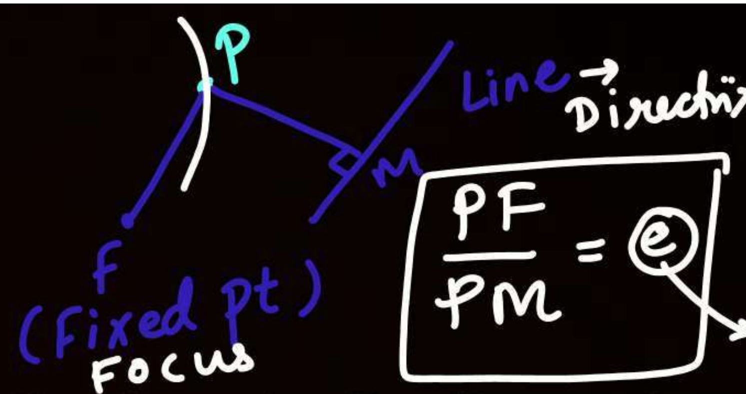
Today's Targets

1. Parabola Definition
2. Standard Equations of Parabola
3. Shifted Forms of Parabola
4. Tangents & Normals to Parabola
5. Chords of Parabola
6. Properties of Parabola
7. PYQS- Extra

Conic Section

$\nearrow P$
$\rightarrow E$
$\searrow H$



$$\boxed{\frac{PF}{PM} = e}$$

FUNDAMENTAL PRINCIPLE OF CONIC

The locus of a point which moves in a plane such that the ratio of its distance from a **fixed point** to its distance from a **fixed straight line** is always a **constant**.

(a) The fixed point is called the **focus**.

(b) The fixed straight line is called the **directrix**.

(c) The constant ratio is called the **eccentricity** denoted by **e**.

(Fixed pt does not lie on Fixed line)

If $e = 1 \Rightarrow$ Parabola

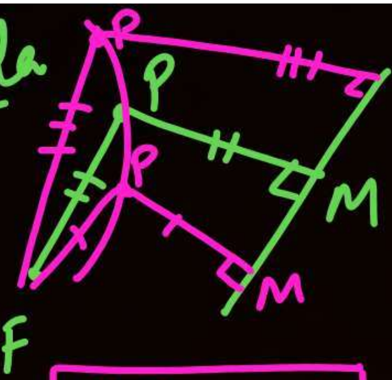
If $e > 1 \Rightarrow$ Hyperbola

If $0 < e < 1 \Rightarrow$ Ellipse

For Parabola $e = 1$

$$\frac{PF}{PM} = 1 \Rightarrow \boxed{PF = PM}$$

For Parabola



$$\boxed{PF = PM}$$

General 2° Curve

$$ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$$

For conic $\boxed{\Delta \neq 0}$

$$\boxed{\Delta = abc + 2fgh - af^2 - bg^2 - ch^2}$$

$\boxed{\Delta \neq 0}$

1) For Parabola $\boxed{h^2 = ab}$
2) For hyperbola $h^2 > ab$
3) For ellipse $h^2 < ab$

Parabola $\longrightarrow$ $\boxed{\underbrace{x^2 + 4y^2 - 4xy}_{(x-2y)^2} + 3x + 2y + 1 = 0}$

$a = 1,\ b = 4,\ 2h = -4 \Rightarrow h = -2$

$2g = 3,\ 2f = 2,\ c = 1$

$\left.\begin{array}{l} h^2 = 4 \\ ab = 4 \end{array}\right\}$

QUESTION

If equation $\sqrt{(3x-2)^2+(3y+5)^2}=\lambda|3x-4y+7|$ represents parabola then find λ. [Ans.

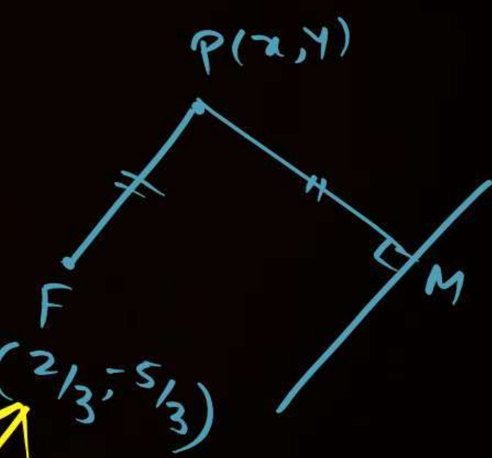
$PF = PM$

$$3\sqrt{\left(x-\tfrac{2}{3}\right)^2+\left(y+\tfrac{5}{3}\right)^2}=5\lambda\left|\frac{3x-4y+7}{\sqrt{3^2+4^2}}\right|$$

$$\underbrace{\sqrt{\left(x-\tfrac{2}{3}\right)^2+\left(y+\tfrac{5}{3}\right)^2}}_{PF}=\frac{5\lambda}{3}\underbrace{\left|\frac{3x-4y+7}{\sqrt{3^2+4^2}}\right|}_{PM}$$

$$\Rightarrow PF = \frac{5\lambda}{3}PM \quad \frac{5\lambda}{3} \to 1$$

$$\Rightarrow \frac{5\lambda}{3}=1 \Rightarrow \boxed{\lambda = 3/5}$$



Focus (2/3, −5/3)

$3x-4y+7=0 \rightarrow$ Directrix

Note that

In the equation of a parabola, $\underbrace{2^0 \text{ terms will always form a complete square}}$.

$$ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$$

For Parabola →

$$h^2 = ab$$

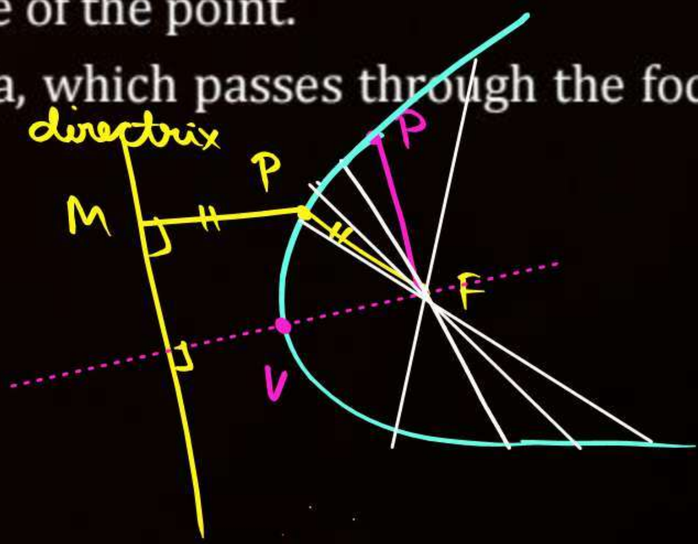
$$h = \pm\sqrt{ab}$$

$$ax^2 + by^2 \pm 2\sqrt{ab}\,xy$$

$$\left(\sqrt{a}x \pm \sqrt{b}y\right)^2$$

Important Definitions_Conic Section

1. **Axis of a Conic:** The line passing through the focus & perpendicular to the directrix is called the Axis of the Conic.
2. **Vertex of a Conic:** Point of intersection of a conic with its axis is called a vertex.
3. **Focal distance:** PF The distance of a point on the parabola from the focus is called the focal distance of the point.
4. **Focal chord:** A chord of the parabola, which passes through the focus is called a focal chord.



Important Definitions_Conic Section

5. Double ordinate: A chord of the parabola perpendicular to the axis of the symmetry is called a double ordinate.

6. Latus rectum: A double ordinate passing through the focus or a focalchord perpendicular to the axis of parabola is called the Latus Rectum.

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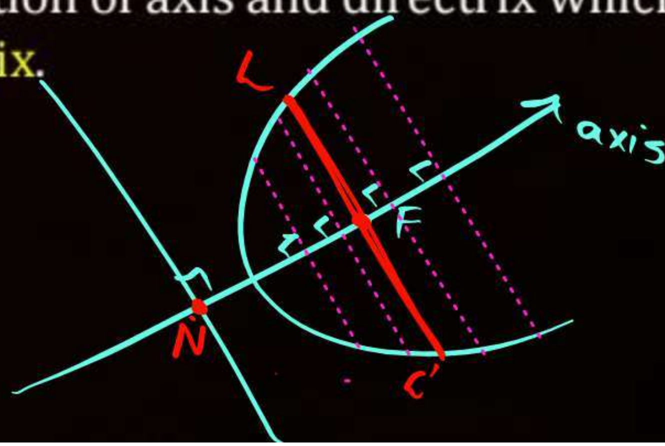
To be precise the length of this chord is called the Latus Rectum.

7. Foot of directrix: The point of intersection of axis and directrix which is called Foot of Directrix.

(N)



Standard Equation 1 of Parabola

1) $\boxed{y^2 = 4ax}$

1) Let Focus Lies on x-axis $(a, 0)$

2) Directrix is $\perp$ to x-axis

3) Let Parabola passes through $(0,0)$

eq^n of Parabola $PF = PM$

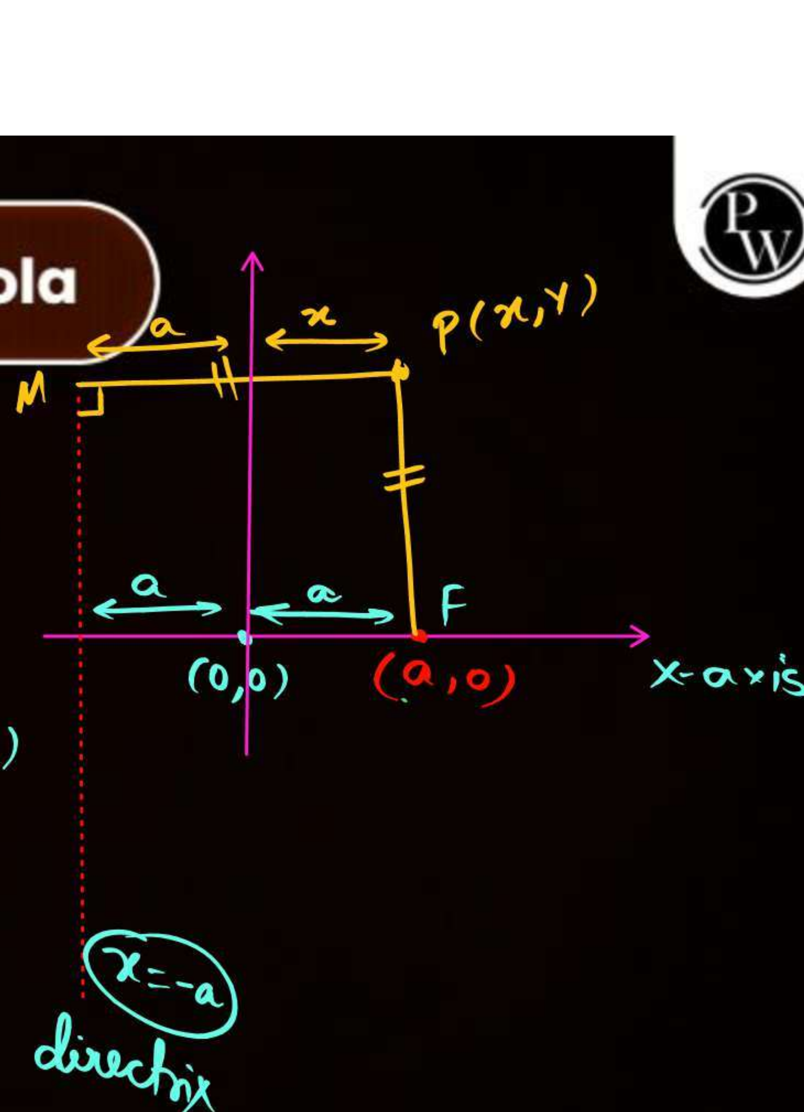
$$\sqrt{(x-a)^2 + y^2} = |x+a|$$

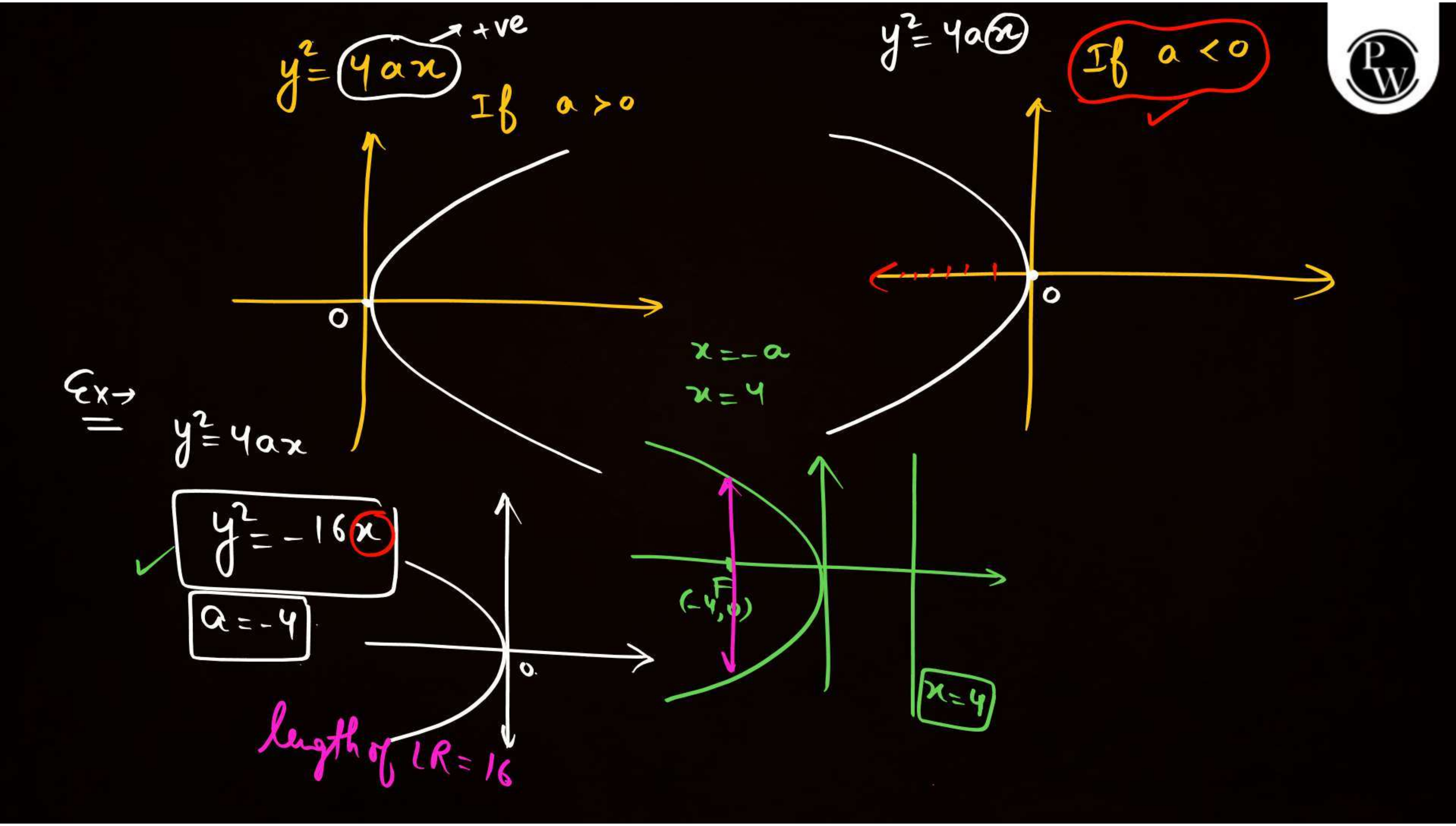
S.B.S

$$(x-a)^2 + y^2 = (x+a)^2$$

$$\cancel{x^2} + \cancel{a^2} - 2ax + y^2 = \cancel{x^2} + \cancel{a^2} + 2ax$$

$$\boxed{y^2 = 4ax}$$





y² = 4ax → +ve
If a > 0
O
y² = 4ax
If a < 0
O
x = -a
x = 4
Ex →
y² = 4ax
y² = -16x
a = -4
O
(-4, 0)
F
x = 4
length of LR = 16

Standard Equation-1

(NCERT)

$y^2 = 4ax$

Axis	x-axis ($y=0$)
Focus	$(a, 0)$
Directrix	$\boxed{x = -a}$
Vertex	$(0, 0)$
Tangent at Vertex	y-axis ($x = 0$)
Foot of Directirx	$(-a, 0)$
Equation of latus Rectum	$\boxed{x = a}$
Ends of latus Rectum	$(a, 2a)$ & $(a, -2a)$
Length of latus Rectum	$\|4a\|$
Parametric Coordinates	

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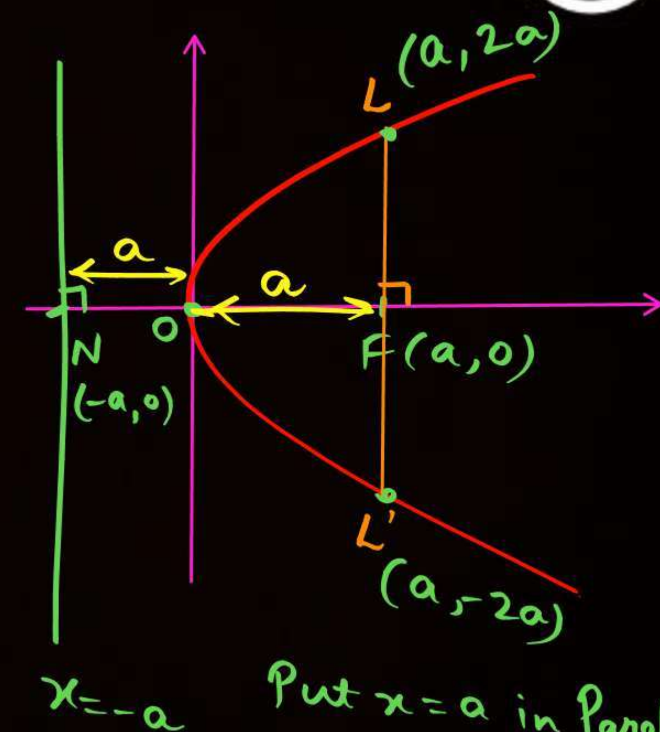


