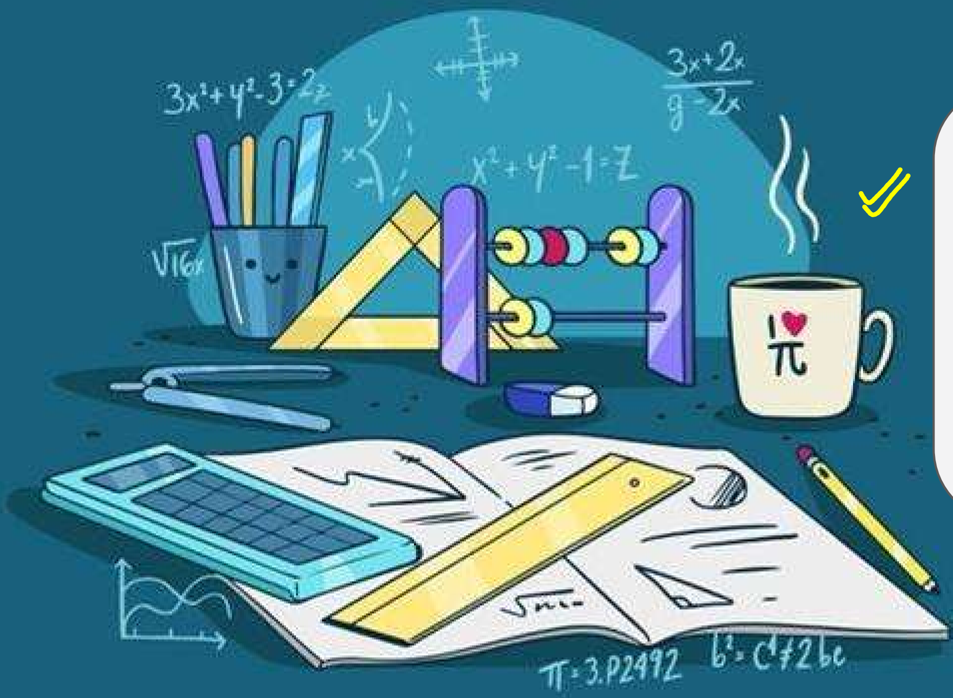
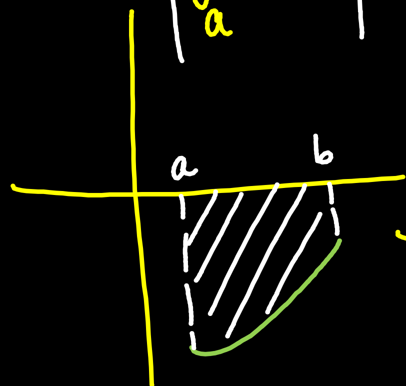
Unacademy Atoms

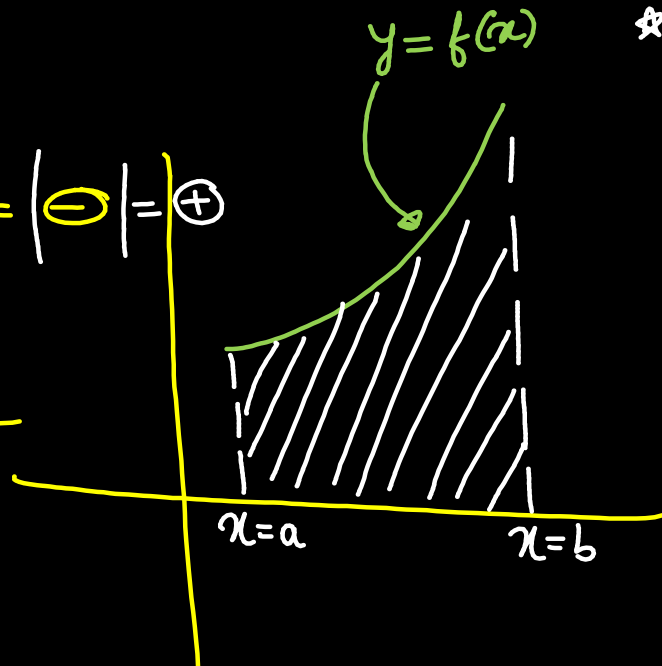


Area Under the Curve

Area under the curve

$$\left|\int_a^b y\,dx\right| = |\ominus| = \oplus$$





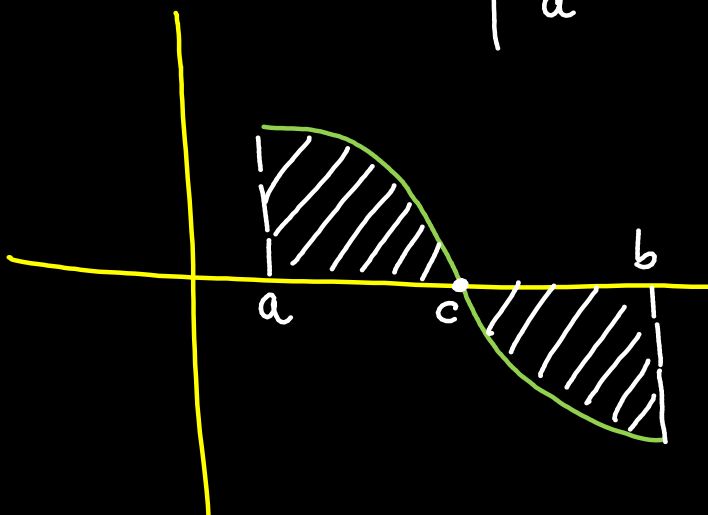
* Area is always $\oplus$
* Area is always added.

