Important Terms

Random Experiment

Experiment which satisfies the following conditions:-

1. It should have **more than one outcome**
2. Outcomes are **non predictable**

Examples of Random Experiment

$\{1,2,3,4,5,6\}$

$\{T, H\}$

Sample Space

It is the **set of all possible outcomes** of an random experiment

Ex

S = {H, T}

$\boxed{n(S) = 2}$

S = {HH, HT, TH, TT}

$\boxed{n(S) = 4}$

S = {1, 2, 3, 4, 5, 6}

Coins

Write sample space when three ~~Dices~~ are thrown ?

#NVStyle

3 coins

$\boxed{n(S) = 8}$

~~S = { HHH, HHT,~~ HTH, TTT,

$$S = \left\{ \begin{array}{ll} HHH & THH \\ HHT & THT \\ HTH & TTH \\ HTT & TTT \end{array} \right\}$$

Write sample space when four ~~Dices~~ Coins are thrown ?

{ HHHH THHH
HHHT THHT
HHTH THTH
HHTT THTT
HTHH TTHH
HTHT TTHT
HTTH TTTH
HTTT TTTT }

4 Coin

$n(S) = 2^4$

$= 16$

#NVStyle

Unacademy Atoms

Number of elements in sample space if n Coins are tossed?

Random Experiment	n(S)
1 Coin is tossed	2
2 Coin are tossed	2^2
3 Coin are tossed	2^3
.	
.	
.	
n Coin are tossed	$\boxed{2^n}$

Event

A B C

It is the subset of sample space

RE

Getting an odd outcome in throwing dice.

$\boxed{S = \{1, 2, 3, 4, 5, 6\}}$

$\boxed{A = \{1, 3, 5\}}$ ← $\boxed{\text{odd}}$

$B = \{2, 4, 6\}$

$C = \{2, 3, 5\}$

$\boxed{A' = \{2, 4, 6\}}$

Complement of an Event

The complement of an event ‘A’ with respect to a sample space S are the set of all elements of ‘S’ which are **not in A.** It are usually denoted by $\boxed{A', \bar{A} \text{ or } A^C.}$

$$\boxed{A' / A^c / \bar{A}}$$

$\boxed{P(A) + P(\bar{A}) = 1}$

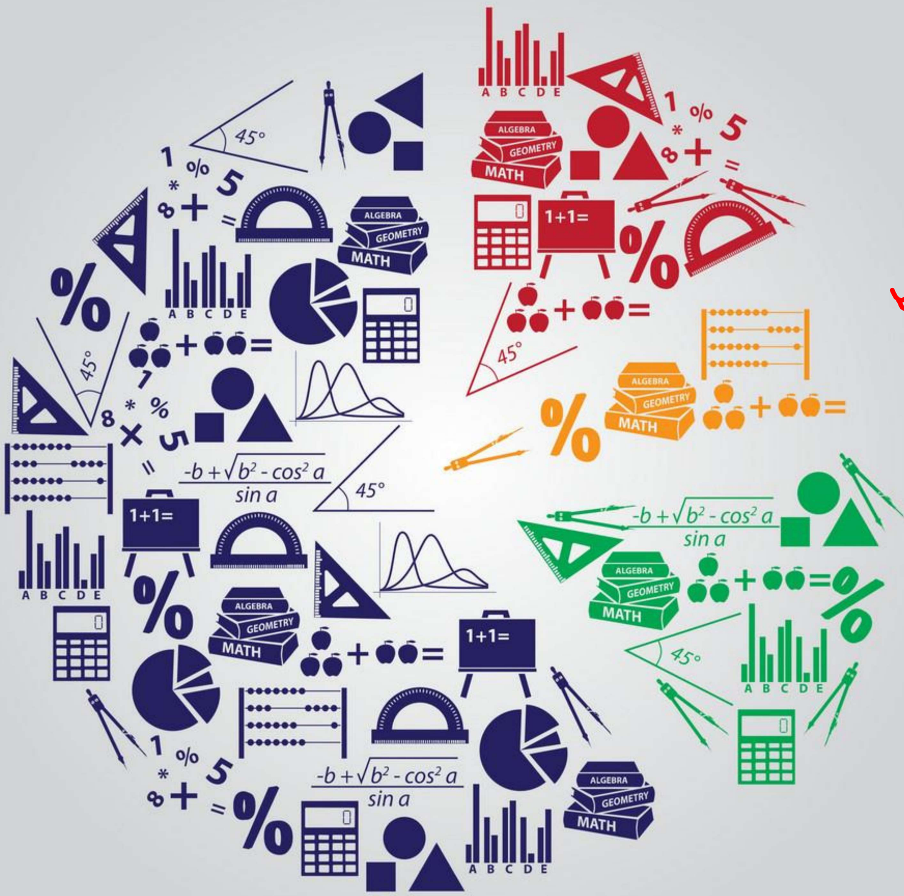
$P(A) \rightarrow$

$$\boxed{P(A) + P(A') = 1}$$

$$P(A) = \frac{2}{5}$$

$$P(A') = 1 - \frac{2}{5}$$

Unacademy Atoms



Classical Definition of Probability

Classical Definition of Probability

$$\boxed{P(A) = \frac{\text{Number of favorable outcomes}}{\text{Total Number of Outcomes}}}$$

$$\underbrace{0 \leq P(A) \leq 1}$$

$$\boxed{P = \frac{\text{fav.}}{\text{Total}}}$$

0 1

Odds in Favour and Odd Against

* Odds in favour of an event = $\dfrac{\text{Number of favorable cases}}{\text{Number of unfavorable cases}} = \dfrac{\text{fav.}}{\text{un. fav.}}$

* Odds against in an event = $\dfrac{\text{Number of unfavorable cases}}{\text{Number of favorable cases}} = \dfrac{\text{Un fav.}}{\text{fav.}}$

$$P(A) = \frac{1}{100} = \frac{\text{fav.}}{\text{Total}}$$

$\boxed{\text{unfav} = 99}$

odd against $\Rightarrow \frac{\text{un fav.}}{\text{fav.}} \Rightarrow \boxed{99 : 1}$

odds in favor $\Rightarrow \frac{\text{fav.}}{\text{un fav.}} \Rightarrow \boxed{1 : 99}$

Q-2 odds in favor $\Rightarrow$ 2 : 3

fav. (2) unfav. (3)

$$P = \frac{2}{5}$$

Unacademy Atoms

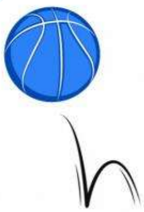
(ph d.)

Double dice Problems

$n(S) = 36$

Outputs of the First Dice

Outputs of the Second Dice	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)



2 Dice

sum = 2
⋮
sum = 12

(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

S=2
$\boxed{S=3}$
S=4
S=5
S=6
S=7

S=8 S=9 S=10 S=11 S=12

$$P(\text{Sum}=5) = \frac{4}{36}$$

$$P(\text{Sum}=10) = \frac{3}{36}$$

#NVStyle

Swamiji Tilak

2	12	1
3	11	2
4	10	3
5	9	4
6	8	5
7		6

$$\boxed{P(\text{sum}=10) = \frac{3}{36}}$$

Find the Probability of getting sum of 7 if a pair of dice are thrown

$$\boxed{P(\text{sum}=7) = \frac{6}{36} = \frac{1}{6}}$$

Unacademy Atoms

Two dices are rolled. If both dices have six faces numbered 1, 2, 3, 5, 7 and 11, then the probability that the sum of the numbers on the top faces is $\boxed{\text{less than or equal to 8}}$ is :