Put $x = a$ in Parabola

$y^2 = 4a \cdot a$

$y^2 = 4a^2$

$y = \pm 2a$

QUESTION

Find every thing for the parabola $\boxed{y^2 = 4x + 4y.}$

Axis	$y = 2$
Focus	$(0, 2)$
Directrix $\longrightarrow$	$\boxed{x = -2}$
Vertex	$(-1, 2)$
Tangent at Vertex	
Foot of Directrix	
Equation of latus Rectum	
Ends of latus Rectum	$(0,0)$ & $(0,4)$
Length of latus Rectum	$4a = 4$

completing the square

$$y^2 - 4y = 4x$$
$$(y-2)^2 - 4 = 4x$$
$$(y-2)^2 = 4x + 4$$
$$\underbrace{(y-2)}_{Y}{}^2 = 4\underbrace{(x+1)}_{X}$$

$$\boxed{Y^2 = 4X}$$ where $$\boxed{\begin{matrix} Y = y - 2 \\ X = x + 1 \end{matrix}}$$

$4a = 4 \Rightarrow \boxed{a = 1}$

Focus $(a, 0) = (1, 0)$

$X = 1$, $Y = 0$

$x + 1 = 1$ & $y - 2 = 0$

$\Rightarrow x = 0$, $y = 2$

Vertex

$X = 0 = x + 1$

$Y = 0 = y - 2$

Directrix

$X = -a$

$x + 1 = -1$

$\boxed{x = -2}$

Axis

$Y = 0$

$y = 2$

Ends of LR

$(a, \pm 2a)$

$X = a \quad , \quad Y = \pm 2a$

$x + \cancel{1} = \cancel{1} \quad , \quad y - 2 = \pm 2$

$x = 0 \quad , \quad y = 2 \pm 2$

$y = 0, 4$

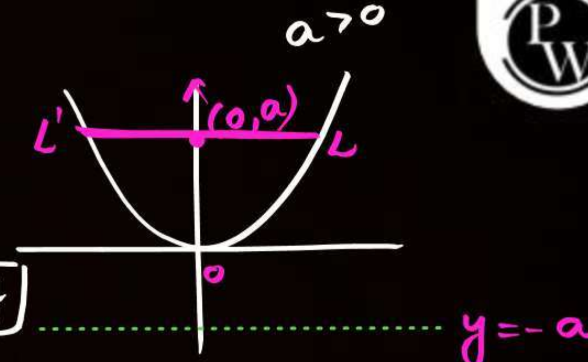
$(0, 0)$

$(0, 4)$

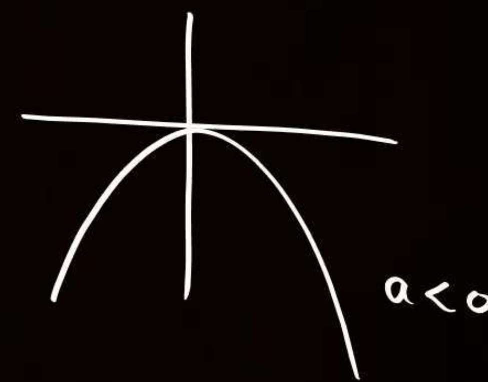
Standard Equation-2

$\boxed{x^2 = 4ay}$

Axis	$\boxed{x = 0}$
Focus	$(0, a)$
Directrix	$\boxed{y = -a}$
Vertex	$(0, 0)$
Tangent at Vertex	$y = 0$
Foot of Directirx	$(0, -a)$
Equation of latus Rectum	$y = a$
Ends of latus Rectum	$(2a, a)$ & $(-2a, a)$
Length of latus Rectum	$\lvert 4a \rvert$
Parametric Coordinates	

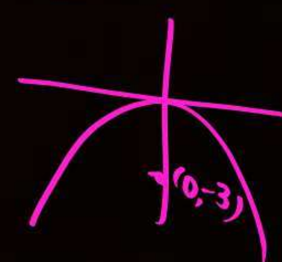


$\boxed{x \leftrightarrow y}$



$\boxed{x^2 = -12y}$

$\boxed{a = -3}$



QUESTION

Find every thing for a parabola $\underbrace{x^2 + 2y = 8x - 7}$.

Axis	
Focus $\longrightarrow (0, a)$	$\longrightarrow (4, 4)$
Directrix	
Vertex	
Tangent at Vertex	
Foot of Directrix	
Equation of latus Rectum	
Ends of latus Rectum	
Length of latus Rectum	

$$x^2 - 8x = -2y - 7$$
$$(x-4)^2 - 16 = -2y - 7$$
$$(x-4)^2 = -2y + 9$$
$$\underbrace{(x-4)^2}_{X} = -2\left(\underbrace{y - 9/2}_{Y}\right)$$
$$\boxed{X^2 = -2Y}$$

$X = x - 4$

$Y = y - 9/2$

$4a = -2$

$a = -\frac{1}{2}$

Focus

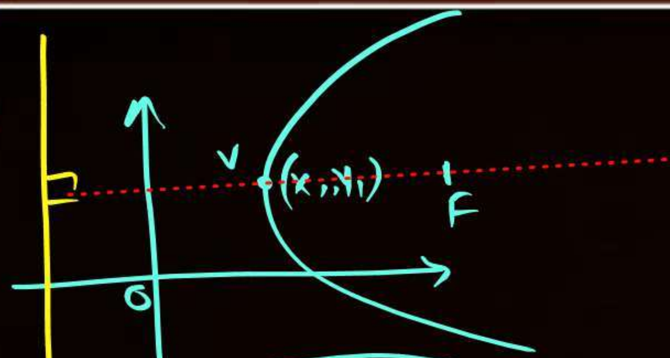
$X = 0$ & $Y = a$

$x - 4 = 0$ & $y - \frac{9}{2} = -\frac{1}{2}$

$x = 4$ & $y = 4$

Shifted Forms of Parabola

1)

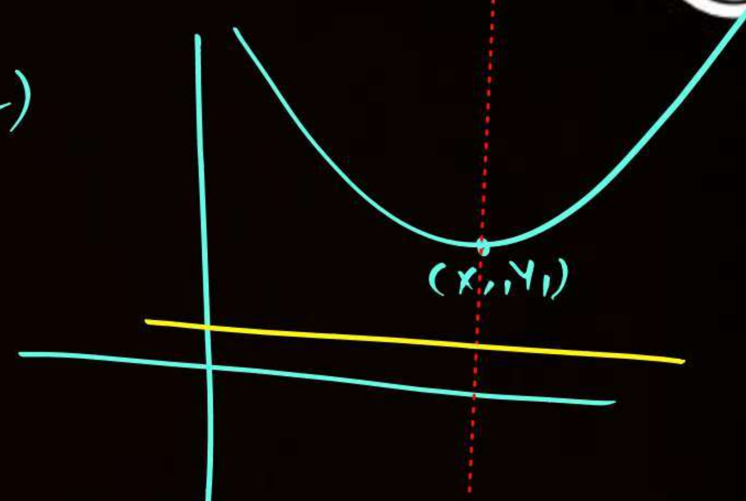


$$\boxed{(y-y_1)^2 = 4a(x-x_1)}$$

where (x_1, y_1) is Vertex

Axis is $\parallel$ to X-axis

2)



$$(x-x_1)^2 = 4a(y-y_1)$$

Axis is $\parallel$ to Y-axis

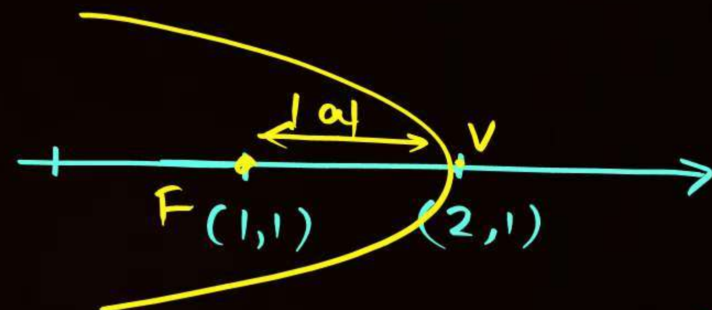
QUESTION

Find the equation of the parabola whose focus is (1,1) and whose vertex is (2,1). Also find its axis and latus rectum.

$\boxed{FV = 1} = |a|$

$\boxed{|a| = 1}$

$\boxed{a = -1}$



axis of Parabola
(y = 1)

⇒ Parallel to x-axis

$$(y - y_1)^2 = 4a(x - x_1)$$

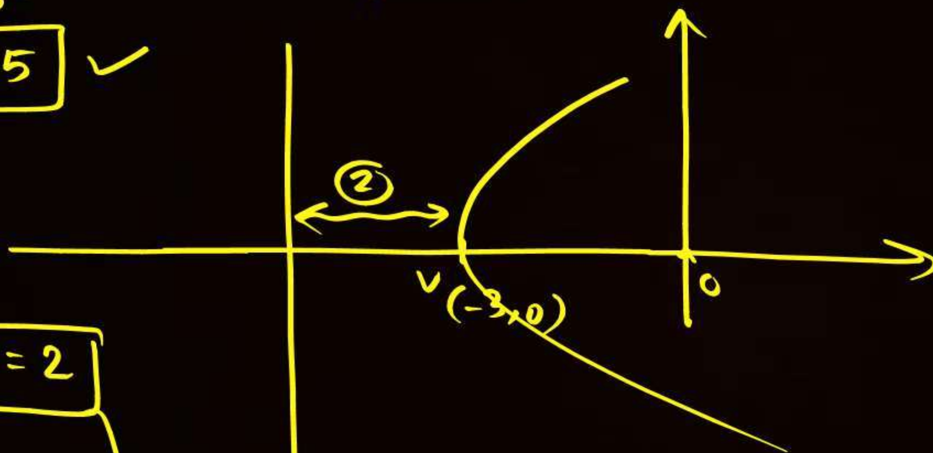
$$\boxed{(y-1)^2 = 4a(x-2)}$$

$$\boxed{(y-1)^2 = -4(x-2)}$$

QUESTION

If the vertex of a parabola is the point $(-3,0)$ and the directrix is the line $\underbrace{x+5=0}$, then find its equation.

$\boxed{x=-5}$ ✓



$\boxed{a=2}$

$x=-5$

$$(y-y_1)^2=4a(x-x_1)$$

$$(y-0)^2=4a(x+3)$$

$$y^2=8(x+3)$$

JEE Main-2019

[Ans. D]

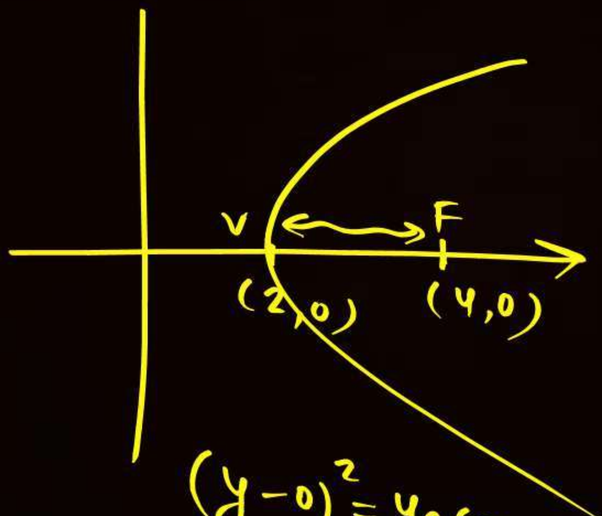
Axis of a parabola lies along x-axis. If its vertex and focus are at distance 2 and 4 respectively from the origin, on the positive x-axis then which of the following points does not lie on it?

(A) $(4, -4)$

(B) $(5, 2\sqrt{6})$

(C) $(6, 4\sqrt{2})$

(D) $(8, 6)$ ✓



$(y-0)^2 = 4a(x-2)$

$y^2 = 4a(x-2)$

$y^2 = 8(x-2)$

$(6)^2 \neq 8(8-2)$

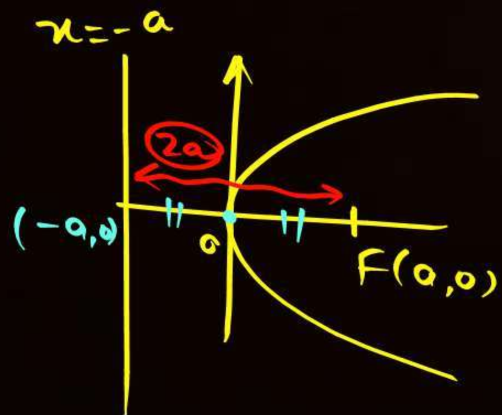
$36 \neq 8 \times 6$

Note that

(i) Perpendicular distance between focus and directrix is equal to semi latus rectum.

(ii) Vertex is middle point of the line joining Focus & the Foot of directrix.

(iii) Two parabolas are said to be equal if they have the same length of latus rectum.



$l.R = 4a$

QUESTION

If equation $\sqrt{(3x-2)^2+(3y+5)^2} = \lambda|3x-4y+7|$ represents parabola then find λ and also find the length of latus rectum. [Ans. [redacted]

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Focus $\left(\frac{2}{3}, -\frac{5}{3}\right)$

Directrix $3x-4y+7=0$

$$d = \left|\frac{3\left(\frac{2}{3}\right)+4\times\frac{5}{3}+7}{5}\right|$$

$$2a = \frac{\frac{26}{3}+7}{5}$$

$$2a = \frac{26+21}{15}$$

$$2a = \frac{47}{15}$$

$$4a = \frac{47\times 2}{15}$$

QUESTION [JEE Main 2022 (June-II)]

If vertex of a parabola is $(2, -1)$ and the equation of its directrix is $4x - 3y = 21$, [Ans. B]
then the length of its latus rectum is

(A) 2

(B) 8 ✓

(C) 12

(D) 16

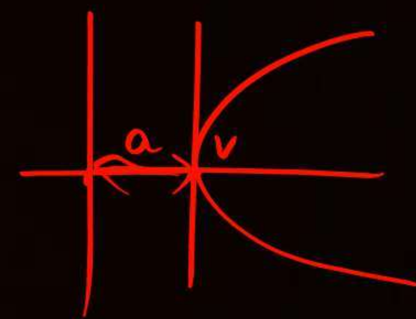
$$d = \left|\frac{4\times 2+3-21}{5}\right|$$

$$d = \left|\frac{11-21}{5}\right|$$

$$a = 10/5 = 2$$

$$\Rightarrow LR = 4a$$

$$= 8$$



JEE Main-2022

If the length of the latus rectum of a parabola, whose focus is (a, a) and the tangent at its vertex is $x + y = a$, is 16, then $|a|$ is equal to:

(A) $2\sqrt{2}$

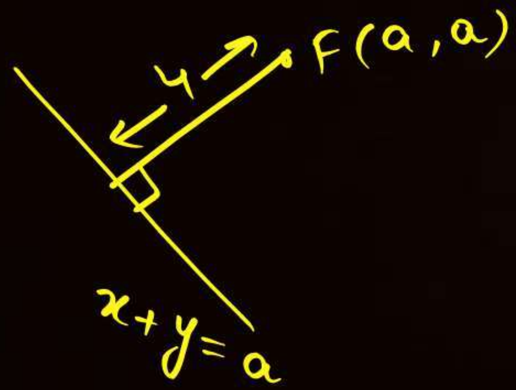
(B) $2\sqrt{3}$

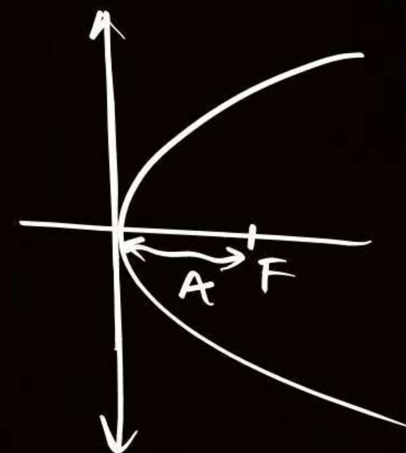
(C) $4\sqrt{2}$ ✓

(D) 4

$4A = 16$

$\boxed{A = 4}$





$$\left|\frac{a + \cancel{a} - \cancel{a}}{\sqrt{2}}\right| = 4$$

$$|a| = 4\sqrt{2}.$$

QUESTION [JEE Main 2022 (June-II)]

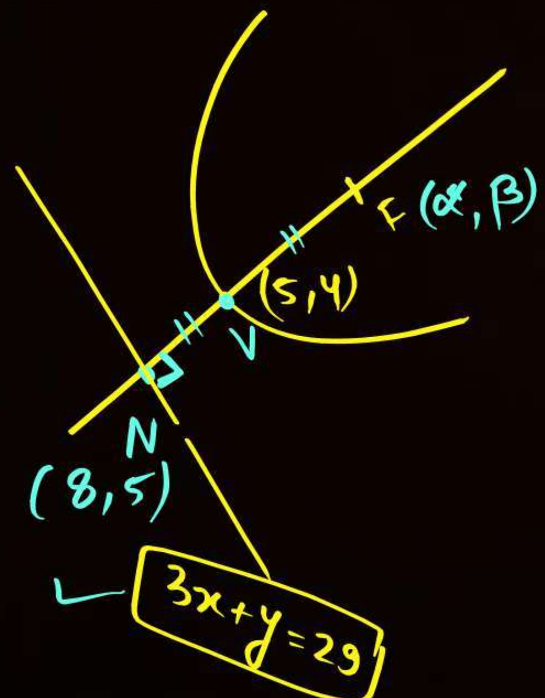
If the equation of the parabola, whose vertex is at (5, 4) and the directrix is $3x + y - 29 = 0$, is $x^2 + ay^2 + bxy + cx + dy + k = 0$ then $a + b + c + d + k$ is equal to [Ans. D]

(A) 575

(B) – 575

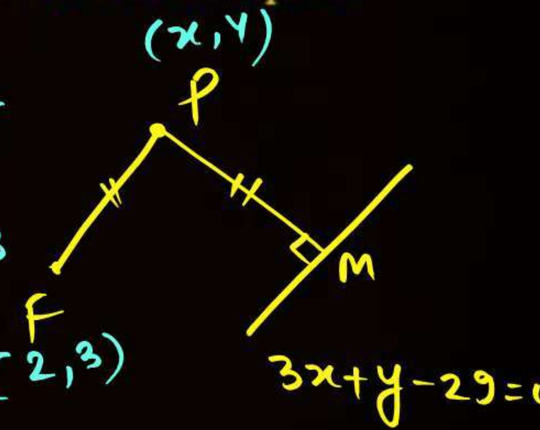
(C) 576

(D) – 576



$$\frac{\alpha+8}{2} = 5 \Rightarrow \alpha = 2$$

$$\frac{\beta+5}{2} = 4 \Rightarrow \beta = 3$$



eqn of Parabola

$$\sqrt{(x-2)^2+(y-3)^2} = \left|\frac{3x+y-29}{\sqrt{10}}\right|$$

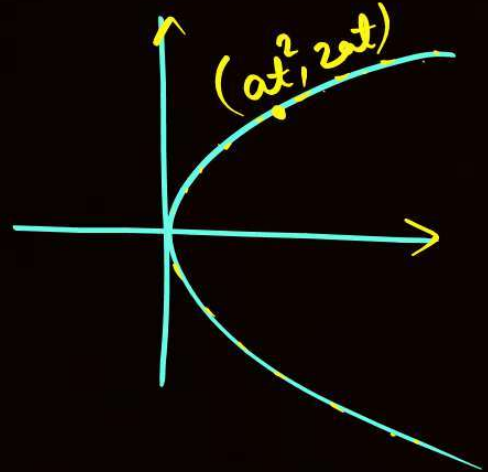
S.B.S.

