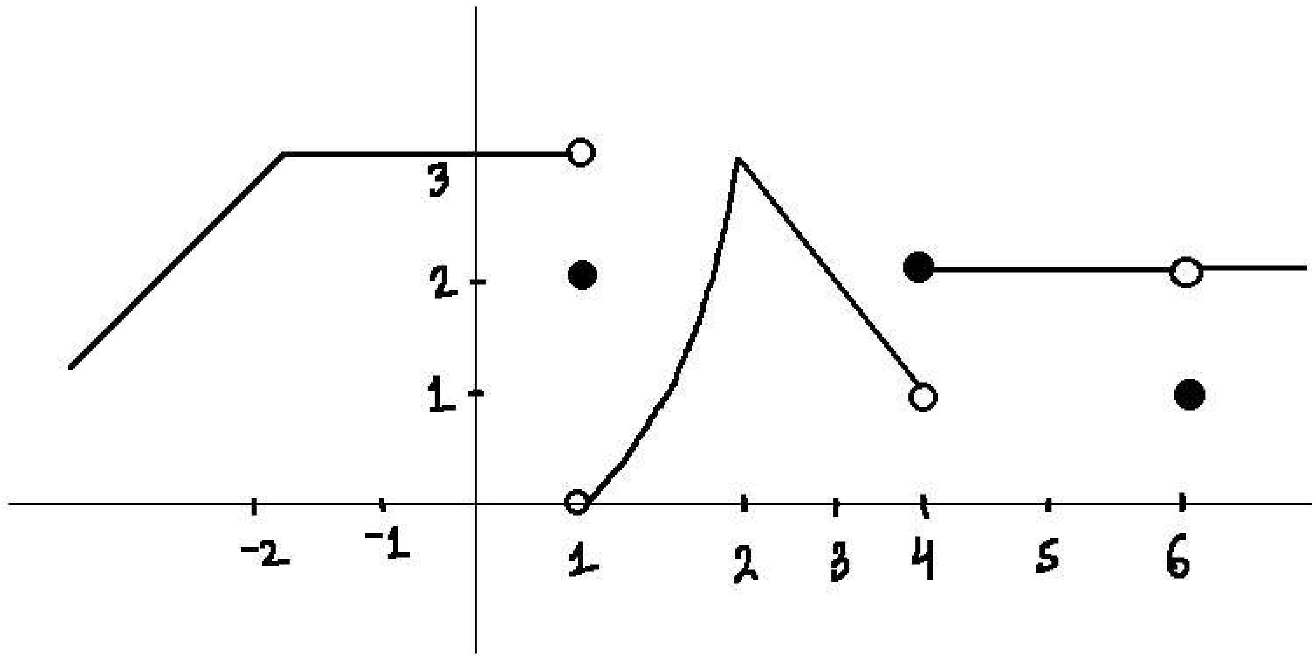
Concept of Limits

Understanding Limits from Graph



Indeterminate Forms

$$\lim_{x \to \infty} \text{ and } \lim_{x \to -\infty}$$

NVStyle

$$\lim_{x\to\infty} \frac{\text{Polynomial}}{\text{Polynomial}}$$

If deg(Nʳ) > deg(Dʳ)

If deg(Nʳ) = deg(Dʳ)

If deg(Nʳ) < deg(Dʳ)

Shortcut

Evaluate

1. $\lim_{x \to \infty} \frac{2x + 3}{5x - 4}$

2. $\lim_{x \to \infty} \frac{2x^2 - x + 1}{3x^2 + 5x - 6}$

Standard Forms

Remember

Remember

$\lim_{x \to 0} \frac{\sin^2(\pi \cos^4 x)}{x^4}$ is equal to :

(JEE Main 2021)

A. π^2

B. $2\pi^2$

C. $4\pi^2$

D. 4π

The value of the limit $\lim_{\theta \to 0} \frac{\tan(\pi \cos^2 \theta)}{\sin(2\pi \sin^2 \theta)}$ is equal to :

(JEE Main 2021)

A. -1/2

B. -1/4

C. 0

D. 1/4

$$\lim_{x\to 0} \frac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}} \text{ equals :}$$

(JEE Main 2019)

A. $4\sqrt{2}$

B. $\sqrt{2}$

C. $2\sqrt{2}$

D. 4

$$\lim_{x \to 0} \frac{x \cot(4x)}{\sin^2 x \cot^2(2x)}$$ is equal to :

A. 0

B. 2

C. 4

D. 1

(JEE Main 2019)

If $\lim_{x \to 0} \left\{ \frac{1}{x^8} \left(1 - \cos\frac{x^2}{2} - \cos\frac{x^2}{4} + \cos\frac{x^2}{2} \cos\frac{x^2}{4} \right) \right\} = 2^{-k}$, then the value of k is_

(JEE Main 2020)

Let f : R ⟶ R be a function such that f(2) = 4 and f' (2) = 1.

