Definite Integration

Definite Integration

let f(x) be a continuous function defined on [a, b].

$$\boxed{\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a)}$$

$$\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a)$$

$$\int_1^2 e^x dx = [e^x]_1^2 = \underline{e^2 - e^1}$$

The value of b > 3 for which

$$12\int_3^b \underbrace{\frac{1}{(x^2-1)(x^2-4)}}_{x^2 = y} dx = \log_e\left(\frac{49}{40}\right)$$, is equal to

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$$\frac{1}{3}\int \frac{(y-1)-(y-4)}{(y-1)(y-4)}$$

$$\frac{1}{3}\left(\int \frac{1}{x^2-4} - \int \frac{1}{x^2-1}\right)$$

$$\overset{4}{\cancel{12}} \times \frac{1}{\cancel{3}}\left(\frac{1}{2(2)}\ln\frac{x-2}{x+2} - \frac{1}{2(1)}\ln\frac{x-1}{x+1}\right)$$

$$\frac{12}{3}\int_3^b \frac{(x^2-1)-(x^2-4)}{(x^2-1)(x^2-4)}dx$$

$$4\left(\int_3^b \frac{dx}{x^2-4} - \int_3^b \frac{dx}{x^2-1}\right)$$

$$\left[\ln\left(\frac{x-2}{x+2}\right) - 2\ln\left(\frac{x-1}{x+1}\right)\right]_3^6 = \ln\left(\frac{49}{40}\right)$$

$$\left(\ln\left(\frac{b-2}{b+2}\right) - \ln\left(\frac{b-1}{b+1}\right)^2\right) - \left(\ln\frac{4}{5}\right) = \ln\frac{49}{40}$$

$$\cancel{\ln}\left(\left(\frac{b-2}{b+2}\right)\frac{(b+1)^2}{(b-1)^2}\right) = \cancel{\ln}\left(\frac{49}{40}\cdot\frac{4}{5}\right)$$

#chalaki $b=6$

$$\frac{(b-2)(b+1)^2}{(b+2)(b-1)^2} = \frac{49}{50}$$

$$\frac{4\ 7^2}{8\ 5^2}$$

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Q The integral $\frac{24}{\pi}\int_0^{\sqrt{2}} \frac{(2-x^2)dx}{(2+x^2)\sqrt{4+x^4}}$ is equal to ____.

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$$\Rightarrow \int \frac{(2-x^2)dx}{(2+x^2)\sqrt{4+x^4}}$$

Put $x+\frac{2}{x}=t$

$\left(1-\frac{2}{x^2}\right)dx = dt$

$$\Rightarrow \int \frac{\frac{(2-x^2)}{x^2}\,dx}{\frac{(2+x^2)}{x}\sqrt{\frac{4+x^4}{x^2}}}$$

$$\Rightarrow \int \frac{\boxed{\left(\frac{2}{x^2}-1\right)dx}}{\left(\frac{2}{x}+x\right)\sqrt{\left(x+\frac{2}{x}\right)^2-4}}$$

$$\int \frac{-dt}{t\sqrt{t^2-2^2}}$$

$$\Rightarrow -\frac{1}{2}\sec^{-1}\left(\frac{t}{2}\right)$$

$0+\frac{2}{0}$

$$-\frac{1}{2}\left[\sec^{-1}\left(\frac{x+\frac{2}{x}}{2}\right)\right]_0^{\sqrt{2}}$$

$$-\frac{1}{2}\left\{\sec^{-1}\sqrt{2}-\sec^{-1}\infty\right]$$

$$-\frac{1}{2}\left\{\frac{\pi}{4}-\frac{\pi}{2}\right\}$$

$$\Rightarrow \frac{\pi}{8}$$

$$\frac{24}{\pi}\times\frac{\pi}{8}\Rightarrow 3$$

Q

Unacademy Atoms

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Let $g : (0, \infty) \to R$ be a differentiable function such that

$$\int\left(\frac{x(\cos x - \sin x)}{e^x + 1} + \frac{g(x)(e^x + 1 - xe^x)}{(e^x + 1)^2}\right)dx = \frac{xg(x)}{e^x + 1} + c,$$

for all $x > 0$, where c is an arbitrary constant. Then.

A. g is decreasing in $\left(0, \frac{\pi}{4}\right)$

B. g' is increasing in $\left(0, \frac{\pi}{4}\right)$

C. $g + g'$ is increasing in $\left(0, \frac{\pi}{2}\right)$

D. $g - g'$ is increasing in $\left(0, \frac{\pi}{2}\right)$

$$\frac{x(\cos x-\sin x)}{e^x+1}+\cancel{\frac{g(x)}{e^x+1}}-\cancel{\frac{x e^x g(x)}{(e^x+1)^2}}=\frac{\overbrace{(x g'(x)+g(x))(e^x+1)}-\overbrace{e^x\cdot x\, g(x)}}{(e^x+1)^2}$$

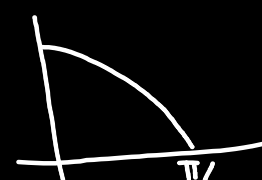
$$=\frac{x\, g'(x)}{e^x+1}+\cancel{\frac{g(x)}{e^x+1}}-\cancel{\frac{e^x\cdot x\, g(x)}{(e^x+1)^2}}$$

$$g'(x)=\cos x-\sin x$$

✓ $g(x)=\underline{\sin x+\cos x}+C$

$$g(x)+g'(x)=2\cos x+C$$

$$g(x)-g'(x)=2\underline{\sin x}-c$$



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Q

If $b_n = \int_0^{\frac{\pi}{2}} \frac{\cos^2 nx}{\sin x} dx,\ n \in N$, then

A. $b_3 - b_2, b_4 - b_3, b_5 - b_4$ are in an A.P. with common difference -2

B. $\frac{1}{b_3 - b_2}, \frac{1}{b_4 - b_3}, \frac{1}{b_5 - b_4}$ are in an A.P. with common difference 2

C. $b_3 - b_2, b_4 - b_3, b_5 - b_4$ are in a G.P.

D. $\frac{1}{b_3 - b_2}, \frac{1}{b_4 - b_3}, \frac{1}{b_5 - b_4}$ are in an A.P. with common difference -2

$$b_{n+1} - b_n = \int_0^{\pi/2} \frac{\cos^2 (n+1)x}{\sin x} - \int_0^{\pi/2} \frac{\cos^2 nx}{\sin x}\, dx$$

