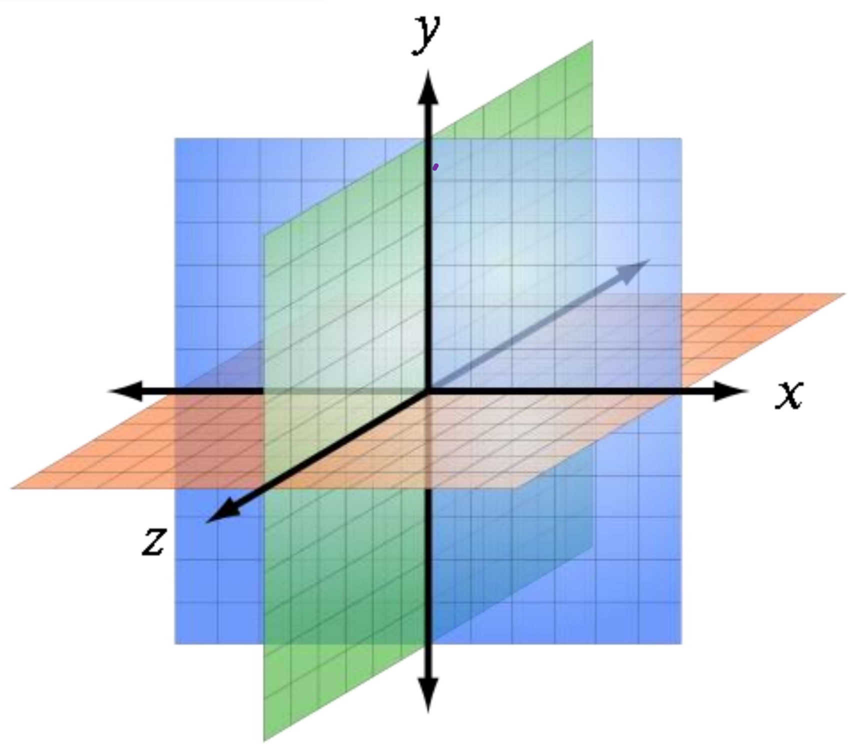
Unacademy Atoms

Cartesian Coordinate System

Cartesian Coordinate System in 3D

- ➢ 3 Coordinate Axes ✓
- ➢ 3 Coordinate Planes (xy , yz , zx)
- ➢ 8 Octants

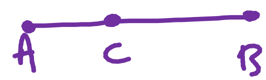


Distance formulae

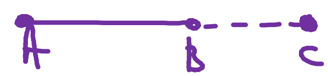
Distance between the points (x_1, y_1, z_1) and (x_2, y_2 , z_2) is equal to $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$

Section formulae

1 Coordinates a point P which divides line joining A (x_1, y_1, z_1) and (x_2, y_2, z_2) in the ratio m : n internally is given by $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}, \frac{mz_2+nz_1}{m+n}\right)$



2 Coordinates a point P which divides line joining A (x_1, y_1, z_1) and (x_2, y_2, z_2) in the ratio m : n externally is given by $\left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}, \frac{mz_2-nz_1}{m-n}\right)$

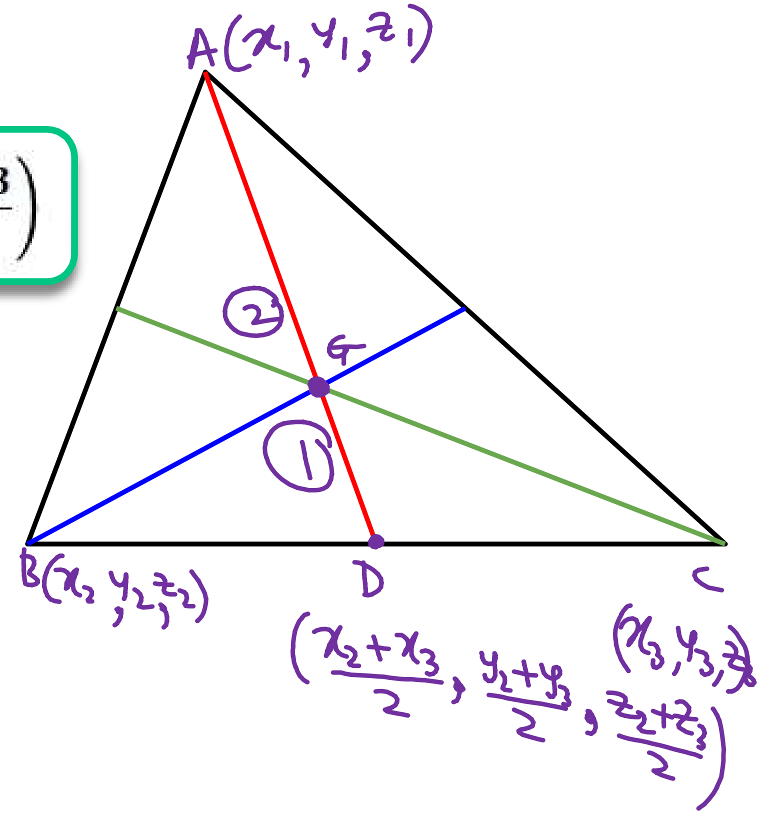


3 Coordinates of midpoint of line joining A (x_1, y_1, z_1) and B (x_2, y_2, z_2) is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$

Coordinates of Centroid

$$G \equiv \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}\right)$$

$$G \equiv \left(\frac{\cancel{2}\left(\frac{x_2+x_3}{\cancel{2}}\right) + 1(x_1)}{2+1}, \dots, \dots\right)$$

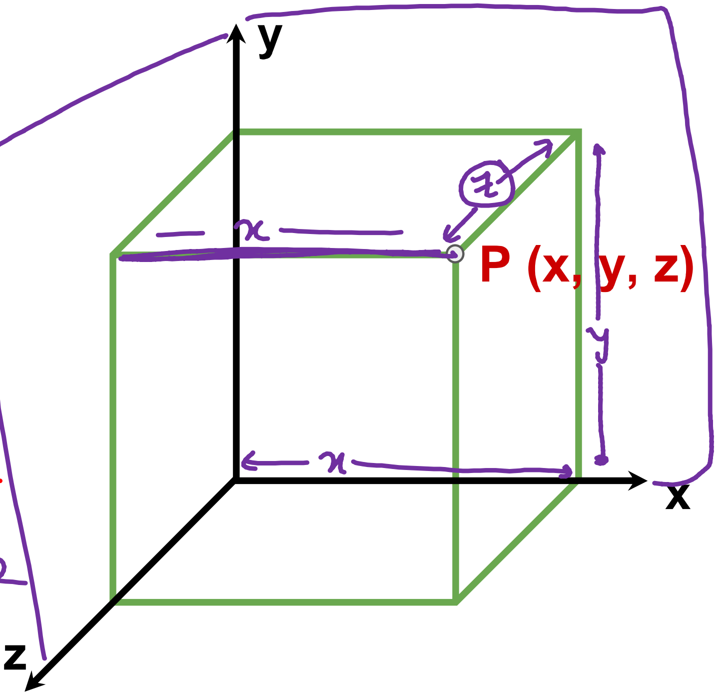


Distance of a point from Coordinate Planes

Distance of Point P from xy plane = | z |
Distance of Point P from yz plane = | x |
Distance of Point P from xz plane = | y |

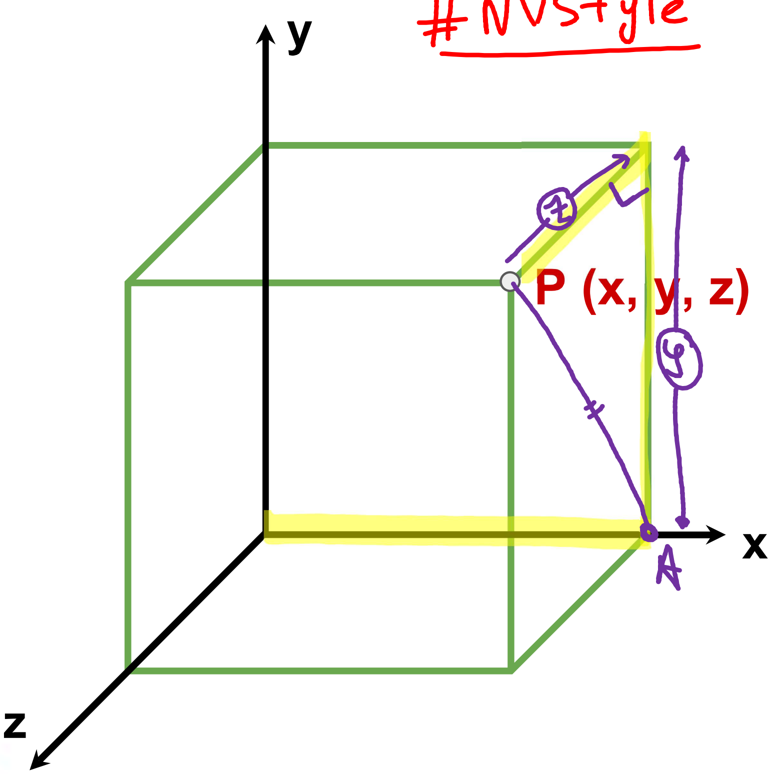
#NVStyle

P(2, −3, −1)   dis xy plane ⇒ 1
〃 yz plane ⇒ 2
〃 xz plane ⇒ 3



Distance of a point from Coordinate axes

#NVStyle



$$PA = \sqrt{y^2 + z^2}$$
$$PB = \sqrt{z^2 + x^2}$$
$$PC = \sqrt{x^2 + y^2}$$

P(1 ,−2 , 3)

dis of P from x axis $\Rightarrow \sqrt{y^2+z^2} \Rightarrow \sqrt{2^2+3^2}$

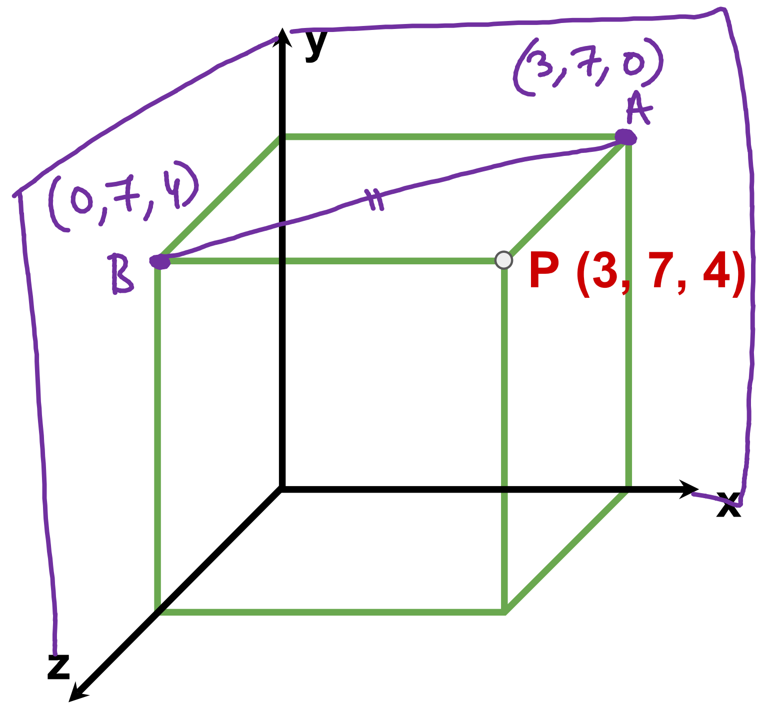
" " " " y axis $\Rightarrow \sqrt{x^2+z^2} = \sqrt{10}$

If A and B are foot of perpendicular from **(3, 7, 4)** on the **XY** plane and **YZ** plane respectively, then find the length of **AB**.

A (3, 7, 0)

B (0, 7, 4)

$AB = \sqrt{(3-0)^2 + 0^2 + 4^2} = 5$



Direction Ratios and Direction Cosines

Direction Cosines (DCs)

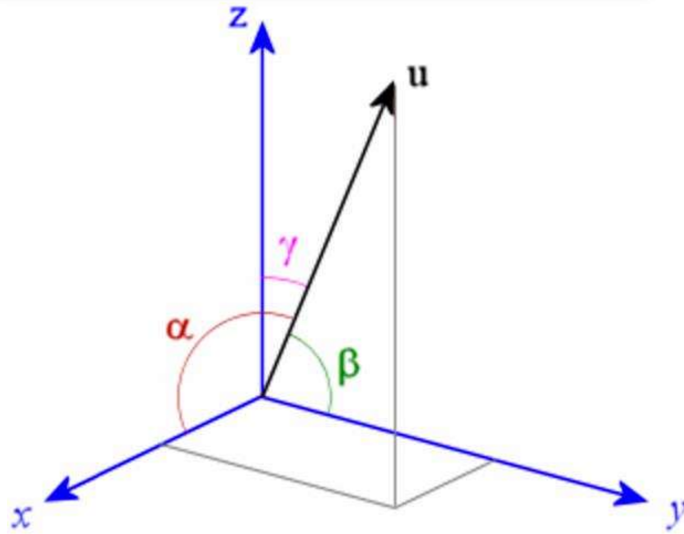
If α, β, γ are the angles which vector makes with positive direction of the x, y, z axes respectively then their cosines $\cos\alpha$, $\cos\beta$, $\cos\gamma$ are called the direction cosines of the vector and are generally denoted by l, m, n respectively.

Thus $l = \cos\alpha$, $m = \cos\beta$, $n = \cos\gamma$

α , β , γ

$\underline{\cos\alpha}$ $\underline{\cos\beta}$ $\underline{\cos\gamma}$

l m n



Important Results

1. $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

2. $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

$\boxed{l^2 + m^2 + n^2 = 1}$

$l\hat{i} + m\hat{j} + n\hat{k} \Rightarrow$ unit vector

$1 - \sin^2\alpha + 1 - \sin^2\beta + \cancel{1} - \sin^2\gamma = \cancel{1}$

$\boxed{\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2}$

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Direction Ratios (DRs)

Direction ratios are multiples of direction cosines

Q. $DC = \left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right)$ $\quad l^2+m^2+n^2=1$

$DR = \lambda\left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right)$ $\quad$ Direction Vector.