<u>w.r.t x axis</u>

$$\left|\int_a^b y\,dx\right| \Rightarrow \oplus$$

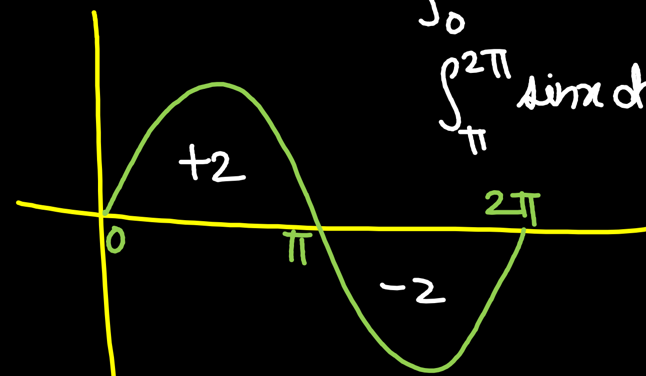
Area under the curve

$$\text{Area} = \left|\int_a^c y\,dx\right| + \left|\int_c^b y\,dx\right|$$



$$\int_0^{\pi} \sin x\,dx = 2$$

$$\int_{\pi}^{2\pi} \sin x\,dx = -2$$

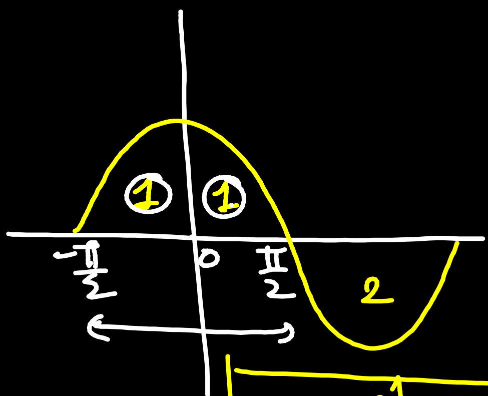


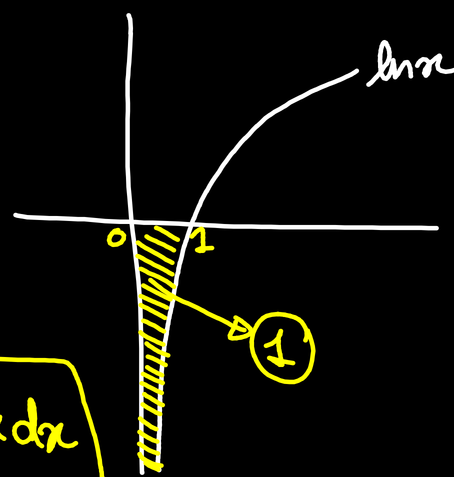
Area from 0 to 2π → 4

$$\int_0^{2\pi} \sin x\,dx = 0$$

Area under the curve

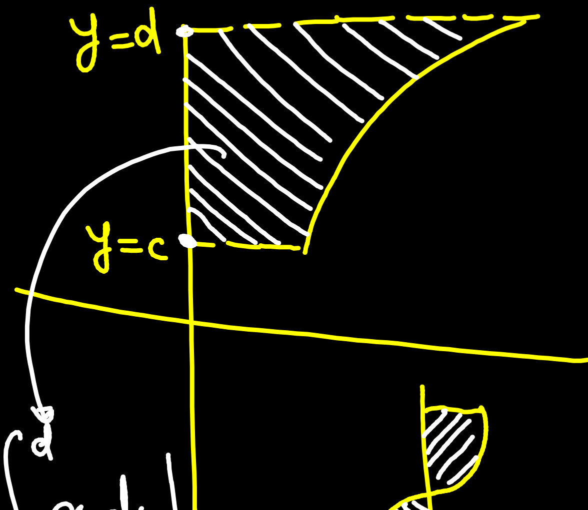
$$\int \ln x\,dx = x\ln x - x$$



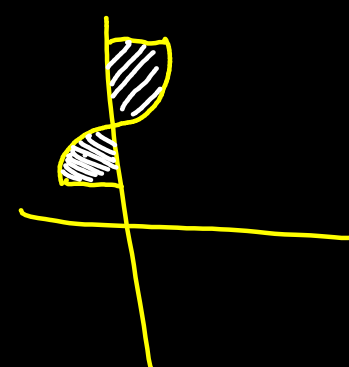


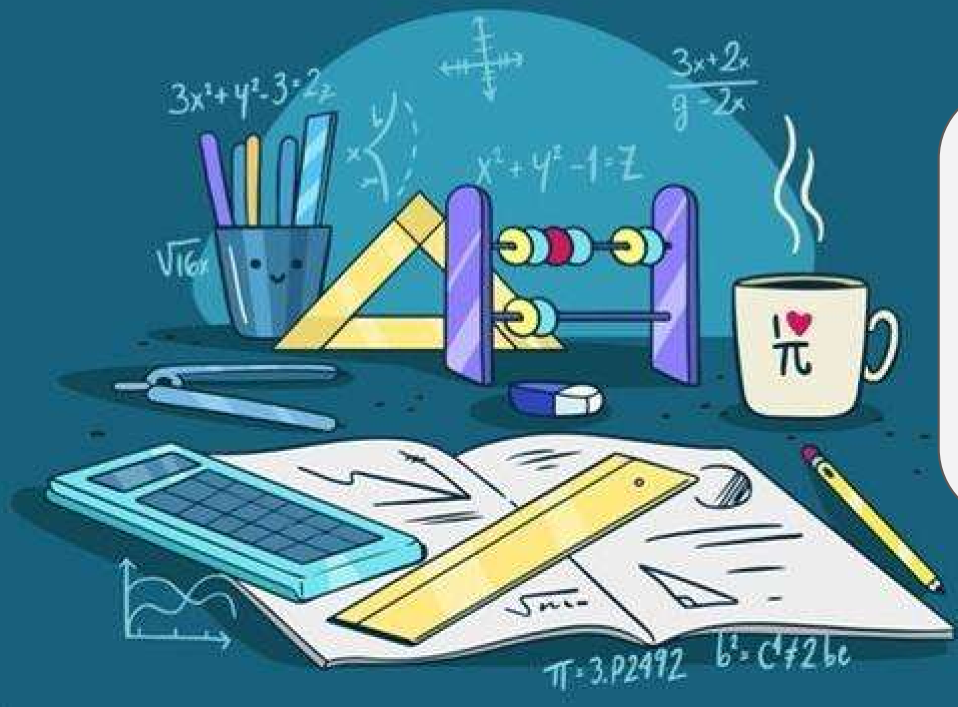
$$\boxed{-1 = \int_0^1 \ln x\,dx}$$

$$\boxed{1 = A}$$



$$\left|\int_c^d x\,dy\right|$$



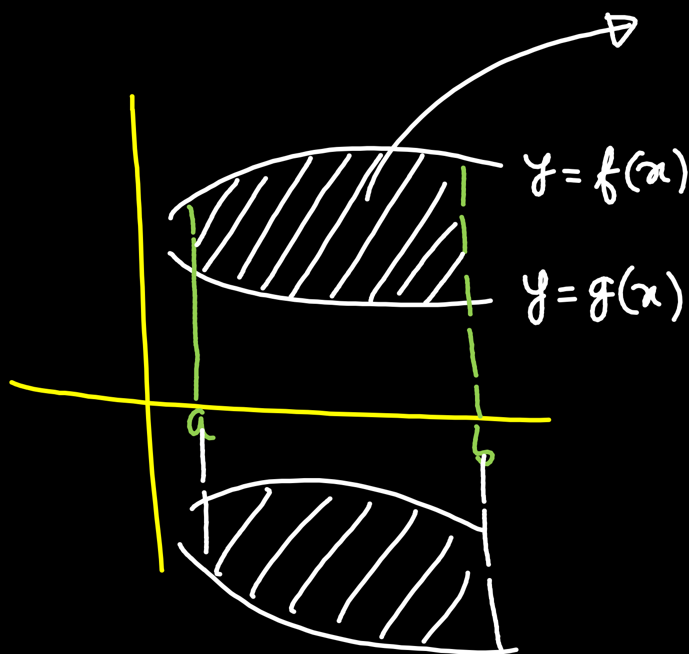


Area Between Two Curves

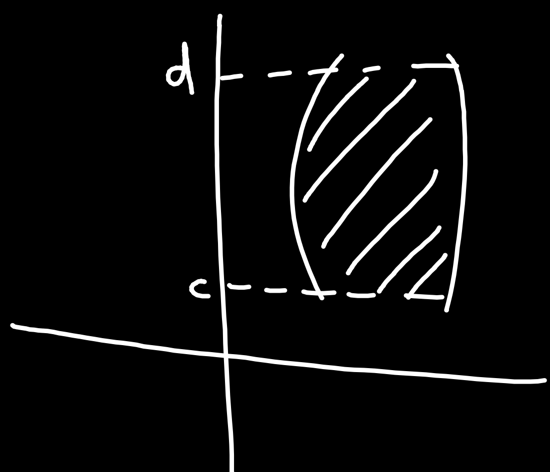
Area ~~under~~ the curve

between

$$\int_a^b \left(y_{upper} - y_{Lower} \right) dx$$



Area between the curve



$$\int_c^d (x_{Right} - x_{Left})\, dy$$

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Q The area (in sq. units) of the region enclosed between the parabola $y^2 = 2x$ and the line $x + y = 4$ is ______.

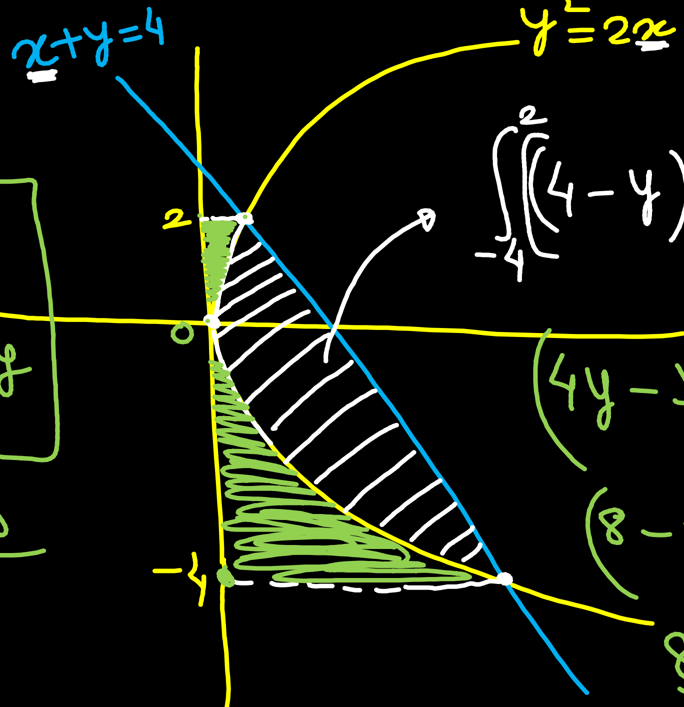
$\frac{2\cancel{4}\times4\times4}{\cancel{8}\,3}$