Parametric Co-ordinates & Focal Distance

$y^2 = 4ax$

$$\boxed{\begin{aligned} x &= at^2 \\ y &= 2at \end{aligned}}$$

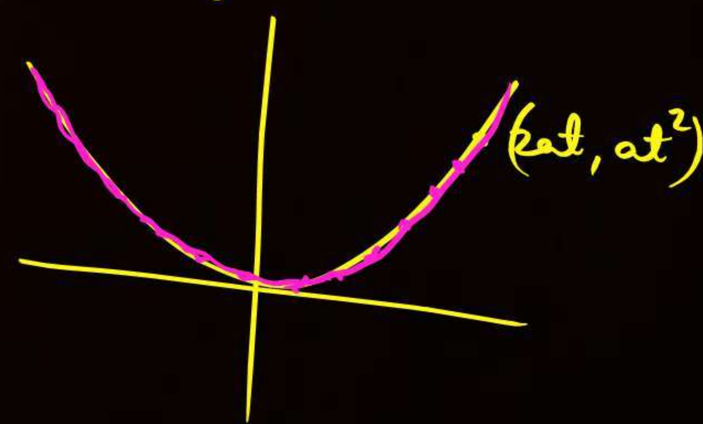
$$\boxed{t \in R}$$



$x^2 = 4ay$

$x = 2at$

$y = at^2$



JEE Main-2021

$\boxed{\text{Locus} \rightarrow \text{study}}$ ✓

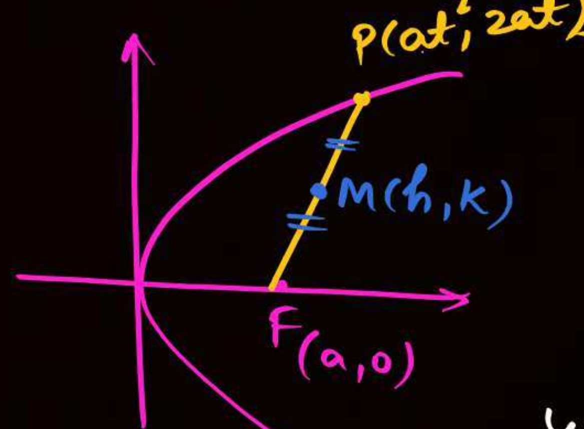
The locus of the mid-point of the line segment joining the focus of the parabola $y^2 = 4ax$ to a moving point of the parabola, is another parabola whose directrix

(A) $x = a$

(B) $x = 0$ ✓

(C) $x = -\frac{a}{2}$

(D) $\frac{a}{2}$



$2h = at^2 + a \Rightarrow 2h = at^2 + a$

$\cancel{2}k = \cancel{2}at + 0$

$\Rightarrow k = at$

$t = k/a$

$2h = a(k/a)^2 + a$

$2h = k^2/a + a$

$(2h - a) = k^2/a$

$k^2 = a(2h - a)$

$y^2 = a(2x - a)$

$\boxed{y^2 = 2a(x - a/2)}$ ✓

$y^2 = 2aX$

$4A = 2a$

$A = a/2$

$X = -A$

$x - a/2 = -a/2$

$x = 0$

Line and a Parabola

$$(mx+c)^2 = 4ax$$

$$m^2x^2 + 2mcx + c^2 = 4ax$$

$$\boxed{m^2x^2 + (2mc-4a)x + c^2 = 0}$$

$\boxed{D=0} \rightarrow$ For Tangent

$$(2mc-4a)^2 - 4m^2c^2 = 0$$

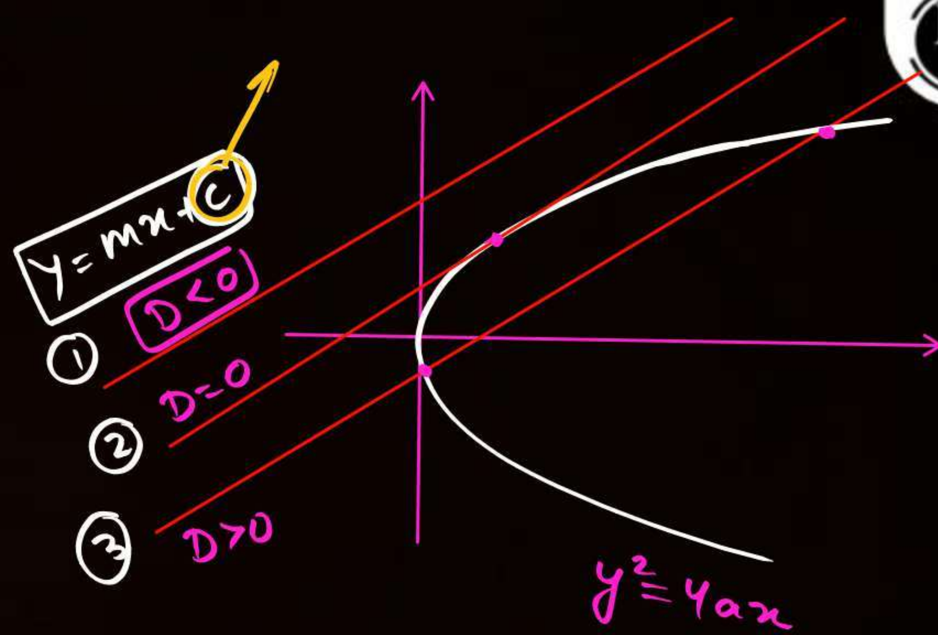
$$(mc-2a)^2 - m^2c^2 = 0$$

$$\cancel{m^2c^2} + 4a^2 - 4acm - \cancel{m^2c^2} = 0$$

$$\cancel{4}a^{\cancel{2}} = \cancel{4}acm$$

$$a = cm$$

$$\boxed{c = a/m}$$

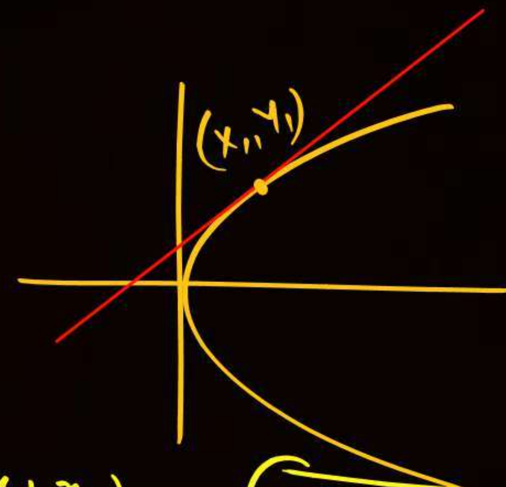


eq^n of Tangent $\boxed{y = mx + a/m}$ ✓

Equation of Tangents to $y^2 = 4ax$

1. **Slope Form of Tangent:**

$\# \quad \boxed{y = mx + a/m}$ ✓ $m \in R - \{0\}$



2. **Cartesian form of Tangent (x_1, y_1):**

$$\boxed{yy_1 = 2a(x + x_1)}$$

$$y^2 = 4ax$$
$$yy_1 = 4a\left(\frac{x + x_1}{2}\right)$$
$$yy_1 = 2a(x + x_1)$$

$$x^2 \to xx_1$$
$$y^2 \to yy_1$$
$$x \to \frac{x + x_1}{2}$$
$$y \to \frac{y + y_1}{2}$$
$$c \to c^2$$

3. **Parametric form of Tangent:**

$$x_1 = at^2, \quad y_1 = 2at$$
$$y \cdot \cancel{2a}t = \cancel{2}a(x + at^2)$$
$$\#\# \quad \boxed{ty = x + at^2}$$

JEE Main-2021 (Feb.-I)

$4a = 6$

$a = 3/2$

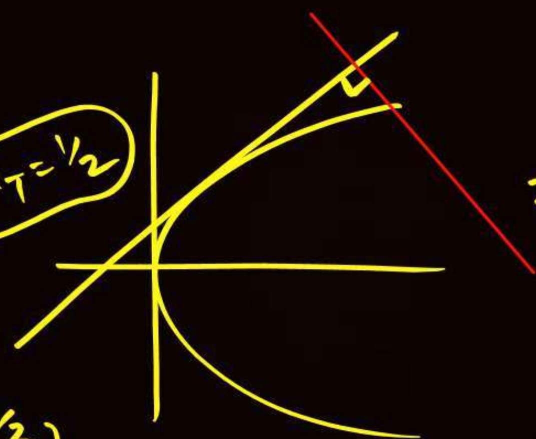
A tangent is drawn to the parabola $y^2 = 6x$ which is perpendicular to the line $2x + y = 1$. Which of the following points does NOT lie on it? [Ans. D]

(A) (0, 3)

(B) (−6, 0)

(C) (4, 5)

(D) (5, 4) ✓



$m_T = 1/2$

$2x + y = 1$

$\boxed{m = -2}$

$m = 1/2$

$y = mx + a/m$

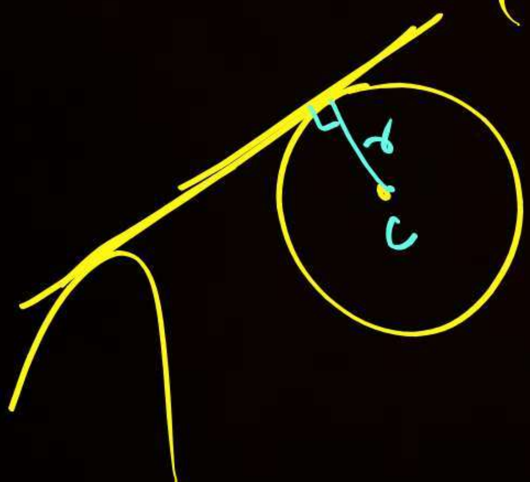
$y = \frac{1}{2}x + \frac{3}{2 \times 1/2}$

$\boxed{y = \frac{x}{2} + 3}$

Finding Eq of Common Tangents

Cirle V/s Conic

1) Write eqⁿ of tangent to conic
('t' form)



2) dis of centre from Tangent = r

Conic V/s Conic

1) Write "slope" form of tangent to both the conics.

2) Compare these eqⁿ.

JEE Main-2023

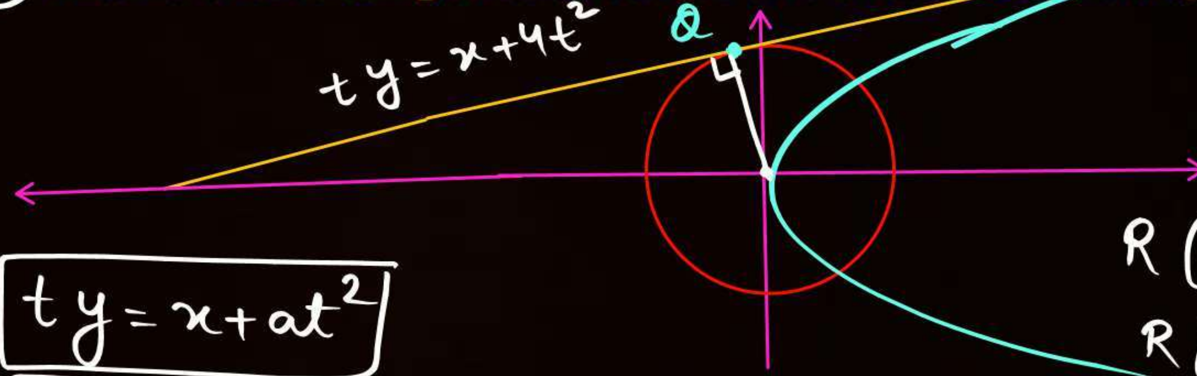
$\boxed{a=4}$

$S_1 = 4^2 + 8^2 - 8$

$S_1 = 16 + 64 - 8 = 8 + 64 = 72$

[Ans. D]

Let A be a point on the x-axis. Common tangents are drawn from A to curves $x^2 + y^2 = 8$ and $y^2 = 16x$. If one these tangents touches the two curves at Q and R, then $(QR)^2$ is equal to



(A) 64

(B) 76

(C) 81

(D) 72 ✓

$\boxed{ty = x + at^2}$

$\boxed{ty = x + 4t^2}$

$\left|\frac{4t^2}{\sqrt{1+t^2}}\right| = 2\sqrt{2} \Rightarrow \left|\frac{\sqrt{2}t^2}{\sqrt{1+t^2}}\right| = 1$

$\boxed{2t^4 = 1 + t^2}$

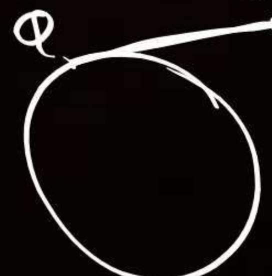
$t^2 = 1$

$\boxed{t = \pm 1}$

$R\,[at^2, 2at]$

$R\,[4t^2, 8t]$

$R\,[4, 8]$



$(QR) = \sqrt{S_1}$

$(QR)^2 = S_1$

$= 72$

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$ty = x + t^2$ $(4, 0)$

Let a common tangent to the curves $y^2 = 4x$ and $(x - 4)^2 + y^2 = 16$ touch the curves at the points P and Q. Then $(PQ)^2$ is equal to ………….. [Ans. 32]

HW

QUESTION [JEE Main 2022 (June-I)]

If $y = m_1x + c_1$ and $y = m_2x + c_2$, $m_1 \neq m_2$ are two common tangents of circle $x^2 + y^2 = 2$ and parabola $y^2 = x$, then the value of $8|m_1m_2|$ is equal to [Ans.

(A) $3 + 4\sqrt{2}$

(B) $-5 + 6\sqrt{2}$

(C) $-4 + 3\sqrt{2}$ ✓

(D) $7 + 6\sqrt{2}$

$r^2 \Rightarrow r = \sqrt{2}$

$4a = 1 \Rightarrow \boxed{a = 1/4}$

$y = mx + a/m$

$\boxed{y = mx + \frac{1}{4m}}$

$\left|\frac{\frac{1}{4m}}{\sqrt{1+m^2}}\right| = \sqrt{2}$

$\frac{1}{16m^2(1+m^2)} = 2$

$\boxed{32m^2(1+m^2) = 1}$

$m^2 = t$

$32t(t+1) = 1$

$\boxed{32t^2 + 32t - 1 = 0}$

$t = \frac{-32 \pm \sqrt{32^2 + 4 \times 32}}{2 \times 32}$

$t = \frac{-32 \pm \sqrt{32 \times 6}}{2 \times 32}$

$t = \frac{-\cancel{32}^{4} \pm \cancel{4}\sqrt{2 \times 6}}{2 \times 32}$

$m^2 = \frac{-4 \pm 3\sqrt{2}}{8}$

$m^2 = \frac{3\sqrt{2} - 4}{8} \rightarrow c$

$m^2 = c$

$m^2 - c = 0$ m_1, m_2

$m_1 m_2 = -c$

$|m_1 m_2| = \left|\frac{3\sqrt{2} - 4}{8}\right|$

JEE Main-2022 (July-II)

Two tangent lines I_1, and I_2, are drawn from the point $(2, 0)$ to the parabola $2y^2 = -x$. If the lines I_1 and I_2 are also tangent to the circle $(x-5)^2 + y^2 = r$, then $17r$ is equal to

[Ans. 9]

$$ty = x + at^2$$

$$y^2 = -x/2$$

$$4a = -1/2$$

$$\boxed{a = -1/8}$$

$$ty = x - \frac{1}{8}t^2$$

$$(2,0)$$

$$0 = 2 - \frac{1}{8}t^2$$

$$t^2 = 16$$

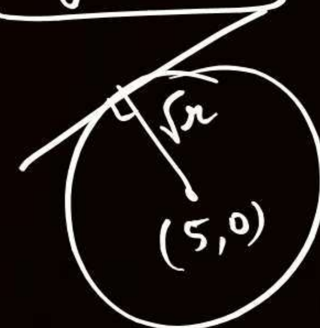
$$\boxed{t = \pm 4}$$

$$t = 4$$

$$4y = x - \frac{1}{8}(4)^2$$

$$4y = x - 2$$

$$\boxed{4y = x - 2}$$



$$\left|\frac{5-2}{\sqrt{17}}\right| = \sqrt{r}$$

$$\frac{3}{\sqrt{17}} = \sqrt{r} \Rightarrow \frac{9}{17} = r$$

JEE Main-2022 (July-I)

HW

[Ans. D]

Let P(a, b) be a point on the parabola $y^2 = 8x$ such that the tangent at P passes through the centre of the circle $x^2 + y^2 - 10x - 14y + 65 = 0$. Let A be the product of all possible values of a and B be the product of all possible values of b. Then the value of A + B is equal to:

