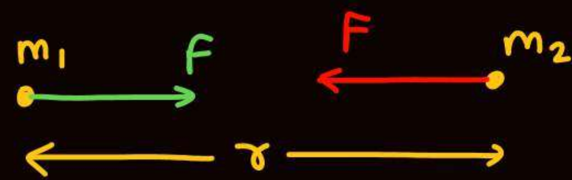
Today's Goal

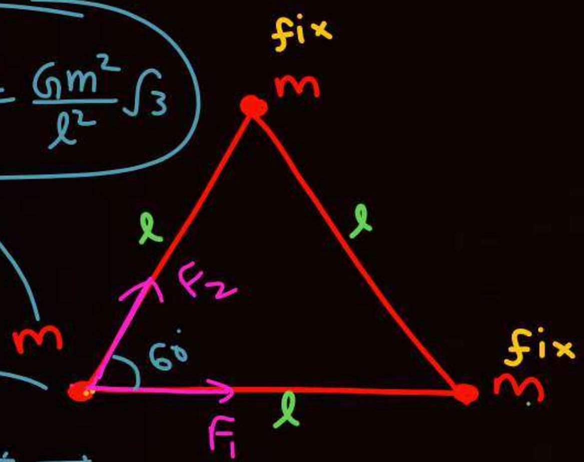
1. Gravitation
2.
3.
4.



$$F \propto \frac{m_1 m_2}{r^2}$$

$$\boxed{F = \frac{G m_1 m_2}{r^2}}$$

$$F_{net} = F\sqrt{3} = \frac{Gm^2}{l^2}\sqrt{3}$$

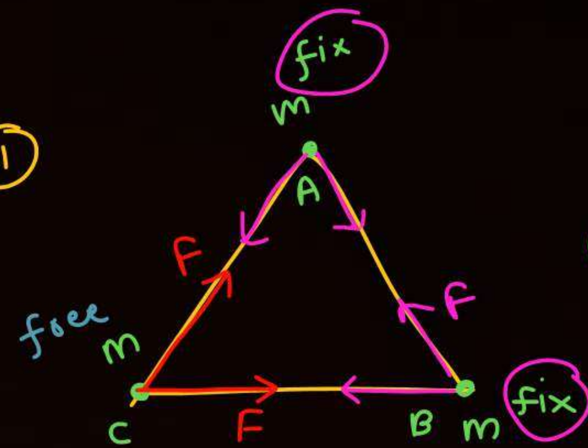


$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2$$

$$F_1 = \frac{Gmm}{l^2} = F$$

$$F_2 = \frac{Gmm}{l^2} = F$$

(1)



(1) $$(F_{net})_{on\ 'c'} = \frac{Gm^2}{l^2}\sqrt{3}$$

$$a_1 = a_{cc} \text{ of mass at 'c'} = \frac{F_{net}}{m} = \frac{Gm\sqrt{3}}{l^2}$$

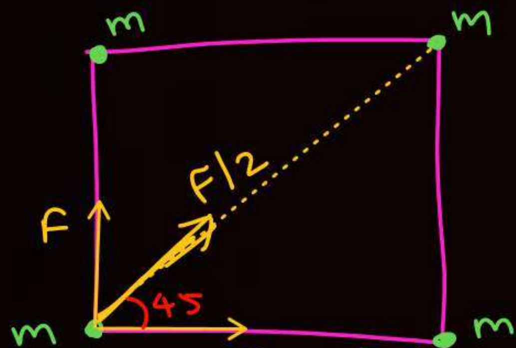
$$a_{com} = \frac{ma_1 + 0 + 0}{m+m+m} = \frac{a}{3}$$

$$F_{net} = M_{total}\, a_{com}$$

$$\| \quad F$$

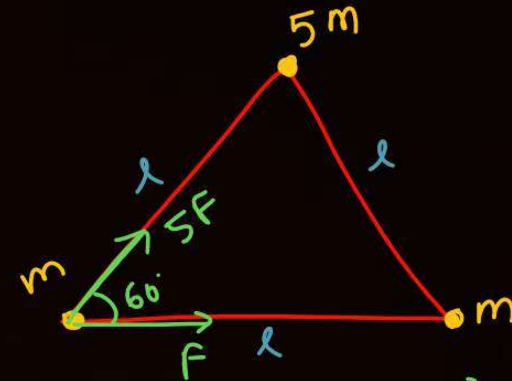
$$= 3m \times \frac{a}{3}$$

बेशर्म



$$F = \frac{Gm^2}{l^2}$$

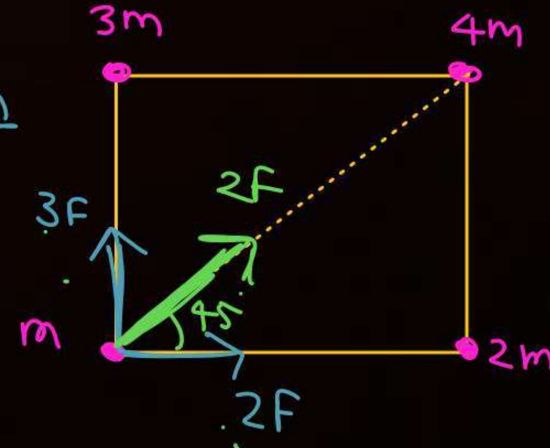
$$F_{net} = F\sqrt{2} + \frac{F}{2}$$



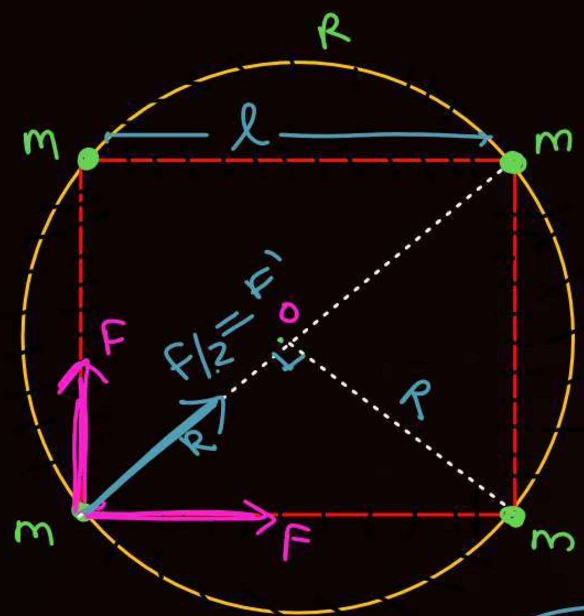
$$F = \frac{Gm^2}{l^2}$$

$$F_{net} = \sqrt{F^2 + (5F)^2 + 2 \times F \times 5F \cos 60}$$

$$F = \frac{Gmm}{l^2}$$



Q_2



$$F = \frac{Gm^2}{l^2}$$

$$l = R\sqrt{2}$$

$$F' = \frac{Gmm}{(l\sqrt{2})^2} = \frac{Gm^2}{2l^2} = \frac{F}{2}$$

$$F_{net} = F\sqrt{2} + \frac{F}{2} = \frac{mv^2}{R}$$

$$= mRW^2$$

q_1 q_2

m_1 m_2

* $F = \dfrac{K q_1 q_2}{r^2}$ $K = \dfrac{1}{4\pi\epsilon_0}$

$F = \dfrac{G m_1 m_2}{r^2}$

* $E.F = \lim\limits_{q_0 \to 0} \dfrac{\vec{F}}{q_0}$

$G.F = E_g = \lim\limits_{m \to 0} \dfrac{\vec{F}}{m_0}$

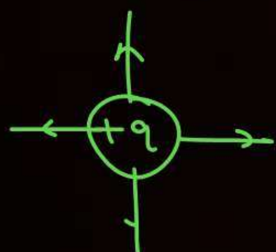
$q \longrightarrow m$

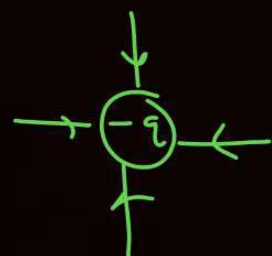
$K \longrightarrow G$

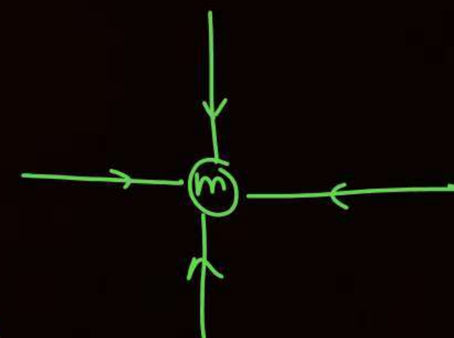
$\dfrac{1}{4\pi\epsilon_0} \longrightarrow G$

$\epsilon_0 \longrightarrow \dfrac{1}{4\pi G}$

$\dfrac{1}{\epsilon_0} \longrightarrow 4G\pi$







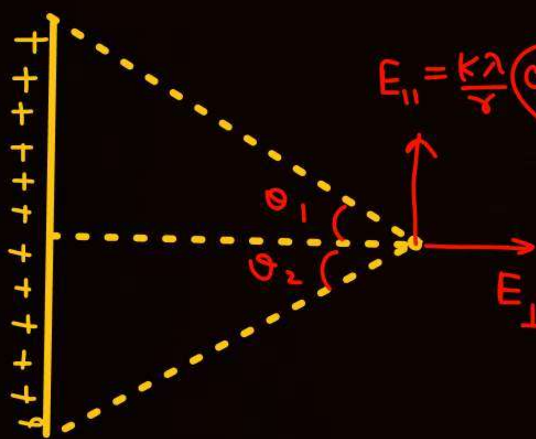
dipole
$-q$
Conductor } X

(1)

m

$E_g = \frac{Gm}{r^2}$

(2)

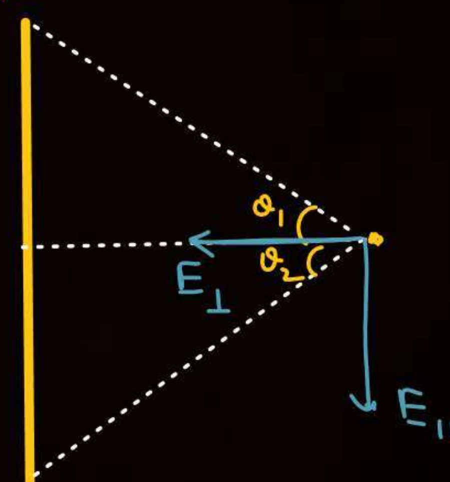


$E_{\parallel} = \frac{k\lambda}{r}(\cos\theta_1 - \cos\theta_2)$

$E_{\perp} = \frac{k\lambda}{r}(\sin\theta_1 + \sin\theta_2)$

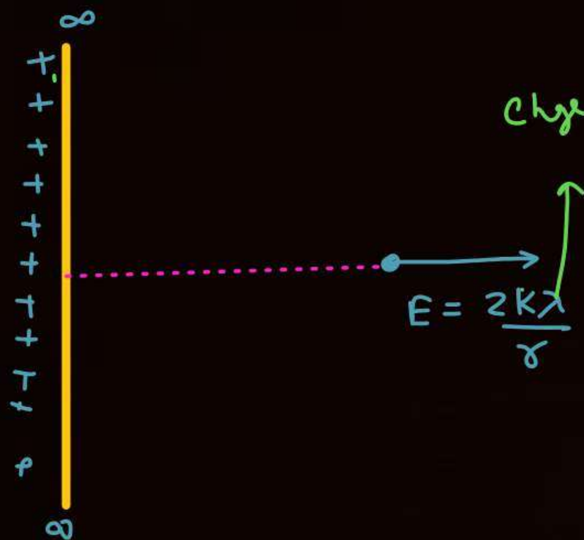
$+m$

$\frac{m}{L} = \lambda$

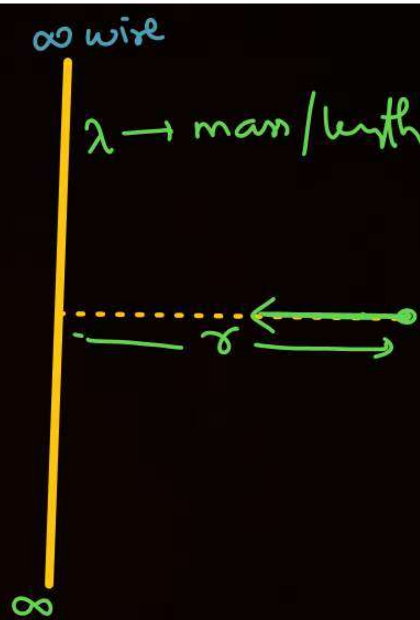


$E_{\parallel} = \frac{G\lambda}{r}(\cos\theta_1 - \cos\theta_2)$

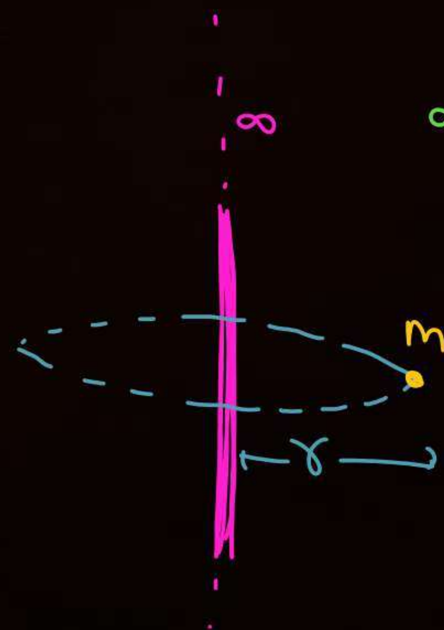
③



$E = \frac{2k\lambda}{r}$

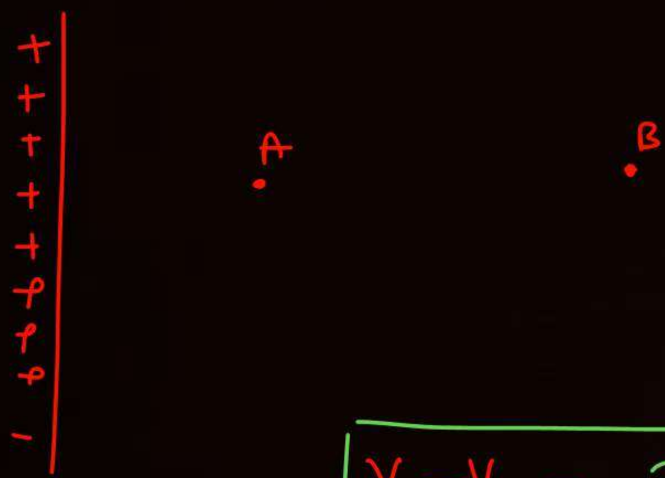


$$\frac{2G\lambda}{r}$$



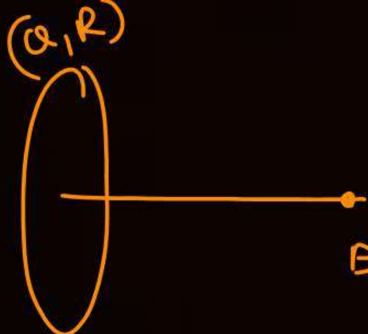
$$F = m \times \frac{2G\lambda}{\cancel{r}} = \frac{mv^2}{\cancel{r}}$$

$$v = \sqrt{2G\lambda}$$

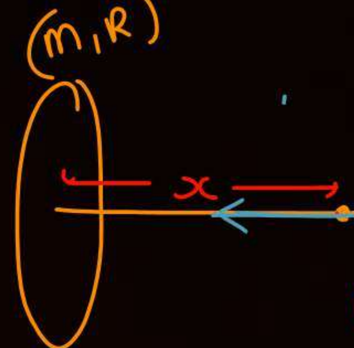


$$\boxed{V_A - V_B = 2k\lambda \ln \frac{r_2}{r_1}}$$

*

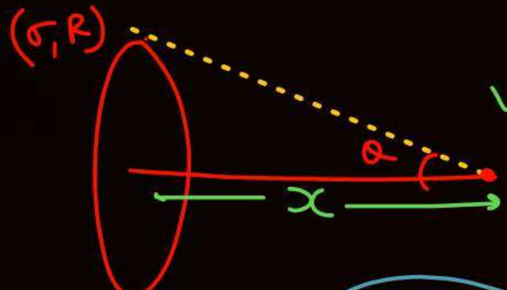


$$E = \frac{KQx}{(R^2+x^2)^{3/2}}$$



$$E_g = \frac{GMx}{(R^2+x^2)^{3/2}}$$

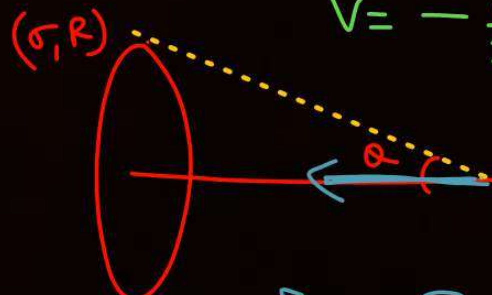
Disc



$$V = \frac{\sigma}{2\epsilon_0}(1-\cos\theta)\sqrt{R^2+x^2}$$

$$E = \frac{\sigma}{2\epsilon_0}(1-\cos\theta)$$

∞ Sheet $\Rightarrow$ $E = \frac{\sigma}{2\epsilon_0}$



$$V = -\frac{\sigma}{2} 4\pi G (1-\cos\theta) \times \sqrt{R^2+x^2}$$

$$E_g = \frac{\sigma}{2} 4\pi G (1-\cos\theta)$$

∞ Sheet $\Rightarrow$ $E_g = \frac{\sigma}{2} 4\pi G$

hollow sphere

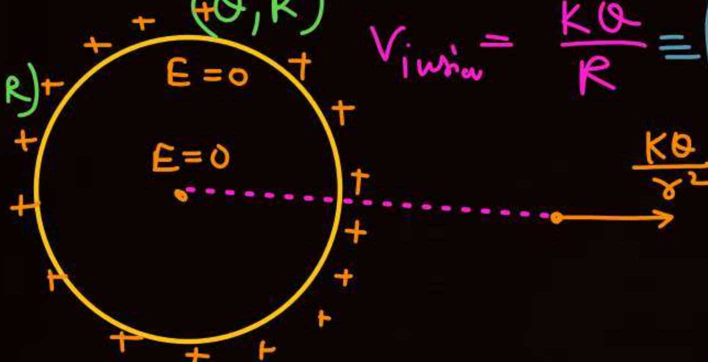
$V_{outside} = \frac{KQ}{r} \equiv -\frac{Gm}{r}$