$$= \int_0^{\pi/2} \frac{\cos^2((n+1)x) - \cos^2(nx)}{\sin x}\, dx$$

$$\frac{\cancel{2}nx + \cancel{2}x + \cancel{2}nx}{\cancel{2}} = \int_0^{\pi/2} \frac{\frac{\cancel{1} + \cos(2nx+2x)}{\cancel{2}} - \frac{\cancel{1} + \cos(2nx)}{\cancel{2}}}{\sin x}$$

$$\underline{2nx + x}$$

$$= \int_0^{\pi/2} \frac{\cos(2nx+2x) - \cos(2nx)}{2 \sin x}$$

$$= \int_0^{\pi/2} \frac{-\cancel{2}\sin(2nx+x)\,\cancel{\sin(x)}}{\cancel{2}\,\cancel{\sin x}}$$

$$(-7)-(-5) = -2$$

$$-5, -7, -9$$

$$= \int_0^{\pi/2} -\sin((2n+1)x)\,dx$$

$$b_{n+1} - b_n = \frac{-1}{2n+1}$$

$$= \left[\frac{\cos(2n+1)x}{(2n+1)}\right]_0^{\pi/2}$$

$$\frac{1}{b_{n+1}-b_n} = -(2n+1)$$

$$= (0) - \left(\frac{1}{2n+1}\right)$$

$$= \frac{-1}{2n+1}$$

$n=2$ $\quad \frac{1}{b_3-b_2} = -5$

$n=3$ $\quad \frac{1}{b_4-b_3} = -7$

$n=4$ $\quad \frac{1}{b_5-b_4} = -9$

Q

Let $f : \mathbf{R} \to \mathbf{R}$ be a differentiable function such that $f\left(\frac{\pi}{4}\right) = \sqrt{2}, f\left(\frac{\pi}{2}\right) = 0$ and $f'\left(\frac{\pi}{2}\right) = 1$ and let $\boxed{g(x) = \int_x^{\pi/4} \left(f'(t)\sec t + \tan t \sec t f(t)\right) dt}$ for $x \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right)$. Then $\lim\limits_{x \to \left(\frac{\pi}{2}\right)^-} g(x)$ is equal to

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(Repeated)

(2021)

A. 2

B. 3

C. 4

D. –3

$\int \underbrace{f'(t)}_{\text{II}} \underbrace{\sec t}_{\text{I}} + \int \tan t \sec t \, f(t)$

$\sec t \, f(t) - \cancel{\int \sec t \tan t \, f(t)} + \cancel{\int \tan t \sec t \, f(t)}$

$$g(x) = \int_{x}^{\pi/4} d\,(f(t)\cdot \sec t)$$

$$\int dx = x$$

$$\int d\square = \square$$

$$= \Big[f(t)\cdot \sec t \Big]_{x}^{\pi/4}$$

$$= f\left(\tfrac{\pi}{4}\right) \sec\tfrac{\pi}{4} - f(x)\sec x$$

$$= \sqrt{2}\cdot\sqrt{2} - \frac{f(x)}{\cos x}$$

$$\lim_{x\to\frac{\pi}{2}^-} \boxed{g(x) = 2 - \frac{f(x)}{\cos x}}$$

$$= 2 - \lim_{x\to\frac{\pi}{2}^-} \frac{f(x)}{\cos x}$$

$$= 2 + \frac{f'(x)}{\sin x}$$

$$= 2 + \frac{f'\left(\frac{\pi}{2}\right)}{\sin\left(\frac{\pi}{2}\right)}$$

$$= 2 + 1$$

$$= 3$$

Properties of Definite Integration

Properties of Definite Integration

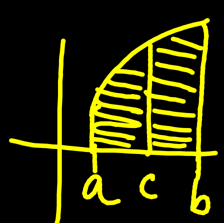
$y = [x]$

$y = |x-2|$

P - 1

$$\int_a^b f(x)dx = \int_a^b f(t)dt$$

Naam में Kya Rakha है !



P - 2

$$\int_a^b f(x)dx = -\int_b^a f(x)dt$$

→ discontinuous

→ f^n → definite

P - 3

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

Integral is broken at points of discontinuity or at the points where definition of ‘f’ changes

Q $\int_0^5 \cos\left(\pi\left(x-\left[\frac{x}{2}\right]\right)\right)dx,$ $\left[\frac{x}{2}\right]$ $\frac{x}{2} \to$ Int

Where [t] denotes greatest integer less than or equal to t, is equal to :

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$0 < x < 5$

$0 < \frac{x}{2} < 2.5$

$\therefore \frac{x}{2} = 1, 2$

$\therefore \boxed{x = 2, 4}$

$4 < x < 5$

$2 < \frac{x}{2} < 2.5$

A. -3 $\cos(\pi - \pi x)$

B. -2

C. 2

D. 0

$\int_0^2 \cos\pi(x-0)\,dx + \int_2^4 \cos\pi(x-1)\,dx + \int_4^5 \cos(\pi)(x-2)\,dx$

$\cos(\pi x - 2\pi)$

$= \cos(2\pi - \pi x)$

$$\int_0^2 \cos(\pi x)\,dx - \int_2^4 \cos(\pi x)\,dx + \int_4^5 \cos(\pi x)\,dx$$

$$\cancel{\left[\frac{\sin(\pi x)}{\pi}\right]_0^2} - \cancel{\left[\frac{\sin(\pi x)}{\pi}\right]_2^4} + \cancel{\left[\frac{\sin(\pi x)}{\pi}\right]_4^5}$$

$$= 0 - 0 + 0$$

$$= \boxed{0}$$

Q

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Let $f:\mathbb{R}\to\mathbb{R}$ be a function defined as

$$f(x)=a\sin\left(\frac{\pi[x]}{2}\right)+[2-x],\ a\in\mathbb{R}$$, where [t]

is the greatest integer less than or equal to t. If $\boxed{\lim_{x\to(-1)} f(x)}$ exists, then the value of $\int_0^4 f(x)\,dx$ is equal to :