(A) 0

(B) 25

(C) 40

(D) 65

Equation of Tangents to $x^2 = 4ay$

1. **Slope Form of Tangent:** $y = mx - am^2$

2. **Cartesian form of Tangent (x_1, y_1):** $x x_1 = 2a(y + y_1)$

3. **Parametric form of Tangent:** $t x = y + at^2$

Question

Find the equation of Common tangents to $\underbrace{y^2 = 4x}$ & $\underbrace{x^2 = 4y}$

$$y = mx + \frac{1}{m} \longrightarrow (1)$$

$$y = mx - m^2 \longrightarrow (2)$$

$$\frac{1}{1} = \frac{m}{m} = \frac{1/m}{-m^2}$$

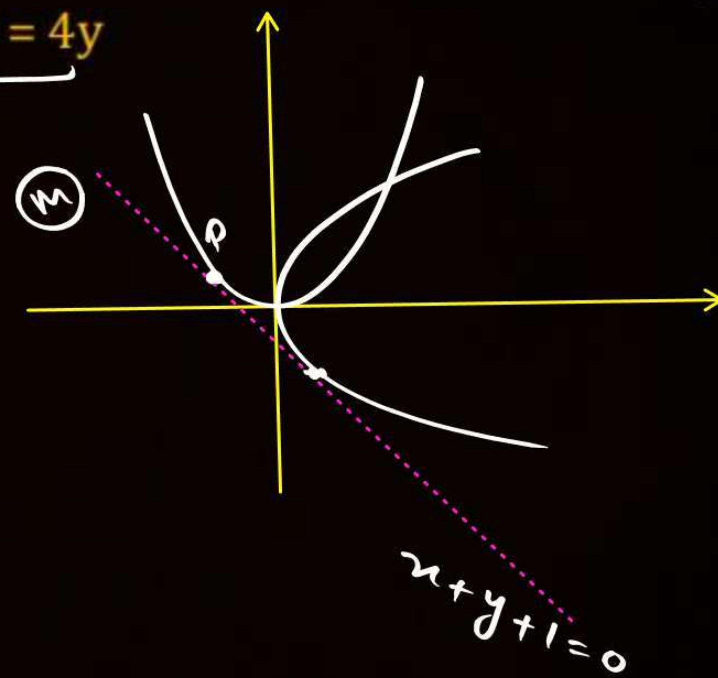
$$\frac{1}{m} = -m^2$$

$$m^3 = -1$$

$$\boxed{m = -1}$$

$$y = -x - 1$$

$$\Rightarrow \boxed{y + x + 1 = 0}$$



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$\boxed{x^2 = 4ay}$

$4a = 1$, $a = 1/4$ $\qquad$ $4a = -1$, $a = -1/4$

The equation of a common tangent to the parabolas $y = x^2$ and $y = -(x-2)^2$ is

(A) $y = 4(x-2)$

(B) $y = 4(x-1)$ ✓

(C) $y = 4(x+1)$

(D) $y = 4(x+2)$

$x^2 = y$

$y = mx - am^2$

$y = mx - \frac{1}{4}m^2$

$y = mx - m^2/4$

$-m^2/4 = -2m + m^2/4$

$\cancel{2}m = \cancel{2}\frac{m^2}{4}$

$4m = m^2$

$m = 0$ or $m = 4$

$(x-2)^2 = -y$

$y = m(x-2) - am^2$

$y = m(x-2) + \frac{1}{4}m^2$

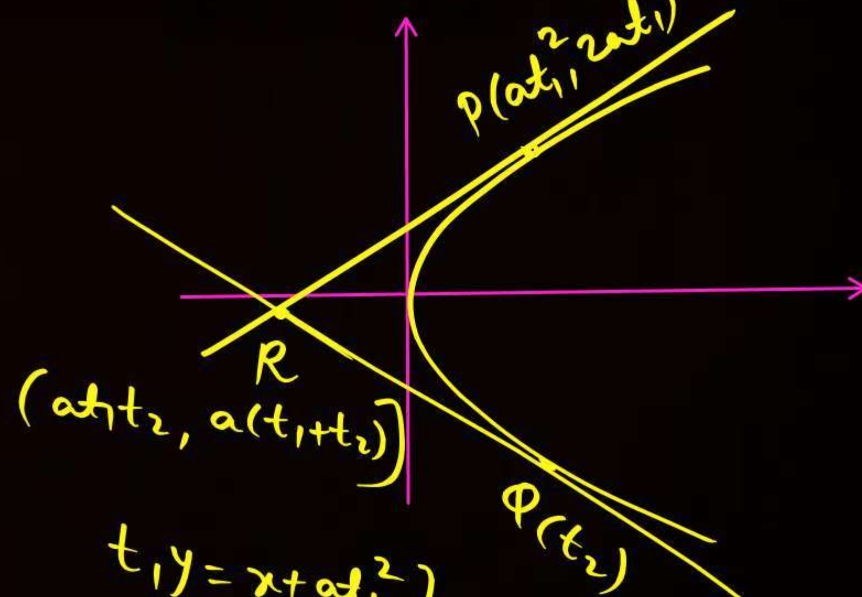
$y = mx - 2m + \frac{1}{4}m^2$

$y = 4x - \frac{1}{4}(4)^2$

$\boxed{y = 4x - 4}$

Point of Intersection of Tangents Drawn at $P(t_1)$ & $Q(t_2)$

$$\boxed{x = at_1t_2 \text{ and } y = a(t_1 + t_2)}$$



$$\left.\begin{aligned} t_1 y &= x + at_1^2 \\ t_2 y &= x + at_2^2 \end{aligned}\right\} \text{Solve}$$

JEE Main-2023

$(a, 0)$ $\quad \boxed{a=3} \quad y^2 = 4ax$

$x = -a \quad \boxed{y^2 = 12x}$

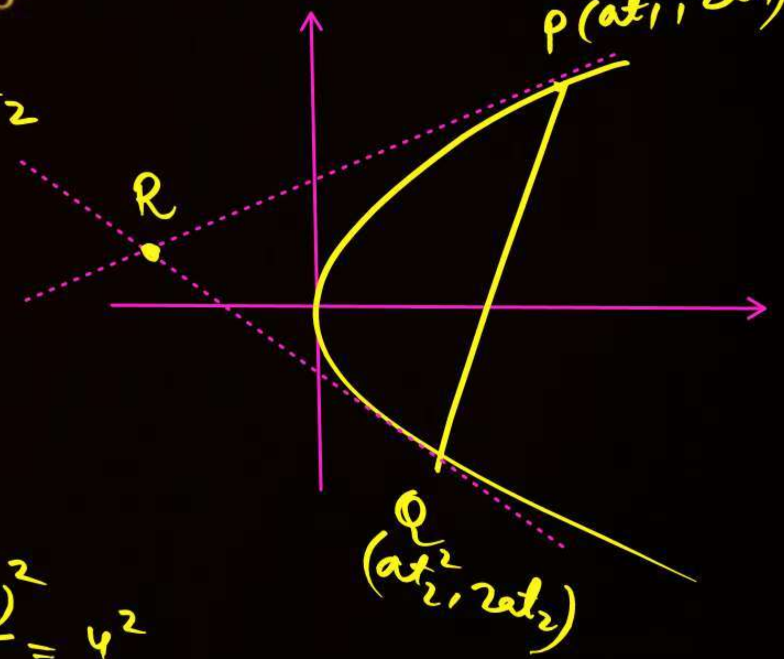
The ordinates of the points P and Q on the parabola with focus (3, 0) and directrix $\underbrace{x = -3}$ are in the ratio 3 : 1. If R(α, β) is the point of intersection of the tangents to the parabola at P and Q, then $\frac{\beta^2}{\alpha}$ is equal to **[Ans. 16]**

$$\frac{\cancel{2a}t_1}{\cancel{2a}t_2} = \frac{3}{1} \Rightarrow \boxed{\frac{t_1}{t_2} = 3} \Rightarrow t_1 = 3t_2$$

$$\alpha = at_1t_2 = 3\cdot 3t_2t_2 = \boxed{9t_2^2 = \alpha}$$

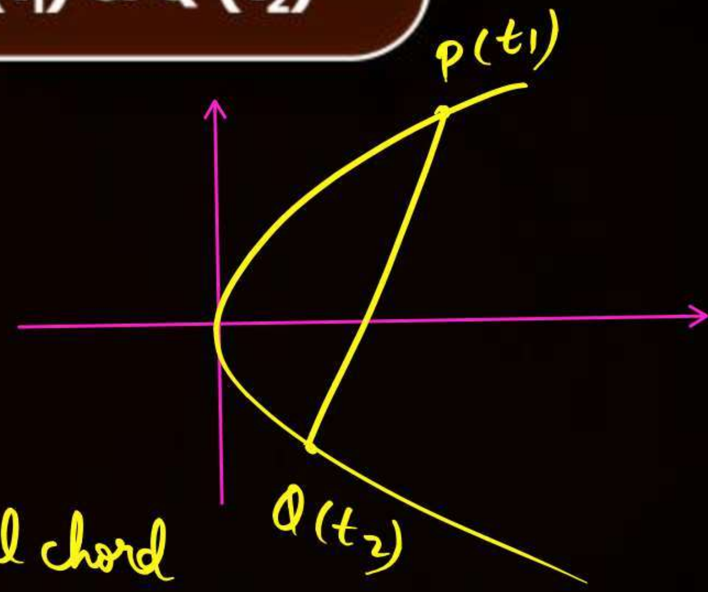
$$\beta = a(t_1+t_2) = 3[3t_2+t_2] = 12t_2 = \beta$$

$$\frac{\beta^2}{\alpha} = \frac{(12t_2)^2}{9t_2^2} = \frac{(12)^2}{9} = \frac{(4\times\cancel{3})^2}{\cancel{3}^2} = 4^2$$



Equation of Chord Joining $P(t_1)$ & $Q(t_2)$

$y^2 = 4ax$



Chord Joining $P(t_1)$ & $Q(t_2)$ is given by

$$\boxed{2x - (t_1 + t_2)y + 2a\,t_1\,t_2 = 0}$$

$$\boxed{m = \frac{2}{t_1 + t_2}}$$

For PQ to be a focal chord

(a,0) satisfies

$2a + 2at_1t_2 = 0$

$\boxed{t_1 t_2 = -1}$

Question

$a = 3$

If the line $y = 3x + c$ touches the parabola $y^2 = 12x$ at point P, then find the equation of the tangent at point Q, where PQ is a focal chord.

$ty = x + at^2$

$y = \frac{x}{t} + at$

$\boxed{m = \frac{1}{t}} = 3$

$\Rightarrow t_1 = \frac{1}{3}$

$\boxed{t_1 t_2 = -1}$

$\boxed{t_2 = -3}$

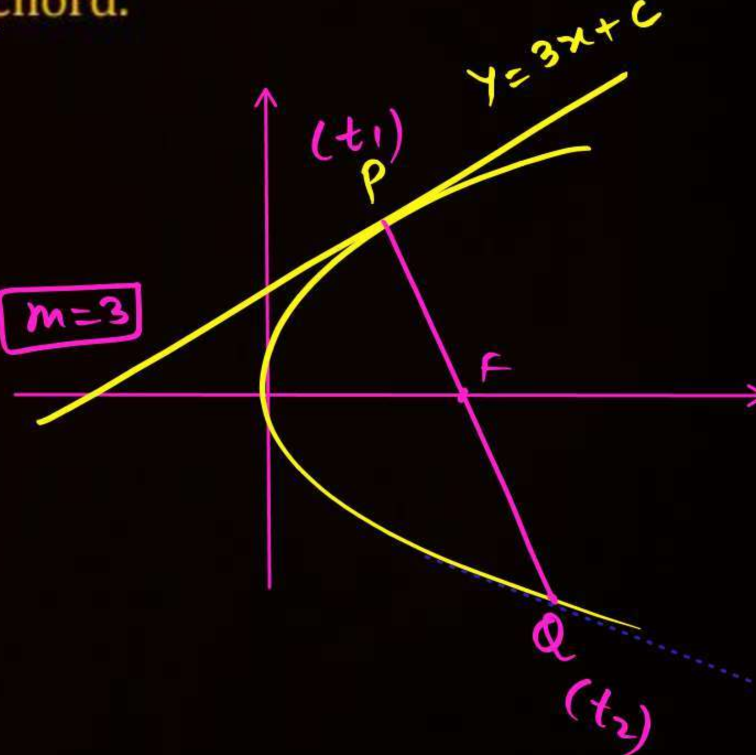
$t_2 y = x + a t_2^2$

$-3y = x + 3(-3)^2$

$-3y = x + 27.$

$\boxed{x + 3y + 27 = 0}$

$\boxed{m = 3}$



Note that

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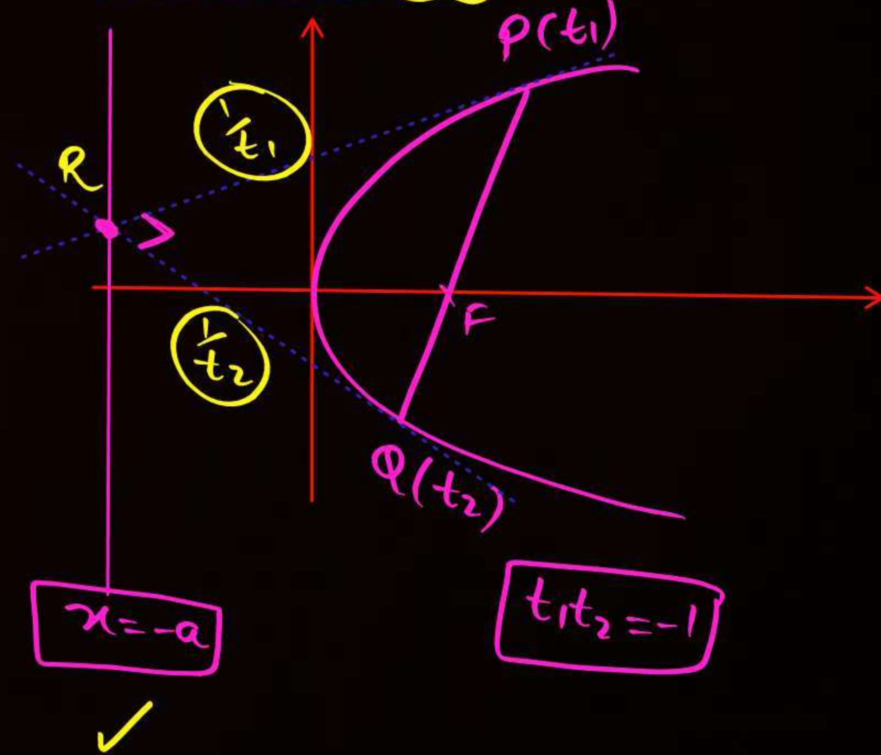
Tangents drawn at the ends of any focal chord are always Perpendicular and they always meet on the **Directrix**.

$$m_1 m_2 = \frac{1}{t_1}\cdot\frac{1}{t_2} = \frac{1}{t_1 t_2}$$

$$\boxed{m_1 m_2 = -1}$$

$$R\left[a\overset{-1}{t_1 t_2},\ a(t_1+t_2)\right]$$

$$R\left[-a,\ \sim\right]$$

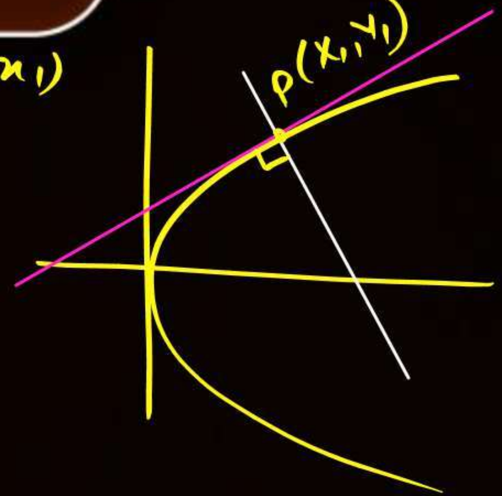


Equations of Normals to $y^2 = 4ax$

$yy_1 = 2a(x+x_1)$

$m_T = 2a/y_1$

$m_N = -y_1/2a$



1. Cartesian form of Normal at (x_1, y_1):

$$y - y_1 = -\frac{y_1}{2a}(x - x_1)$$

$x_1 = at^2, \quad y_1 = 2at$

2. Parametric form of Normal:

$$\boxed{y + tx = 2at + at^3}$$

3. Slope Form of Normal:

$$\boxed{y = mx - 2am - am^3}$$

Equations of Normals to $x^2 = 4ay$

1. **Cartesian form of Normal at (x_1, y_1):**

$$y - y_1 = -\frac{2a}{x_1}(x - x_1)$$

2. **Parametric form of Normal:**

$$\boxed{x + ty = 2at + at^3}$$

3. **Slope Form of Normal:**

$$\boxed{y = mx + 2a + \frac{a}{m^2}}$$

QUESTION [JEE Main 2022 (June-I)]

$4a = 6$

$a = 3/2$

[Ans. B]

Let the normal at the point P on the parabola $y^2 = 6x$ pass through the point $(5, -8)$. If the tangent at P to the parabola intersects its directrix at the point Q, then the ordinate of the point Q is:

(A) -3

(B) $-\frac{9}{4}$ ✓

(C) $-\frac{5}{2}$

(D) -2

$y + tx = 2at + at^3$

$(5, -8)$

$-8 + 5t = 2at + at^3$

$-8 + 5t = 3t + \frac{3}{2}t^3$

$\boxed{-8 + 2t = \frac{3}{2}t^3}$

$\boxed{t = -2} \rightarrow$ satisfies

$-8 - 4 = \frac{3}{2}(-2)^3$

$\boxed{-12 = -12}$

eq^n of T:

$ty = x + at^2$

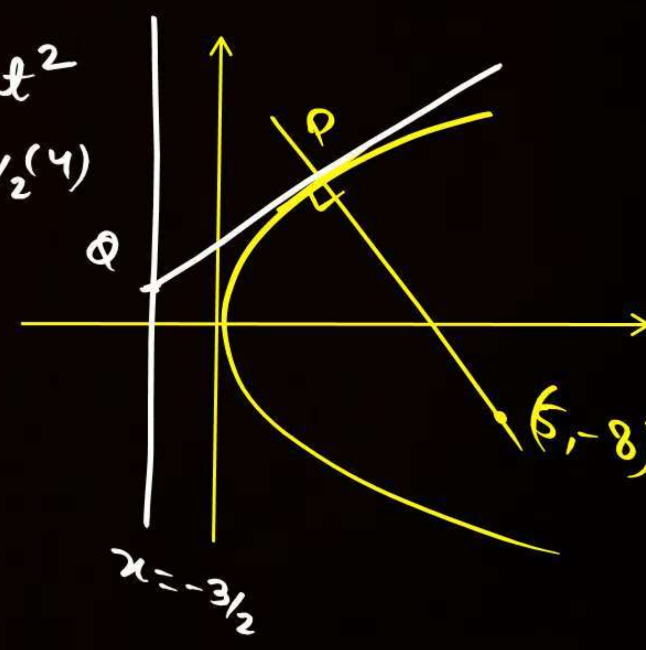
$-2y = x + \frac{3}{2}(4)$

$-2y = x + 6$

$-2y = -\frac{3}{2} + 6$

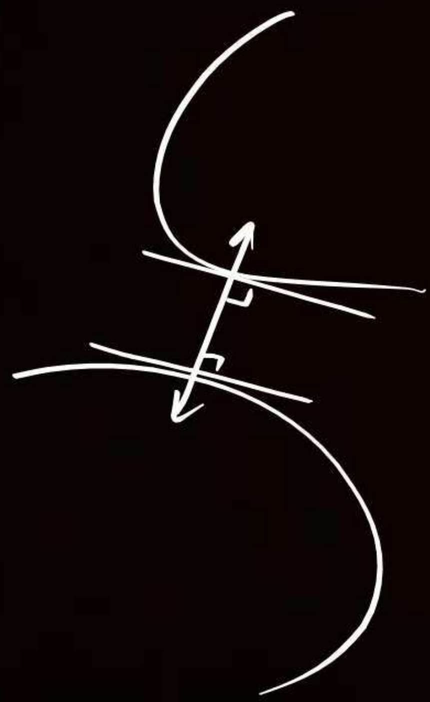
$-2y = \frac{9}{2}$

$y = -\frac{9}{4}$.