$V_{surface} = \frac{KQ}{R} \equiv -\frac{Gm}{R}$

$V_{inside} = \frac{KQ}{R} \equiv -\frac{Gm}{R}$

(Q, R)

$E_{inside} = 0, \ (r<R)$



outside

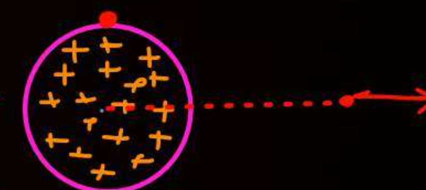
$E = \frac{KQ}{r^2}$

surface $E = \frac{KQ}{R^2}$

solid sphere $\equiv$ Earth

outside $r>R$

$E = \frac{KQ}{r^2} \equiv \frac{Gm}{r^2}$



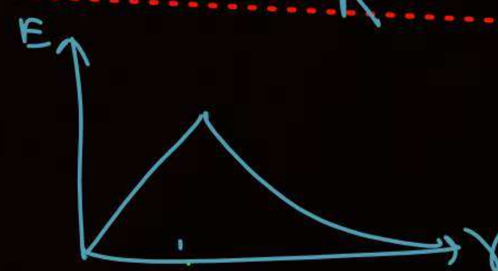
surface $E = \frac{KQ}{R^2} \equiv \frac{Gm}{R^2}$

inside

$\vec{E} = \frac{KQ\vec{r}}{R^3} = \rho\frac{\vec{r}}{3\epsilon_0} \equiv \frac{\rho r 4\pi G}{3}$

$\frac{Gmr}{R^3}$

$E_g = 0, \quad \frac{Gm}{R^2}, \quad \frac{Gm}{r^2}$



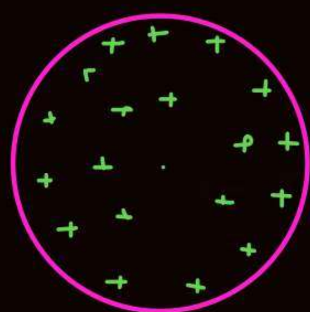
Solid sphere

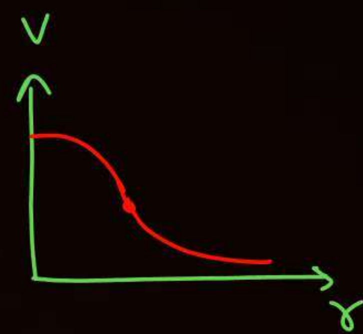
① outside $V = \frac{KQ}{r}$

② surface $V = \frac{KQ}{R}$

③ Inside $V = \frac{KQ(3R^2 - r^2)}{2R^3}$

④ At center $V_{center} = \frac{3}{2}\frac{KQ}{R}$





Gravitation earth

outside $V = -\frac{GM}{r}$

Surface $V = -\frac{GM}{R}$

Inside $V = -\frac{GM}{2R^3}(3R^2 - r^2)$

Center $V = -\frac{3}{2}\frac{GM}{R}$

Electrostatics

① Velocity of charge when it reaches at center of sphere. $\equiv V_{max}$

$$K_i + U_i = K_f + U_f$$

$$0 + qV_A = \frac{1}{2}mv^2 + qV_{center}$$

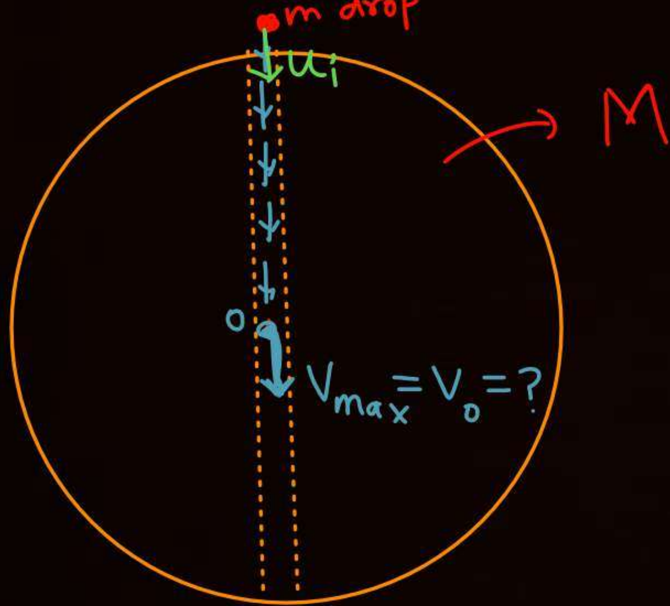
$$q\frac{KQ}{R} = \frac{1}{2}mV^2 + q\left(\frac{3}{2}\frac{KQ}{R}\right)$$

ⓑ find speed of charge particle when it reaches at a distance R/2 from center of sphere

$$K_i + U_i = K_f + U_f$$

$$0 + q\frac{KQ}{R} = \frac{1}{2}mV_B^2 + q\left(\frac{KQ\left(3R^2 - \left(\frac{R}{2}\right)^2\right)}{2R^3}\right)$$

①

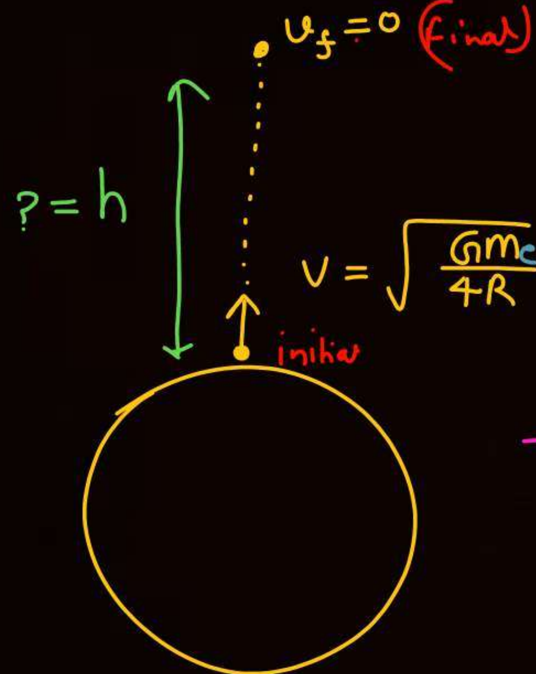


$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2} m u_i^2 + m\left(-\frac{GM_e}{R}\right) = \frac{1}{2} m V_{max}^2 + m\left(-\frac{3}{2}\frac{GM_e}{R}\right)$$

$$T = 2\pi\sqrt{\frac{R}{g}}$$

(2)



$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2} m \frac{GM_e}{4R} + m\left[-\frac{GM_e}{R}\right] = 0 + m\left[-\frac{GM_e}{h+R}\right]$$

$$-\frac{7\cancel{GM_e m}}{8R} = -\frac{\cancel{GM_e m}}{(h+R)}$$

$$7(h+R) = 8R$$

$$h = R/7$$

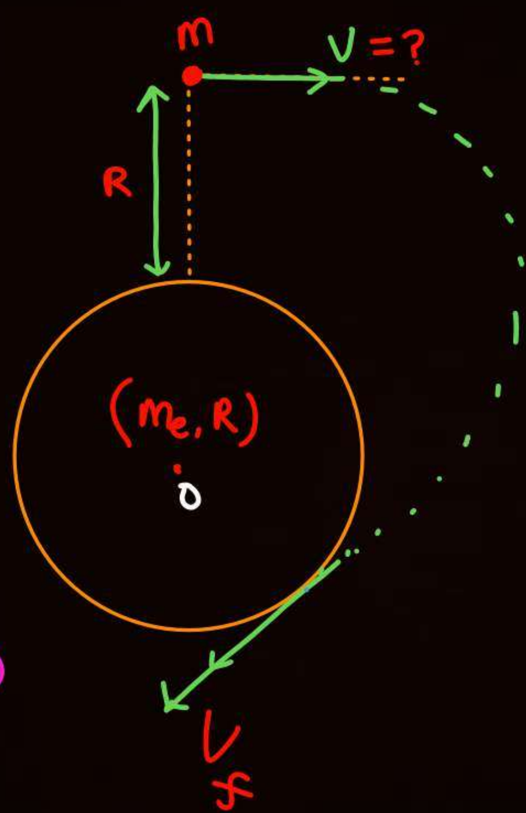
$$r = \frac{8R}{7}$$

from center of earth

Q

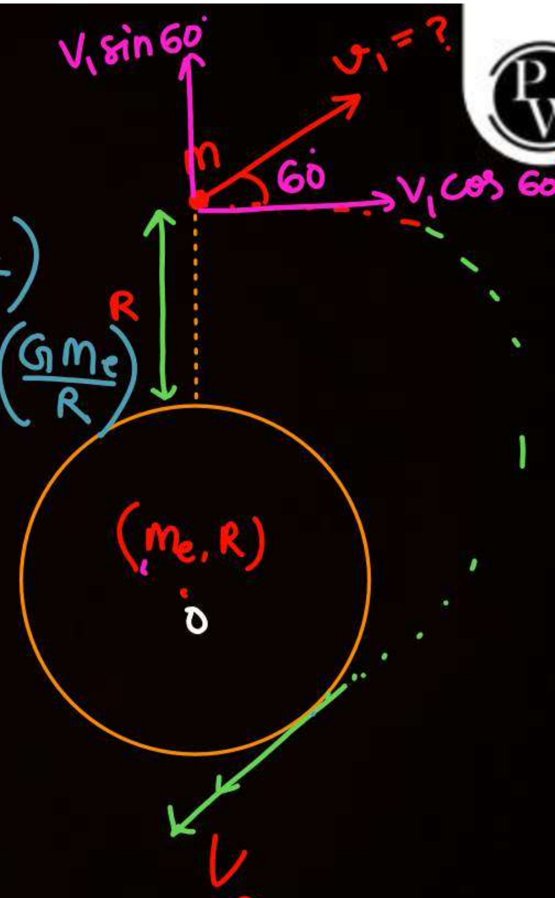
$$\frac{1}{2}mv^2 - m\left(\frac{GM_e}{2R}\right) = \frac{1}{2}mv_f^2 - m\left(\frac{GM_e}{R}\right) \quad \text{—— (1)}$$

$$mv2R = mv_fR \quad \text{—— (2)}$$



(1)

$$\frac{1}{2}mv_1^2 - m\left(\frac{GM_e}{2R}\right) = \frac{1}{2}mv_f^2 - m\left(\frac{GM_e}{R}\right)$$



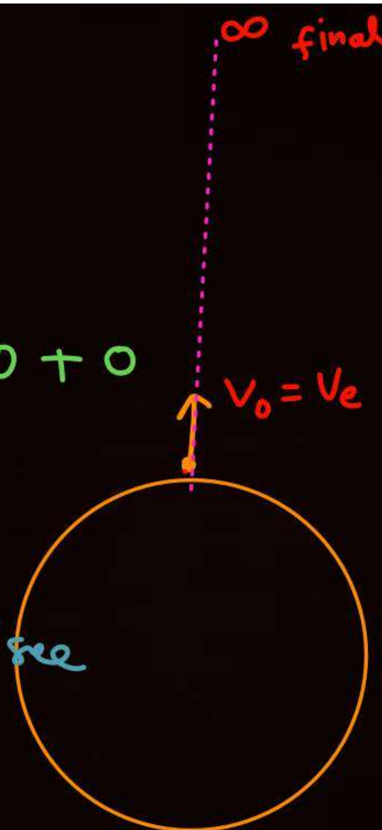
$$mV\cos 60\, R = mV_f R \quad \text{—— (2)}$$

Q $K_i + U_i = K_f + U_f$

$$\frac{1}{2}mv_e^2 + m\left(-\frac{GM_e}{R}\right) = 0 + 0$$

$$V_e = \sqrt{\frac{2GM_e}{R_e}} = 11.2 \text{ km/sec}$$

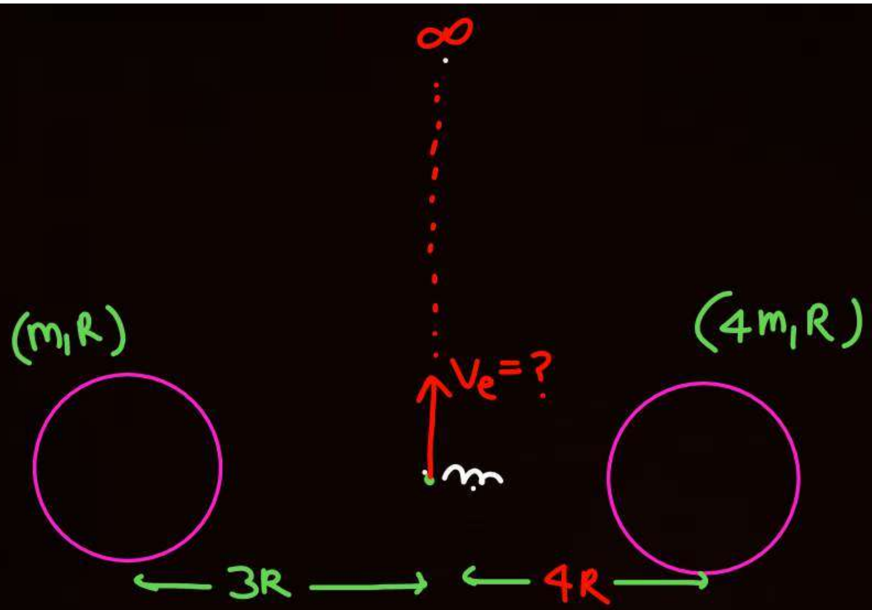
Escape Velocity



$$V_e = \sqrt{\frac{2GM \cdot R}{R \cdot R}}$$

$$\boxed{V_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}}$$

Q

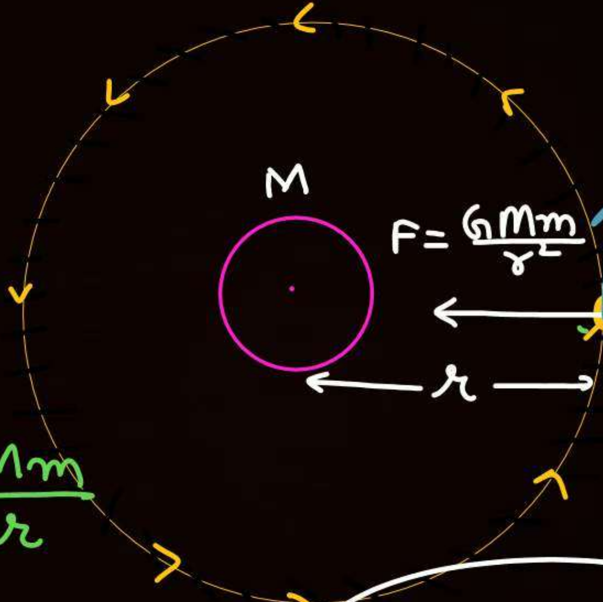


$$\frac{1}{2} m V_e^2 + m\left[-\frac{Gm}{3R} - \frac{G4m}{4R}\right] = 0 + 0$$

1. Variation of g
2. Escape velocity
3. Sattllite motn
4. $\dfrac{G m_1 m_2}{r^2} \equiv$
5. Kepler's Law
6. G.P.E

P.E

Sattelite motion.



