TOPICS *to be covered*

- Determinants
- Matrices
- Relations and Functions
- Inverse Trigonometric Functions
- Limits
- Continuity & Differentiability
- Application of Derivatives
- Integration
- Area Under Curve
- Differential Equations
- Vectors
- 3D Geometry
- Probability

Determinants

1) Properties of Det →
2) diff of det
3) Cramer's Rule

QUESTION [JEE MAIN – 2024 (II) (27 Jan)]

[Ans. B]

The values of α, for which $\begin{vmatrix} 1 & \frac{3}{2} & \alpha+\frac{3}{2} \\ 1 & \frac{1}{3} & \alpha+\frac{1}{3} \\ 2\alpha+3 & 3\alpha+1 & 0 \end{vmatrix} = 0$, lie in the interval

(A) $(-2, 1)$

(B) $(-3, 0)$

(C) $\left(-\frac{3}{2}, \frac{3}{2}\right)$

(D) $(0, 3)$

$R_1 \to R_1 - R_0$

$$\begin{vmatrix} 0 & 3/2 - 1/3 & 3/2 - 1/3 \\ 1 & 1/3 & \alpha + 1/3 \\ 2\alpha+3 & 3\alpha+1 & 0 \end{vmatrix} = 0$$

$$\begin{vmatrix} 0 & 1 & 1 \\ 1 & 1/3 & \alpha + 1/3 \\ 2\alpha+3 & 3\alpha+1 & 0 \end{vmatrix} = 0$$

$(2\alpha+3)(\alpha+1/3) + \left[3\alpha+1-\frac{1}{3}(2\alpha+3)\right] = 0$

$2\alpha^2+3\alpha+\cancel{\frac{2\alpha}{3}}+1+3\alpha+1-\cancel{\frac{2\alpha}{3}}-1=0$

$$\boxed{2\alpha^2+6\alpha+1=0}$$

$$f(\alpha) = 2\alpha^2 + 6\alpha + 1 = 0$$

$$f'(\alpha) = 0 \Rightarrow 4\alpha + 6 = 0$$

$$\alpha = -3/2$$

$$\alpha = \frac{-6 \pm \sqrt{36-8}}{2\times 2}$$

$$\alpha = \frac{-6 \pm \sqrt{28}}{2\times 2}$$

$$\boxed{\alpha = \frac{-3 \pm \sqrt{7}}{2}}$$

QUESTION [JEE MAIN – 2024 (I) (30 Jan)]

If $f(x) = \begin{vmatrix} 2\cos^4 x & 2\sin^4 x & 3+\sin^2 2x \\ 3+2\cos^4 x & 2\sin^4 x & \sin^2 2x \\ 2\cos^4 x & 3+2\sin^4 x & \sin^2 2x \end{vmatrix}$ then $\frac{1}{5}f'(0)$ is equal to

[Ans. A]

(A) 0

(B) 1

(C) 2

(D) 6

$R_1 \to R_1 - R_2$ & $R_2 \to R_2 - R_3$

$f(x) = \begin{vmatrix} -3 & 0 & 3 \\ 3 & -3 & 0 \\ 2\cos^4 x & 3+2\sin^4 x & \sin^2 2x \end{vmatrix}$

$= 9\begin{vmatrix} -1 & 0 & 1 \\ 1 & -1 & 0 \\ 2\cos^4 x & 3+2\sin^4 x & \sin^2 2x \end{vmatrix}$

$f(x) = 9\left[\sin^2 2x + 3 + 2\sin^4 x + 2\cos^4 x\right]$

$= 9\left[\sin^2 2x + 3 + 2(\underbrace{\sin^4 x + \cos^4 x})\right]$

$= 9\left[\sin^2 2x + 3 + 2[1 - 2\sin^2 x\cos^2 x]\right]$

$= 9\left[\cancel{\sin^2 2x} + 5 - \cancel{4\sin^2 x\cos^2 x}\right]$

$= \boxed{45}$

QUESTION [JEE MAIN – 2024 (I) (31 Jan)]

$$f(0) = \begin{vmatrix} 0 & 1 & 1 \\ 2 & 0 & 6 \\ 0 & 4 & -2 \end{vmatrix}$$

If $f(x) = \begin{vmatrix} x^3 & 2x^2+1 & 1+3x \\ 3x^2+2 & 2x & x^3+6 \\ x^3-x & 4 & x^2-2 \end{vmatrix}$ for all $x \in \mathbb{R}$, $= 4 + 1\,[8] = 12$

[Ans. C]

then $2f(0) + f'(0)$ is equal to $24+18=42$

(A) 48

(B) 24

(C) 42

(D) 18

$$f'(x) = \begin{vmatrix} 3x^2 & 4x & 3 \\ & & \\ & & \end{vmatrix} + \begin{vmatrix} & & \\ 6x & 2 & 3x^2 \\ & & \end{vmatrix} + \begin{vmatrix} & & \\ & & \\ 3x^2-1 & 0 & 2x \end{vmatrix}$$

$$f'(0) = \underbrace{\begin{vmatrix} 0 & 0 & 3 \\ 2 & 0 & 6 \\ 0 & 4 & -2 \end{vmatrix}}_{24} + \underbrace{\begin{vmatrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 4 & -2 \end{vmatrix}}_{0} + \underbrace{\begin{vmatrix} 0 & 1 & 1 \\ 2 & 0 & 6 \\ -1 & 0 & 0 \end{vmatrix}}_{-6}$$

$= 18$

Determinants

$$\boxed{x=\frac{D_1}{D},\ y=\frac{D_2}{D},\ z=\frac{D_3}{D}}$$

Cramer's Rule

$$\boxed{\begin{aligned} a_1x+b_1y+c_1z&=d_1\\ a_2x+b_2y+c_2z&=d_2\\ a_3x+b_3y+c_3z&=d_3\end{aligned}}$$

$$D=\begin{vmatrix} a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3\end{vmatrix},\quad D_1=\begin{vmatrix} d_1 & b_1 & c_1\\ d_2 & b_2 & c_2\\ d_3 & b_3 & c_3\end{vmatrix}$$

$$D_2=\begin{vmatrix} a_1 & d_1 & c_1\\ a_2 & d_2 & c_2\\ a_3 & d_3 & c_3\end{vmatrix}$$

$$D_3=\begin{vmatrix} a_1 & b_1 & d_1\\ a_2 & b_2 & d_2\\ a_3 & b_3 & d_3\end{vmatrix}$$

Nature of Solutions

Case-1

If $D \neq 0$ & $D_1 = D_2 = D_3 = 0$ then Unique trivial Solution.

Case-2

If $D \neq 0$ & at-least one of D_1, D_2 or $D_3 \neq 0$ then Unique non-trivial Solution.

Case-3 → Inconsistent

If $D = 0$ & at-least one of D_1, D_2 or $D_3 \neq 0$ then no Solution.

Case-4

If $D = 0$ & $D_1 = D_2 = D_3 = 0$ then Infinite Solution.

QUESTION [JEE MAIN – 2024 (II) (01 Feb)]

[Ans. B]

Let the system of equations

$x + 2y + 3z = 5,$

$2x + 3y + z = 9,$

$4x + 3y + \lambda z = \mu$

have infinite number of solutions. Then $\lambda + 2\mu$ is equal to :

(A) 28

(B) 17

(C) 22

(D) 15

$$D = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 4 & 3 & \lambda \end{vmatrix} = 0$$

$$(3\lambda - 3) - 2(2\lambda - 4) + 3(6 - 12) = 0$$

$$3\lambda - 3 - 4\lambda + 8 - 18 = 0$$

$$\boxed{\lambda = -13}$$

$$D_3 = \begin{vmatrix} 1 & 2 & 5 \\ 2 & 3 & 9 \\ 4 & 3 & \mu \end{vmatrix} = 0$$

$$(3\mu - 27) - 2[2\mu - 36] + 5[-6] = 0$$

$$-\mu - 27 + 72 - 30 = 0$$

$$-\mu + 42 - 27 = 0$$

$$\boxed{\mu = 15}$$

$$-13 + 30$$

Ans $= 17$

QUESTION [JEE MAIN – 2024 (II) (29 Jan)]

$\boxed{2b = a + c}$

$P(1,-2)$ ✓

[Ans. 113]

$(1,-2)$

Let for any three distinct consecutive terms a, b, c of an A.P, the lines $\boxed{ax + by + c = 0}$ be concurrent at the point P and $Q(\alpha, \beta)$ be a point such that the system of equations

$(1,-2)$

$x + y + z = 6,$

$2x + 5y + \alpha z = \beta$ and

$x + 2y + 3z = 4,$

has infinitely many solutions. Then $(PQ)^2$ is equal to

$$D_3 = \begin{vmatrix} 1 & 1 & 6 \\ 2 & 5 & \beta \\ 1 & 2 & 4 \end{vmatrix} = 0.$$

$20 - 2\beta - 1[8 - \beta] + 6[-1] = 0$

$20 - 2\beta - 8 + \beta - 6 = 0.$

$\boxed{\beta = 6}$

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 5 & \alpha \\ 1 & 2 & 3 \end{vmatrix} = 0$$

$(15 - 2\alpha) - 1[6 - \alpha] + (-1) = 0.$

$15 - 2\alpha - 6 + \alpha - 1 = 0.$

$\boxed{8 = \alpha}$ ✓

$P(1,-2)$

$Q(8,6)$

$(PQ)^2 = 7^2 + 8^2$

$= 49 + 64 = 113$

QUESTION [JEE MAIN – 2024 (II) (30 Jan)]

Consider the system of linear equations

[Ans. [redacted]

$x + y + z = 5,$

$x + 2y + \lambda^2 z = 9,$

$x + 3y + \lambda z = \mu,$

where $\lambda, \mu \in R$. Then, which of the following statement is NOT correct?

(A) System has infinite number of solution if $\lambda = 1$ and $\mu = 13$

(B) System is inconsistent if $\lambda = 1$ and $\mu \neq 13$

(C) System is consistent if $\lambda \neq 1$ and $\mu = 13$

(D) System has unique solution if $\lambda \neq 1$ and $\mu \neq 13$

Matrices

1) Multiplication

2) Inverse & Adj.

3) Symm & Skew

QUESTION [JEE MAIN – 2024 (I) (01 Feb)]

$|ABC| = |A||B||C|$.

If $A = \begin{bmatrix} \sqrt{2} & 1 \\ -1 & \sqrt{2} \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, $C = ABA^T$ and $\underbrace{X = A^TC^2A}_{\text{det both sides}}$, then det X is equal to [Ans. B]

$|B| = 1$

(A) 243

(B) 729

(C) 27

(D) 891

$\boxed{|A| = 3}$

$|C| = |A||B||A|$

$= 3 \cdot 1 \cdot 3$

$= 9$

det both sides

$|X| = |A^T||C|^2|A|$

$= |A|^2|C|^2$

$= 3^2 \cdot 9^2$

$= 9^3$

$= 729$

Wrong Approach

$X = A^TABA^TA^TABA^TA$

$A^TA = \underbrace{\begin{bmatrix} \sqrt{2} & -1 \\ 1 & \sqrt{2} \end{bmatrix}\begin{bmatrix} \sqrt{2} & 1 \\ -1 & \sqrt{2} \end{bmatrix}}_{} \neq I$

QUESTION [JEE MAIN – 2024 (I) (27 Jan)]

[Ans. D]

Consider the matrix $f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Given below are two statements :

Statement I : $f(-x)$ is the inverse of the matrix $f(x)$.

Statement II : $f(x)f(y) = f(x+y)$.

In the light of the above statements, choose the correct answer from the options given below

(A) Statement I is false but Statement II is true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Both Statement I and Statement II are true

$f(-x) =$

$$\underbrace{\begin{bmatrix} \cos x & \sin x & 0 \\ -\sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{f(-x)} = ? = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$f(x)f(y) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ & & \\ & & \end{bmatrix}$$

QUESTION [JEE MAIN – 2024 (II) (27 Jan)]

characteristic Eq^n

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

#

Let A be a 2×2 real matrix and I be the identity matrix of order 2. If the roots of the equation $\boxed{|A - xI| = 0}$ be –1 and 3, then the sum of the diagonal elements of the matrix A^2 is

[Ans. 10]

Sum of roots = $a+d$

Product of roots = $ad - bc = |A|$

$$\boxed{\begin{aligned} a+d &= 2 \\ ad-bc &= -3 \end{aligned}}$$

$$\boxed{x^2 - 2x - 3 = 0}$$

$x = A$

$A^2 - 2A - 3I = 0$

$$\boxed{A^2 = 2A + 3I}$$

$$tr(A^2) = 2\,t\underbrace{r(A)} + 3\,t\underbrace{r(I)}$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$2 \cdot 2 + 3 \times 2$

$= 4 + 6 = 10$

Proof → $|A - xI| = 0$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} - x\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{vmatrix} a-x & b \\ c & d-x \end{vmatrix} = 0$$

$$(a-x)(d-x) - bc = 0$$

$$ad - ax - dx + x^2 - bc = 0$$

$$x^2 - \underbrace{(a+d)}_{\text{sum}}x + \underbrace{(ad-bc)}_{\text{Product}} = 0$$

$tr(A) \rightarrow$ sum of dia elements

QUESTION [JEE MAIN – 2024 (I) (29 Jan)]

$|KA| = K^n |A|$

[Ans. B}

Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \alpha & \beta \\ 0 & \beta & \alpha \end{bmatrix}$ and $|2A|^3 = 2^{21}$ where $\alpha, \beta \in Z$, Then $\underbrace{\text{a value of}}$ α is

(A) 3 $|A| = \alpha^2 - \beta^2$

(B) 5 ✓

(C) 17

(D) 9

$|2A|^3 = 2^{21}$

$|2A| = 2^7$

$2^3 |A| = 2^7$

$\boxed{|A| = 2^4}$

$|A| = 16$

$\boxed{\alpha^2 - \beta^2 = 16}$

$\left.\begin{array}{l} \alpha = 5 \\ \beta = 3 \end{array}\right\}$ ✓

QUESTION [JEE MAIN – 2024 (I) (29 Jan)]

$\boxed{(A+B)^2 = A^2 + B^2 + AB + BA}$

Let A be a square matrix such that $\underbrace{AA^T = I}$. Then $\frac{1}{2}A\left[(\underbrace{A + A^T})^2 + (A - A^T)^2\right]$ is equal to

[Ans. D]

(A) $A^2 + I$

(B) $A^3 + I$

(C) $A^2 + A^T$

(D) $A^3 + A^T$

$\frac{1}{2}A\left[A^2 + (A^T)^2 + \cancel{AA^T} + \cancel{A^TA} + A^2 + (A^T)^2 - \cancel{AA^T} - \cancel{A^TA}\right]$

$\frac{1}{\cancel{2}}A\left[\cancel{2}A^2 + \cancel{2}(A^T)^2\right]$

$A^3 + \underbrace{AA^T}_{I}A^T$

$A^3 + A^T$

QUESTION [JEE MAIN – 2024 (II) (29 Jan)]

$\boxed{I = P^{-1}P = PP^{-1}}$ $\boxed{|P^{-1}| = \frac{1}{|P|}}$

[Ans. A]

Let $A = \begin{bmatrix} 2 & 1 & 2 \\ 6 & 2 & 11 \\ 3 & 3 & 2 \end{bmatrix}$ and $P = \begin{bmatrix} 1 & 2 & 0 \\ 5 & 0 & 2 \\ 7 & 1 & 5 \end{bmatrix}$.