Min Distance Between 2 Curves

Min distance always occurs ~~along~~ along common normal



JEE Main-2021 (Feb.-II)

$$\frac{dy}{dx} = \boxed{2x = 4}$$

$$\boxed{x = 2}$$

If P is a point on the parabola $y = x^2 + 4$ which is closest to the straight line $y = 4x - 1$, then co-ordinates of P are: [Ans. D]

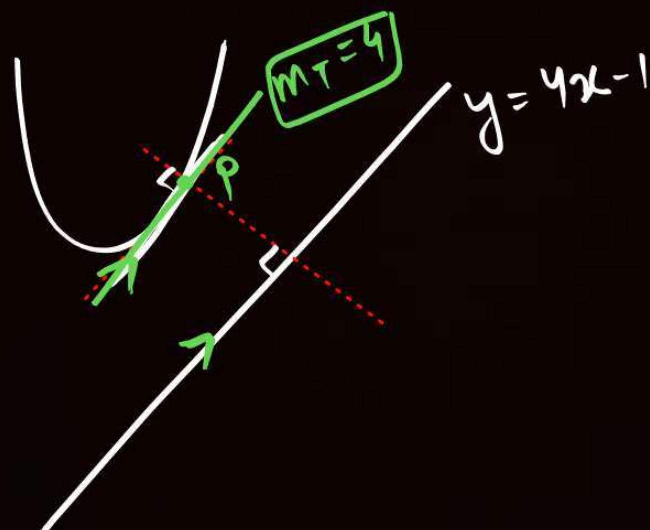
$$y = 2^2 + 4 = 8$$

(A) $(-2, 8)$

(B) $(1, 5)$

(C) $(3, 13)$

(D) $(2, 8)$ ✓



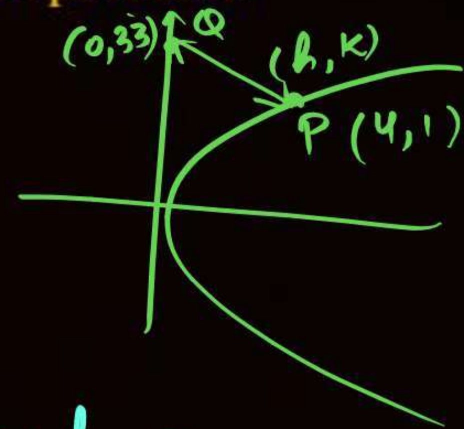
JEE Main-2023

$\boxed{y^2 = x/4}$ $\quad 4a = 1/4$

$a = 1/16$

[Ans. D]

If P(h, k) be point on the parabola $x = 4y^2$, which is nearest to the point Q(0, 33), then the distance of P from the directrix of the parabola $\underbrace{y^2 = 4(x + y)}$ is equal to :



$y + tx = 2at + at^3$

$33 = 2at + at^3$

$$\boxed{33 = \frac{t}{8} + \frac{t^3}{16}}$$

$t = 8$

$h = at^2 = \frac{1}{16} \times 8^2 = 4$

$k = 2at = 2 \times \frac{1}{16} \times 8 = 1$

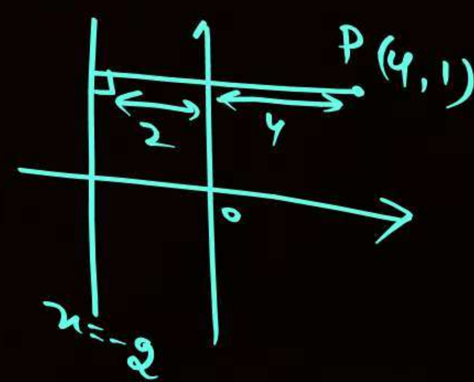
(A) 2

(B) 4

(C) 8

(D) 6 ✓

Directrix is $x = -2$



JEE ADV-2023

→ $\boxed{\text{Try}}$

$$\underbrace{a^2 m(1+m^2)}_{} = 4\times 3\times 10$$

$a^2 = 4 \Rightarrow a = 2$ $\quad$ $m(1+m^2) = 3\times 10$

$m = 3$

Let P be a point on the parabola $y^2 = 4ax$, where $a \geq 0$. The normal to the parabola at P meets x-axis at a point Q. The area of the triangle PFQ, where F is the focus of the parabola, is 120. If the slope m of the normal and a are both positive integers, then the pair (a, m) is [Ans.

(A) $(2, 3)$ $\quad \boxed{m = -t}$

$t = -m$

(B) [redacted]

(C) [redacted]

(D) [redacted]

$$y + tx = 2at + at^3$$

$y = 0$

$tx = 2at + at^3$

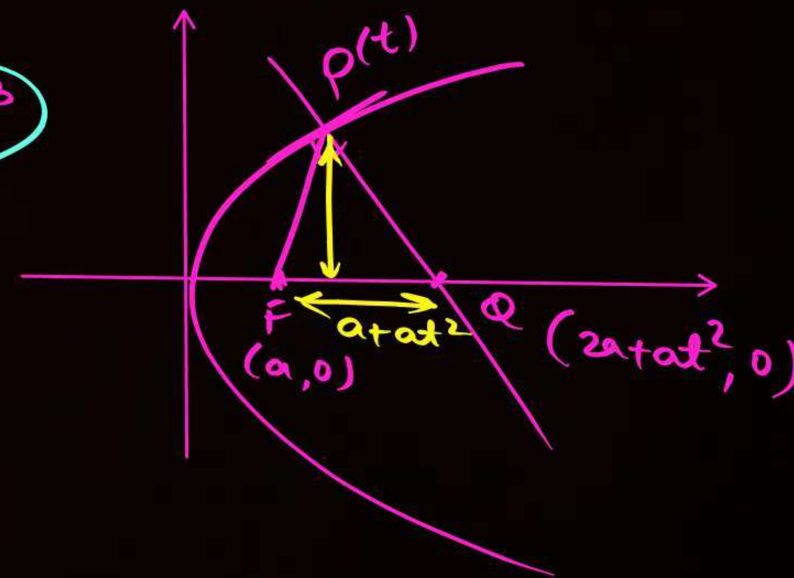
$x = 2a + at^2$

$$\left|\frac{1}{\cancel{2}}\cdot \cancel{2}at\,(a + at^2)\right| = 120$$

$$|a^2 t(1+t^2)| = 120.$$

$$|a^2(-m)(1+m^2)| = 120$$

$$a^2 m(1+m^2) = 120$$



Note that

Normal chord

If the normal to the parabola $y^2 = 4ax$ at the point t_1, meets the parabola again at point t_2, then $t_2 = -t_1 - \frac{2}{t_1}$.

$$\boxed{t_2 = -t_1 - 2/t_1}$$

✓

$$m_{PQ} = \frac{2}{t_1+t_2}$$

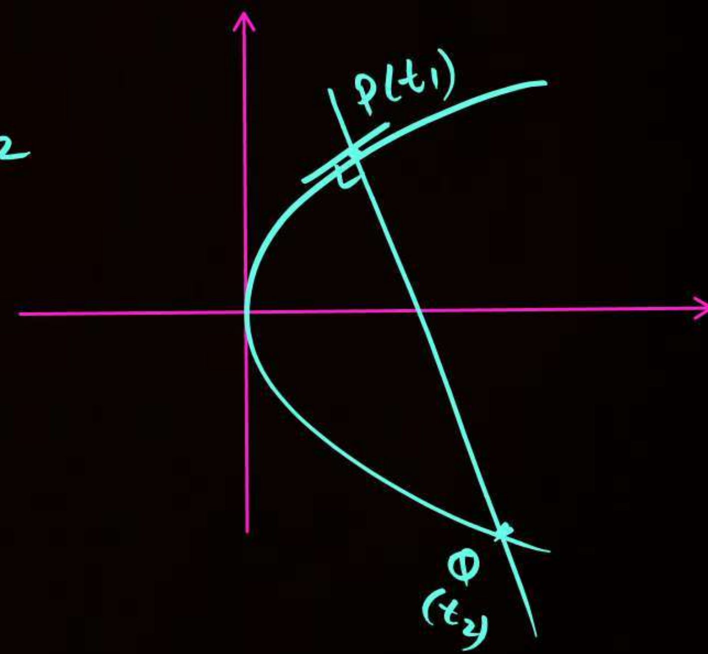
$$m_N = -t_1$$

equate

$$\frac{2}{t_1+t_2} = -t_1$$

$$-\frac{2}{t_1} = t_1 + t_2$$

$$t_2 = -t_1 - 2/t_1$$



Question

$\boxed{a=3}$

If the normal to $y^2 = 12x$ at $P(3, 6)$ meets the parabola again at a point Q. Find the equation of circles with PQ as diameter.

$6 = 2at_1$

$\boxed{t_1 = 1}$

$t_2 = -t_1 - 2/t_1$

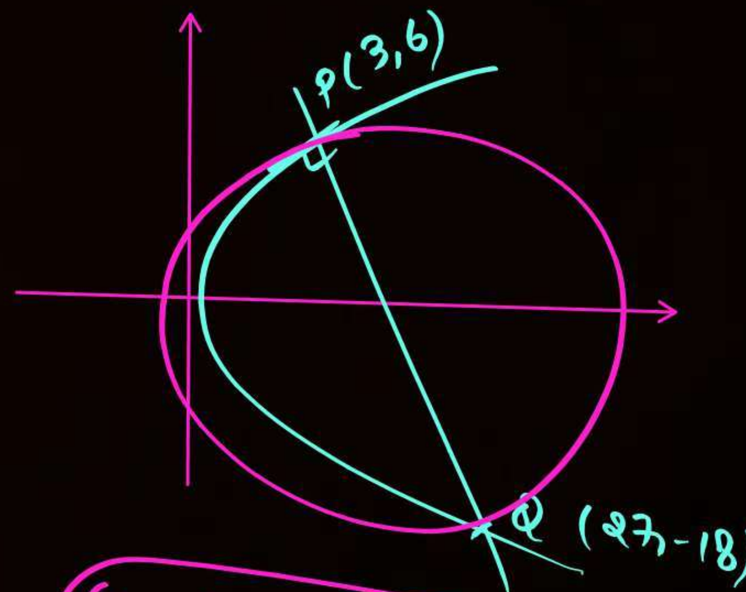
$t_2 = -1 - 2/1$

$\boxed{t_2 = -3}$

$Q\,[at_2^2,\ 2at_2]$

$Q\,[3(-3)^2,\ 2\times3(-3)]$

$Q\,(27, -18)$



$(x-3)(x-27) + (y-6)(y+18) = 0$

Co-Normal Points

$$y + tx = 2at + at^3$$

$$(x_1, y_1)$$

$$y_1 + tx_1 = 2at + at^3$$

$$\boxed{at^3 + (2a - x_1)t - y_1 = 0} \quad t_1, t_2, t_3$$

The feet of 3 Concurrent normals are called co-normal Points.

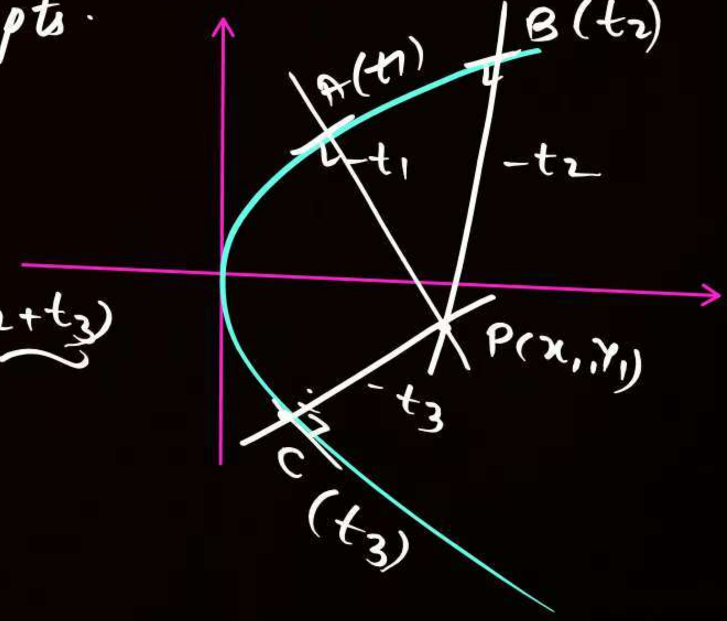
here A, B, C are co-normal pts.

$$\boxed{t_1 + t_2 + t_3 = 0}$$

$$m_1 + m_2 + m_3 = -\underbrace{(t_1 + t_2 + t_3)}$$

$$= 0$$

$$\boxed{y_1 + y_2 + y_3 = 0}$$



Co-Normal Points

$m_N = -t$
$m_1 = -t_1$

- If $P(t_1), Q(t_2)$ and $R(t_3)$ are co-normal points of a parabola $y^2 = 4ax$ then $\boxed{t_1 + t_2 + t_3 = 0}$.
- Algebraic sum of slopes of the three concurrent normal is zero.
- Algebraic sum of ordinates of the three co-normal points on the parabola is zero.
- Centroid of the Δ formed by three co-normal points lies on the axis of parabola (x-axis).

Question

The parabola $y^2 = 4ax$ and the line $lx + my = n$ meet at two distinct points A and B. Normals drawn to the parabola at these points A and B intersect at a point P. Now a third normal is drawn passing through the point P. Find the coordinates of feet of this 3rd normal.

$$\boxed{y_1 + y_2 + y_3 = 0}$$

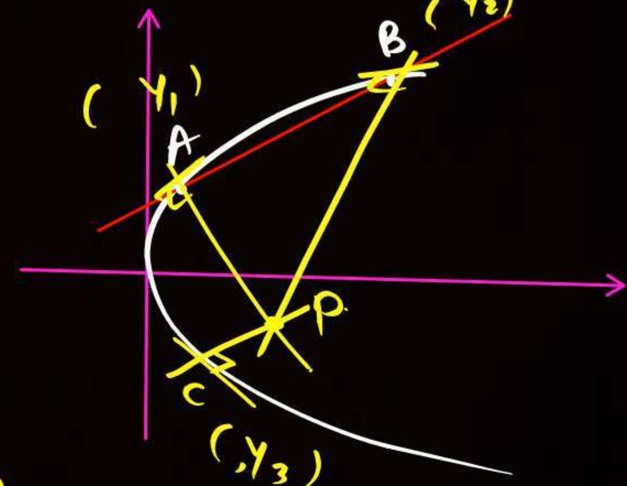
$$y_3 = -(y_1 + y_2)$$

$$y^2 = 4a\left[\frac{n - my}{l}\right]$$

$$ly^2 + 4amy - 4an = 0 \quad \begin{matrix} y_1 \\ y_2 \end{matrix}$$

$$y_1 + y_2 = -\frac{4am}{l}$$

$$\boxed{y_3 = \frac{4am}{l}}$$



$$\left(\frac{4am}{l}\right)^2 = 4ax$$

$$\frac{4am^2}{l^2} = x$$

$$C\left[\frac{4am^2}{l^2}, \frac{4am}{l}\right]$$

Note that

#

If P and R are the points t_1 and t_2 on the parabola $y^2 = 4ax$ such that the normal to the parabola at P and R meet at Q (a point on the parabola) then $\boxed{t_1 t_2 = 2}$ and the line PR passes through a fixed point on the axis of the curve.

