ALTERNATING CURRENT

PHYSICS WALLAH

ALTERNATING CURRENT

"If the direction of current in a resistor or any other element changes alternately, the current is called an alternating current"

ROOT MEAN SQUARE CURRENT

$I_{rms} = \sqrt{\overline{I^2}} = \sqrt{\frac{I_o^2}{2}} = \frac{I_o}{\sqrt{2}}$

$I_{rms} = \frac{I_o}{\sqrt{2}}$ $\quad V_{rms} = \frac{V_o}{\sqrt{2}}$

Average value of ac is defined for positive or negative half cycle

$\bar{I} = \frac{2I_o}{\pi}$ $\quad \bar{V} = \frac{2V_o}{\pi}$

AVERAGE AND RMS VALUE OF AC

If the current or voltage is sinusoidal than it can be expressed as

$i = i_o \sin(\omega t + \Phi)$

$v = v_o \sin(\omega t + \Phi)$

$i_o \rightarrow$ Peak current or current amplitude

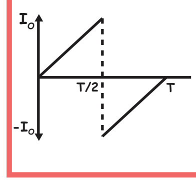
$v_o \rightarrow$ Peak voltage or voltage amplitude

$\omega = \frac{2\pi}{T} = 2\pi f$ $\quad$ T: Time period

f: frequency (Hz or cycle/sec)

$(\omega t + \Phi)$: Total phase

SAWTOOTH FUNCTION



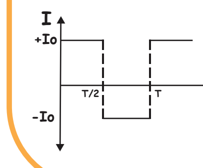
For half cycle $\bar{I} = \frac{I_o}{2}$

For full cycle $\bar{I} = 0$

Mean square current $\overline{I^2} = \frac{I_o^2}{3}$

rms current $I_{rms} = \frac{I_o}{\sqrt{3}}$

RECTANGULAR FUNCTION



For half cycle $\bar{I} = Io$

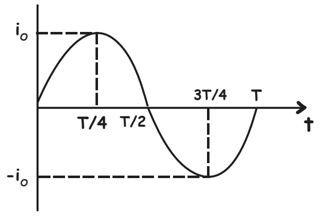
For full cycle $\bar{I} = 0$

Mean square current $\overline{I^2} = Io^2$

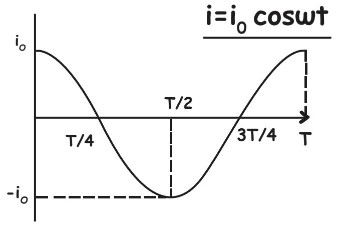
rms current $I_{rms} = Io$

GENERAL GRAPH

if $i = i_o \sin\omega t$



$i = i_o \cos\omega t$



⇒ for measuring ac hot wire instruments are used

AVERAGE HEAT PRODUCED DURING A CYCLE OF AC

$H_{avg} = \frac{1}{2} Io^2 R = I_{rms}^2 R$

Keep in mind

⇒ rms value is also called virtual value or effective value

⇒ AC ammeter and voltmeter always measure rms value

⇒ Values printed on ac circuits are rms values

⇒ In houses ac is supplied at 220V which is the rms of voltage

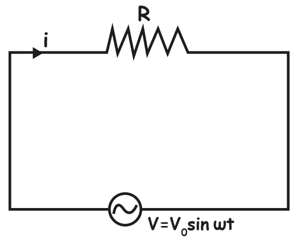
⇒ Peak value is $220\sqrt{2} = 311V$

⇒ Frequency in general is 50Hz

⇒ $\omega = 2\pi f = 100\pi$ rad/sec (314 rad/sec)

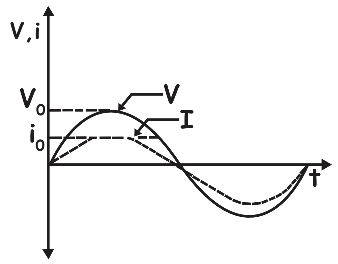
SINGLE COMPONENT CIRCUITS

Resistor only



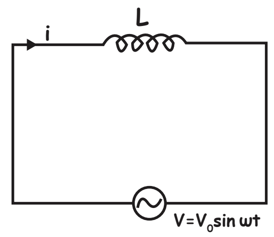
1. $V = V_o \sin\omega t$
2. $i = i_o \sin\omega t$
3. V&i are in phase
4. 
5. $\Phi = 0, \cos\Phi = 1$