The sum of the prime factors of $|P^{-1}AP - 2I|$ is equal to

(A) 26

(B) 27

(C) 66

(D) 23

$A - 2I = \begin{bmatrix} 2 & 1 & 2 \\ 6 & 2 & 11 \\ 3 & 3 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

$|P^{-1}AP - 2P^{-1}P|$

$|P^{-1}|\,|AP - 2P|$

$\cancel{|P^{-1}|}\,|A - 2I|\,\cancel{|P|}$

$|A - 2I| = ?$

$\begin{bmatrix} 0 & 1 & 2 \\ 6 & 0 & 11 \\ 3 & 3 & 0 \end{bmatrix}$

$-1(-33) + 2(18)$

$= 33 + 36$

$= \boxed{69}$ $\to 3$, $\to 23$

QUESTION [JEE MAIN – 2023 (II) (25 Jan)]

$\boxed{A^T = A}$ $\boxed{B^T = -B}$ $C^T = -C$

$B^{odd} \to$ skew

Let A, B, C be 3 × 3 matrices such that A is symmetric and B and C are skew-symmetric. Consider the statements [Ans. A]

$B^{even} \to$ symm ✓

(S1) $A^{13}B^{26} - B^{26}A^{13}$ is symmetric

(S2) $A^{26}C^{13} - C^{13}A^{26}$ is symmetric

Then,

X Y

(A) Only S_2 is true

(B) Only S_1 is true

(C) Both S_1 and S_2 are false

(D) Both S_1 and S_2 are true

$A^{13} = X \longrightarrow$ symm

$B^{26} = Y \to$ symm

S_1: $Z = XY - YX$

$Z^T = (XY)^T - (YX)^T$

$= Y^TX^T - X^TY^T$

$= YX - XY$

$= -(XY - YX)$

$\boxed{Z^T = -Z}$

$\Rightarrow Z$ is skew

$$\boxed{\begin{aligned} &(\text{symm})^n \longrightarrow \text{symm} \\ &(\text{skew})^{odd} \longrightarrow \text{skew} \\ &(\text{skew})^{Even} \longrightarrow \text{symm} \end{aligned}}$$

(2) $XY - YX = Z$

$Y^TX^T - X^TY^T = Z^T$

$-YX + XY = Z^T$

$\boxed{Z^T = Z}$

$X \to$ symm

$Y \to$ skew

Properties of Adjoint of a Matrix

Property 1 For any square matrix A of order n,

$$A(\text{adj.} A) = (\text{adj.} A). A = |A| I_n$$

Property 2 Let A be a non – singular matrix of order n. Then

$$|adj. A| = |A|^{n-1}$$

Property 3 If A is a non singular square matrix of order n, then

$$adj\ (adj\ A| = |A|^{n-2} A$$

Property 4 If A is a non singular square matrix of order n, then

$$|adj\ (adj\ A| = |A|^{(n-1)^2}$$

Property 5 If A is and B are non singular square matrices, then

$$adj\ (AB) = (adj\ B)\ (adj\ A)$$

Property 6 If A is non singular square matrix, then $adj\ A^T = (adj\ A)^T$

QUESTION [JEE MAIN – 2023 (II) (24 Jan)]

Let A be a 3×3 matrix such that $\underbrace{|\text{adj}\,(\text{adj}\,(\text{adj}\,A))|}_{|A|^{(n-1)^3}} = 12^4$. Then $\underbrace{|A^{-1}\,\text{adj}\,A|}$ is equal to

[Ans. A]

(A) $2\sqrt{3}$

(B) $\sqrt{6}$

(C) 12

(D) 0

$|A^{-1}| \cdot |\text{adj}\,A|$

$\frac{1}{|A|} \cdot |A|^{n-1}$

$|A|^{8} = 12^{4}$

$|A|^{2} = 12$

$|A| = \pm\sqrt{12}$

$\frac{1}{|A|} \cdot |A|^{2} = |A| = \sqrt{12}$

QUESTION [JEE MAIN – 2024 (II) (29 Jan)]

Let A be a 3×3 matrix and $\det(A) = 2$. If $n = \det(\underbrace{\text{adj}(\text{adj}(\ldots\ldots(\text{adj}\,A))}_{2024-\text{times}}))$, then the remainder when n is divided by 9 is equal to

[Ans. 7]

$|A| = 2$

$\frac{16 \times 9I + 16}{9}$ $\frac{16}{9}$

$n = |A|^{(n-1)^{2024}}$

$|A|^{2^{2024}}$

$n = 2^{2^{2024}}$

$2^{2024} = 2^2 \cdot 2^{2022}$

$= 4 \cdot (8)^{674}$

$= 4(9-1)^{674}$

$2^{2024} = 4[9k+1]$

$= \boxed{36k+4}$

$n = (2)^{36k+4} = 2^4 \cdot 2^{36k}$

$= 16(2^3)^{12k}$

$= 16[9I+1] = 16(9-1)^{12k}$

Relations and Functions

Relations

- Types of Relations.

Functions

- Types of Functions, Domain & Range Questions
- P n C Questions on number of Functions
- Composite & Inverse of a Functions
- GIF & Fractional Functions

QUESTION [JEE MAIN – 2024 (II) (01 Feb)]

$R_1 \rightarrow$ not Equivalence

[Ans. B]

Consider the relations R_1 and R_2 defined as $\boxed{aR_1b \Leftrightarrow a^2 + b^2 = 1}$ for all a, b $\in \mathbb{R}$ and $(a, b)R_2(c, d) \Leftrightarrow a + d = b + c$ for all $(a, b), (c, d) \in \mathbb{N} \times \mathbb{N}$. Then :

(A) Only R_1 is an equivalence relation

(B) Only R_2 is an equivalence relation

(C) R_1 and R_2 both are equivalence relations

(D) Neither R_1 nor R_2 is an equivalence relation

$\cancel{a^2 + b^2} = 1$

$(a, b)\, R_2\, (c, d) \Rightarrow a + d = b + c$

$(a, b)\, R\, (a, b) \Rightarrow$ Reflexive

$(a, b)\, R\, (c, d) \Rightarrow a + d = b + c$

$(c, d)\, R\, (a, b) \Rightarrow$ Symm

$(a,b) \to (c,d) \Rightarrow \boxed{a+\cancel{d}=b+\cancel{c}}$

$(c,d) \to (e,f) \Rightarrow \boxed{\cancel{c}+f=\cancel{d}+e}$

add

$a+f=b+e$

check $(a,b) \longrightarrow (e,f) = ?$

$a+f = b+e$?

✓

R_2 is equivalence

QUESTION [JEE MAIN – 2024 (I) (29 Jan)]

Let R be a relation on $Z \times Z$ defined by $(a, b)\ R\ (c, d)$ if and only if $ad - bc$ is divisible by 5. Then R is

[Ans. A]

$(a, b) \rightarrow (c, d) \Rightarrow ad - bc = 5K$

(A) Reflexive and symmetric but not transitive

(B) Reflexive and transitive but not symmetric

(C) Reflexive but neither symmetric nor transitive

(D) Reflexive, symmetric and transitive

$(a, b) \rightarrow (a, b)$ $\quad ab - ab = 0$

Reflexive

$(c, d) \rightarrow (a, b)$

$bc - ad = 5K$

Symm.

$(a, b) \rightarrow (c, d) \Rightarrow$ $ad - bc = 5K_1$

$(c, d) \rightarrow (e, f) \Rightarrow$ $cf - de = 5K_2$

$(a, b) \rightarrow (e, f)$

$\underbrace{af} - \underbrace{be} = ?$ Cant say.

QUESTION [JEE MAIN – 2024 (II) (30 Jan)]

The number of symmetric relations defined on the set $\{1, 2, 3, 4\}$ which are not reflexive is

[Ans. [redacted]

QUESTION [JEE MAIN – 2024 (I) (01 Feb)]

Let $A = \{1, 2, 3, \ldots\ldots, 20\}$. Let R_1 and R_2 two relation on A such that

$R_1 = \{(a, b) : b \text{ is divisible by } a\}$

$R_2 = \{(a, b) : a \text{ is an integral multiple of } b\}$.

Then, number of elements in $R_1 - R_2$ is equal to

[Ans. [redacted]

QUESTION [JEE MAIN – 2024 (II) (01 Feb)]

[Ans. C]

If the domain of the function $f(x) = \frac{\sqrt{x^2-25}}{(4-x^2)} + \log_{10}(x^2+2x-15)$ is $(-\infty, \alpha) \cup [\beta, \infty)$, then $\alpha^2 + \beta^3$ is equal to

(A) 140

(B) 175

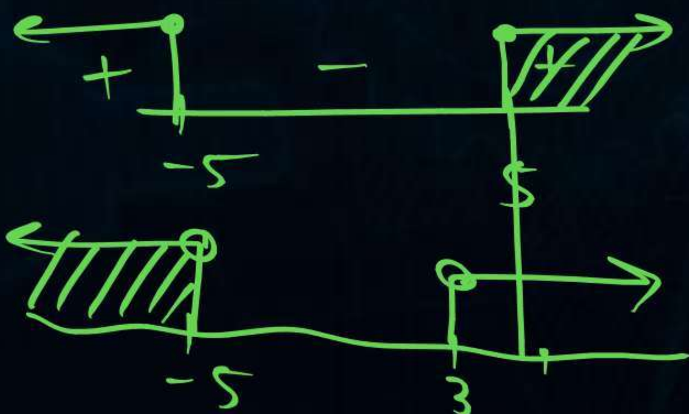
(C) 150 ✓

(D) 125

$x \neq \pm 2$

$x^2 - 25 \geq 0$

$(x-5)(x+5) \geq 0$



& $x^2 + 2x - 15 > 0$

$(x+5)(x-3) > 0$

$(-\infty, -5) \cup [5, \infty)$.

$\alpha = -5$, $\beta = 5$

$25 + 125 = 150$

QUESTION [JEE MAIN – 2024 (II) (30 Jan)]

[Ans. B]

$15-3=12$

If the domain of the function $f(x)=\log_e\left(\frac{2x+3}{4x^2+x-3}\right)+\cos^{-1}\left(\frac{2x-1}{x+2}\right)$ is $(\alpha,\beta]$, then the value of $5\beta-4\alpha$ is equal to

>0 $\quad(-1,1)$

(A) 10

(B) 12

(C) 11

(D) 9

$$\frac{2x+3}{4x^2+x-3}>0$$

$$\frac{2x+3}{4x^2+4x-3x-3}>0$$

$$\frac{2x+3}{(4x-3)(x+1)}>0$$

$\alpha=3/4$

$\beta=3$

$$\underbrace{-1\le\frac{2x-1}{x+2}}\ \underbrace{\le 1}$$

$$\frac{2x-1}{x+2}+1\geq 0$$

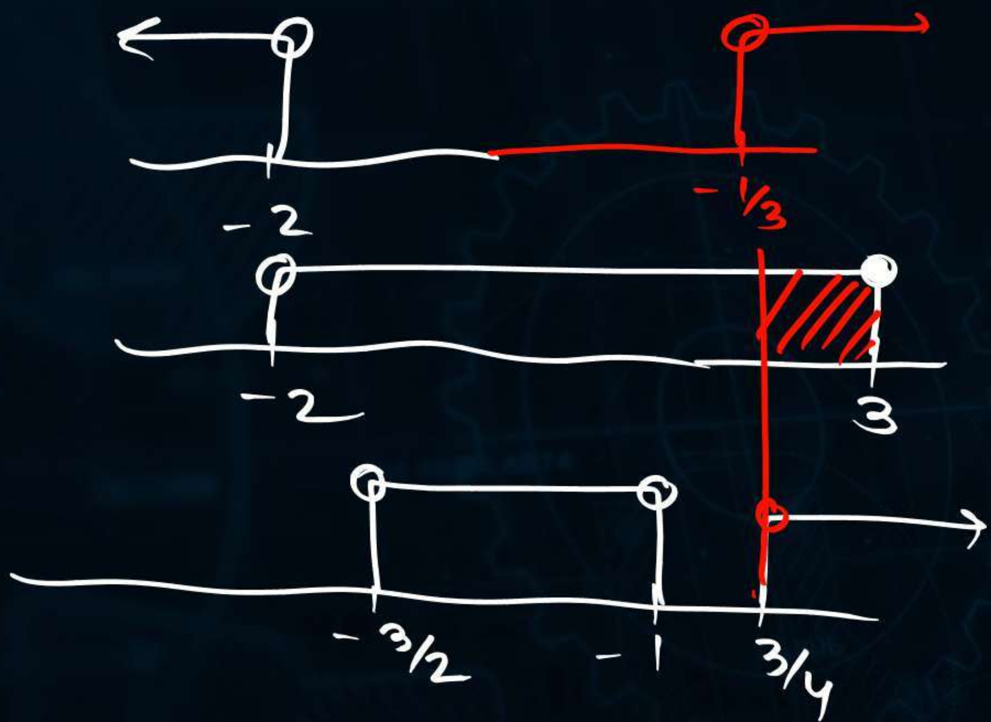
$$\frac{2x-1+x+2}{x+2}\geq 0$$

$$\frac{3x+1}{x+2}\geq 0$$

$$\frac{2x-1}{x+2}-1\le 0$$

$$\frac{2x-1-x-2}{x+2}\le 0$$

$$\frac{x-3}{x+2}\le 0$$



$$\left(\frac{3}{4}, 3\right]$$

QUESTION [JEE MAIN – 2024 (I) (30 Jan)]

HW

If the domain of the function $f(x) = \cos^{-1}\left(\frac{2-|x|}{4}\right) + \{\log_e(3-x)\}^{-1}$ is $[-\alpha, \beta) - \{\gamma\}$, then $\alpha + \beta + \gamma$ is equal to :

[Ans. C]

(A) 12

(B) 9

(C) 11

(D) 8

QUESTION [JEE MAIN – 2024 (I) (27 Jan)]

The function $f : \boxed{N - \{1\}} \rightarrow N$; defined by $f(n)$ = the highest prime factor of n, is: [Ans. D]

$f(5) = 5$

$f(15) = 5$

$\boxed{f(10) = 5}$

$f(121) = 11$

(A) both one-one and onto

(B) one-one only

(C) onto only

(D) neither one-one nor onto

QUESTION [JEE MAIN – 2024 (I) (31 Jan)]

If $\boxed{f(x) = \frac{4x+3}{6x-4}}$, $x \neq \frac{2}{3}$ and $\boxed{(fof)(x) = g(x)}$, where $g : \mathbb{R} - \left\{\frac{2}{3}\right\} \rightarrow \mathbb{R} - \left\{\frac{2}{3}\right\}$, then $\underline{(gogog)}(4)$ is equal to

[Ans. D]

(A) $-\frac{19}{20}$

(B) $\frac{19}{20}$

(C) -4

(D) 4 ✓

$$ff(x) = \frac{4f(x)+3}{6f(x)-4}$$

$$= \frac{4\left[\frac{4x+3}{6x-4}\right]+3}{6\left[\frac{4x+3}{6x-4}\right]-4}$$

$$= \frac{16x+\cancel{12}+18x-\cancel{12}}{\cancel{24x}+18-\cancel{24x}+16}$$

$$ff(x) = \frac{\cancel{34}x}{\cancel{34}} = x$$

$$\boxed{g(x) = x}$$

$$g(gog(x)) = g(g(x))$$

$$= g(x)$$

$$= x$$

$$\boxed{gogog(x) = x}$$

QUESTION [JEE MAIN – 2024 (II) (27 Jan)]

[Ans. A]

Let $f : R - \left\{\frac{-1}{2}\right\} \to R$ and $g : R - \left\{\frac{-5}{2}\right\} \to R$ be defined as

$f(x) = \frac{2x+3}{2x+1}$ and $g(x) = \frac{|x|+1}{2x+5}$. Then, the domain of the function fog is :

$x \to g(x)$

(A) $R - \left\{-\frac{5}{2}\right\}$

(B) R

(C) $R - \left\{-\frac{5}{2}, -\frac{7}{4}\right\}$

(D) $R - \left\{-\frac{7}{4}\right\}$

$$f(g(x)) = \frac{2g(x)+3}{2g(x)+1}$$

$\boxed{g(x) \text{ must be defined}}$

& $\boxed{g(x) \neq -\tfrac{1}{2}}$ $\qquad \boxed{x \in R - \{-5/2\}}$

$$\frac{|x|+1}{2x+5} = -\frac{1}{2} \Rightarrow -2|x| - 2 = 2x + 5$$

$x < 0 \Rightarrow$

$|x|\ \cancel{2x} - 2 = \cancel{2x} + 5$

ϕ

$\boxed{x > 0}$

$-2x - 2 = 2x + 5$

$4x = -7$

$\boxed{x = -7/4}$

Reject

QUESTION [JEE MAIN – 2024 (I) (29 Jan)]

HW

[Ans. [redacted]

If $f(x) = \begin{cases} 2+2x, & -1 \le x < 0 \\ 1-\frac{x}{3}, & 0 \le x \le 3 \end{cases}$; $\quad g(x) = \begin{cases} -x, & -3 \le x \le 0 \\ x, & 0 < x \le 1 \end{cases}$,

then range of (fog)(x) is

$f(g(x)) \longrightarrow$ graph $\longrightarrow$ Range.