PQ → normal chord

$$\boxed{t_3 = -t_1 - \frac{2}{t_1}}$$

RQ → normal chord

$$\boxed{t_3 = -t_2 - \frac{2}{t_2}}$$

equate

$$-t_1 - \frac{2}{t_1} = -t_2 - \frac{2}{t_2}$$

$$t_1 + \frac{2}{t_1} = t_2 + \frac{2}{t_2}$$

$$t_1 - t_2 = \frac{2}{t_2} - \frac{2}{t_1}$$

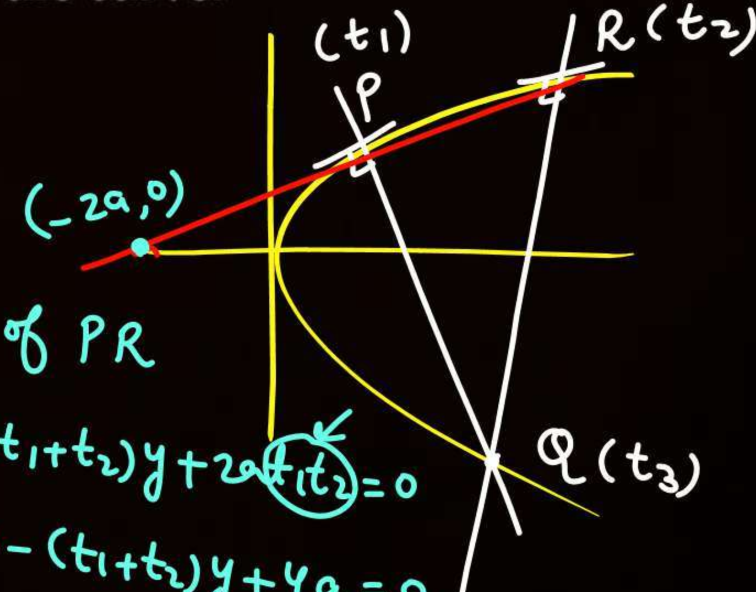
$$\boxed{t_1 t_2 = 2}$$

eqn of PR

$$2x - (t_1+t_2)y + 2a t_1 t_2 = 0$$

$$2x - (t_1+t_2)y + 4a = 0.$$

$(-2a, 0)$



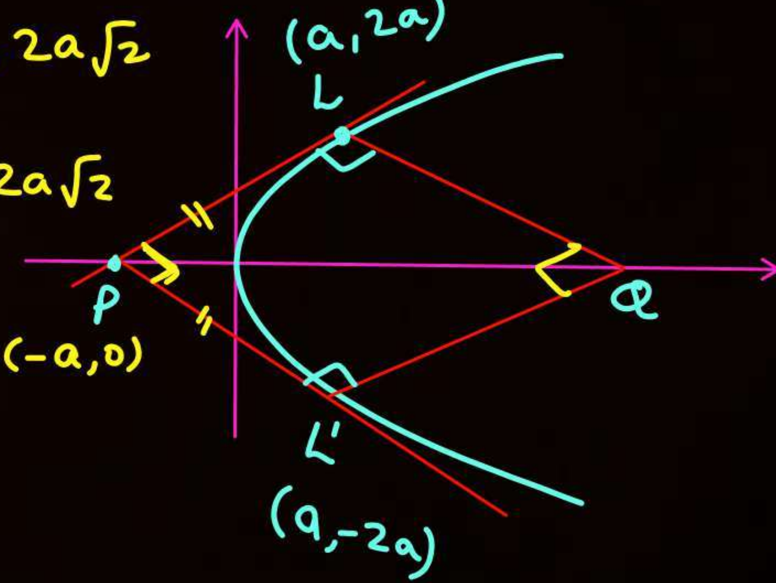
Note that

Tangents & Normals drawn at the ends of latus rectum enclose a square of side length $(2\sqrt{2}a)$

$$PL = \sqrt{(2a)^2 + (2a)^2} = 2a\sqrt{2}$$

$$PL' = \sqrt{(2a)^2 + (2a)^2} = 2a\sqrt{2}$$

$\Rightarrow \boxed{\text{Square}}$ ✓



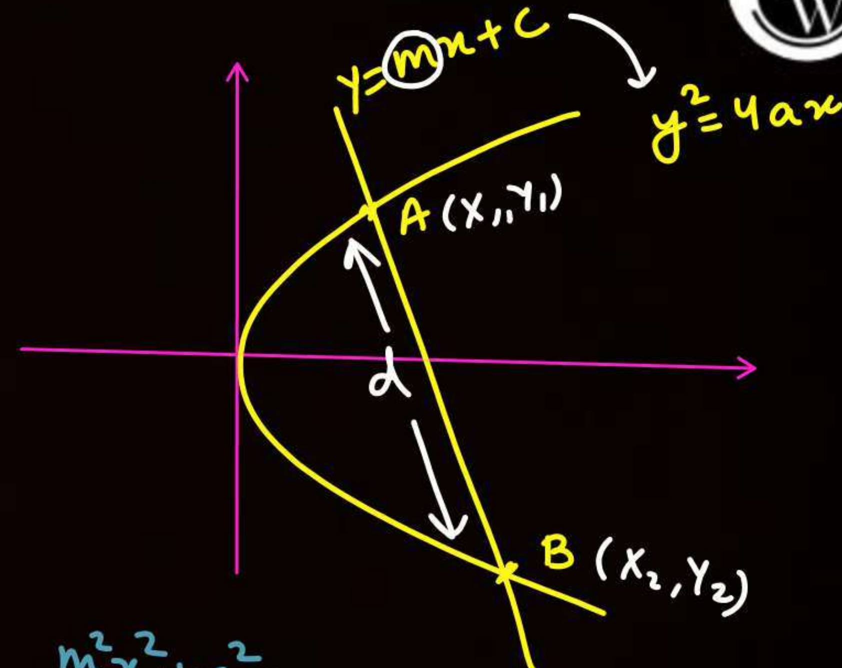
Length of a Chord

$$(mx+c)^2 = 4ax$$

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$d = |x_2-x_1|\sqrt{1+\left(\frac{y_2-y_1}{x_2-x_1}\right)^2}$$

$$d = |x_2-x_1|\sqrt{1+m^2}$$



$$m^2x^2+c^2+2mxc = 4ax$$

$$m^2x^2+2mxc-4ax+c^2=0 \quad \begin{matrix} x_1 \\ x_2 \end{matrix}$$

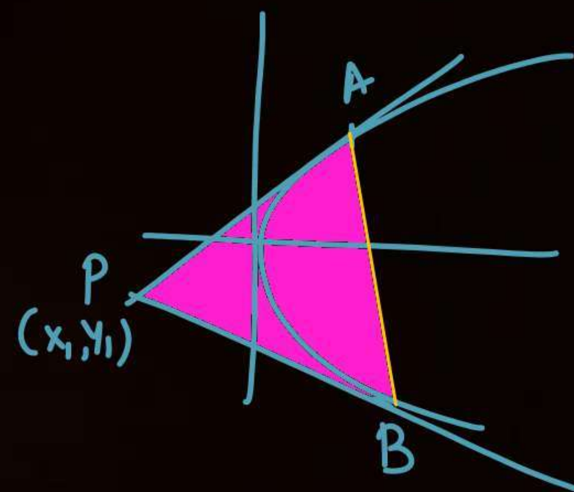
$$|x_1-x_2| = \frac{\sqrt{D}}{m^2}$$

Chords of Parabola

Chord of Contact

$$\boxed{T=0}$$

$$\boxed{yy_1 = 2a(x+x_1)}$$



$$\text{Area of } \triangle PAB = \frac{(y_1^2 - 4ax_1)^{3/2}}{2a}$$

$$= \frac{(S_1)^{3/2}}{2a}$$

Question

Find the locus of point whose chord of contact with respect to the parabola $y^2 = 4bx$ is the tangent of the parabola $y^2 = 4ax$.

$(h, k) \rightarrow \underbrace{\text{C.O.C}}$

$$\boxed{yk = 2b(x+h)} \rightarrow (1)$$

$$\boxed{y = mx + a/m} \rightarrow (2)$$

$$\underbrace{\frac{k}{1} = \frac{2b}{m}} = \underbrace{\frac{2bh}{a/m}}$$

$$m = \frac{2b}{k} \qquad k = \frac{2bh}{a} m$$

$$k = \frac{2bh}{a} \cdot \frac{2b}{k}$$

$$k^2 = \frac{4b^2}{a} h$$

$$\boxed{y^2 = \frac{4b^2}{a} x} \checkmark$$

NOTE

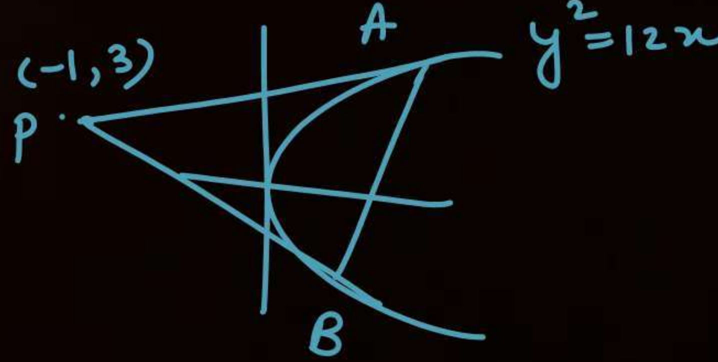
The area of the triangle formed by the tangents from the point (x_1, y_1) and the chord of contact is $\boxed{\dfrac{(y_1^2 - 4ax_1)^{3/2}}{2a} \text{ i.e. } \dfrac{(S_1)^{3/2}}{2a}}$

$y^2 - 12x = 0$

$S_1 : (3)^2 - 12(-1)$

$= 9 + 12 = 21$

$$\boxed{\text{Area} = \frac{(21)^{3/2}}{2 \times 3}}$$



JEE Main-2022 (July-I)

$y^2 = 2(x - 3/2)$

If the tangents drawn at the points P and Q on the parabola $y^2 = 2x - 3$ intersect at the point R(0, 1), then the orthocentre of the triangle PQR is: [Ans. B]

(A) (0, 1)

(B) (2, –1) ✓

(C) (6, 3)

(D) (2, 1)

eqⁿ of PQ:

$yy_1 = (x + x_1) - 3$

$x_1 = 0,\ y_1 = 1$

$\boxed{y = x - 3}$

$(x-3)^2 = 2x - 3$

$x^2 + 9 - 6x = 2x - 3$

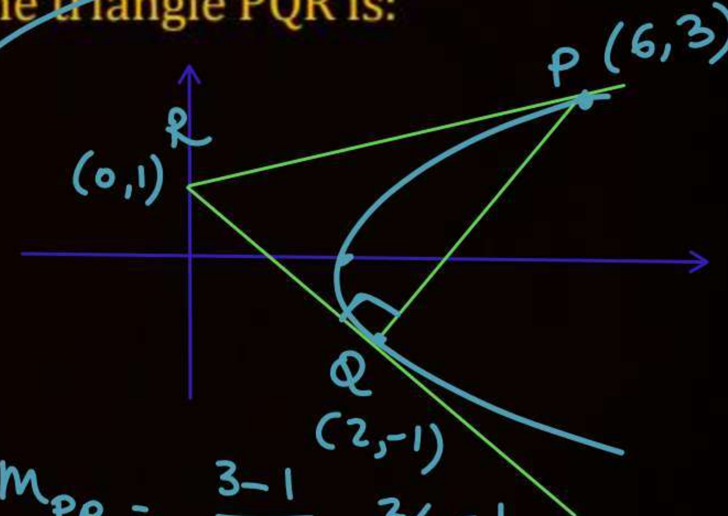
$x^2 - 8x + 12 = 0$

$(x-6)(x-2) = 0$

$x = 2$ or 6

(2, –1)

(6, 3)



$m_{PR} = \frac{3-1}{6-0} = 2/6 = 1/3$

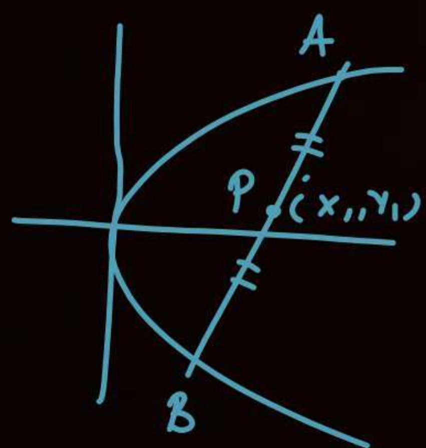
$m_{QR} = \frac{2}{0-2} = -1$

$m_{PQ} = \frac{3+1}{6-2} = 1$

$\Rightarrow QR \perp PQ$

Chord with a Given Mid Point

$$\boxed{T = S_1} \rightarrow \begin{cases} T : yy_1 - 2a(x + x_1) \\ S_1 : y_1^2 - 4ax_1 \end{cases}$$



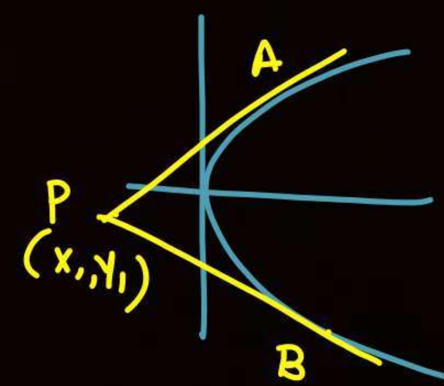
Equation of Pair of Tangents

The equation of the pair of tangents which can be drawn from any point $\mathbf{P(x_1, y_1)}$ outside the parabola to the parabola $\mathbf{y^2 = 4ax}$ is given by: $\mathbf{SS_1 = T^2}$ where:

$S:\ y^2 - 4ax$

$S_1:\ y_1^2 - 4ax_1$

$T:\ yy_1 - 2a(x + x_1)$



JEE ADV-2017

$a = 4$

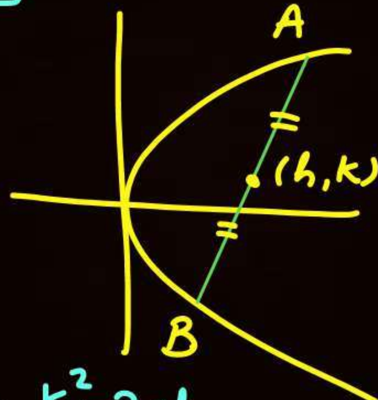
If a chord, which is not a tangent, of the parabola $y^2 = 16x$ has the equation $\boxed{2x + y = p}$, and midpoint (h, k) then which of the following is (are) possible value (s) of p, h and k? [Ans. B]

(A) $p = -1, h = 1, k = -3$ ✗

(B) $p = 2, h = 3, k = -4$ ✓

(C) $p = -2, h = 2, k = -4$ ✗

(D) $p = 5, h = 4, k = -3$ ✗



$T = S_1$

$yy_1 - 2a(x + x_1) = y_1^2 - 4ax_1$

$yk - 2a(x + h) = k^2 - 4ah$

$yk - 2ax - \cancel{2ah} = k^2 - \cancel{4}ah$

$\boxed{yk - 2ax = k^2 - 2ah}$

$y + 2x = p$

$\frac{k}{1} = \frac{-2a}{2} = \frac{k^2 - 2ah}{p}$

$\boxed{k = -4}$

$-a = \frac{k^2 - 2ah}{p}$

$-4p = k^2 - 8h$

$-4p = 16 - 8h$

$-p = 4 - 2h$

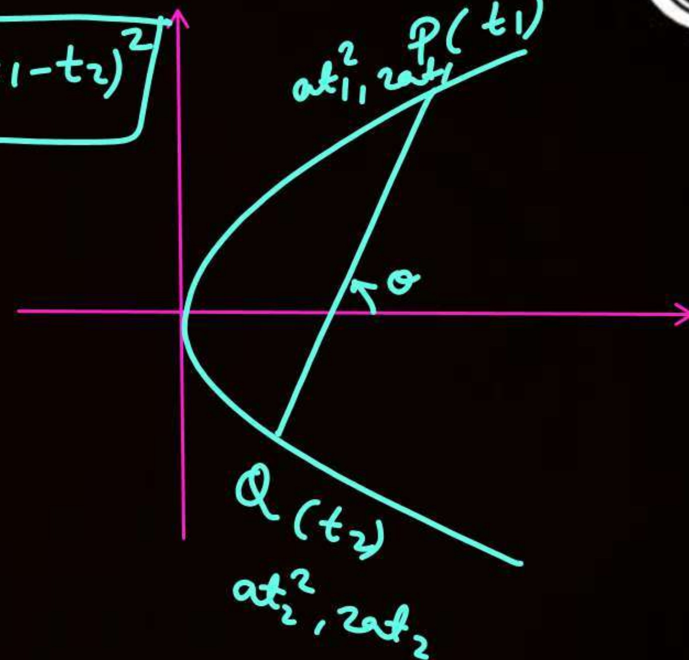
$\boxed{2h = p + 4}$

Note that

$$(PQ)^2 = (at_1^2 - at_2^2)^2 + (2at_1 - 2at_2)^2$$

$t_1 t_2 = -1$

$$\boxed{PQ = a(t_1 - t_2)^2}$$



Length of Focal Chord is $= \boxed{a(t_1 - t_2)^2}$

#

$= \underbrace{4a \operatorname{cosec}^2 \theta}$

Where θ is the inclination of the focal chord

$$\boxed{m = \tan\theta}$$

$$\frac{2}{t_1 + t_2} = \tan\theta$$

$$\boxed{t_1 + t_2 = 2\cot\theta}$$

$$\boxed{t_1 t_2 = -1}$$

$$PQ = a(t_1 - t_2)^2$$

$$a\left[(t_1 + t_2)^2 - 4t_1 t_2\right]$$

$$a\left[4\cot^2\theta + 4\right]$$

JEE Main-2019

$a = 4$

If one end of a focal chord of the parabola $y^2 = 16x$ is at $(1, 4)$, then the length of this focal chord is

[Ans. D]

(A) 24

(B) 20

(C) 22

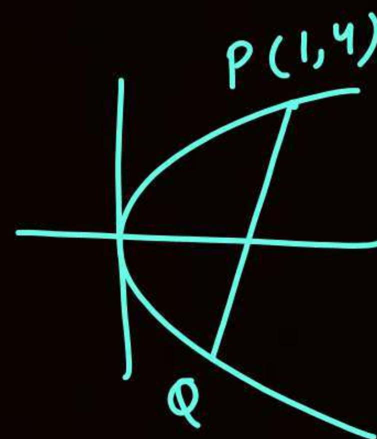
(D) 25 ✓

$4 = 2at_1$

$\cancel{4} = 2 \times \cancel{4}\, t_1$

$\Rightarrow \boxed{t_1 = 1/2}$

$\boxed{t_2 = -2}$



$l = a(t_1 - t_2)^2$

$= 4\left[\frac{1}{2} + 2\right]^2$

$= 4 \times (5/2)^2$

$= 25$

JEE Main-2023

If the x-intercept of a focal chord of the parabola $y^2 = 8x + 4y + 4$ is 3, then the length of this chord is equal to _____. [Ans. 16]

