda_2}$$

$$f_1 - f_2 = 12$$

$$v\left[\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right] = 12$$

$$v = \frac{12}{\left[\frac{100}{50} - \frac{100}{51}\right]}$$

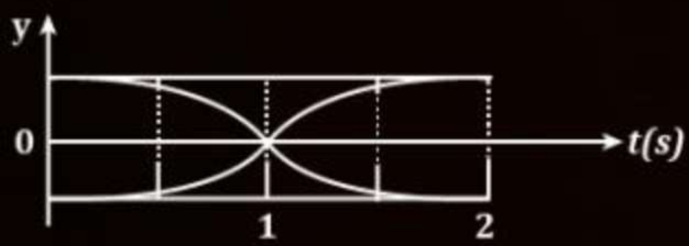
$$v = \frac{\cancel{12}\,6}{\cancel{100}\,\cancel{2}\left[\frac{51-50}{\cancel{50}\times 51}\right]}$$

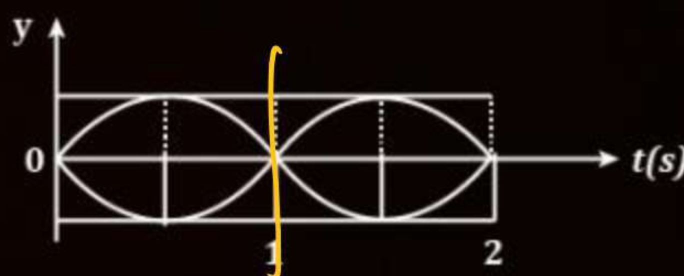
$$= \frac{6\times 51}{1} = 306\text{ m/s}$$

QUESTION

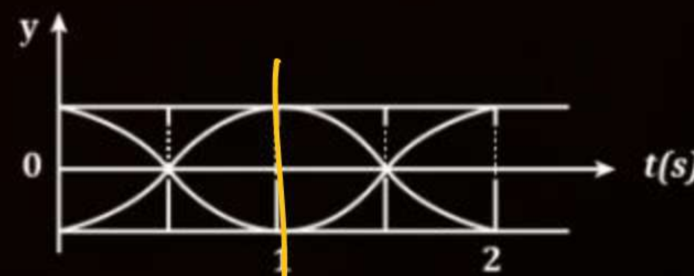
$f_{beat} = 2\ Hz.$ $T = \frac{1}{f} = \frac{1}{2}\ s.$

The correct figure that shows, schematically, the wave pattern produced by superposition of two waves of frequencies 9 Hz and 11 Hz is: **[10 April, 2019 (S-II)]**

(A) 

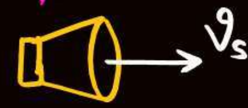

(B) 


(C) 


(D) 

# Doppler's Effect

(JEE Adv.)





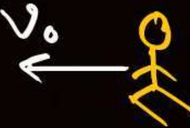
Operating System

O → observer

S → Source

$$f = f_0\left[\frac{c + v_0}{c - v_s}\right]$$

c = speed of sound in air


$$f = f_0\left[\frac{c + v_0}{c + v_s}\right]$$

**QUESTION**

Two cars moving in opposite directions approach each other with speed of 22 m/s and 16.5 m/s respectively. The driver of the first car blows a home having a frequency 400 Hz. The frequency heard by the driver of the second car is [velocity of sound 340 m/s]