(A) $[0, 1)$

(B) $[0, 3)$

(C) $[0, 1]$

(D) $(0, 1]$

QUESTION [JEE MAIN – 2024 (I) (29 Jan)]

Consider the function $f : \left[\frac{1}{2}, 1\right] \to \mathbb{R}$ defined by $f(x) = 4\sqrt{2}x^3 - 3\sqrt{2}x - 1$. [Ans. D]

Consider the statements

(I) The curve $y = f(x)$ intersects the x-axis exactly at one point.

(II) The curve $y = f(x)$ intersects the x-axis at $x = \cos\frac{\pi}{12}$. Then

(A) Only (I) is correct.

(B) Both (I) and (II) are incorrect.

(C) Only (II) is correct.

(D) Both (I) and (II) are correct.

$f'(x) = 12\sqrt{2}x^2 - 3\sqrt{2} = 0$

$3\sqrt{2}[4x^2 - 1] = 0$

$3\sqrt{2}(2x-1)(2x+1)$

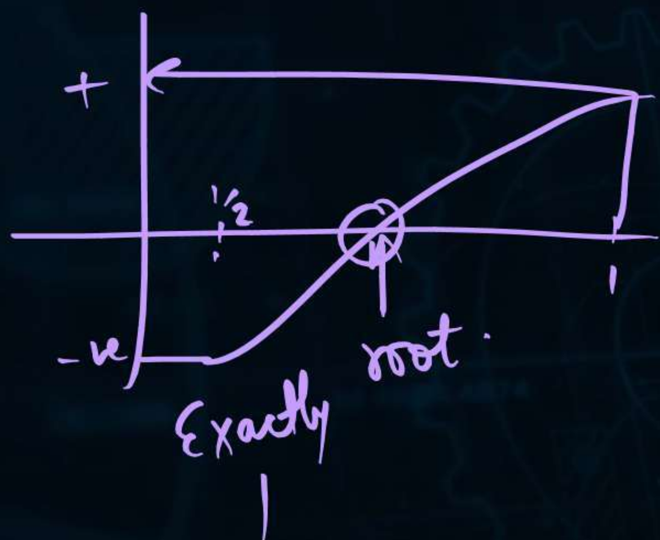
$+ \quad -\tfrac{1}{2} \quad - \quad \tfrac{1}{2} \quad +$

$f'(x) > 0 \ \forall \ x > \tfrac{1}{2}$

$\Rightarrow f$ is $\uparrow$ in $x \in (\tfrac{1}{2}, 1)$

$f(\tfrac{1}{2}) = \frac{4\sqrt{2}}{8} - \frac{3\sqrt{2}}{2} - 1 = -ve.$

$f(1) = 4\sqrt{2} - 3\sqrt{2} - 1 = +ve$



$$4\sqrt{2}x^3 - 3\sqrt{2}x - 1 = 0$$

$$\sqrt{2}(4x^3 - 3x) = 1$$

$$x = \cos\theta$$

$$\sqrt{2}[\underbrace{4\cos^3\theta - 3\cos\theta}] = 1$$

$$\cos 3\theta = \frac{1}{\sqrt{2}}$$

$$3\theta = \pi/4$$

$$\theta = \pi/12$$

$$x = \cos \pi/12$$

QUESTION [JEE MAIN – 2024 (II) (31 Jan)]

If the function $f:(-\infty,-1] \to (a,b]$ defined by $f(x)=e^{x^3-3x+1}$ is one-one and onto, then the distance of the point $P(2b+4, a+2)$ from the line $x+e^{-3}y=4$ is :

[Ans. A]

(A) $2\sqrt{1+e^6}$

(B) $\sqrt{1+e^6}$

(C) $3\sqrt{1+e^6}$

(D) $4\sqrt{1+e^6}$

$f'(x) = \underbrace{e^{x^3-3x+1}[3x^2-3]}_{+ve}$

$f(-\infty) = e^{-\infty} \to 0$

$f(-1) = e^{-1+3+1} = e^3$

$\Rightarrow y \in (0, e^3]$

$\boxed{a=0,\ b=e^3}$

$P[2e^3+4, 2]$

$d = \left|\dfrac{2e^3+\cancel{4}+e^{-3}\,2-\cancel{4}}{\sqrt{1+e^{-6}}}\right|$

$= \dfrac{2(e^3+e^{-3})}{\sqrt{1+e^{-6}}} = \dfrac{2(e^6+1)}{\sqrt{e^6+1}}$

QUESTION [JEE MAIN – 2023 (II) (24 Jan)]

$\frac{2022}{2023} = 1 - \frac{1}{2023}$

$= 1 - x$

If $\boxed{f(x) = \frac{2^{2x}}{2^{2x}+2}}$, $x \in R$ then $f\left(\frac{1}{2023}\right) + f\left(\frac{2}{2023}\right) + \ldots.. f\left(\frac{2022}{2023}\right)$ is equal to [Ans. D]

$\frac{1}{2023} \rightarrow x$, $\frac{2022}{2023} \rightarrow 1-x$

(A) 2011

(B) 1010

(C) 2010

(D) 1011

$f(x) = \frac{4^x}{4^x+2} \longrightarrow (1)$

$f(x) + f(1-x)$

$x \rightarrow 1-x$

$f(1-x) = \frac{4^{1-x}}{4^{1-x}+2} = \frac{4}{4+2.4^x} = \frac{2}{2+4^x} \longrightarrow (2)$

$\boxed{f(x) + f(1-x) = 1}$

QUESTION [JEE MAIN – 2024 (I) (31 Jan)]

$\rightarrow f(x)+f(1-x)=1$

$f(a)+f(1-a)=1$

Let $f:\mathbb{R}\to\mathbb{R}$ be a function defined by $\boxed{f(x)=\frac{4^x}{4^x+2}}$ and [Ans. 5]

$M=\int_{f(a)}^{f(1-a)} x\sin^4(x(1-x))\,dx$ $\Rightarrow M=\int (1-x)\sin^4((1-x)x)\,dx$

$N=\int_{f(a)}^{f(1-a)} \sin^4(x(1-x))dx\,;\,a\neq\frac{1}{2}.$ $M=\underbrace{\int \sin^4(\;)dx}_{N}-\underbrace{\int x\sin^4(\;)dx}_{M}$

If $\alpha M=\beta N$, $\alpha,\beta\in\mathbb{N}$, then the least value of $\alpha^2+\beta^2$ is equal to 5

$M=N-M$

$2M=N$

$\boxed{\alpha=2,\ \beta=1}$

QUESTION [JEE MAIN – 2023 (II) (24 Jan)]

Let f(x) be a function such that $\boxed{f(x+y) = f(x) \cdot f(y)}$ for all $x, y \in \mathbb{N}$. [Ans. C]

If $f(1) = 3$ and $\sum_{k=1}^{n} f(k) = 3279$, then the value of n is

(A) 6

(B) 8

(C) 7 ✓

(D) 9

$f(x) = a^x$

$f(1) = a^1 = 3$

$a = 3$

$\boxed{f(x) = 3^x}$

$f(1) + f(2) + f(3) + \dots f(n) = 3279$

$3^1 + 3^2 + 3^3 + \dots + 3^n = 3279$

$\frac{3(3^n - 1)}{(3-1)} = 3279$

$\frac{3^n - 1}{2} = 1093$

$3^n - 1 = 2186$

$3^n = 2187$

$3^n = 729 . 3$

$3^n = 3^6 . 3^1$

$3^n = 3^7$

$\boxed{n = 7}$

Some Common Functional Identities

(i) $f(xy) = f(x) + f(y) \Rightarrow f(x) = k \ln x$ or $f(x) = 0$

(ii) $f(xy) = f(x) \cdot f(y) \Rightarrow f(x) = x^n, n \in R$

(iii) $f(x + y) = f(x) \cdot f(y) \Rightarrow f(x) = a^x$.

(iv) $f(x + y) = f(x) + f(y) \Rightarrow f(x) = kx$, where k is a constant.

(v) If f is a polynomial function satisfying

$$f(x) \cdot f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right) \Rightarrow f(x) = 1 \pm x^n \text{ where } n \in N$$

ITF

Inverse Trigonometric Functions

- Domain & Range of ITF
- Properties of ITF
- Solving Inverse Trigo Equations.
- Telescoping Series Problems

QUESTION [JEE MAIN – 2024 (II) (27 Jan)]

Considering only the principal values of inverse trigonometric functions, the number of positive real values of x satisfying $\tan^{-1}(x) + \tan^{-1}(2x) = \frac{\pi}{4}$ is :

[Ans. B]

(A) more than 2

(B) 1

(C) 2

(D) 0

$$\tan^{-1} \frac{x + 2x}{1 - 2x^2} = \pi/4$$

$$\frac{3x}{1 - 2x^2} = 1$$

$$3x = 1 - 2x^2$$

$$2x^2 + 3x - 1 = 0$$

$$\frac{-3 \pm \sqrt{9+8}}{2}$$

QUESTION [JEE MAIN – 2024 (II) (29 Jan)]

[Ans. C]

Let $x = \frac{m}{n}$ (m, n are co-prime natural numbers) be a solution of the equation $\cos(2\underbrace{\sin^{-1}x}_{\theta}) = \frac{1}{9}$ and let $\alpha, \beta(\alpha > \beta)$ be the roots of the equation $mx^2 - nx - m + n = 0$ Then the point (α, β) lies on the line

(A) $3x - 2y = -2$

(B) $3x + 2y = 2$

(C) $5x + 8y = 9$

(D) $5x - 8y = -9$

$\sin^{-1}x = \theta$

$x = \sin\theta$

$\cos 2\theta = \frac{1}{9}$

$1 - 2\sin^2\theta = \frac{1}{9}$

$1 - 2x^2 = \frac{1}{9}$

$2x^2 = 8/9$

$x^2 = 4/9$

$x = 2/3$

$\boxed{\begin{matrix} m = 2 \\ n = 3 \end{matrix}}$

$2x^2 - 3x + 1 = 0$

$2x^2 - 2x - x + 1 = 0$

$(2x-1)(x-1) = 0$

$x = \frac{1}{2}, 1$

$\alpha = 1, \beta = \frac{1}{2}$

$\leftarrow P(1, \frac{1}{2})$

QUESTION [JEE MAIN – 2024 (I) (31 Jan)]

For $\alpha, \beta, \gamma \neq 0$, if $\boxed{\sin^{-1}\alpha + \sin^{-1}\beta + \sin^{-1}\gamma = \pi}$ and $\boxed{(\alpha+\beta+\gamma)(\alpha-\gamma+\beta) = 3\alpha\beta}$, then γ equals

[Ans. A]

(A) $\frac{\sqrt{3}}{2}$ ✓

(B) $\frac{1}{\sqrt{2}}$

(C) $\frac{\sqrt{3}-1}{2\sqrt{2}}$

(D) $\sqrt{3}$

$(\alpha+\beta)^2 - \gamma^2 = 3\alpha\beta$

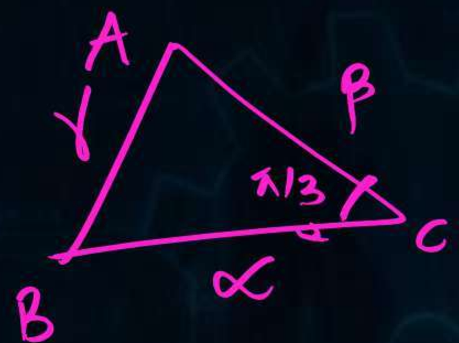
$\alpha^2+\beta^2+\cancel{2\alpha\beta}-\gamma^2 = 3\alpha\beta$

$\alpha^2+\beta^2-\gamma^2 = \alpha\beta$

$\boxed{\frac{\alpha^2+\beta^2-\gamma^2}{2\alpha\beta}} = \frac{1}{2}$

$\cos C = \frac{1}{2}$

$C = \pi/3$



$\sin^{-1}\gamma = \pi/3$

$\gamma = \sin\pi/3$

$\gamma = \sqrt{3}/2$

QUESTION [JEE MAIN – 2024 (II) (31 Jan)]

If $a = \sin^{-1}(\sin(5))$ and $b = \cos^{-1}(\cos(5))$, then $a^2 + b^2$ is equal to

[Ans. B]

(A) $4\pi^2 + 25$

(B) $8\pi^2 - 40\pi + 50$

(C) $4\pi^2 - 20\pi + 50$

(D) 25

$$\sin^{-1}\sin 5 = \boxed{5 - 2\pi = a}$$
$$\cos^{-1}\cos 5 = \boxed{2\pi - 5 = b}$$

$$a^2 + b^2 = (5-2\pi)^2 + (2\pi-5)^2$$
$$= 2(4\pi^2 + 25 - 20\pi)$$

Limits

QUESTION [JEE MAIN – 2024 (I) (27 Jan)]

[Ans. B]

If $a = \lim\limits_{x\to 0} \frac{\sqrt{1+\sqrt{1+x^4}}-\sqrt{2}}{x^4}$ and $b = \lim\limits_{x\to 0} \frac{\overset{1}{\sin^2 x}\cdot x^2}{\sqrt{2}-\sqrt{1+\cos x}}$, then the value of ab^3 is :

(A) 36

(B) 32 ✓

(C) 25

(D) 30

$$b = \frac{(x^2)}{(\sqrt{2}-\sqrt{1+\cos x})}\cdot\frac{\sqrt{2}+\sqrt{1+\cos x}}{(\sqrt{2}+\sqrt{1+\cos x})}$$

$$\frac{2\sqrt{2}\,\cancel{x^2}}{\frac{1-\cos x}{x^2}\cdot\cancel{x^2}}$$

$$b = 4\sqrt{2}$$

$$ab^3 = \frac{1}{4\sqrt{2}}\cdot(4\sqrt{2})^3 = (4\sqrt{2})^2 = 16\times 2$$

$$a = \frac{\sqrt{1+\sqrt{1+x^4}} - \sqrt{2}}{x^4}\left(\frac{\sqrt{1+\sqrt{1+x^4}}+\sqrt{2}}{11}\right)$$

$$\frac{\left(1+\sqrt{1+x^4}\right) - 2}{x^4(2\sqrt{2})} = \frac{\left(\sqrt{1+x^4}-1\right)}{x^4(2\sqrt{2})} \cdot \frac{\left(\sqrt{1+x^4}+1\right)}{\left(\underbrace{\sqrt{1+x^4}+1}\right)}$$

$$= \frac{\cancel{1}+x^4-\cancel{1}}{x^4(2\sqrt{2})\cdot 2}$$

$$a = \frac{1}{4\sqrt{2}}$$

QUESTION [JEE MAIN – 2024 (I) (01 Feb)]

Let $\{x\}$ denote the fractional part of x and $f(x) = \frac{\cos^{-1}(1-\{x\}^2)\sin^{-1}(1-\{x\})}{\{x\}-\{x\}^3}, x \neq 0.$ [Ans. 18]

If L and R respectively denotes the left hand limit and the right hand limit of f(x) at x = 0, then $\frac{32}{\pi^2}(L^2+R^2)$ is equal to ______

$$\lim_{x\to 0^+} \frac{\cos^{-1}(1-x^2)\sin^{-1}(1-x)}{x-x^3}$$

$$\{x\} = x - [x]$$

$$x \to 0^+ \Rightarrow \{x\} = x - 0 = x$$

$$x \to 0^- \Rightarrow \{x\} = x - (-1) = x+1$$

$$\lim_{x\to 0} \frac{\cos^{-1}(1-x^2)\overbrace{\sin^{-1}(1-x)}^{\pi/2}}{x\,\underbrace{(1-x^2)}_{1}}$$

$$\cos^{-1}(1-x^2) = \theta$$

$$1-x^2 = \cos\theta$$

$$x^2 = 1-\cos\theta$$

$$\frac{1}{\frac{1}{\sqrt{2}}}$$

$$\frac{\pi}{2}\lim_{x\to 0}\frac{\cos^{-1}(1-x^2)}{x} = \frac{\pi}{2}\lim_{\theta\to 0}\frac{\theta}{\sqrt{1-\cos\theta}} = \frac{1}{\sqrt{\frac{1-\cos\theta}{\theta^2}}} = \frac{\pi}{2}\sqrt{2} = \boxed{\frac{\pi}{\sqrt{2}} = R}$$

$$L = \lim_{x\to 0^-} \left(\frac{\cos^{-1}(1-(x+1)^2)\,\sin^{-1}(1-(1+x))}{(1+x)\left[1-(1+x)^2\right]} \right)$$

