SETS

01 Introduction

A set is collection of well-defined distinguished objects. By well defined we mean that there should be no ambiguity regarding the inclusion and exclusion of the objects. For example a collection of scariest movies can't be considered as a set, because it will differ from person to person

02 Cardinal Number

The number of elements in a finite set is represented by n(A), known as cardinal number.
Eg .: A = {a, b, c, d, e} Then, n(A) = 5

03 Subset

Let A and B be two sets. If every element of A is an element of B, then A is called a subset of B and written as $A \subset B$ or $B \supset A$ (read as 'A' is contained in 'B' or 'B' contains 'A'). B is called superset of A.

Note:
- Every set is a subset and superset of itself.
- If A is not a subset of B, we write $A \not\subset B$.
- The empty set is the subset of every set.
- If A is a set with n(A) = m, then no. of subset of A are 2^n and the number of proper subsets of A are $2^n - 1$

Eg. Let A={3, 4}, then subsets of A are ϕ, {3}, {4}, {3,4}. Here, n(A) = 2 and number of subsets of A = 2^2 = 4

04 Types of Sets

Empty set or Null set

A set which has no element is called null set. It is denoted by symbol ϕ or {}.

Equivalent set

Two finite sets A and B are said to be equivalent, if n(A) = n(B). Clearly, equal set are equivalent but equivalent set need not to be equal.

Singleton set

A set having one element is called singleton set.

Finite and Infinite set

A set which has finite number of elements is called a finite set. Otherwise, it is called an infinite set.

Power set

The set of all subset of a given set A is called power set of A and denoted by P(A).

Equal set

Two sets A and B are said to be equal, written as A=B, if every element of A is in B and every element of B is in A.

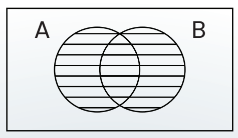
05 Operations on Sets

Difference of two sets

If A and B are two sets, then their difference A-B is defined as: $A-B=\{x : x \in A \text{ and } x \notin B\}$ Similarly, $B-A=\{x : x \in B \text{ and } x \notin A\}$

Union

The union of two sets A and B, written as $A \cup B$ (read as A union B) is the set of all elements which are either in A or in B or in both. Thus, $A \cup B=\{x : x \in A \text{ or } x \in B\}$

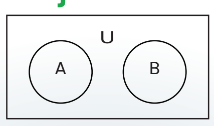


clearly, $x \in A \cup B \Rightarrow x \in A$ or $x \in B$ and $x \notin A \cup B \Rightarrow x \notin A$ and $x \notin B$

Symmetric Difference

The symmetric difference of two sets A and B, denoted by $A \Delta B$, in defined as $(A \Delta B)=(A-B) \cup (B-A)$

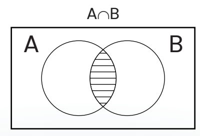
Disjoint sets



Two sets A and B are said to be disjoint, if $A \cap B = \varphi$ i. e, A and B have no common element.

Intersection

The intersection of two sets A and B, written as $A \cap B$ (read as 'A' intersection 'B') is the set consisting of all the common elements of A and B.



Thus, $A \cap B=\{x : x \in A \text{ and } x \in B\}$
Clearly, $x \in A \cap B \Rightarrow \{x \in A \text{ and } x \in B\}$ and $x \notin A \cap B \Rightarrow \{x \notin A \text{ or } x \notin B\}$.

Compliment of set

If U is a universal set and A is a subset of U, then complement of A is the set which contains those elements of U, which are not present in A and is denoted by A' or A^c. Thus, $A^c = \{x : x \in U \text{ and } x \notin A\}$

Properties of complement

Complement law:
(i) $A \cup A' = U$ (ii) $A \cap A' = \varphi$

De morgan's Law:
(i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$

Double Complement law:
$(A')' = A$

Law of empty set and universal set $\varphi' = U$ and $U' = \varphi$

06

Results on Operation of Sets:

1. $A \subseteq A \cup B$, $B \subseteq A \cup B$, $A \cap B \subseteq A$, $A \cap B \subseteq B$.
2. $A - B = A \cap B^c$
3. $(A \cup B) \cap (A \cup B') = A$
4. $(A - B) \cup B = A \cup B'$
5. $(A - B) \cap B = \varphi$
6. $A \subseteq B \Leftrightarrow B' \subseteq A'$
7. $A - B = B' - A'$
8. $A \cup B = (A - B) \cup (B - A) \cup (A \cap B)$
9. $A - B = B - A \Leftrightarrow A = B$
10. $A \cup B = A \cap B \Leftrightarrow A = B$.

Cardinal Number of Some Sets:

1. $n(A') = n(U) - n(A)$.
2. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
3. $n(A \cap B) = n(A) - n(A \cap B)$
4. $n(A \cup B) = n(A) + n(B)$. [If A and B disjoint]
5. $n(A' \cap B') = n(A \cup B)' = n(U) - n(A \cup B)$
6. $n(A' \cup B') = n(A \cap B)' = n(U) - n(A \cap B)$
7. $n(A - B) = n(A) - n(A \cap B)$
8. $n(A \cap B) = n(A \cup B) - n(A \cap B') - n(A' \cap B)$
9. $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$
10. $n(A_1 \cup A_2 \cup A_3 ... \cup A_n) = n(A_1) + n(A_2) + ... + n(A_n)$.
[If $A_1, A_2, ..., A_n$ are disjoint sets]

07 Cartesian Product

$A \times B = \{(a, b) : a \in A, b \in B\}$.
$A \times B \neq B \times A$.
If A has p elements & B has q elements then A × B has pq elements.

08 Important results on Cartesian product:

If A,B,C are three sets.
1. $A \times (B \cup C) = (A \times B) \cup (A \times C)$
2. $A \times (B \cap C) = (A \times B) \cap (A \times C)$
3. $A \times (B - C) = (A \times B) - (A \times C)$
4. $(A \times B) \cap (S \times T) = (A \cap S) \times (B \cap T)$
5. If $A \subseteq B$, $(A \times C) \subseteq (B \times C)$.
6. If $A \subseteq B$, $(A \times B) \cap (B \times A) = A \times A$.
7. If $A \subseteq B$ & $C \subseteq D$, then $A \times C \subseteq B \times D$.

09 Congruence

Let m be a positive integer, then the two integers a & b are said to be congruent modulo m if a - b is divisible by m. i.e. $a - b = m\lambda$.
$a \equiv b \pmod{m}$.