Quadratic Equations

* Definition

→ If $p(x)$ is a quadratic polynomial, then $\boxed{p(x) = 0}$ is called an Quadratic Equation.

Degree of $p(x) = 2$

Ex: $\boxed{2x^2 + 5x - 6 = 0}$

$\boxed{7x^2 + 3x = 0}$

$\boxed{x^2 = 0}$

Quadratic Equations

* General / Standard form of a Quadratic Equation :-

→ $\boxed{ax^2 + bx + c = 0}$ (in variable 'x')

→ $\boxed{ay^2 + by + c = 0}$ (in variable 'y')

$ax^2 + bx + c = 0$

Standard form of a Quadratic Polynomial

where,

a, b & c → Real Nos.

& $a \neq 0$

Quadratic Equations

* $\boxed{\text{Note}}$:

→ To analyse any Q.E,

a) Simplification ✓.

b) Standard form ✓.

$\because$ QE ⇒ QP = 0

Hence, rules of being a polynomial must be satisfied.

QUESTION

Check whether the following are quadratic equations:

(i) $(x - 2)^2 + 1 = 2x - 3$

$\boxed{Sol^n}$

$\Rightarrow x^2 + 4 - 4x + 1 = 2x - 3$

$\Rightarrow \boxed{x^2 - 6x + 8 = 0}$

QE

QUESTION

Check whether the following are quadratic equations:

(ii) $x(x + 1) + 8 = (x + 2)(x - 2)$

Soln

$\Rightarrow \cancel{x^2} + x + 8 = \cancel{x^2} - 4$

$\Rightarrow \boxed{x + 12 = 0}$

L.E

QUESTION

Check whether the following are quadratic equations:

(iv) $(x + 2)^3 = x^3 - 4$

Sol^n

$\Rightarrow \cancel{x^3} + 8 + 6x^2 + 12x = \cancel{x^3} - 4$

$\Rightarrow 6x^2 + 12x + 12 = 0$

$\Rightarrow 6(x^2 + 2x + 2) = 0$

$\Rightarrow \boxed{x^2 + 2x + 2 = 0}$

QE

QUESTION

Check whether the following are quadratic equations:

(v) $(2x-1)(x-3)=(x+5)(x-1)$

Sol^n

$\Rightarrow 2x^2-6x-x+3=x^2-x+5x-5$

$\Rightarrow 2x^2-7x+3=x^2+4x-5$

$\Rightarrow \boxed{x^2-11x+8=0}$

QE

Quadratic Equations

* **Meaning of Solution / Roots of a Quadratic Equation :-**

→ If '$p(x) = 0$' is a Q.E, then the zeros of the polynomial $p(x)$ are called roots / solⁿ of the Q.E $p(x) = 0$.

→ Hence, if $\boxed{x = \alpha}$ is a root of $p(x) = 0$, then $\boxed{p(\alpha) = 0}$.
ie, the root must satisfy the equation.

→ **Ex:** If 'k' is a root of eqⁿ $\boxed{x^2 - 2x + 1 = 0}$.
Then, ... $\boxed{(k)^2 - 2k + 1 = 0}$

Quadratic Equations

Roots ← Q.E → Q. Polynomial = 0

Zeros

Quadratic Equations

* How to find sol^n / roots of a Quadratic Eq^n ?

Factorisation Method

Quadratic formula

(Shreedharacharya's Rule)

QUESTION

$$6 \to 6 \times 1,\ 2 \times 3$$

Find the roots of the equation $2x^2 - 5x + 3 = 0$. $\xrightarrow{\text{Roots}}$ $1, \frac{3}{2}$

Method - I Splitting / Factorisation

Sol^n

$$2x^2 - 5x + 3 = 0$$

$$\Rightarrow 2x^2 - 2x - 3x + 3 = 0$$

$$\Rightarrow 2x(x-1) - 3(x-1) = 0$$

$$\Rightarrow (x-1)(2x-3) = 0$$

$x - 1 = 0$	$2x - 3 = 0$
$x = 1$	$x = \frac{3}{2}$

QUESTION

Find the roots of the equation $2x^2 - 5x + 3 = 0$.