$$\frac{GM\cancel{m}}{r^2} = \frac{\cancel{m}v^2}{r}$$

① $v = \sqrt{\frac{GM}{r}} = v_0$

② $K.E. = \frac{1}{2}mv_0^2 = \frac{1}{2}\frac{GMm}{r}$

$P.E. = -\frac{GMm}{r}$

$T.E. = T.M.E. = KE + PE = -\frac{GMm}{2r}$

$K.E. = |T.E.| = \frac{|P.E|}{2}$

$v_0 = \sqrt{\frac{GM}{r}} = \frac{2\pi r}{T}$

$$\boxed{T = 2\pi\sqrt{\frac{r^3}{GM}}}$$

$\omega = \frac{2\pi}{T} = \sqrt{\frac{GM}{r^3}}$

sattellite
r_1 orbit
$(T.E.)_i$

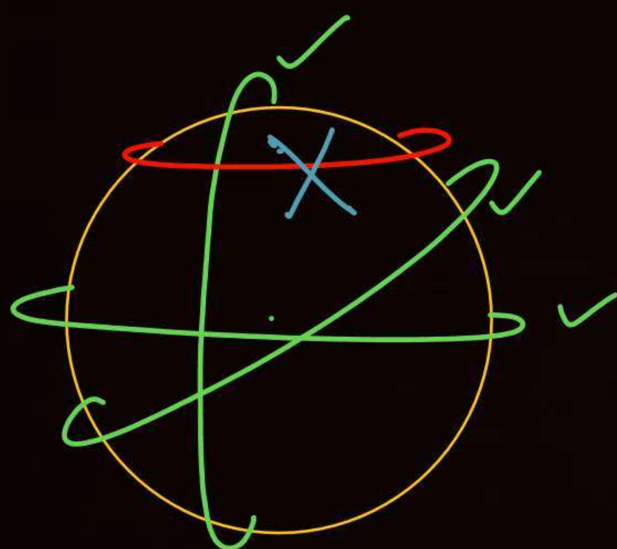
$\xrightarrow[\text{Energy required}]{\text{orbit change}}$

r_2
$(T.E.)_f$

$$\Delta E = (T.E.)_f - (T.E.)_i$$

$$= \left(-\frac{GMm}{2r_2}\right) - \left(-\frac{GMm}{2r_1}\right)$$

sattellite



Geostationary Sattelite

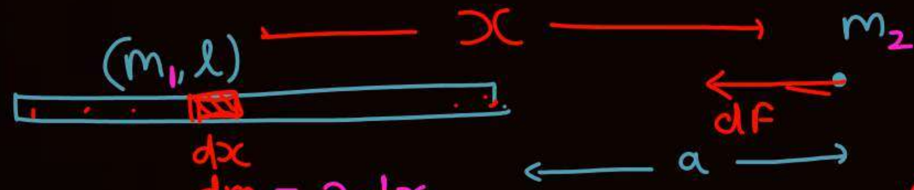
$T = 24$ hour

$r \simeq 42600$ km from center of earth

surface $h \approx 36000$ km

Q

net force on point mass



$dm = \lambda dx$

$$dm = \frac{m_1}{l} dx$$

$\frac{Gmm}{l^2}$ X

$\frac{Gmm}{a^2}$ X

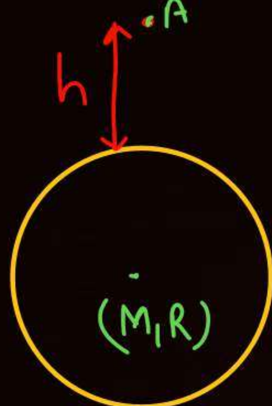
$\frac{Gmm}{\left(a+\frac{l}{2}\right)^2}$ X

$$\int dF = \int_a^{a+l} \frac{G m_2 \times \lambda}{x^2} dx$$

$$= \frac{G m_1 m_2}{l} \int_a^{a+l} \frac{1}{x^2} dx = \checkmark$$

Earth Variation of 'g'

Outside



$$(E_g)_A = \frac{Gm}{r^2} = \frac{Gm}{(R+h)^2} = g_A$$

If $h << R$

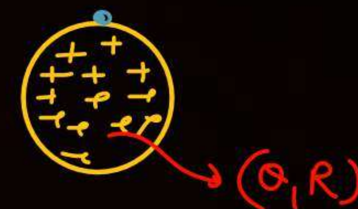
$$g_A = Gm(R+h)^{-2}$$

$$= \frac{Gm}{R^2}\left(1+\frac{h}{R}\right)^{-2}$$

$$g_A \simeq \frac{Gm}{R^2}\left(1-\frac{2h}{R}\right) = g_0\left(1-\frac{2h}{R}\right)$$

(2) At surface

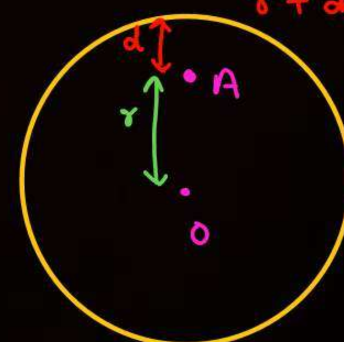
$$g_0 = \frac{Gm}{R^2}$$



$g \simeq 9.8\ m/s^2 = 10\ m/s^2$ (earth)

(3) Inside

$r + d = R$



$$g_A = \frac{Gmr}{R^3}$$

$$g_A = \frac{Gm(R-d)}{R^3}$$

$$g_A = \frac{Gm}{R^2}\left(1-\frac{d}{R}\right) = g_0\left(1-d/R\right)$$

(1)

$g_0 = \frac{GM}{R^2}$ (at surface)

height $\Rightarrow$ $\boxed{g = \frac{GM}{(R+h)^2}}$

Approx $\Rightarrow$ $g' = g_0\left(1 - \frac{2h}{R}\right)$

depth $g = g_0\left(1 - \frac{d}{R}\right)$

Mains freq

अगर height ज्यादा given हो

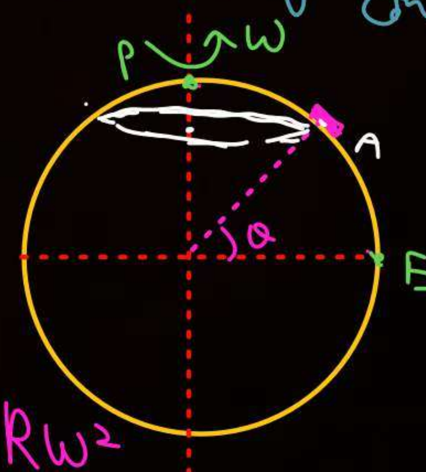
या ना given हो information $\equiv g = \frac{GM}{(R+h)^2}$

(II) (Variation of g when rotation of earth is considered)

$g_A = g_0 - R\omega^2\cos^2\theta$

At pole $\theta = 90$, $g_A = g_0$

At equator $\theta = 0$, $g_E = g_0 - R\omega^2$



Electrost.

$\vec{F} = q\vec{E}$

$\vec{F} = m\vec{E_g}$

Electrost. P.E.

q_1 • ⟵ r ⟶ • q_2

$$\boxed{U = \frac{K q_1 q_2}{r}}$$

q_1, q_2 ⟹ with sign

$$V_A = \frac{U}{q_2} = \frac{K q_1}{r}$$

q • ⟵ r ⟶ • A

Grav. P.E.

m_1 • ⟵ r ⟶ • m_2

$$\boxed{U = -\frac{G m_1 m_2}{r}}$$

Grav. Pot. at 'A' due to point mass $= -\frac{G m_1}{r}$

m_1 • ⟵ r ⟶ • A

$+q$ .... A

$\longleftarrow r \longrightarrow$

$E = \frac{Kq}{r^2}$

$V_A = \frac{Kq}{r}$

m .... A

r

$E_g = \frac{Gm}{r^2}$

$V_A = -\frac{Gm}{r}$

$\vec{F} = q\vec{E} \longrightarrow$

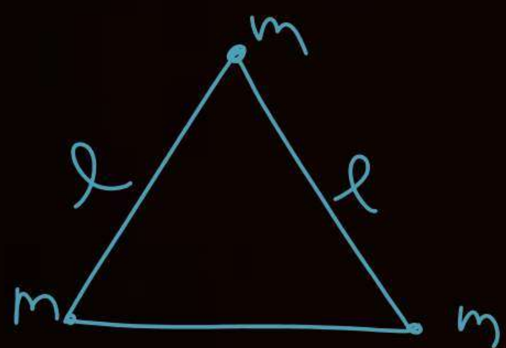
$\boxed{U = qV}$

$\boxed{\begin{aligned} \vec{F} &= m\vec{E_g} \\ F &= mg \end{aligned}}$

$\boxed{U_g = mV}$

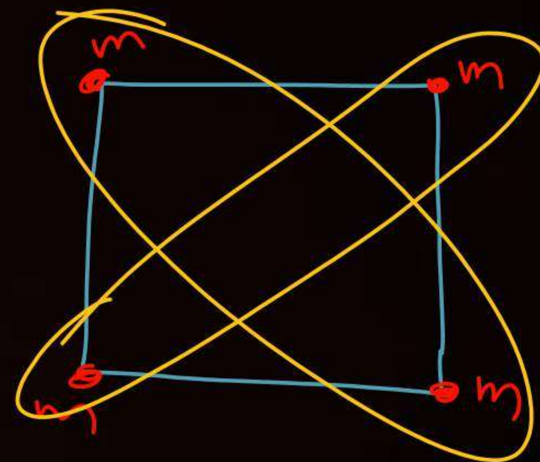
Grav. poten

Q



$$U = -\frac{Gmm}{l} \times 3$$

Q



$$T.P.E. = -\frac{Gmm}{l} \times 4 - \frac{Gmm}{l\sqrt{2}} 2$$

Calculate the distance from the surface of the earth at which above and below the surface acceleration due to gravity is the same.

5

$$\frac{\cancel{Gm}}{(R+h)^2} = \frac{\cancel{Gm}}{R^2}\left(1-\frac{h}{R}\right)$$

$$h = (\sqrt{5}-1)\frac{R}{2}$$

Ans. $h = \dfrac{\sqrt{5}-1}{2}R$

An object is projected vertically upward from the surface of the earth of mass M with a velocity such that the maximum height reached is eight times the radius R of the earth. Calculate:

(i) the initial speed of projection

(ii) the speed at half the maximum height.

Ans. *(i)* $\frac{4}{3}\sqrt{\frac{Gm}{R}}$, *(ii)* $\frac{2}{3}\sqrt{\frac{2Gm}{5R}}$

A satellite of mass m is orbiting the earth in a circular orbit of radius r. It starts losing energy due to small air resistance at the rate of C J/ s. Then the time taken for the satellite to reach the earth is

if एक sec ≡ 100 J

$$(T.E)_i = -\frac{GMm}{2r} \Rightarrow 1000$$

⇓ 900 खो गई

$$(T.E)_f = -\frac{GMm}{2R} \equiv 100$$

$$t = \frac{\left|\frac{GMm}{2}\left[\frac{1}{r}-\frac{1}{R}\right]\right|}{C}$$

Ans. $t = \frac{GMm}{2C}\left(\frac{1}{R_e}-\frac{1}{r}\right)$

A small body of mass m is projected with a velocity just sufficient to make it reach from the surface of a planet (of radius 2R and mass 3M) to the surface of another planet (of radius R and mass M). The distance between the centers of the two spherical planets is 6R. The distance of the body from the center of bigger planet is 'x' at any moment. During the journey, find the distance x where the speed of the body is (a) ~~maximum~~ (b) minimum. Assume motion of body along the line joining centres of planets.

Ans. 2R, $3R[3-\sqrt{3}$

Two identical satellites are at the heights R and 7R from the Earth's surface. Then which of the following statement is incorrect. (R = radius of the Earth)

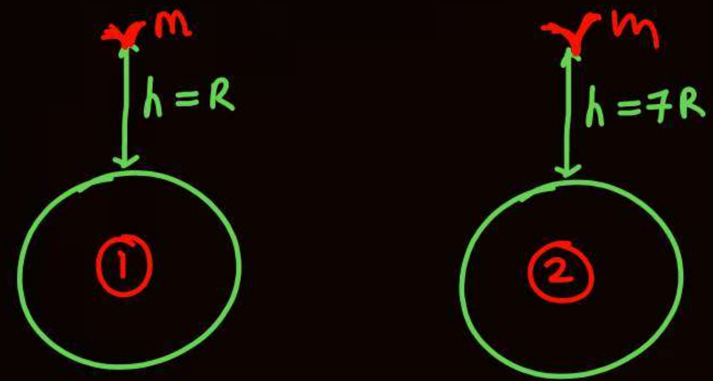
(A) Ratio of total energy of both is 5

(B) Ratio of kinetic energy of both is 4

(C) Ratio of potential energy of both 4

(D) Ratio of total energy of both is 4 and ratio of magnitude of potential to kinetic energy is 2

Ans. (A)



$$v_0 = \sqrt{\frac{GM}{r}} = \frac{2\pi r}{T}$$

$$T = 2\pi\sqrt{\frac{r^3}{GM}}$$

$$T \propto r^{3/2}$$

(1) $$\frac{(v_0)_2}{(v_0)_1} = \frac{\sqrt{\frac{GM}{r_2}}}{\sqrt{\frac{GM}{r_1}}} = \sqrt{\frac{r_1}{r_2}} = \sqrt{\frac{R+R}{7R+R}} = \frac{1}{2}$$

(2) $$\frac{T_2}{T_1} = \left(\frac{r_2}{r_1}\right)^{3/2} = \left(\frac{8R}{2R}\right)^{3/2} = 8$$

(3) $$\frac{\omega_2}{\omega_1} = \frac{T_1}{T_2} = \frac{1}{8}$$

(4) $$\frac{(K.E)_2}{(K.E)_1} = \frac{\frac{1}{2}m(V_0)_2^2}{\frac{1}{2}m(V_0)_1^2} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

(5) $$\left|\frac{(T.E)_2}{(T.E)_1}\right| = \left|\frac{-\frac{GMm}{2r_2}}{-\frac{GMm}{2r_1}}\right| = \frac{r_1}{r_2} = \frac{2R}{8R} = \frac{1}{4}$$

The escape velocity for a planet is v_e. A tunnel is dug along a diameter of the planet and a small body is dropped into it at the surface. When the body reaches the centre of the planet, its speed will be

(A) v_e (B) $\frac{v_e}{\sqrt{2}}$ (C) $\frac{v_e}{2}$ (D) 0

H/W

Ans. (B)

A satellite of mass m, initially at rest on the earth, is launched into a circular orbit at a height equal to the radius of the earth. The minimum energy required is

(A) $\frac{\sqrt{3}}{4}$mgR (B) $\frac{1}{2}$mgR (C) $\frac{1}{4}$mgR (D) $\frac{3}{4}$mgR

$$(T.E.)_f = -\frac{GMm}{2r} = -\frac{GMm}{2(2R)}$$

$$(T.E)_i = K.E. + P.E. = 0 - \frac{GMm}{R}$$

Ans $(T.E)_f - (T.E)_i = \left(-\frac{GMm}{4R}\right) - \left(-\frac{GMm}{R}\right)$

$$= \frac{3}{4}\frac{GMmR}{RR} = \frac{3}{4}gmR$$

Ans. (D)

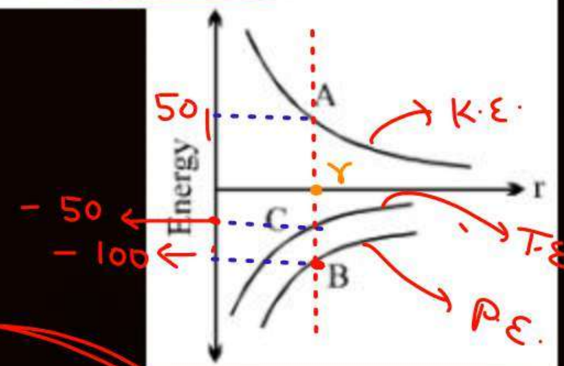
The figure shows the variation of energy with the orbit radius of a body in circular planetary motion. Find the correct statement about the curves A, B and C

(A) A shows the kinetic energy, B the total energy and C the potential energy of the system.

(B) C shows the total energy, B the kinetic energy and A the potential energy of the system.

(C) C and A are kinetic and potential energies respectively and B is the total energy of the system.