$y^2 = 2(4-y)$

$y^2 + 2y - 8 = 0$

$(y+4)(y-2) = 0$

$y = -4, 2$



$\int_{-4}^{2}\left[(4-y)-\frac{y^2}{2}\right]dy$

$\left(4y-\frac{y^2}{2}-\frac{y^3}{6}\right)_{-4}^{2}$

$\left(8-2-\frac{8}{6}\right)-\left(-16-8+\frac{32}{3}\right)$

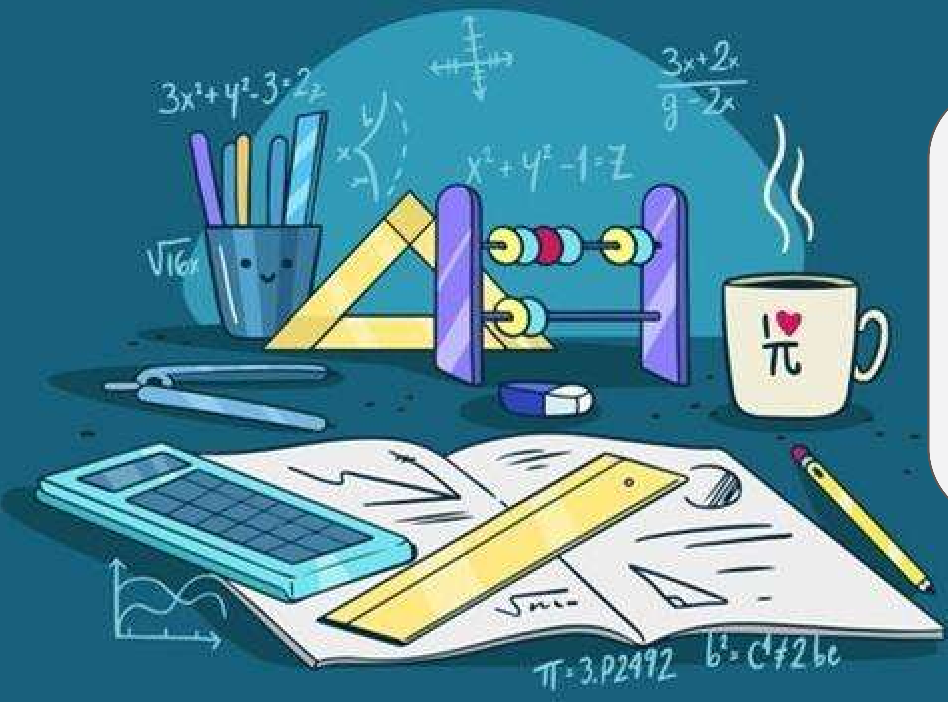
$8-2+16+8-\frac{4}{3}-\frac{32}{3}$

$30-12=18$

□ABCD

$-\int_{-4}^{2}\frac{y^2}{2}\,dy$

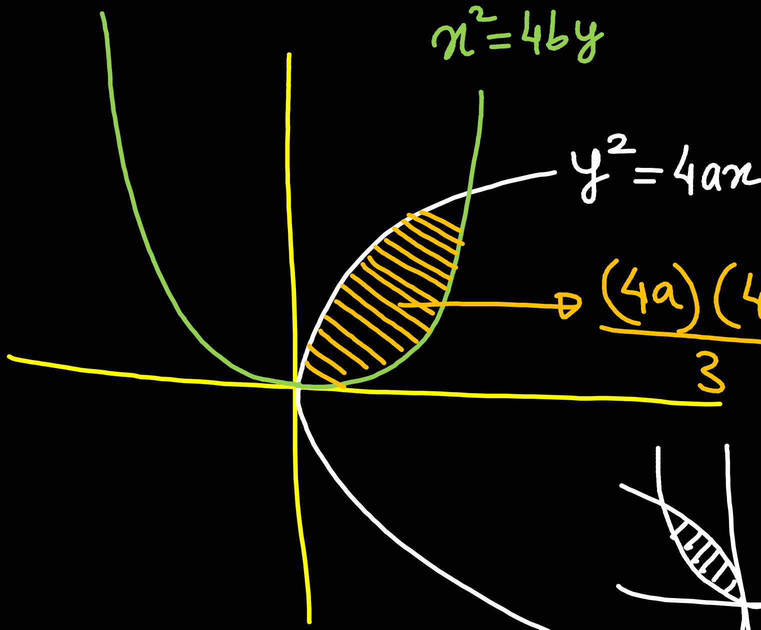
18: Ans



Shortcuts
#NVStyle

Shortcuts #NVStyle

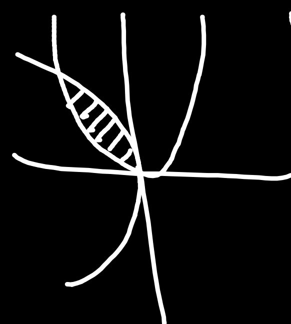
SC1



$$\frac{(4a)(4b)}{3} = \frac{16ab}{3}$$

$$y^2=(12)x$$
$$x^2=(16)y$$

$$A = \frac{(12)(16)}{3}$$



$$y^2=\pm 12x$$
$$x^2=\pm 16y$$

Shortcuts #NVStyle

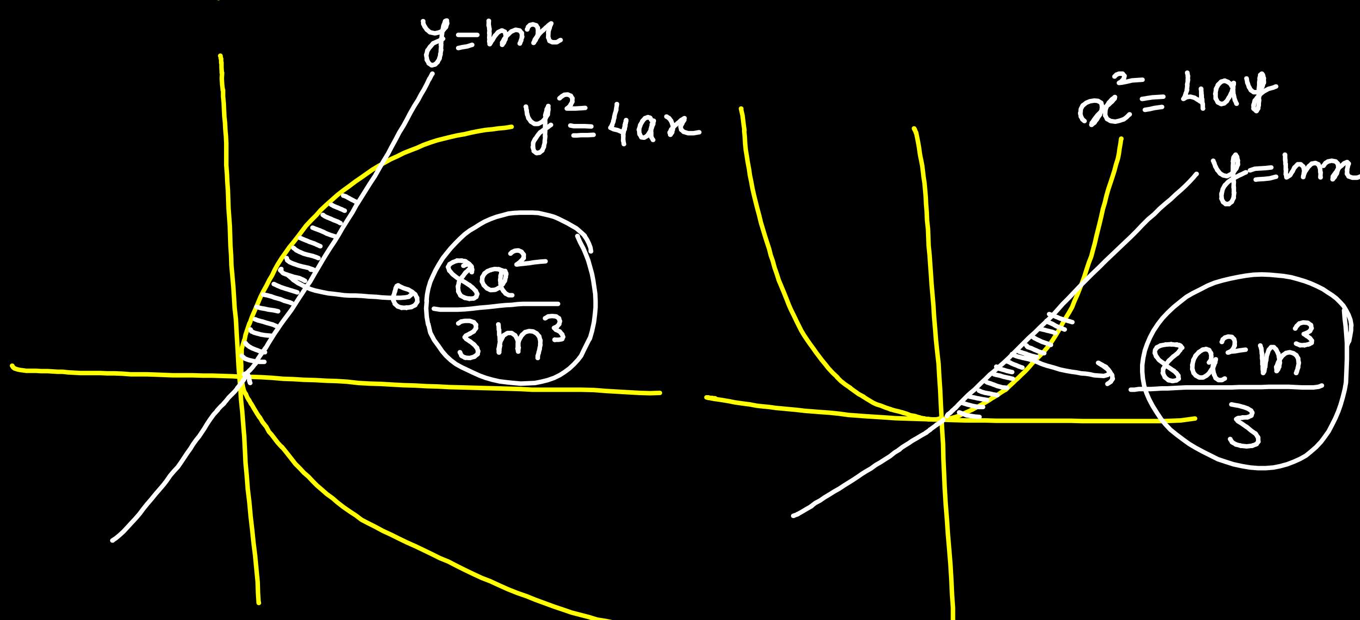
$$(y-3)^2 = 12(x-2) \longrightarrow (2,3)$$

$$(x-2)^2 = 16(y-3) \longrightarrow (2,3)$$

$$\boxed{A = \frac{(12)(16)}{3} = 64}$$

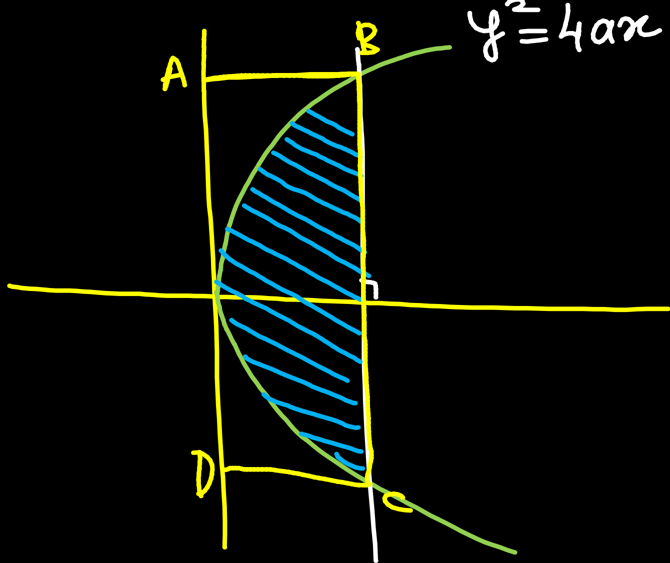
Shortcuts #NVStyle

#SC2

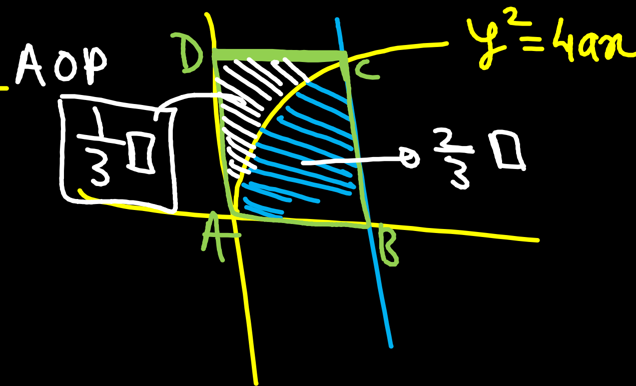


Shortcuts #NVStyle

#SC3



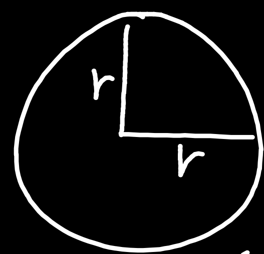
$$\boxed{\text{BLUE} = \frac{2}{3} \times \square ABCD}$$



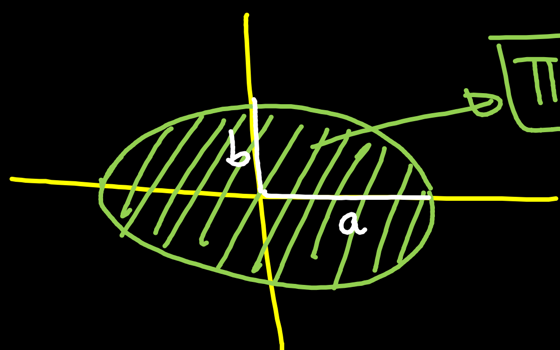
Shortcuts #NVStyle

#SC4

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



$\pi(r)(r)$



$\boxed{\pi ab}$

Shortcuts #NVStyle

Shortcuts #NVStyle

Q Let S be the region bounded by the curves $y = x^3$ and $y^2 = x$. The curve $y = 2|x|$ divides S into two regions of areas R_1 and R_2.

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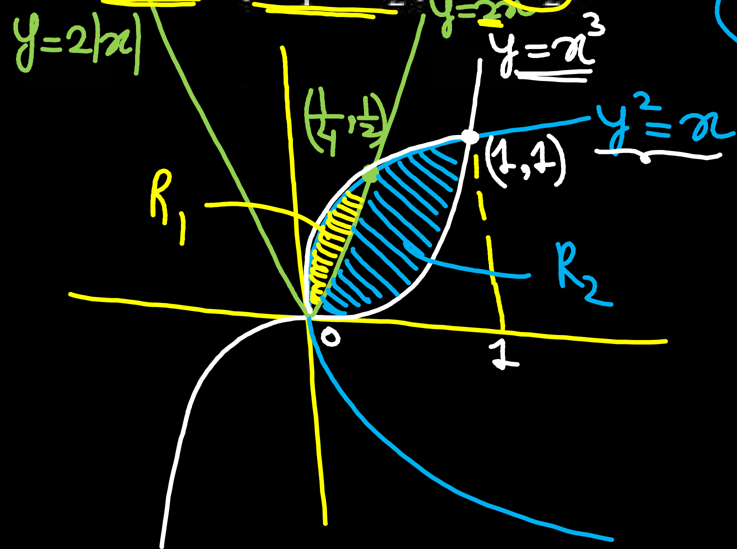
If $\max\{R_1, R_2\} = R_2$, then $\dfrac{R_2}{R_1}$ is equal to 19.



