LAWS OF MOTION

INERTIA

A body cannot change its state of rest or uniform motion along a straight line. This property is called inertia. Inertia has no unit and no dimension.

INERTIA
- Inertia of Rest
- Inertia of Motion
- Inertia of Direction

- Inertia of Rest
Inability to change state of rest by itself.
- Inertia of Motion
Inability of a body to change its state of uniform motion by itself.
- Inertia of Direction
Inability of a body to change direction of motion by itself.

Newton's Second Law

F_{net} = Rate of change of linear momentum.

Instantaneous: $\vec{F} = \frac{d\vec{p}}{dt}$

Average: $\vec{F}_{av} = \frac{\Delta\vec{P}}{\Delta t} = \frac{\vec{P}_f - \vec{P}_i}{\Delta t}$

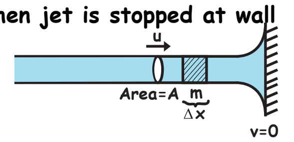
MOMENTUM

$\vec{P} = m\vec{v}$

- It is a vector quantity having direction same as that of velocity
- Unit is kg m/s.

LIQUID JETS

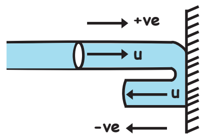
When jet is stopped at wall



$F_{jet} = \frac{P_f - P_i}{\Delta t} = \frac{m \times 0 - mu}{\Delta t} = \frac{-mu}{\Delta t} = -\rho A u^2 (m = \rho A \Delta x, \frac{\Delta x}{\Delta t} = u)$

$F_{wall} = \rho A u^2$

When liquid jet bounce back

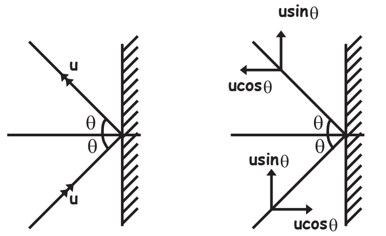


$F_{jet} = \frac{P_f - P_i}{\Delta t} = \frac{-2mu}{\Delta t}$

$F_{jet} = -2\rho A u^2$

$F_{wall} = 2\rho A u^2$

When liquid jet strikes obliquely



$F_{jet} = -2\rho A u^2 \cos\theta$ $F_{wall} = 2\rho A u^2 \cos\theta$

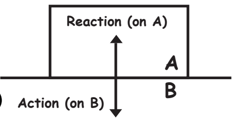
Change in momentum $= -2mu \cos\theta$

NEWTON'S THIRD LAW

- To every action, there is always an equal (in magnitude) and opposite (in direction) reaction.
- Forces in nature always occur in pairs.
- A single isolated force is not possible.
- Counter force experienced by a body- reaction
- Action and reaction never act on the same body

* Force exerted on body A by body B (action)
* force exerted on body B by body A (reaction)

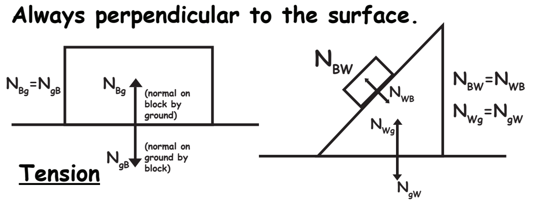
$\vec{F}_{Action} = -\vec{F}_{Reaction}$



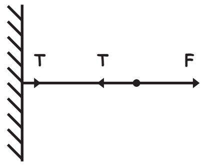
Some common forces

1. Normal Reaction
- Occurs when two surfaces are in contact with each other

Always perpendicular to the surface.

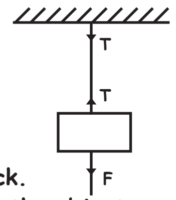


Tension



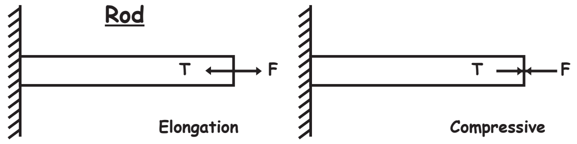
Restoring force developed when a longitudinal force is applied on a body

Ideal Rope



*Massless
*Tension is same everywhere
*Opposes only elongation
*On compression it becomes slack.
*Tension always acts away from the object.