$= (2, 2, 1)$ ✓ $\Longrightarrow \boxed{2\hat{i}+2\hat{j}+\hat{k}}$

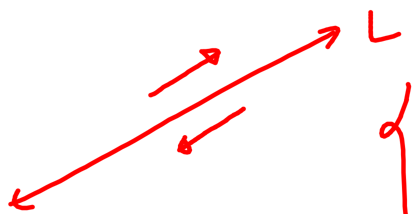
$= (-2, -2, -1)$ ✓

$= (4, 4, 2)$ ✓

$\vdots$

Important Results

1 Direction ratios of a line is not unique but infinite in number but direction cosines will be for a line will be only two.
(l, m, n or -l, -m, -n)



$$\begin{cases} (l, m, n) \\ (-l, -m, -n) \end{cases}$$

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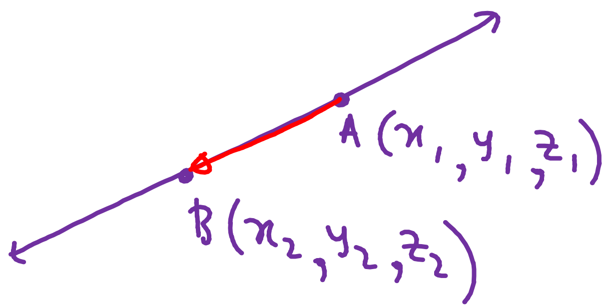
Important Results

2 A vector along the line with direction ratios a, b, c can be ai + bj + ck

Important Results

3 Direction ratios of a line joining two points A and B are proportional to $x_2 - x_1$, $y_2 - y_1$, $z_2 - z_1$



$\overrightarrow{AB}$ or $\overrightarrow{BA}$

$\overrightarrow{AB} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

DV

DR :- $(x_2 - x_1, \; y_2 - y_1, \; z_2 - z_1)$

A line AB in three-dimensional space makes 45° and 120° with the positive x-axis and the positive y-axis respectively. If AB makes an acute angle θ with the positive z-axis, then θ equals

A. 45° **B.** 60° **C.** 75° **D.** 30°

2010

$\alpha = 45^\circ \quad \beta = 120^\circ \quad \gamma = ?$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\cos^2 45 + \cos^2 120^\circ + \cos^2\gamma = 1$

$\frac{1}{2} + \frac{1}{4} + \cos^2\gamma = 1$

$\boxed{\cos\gamma = \frac{1}{2}}$

Equation of Straight Line

Equation of Line: Vector Form

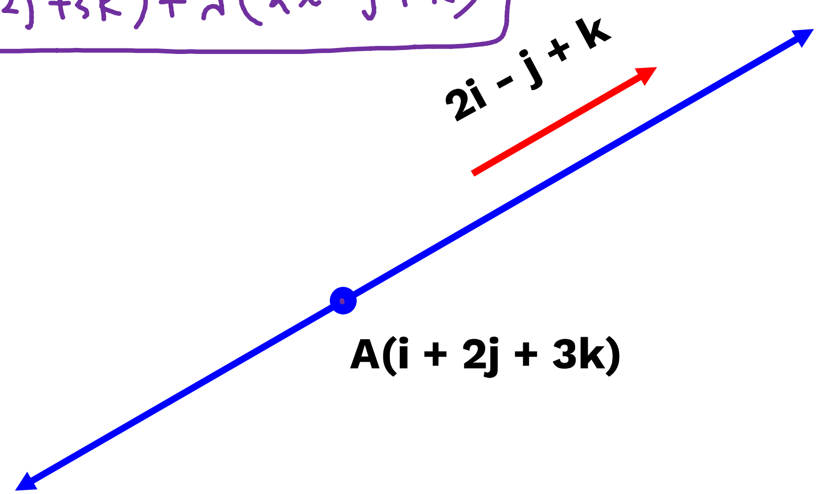
1 Passing through a point and parallel to given direction

point :- ✓

dir^n :- ✓

$$\boxed{\vec{r} = \underbrace{\vec{a}}_{\text{Point}} + \lambda \underbrace{\vec{b}}_{\text{dir}^n}}$$

$$\boxed{\vec{r} = (\hat{i}+2\hat{j}+3\hat{k}) + \lambda(2\hat{i}-\hat{j}+k)}$$



Equation of Line: Cartesian Form

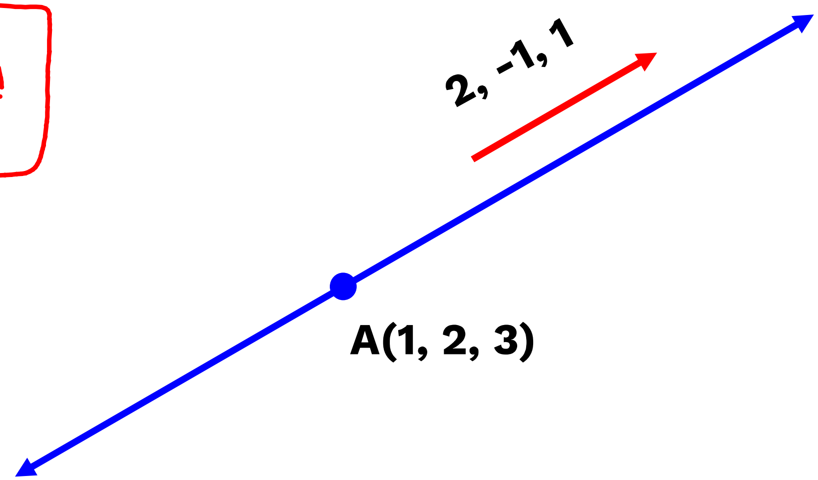
1 Passing through a point and given DRs or DCs

$(x_1, y_1, z_1) \rightarrow$ point on S.L.

$$\boxed{\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}}$$

$(a, b, c) = $ D.R.

$$\boxed{\frac{x-1}{2} = \frac{y-2}{-1} = \frac{z-3}{1}}$$



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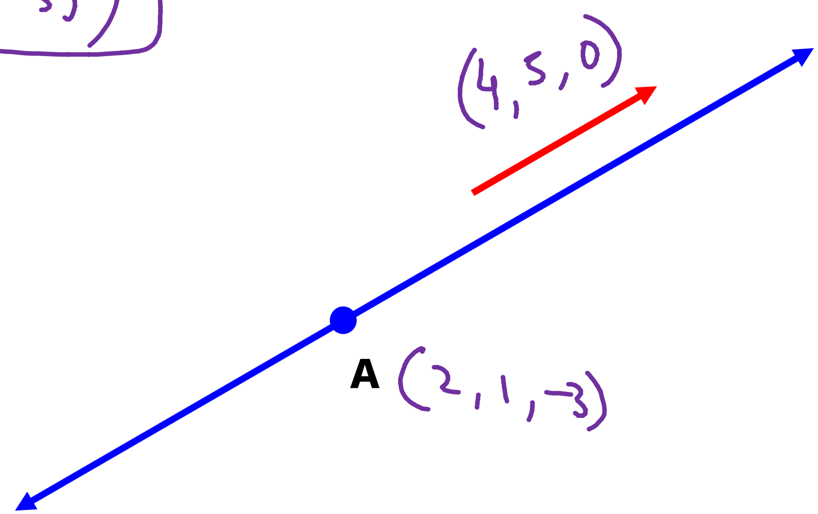
Find the equation of the line through the point **2i + j - 3k** and parallel to the vector **4i + 5j** in

I. Vector Form

II. Cartesian Form

I. $$\boxed{\vec{r} = (2\hat{i} + \hat{j} - 3\hat{k}) + \lambda(4\hat{i} + 5\hat{j})}$$

II. $$\boxed{\frac{x-2}{4} = \frac{y-1}{5} = \frac{z+3}{0}}$$



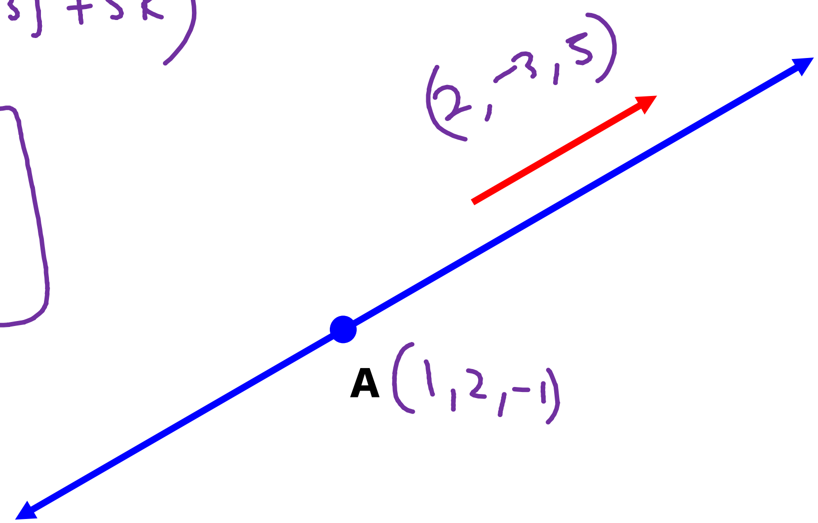
Find the equation of the line through the point (1, 2, -1) and having DRs 2, -3 and 5

I. Vector Form

II. Cartesian Form

I $\vec{r} = (\hat{i} + 2\hat{j} - \hat{k}) + \lambda(2\hat{i} - 3\hat{j} + 5\hat{k})$

II $\boxed{\dfrac{x-1}{2} = \dfrac{y-2}{-3} = \dfrac{z+1}{5} = \lambda}$



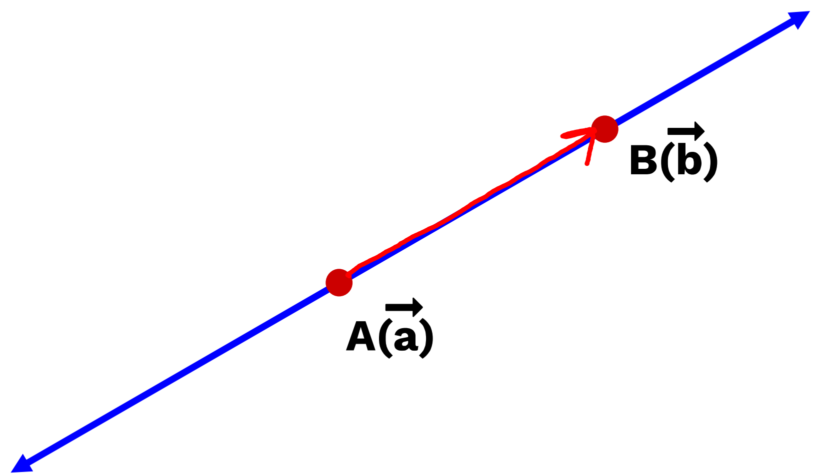
Equation of Line: Vector Form

2 Passing through a two points

point :- $\vec{a}$ / $\vec{b}$

dirⁿ :- $\vec{AB} = \vec{b} - \vec{a}$

$$\boxed{\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})}$$



Find the equation of straight line passing through points A (6, -7, -1) and B (2, -3, 1) in

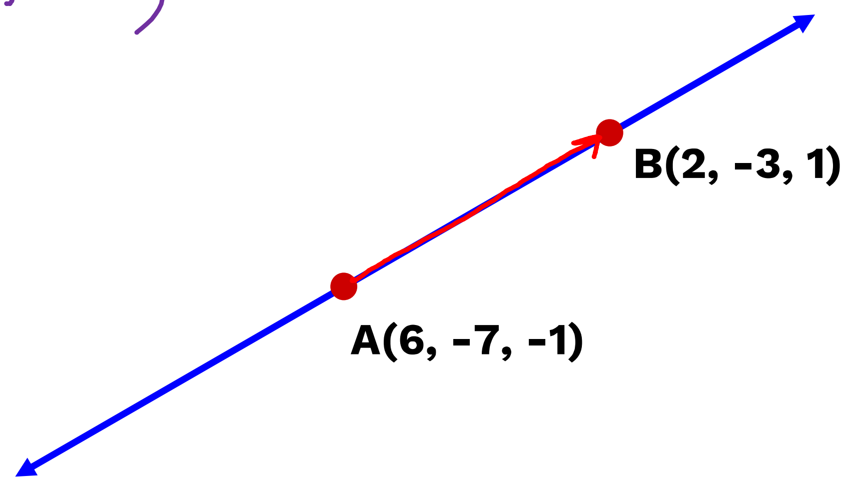
I. Vector Form
II. Cartesian Form

point $= (2, -3, 1)$

$\text{dir}^n = \overrightarrow{AB} = (-4, 4, 2)$

(I) $\vec{r} = 2\hat{i} - 3\hat{j} + \hat{k} + \lambda(-4\hat{i} + 4\hat{j} + 2k)$

(II) $\dfrac{x-2}{-4} = \dfrac{y+3}{4} = \dfrac{z-1}{2}$



Angle Between Two Straight Lines

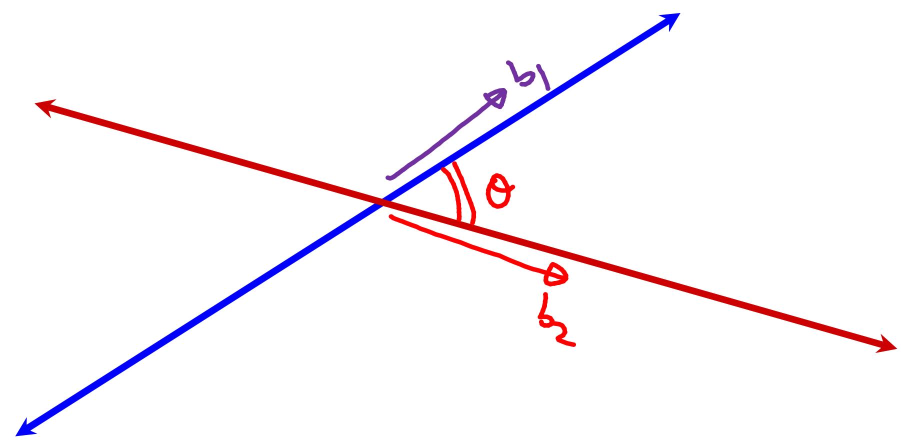
Angle Between Two Lines

Angle between two lines = Angle between their Direction Vectors

$L_1 : \vec{r} = \vec{a_1} + \lambda \vec{b_1}$

$L_2 : \vec{r} = \vec{a_2} + \mu \vec{b_2}$

$$\boxed{\cos\theta = \frac{\vec{b_1} \cdot \vec{b_2}}{|b_1||b_2|}}$$



Find the angle between the pair of lines given by

$$\vec{r} = 3\hat{i}+2\hat{j}-4\hat{k}+\lambda(\hat{i}+2\hat{j}+2\hat{k}) \quad L_1$$

$$\vec{r} = 5\hat{i}-2\hat{j}+\mu(3\hat{i}+2\hat{j}+6\hat{k}) \quad L_2$$

$\Rightarrow (1,2,2)$

$\Rightarrow (2,4,4)$

$\frac{1}{2} = \frac{2}{4} = \frac{2}{4}$

$$\cos\theta = \frac{(1,2,2)\cdot(3,2,6)}{\sqrt{1+4+4}\sqrt{9+4+36}}$$

$$= \frac{3+4+12}{3\times 7}$$

$$\cos\theta = \frac{19}{21} \Longrightarrow \boxed{\theta = \cos^{-1}\left(\frac{19}{21}\right)}$$