Method – II   Shreedharacharya's rule / Quadratic formula

$\rightarrow a = 2, b = -5, c = 3$

$D = b^2 - 4ac$ $ax^2 + bx + c = 0$

$= (-5)^2 - (4 \times 2 \times 3)$

$= 25 - 24$

$\boxed{D = 1}$

$x = \dfrac{-b \pm \sqrt{D}}{2a}$

$\rightarrow \dfrac{-b + \sqrt{D}}{2a}$ ✓

$\rightarrow \dfrac{-b - \sqrt{D}}{2a}$ ✓

$\dfrac{5 + \sqrt{1}}{4} = \dfrac{5+1}{4} = \dfrac{\cancel{6}^{3}}{\cancel{4}_{2}} = \dfrac{3}{2}$ ✓

$\dfrac{5 - \sqrt{1}}{4} = \dfrac{5-1}{4} = \dfrac{\cancel{4}}{\cancel{4}} = 1$ ✓

QUESTION

Middle term → root wala

gF

Find the roots of the quadratic equation $3x^2 - 2\sqrt{6}x + 2 = 0$

$\boxed{\text{Sol}^n}$

$3x^2 - 2\sqrt{6}\,x + 2 = 0$

$a = 3,\ b = -2\sqrt{6}$ & $c = 2$

$x = \dfrac{-b \pm \sqrt{D}}{2a}$

$D = b^2 - 4ac$

$= (-2\sqrt{6})^2 - (4 \times 3 \times 2)$

$= 24 - 24$

$= \underline{\underline{0}}$

x — $-\dfrac{b}{2a} \rightarrow \dfrac{\cancel{2}\sqrt{6}}{\cancel{6}\,3} = \dfrac{\sqrt{6}}{3}, \dfrac{\sqrt{6}}{3}$

Roots

$\dfrac{-b + \sqrt{\cancel{D}}}{2a}$ $\quad$ $\dfrac{-b - \sqrt{\cancel{D}}}{2a}$

$\dfrac{-b}{2a}$ $\quad$ $\dfrac{-b}{2a}$

QUESTION

coeff fract → int coeff

Find the roots of the following quadratic equations ~~by factorisation~~

$2x^2 - x + 1/8 = 0$

$\boxed{Sol^n}$

$\frac{2x^2}{1} - \frac{x}{1} + \frac{1}{8} = 0$

$\Rightarrow \frac{16x^2 - 8x + 1}{8} = 0$

$-\frac{b}{2a}$

$\Rightarrow \boxed{16x^2 - 8x + 1 = 0}$

$\frac{\cancel{8}}{\cancel{32}_4}$

$(4x)^2 + (1)^2 - 2 \times (4x) \times (1) = 0$

$\Rightarrow (4x-1)^2 = 0$

$\frac{1}{4}$

$\Rightarrow (4x-1)(4x-1) = 0$

$4x - 1 = 0$ | $4x - 1 = 0$

$x = \frac{1}{4}$ | $x = \frac{1}{4}$

QUESTION

Find the roots of the following quadratic equations ~~by factorisation.~~
$100x^2 - 20x + 1=0$

Sol^n

$100x^2 - 20x + 1 = 0$

$\Rightarrow (10x)^2 + (1)^2 - 2\times(10x)\times(1) = 0$

$\Rightarrow (10x-1)^2 = 0$

$\Rightarrow \boxed{x = \frac{1}{10}, \frac{1}{10}}$

Quadratic Equations

3 cases

✓ Every QE has exactly 2 roots.

✓ Every QE has atmost 2 real roots.

* Nature of Roots :- $\underline{R_1}$, $\underline{R_2}$ | $\underline{R}$, $\underline{R}$ { $\underline{\times}$, $\underline{\times}$

* With the help of Discriminant $\boxed{(D) = b^2 - 4ac}$, without actually solving whole Q.E we can determine how will be roots like....

Real $(D \geq 0)$

Roots

- Real & distinct — $D > 0$ (+)
- Real & equal — $D = 0$ — $-\frac{b}{2a}, -\frac{b}{2a}$
- Non real — $D < 0$ (−)

QUESTION

Find the discriminant of the quadratic equation $2x^2 - 4x + 3 = 0$, and hence find the nature of its roots.