$y^2 - 4y = 8x + 4$

$(y-2)^2 = 8x + 8$

$(y-2)^2 = 8(x+1)$

$Y^2 = 8X$

Focus $X = 2$, $Y = 0$

$x+1 = 2$, $y-2 = 0$

$x = 1$, $y = 2$

focus $(1, 2)$

Focal chord $(3, 0) \rightarrow$

$(1, 2)$

$m \Rightarrow \tan\theta = \frac{2-0}{1-3} = \frac{2}{-2} = -1$

$\boxed{\theta = 135^\circ}$

$l = 4a \operatorname{cosec}^2\theta$

$= 4 \times 2 \operatorname{cosec}^2 135^\circ$

$= 4 \times 2 \times 2$

$= 16$

Note that

Valid for P, E & H

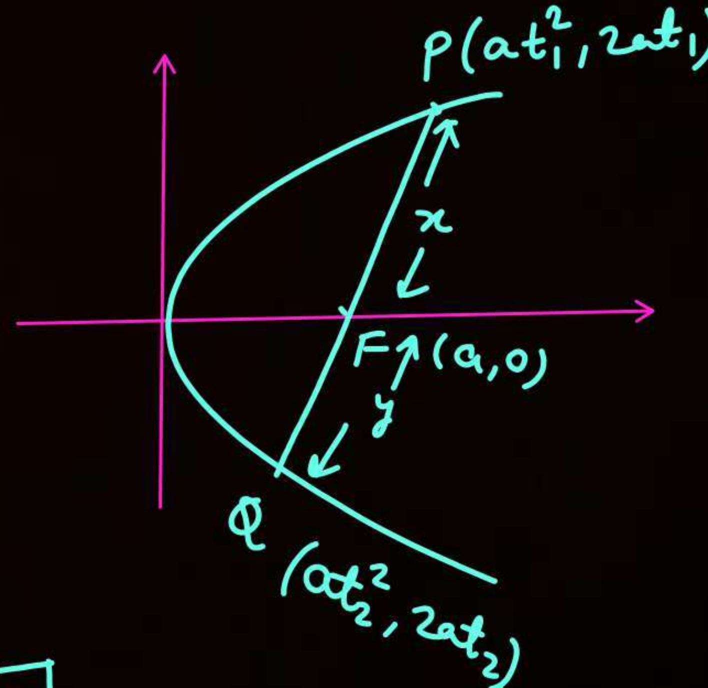
$\boxed{t_1 t_2 = -1}$

$PF = x = a + at_1^2$

$QF = y = a + at_2^2$

Semi Latus Rectum is the Harmonic Mean between any 2 Segments of a Focal Chord.

$$2a = \frac{2xy}{x+y}$$



RHS:
$$\frac{2(a+at_1^2)(a+at_2^2)}{a+at_1^2+a+at_2^2}$$

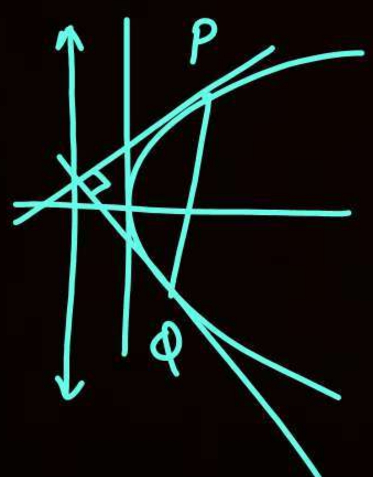
$$= \frac{2a^{\cancel{2}}(1+t_1^2)(1+t_2^2)}{\cancel{a}(t_1^2+t_2^2+2)}$$

$$= \frac{2a(\cancel{1+t_1^2+t_2^2+(t_1t_2)^2})}{\cancel{(t_1^2+t_2^2+2)}} = \boxed{2a}$$

Properties of Parabola

Property-1

Tangents drawn at the ends of any focal chord are always Perpendicular and they always meet on the **Directrix**.

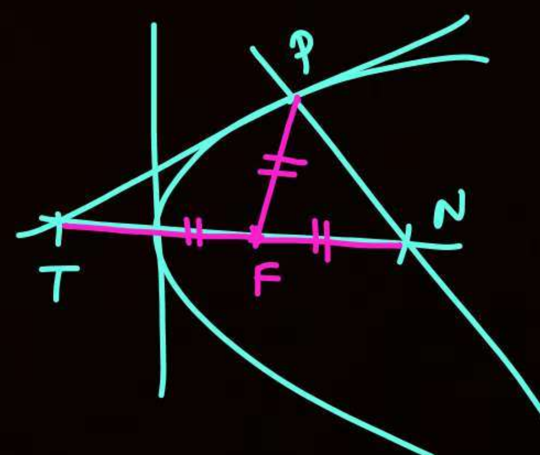


Properties of Parabola

Property-2

If the tangent and normal drawn at any point P to a parabola meets the axis at T & N respectively then PF = TF = FN , where F is the focus of the parabola.

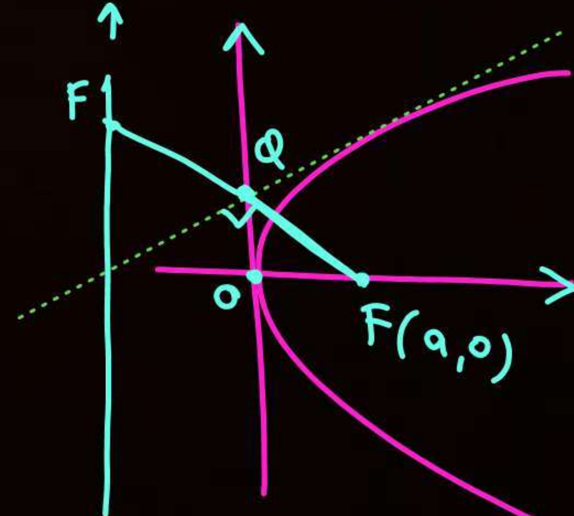
$$\boxed{PF = a + at^2}$$



Properties of Parabola

Property-3

The foot of the perpendiculars from the focus, to any tangent, lies on the tangent at the vertex.



Properties of Parabola

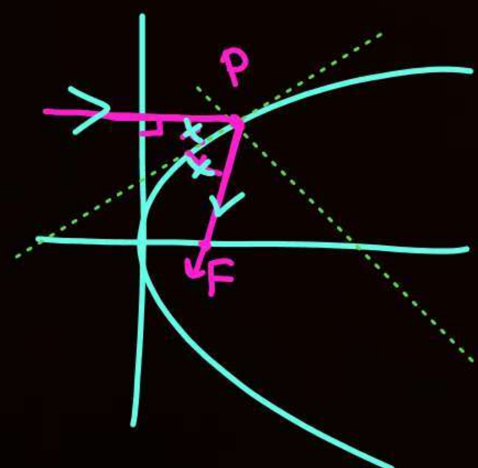
Property-4

The reflection of the focus **F**, in the tangent, **T**, at a point **P** on the curve lies on the directrix.

Properties of Parabola

Property-5

The tangent and the normal at a point **P** bisect the angles between the focal radius of **P** and the abscissa of **P**.

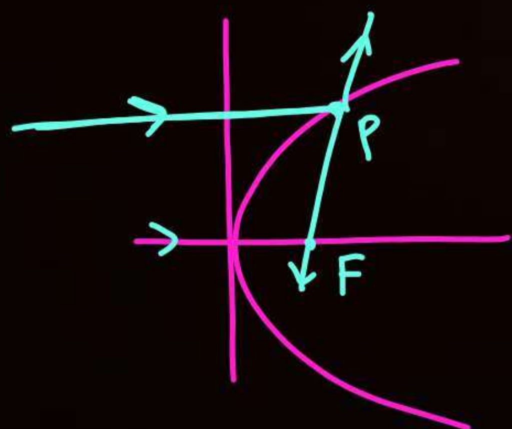


Properties of Parabola

Reflection Property

Property-6

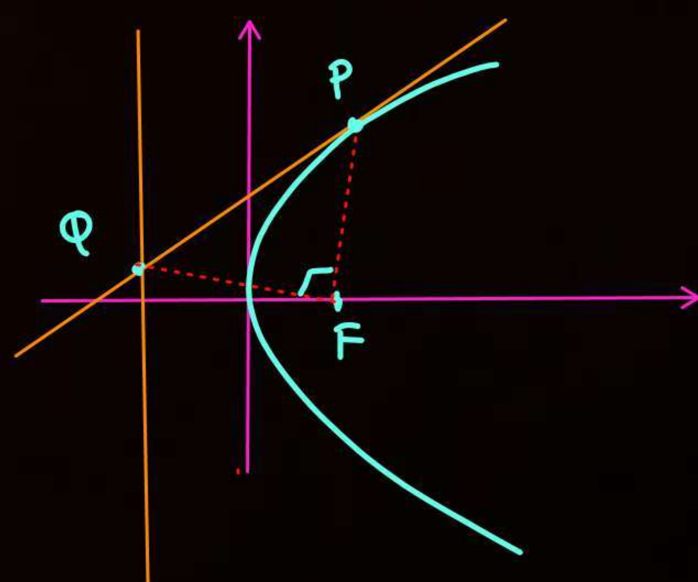
If incident ray is parallel to axis of the parabola then after getting reflected from the surface of the parabola, reflected ray passes through focus of the parabola.



Properties of Parabola

Property-7 (P, E & H)

The portion of a tangent to a parabola cut off between the directrix and the point of tangency subtends a right angle at the focus.

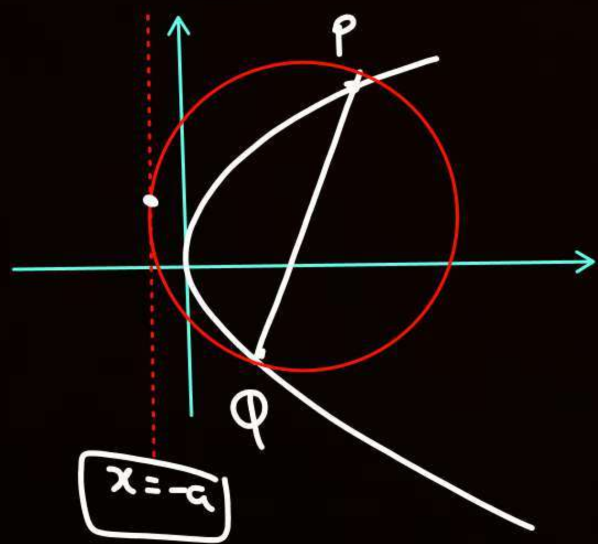


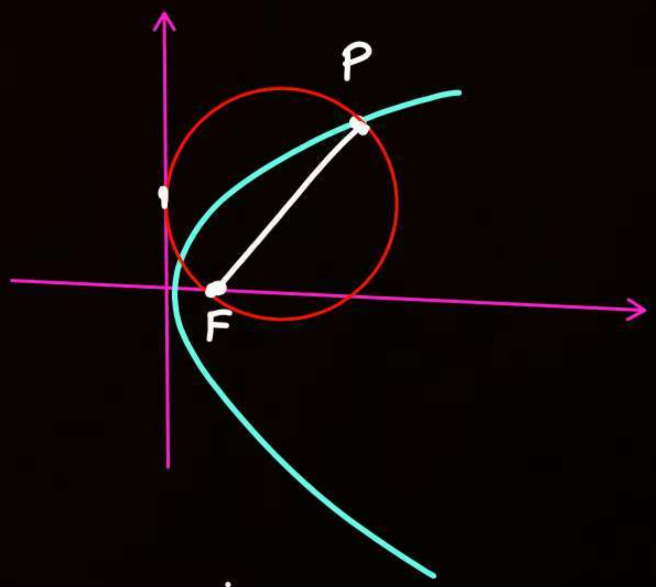
Properties of Parabola

Property-8

(a) Circle with Focal Distance as diameter touches the tangent at vertex

(b) Circle with Focal Chord as diameter touches the directrix.





JEE ADV-2021

$\boxed{a=2}$ $(PF) = \sqrt{4^2+4^2} = 4\sqrt{2}$

Let E denote the parabola $y^2 = 8x$. Let $P = (-2, 4)$ and let Q and Q' be two distinct points on E such that the lines PQ and PQ' are tangents to E. Let F be the focus of E. Then which of the following statement is (are) True?

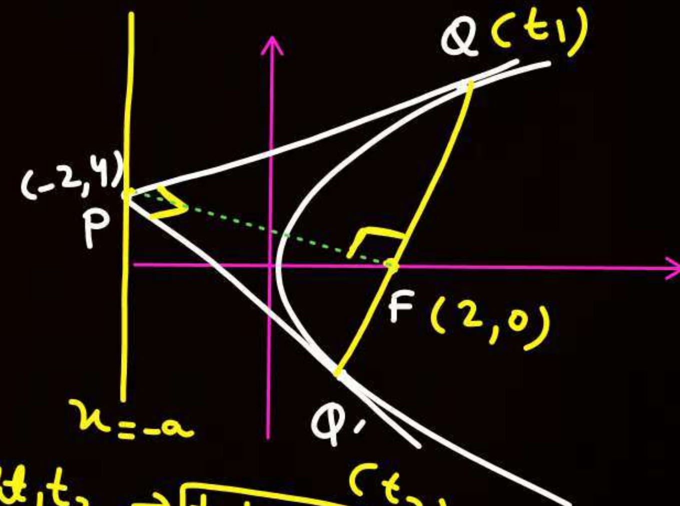
[Ans. A, B, D]

(A) The triangle PFQ is a right- angled triangle

(B) The triangle QPQ' is a right- angled triangle

(C) The distance between P and F is $5\sqrt{2}$

(D) F lies on the line joining Q and Q'



$-\cancel{a} = \cancel{a}t_1t_2 \Rightarrow \boxed{t_1t_2=-1}$

$4 = a(t_1+t_2)$

$\boxed{t_1+t_2=2}$

Properties of Parabola

Property-9

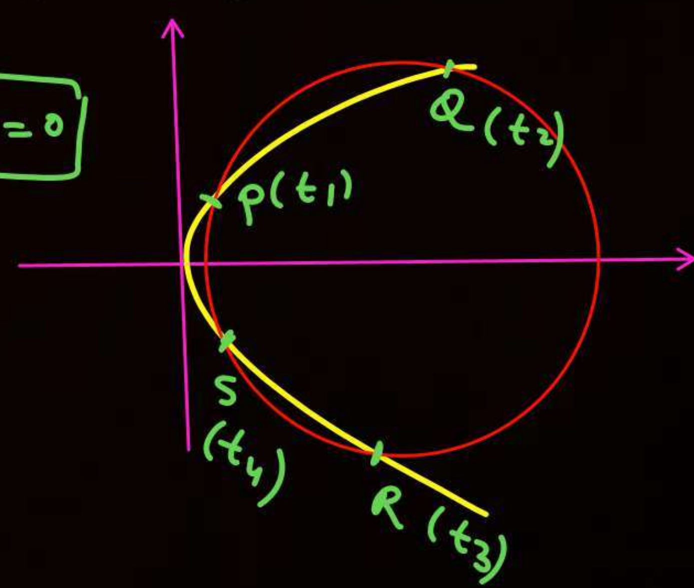
If P (t_1), Q (t_2), R(t_3), S(t_4) are 4 concyclic points on parabola $y^2 = 4ax$ then $t_1 + t_2 + t_3 + t_4 = 0$

$$\boxed{t_1 + t_2 + t_3 + t_4 = 0}$$

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$$(at^2, 2at)$$

$$\boxed{\text{coeff of } t^3 = 0} \checkmark$$



Question

AB is a focal chord of $y^2 = 4ax$, where $A = (4a, 4a)$. BC is the normal chord at B. The circle ABC cuts the curve at D. The co-ordinates of D are (p, q) then $(p + q)/4a$ is

HW

[Ans. 6]

PYQs

JEE Main-2023

$m = \frac{1}{2\sqrt{2}}$

The distance of the point $(6, -2\sqrt{2})$ from the common tangent $y = mx + c$, $m > 0$, of the curves $x = 2y^2$ and $x = 1 + y^2$ is

[Ans. B]

(A) $\frac{1}{3}$

(B) 5

(C) $\frac{14}{3}$

(D) $5\sqrt{3}$

$y^2 = x/2$

$4a = 1/2$

$y = mx + a/m$

$\boxed{y = mx + \frac{1}{8m}}$

$\frac{1}{8m} = -m + \frac{2}{8m}$

$m = \frac{1}{8m}$

$m^2 = \frac{1}{8}$

$m = \frac{1}{2\sqrt{2}}$

$y^2 = (x-1)$

$4a = 1$

$y = m(x-1) + \frac{1}{4m}$

$y = mx - m + \frac{1}{4m}$

$y = \frac{1}{2\sqrt{2}}x + \frac{1}{8} \cdot 2\sqrt{2}$

$y = \frac{x}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$

$2\sqrt{2}y = x + 1$

$\boxed{x - 2\sqrt{2}y + 1 = 0}$

$d = \left|\frac{6 + 2\sqrt{2} \cdot 2\sqrt{2} + 1}{3}\right|$

$\boxed{d = 5}$

JEE Main-2022 (July-I)

$f'(x)<0$ $\qquad$ $f'(x)>0$

Let the function $\boxed{f(x) = 2x^2 - \log_e x,}$ $x > 0$ be decreasing in $\boxed{(0, a) \text{ and increasing}}$ in $(a, 4)$. A tangent to the parabola $y^2 = 4ax$ at a point P on it passes through the point $(8a, 8a - 1)$ but does not pass through the point $(-1/a, 0)$. If the equation of the normal at P is $\frac{x}{\alpha} + \frac{y}{\beta} = 1$, then $\alpha + \beta$ is equal to-

[Ans. 45]

$f'(x) = 4x - \frac{1}{x}$

$= \frac{4x^2 - 1}{x}$

$= \frac{(2x-1)(2x+1)}{x}$

$-$ $\quad$ $+$

0 $\quad$ $\frac{1}{2} \to a$

$\boxed{a = \frac{1}{2}}$ ✓

$y^2 = 4ax$

$Q[4, 3]$

$R[-2, 0]$ ×

$ty = x + \frac{at^2}{2}$

$3t = 4 + \frac{t^2}{2}$

$6t = 8 + t^2$

$t^2 - 6t + 8 = 0$

$t = 2$ or 4

$t \neq 2$ $\qquad$ $t = 4$

$\cancel{2y = x + 2}$ $\qquad$ $4y = x + 8$ ✓

$$a = \tfrac{1}{2}, \quad t = 4$$

$$y + tx = 2at + at^3$$

$$y + 4x = 2 \times \tfrac{1}{2} \times 4 + \tfrac{1}{2} \times 4^3$$

$$y + 4x = 4 + 32$$

$$\boxed{y + 4x = 36}$$

$$\frac{x}{9} + \frac{y}{36} = 1$$

$$\alpha + \beta = 9 + 36$$
$$= 45$$

JEE Main-2023

$x = 0 \quad y = \frac{\lambda}{\lambda - 1} \qquad \frac{\lambda}{\lambda-1} = \frac{4}{\lambda} \qquad y = 4/\lambda$