Then, the value of $\lim_{x\to2} \frac{x^2 f(2)-4f(x)}{x-2}$ is

(JEE Main 2021)

A. 4

B. 8

C. 16

D. 12

Let $f(x) = x^6 + 2x^4 + x^3 + 2x + 3$, $x \in R$. Then the natural number n for which $\lim_{x \to 1} \frac{x^n f(1) - f(x)}{x-1} = 44$ is .

(JEE Main 2021)

$$\lim_{x \to 2} \frac{3^x + 3^{3-x} - 12}{3^{-\frac{x}{2}} - 3^{1-x}}$$ is equal to

[JEE Main 2020]

$$\lim_{x \to \frac{\pi}{2}} \left(\tan^2 x \left(\left(2\sin^2 x + 3\sin x + 4\right)^{\frac{1}{2}} - \left(\sin^2 x + 6\sin x + 2\right)^{\frac{1}{2}} \right) \right)$$

is equal to

(JEE Main 2022)

A. $\frac{1}{12}$

B. $-\frac{1}{18}$

C. $-\frac{1}{12}$

D. $-\frac{1}{6}$

$$\lim_{x \to \frac{1}{\sqrt{2}}} \frac{\sin(\cos^{-1} x) - x}{1 - \tan(\cos^{-1} x)} \text{ is equal to :}$$

(JEE Main 2022)

A. $\sqrt{2}$

B. $-\sqrt{2}$

C. $\frac{1}{\sqrt{2}}$

D. $-\frac{1}{\sqrt{2}}$

$$\lim_{x\to 0}\frac{\cos(\sin x)-\cos x}{x^4}$$ is equal to :

(JEE Main 2022)

A. 1/3

B. 1/4

C. 1/6

d. 1/12

Let [t] denote the greatest integer $\leq$ t and {t} denote the fractional part of t. Then integral value of α for which the left hand limit of the function

(JEE Main 2022)

$$f(x)=[1+x]+\frac{\alpha^{2[x]+\{x\}}+[x]-1}{2[x]+\{x\}}$$ at $x = 0$ is equal to

$\alpha-\frac{4}{3}$ is ____

The value of $\lim_{n\to\infty} 6\tan\left\{\sum_{r=1}^{n}\tan^{-1}\left(\frac{1}{r^2+3r+3}\right)\right\}$ *(JEE Main 2022)*

is equal to

A. 1

B. 2

C. 3

C. 6

If $\lim_{x \to 1} \frac{\sin(3x^2 - 4x + 1) - x^2 + 1}{2x^3 - 7x^2 + ax + b} = -2$, then the value of (a – b) is equal to

(JEE Main 2022)

The value of $\lim_{x \to 1} \frac{(x^2-1)\sin^2(\pi x)}{x^4-2x^3+2x-1}$ is equal to:

(JEE Main 2022)

A. $\frac{\pi^2}{6}$

B. $\frac{\pi^2}{3}$

C. $\frac{\pi^2}{2}$

D. π^2

$$\lim_{x \to \frac{\pi}{4}} \frac{8\sqrt{2} - (\cos x + \sin x)^7}{\sqrt{2} - \sqrt{2}\sin 2x}$$ is equal to

(JEE Main 2022)

A. 14

B. 7

C. 14√2

D. 7√2

$0 \times \infty$ and $\infty - \infty$ Form

Method to solve ∞-∞, 0x∞

S - 1 Convert to $\frac{0}{0}$ or $\frac{\infty}{\infty}$

S - 2 Use factorization/ rationalization/ L'Hopital

Evaluate $\lim_{x \to \infty} \left(\sqrt{25x^2 - 3x} - 5x \right)$

If $\lim_{x\to\infty}\left(\sqrt{x^2-x+1}-ax\right)=b$, then the ordered pair (a,b) is:

(JEE Main 2021)