$\left.\begin{array}{l} y = 2x \\ y^2 = x \end{array}\right\}$ Solve

$4x^2 = x$

$\therefore \boxed{x = \frac{1}{4}}$ $\boxed{y = \frac{1}{2}}$

#NVTip

$$Y = \frac{8\left(\frac{1}{4}\right)\left(\frac{1}{4}\right)}{3\cancel{(2)}\cancel{(2)}\cancel{(2)}} = \boxed{\frac{1}{48}} = R_1$$

$$Ar(S) = \int_0^1 (y_u - y_L)\,dx$$

$$= \int_0^1 (\sqrt{x} - x^3)\,dx$$

$$= \left(\frac{2x^{3/2}}{3} - \frac{x^4}{4}\right)_0^1$$

$$= \frac{2}{3} - \frac{1}{4} = \frac{5\times4}{12\times4} = \frac{20}{48}$$

$$BLUE = \frac{20}{48} - \frac{1}{48}$$

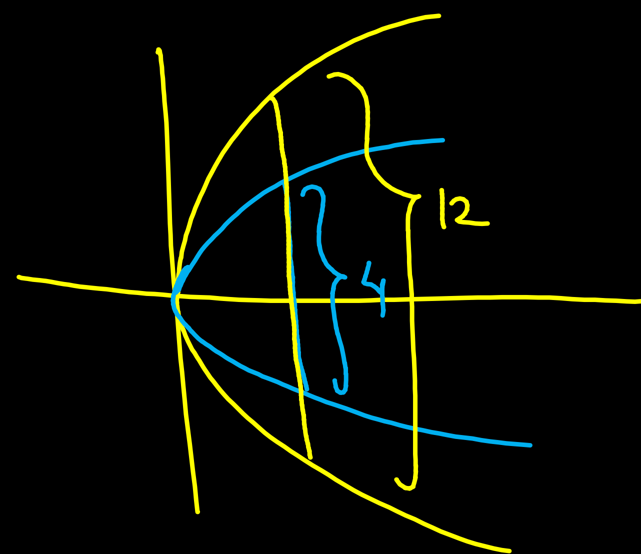
$$= \frac{19}{48} = R_2$$

Concept $LR = 4a$

$y^2 = \underline{12}x \qquad LR = 12$

$y^2 = \underline{4}x \qquad \boxed{LR = 4}$

$LR \uparrow$ para $\uparrow$



Q

The area of the region enclosed between the parabolas $y^2 = 2x - 1$ and $y^2 = 4x - 3$ is

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A. $\frac{1}{3}$

B. $\frac{1}{6}$

C. $\frac{2}{3}$

D. $\frac{3}{4}$

$x = 1 \quad y^2 = 1$

$y = \pm 1$

$2x - 1 = 4x - 3$

$2 = 2x$

$x = 1$

$x = \frac{y^2+1}{2}$

$y^2 = 2\left(x - \frac{1}{2}\right) \rightarrow V\left(\frac{1}{2}, 0\right)$

$y^2 = 4\left(x - \frac{3}{4}\right) \rightarrow V\left(\frac{3}{4}, 0\right)$

$y^2 = 4x - 3$

$\frac{y^2+3}{4} = x$

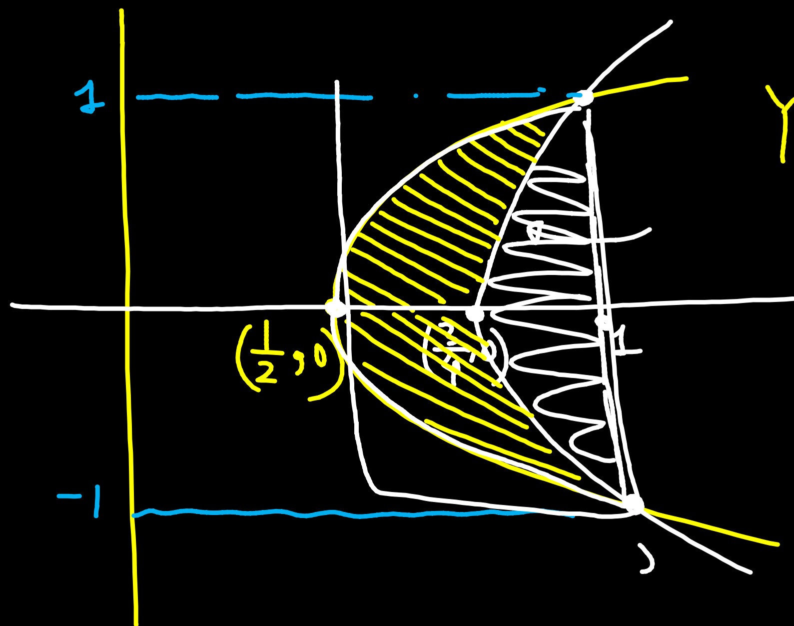
$$\frac{2}{3}\times(1)-\frac{2}{3}\left(\frac{1}{4}\times 2\right)$$

$$\text{GREEN}=\frac{2}{3}\times\frac{1}{4}\times 2=\frac{1}{3}$$

$$Y+G=\frac{2}{3}\times 2\times\frac{1}{2}=\frac{2}{3}$$

$$Y=\frac{2}{3}-\frac{1}{3}$$

$$Y=1/3$$



$$\int_{-1}^{1}(x_R - x_L)\,dy$$

$$\Rightarrow \int_{-1}^{1}\left(\frac{y^2+3}{4} - \frac{y^2+1}{2}\right)dy = \frac{1}{3}$$

Q The area of the region bounded by $y^2 = 8x$ and $y^2 = 16(3 - x)$ is equal to :-

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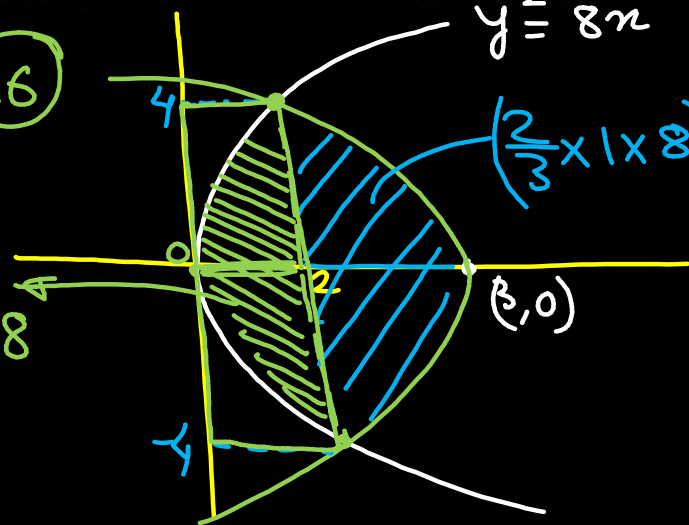
$y^2 = 16$

A. $\frac{32}{3}$ **B.** $\frac{40}{3}$ **C.** 16 **D.** 19

$\frac{48}{3} = 16$



$\left(\frac{2}{3} \times 1 \times 8\right) = \frac{16}{3}$

$\frac{2}{3} \times 2 \times 8$

$= \frac{32}{3}$

$\underbrace{y^2 = -16(x-3)}_{(3,0)}$

$-\cancel{16}^{\,2}(x-3) = \cancel{8}x$

$-2x + 6 = x$

$6 = 3x$

$x = 2$

The area of the region

$S = \{(x, y) : y^2 \leq 8x, y \geq \sqrt{2}x, x \geq 1\}$ is

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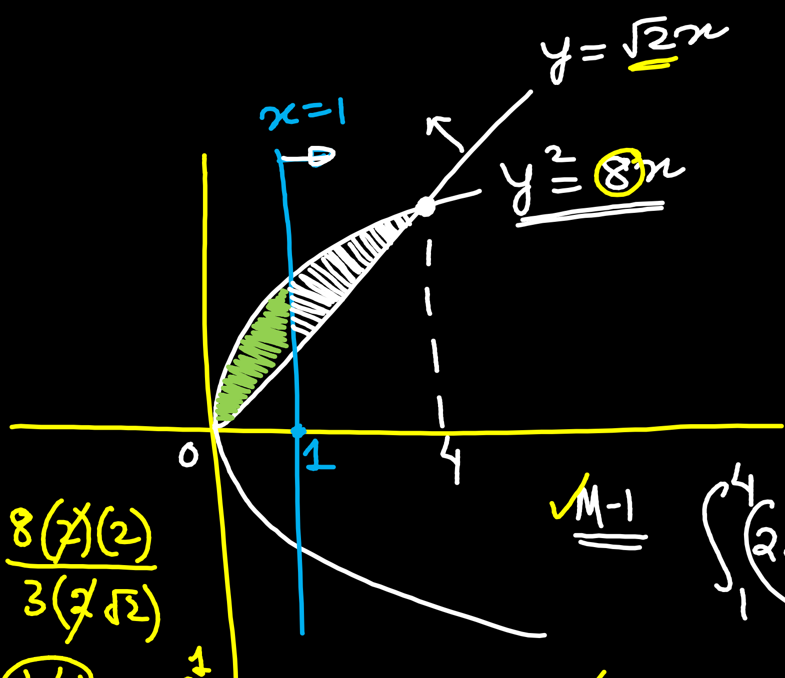
A. $\frac{13\sqrt{2}}{6}$

$y^2 = 8x$
$y = \sqrt{2}x$
$x = 1$
Plot

B. $\frac{11\sqrt{2}}{6}$

C. $\frac{5\sqrt{2}}{6}$

D. $\frac{19\sqrt{2}}{6}$



$$2x^2 = 8x$$
$$x = 4$$

$$y^2 \le 8x \qquad y^2 - 8x \le 0$$
$$y \ge \sqrt{2}\,x \qquad S_1 \le 0$$
$$x \ge 1$$

✓ M-1 $\displaystyle\int_1^4 \left(2\sqrt{2}\sqrt{x} - \sqrt{2}\,x\right)dx = $ ✓

$$W = \frac{8\cancel{(2)}(2)}{3(\cancel{2}\sqrt{2})}$$

$$\frac{16}{3\sqrt{2}} - \underbrace{\int_0^1 \left(2\sqrt{2}\sqrt{x} - \sqrt{2}\,x\right)dx}$$

✓ M-2

Q

The area enclosed by $y^2 = 8x$ and $y = \sqrt{2}\,x$ that lies outside the triangle formed by $y = \sqrt{2}x, x = 1, y = 2\sqrt{2}$, is equal to :