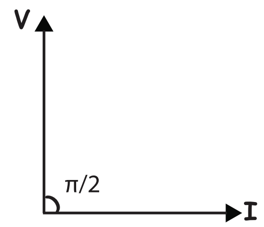
   $\bar{P} = \varepsilon_{rms} I_{rms}$



6. $i_o = \frac{V_o}{R}$ & $i_{rms} = \frac{V_{rms}}{X_L}$

Inductor only



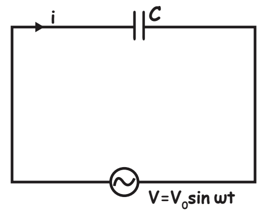
1. $V = V_o \sin\omega t$
2. $i = i_o \sin(\omega t - \pi/2)$
3. or current leads to the voltage by $\pi/2$
4. 
5. $\Phi = \pi/2, \cos\Phi = 0$
6. $\bar{P} = 0$ (wattless circuits)
7. Inductive reactance (X_L)

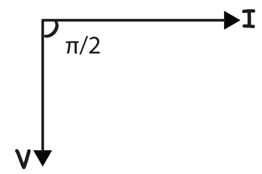
   $X_L = L\omega$

   Unit-ohm(Ω)

   plays role of resistance
8. $i_o = \frac{V_o}{X_L}$ & $i_{rms} = \frac{V_{rms}}{X_c}$

Capacitor only



1. $V = V_o \sin\omega t$
2. $i = i_o \sin(\omega t + \pi/2)$
3. Current leads the voltage by $\pi/2$
4. 
5. $\Phi = \pi/2, \cos\Phi = 0$
6. $\bar{P} = 0$ (wattless circuits)
7. Inductive reactance

   $X_c = \frac{1}{c\omega}$

   Unit-ohm(Ω)

   plays role of resistance
8. $i_o = \frac{V_o}{X_c}$ & $i_{rms} = \frac{V_{rms}}{X_c}$

SUMMARY

	Z (Impedance)	Φ
1. R only	R	0
2. L only	$X_L = \omega L$	$-\pi/2$
3. C only	$X_c = \frac{1}{c\omega}$	$\pi/2$

AVERAGE VALUE OF AC FOR ONE TIME PERIOD

$\bar{I} = \frac{\int_0^T I dt}{\int_0^T dt} = \frac{\int_0^T I_o \sin\omega t \, dt}{\int_0^T dt} = \frac{\text{area of I-t graph}}{\text{time}}$

$\bar{I} = 0$ for $0 \rightarrow T$ for a sinusoidal ac wave.

The average value of sin or cos function for one time period or n time periods (n=1,2...) is zero

Keep in mind

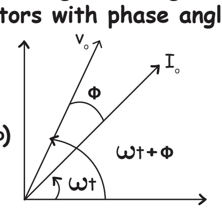
Long period is equivalent to one time period

PHASOR DIAGRAM

Diagram representing ac voltage or current as vectors with phase angle between them.

$I = I_o \sin(\omega t)$

$v = v_o \sin(\omega t + \Phi)$



Mean square current for one time Period

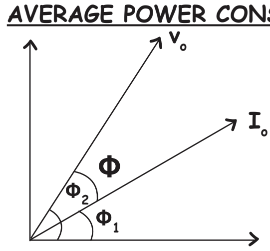
$\overline{I^2} = \frac{\int_0^T I^2 dt}{\int_0^T dt} = \frac{I_o^2}{2}$

Remember

The average value of square of sin or cosine function for one time period is $\frac{1}{2}$

$\boxed{\begin{array}{l} \int_0^T \sin^2 k\omega t = \frac{1}{2} \times T \\ \int_0^{nT} \sin^2 k\omega t = \left(\frac{n}{2}\right) \times T \end{array}}$

AVERAGE POWER CONSUMPTION



$P = VI$

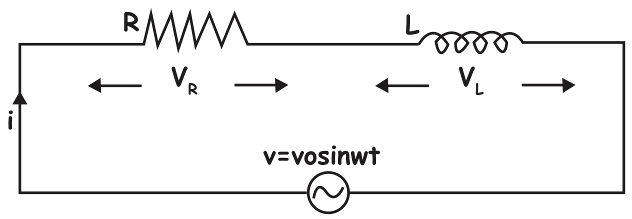
$\boxed{\bar{P}_{avg} = V_{rms} I_{rms} \cos\Phi}$

for $\cos\Phi = 1$ or $\Phi = 0°$

$\bar{P}_{avg} = V_{rms} I_{rms}$

SERIES AC CIRCUITS

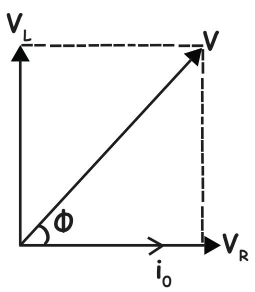
1) R-L CIRCUITS



1. $V = V_o \sin\omega t$ $\quad V_R = i_o R$, $V_L = i_o X_L$
2. Voltage phasor diagram

