CHAPTER

1 Electric Charges and Fields

Electric Charge

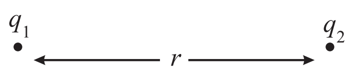
Charge is a property of matter due to which two bodies interact with each other electromagnetically. There are two kinds of charges- positive and negative. SI unit is coulomb. Charge is quantized and additive.

Coulomb's Law

Force between two charges

$$\vec{F} = \frac{1}{4\pi \in_0 \in_r} \frac{q_1 q_2}{r^2} \hat{r}$$

$\in_r$ = dielectric constant of the medium, $\in_0$ = permittivity of free space



❖ Coulomb's Law is applicable only for static point charges.

Principle of Superposition

Force on a point charge due to many charges is given by

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ...$$

❖ The force due to one charge is not affected by the presence of other charges.

Electric Field or Electric Field Intensity

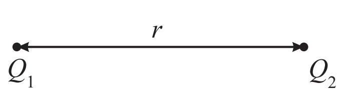
$$\vec{E} = \frac{\vec{F}}{q}$$

Unit is N/C or V/m.

Electric Field due to Point Charge Q

$$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0} = \frac{1}{4\pi \in_0} \frac{Q}{r^2} \hat{r}$$

Null Point for Two Charges



If $|Q_1| > |Q_2|$

$\Rightarrow$ Null point is m ear Q_2

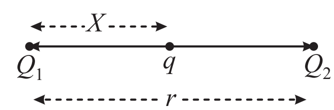
$$x = \frac{\sqrt{Q_1}\, r}{\sqrt{Q_1} \pm \sqrt{Q_2}}$$

$x \rightarrow$ distance of null point from Q_1 charge

(+) for like charges [null point will be in between Q_1 & Q_2]

μ_s (–) for unlike charges [null point will be outside Q_1 & Q_2 and near weaker charge]

Equilibrium of Three Point Charges



(i) Q_1 & Q_2 must be of like nature.

(ii) q should be of unlike nature.

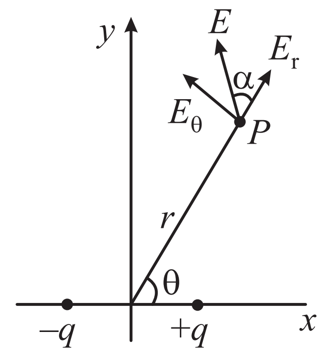
$$x = \frac{\sqrt{Q_1}}{\sqrt{Q_1} + \sqrt{Q_2}} r \text{ and } q = \frac{-Q_1 Q_2}{\left(\sqrt{Q_1} + \sqrt{Q_2}\right)^2}$$

Electric Dipole

❖ Electric dipole moment $\vec{p} = q\vec{d}$, where $\vec{d}$ is distance from negative to positive charge.

❖ Torque on dipole placed in a uniform electric field $\vec{\tau} = \vec{p} \times \vec{E}$

❖ Electric field at a general point due to a dipole.



Electric field: $E = \frac{1}{4\pi \in_0} \frac{p\sqrt{1 + 3\cos^2\theta}}{r^3}$

Direction: $\tan\alpha = \frac{E_\theta}{E_r} = \frac{1}{2}\tan\theta$

❖ Electric field at an axial point (or End-on) of dipole

$$\vec{E} = \frac{1}{4\pi \in_0} \frac{2\vec{p}}{r^3}$$

❖ Electric field at an equatorial position (Broad-on) of dipole

$$\vec{E} = \frac{1}{4\pi \in_0} \frac{(-\vec{p})}{r^3}$$

Electric Lines of Force

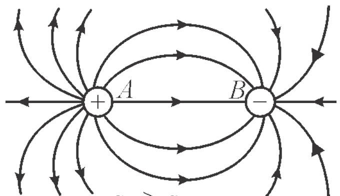
Electric lines of electrostatic field have following properties.

(i) Imaginary

(ii) Can never cross each other

(iii) Can never form closed loops

(iv) The number of lines originating or terminating on a charge is proportional to the magnitude of charge.



$q_A > q_B$

(v) Lines of force end or start normally at the surface of a con ductor.

(vi) If there is no electric field there will be no lines of force.

(vii) Lines of force per unit area normal to the area at a point represents magnitude of intensity. Crowded lines represent strong field while distant lines represent weak field.

(viii) Tangent to the line of force at a point in an electric field gives the direction of Electric Field.

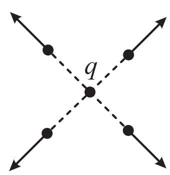
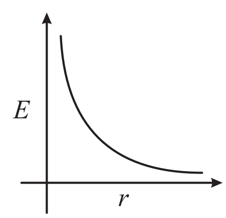
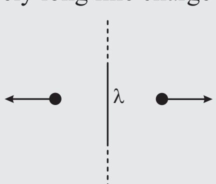
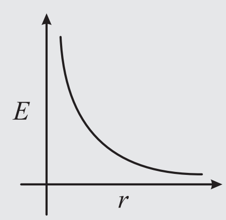
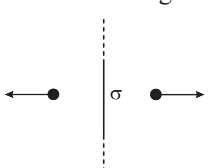
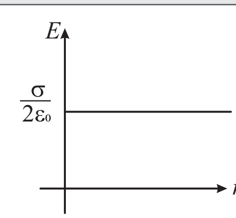
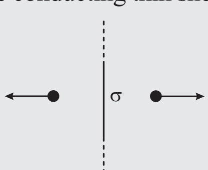
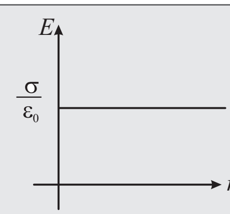
Gauss' Law