A. -1

B. -2

C. 1

D. 2

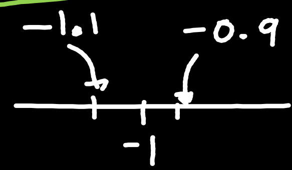
$$\int_0^1 1\,dx+\int_1^2(-1)\,dx+\int_2^3(-1)\,dx+\int_3^4(-1)\,dx$$

$$=\cancel{1}-\cancel{1}-1-1$$

$$=-2$$

$$\boxed{f(x) = -\sin\left(\frac{\pi[x]}{2}\right) + 2 + [-x]}$$

$$\lim_{x \to (-1)} f(x) = \text{Exist}$$



$$LHL = RHL$$

$$f(-1.1) = f(-0.9)$$

$$\cancel{a \sin\left(\frac{\pi(-2)}{2}\right)} + \cancel{2} + 1 = a \sin\left(\frac{\pi(-1)}{2}\right) + \cancel{2} + 0$$

$$1 = -a$$

$$\therefore \boxed{a = -1}$$

$$f(1.1) = -\sin\frac{\pi}{2} + 2 + [-1.1]$$

$$= -1 + \cancel{2} - \cancel{2}$$

$$= -1$$

$$f(2.2) = \cancel{-\sin\left(\frac{\pi(2)}{2}\right)} + 2 + [-2.2]$$

$$= 2 - 3 = -1$$

$$f(3.1)$$

$$= -\sin\frac{3\pi}{2} + 2 + [-3.1]$$

$$= 1 + 2 - 4$$

$$= -1$$

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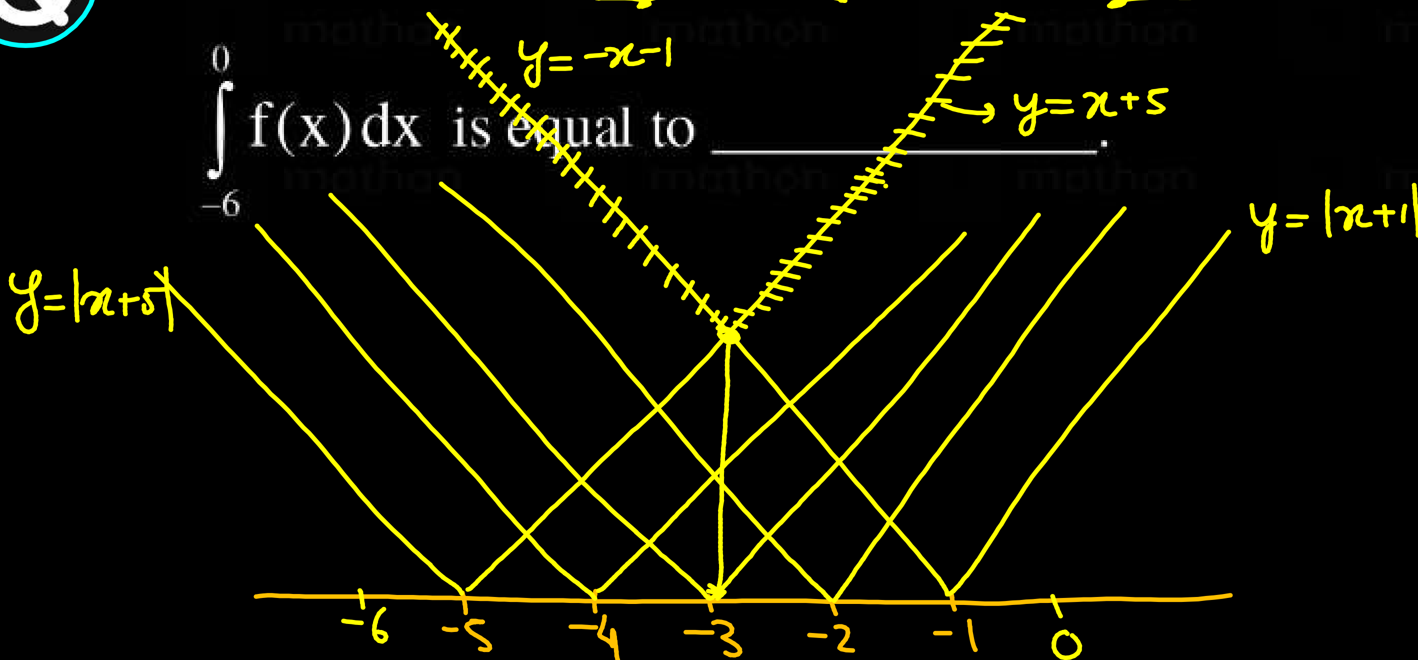
Q Let $f(x) = \max\{|x+1|, |x+2|, \ldots, |x+5|\}$. Then

$\int_{-6}^{0} f(x)\,dx$ is equal to ______.



$$\int_{-6}^{-3}(-x-1)\,dx + \int_{-3}^{0}(x+5)\,dx$$

$$\left(-\frac{x^2}{2}-x\right)_{-6}^{-3} + \left(\frac{x^2}{2}+5x\right)_{-3}^{0}$$

$$\Rightarrow \left(-\frac{9}{2}+3\right)-\left(-18+6\right)+\left(0\right)-\left(\frac{9}{2}-15\right)$$

$$\Rightarrow -\cancel{9}+3+18-\cancel{6}+\cancel{15}$$

$$\Rightarrow 21$$

Q

Let $f(x) = \min\{[x-1], [x-2], \ldots, [x-10]\}$ where $[t]$ denotes the greatest integer $\leq t$. Then $\int_0^{10} f(x)dx + \int_0^{10} (f(x))^2 dx + \int_0^{10} |f(x)| dx$ is equal to ____.

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Homework

Q

Let [t] denote the greatest integer less than or equal to t. Then, the value of the integral $\int_0^1 [\underbrace{-8x^2+6x}-1]\,dx$ is equal to