A. (1, 1/2)

B. (1, -1/2)

C. (-1, 1/2)

D. (-1, -1/2)

If $\lim_{n \to \infty}\left(\sqrt{n^2 - n - 1} + n\alpha + \beta\right) = 0$ then $8(\alpha + \beta)$ is equal to :

(JEE Main 2022)

A. 4

B. -8

C. -4

D. 8

If $\lim_{x \to 1} \frac{x^2 - ax + b}{x - 1} = 5$, then $a + b$ is equal to :

A. -4

B. 5

C. -7

D. 1

(JEE Main 2019)

Binomial Approximation

Binomial Approximation

$$(1+x)^n \approx 1 + nx$$

$$(1+x)^n \approx 1 + nx + \frac{n(n-1)}{2}x^2$$

Let L = $\lim\limits_{x\to 0} \dfrac{a - \sqrt{a^2 - x^2} - \dfrac{x^2}{4}}{x^4}$, $a > 0$f L is finite, then

A. a = 2

B. a = 1

C. L = 1/64

D. L = 1/32

[JEE Adv 2009]

Super Table

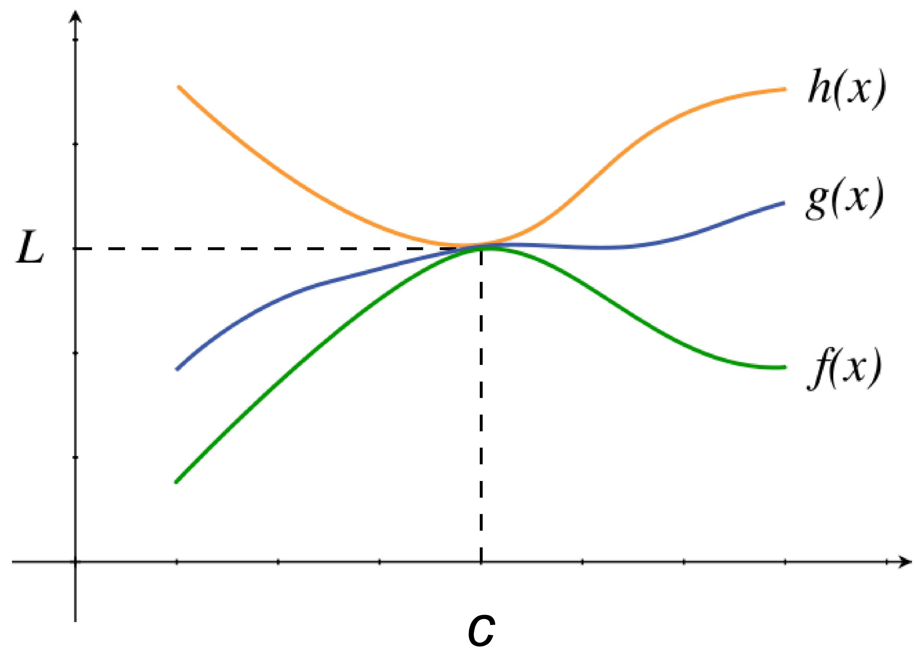
Super Table

f(x)	g(x)	f ± g	f.g and f/g
Exist	Exist	Exist	Exist
Exist	D.N.E	D.N.E	May Exist
D.N.E	D.N.E	May Exist	May Exist

Squeeze (or Sandwich) Theorem

If f(x) ≤ g(x) ≤ h(x) and $\lim_{x \to c} f(x) = L$ and $\lim_{x \to c} h(x) = L$

then $\lim_{x \to c} g(x) = L$



If $4x - 9 \le f(x) \le x^2 - 4x + 7 \ \forall\ x \ge 0$ find $\lim\limits_{x \to 4} f(x)$

A. -7

B. 7

C. 1/7

D. D.N.E.

$$\lim_{x\to\infty} \frac{\{x\}}{x}$$

$$\lim_{x\to\infty} \frac{[x]}{x}$$

The value of $\lim_{n\to\infty} \frac{[r]+[2r]+\ldots\ldots+[nr]}{n^2}$, where r is non-zero real numbers and [r] denotes the greatest integer less than or equal to r, is equal to :

(JEE Main 2021)