(D) A and B are kinetic and potential energies and C is the total energy of the system.



$A \longrightarrow K.E. \Rightarrow > 0$

$P.E. = -\frac{GMm}{R} = -100$

$K.E. = \frac{GMm}{2r} = 50$

$T.E. = -\frac{GMm}{2r} = -50$

$A \longrightarrow K.E.$
$B \longrightarrow P.E.$
$C \longrightarrow T.E.$

Ans. (D)

The fractional change in the value of free–fall acceleration g for a particle when it is lifted from the surface to an elevation h (h<<R) is

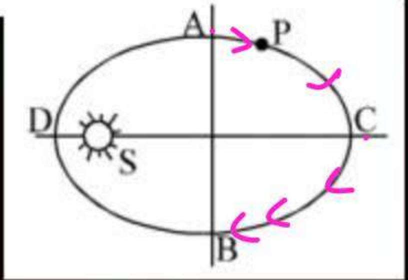
(A) $\frac{h}{R}$ (B) $\frac{2h}{R}$ (C) $-\frac{2h}{R}$ (D) $-\frac{h}{R}$

H/W

Ans. (C)

Paragraph for Question No. 10 and 11

Figure shows the orbit of a planet P around the sun S. AB and CD are the minor and major axes of the ellipse.



10. If t_1 is the time taken by the planet to travel along ACB and t_2 the time along BDA, then

(A) $t_1 = t_2$

(B) $t_1 > t_2$

(C) $t_1 < t_2$

(D) nothing can be concluded

Ans. (B)

11. If U is the potential energy and K kinetic energy then $|U| > |K|$ at

(A) Only D (B) Only C (C) both D & C (D) neither D nor C

Ans. (C)

Two bodies of masses m and 4m are placed at a distance r. The gravitational potential at a point on the line joining them where the gravitational field is zero is :- [AIEEE - 2011]

(1) $-\frac{6Gm}{r}$ (2) $-\frac{9Gm}{r}$ (3) zero (4) $-\frac{4Gm}{r}$

Ans. (2)

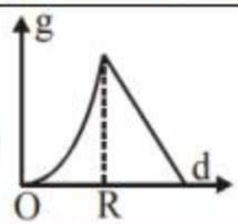
Two particles of equal mass 'm' go around a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle with respect to their centre of mass is:-

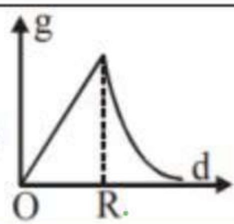
[AIEEE-2011]

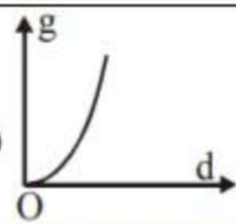
(1) $\sqrt{\frac{Gm}{R}}$ (2) $\sqrt{\frac{Gm}{4R}}$ (3) $\sqrt{\frac{Gm}{3R}}$ (4) $\sqrt{\frac{Gm}{2R}}$

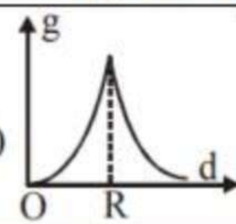
Ans. (2)

The variation of acceleration due to gravity g with distance d from centre of the earth is best represented by (R = Earth's radius) :- **[JEE-Main 2017]**

(1) 

(2) 

(3) 

(4) 

Ans. (2)

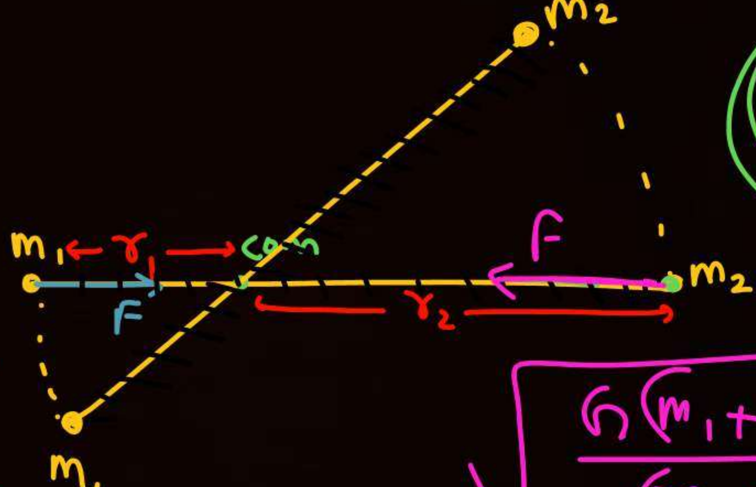
Binary star

JA → 19ws

$$T = 2\pi\sqrt{\frac{r^3}{Gm}}$$

$$F = \frac{G\cancel{m_1}m_2}{(r_1+r_2)^2} = \cancel{m_1} r_1 \omega^2$$

$$\frac{Gm_1\cancel{m_2}}{(r_1+r_2)^2} = \cancel{m_2} r_2 \omega^2$$



$$\frac{Gm_2}{(r_1+r_2)^2} + \frac{Gm_1}{(r_1+r_2)^2} = (r_1+r_2)\omega^2$$

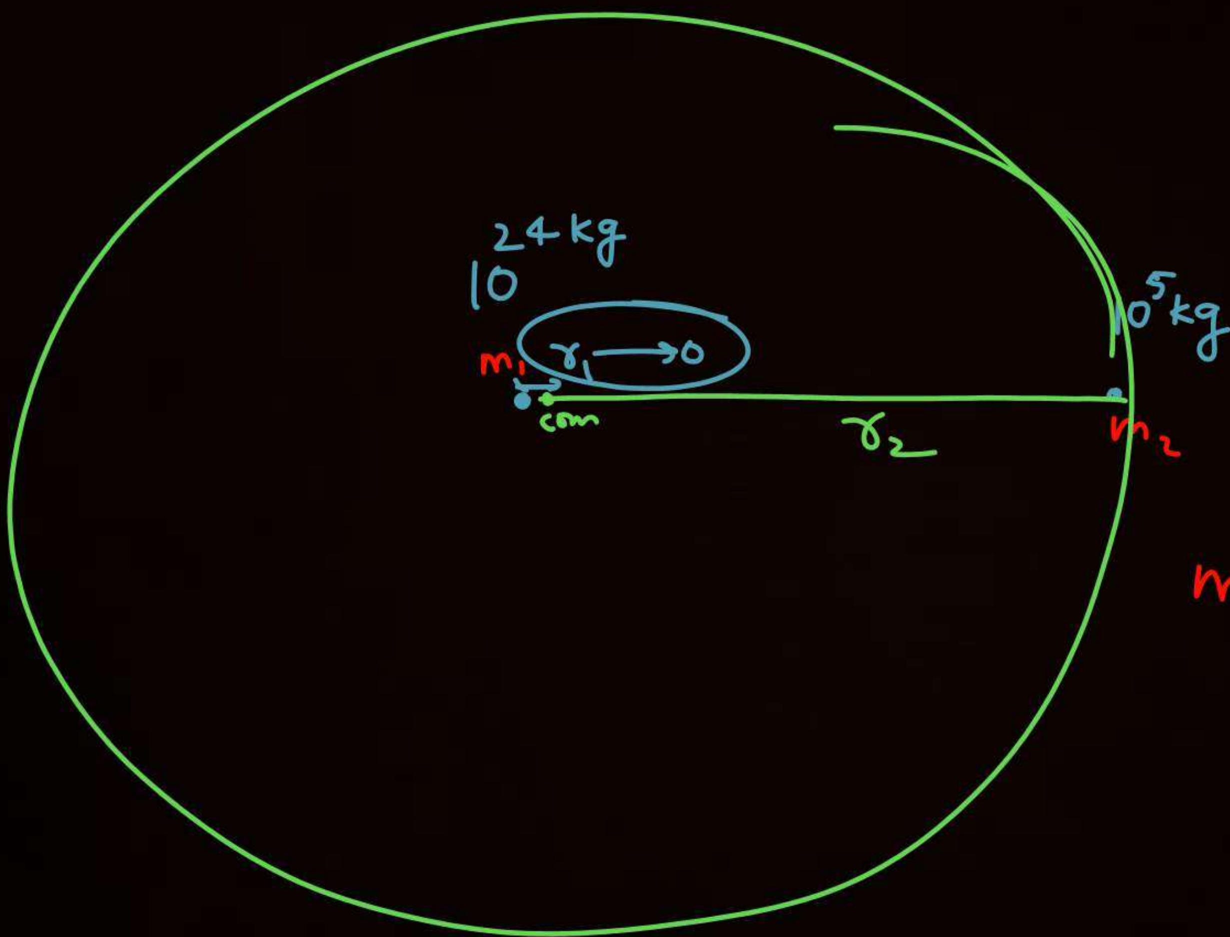
$$\sqrt{\frac{G(m_1+m_2)}{(r_1+r_2)^3}} = \omega$$

$$T = 2\pi\sqrt{\frac{(r_1+r_2)^3}{G(m_1+m_2)}}$$

If $m_1 >>> m_2$

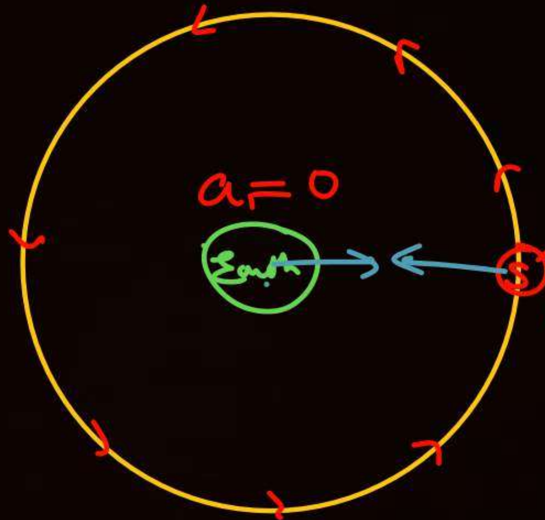
$r_1 <<< r_2 = r$

$$T = 2\pi\sqrt{\frac{r^3}{Gm}}$$



$m_1 >>>>>>>> m_2$

$$a_{com} = \frac{m_1 \overset{\to 0}{a_1} + m_2 a_2}{m_1 + m_2}$$



$\to a_{centripital}$

$a_{com} \neq 0$

$\parallel$

$F_{net} \neq 0$

Mass earth >>>>> m_{sattl}

QUESTION

If R is the radius of the earth and the acceleration due to gravity on the surface of earth is $g = \pi^2\ \mathrm{m/s^2}$, then the length of the second's pendulum at a height $h = 2R$ from the surface of earth will be,: **(01 Feb 2024 - Shift 1)**

(1) $\frac{2}{9}$ m

(2) $\frac{1}{9}$ m

(3) $\frac{4}{9}$ m

(4) $\frac{8}{9}$ m

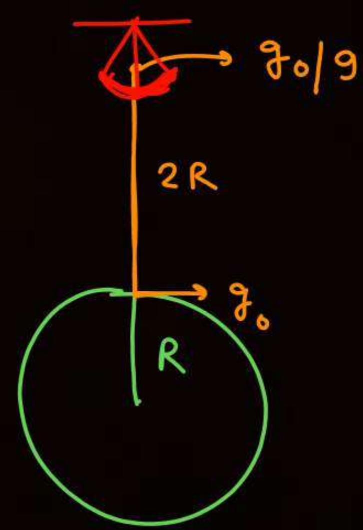
$$T = 2\pi\sqrt{\frac{l}{g}}$$

2 sec

$$\cancel{2} = \cancel{2}\pi\sqrt{\frac{l \times 9}{g_0}}$$

$$1 = \cancel{\pi^2} \times \frac{9l}{\cancel{g_0}}$$

$$l = \frac{1}{9}$$



Ans : (2)

QUESTION

A light planet is revolving around a massive star in a circular orbit of radius R with a period of revolution T. If the force of attraction between planet and star is proportional to $R^{-3/2}$ then choose the correct option: **(01 Feb 2024 - Shift 2)**

(1) $T^2 \propto R^{5/2}$

(2) $T^2 \propto R^{7/2}$

(3) $T^2 \propto R^{3/2}$

(4) $T^2 \propto R^3$

$$F = \frac{Gm_1\cancel{m_2}}{R^{3/2}} = \frac{\cancel{m_2}v^2}{R}$$

$$v = \sqrt{\frac{Gm_1}{R^{\frac{1}{2}}}}$$

$$v \propto R^{-\frac{1}{4}}$$

$$v = \frac{2\pi R}{T} \propto R^{-\frac{1}{4}}$$

$$T \propto R^{1+\frac{1}{4}} \propto R^{5/4}$$

$$T^2 \propto R^{5/2}$$

Ans : (1)

QUESTION

The acceleration due to gravity on the surface of earth is g. If the diameter of earth reduces to half of its original value and mass remains constant, then acceleration due to gravity on the surface of earth would be: **(27 Jan 2024 - Shift 1)**

$$g = \frac{GM}{R^2}$$

$R \longrightarrow R/2$

$M \longrightarrow$ Same

(1) $g/4$

(2) $2g$

(3) $g/2$

(4) $4g$

Ans : (4)

QUESTION

$\omega_1 > \omega_2$

$T_1 < T_2$

Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A): The angular speed (T_1) of the moon (≡ 27 days) in its orbit about the earth is more than the angular speed of the (T_2) earth in its orbit about the sun. ≃ 365 days

Reason (R): The moon takes less time to move around the earth than the time taken by the earth to move around the sun.

In the light of the above statements, choose the most appropriate answer from the options given below:

(27 Jan 2024 - Shift 2)

1. (A) is correct but (R) is not correct
2. Both (A) and (R) are correct and (R) is the correct explanation of (A)
3. Both (A) and (R) are correct but (R) is not the correct explanation of (A)
4. (A) is not correct but (R) is correct

Ans : (2)

QUESTION

Same

At what distance above and below the surface of the earth a body will have same weight, (take radius of earth as R). **(29 Jan 2024 - Shift 1)**

(1) $\sqrt{5}R - R$

(2) $\dfrac{\sqrt{3}R - R}{2}$

(3) $\dfrac{R}{2}$

(4) $\dfrac{\sqrt{5}R - R}{2}$

Ans : (4)

QUESTION

A planet takes 200 days to complete one revolution around the Sun. If the distance of the planet from Sun is reduced to one fourth of the original distance, how many days will it take to complete one revolution? **(29 Jan 2024 - Shift 2)**

(1) 25

(2) 50

(3) 100

(4) 20

$$T = \sqrt{\frac{r^3}{Gm}} \times 2\pi$$

$$T^2 \propto r^3$$

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$$

$$\left(\frac{200}{T_2}\right)^2 = \left(\frac{r}{r/4}\right)^3$$

Ans : (1)

QUESTION

Ans $r - 6400000$

The gravitational potential at a point above the surface of earth is -5.12×10^7 J/kg and the acceleration due to gravity at that point is 6.4 m/s^2. Assume that the mean radius of earth to be 6400 km. The height of this point above the earth's surface is:

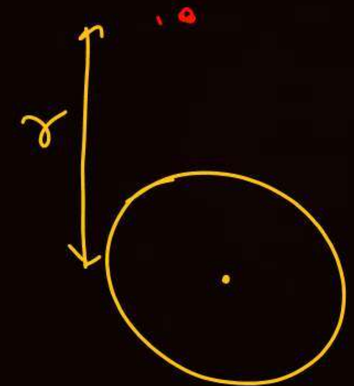
(30 Jan 2024 - Shift 1)

(1) 1600 km

(2) 540 km

(3) 1200 km

(4) 1000 km



$$\frac{GM}{r^2} = 6.4 = \frac{GM}{r}\cdot\frac{1}{r}$$

$$\cancel{-}\frac{GM}{r} = \cancel{-}5.12 \times 10^7$$

Ans : (1)

QUESTION

$\frac{11.2 \times 1.4}{2} = 11.2 \times .7 = 7.84$

Escape velocity of a body from earth is 11.2 km/s. If the radius of a planet be one-third the radius of earth and mass be one-sixth that of earth, the escape velocity from the plate is:

(30 Jan 2024 - Shift 2)

$$v_e = \sqrt{\frac{2GM}{R_e}}$$

$M \to M/6$, $R_e \to R/3$

Earth	Planet
R	R/3
m	m/6

$v_e \to \frac{1}{\sqrt{2}}$ times $\equiv \frac{1}{\sqrt{2}} \times 11.2$

(1) 11.2 km/s

(2) 8.4 km/s

(3) 4.2 km/s

(4) 7.9 km/s

Ans : (4)

QUESTION

HW

Four identical particles of mass m are kept at the four corners of a square. If the gravitational force exerted on one of the masses by the other masses is $\left(\frac{2\sqrt{2}+1}{32}\right)\frac{Gm^2}{L^2}$, the length of the sides of the square is: **(31 Jan 2024 - Shift 1)**

(1) L/2

(2) 4L

(3) 3L

(4) 2L

Ans : (2)

QUESTION

The mass of the moon is 1/144 times the mass of a planet and its diameter 1/16 times the diameter of a planet. If the escape velocity on the planet is v, the escape velocity on the moon will be:

(31 Jan 2024 - Shift 2)

① plant	② moon
m	$\frac{m}{144}$
R	$\frac{R}{16}$

$$\frac{V_1}{V_2} =$$

1. v/3
2. v/4
3. v/12
4. v/6

Ans : (1)

Newton's law of Gravitation

It states that every body in the universe attracts every other body with a force which is directly proportional to the product of their masses and is inversely proportional to the square of the distance between them.