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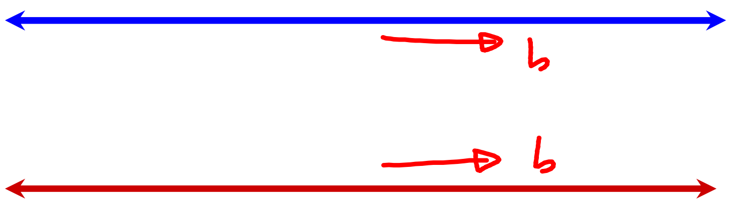
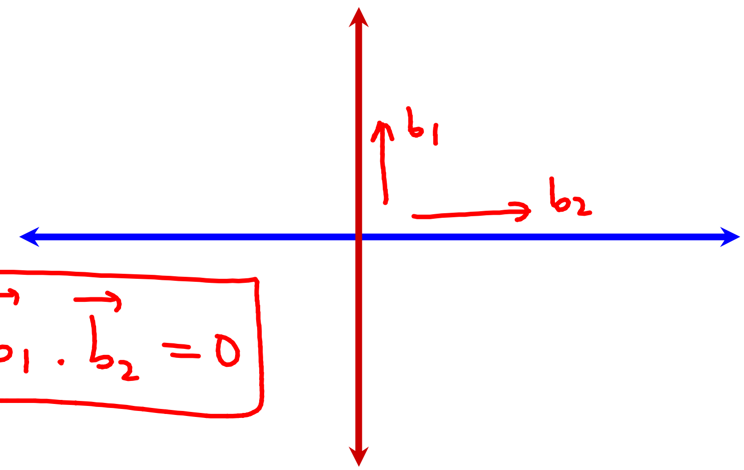
Angle Between Two Lines





$$\boxed{\vec{b_1} \cdot \vec{b_2} = 0}$$

Find the value of λ so that the following lines are perpendicular

$$\frac{x-5}{5\lambda+2}=\frac{2-y}{5}=\frac{1-z}{-1}$$

$$\frac{x}{1}=\frac{2y+1}{4\lambda}=\frac{1-z}{-3}$$

$$\frac{x-5}{5\lambda+2}=\frac{y-2}{-5}=\frac{z-1}{1}$$

$$\frac{x}{1}=\frac{y+\frac{1}{2}}{2\lambda}=\frac{z-1}{3}$$

$$1(5\lambda+2)-5(2\lambda)+3=0$$

$$5\lambda+2-10\lambda+3=0$$

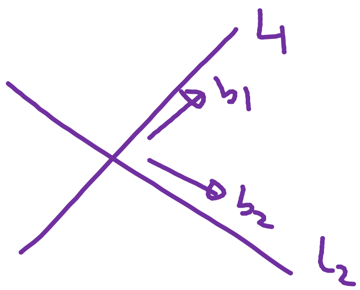
$$\boxed{\lambda=1}$$

Consider the lines $L_1 : \frac{x+1}{3} = \frac{y+2}{1} = \frac{z+1}{2} \quad L_2 : \frac{x-2}{1} = \frac{y+2}{2} = \frac{z-3}{3}$

The unit vector perpendicular to both L_1 and L_2 is

A. $\frac{-\hat{i}+7\hat{j}+7\hat{k}}{\sqrt{99}}$ ✓ **B.** $\frac{-\hat{i}-7\hat{j}+5\hat{k}}{5\sqrt{3}}$ **C.** $\frac{-\hat{i}+7\hat{j}+5\hat{k}}{5\sqrt{3}}$ **D.** $\frac{7\hat{i}-7\hat{j}-\hat{k}}{\sqrt{99}}$

[JEE Adv. 2018]



$$= \vec{b_1} \times \vec{b_2}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{vmatrix} = \frac{\hat{i}(-1) - \hat{j}(7) + k(5)}{\sqrt{1^2+7^2+5^2}}$$

✓ Parametric Coordinates

Line

Find parametric coordinates of straight lines

CF **I.** $\frac{x+3}{3}=\frac{y-1}{5}=\frac{z+3}{4}=\lambda$ PC $(3\lambda-3,\ 5\lambda+1,\ 4\lambda-3)$

VF **II.** $\vec{r}=\hat{i}+\hat{j}+\lambda(2\hat{i}-\hat{j}+\hat{k})$

PC $(1+2\lambda,\ 1-\lambda,\ \lambda)$

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Point of Intersection of Two Lines

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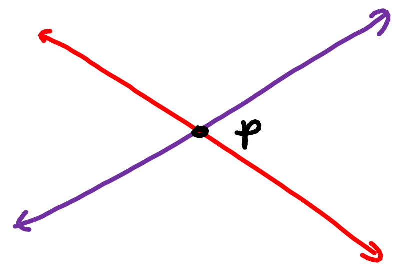
Point of Intersection of two Lines

Find the point of intersection of L_1 and L_2

$$L_1 : \frac{x-2}{6} = \frac{y-1}{-8} = \frac{z+2}{10} = \lambda = \frac{1}{2}$$

$$\boxed{L_2 : \frac{x+1}{6} = \frac{y+10}{7} = \frac{z-5}{-2} = \mu = 1}$$

P.O.I.
$(5, -3, 3)$



$6\lambda + 2 = 6\mu - 1$ | $-8\lambda + 1 = 7\mu - 10$ | $\underbrace{10\lambda - 2} = \underbrace{-2\mu + 5}$

$\lambda = \frac{1}{2} \quad \mu = 1$

LHS = 3

RHS = 3

LHS = RHS

L_1 and L_2 intersecting

Foot of Perpendicular/ Image from Point to Line

Find the foot of perpendicular from the point (0, 2, 3) on the line $\frac{x+3}{5}=\frac{y-1}{2}=\frac{z+4}{3}$. Find the length of perpendicular.

Method :- $\overrightarrow{PQ}\cdot(5,2,3)=0$

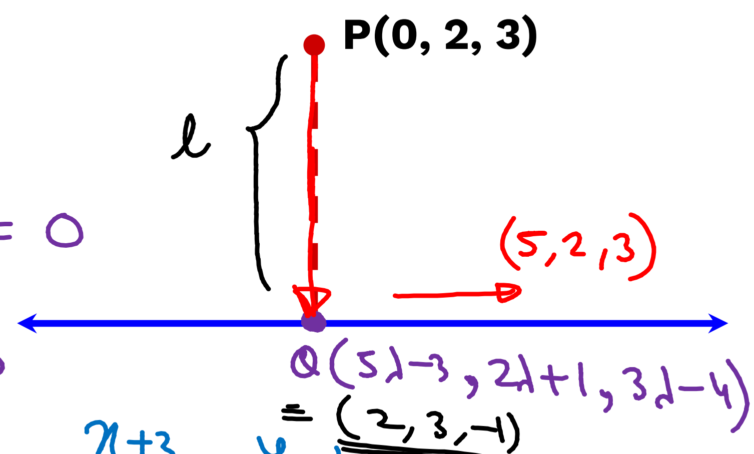
$(5\lambda-3,\ 2\lambda-1,\ 3\lambda-7)\cdot(5,2,3)=0$

$5(5\lambda-3)+2(2\lambda-1)+3(3\lambda-7)=0$

$25\lambda+4\lambda+9\lambda-15-2-21=0$

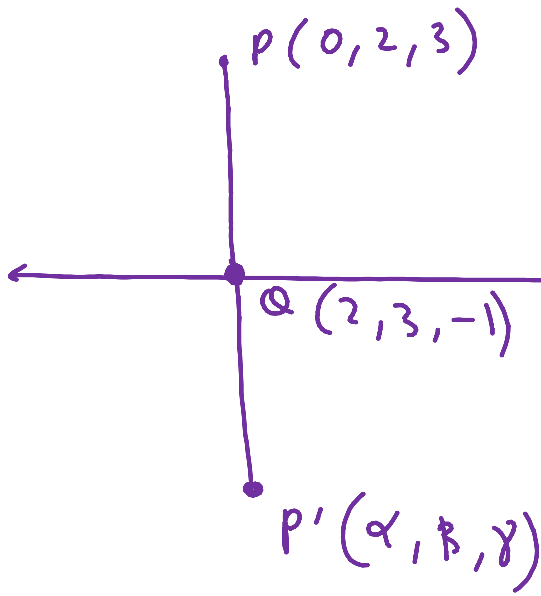
$38\lambda-38=0$

$\boxed{\lambda=1}$



$\frac{x+3}{5}=\frac{y-1}{2}=\frac{z+4}{3}=\lambda$

Image of Point in Line



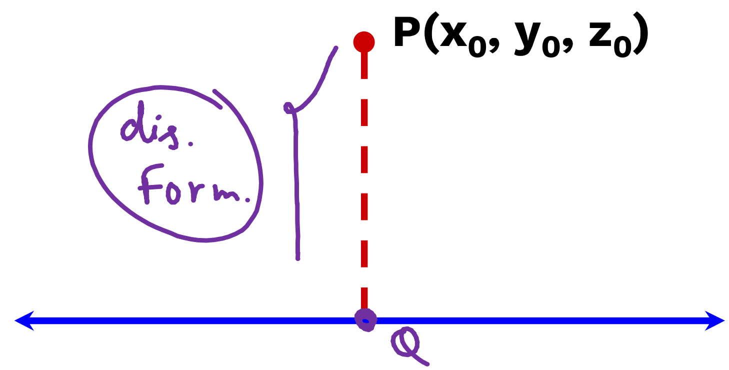
$$\left(\frac{\alpha+0}{2}, \frac{\beta+2}{2}, \frac{\gamma+3}{2}\right) \equiv (2,3,-1)$$

$$\boxed{\alpha = 4 \quad \beta = 4 \quad \gamma = -5}$$

Perpendicular Distance of Point from Line

Perpendicular Distance of Point from Line

M-1 Find Foot of Perpendicular then use Distance Formula



$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$$

Perpendicular Distance of Point from Line

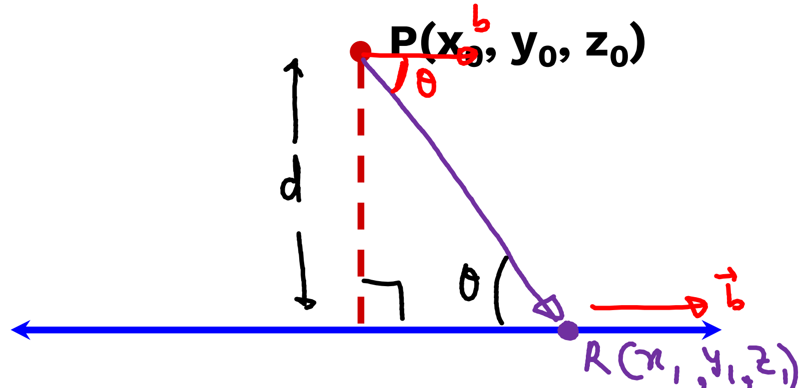
M-2 Perpendicular Distance = $\dfrac{|\vec{PR} \times \vec{b}|}{|\vec{b}|}$

$\vec{b} \wedge \vec{PR} = \theta$

$\dfrac{d}{PR} = \sin\theta$

$d = |PR| \sin\theta$

$= \cancel{|PR|} \dfrac{\vec{PR} \times \vec{b}}{\cancel{|PR|}\,|b|}$



$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

Find distance between point P(0, 2, 3) and line $\dfrac{x-3}{2} = \dfrac{y-1}{1} = \dfrac{z+1}{2}$

$P(0, 2, 3)$

$R(3, 1, -1)$

$\vec{PR} = (3, -1, -4)$

$\vec{b} = (2, 1, 2)$

$$\boxed{P.d = \frac{15}{3} = 5 \text{ units}}$$

$$\vec{PR} \times \vec{b} = \begin{vmatrix} i & j & k \\ 3 & -1 & -4 \\ 2 & 1 & 2 \end{vmatrix}$$

$$= i(2) - j(14) + k(5)$$

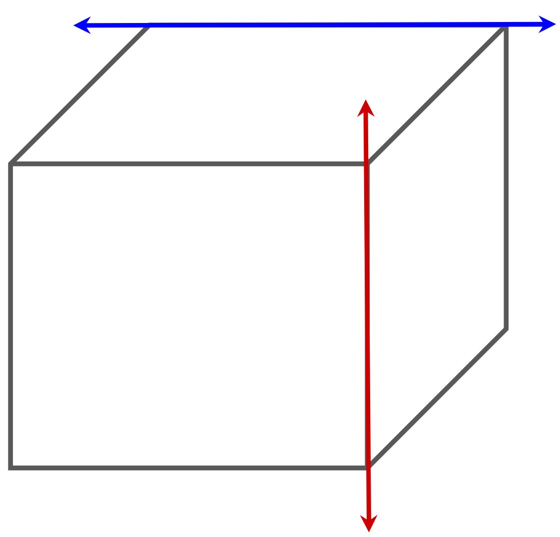
$$|PR \times b| = \sqrt{2^2 + 14^2 + 5^2} = \sqrt{4 + 196 + 25} = 15$$

$$|b| = \sqrt{2^2 + 1^2 + 2^2} = 3$$

Skew Lines

Skew Lines

Non parallel, non intersecting lines are called skew lines

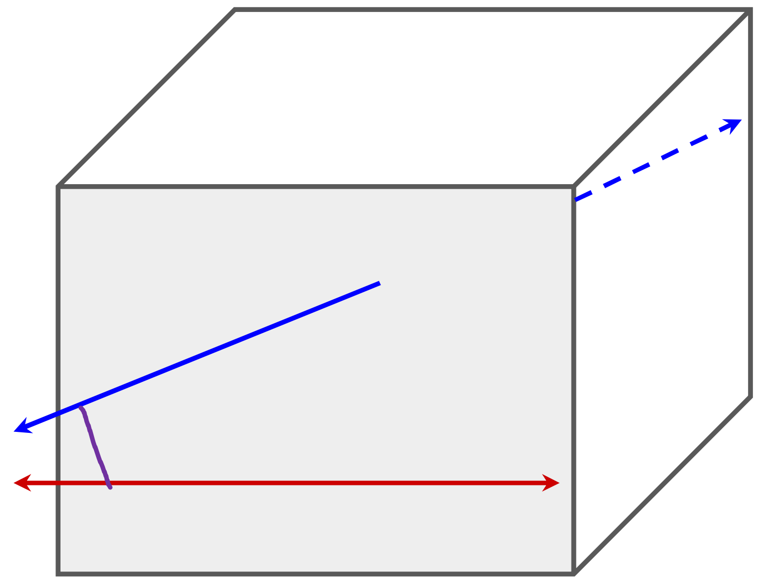


Shortest Distance between two Skew Lines

Shortest distance = $$\frac{\left|(\overrightarrow{a_2} - \overrightarrow{a_1}).(\overrightarrow{b_1} \times \overrightarrow{b_2})\right|}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|}$$

$$\vec{r} = \vec{a_1} + \lambda \vec{b_1}$$

$$\vec{r} = \vec{a_2} + \mu \vec{b_2}$$



Consider the lines $L_1: \frac{x+1}{3} = \frac{y+2}{1} = \frac{z+1}{2}$ $L_2: \frac{x-2}{1} = \frac{y+2}{2} = \frac{z-3}{3}$