Rod



can support both elongation and compression

SINGLE BLOCK

Horizontal Force

Acceleration is along x-axis only

Along y-axis $a_y = 0$,

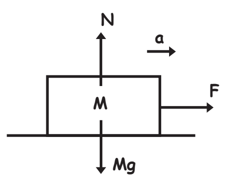
$N - Mg = Ma_y$

$\because a_y = 0$

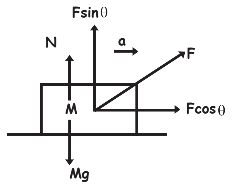
$N = Mg$

Along x-axis $a = \frac{F}{M}$

$F - 0 = Ma$



Inclined Forces



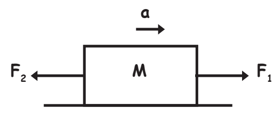
- If, $F\sin\theta < Mg$ block remains in contact with ground & accelerates horizontally
- If, $F\sin\theta = Mg$ block just leaves contact with ground
- If, $F\sin\theta > Mg$ the block leaves contact with ground and it begins to accelerate obliquely.

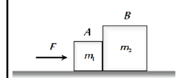
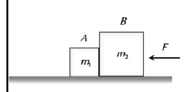
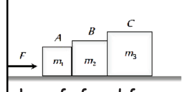
MOTION OF CONNECTED BODIES

$F_1 > F_2$

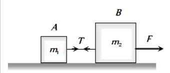
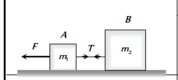
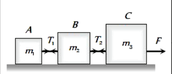
$F_1 - F_2 = Ma$

$a = \frac{F_1 - F_2}{M}$



Condition	Free body diagram	Equation	Force and acceleration
		$F - f = m_1 a$	$a = \frac{F}{m_1 + m_2}$
		$f = m_2 a$	$f = \frac{m_2 F}{m_1 + m_2}$
		$f = m_1 a$	$a = \frac{F}{m_1 + m_2}$
		$F - f = m_2 a$	$f = \frac{m_1 F}{m_1 + m_2}$
		$F - f_1 = m_1 a$	$a = \frac{F}{m_1 + m_2 + m_3}$
		$f_1 - f_2 = m_2 a$	$f_1 = \frac{(m_2 + m_3)F}{m_1 + m_2 + m_3}$
here, f, f_1 and f_2 are normal reactions between blocks		$f_2 = m_3 a$	$f_2 = \frac{m_3 F}{m_1 + m_2 + m_3}$

MOTION OF BLOCKS CONNECTED BY MASSLESS STRING

Condition	Free body diagram	Equation	Tension and acceleration
		$T = m_1 a$	$a = \frac{F}{m_1 + m_2}$
		$F - T = m_2 a$	$T = \frac{m_1 F}{m_1 + m_2}$
		$F - T = m_1 a$	$a = \frac{F}{m_1 + m_2}$
		$T = m_2 a$	$T = \frac{m_2 F}{m_1 + m_2}$
		$T_1 = m_1 a$	$a = \frac{F}{m_1 + m_2 + m_3}$
		$T_2 - T_1 = m_2 a$	$T_1 = \frac{m_1 F}{m_1 + m_2 + m_3}$
		$F - T_2 = m_3 a$	$T_2 = \frac{(m_1 + m_2)F}{m_1 + m_2 + m_3}$

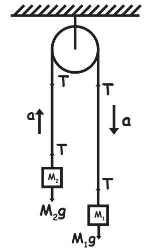
PULLEY-BLOCK SYSTEM

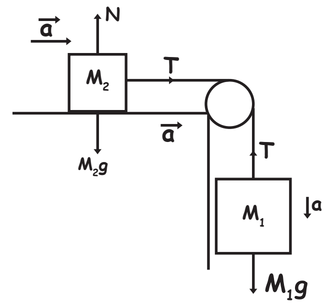
Ideal pulley

$M_1 > M_2$

$a = \frac{F}{M} = \frac{M_1 - M_2}{M_1 + M_2} g$

$T = \frac{2M_1 M_2}{M_1 + M_2} g$

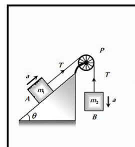
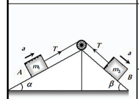
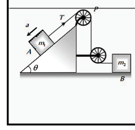