A. $\frac{4}{9}$ B. $\frac{17}{36}$ C. $\frac{5}{12}$ D. $\frac{1}{2}$

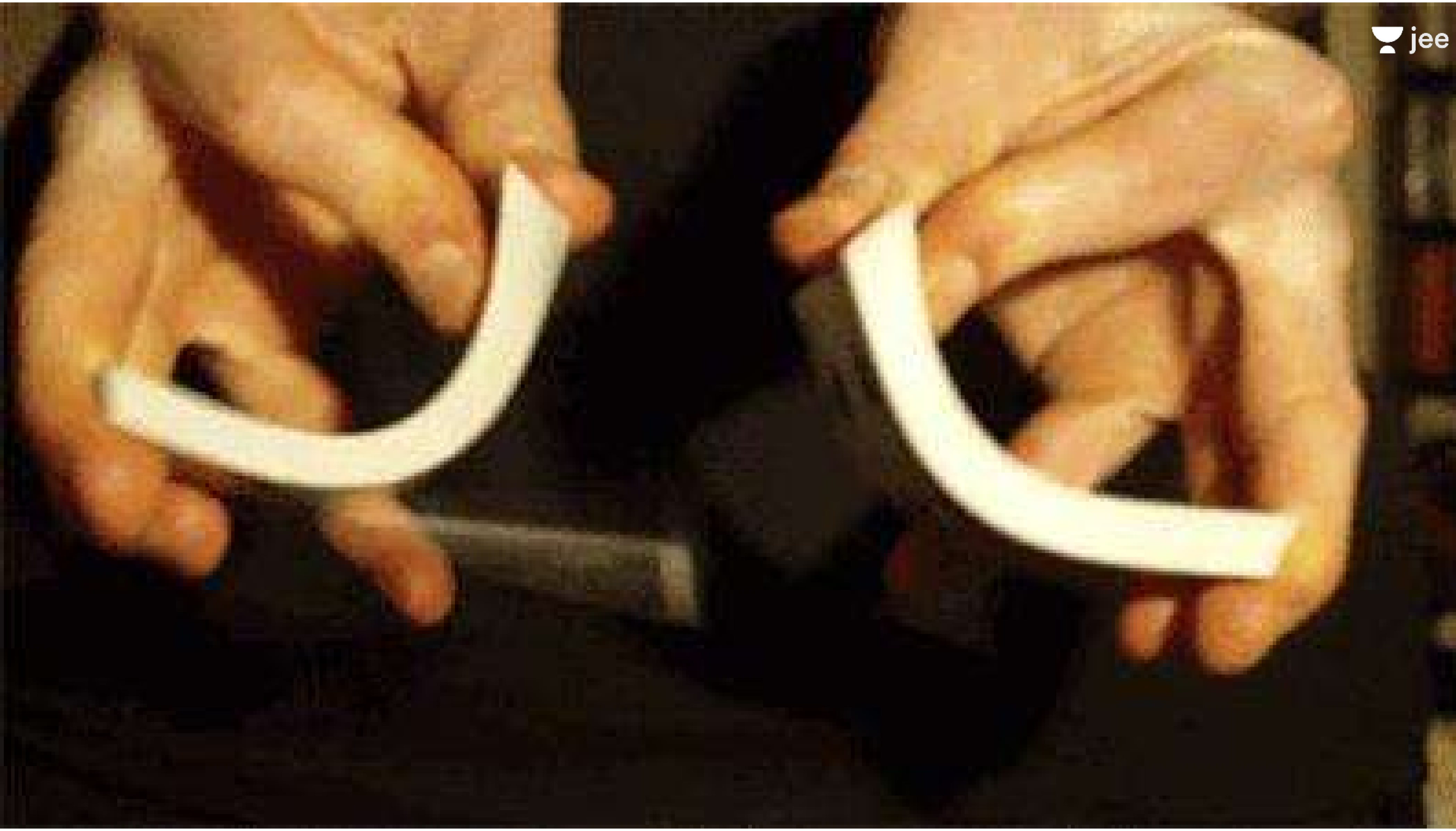
JEE Main 2021

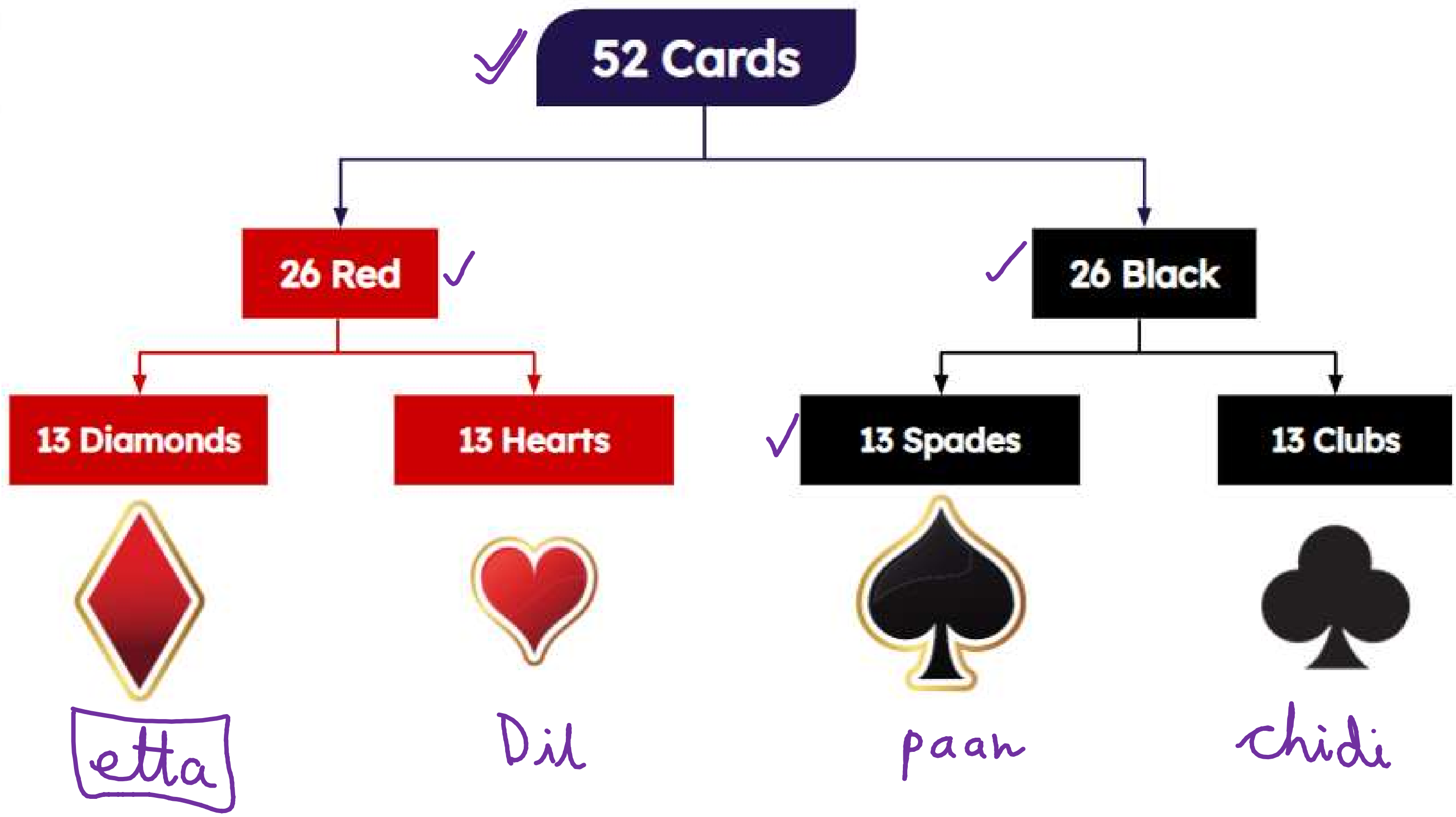
$\frac{17}{36}$

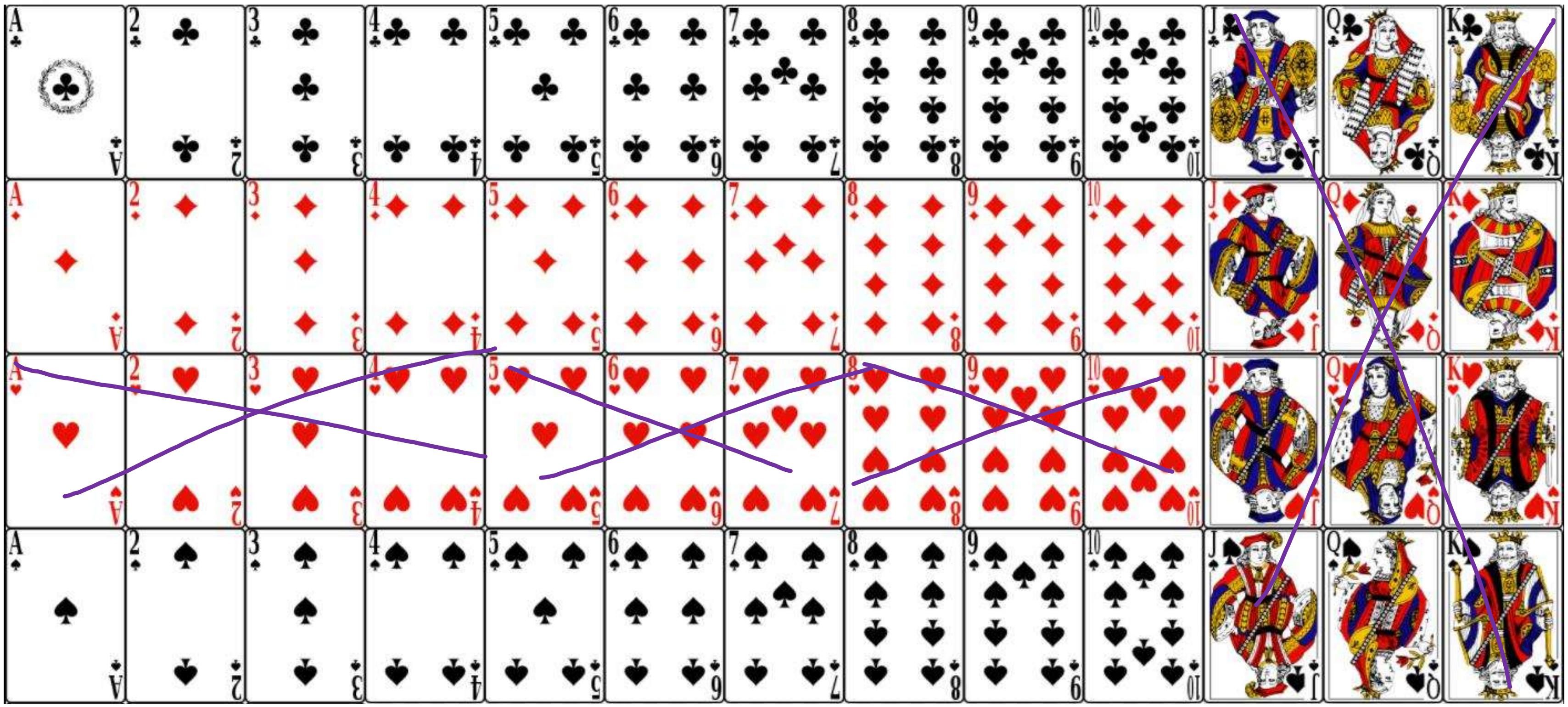
$P(\text{Sum} \leq 8)$

	1	2	3	5	7	11
1	2	3	4	6	8	X
2	3	4	5	7	X	X
3	4	5	6	8	X	X
5	6	7	8	X	X	X
7	8	X	X	X	X	X
11	X	X	X	X	X	X

Deck of Card




52 Cards
26 Red
26 Black
13 Diamonds
13 Hearts
13 Spades
13 Clubs
etta
Dil
paan
chidi



(i) Face Cards: K, Q and J 12

(ii) Honours Cards: A, K, Q and J 16

(iii) Knave Cards: 10, J and Q 12

A card are drawn randomly from a well shuffled pack of 52 cards. The probability that the drawn card is "**neither a heart nor a face card**".

$$P = \frac{30}{52}$$

$$\frac{52 - 12 - 10}{}$$

Questions Based on P & C

Words with or without meaning are to be formed using all the letters of the word EXAMINATION. The probability that the letter **M appears at the fourth position** in any such word is:

A. $\frac{1}{66}$ B. $\frac{1}{11}$ C. $\frac{1}{9}$ D. $\frac{2}{11}$

(20 July 2021 Shift 1)

$$\boxed{\text{Total} \Rightarrow \frac{11!}{2!\,2!\,2!}}$$

E X A M I N A T I O N

A A I I N N E X M T O

_ _ _ M _ _ _ _ _ _ _

$$\boxed{\text{fav} \Rightarrow \frac{10!}{2!\,2!\,2!}}$$

$$P = \frac{\dfrac{10!}{\cancel{2!\,2!\,2!}}}{\dfrac{11!}{\cancel{2!\,2!\,2!}}} = \frac{1}{11}$$

Let 9 distinct balls be distributed among 4 boxes, B_1, B_2, B_3 and B_4. If the probability than B_3 contains exactly 3 balls is $k\left(\frac{3}{4}\right)^9$ Then k lies in the set:

$\left|\frac{28}{9} - 3\right|$

$b_1 \; b_2 \; b_3 \; \ldots \; b_9$

3 balls

$B_1 \quad B_2 \quad B_3 \quad B_4$

$\boxed{\text{Total} \Rightarrow 4^9}$

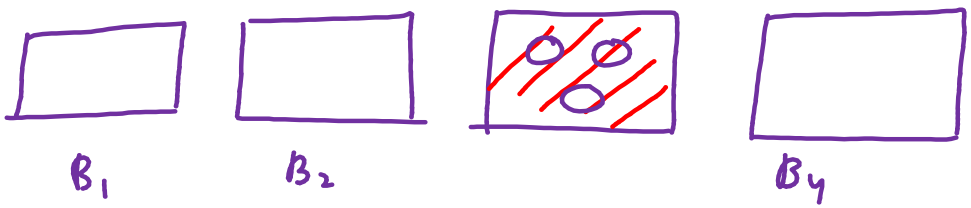
A. $\{x \in \mathbf{R} : |x-3| < 1\}$

B. $\{x \in \mathbf{R} : |x-2| \leq 1\}$

C. $\{x \in \mathbf{R} : |x-1| < 1\}$

D. $\{x \in \mathbf{R} : |x-5| \leq 1\}$

(20 July 2021 Shift 1)



6 ball

$$^9C_3 = \frac{9!}{3!\,6!} = \frac{\cancel{9}^{3} \times \cancel{8}^{4} \times 7}{\cancel{3} \times \cancel{2}}$$

$$\boxed{^9C_3 \times 1 \times 3^6}$$

$$K = \frac{\cancel{3} \times 4 \times 7}{\cancel{3} \times 3 \times 3}$$

$$K = \frac{28}{9}$$

$$P = \frac{^9C_3 \times 3^6 \times 3^3}{3^3\, 4^9}$$

$$P = \frac{^9C_3}{3^3}\left(\frac{3}{4}\right)^9$$

$$= K\left(\frac{3}{4}\right)^9$$

Unacademy Atoms

Let A denote the event that a 6 - digit integer formed by 0, 1, 2, 3, 4, 5, 6 without repetitions, be divisible by 3. Then probability of event A is equal to :

RepeX

A. $\frac{9}{56}$ B. $\frac{4}{9}$ C. $\frac{3}{7}$ D. $\frac{11}{27}$

(16 Mar 2021 Shift 2)

$\boxed{0, 1, 2, 3, 4, 5, 6}$

$\boxed{\text{Total} \Rightarrow 6 \times 6!}$

3 0

$6 \times 6 \times 5 \times 4 \times 3 \times 2$

fav:- $\boxed{\text{sum of digits} = 3K}$

C-1 $\boxed{1, 2, 3, 4, 5, 6}$ — — — — — — $6!$

C-2 $\boxed{0, 1, 3, 4, 5, 6}$ $\underset{5}{\overset{2}{\_}}\ \underset{5}{\_}\ \underset{4}{\_}\ \underset{3}{\_}\ \underset{2}{\_}\ \underset{1}{\_}$ $5 \times 5!$

C-3 $\boxed{0, 1, 2, 3, 4, 5}$ — — — — — — $5 \times 5!$

$$p = \frac{10 \times \cancel{5!} + 6 \times \cancel{5!}}{6 \times 6 \times \cancel{5!}} = \frac{\cancel{16}^{4}}{\cancel{6}_{3} \times \cancel{6}_{3}} = \frac{4}{9}$$

$$\text{fav} \Rightarrow 5 \times 5! + 5 \times 5! + 6!$$

JEE MAIN 2021

A seven digit number is formed using digit 3, 3, 4, 4, 4, 5, 5. The probability, that number so formed is divisible by 2, is :

A. $\frac{6}{7}$ B. $\frac{4}{7}$ C. $\frac{3}{7}$ D. $\frac{1}{7}$

$\_\ \_\ \_\ \_\ \_\ \_\ \underset{\text{fix}}{4}$

$\boxed{3, 3, \cancel{4}, 4, 4, 5, 5}$

$\boxed{3, 3, 4, 4, 5, 5}$

$$\boxed{\frac{7!}{2!\,3!\,2!} = \text{Total}}$$

$$\boxed{\text{fav} = \frac{6!}{2!\,2!\,2!}}$$

$$\frac{\dfrac{6!}{\cancel{2!}\,2!\,\cancel{2!}}}{\dfrac{7!}{\cancel{2!}\,3!\,\cancel{2!}}} = \frac{6! \times 3!}{7! \times 2!} = \frac{3}{7}$$

Unacademy Atoms

Types of Events

Events

Examples

1. Getting 7 on a throw of single dice (Impossible)
2. Getting a number less than 7 on a throw of single dice (Sure)

$P = 0$ $P = 1$

Simple Event VS Compound Event

Simple Event	Compound Event
$\boxed{n(E) = 1}$	$n(E) > 1$
RE: Tossing 2 ~~Dices~~ Coins	RE: Tossing 2 Dices
A : getting both heads	B : getting at least one head
$A = \{HH\}$	$B = \{HH, HT, TH\}$

Types of Events ☆ (IIT)

- Equally Likely
- Mutually Exclusive/Disjoint
- Exhaustive

Equally Likely

- Events are equally likely if they have same probability of occurrence.

$\boxed{P(A) = P(B)}$

$\boxed{P(H) = P(T)}$

Example:

1. 'Getting odd outcome' and 'getting even outcome' in single throw of a fair dice.
2. 'Getting head' and 'getting tail' on the toss of fair ~~Dice~~ Coin.