The shortest distance between L_1 and L_2 is

A. 0 **B.** $17 / \sqrt{3}$ **C.** $41 / 5\sqrt{3}$ **D.** $17 / 5\sqrt{3}$

[JEE Adv. 2018]

$\vec{a}_2 = (2, -2, 3)$

$\vec{a}_1 = (-1, -2, -1)$

$\vec{a}_2 - \vec{a}_1 = (3, 0, 4)$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} i & j & k \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{vmatrix}$$

$$= \hat{i}(-1) - \hat{j}(7) + \hat{k}(5)$$

$$= (-1, -7, 5)$$

$$S.d = \frac{(a_2 - a_1)\cdot \vec{b_1} \times \vec{b_2}}{|b_1 \times b_2|}$$

$$= \frac{(3,0,4)\cdot(-1,-7,5)}{\sqrt{1+49+25}}$$

$$= \left| \frac{-3+20}{5\sqrt{3}} \right|$$

$$= \frac{17}{5\sqrt{3}}$$

If the shortest distance between the lines

$\vec{r}_1 = \alpha\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} + 2\hat{k}), \lambda \in \mathbf{R}, \alpha > 0$ and

$\vec{r}_2 = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k})$, $\mu \in R$ is 9, then α is equal to _

[20 July 2021 Shift 1]

$S.d = 9$

$\vec{a}_2 = (-4, 0, -1)$

$\vec{a}_1 = (\alpha, 2, 2)$

$\vec{a}_1 - \vec{a}_2 = (\alpha+4, 2, 3)$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} i & j & k \\ 1 & -2 & 2 \\ 3 & -2 & -2 \end{vmatrix}$$

$$= \hat{i}(8) - \hat{j}(-8) + \hat{k}(4)$$

$$= (8, 8, 4)$$

$$= (2, 2, 1)$$

$$s.d = \frac{(\vec{a}_1 - \vec{a}_2)\cdot(\vec{b}_1 \times \vec{b}_2)}{|b_1 \times b_2|}$$

$$= \left| \frac{(\alpha+4, 2, 3)\cdot(2, 2, 1)}{3} \right| = 9$$

$$|2(\alpha+4) + 4 + 3| = 27$$

$$2\alpha + 15 = \pm 27$$

$$2\alpha + 15 = 27$$

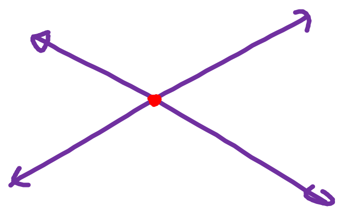
$$\boxed{\alpha = 6} \checkmark$$

$$\cancel{2\alpha + 15 = -27}$$

$$\cancel{\alpha = \text{Rej}}$$

Condition for 2 Lines to Intersect

If shortest distance between 2 lines = 0 ⇒ they are intersecting



$\boxed{\text{if } S.d = 0 \Rightarrow \text{intersecting}}$

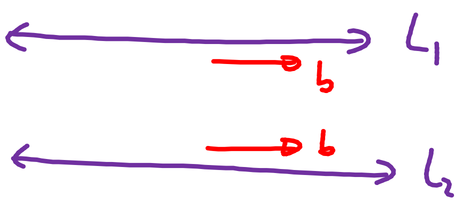
$(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = 0$

$$\boxed{[\vec{a}_2 - \vec{a}_1 \quad \vec{b}_1 \quad \vec{b}_2] = 0}$$

Shortest Distance between two Parallel Lines

Shortest Distance between two Parallel Lines

$$\text{Shortest distance between parallel lines} = \frac{\left|(\vec{a_2} - \vec{a_1}) \times \vec{b}\right|}{|\vec{b}|}$$



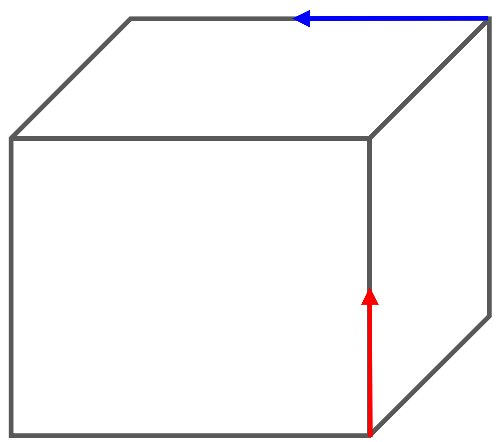
$$\boxed{\begin{aligned} \vec{r} &= \vec{a_1} + \lambda \vec{b} \\ \vec{r} &= a_2 + \mu \vec{b} \end{aligned}}$$

$$\frac{\left|(a_2 - a_1) \times b\right|}{|b|}$$

Condition for Coplanarity

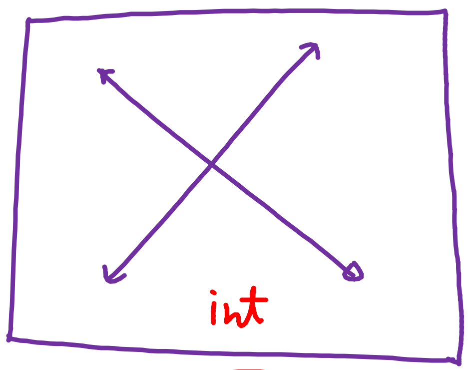
Condition of Coplanarity (for vectors)

1. Two non-zero vectors are always coplanar
2. Three non-zero vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar if $[\vec{a}\ \vec{b}\ \vec{c}] = 0$

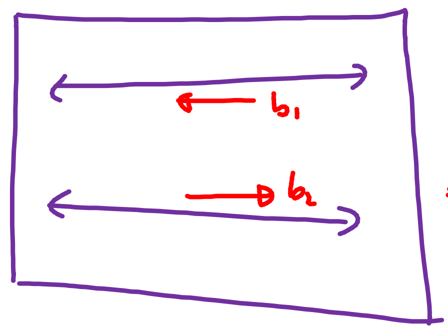


Condition of Coplanarity (for lines)

Two lines are COPLANAR if either they are Intersecting or Parallel



$\boxed{S.d = 0}$



$\vec{b}_1 \parallel \vec{b}_2$

$\boxed{\text{Ratio} = \text{same}}$

$$\frac{x-k}{1}=\frac{y-2}{2}=\frac{z-3}{3} \text{ and } \frac{x+1}{3}=\frac{y+2}{2}=\frac{z+3}{1}$$

are co-planar, then the value of k is

[25 July 2021 Shift 2]

$\Downarrow$

int OR ~~parallel~~

$a_2 = (-1, -2, -3)$

$a_1 = (k, 2, 3)$

$\vec{a}_1 - \vec{a}_2 = (k+1, 4, 6)$

$$[a_2 - a_1 \quad b_1 \quad b_2] = 0$$

$$\begin{vmatrix} k+1 & 4 & 6 \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{vmatrix} = 0 \Longrightarrow k = 1$$

Two lines $L_1 : x = 5, \frac{y}{3-\alpha} = \frac{z}{-2}$ and $L_2 : x = \alpha, \frac{y}{-1} = \frac{z}{2-\alpha}$ are coplanar. Then $\boldsymbol{\alpha}$ can take value(s)

AD

A. 1 **B.** 2 **C.** 3 **D.** 4

[JEE Adv. 2013]

$L_1 : \frac{x-5}{0} = \frac{y}{3-\alpha} = \frac{z}{-2}$

$L_2 : \frac{x-\alpha}{0} = \frac{y}{-1} = \frac{z}{2-\alpha}$

$\vec{a}_2 = (\alpha, 0, 0)$

$\vec{a}_1 = (5, 0, 0)$

$\vec{a}_2 - \vec{a}_1 = (\alpha - 5, 0, 0)$

$$S.d = 0$$

$$[a_2 - a_1 \quad b_1 \quad b_2] = 0$$

$$\begin{vmatrix} \alpha - 5 & 0 & 0 \\ 0 & 3-\alpha & -2 \\ 0 & -1 & 2-\alpha \end{vmatrix} = 0$$

$$(\alpha - 5)\{(3-\alpha)(2-\alpha) - 2\} = 0$$

$$(\alpha - 5)\{\alpha^2 - 5\alpha + 4\} = 0$$

$$(\alpha - 5)(\alpha - 1)(\alpha - 4) = 0$$

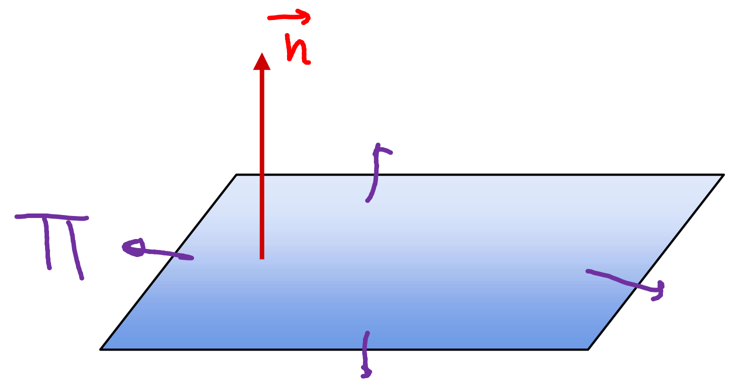
$$\boxed{\alpha = 1, 4, 5}$$

Equation of Planes

Definition of Plane

Flat surface perpendicular to a fixed vector.
This flat surface is called **Plane** and fixed vector is called **Normal**

Representation of Plane : P, $\prod$, $\boldsymbol{\sigma}$



General Equation of Plane

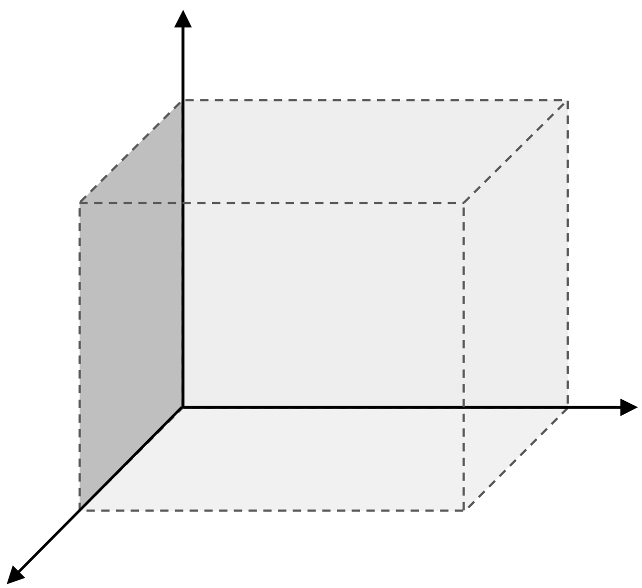
$$\boxed{ax + by + cz + d = 0}$$

$$\boxed{2x + 3y + 5z - 3 = 0}$$

Plane

Note

1. Equation of xy plane is $z=0$
2. Equation of yz plane is $x=0$
3. Equation of zx plane is $y=0$



Form

If point lying on plane is A($\vec{a}$)

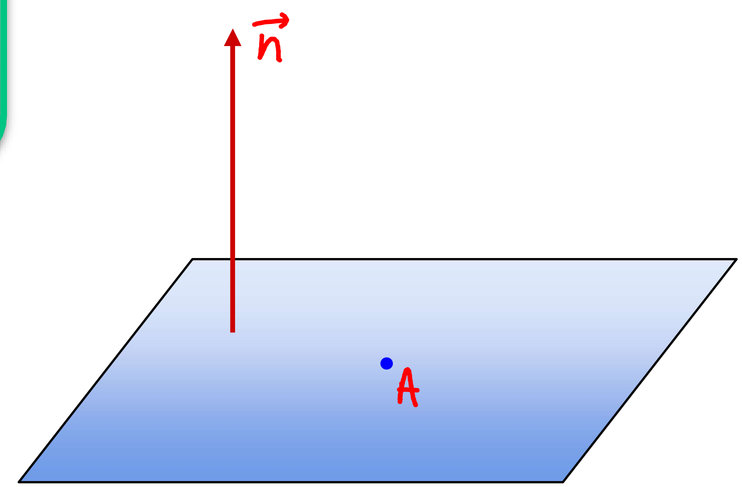
and normal vector to plane is $\vec{n}$

then Equation of Plane is given by $\boxed{\vec{r}.\vec{n} = \vec{a}.\vec{n}}$

① point = $\vec{a}$

② $\vec{n}$ = $\vec{n}$

$\boxed{\vec{r}.\vec{n} = \vec{a}.n}$



Find the equation of plane passing through ***(2,0,1)*** and normal to ***2i + j - k*** in

I. Vector Form

II. Cartesian Form

Point $\vec{a} = (2,0,1)$

$\vec{n} = (2,1,-1)$

$\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n} \longrightarrow$ Normal vector

VF $\boxed{\vec{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) = 3}$

CF $\boxed{2x + y - z = 3}$ (given)