$\pi/2$

$$\frac{\pi}{2}\left(\frac{\sin^{-1}(-x)}{-2x-x^2}\right)$$

$$\frac{\pi}{2}\left(\frac{\sin^{-1}(-x)}{(-x)(x+2)}\right) \to 1$$

$$L = \frac{\pi}{2(x+2)} = \frac{\pi}{4}.$$

$$\frac{32}{\pi^2}\left[\frac{\pi^2}{16} + \frac{\pi^2}{2}\right]$$

$$\frac{32}{\pi^2}\left[\frac{\pi^2 + 8\pi^2}{16}\right]$$

$$\frac{32}{\cancel{\pi^2}} \cdot \frac{9\cancel{\pi^2}}{16} = 18$$

QUESTION [JEE MAIN – 2024 (II) (27 Jan)]

If $\lim\limits_{x\to 0} \dfrac{3+\alpha \sin x+\beta \cos x+\log_e(1-x)}{3\tan^2 x} = \dfrac{1}{3}$, then $2\alpha-\beta$ is equal to : [Ans. C]

$\dfrac{\tan^2 x}{x^2} \to 1$, x^2

$2\alpha - \beta = 2\times 1 + 3$

(A) 2

(B) 7

(C) 5 ✓

(D) 1

$$\lim_{x\to 0}\left(\frac{\boxed{3+\alpha\sin x+\beta\cos x+\ln(1-x)}}{x^2}\right) = 1$$

$3+\beta+\ln 1 = 0$

$\boxed{\beta = -3}$

$\alpha + 0 - 1 = 0$

$\boxed{\alpha = 1}$

LH

$$\frac{\alpha\cos x + 3\sin x + \frac{1}{1-x}(-1)}{2x} = 1$$

QUESTION [JEE MAIN – 2024 (II) (31 Jan)]

HW

If $\lim\limits_{x\to 0} \dfrac{ax^2e^x - b\log_e(1+x) + cxe^{-x}}{x^2 \sin x} = 1$, then $16(a^2+b^2+c^2)$ is equal to

[Ans. [redacted]

Continuity & Differentiability

Continuity & Differentiability

- Continuity & Differentiability at a Point
- Problem on Functional Identities
- Properties of Continuous & Differentiable Functions

MOD

- Parametric Differentiation,
- Differentiation of ITF.
- Higher order Derivatives
- Differentiation of f(x) w.r.t g(x)

QUESTION [JEE MAIN – 2024 (I) (27 Jan)]

$-a/2 = 2$

$\boxed{a = -4}$

[Ans. D]

Consider the function, $f(x) = \begin{cases} \frac{a(7x-12-x^2)}{b|x^2-7x+12|}, & x < 3 \\ 2^{\frac{\sin(x-3)}{x-[x]}}, & x > 3 \\ \boxed{b,} & x = 3, \end{cases}$ where $[x]$ denotes the greatest integer less than or equal to x. If S denotes the set all ordered pairs (a, b) such that f(x) is continuous at x = 3, then the number of elements in S is:

(A) 2

(B) Infinitely many

(C) 4

(D) 1

RHL $\lim_{x \to 3^+} 2^{\frac{\sin(x-3)}{x-3}} = 2^1 = \boxed{2 = b}$

$\underset{+}{\longleftarrow 3} \overset{-}{\quad} \underset{4}{} \overset{+}{}$

LHL: $\lim_{x \to 3^-} \frac{-a(x^2-7x+12)}{b|x^2-7x+12|} = -\frac{a}{b} \frac{(x-3)(x-4)}{|\underbrace{(x-3)(x-4)}_{+}|} = -\frac{a}{b} = -a/2$

QUESTION [JEE MAIN – 2024 (II) (30 Jan)]

diff $\Rightarrow$ cont

[Ans. D]

Let a and b be real constants such that the function f defined by $f(x) = \begin{cases} x^2 + 3x + a, & x \le 1 \\ bx + 2, & x > 1 \end{cases}$ be $\underbrace{\text{differentiable}}$ on R. Then, the value of $\int_{-2}^{2} f(x)dx$ equals

$f'(x) = \begin{cases} 2x+3 \\ b \end{cases}$

$\boxed{5 = b}$

(A) $\frac{15}{6}$

(B) $\frac{19}{6}$

(C) 21

(D) 17 ✓

$1+3+a = b+2$

$\boxed{b = a+2}$

$\boxed{a = 3}$

$\int_{-2}^{2} f(x)dx = \int_{-2}^{1}(x^2+3x+3)dx + \int_{1}^{2}(5x+2)dx$

$x^3/3 + \frac{3x^2}{2} + 3x + \frac{5x^2}{2} + 2x$

$\frac{1}{3}[1+8] + 3/2(-3) + 9 + 5/2[3] + 2$

$= 3 - 9/2 + 9 + 15/2 + 2 = 17$

QUESTION [JEE MAIN – 2024 (I) (01 Feb)]

[Ans. D]

Let f: R → R be defined as : $f(x) = \begin{cases} \dfrac{a - b\cos 2x}{x^2}; & x < 0 \\ x^2 + cx + 2; & 0 \le x \le 1 \\ 2x + 1; & x > 1 \end{cases}$

If f is continuous everywhere in **R** and m is the number of points where f is NOT differential then m + a + b + c equals :

(A) 1

(B) 4

(C) 3

(D) 2

$\frac{1-\cos 2x}{x^2}$

$\boxed{\begin{matrix} a = b = 1 \\ c = 0 \end{matrix}}$

$3 = 1 + c + 2 \Rightarrow \boxed{c = 0}$

$\lim_{x \to 0^-} \left(\frac{a - b\cos 2x}{x^2} \right) = 2$

$a - b = 0 \Rightarrow \boxed{a = b}$

$\lim_{x \to 0^-} \frac{a(1 - \cos 2x)}{x^2} = 2a \frac{\sin^2 x}{x^2} = 2 \Rightarrow \boxed{a = 1}$

$f'(1^+) = 2$

$f'(1^-) = 2x = 2$

$f'(0^+) = 0$

$f'(0^-) = 0$

$\Rightarrow$ diff $\Rightarrow \boxed{m = 0}$

LHD: $$f'(0^-) = \frac{f(0-h) - f(0)}{-h}$$

$$= \frac{\frac{1-\cos 2h}{h^2} - 2}{-h}$$

$$= \frac{2\sin^2 h - 2h^2}{-h^3}$$

$$= 2\underbrace{\left(\frac{\sin h - h}{-h^2}\right)}\underbrace{\left(\frac{\sin h + h}{h}\right)} \rightarrow 2$$

$$= -4 \lim_{h \to 0}\left(\frac{\sin h - h}{h^2}\right) = -4\left(\frac{\cos h - 1}{2h}\right) \rightarrow 0.$$

QUESTION [JEE MAIN – 2024 (I) (30 Jan)]

HW

[Ans. 15]

If the function $f(x) = \begin{cases} \frac{1}{|x|}, & |x| \geq 2 \\ ax^2 + 2b, & |x| < 2 \end{cases}$ is differentiable on R, then $48(a + b)$ is equal to ______

QUESTION [JEE MAIN – 2024 (II) (31 Jan)]

$\ln x < 0$

Consider the function $f : (0, \infty) \to \mathbb{R}$ defined by $f(x) = e^{-|\log_e x|}$. If m and n be respectively the number of points at which f is not continuous and f is not differentiable, then m + n is

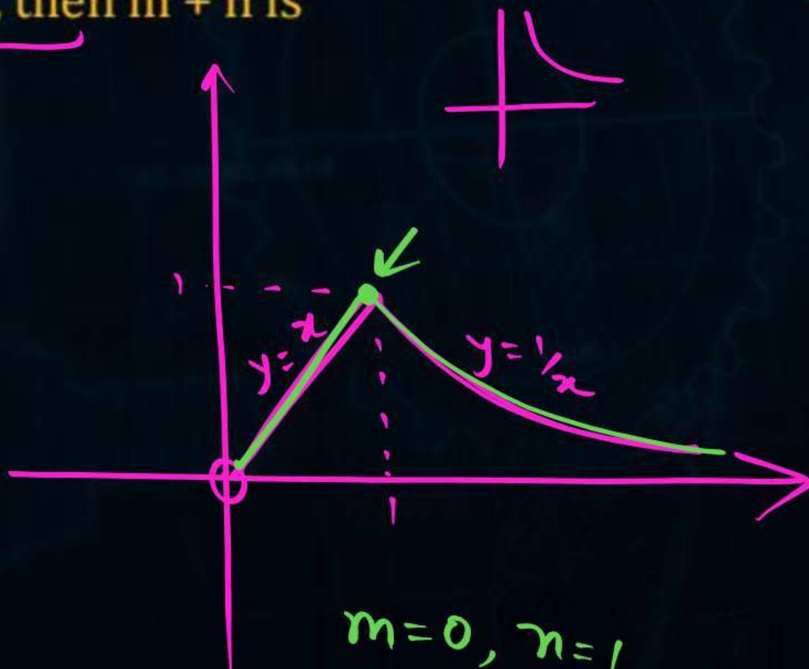
[Ans. C]

(A) 0

(B) 3

(C) 1

(D) 2



$m = 0,\ n = 1$

$f(x) = e^{-|\ln x|} = e^{-\ln x} = 1/x$

if $\ln x > 0$

$\Rightarrow \boxed{x > 1}$

1) $f(x) = 1/x,\ x > 1$

2) $f(x) = e^{\ln x} = x,\ x < 1$

QUESTION [JEE MAIN – 2024 (II) (01 Feb)]

$$\boxed{\begin{matrix} m = 0 \\ n = 3 \end{matrix}}$$

Let $f(x) = |2x^2 + 5|x| - 3|$, $x \in R$. If m and n denote the number of points where f is not continuous and not differentiable respectively, then m + n is equal to :

[Ans. D]

(A) 5

(B) 2

(C) 0

(D) 3 ✓

$2x^2 + 5x - 3$

$2x^2 + 6x - x - 3$

$(2x-1)(x+3) = 0$

$x = -3, \frac{1}{2}$

$g(x) = |2x^2 + 5x - 3|$

$g(|x|) = |2|x|^2 + 5|x| - 3|$

QUESTION [JEE MAIN – 2024 (II) (01 Feb)]

If $y = \boxed{\dfrac{(\sqrt{x}+1)(x^2-\sqrt{x})}{x\sqrt{x}+x+\sqrt{x}}} + \dfrac{1}{15}\underbrace{(3\cos^2 x - 5)\cos^3 x}$, then $96y'\left(\dfrac{\pi}{6}\right)$ is equal to : [Ans. 105]

$$x^{3/2} - 1 = (x^{1/2} - 1)(x + 1 + x^{1/2})$$

$$96 \times \frac{35}{32} = 3 \times 35 = 105$$

$$y = \frac{(\sqrt{x}+1)\cancel{\sqrt{x}}(x^{3/2}-1)}{\cancel{\sqrt{x}}(x+\sqrt{x}+1)} + \downarrow$$

$$\frac{(\sqrt{x}+1)(x^{1/2}-1)\cancel{(x+1+\sqrt{x})}}{\cancel{(x+\sqrt{x}+1)}}$$

$$(x-1)$$

$$\boxed{y = x - 1 + \tfrac{1}{5}\cos^5 x - \frac{\cos^3 x}{3}}$$

$$y' = 1 + \cos^4 x(-\sin x) + \cos^2 x \cdot \sin x$$

$$y'(\pi/6) = 1 + \left(\tfrac{3}{4}\right)^2 \cdot \left(-\tfrac{1}{2}\right) + \tfrac{3}{4} \cdot \tfrac{1}{2}$$

$$1 - \frac{9}{32} + \frac{3}{8}$$

$$\frac{32 - 9 + 12}{32} = \frac{35}{32}$$

QUESTION [JEE MAIN – 2024 (I) (27 Jan)]

Let $f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3), x \in R$.

Then $f'(10)$ is equal to ______

[Ans. 202]

$f'(1) \to a$, $f''(2) \to b$, $f'''(3) \to c$

$f(x) = x^3 + ax^2 + bx + c$

$\boxed{f'(x) = 3x^2 + 2ax + b} \Rightarrow 3 + 2a + b = a$

$\boxed{a + b = -3}$

$f''(x) = \boxed{6x + 2a} \Rightarrow \boxed{12 + 2a = b}$

$\boxed{f'''(x) = 6} \Rightarrow \boxed{c = 6}$

$12 + 2a = -3 - a$

$3a = -15$

$\boxed{a = -5}$

$\boxed{b = 2}$

$f'(10) = 300 + 20a + b$

$= 300 - 100 + 2$

$= 202$

AOD

Application of Derivatives

- Monotonicity, Maxima-Minima
- Geometrical Problems
- Finding Range using Maxima Minima

QUESTION [JEE MAIN – 2024 (II) (27 Jan)]

$f'(x) \uparrow$

$f'(a) > f'(b) \Rightarrow a > b$

Let $\boxed{g(x) = 3f\left(\frac{x}{3}\right) + f(3 - x)}$ and $f''(x) > 0$ for all $x \in (0,3)$. If g is decreasing in $(0, \alpha)$ and increasing in $(\alpha, 3)$, then 8α is : [Ans. C]

(A) 24

(B) 0

(C) 18

(D) 20

$g'(x) < 0$

$g'(x) = \cancel{3} f'(x/3) \cdot \frac{1}{\cancel{3}} + f'(3-x) \cdot (-1)$

$= f'(x/3) - f'(3-x) < 0$

$f'(x/3) < f'(3-x)$

$\Rightarrow x/3 < 3-x$

$4x/3 < 3 \quad \boxed{x < 9/4}$

$\alpha = 9/4$

$8\alpha = 18$

QUESTION [JEE MAIN – 2024 (I) (30 Jan)]

[Ans. A]

Let $g : R \to R$ be a non constant twice differentiable function such that $g'\left(\frac{1}{2}\right) = g'\left(\frac{3}{2}\right)$. If a real valued function f is defined as

$f(x) = \frac{1}{2}[g(x) + g(2 - x)]$ then

$$f'(x) = \frac{1}{2}\left[g'(x) - g'(2-x)\right]$$

$$f'\left(\tfrac{1}{2}\right) = \frac{1}{2}\left[\cancel{g'\left(\tfrac{1}{2}\right)} - \cancel{g'\left(\tfrac{3}{2}\right)}\right] = 0$$

(A) $f''(x) = 0$ for atleast two x in $(0, 2)$

(B) $f'\left(\frac{3}{2}\right) + f'\left(\frac{1}{2}\right) = 1$

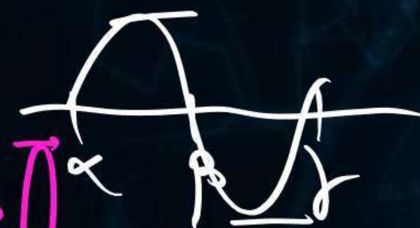
(C) $f''(x) = 0$ for no x in $(0, 1)$

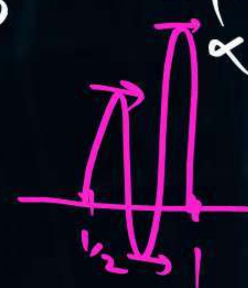
(D) $f''(x) = 0$ for exactly one x in $(0, 1)$

$$\boxed{f'\left(\tfrac{1}{2}\right) = 0}$$

$$f'\left(\tfrac{3}{2}\right) = 0$$

$$f'(1) = 0$$





QUESTION [JEE MAIN – 2024 (II) (30 Jan)]

$392 + 216$
$= 608$

Let $\boxed{f(x) = (x+3)^2 (x-2)^3}$, $\boxed{x \in [-4, 4]}$. If M and m are the maximum and minimum values of f, respectively in [–4, 4], then the value of M – m is

[Ans. C]

(A) 108

(B) 392

(C) 608 ✓

(D) 600

$$f'(x) = (x+3)^2 \cdot 3(x-2)^2 + (x-2)^3 \cdot 2(x+3) = 0$$

$$= (x+3)(x-2)^2 [3(x+3) + 2(x-2)] = 0$$

$$(x+3)(x-2)^2 [5x+5] = 0$$

$$x = -3, 2, -1$$

$$f(-1) = 2^2 (-3)^3 = -27 \times 4$$

$$f(4) = 7^2 \cdot 2^3 =$$

$$f(-4) = (1)^2 (-6)^3 = -216.$$

x	y
-3	0
2	0
-1	-27×4
-4	$-216 \rightarrow m$
4	49×8
	M

$400 - 8 = 392$

QUESTION [JEE MAIN – 2024 (I) (01 Feb)]

If $5f(x) + 4f\left(\frac{1}{x}\right) = x^2 - 2$, $\forall x \neq 0$ and $y = 9x^2 f(x)$, then y is strictly increasing in:

[Ans. B]

(A) $\left(0, \frac{1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right)$

(B) $\left(-\frac{1}{\sqrt{5}}, 0\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right)$

(C) $\left(-\frac{1}{\sqrt{5}}, 0\right) \cup \left(0, \frac{1}{\sqrt{5}}\right)$

(D) $\left(-\infty, \frac{1}{\sqrt{5}}\right) \cup \left(0, \frac{1}{\sqrt{5}}\right)$

$\boxed{5f(x) + 4f(1/x) = x^2 - 2}$

$x \to 1/x$

$\boxed{5f(1/x) + 4f(x) = \frac{1}{x^2} - 2}$

Eliminate $f(1/x)$ to get $f(x) = \checkmark$

Integration

Indefinite Integration

- By substitution, Parts, Integrals by Creating Negative Powers & Algebraic Twins.