RE : Dice

$S = \{1, 2, 3, 4, 5, 6\}$

$A = \{1, 3, 5\}$

$B = \{2, 4, 6\}$

$$\boxed{P(A) = \frac{3}{6} \quad P(B) = \frac{3}{6}}$$

Mutually Exclusive / Disjoint

- Two events A and B are said to be mutually exclusive or disjoint if their simultaneous occurrence are impossible
- If A and B are mutually exclusive then $\boxed{A \cap B = \phi}$ ME ✓

$A \cap B \neq \phi$ ME ✗

Example:

RE: throwing a dice

A: getting odd number

B: getting even number

$S = \{1, 2, 3, 4, 5, 6\}$

$A = \{1, 3, 5\}$

$B = \{2, 4, 6\}$

$\boxed{A \cap B = \phi}$

$\boxed{\text{ME} ✓}$

Mutually Exclusive / Disjoint

Question 1:

RE: throwing a dice

A: getting prime number = {2, 3, 5}

B: getting even number = {2, 4, 6}

C: getting multiple of 3 = {3, 6}

ME

A and B ×

B and C ×

A and C ×

Question 2:

RE: drawing one card from a pack of 52 cards

A: getting ace A♢ A♡ A♤ A♧

B: getting red card

ME ×

Exhaustive Events

- Events whose union are equal to sample space
- If A, B and C are exhaustive then $\boxed{\mathbf{A \cup B \cup C = S}}$

$\boxed{A \cup B \neq S}$

Exh nahi hai

Example:

RE: Throwing a dice

A: getting even number $= \{2, 4, 6\}$

B: getting prime number $= \{2, 3, 5\}$

C: getting number less than 4 $= \{1, 2, 3\}$

$A \cup B \cup C = \{1, 2, 3, 4, 5, 6\}$

$= S$

Dependent and Independent Events

- Two events A and B are independent if occurrence or non occurrence of A has no effect on occurrence or non occurrence of B

Example:

1. A dice are thrown and a Dice are thrown, than getting even number on dice and getting head on Dice are independent
2. If it rains then crop will be good (dependent)

Four persons independently solve a certain problem correctly with probabilities 1/2, 3/4, 1/4, 1/8. Then the probability that the problem are solved correctly by ~~at least one~~ of them are

Total – No

A. 235/256

B. 21/256

C. 3/256

D. 253/256

(JEE Adv. 2013)

A → 1/2, B → 3/4, C → 1/4, D → 1/8

P (atleast one of them will solve correctly) = 1 – P (None)

$$= 1 - \left(1-\frac{1}{2}\right)\left(1-\frac{3}{4}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{8}\right)$$

$$= 1 - \frac{1}{2}\times\frac{1}{4}\times\frac{3}{4}\times\frac{7}{8}$$

$$= \frac{235}{256}$$

Four persons can hit a target correct with probabilities 1/2, 1/3, 1/4, 1/8. If all hit at the target independently, then the probability that the target would be hit, are

(JEE M 2019)

A. 25/192

B. 7/32

C. 1/192

D. 25/32

A B C D

$\downarrow$ $\downarrow$ $\downarrow$ $\downarrow$

1/2 1/3 1/4 1/8

$$\Rightarrow 1 - \frac{1}{\cancel{2}} \times \frac{\cancel{2}}{\cancel{3}} \times \frac{\cancel{3}}{4} \times \frac{7}{8}$$

$$\Rightarrow 1 - \frac{7}{32}$$

$$\Rightarrow \frac{25}{32}$$

Addition theorem on Probability

Addition theorem on Probability

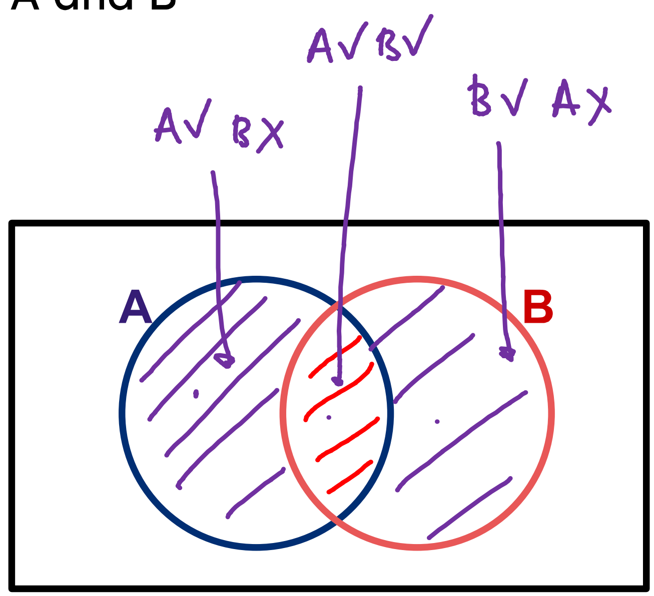
If A and B are two events associated with an experiment then

1. $P(A \cup B)$ are probability of occurrence of **at least one event**
2. $P(A \cap B)$ are probability of occurrence of both A and B
3. $P(A)$ are probability of occurrence of A
4. $P(B)$ are probability of occurrence of B

Both

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

A and B



1

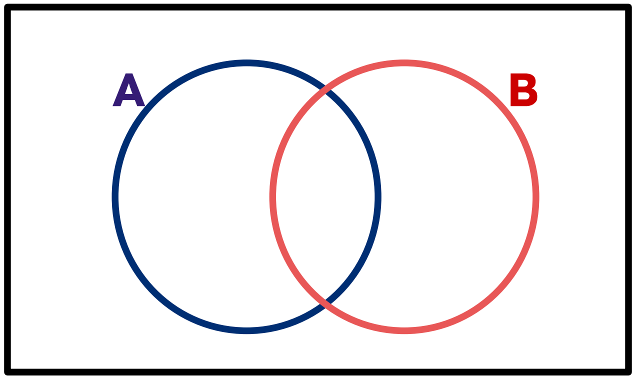
P(at least one event will occur)

= P(AUB)

= P(A) + P(B) - P(A ∩ B)

$P(A \text{ or } B) = P(A \cup B)$

$P(\text{atleast one}) = P(A \cup B)$



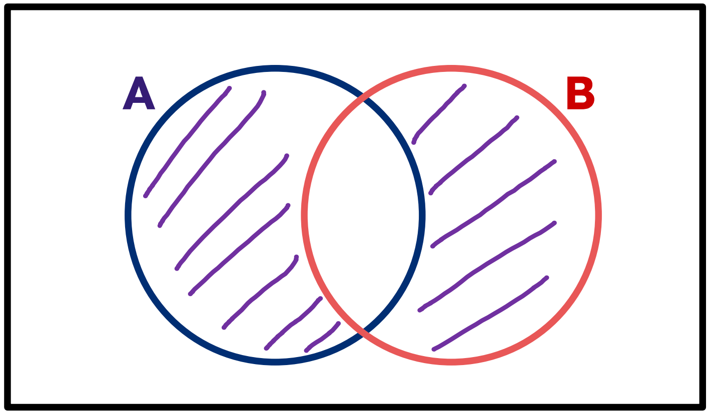
2

P(exactly one event will occur)

= **P(A) + P(B) - 2 P(A∩B)**

dot- cross

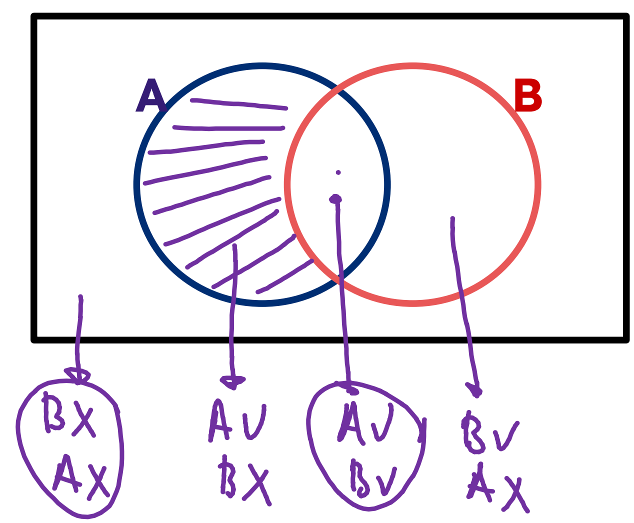
$P(A) + P(B) - 2P(A \cap B)$



3

P(only A occurs)

$= P(A) - P(A \cap B)$



Note:

I. If A and B are **mutually exclusive** events then

$\boxed{P(A \cup B) = P(A) + P(B)}$ $\{\because P(A \cap B) = 0\}$

II. If A and B are **exhaustive events** then $P(A \cup B) = 1$

★ $\boxed{ME}$ $A \cap B = \phi$

$n(A \cap B) = 0$

$\boxed{P(A \cap B) = 0}$

A and B → Exh.

★ $n(A \cup B) = n(S)$

$P(A \cup B) = \frac{\cancel{n(A \cup B)}}{\cancel{n(S)}} = 1$

Let A and B be two events such that the probability that exactly one of them occurs is $\boxed{2/5}$ and the probability that A or B occurs is 1/2 , then the probability of both of them occur together is

$P(A \cap B) = ?$

A. 1/10

B. 2/9

C. 1/8

D. 1/12

(JEE Main 2020 - 8 Jan)

$$\boxed{\cancel{P(A)} + \cancel{P(B)} - 2\,P(A \cap B) = \frac{2}{5}} \quad \text{— (1)}$$

$$P(A \cup B) = \frac{1}{2}$$

$$\boxed{\cancel{P(A)} + \cancel{P(B)} - P(A \cap B) = \frac{1}{2}} \quad \text{— (2)}$$

$$- \quad - \quad + \quad -$$

$$-P(A \cap B) = \frac{2}{5} - \frac{1}{2}$$

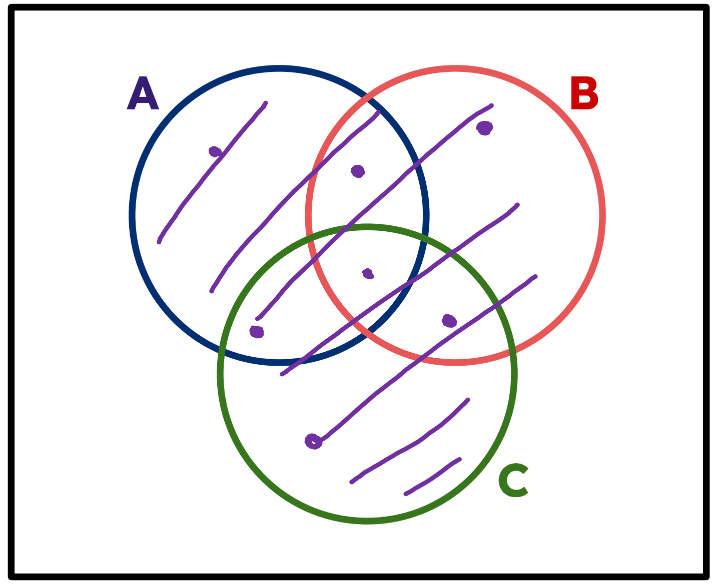
$$\boxed{P(A \cap B) = \frac{1}{10}}$$

1

$P(A \cup B \cup C) =$

$P(A) + P(B) + P(C) - P(A \cancel{\cap} B) - P(B \cancel{\cap} C) - P(A \cancel{\cap} C) + P(\cancel{A \cap B} \cap C)$

Kese?