$\boxed{\vec{n} = (2,1,-1)}$

Note

If $\boxed{ax + by + cz + d = 0}$ is the equation of plane, then

a, b, c are direction ratios of normal or

$a\hat{i} + b\hat{j} + c\hat{k}$ is the normal vector

If the Equation of Plane is ***2x + y + 2z = 5***, then find

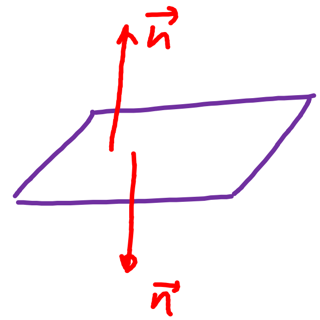
I. Normal Vector

II. DRs of Normal $\Rightarrow \lambda(2, 1, 2)$

III. DCs of Normal $\Rightarrow \pm\left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)$

IV. Equation of plane in Vector Form

$$\boxed{\vec{r}\cdot(2\hat{i}+\hat{j}+2\hat{k}) = 5}$$



$$2x + y + 2z = 5$$

$$\sqrt{4+1+4}$$

$$\vec{n} = \pm(2\hat{i} + \hat{j} + 2\hat{k})$$

$$DC = \pm\left(\frac{2\hat{i} + \hat{j} + 2\hat{k}}{3}\right)$$

Unacademy Atoms

2. Three Points Form

Equation of plane passing through $A(\vec{a})$, $B(\vec{b})$, $C(\vec{c})$ is given by

$$[\vec{r}-\vec{a} \quad \vec{b}-\vec{a} \quad \vec{c}-\vec{a}]=0$$

Eqn. of plane

① point $= \vec{a}$

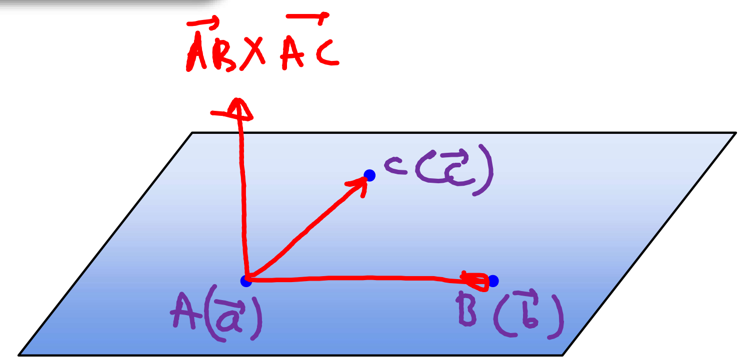
② $\vec{n} = \vec{AB} \times \vec{AC}$

$$\vec{r}\cdot\vec{n} - \vec{a}\cdot\vec{n} = 0$$

$$(\vec{r}-\vec{a})\cdot\vec{n} = 0$$

$$(\vec{r}-\vec{a})\cdot(\vec{AB}\times\vec{AC}) = 0$$

$$[r-a \quad \vec{AB} \quad \vec{AC}] = 0$$



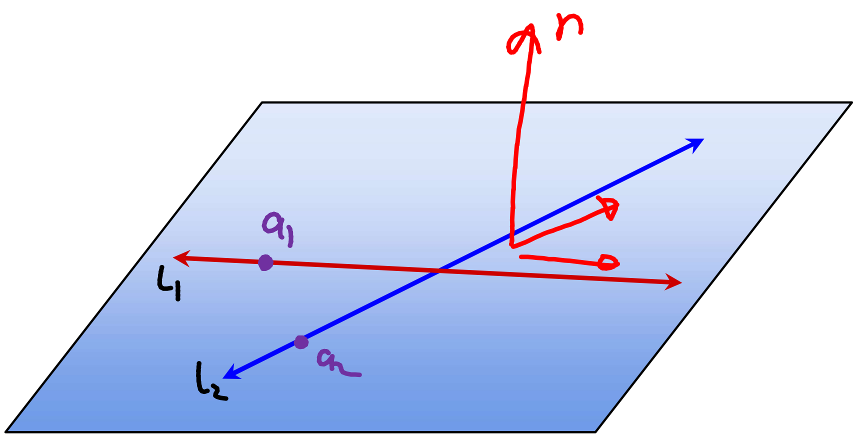
3. Plane Containing Two Intersecting Lines

Equation of plane containing

$L_1 : \vec{r} = \overrightarrow{a_1} + \lambda \overrightarrow{b_1}$

$L_2 : \vec{r} = \overrightarrow{a_2} + \lambda \overrightarrow{b_2}$

is given by $[\vec{r} - \overrightarrow{a_1} \quad \overrightarrow{b_1} \quad \overrightarrow{b_2}] = 0$



Eqn of plane

① point = $\vec{a}_1$ / $\vec{a}_2$

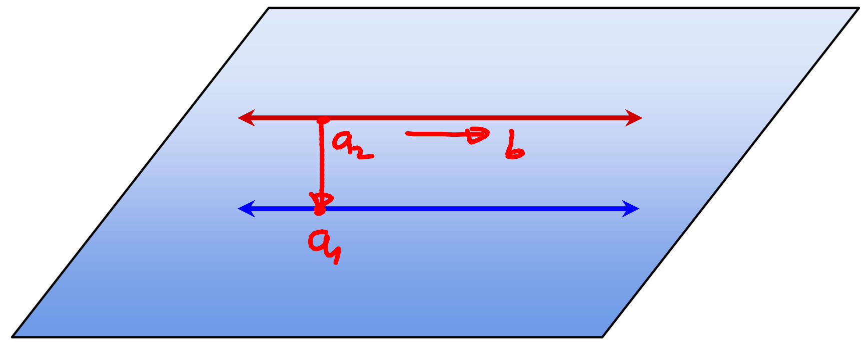
② normal = $\vec{b}_1 \times \vec{b}_2$

4. Equation of Plane Containing Two Parallel Lines

$$\boxed{\begin{aligned} \vec{r} &= \vec{a_1} + \lambda \vec{b} \\ \vec{r} &= a_2 + \mu \vec{b} \end{aligned}}$$

point $= \vec{a}_1 \ / \ \vec{a}_2$

$$\vec{n} = (a_2 - a_1) \times \vec{b}$$

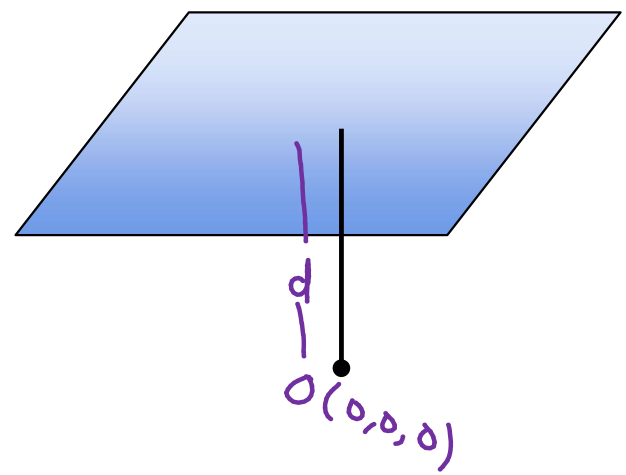


5. Equation of Plane: Normal Form

If unit vector normal to the plane is $\hat{n}$, and perpendicular distance of plane from origin is d then equation of plane is given by $\vec{r}.\hat{n} = d$

$\hat{n}$ d

$$\boxed{\vec{r}\cdot\hat{n} = d}$$



Find the vector equation of plane which is at a distance of **8 units** from the origin and which is normal to the vector **2i + j + 2k**.

$d = 8$

$\vec{n} = 2\hat{i} + \hat{j} + 2\hat{k}$

$\hat{n} = \left(\dfrac{2\hat{i} + \hat{j} + 2\hat{k}}{3}\right)$

Eqn of Plane $\vec{r} \cdot \hat{n} = d$

$$\boxed{\vec{r} \cdot \left(\dfrac{2\hat{i} + \hat{j} + 2\hat{k}}{3}\right) = 8}$$

Unacademy Atoms

6. Intercept form

If x, y and z intercept of a plane is a, b, c

then Equation of Plane is given by $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

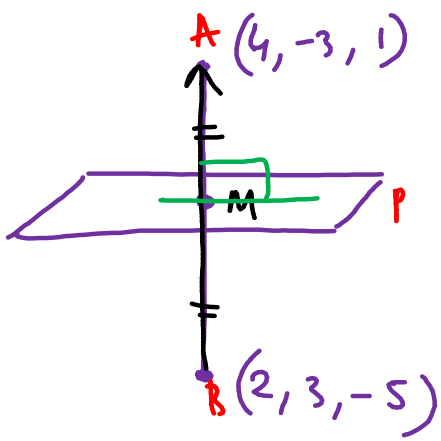
$$\boxed{\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1}$$

$$\frac{x}{2}+\frac{y}{3}+\frac{z}{5}=1$$

Unacademy Atoms

Let the plane $ax + by + cz + d = 0$ bisect the line joining the points (4, -3, 1) and (2, 3, -5) at the right angles. If a, b, c, d are integers, then the minimum value of $(a^2 + b^2 + c^2 + d^2)$ is

[18 Mar 2021 shift 1]



Eqn. of Plane

① point = $(3, 0, -2)$

② $\vec{n} = \vec{BA} = (2, -6, 6)$

$= (1, -3, 3)$

$$\boxed{\vec{r}\cdot\vec{n} = \vec{a}\cdot\vec{n}}$$

$$(x, y, z)\cdot(1, -3, 3) = (3, 0, -2)\cdot(1, -3, 3)$$

$$\boxed{x - 3y + 3z = -3}$$

$ax + by + cz + d = 0$

$\boxed{x - 3y + 3z + 3 = 0}$ ✓

$a = 1 \quad b = -3 \quad c = 3 \quad d = 3$

$\boxed{a^2 + b^2 + c^2 + d^2 = 28}$

Unacademy Atoms

Let $\alpha, \beta, \gamma, \delta$ be real numbers such that $\alpha^2+\beta^2+\gamma^2 \neq 0$ and $\boxed{\alpha+\gamma=1}$. Suppose the point $(3,2,-1)$ is the mirror image of the point $(1,0,-1)$ with respect to the plane $\alpha x+\beta y+\gamma z=\delta$. Then which of the following statements is/are TRUE?

[JEE Adv. 2020]

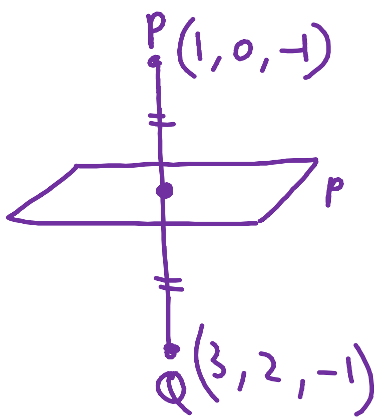
(A) $\overset{1}{\alpha}+\overset{1}{\beta}=2$

(B) $\delta-\gamma=3$

(C) $\overset{3}{\delta}+\overset{1}{\beta}=4$

(D) $\alpha+\beta+\gamma=\delta$

ABC



Eqn of Plane

① point $=(2,1,-1)$

② $\vec{n}=(2,2,0)$

$$\vec{r}\cdot\vec{n}=\vec{a}\cdot\vec{n}$$

$$(x,y,z)\cdot(2,2,0)=(2,1,-1)\cdot(2,2,0)$$

$$2x+2y=6$$

$$x + y = 3$$

$$\alpha x + \beta y + \gamma z = \delta$$

$$\alpha = 1 \quad \beta = 1 \quad \gamma = 0 \quad \delta = 3$$

$$\alpha + \gamma = 1$$

Unacademy Atoms

The equation of the plane passing through the point (1, 1, 1) and perpendicular to the planes $2x+y-2z=5$ and $3x-6y-2z=7$, is

[JEE Adv. 2017]

(a) $14x+2y-15z=1$
(b) $14x-2y+15z=27$
(c) $14x+2y+15z=31$ ✓
(d) $-14x+2y+15z=3$

point = (1, 1, 1)

normal $= \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} i & j & k \\ 2 & 1 & -2 \\ 3 & -6 & -2 \end{vmatrix} = \hat{i}(-14) - \hat{j}(2) + \hat{k}(-15)$

$= (-14, -2, -15)$

$= (14, 2, 15)$

$\vec{r}\cdot\vec{n} = \vec{a}\cdot\vec{n}$

$$\boxed{14x+2y+15z=31}$$

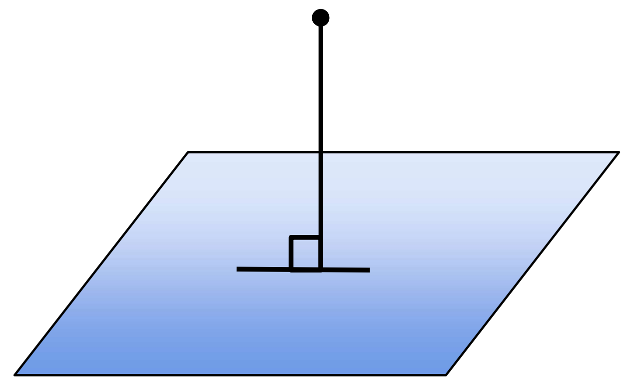
Perpendicular Distance of Point from Plane

Unacademy Atoms

Perpendicular Distance Of A Point From A Plane

Perpendicular distance of (x_1, y_1, z_1) from $ax + by + cz + d = 0$ is

$$\boxed{\frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}}$$



A plane which is perpendicular to two planes $2x-2y+z=0$ and $x-y+2z=4$, passes through $(1,-2,1)$. The distance of the plane from the point $(1,2,2)$ is

(a) 0 (b) 1 (c) $\sqrt{2}$ (d) $2\sqrt{2}$ ✓

[JEE Adv. 2006]

plane

① point = $(1,-2,1)$

② Normal = $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -2 & 1 \\ 1 & -1 & 2 \end{vmatrix} = \hat{i}(-3) - \hat{j}(3) + \hat{k}(0)$

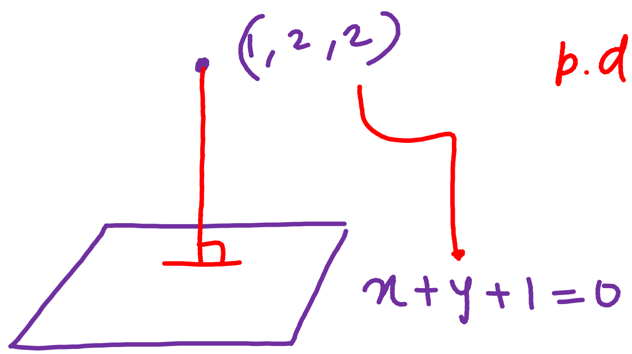
$= (-3,-3,0)$

$= (1,1,0)$

$\vec{r}\cdot\vec{n} = \vec{a}\cdot\vec{n}$

$(x,y,z)\cdot(1,1,0) = (1,-2,1)\cdot(1,1,0)$

$\boxed{x+y=-1}$

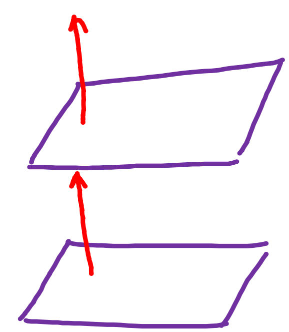


$$p.d = \left| \frac{1+2+1}{\sqrt{1^2+1^2+0^2}} \right| = \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

Distance Between Two Parallel Planes

Distance between Two Parallel Planes

Distance between two parallel planes $ax + by + cz + d_1 = 0$ and $ax + by + cz + d_2 = 0$ is given by $\boxed{\dfrac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}}$



If the distance between the plane $\underset{P_1}{Ax - 2y + z = d}$ and the plane (P_2) containing the lines (L_1) $\boxed{\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}}$ and (L_2) $\boxed{\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}}$ is $\sqrt{6}$, then find $|d|$.