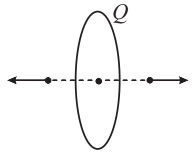
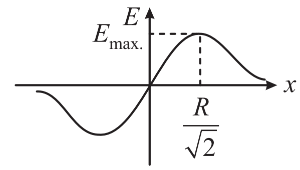
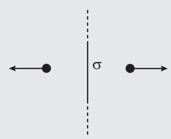
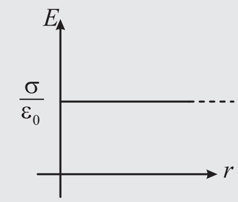
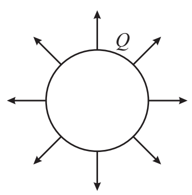
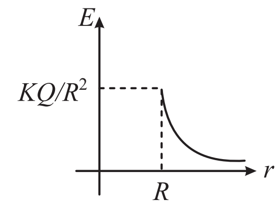
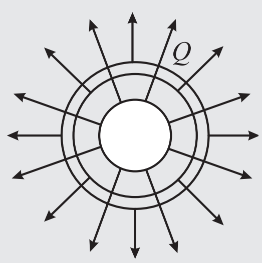
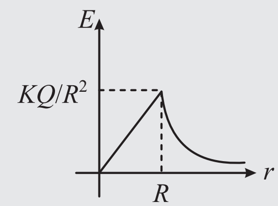
Electric flux: $\phi = \int \vec{E}.\vec{ds}$

Expression for Gauss' Law: $\oint \vec{E}.\vec{ds} = \frac{\sum q_{\text{enlosed}}}{\in_0}$

(Applicable only on closed surface)

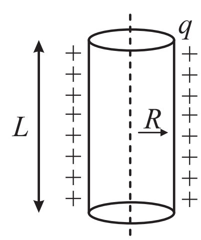
Net flux emerging out of a closed surface is $\frac{q_{en}}{\varepsilon_0}$

Name/Type	Formula	Note	Graph
Point charge 	$\vec{E} = \frac{kq}{\lvert \vec{r} \rvert^2}\hat{r}$	❖ q is source charge. ❖ $\vec{\ }$ is vector drawn from source charge to the point. ❖ Outwards due to + ve charges and inwards due to – ve charges.	
Infinitely long line charge 	$\frac{\lambda}{2\pi\varepsilon_0 r}\hat{r}$ or $\frac{2k\lambda}{r}\hat{r}$	❖ λ is linear charge density (assumed uniform) ❖ r is perpendicular distance of point from line charge. ❖ $\hat{r}$ is radial unit vector drawn from the line charge to the point.	
Infinite non-conducting thin sheet 	$\frac{\sigma}{2\varepsilon_0}\hat{n}$	❖ σ is surface charge density. (assumed uniform) ❖ $\hat{n}$ is unit normal vector	
Infinite conducting thin sheet 	$\frac{\sigma}{\varepsilon_0}\hat{n}$	❖ σ is surface charge density. (assumed uniform) ❖ $\hat{n}$ is unit normal vector	

Uniformly charged ring 	$E = \dfrac{kQx}{\left(R^2 + x^2\right)^{3/2}}$ $E_{centre} = 0$	❖ Q is total charge of the ring ❖ x = distance of point on the axis from center of the ring. ❖ Electric field is always along the axis. (away from ring if Q is +ve, towards ring if Q is –ve.)	
Infinitely large charged conducting sheet 	$\dfrac{\sigma}{\varepsilon_0}\hat{n}$	❖ σ is the surface charge density (assumed uniform) ❖ $\hat{n}$ is the unit vector perpendicular to the sheet.	
Uniformly charged hollow conducting/non-conducting/ solid conducting sphere 	(i) for $r \geq R$ $\vec{E} = \dfrac{kQ}{\lvert\vec{r}\rvert^2}\hat{r}$ (ii) For r < R $E = 0$	❖ R is radius of the sphere. ❖ $\vec{r}$ is vector drawn from center of sphere to the point. ❖ Sphere acts like a point charge placed at center for points outside the sphere. ❖ $\vec{E}$ is always along radial direction. ❖ Q is total charge (= $\sigma 4\pi R^2$). (σ = surface charge density)	
Uniformly charged solid non-conducting sphere (insulating material) 	(i) for $r \geq R$ $\vec{E} = \dfrac{kQ}{\lvert\vec{r}\rvert^2}\hat{r}$ (ii) for $r \leq R$ $\vec{E} = \dfrac{kQ\lvert\vec{r}\rvert}{R^3}\hat{r}$	❖ $\vec{r}$ is vector drawn from center of sphere to the point ❖ Sphere acts like a point charge placed at the center of points outside the sphere ❖ $\vec{E}$ is always along radial direction ❖ Q is total charge $\left(\rho \cdot \dfrac{4}{3}\pi R^3\right)$. (ρ = volume charge density) ❖ Inside the sphere $E \propto r$. ❖ Outside the sphere $E \propto 1/r^2$.	

For a charged Long Conducting Cylinder of length L

- For $r \geq R : E = \dfrac{q}{2\pi \in_0 rL}$
- For $r < R : E = 0$



- Electric field intensity at a point near a charged conductor $E = \dfrac{\sigma}{\in_0}$
- Electrostatic pressure on a charged conductor, $P = \dfrac{\sigma^2}{2\in_0}$
- Energy density in electric field $U = \dfrac{\in_0}{2}E^2$