(A) 350 Hz

(B) 361 Hz

(C) 411 Hz

(D) 448 Hz

$$f = f_0\left[\frac{c+v_0}{c-v_s}\right]$$

$$= 400\left[\frac{340+16.5}{340-22}\right] = 400 \times \frac{356.5}{318}$$

QUESTION

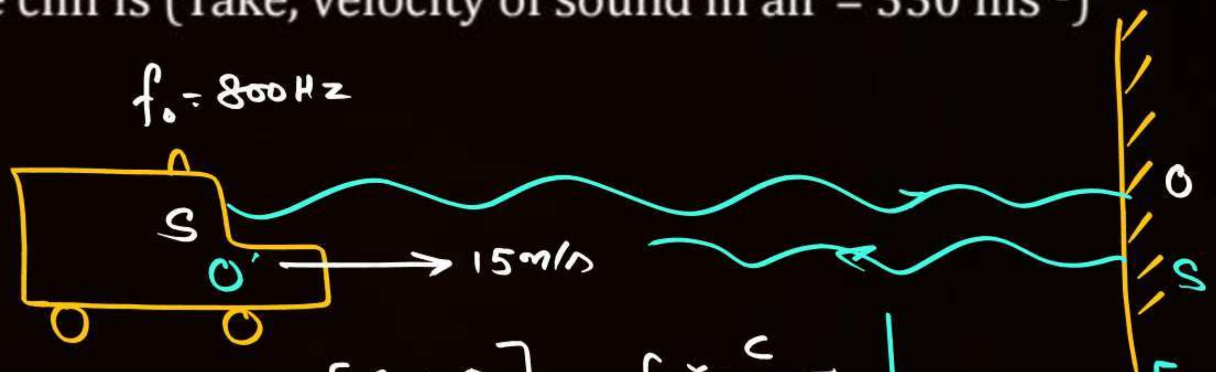
A siren emitting a sound of frequency 800 Hz moves away from an observer towards a cliff at a speed of $15 ms^{-1}$. Then, the frequency of sound that the observer hears in the echo reflected from the cliff is (Take, velocity of sound in air = $330\ ms^{-1}$)

(A) 800 Hz

(B) 838 Hz

(C) 885 Hz

(D) 765 Hz




$$f_1 = f_0\left[\frac{c+0}{c-15}\right] = f_0 \times \frac{c}{c-15}$$

$$f_2 = f_1\left[\frac{c+15}{c}\right]$$

$$f_2 = 800 \times \frac{\cancel{c}}{c-15} \times \frac{c+15}{\cancel{c}}$$

$$f_2 = 800 \times \frac{\overset{609}{\cancel{345}}}{\underset{603}{\cancel{315}}}$$

# Sound Properties

Loudness →

$$\boxed{\text{Loudness} = 10 \log \frac{I}{I_0}} \ dB$$

decibell

$$I_0 = 10^{-12} \frac{W}{m^2}$$

QUESTION

A small speaker delivers $\boxed{2W}$ of audio output. At what distance from the speaker will one detect $\boxed{120 \text{ dB}}$ intensity sound? [Given reference intensity of sound as $10^{-12} W/m^2$]

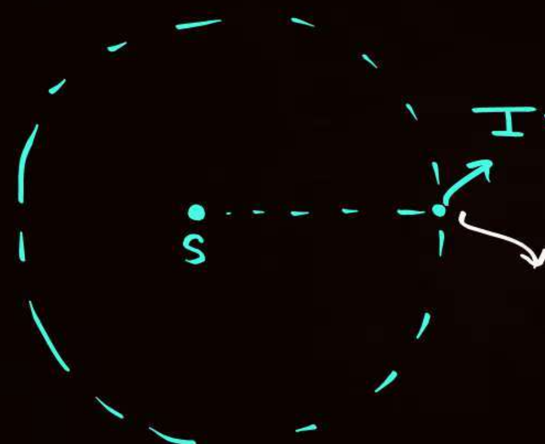
[12 April 2019 II]

(A) 40 cm

(B) 20 cm

(C) 10 cm

(D) 30 cm




$$I = \frac{E}{A} = \frac{2}{4\pi r^2}$$

$$120 = 10 \log_{10} \frac{I}{10^{-12}}$$

$$\Rightarrow 10^{12} = \frac{I}{10^{-12}} \Rightarrow I = 1$$

$$\Rightarrow \frac{2}{4\pi r^2} = 1$$

$$4\pi r^2 = 2$$

$$r = \sqrt{\frac{\cancel{2}}{\cancel{4}\pi}}$$

$$r = \frac{100}{\sqrt{2\pi}}$$

$$= \frac{100}{\sqrt{6.28}}$$

$$= \frac{100}{2.5}$$

$$= 40$$

## Homework

DPP + class Q