[JEE Adv. 2010]

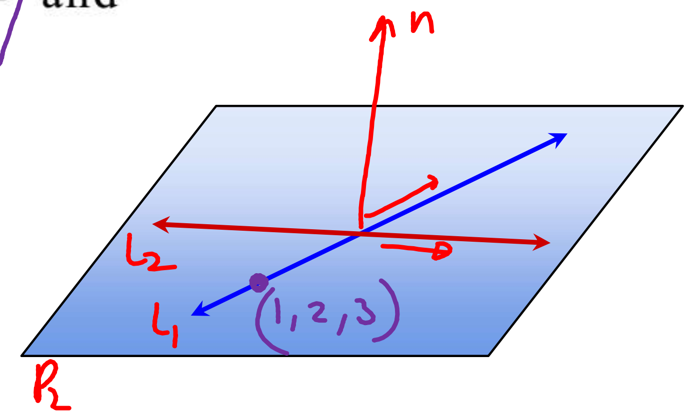
P_1 & $P_2 \to \sqrt{6}$

Eqn of P_2 :-

① Point = $(1, 2, 3)$

② Normal = $\begin{vmatrix} \hat{i} & \hat{j} & k \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = \hat{i}(-1) - \hat{j}(-2) + k(-1)$

$= (-1, 2, -1)$



$$\vec{r}\cdot\vec{n} = \vec{a}\cdot\vec{n}$$

$$(x, y, z)\cdot(-1, 2, -1) = (1, 2, 3)\cdot(-1, 2, -1)$$

P_2 $\boxed{-x + 2y - z = 0}$

P_1 $\boxed{Ax - 2y + z = d}$

P_2 $\boxed{x - 2y + z = 0}$

$A = 1$

P_1 and P_2 — int OR Parallel

$$\left|\frac{d - 0}{\sqrt{1^2 + 2^2 + 1^2}}\right| = \sqrt{6}$$

$$\boxed{|d| = 6}$$

The equation of the planes parallel to the plane $x - 2y + 2z - 3 = 0$ which are at unit distance from the point $(1, 2, 3)$ is $ax + by + cz + d = 0$. If $(b - d) = K(c - a)$, then the positive value of K is

[18 Mar 2021 Shift 1]

given plane $x - 2y + 2z - 3 = 0$

$P : x - 2y + 2z + \lambda = 0 \qquad (1, 2, 3)$

$$p.d = \left| \frac{1 - 4 + 6 + \lambda}{\sqrt{1^2 + 2^2 + 2^2}} \right| = 1$$

$|3 + \lambda| = 3$

$3 + \lambda = \pm 3$

$\boxed{\lambda = 0 ; \lambda = -6}$

$x - 2y + 2z = 0$

$x - 2y + 2z - 6 = 0$

$ax + by + cz + d = 0$

$a = 1 \quad b = -2$

$c = 2 \quad d = 0$

$$K = \frac{(b - d)}{(c - a)} = \frac{-2}{1}$$

Rej

$a = 1 \quad b = -2$

$c = 2 \quad d = -6$

$$\boxed{K = \frac{b - d}{(c - a)} = \frac{4}{1}}$$

$$\boxed{K = 4}$$

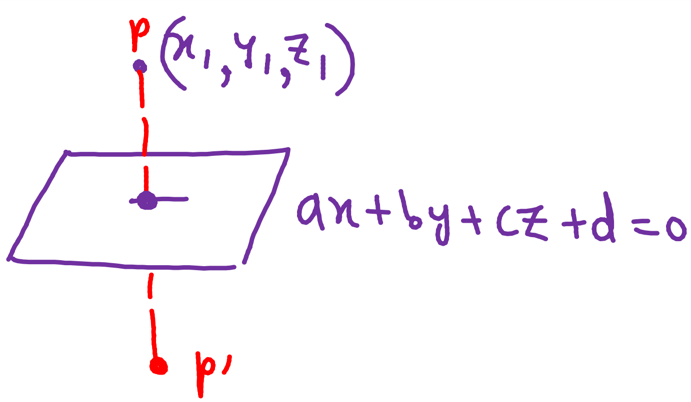
S.C

Foot of Perpendicular / Image of Point in Plane

Image of Point in Plane

Image of the point (x_1, y_1, z_1) in the plane $ax + by + cz + d = 0$

is given by $\boxed{\dfrac{x-x_1}{a} = \dfrac{y-y_1}{b} = \dfrac{z-z_1}{c} = -2\left(\dfrac{ax_1+by_1+cz_1+d}{a^2+b^2+c^2}\right)}$



Foot of Perpendicular from Point on the Plane

Foot of perpendicular of point (x_1, y_1, z_1) on the plane $ax + by + cz + d = 0$

is given by $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c} = -\left(\frac{ax_1+by_1+cz_1+d}{a^2+b^2+c^2}\right)$

If the mirror image of the point (1,3,5) with respect to the plane $4x - 5y + 2z = 8$ is (α, β, γ), then $5(\alpha + \beta + \gamma)$ equals:

A. 47 B. 39 C. 43 D. 41

[26 Feb 2021 Shift 2]

$$\frac{x-1}{4} = \frac{y-3}{-5} = \frac{z-5}{2} = -2\left(\frac{4-15+10-8}{16+25+4}\right)$$

$\frac{4}{5}+5$

$$\frac{x-1}{4} = \frac{y-3}{-5} = \frac{z-5}{2} = \frac{2}{5}$$

$$(\alpha, \beta, \gamma) = \left(\frac{13}{5}, 1, \frac{29}{5}\right)$$

If the distance of the point $P(1, -2, 1)$ from the plane $x + 2y - 2z = \alpha$, where $\alpha > 0$, is 5, then the foot of the perpendicular from P to the plane is

(a) $\left(\frac{8}{3}, \frac{4}{3}, -\frac{7}{3}\right)$ (b) $\left(\frac{4}{3}, -\frac{4}{3}, \frac{1}{3}\right)$

(c) $\left(\frac{1}{3}, \frac{2}{3}, \frac{10}{3}\right)$ (d) $\left(\frac{2}{3}, -\frac{1}{3}, \frac{5}{2}\right)$

[JEE Adv. 2010]

$P(1, -2, 1)$

$x + 2y - 2z - \alpha = 0$

$$\left|\frac{1 + (-4) - 2 - \alpha}{\sqrt{1^2 + 2^2 + 2^2}}\right| = 5 \Rightarrow \left|\frac{+5 + \alpha}{3}\right| = 5$$

$|5 + \alpha| = 15$

$5 + \alpha = \pm 15$

$\boxed{\alpha = 10}$ $\boxed{\cancel{\alpha = -20}}$

$P(1,-2,1)$

$x+2y-2z-10=0$

$\frac{15}{9}=5/3$

$$\frac{x-1}{1}=\frac{y+2}{2}=\frac{z-1}{-2}=-1\left(\frac{1-4-2-10}{9}\right)$$

$$\frac{x-1}{1}=\frac{y+2}{2}=\frac{z-1}{-2}=\frac{5}{3}$$

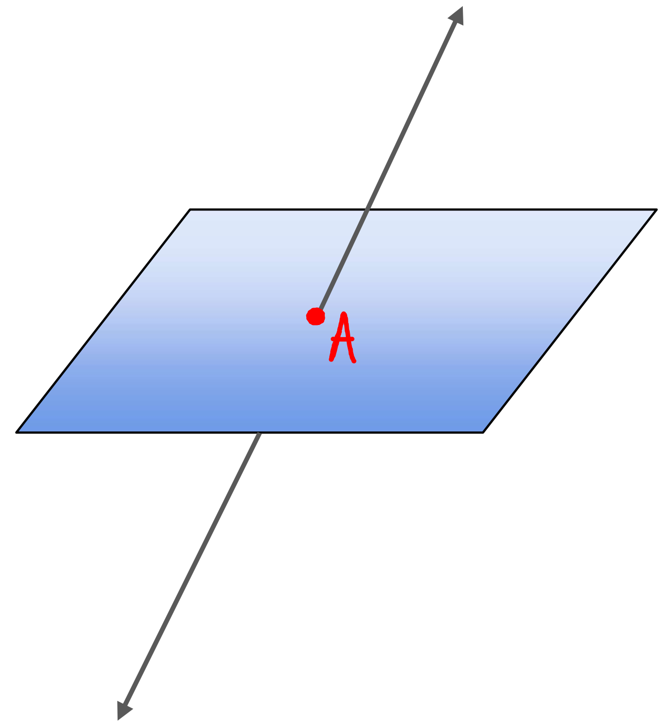
$$x=\frac{8}{3}$$

Solving Line and Plane

Solving Line and Plane

Method:

Take a general point on line and substitute it in plane



The distance of the point (1, 1, 9) from the point of intersection of the line $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2}$ and the plane x + y + z = 17 is :

JEE M 2024

(1) $\sqrt{38}$

(2) $19\sqrt{2}$

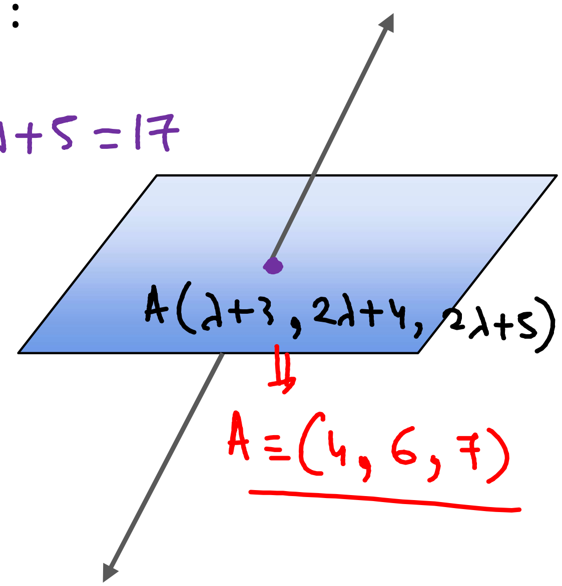
(3) $2\sqrt{19}$

(4) 38

$\lambda + 3 + 2\lambda + 4 + 2\lambda + 5 = 17$

$5\lambda = 5$

$\lambda = 1$



$A = (4, 6, 7)$

$\left.\begin{matrix}(1,1,9)\\(4,6,7)\end{matrix}\right\}$ $d = \sqrt{3^2 + 5^2 + 2^2}$

$= \sqrt{9 + 25 + 4}$

$= \sqrt{38}$ //

Unacademy Atoms

Projection of Line Segment on Plane

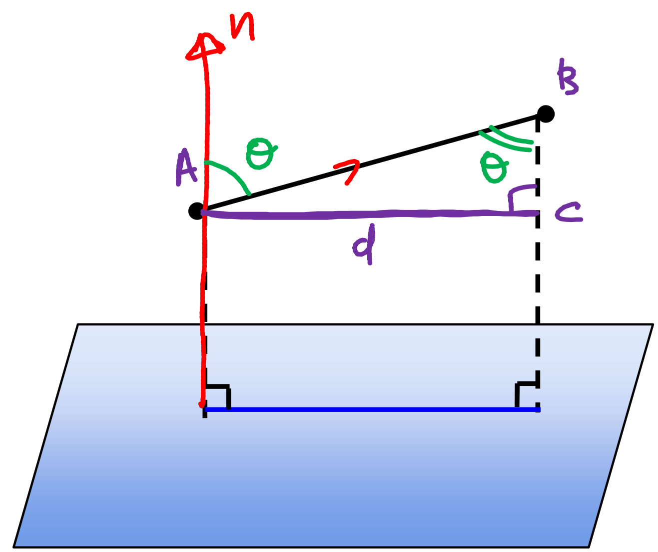
Projection of Line Segment on Plane

$$\frac{d}{|AB|} = \sin\theta$$

$$d = |AB| \sin\theta$$

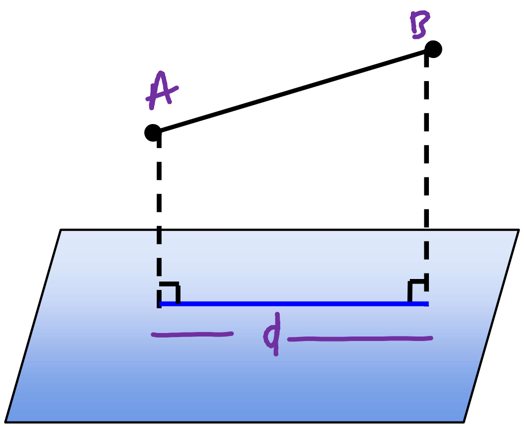
$$= \cancel{|AB|} \frac{\overrightarrow{AB} \times \vec{n}}{\cancel{|AB|}\, |\vec{n}|}$$

$$\boxed{d = \frac{|\overrightarrow{AB} \times \vec{n}|}{|\vec{n}|}}$$



The length of the projection of the line segment joining the points $\overset{A}{(5, -1, 4)}$ and $\overset{B}{(4, -1, 3)}$ on the plane, $x + y + z = 7$ is:

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ ✓(c) $\sqrt{\frac{2}{3}}$ (d) $\frac{2}{\sqrt{3}}$



[JEE M 2018]

$\vec{AB} \equiv (-1, 0, -1)$

$\vec{n} = (1, 1, 1)$ $\boxed{|\vec{n}| = \sqrt{3}}$

$\vec{AB} \times \vec{n} = \begin{vmatrix} i & j & k \\ -1 & 0 & -1 \\ 1 & 1 & 1 \end{vmatrix} = \hat{i}(1) - \hat{j}(0) + k(-1)$

$= (1, 0, -1)$

$|\vec{AB} \times \vec{n}| = \sqrt{1^2 + 0^2 + 1^2} = \underline{\sqrt{2}}$

Angle between Plane-Plane, Line-Line and Line-Plane

Angle Between Two Planes

Angle between two planes is same as angle between their normal

$$\boxed{\cos\theta = \frac{\overrightarrow{n_1}.\overrightarrow{n_2}}{|\overrightarrow{n_1}||\overrightarrow{n_2}|}}$$

Unacademy Atoms

Angle Between Two Planes

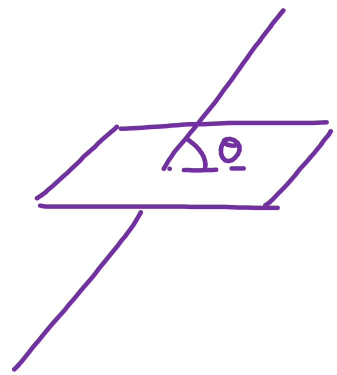
Angle between two lines is same as angle between direction vectors

$$\cos\theta = \frac{\overrightarrow{b_1}.\overrightarrow{b_2}}{|\overrightarrow{b_1}||\overrightarrow{b_2}|}$$

Angle Between Line and Plane

$$\boxed{\sin\theta = \frac{\vec{n}.\vec{b}}{|\vec{n}||\vec{b}|}}$$

$$\boxed{\sin\theta = \frac{\vec{n} \cdot \vec{b}}{|n|\,|b|}}$$



Parallel and Perpendicular Planes

Two plane $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ are

1. Parallel if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \neq \frac{d_1}{d_2}$
2. Identical if $\boxed{\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{d_1}{d_2}}$
3. Perpendicular if $\boxed{a_1a_2 + b_1b_2 + c_1c_2 = 0}$

$P_1 \perp P_2$

$\boxed{90^\circ}$

$\boxed{\vec{n_1} \cdot \vec{n_2} = 0}$

Consider the three planes

$P_1 : 3x + 15y + 21z = 9$

$P_2 : x - 3y - z = 5$, and

$P_3 : 2x + 10y + 14z = 5$

$$\boxed{\frac{3}{2} = \frac{15}{10} = \frac{21}{14} \neq \frac{9}{5}}$$

Then, which one of the following is true ?