$a = \frac{M_1 g}{M_1 + M_2}$

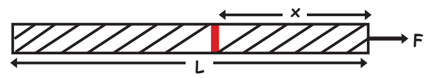
$T = \frac{M_1 M_2}{M_1 + M_2} g$

INCLINED PLANE + PULLEY

		$m_1 a = T - m_1 g \sin\theta$	$a = \left[\frac{m_2 - m_1 \sin\theta}{m_1 + m_2}\right] g$
		$m_2 a = m_2 g - T$	$T = \frac{m_1 m_2 (1 + \sin\theta)}{m_1 + m_2} g$
		$T - m_1 g \sin\alpha = m_1 a$	$a = \frac{(m_2 \sin\beta - m_1 \sin\alpha)}{m_1 + m_2} g$
		$m_2 a = m_2 g \sin\beta - T$	$T = \frac{m_1 m_2 (\sin\alpha + \sin\beta)}{m_1 + m_2} g$
		$m_1 g \sin\theta - T = m_1 a$ $T = m_2 a$	$a = \frac{m_1 g \sin\theta}{m_1 + m_2}$ $T = \frac{m_1 m_2 \sin\theta g}{m_1 + m_2}$

THICK ROPE

Tension will be different at different points.



Mass per unit length $= \frac{M}{L}$

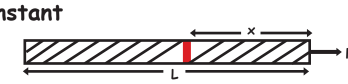
Mass of x length of rope $= \frac{M}{L} x$

Note :

Mass of given length $= \frac{\text{total mass}}{\text{total length}} \times$ given length

$\frac{\text{mass}}{\text{length}}$ = constant

$a_{rope} = \frac{F}{M}$



For(L-x) rope length, $\frac{M}{L}(L-x) = m_2$ $\frac{M}{L}x = m_1$

$\vec{F} = m\vec{a} \Rightarrow T = m_2 \frac{F}{M}$

LIFT PROBLEMS

Apparent weight of body in a lift

Reading of weighing machine = reaction force exerted b weighing machine

Apparent weight, $(W_{apparent})$ = Reaction force (R)

Case 1: Lift is at rest

$R = mg$, $W_{apparent} = W_{actual} = mg$

Case 2: Lift moving up or down with constant velocity

$R = mg$, $W_{apparent} = W_{actual} = mg$

Case 3: Accelerated upward at a rate of 'a'

$R - mg = ma \Rightarrow R = m(g+a) = W_{app}$

$W_{apparent} > W_{actual}$ → Feels over weight

Accelerated upward at a rate of 'g'

$R - mg = mg$, $R = 2mg$, $W_{app} = 2 \times W_{act}$

Case 4: Accelerated downward at a rate of 'a'

$mg - R = ma$, $R = m(g-a) = W_{app}$, $W_{app} < W_{act}$

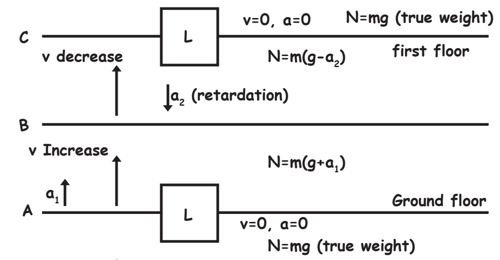
Accelerated downward at a rate of 'g' [Freefall]