Definite Integration

- Properties of Definite Integration,
- Newton Leibnitz Law,
- Solving Limit of a sum.

#

QUESTION [JEE MAIN – 2024 (II) (27 Jan)]

The integral $\int \frac{(x^8-x^2)dx}{(x^{12}+3x^6+1)\tan^{-1}\left(x^3+\frac{1}{x^3}\right)}$ equal to : [Ans. A]

(A) $\log_e\left(\left|\tan^{-1}\left(x^3+\frac{1}{x^3}\right)\right|\right)^{1/3}+C$

(B) $\log_e\left(\left|\tan^{-1}\left(x^3+\frac{1}{x^3}\right)\right|\right)^{1/2}+C$

(C) $\log_e\left(\left|\tan^{-1}\left(x^3+\frac{1}{x^3}\right)\right|\right)+C$

(D) $\log_e\left(\left|\tan^{-1}\left(x^3+\frac{1}{x^3}\right)\right|\right)^{3}+C$

$$\frac{x^8-x^2}{x^6\left[x^6+\frac{1}{x^6}+1\right]\tan^{-1}\left(x^3+\frac{1}{x^3}\right)}dx$$

$$\int\frac{(x^2-x^{-4})\,dx}{\left(x^6+\frac{1}{x^6}+3\right)\tan^{-1}\left(x^3+\frac{1}{x^3}\right)}$$

$x^3+\frac{1}{x^3}=t$

$(3x^2-3x^{-4})dx=dt$

$(x^2-x^{-4})dx=dt/3$

$$\int\frac{dt/3}{(t^2+1)(\tan^{-1}t)}$$

$\tan^{-1}t=z$

$$\frac{1}{3}\int\frac{dz}{z}=\frac{1}{3}\ln\left(\tan^{-1}\left(x^3+\frac{1}{x^3}\right)\right)+C$$

QUESTION [JEE MAIN – 2024 (I) (29 Jan)]

For $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, if $y(x) = \int \frac{\operatorname{cosec} x + \sin x}{\operatorname{cosec} x \sec x + \tan x \sin^2 x} dx$ and $\lim_{x \to \left(\frac{\pi}{2}\right)^-} y(x) = 0$ then $y\left(\frac{\pi}{4}\right)$ is equal to [Ans. D]

(A) $\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

(B) $\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

(C) $-\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

(D) $\frac{1}{\sqrt{2}}\tan^{-1}\left(-\frac{1}{2}\right)$

$$\int \frac{\frac{1}{\sin x} + \sin x}{\frac{1}{\sin x \cos x} + \frac{\sin^3 x}{\cos x}} dx$$

$$\int \frac{(1 + \sin^2 x)\cos x \, dx}{1 + \sin^4 x}$$

$\boxed{\sin x = t}$

$$\int \frac{(1+t^2)dt}{1+t^4}.$$

$$\int \frac{\left(1 + \frac{1}{t^2}\right)dt}{t^2 + \frac{1}{t^2}}$$

$$\int \frac{\left(1 + \frac{1}{t^2}\right)dt}{\left(t - \frac{1}{t}\right)^2 + 2}$$

$t - \frac{1}{t} = z$

$$\int \frac{dz}{z^2+2}$$

$$\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t-\frac{1}{t}}{\sqrt{2}}\right) + c$$

$$y(x) = \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{\sin x - \frac{1}{\sin x}}{\sqrt{2}}\right) + c$$

$x \to \pi/2$

$$\boxed{c = 0}$$

$$y(\pi/4) = \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{\frac{1}{\sqrt{2}} - \sqrt{2}}{\sqrt{2}}\right)$$

$$= \frac{1}{\sqrt{2}} \tan^{-1}\left(-\frac{1}{2}\right)$$

QUESTION [JEE MAIN – 2024 (II) (30 Jan)]

$\boxed{\begin{matrix}\alpha = 36 \\ \beta = -10\end{matrix}}$

Let $y = f(x)$ be a thrice differentiable function in $(-5, 5)$. Let the tangents to the curve $y = f(x)$ at $(1, f(1))$ and $(3, f(3))$ make angles $\frac{\pi}{6}$ and $\frac{\pi}{4}$, respectively with positive x-axis. If $27\int_1^3 ((f'(t))^2 + 1)f''(t)dt = \alpha + \beta\sqrt{3}$ where α, β are integers, then the value of $\alpha + \beta$ equals

[Ans. B]

(A) – 14

(B) 26

(C) – 16

(D) 36

$$\boxed{\begin{matrix} f'(1) = \frac{1}{\sqrt{3}} \\ f'(3) = 1 \end{matrix}}$$

$$27\int_1^3 ((f'(t))^2 + 1) f''(t)\,dt$$

$$\boxed{f'(t) = z}$$

$$27\int_{1/\sqrt{3}}^{1} (z^2+1)\,dz = 27\left[\frac{z^3}{3} + z\right]$$

$$= 9z^3 + 27z \Big|_{1/\sqrt{3}}^{1}$$

$$36 - \left[\frac{9}{3\sqrt{3}} + \frac{27}{\sqrt{3}}\right]$$

$$36 - \left(\frac{30}{\sqrt{3}}\right)$$

$$= 36 - 10\sqrt{3}$$

QUESTION [JEE MAIN – 2024 (II) (31 Jan)]

Let f, g: (0, ∞) → R be two functions defined by

$\boxed{f(x) = \int_{-x}^{x} (|t| - t^2)e^{-t^2}\,dt}$ and $\boxed{g(x) = \int_{0}^{x^2} t^{1/2}e^{-t}\,dt}$.

Then the value of $9\left(f\left(\sqrt{\log_e 9}\right) + g\left(\sqrt{\log_e 9}\right)\right)$ is equal to

(A) 6

(B) 9

(C) 8 ✓

(D) 10

[Ans. C]

$$f(x) = 2\int_0^x (t - t^2)e^{-t^2}\,dt$$

$$\boxed{t^2 = z}$$

$$f(x) = \int_0^x (1-t)e^{-t^2}\,2t\,dt$$

$$f(x) = \int_0^{x^2} (1-\sqrt{z})e^{-z}\,dz$$

$$f(x) = \int_0^{x^2} e^{-z}\,dz - \underbrace{\int_0^{x^2} \sqrt{z}\,e^{-z}\,dz}_{g(x)}$$

$$f(x) = \frac{e^{-z}}{-1}\Big|_0^{x^2} - g(x)$$

$$f(x) + g(x) = e^0 - e^{-x^2}$$

$$= 1 - e^{-x^2}$$

$$x = \sqrt{\ln 9}$$

$$f(x) + g(x) = 1 - e^{-\ln 9}$$

$$= 1 - \frac{1}{9} = 8/9$$

QUESTION [JEE MAIN – 2024 (I) (31 Jan)]

$p = 0^2 + 2^2 = 4 \Rightarrow p^2 = 16$

$q = 5 + \frac{1}{2} = \frac{11}{2} \quad 2q = 11$

[Ans. 27]

Let $S = (-1, \infty)$ and $f : S \to \mathbb{R}$ be defined as

$$f(x) = \int_{-1}^{x} (e^t - 1)^{11}(2t - 1)^5(t - 2)^7(t - 3)^{12}(2t - 10)^{61}\,dt$$

Let p = sum of square of the values of x, where f(x) attains local maxima on S. and q = sum of the values of x, where f(x) attains local minima on S. Then, the value of $p^2 + 2q$ is ___

$$f'(x) = (e^x - 1)^{11}(2x-1)^5(x-2)^7(x-3)^{12}(2x-10)^{61} = 0$$

$+ \quad - \quad + \quad - \quad - \quad +$

$0 \quad \frac{1}{2} \quad 2 \quad 3 \quad 5$

0: max, $\frac{1}{2}$: min, 2: max, 5: min

QUESTION [JEE MAIN – 2024 (II) (29 Jan)]

[Ans. 6]

If $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sqrt{1-\sin 2x}\,dx = \alpha + \beta\sqrt{2} + \gamma\sqrt{3}$,

where α, β and γ are rational numbers, then $3\alpha + 4\beta - \gamma$ is equal to

$$\int \sqrt{(\cos x - \sin x)^2}\,dx$$

$$\int_{\pi/6}^{\pi/3} |\cos x - \sin x|\,dx$$

$$\int_{\pi/6}^{\pi/4} (\cos x - \sin x)\,dx + \int_{\pi/4}^{\pi/3} (\sin x - \cos x)\,dx = \text{Ans}$$

QUESTION [JEE MAIN – 2024 (I) (01 Feb)]

$$2I = \int_{-\pi/2}^{\pi/2} \frac{8\sqrt{2}\cos x\,dx}{1+\sin^4 x}$$

[Ans. 8]

$I =$ If $\displaystyle\int_{-\pi/2}^{\pi/2} \frac{8\sqrt{2}\cos x\,dx}{\underbrace{(1+e^{\sin x})}(1+\sin^4 x)} = \alpha\pi + \beta\log_e(3+2\sqrt{2})$, → (1)

where α, β are integers, then $\alpha^2+\beta^2$ equals

$x \to -x$

$$I = \int_{-\pi/2}^{\pi/2} \frac{1}{1+e^{-\sin x}} \frac{8\sqrt{2}\cos x}{1+\sin^4 x}$$

$$\int \frac{e^{\sin x}}{1+e^{\sin x}} \frac{8\sqrt{2}\cos x}{1+\sin^4 x} dx \to (2)$$

$$\cancel{2}I = \cancel{2}\int_0^{\pi/2} \frac{8\sqrt{2}\cos x\,dx}{1+\sin^4 x}$$

$\sin x = t$

$$I = \int_0^1 \frac{8\sqrt{2}\,dt}{1+t^4}$$

$$I = 4\sqrt{2}\int \frac{(1+t^2)+(1-t^2)}{(1+t^4)} dx.$$

QUESTION [JEE MAIN – 2024 (II) (27 Jan)]

$g(-x) = -g(x)$

Let $f(x) = \int_0^x g(t)\log_e\left(\frac{1-t}{1+t}\right)dt$, where g is $\underbrace{\text{a continuous odd}}$ function. [Ans. 2]

If $\int_{-\pi/2}^{\pi/2}\left(f(x) + \frac{x^2\cos x}{1+e^x}\right)dx = \left(\frac{\pi}{\alpha}\right)^2 - \alpha$, then α is equal to

$\boxed{t = -z}$ ✓

$$f(x) = \int_0^{-x} g(-z).\ln\left(\frac{1+z}{1-z}\right)(-dz)$$

$$f(x) = -\underbrace{\int_0^{-x} g(z)\ln\left(\frac{1-z}{1+z}\right)dz}_{f(-x)}$$

$\boxed{f(x) = -f(-x)}$

$\Rightarrow f(x)$ is odd

$$I = \int_{-\pi/2}^{\pi/2} \frac{x^2\cos x}{1+e^x}dx$$

$$2I = \int_{-\pi/2}^{\pi/2} x^2\cos x\,dx \Rightarrow I = \int_0^{\pi/2} x^2\cos x$$

QUESTION [JEE MAIN – 2024 (I) (01 Feb)]

The value of the integral $\int_0^{\frac{\pi}{4}} \frac{x\,dx}{\sin^4(2x)+\cos^4(2x)}$ equals :

[Ans. C]

(A) $\frac{\sqrt{2}\pi^2}{8}$

(B) $\frac{\sqrt{2}\pi^2}{16}$

(C) $\frac{\sqrt{2}\pi^2}{32}$

(D) $\frac{\sqrt{2}\pi^2}{64}$

$2x = \theta$

$dx = d\theta/2$

$$I = \int_0^{\pi/2} \frac{\frac{\theta}{2}\frac{d\theta}{2}}{\sin^4\theta+\cos^4\theta}$$

$$= \frac{1}{4}\int_0^{\pi/2} \frac{\theta\, d\theta}{\sin^4\theta+\cos^4\theta}$$

$$I = \frac{1}{4}\int \frac{(\pi/2-\theta)}{\sin^4\theta+\cos^4\theta}\, d\theta$$

$$2I = \frac{1}{4}\frac{\pi/2}{\sin^4+\cos^4\theta}\, d\theta$$

$$I = \frac{\pi}{16}\int_0^{\pi/2} \frac{d\theta}{\sin^4\theta+\cos^4\theta}$$

$$\frac{\pi}{16}\int_0^{\pi/2} \frac{d\theta}{1-2\sin^2\theta\cos^2\theta}$$

$$\frac{\pi}{16}\int_0^{\pi/2} \frac{d\theta}{1-\frac{\sin^2 2\theta}{2}}$$

$$\int \frac{dx}{a+b\sin^2 x}$$

divide $\cos^2 x$

by

Solving Limit of a Sum

Steps to convert limit of a sum into a Definite Integral.

Step I: Express the given series in the form $\lim\limits_{n\to\infty}\sum$

Step II: Replace $\frac{r}{n}$ by x and $\frac{1}{n}$ by dx and $\lim\limits_{n\to\infty}\sum$ by the sign of $\int$

Step III: Find the range of x for definite integral by finding the limiting values of $\frac{r}{n}$ for the first and the last term of r respectively.

