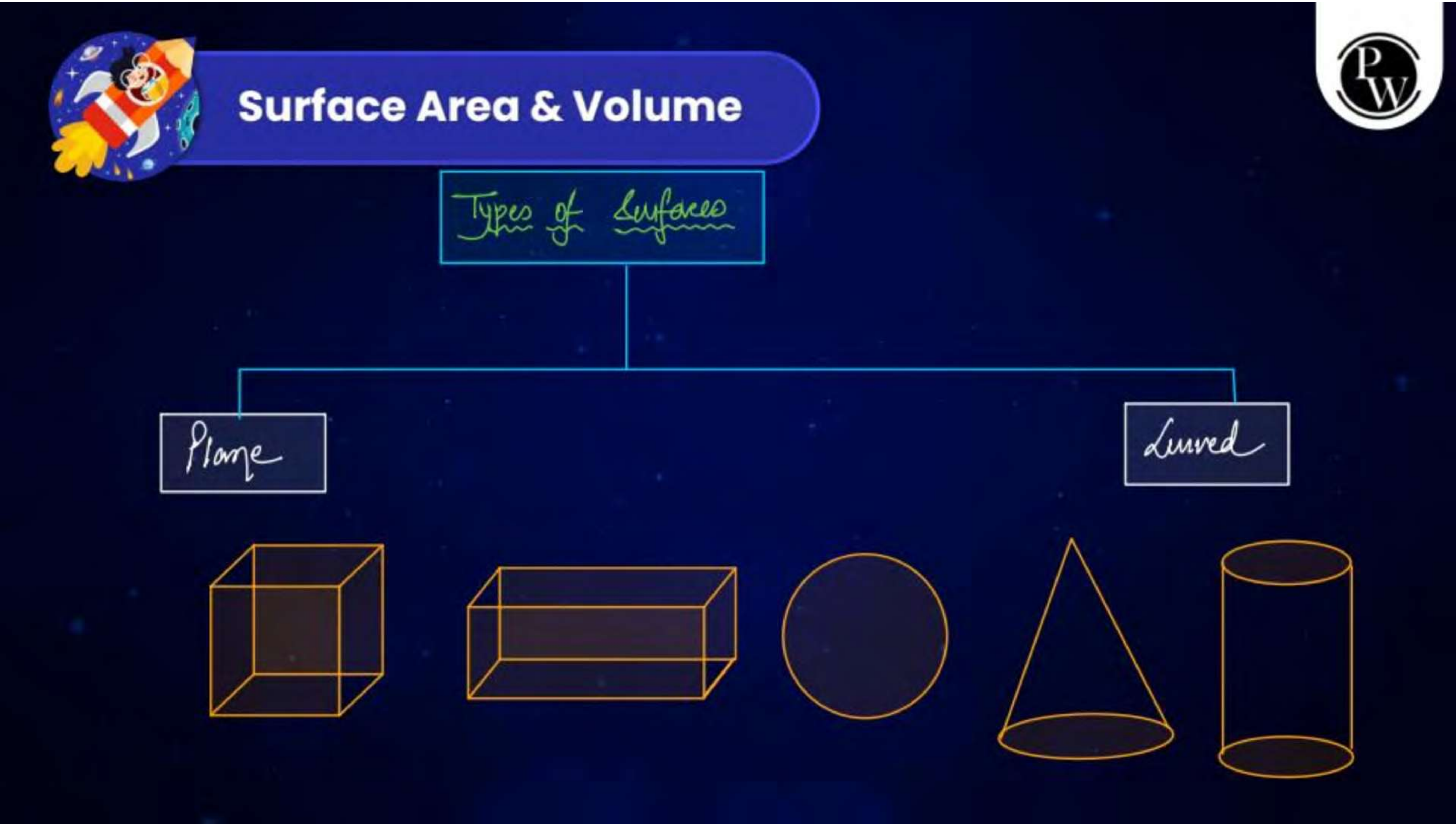
Surface Area & Volume

i) Volume :-

→ The measurement of amount of space occupied by something.

→ Unit → (m^3, cm^3, mm^3, ...) cm^3 / cu. cm

($Unit)^3$ / cu. units

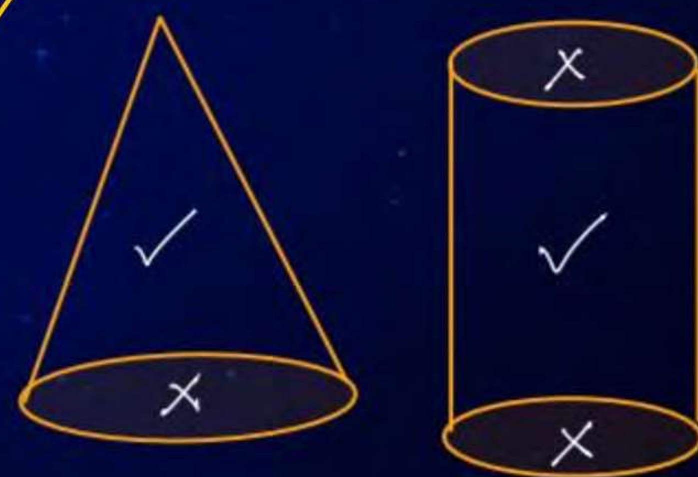

Surface Area & Volume
Types of Surfaces
Plane
Curved

Surface Area & Volume

ii) $\boxed{\text{Curved Surface Area}}$:—

→ Combined area of all the curved surfaces of the object.

→ Unit → (m^2, cm^2, mm^2, ...)
$(\text{Unit})^2$ / sq. units



Surface Area & Volume

(ii) **Lateral Surface Area (L.S.A)** :-

→ Area of 4 walls of a cube/cuboid.

→ Unit → (m^2, cm^2, mm^2, ...)
($Unit)^2$ / sq. units

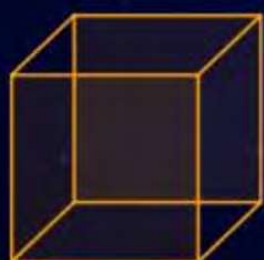
Surface Area & Volume

iii) Total Surface Area (T.S.A)   S.A → Surface Area

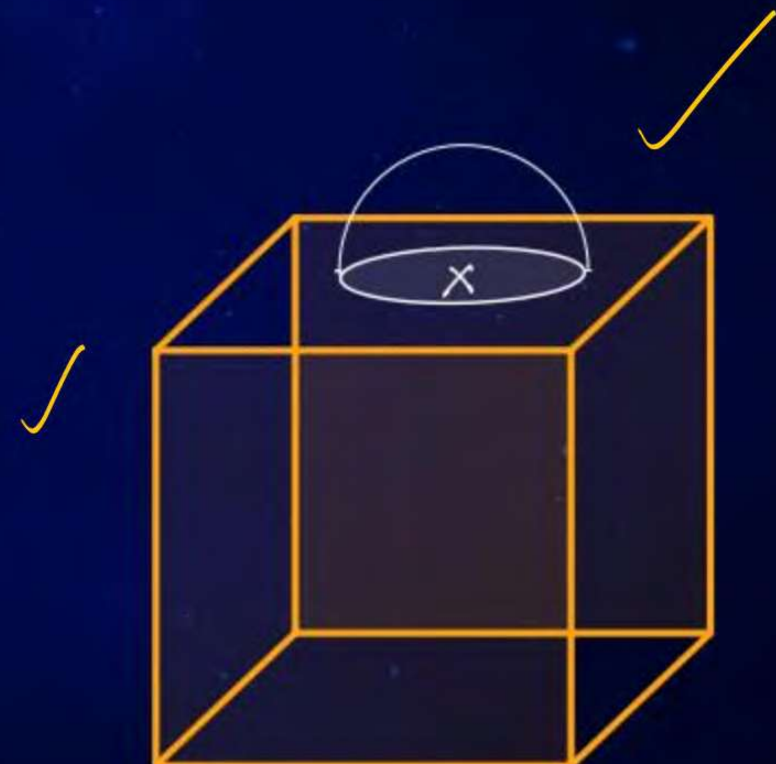
→ Combined area of all the surfaces that can be touched (Plane or Curved).