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A. -1

B. $-\frac{5}{4}$

C. $\frac{\sqrt{17}-13}{8}$

D. $\frac{\sqrt{17}-16}{8}$

$y = -8x^2+6x-1$

$y = -(4x-1)(2x-1)$

$\int_0^1 [\underbrace{-8x^2+6x}-1]\,dx$

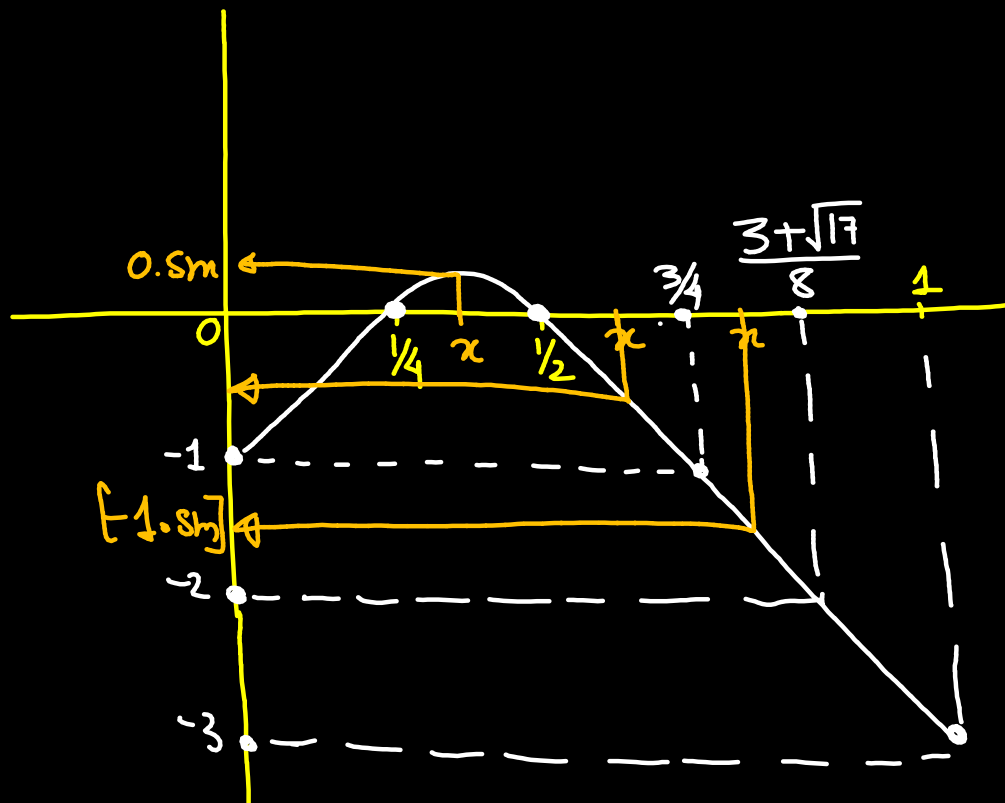
$\underbrace{\int_0^{1/4}(-1)\,dx} + \cancel{\int_{1/4}^{1/2} 0\,dx} + \int_{1/2}^{3/4}(-1)\,dx + \int_{3/4}^{\frac{3+\sqrt{17}}{8}}(-2)\,dx + \int_{\frac{3+\sqrt{17}}{8}}^{1}(-3)\,dx$

$\boxed{-\frac{1}{4} - \frac{1}{4} - 2\left(\frac{3+\sqrt{17}}{8} - \frac{3}{4}\right) - 3\left(1 - \frac{3+\sqrt{17}}{8}\right)}$

$$y = -(4x-1)(2x-1)$$

$$-(3)(1)$$

x	0	$1/4$	$1/2$	1
y	-1	0	0	-3



$$-8x^2+6x-\cancel{1} = \cancel{-1}$$

$$-8x+6=6$$

$$x = 6/8 = 3/4$$

$$-8x^2+6x-1=-2$$

$$-8x^2+6x+1=0$$

$$8x^2-6x-1=0$$

$$x = \frac{6 \pm \sqrt{36+32}}{16}$$

$$x = \frac{6 \pm 2\sqrt{17}}{16} = \frac{3 \pm \sqrt{17}}{8}$$

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Q

$$\int_0^2 \left(\left| 2x^2 - 3x \right| + \left[x - \frac{1}{2} \right] \right) dx,$$

where [t] is the greatest integer function, is equal to:

$2x^2 - 3x = 0$

$x(2x-3) = 0$

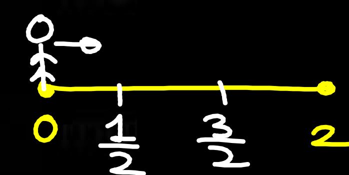
$x = 0, \frac{3}{2}$

$\frac{1}{2} < x < \frac{3}{2}$

$0 < x - \frac{1}{2} < 1$

$x - \frac{1}{2} \Rightarrow \text{Int}$

0



A. $\frac{7}{6}$ B. $\frac{19}{12}$ C. $\frac{31}{12}$ D. $\frac{3}{2}$

$$\int_0^2 \left|2x^2 - 3x\right| dx + \int_0^2 \left[x - \frac{1}{2}\right] dx$$

$$\int_0^{3/2} (-2x^2 + 3x)\,dx + \int_{3/2}^{2} (2x^2 - 3x)\,dx + \int_0^{1/2} (-1)\,dx + \int_{1/2}^{3/2} 0\,dx + \int_{3/2}^{2} 1\,dx$$

Properties of Definite Integration

P - 4

$$\int_{-a}^{a} f(x)dx = \int_{0}^{a} \big(f(x) + f(-x)\big) = \begin{cases} 0 & \text{if } f(x) \text{ is odd} \\ 2\int_{0}^{a} f(x)dx & \text{if } f(x) \text{ is even} \end{cases}$$

Odd Even

$x \to -x$

$$\int_{-a}^{a} f(x)dx = \begin{cases} 0 & f(x) \to \text{odd} \\ 2\int_{0}^{a} f(x)dx & f(x) \to \text{Even} \end{cases}$$

Properties of Definite Integration

#Baahubali

P - 5

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

$\boxed{x \to a+b-x}$

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

Properties of Definite Integration

#Kattappa

P - 6

$$\int_0^{2a} f(x)dx = \int_0^{a} f(x)dx + \int_0^{a} f(2a-x)dx$$

$$\int_0^{\boxed{2a}} f(x)\,dx = \int_0^{a} \left(f(x) + f(2a-x)\right) dx$$

Important Results

$$I = \int_0^{\pi/2} \ln \sin x \, dx = \int_0^{\pi/2} \ln \cos x \, dx = \int_0^{\pi/2} \ln \sin 2x \, dx = -\frac{\pi}{2} \ln 2$$

$$I = \int_0^{\pi/2} \ln(\sin x)\, dx$$

$$I = \int_0^{\pi/2} \ln(\cos x)\, dx$$

$$2I = \int_0^{\pi/2} \left(\ln(\sin 2x) - \ln 2\right) dx$$

$$2I = I - \ln 2\left(\frac{\pi}{2}\right)$$

$$\boxed{I = -\ln 2 \times \frac{\pi}{2}}$$

Q

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The value of the integral

$$\frac{1}{2}\int_{-\pi/2}^{\pi/2} \frac{dx}{\left(\sin^6 x+\cos^6 x\right)}$$ is equal to