Sol^n

$$2x^2 - 4x + 3 = 0$$

$$\Rightarrow a = 2, b = -4, c = 3$$

$$\Rightarrow D = b^2 - 4ac$$

$$= (-4)^2 - (4 \times 2 \times 3)$$

$$= 16 - 24$$

$$\boxed{D = -8}$$

$D < 0 \rightarrow$ non-real roots

QUESTION

Find the discriminant of the equation 3x² - 2x + 1/3 = 0 and hence find the nature of its roots. Find them, if they are real.

Sol^n

$$3x^2 - 2x + \frac{1}{3} = 0$$

$$a = 3,\ b = -2,\ c = \frac{1}{3}$$

$$D = b^2 - 4ac$$

$$= (-2)^2 - \left(4 \times \cancel{3} \times \frac{1}{\cancel{3}}\right)$$

$$= 4 - 4$$

$$\boxed{D = 0} \longrightarrow \text{Real \& equal} \xrightarrow{\text{roots}} -\frac{b}{2a}$$

$$\Rightarrow \frac{\cancel{2}}{\cancel{6}\,3} = \boxed{\frac{1}{3}, \frac{1}{3}}$$

QUESTION

Find the nature of the roots of the following quadratic equation. If the real roots exist, find them:

$2x^2 - 3x + 5 = 0$

Sol^n $2x^2 - 3x + 5 = 0$

$D = (-3)^2 - (4 \times 2 \times 5)$

$= 9 - 40$

$\boxed{D = -31}$

$D < 0 \longrightarrow$ non real roots

QUESTION

$$\begin{array}{r|l} 2 & 24 \\ \hline 3 & 12 \\ \hline 2 & 6 \\ \hline & 3 \end{array}$$

2

Find the values of k for each of the following quadratic equations, so that they have two equal roots.

(i) $2x^2 + kx + 3 = 0$

$\boxed{Sol^n}$

$D = 0$

$\Rightarrow (k)^2 - (4 \times 2 \times 3) = 0$

$\Rightarrow k^2 - 24 = 0$

$\Rightarrow \boxed{k^2 = 24}$

$\Rightarrow k$

$+\sqrt{24}$ $\quad$ $-\sqrt{24}$

$2\sqrt{6}$ $\quad$ $-2\sqrt{6}$

Check Real & equal roots

$2x^2 + 2\sqrt{6}x + 3 = 0$ equal root

$D = (2\sqrt{6})^2 - (4 \times 2 \times 3)$

$= 24 - 24$

$= \underline{\underline{0}}$

$2x^2 - 2\sqrt{6}x + 3 = 0$

$D = (-2\sqrt{6})^2 - (4 \times 2 \times 3)$

$= 24 - 24$

$= \underline{\underline{0}}$

$x^2 = k$

$x = \pm\sqrt{k}$

$\sqrt{k}$ $\quad$ $-\sqrt{k}$

QUESTION

$\boxed{k = 6}$

Find the values of k for each of the following quadratic equations, so that they have two equal roots.

(ii) $kx(x-2)+6=0$

$$\Rightarrow \boxed{kx^2-2kx+6=0}$$

$$D=0$$

$$\Rightarrow (-2k)^2-(4\times k\times 6)=0$$

$$\Rightarrow 4k^2-24k=0$$

$$\Rightarrow 4k(k-6)=0$$

$$\Rightarrow k(k-6)=0$$

$$\cancel{k=0} \quad | \quad k=6$$

Check

$6=0$ ✗

$$6x^2-12x+6=0$$

$$6(x^2-2x+1)=0$$

$$x^2-2x+1=0$$

Quadratic Equations

* Word Problems

- Linear Eq^n Approach
- Q.E approach

QUESTION

Represent the following situations mathematically:

(i) John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. We would like to find out how many marbles they had to start with.