$F \propto m_1 m_2$ and $F \propto \frac{1}{r^2}$ so $F \propto \frac{m_1 m_2}{r^2}$

$\therefore F = -\frac{G m_1 m_2}{r^2}\hat{r}$ [G = Universal gravitational constant]

Question

Two particles of masses 1 kg and 2 kg are placed at a separation of 50 cm. Assuming that the only forces acting on the particles are their mutual gravitation, find the initial acceleration of heavier particle.

m_1 m_2

$$\frac{G m_1 m_2}{r^2 \times m_2}$$

Ans. $2.65 \times 10^{-10}\ \mathrm{ms^{-2}}$

Analogy betwen Electrostatics & Gravitation

(1) Point Charge

(a) $E = \frac{kQ}{r^2}$

(b) $V = \frac{KQ}{r}$

Point Mass

$g = \frac{GM}{r^2}$

$V_G = \frac{-GM}{r}$

(2) Uniform charged ring

(a) $E = \frac{kQx}{(r^2+x^2)^{3/2}}$ on axis

E is max. when $x = \frac{r}{\sqrt{2}}$

(b) $V = \frac{KQ}{\sqrt{r^2+x^2}}$ on axis, $\frac{kQ}{r}$ at center

Ring of uniform mass distribution

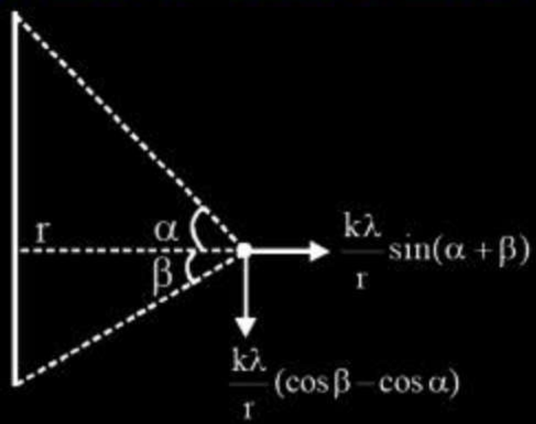
$g = \frac{GMx}{(r^2+x^2)^{3/2}}$ on axis

g is max. when $x = \frac{r}{\sqrt{2}}$

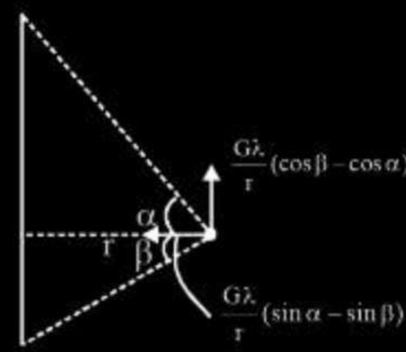
$V_G = \frac{-GM}{\sqrt{r^2+x^2}}$ on axis, $-\frac{GM}{r}$ at center

Analogy betwen Electrostatics & Gravitation

(3) Uniform linear charge

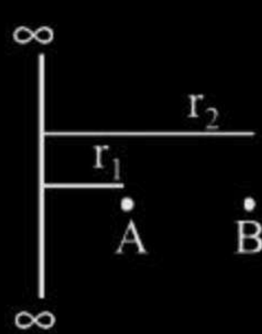


Uniform linear mass



(λ=mass per unit length)

(4) Infinite Linear charge



Infinite linear mass

Analogy betwen Electrostatics & Gravitation

(a) $E = \frac{2K\lambda}{r}$

$g = \frac{2G\lambda}{r}$

(b) $V_B - V_A = -2\,K\lambda \ln\frac{r_2}{r_1}$

$V_B - V_A = 2\,G\lambda \ln\left(\frac{r_2}{r_1}\right)$

(5) Infinite Sheet of charge

$E = \frac{\sigma}{2\varepsilon_0}$

Infinite Sheet of mass

$g = \frac{\sigma}{2} \times 4\pi G = 2\pi G\sigma$

(σ = mass per unit area)

* Notice gravitational force is always attractive and hence gravitational potential is always –ve. (for a repulsive force potential is postive). This can be explained from the sign of W_{ext} in moving the test charge from ∞ to the point under consideration.

** Since $\vec{g}$ points from B towards A potential increases as we move from A to B. Just like electric potentical gravitational potential also increases opposite to field direction.

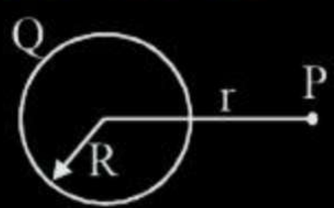
Analogy betwen Electrostatics & Gravitation

(6) Uniformly charged hollow shpere

Charge Q, radius R

distance of field point from center r

Case I $r > R$



$$E = \frac{kQ}{r^2}$$

$$V = \frac{kQ}{r}$$

Hollow sphere of uniform mass

Mass M , radius R

distance of field point from center r

$$g = \frac{GM}{r^2}$$

$$V_G = -\frac{GM}{r}$$

Analogy betwen Electrostatics & Gravitation

Case II r < R

$g = 0$

$E = 0$

$V_G = -\frac{GM}{r}$

$V = \frac{kQ}{r}$

(7) Electrostatics self energy of uniformly charged thin spherical shell.

$$\boxed{U = \frac{KQ^2}{2R}}$$

SS $\frac{3}{5}\frac{KQ^2}{R}$

Gravitational self energy of uniform thin spherical shell.

$$U = \frac{GM^2}{2R}$$

Analogy betwen Electrostatics & Gravitation

PW

(8) Uniformly charged solid sphere

$E = \frac{kQ}{r^2}, r > R$

$\frac{kQr}{R^3}, r < R$

$V = \frac{KQ}{r}, r > R$

$\frac{KQ}{2R^3}(3R^3 - r^2), r > R$

Uniformly solid sphere

$g = \frac{GM}{r^2}, r > R$

$\frac{GM}{R^3} r, r < R$

$V_a = -\frac{GM}{r}, r > R$

$\frac{-GM}{2R^3}(3R^3 - r^2), r > R$

(9) Electrostatics self energy of Uniformly charged solid sphere.

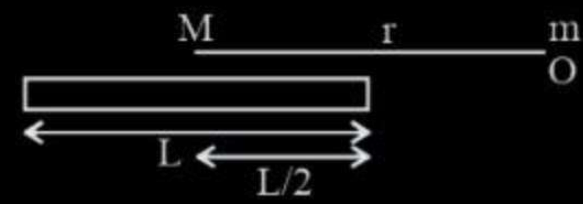
$U - \frac{3}{5}\frac{KQ^2}{R}$

Gravitational self energy of uniform solid sphere.

$U - \frac{3}{5}\frac{GM^2}{R}$

Question

Find gravitational force between the point mass & the rod of uniform mass.



Ans. $\dfrac{GMm}{\left(r+\frac{1}{2}\right)\left(r-\frac{1}{2}\right)}$

Question

Find ratio of gravitational field on the surface of two planets which are of uniform mass density & have radius R_1 & R_2 if

(a) They are of same mass

(b) They are of same density

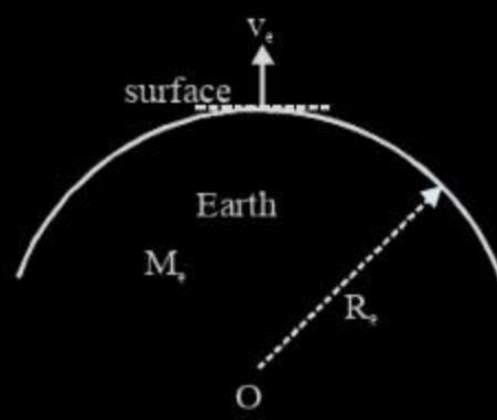
Ans. (a) $\frac{g_1}{g_2} = \frac{R_2^2}{R_1^2}$ (b) $\frac{g_1}{g_2} = \frac{R_1}{R_2}$

❑ Escape speed (v_e)

Minimum speed required for an object at Earth's surface so that it just escapes the Earth's gravitational field.

❑ Escape energy

Minimum energy given to a particle in form of kinetic energy so that it can just escape from Earth's gravitational field.



Escape energy $= \frac{GM_e m}{R_e}$ (-ve of PE of Earth's surface)

Escape energy = Kinetic Energy $\Rightarrow \frac{GM_e m}{R_e} = \frac{1}{2}mv_e^2 \Rightarrow v_e = \sqrt{\frac{2GM_e}{R_e}}$

- $v_e = \sqrt{\frac{2GM_e}{R_e}}$ (In form of mass) If M = constant $v_e \propto \frac{1}{\sqrt{R_e}}$

- $v_e = \sqrt{2gR_e}$ (In form of g) If g = constant $v_e \propto \sqrt{R_e}$
- $v_e = R_e\sqrt{\frac{8\pi G.\rho}{3}}$ (In form of density) If ρ = constant $v_e \propto R_e$
- Escape velocity does not depend on mass of body, angle of projection or direction of projection. $v_e \propto m^0$ and $v_e \propto \theta^0$
- Escape velocity at : Earth surface $v_e = 11.2$ km/s Moon surface $v_e = 2.31$ km/s
- Atmosphere on Moon is absent because root mean square velocity of gas particle is greater then escape velocity. $v_{rms} > v_e$

Question

Find the minimum speed with which an object should be projected vertically upward from earth's surface to reach a height equal to radius of earth, R_e.

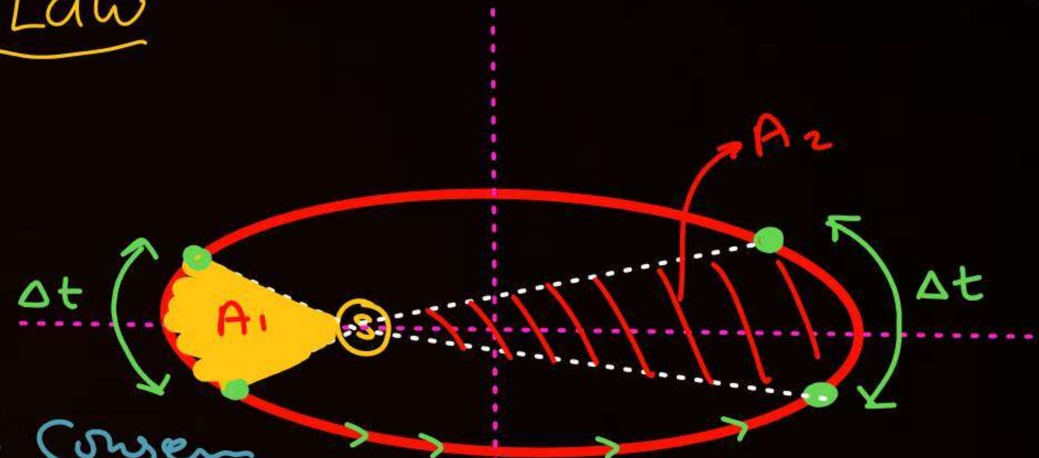
Ans. $\sqrt{\frac{GM}{R_e}}$

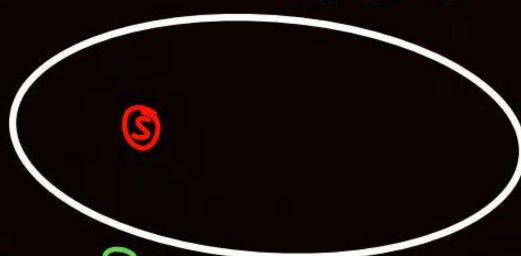
Kepler's Law

① Elliptical

② $(\tau)_{abt\ sun} = 0$

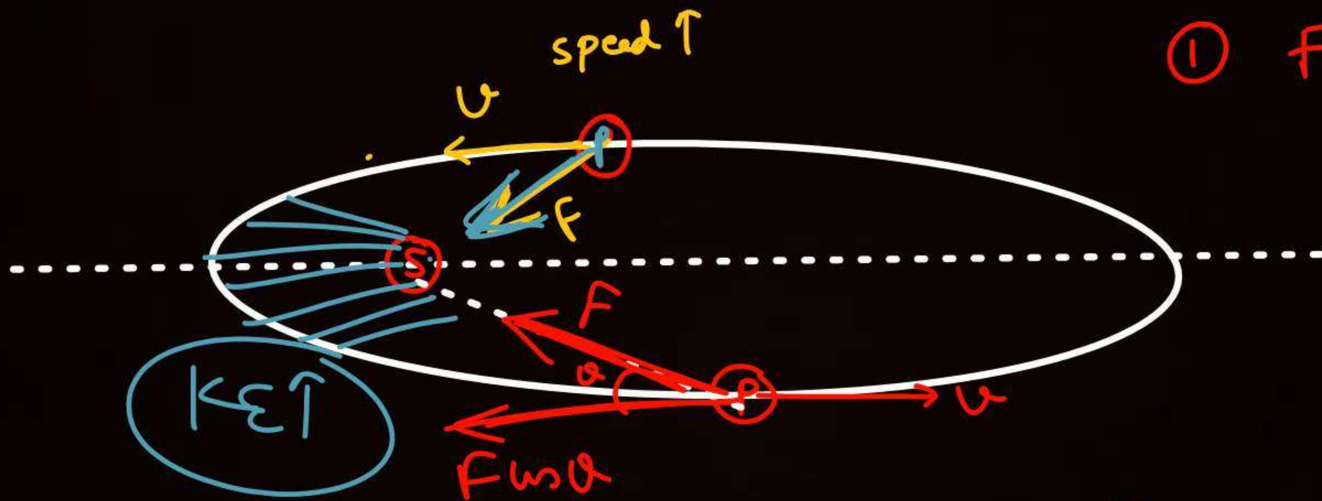
$\vec{L}_{abt\ sun} = \text{Consen}$





$mvr = \text{Cons}$

③ $T^2 \propto r^3$



① Force ✓

$T.E = PE + K.E$

②

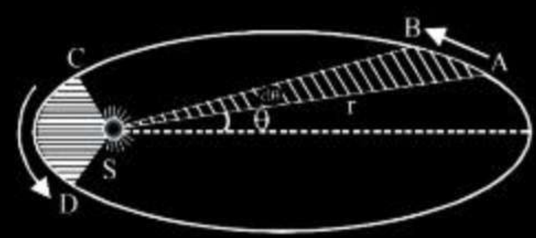
$r\downarrow$ $PE\uparrow$ $PE\downarrow$ $KE\uparrow$

Kepler's Laws

Kepler found important regularities in the motion of the planets. These regularities are known as 'Kepler's three laws of planetary motion'.

(a) First Law (Law of Orbits) : All planets move around the Sun in elliptical orbits, having the Sun at one focus of the orbit.

(b) Second Law (Law of Areas) : A line joining any planet to the Sun sweeps out equal areas in equal times, that is, the areal speed of the planet remains constant. According to the second law, when the planet is nearest the Sun, then its speed is maximum and when it is farthest from the Sun, then its speed is minimum. In figure if a planet moves from A to B in a given time–interval, and from C to D in the same time–interval, then the areas ASB and CSD will be equal

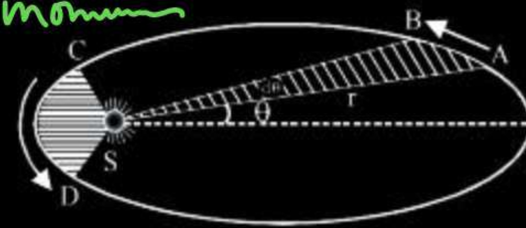


Kepler's Laws

$$\frac{dA}{dt} = \frac{J}{2m} \quad .....(iii)$$

$$\frac{dA}{dt} = \frac{J}{2m}$$

→ Angular momentum



Now, the areal speed dA/dt of the planet is constant, according to Kepler's second law. Therefore, according to eq. (iii), the angular momentum J of the planet is also constant, that is, the angular momentum of the planet is conserved. Thus, Kepler's second law is equivalent to conservation of angular momentum.