   $V = \sqrt{V_R^2 + V_L^2}$

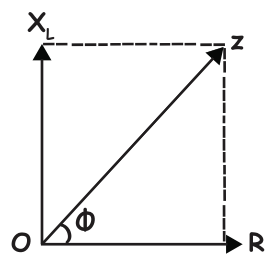
   $\tan\Phi = \left[\frac{V_L}{V_R}\right]$



3. $i = i_o \sin(\omega t - \Phi)$
4. $\frac{V_o}{i_o} = \frac{V_{rms}}{i_{rms}} = Z = \sqrt{R^2 + X_L^2}$
5. Impedance phasor

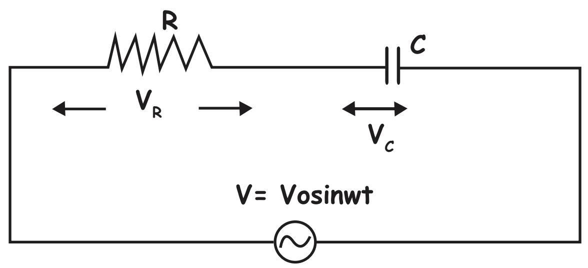
   $Z = \sqrt{R^2 + I^2}$

   $\tan\Phi = \frac{X_L}{R}$



6. $i_o = \frac{V_o}{Z}$

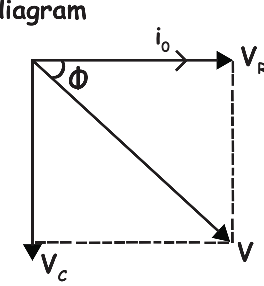
2) R-C CIRCUITS



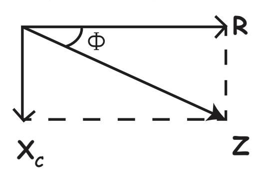
1. $V = V_o \sin\omega t$ $\quad V_R = i_o R$, $V_C = i_o X_C$
2. Voltage phasor diagram

   $V = \sqrt{V_R^2 + V_c^2}$

   $\tan\Phi = \left[\frac{V_c}{V_R}\right]$



3. $i = i_o \sin(\omega t + \Phi)$
4. Impedance phasor



   $Z = \sqrt{R^2 + X_c^2}$

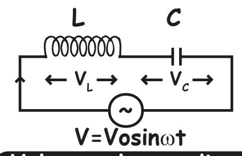
   $\tan\Phi = \left(\frac{X_c}{R}\right)$

5. $i_o = \left(\frac{V_o}{Z}\right)$

   or $i_{rms} = \frac{V_{rms}}{Z}$

i

3) L-C CIRCUIT

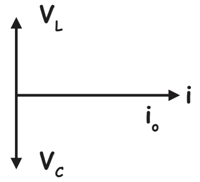


$V = V_0 \sin\omega t$

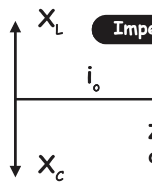
$V_L = i_0 X_L,\ V_C = i_0 X_C$

Voltage phasor diagram

$V = V_L \sim V_C$ [ie, $(V_L - V_C)$ or $(V_C - V_L)$]



Impedance Phasor Diagram



$Z = X_L \sim X_C$ [$(X_L - X_C)$ or $(X_C - X_L)$]

if $X_L > X_C$, Voltage leads the current by $\frac{\pi}{2}$

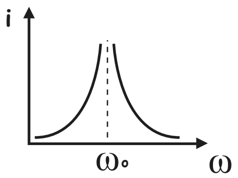
if $X_C > X_L$, current leads the voltage by $\frac{\pi}{2}$