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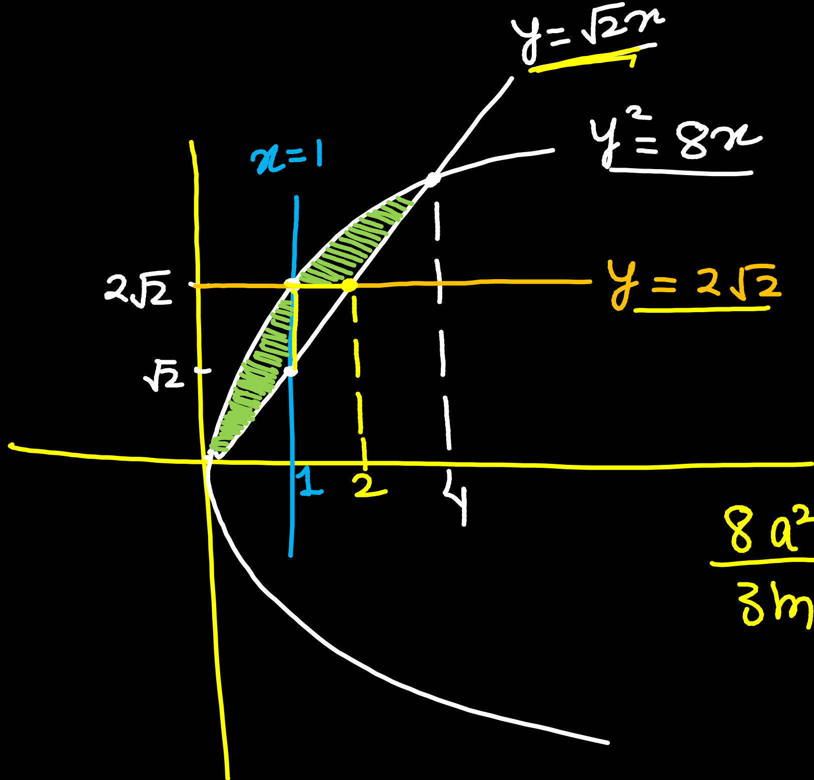
A. $\frac{16\sqrt{2}}{6}$

B. $\frac{11\sqrt{2}}{6}$

C. $\frac{13\sqrt{2}}{6}$

D. $\frac{5\sqrt{2}}{6}$

$$2\cancel{\sqrt{2}} = \cancel{\sqrt{2}}\,x$$



$$\frac{16}{3\sqrt{2}} - \frac{1}{2}\times 1\times\sqrt{2} = \frac{16}{3\sqrt{2}} - \frac{1}{\sqrt{2}}$$

$$= \frac{13}{3\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$$

$$\frac{8a^2}{3m^3} = \frac{8(\cancel{2})(2)}{3(\cancel{2}\sqrt{2})} = \frac{16}{3\sqrt{2}}$$

$$= \frac{13\sqrt{2}}{6}$$

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Q

For real numbers a,b (a > b > 0), let

$$Area\left\{(x,y): \underbrace{x^2+y^2 \le a^2}_{\text{in}} \text{and} \underbrace{\frac{x^2}{a^2}+\frac{y^2}{b^2} \ge 1}_{\text{out}}\right\} = 30\pi$$

and

$$Area\left\{(x,y): \underbrace{x^2+y^2 \ge b^2}^{\text{out}} \text{and} \underbrace{\frac{x^2}{a^2}+\frac{y^2}{b^2} \le 1}^{\text{in}}\right\} = 18\pi$$

Then the value of (a–b)^2 is equal to 12

$$\cancel{\pi} a^2 - \cancel{\pi} ab = 30\cancel{\pi}$$

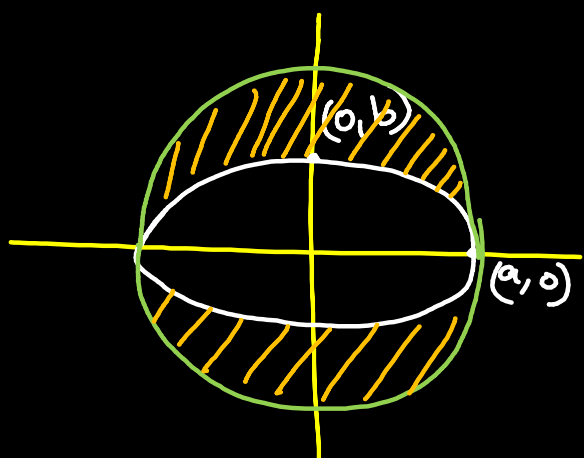
$$a^2 - ab = 30$$

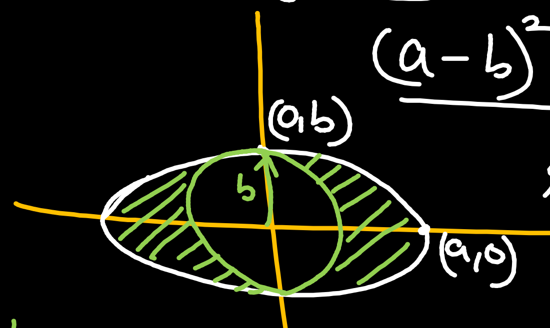
$$-ab + b^2 = -18$$

$$a^2 - 2ab + b^2 = 12$$

$$(a-b)^2 = 12$$

Ans : 12





$$\cancel{\pi} ab - \cancel{\pi} b^2 = 18\cancel{\pi}$$

Q

The area of the region given by

$y = x + 2$ $\quad$ $y = 4 - 3x$

$A = \{(x, y) : x^2 \le y \le \min\{x + 2, 4 - 3x\}\}$ is :

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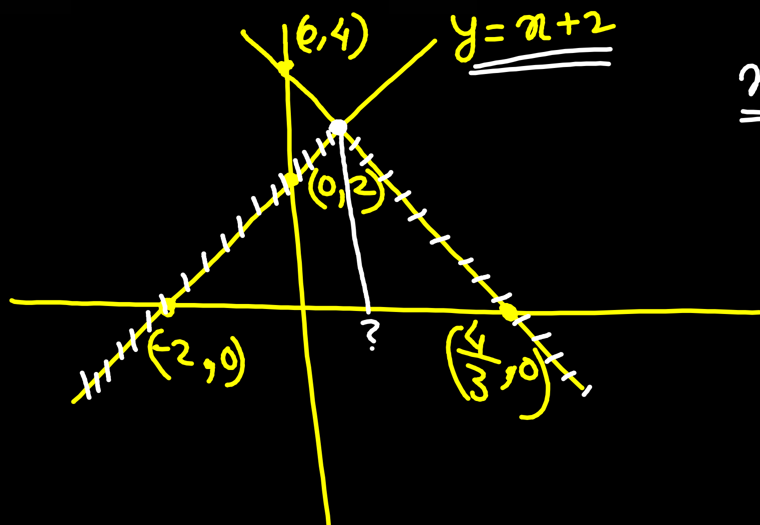
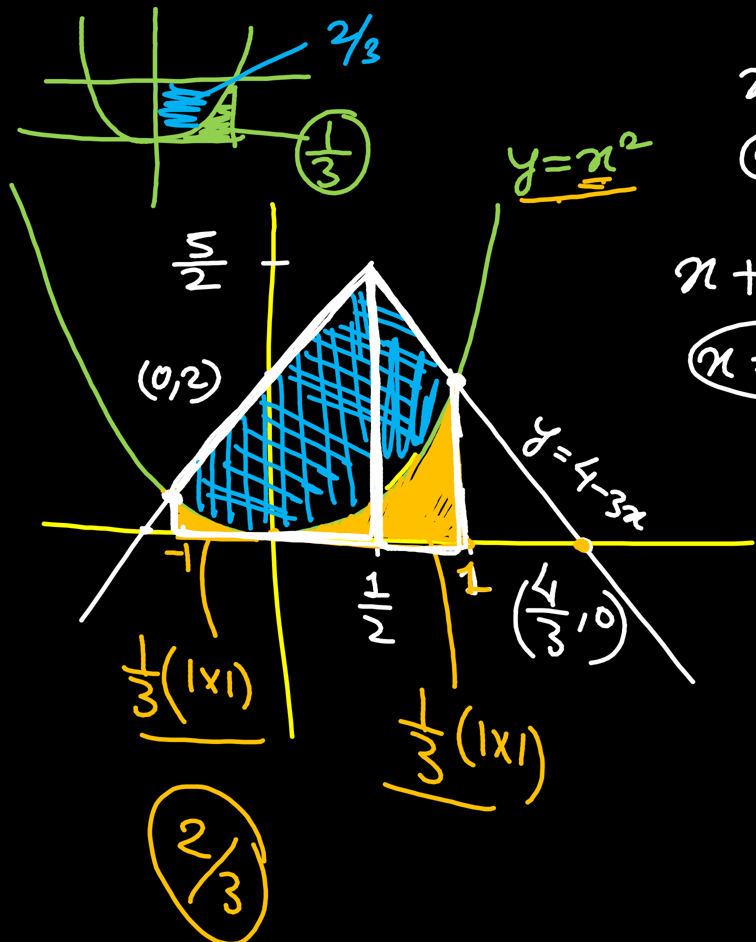
A. $\frac{31}{8}$ **B.** $\frac{17}{6}$ **C.** $\frac{19}{6}$ **D.** $\frac{27}{8}$



$x + 2 = 4 - 3x$

$4x = 2$

$x = 1/2$



$x^2 = 4 - 3x$

$x = 1$

$x + 2 = x^2$

$x = -1$

$$\text{Req Area} = \underbrace{\int_{-1}^{\frac{1}{2}} (x + 2 - x^2)\,dx}_{} + \underbrace{\int_{\frac{1}{2}}^{1} (4 - 3x - x^2)\,dx}_{}$$