→ Unit → (m^2, cm^2, mm^2, ...)

$(\text{Unit})^2$ / sq. units



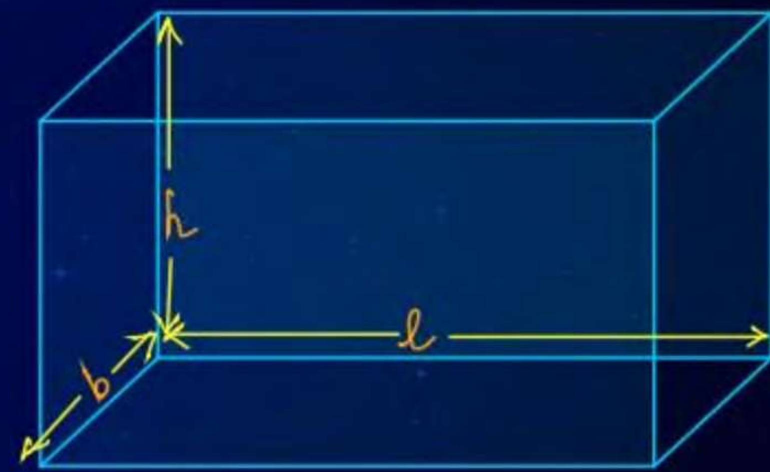
Surface Area & Volume



Surface Area & Volume

* $\boxed{\text{Cuboid}}$:-

Let length be 'l' units, breadth (width) be 'b' units &
height be 'h' units.



i) Volume = $\boxed{lbh \ (\text{unit})^3}$

ii) Diagonal = $\boxed{\sqrt{l^2+b^2+h^2} \text{ units}}$

iii) TSA = $\boxed{2(lb+bh+lh)(\text{unit})^2}$

iv) Area of 4 walls of a room = $\boxed{2(l+b)h \text{ sq. units}}$

Surface Area & Volume

Note:
The same can be obtained when we put
$l = a$
$b = a$
& $h = a$

* **Cube** :-

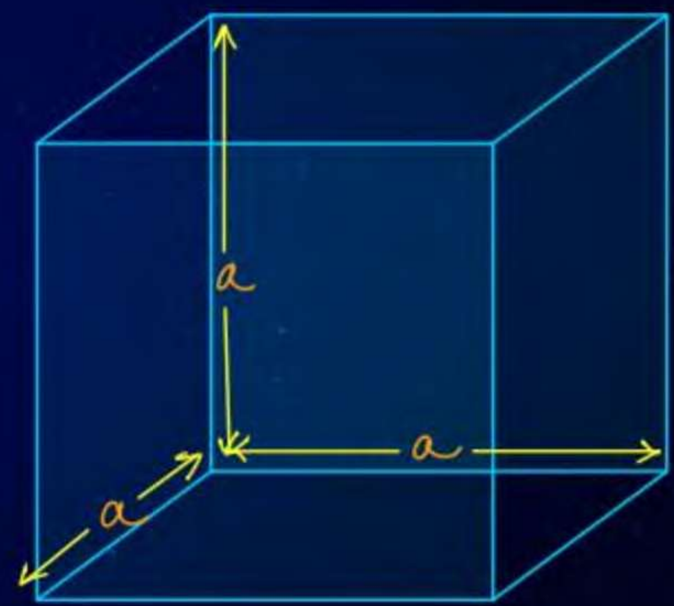
Let edge length of a cube be 'a' units.