Unacademy Atoms

Note:

I. If A, B and C are **mutually exclusive** events then $P(A \cup B \cup C) = P(A) + P(B) + P(C)$

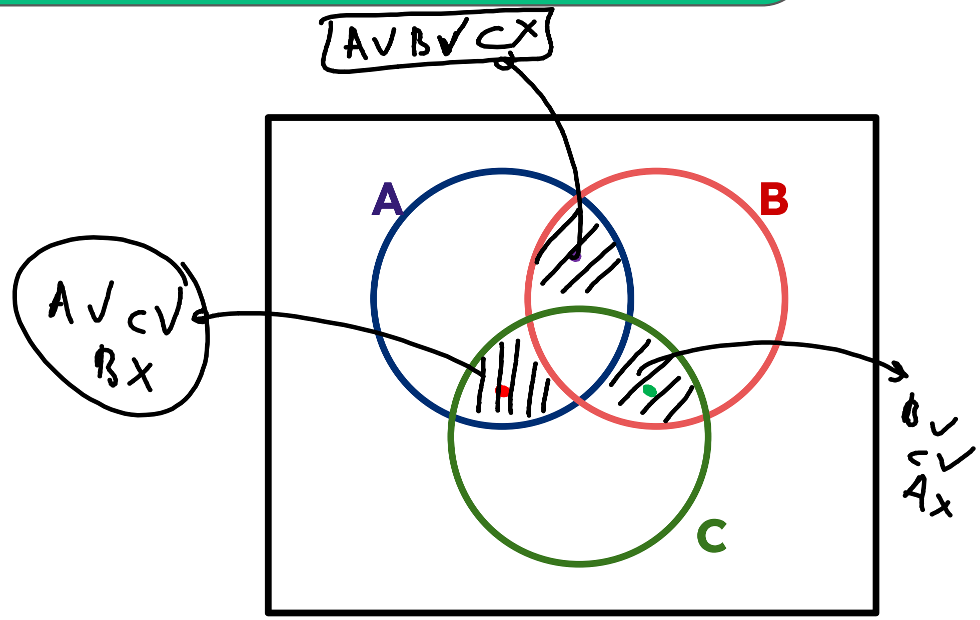
II. If A and B are **exhaustive events** then $P(A \cup B \cup C) = 1$

ME $\cap$ $A \cup B \cup C = S$

2

P(exactly two events A, B, C occur)

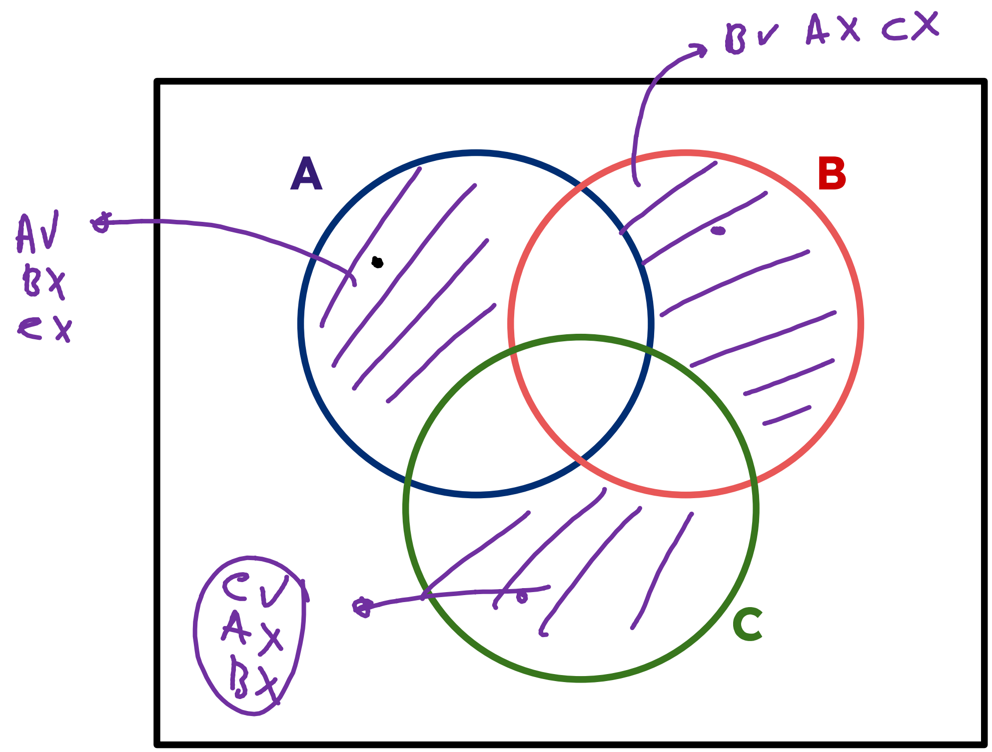
$= P(A \cap B) + P(B \cap C) + P(C \cap A) - 3\,P(A \cap B \cap C)$



3

P(exactly one of the events A, B, C occur)

$P(A) + P(B) + P(C) - 2P(A \cap B) - 2P(B \cap C) - 2P(A \cap C) + 3P(A \cap B \cap C)$



For three events A, B and C

P(Exactly one of A or B occurs) = P (Exactly one of B or C occurs) = P(Exactly one of C or A occurs) = 1/4 and P(All the three events occur simultaneously) = 1/16

Then the probability that at least one of the events occurs, are:

$\boxed{P(A\cap B\cap C) = \frac{1}{16}}$

$\boxed{P(A\cup B\cup C) = ?}$

A. 3/16 **B.** 7/32

C. 7/16 **D.** 7/64

(JEE M 2017)

$P(A) + P(B) - 2P(A\cap B) = \frac{1}{4}$

$P(B) + P(C) - 2P(B\cap C) = \frac{1}{4}$

$P(C) + P(A) - 2P(C\cap A) = \frac{1}{4}$

$\left(P(A)+P(B)+P(C)\right) - \left(P(A\cap B)+P(B\cap C)+P(C\cap A)\right) = \frac{3}{4\times 2}$

$$\boxed{P(A) + P(B) + P(C) - P(A\cap B) - P(B\cap C) - P(C\cap A) + P(A\cap B\cap C)} = \frac{3}{8} + \frac{1}{16}$$

$$P(A\cup B\cup C) = \frac{3}{8} + \frac{1}{16}$$

$$= \frac{7}{16}$$

The probabilities of three events A, B and C are given by $P(A) = 0.6$, $P(B) = 0.4$ and $P(C) = 0.5$ if $P(A \cup B) = 0.8$, $P(A \cap C) = 0.3$, $P(A \cap B \cap C) = 0.2$, $P(B \cap C) = \beta$ and $P(A \cup B \cup C) = \alpha$, where $0.85 \le \alpha \le 0.95$, then β lies in the interval :

A. [0.25, 0.35] B. [0.35, 0.36]

C. [0.36, 0.40] D. [0.20, 0.25]

(JEE Main 2020 6 Sep)

$\boxed{\text{\# Simpul}}$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

$$\alpha = 0.6 + 0.4 + 0.5 - 0.2 - \beta - 0.3 + 0.2$$

$$\alpha = 1.2 - \beta$$

$$\beta = 1.20 - \alpha$$

0.85

0.95

$\beta = 0.35$

$\beta = 0.25$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

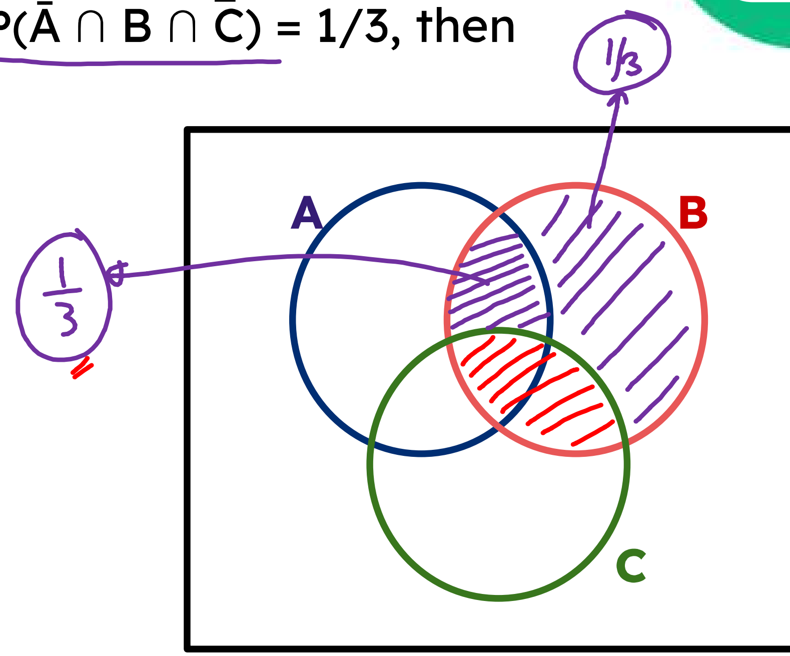
$$0.8 = 0.6 + 0.4 - P(A \cap B)$$

$$\underline{P(A \cap B) = 0.2}$$

Unacademy Atoms

If P(B) = 3/4, P(A ∩ B ∩ $\bar{C}$) = 1/3 and P($\bar{A}$ ∩ B ∩ $\bar{C}$) = 1/3, then P(B ∩ C) are

A. 1/12
B. 1/6
C. 1/15
D. 1/9



10 Sec

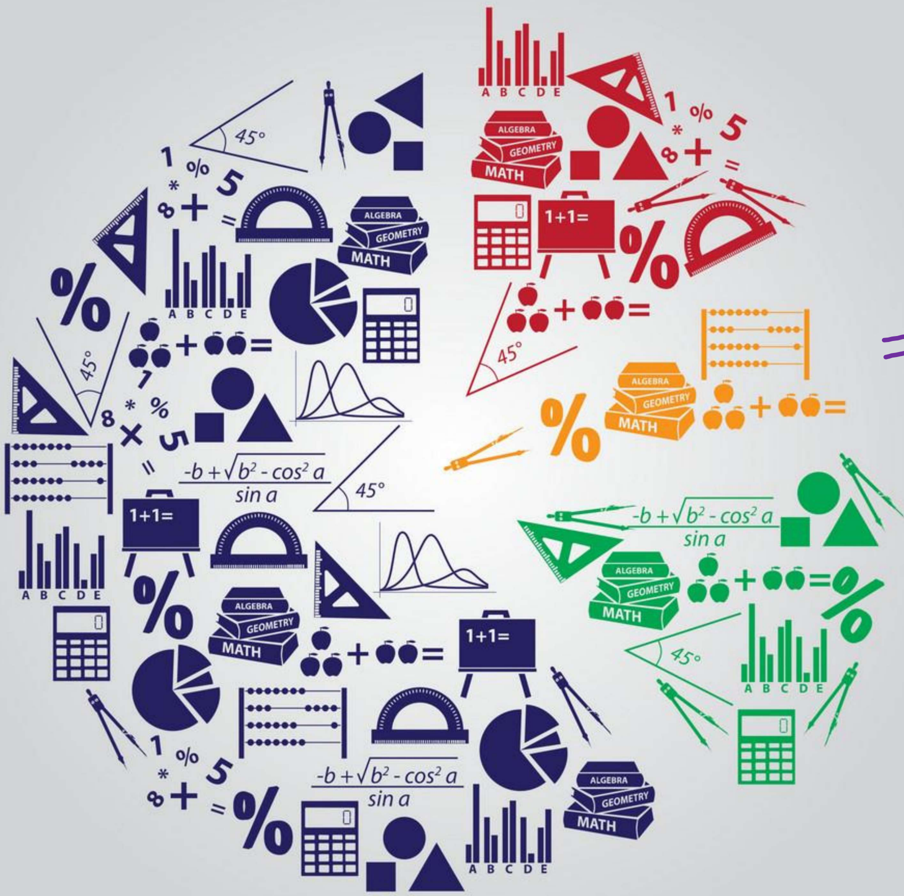
$$P(B\cap C) = \frac{3}{4} - \frac{1}{3} - \frac{1}{3}$$

$$= \frac{3}{4} - \frac{2}{3}$$

$$= \frac{1}{12}$$

(JEE 2003)

Unacademy Atoms



#. CONDITIONAL PROBABILITY

Illustration

A: "odd outcome" = {1, 3, 5}

B: "Prime outcome" = {2, 3, 5}

S: Sample space = {1, 2, 3, 4, 5, 6}

Logical

CP Extra info :- B → Kar chuka hai

$P = 2/3$

$S = \{2, 3, 5\}$

$fav = \{3, 5\}$

M.2 formula

$\boxed{S = \{1, 2, 3, 4, 5, 6\}}$

$A = \{1, 3, 5\}$

$B = \{2, 3, 5\}$

$\boxed{A \cap B = \{3, 5\}}$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$P(B) = \frac{n(B)}{n(S)}$$

$$= \frac{\frac{2}{\cancel{6}}}{\frac{3}{\cancel{6}}} = \frac{2}{3}$$

Definition

$$P(X/Y) = \frac{P(X \cap Y)}{P(Y)}$$

Let A and B be two events associated with a same sample space S. the conditional probability of an event A given B, where B has already

occurred, are denoted c $P(A/B)$ or $P\left(\frac{A}{B}\right)$

$$\boxed{P(B/A) = \frac{P(B \cap A)}{P(A)}}$$

pro.

Ho chuka

And are defined a $\boxed{P(A/B) = \frac{P(A \cap B)}{P(B)}}$, where P(B) ≠ 0

given

Roll a fair die twice. Let A be the event that the sum of the two rolls equals six, and let B be the event that the same number comes up twice. What are P(A/B)

A. 1/6 B. 5/36

Ho chuka!

