

Charge

Quantization of charge

$$Q = \pm ne \quad Q = \text{Total charge}$$

$$n = 1, 2, 3, \dots$$

$$e = 1.6 \times 10^{-19} \text{C}$$

Additivity of charge

$$Q' = Q_1 + Q_2$$

Redistribution of charge



$$Q' = \frac{Q_1 + Q_2}{2}$$

Q' = Charge on each shell after redistribution

Charge Density

Linear Charge density, $\lambda = \frac{Q}{L}$ Unit = $\frac{C}{m}$

Surface Charge density, $\sigma = \frac{Q}{S}$ Unit = $\frac{C}{m^2}$

Volume Charge density, $\rho = \frac{Q}{V}$ Unit = $\frac{C}{m^3}$

Q = Total charge V = Volume
 L = Length S = Area



If a charge on the body is 1 nC, then how many electrons are present on the body?

- a) 1.6×10^{19} b) 6.25×10^9
c) 6.25×10^{27} d) 6.25×10^{28}

Coulomb's Law

$$Q_1 \xrightarrow{F} Q_2 \quad F = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{r^2}$$

ϵ_0 = Permittivity of free space

$$[\epsilon_0] = \frac{[Q_1][Q_2]}{[r^2][F]} = \frac{[AT][AT]}{[L^2][MLT^{-2}]} = M^{-1}L^{-3}T^4A^2$$

$$Q_1 \xrightarrow{F} Q_2 \quad F_{\text{med}} = \frac{F_{\text{air}}}{k}$$

k = dielectric constant of the medium

Superposition

- Direction:
- Like - Towards the point at which force has to be evaluated (repulsion)
 - Unlike - Away from the point at which force has to be evaluated (attraction)

General rule
 $F_{\text{net}} = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$

When $\theta = 60^\circ$
 $F_{\text{net}} = \sqrt{3}F$

When $\theta = 90^\circ$
 $F_{\text{net}} = \sqrt{2}F$

When $\theta = 120^\circ$
 $F_{\text{net}} = F$

Equilibrium of Charges

Calculation of Charge

$$Q_1 \xrightarrow{r_1} Q_2 \xrightarrow{r_2} Q_3$$

$$\frac{Q_1}{Q_2} = \left(\frac{r_1}{r_2}\right)^2 \quad q \text{ in equilibrium}$$

$$q = -\left(\frac{r_1}{r_1 + r_2}\right)^2 Q_2 \quad Q_1 \text{ in equilibrium}$$

$$q = -\left(\frac{r_2}{r_1 + r_2}\right)^2 Q_1 \quad Q_2 \text{ in equilibrium}$$



A charge is placed at the centre of the line joining two equal charges Q . The system of the three charges will be in equilibrium if q is equal to

- a) $-Q/2$ c) $+Q/4$
b) $-Q/4$ d) $+Q/2$

Charge on pendulum

$$\tan\theta = \frac{qE}{mg}$$

$$\sin\theta = \frac{r}{2l}$$

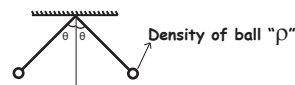
$$r = 2l \sin\theta$$

if θ is very small

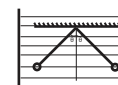
$$\tan\theta \approx \sin\theta$$

$$\frac{r}{2l} = \frac{qE}{mg}$$

$$\frac{r}{2l} = \frac{kq^2/r^2}{mg} \quad r^3 \propto q^2$$



it θ does not change on submerging in liquid
Dielectric constant of liquid,



density of liquid = σ

$$k = \frac{\rho}{\rho - \sigma}$$

Electric Field

Electric field at a point, due to point charge $E = \frac{kq}{r^2}$

$$K = \frac{1}{4\pi\epsilon_0}$$

Superposition

General rule
 $E_{\text{net}} = \sqrt{E_1^2 + E_2^2 + 2E_1E_2\cos\theta}$

$E_1 \rightarrow E_{\text{net}} = E_1 + E_2$

$E_1 \leftarrow E_{\text{net}} = |E_1 - E_2|$

$E_{\text{net}} = \sqrt{E_1^2 + E_2^2 + 2E_1E_2\cos\theta}$
If, $E_1 = E_2 = E$ Then, $E_{\text{net}} = \sqrt{3}E$

$E_{\text{net}} = \sqrt{E_1^2 + E_2^2}$
If, $E_1 = E_2 = E$ Then, $E_{\text{net}} = \sqrt{2}E$

$E_{\text{net}} = \sqrt{E_1^2 + E_2^2 - E_1E_2}$
If, $E_1 = E_2 = E$ Then, $E_{\text{net}} = E$

Direction

- Positive charge:- Towards the point at which electric field has to be evaluated
- Negative charge:- Away from the point at which electric field has to be evaluated

Neutral Point

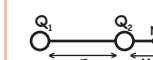
Like Charges

$$x_1 = \frac{\sqrt{Q_1}r}{\sqrt{Q_1} + \sqrt{Q_2}}$$

$$x_2 = \frac{\sqrt{Q_2}r}{\sqrt{Q_1} + \sqrt{Q_2}}$$

Unlike Charges

Outside closer to smaller charge



$$|Q_2| < |Q_1|$$

$$x = \frac{\sqrt{Q_2}r}{\sqrt{Q_1} - \sqrt{Q_2}}$$

Distance from $Q_1 = x + r$



Two point charges $+8q$ and $-2q$ are located at $x = 0$ and $x = L$ respectively. The location of a point on the x axis at which the net electric field due to these two point charges is zero is:

- a) $8L$
b) $4L$
c) $2L$
d) $L/4$

Charged particle released in an electric field

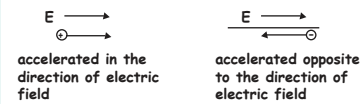
1) Force, $F = qE$

2) Acceleration, $a = \frac{qE}{m}$

3) Velocity, $V = \frac{qE}{m}t$

4) Velocity, $V = \sqrt{\frac{2qEx}{m}}$

5) Kinetic energy, $K.E = \frac{q^2E^2t^2}{2m}$



$$V = \sqrt{V_x^2 + V_y^2}$$

$$V_x = \frac{qEt}{m}$$

$$V = \sqrt{U^2 + \left(\frac{qEt}{m}\right)^2}$$

accelerated in the direction of field and perpendicular to initial velocity

$$\frac{M_p}{M_e} = 1837, \quad \frac{e}{m} = 1.7 \times 10^{-11}$$

$$\frac{1}{2}at^2 = h = \text{Constant}$$

$$\frac{1}{2} \frac{q^2E^2}{m} t^2 = h$$

$$t^2 \propto m$$

$$\frac{t_p}{t_e} = \left(\frac{m_p}{m_e}\right)^{1/2}$$

$$\Rightarrow t_p > t_e$$

Time period of Charged Pendulum in an electric field

$T = 2\pi \sqrt{\frac{l}{(g - \frac{QE}{m})}}$
Time period will increase

$T = 2\pi \sqrt{\frac{l}{(g + \frac{QE}{m})}}$
Time period will decrease

$T = 2\pi \sqrt{\frac{l}{(g^2 + (\frac{QE}{m})^2)}}$
Time period will decrease

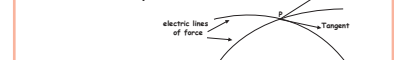
Electric field inside a dielectric medium

$$E_{\text{net}} = \frac{E}{k}$$

$$E_{\text{net}} = E - E_{\text{induced}}, \quad E_{\text{induced}} = E - E_{\text{net}} = E(1 - \frac{1}{k})$$

Properties of field lines

- Start from positive charge and end on negative charge
- Never intersect each other. If they intersect there will be 2 directions for electric field which is not possible



- Always perpendicular to Conducting surface

- $E \propto$ Electric field line density



- Never form closed loops (Conservative force)

- $q \propto$ no. of field lines
 $|q_2| > |q_1|$

Electric lines of force about negative point charge are:

- circular, anticlockwise
- circular, clockwise
- radial, inward
- radial, outward

Electric flux

Flux is proportional to total no. of field lines passing through an area

$$\Phi = \int \mathbf{E} \cdot d\mathbf{s} \cos\theta$$

$$\Phi = \int \mathbf{E} \cdot d\mathbf{s}$$

Gauss Law:- $\Phi = \frac{q}{\epsilon_0} = \oint \mathbf{E} \cdot d\mathbf{s} \cos\theta$

Zero flux:- $\Phi = \frac{q_{\text{net}}}{\epsilon_0} = 0$, where $q_{\text{net}} = 0$

Electric flux for Cube

- No charge inside the cube

$$\Phi = \frac{q}{\epsilon_0} = 0$$

- Charge placed at the center

$$\Phi_{\text{total}} = \frac{q}{\epsilon_0}$$

$$\Phi_{\text{one side}} = \frac{q}{6\epsilon_0}$$

- Charge placed at the face

$$\Phi_{\text{cube}} = \frac{q}{2\epsilon_0}$$

- Charge placed at the corner

$$\Phi_{\text{cube}} = \frac{q}{8\epsilon_0}$$

$$\Phi_{\text{one face}} = \frac{q}{8\epsilon_0} \times \frac{1}{3} = \frac{q}{24\epsilon_0}$$

- Charge placed at the edge

$$\Phi_{\text{cube}} = \frac{q}{4\epsilon_0}$$

$$\Phi_{\text{face}} = \frac{q}{\epsilon_0} \times \frac{1}{4} = \frac{q}{16\epsilon_0}$$

- Flux through curved surface

$$\Phi_{\text{effective}} = \Phi_{\text{curve}} + 2\Phi_{\text{cross section}}$$

Application of Gauss's Theorem

- Point charge $E = \frac{kq}{r^2}$

- Metal sphere/Hollow sphere
 $E_{\text{surface}} = \frac{kQ}{R^2}$
 $E_{\text{outside}} = \frac{kQ}{r^2}$
 $E_{\text{inside}} = 0$

- Non-Conducting sphere

$$E_{\text{inside}} = \frac{kQr}{R^3}$$

$$E_{\text{surface}} = \frac{kQ}{R^2}$$

$$E_{\text{outside}} = \frac{kQ}{r^2}$$

- Conducting sheet

$$E = \frac{\sigma}{\epsilon_0}$$

- Non-conducting sheet

$$E = \frac{\sigma}{2\epsilon_0}$$

$Q_{\text{outer}}, Q_{\text{inner}}, Q_{\text{plate}}$
 $Q_{\text{inner}} = Q_{\text{plate}} - Q_{\text{outer}}$

- Electric field due to a finite linear charge distribution

$$E = \frac{2k\lambda}{r} \sin(\theta/2)$$

- Electric field due to a infinite linear charge distribution

$$E = \frac{2k\lambda}{r}$$

- Electric field due to circular arc at its center

$$E_o = \frac{2k\lambda}{r} \sin(\theta/2)$$

eg: For a semicircle $\theta = 180^\circ$

$$E_o = \frac{2k\lambda}{r} \sin(90^\circ)$$

$$= \frac{2k\lambda}{r}$$

- Electric field at the center of a circular ring

$$E_o = 0$$

- Electric field due to a circular ring of charge

$$E = \frac{1}{4\pi\epsilon_0} \frac{qx}{(x^2 + r^2)^{3/2}}$$

(For large distance)

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{x^2}$$