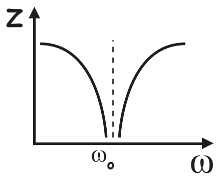
if $X_L = X_C$, $Z = 0$, $i = \infty$

or,

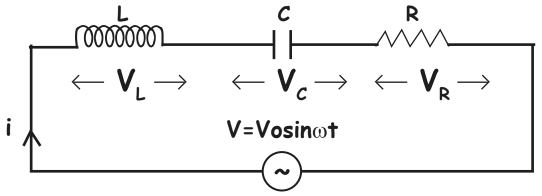
$\omega L = \frac{1}{\omega C}$

$\omega = \frac{1}{\sqrt{LC}} = \omega_0$





L-C-R Series Circuit

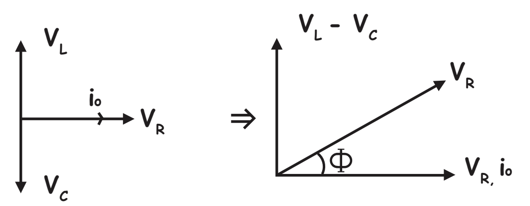


$V = V_0 \sin\omega t$

$V_R = i_0 R$ $V_L = i_0 X_L$, $V_C = i_0 X_C$

Assuming $V_L > V_C$ for drawing phasor

Voltage phasor diagram

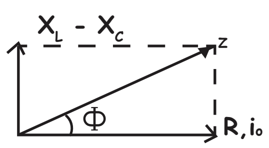


$V = \sqrt{V_R^2 + (V_L - V_C)^2}$

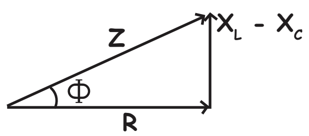
Here $i = i_0 \sin(\omega t - \Phi)$

(since V_L is leading)

Impedance Triangle



$Z = \sqrt{R^2 + (X_L - X_C)^2}$



$\tan\Phi = \frac{X_L - X_C}{R}$

RESONANCE IN LCR SERIES CIRCUIT

In series resonance, impedance of circuit is minimum & equal to resistance ⇒Z= R, and curent is maximum

Condition for resonance

$X_L = X_C \Rightarrow L\omega = \frac{1}{C\omega}$

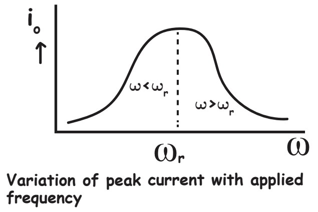
$\omega = \omega_r = \frac{1}{\sqrt{LC}}$ rad / sec

$\omega_r \rightarrow$ resonant frequency (angular)

$f = f_r = \frac{1}{2\pi\sqrt{LC}}$ H_z

f_r = resonant frequency

GRAPH



Variation of peak current with applied frequency

$\omega < \omega_r$, $X_C > X_L$ current leads

$\omega > \omega_r$, $X_L > X_C$ current lags

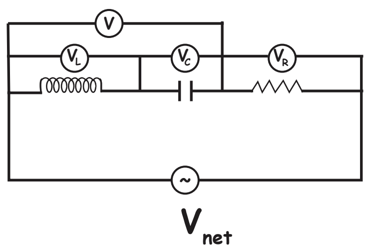
In resonance

$V = V_R$ (applied voltage = voltage across resistance)

$Z = R$ (impedance is minimum and equal to resistance)

Voltmeter connected across V_L & V_C will show the same reading

Voltmeter connected commonly across inductor & capacitor shows no reading



Here V = 0

$V_L = V_C$

$V_N = V_R$

$i_{max} = \frac{V_{net}}{Z} = \frac{V_R}{R}$

APPLICATION OF RESONANT CIRCUIT

Tuning mechanism of a radio or TV set

1. Antenna of radio accepts signals

2. Signal acts as an AC source in tuning the radio

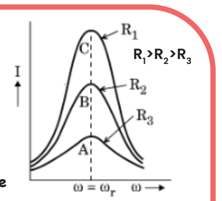
3. In tuning, capacitance of capacitor is varied such that the resonant frequency of the circuit becomes nearly equal to the frequency of the radio signal received.

So, the simple is largely amplified and distinctly heard

QUALITY FACTOR

$Q = \frac{\omega_r L}{R} = \frac{1}{\omega_r CR} = \frac{1}{R}\sqrt{\frac{L}{C}}$

or $Q = \left[\frac{\text{Voltage across C or L}}{\text{applied voltage}}\right]_{resonance}$



Less sharp the resonance, less is the selectivity of the circuit. If the Quality factor is large, R is low or L is large, the circuit is more selective.

Sharpness of Resonance

Sharpness= $Q = \frac{\omega_r}{2\Delta\omega}$; $2\Delta\omega$ -bandwidth

smaller $\Delta\omega$, sharper or narrower the resonance.