$mg - R = mg$, $R = mg - mg = 0$, $W_{app} = 0$

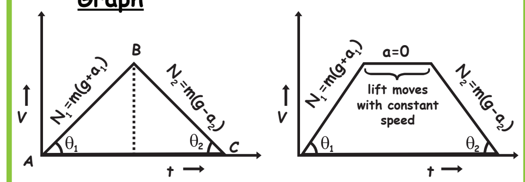
If $a > g$:

body leaves contact from ground and begins free-fall

Lift moves from ground floor to first floor



Graph



Acceleration, $a_1 = \tan\theta_1$

Retardation, $a_2 = \tan\theta_2$

FRAME OF REFERENCE & PSEUDO FORCE

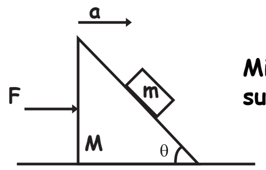
Frame of Reference

A frame in which observer is situated and makes his observation

Inertial frame of reference	Non-Inertial frame of reference
At rest or moving with uniform velocity along straight line. i.e unaccelerated	Accelerated frame of reference.
Newton's law of motion hold's $\vec{F}_{net} = m\vec{a}$	Newton's law of motion not applicable. $\vec{F}_{net} + \vec{F}_{pseudo} = m\vec{a}$ $\vec{F}_{pseudo} = -m\vec{a}$

LAWS OF MOTION

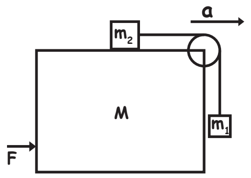
RELATIVE SLIPPING



Minimum force required to push the inclined plane such that "m" does not slip with respect to "M"

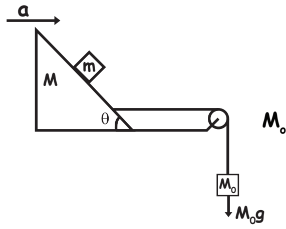
$F = (m+M)\, g\tan\theta$, $a = g\tan\theta$

Minimum acceleration of "M" such that there is no relative slipping.



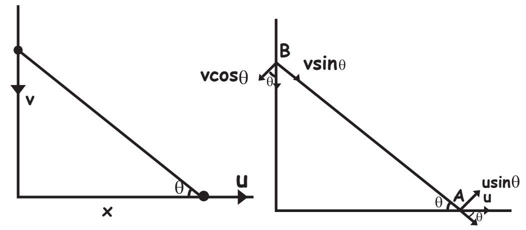
$a = \left(\frac{m_1}{m_2}\right) g$

Minimum mass M_0 such that there is no relative slipping



$M_0 = \frac{M+m}{\cot\theta - 1}$

ROD SLIDING ON A WALL



velocity of B towards A = $v\sin\theta$

velocity of A away from B = $u\cos\theta$

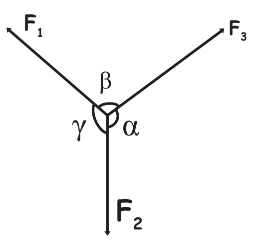
these velocities should be equal,

$\Rightarrow v\sin\theta = u\cos\theta$

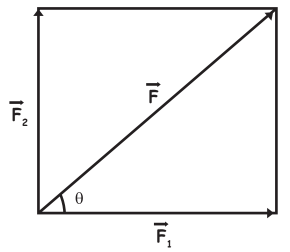
$\Rightarrow v = u\cot\theta$

EQUILLIBRIUM & LAMI'S THEOREM

$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$



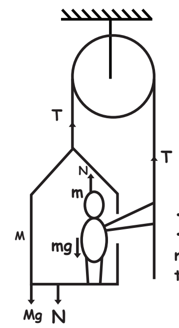
PARALLELOGRAM LAW



$\vec{F} = \vec{F_1} + \vec{F_2}$, $F = \sqrt{F_1^2 + F_2^2}$, $\tan\theta = \frac{F_2}{F_1}$

MAN-CAGE PROBLEM

Man holds the cage stationary



$T = \left(\frac{m+M}{2}\right) g$

$N = \left(\frac{m-M}{2}\right) g$

- m<M
- if m>M

man cannot hold the cage stationary

IMPULSE

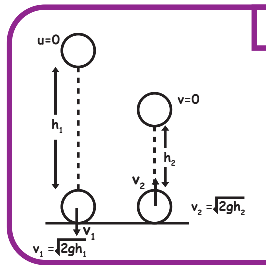
If a large force acting for short period of time, there will be a sudden change in momentum