A. 2π

B. 0

C. π

D. $\frac{\pi}{2}$

$$\frac{1}{\cancel{2}}\times\cancel{2}\times\int_0^{\pi/2}\frac{dx}{\sin^6 x+\cos^6 x}$$

$$\Rightarrow \int_0^{\pi/2}\frac{dx}{1-3\sin^2 x\cos^2 x}$$

$$\int_0^{\pi/2} \frac{\boxed{\sec^2 x}\, \sec^2 x\, dx}{\frac{1-3\sin^2 x\cos^2 x}{\cos^4 x}}$$

$$\int_0^{\pi/2} \frac{(1+\tan^2 x)\,\boxed{\sec^2 x\, dx}}{(1+\tan^2 x)^2 - 3\tan^2 x}$$

$$\int_0^{\infty} \frac{(1+t^2)\,dt}{(1+t^2)^2 - 3t^2}$$

$$= \int_0^{\infty} \frac{\frac{(1+t^2)\,dt}{t^2}}{\frac{t^4 - t^2 + 1}{t^2}}$$

$$\int_0^{\infty} \frac{\left(\frac{1}{t^2}+1\right)dt}{\left(t-\frac{1}{t}\right)^2+1}$$

$$t - \frac{1}{t} = z$$

$$\left(1+\frac{1}{t^2}\right)dt = dz$$

$$\int \frac{dz}{z^2+1}$$

$$\left(\tan^{-1}\left(t-\frac{1}{t}\right)\right)_0^{\infty}$$

$$\tan^{-1}(\infty) - \tan^{-1}(-\infty)$$

$$\frac{\pi}{2} + \frac{\pi}{2} \Rightarrow \pi$$

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The value of $\displaystyle\int_0^{\pi} \frac{\cancel{e^{\cos x}}\sin x}{(1+\cos^2 x)\left(\cancel{e^{\cos x}+e^{-\cos x}}\right)}dx$ is equal to

$x \to \pi - x$

A. $\frac{\pi^2}{4}$

B. $\frac{\pi^2}{2}$

C. $\frac{\pi}{4}$ ✓

D. $\frac{\pi}{2}$

$\frac{1}{2}\int_0^{\pi} \frac{\sin x\,dx}{1+\cos^2 x}$

$\int^{\pi}$

$\cos x = t$

$-\sin x\,dx = dt$

$\frac{1}{2}\int_{-1}^{1} \frac{+dt}{1+t^2}$

$\frac{1}{2}\left(\tan^{-1}(t)\right)_{-1}^{1}$

$\frac{1}{2}\left(\frac{\pi}{4}+\frac{\pi}{4}\right) \Rightarrow \frac{\pi}{4}$

Q

The value of the integral $\int_{-2}^{2} \frac{\left|x^3+x\right|}{\left(e^{x|x|}+1\right)}dx$ is equal to :

A. $5e^2$

B. $3e^{-2}$

C. 4

D. 6

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$\cancel{\frac{1}{2}} \times \cancel{2} \int_0^2 \underbrace{|x^3+x|}\, dx$

$\Rightarrow \int_0^2 |x^3+x|\, dx$

$\Rightarrow \int_0^2 (x)(x^2+1)\, dx$

$= \int_0^2 (x^3+x)\, dx$

$\left[\frac{x^4}{4}+\frac{x^2}{2}\right]_0^2$

$\Rightarrow 4+2$

$\Rightarrow 6$

Q

The integral $\int_0^{\frac{\pi}{2}} \frac{1}{3+2\underline{\sin x}+\underline{\cos x}}dx$ is equal to:

Type-2

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A. $\tan^{-1}(2)$

B. $\tan^{-1}(2)-\frac{\pi}{4}$

C. $\frac{1}{2}\tan^{-1}(2)-\frac{\pi}{8}$

D. $\frac{1}{2}$

$\tan\left(\frac{x}{2}\right) = t$

$$\int_0^1 \frac{2dt}{3(1+t^2)+2(2t)+(1-t^2)} = \int_0^1 \frac{2dt}{2t^2+4t+4}$$

$$= \int_0^1 \frac{dt}{(t+1)^2+1}$$

$$\left(\tan^{-1}(t+1)\right)_0^1$$

$$\underbrace{\tan^{-1}2 - \frac{\pi}{4}}$$

Properties of Definite Integration

Unacademy Atoms

#NVStyle

P - 7 If f(x) is periodic $\rightarrow T$

No. of Cycle $\Rightarrow \left(\frac{UL-LL}{T}\right)$

Uncle

$\frac{nT-mT}{T}$

aunty

$\frac{nT-0}{T} = n$

$\frac{nT}{T}$

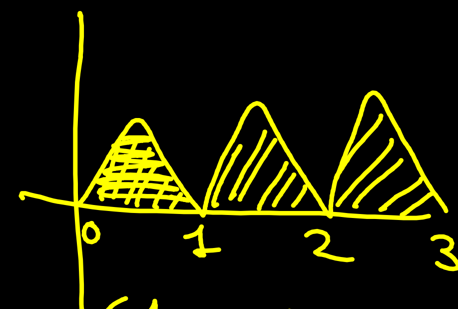
1. $\int_0^{nT} f(x)dx = n\int_0^{T} f(x)dx$

2. $\int_a^{a+nT} f(x)\,dx = n\int_0^{T} f(x)dx\,,\ n\epsilon Z,\ a\epsilon R$

3. $\int_{mT}^{nT} f(x)\,dx = (n-m)\int_0^{T} f(x)dx\quad m,n\epsilon Z$

4. $\int_{\cancel{nT}}^{a+\cancel{nT}} f(x)\,dx = \int_0^{a} f(x)dx\,,\ n\epsilon Z\,, a\epsilon R$

5. $\int_{a+\cancel{nT}}^{b+\cancel{nT}} f(x)\,dx = \int_a^{b} f(x)dx\,,\ n\epsilon Z\,, a, b\epsilon R$



$= 3\left(\int_0^1 f(x)dx\right)$

Q

$$\int_0^{20\pi} \big(\underbrace{|\sin x| + |\cos x|}_{\text{period} = \pi/2}\big)^2 dx \text{ is equal to :-}$$