$\lambda^2 = 4\lambda - 4$

$\lambda^2 - 4\lambda + 4 = 0$

The equations of the sides AB and AC of a triangle ABC are $(\lambda + 1)x + \lambda y = 4$ and $\lambda x + (1 - \lambda)y + \lambda = 0$ respectively. Its vertex A is on the y-axis and its orthocentre is (1, 2). The length of the tangent from the point C to the part of the parabola $y^2 = 6x$ is the first quadrant is **[Ans. B]**

$(\lambda - 2)^2 = 0$

$\boxed{\lambda = 2}$

(A) $\sqrt{6}$

(B) $2\sqrt{2}$

(C) 2

(D) 4

AC: $2x - y + 2 = 0$

$\boxed{2x + 2 = y}$

AB: $3x + 2y = 4$

eq^n of CF:

$y - 2 = \frac{2}{3}(x - 1)$

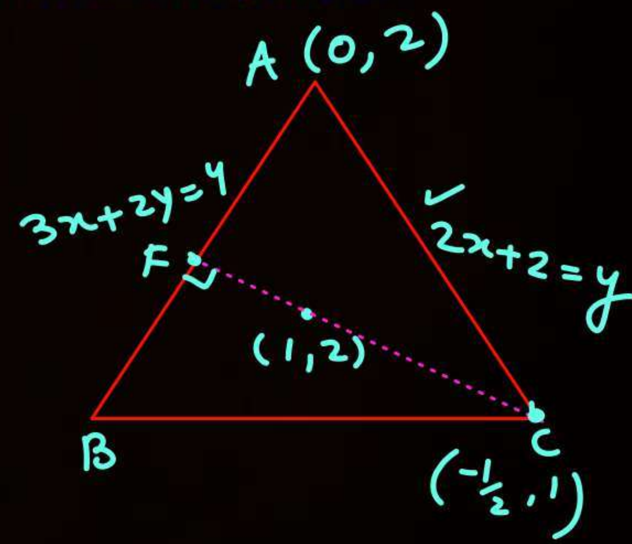
$3y - 6 = 2x - 2$

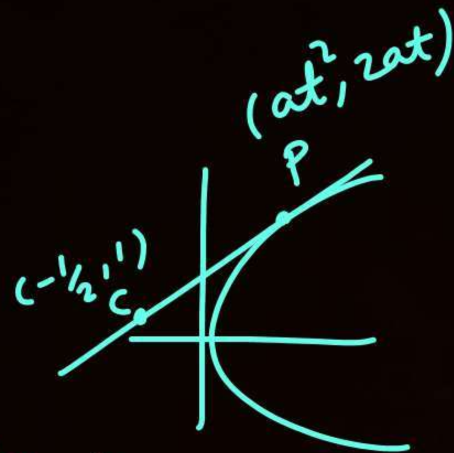
$\boxed{3y = 2x + 4}$

$3y = y - 2 + 4$

$2y = 2$

$\boxed{y = 1}$





$ty = x + at^2$

$t = -\frac{1}{2} + at^2$

$6x = 4a$

$a = 3/2$

$t = -\frac{1}{2} + \frac{3}{2}t^2$

$2t = -1 + 3t^2$

$3t^2 - 2t - 1 = 0.$

$3t^2 - 3t + t - 1 = 0.$

$(3t+1)(t-1) = 0$

$t = -\frac{1}{3}$ or 1

$P\,[a, 2a]$

$[3/2, 3]$

QUESTION [JEE Main 2022 (June-II)]

Let P : $y^2 = 4ax$, $a > 0$ be a parabola with focus S. Let the tangents to the parabola P make an angle of $\frac{\pi}{4}$ with the line $y = 3x + 5$ touch the parabola P at A and B. then the value of a for which A, B and S are collinear is: [Ans. D]

(A) 8 only

(B) 2 only

(C) $\frac{1}{4}$ only

(D) any $a > 0$

$$\tan 45^\circ = \left|\frac{\frac{1}{t} - 3}{1 + \frac{3}{t}}\right|$$

$$\pm 1 = \frac{1 - 3t}{t + 3}$$

(+) $t + 3 = 1 - 3t$

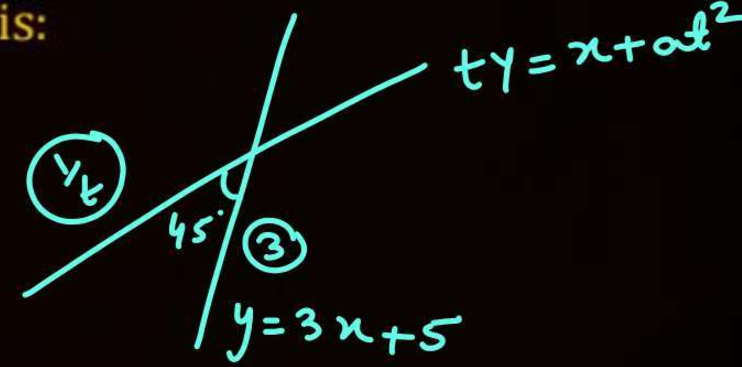
$4t = -2 \Rightarrow \boxed{t_1 = -\frac{1}{2}}$

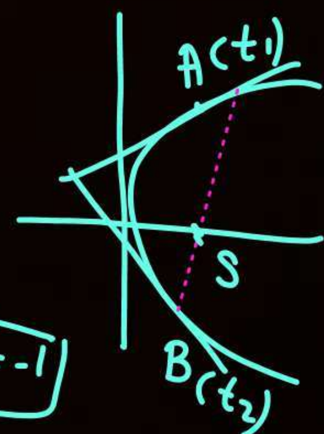
(−) $-t - 3 = 1 - 3t$

$2t = 4$

$\boxed{t_2 = 2}$

$\boxed{t_1 t_2 = -1}$





JEE Main-2023

$4a = 20$

$a = 5$

Let R be the focus of the parabola $y^2 = 20x$ and the line $y = mx + c$ intersect the parabola at two points P and Q. Let the points G(10, 10) be the centroid of the triangle PQR. If $\boxed{c - m = 6}$ then PQ^2 is

[Ans. B]

(A) 296

(B) 325

(C) 317

(D) 346

$$\frac{y_1 + y_2 + 0}{3} = 10$$

$$\boxed{y_1 + y_2 = 30}$$

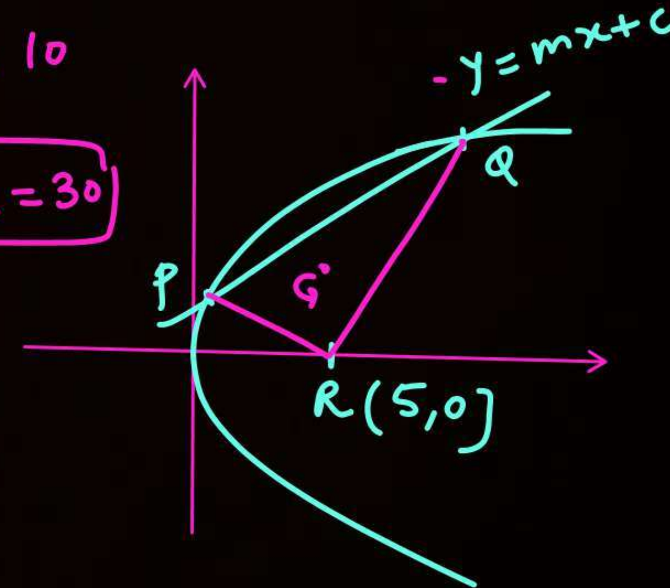
$$y^2 = 20\left[\frac{y - c}{m}\right]$$

$$my^2 - 20y + 20c = 0 \quad y_1, y_2$$

$$y_1 + y_2 = \frac{20}{m} = 30 \Rightarrow \boxed{m = 2/3}$$

$$c - m = 6$$

$$c = 6 + 2/3 = 20/3$$



$$PQ = |x_2 - x_1|\sqrt{1+(2/3)^2}$$

$$= |x_2 - x_1| \frac{\sqrt{13}}{3}$$

$$PQ = 15 \cdot \frac{\sqrt{13}}{3}$$

$$PQ = 5\sqrt{13}$$

$$(PQ)^2 = 25 \times 13$$

$$= 250 + 75$$

$$= \boxed{325}$$

$$y = mx + c$$

$$y^2 = 20x$$

$$(mx+c)^2 = 20x$$

$$\left(\frac{2}{3}x + \frac{20}{3}\right)^2 = 20x$$

$$\frac{4}{9}(x+10)^2 = 20x$$

$$(x+10)^2 = 45x$$

$$x^2 + 20x + 100 = 45x$$

$$x^2 - 25x + 100 = 0$$

$$D = 25^2 - 400$$

$$= 225$$

$$x_1 - x_2 = \frac{\sqrt{D}}{1} = \sqrt{225}$$

$$= 15$$

QUESTION [JEE Main 2022 (June-II)]

$m_{PV} = \dfrac{\frac{1}{4}+6}{\frac{1}{2}+2} = \dfrac{25/4}{5/2} = 5/2$

If the line y = 4 + kx, k > 0, is the tangent to the parabola y = x – x² at the point P and V is the vertex of the parabola, then the slope of the line through P and V is: **[Ans. C]**

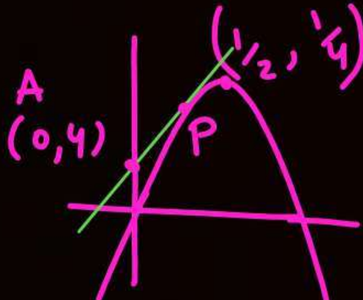
(A) $\frac{3}{2}$

(B) $\frac{26}{9}$

(C) $\frac{5}{2}$

(D) $\frac{23}{6}$

$\frac{dy}{dx} = 0 \quad 1 - 2x = 0$



$P[\alpha, \alpha - \alpha^2]$

$m_{PA} = \dfrac{\alpha - \alpha^2 - 4}{\alpha}$

$K = 1 + 4$
$= 5$

$P[-2, -2-4]$

$P[-2, -6]$

$V(\frac{1}{2}, \frac{1}{4})$

$\frac{dy}{dx} = 1 - 2x = K$

$1 - 2\alpha = K$

$1 - 2\alpha = \dfrac{\alpha - \alpha^2 - 4}{\alpha}$

$\cancel{\alpha} - 2\alpha^2 = \cancel{\alpha} - \alpha^2 - 4$

$\Rightarrow \alpha^2 = 4$

$\alpha = \pm 2$

$\alpha = 2 \quad \alpha = -2$
$\times$

JEE Main-2023

A $\begin{matrix} 4R \\ 6B \end{matrix}$ B $\begin{matrix} 5R \\ 5B \end{matrix}$ C $\begin{matrix} \lambda R \\ 4B \end{matrix}$

The urns A, B and C contain 4 red, 6 black; 5 red, 5 black and λ red, 4 black balls respectively. One of the urns is selected at random and a ball is drawn. If the ball drawn is red and the probability that it is drawn from urn C is 0.4 then the square of the length of the side of the largest equilateral triangle, inscribed in the parabola $y^2 = \lambda x$ with one vertex at the vertex of the parabola is

[Ans. 432]

A : $P(A) = 1/3$ $\quad P(E/A) = 4/10$

B : $P(B) = 1/3$ $\quad P(E/B) = 5/10$

C : $P(C) = 1/3$ $\quad P(E/C) = \frac{\lambda}{\lambda+4}$

E : Red Ball

$P(C/E) = 0.4$

$$P(C/E) = \frac{\lambda/\lambda+4}{\frac{4}{10}+\frac{5}{10}+\frac{\lambda}{\lambda+4}} = 0.4$$

$$\frac{\lambda/\lambda+4}{\frac{9}{10}+\frac{\lambda}{\lambda+4}} = 0.4$$

$$\frac{10\lambda}{9(\lambda+4)+10\lambda} = 2/5$$

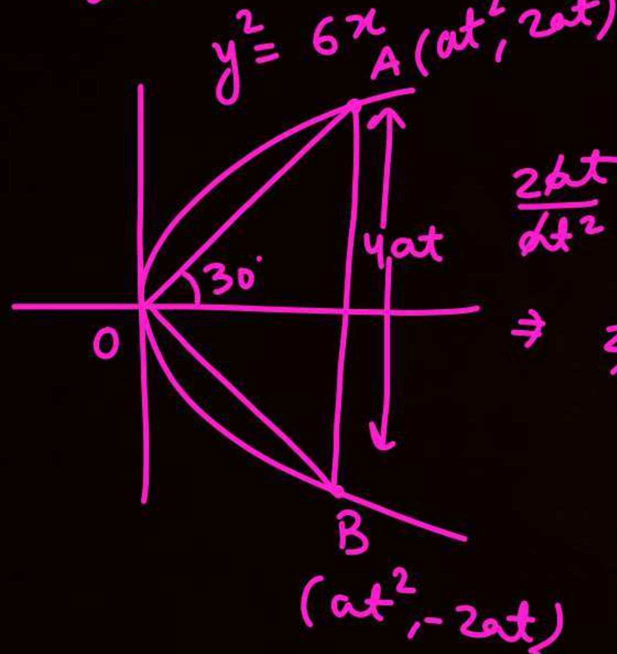
$$\frac{10\lambda}{19\lambda+36} = 2/5$$

$$50\lambda = 38\lambda + 72 \Rightarrow 12\lambda = 72$$

$$\boxed{\lambda = 6}$$

$4a = 6$

$a = 3/2$



$$\frac{2\cancel{a}t}{\cancel{a}t^2} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{2}{t} = \frac{1}{\sqrt{3}}$$

$$\boxed{t = 2\sqrt{3}}$$

$$\begin{array}{r} 144 \\ \times\ 3 \\ \hline 432 \\ \hline \end{array}$$

$$\ell = 4at$$

$$= 4 \times \cancel{2}\sqrt{3} \times 3/\cancel{2}$$

$$\ell = 12\sqrt{3}$$

$$\ell^2 = 12^2 \times 3$$

JEE Main-2023

HW

Let PQ be a focal chord of the parabola $y^2 = 36x$ of length 100, making an acute angle with the positive x-axis. Let the ordinate of P be positive and M be the point on the line segment PQ such that PM : MQ = 3 : 1. Then which of the following points does NOT on the line passing through M and perpendicular to the line PQ?

[Ans. D]

(A) $(-6,45)$

(B) $(6,29)$

(C) $(3,33)$

(D) $(-3,43)$

QUESTION [JEE Main 2022 (June I)]

Let $x^2 + y^2 + Ax + By + C = 0$ be a circle passing through $(0, 6)$ and touching the parabola $y = x^2$ at $(2, 4)$. Then $\underbrace{A + C}$ is equal to______

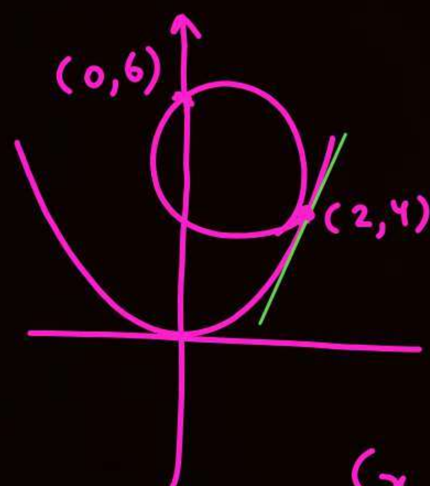
[Ans. A]

(A) 16 ✓

(B) 88/5

(C) 72

(D) –8



$x^2 = y$

$xx_1 = \frac{y+y_1}{2}$

$x \times 2 = \frac{y+4}{2}$

$\boxed{4x = y+4}$

$(x-2)^2 + (y-4)^2 + \lambda(y+4-4x) = 0$

$(0,6)$

$4 + 4 + \lambda[10-0] = 0$

$\lambda = -8/10$

$\lambda = -4/5$

$(x-2)^2 + (y-4)^2 - \frac{4}{5}(y+4-4x)$

$x^2 + y^2 - 4x + 4 + 16 - 8y - \frac{4y}{5} - \frac{16}{5} + \frac{16x}{5} = 0$

$x^2 + y^2 + \left(\underbrace{\frac{16}{5}} - 4\right)x + \left(20 - \frac{16}{5}\right) - 8y - 4y/5 = 0$

$\left.\begin{array}{l} A = -4/5 \\ C = 84/5 \end{array}\right\} \quad A + C = \frac{80}{5} = 16$

QUESTION [JEE Main 2022 (June I)]

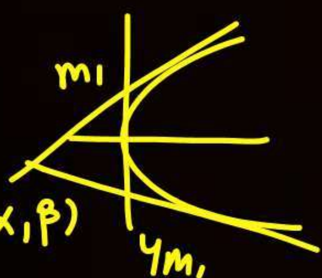
$25(2\alpha+1)^2[1+100]$

$25\times 101\times(4\alpha^2+4\alpha+1)$

$25\times 101\left(\frac{4}{25}+1\right) = 101\times 29$

If two tangents drawn from a point (α, β) lying on the ellipse $25x^2 + 4y^2 = 1$ to the parabola $y^2 = 4x$ are such that the slope of one tangent is four times the other, then the value of $(10\alpha + 5)^2 + (16\beta^2 + 50)^2$ equals ______ [Ans. 2929]

$\boxed{25\alpha^2+4\beta^2=1} \rightarrow (1)$



$y = mx + \frac{1}{m}$

(α, β)

$\beta = m\alpha + \frac{1}{m}$

$\beta m = m^2\alpha + 1$

$m^2\alpha - \beta m + 1 = 0 < m_1, 4m_1$

$5m_1 = \beta/\alpha \Rightarrow m = \beta/5\alpha$

$4m_1^2 = \frac{1}{\alpha}$

$4\left(\beta/5\alpha\right)^2 = \frac{1}{\alpha}$

$\frac{4\beta^2}{25\alpha^{\cancel{2}}} = \frac{1}{\cancel{\alpha}}$

$\Rightarrow \boxed{4\beta^2 = 25\alpha} \rightarrow (2)$

$25\alpha^2 + 25\alpha = 1$

$25(\alpha^2+\alpha) = 1 \Rightarrow \alpha^2+\alpha = \frac{1}{25}$

$(10\alpha+5)^2 + (100\alpha+50)^2$

$25(2\alpha+1)^2 + 2500(2\alpha+1)^2$

Home Work

1) Learn
2) Redo PYQS
3) DPP / PYQS → 2020-2021