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Q The odd natural number a such that the area of the region bounded by $y = 1$, $y = 3$, $x = 0$, $x = y^a$ is $\frac{364}{3}$, equal to :

y axis

$x = y^a$

$a = \text{odd } N$

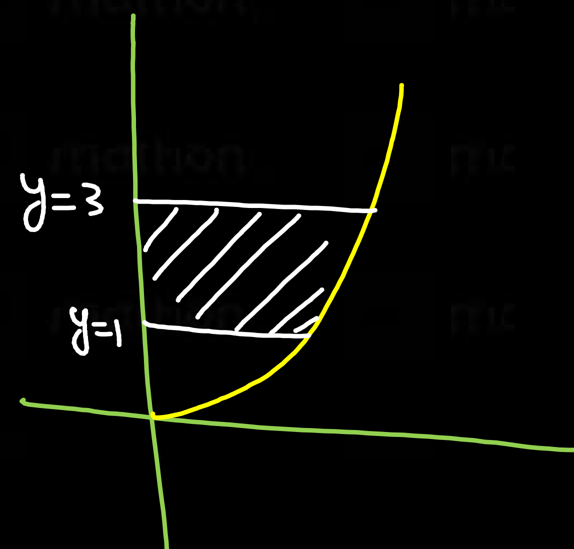
A. 3

B. 5

C. 7

D. 9

$$\int_1^3 y^a \, dy = \frac{364}{3}$$



$$\left\{\frac{y^{a+1}}{a+1}\right\}_1^3 = \frac{364}{3}$$

$$\frac{3^{a+1}-1}{a+1} = \frac{364\times 2}{3\times 2} = \frac{728}{6}$$

$$\frac{3^{a+1}-1}{a+1} = \frac{729-1}{6}$$

$$\boxed{a=5}$$

Q

The area bounded by the curves $y = |x^2-1|$ and $y = 1$ is

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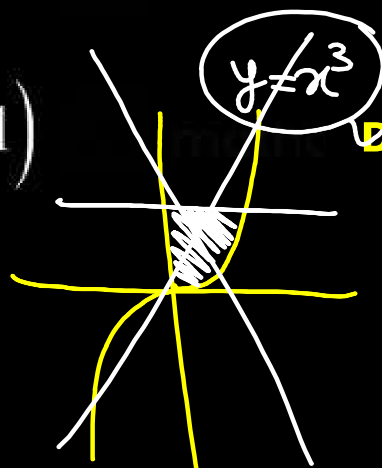
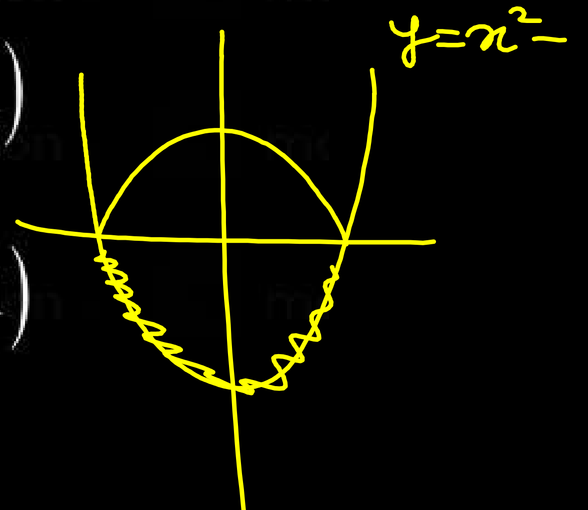
A. $\frac{2}{3}\left(\sqrt{2}+1\right)$

B. $\frac{4}{3}\left(\sqrt{2}-1\right)$

C. $2\left(\sqrt{2}-1\right)$

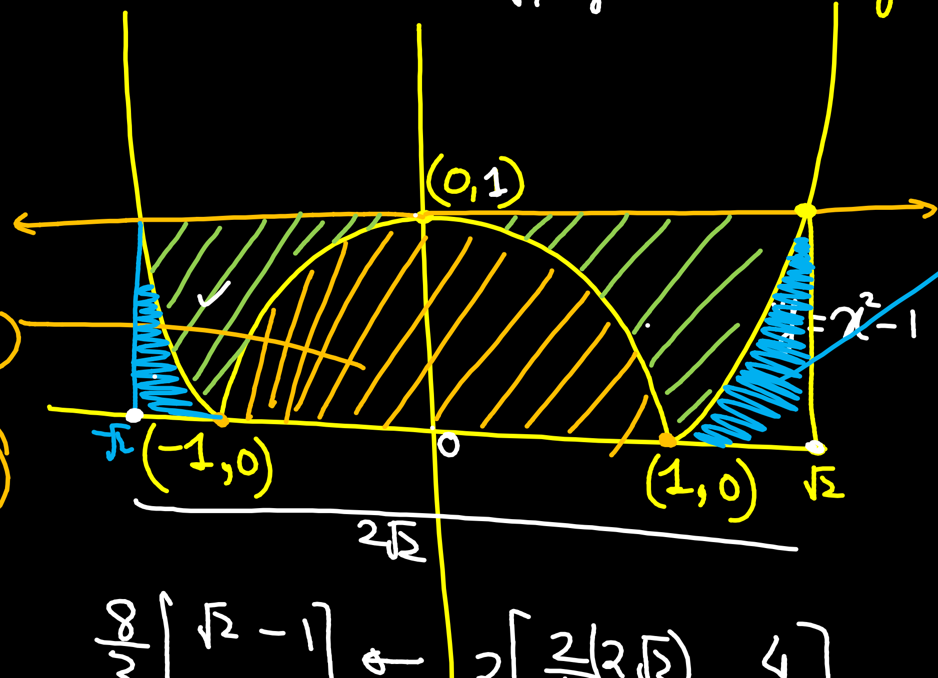
D. $\frac{8}{3}\left(\sqrt{2}-1\right)$





$G = 2\sqrt{2} - 2B - 0$ $\quad y = |x^2 - 1|$ $\quad y + 1 = x^2$

$\sqrt{1-y} = x$ $\quad y = |x^2 - 1|$



$$\boxed{\int_1^{\sqrt{2}} (x^2 - 1)\,dx = B}$$

$\frac{2}{3}(2\times 1)$

$= 4/3$

$\Rightarrow 2\int_0^1 (x_R - x_L)\,dy$

$\Rightarrow 2\int_0^1 \left(\sqrt{y+1} - \sqrt{1-y}\right) dy$

$\Rightarrow 2\left(\frac{2(y+1)^{3/2}}{3} + \frac{2(1-y)^{3/2}}{3}\right)_0^1$

$\frac{8}{3}\left[\sqrt{2} - 1\right] \leftarrow 2\left[\frac{2}{3}(2\sqrt{2}) - \frac{4}{3}\right]$

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Q The area of the region enclosed by

$y \le 4x^2, x^2 \le 9y$ and $y \le 4$, is equal to :

$0 \le 4$

A. $\frac{40}{3}$

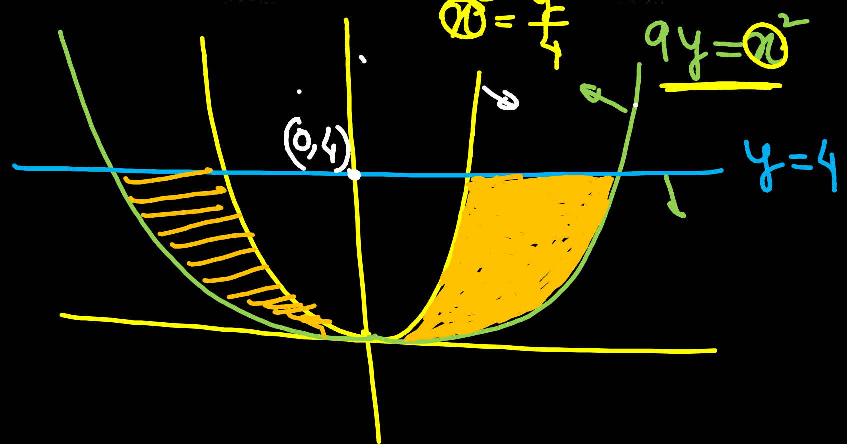
B. $\frac{56}{3}$

C. $\frac{112}{3}$

D. $\frac{80}{3}$ ✓

up $\boxed{\frac{y}{4} = x^2}$

up $\boxed{9y = x^2}$



$y \le 4x^2$

$4 \le 0$

$$\Rightarrow 2\int_0^4 (x_R - x_L)\,dy$$

$$\Rightarrow 2\int_0^4 \left(3\sqrt{y} - \frac{\sqrt{y}}{2}\right)dy$$

$$\Rightarrow 2\times\frac{5}{2}\int_0^4 \sqrt{y}\,dy$$

$$\Rightarrow 5\left(\frac{2y^{3/2}}{3}\right)_0^4 \Rightarrow \frac{10}{3}(8) \Rightarrow 80/3$$

Q

Consider a curve $y = y(x)$ in the first quadrant as shown in the figure. Let the area A_1 is twice the area A_2. Then the normal to the curve perpendicular to the line $2x - 12y = 15$ does **NOT** pass through the point.

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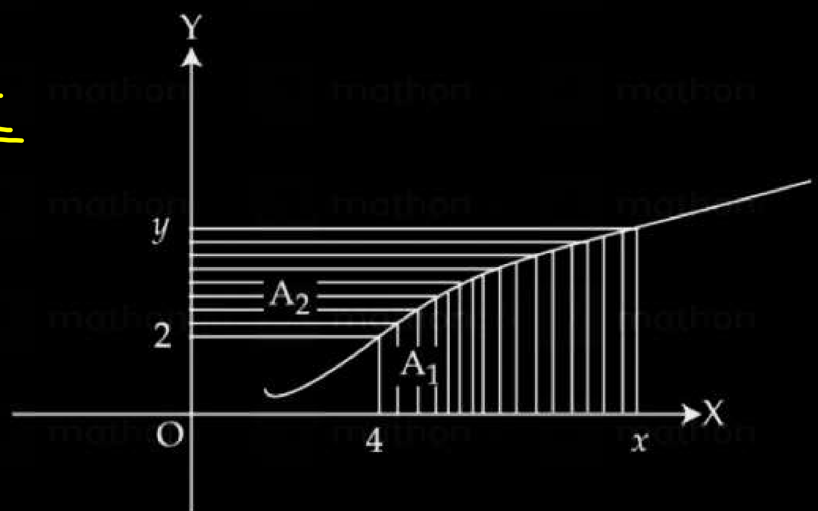
H.W.