C. 1/5 D. None

$A = \{(1,5), (5,1), (2,4), (4,2), \boxed{(3,3)}\}$

$B = \{(1,1), (2,2), \boxed{(3,3)}, (4,4), (5,5), (6,6)\}$

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1}{6}$

$1/6$

New $S = \{(1,1), (2,2), \boxed{(3,3)}, (4,4), (5,5), (6,6)\}$

Unacademy Atoms

Let X and Y be two events such that $P(X) = 1/3$, $P(X/Y) = 1/2$ and $P(Y/X) = 2/5$, Then

mcq

~~A.~~ $P(Y) = 4/15$

~~B.~~ $P(X' \mid Y) = 1/2$

~~C.~~ $P(X \cap Y) = 1/5$

~~D.~~ $\boxed{P(X \cup Y) = 2/5}$

(JEE Adv. 2017)

AB

$P(X/Y) = \frac{1}{2}$

$\frac{\boxed{P(X \cap Y)}}{P(Y)} = \frac{1}{2}$

$\frac{2}{15} \times 2 = P(Y)$

$\boxed{P(Y) = \frac{4}{15}}$

$P(Y/X) = \frac{2}{5}$

$\frac{P(Y \cap X)}{P(X)} = \frac{2}{5}$

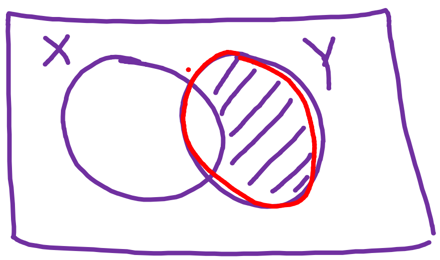
$P(Y \cap X) = \frac{2}{5} \times \frac{1}{3}$

$\boxed{P(Y \cap X) = \frac{2}{15}}$

$$P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$$

$$= \frac{5 \times 1}{5 \times 3} + \frac{4}{15} - \frac{2}{15}$$

$$= \frac{7}{15}$$



$$P(X'/Y)$$

$$\Rightarrow \frac{P(X' \cap Y)}{P(Y)}$$

$$\Rightarrow \frac{P(Y) - P(X \cap Y)}{P(Y)}$$

$$\Rightarrow \frac{\frac{4}{15} - \frac{2}{15}}{\frac{4}{15}}$$

$$\Rightarrow \frac{1}{2}$$

Unacademy Atoms

Multiplication Theorem on Probability

Multiplication Theorem

1 $P(A \cap B) = P(A).\ P(B/A)$

C.P. $\boxed{P(B/A) = \frac{P(A \cap B)}{P(A)}}$

M.T. $\boxed{P(A) \cdot P(B/A) = P(A \cap B)}$

$\boxed{P(A)\ P(B) = P(A \cap B)}$

If A and B be two independent events, then $P(A/B) = P(A)$

If A and B be two independent events, then
$\boxed{P(A \cap B) = P(A).P(B)}$

A and B $\rightarrow$ independent

$\boxed{P(A/B) = P(A)}$ $\boxed{P(B/A) = P(B)}$

Ho chuka

Independent Events

If A, B and C be two independent events, then **$P(A \cap B \cap C) = P(A). P(B). P(C)$**

$$P(A \cap B) = P(A) . P(B)$$

$$\boxed{P(A \cap B \cap C) = P(A)\, P(B)\, P(C)}$$

Unacademy Atoms

Note :

If A and B be independent events, then (A and B'), (A' and B) and (A' and B') **are also independent** events.

$$P(A \cap B') = P(A).P(B')$$

$$P(A' \cap B) = P(A').P(B)$$

$$P(A' \cap B') = P(A').P(B')$$

A and B → indep

⇓

A' and B
A and B'
A' and B'

indep.

Unacademy Atoms

Questions

Let A and B be two independent events such that $P(A)=\frac{1}{3}$ and $P(B)=\frac{1}{6}$. Then, which of the following is True ?

A. $P\left(\frac{A}{B}\right)=\frac{1}{6}$

B. $P\left(\frac{A}{B}\right)=\frac{1}{3}$

C. $P\left(\frac{A}{B}\right)=\frac{2}{3}$

D. $P\left(\frac{A}{B}\right)=\frac{5}{6}$

(JEE Main 2020 - 8

$$\boxed{\begin{aligned} P(A/B) &= P(A) \\ P(B/A) &= P(B) \end{aligned}}$$

Let A and B be two events such that $P(\overline{A \cup B}) = 1/6$, $P(A \cap B) = 1/4$, and $P(\bar{A}) = 1/4$, where $\bar{A}$ stands for complement of event A. then events A and B are

A. Equally likely and mutually exclusive
B. Equally likely but not independent
C. Independent but not equally likely
D. Mutually exclusive and independent

(JEE 2005, 2014)

$$\boxed{\begin{aligned} P(A \cup B) &= \frac{5}{6} \\ P(A \cap B) &= \frac{1}{4} \\ P(A) &= \frac{3}{4} \end{aligned}}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\frac{5}{6} = \frac{3}{4} + P(B) - \frac{1}{4}$$

$$\frac{5}{6} - \frac{1}{2} = P(B)$$

$$\boxed{\frac{1}{3} = P(B)}$$

E.L $P(A) = P(B)$

M.E $A \cap B = \phi$

$$\boxed{P(A \cap B) = 0}$$

Ext $P(A \cup B) = 1$

Indep $\boxed{P(A \cap B) = P(A)\,P(B)}$

Indep.

$$P(A \cap B) = \underline{P(A)}\, P(B)$$

$$\frac{1}{4} = \frac{\cancel{3}}{4} \times \frac{1}{\cancel{3}}$$

$$\underline{\frac{1}{4} = \frac{1}{4}}$$

Let E and F be two independent events. The probability that exactly one of them occurs are 11/25 and the probability of none of them occurring are 2/25. If P(T) denoted the probability of occurrence of the event T, then

AD

A. P(E) = 4/5, P(F) = 3/5

~~B.~~ P(E) = 1/5, P(F) = 2/5

~~C.~~ P(E) = 2/5, P(F) = 1/5

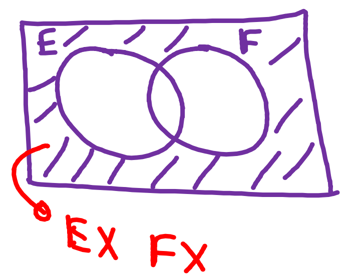
D. P(E) = 3/5, P(F) = 4/5

(JEE Adv. 2011)

$\boxed{P(E)+P(F)-2P(E\cap F)=\frac{11}{25}}$

$\boxed{1-P(E\cup F)=\frac{2}{25}}$

$\boxed{P(E\cup F)=\frac{23}{25}}$



$$\cancel{P(E)} + \cancel{P(F)} - P(E\cap F) = \frac{23}{25}$$

$$\cancel{P(E)} + \cancel{P(F)} - 2P(E\cap F) = \frac{11}{25}$$

$$- \quad - \quad + \quad -$$

$$P(E\cap F) = \frac{12}{25}$$

$$\boxed{P(E)\cdot P(F) = \frac{12}{25}}$$

(JEE Adv. 2019)

Let S be the sample space of all 3 x 3 matrices with entries from the set {0, 1}. Let the events E_1 and E_2 be given by $E_1 = \{A \in S : \det A = 0\}$ and $E_2 = \{A \in S :$ sum of entries of A are $7\}$. If a matrix are chosen at random from S, then the conditional probability $P(E_1/E_2)$ equals ____.

$$S \Rightarrow \begin{bmatrix} 0/1 & 0/1 & 0/1 \\ 0/1 & 0/1 & 0/1 \\ 0/1 & 0/1 & 0/1 \end{bmatrix}_{3\times3}$$

$$\boxed{n(S) = 2^9 = 512}$$

$$E_1 \Rightarrow \boxed{\det(A) = 0}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}_{3\times3}$$

$E_2 \Rightarrow$ Sum of entries $= 7$

$(1, 1, 1, 1, 1, 1, 1, 0, 0)$

$$\boxed{n(E_2) = {}^9C_2 \times 1 \times 1}$$

$$P(E_1/E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)}$$

$$= \frac{P(\text{ det} = 0 \text{ and sum of entries} = 7)}{P(\text{sum of entris} = 7)} = \frac{18}{{}^{9}C_2} = \frac{18}{36}$$

$$= \boxed{0.5}$$

$$\begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}, \begin{vmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}, \begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} \quad 3$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{vmatrix} \quad \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{vmatrix} \quad \begin{vmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix} \quad 3$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{vmatrix} \quad \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{vmatrix} \quad \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix} \quad 3$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{vmatrix}, \begin{vmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix}, \begin{vmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{vmatrix} \quad 3$$

$$\begin{vmatrix} 1 & & 1 \\ 1 & & 1 \\ 1 & & 1 \end{vmatrix} \quad 3$$

$$\begin{vmatrix} & 1 & 1 \\ & 1 & 1 \\ & 1 & 1 \end{vmatrix} \quad 3$$

Let X and Y be two events such that $P(X \mid Y) = 1/2$, $P(Y/X) = 1/3$, and $P(X \cap Y) = 1/6$. Which of the following are (are) correct?

A. $P(X \cup Y) = 2/3$
B. X and Y are independent
C. X and Y are not independent
D. $P(X^c \cap Y) = 1/2$

(JEE Adv. 2012)

AB

$$\frac{\boxed{P(X \cap Y)}}{P(Y)} = \frac{1}{2} \qquad \frac{\boxed{P(Y \cap X)}}{P(X)} = \frac{1}{3}$$

$$P(Y) = \frac{1}{3} \qquad \boxed{P(X) = \frac{1}{2}}$$

$$P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$$

$$= \frac{1}{2} + \frac{1}{3} - \frac{1}{6} = \frac{2}{3}$$

$$P(X \cap Y) = P(X) \cdot P(Y)$$

$$\underbrace{\frac{1}{6} = \frac{1}{2} \times \frac{1}{3}}$$

$$P(X^c \cap Y)$$

$$\Rightarrow P(X^c)\, P(Y)$$

$$\Rightarrow \left(1 - \frac{1}{2}\right)\left(\frac{1}{3}\right)$$

$$\Rightarrow \frac{1}{6}$$

★ 2020 Adv.

Infinite G.P.

★ $\boxed{\text{R.E} \rightarrow \infty \text{ times}}$

A six faced fair die are thrown until 1 comes, then the probability that 1 comes in even number of trails are

A. 5/11 B. 5/6

C. 6/11 D. 1/6

(IIT - JEE 2005)

$\frac{5}{6}\times\frac{1}{6} + \left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^5\left(\frac{1}{6}\right) + \cdots\cdots$

C-1 1x 1✓ $\longrightarrow \frac{5}{6}\times\frac{1}{6}$

C-2 1x 1x 1x 1✓ $\longrightarrow \left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right)$

C-3 1x 1x 1x 1x 1x 1✓ $\left(\frac{5}{6}\right)^5\left(\frac{1}{6}\right)$

$\frac{a}{1-r}$

$\Rightarrow \frac{\frac{5}{6}\times\frac{1}{6}}{1-\frac{25}{36}}$

$\Rightarrow \frac{5}{11}$

A dice is thrown until either a 4 or 6 appears. Find the probability that number 6 occurs before 4?

A. 1/2

B. 5/6

C. 1/6

D. 1/3

fav : 6 before 4

dice → 4 OR 6 STOP!

{1,2,3,5}

C-1 6 → $\frac{1}{6}$

C-2 4x6x 6 → $\frac{4}{6} \times \frac{1}{6}$

C-3 4x6x 4x6x 6 → $\left(\frac{4}{6}\right)^2 \left(\frac{1}{6}\right)$

⋮

$$\frac{\frac{1}{6}}{1-\frac{4}{6}} = \frac{1}{2}$$

A pair of unbiased dice are rolled together till a sum of "either 5 or 7" are obtained. Then find the probability that 5 comes before 7.

A. 5/11 **B.** 5/6

C. 6/11 ~~**D.**~~ 2/5

far Sum=5 > Sum = 7

4+6

$\boxed{\text{Sum} = 5 \quad \text{OR} \quad \text{Sum} = 7}$ STOP!

C-1 S = 5 $\longrightarrow \frac{4}{36}$

C-2 S=5,7 × S = 5 $\longrightarrow \left(\frac{26}{36}\right)\left(\frac{4}{36}\right)$

C-3 S=5,7 × S=5,7 × S = 5 $\longrightarrow \left(\frac{26}{36}\right)^2\left(\frac{4}{36}\right)$

$$\frac{\frac{4}{36}}{1-\frac{26}{36}}$$

$$\frac{4}{10} = 2/5$$

$P(\underline{\text{Sum} \neq 5}, \text{Sum} \neq 7)$

$\underline{36-10}$

2	12
3	11
4	10
~~5~~	~~9~~
6	8
~~7~~	

$\to$ 4

$\to$ 6

Paragraph Question 1

g.p.

A fair die are thrown repeatedly until a six are obtained. Let X denote the number of toss required.

1/2/3/4/5

The probability that $\boxed{X = 3}$ equals

6× 6× 6✓

A. 25/216

B. 25/36

C. 5/36

D. 125/216

$\boxed{\frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}}$

Adv. (2009)

$\boxed{\text{fair die}} \longrightarrow$ Untill $\boxed{6}$ STOP!

Paragraph Question 2

A fair die are thrown repeatedly until a six are obtained. Let X denote the number of toss required.

The probability that $X \geq 3$ equals

A. 125/216 ~~B.~~ 25/36

C. 5/36 D. 25/216

(2009)

$$\underbrace{P(X=3)} + P(X=4) + P(X=5) + \cdots$$

$$\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right) \cdots\cdots$$

$$\frac{\left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right)}{1-\frac{5}{6}} = \frac{25}{36}$$

Paragraph Question 3

A fair die are thrown repeatedly until a six are obtained. Let X denote the number of toss required.