i) Volume $= a^3$ cu. units

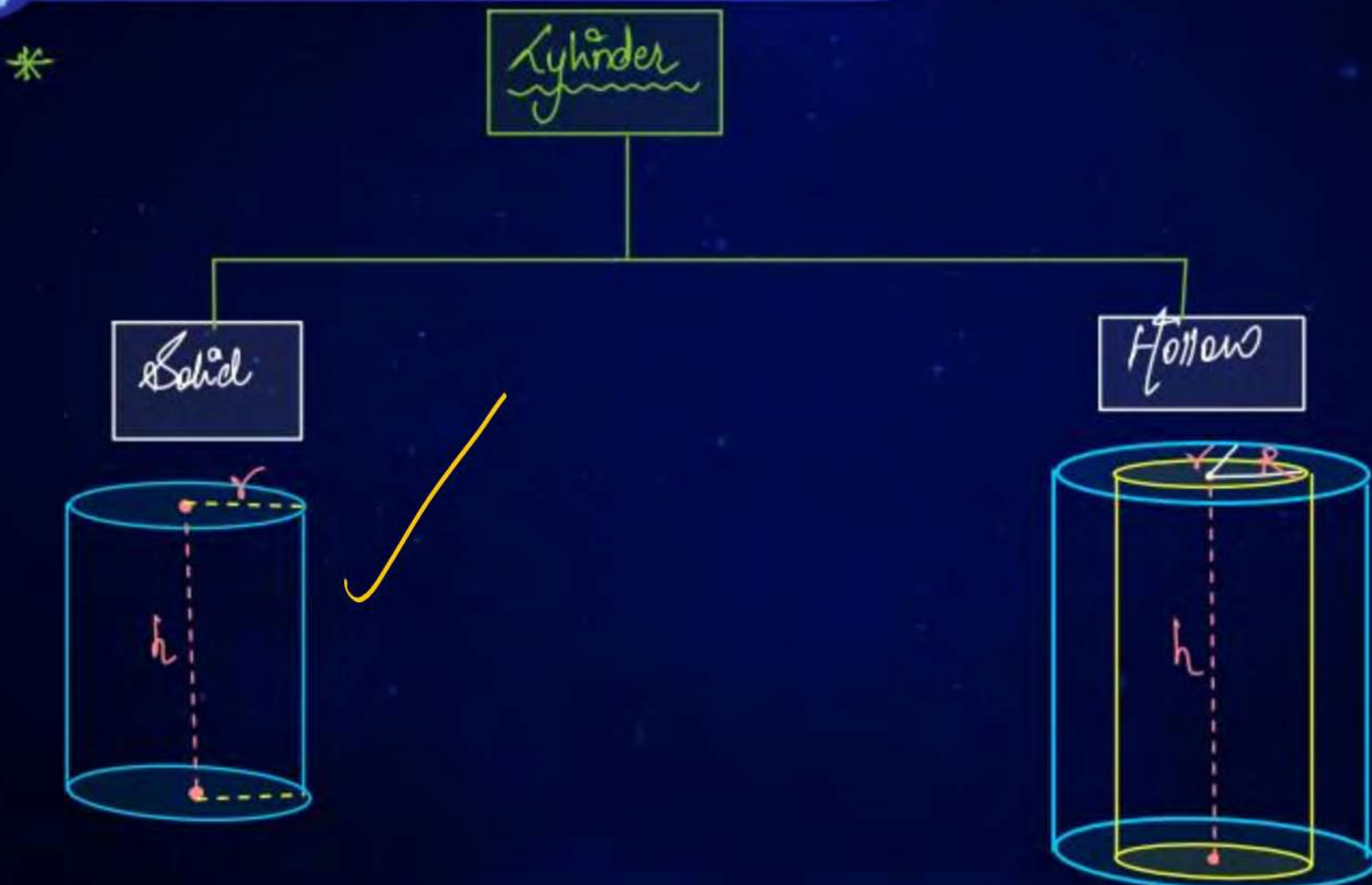
ii) Diagonal $= \sqrt{3}\, a$ units

iii) TSA $= 6a^2$ sq. units

iv) LSA $= 4a^2$ sq. units



Surface Area & Volume



Surface Area & Volume

* Cylinder (Solid) → Right circular

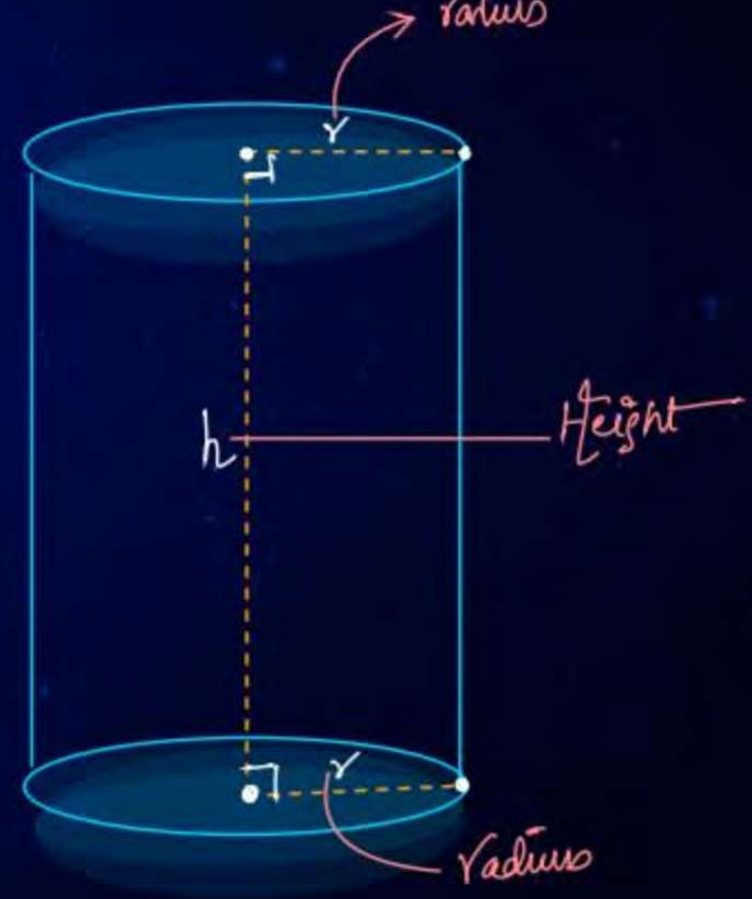
Let height be 'h' units & radius of top/base/circular part = r units.

i) Area of each end (top/base) =
area of circular part = πr^2 (unit)^2.

ii) CSA = $2\pi rh$ (unit)^2 [Perimeter of end × height]

iii) TSA = $2\pi r(h+r)$ (unit)^2 [CSA + ends area]

iv) Volume = $\pi r^2 h$ (unit)^3. [Area of end × Height]



Surface Area & Volume

* Cone (Solid) → Right Circular

Let radius = r, height = h & slant height = l.

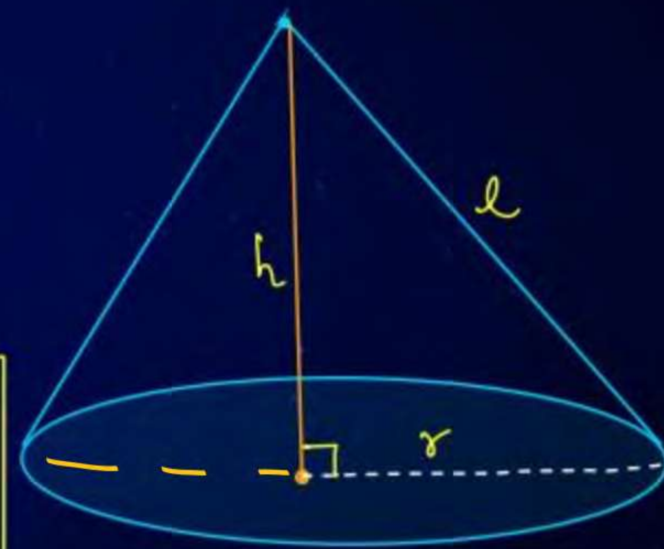
i) $\boxed{h^2 + r^2 = l^2}$

ii) $\boxed{CSA = \pi r l \text{ sq. units}}$

iii) TSA = CSA + base area = $\pi r(l+r)$ sq units

iv) Volume = $\frac{1}{3}\pi r^2 h$ sq units

= $\frac{1}{3}$ × area of base × h



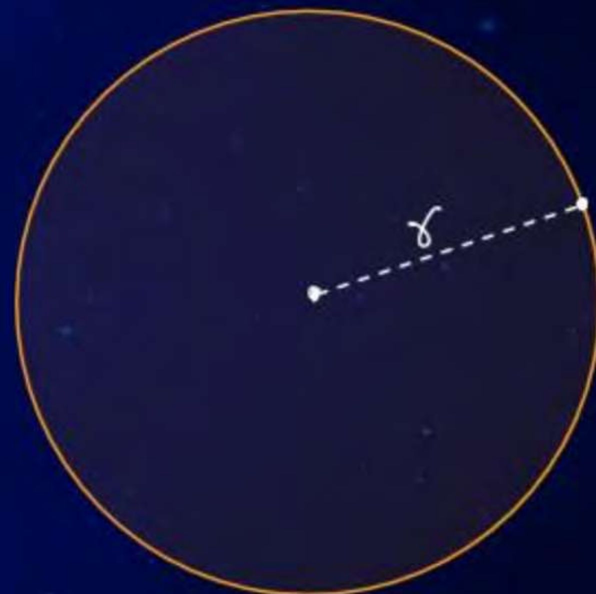
Surface Area & Volume

* Sphere (Solid)

→ Let radius be 'r' units

i) TSA/SA/CSA $= 4\pi r^2$ sq. units

ii) Volume $= \frac{4}{3}\pi r^3$ cu. units



Surface Area & Volume

* Hemisphere :- (Solid)

→ Let radius be 'r' units.

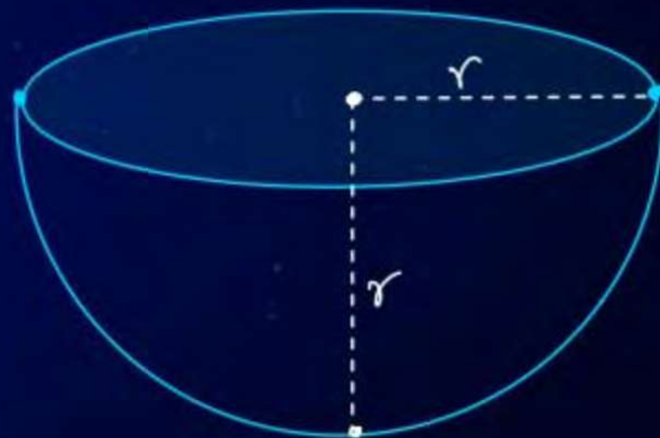
i) $CSA = \boxed{2\pi r^2}$ sq. units

ii) $TSA = CSA + \text{area of base}$

$= 2\pi r^2 + \pi r^2$

$= \boxed{3\pi r^2}$ sq. units

iii) $\text{Volume} = \boxed{\frac{2}{3}\pi r^3}$ cu. units



Surface Area & Volume

→ $\boxed{\text{10 for each step}}$

* Conversion Table

$\boxed{1\,cm^3 = 1\,ml}$

$\times(10)^n$ decreasing ↓

k (kilo)
h (hecto)
da (deca)
meter
d (deci)
c (centi)
m (milli)