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A. $10(\pi+4)$

B. (B) $10(\pi+2)$

C. $20(\pi-2)$

D. $\cancel{\text{(D)}}$ $20(\pi+2)$ ✓

$f(x+T) = f(x)$

↑ period

No. of Cycles $= \dfrac{20\pi - 0}{(\pi/2)} = 40$

$f\left(x+\frac{\pi}{2}\right) = f(x)$

$f\left(x+\frac{\pi}{4}\right) \neq f(x)$

$$40 \times \int_0^{\pi/2} (\sin x + \cos x)^2 dx$$

$$40 \int_0^{\pi/2} (1 + \sin 2x)\, dx$$

$$40 \left(x - \frac{\cos 2x}{2} \right)_0^{\pi/2}$$

$$40 \left(\frac{\pi}{2} + \frac{1}{2} \right) - \left(-\frac{1}{2} \right)$$

$$40 \left(\frac{\pi}{2} + 1 \right)$$

$$\underline{\underline{20(\pi + 2)}}$$

Unacademy Atoms

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Q

Let $[t]$ denote the greatest integer less than or equal to t. Then the value of the integral $\int_{-3}^{101}\left([\sin(\pi x)]+e^{[\cos(2\pi x)]}\right)dx$ is equal to

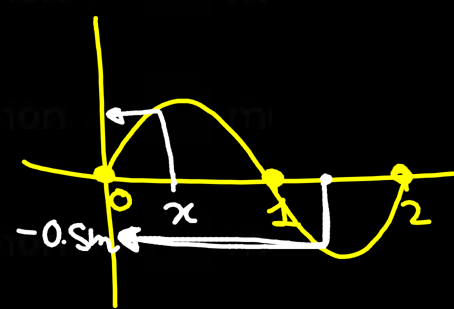
A. $\frac{52(1-e)}{e}$

B. $\frac{52}{e}$

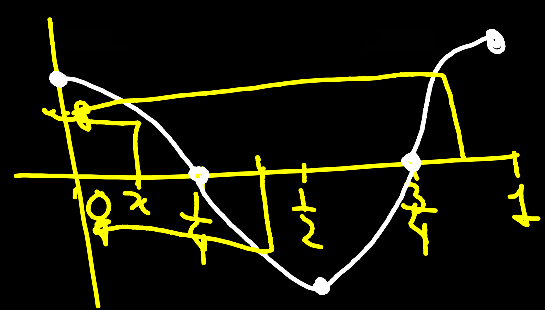
C. $\frac{52(2+e)}{e}$

D. $\frac{104}{e}$

$y = \sin \pi x$



$y = \cos 2\pi x$



$\frac{101-(-3)}{2}$

$$\underbrace{\int_{-3}^{101}[\sin \pi x]\,dx} + \underbrace{\int_{-3}^{101} e^{[\cos 2\pi x]}\,dx}$$

$\frac{101-(-3)}{1}$

$\sin x \rightarrow 2\pi$

$\sin(ax+b) \rightarrow \frac{2\pi}{|a|}$

$\sin(\pi x) \rightarrow \frac{2\pi}{\pi} = 2$

$\cos(2\pi x) \rightarrow \frac{2\pi}{2\pi} = 1$

$$52\int_0^2 [\sin \pi x]\,dx + 104\int_0^1 e^{[\cos 2\pi x]}\,dx$$

$$52\left\{\int_0^1 \cancel{0}\,dx + \int_1^2 (-1)\,dx\right\} + 104\left\{\int_0^{1/4} 1\,dx + \int_{1/4}^{3/4} e^{-1}\,dx + \int_{3/4}^{1} 1\,dx\right\}$$

$$-52 + 104\left\{\frac{1}{2} + \frac{1}{2e}\right\} \Rightarrow \frac{52}{e}$$

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Q

Let $f : \mathbf{R} \to \mathbf{R}$ be continuous function satisfying $f(x) + f(x+k) = n$ for all $x \in \mathbf{R}$ where $k > 0$ and n is a positive integer. If $I_1 = \int_0^{4nk} f(x)dx$ and $I_2 = \int_{-k}^{3k} f(x)dx$, then

$f \to$ periodic $\boxed{T=2K}$

$\frac{4nK-0}{2K} = 2n$

$\frac{3K-(-K)}{2K} = 2$

A. $I_1 + 2I_2 = 4nk$

B. $I_1 + 2I_2 = 2nk$

C. $I_1 + nI_2 = 4n^2k$

D. $I_1 + nI_2 = 6n^2k$

Shortcut

$f(x) + f(x + \square) = \text{Const.}$

f^n periodic

Period $= 2\square$

$x+k=t$

$$I_1 = 2n\int_0^{2k} f(x)\,dx$$

$+$

$$nI_2 = 2n\int_0^{2k} f(x)\,dx$$

$$I_1 + nI_2 = 4n\underbrace{\int_0^{2k} f(x)\,dx}_{?}$$

$$= 4n \times nk$$

$$= 4n^2k$$

$$\int_0^k f(x) + \int_0^k f(x+k) = \int_0^k n\,dx$$

$$\int_0^k f(x)\,dx + \int_k^{2k} f(t)\,dt = nk$$

$$\int_0^{2k} f(t)\,dt = nk$$

$$A = \underbrace{\int_{-2}^{5} f(t)\,dt} \rightarrow \text{Numerical Value}$$

Definite Integration as Constant

Q

Let f be a real valued continuous function on $[0,1]$ and $\boxed{f(x)=x+\int_0^1 (x-t)f(t)\,dt.}$ Then which of the following points (x,y) lies on the curve $y=f(x)$?

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A. (2, 4)

B. (1, 2)

C. (4, 17)

D. (6, 8)

$$f(x) = x + x\underbrace{\int_0^1 f(t)\,dt}_{A} - \underbrace{\int_0^1 t f(t)\,dt}_{B}$$

$$f(x) = x + Ax - B$$

$$\boxed{f(x) = (1+A)x - B}$$

$$A = \int_0^1 \underbrace{f(t)}\,dt$$

$$= \int_0^1 \big((1+A)t - B\big)\,dt$$

$$= \left[(1+A)\frac{t^2}{2} - Bt\right]_0^1$$

$$A = \frac{1+A}{2} - B$$

$$2A = 1 + A - 2B$$

$$\boxed{A = 1-2B}$$

$$B = \int_0^1 t \cdot \underbrace{f(t)}\,dt$$

$$= \int_0^1 \big((1+A)t^2 - Bt\big)\,dt$$

$$= \left[\frac{(1+A)\,t^3}{3} - \frac{Bt^2}{2}\right]_0^1$$

$$6\left(B = \frac{1+A}{3} - \frac{B}{2}\right)$$

$$6B = 2 + 2A - 3B$$

$$\boxed{9B = 2+2A}$$

$A =$ $B =$

Q

Let $f(\theta) = \sin\theta + \int_{-\pi/2}^{\pi/2} (\sin\theta + t\cos\theta) f(t)\,dt$. Then the value of $\left|\int_0^{\pi/2} f(\theta)\,d\theta\right|$ is ________.