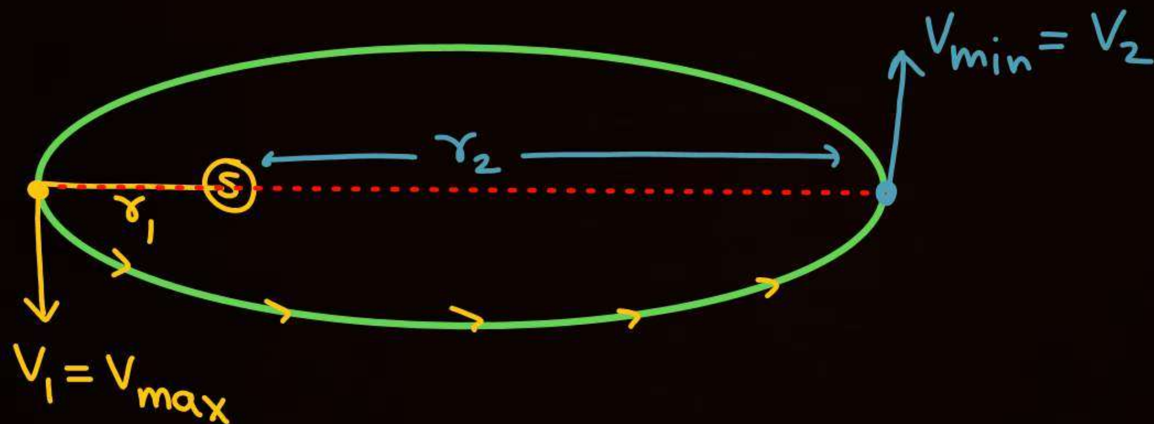
(c) Third Law : (Law of Periods) : The square of the period of revolution (time of one complete revolution) of any planet around the Sun is directly proportional to the cube of the semi–major axis of its elliptical orbit.

$T^2 \propto a^3$

So it is clear through this rule that the farthest planet from the Sun has largest period of revolution. The period of revolution of the closest planet Mercury is 88 days, while that of the farthest dwarf planet Pluto is 248 years.

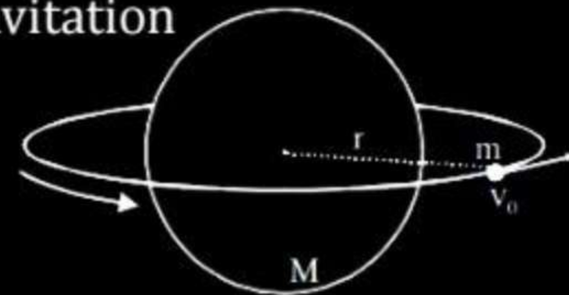
3-4 बार

$$\boxed{m_1 v_1 r_1 = m_2 v_2 r_2}$$



- **Satellite motion:** A light body revolving round a heavier body due to gravitational attraction, is called satellite. Earth is a satellite of the Sun while Moon is satellite of Earth.

- **Orbital velocity (v_0) :** A satellite of mass m moving in an orbit of radius r with speed v_0 then required centripetal force is provided by gravitation



$$F_{cp} = F_g \Rightarrow \frac{mv_0^2}{r} = \frac{GMm}{r^2} \Rightarrow v_0 = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{(R_e+h)}} \quad (r = R_e + h)$$

For a satellite very close to the Earth surface $h << R_e$ $\therefore r = R_e$

$$v_0 = \sqrt{\frac{GM}{R_e}} = \sqrt{gR_e} = 8 km/s$$

- If a body is taken at some height from Earth and given horizontal velocity of magnitude 8 km/sec then the body becomes satellite of Earth.
- v_o depends upon : Mass of planet, Radius of circular orbit of satellite, g (at planet), Density of planet

- If orbital velocity of a near by satellite becomes $\sqrt{2}\,v_o$ (or increased by 41.4%, or K.E. is doubled) then the satellite escapes from gravitational field of Earth.

Time Period of a Satellite $T = \frac{2\pi r}{v_0} = \frac{2\pi r^{3/2}}{\sqrt{GM}} = \frac{2\pi r^{3/2}}{R\sqrt{g}} \Rightarrow T^2 = \frac{4\pi^2}{GM} r^3$

$\Rightarrow T^2 \propto r^3 (r = R + h)$

For Geostationary Satellite T = 24 hr, h = 36,000 km $\simeq 6R_e (r \simeq 7R_e), v_0 = 3.1 k\, m/s$

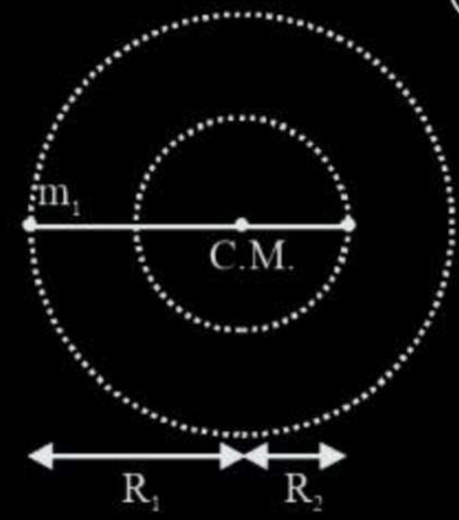
For Near by satellite $v_0 = \sqrt{\frac{GM_e}{R_e}} = 8K\, m/s$

$T_{Ns} = 2\pi\sqrt{\frac{R_e}{g}}$ = 84 minute = 1 hour 24 minute = 1.4 hr = 5063 s

In terms of density $T_{Ns} = \frac{2\pi (R_e)^{1/2}}{(G \times 4/3\pi R_e \times \rho)^{1/2}} = \sqrt{\frac{3\pi}{G\rho}}$

❑ Binary star system:

Figure shows two particles moving due to mutually attractive gravitational force about center of mass. Since there is no external force CM of system remains fixed and time period of revolution must be same. Both bodies have comparable mass and both are moving in circular orbit centre of mass as shown in diagram

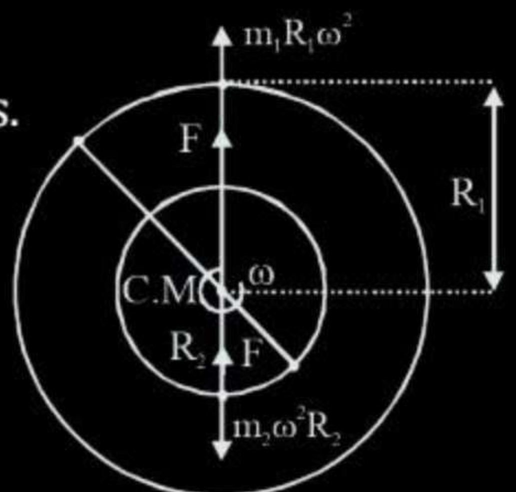


$$\omega = \sqrt{\frac{G(m_1+m_2)}{R^3}}$$

Angular momentum of the system about centre of mass.

$$L = \left(\frac{m_1 m_2}{m_1+m_2}\right) R^2 \omega$$

$$\text{Kinetic energy} = \frac{1}{2}\left(\frac{m_1 m_2}{m_1+m_2}\right) R^2 \omega^2$$



Question

A satellite of mass m is orbiting the earth in a circular orbit of radius r. It starts losing energy due to small air resistance at the rate of C J/s. Then the time taken for the satellite to reach the earth is ________.

Ans. $t = \frac{GMm}{2C}\left(\frac{1}{R_e} - \frac{1}{r}\right)$

Question

A spaceship is sent to investigate a planet of mass M and radius R. While hanging motionless in space at a distance 5R from the centre of the planet, the spaceship fires an instrument package with speed v_0 as shown in the figure. The package has mass m, which is much smaller than the mass of the spaceship. The package just grazes the surface of the planet and at that moment it's speed is v. $v_0 = \sqrt{\frac{xGM}{75R}}$. Fill x in OMR sheet.

$$\frac{1}{2}mv_0^2 - m\frac{GM}{5R} = \frac{1}{2}mv_f^2 - m\frac{GM}{R} \quad \text{—— (1)}$$

$$\cancel{m} \cdot V_0 \sin 53 \cdot \times 5R = \cancel{m} \cdot V_f \cdot R \quad \text{—— (2)}$$

Ans. 8

Question

Planet A has mass M and radius R. Planet B has half the mass and half the radius of Planet A. If the escape velocities from the Planets A and B are v_A and v_B, respectively, then $\frac{v_A}{v_B} = \frac{n}{4}$. The value of n is: **(JEE Main-2020)**

ε

(A) 4

(B) 1

(C) 2

(D) 3

Ans : (A)

Question

K JA gauss Law

The mass density of a spherical galaxy varies as $\frac{K}{r}$ over a large distance 'r' from its centre. In that region a small star is in circular orbit of radius R. Then the period of revolution T depends on R as: (JEE Main-2020)

Elcdrost.

(A) $T \propto R$

(B) $T^2 \propto \frac{1}{R^3}$

(C) $T^2 \propto R^2$

(D) $T^2 \propto R^3$

Ans : (C)

Question

The height 'h' at which the weight of a body will be the same as that at the same depth 'h' from the surface of the earth is (Radius of the earth is R and effect of the rotation of the earth is neglected): **(JEE Main-2020)**

(A) $\frac{\sqrt{5}R - R}{2}$

Repeat

(B) $\frac{\sqrt{5}}{2}R - R$

(C) $\frac{R}{2}$

(D) $\frac{\sqrt{3}R - R}{2}$

Ans : (A)

Question

Ring $\quad \dfrac{Gmx}{(R^2+x^2)^{3/2}} = \dfrac{A\ x}{(R^2+x^2)^{3/2}}$

On the x-axis and a distance x from the origin, the gravitational field due to a mass distribution is given by $\frac{Ax}{(x^2+a^2)^{3/2}}$ in the x-direction. The magnitude of gravitational potential on the x-axis at a distance x, taking its value to be zero at infinity, is:

(JEE Main-2020)

$-\dfrac{Gm}{\sqrt{R^2+x^2}}$

(A) $\dfrac{A}{(x^2+a^2)^{1/2}}$

(B) $\dfrac{A}{(x^2+a^2)^{3/2}}$

(C) $(x^2+a^2)^{3/2}$

(D) $(x^2+a^2)^{1/2}$

Ans : (A)

Question

The value of the acceleration due to gravity is g_1 at a height $h = \frac{R}{2}$ (R = radius of the earth) from the surface of the earth. It is again equal to g_1 at a depth d below the surface of the earth. The ratio $\left(\frac{d}{R}\right)$ equals: **(JEE Main-2020)**

(A) $\frac{7}{9}$

(B) $\frac{4}{9}$

(C) $\frac{1}{3}$

(D) $\frac{5}{9}$

$$\frac{Gm}{\left(R+\frac{R}{2}\right)^2} = \frac{Gm(R-d)}{R^3}$$

Ans : (D)

Question

$g = g_0 - R\omega^2 \cos^2\theta$ $\quad g_0$

The acceleration due to gravity on the earth's surface at the poles is g and angular velocity of the earth about the axis passing through the pole is ω. An object is weighed at the equator and at a height h above the poles by using a spring balance. If the weights are found to be same, then h is: (h<<R, where R is the radius of the earth)

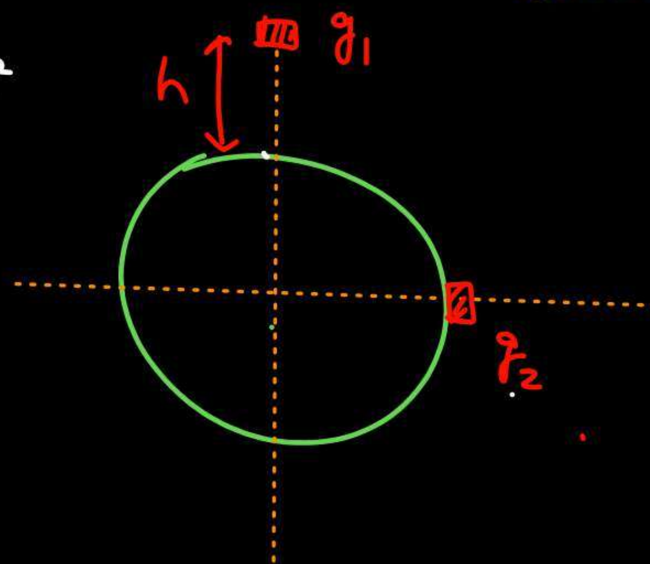
(JEE Main-2020)

$g_1 = g_2$

$g_0\left(1 - \frac{2h}{R}\right) = g_0 - R\omega^2$

$+g_0 \frac{2h}{R} = +R\omega^2$

$$h = \frac{R^2\omega^2}{2g_0}$$



(A) $\frac{R^2\omega^2}{8g}$

(B) $\frac{R^2\omega^2}{4g}$

(C) $\frac{R^2\omega^2}{g}$

(D) $\frac{R^2\omega^2}{2g}$

Ans : (D)

Question

3rd time

A satellite is in an elliptical orbit around a planet P. It is observed that the velocity of the satellite when it is farthest from the planet is 6 times less than that when it is closest to the planet. The ratio of distances between the satellite and the planet at closest and farthest points is:

(JEE Main-2020)

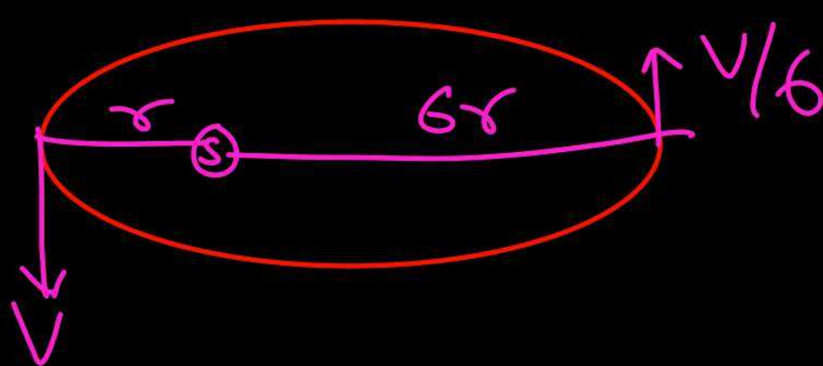
(A) 1 : 6

(B) 3 : 4

(C) 1 : 3

(D) 1 : 2

$$m v_1 r_1 = m v_2 r_2$$



Ans : (A)

Question

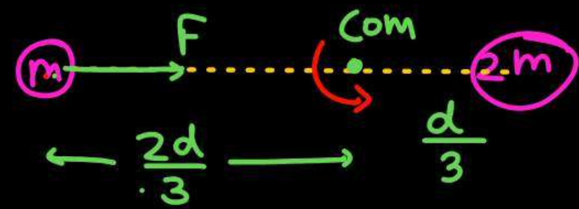
Two stars of masses m and 2m at a distance d rotate about their common centre of mass in free space. The period of revolution is: **(JEE Main-2021)**

(A) $\frac{1}{2\pi}\sqrt{\frac{d^3}{3Gm}}$

(B) $2\pi\sqrt{\frac{d^3}{3Gm}}$

(C) $\frac{1}{2\pi}\sqrt{\frac{3Gm}{d^3}}$

(D) $2\pi\sqrt{\frac{3Gm}{d^3}}$



$$\frac{Gm\,2m}{d^2} = m\frac{2d}{3}\omega^2$$

Ans : (B)

Question

Four identical particles of equal masses 1kg made to move along the circumference of a circle of radius 1 m under the action of their own mutual gravitational attraction. The speed of each particle will be : (JEE Main-2021)

Same

(A) $\sqrt{\frac{G}{2}(1+2\sqrt{2})}$

(B) $\sqrt{G(1+2\sqrt{2})}$

(C) $\sqrt{\frac{G}{2}(2\sqrt{2}-1)}$

(D) $\sqrt{\frac{(1+2\sqrt{2})}{2}G}$

Ans : (D)

Question

Consider two satellites S_1 and S_2 with periods of revolution 1 hr. and 8hr. respectively revolving around a planet in circular orbits. The ratio of angular velocity of satellite S_1 to the angular velocity of satellites S_2 is: **(JEE Main-2021)**