A. r/2

B. r

C. 2r

D. 0

Evaluate $\lim\limits_{x \to \infty} \dfrac{x + 7 \sin x}{-2x + 13}$

Evaluate $\lim\limits_{x \to \infty} \dfrac{1}{1+n^2} + \dfrac{2}{2+n^2} + \cdots + \dfrac{n}{n+n^2}$

Expansions

Maclaurin series

$$f(x) = f(0) + f'(0)\,x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots$$

Expansions

$$a^x = 1 + \frac{x\,\ln a}{1!} + \frac{x^2\,\ln^2 a}{2!} + \frac{x^3\,\ln^3 a}{3!} + \ \ldots .$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ \ldots$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ \ldots .$$

Expansions

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \ldots$$

Expansions

$$\sin^{-1}x = x + \frac{1^2}{3!}x^3 + \frac{1^2.3^2}{5!}x^5 + \frac{1^2.3^2.5^2}{7!}x^7 + \cdots$$

$$\sec^{-1}x = 1 + \frac{x^2}{2!} + \frac{5x^4}{4!} + \frac{61x^6}{6!} + \cdots$$

$$\tan^{-1}x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ \ldots.$$

If $\lim_{x \to 0} \frac{\sin^{-1} x - \tan^{-1} x}{3x^3}$ is equal to L, then the value of $(6\,L + 1)$ is

(JEE Main 2021)

A. 1/6

B. 1/2

C. 6

D. 2

$$= \lim_{x \to 0} \frac{\sin^{-1} x - \tan^{-1} x}{3x^3} = L$$

$$= \lim_{x \to 0} \frac{\left(\cancel{x} + \frac{\cancel{x^3}}{6}\right) - \left(\cancel{x} - \frac{\cancel{x^3}}{3}\right)}{3\cancel{x^3}}$$

$$= \frac{\frac{1}{6} + \frac{1 \times 2}{3 \times 2}}{3}$$

$$= \frac{1}{6}$$

#NVTip

$6L + 1$

$= 6\left(\frac{1}{6}\right) + 1$

$= \boxed{2}$

If $\lim_{x\to 0} \dfrac{ae^x - b\cos x + ce^{-x}}{x \frac{\sin x}{x} \cdot x} = 2$, then $a + b + c$ is equal to______

(JEE Main 2021)

$$\lim_{x\to 0} \frac{ae^x - b\cos x + ce^{-x}}{x^2} = 2$$

$$\lim_{x\to 0} \frac{a\left(1 + x + \frac{x^2}{2}\right) - b\left(1 - \frac{x^2}{2}\right) + c\left(1 - x + \frac{x^2}{2}\right)}{x^2}$$

$$\frac{0}{(\to 0)^2} = 0$$

$$\lim_{x\to 0} \frac{\cancel{(a-b+c)}^{0} + \cancel{(a-c)}^{0}x + \left(\frac{a}{2} + \frac{b}{2} + \frac{c}{2}\right)x^2 + \ldots}{1x^2} = 2$$

$$a^2 + b^2 + c^2 = ?$$

$$\boxed{\begin{array}{l} * \ a - b + c = 0 \\ * \ a - c = 0 \\ * \ \frac{a}{2} + \frac{b}{2} + \frac{c}{2} = 2 \end{array}} \implies \boxed{a + b + c = 4}$$

M-2

$$\lim_{x\to 0} \frac{ae^{x} - b\cos x + ce^{-x}}{x^2} \Rightarrow \frac{a-b+c}{0}$$

$$\boxed{a-b+c=0} \quad (1)$$

$$\lim_{x\to 0} \frac{ae^{x} + b\sin x - ce^{-x}}{2x} \Rightarrow \frac{a-c}{0}$$

$$\boxed{a-c=0} \quad (2)$$

$$\lim_{x\to 0} \frac{ae^{x} + b\cos x + ce^{-x}}{2}$$

$$\frac{a+b+c}{2} = 2$$

$$\boxed{a+b+c=4} \quad (3)$$

If $\lim_{x\to 0} \frac{\alpha x e^{x} - \beta \log_e(1+x) + \gamma x^2 e^{-x}}{x \sin^2 x} = 10$, $\alpha, \beta, \gamma \in \mathbf{R}$, then the value of $\alpha + \beta + \gamma$ is

x^2 x^2

(JEE Main 2021)

$$\lim_{x\to 0} \frac{\alpha x e^{x} - \beta \ln(1+x) + \gamma x^2 e^{-x}}{x^3}$$

$$= \lim_{x\to 0} \frac{\alpha x\left(1 + x + \frac{x^2}{2}\right) - \beta\left(x - \frac{x^2}{2} + \frac{x^3}{3}\right) + \gamma x^2 (1 - x)}{x^3}$$

$$= \lim_{x\to 0} \frac{\cancel{(\alpha - \beta)}^{\,0} x + \cancel{\left(\alpha + \frac{\beta}{2} + \gamma\right)}^{\,0} x^2 + \left(\frac{\alpha}{2} - \frac{\beta}{3} - \gamma\right) x^3 + \ldots}{1 x^3}$$