A. P_1 and P_3 are parallel.

B. P_2 and P_3 are parallel.

C. P_1 and P_2 are parallel.

D. P_1, P_2 and P_3 all are parallel.

[26 Feb 2021 Shift 1]

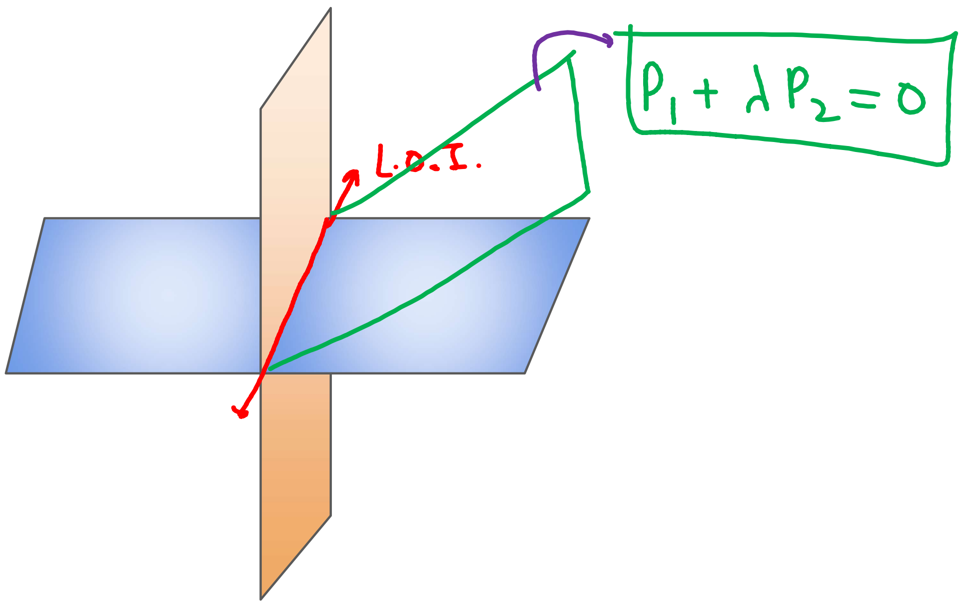
Unacademy Atoms

Unacademy Atoms

Family of Planes

Family of Planes

Equation of Plane passing through the line of intersection of planes P_1 and P_2 is given by $P_1 + \lambda P_2 = 0$



[08 Apr 2019 Shift 1]

The equation of a plane containing the line of intersection of the planes $2x - y - 4 = 0$ and $y + 2z - 4 = 0$ and passing through the point $(1, 1, 0)$ is :

A. $x - 3y - 2z = -2$

B. $2x - z = 2$

C. $x - y - z = 0$

D. $x + 3y + z = 4$

$$(2x - y - 4) - 1(y + 2z - 4) = 0$$

$$(2x - y - 4) + \lambda(y + 2z - 4) = 0$$

$$2x + (-1+\lambda)y + (2\lambda)z + (-4-4\lambda) = 0$$

$$2(1) + (-1+\lambda)1 + 0 - 4 - 4\lambda = 0$$

$$2 - 1 + \lambda - 4 - 4\lambda = 0$$

$$-3\lambda - 3 = 0$$

$$\boxed{\lambda = -1}$$

If the equation of the plane passing through the line of intersection of the planes $2x - 7y + 4z - 3 = 0$, $3x - 5y + 4z + 11 = 0$ and the point $(-2, 1, 3)$ is $ax + by + cz - 7 = 0$, then the value of $2a + b + c - 7$ is

[17 Mar 2021 Shift 1]

$$(2x - 7y + 4z - 3) + \lambda(3x - 5y + 4z + 11) = 0$$

$$(-4 - 7 + 12 - 3) + \lambda(-\cancel{6} - \cancel{5} + 12 + \cancel{11}) = 0$$

$$(-2) + \lambda(12) = 0$$

$$\boxed{\lambda = 1/6}$$

$$6\left\{P_1 + \frac{1}{6}P_2 = 0\right\}$$

$$6P_1 + P_2 = 0$$

$$6(2x - 7y + 4z - 3) + (3x - 5y + 4z + 11) = 0$$

$$15x - 47y + 28z - 7 = 0$$

$a = 15 \quad b = -47 \quad c = 28$

$$2a + b + c - 7$$

$$= 30 - 47 + 28 - 7 = 4$$

Angle Bisector

Unacademy Atoms

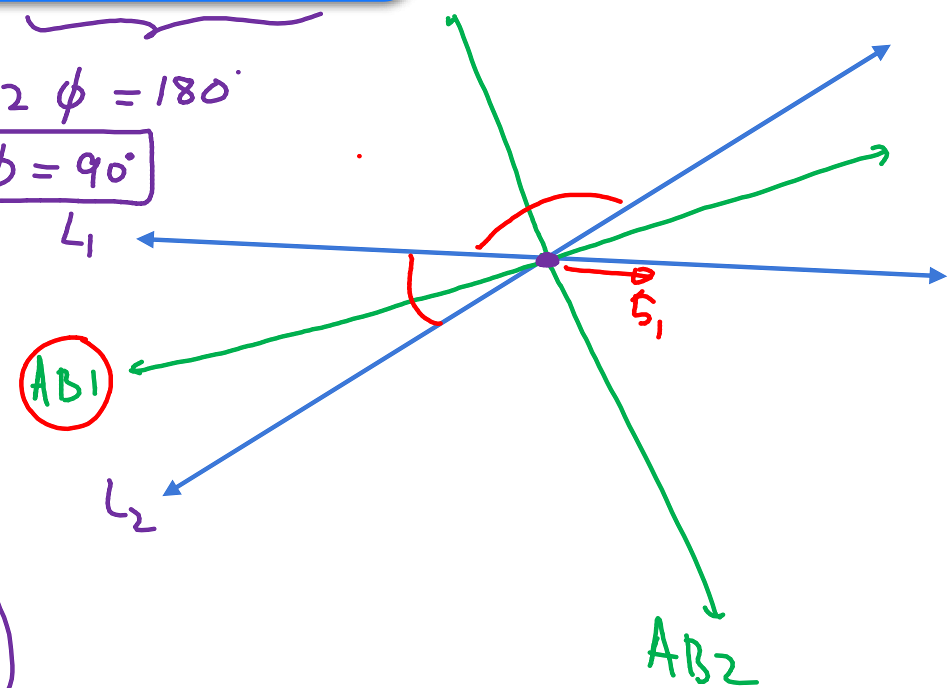
Bisector of Angle between Two Lines

$L_1 : \vec{r} = \vec{a_1} + \lambda \vec{b_1}$ ✓

$L_2 : \vec{r} = \vec{a_2} + \mu \vec{b_2}$ ✓

$2\theta + 2\phi = 180^\circ$

$\boxed{\theta + \phi = 90^\circ}$



$\boxed{AB1 \perp AB2}$

AB1 P.O.I $\hat{b}_1 + \hat{b}_2$

AB2 P.O.I $\hat{b}_1 - \hat{b}_2$

Bisector of Angle between Two Lines

	Acute Angle Bisector	Obtuse Angle Bisector
$\boxed{b_1 . b_2 > 0}$ +	$\widehat{b_1} + \widehat{b_2}$	$\widehat{b_1} - \widehat{b_2}$
$\boxed{b_1 . b_2 < 0}$	$\widehat{b_1} - \widehat{b_2}$	$\widehat{b_1} + \widehat{b_2}$

$b_1 . b_2 = +$

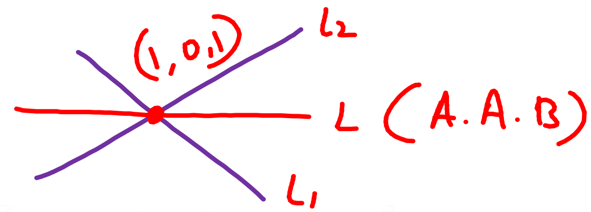
Let L_1 and L_2 be the following straight lines.

$$\boxed{L_1: \frac{x-1}{1} = \frac{y}{-1} = \frac{z-1}{3}} \text{ and } \boxed{L_2: \frac{x-1}{-3} = \frac{y}{-1} = \frac{z-1}{1}}$$

$\boxed{\text{P.O.I } (1, 0, 1)}$

Suppose the straight line

$\boxed{\text{A.A.B } L_1 \,\&\, L_2} \Leftarrow \boxed{L: \frac{x-\alpha}{l} = \frac{y-1}{m} = \frac{z-\gamma}{-2}}$



AB

lies in the plane containing L_1 and L_2, and passes through the point of intersection of L_1 and L_2. If the line L bisects the acute angle between the lines L_1 and L_2, then which of the following statements is/are TRUE?

A. $\alpha - \gamma = 3$ ✓

B. $l + m = 2$ ✓ $1+1$

C. $\alpha - \gamma = 1$ ✗

D. $l + m = 0$ ✗

[JEE Adv. 2020]

$$\vec{b}_1 \cdot \vec{b}_2 = (1, -1, 3) \cdot (-3, -1, 1)$$

$$= -\cancel{3} + 1 + \cancel{3}$$

$$= +$$

$$\boxed{\frac{1-\alpha}{1} = \frac{-1}{1} = \frac{1-\gamma}{-2}}$$

$\alpha = 2 \quad \gamma = -1$

dir^n of L : $\hat{b}_1 + \hat{b}_2$

$$= \frac{(1, -1, 3)}{\sqrt{11}} + \frac{(-3, -1, 1)}{\sqrt{11}}$$

$$= \left(\frac{-2}{\sqrt{11}}, \frac{-2}{\sqrt{11}}, \frac{4}{\sqrt{11}}\right)$$

$$= (-2, -2, 4)$$

$$= (1, 1, -2)$$

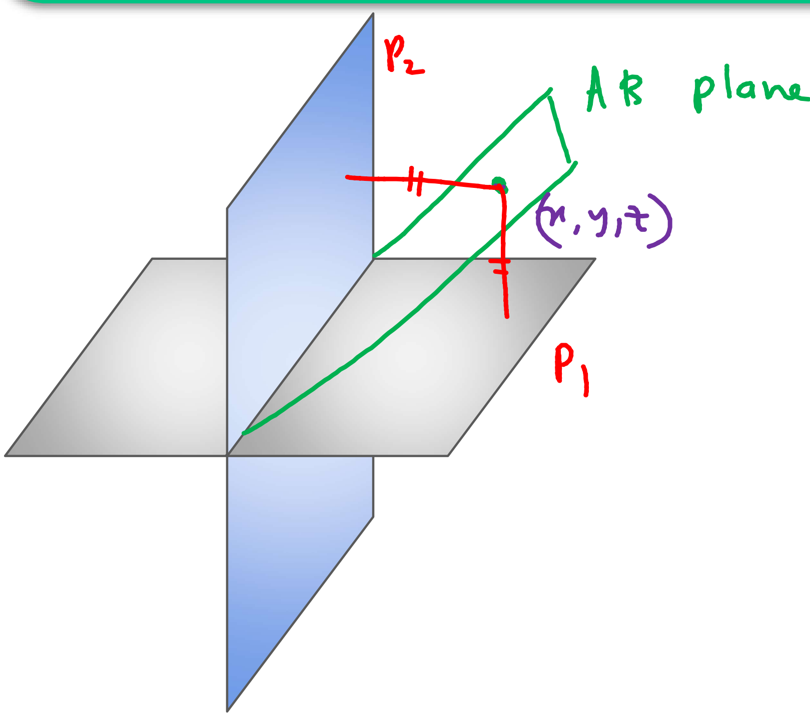
$$= (l, m, -2)$$

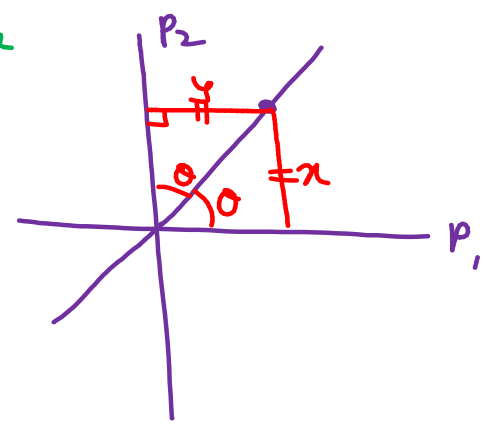
$l = 1$
$m = 1$

Bisector of Angle between Two Planes

S.L.

$$\left(\frac{a_1x+b_1y+c_1z+d_1}{\sqrt{a_1^2+b_1^2+c_1^2}}\right)=\pm\left(\frac{a_2x+b_2y+c_2z+d_2}{\sqrt{a_2^2+b_2^2+c_2^2}}\right)$$





Bisectors

1. Make constant terms positive

2.