↑ $\div(10)^n$ increasing

Surface Area & Volume

i) $6\,mm = \dfrac{6}{(10)^3}\,m$

ii) $5\,dm = \dfrac{5}{(10)^2}\,dam$

iii) $101\,km = 101 \times (10)^5\,cm$

iv) $9\,mm = \_\_\_\_\_\_\,hm$

v) $100\,dm = \_\_\_\_\_\_\,km$

vi) $1\,km = \_\_\_\_\_\_\,mm$

QUESTION

$l^2 = h^2 + r^2$

$l = \sqrt{h^2 + r^2}$

$= \sqrt{\left(\frac{13}{4}\right)^2 + \left(\frac{7}{4}\right)^2}$

$\sqrt{\frac{169 + 49}{16}}$

$\frac{\sqrt{218}}{4}$

218

$h + r = 5$

$h = 5 - r$

$= 5 - \frac{7}{4}$

$\frac{13}{4}$ cm

Rasheed got a playing top (lattu) as his birthday present, which surprisingly had no colour on it. He wanted to colour it with his crayons. The top is shaped like a cone surmounted by a hemisphere (see Fig). The entire top is 5 cm in height and the diameter of the top is 3.5 cm. Find the area he has to colour. (Take π = 22/7)

Solⁿ The area he has to colour = T.S.A of this solid

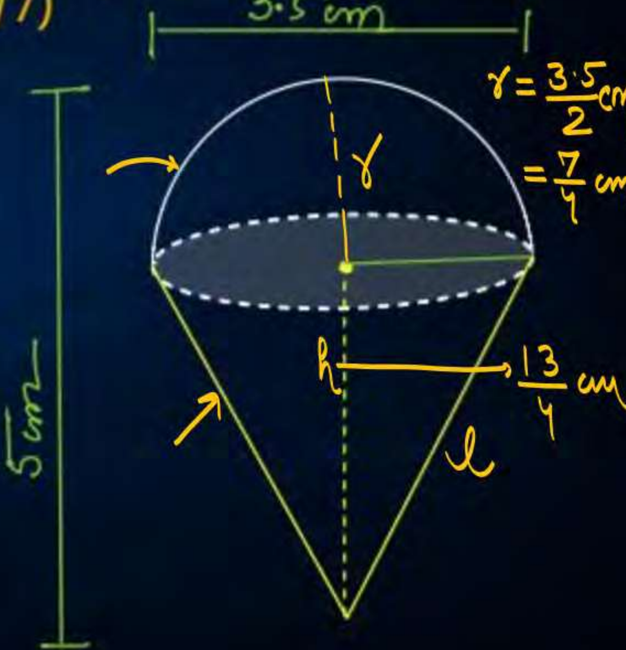
$= C.S.A_h + C.S.A_{co}$

$= 2\pi r^2 + \pi r l$

$= \pi r (2r + l)$

$= \left[\frac{22}{7} \times \frac{7}{4} \times \left(\frac{7}{2} + \frac{\sqrt{218}}{4}\right)\right] \text{cm}^2$

$\approx 39.6 \text{ cm}^2$



QUESTION

$r \to 2.1 = \frac{21}{10}$

63

126×11

13.86

The decorative block shown in Fig. is made of two solids - a cube and a hemisphere. The base of the block is a cube with edge 5 cm, and the hemisphere fixed on the top has a diameter of 4.2 cm. Find the total surface area of the block. (Take π = 22/7)

Sol^n

$r = 2.1\,cm$

$$T.S.A = 5a^2 + C.S.A_h + (a^2 - \pi r^2)$$

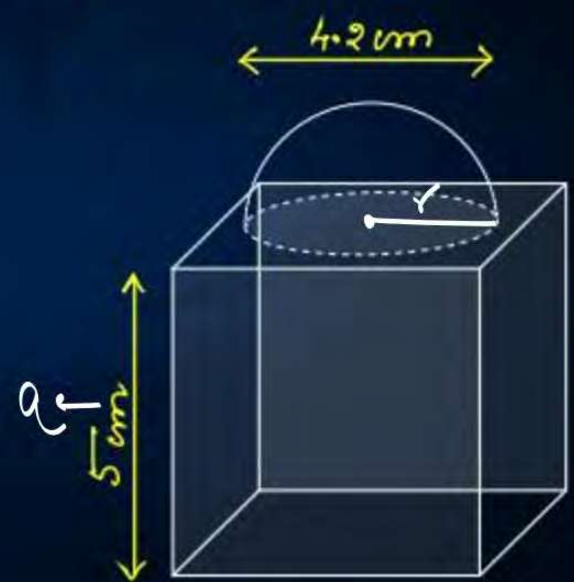
$$= 5a^2 + 2\pi r^2 + a^2 - \pi r^2$$

$$= 6a^2 + \pi r^2$$

$$= \left[6 \times (5)^2 + \left(\frac{\overset{2\times 11}{\cancel{22}}}{\cancel{7}} \times \frac{\overset{3}{\cancel{21}}}{10} \times \frac{21}{10}\right)\right] cm^2$$

$$= (150 + 13.86)\,cm^2$$

$$= \boxed{163.86\,cm^2}$$



QUESTION

$2h_2 + r$

$40 + \frac{3}{2} \rightarrow \frac{83}{2}$

$$\begin{array}{r} 157 \\ \times 83 \\ \hline 471 \\ 1256\times \\ \hline 13031 \end{array} \qquad \begin{array}{r} 13031 \\ \times 3 \\ \hline 39093 \\ \hline 2 \end{array} \quad 195.46.5$$

A wooden toy rocket is in the shape of a cone mounted on a cylinder, as shown in Fig. 12.8. The height of the entire rocket is 26 cm, while the height of the conical part is 6 cm. The base of the conical portion has a diameter of 5 cm, while the base diameter of the cylindrical portion is 3 cm. If the conical portion is to be painted orange and the cylindrical portion yellow, find the area of the rocket painted with each of these colours. (Take π = 3.14)

$\boxed{Sol^n}$

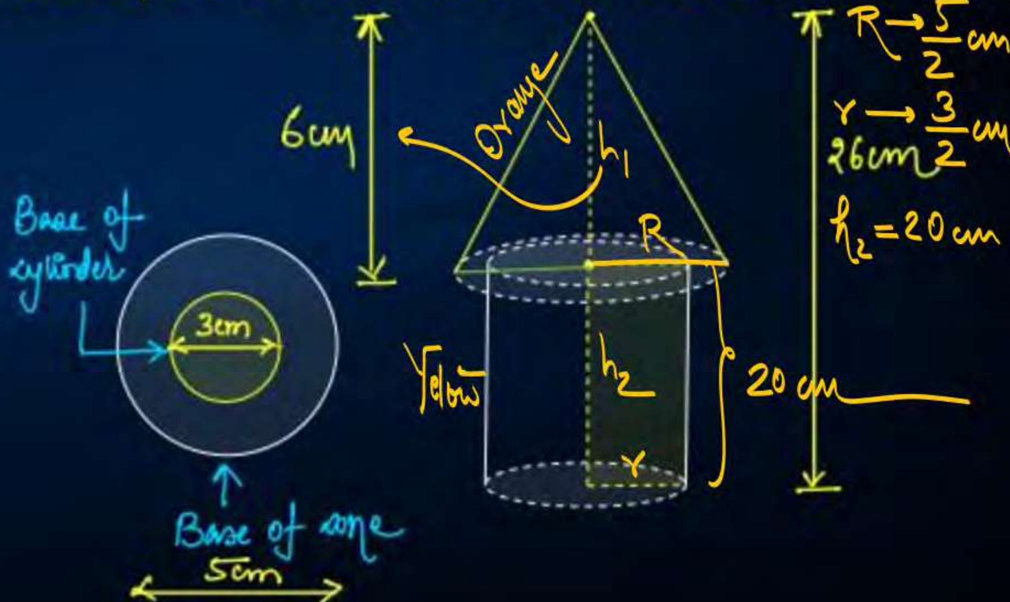
$$\text{Area}_{\text{yellow}} = C.S.A_{cy} + \pi r^2$$