- ★ $\alpha - \beta = 0$
- ★ $\alpha + \frac{\beta}{2} + \gamma = 0$
- ★ $\frac{\alpha}{2} - \frac{\beta}{3} - \gamma = 10$

$\alpha = \beta = 6$

$\gamma = \frac{-3\alpha}{2} = \frac{-3}{2}(6) = -9$

$\frac{\alpha}{2} - \frac{\alpha}{3} + \frac{3\alpha}{2} = 10$

$\frac{3\alpha - 2\alpha + 9\alpha}{6} = 10$

$\boxed{\alpha = 6}$

$\alpha = \beta = 6$

$\gamma = -9$

$\alpha + \beta + \gamma$

$= 6 + 6 - 9$

$= 3$

If $\lim_{x\to 0} \frac{ax-(e^{4x}-1)}{ax\frac{(e^{4x}-1)}{4x}.4x}$ exists and is equal to b, then the value of $a-2b$ is

(JEE Main 2021)

$$\lim_{x\to 0} \frac{ax - e^{4x} + 1}{4ax^2} \Rightarrow \frac{0-1+1}{0}$$

$$\lim_{x\to 0} \frac{a - 4e^{4x}}{8ax} \Rightarrow \frac{a-4}{0} \quad \therefore \boxed{a=4}$$

$$\lim_{x\to 0} \frac{-4(4e^{4x})}{8a} \Rightarrow \frac{-16}{8(4)} = b$$

$$\boxed{b = \frac{-1}{2}}$$

$$\Rightarrow 4 - 2\left(\frac{-1}{2}\right)$$

$$\Rightarrow 5$$

Let $\beta = \lim_{x\to 0} \frac{\alpha x - (e^{3x} - 1)}{\alpha x (e^{3x} - 1)}$ for some $\alpha \in \mathbb{R}$. Then the value of $\alpha + \beta$ is :

(JEE Main 2022)

$3x$

$3 - \frac{1}{2} = 5/2$

A. 14/5

B. 3/2

C. 5/2

D. 7/2

$$\beta = \lim_{x\to 0} \frac{\alpha x - e^{3x} + 1}{3\alpha x^2}$$

$\alpha = 3$

$$\beta = \lim_{x\to 0} \frac{\alpha x - \left(\cancel{1} + 3x + \frac{(3x)^2}{2}\right) + \cancel{1}}{3\alpha x^2}$$

$$\beta = \lim_{x\to 0} \frac{(\cancel{\alpha - 3})x - \frac{9}{2}x^2}{3\alpha x^2} = \frac{-9/2}{\cancel{3\alpha}} = \frac{-1}{2}$$

If $\lim\limits_{x\to 0} \dfrac{\alpha e^{x} + \beta e^{-x} + \gamma \sin x}{x\,\dfrac{\sin^2 x}{x^2}\,x^2} = \dfrac{2}{3}$,

where α, β, γ ∈ R, then which of the following is NOT correct ?

(JEE Main 2022)

$$\lim_{x\to 0}\frac{\alpha\left(1+x+\frac{x^2}{2}+\frac{x^3}{6}\right)+\beta\left(1-x+\frac{x^2}{2}-\frac{x^3}{6}\right)+\gamma\left(x-\frac{x^3}{6}\right)}{x^3}$$

$$\lim_{x\to 0}\frac{\overset{0}{\cancel{(\alpha+\beta)}}+\overset{0}{\cancel{(\alpha-\beta+\gamma)}}\,x+\overset{0}{\cancel{\left(\frac{\alpha}{2}+\frac{\beta}{2}\right)}}\,x^2+\left(\frac{\alpha}{6}-\frac{\beta}{6}-\frac{\gamma}{6}\right)x^3}{1x^3}$$