The conditional Probability that X ≥ 6 given X>3 equals

A. 125/216 **B.** 25/36

C. 5/36 **D.** 25/216

(2009)

$\boxed{\text{12 marks in 10 min}}$

$$P(X \geq 6 / X > 3) = \frac{P(X \geq 6 \cap X > 3)}{P(X > 3)}$$

$$= \frac{P(X \geq 6)}{P(X > 3)} = \frac{\left(\frac{5}{6}\right)^5}{\left(\frac{5}{6}\right)^3} = \frac{25}{36}$$

$$P(X \geq 6) = P(X=6) + P(X=7) + P(X=8) \cdots\cdots$$

$$= \left(\frac{5}{6}\right)^5 \left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^6 \left(\frac{1}{6}\right) + \cdots$$

$$= \frac{\left(\frac{5}{6}\right)^5 \left(\frac{1}{6}\right)}{1 - \frac{5}{6}} = \left(\frac{5}{6}\right)^5$$

$$P(X > 3) = P(X=4) + P(X=5) + \cdots\cdots$$

$$= \left(\frac{5}{6}\right)^3 \left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^4 \left(\frac{1}{6}\right) + \cdots$$

$$= \frac{\left(\frac{5}{6}\right)^3 \left(\frac{1}{6}\right)}{1 - \frac{5}{6}} = \left(\frac{5}{6}\right)^3$$

Two fair dice, each with faces numbered 1, 2, 3, 4, 5 and 6, are rolled together and the sum of the numbers on the faces are observed. This process are repeated $\boxed{\text{till}}$ the sum is either a prime number or a perfect square. Suppose the sum turns out to be a perfect square before it turns out to be a prime number. If p is the probability that this perfect square is an odd number, then the value of 14p are ____.

(A)

(JEE Adv. 2020)

RE :- sum = prime OR sum = p. sq. $\longrightarrow$ STOP !

✓ A : sum = p.sq > sum = prime.

✓ B : p.sq = odd

$\boxed{\text{Sum} = 2 \text{ to } 12}$

- Sum = prime = {2, 3, 5, 7, 11}
- Sum = p.sq = {4, 9}

$\boxed{1+3+5+5}$

~~2 12~~ $\rightarrow$ 1

3 11

~~4 10~~ $\rightarrow$ 3

~~5 9~~ $\rightarrow$ 4

~~6 8~~ $\rightarrow$ 5

7

$$P(B/A) = \frac{P(B \cap A)}{P(A)}$$

$\boxed{q = \text{sum}}$

$$= \frac{P(\boxed{p.\,Sq = \text{odd}} \text{ and } p.\,Sq > \text{prime})}{\boxed{P(p.\,sq > \text{prime})}}$$

$$p = \frac{\frac{4}{\cancel{22}}}{\left(\frac{7}{\cancel{22}}\right)}$$

$$p = \frac{4}{7}$$

$$14 \times \frac{4}{7} = \underline{8 \text{ Ans}}$$

$P(\text{p.sq} > \text{prime})$

C-1 $\boxed{\text{p.sq}} \longrightarrow \frac{7}{36}$

C-2 ~~p.sq~~ ~~prime~~ p.sq $\longrightarrow \left(\frac{14}{36}\right) \times \frac{7}{36}$

C-3 ~~p.sq~~ ~~prime~~ ~~p.sq~~ ~~prim~~ p.sq $\rightarrow \left(\frac{14}{36}\right)^2 \left(\frac{7}{36}\right)$

⋮

$$\frac{\frac{7}{36}}{1 - \frac{14}{36}} = \frac{7}{22}$$

N^2 $\quad \underline{p.sq = 9}$ and p.sq > prime.

C-1 $\underline{S=9} \longrightarrow \left(\frac{4}{36}\right)$

C-2 $\underline{\cancel{p.sq}\ \cancel{prime}}$ $\underline{S=9} \longrightarrow \frac{14}{36} \times \frac{4}{36}$

C-3 $\underline{\cancel{p.sq}\ \cancel{prime}}$ $\underline{\cancel{p.sq}\ \cancel{prime}}$ $\underline{S=9} \longrightarrow \left(\frac{14}{36}\right)^2 \frac{4}{36}$

⋮

$$\frac{\frac{4}{36}}{1-\frac{14}{36}} = \frac{4}{22}$$

Total Probability Theorem

End → T.P.T.

Middle → B.T.
(End → given)

Example 1:

A box contains three Dices, one Dice are fair, one Dice are two-headed, and one Dice are weighted (P(H) = ⅓). A Dice are selected at random and thrown. Find the probability that head appears.

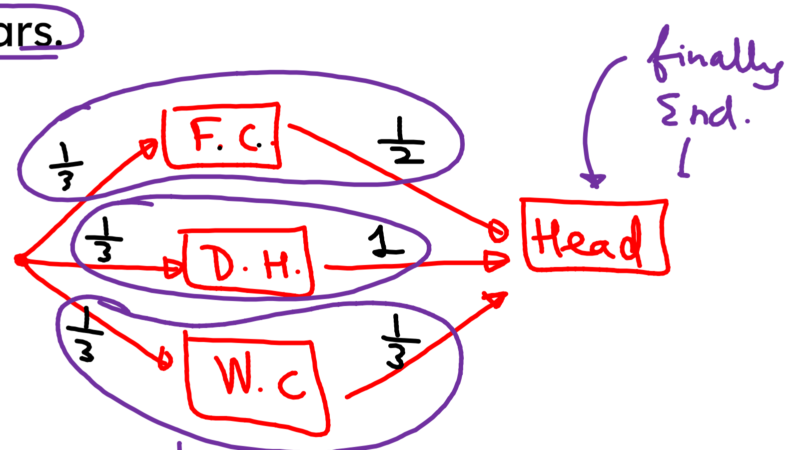
1 Coin

→ Fair

→ Doubly headed

→ Weighted ✓

$P(H) = \frac{1}{3}$ $\quad P(T) = \frac{2}{3}$



$$\boxed{\frac{1}{3}\times\frac{1}{2}+\frac{1}{3}\times 1+\frac{1}{3}\times\frac{1}{3} = ?}$$

Total Probability Theorem

Let E_1, E_2, ……E_n be n mutually exclusive and exhaustive events, with non-zero probabilities, of a random experiment. If A be any arbitrary event of the sample space of the above random experiment with P(A) > 0, then

$$\boxed{P(A) = P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + \ldots\ldots\ldots P(E_n)P\left(\frac{A}{E_n}\right)}$$

B.T.

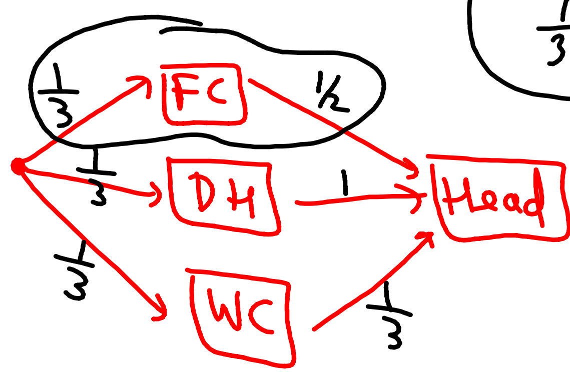
Bayes' Theorem

Illustration

A box contains three ~~Dices~~, one ~~Dice~~ Coin are fair, one Dice are two-headed, and one Dice are weighted (P(H) = ⅓). A ~~Dice~~ are selected at random and thrown. Find the probability that head appears.

$\boxed{1 \text{ Coin}} \rightarrow \text{toss} \rightarrow \text{head}$ (given)

- → Fair
- → Doubly headed
- → Weighted



$$\frac{\frac{1}{3}\times\frac{1}{2}}{\frac{1}{3}\times\frac{1}{2}+\frac{1}{3}\times 1+\frac{1}{3}\times\frac{1}{3}} = P$$

Bayes Theorem

If an event A can occur only with one of the n pairwise mutually exclusive and exhaustive events B_1, B_2, ...B_n & if the conditional probabilities of the events.

$P(A/B_1)$, $P(A/B_2)$ ...... $P(A/B_n)$ are known the,

#

$$\boxed{P(B_i/A) = \frac{p(B_i)P(A/B_i)}{\sum_{i=1}^{n} P(B_i)P(A/B_i)}}$$

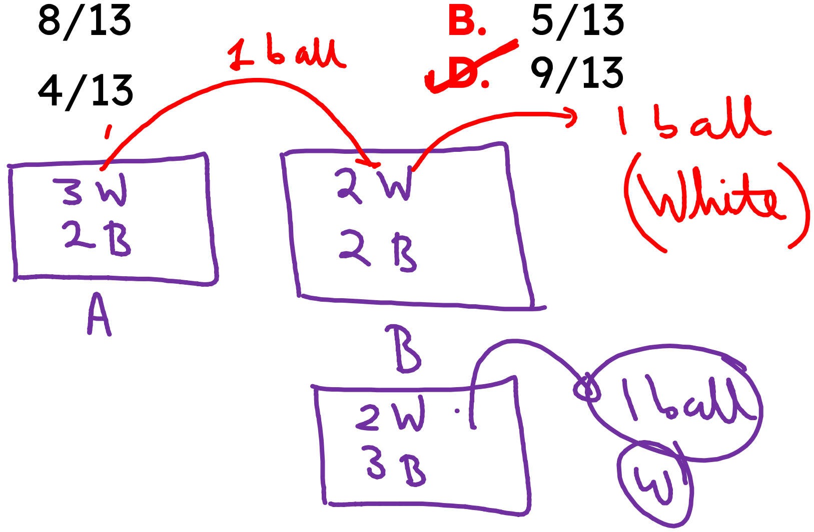
Bag A contains 3 white and 2 black balls. Bag B contains 2 white and 2 black balls. One ball are drawn at random from A and transferred to B. One ball are selected at random from B and are found to be white. The probability that the transferred ball are white are

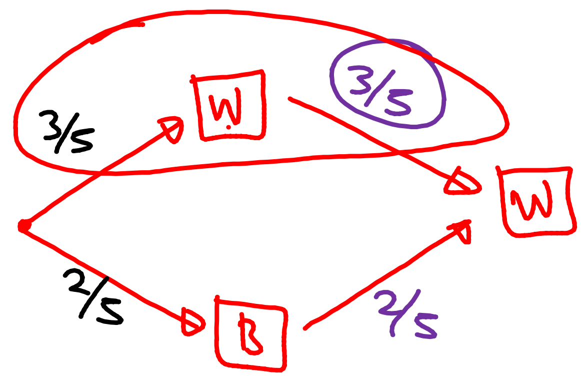
A. 8/13

B. 5/13

C. 4/13

D. 9/13

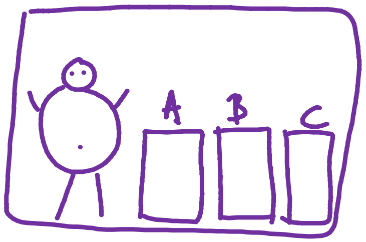


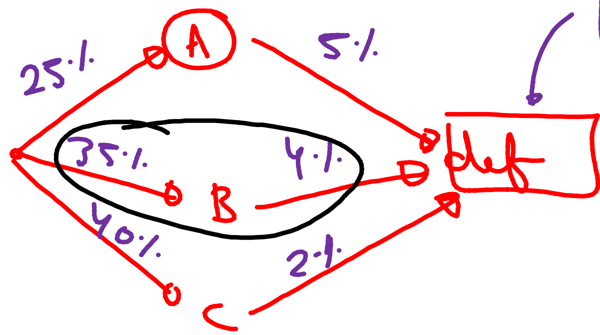


$$\frac{\frac{3}{5}\times\frac{3}{5}}{\frac{3}{5}\times\frac{3}{5}+\frac{2}{5}\times\frac{2}{5}} = \frac{9}{13}$$

In a factory which manufactures bolts, machines A, B and C manufacture respectively 25%, 35% and 40% of the bolts. Of their outputs, 5, 4 and 2 percent are respectively defective bolt. A bolts are drawn at random from the product and are found to be defective. What are the probability that it are manufactured by the machine B?

given def





25% 35% 40%

def:- 5% 4% 2%

$$P = \frac{35 \times 4}{25 \times 5 + 35 \times 4 + 40 \times 2}$$

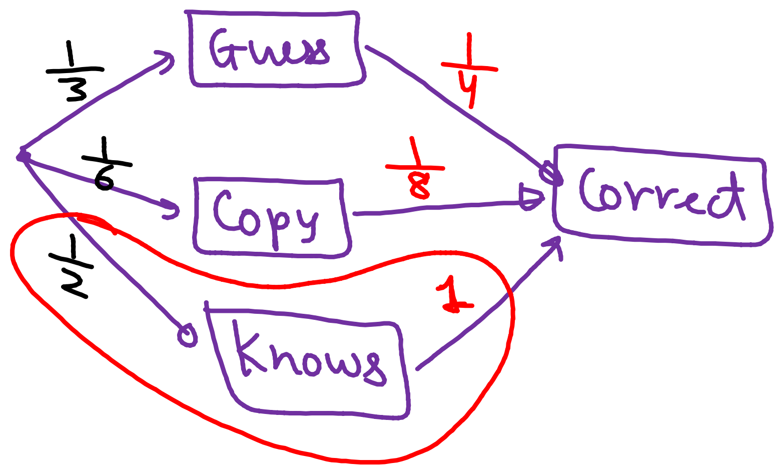
In a test, an examinee either guesses or copies or knows the answer for a multiple choice question having FOUR choices of which exactly one are correct. The probability that he makes a guess are 1/3 and the probability for copying are 1/6. The probability that hare answer are correct, given that he copied it are 1/8. The probability that he knew the answer, given that hare answer are correct are

BT/TPT

A. 5/29 **B.** 9/29

C. $\boxed{24/29}$ **D.** 20/29

$$P = \frac{\frac{1}{2} \times 1}{\frac{1}{3} \times \frac{1}{4} + \frac{1}{6} \times \frac{1}{8} + \frac{1}{2} \times 1}$$



Adv 2020

Binomial Probability

Binomial probability

Let an experiment has n-independent trials, and each of the trial has two possible outcomes

$\boxed{\text{Kal} \rightarrow ?}$

I. Success $\rightarrow p$
II. Failure

$\boxed{\text{RE} \rightarrow n \text{ times}}$

p → Probability of getting success
q → Probability of getting failure
such that $\boxed{p + q = 1}$

$$\boxed{P(X=r) = {}^nC_r\, p^r\, q^{n-r}}$$

No. of Success

Then, **P(Exactly r successes) = P(X = r) = $^nC_r\, p^r\, q^{n-r}$**

A pair of dice are thrown 6 times, getting a doublet are considered success. Compute the probability of

I. No success

II. Exactly one success

III. At least one success

IV. At most one success

$\boxed{n = 6}$

$$\boxed{\begin{aligned} p &= \frac{1}{6} \\ q &= \frac{5}{6} \end{aligned}}$$

I $P(X = 0)$

$$= {}^6C_0 \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^6$$

$$= \left(\frac{5}{6}\right)^6$$

II $P(X = 1)$

$$= {}^6C_1 \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^5$$

$P(x \geq 1)$

$= P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5) + P(X=6)$

$= 1 - P(X=0)$

$= 1 - {}^6C_0 \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^6$

IV $\quad P(X=0) + P(X=1)$

$${}^6C_0 \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^6 + {}^6C_1 \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^5$$

In a hurdle race a man has to clear 9 hurdles. Probability that he clears a hurdle 2/3 and the probability that he knocks down the hurdle are 1/3. Find the probability that he knocks down less than 2 hurdles.