$\vec{F}_{imp} = \frac{d\vec{p}}{dt}$

$\vec{P_f} - \vec{P_i} = \int \vec{F}_{imp}\, dt$ = area under F-t graph

$\vec{I}$ = Impulse = $\vec{P_f} - \vec{P_i} = \int_0^t \vec{F}_{imp}\, dt$ = area of F-t graph

Case-1



Impulse

$I = P_f - P_i$

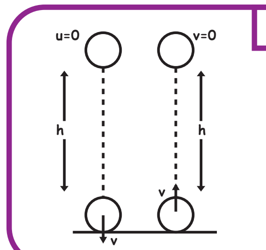
$= mV_2 - (-mV_1)$

$I = m\,(V_1 + V_2)$

Average impulsive force

$F_{avg} = \frac{\text{impulse}}{t_0} = \frac{\Delta P}{t_0} = \frac{m(v_1+v_2)}{t_0}$

Case-2



Impulse

$m(v+v) = J$

$= m(2v)$

$J = 2mv$

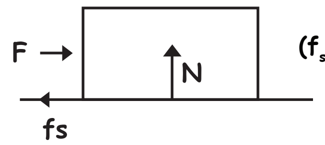
Average impulsive force

$F_{avg} = \frac{\Delta P}{\Delta t} = \frac{2mv}{t_0}$

FRICTION

Static friction

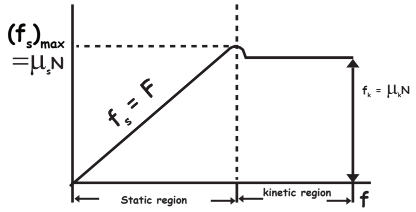
- It is a self adjusting force.
- The opposing force that comes into play, when object tends to slip over the surface of other object, but slipping has not yet started.
- As applied force increases static friction also increases.
- The body doesn't move until a maximum value of static friction is attained
- This value is called limiting friction or $(f_s)_{max}$



$(f_s)_{max} = \mu_s N$

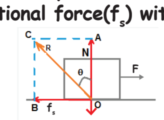
Kinetic friction

If the applied force is increased further and slipping between surfaces start, the friction opposing the slipping is called kinetic friction.



ANGLE OF FRICTION

Angle(θ) made by resultant of normal (N) & frictional force(f_s) with normal



$R = \sqrt{N^2 + f_s^2}$ R is resultant

$\tan\theta = \frac{f_s}{N}$

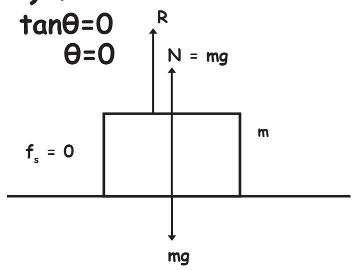
When $f_s = (f_s)_{max}$ $R = N\sqrt{1+\mu_s^2}$

$\tan\theta = \mu_s$, μ_s is Coefficient of friction

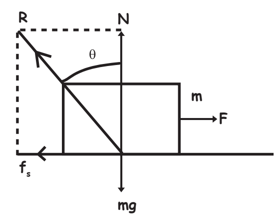
i) $f = 0$

$\tan\theta = 0$

$\theta = 0$



ii) $0 < f_s < (f_s)_{max}$

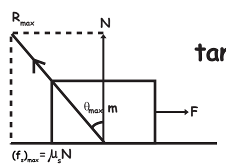


$0 \le \tan\theta \le \mu_s$

$N < R < N\sqrt{1+\mu_s^2}$

iii) $f_s = (f_s)_{max}$

$R_{max} = N\sqrt{1+\mu_s}$

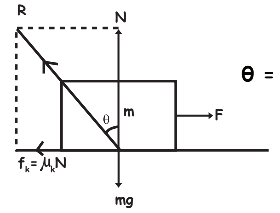


$\tan\theta_{max} = \mu_s$

iv) When slipping has started,

$f = f_k$

$R_{max} = N\sqrt{1+\mu_k^2}$



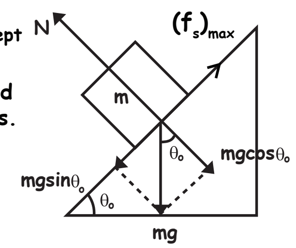
θ = Angle of kinetic friction

ANGLE OF REPOSE

Angle made by inclined plane such that a block kept on it just begins to slide

Depends only on μ_s and is independent of mass.