A. $\alpha^2+\beta^2+\gamma^2=6$

B. $\alpha\beta+\beta\gamma+\gamma\alpha+1=0$

C. $\alpha\beta^2+\beta\gamma^2+\gamma\alpha^2+3=0$

D. $\alpha^2-\beta^2+\gamma^2=4$

$$\boxed{\begin{aligned} &\checkmark\ \alpha+\beta = 0 \\ &\checkmark\ \alpha-\beta+\gamma = 0 \\ &\checkmark\ \alpha-\beta-\gamma = 4 \end{aligned}}$$

$\alpha = \checkmark$

$\beta = \checkmark$

$\gamma = \checkmark$

Let e denote the base of natural logarithm. The value of real number a for which the right hand limit

$$\lim_{x\to 0^+} \frac{\boxed{(1-x)^{\frac{1}{x}}} - e^{-1}}{x^a}$$

is equal to a nonzero ~~real number~~, is 1 __________

[JEE Adv 2021]

$\boxed{a<1}$ $\boxed{\text{Lim}=0}$

$\boxed{a=1}$

$\boxed{a>1}$ Limit = DNE

$$\lim_{x\to 0^+} \frac{\frac{1}{e}\left(\cancel{1} - \frac{x}{2} - \frac{5x^2}{24} \cdots\right) - \cancel{\frac{1}{e}}}{x^a}$$

$$\lim_{x\to 0^+} \frac{\left(\frac{-1}{2e}\right)x + \left(\frac{-5}{24e}\right)x^2 + \cdots}{x^{-1}} = \text{Non-Zero Real Number}$$

$$\left(\frac{-1}{2e}\right)\cancel{x^2} + \left(\frac{-5}{24e}\right)\cancel{x^3}$$

Expansions

(1) $$\boxed{(1+x)^{\frac{1}{x}} = e\left(1 - \frac{x}{2} + \frac{11x^2}{24} + \cdots\right)}$$

(2) $$\boxed{(1-x)^{\frac{1}{x}} = \frac{1}{e}\left(1 - \frac{x}{2} - \frac{5x^2}{24} + \cdots\right)}$$

✓ 0^0 and ∞^0 Form

Form : $0^0, \infty^0$

Method: Take log both sides

$\log_e$

$$\lim_{x\to 0} x^x \quad (0^0)$$

$$y = \lim_{x\to 0} x^x \Rightarrow 1$$

$$0^0 = 1$$

$$\ln y = \lim_{x\to 0} x \ln x \quad (0\times\infty)$$

$$\ln y = \lim_{x\to 0} \frac{\ln x}{\left(\frac{1}{x}\right)} \quad \left(\frac{\infty}{\infty}\right)$$

$$= \lim_{x\to 0} \frac{\left(\frac{1}{x}\right)}{\left(\frac{-1}{x^2}\right)}$$

$$= \lim_{x\to 0} (-x)$$

$$= 0$$

$\therefore \ln y = 0$

$\therefore y = e^0$

$$\boxed{y = 1}$$

$$\lim_{x\to 0} x^{\tan x} \quad (0^0)$$

$$y = \lim_{x\to 0} x^{\tan x}$$

$$\Rightarrow \ln y = \lim_{x\to 0} \tan x \cdot \ln x \quad (0\times\infty)$$

$$\Rightarrow \ln y = \lim_{x\to 0} \frac{\ln x}{\cot x} \quad \left(\frac{\infty}{\infty}\right)$$

$$= \lim_{x\to 0} \frac{\frac{1}{x}}{-\operatorname{cosec}^2 x} = -\frac{\sin^2 x}{x} = \lim_{x\to 0}(-\sin x) = 0$$

$$\ln y = 0$$

$$\therefore y = 1$$

1^∞ Form

Form : 1^{∞}

$$\lim_{x\to a} f(x)^{g(x)} \qquad 1^{\infty}$$

$$\boxed{\lim_{x\to a} \left(f(x)-1\right) g(x)}$$

$\Rightarrow \quad e$

Evaluate $\lim_{x \to 0} (1 + x)^{\operatorname{cosec} x}$ $\boxed{1^{\infty}}$

A. 1

B. e

C. e^2

D. e^{-1}

$$e^{\lim_{x \to 0} (\cancel{1} + x - \cancel{1}) \operatorname{cosec} x}$$

$$= e^{\lim_{x \to 0} \frac{x}{\sin x}}$$

$$= e^{1}$$

If $\boxed{\alpha = \lim_{x\to\pi/4} \frac{\tan^3 x - \tan x}{\cos\left(x+\frac{\pi}{4}\right)}}$ and $\boxed{\beta = \lim_{x\to 0}(\cos x)^{\cot x}}$ are 1^∞ the roots of the equation, $ax^2 + bx - 4 = 0$, then the ordered pair (a, b) is :