(1) (6, 21)
(2) (8, 9)
(3) (10, –4)
(4) (12, –15)

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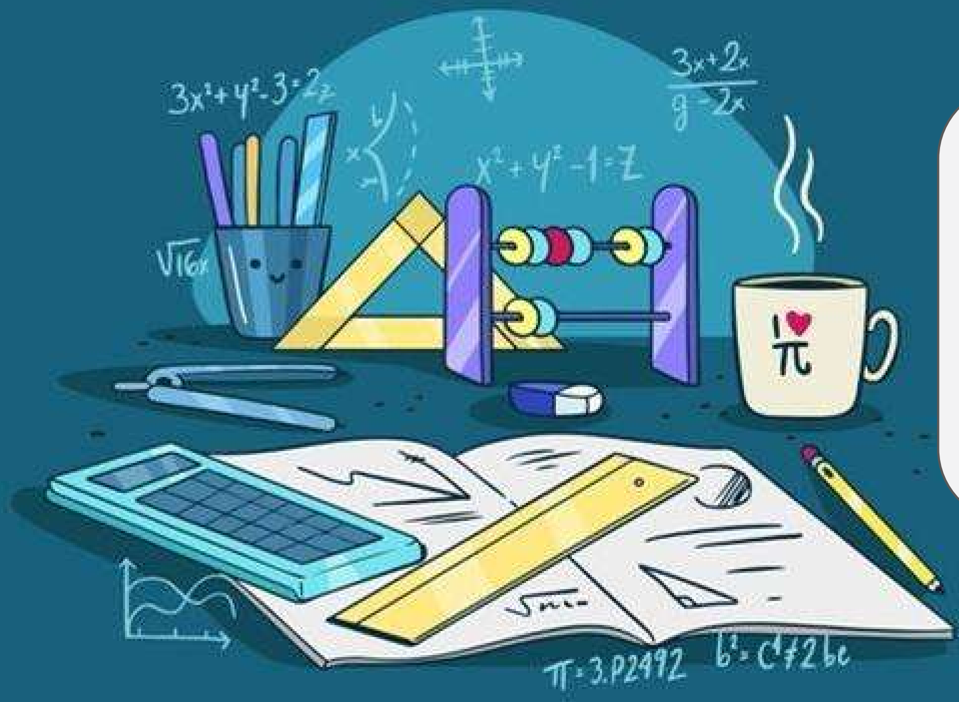


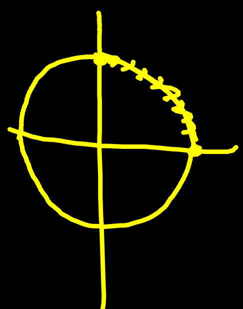
Curve Sketching

Curve Sketching

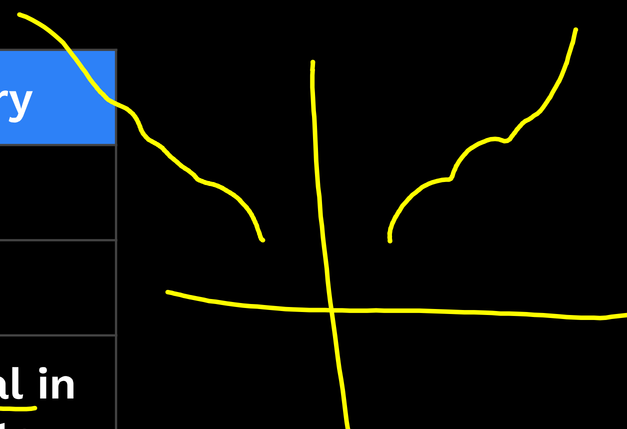
1 Check Symmetry

$$\boxed{x^2 + y^3 = 3}$$

$x^2 + y^2 = 1$



Replace	Symmetry
1. $\boxed{x \to -x}$	y axis
2. $\boxed{y \to -y}$	x axis
3. $\boxed{x \to -x,\ y \to -y}$	Symmetrical in all quadrants

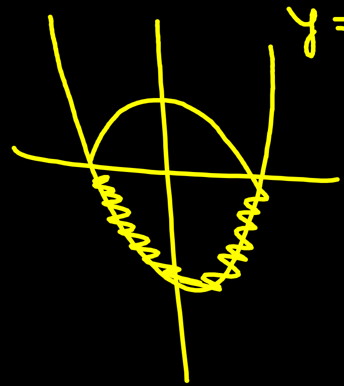


Curve Sketching

2 Use Graphical Transformation

$y = x^2 - 1$

$y = |x^2 - 1|$

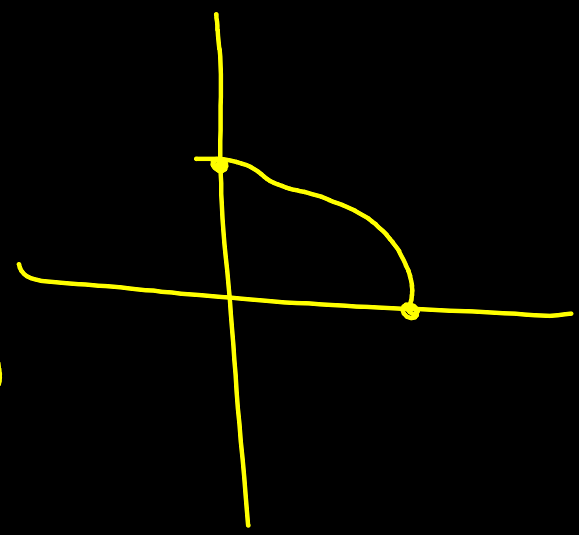


Curve Sketching

3 **Find the points where the curve crosses the x-axis and y-axis**

Kuch point

$$\boxed{\begin{array}{ll} x=0 & y=? \\ x=? & y=0 \end{array}}$$



Curve Sketching

4 **Find dy/dx and examine, if possible, the intervals where f(x) is increasing or decreasing and also its stationary points.**

$\boxed{\text{AOD}}$

$\frac{dy}{dx} = \boxed{}$

0 ✓ ↑ ↓

max mini POI

Curve Sketching

5 Examine y when $x \longrightarrow \infty$ or $x \longrightarrow -\infty$

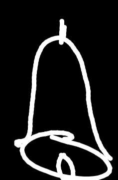
① $x \to (-x)$ $\quad$ y axis = M.I

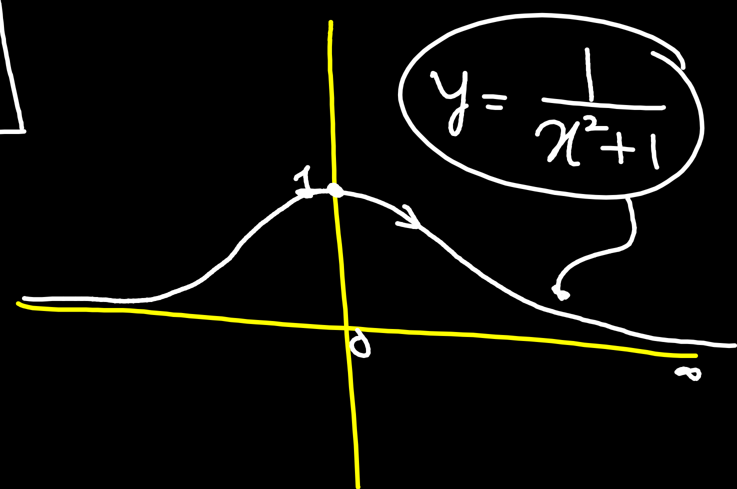
$$\boxed{y = \frac{1}{x^2+1}}$$

$(0,1)$

$(\infty, 0)$

$\boxed{\text{bell}}$

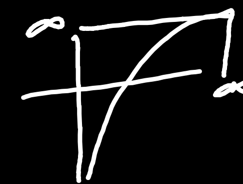




The area of the region enclosed by the curves **y = x log x** and **y = 2x - 2x²** is $y = 2x(1-x)$

A. 7 / 12 sq. units $y=\frac{1}{e}(-1)$

B. 1 / 2 sq. units



C. 5 / 12 sq. units

D. None of these

$\boxed{y = x\ln x}$ 1

① Sym → X

② $\boxed{x>0}$ Domain

$x\in(0,\infty)$

③ GTX

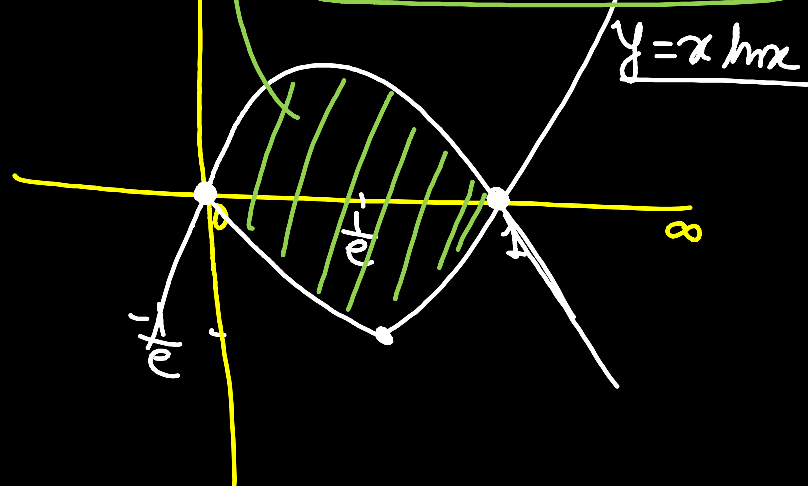
$$\boxed{\int_0^1 (2x-2x^2-x\ln x)\,dx}$$

$\lim_{x\to\infty} x\ln x$

$\infty \ln\infty$

$\infty\times\infty$

∞



(4) $\frac{dy}{dx} = x\frac{1}{x} + \ln x$

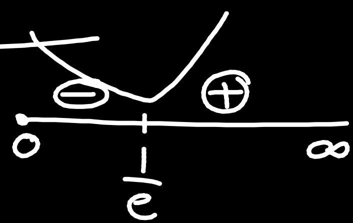
$\frac{dy}{dx} = 1 + \ln x$

$1 + \ln x = 0$

$\ln x = -1$

$x = e^{-1}$

$x = \frac{1}{e}$



$$\int_0^1 \left(2x - 2x^2 - \underset{②}{x}\,\underset{①}{\ln x}\right) dx$$

$\Rightarrow$?

$$\underbrace{x^2 - \frac{2x^3}{3} - \int_0^1 \underset{②}{x}\,\underset{①}{\ln x}\,dx}$$