(A) 8 : 1

(B) 1 : 4

(C) 2 : 1

(D) 1 : 8

Easy

$T^2 \propto r^3$

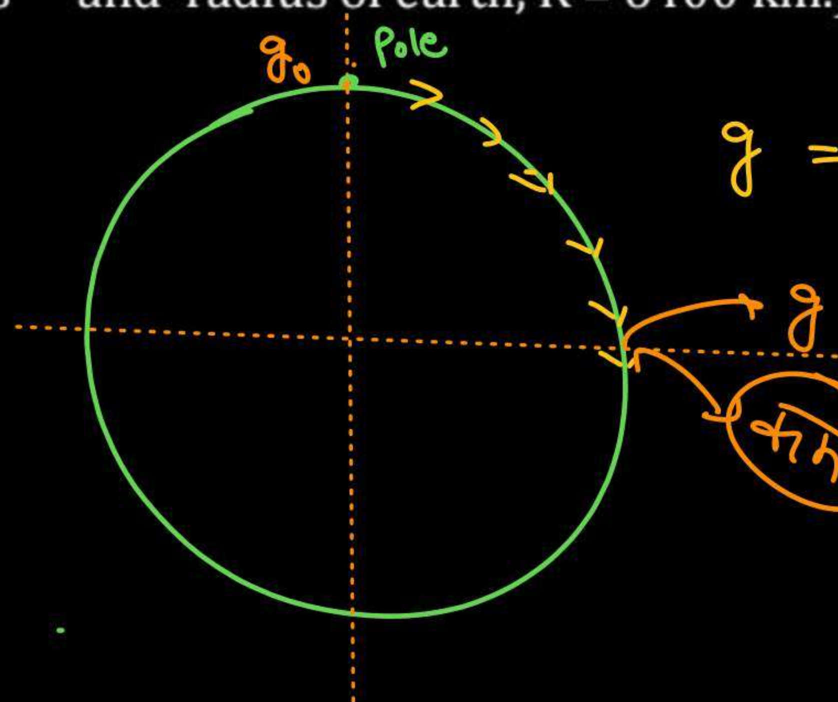
$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3$

$\frac{r_1}{r_2} = \frac{1}{4}$

Ans : (A)

Question

A body weighs 49 N on a spring balance at the north pole. What will be its weight recorded on the same weighing machine, if it is shifted to the equator?
(Use $g = \frac{GM}{R^2} = 9.8\,ms^{-2}$ and radius of earth, $R = 6400$ km.) **(JEE Main-2021)**



$$g = g_0 - R\omega^2\cos^2\theta$$

$$g = g_0 - R\omega^2$$

(A) 49 N

(B) 48.83 N

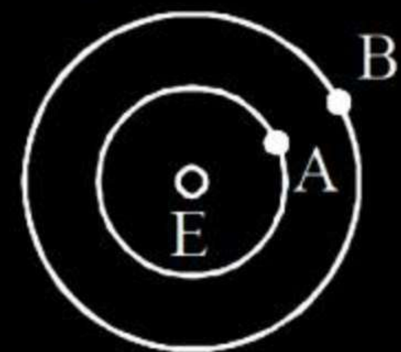
(C) 49.83 N

(D) 49.17 N

Ans : (B)

Question

Two satellites A and B of masses 200kg and 400kg are revolving round the earth at height of 600 km and 1600 km respectively. If T_A and T_B are the time periods of A and B respectively then the value of $T_B - T_A$:
[Given : radius of earth = 6400km, mass of earth = 6×10^{24} kg] **(JEE Main-2021)**



(A) 1.33×10^3 s

(B) 3.33×10^2 s

(C) 4.24×10^3 s

(D) 4.24×10^2 s

Ans : (A)

Question

A solid sphere of radius R gravitationally attracts a particle placed at 3R form its centre with a force F_1. Now a spherical cavity of radius $\left(\frac{R}{2}\right)$ is made in the sphere (as shown in figure) and the force becomes F_2. The value of $F_1 : F_2$ is:

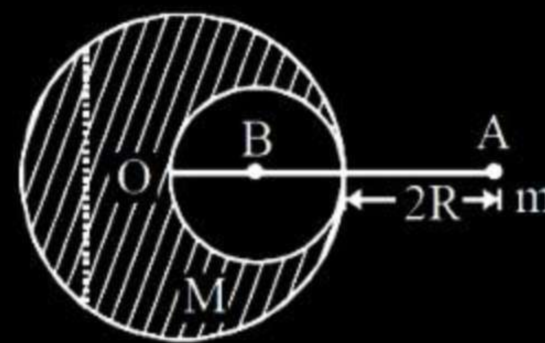
(JEE Main-2021)

(A) $25 : 36$

(B) $36 : 25$

(C) $50 : 41$

(D) $41 : 50$



Ans : (C)

Question

Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A : The escape velocities of planet A and B are same. But A and B are of unequal mass.

Reason R : The product of their mass and radius must be same, $M_1R_1 = M_2R_2$

In the light of the above statements, choose the most appropriate answer from the options given below : **(JEE Main-2021)**

(A) Both A and R are correct but R is NOT the correct explanation of A

(B) A is correct but R is not correct

(C) Both A and R are correct and R is the correct explanation of A

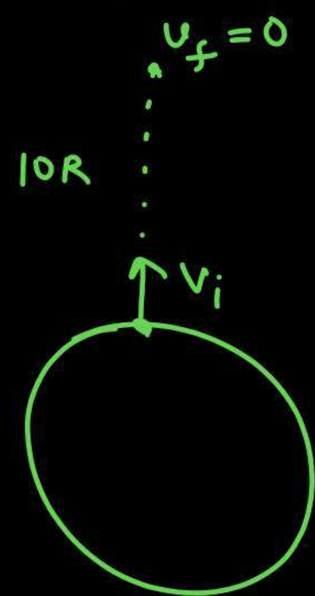
(D) A is not correct but R is correct

Ans : (B)

Question

The initial velocity vi required to project a body vertically upward from the surface of the earth to reach a height of 10R, where R is the radius of the earth, may be described in terms of escape velocity v_e such that $v_i = \sqrt{\frac{x}{y}} \times v_e$. The value of x will be ______.

(JEE Main-2021)



$$v_e = \sqrt{\frac{Gm}{R}}$$

Ans : (10)

Question

Assume that a tunnel is dug along a chord of the earth, at a perpendicular distance (R/2) from the earth's centre, where 'R' is the radius of the Earth. The wall of the tunnel is frictionless. If a particle is released in this tunnel, it will execute a simple harmonic motion with a time period: **(JEE Main-2021)**

(A) $\frac{2\pi R}{g}$

(B) $\frac{g}{2\pi R}$

(C) $\frac{1}{2\pi}\sqrt{\frac{g}{R}}$

(D) $2\pi\sqrt{\frac{R}{g}}$

Ans : (D)

Question

A planet revolving in elliptical orbit has:
(A) a constant velocity of revolution.
(B) has the least velocity when it is nearest to the sun.
(C) its areal velocity is directly proportional to its velocity.
(D) areal velocity is inversely proportional to its velocity.
(E) to follow a trajectory such that the areal velocity is constant.
Choose the correct answer from the options given below : **(JEE Main-2021)**

(A) A only

(B) D only

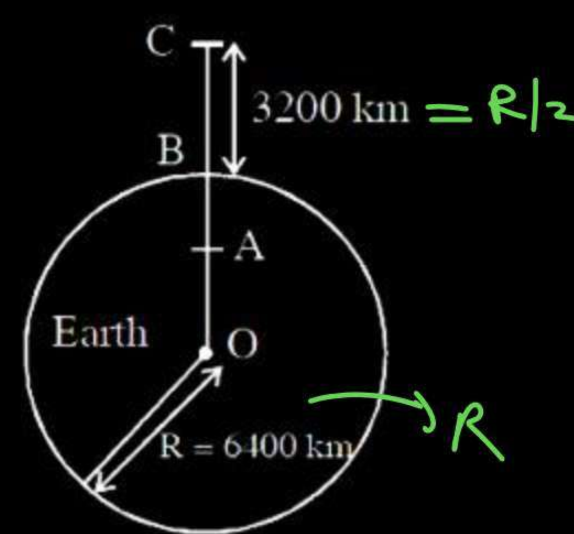
(C) C only

(D) E only

Ans : (D)

Question

In the reported figure of earth, the value of acceleration due to gravity is same at point A and C but it is smaller than that of its value at point B (surface of the earth). The value of OA : AB will be x : y. The value of x is .... **(JEE Main-2021)**



Ans : (4)

Question

The maximum and minimum distances of a comet from the Sun are 1.6×10^{12} m and 8.0×10^{10} m respectively. If the speed of the comet at the nearest point is $6 \times 10^4\ ms^{-1}$, the speed at the farthest point is: **(JEE Main-2021)**

$\to V_{max}$

(A) 1.5×10^3 m/s

(B) 6.0×10^3 m/s

(C) 3.0×10^3 m/s

(D) 4.5×10^3 m/s

$$\cancel{m} v_1 r_1 = \cancel{m} v_2 r_2$$

$$V_{max} r_{min} = V_{min} r_{max}$$

$$6 \times 10^4 \times \cancel{8} \times 10^{10} = V_{min} \times \cancel{1.6}^{\,.2} \times 10^{12}$$

$$V_n = \frac{6 \times 10^4}{20}$$

Ans : (C)

Question

If one wants to remove all the mass of the earth to infinity in order to break it up completely. The amount of energy that needs to be supplied will be $\frac{x}{5}\frac{Gm^2}{R}$ where x is ______.

(Round off to the Nearest Integer)

(M is the mass of earth, R is the radius of earth, G is the gravitational constant)

(JEE Main-2021)

Ans : (3)

Question

A geostationary satellite is orbiting around an arbitrary planet 'P' at a height of 11R above the surface of 'P', R being the radius of 'P'. The time period of another satellite in hours at a height of 2R from the surface of 'P' is ________. 'P' has the time period of 24 hours.

(JEE Main-2021)

(A) $6\sqrt{2}$

(B) $\frac{6}{\sqrt{2}}$

(C) 3

(D) 5

Ans : (C)

Question

The time period of a satellite in a circular orbit of radius R is T. The period of another satellite in a circular orbit of radius 9R is: **(JEE Main-2021)**

(A) 9 T

(B) 27 T

(C) 12 T

(D) 3 T

Ans : (B)

Question

The angular momentum of a planet of mass M moving around the sun in an elliptical orbit is $\vec{L}$. The magnitude of the areal velocity of the planet is :

(JEE Main-2021)

Easy

$\frac{J}{2m}$

(A) $\frac{4L}{m}$

(B) $\frac{L}{M}$

(C) $\frac{2L}{M}$

(D) $\frac{L}{2M}$

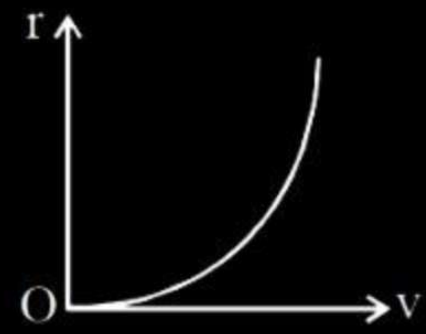
Ans : (D)

Question

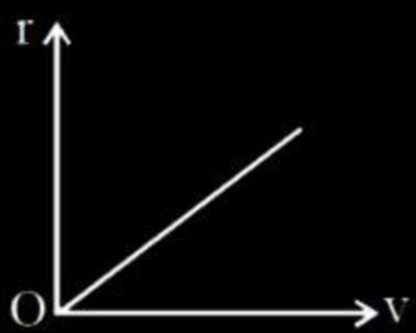
A particle of mass m moves in a circular orbit under the central potential field, $U(r) = \frac{-C}{r}$. Where C is a positive constant. The correct radius – velocity graph of the particle's motion is: **(JEE Main-2021)**

(A)

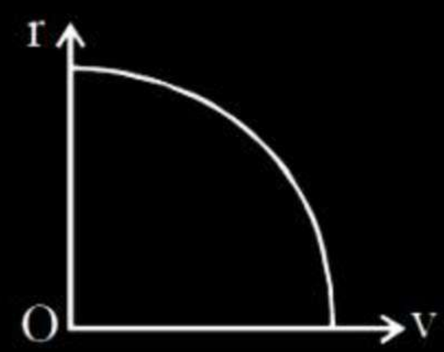
(B)



(C)



(D)



Ans : (A)

Question

A satellite is launched into a circular orbit of radius R around earth, while a second satellite is launched into a circular orbit of radius 1.02 R. The percentage difference in the time periods of the two satellites is: **(JEE Main-2021)**

(A) 1.5

(B) 2.0

(C) 0.7

(D) 3.0

Ans : (D)

Question

Consider a binary star system of star A and star B with masses m_A and m_B revolving in a circular orbit of radii r_A and r_B, respectively. If T_A and T_B are the time period of star A and star B, respectively, then: **(JEE Main-2021)**

(A) $\frac{T_A}{T_B} = \left(\frac{r_A}{r_B}\right)^{\frac{3}{2}}$

(B) $T_A = T_B$

(C) $T_A > T_B$ (if $m_A > m_B$)

(D) $T_A > T_B$ (if $r_A > r_B$)

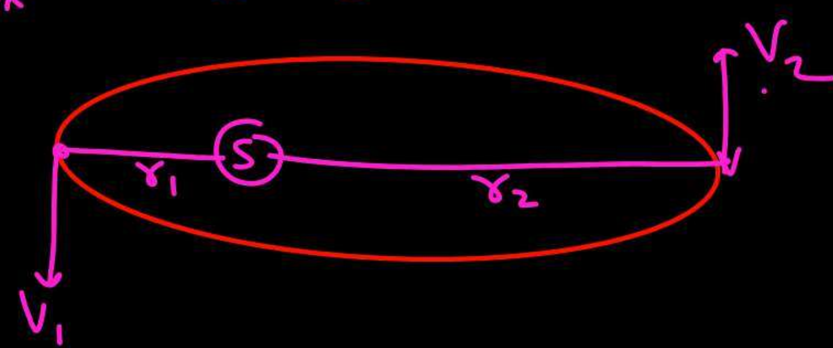
Ans : (B)

Question

The minimum and maximum distances of a planet revolving around the Sun are x_1 and x_2. If the minimum speed of the planet on its trajectory is v_0 then its maximum speed will be:

(JEE Main-2021)

$$x_1 v_{max} = x_2 v_0$$



(A) $\frac{v_0 x_1^2}{x_2^2}$

(B) $\frac{v_0 x_2^2}{x_1^2}$

(C) $\frac{v_0 x_1}{x_2}$

(D) $\frac{v_0 x_2}{x_1}$

Ans : (D)

Question

Consider a planet in some solar system which has a mass double the mass of earth and density equal to the average density of earth. If the weight of an object on earth is W, the weight of the same object on that planet will be: **(JEE Main-2021)**

(A) 2W

(B) W

(C) $2^{\frac{1}{3}}W$

(D) $\sqrt{2}W$

Ans : (C)

Question

Two identical particles of mass 1 kg each go round a circle of radius R, under the action of their mutual gravitational attraction. The angular speed of each particle is:

(JEE Main-2021)

(A) $\sqrt{\frac{G}{2R^3}}$

(B) $\frac{1}{2}\sqrt{\frac{G}{R^3}}$

(C) $\frac{1}{2R}\sqrt{\frac{1}{G}}$

(D) $\sqrt{\frac{2G}{R^3}}$

Ans : (B)

Question

The planet Mars has two moons, if one of them has a period 7 hours, 30 minutes and an orbital radius of 9.0×10^3 km. Find the mass of Mars.