(JEE Main 2021)

A. (1, -3)

B. (-1, 3)

C. (-1, -3)

D. (1, 3)

$$\alpha = \lim_{x\to\frac{\pi}{4}} \frac{\tan^3 x - \tan x}{\cos\left(x+\frac{\pi}{4}\right)}$$

$$= \lim_{x\to\frac{\pi}{4}} \frac{3\tan^2 x \sec^2 x - \sec^2 x}{-\sin\left(x+\frac{\pi}{4}\right)}$$

$$= \frac{3(2)-(2)}{-1}$$

$$= -4$$

$$\beta = e^{\lim_{x\to 0}\left(\frac{\cos x - 1}{x^2}\right)\cot x \, x^2}$$

$$= e^{\lim_{x\to 0} \frac{-1}{2} \times \frac{x^2}{\tan x}}$$

$$= e^{\lim_{x\to 0} \frac{-1}{2} \times x}$$

$$= e^0 = 1$$

$ax^2+bx-4=0$ roots -4, 1

$\frac{-4}{a} = -4 \quad \therefore \boxed{a=1}$

$\frac{-b}{a} = -3 \quad \therefore \boxed{b=3}$

Limits Involving G.I.F.

JEE Level

1. $\lim_{x\to 0}\left[\frac{\sin x}{x}\right] = [0.9999] = 0$

2. $\lim_{x\to 0}\left[\frac{x}{\sin x}\right] = [1.0001] = 1$

3. $\left[\lim_{x\to 0}\frac{\sin x}{x}\right] = [1] = 1$

CBSE

4. $\lim_{x\to 0}\frac{\sin x}{x} = 1$

$\sin x \quad x \quad \tan x$

☆ $\lim_{x\to 0}\left[2024\frac{\sin x}{x}\right] = 2023$

$\sin x < x$

$\frac{\sin x}{x} < 1$

$x > \sin x$

$\dfrac{x}{\sin x} > 1$

$\sin x \simeq x \simeq \tan x$

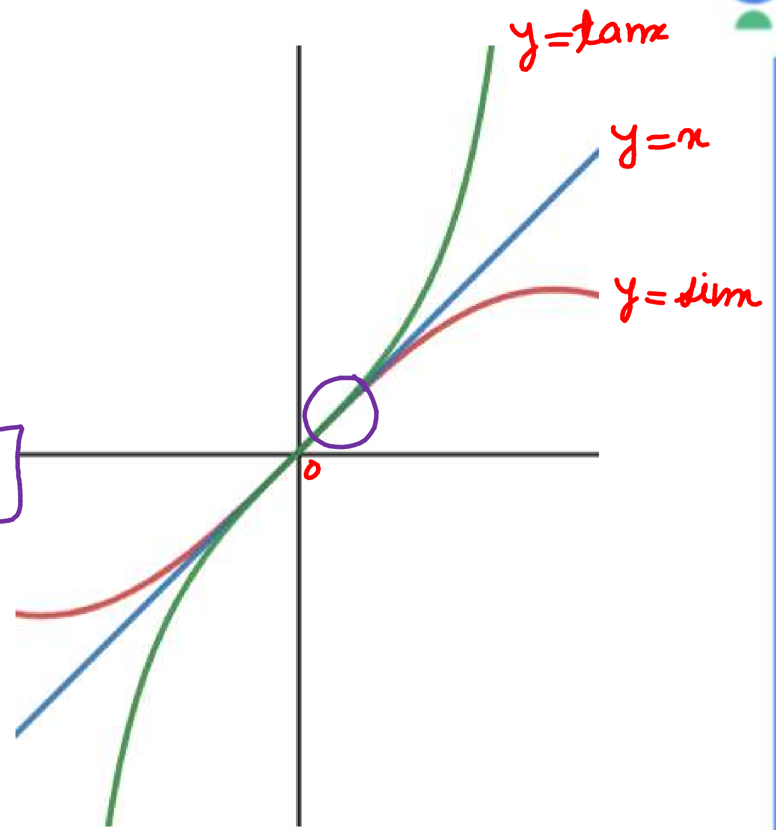
Exact value $\boxed{\tan x > \underbrace{x > \sin x}}$