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$$f(\theta) = \sin\theta + \sin\theta \underbrace{\int_{-\pi/2}^{\pi/2} f(t)\,dt}_{A} + \cos\theta \underbrace{\int_{-\pi/2}^{\pi/2} t\, f(t)\,dt}_{B}$$

$$f(\theta) = \sin\theta + A\sin\theta + B\cos\theta$$

$$\boxed{f(\theta) = (1+A)\sin\theta + B\cos\theta}$$

$$f(\theta) = \left(1 - \frac{4}{3}\right)\sin\theta - \frac{2}{3}\cos\theta = -\frac{1}{3}\sin\theta - \frac{2}{3}\cos\theta$$

$$A = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\cancel{(1+A)\sin t} + B\cos t \right) dt$$

$$= B \int_{-\pi/2}^{\pi/2} \cos t \, dt$$

$$\boxed{A = 2B} \quad — (1)$$

$$B = 2 + 2(2B)$$

$$\boxed{\frac{-2}{3} = B} \qquad \boxed{A = \frac{-4}{3}}$$

$$B = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left((1+A)\, t \sin t + \cancel{B\, t \cos t} \right) dt$$

$$= (1+A) \times 2 \times \int_0^{\pi/2} t \sin t \, dt$$

$$= 2(1+A) \left(-t\cos t + \sin t \right)_0^{\pi/2}$$

$$= 2(1+A)(1)$$

$$\boxed{B = 2 + 2A} \quad — (2)$$

$$\left| \int_0^{\pi/2} \left(-\frac{1}{3}\sin\theta - \frac{2}{3}\cos\theta \right) d\theta \right|$$

$$\left| -\frac{1}{3}(1) - \frac{2}{3}(1) \right|$$

$$= 1$$

Unacademy Atoms

Leibniz Rule

Derivatives of Antiderivatives (Leibniz Rule)

Unacademy Atoms

If f is continuous then

$t \to x^2$

$t \to x$

$$\frac{d}{dx}\left(\int_x^{x^2} x\, e^t\, dt\right) = \frac{d}{dx}\left(x \cdot \int_x^{x^2} e^t\, dt\right)$$

$$\frac{d}{dx}\left(\int_{g(x)}^{h(x)} f(t)\, dt\right) = f\big(h(x)\big).\ h'(x) - f\big(g(x)\big).g'(x)$$

Ex $$\frac{d}{dx}\left(\int_x^{x^2} \boxed{\sin t}\, dt\right) = \sin(x^2) \cdot 2x - \sin(x) \cdot (1)$$

$$= 2x \sin(x^2) - \sin x$$

$$\frac{d}{dx}\left(\int_x^{\sin x} (t^2+1)\, dt\right) = (\sin^2 x + 1) \cdot \cos x - (x^2+1)(1)$$

Q $\lim_{x \to 0} \frac{\int_0^{x^2} (\sin \sqrt{t})dt}{x^3}$ is equal to:

(1) $\frac{2}{3}$

(2) 0

(3) $\frac{1}{15}$

(4) $\frac{3}{2}$

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$$\lim_{x \to 0^+} \frac{\frac{d}{dx}\left(\int_0^{x^2} \sin \sqrt{t}\, dt\right)}{\frac{d}{dx}(x^3)}$$

$$\lim_{x \to 0^+} \frac{\sin \sqrt{x^2} \cdot 2}{3x}$$

$$\lim_{x \to 0^+} \frac{\sin x \cdot 2}{3x} = \frac{2}{3}$$

$$\lim_{n\to\infty}\sum_{r=1}^{n}\frac{r/n}{1+\left(\frac{r}{n}\right)^2}\cdot\frac{1}{n} \to \int_0^1 \frac{x}{1+x^2}\,dx$$

$$\sum_{\cdots}^{\cdots} \longrightarrow \int_{\cdots}^{\cdots}$$

$$\frac{r}{n} \to x$$

$$\frac{1}{n} \to dx$$

$$\frac{1}{n} < \frac{r}{n} < \frac{n}{n}$$

$$0 < x < 1$$

Definite Integration as Limit of Sum

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Q If $\boxed{a = \lim_{n\to\infty}\sum_{k=1}^{n}\frac{2n}{n^2+k^2}}$ and $f(x) = \sqrt{\frac{1-\cos x}{1+\cos x}}, x \in (0,1)$, then :

A. $2\sqrt{2}f\left(\frac{a}{2}\right) = f'\left(\frac{a}{2}\right)$

B. $f\left(\frac{a}{2}\right)f'\left(\frac{a}{2}\right) = \sqrt{2}$

C. ✓ $\sqrt{2}f\left(\frac{a}{2}\right) = f'\left(\frac{a}{2}\right)$

D. $f\left(\frac{a}{2}\right) = \sqrt{2}f'\left(\frac{a}{2}\right)$

$$a = \lim_{n\to\infty}\sum_{k=1}^{n}\frac{2\cancel{n}}{\left(\frac{n^2+k^2}{n^2}\right)\cdot n^{\cancel{2}}}$$

$$a = \lim_{n\to\infty}\sum_{k=1}^{n}\frac{2}{1+\left(\frac{k}{n}\right)^2}\cdot\frac{1}{n}$$

$$a = \int_0^1 \frac{2}{1+x^2}dx = 2\left(\tan^{-1}x\right)_0^1$$

$$\boxed{a = \pi/2}$$

$a = \frac{\pi}{2}$

$\frac{a}{2} = \frac{\pi}{4}$

$$f(x) = \sqrt{\frac{2\sin^2 x/2}{2\cos^2 \frac{x}{2}}} = \tan\left(\frac{x}{2}\right)$$

$$f(x) = \tan\left(\frac{x}{2}\right) \qquad f\left(\frac{a}{2}\right) = \tan\left(\frac{a}{4}\right) = \tan\left(\frac{\pi}{8}\right) = \sqrt{2} - 1$$

$$f'(x) = \frac{1}{2}\sec^2\left(\frac{x}{2}\right) \qquad f'\left(\frac{a}{2}\right) = \frac{1}{2}\sec^2\left(\frac{\pi}{8}\right)$$

$$f\left(\frac{a}{2}\right) = \sqrt{2} - 1 \qquad = \frac{1}{2}\left(1 + \tan^2\frac{\pi}{8}\right)$$

$$f'\left(\frac{a}{2}\right) = \sqrt{2}\, f\left(\frac{a}{2}\right) \qquad = \frac{1}{2}\left(1 + (\sqrt{2}-1)^2\right)$$

$$= \frac{1}{2}\left(4 - 2\sqrt{2}\right)$$

$$= 2 - \sqrt{2}$$

Q

$$\lim_{n\to\infty}\left(\frac{n^2}{(n^2+1)(n+1)}+\frac{n^2}{(n^2+4)(n+2)}+\frac{n^2}{(n^2+9)(n+3)}+\ldots+\frac{n^2}{(n^2+n^2)(n+n)}\right)$$

is equal to

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A. $\frac{\pi}{8}+\frac{1}{4}\log_e 2$