$\boxed{n = 9}$

clear $\rightarrow \frac{2}{3} = q$

Knock $\rightarrow \frac{1}{3} = p$

$\Rightarrow P(X < 2)$

$\Rightarrow P(X=0) + P(X=1)$

$\Rightarrow {}^9C_0 \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^9 + {}^9C_1 \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^8$

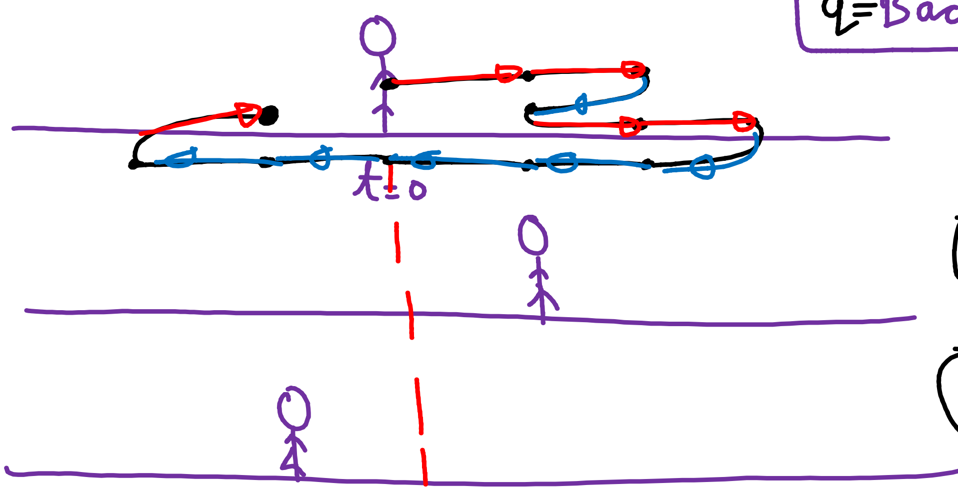
SRK

A drunkard takes a step forward or backward. The probability that he takes a step forward are 0.4. Find the probability that at the end of 11 steps he are one step away from the starting point.

$n = 11$

$p = \text{Forward} \rightarrow 0.4$

$q = \text{Backward} \rightarrow 0.6$



$6F + 5B$

$5F + 6B$

$$P(x=6) + P(x=5)$$

$$\underline{{}^{11}C_6 (0.4)^6 (0.6)^5 + {}^{11}C_5 (0.4)^5 (0.6)^6}$$

A fair coin is tossed n-times such that the probability of getting at least one head is at least 0.9. Then the minimum value of n is_

$n = n$

head $\rightarrow \frac{1}{2} = p$

tail $\rightarrow \frac{1}{2} = q$

4 Ans

(25 July 2021 Shift 2)

$P(X \geq 1) \geq 0.9$

$1 - P(X=0) \geq 0.9$

$0.1 \geq P(X=0)$

$\frac{1}{10} \geq {}^{n}C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^n$

$\frac{1}{10} \geq \frac{1}{2^n}$

$\boxed{2^n \geq 10}$

$2^1 \leq 10$

$2^2 \leq 10$

$2^3 \leq 10$

$2^4 \geq 10$

The minimum number of times a fair coin needs to be tosses, so that the probability of getting at least two heads is at least 0.96, are

(JEE Adv. 2015)

n times $\quad n_{min} = ?$

head $= p = \frac{1}{2}$

tail $= q = \frac{1}{2}$

$P(X \geq 2) \geq 0.96$

$1 - [P(X=0) + P(X=1)] \geq 0.96$

$1 - 0.96 \geq P(X=0) + P(X=1)$

$\frac{4}{100} \geq {}^nC_0 \left(\frac{1}{2}\right)^n + {}^nC_1 \left(\frac{1}{2}\right)^n$

8

$$\frac{1}{25} \geq \frac{1}{2^n} + \frac{n}{2^n}$$

$$2^n \geq 25(1+n)$$

$n=1$ $\quad 2^1 < 25(1+1)$

$n=2$ $\quad 2^2 < 25(1+2)$

⋮

$n=4$ $\quad 2^4 < 25(1+4)$

$n=5$ $\quad 2^5 < 25(1+5)$

$n=6$ $\quad 2^6 < 25(1+6)$

$n=7$ $\quad 2^7 < 25(1+7)$

$n=8$ $\quad 2^8 > 25(1+8)$

256 $\quad$ 225

Unacademy Atoms

The probability that a missiles hits a target successfully is 0.75. In order to destroy the target completely, at least three successful hits are required. Then the minimum number of missiles that have to be fired so that the probability of completely destroying the target is $\boxed{\text{NOT less than 0.95}}$ are ______.

(JEE Adv. 2020)

$$\boxed{\begin{aligned} \text{hit} &= \text{success} = \frac{3}{4} = p \\ \text{NOT hit} &= \text{failure} = \frac{1}{4} = q \end{aligned}}$$

$\boxed{n \text{ times}}$

$n_{min} = ?$

mini 3 hits = destroyed.

$$P(X \geq 3) \geq 0.95$$

$$1 - \left(P(X=0) + P(X=1) + P(X=2)\right) \geq 0.95$$

Unacademy Atoms

$$0.05 \geqslant P(X=0) + P(X=1) + P(X=2)$$

$$\frac{1}{20} \geqslant {}^{n}C_0 \left(\frac{3}{4}\right)^0 \left(\frac{1}{4}\right)^n + {}^{n}C_1 \left(\frac{3}{4}\right)^1 \left(\frac{1}{4}\right)^{n-1} + {}^{n}C_2 \left(\frac{3}{4}\right)^2 \left(\frac{1}{4}\right)^{n-2}$$

$$\frac{1}{20} \geqslant \frac{1 + 3n + 9\,{}^{n}C_2}{4^n}$$

$$\boxed{\text{Ans : } 6}$$

$$\boxed{4^n \geqslant 20\left(1 + 3n + 9 \cdot {}^{n}C_2\right)}$$

$n=2$ $\leqslant$

$n=3$ $\leqslant$

$\vdots$ $\leqslant$

$n=6$ $\geqslant$

Probability Distribution Table

Three balls are drawn one by one without replacement from a bag containing 5 white and 4 red balls. Find the probability distribution of the number of red balls drawn.

5 W
4 R

$\boxed{X = \text{no. of Red Balls}}$

$\boxed{\sum p_i = 1}$

X_i	p_i	$p_i x_i$	$p_i x_i^2$
0	$\frac{5}{42}$	0	0
1	$\frac{20}{42}$	$\frac{20}{42}$	$\frac{20}{42}$
2	$\frac{15}{42}$	$\frac{30}{42}$	$\frac{60}{42}$
3	$\frac{2}{42}$	$\frac{6}{42}$	$\frac{18}{42}$
	$\sum p_i = 1$	$\sum p_i x_i = \frac{56}{42} = \mu$	$\sum p_i x_i^2 = \frac{98}{42}$

$$P(X=0) = WWW$$

$$= \frac{5}{\cancel{9}\,3} \times \frac{4}{\cancel{8}\,2} \times \frac{\cancel{3}}{7} = \frac{5}{42}$$

$$P(x=1) \Rightarrow RWW + \underline{WRW} + \underline{WWR}$$

$$\Rightarrow \left(\frac{4}{\cancel{9}\,3} \times \frac{5}{\cancel{8}\,2} \times \frac{\overset{2}{\cancel{4}}}{7}\right) \times \cancel{3} \Rightarrow \frac{10}{21} = \frac{20}{42}$$

$$P(X=2) \Rightarrow RRW + RWR + WRR$$

$$\Rightarrow \left(\frac{4}{\cancel{9}} \times \frac{\cancel{3}}{\cancel{8}\,2} \times \frac{5}{7}\right) \times \cancel{3} = \frac{5}{14} = \frac{15}{42}$$

$$P(x=3)$$

$$\Rightarrow RRR$$

$$\Rightarrow \frac{\cancel{4}}{9} \times \frac{3}{\cancel{8}} \times \frac{\cancel{2}}{7}$$

$$\Rightarrow \frac{1}{21}$$

$$\Rightarrow \frac{2}{42}$$

Mean and Variance of a Probability Distribution:

1. Mean :

$$\boxed{\mu = \sum p_i x_i}$$

$$\boxed{\mu = \frac{\sum f_i x_i}{\sum f_i} = \frac{\sum p_i x_i}{\sum p_i} = \sum p_i x_i}$$

2. Variance:

$$\boxed{\sigma^2 = \left(\sum p_i x_i^2\right) - \mu^2}$$

$$\boxed{Var = \sigma^2 = \frac{98}{42} - \left(\frac{56}{42}\right)^2}$$

3. Standard Deviation :

$$\sigma = \sqrt{\text{Variance}}$$

s.d.

Two cards are drawn successively with replacement from a well-shuffled deck of 52 cards. Let X denote the random variable of number of aces obtained in the two drawn cards.
Then P(X = 1) + P(X = 2) equals:

A. 49/169 **B.** 52/169 **C.** 24/169 **D.** 25/169

(JEE M 2019)

$P(Ace) = \frac{4}{52} = \frac{1}{13}$

$\boxed{X = \text{No. of Aces}}$

$P(X=1) + P(X=2)$

$A\bar{A} + \bar{A}A + AA$

$= \left(\frac{1}{13}\right)\left(\frac{12}{13}\right) + \left(\frac{12}{13}\right)\left(\frac{1}{13}\right) + \left(\frac{1}{13}\right)\left(\frac{1}{13}\right)$

$= \frac{25}{169}$

only.

JEE Main

Binomial Probability D~~are~~tribution

Binomial Probability Distribution (BPD)

Let an experiment has n independent trials and each of the trial has two possible outcomes i.e. success or failure.

If random variable (X_i) = number of successes
then probability of getting exactly 'r' successes are $\boxed{P(X = r) = {}^nC_r\, p^r.q^{n-r}}$

where p = probability of success
and q = probability of failure

Mean of BPD : $\boxed{np}$

Variance of BPD : npq

Standard Deviation of BPD : $\sqrt{npq}$

$$\boxed{\begin{array}{l} * \ p+q=1 \\ * \ P(X=r) = {}^{n}C_{r}\, p^{r} q^{n-r} \end{array}}$$

A box contains 15 green and 10 yellow balls. If 10 balls are randomly drawn, one-by-one, with replacement, then the variance of the number of green balls drawn are:

A. 6/25 **B.** 12/5 **C.** 6 **D.** 4

(JEE M 2017)

$\boxed{\begin{array}{l} 15\ G \\ 10\ Y \end{array}}$

$$\text{Var} = npq$$

$$= 10 \times \left(\frac{15}{25}\right)\left(\frac{10}{25}\right)$$

$$= \overset{2}{\cancel{10}} \times \frac{3}{\cancel{5}} \times \frac{2}{5}$$

$$= 12/5$$

The mean and the variance of a binomial distribution are 4 and 2 respectively. Then the probability of exactly 2 successes are

A. 28/256 B. 219/256 C. 128/256 D. 37/256

(2004)

① $np = 4 \Rightarrow n\left(\frac{1}{2}\right) = 4$

② $npq = 2$

$n = 8$

$\boxed{q = \frac{1}{2}}$

$\boxed{p = \frac{1}{2}}$

$$P(X=2) = {}^{8}C_{2}\left(\frac{1}{2}\right)^{2}\left(\frac{1}{2}\right)^{6}$$

$$= \frac{28}{2^{8}} = \frac{28}{256}$$

The mean and variance of a random variable X having binomial distribution are 4 and 2 respectively, then P(X = 1) are

A. 1/4 **B.** 1/32 **C.** 1/16 **D.** 1/8

H.W.