QUESTION [JEE MAIN – 2024 (I) (30 Jan)]

The value of $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{n^3}{(n^2+k^2)(n^2+3k^2)}$ is:

(A) $\frac{(2\sqrt{3}+3)\pi}{24}$

(B) $\frac{13\pi}{8(4\sqrt{3}+3)}$

(C) $\frac{13(2\sqrt{3}-3)\pi}{8}$ ∞

(D) $\frac{\pi}{8(2\sqrt{3}+3)}$

[Ans. B]

$1 \le r \le n$

$0 \le r/n \le 1$

$$\sum_{r=1}^{n}\frac{n^3}{(n^2+r^2)(n^2+3r^2)}$$

$$\frac{\cancel{n^3}}{\cancel{n^2}\left[1+\frac{r^2}{n^2}\right]\cdot \cancel{n^2}\left[1+3\frac{r^2}{n^2}\right]}$$

$$\int_0^1\frac{dx}{(1+x^2)(1+3x^2)}$$

$$\frac{1}{2}\int_0^1\frac{3(1+x^2)-(1+3x^2)}{(1+x^2)(1+3x^2)}$$

$$\frac{1}{2}\int_0^1\frac{3}{1+3x^2}dx-\frac{1}{2}\int_0^1\frac{dx}{1+x^2}$$

$$\frac{1}{2}\int\frac{dx}{x^2+1/3}-\frac{1}{2}\tan^{-1}x\Big|_0^1$$

$$\frac{1}{2}\cdot\frac{1}{\frac{1}{\sqrt{3}}}\tan^{-1}\frac{x\sqrt{3}}{1}\Big|_0^1-\frac{1}{2}\cdot\pi/4$$

$$\frac{\sqrt{3}}{2}\cdot\pi/3-\pi/8$$

$$\frac{4\sqrt{3}\pi-3\pi}{24}$$

$$\frac{(4\sqrt{3}-3)\pi}{24} \quad \frac{4\sqrt{3}+3}{4\sqrt{3}+3}$$

$$\frac{(48-9)\pi}{24(4\sqrt{3}+3)}$$

$$\frac{39\pi}{24(4\sqrt{3}+3)}$$

$$\frac{13\pi}{8(4\sqrt{3}+3)}$$

QUESTION [JEE MAIN – 2023 (II) (30 Jan)]

HW

$$\lim_{n\to\infty} \frac{3}{n}\left\{4+\left(2+\frac{1}{n}\right)^2+\left(2+\frac{2}{n}\right)^2+\cdots+\left(3-\frac{1}{n}\right)^2\right\}$$ is equal to

[Ans. [redacted]

(A) 12

(B) $\frac{19}{3}$

(C) 0

(D) 19

Newton Leibnitz Law

Derivative of a Function Expressed in Antiderivative Form

If $h(x)$ and $g(x)$ are differentiable functions of x then,

$$\frac{d}{dx}\int_{g(x)}^{h(x)} f(t)dt = f[h(x)].h'(x) - f[g(x)].g'(x)$$

QUESTION [JEE MAIN – 2024 (I) (30 Jan)]

Let $f : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \rightarrow R$ be a differentiable function such that $f(0) = \frac{1}{2}$. [Ans. B]

If the $\lim_{x \to 0} \frac{x \int_0^x f(t)dt}{e^{x^2} - 1} = \alpha$, then $8\alpha^2$ is equal to :

$\frac{e^{x^2}-1}{x^2} x^2$

$\downarrow$

1

(A) 16

(B) 2 ✓

(C) 1

(D) 4

$$\frac{\int_0^x f(t)\,dt}{x}$$

$$\lim_{x \to 0} \frac{f(x)}{1} = f(0) = \frac{1}{2}$$

$\alpha = \frac{1}{2}$

$\alpha^2 = \frac{1}{4}$

$8\alpha^2 = 2$

QUESTION [JEE MAIN – 2024 (I) (29 Jan)]

[Ans. C]

$$\lim_{x \to \frac{\pi}{2}} \left(\frac{1}{\left(x-\frac{\pi}{2}\right)^2} \int_{x^3}^{\left(\frac{\pi}{2}\right)^3} \cos(t)^{\frac{1}{3}} \, dt \right)$$ is equal to

(A) $\frac{3\pi}{8}$

(B) $\frac{3\pi^2}{4}$

(C) $\frac{3\pi^2}{8}$

(D) $\frac{3\pi}{4}$

$$-\frac{(\cos x)\, 3x^2}{2\left(x-\pi/2\right)}$$

$$1 \leftarrow \frac{\sin\left(x-\pi/2\right)}{2\left(x-\pi/2\right)}\, 3x^2$$

$$= \frac{3}{2}\left(\pi/2\right)^2 = \frac{3\pi^2}{8}$$

QUESTION [JEE MAIN – 2024 (II) (31 Jan)]

[Ans. D]

Let a be the sum of all coefficients in the expansion of

$\boxed{a = 1}$ ✓

$(1-2x+2x^2)^{2023}(3-4x^2+2x^3)^{2024}$ and $b = \lim_{x\to 0}\left(\frac{\int_0^x \frac{\log(1+t)}{t^{2024}+1}dt}{x^2}\right)$.

If the equations $cx^2+dx+e=0$ and $2bx^2+ax+4=0$ have a common root, where c, d, e ∈ R, then d : c : e equals

(A) 2 : 1 : 4

(B) 4 : 1 : 4

(C) 1 : 2 : 4

(D) 1 : 1 : 4

$b = \dfrac{\frac{\ln(1+x)}{x^{2024}+1}}{2x} = \dfrac{1}{2(x^{2024}+1)} = \dfrac{1}{2}$

$\boxed{x^2+x+4=0}$ → non real

$cx^2+dx+e=0$

$\frac{c}{1} = \frac{d}{1} = \frac{e}{4} = K$

$c = K,\ d = K$

$e = 4K$

Area Under Curve

Area Under Curve (Application of Integrals)

- Area bounded between 2 Curves.
- Region Based Questions (Inequalities)
- The area bounded by $y^2 = 4ax$ & $x^2 = 4ay$ is $16ab/3$
- The area bounded by $y^2 = 4ax$ & $Y = mx$ is $\frac{8a^2}{3m^3}$

QUESTION [JEE MAIN – 2024 (II) (27 Jan)]

$18-9$ $\quad 6x-x^2$

$6-2x$

If the area of the region $\{(x, y): 0 \leq y \leq \min\{2x,\ 6x - x^2\}\}$ is A, then 12A is equal to

[Ans. 304]

$6x - x^2 = 2x$

$x^2 = 4x$

$\boxed{x = 4}$

$$A = 16 + \int_4^6 (6x - x^2)\,dx$$

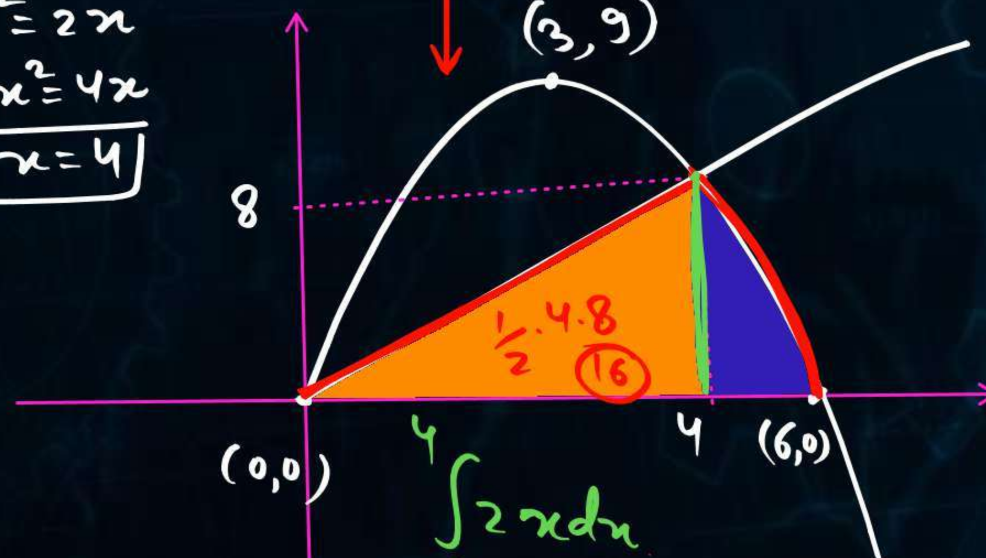
$$A = 16 + 3x^2\Big|_4^6 - \frac{x^3}{3}\Big|_4^6$$

$$16 + 3[20] - \frac{1}{3}[216 - 64]$$

$$= 76 - \frac{1}{3} \cdot 152$$

$$= 76\left(1 - \frac{2}{3}\right) = \frac{76}{3} = A$$

$$\Rightarrow 12A = 76 \times 4 = 304$$



$$\int_0^4 2x\,dx$$

$$x^2\Big|_0^4 = 16 - 0 = 16$$

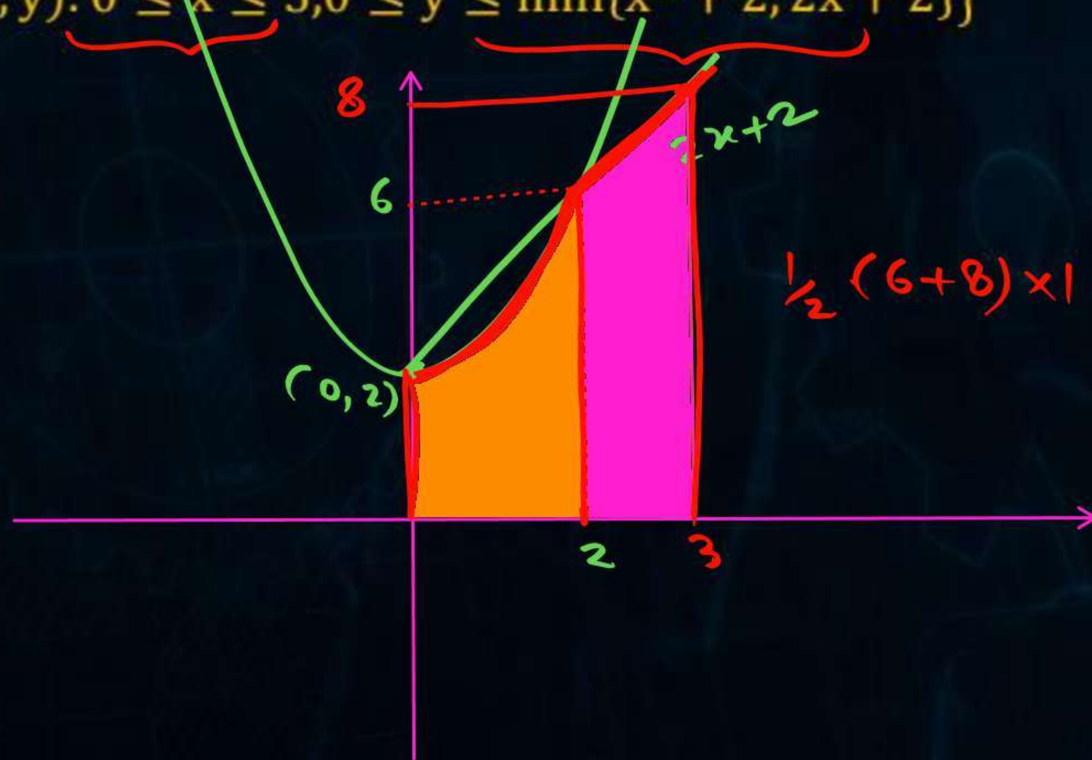
QUESTION [JEE MAIN – 2024 (II) (29 Jan)]

Let the area of the region $\{(x, y): \underbrace{0 \le x \le 3}, 0 \le y \le \underbrace{\min\{x^2 + 2, 2x + 2\}}\}$ be A. Then 12A is equal to

[Ans. 164]

$x^2 + \cancel{2} = 2x + \cancel{2}$

$\boxed{x = 2}$



$\frac{1}{2}(6+8)\times 1 = 7$

$$A = \int_0^2 (x^2+2)\,dx + 7$$

$$= \frac{x^3}{3} + 2x \Big|_0^2 + 7$$

$$= 8/3 + 11 = \frac{41}{3} = A$$

$$12A = 4\times 41$$
$$= 164$$

QUESTION [JEE MAIN – 2024 (I) (30 Jan)]

The area (in square units) of the region bounded by the parabola $y^2 = 4(x-2)$ and the line $y = 2x - 8$

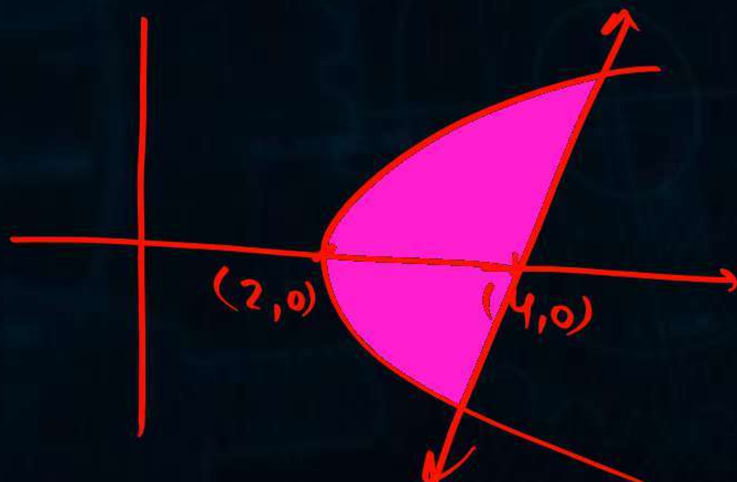
[Ans. [redacted]

(A) 8

(B) 9

(C) 6

(D) 7



Differential Equations

QUESTION [JEE MAIN – 2024 (II) (31 Jan)]

The temperature T(t) of a body at time $t = 0$ is $160°F$ and it decreases continuously as per the differential equation $\boxed{\frac{dT}{dt} = -K(T - 80)}$, where K is positive constant. If $T(15) = 120°F$, then $T(45)$ is equal to

[Ans. C]

(A) 85°F

(B) 95°F

(C) 90°F

(D) 80°F

$$\int \frac{dT}{T-80} = -\int K\,dt$$

$$\ln(T-80) = -Kt + c$$

$$\ln 80 = -K\times 0 + c$$

$$c = \ln 80$$

$$\ln(T-80) = -Kt + \ln 80$$

$$\ln\left(\frac{T-80}{80}\right) = -Kt$$

$$\frac{T-80}{80} = e^{-Kt}$$

$$T - 80 = 80(e^{-Kt})$$

$$\boxed{T = 80(1+e^{-Kt})}$$

$$120 = 80(1+e^{-K\cdot 15})$$

$$\frac{12\cancel{0}}{8\cancel{0}} = 1 + e^{-15K}$$

$$3/2 - 1 = e^{-15K}$$

$$\boxed{e^{-15K} = 1/2} \Rightarrow e^{-45K} = 1/8$$

$$T(45) = 80\left(1 + e^{-45K}\right)$$

$$80\left(1 + \frac{1}{8}\right)$$

$$80 \times \frac{9}{8}$$

$$= 90.$$

QUESTION [JEE MAIN – 2024 (II) (01 Feb)]

Let α be a non-zero real number. Suppose $f: R \to R$ is a differentiable function such that $f(0) = 2$ and $\lim_{x \to -\infty} f(x) = 1$. If $f'(x) = \alpha f(x) + 3$, for all $x \in R$, then $f(-\log_e 2)$ is equal to

[Ans. C]

(A) 3

(B) 5

(C) 9

(D) 7

$$\boxed{dy/dx = \alpha y + 3}$$

$$\int \frac{dy}{\alpha y + 3} = \int dx$$

$$\frac{\ln(\alpha y + 3)}{\alpha} = x + c$$

$$\frac{\ln(\alpha 2 + 3)}{\alpha} = c$$

$$c\alpha = \ln(2\alpha + 3)$$

$$\ln(\alpha y + 3) = \alpha x + c\alpha$$

$$\ln(\alpha y + 3) = \alpha x + \ln(2\alpha + 3)$$

$$\ln\frac{(\alpha y + 3)}{(2\alpha + 3)} = \alpha x$$

$$\boxed{\frac{\alpha y + 3}{2\alpha + 3} = e^{\alpha x}}$$

$x \to -\infty$

$$\alpha(1) + 3 = 0 \Rightarrow \boxed{\alpha = -3}$$

$$\frac{-3y+3}{-3} = e^{-3x}$$

$$\frac{-3y+3}{-3} = e^{-3(-\ln 2)}$$

$$\frac{-3y+3}{-3} = 8$$

$$\frac{-y+1}{-1} = 8$$

$$-y+1 = -8$$

$$\boxed{y=9}$$

QUESTION [JEE MAIN – 2024 (I) (31 Jan)]

Let $y = y(x)$ be the solution of the differential equation

[Ans. A]

$\frac{dy}{dx} = \frac{(\tan x)+y}{\sin x(\sec x - \sin x \tan x)}$, $x \in \left(0, \frac{\pi}{2}\right)$ satisfying the condition $y\left(\frac{\pi}{4}\right) = 2$.