B. $\frac{\pi}{4}+\frac{1}{8}\log_e 2$

C. $\frac{\pi}{4}-\frac{1}{8}\log_e 2$

D. $\frac{\pi}{8}+\log_e \sqrt{2}$

$$\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{\underbrace{(n^2+r^2)}_{n^2}\underbrace{(n+r)}_{n}}\cdot\frac{1}{n} \longrightarrow \int_0^1 \frac{1}{(1+x^2)(1+x)}\,dx$$

$$\int_0^1 \frac{dx}{(1+x^2)(1+x)}$$

$$\frac{1}{(1+x^2)(1+x)} = \frac{A}{1+x} + \frac{Bx+C}{1+x^2}$$

$$1 = A(1+x^2) + (Bx+C)(x+1)$$

$x=-1$: $1 = 2A \longrightarrow \boxed{A = 1/2}$

$x=0$: $1 = \frac{1}{2} + C \longrightarrow \boxed{C = 1/2}$

$x=1$: $\cancel{1} = \cancel{1} + 2B + 1$ $\boxed{B = \frac{-1}{2}}$

$$\int \frac{\frac{1}{2}}{x+1} + \int \frac{\frac{-1}{2}x + \frac{1}{2}}{x^2+1}$$

$$\frac{1}{2}\ln(x+1) - \frac{1}{4}\int \frac{\boxed{2x\,dx}}{\boxed{x^2+1}} + \frac{1}{2}\int \frac{dx}{x^2+1}$$

$$\left[\frac{1}{2}\ln(x+1) - \frac{1}{4}\ln(x^2+1) + \frac{1}{2}\tan^{-1}(x)\right]_0^1$$

$$\Rightarrow \frac{1}{2}\ln 2 - \frac{1}{4}\ln 2 + \frac{\pi}{8}$$

$$\Rightarrow \frac{1}{4}\ln 2 + \frac{\pi}{8}$$

Walli’s Theorem

Walli's Theorem

$$\int_0^{\pi/2} \sin^m x \,.\, \cos^n x \, dx = \frac{[(n-1)(n-3)\ldots.1 \text{ or } 2][(m-1)(m-3)\ldots 1 \text{ or } 2]}{(m+n)(m+n-2)\ldots 1 \text{ or } 2} K$$

(m, n are non-negative integer)

$$\text{wher } K = \begin{cases} \dfrac{\pi}{2} & \text{if } \underline{m, n \text{ both are even}} \\ 1 & \underline{\text{otherwise}} \end{cases}$$

$\boxed{\#\text{TRAIN}}$

Evaluate $\int_0^{\pi/2} \sin^6 x \cdot \cos^4 x \, dx$

Kab? ① $0 \to \frac{\pi}{2}$

STOP → 1 OR 2

A. $3\pi/512$ **B.** $3\pi/216$ **C.** $\pi/512$ **D.** $\pi/216$

$$\Rightarrow \frac{(5 \times 3 \times 1)(3 \times 1)}{(10 \times 8 \times 6 \times 4 \times 2)}\left(\frac{\pi}{2}\right)$$

Q Evaluate $\int_0^{\frac{\pi}{2}} \sin^5 x \, dx \cdot (\cos x)^0$

$\underline{5 + 0}$

A. 8/15 ✓ **B.** 4/15 **C.** 16/15 **D.** 7/15

$$\Rightarrow \frac{(4 \times 2)}{(5 \times 3 \times 1)} (1)$$

$$\Rightarrow \frac{8}{15}$$

$$\Rightarrow \int_0^{\pi/2} \sin^5 x \, dx$$

$$\Rightarrow \frac{8}{15}$$

Int. By parts

Reduction Formula

Q

If $I_1 = \int_0^1 (1-x^{50})^{100} dx$ and $I_2 = \int_0^1 (1-x^{50})^{101} dx$

such that $I_2 = \alpha I_1$ then α equals to :

[JEE Main 2020]

$$\boxed{\alpha = \frac{I_2}{I_1}}$$

A. 5049/5050

B. 5050/5049

C. 5050/5051

D. 5051/5050

C

$$I_2 = \int_0^1 \underbrace{(1-x^{50})^{101}}_{\text{I}} \cdot \underbrace{1}_{\text{II}} \cdot dx$$

$$I_2 = \cancel{\left[(1-x^{50})^{101} \cdot x\right]_0^1} + \int_0^1 101 (1-x^{50})^{100}\, 50 x^{49} \cdot x \, dx$$

$$I_2 = -5050 \int_0^1 (1-x^{50})^{100} (1-x^{50}-1) \, dx$$

$$I_2 = -5050 \left\{ \int_0^1 (1-x^{50})^{101} - \int (1-x^{50})^{100} dx \right\}$$

$$I_2 = -5050 \{ I_2 - I_1 \}$$

$$I_2 = -5050 I_2 + 5050 I_1$$

$$5051 I_2 = 5050 I_1$$

$$\boxed{\frac{I_2}{I_1} = \frac{5050}{5051}}$$

If

$$\underbrace{n(2n+1)\int_0^1 (1-x^n)^{2n} dx}_{I_2} = 1177 \underbrace{\int_0^1 (1-x^n)^{2n+1} dx}_{I_1}, \quad \text{then}$$

$n \in N$ is equal to ______

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Ratio $= \dfrac{I_2}{I_1} = \dfrac{1177}{n(2n+1)}$

$$I_1 = \int_0^1 \underbrace{(1-x^n)^{2n+1}}_{\text{I}} \cdot \underbrace{1}_{\text{II}} \cdot dx$$