2	324
2	162
3	81
3	27
3	9
	3

36 8 9

Sol^n

John $\longrightarrow$ x marbles $\longrightarrow$ $(x-5)$

Jivant $\longrightarrow$ y marbles $\longrightarrow$ $(y-5)$

36 & 9

$$\boxed{x+y=45}$$

$$\boxed{(x-5)(y-5)=124}$$

$$x=(45-y)$$

$$(45-y-5)(y-5)=124$$

$$\Rightarrow (40-y)(y-5)=124$$

$$\Rightarrow 40y-200-y^2+5y=124$$

$$\Rightarrow 45y-y^2-324=0$$

$$\Rightarrow \boxed{y^2-45y+324=0}$$

$$y^2-45y+324=0$$

$$y^2-36y-9y+324=0$$

$$y(y-36)-9(y-36)=0$$

$$(y-36)(y-9)=0$$

$y=36$, $x=9$ | $y=9$, $x=36$

QUESTION

25, 30

A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was Rs. 750. We would like to find out the number of toys produced on that day.

Sol^n

No. of toys produced in a day = x

Price of each toy (in Rs) = y

$y = x - 55$ ✗

$\boxed{y = (55 - x)}$ ✓

T.C = 750

$\boxed{xy = 750}$

$x \cdot (55 - x) = 750$

$\Rightarrow 55x - x^2 = 750$

$\Rightarrow \boxed{x^2 - 55x + 750 = 0}$

$\Rightarrow x^2 - 25x - 30x + 750 = 0$

$\Rightarrow x(x-25) - 30(x-25) = 0$

$\Rightarrow (x-25)(x-30) = 0$

$x = 25$, $y = 30$

$x = 30$, $y = 25$

QUESTION

33 & 32

The area of a rectangular plot is 528 m^2. The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.

$\underline{Sol^n}$

Length $= x$ m

Breadth $= y$ m

$\Rightarrow \boxed{x \cdot y = 528}$

$\Rightarrow \boxed{x = 2y + 1}$

$\Rightarrow y(2y+1) = 528$

$\Rightarrow \boxed{2y^2 + y - 528 = 0}$

$2y^2 + 33y - 32y - 528 = 0$

$y(2y+33) - 16(2y+33) = 0$

$(2y+33)(y-16) = 0$

$\cancel{y = -\frac{33}{2}}$ $\quad$ $y = 16$ m

$x = 33$ m

Ar $= 528$ m^2

2	528
2	264
2	132
2	66
3	33
	11

QUESTION

The product of two consecutive positive integers is 306. We need to find the integers.

Sol^n

S: x L: $x+1$

$x\cdot(x+1) = 306$

$\Rightarrow \boxed{x^2 + x - 306 = 0}$

$\Rightarrow x^2 + 18x - 17x - 306 = 0$

$\Rightarrow x(x+18) - 17(x+18) = 0$

$\Rightarrow (x+18)(x-17) = 0$

~~$x = -18$~~

$x = 17$

$x + 1 = 18$

2	306
3	153
3	51
	17

QUESTION

Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age. → 7 yrs

Present

Future

$$\begin{array}{r} 360 \\ -87 \\ \hline 273 \end{array}$$

$$\begin{array}{r|l} 3 & 273 \\ \hline 7 & 91 \\ \hline & 13 \end{array}$$

39 & 7

	Past	Present	Future (+3)
Rohan		x	$x+3$
Mom		y	$y+3$

Sol^n

$$\boxed{y - x = 26} \rightarrow y = 26 + x$$

$$\boxed{(x+3)(y+3) = 360}$$

$$\Rightarrow (x+3)(26+x+3) = 360$$

$$\Rightarrow (x+3)(x+29) = 360$$

$$\Rightarrow x^2 + 29x + 3x + 87 - 360 = 0$$

$$\Rightarrow \boxed{x^2 + 32x - 273 = 0}$$

$$x^2 + 39x - 7x - 273 = 0$$

$$\Rightarrow x(x+39) - 7(x+39) = 0$$

$$\Rightarrow (x+39)(x-7) = 0$$

$$\cancel{x = -39} \qquad x = 7 \text{ yrs}$$

$$y = 33 \text{ yrs}$$

$$10 \times 36$$

QUESTION

120
96

A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train.