$$= 2\pi r h_2 + \pi r^2$$

$$= \pi r(2h_2 + r)$$

$$= \left(\frac{\cancel{314}^{157}}{100} \times \frac{3}{\cancel{2}} \times \frac{83}{2}\right) cm^2$$

$$= 195.465\ cm^2$$



QUESTION

$l^2 = h_1^2 + R^2$

$l = \sqrt{h_1^2 + R^2}$

$= \sqrt{(6)^2 + \left(\frac{5}{2}\right)^2}$

$\sqrt{36 + \frac{25}{4}}$

$\sqrt{\frac{144 + 25}{4}}$

$\sqrt{\frac{169}{4}}$

$= \frac{13}{2}$ cm $\rightarrow l$

A wooden toy rocket is in the shape of a cone mounted on a cylinder, as shown in Fig. 12.8. The height of the entire rocket is 26 cm, while the height of the conical part is 6 cm. The base of the conical portion has a diameter of 5 cm, while the base diameter of the cylindrical portion is 3 cm. If the conical portion is to be painted orange and the cylindrical portion yellow, find the area of the rocket painted with each of these colours. (Take π = 3.14)

$\boxed{Sol^n}$

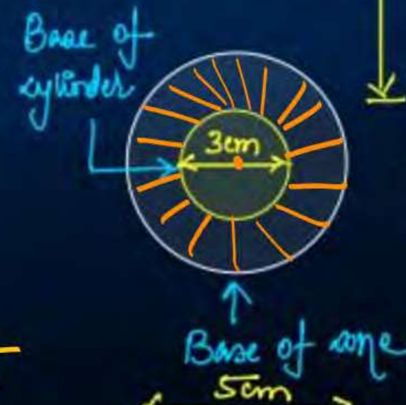
$Ar_{orange} = C.S.A_{co} +$ ring like st. area

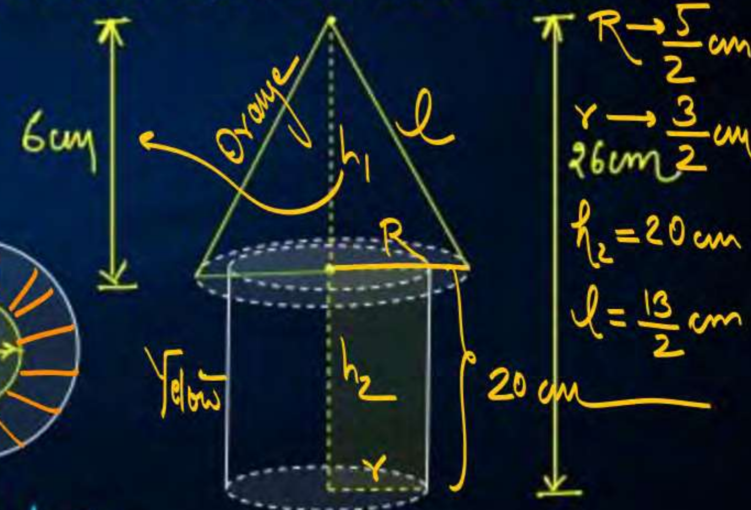
$= \pi R l + (\pi R^2 - \pi r^2)$

$= \pi R l + \pi R^2 - \pi r^2$

$= \pi [R l + R^2 - r^2]$

$= \frac{314}{100} \times \left[\frac{5}{2} \times \frac{13}{2} + \frac{25}{4} - \frac{9}{4}\right] cm^2$





$$= \frac{314}{100}\left[\frac{65}{4} + \frac{25}{4} - \frac{9}{4}\right]$$

$$= \left(\frac{\cancel{314}^{157}}{100} \times \frac{81}{\cancel{4}_{2}}\right) cm^2$$

$$= 63.585\ cm^2$$

$$\begin{array}{r} 157 \\ \times 81 \\ \hline 157 \\ 1256\times \\ \hline \cancel{12717} \\ \hline \cancel{2} \end{array}$$

63.58 .5

QUESTION

Unless stated otherwise, Use $\pi = 22/7$.

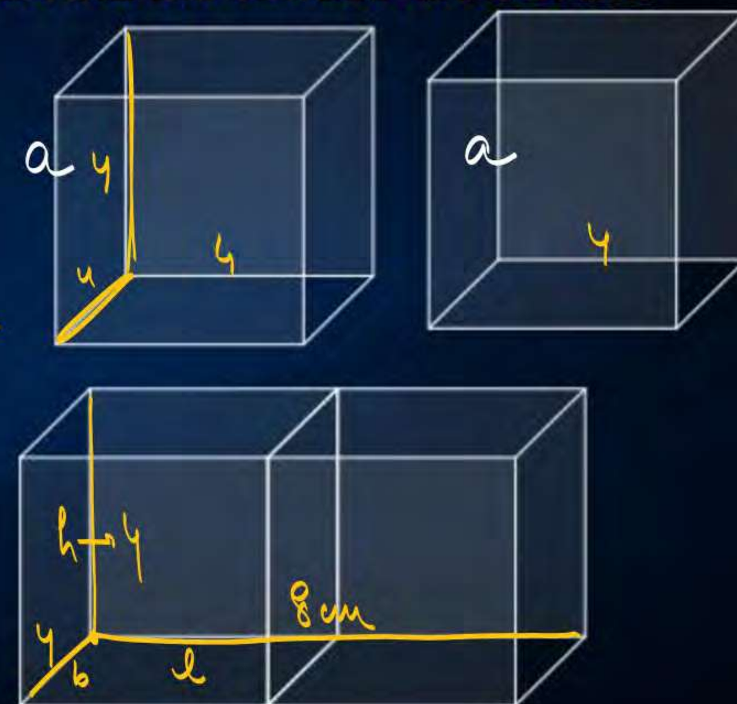
2 cubes each of volume $64\ cm^3$ are joined end to end. Find the surface area of the resulting cuboid.

$$\left.\begin{aligned} l &= 8\ cm \\ b &= 4\ cm \\ h &= 4\ cm \end{aligned}\right\} 2(lb + bh + lh)$$

Sol^n

$$a^3 = 64$$
$$a = (64)^{1/3}$$
$$a = 4\ cm$$

$$T.S.A = 10a^2$$
$$= 10 \times (4)^2$$
$$= 160\ cm^2 \text{ Ans}$$



QUESTION

52×11

$h + r = 13$

$\Rightarrow h = 13 - r$

$h = 6\text{ cm}$

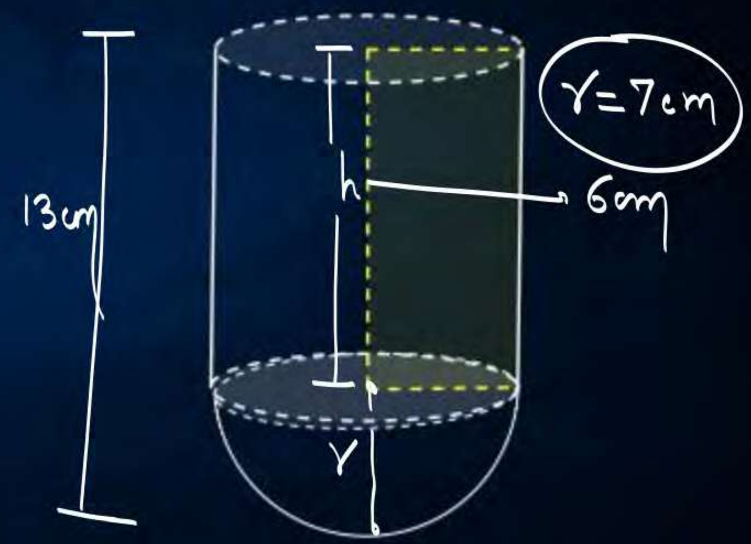
Unless stated otherwise, Use π = 22/7.