$\tan\theta_0 = \mu_s$



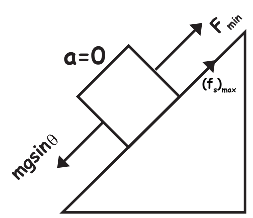
Variation of R

As angle of inclined plane increases, R remains constant (=mg) and when sliding starts R starts decreasing.

Variation of angle of friction.

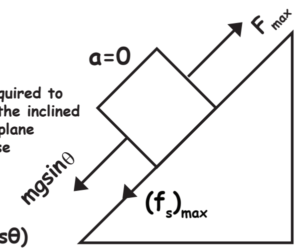
As angle of inclined plane increases, angle of friction will also increase and as sliding starts its value becomes constant and $\tan\theta = \mu_k$

Minimum & Maximum force (applied parallel to inclined plane)



$F_{min} = mg\,(\sin\theta - \mu_s\cos\theta)$

F_{min} is the minimum force required to keep the block stationary if the angle of inclined plane is greater than angle of repose

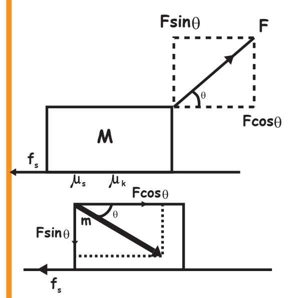


F_{max} is the maximum force required to keep the block stationary on the inclined plane if the angle of inclined plane is greater than angle of repose

$F_{max} = mg\,(\sin\theta + \mu_s\cos\theta)$

$mg(\sin\theta - \mu_s\cos\theta) \le F \le mg\,(\sin\theta + \mu_s\cos\theta)$

PULLING FORCE & PUSHING FORCE



$F\cos\theta \ge \mu_s(mg - F\sin\theta)$

$F_{pulling} = \frac{\mu_s mg}{\cos\theta + \mu_s\sin\theta}$

$F \ge \frac{\mu_s mg}{\cos\theta + \mu_s\sin\theta}$

$F\cos\theta \ge \mu_s(mg + F\sin\theta)$

$F \ge \frac{\mu_s mg}{\cos\theta - \mu_s\sin\theta}$

$F_{pushing} = \frac{\mu_s mg}{\cos\theta - \mu_s\sin\theta}$

$\boxed{F_{pushing} > F_{pulling}}$

HORIZONTAL TRUCK BOX

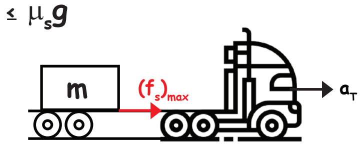
Case-1

Box does not slip.

$f_s \le \mu_s N$

$\Rightarrow ma_T \le \mu_s mg$

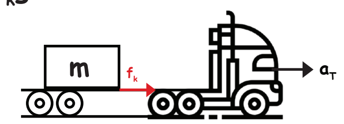
$\Rightarrow a_T \le \mu_s g$



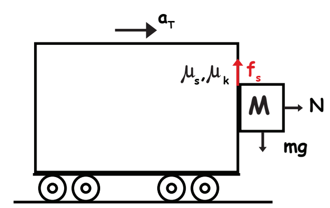
Case-2

Box slips

$a_T \ge \mu_k g$



VERTICAL TRUCK BOX



Case-1

Box does not slip.

$f_s \le \mu_s N$

$\Rightarrow mg \le \mu_s ma_T$

$\Rightarrow \mu_s \ge \frac{g}{a_T}$

Case-2

Box slips

$\mu_k \le \frac{g}{a_T}$