Then, $y\left(\frac{\pi}{3}\right)$ is

(A) $\sqrt{3}(2 + \log_e \sqrt{3})$

(B) $\frac{\sqrt{3}}{2}(2 + \log_e 3)$

(C) $\sqrt{3}(1 + 2\log_e 3)$

(D) $\sqrt{3}(2 + \log_e 3)$

$$\frac{dy}{dx} = \frac{\tan x + y}{\sin x\left[\frac{1}{\cos x} - \frac{\sin^2 x}{\cos x}\right]}$$

$$\frac{dy}{dx} = \frac{\tan x + y}{\tan x\left[\cos^2 x\right]}$$

$$\frac{dy}{dx} = \frac{y\sec^2 x}{\tan x} + \sec^2 x$$

$$\frac{dy}{dx} - \frac{y\sec^2 x}{\tan x} = \sec^2 x$$

$$IF = e^{-\int \frac{\sec^2 x}{\tan x} dx}$$

$$e^{-\ln(\tan x)}$$

$$= \frac{1}{\tan x}$$

$$\frac{y}{\tan x} = \int \frac{1}{\tan x} \cdot \sec^2 x \, dx$$

$$= \ln(\tan x) + c$$

$$\frac{y}{\tan x} = \ln(\tan x) + C$$

$$\frac{y}{1} = \ln 1 + C$$

$$\boxed{2 = C}$$

$$\boxed{\frac{y}{\tan x} = \ln(\tan x) + 2}$$

$$\frac{y}{\sqrt{3}} = (\ln\sqrt{3}) + 2$$

QUESTION [JEE MAIN – 2024 (II) (01 Feb)]

If $\frac{dx}{dy}=\frac{1+x-y^2}{y}$, $x(1)=1$, then $5x(\underbrace{2}_{1})$ is equal to : [Ans. 5]

$$\frac{dx}{dy}-x/y=\frac{1-y^2}{y}$$

$$IF = e^{-\int \frac{1}{y}dy}=e^{-\ln y}=\frac{1}{y}$$

$$x/y=\int \frac{1-y^2}{y}\cdot\frac{1}{y}\,dy$$

$$x/y=\int\left(\frac{1}{y^2}-1\right)dy$$

$$\boxed{x/y=-\frac{1}{y}-y+c}$$

$$x/y+\frac{1}{y}+y=3$$

$$\frac{x}{2}+\frac{1}{2}+2=3$$

$$x/2=\frac{1}{2}$$

$$\boxed{x=1}$$

QUESTION [JEE MAIN – 2024 (I) (01 Feb)]

If $x = x(t)$ is the solution of the differential equation $(t+1)dx = (2x + (t+1)^4)dt$, $x(0) = 2$, then, $x(1)$ equals

[Ans. [redacted]

QUESTION [JEE MAIN – 2024 (II) (29 Jan)]

If $\boxed{\sin\left(\frac{y}{x}\right) = \log_e|x| + \frac{\alpha}{2}}$ is the solution of the differential equation $\boxed{x\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x}$ and $y(1) = \frac{\pi}{3}$, then α^2 is equal to

[Ans. A]

(A) 3 ✓

(B) 12

(C) 4

(D) 9

$x = 1$

$y = \pi/3$

$\sin \pi/3 = \cancel{\ln 1} + \alpha/2$

$\sqrt{3}/\cancel{2} = \alpha/\cancel{2}$

$\boxed{\alpha = \sqrt{3}}$

$\boxed{M-2}$

$\cos(y/x)\, dy/dx = y/x \cos(y/x) + 1$

$y = vx$

$\cos v\left[v + x\frac{dv}{dx}\right] = v\cos v + 1$

$x\cos v \frac{dv}{dx} = 1$

$\int \cos v\, dv = \int \frac{dx}{x}$

$\sin v = \ln x + c$

Equation Reducible to Homogenous D.E.

Consider $\boxed{\frac{dy}{dx} = \frac{a_1x+b_1y+c_1}{a_2x+b_2y+c_2}}$

Case I: If given lines are parallel $\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ then substitute

$\boxed{a_1x + b_1y = t}$

Case II: If $a_2 + b_1 = 0$, then a simple cross multiplication and substituting d(xy) for xdy + ydx and integrating term by term, yield the results easily.

Case-III: If the lines are neither parallel nor $a_2 + b_1 = 0$, then the equation is reducible to homogeneous

QUESTION [JEE MAIN – 2024 (I) (27 Jan)]

[Ans. [redacted]

If the solution of the differential equation

$(2x + 3y - 2)dx + (4x + 6y - 7)dy = 0, y(0) = 3$, is

$\alpha x + \beta y + 3\log_e|2x + 3y - \gamma| = 6$, then $\alpha + 2\beta + 3\gamma$ is equal to

HW

QUESTION [JEE MAIN – 2022 (I) (27 June)]

$x^2+y^2+2x-2y+\frac{2d}{a}=0$, $\frac{2d}{a} \to K$

$\boxed{x^2+y^2+2x-2y+K=0}$

Let $\frac{dy}{dx}=\frac{ax-by+a}{bx+cy+a}$, where a, b, c are constants, represent a circle passing through the point (2, 5). Then the shortest distance of the point (11, 6) from this circle is : **[Ans. B]**

(A) 10

(B) 8

(C) 7

(D) 5

$bx\,dy + cy\,dy + a\,dy = ax\,dx - by\,dx + a\,dx$

$\int b\,d(xy) + \int cy\,dy + \int a\,dy = \int ax\,dx + \int a\,dx$

$bxy + \frac{cy^2}{2} + ay = \frac{ax^2}{2} + ax + d$

$b = 0$

$\frac{ax^2}{2} + ax - ay - \frac{cy^2}{2} + d = 0$

$\frac{a}{2} = -\frac{c}{2}$

$\boxed{a=-c}$

$\boxed{\frac{ax^2}{2} + \frac{ay^2}{2} + ax - ay + d = 0}$

QUESTION [JEE MAIN – 2024 (II) (27 Jan)]

If the solution curve, of the differential equation $\boxed{\frac{dy}{dx} = \frac{x+y-2}{x-y}}$ passing through the point $(2,1)$ is $\tan^{-1}\left(\frac{y-1}{x-1}\right) - \frac{1}{\beta}\log_e\left(\alpha + \left(\frac{y-1}{x-1}\right)^2\right) = \log_e|x-1|$, then $5\beta + \alpha$ is equal to

[Ans. 11]

Vectors

QUESTION [JEE MAIN – 2024 (I) (30 Jan)]

Let A(2,3,5) and C(−3,4,−2) be opposite vertices of a parallelogram ABCD if the diagonal $\overrightarrow{BD} = \hat{i} + 2\hat{j} + 3\hat{k}$ then the area of the parallelogram is equal to

[Ans. B]

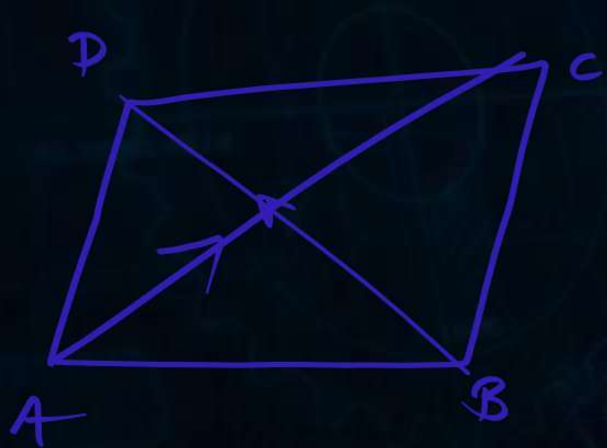
(A) $\frac{1}{2}\sqrt{410}$

(B) $\frac{1}{2}\sqrt{474}$

(C) $\frac{1}{2}\sqrt{586}$

(D) $\frac{1}{2}\sqrt{306}$

$A = \frac{1}{2}|\vec{d_1} \times \vec{d_2}|$



$\vec{AC} = -5\hat{i} + \hat{j} - 7\hat{k}$

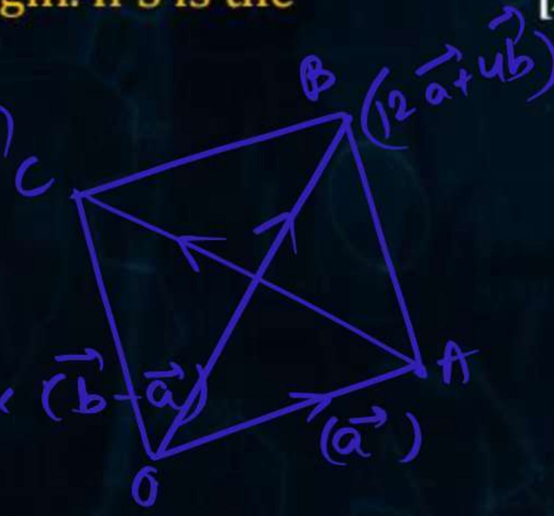
$\vec{BD} = \hat{i} + 2\hat{j} + 3\hat{k}$

QUESTION [JEE MAIN – 2024 (II) (29 Jan)]

[Ans. D]

Let $\overrightarrow{OA} = \vec{a}, \overrightarrow{OB} = 12\vec{a} + 4\vec{b}$ and $\overrightarrow{OC} = \vec{b}$, where O is the origin. If S is the parallelogram with adjacent sides OA and OC, then

$$\frac{\text{area of the quadrilateral OABC}}{\text{area of s is equal to}}$$ is equal to ______.



$S = |\vec{a} \times \vec{b}|$

area of Quad OABC $= \frac{1}{2}(12\vec{a}+4\vec{b}) \times (\vec{b}-\vec{a})$

$= 2(3\vec{a}+\vec{b}) \times (\vec{b}-\vec{a})$

$= 2(3\vec{a}\times\vec{b} + \vec{a}\times\vec{b})$

$= 8(\vec{a}\times\vec{b})$

(A) 6

(B) 10

(C) 7

(D) 8 ✓

QUESTION [JEE MAIN – 2024 (I) (31 Jan)]

Let $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = 4\hat{i} + \hat{j} + 7\hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} + 4\hat{k}$ be three vectors. If a vectors $\vec{p}$ satisfies $\vec{p} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{p} \cdot \vec{a} = 0$, then $\vec{p} \cdot (\hat{i} - \hat{j} - \hat{k})$ is equal to

[Ans. D]

(A) 24

(B) 36

(C) 28

(D) 32

$$\vec{p} \times \vec{b} - \vec{c} \times \vec{b} = 0$$
$$(\vec{p} - \vec{c}) \times \vec{b} = 0$$
$$\vec{p} - \vec{c} = \lambda \vec{b}$$
$$\vec{p} = \vec{c} + \lambda \vec{b}$$
$$\vec{p} = (\hat{i} - 3\hat{j} + 4\hat{k}) + \lambda(4\hat{i} + \hat{j} + 7\hat{k})$$
$$\vec{p} = (1+4\lambda)\hat{i} + (\lambda - 3)\hat{j} + (7\lambda + 4)\hat{k}$$
$$\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$$
$$\vec{p} \cdot \vec{a} = 0$$
$$\lambda = $$

QUESTION [JEE MAIN – 2023 (I) (31 Jan)]

Let $\vec{a}$ and b be two vector such that $|\vec{a}| = \sqrt{14}$, $|\vec{b}| = \sqrt{6}$ and $|\vec{a} \times \vec{b}| = \sqrt{48}$. Then $(\vec{a} \cdot \vec{b})^2$ is equal to

[Ans. 36]

$$|\vec{a}|^2 |\vec{b}|^2 = (\vec{a} \cdot \vec{b})^2 + |\vec{a} \times \vec{b}|^2$$

$$14 \times 6 = x + 48$$

$$x = 84 - 48$$

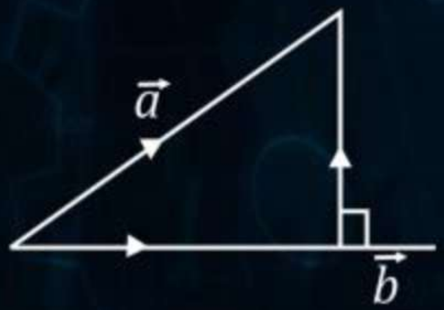
$$= \boxed{36}$$

Projection of $\vec{a}$ along $\vec{b}$

The scalar Projection of $\vec{a}$ along $\vec{b} = \left(\frac{\vec{a}.\vec{b}}{\vec{b}^2}\right)$

The vector Projection of $\vec{a}$ along $\vec{b} = \left(\frac{\vec{a}.\vec{b}}{\vec{b}^2}\right)\vec{b}$

The vector Projection of $\vec{a}$ perpendicular to $\vec{b}$ is $\vec{a} - \left(\frac{\vec{a}.\vec{b}}{\vec{b}^2}\right)\vec{b}$



QUESTION [JEE MAIN – 2024 (II) (01 Feb)]

Consider a $\triangle ABC$ where $A(1,3,2), B(-2,8,0)$ and $C(3,6,7)$. If the angle bisector of $\angle BAC$ meets the line BC at D, then the length of the projection of the vector $\overrightarrow{AD}$ on the vector $\overrightarrow{AC}$ is:

[Ans. A]

(A) $\dfrac{37}{2\sqrt{38}}$

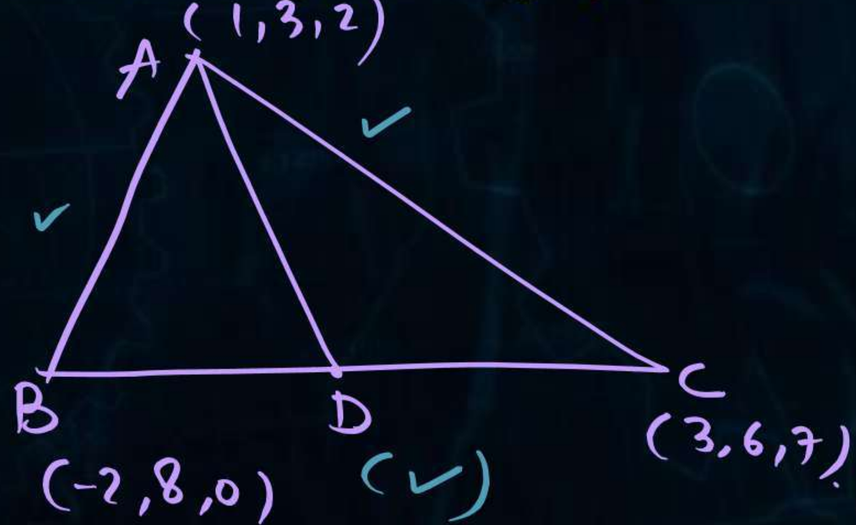
(B) $\dfrac{\sqrt{38}}{2}$

(C) $\dfrac{39}{2\sqrt{38}}$

(D) $\sqrt{19}$

$\dfrac{AB}{AC} = \dfrac{BD}{DC}$

$\overrightarrow{AD} \cdot \hat{AC} = \text{Ans}$



Linear Combination

Linear Combination in Plane

If the vectors $\vec{a}$ & $\vec{b}$ are two non zero non collinear vectors then a vector $\vec{r}$ lying in the plane of $\vec{a}$ & $\vec{b}$ can be written as $\boxed{\vec{r} = x\,\vec{a} + y\,\vec{b}}$, where x & y are scalars.

Linear Combination in Space (3D)

If the vectors $\vec{a}$, $\vec{b}$ & $\vec{c}$ are three non zero, non collinear, non coplanar vectors then a vector $\vec{r}$ lying in the Space can be written as $\boxed{\vec{r} = x\,\vec{a} + y\,\vec{b} + z\,\vec{c}}$, where x , y & z are scalars.