A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel.

7 cm

$\boxed{\text{Sol}^n}$

$$\text{Inner SA} = \text{C.S.A}_{cy} + \text{C.S.A}_{h}$$
$$= 2\pi rh + 2\pi r^2$$
$$= 2\pi r(h + r)$$
$$= \left(2 \times \frac{22}{\cancel{7}} \times \cancel{7} \times 13\right)\text{cm}^2$$
$$= 572\text{ cm}^2$$

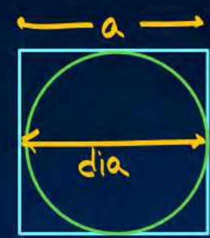


QUESTION

Unless stated otherwise, Use π = 22/7.

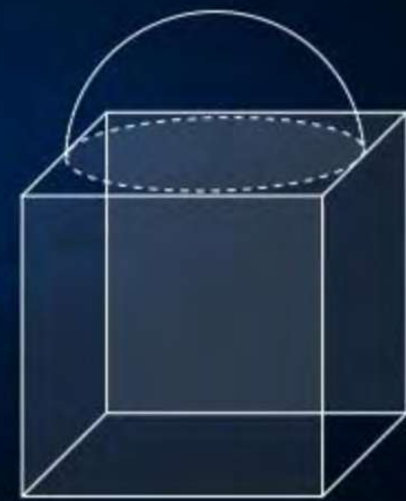
A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have?

$\boxed{\text{Sol}^n}$



$$\text{dia} = a$$
$$= 7\,\text{cm}$$

Ans



$$6 + \frac{\cancel{22}^{11}}{7 \times \cancel{4}_{2}}$$

$$6 + \frac{11}{14} = \frac{84+11}{14} = \frac{95}{14}$$

QUESTION

Unless stated otherwise, Use π = 22/7.

A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter l of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

Sol^n

$$S.A_{\text{remaining solid}} \Rightarrow 5a^2 + CSA_h + (A_{sq} - A_{circle})$$

$r \rightarrow \frac{l}{2}$

$$= 5l^2 + 2\pi r^2 + (l^2 - \pi r^2)$$

$$= 5l^2 + 2\pi r^2 + l^2 - \pi r^2$$

$$= 6l^2 + \pi r^2$$

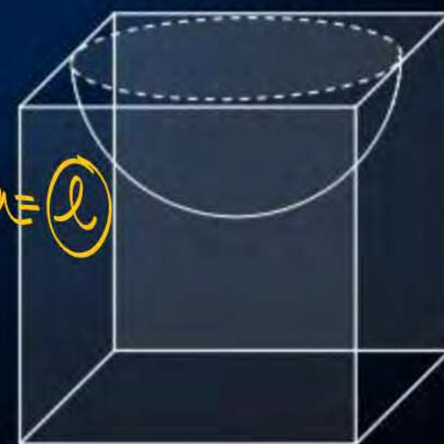
$$= 6l^2 + \pi \times \frac{l^2}{4}$$

$$= 6l^2 + \frac{\pi l^2}{4}$$

$$l^2\left(6 + \frac{\pi}{4}\right)$$

$$\boxed{\frac{95}{14} l^2 \text{ sq. units}}$$

$a = l$



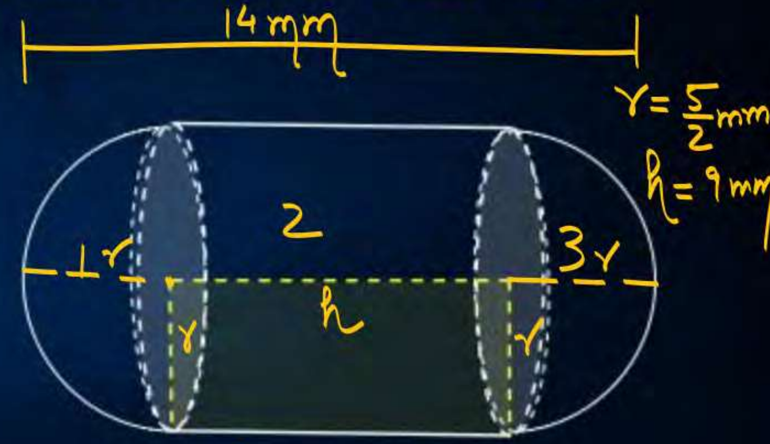
QUESTION

$9+5$

14

Unless stated otherwise, Use π = 22/7.

A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends (see Fig. 12.10). The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area.



Sol^n

$$T.S.A_{capsule} = CSA_{cy} + 2\,CSA_{h}$$

$$= 2\pi r h + 2\times 2\pi r^2$$

$$= \underline{2\pi r}h + 4\pi r^2$$

$$= 2\pi r\,(\underline{h+2r})$$

$$= \left(2\times \frac{\cancel{22}^{2\times 11}}{\cancel{7}}\times\frac{5}{\cancel{2}}\times \cancel{14}^{2}\right)\text{mm}^2$$

$$= 220\ \text{mm}^2$$

$h + 2r = 14$

$h = 14 - 2r$

$= 14 - 5$

$h = 9$ mm

QUESTION

$\frac{2.8 + 4.2}{7}$ $\qquad$ $T.C = R \times Q$

Unless stated otherwise, Use π = 22/7.

A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of ₹500 per m². (Note that the base of the tent will not be covered with canvas.)

Sol^n Total area of the canvas used $= CSA_{co} + CSA_{cy}$

$= \pi r l + 2\pi r h_1$

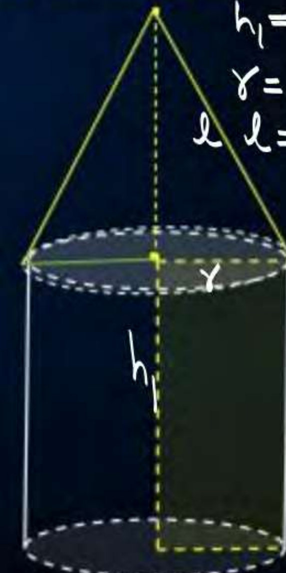
$= \pi r (l + 2h_1)$

$= \left(\frac{22}{\cancel{7}} \times 2 \times \cancel{7}\right) m^2$

$= \boxed{44\, m^2}$

Total cost $= 500 \times 44$

$= 22000/-$



$h_1 = 2.1\, m$

$r = 2\, m$

$l = 2.8\, m$

QUESTION

$l = \sqrt{h^2 + r^2}$

$= \sqrt{\left(\frac{24}{10}\right)^2 + \left(\frac{7}{10}\right)^2}$

$\sqrt{\frac{24^2}{100} + \frac{7^2}{100}}$

$\sqrt{\frac{25^2}{100}} \quad \frac{25}{10} = l \quad \frac{\cancel{25}\,7}{\cancel{10}\,5} = \frac{7}{5}$

Unless stated otherwise, Use π = 22/7.