Originally, Speed → x kmph → 40 kmph
Time → y hr

480 km

New, S → $(x-8)$ kmph
T → $(y+3)$ hr

$\frac{32}{96}$

Sol^n

2, 2, 2 → 3

$$\begin{array}{r|l} 2 & 480 \\ \hline 2 & 240 \\ \hline 2 & 120 \\ \hline 2 & 60 \\ \hline 2 & 30 \\ \hline 3 & 15 \\ \hline & 5 \end{array}$$

$$\boxed{xy = 480} \rightarrow y = \frac{480}{x}$$

$$\boxed{(x-8)(y+3) = 480}$$

$$(x-8)\left(\frac{480}{x}+3\right) = 480$$

$$\Rightarrow \cancel{480} + 3x - \frac{8\times480}{x} - 24 = \cancel{480}$$

$$\Rightarrow \frac{3x^2-(8\times480)-24x}{x} = 0$$

$$\Rightarrow \boxed{3x^2-24x-(8\times480)=0}$$

$$3x^2-120x+96x-(8\times480)=0$$

$$3x(x-40)+96(x-40)=0$$

$$(x-40)(3x+96)=0$$

$$x=40 \qquad \cancel{x=-\frac{96}{3}}$$

QUESTION

14, 13

Find two numbers whose sum is 27 and product is 182.

Sol^n No$_1 \to x$

No$_2 \to y$

$\boxed{x+y=27} \to x = 27-y$

$\boxed{xy=182}$

$y(27-y)=182$

$\Rightarrow 27y-y^2=182$

$\Rightarrow \boxed{y^2-27y+182=0}$

$y^2-14y-13y+182=0$

$y(y-14)-13(y-14)=0$

$\Rightarrow (y-14)(y-13)=0$

$y=14$	$y=13$
$x=13$	$x=14$

$$\begin{array}{r|l} 2 & 182 \\ \hline 7 & 91 \\ \hline & 13 \end{array}$$

QUESTION

Find two consecutive positive integers, sum of whose squares is 365.

QUESTION

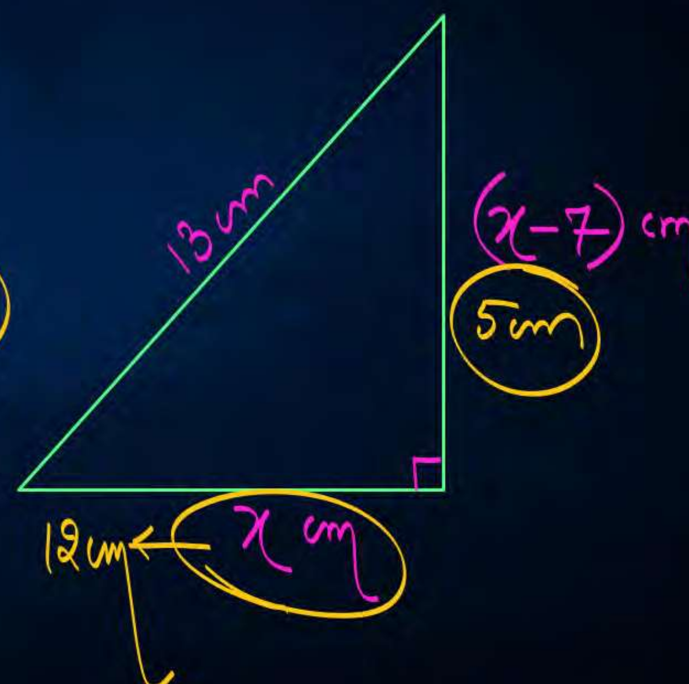
The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

Sol^n

$$x^2+(x-7)^2=169$$
$$\Rightarrow x^2+x^2+49-14x=169$$
$$\Rightarrow 2x^2-14x-120=0$$
$$\Rightarrow \boxed{x^2-7x-60=0}$$
$$\Rightarrow x^2-12x+5x-60=0$$
$$\Rightarrow x(x-12)+5(x-12)=0$$
$$(x-12)(x+5)=0$$

2	60
2	30
3	15
	5

$x=12$ ✓ $\cancel{x=-5}$



QUESTION

Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is 800 m^2? If so, find its length and breadth.

Sol^n Length $= x$ m

Breadth $= y$ m

$\boxed{x \cdot y = 800}$

$\boxed{x = 2y}$

$\boxed{y^2 - 400 = 0}$

$y \times 2y = 800$

$y^2 = 400$

$y = 20$ m, ~~-20 m~~

$x = 40$ m

$D = 0 - (4 \times 1 \times -400)$

$= 1600$

$D > 0$