QUESTION [JEE MAIN – 2023 (I) (April)]

$(-1 \,\&\, 2)$

An arc PQ of a circle subtends a right angle at its centre O. The mid point of the arc PQ is R. If $\overrightarrow{OP} = \vec{u}$, $\overrightarrow{OR} = \vec{v}$, and $\overrightarrow{OQ} = \alpha\vec{u} + \beta\vec{v}$, then α, β^2, are the roots of the equation

[Ans. B]

(A) $x^2 + x - 2 = 0$

(B) $x^2 - x - 2 = 0$ ✓

(C) $3x^2 - 2x - 1 = 0$

(D) $3x^2 + 2x - 1 = 0$

$\vec{w} = \alpha\vec{u} + \beta\vec{v}$

dot with $\vec{w}$

$w \cdot w = \alpha\,\cancel{\vec{u}\cdot\vec{w}} + \beta\,\vec{v}\cdot\vec{w}$

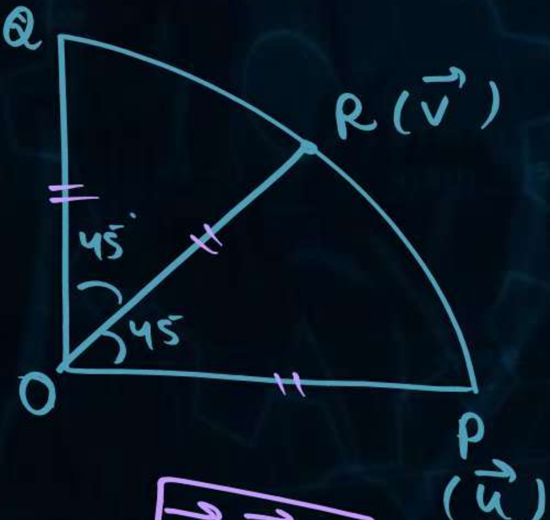
$\cancel{r^2} = \beta \cdot \cancel{r^2}\cos 45$

$\boxed{\beta = \sqrt{2}}$

dot with $\vec{u}$

$0 = \alpha \cancel{r^2} + \beta \cancel{r^2} \cos 45$

$\alpha + 1 = 0 \quad \boxed{\alpha = -1}$ ✓



$\boxed{\vec{u}\cdot\vec{w} = 0}$

$|u| = |w| = |v| = r$

QUESTION [JEE MAIN – 2022 (26 June)]

Let $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}, \vec{b} = 2\hat{i} - 3\hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} + \hat{k}$ be three given vectors. Let $\vec{v}$ be a vector in the plane of $\vec{a}$ and $\vec{b}$ whose projection on $\vec{c}$ is $\frac{2}{\sqrt{3}}$. If $\vec{v} \cdot \hat{j} = 7$, then $\vec{v} \cdot (\hat{i} + \hat{k})$ is equal to :

[Ans. D]

HW

(A) 6

(B) 7

(C) 8

(D) 9

3D Geometry

QUESTION [JEE MAIN – 2024 (I) (29 Jan)]

Let O be the origin and the position vector of A and B be $2\hat{i} + 2\hat{j} + \hat{k}$ and $2\hat{i} + 4\hat{j} + 4\hat{k}$ respectively. If the internal bisector of $\angle AOB$ meets the line AB at C, then the length of OC is.

[Ans. B]

(A) $\frac{2}{3}\sqrt{31}$

(B) $\frac{2}{3}\sqrt{34}$

(C) $\frac{3}{4}\sqrt{34}$

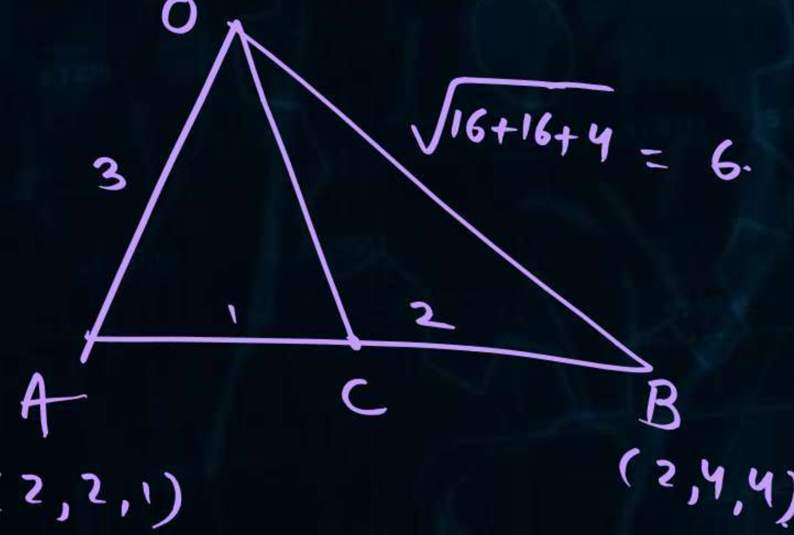
(D) $\frac{3}{2}\sqrt{31}$

$$\frac{OA}{OB} = \frac{AC}{CB} = 3/6 = 1/2$$

$$C\left[\frac{2+4}{3}, \frac{4+4}{3}, \frac{4+2}{3}\right]$$

$$C\left[2, 8/3, 2\right]$$

$$(OC) = \sqrt{4+\frac{64}{9}+4} = \sqrt{8}\sqrt{1+8/9} = \frac{\sqrt{17}}{3}\,2\sqrt{2}$$



QUESTION [JEE MAIN – 2024 (I) (27 Jan)]

The distance, of the point (7, −2, 11) from the line $\frac{x-6}{1}=\frac{y-4}{0}=\frac{z-8}{3}$ $= \mu$ along the line $\frac{x-5}{2}=\frac{y-1}{-3}=\frac{z-5}{6}$, is.

[Ans. B]

(A) 12

(B) 14 ✓

(C) 18

(D) 21

$(PQ) = \sqrt{4^2+6^2+12^2}$

$= 2\sqrt{4+9+36}$

$= 14$

eqⁿ of PQ

$\frac{x-7}{2}=\frac{y+2}{-3}=\frac{z-11}{6}=\lambda$

$x = 2\lambda+7$

$y = -3\lambda-2$

$z = 6\lambda+11$

$2\lambda+7 = \mu+6$

$-3\lambda-2 = 4$

$-3\lambda = 6 \Rightarrow \boxed{\lambda=-2}$

$-4+1 = \mu \Rightarrow \boxed{\mu=-3}$

$x = \mu+6$

$y = 4$

$z = 3\mu+8$

$Q\,[3,4,-1]$

QUESTION [JEE MAIN – 2024 (I) (27 Jan)]

If the shortest distance between the lines $\frac{x-4}{1}=\frac{y+1}{2}=\frac{z}{-3}$ and $\frac{x-\lambda}{2}=\frac{y+1}{4}=\frac{z-2}{-5}$ is $\frac{6}{\sqrt{5}}$, then the sum of all possible values of λ is:

(A) 5

(B) 8

(C) 7

(D) 10

[Ans. B]

$\vec{b_1} = \hat{i} + 2\hat{j} - 3\hat{k}$

$\vec{b_2} = 2\hat{i} + 4\hat{j} - 5\hat{k}$

$$d_{min} = \left| \frac{(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|} \right|$$

$(4, -1, 0)$

$(\lambda, -1, 2)$

$\vec{a_1} - \vec{a_2} = (4-\lambda)\hat{i} - 2\hat{k}$

QUESTION [JEE MAIN – 2024 (I) (30 Jan)]

If d_1 is the shortest distance between the lines $x + 1 = 2y = -12z$, $z = y + 2 = 6z - 6$ and d_2 is the shortest distance between the lines $\frac{x-1}{2} = \frac{y+8}{-7} = \frac{z-4}{5}$, $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-6}{-3}$, then the value of $\frac{32\sqrt{3}\,d_1}{d_2}$ is :

[Ans. 16]

HW

Probability

(2-3) ✓

QUESTION [JEE MAIN – 2024 (II) (01 Feb)]

$P(A) = 5/7$ ✓

Let Ajay will not appear in JEE exam with probability $p = \frac{2}{7}$, while both Ajay and Vijay will appear in the exam with probability $q = \frac{1}{5}$. Then the probability, that Ajay will appear in the exam and Vijay will not appear is :

[Ans. B]

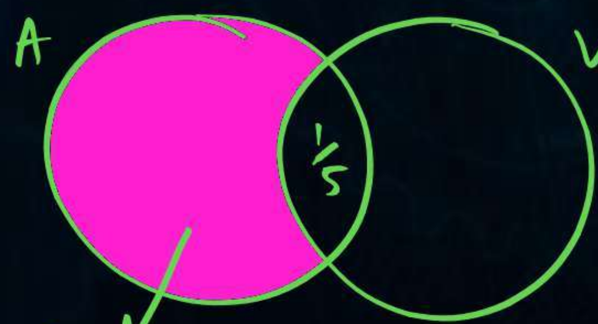
$P(A \cap V) = 1/5$

(A) $\frac{9}{35}$

(B) $\frac{18}{35}$ ✓

(C) $\frac{24}{35}$

(D) $\frac{3}{35}$



$\left(5/7 - 1/5\right) = \frac{25-7}{35} = 18/35$

QUESTION [JEE MAIN – 2024 (I) (31 Jan)]

Two marbles are drawn in succession from a box containing 10 red, 30 white, 20 blue and 15 orange marbles, with replacement being made after each drawing. Then the probability, that first drawn marble is red and second drawn marble is white, is

[Ans. D]

(A) $\frac{2}{25}$

(B) $\frac{4}{25}$

(C) $\frac{2}{3}$

(D) $\frac{4}{75}$ ✓

10R
30W
20B
15O

$\longrightarrow$ 2 marbles

$\frac{10}{75} \times \frac{\cancel{30}^{10}}{\cancel{75}_{25}}$

$\frac{100}{75 \times 25} = \frac{4}{75}$

QUESTION [JEE MAIN – 2024 (I) (29 Jan)]

A fair die is thrown until 2 appears. Then the probability, that 2 appears in even number of throws, is

[Ans. C]

$P(2) = 1/6 \quad P(\bar{2}) = 5/6$

(A) $\frac{5}{6}$

(B) $\frac{1}{6}$

(C) $\frac{5}{11}$ ✓

(D) $\frac{6}{11}$

$P(\bar{2}\,2) + P(\bar{2}\bar{2}\bar{2}\,2) + P(\bar{2}\bar{2}\bar{2}\bar{2}\bar{2}\,2) + \cdots \infty$

$\frac{1}{6}\cdot\frac{5}{6} + (5/6)^3\cdot\frac{1}{6} + (5/6)^5\cdot\frac{1}{6} + \cdots \infty$

$= \frac{5/36}{1-\frac{25}{36}} = \frac{5/\cancel{36}}{11/\cancel{36}} = 5/11$

QUESTION [JEE MAIN – 2024 (I) (27 Jan)]

A fair die is tossed repeatedly until a six is obtained. Let X denote the number of tosses required and let $\underline{a = P(X = 3)}$, $b = P(X \geq 3)$ and $c = P(X \geq 6 \mid X > 3)$. Then $\frac{b+c}{a}$ is equal to ________.

[Ans. 12]

$a = \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{1}{6}$

$a = \frac{25}{216}$

$b = \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^3 \cdot \frac{1}{6} + \dots \infty$

$= \frac{\left(\frac{5}{6}\right)^2 \cdot \cancel{\frac{1}{6}}}{\cancel{\left(1 - \frac{5}{6}\right)}} = \frac{25}{36}$

OR

$\boxed{b = \frac{5}{6} \cdot \frac{5}{6}} = \frac{25}{36}$

$c = \underbrace{\frac{5}{6}}_{\text{IV}} \cdot \underbrace{\frac{5}{6}}_{\text{V}} = \frac{25}{36}$

$$\boxed{\begin{array}{c} \dfrac{50/36}{25/216} \\ 2 \times 6 = 12 \end{array}}$$

QUESTION [JEE MAIN – 2024 (II) (27 Jan)]

An urn contains 6 white and 9 black balls. Two successive draws of 4 balls are made without replacement. The probability, that the first draw gives all white balls and the second draw gives all black balls, is :

[Ans.

(A) $\frac{5}{256}$

(B) $\frac{5}{715}$

(C) $\frac{3}{715}$

(D) $\frac{3}{256}$

6W, 9B $\xrightarrow{\text{4 balls}}$ 2W, 9B $\longrightarrow$ 4 balls

$$\frac{{}^{6}C_4}{{}^{15}C_4} \times \frac{{}^{9}C_4}{{}^{11}C_4} = \text{Ans}$$

QUESTION [JEE MAIN – 2024 (I) (30 Jan)]

Two integers x and y are chosen with replacement from the set $\{0, 1, 2, 3, ..., 10\}$. Then the probability that $|x - y| > 5$ is :

[Ans. A]

(A) $\frac{30}{121}$

(B) $\frac{62}{121}$

(C) $\frac{60}{121}$

(D) $\frac{31}{121}$

$\xleftarrow{\quad 5 \quad}$

$n(S) = 11 \cdot 11 = 121$

Favourable $\rightarrow |x-y| > 5$

$x = 0 \Rightarrow y = 6, 7, 8, 9, 10 \rightarrow 5$

$x = 1, y = 7$ to $10 \rightarrow 4$

$x = 2, y = 8$ to $10 \rightarrow 3$

$x = 3 \rightarrow 2$

$x = 4 \rightarrow 1$

$x = 5 \rightarrow (x)$

$x = 6, y = 0 \rightarrow 1$

$x = 7, y = 0, 1 \rightarrow 2$

$x = 8, y = 0, 1, 2 \rightarrow 3$

$x = 9 \rightarrow 4$

$x = 10 \rightarrow 5$

$\text{Fav} = 2[5+4+3+2+1]$
$= 30$

QUESTION [JEE MAIN – 2024 (II) (30 Jan)]

[Ans. C]

Bag A contains 3 white, 7 red balls and bag B contains 3 white, 2 red balls. One bag is selected at random and a ball is drawn from it. The probability of drawing the ball from the bag A, if the ball drawn in white, is :

(A) $\frac{1}{4}$

(B) $\frac{1}{9}$

(C) $\frac{1}{3}$ ✓

(D) $\frac{3}{10}$

A: 3W, 7R

B: 3W, 2R

$P(W) = \frac{1}{2}\cdot\frac{3}{10} + \frac{1}{2}\cdot\frac{3}{5}$

$P(A/W) = \dfrac{\cancel{\frac{1}{2}}\cdot\frac{3}{10}}{\cancel{\frac{1}{2}}\cdot\frac{3}{10} + \cancel{\frac{1}{2}}\cdot\frac{3}{5}}$

$= \dfrac{3}{3+6} = \frac{3}{9}$

QUESTION [JEE MAIN – 2024 (I) (31 Jan)]

3R
15G

$\frac{18 \times 17}{2}$
9×17
$= 153$

Three rotten apples are accidently mixed with fifteen good apples. Assuming the random variable x to be the number of rotten apples in a draw of two apples, the variance of x is

[Ans. D]

(A) $\frac{37}{153}$

(B) $\frac{57}{153}$

(C) $\frac{47}{153}$

(D) $\frac{40}{153}$

$x = 0 \Rightarrow P = \frac{{}^{15}C_2}{{}^{18}C_2} = \frac{105}{153}$

$x = 1 \Rightarrow \frac{{}^{15}C_1 \cdot {}^{3}C_1}{{}^{18}C_2} = \frac{45}{153}$

$x = 2 \rightarrow \frac{{}^{15}C_0 \cdot {}^{3}C_2}{{}^{18}C_2} = \frac{3}{153}$

x_i	$P(x_i)$	$x_i P(x_i)$	$x_i^2 P(x_i)$
0	$\frac{105}{153}$	0	0
1	$\frac{45}{153}$	$45/153$	$45/153$
2	$3/153$	$6/153$	$12/153$

$\mathrm{Var}(x) = \sum x_i^2 P(x_i) - (\mu)^2$

$= \frac{57}{153} - \frac{1}{9} = \frac{57-17}{153} = \frac{40}{153}$

$\mu = \frac{51}{153} = \frac{1}{3}$

$\frac{57}{153}$

QUESTION [JEE MAIN – 2024 (II) (31 Jan)]

biased

A coin is based so that a head is twice as likely to occur as a tail. If the coin is tossed 3 time, then the probability of getting two tails and one head is :

[Ans. A]

(A) $\frac{2}{9}$

$$\boxed{\begin{array}{l} P(H) = 2/3 \\ P(T) = 1/3 \end{array}}$$

(B) $\frac{1}{9}$

$(TTH) + (HTT) + (THT)$

(C) $\frac{2}{27}$

$\left(\frac{1}{3}\right)^2 \cdot \frac{2}{\cancel{3}} \times \cancel{3}$

(D) $\frac{1}{27}$

$\frac{1}{9} \times 2$