(2003)

Let in a Binomial distribution, consisting of 5 independent trials, probabilities of exactly 1 and 2 successes be 0.4096 and 0.2048 respectively. Then the probability of getting exactly 3 successes is equal to:

(18 Mar 2021 Shift 2)

A. 32/625

B. 80/243

C. 40/243

D. 128/625

$n = 5$

$P(X=1) = 0.4096$

$P(X=2) = 0.2048$

$$\frac{{}^5C_1\, p^1 q^4}{{}^5C_2\, p^2 q^3} = 2$$

$$\frac{5q}{10p} = 2 \Rightarrow \boxed{q = 4p}$$

$$\boxed{q + p = 1}$$

$p = \frac{1}{5}$

$q = \frac{4}{5}$

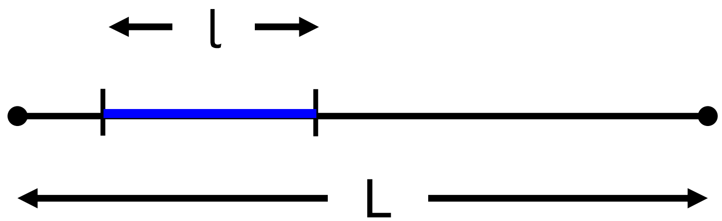
$$P(X=3) = {}^5C_3 \left(\frac{1}{5}\right)^3 \left(\frac{4}{5}\right)^2$$

Geometrical Probability

Geometrical probability (Continuous sample space)

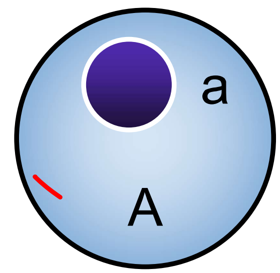
1. One-dimensional Probability:

$$P = \frac{\text{Favourable length}}{\text{Total length}}$$



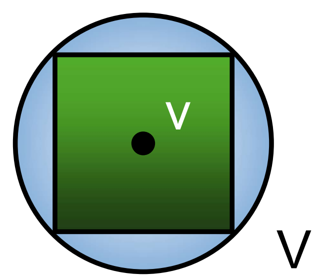
2. Two-dimensional Probability:

$$P = \frac{\text{Favourable area}}{\text{Total area}}$$



3. Three-dimensional Probability:

$$P = \frac{\text{Favourable volume}}{\text{Total volume}}$$

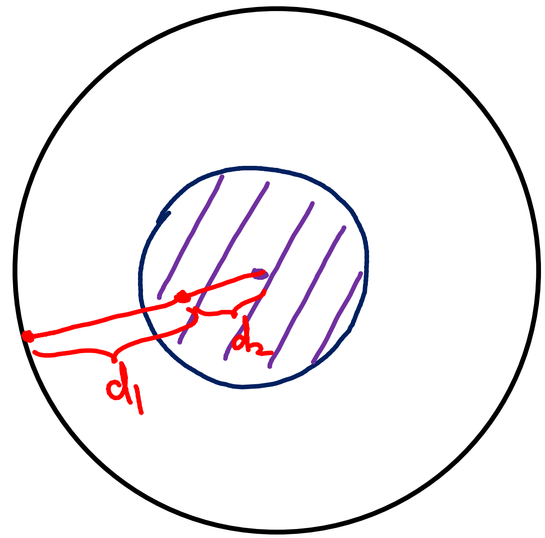


A point is taken inside a circle of radius r find the probability that the point is closer to the centre as a circumference.

$$P = \frac{\text{fav. area}}{\text{Total area}} = \frac{\pi\left(\frac{r}{2}\right)^2}{\pi r^2}$$

$$= \frac{1}{4}$$

$d_2 < d_1$

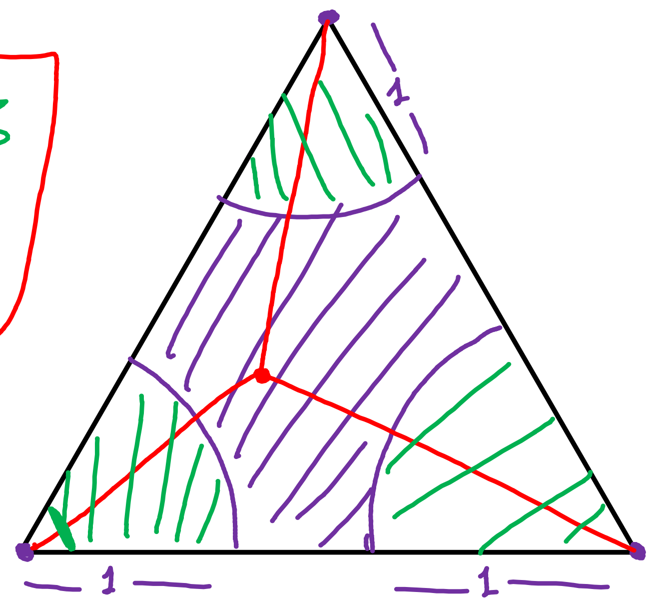


A point is selected randomly inside a equilateral triangle whose length is 3. Find the probability that its distance from any corner is greater than 1.

$\boxed{\frac{1}{2}\theta r^2}$

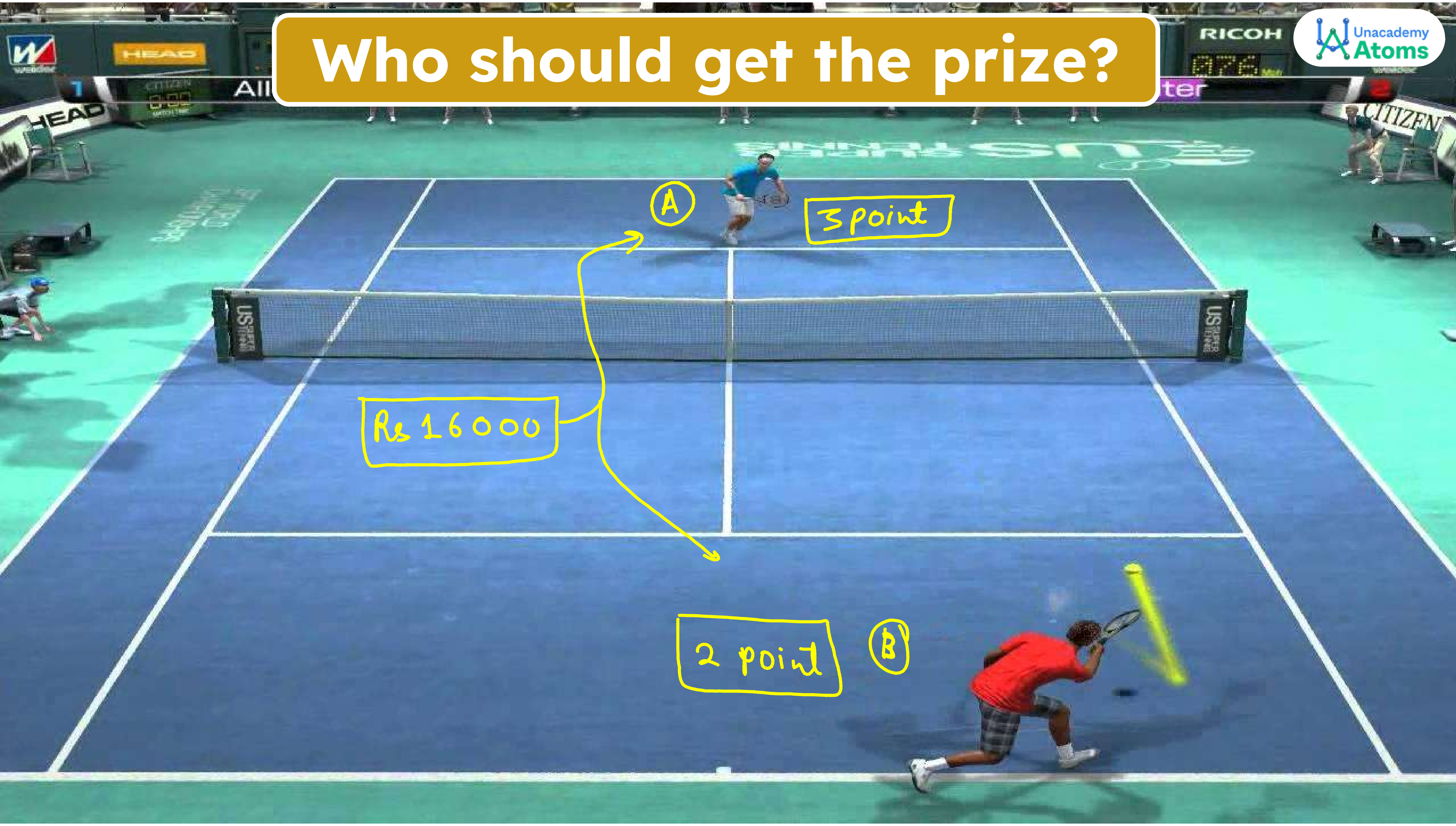
$\boxed{\text{Side} = 3}$

$$\boxed{P = \frac{\frac{\sqrt{3}}{4}(3)^2 - \frac{1}{2}\times\frac{\pi}{3}\times(1)^2\times 3}{\frac{\sqrt{3}}{4}(3)^2}}$$



M.E = prob. × paisa

Mathematical Expectation


Who should get the prize?
Unacademy Atoms
RICOH
HEAD
CITIZEN
A
3 point
Rs 16000
2 point
B

Two players of equal skill A and B are playing a game. They leave off playing (due to some force majeure conditions) when A wants 3 points and B wants 2 to win. If the prize money are Rs. 16000/-. How can the referee divide the money in a fair way.

$$\boxed{\begin{array}{l} A \rightarrow 3 \text{ point} \\ B \rightarrow 2 \text{ point} \end{array}}$$

$P(A \text{ win}) \Rightarrow$ ① A A A $\longrightarrow \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{2}{2}$

② B A A A $\longrightarrow \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

③ A B A A $\longrightarrow \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

④ A A B A $\longrightarrow \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$P(A \text{ win}) = \frac{5}{16}$

$P(A\ win) = \frac{5}{16}$ $P(B\ win) = 1 - \frac{5}{16} = \frac{11}{16}$

$\frac{11}{16} \times 16000$

$$\boxed{ME_B = 11000}$$

$ME_A = P(A\ win) \times 16000$

$= \frac{5}{16} \times 16000$

$= 5000$ Rs

Unacademy Atoms

Coincidence Testimony

Who killed the teacher ?

Coincidence Testimony

Coincidence Testimony

If p_1 and p_2 are the probabilities of speaking the truth of two independent witnesses A and B who give the same statement

$$P(\text{both speaks truth} / \text{Statements Match}) = \boxed{\frac{p_1 p_2}{p_1 p_2 + (1 - p_1)(1 - p_2)}}$$

T T F F

T T

F F

Unacademy Atoms

A speaks the truth 3 out of 4 times, and B 5 out of 6 times. What are the probability that they will contradict each other in stating the same fact.

A. 4/5 **B.** 1/3 **C.** 7/20 **D.** 3/20

A → 3/4

B → 5/6

TF + FT

$$= \frac{3}{4} \times \frac{1}{6} + \frac{1}{4} \times \frac{5}{6}$$

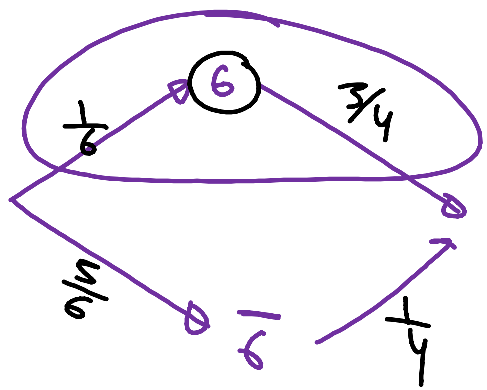
$$= \frac{8}{4 \times 6} = \frac{1}{3}$$

A man is known to speak the truth 3 out if 4 times. He throws a die and reports that it is a six. The probability that it is actually a six is

A. 3/8 B. 1/5 C. 3/4 D. None of these

$P(T) = \frac{3}{4}$

$P(F) = \frac{1}{4}$



$$\frac{\frac{1}{6} \times \frac{3}{4}}{\frac{1}{6} \times \frac{3}{4} + \frac{5}{6} \times \frac{1}{4}} = \frac{3}{8}$$

The probability that A speaks truth are 4/5 , while the probability for B are 3/4. The probability that they contradict each other when asked to speak on a fact are

(2004)

A. 4/5 **B.** 1/5 **C.** 7/20 **D.** 3/20

$A \rightarrow \frac{4}{5}$

$B \rightarrow \frac{3}{4}$

$$A\bar{B} + \bar{A}B$$

$$\frac{4}{5} \times \frac{1}{4} + \frac{1}{5} \times \frac{3}{4}$$

$$\frac{7}{20}$$