From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm².

$h = 2.4\,cm$

$r = \frac{7}{10}\,cm$

Soln

$T.S.A_{\text{rem solid}} \longrightarrow C.S.A_{cy} + C.S.A_{co} + \pi r^2$

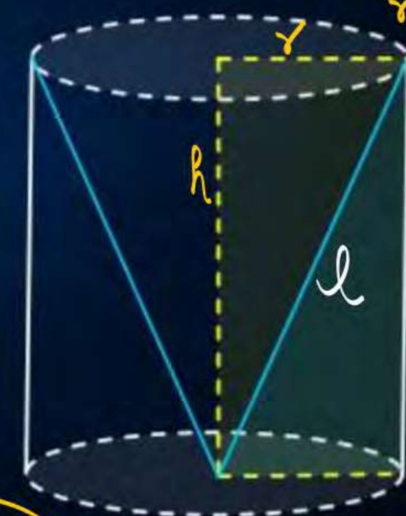
$= 2\pi r h + \pi r l + \pi r^2$

$= \pi r (2h + l + r)$

$= \frac{22}{7} \times \frac{7}{10} \times (4.8 + 0.7 + 2.5)$

$= \left(\frac{22}{\cancel{7}} \times \frac{\cancel{7}}{10} \times 8\right) cm^2 \longrightarrow \frac{176}{10}$

$= 17.6\ cm^2$



QUESTION

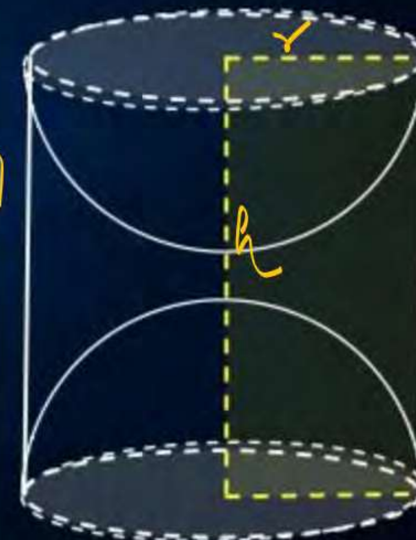
$10+7$

Unless stated otherwise, Use π = 22/7.
A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in Fig. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the total surface area of the article.

$\boxed{Sol^n}$

$$T.S.A = C.S.A_{cy} + 2 \cdot C.S.A_{h}$$
$$= 2\pi rh + 2\times 2\pi r^2$$
$$= 2\pi rh + 4\pi r^2$$
$$= 2\pi r(h+2r)$$
$$= \left(\cancel{2}\times \frac{\overset{2\times 11}{22}}{\cancel{7}} \times \frac{\cancel{7}}{\cancel{2}} \times 17\right) cm^2$$
$$= 34\times 11 \longrightarrow 374\ cm^2$$

$h = 10\ cm$
$r = \frac{7}{2}\ cm$



QUESTION

Capacity $\longrightarrow$ Vol

$1\,cm^3 = 1\,ml$

$\begin{array}{r} 157 \\ \times 25 \\ \hline 785 \\ 314\times \\ \hline 3925 \end{array}$

$\dfrac{\cancel{3925}}{\cancel{2}}$ 1962.5

A juice seller was serving his customers using glasses as shown in Fig. 12.13. The inner diameter of the cylindrical glass was 5 cm, but the bottom of the glass had a hemispherical raised portion which reduced the capacity of the glass. If the height of a glass was 10 cm, find the apparent capacity of the glass and its actual capacity. (Use π = 3.14.)

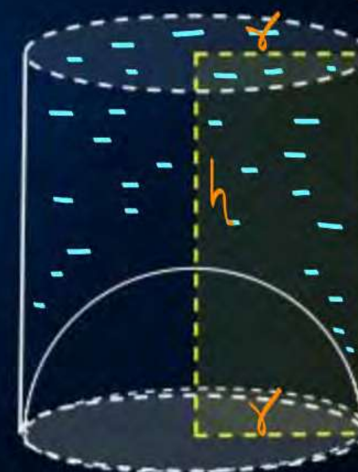
Solⁿ

Apparent Capacity = Vol cyl

$= \pi r^2 h$

$= \left(\frac{\cancel{314}^{157}}{\cancel{100}} \times \frac{5}{\cancel{2}} \times \frac{5}{2} \times \cancel{10}\right) cm^3$

$= 196.25\,cm^3$



$r \longrightarrow \frac{5}{2}\,cm$

$h = 10\,cm$

Actual Capacity $\longrightarrow V_{olcy} - V_h$

$$= \pi r^2 h - \frac{2}{3}\pi r^3$$

$$= \pi r^2 \left(h - \frac{2}{3} r\right)$$

$$= \left(\frac{\overset{157}{\cancel{314}}}{100} \times \frac{5}{\cancel{2}} \times \frac{5}{2} \times \frac{25}{3}\right) cm^3$$

$$\approx \boxed{}$$

$h - \frac{2}{3} r$

$10 - \frac{\cancel{2}}{3} \times \frac{5}{\cancel{2}}$

$10 - \frac{5}{3}$

$\frac{25}{3}$

QUESTION

$2+4$

$$\begin{array}{r} 1256 \\ \times\ 2 \\ \hline 2512 \end{array}$$

A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of the cone is 2 cm and the diameter of the base is 4 cm. Determine the volume of the toy. If a right circular cylinder circumscribes the toy, find the difference of the volumes of the cylinder and the toy. (Take $\pi = 3.14$)

$\boxed{\text{Sol}^n}$

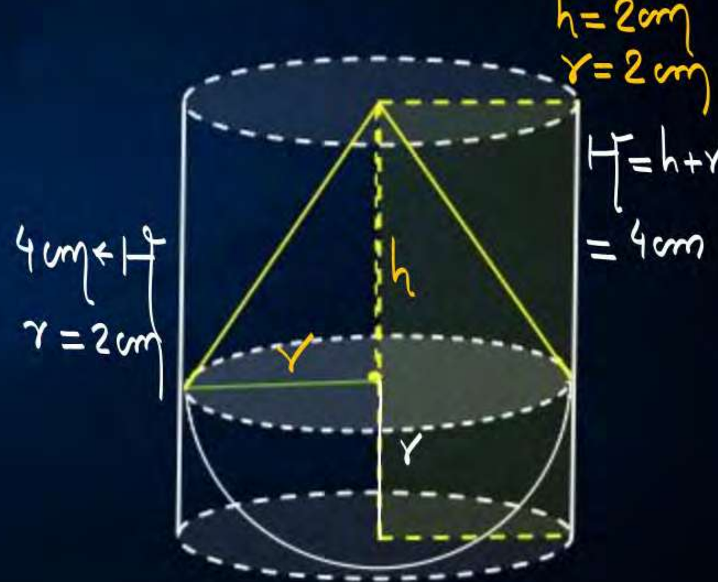
$$V_{Toy} = V_{co} + V_h$$

$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \frac{\pi r^2}{3}\left(h + 2r\right)$$

$$= \left(\frac{314}{100} \times \frac{4}{3} \times 6\right) cm^3$$

$$= 25.12\, cm^3$$



Reqd. Vol $= V_{cy} - V_{Toy}$

$= \pi r^2 H - 25.12$

$= \frac{314}{100} \times 4 \times 4 - 25.12$

$= 50.24 - 25.12$

$= 25.12 \text{ cm}^3$

$$\begin{array}{r} 314 \\ 16 \\ \hline 1884 \\ 314\times \\ \hline 50.24 \end{array}$$

QUESTION

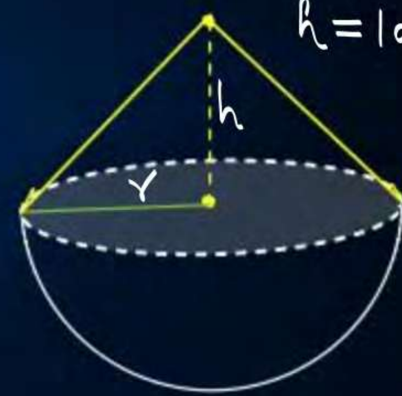
Unless stated otherwise, Use π = 22/7.