POWER IN AC CIRCUIT

Average Power $\bar{P} = V_{rms} I_{rms} \cos\Phi$

$\bar{P} = I^2_{rms} Z \cos\Phi$

Case 1

Purely Resistive circuit - $\Phi = 0$, $\cos\Phi = 1$

Maximum power dissipation

Case 2

Purely inductive or capacitive circuit-

$\Phi = 90^0$ $\cos\Phi = 0$

No power is dissipated even though a current is flowing in the circuit

Case 3

LCR Series circuit

Φ non zero in R-L, C-R, or CLR circuit.

$\bar{P} = V_{rms} I_{rms} \cos\Phi$

Case 4

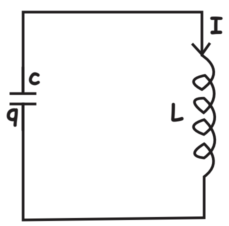
Power dissipation at resonance

$X_L - X_C = 0$ or $\Phi = 0$ ⟹ $\cos\Phi = 1$ ⇒ Z=R

$P = I^2 Z = I^2 R$

Maximum power is dissipated in a circuit at resonance.

LC OSCILLATIONS



Capacitor ⟶ stores electrical energy

Inductor ⟶ stores magnetic energy

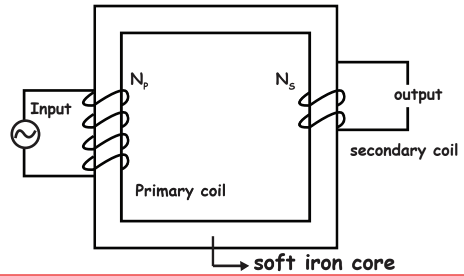
When connected, charge on the capacitor and current in the inductor perform electrical oscillations between each other.

COMPARISON OF LC OSCILLATION WITH A MASS SPRING SYSTEM

Mass spring system	LC Circuit
1. Displacement (x)	1. Charge (q)
2. Velocity $V = \frac{dx}{dt}$	2. Current $I = \frac{dq}{dt}$
3. Acceleration $a = \frac{dv}{dt}$	3. Rate of change of current $= \left(\frac{dI}{dt}\right)$
4. Mass (m), (inertia)	4. Inductance (L), inertia of circuit
5. Force constant K	5. Capacitance (C)
6. Momentum p = mv	6. Magnetic flux $\Phi = LI$
7. Retarding force $-m\frac{dv}{dt}$	7. Self induced emf $\left(-L\frac{dI}{dt}\right)$
8. Differential equation $\frac{d^2x}{dt^2} + \omega^2 x = 0$, $\omega = \sqrt{\frac{k}{m}}$	8. Differential equation $\frac{d^2q}{dt^2} + \omega^2 q = 0$, $\omega = \sqrt{\frac{1}{LC}}$
9. K.E $= \frac{1}{2}mv^2$, Elastic U $= \frac{1}{2}kx^2$	9. Magnetic energy $= \frac{1}{2}LI^2$, Elastic U $= \frac{q^2}{2C}$

TRANSFORMERS

"Device which raises or lowers voltage in ac circuits through mutual induction". Transformer can increase or decrease voltage or current but not both simultaneously.



EQUATIONS

1) $\frac{V_S}{V_P} = \frac{N_S}{N_P} = \frac{I_P}{I_S}$

2) Efficiency $\eta = \frac{P_{out}}{P_{in}} = \frac{V_S I_S}{V_P I_P}$

3) For ideal transformer, η=1

V_s -Voltage in secondary

V_p -Voltage in primary

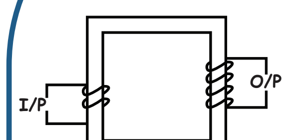
N_s -No of turns in secondary

N_p -No of turns in primary

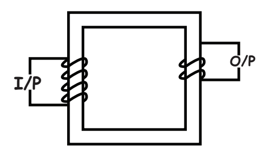
I_p -Current in primary

I_s -Current in secondary

TRANSFORMER TYPES



N_p	<	N_s
V_p	<	V_s
I_p	>	I_s
R_p	<	R_s



N_p	>	N_s
V_p	>	V_s
I_p	<	I_s
R_p	>	R_s

LOSSES IN TRANSFORMER

1) Cu loss (I^2R loss)
→ To minimise, windings are made of thick Cu wires (high resistance)

2) Eddy current loss
→ To minimise Cores are laminated

3) Hysteresis loss
→ select material of narrow hysteresis loop
→ Cores of transformer is made of soft iron

4) Magnetic flux linkage
→ To minimise, secondary winding is kept inside the primary winding

5) Humming loss

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