$\boxed{\dfrac{\sin x}{x} < 1}$

$\left[\dfrac{\sin x}{x}\right] \approx [0.9999]$

$x > \sin x$

$\boxed{1 > \dfrac{\sin x}{x}}$



1. $\lim_{x\to 0}\left[\frac{\tan x}{x}\right] = [1.001] = 1$

2. $\lim_{x\to 0}\left[\frac{x}{\tan x}\right] = [0.999] = 0$

$\tan x > x$

$\frac{\tan x}{x} > 1$

3. $\left[\lim_{x\to 0}\frac{\tan x}{x}\right] = 1$

$a+b+c=?$

$= 2023$

$\tan x > x$

$\frac{\tan x}{x} > 1$

$1 > \frac{x}{\tan x}$

JEE adv

1. $\lim_{x\to 0}\left[2023\frac{\tan x}{x}\right] = a = [2023 \times 1.000001]$

2. $\lim_{x\to 0}\left[2023\frac{x}{\tan x}\right] = b = [2023 \times 0.9999999] = \boxed{2022}$

3. $\left[\lim_{x\to 0} 2023\frac{\tan x}{x}\right] = c = 2023$

1. $\lim_{x\to 0}\left[\frac{\sin^{-1}x}{x}\right]$ $= [1.001] = 1$

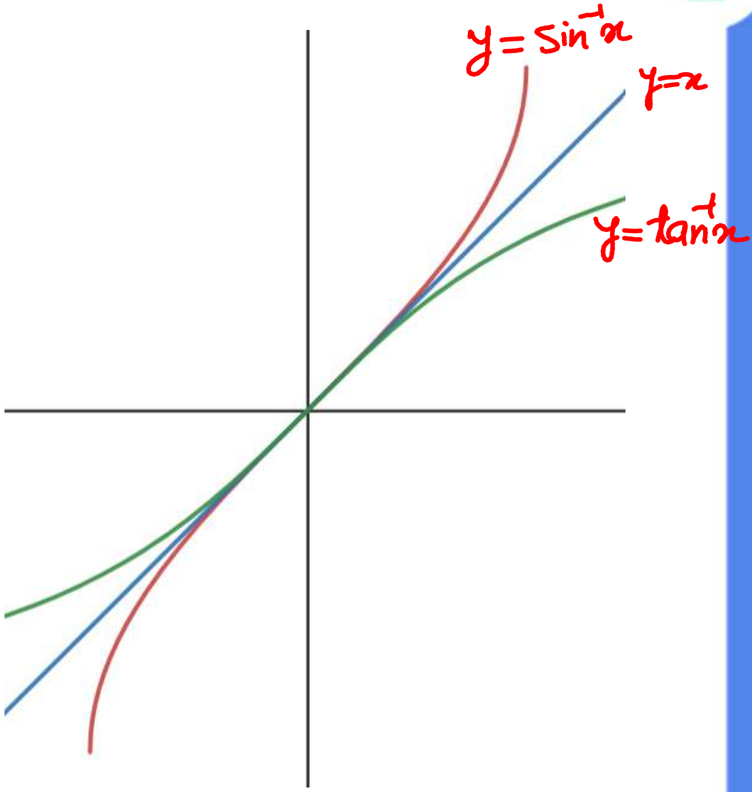
2. $\lim_{x\to 0}\left[\frac{x}{\sin^{-1}x}\right]$ $= [0.99] = 0$

3. $\left[\lim_{x\to 0}\frac{\sin^{-1}x}{x}\right]$ $= 1$

$\sin^{-1}x > x$

$\frac{\sin^{-1}x}{x} > 1$

$$\underbrace{\sin^{-1}x > x > \tan^{-1}x}$$



1. $\lim_{x\to0}\left[\frac{tan^{-1}x}{x}\right]$ = 0

2. $\lim_{x\to0}\left[\frac{x}{tan^{-1}x}\right]$ = 1

3. $\left[\lim_{x\to0}\frac{tan^{-1}x}{x}\right]$ = 1

$\tan^{-1}x < x$

$\frac{\tan^{-1}x}{x} < 1$