	Obtuse Angle Bisector	Acute Angle Bisector
✓ $a_1a_2 + b_1b_2 + c_1c_2 > 0$	+	-
$a_1a_2 + b_1b_2 + c_1c_2 < 0$	-	+

$\vec{n}_1 \cdot \vec{n}_2 = ?$

Equation of plane bisecting the acute angle between the planes $x - y + z - 1 = 0$ and $x + y + z - 2 = 0$

A. $x + z = 3/2$

B. $2y = 1$

C. $x - y - z = 3$

D. $x + 2z = 3$

$\vec{n}_1 \cdot \vec{n}_2 = +$

Acute Angle Bisector

1. $-x + y - z + 1 = 0$ $\boxed{P_1}$ $\qquad (-1, 1, -1) = \vec{n}_1$

$-x - y - z + 2 = 0$ $\boxed{P_2}$ $\qquad (-1, -1, -1) = \vec{n}_2$

$$\left(\frac{-x+y-z+1}{\cancel{\sqrt{1^2+1^2+1^2}}}\right) = -\left(\frac{-x-y-z+2}{\cancel{\sqrt{1^2+1^2+1^2}}}\right)$$

$$-x+y-z+1 = -(-x-y-z+2)$$

$$-x+\cancel{y}-z+1 = x+\cancel{y}+z-2$$

$$\boxed{x+z = 3/2}$$

$$\boxed{\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}}$$

Equation of Straight Line in Unsymmetrical Form

Equation of Straight Line in Symmetrical form

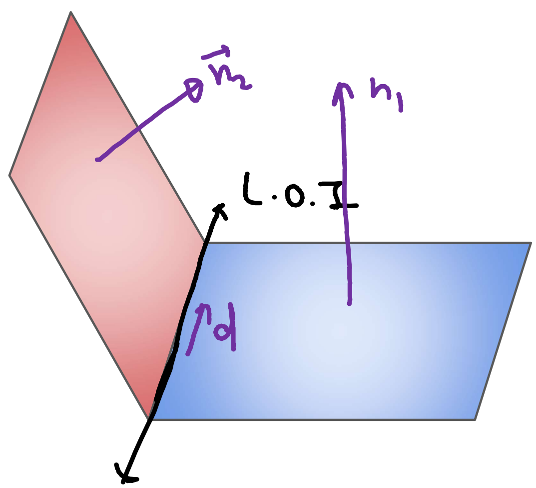
$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$$

(x_1, y_1, z_1) point

(a, b, c) = d.v. / d.R

Equation of Straight Line in Unsymmetrical form

$$\underline{a_1x + b_1y + \cancel{c_1z} + d_1} = \underline{a_2x + b_2y + \cancel{c_2z} + d_2} = 0$$



L.O.I

① point : $\boxed{z = 0 \quad x = \checkmark \quad y = \checkmark}$

② $\boxed{\text{dir}^n = \vec{n}_1 \times \vec{n}_2}$

Consider the planes $3x - 6y - 2z = 15$ and $2x + y - 2z = 5$

F **Statement-1 :** The parametric equations of the line of intersection of the given planes are $x = 3 + 14t$, $y = 1 + 2t$, $z = 15t$. Because

T **Statement-2 :** The vector $14\hat{i} + 2\hat{j} + 15\hat{k}$ is parallel to the lines of intersection of given planes.

[JEE Adv. 2007]

$$\left.\begin{aligned} 3x - 6y - 2z &= 15 \\ 2x + y - 2z &= 5 \end{aligned}\right\} \text{L.O.I.}$$

① Point $z = 0$

$$3x - 6y = 15$$
$$6 \times (2x + y = 5)$$
$$12x + 6y = 30$$
$$15x = 45$$
$$x = 3$$

$$y = -1$$

Point $(3, -1, 0)$

$$\text{dir}^n = n_1 \times n_2$$

$$= \begin{vmatrix} i & j & k \\ 3 & -6 & -2 \\ 2 & 1 & -2 \end{vmatrix}$$

$$= \hat{i}(14) - \hat{j}(-2) + \hat{k}(15)$$

$$= \underline{14\hat{i} + 2\hat{j} + 15\hat{k}}$$

<u>L.O.I.</u>

$$\frac{x-3}{14} = \frac{y+1}{2} = \frac{z-0}{15} = t$$

$$(14t+3,\ \underline{2t-1},\ 15t)$$

Unacademy Atoms

Equation of Sphere

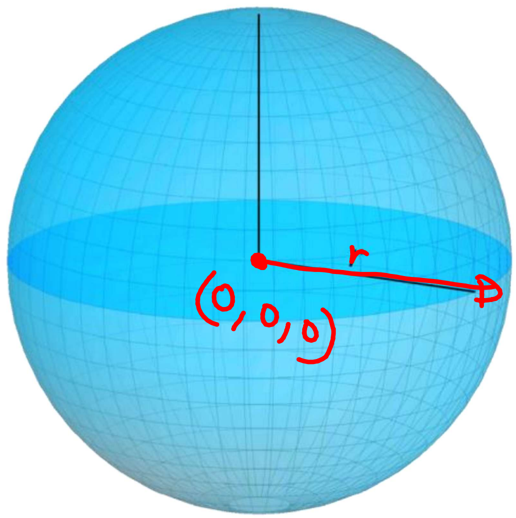
(JEE M)

Equation of Sphere

Equation of Sphere with centre at origin and radius r is given by

$$x^2 + y^2 + z^2 = r^2$$

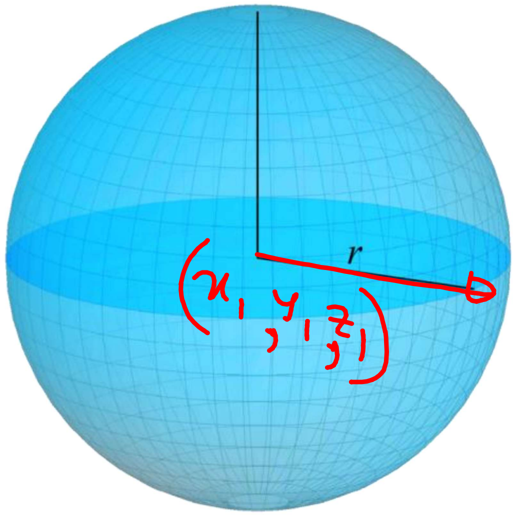
$$\boxed{x^2 + y^2 + z^2 = r^2}$$



Equation of Sphere

Equation of Sphere with centre at (x_1, y_1, z_1) and radius r is given by

$$\boxed{(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2 = r^2}$$



General Equation of Sphere

\# $x^2+y^2+2gx+2fy+c=0$

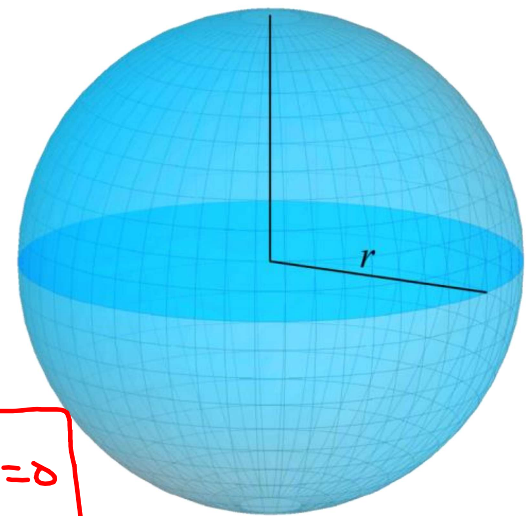
$C(-g,-f)$

$\text{rad}=\sqrt{g^2+f^2-c}$

\# $x^2+y^2+z^2+2gx+2fy+2hz+c=0$

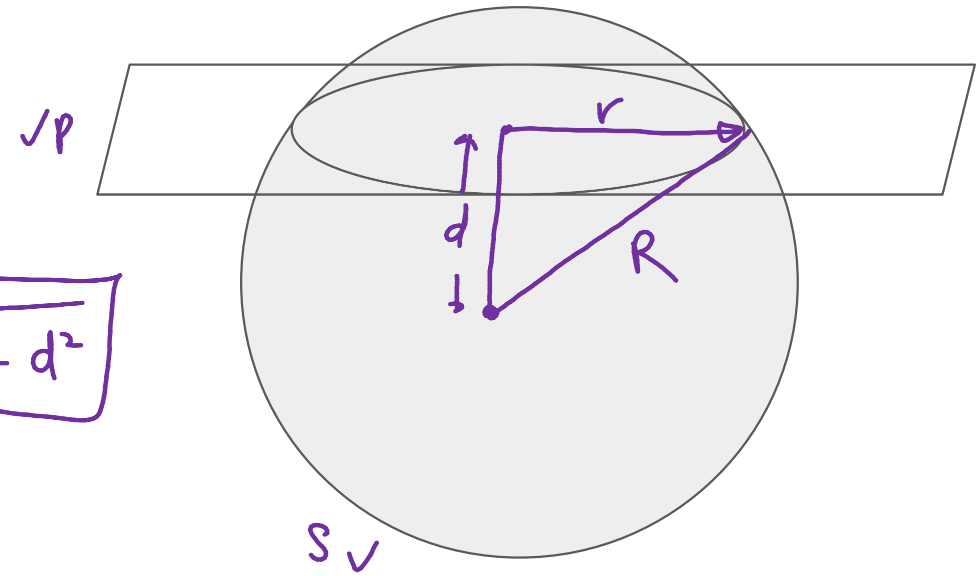
$C(-g,-f,-h)$

$\text{rad}=\sqrt{g^2+f^2+h^2-c}$



Plane and Sphere

JEE m



$$\boxed{r = \sqrt{R^2 - d^2}}$$

Unacademy Atoms

The plane $x + 2y - z = 4$ cuts the sphere $x^2 + y^2 - x + z - 2 = 0$
In a circle of radius

A. 3

B. 1

C. 2

D. $\sqrt{2}$

[2005]

$\boxed{C\left(\frac{1}{2}, 0, \frac{-1}{2}\right)}$

$rad = \sqrt{\left(\frac{1}{2}\right)^2 + 0^2 + \left(\frac{1}{2}\right)^2 - (-2)}$

$R = \sqrt{\frac{1}{2} + 2} = \sqrt{\frac{5}{2}}$

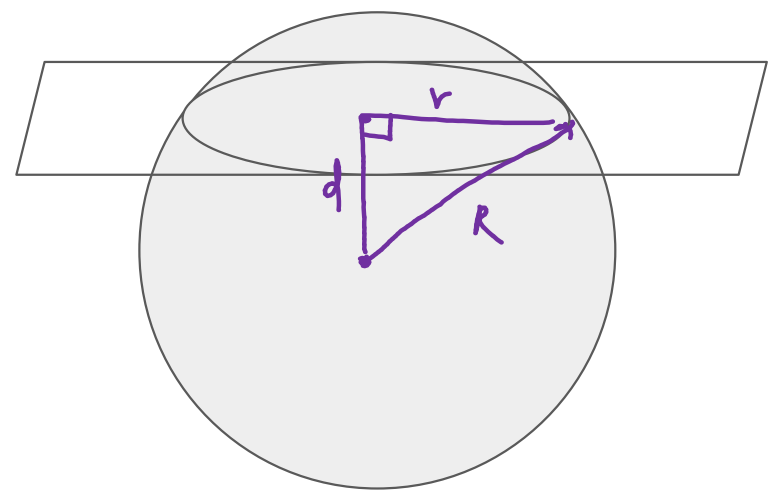
$\left(\frac{1}{2}, 0, \frac{-1}{2}\right)$

$d = \left| \frac{\frac{1}{2} + 0 + \frac{1}{2} - 4}{\sqrt{1^2 + 2^2 + 1^2}} \right|$

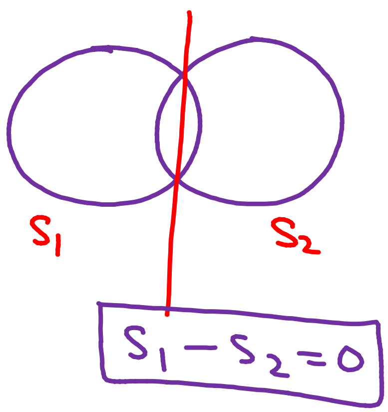
$\boxed{d = \frac{3}{\sqrt{6}}}$

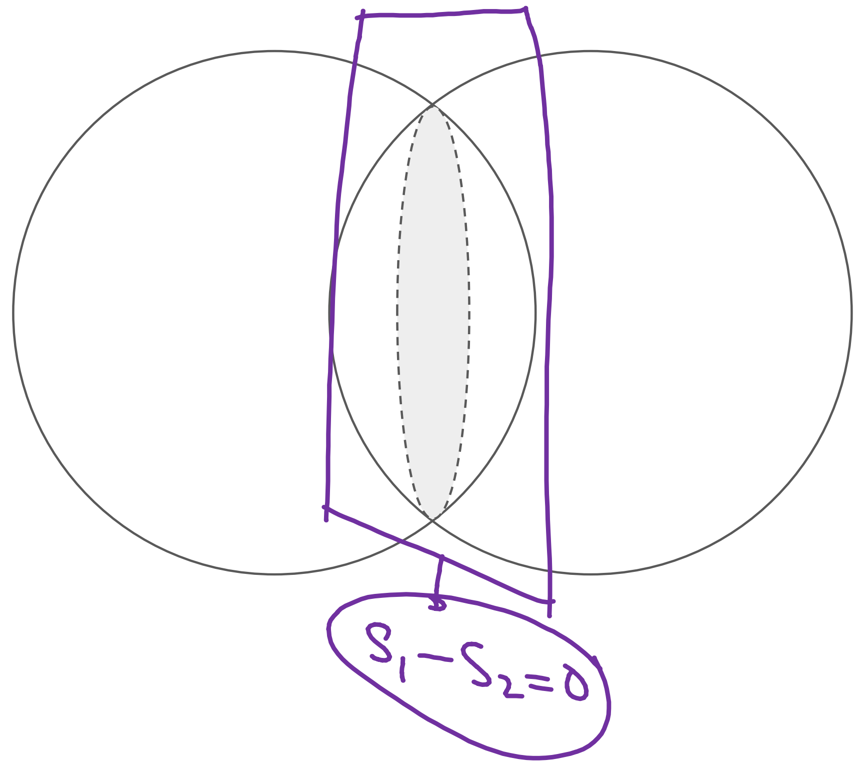
$r = \sqrt{R^2 - d^2}$

$= \sqrt{\frac{5}{2} - \frac{9}{6}} = 1$



Intersection of Two Spheres





Unacademy Atoms

The intersection of the spheres

$\cancel{x^2} + \cancel{y^2} + \cancel{z^2} + 7x - 2y - z = 13$ and

$\cancel{x^2} + \cancel{y^2} + \cancel{z^2} - 3x + 3y + 4z = 8$ is the same as the intersection of one of the sphere and the plane

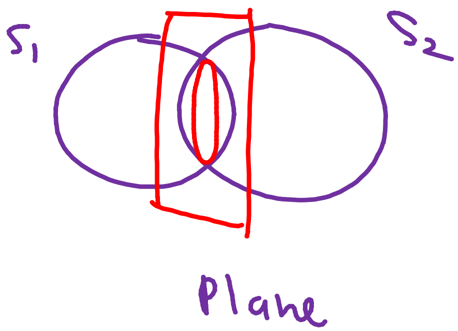
[2004]

A. $2x - y - z = 1$

B. $x - 2y - z = 1$

C. $x - y - 2z = 1$

D. $x - y - z = 1$



$10x - 5y - 5z = 5$

$2x - y - z = 1$