Q

$$4+\frac{8}{3}=\frac{20}{3}$$

para → out

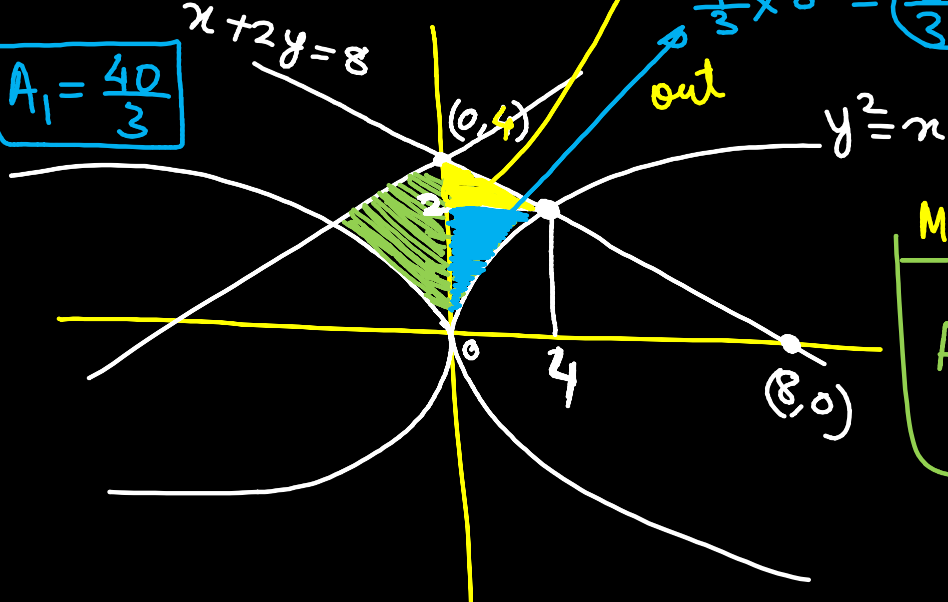
$x=4 \quad y=2$

$$\frac{1}{2}\times 2\times 4 = 4$$

$$\frac{1}{3}\times 8 = \frac{8}{3}$$

out

$$\boxed{A_1=\frac{40}{3}}$$



$$y^2 = 8-2y$$

$$y^2+2y-8=0$$

$$(y+4)(y-2)=0$$

$$y=2,\ \cancel{-4}$$

M-1

$$A_1 = 2\int_0^4\left(\frac{8-x}{2}-\sqrt{x}\right)dx$$

JEE M 2022

Q Let $\boxed{|x| = y^2}$ $\boxed{|x| + 2y = 8}$

$A_2 = 2k^2$

$A_1 = \{(x, y) : \underbrace{|x| \le y^2}_{\boxed{8 \le 0}}, |x| + 2y \le 8\}$ and $27 \times \frac{4\cancel{0}}{3} = \cancel{5}(\cancel{2}k^2)$

$\therefore \boxed{K = 6}$

$A_2 = \{(x, y) : |x| + |y| \le k\}$. If 27 (Area A_1) = 5 (Area A_2), then k is equal to :

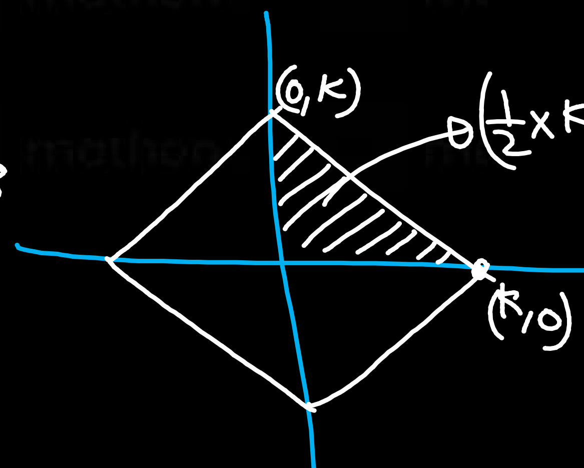
I^{st} $\boxed{y^2 = x}$

$\underbrace{x \to (-x)}_{\text{y axis}}$

$|x| + 2y = 8$

$x + 2y = 8$

SL $(0, 4)$ $(8, 0)$



$\left(\frac{1}{2} \times k \times k\right)4 = 2k^2$

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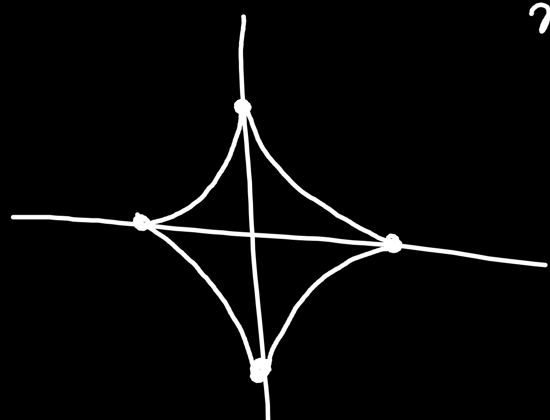
$\frac{256}{\cancel{\pi}}\cdot\frac{9\cancel{\pi}}{64} = 36$ $\qquad y = \left(1 - x^{2/3}\right)^{3/2}$

Q If the area of the region $\left\{(x,y): \underbrace{x^{\frac{2}{3}}+y^{\frac{2}{3}}\le 1}_{0+0\le 1}\ \underbrace{x+y\ge 0}_{0+1\ge 0\ \checkmark},\ y\ge 0\right\}$ is A, then $\frac{256A}{\pi}$ is

$(\cos^2\theta)^{3/2}$

$x^{2/3}+y^{2/3}=1$ — x axis = sym, y-axis = sym



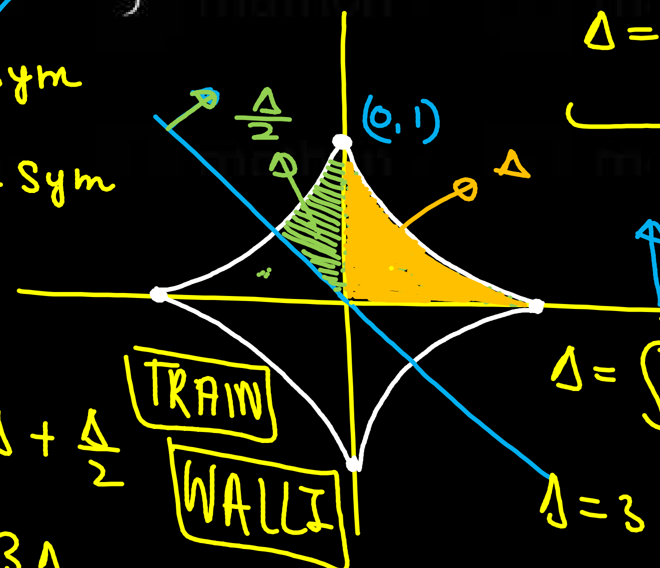
$x+y=0$

$y=-x$

Total $= \Delta + \frac{\Delta}{2}$

$= \frac{3\Delta}{2}$

$= \frac{3}{2}\left(\frac{3\pi}{32}\right) = \frac{9\pi}{64}$



$$\Delta = \int_0^1 \left(1 - x^{2/3}\right)^{3/2} dx$$

$x = \sin^3\theta$

$$\Delta = \int_0^{\pi/2} \cos^3\theta \left(3\sin^2\theta\right)\cos\theta\, d\theta$$

$$\Delta = 3\int_0^{\pi/2} \cos^4\theta \sin^2\theta\, d\theta$$

$$\Delta = \cancel{3}\,\frac{(3\times 1)(1)}{2(\cancel{8}\times 4\times 2)}\left(\frac{\pi}{2}\right) = \frac{3\pi}{32}$$

Unacademy Atoms

Q

The area of the bounded region enclosed by the curve $\underbrace{y = 3 - \left|x - \frac{1}{2}\right| - |x+1|}$ and the x-axis is

JEE M 2022

A. $\frac{9}{4}$

B. $\frac{45}{16}$

C. $\frac{27}{8}$

D. $\frac{63}{16}$

C

$$y = 3 - \left|x-\tfrac{1}{2}\right| - |x+1|$$

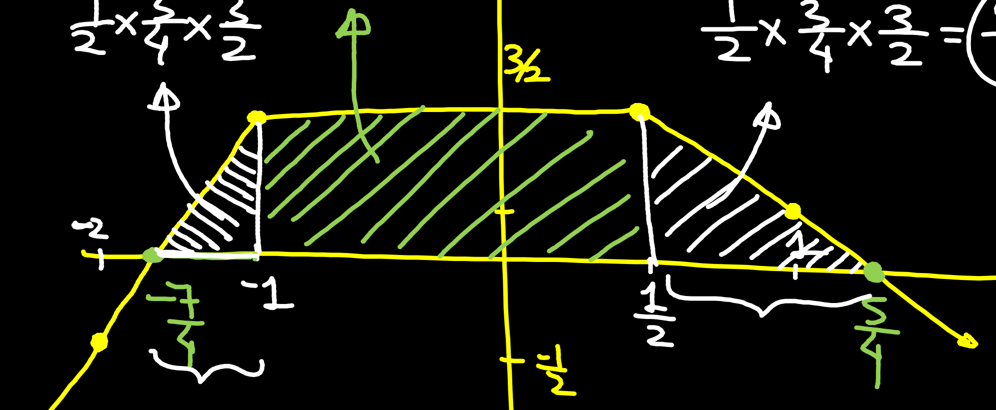
$\frac{9}{16}$

$\|$

$\frac{1}{2}\times\frac{3}{4}\times\frac{3}{2}$

$\frac{3}{2}\times\frac{3}{2} = \frac{9}{4}$

$\frac{1}{2}\times\frac{3}{4}\times\frac{3}{2} = \frac{9}{16}$



x	y
-2	$3-\frac{5}{2}-1 = 2-\frac{5}{2} = \frac{-1}{2}$
-1	$3-3/2 = 3/2$
$\frac{1}{2}$	$3-3/2 = 3/2$
1	$3-\frac{1}{2}-2 = 1/2$

$$|x+1| + \left|x-\tfrac{1}{2}\right| = 3$$

$2x+\frac{1}{2} = 3$ | $-2x-\frac{1}{2} = 3$

$x = 5/4$ | $x = \frac{-7}{4}$

$$\frac{9}{16}\times 2 + \frac{9}{4} = \frac{9}{8} + \frac{9\times 2}{4\times 2} = \frac{27}{8}$$