$\left\{\text{Given } \frac{4\pi^2}{G} = 6 \times 10^{11} N^{-1} m^{-2} kg^2\right\}$ **(JEE Main-2021)**

(A) 5.96×10^{19} kg

(B) 3.25×10^{21} kg

(C) 7.02×10^{25} kg

(D) 6.00×10^{23} kg

Ans : (D)

Question

Inside a uniform spherical shell:

(a) the gravitational field is zero

(b) the gravitational potential is zero

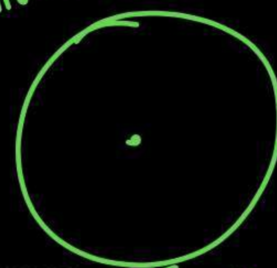
(c) the gravitational field is same everywhere

(d) the gravitation potential is same everywhere

(e) all of the above

Choose the most appropriate answer from the options given below:

hollow



= 0

(JEE Main-2021)

(A) (a), (c) and (d) only

(B) (e) only

(C) (a), (b) and (c) only

(D) (b), (c) and (d) only

Ans : (D)

Question

A mass of 50 kg is placed at the centre of a uniform spherical shell of mass 100 kg and radius 50 m. If the gravitational potential at a point, 25 m from the centre is V kg/m. The value of V is: **(JEE Main-2021)**

(A) $-60\,G$

(B) $+2\,G$

(C) $-20\,G$

(D) $-4\,G$

Ans : (D)

Question

The masses and radii of the earth and moon are (M_1, R_1) and (M_2, R_2) respectively. Their centres are at a distance 'r' apart. Find the minimum escape velocity for a particle of mass 'm' to be projected from the middle of these two masses:

(JEE Main-2021)

(A) $V = \frac{1}{2}\sqrt{\frac{4G(M_1 + M_2)}{r}}$

H/w

(B) $V = \sqrt{\frac{4G(M_1+M_2)}{r}}$

(C) $V = \frac{1}{2}\sqrt{\frac{2G(M_1 + M_2)}{r}}$

(D) $V = \frac{\sqrt{2G}(M_1 + M_2)}{r}$

Ans : (B)

Question

Four particles each of mass M, move along a circle of radius R under the action of their mutual gravitational attraction as shown in figure. The speed of each particle is:

(JEE Main-2021)

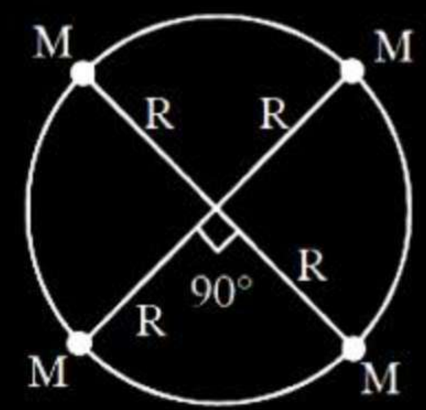
Easy

(A) $\frac{1}{2}\sqrt{\frac{GM}{R(2\sqrt{2}+1)}}$

(B) $\frac{1}{2}\sqrt{\frac{GM}{R}(2\sqrt{2}+1)}$

(C) $\frac{1}{2}\sqrt{\frac{GM}{R}(2\sqrt{2}-1)}$

(D) $\sqrt{\frac{GM}{R}}$



Ans : (B)

Question

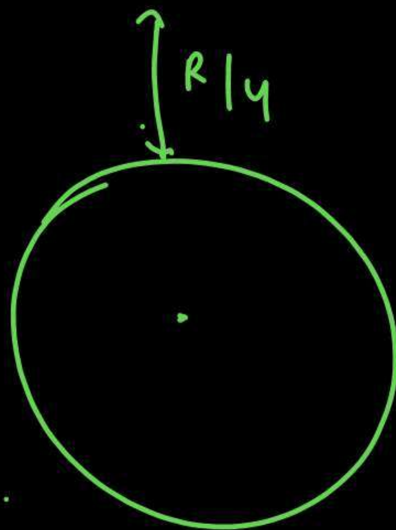
An object is taken to a height above the surface of earth at a distance $\frac{5}{4}R$ from the centre of the earth. Where radius of earth, $R = 6400$ km.
The percentage decrease in the weight of the object will be **(JEE Main-2022)**

(A) 36 %

(B) 50%

(C) 65%

(D) 25%



g

$$g' = \frac{GM}{\left(\frac{5R}{4}\right)^2}$$

Ans : (A)

Question

The percentage decrease in the weight of a rocket, when taken to a height of 32 km above the surface of earth will, be :

(Radius of earth = 6400km) **(JEE Main-2022)**

(A) 1 %

(B) 3%

(C) 4%

(D) 0.S¾

Ans : (A)

Question

A body is projected vertically upwards from the surface of earth with a velocity equal to one third of escape velocity. The maximum height attained by the body will be:

(Take radius of earth = 6400 km and $g = 10\ ms^{-2}$) **(JEE Main-2022)**

(A) 800 km

(B) 1600 km

(C) 2133 km

(D) 4800 km

Ans : (A)

Question

Two satellites A and B having masses in the ratio 4 : 3 are revolving in circular orbits of radii 3 r and 4 r respectively around the earth. The ratio of total mechanical energy of A to B is :

(JEE Main-2022)

(A) 9 : 16

(B) 16 : 9

(C) 1 : 1

(D) 4 : 3

Ans : (B)

Question

If the radius of earth shrinks by 2% while its mass remains same. The acceleration due to gravity on the earth's surface will approximately:

(JEE Main-2022)

(A) decrease by 2%

(B) decrease by 4%

(C) increase by 2%

(D) increase by 4%

$$g = \frac{GM}{R^2}$$

$$\left|\frac{\Delta g}{g}\right| = \frac{2\Delta R}{R}$$

Ans : (D)

Question

If the acceleration due to gravity experienced by a point mass at a height h above the surface of earth is same as that of the acceleration due to gravity at depth ah ($h << R_e$) from the earth surface. The value of a will be. (use $R_e = 6400$ km)

(JEE Main-2022)

Ans : (2)

Question

The approximate height from the surface of earth at which the weight of the body becomes $\frac{1}{2}$ of its weight on the surface of earth is :

[Radius of earth R = 6400 km and $\sqrt{3} = 1.732$]

(JEE Main-2022)

(A) 3840 km

(B) 4685 km

(C) 2133 km

(D) 4267 km

Ans : (B)

Question

The distance between Sun and Earth is R. The duration of year if the distance between Sun and Earth becomes 3R will be : (JEE Main-2022)

(A) $\sqrt{3}$ years

(B) 3 years

(C) 9 years

(D) $3\sqrt{3}$ years

Ans : (D)

Question

The height of any points P above the surface of earth is equal to diameter of earth. The value of acceleration due to gravity at point P will be : (Given g = acceleration due to gravity at the surface of earth)

(JEE Main-2022)

(A) $g/2$

(B) $g/4$

(C) $g/3$

(D) $g/9$

Ans : (D)

Question

Two satellites S_1 and S_2 are revolving in circular orbits around a planet with radius $R_1 = 3200$ km and $R_2 = 800$ km respectively. The ratio of speed of satellite S_1 to the speed of satellite S_2 in their respective orbits would be $\frac{1}{x}$ where x =

(JEE Main-2022)

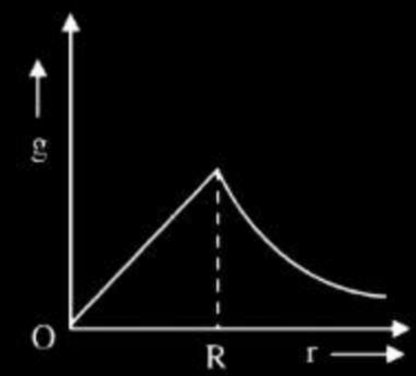
Ans : (2)

Question

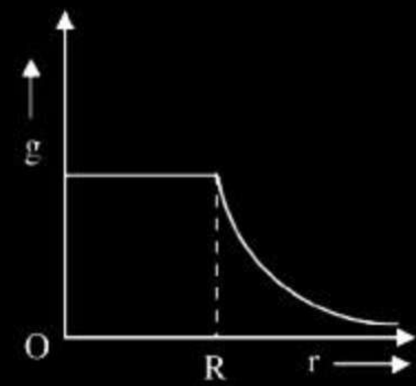
The variation of acceleration due to gravity (g) with distance (r) from the center of the earth is correctly represented by : (given R = radius of earth)

(JEE Main-2022)

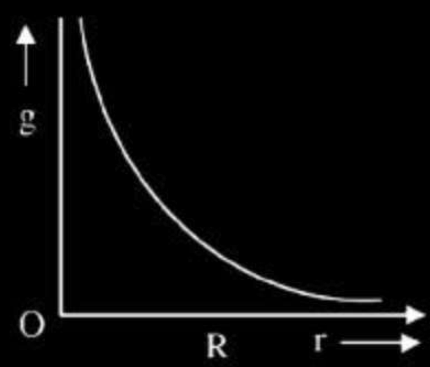
(A)



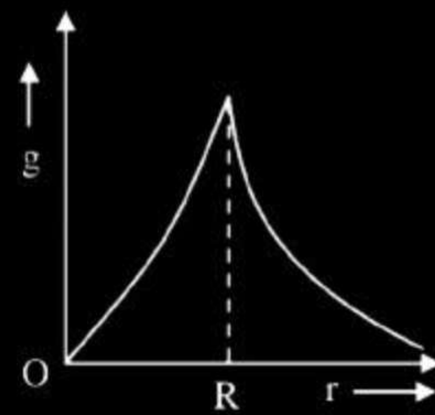
(B)



(C)



(D)



Ans : (A)

Question

Given below are two statements : is labelled **Assertion A** and **Reason R.**

Assertion A : If we move from poles to equator, the direction of acceleration due to gravity of earth always points towards the center of earth without any variation in its magnitude.

Reason R : At equator, the direction of acceleration due to the gravity is towards the center of earth.

In the light of above statements, choose the correct answer from the options given below : **(JEE Main-2022)**

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is NOT the correct explanation of A.

(C) A is true but R is false

(D) A is false but R is true

Ans : (D)

Question

Two planets A and B of equal mass are having their period of revolution T_A and T_B such that $T_A = 2T_B$. These planets are revolving in the circular orbits of radii r_A and r_B respectively. Which out of the following would be the correct relationship of their orbits ?

(JEE Main-2022)

(A) $2r_A^2 = r_B^2$

(B) $r_A^3 = 2r_B^3$

(C) $r_A^3 = 4r_B^3$

(D) $T_A^2 - T_B^2 = \frac{\pi^2}{GM}(r_B^3 - 4r_A^3)$

Ans : (C)

Question

Two objects of equal masses placed at certain distance from each other attracts each other with a force of F. If one-third mass of one object is transferred to the other object, then the new force will be : (JEE Main-2022)

(A) $\frac{2}{9}F$

(B) $\frac{16}{9}F$

(C) $\frac{8}{9}F$

(D) F

Ans : (C)

Question

The escape velocity of a body on a planet 'A' is 12 kms^{-1}. The escape velocity of the body on another planet 'B', whose density is four times and radius is half of the planet 'A', us :

(JEE Main-2022)

(A) 12 kms^{-1}

(B) 24 kms^{-1}

(C) 36 kms^{-1}

(D) 6 kms^{-1}

Ans : (A)

Question

The time period of a satellite revolving around earth in a given orbit is increased to three times its previous value, then approximate new time period of the satellite will be : (JEE Main-2022)

(A) 40 hours

(B) 36 hours

(C) 30 hours

(D) 25 hours

Ans : (B)

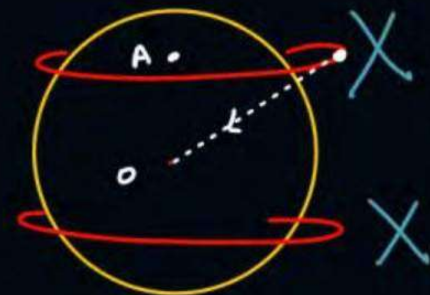
$$\boxed{R_{earth} = 6400\,km}$$

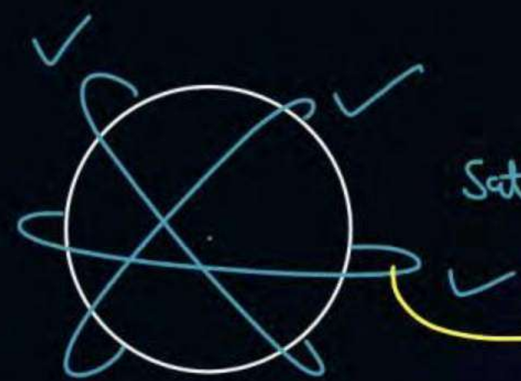
Geostationary sattelite

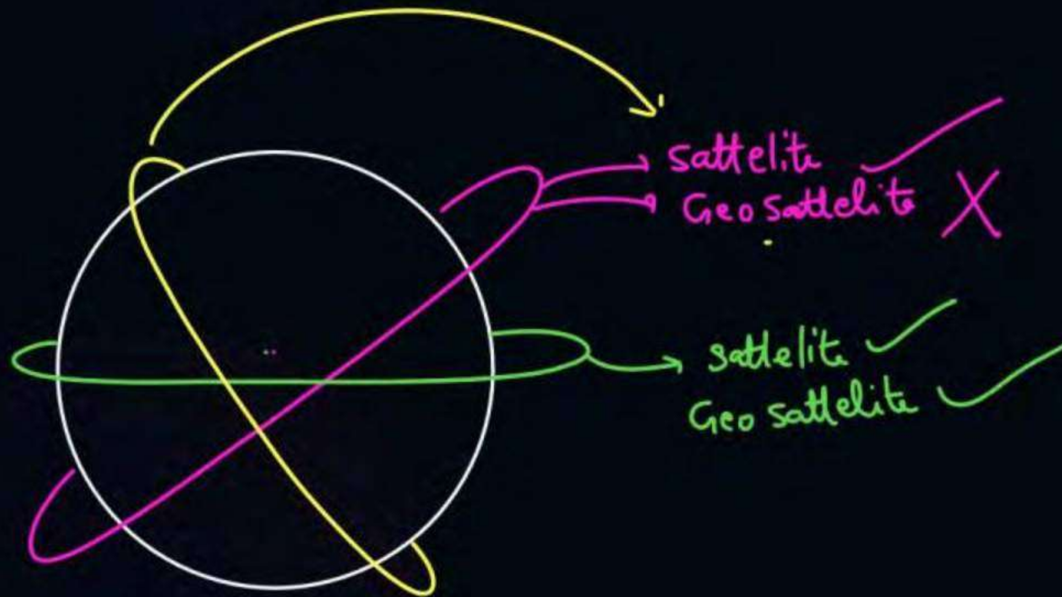
- Which remains at rest wrt earth.
- $T = 24$ hour $=$ Time period of earth
- $T = 2\pi\sqrt{\dfrac{r^3}{Gm_e}}$

Value put $\Rightarrow$ Solve ... $r \approx 42000$ km from centre of earth

- $h = 36000$ Km from earth surface.
- It must be in equitorial plane.
- same sense of rotation as earth have





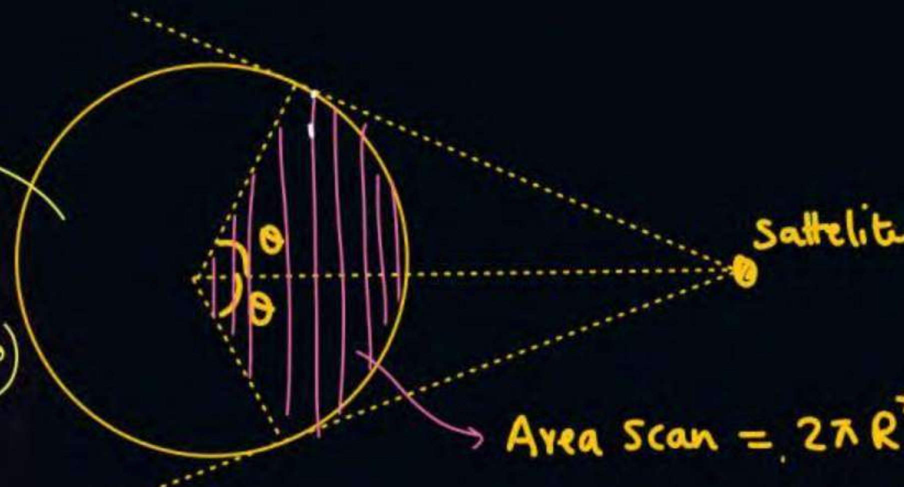


sattelite
Geo sattelite X
sattelite
Geo sattelite

Imp (*)

Area Unscan

$= 4\pi R^2 - 2\pi R^2 (1 - \cos\theta)$



Area Scan $= 2\pi R^2 (1 - \cos\theta)$

Both are moving in circular path of radius r_1 & r_2 abt their com.

$$T = 2\pi\sqrt{\frac{(r_1+r_2)^3}{G(m_1+m_2)}}$$

* $T \longrightarrow$ Same
  $\omega \longrightarrow$ Same

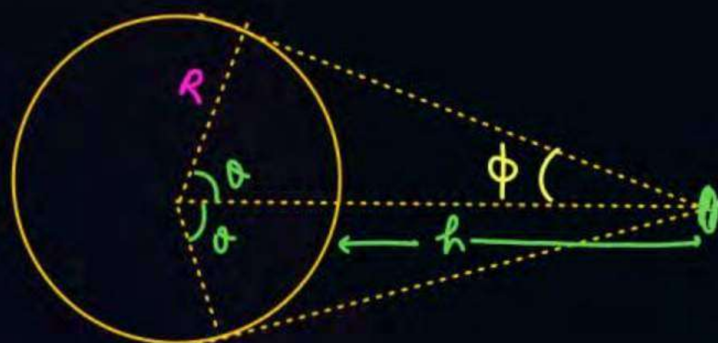
Binary star

पहले $T = 2\pi\sqrt{\dfrac{r^3}{GM}}$

Imp formula

$$T = 2\pi\sqrt{\frac{(r_1+r_2)^3}{G(m_1+m_2)}}, \quad \omega = \sqrt{\frac{G(m_1+m_2)}{(r_1+r_2)^3}}$$

**



Coaltitude → ϕ

$$\sin\phi = \frac{R}{R+h}$$

$$\text{Area Scan} = 2\pi R^2(1-\cos\theta)$$