A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of π.

Soln

$r = 1\,cm$

$h = 1\,cm$



$$V_{solid} = V_{co} + V_h$$

$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \frac{1}{3}\pi \times 1 \times 1 + \frac{2}{3}\pi \times 1$$

$$= \frac{\pi}{3} + \frac{2\pi}{3}$$

$$= \pi\ cm^3$$

QUESTION

$$8+\frac{2}{3}\times 2$$
$$8+\frac{4}{3}=\frac{28}{3}$$

$$h_1+2h_2=12$$
$$h_1=12-2h_2$$
$$=12-(2\times 2)$$
$$12-4$$
$$8\text{ cm}$$

Unless stated otherwise, Use π = 22/7.

Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model to be nearly the same.)

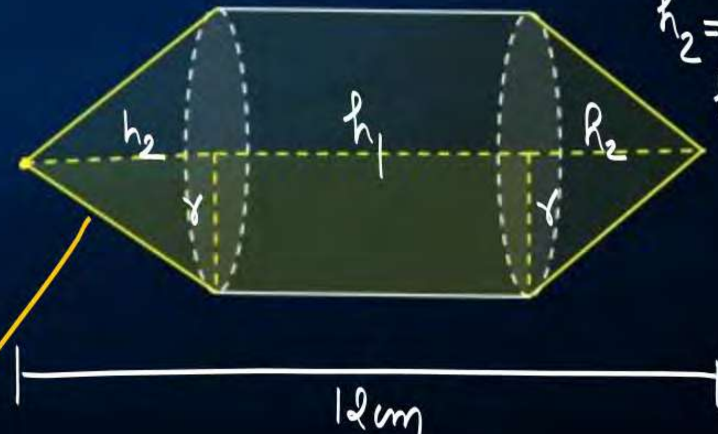
$r=\frac{3}{2}$ cm

$h_2=2$ cm

$h_1=8$ cm

Sol^n

$$V_{air}=V_{cy}+2V_{co}$$
$$=\pi r^2 h_1+2\times\frac{1}{3}\pi r^2 h_2$$
$$=\pi r^2\left(h_1+\frac{2}{3}h_2\right)$$
$$=\left(\frac{22}{7}\times\frac{9}{4}\times\frac{28}{3}\right)\text{cm}^3 \longrightarrow 66\text{ cm}^3$$



QUESTION

$h + 2r = 5$

$h = 5 - 2.8$

$= 2.2$

$h \longrightarrow \frac{\cancel{22}^{11}}{\cancel{10}_{5}} \rightarrow \frac{11}{5}$ cm

$\frac{\cancel{14}^{7}}{\cancel{10}_{5}}$

Unless stated otherwise, Use π = 22/7.

A gulab jamun, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm (see Fig.).

$r = 1.4$ cm $= \frac{7}{5}$ cm

$h = \frac{11}{5}$ cm

Sol^n

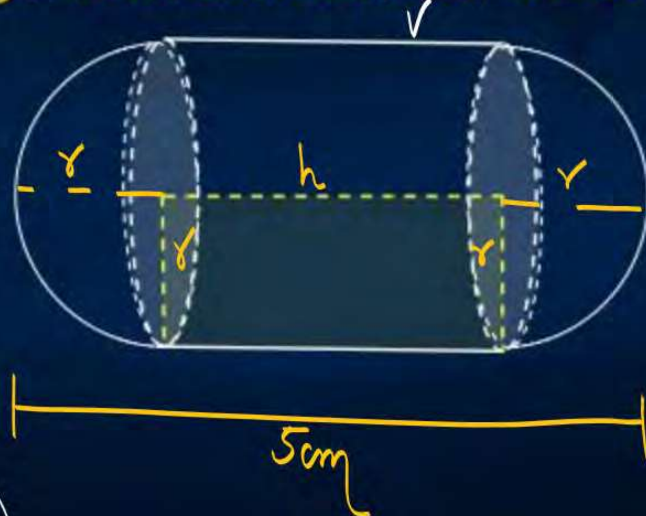
30 % of V

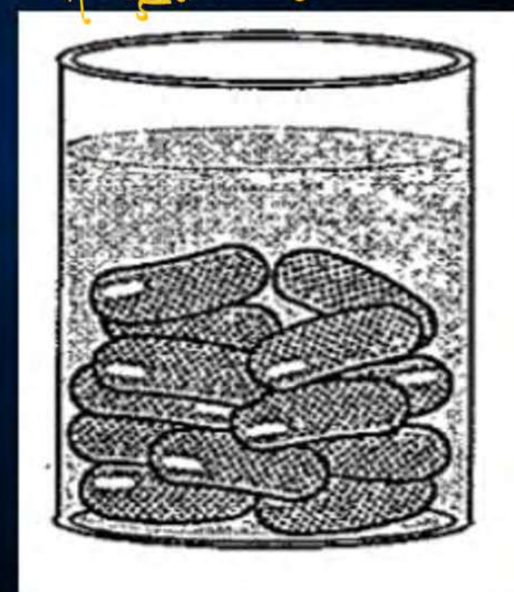
$\frac{3\cancel{0}}{10\cancel{0}} \times V$

$\frac{3V}{10} \rightarrow$ sugar syrup

Total vol of syrup $= 45 \times \frac{3V}{10}$

$= \frac{45}{10} \times 3 \times \left(V_{cy} + 2V_{h}\right)$





$h + \frac{4}{3}r$

$\frac{11}{5} + \frac{4}{3} \times \frac{7}{5} = \frac{11}{5} + \frac{28}{15} \rightarrow \frac{33 + 28}{15} = \frac{61}{15}$

$= \frac{45 \times 3}{10} \times \left(\pi r^2 h + 2 \times \frac{2}{3} \pi r^3 \right)$

$= \frac{45 \times 3}{10} \pi r^2 \left(h + \frac{4}{3} r \right)$

$= \left(\frac{\cancel{45}^{9} \times \cancel{3}}{10} \times \frac{22}{\cancel{7}} \times \frac{\cancel{7}}{\cancel{5}} \times \frac{7}{5} \times \frac{61}{\cancel{15}} \times 4 \right) \text{cm}^3$

2×11

$= \frac{9 \times 22 \times 7 \times 61 \times 4}{1000} = 338.184 \text{ cm}^3$

$$\begin{array}{r} 72 \\ \times\ 7 \\ \hline 504 \\ \times\ 61 \\ \hline 504 \\ 3024\times \\ \hline 30744 \\ \times\ 11 \\ \hline 338.184 \end{array}$$

QUESTION

Unless stated otherwise, Use $\pi = 22/7$.

A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm. Find the volume of wood in the entire stand (see Fig.).

Sol^n