Quadratic Equations

Quadratic Equations

$\to$ deg = 2

$$\boxed{ax^2 + bx + c = 0} \quad a \neq 0 \quad \alpha, \beta$$

$$\boxed{\alpha, \beta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

$D = b^2 - 4ac$

= discriminant

Q.E $\to$ 2 Root (Real / Img)

Quadratic Equations

$$ax^2+bx+c=0 \begin{cases} \alpha \\ \beta \end{cases}$$

① $\boxed{\alpha+\beta = \frac{-b}{a}}$

② $\boxed{\alpha\beta = \frac{c}{a}}$

Factorize

$x^2 \rightarrow$ Coeff $= 1$

✓ $1x^2 - 3x + 2 = 0$ (#NVStyle)

✓ $x^2 - 2x - x + 2 = 0$

✓ $x(x-2) - 1(x-2) = 0$

✓ $(x-2)(x-1) = 0$

✓ $x = 1, 2 \rightarrow$ Roots | Zeros

Quadratic Equations

① $1x^2 - 3x + 2 = 0$

$(x-2)(x-1) = 0$

② $1x^2 - 5x + 4$

$= (x-1)(x-4)$

③ $1x^2 - x - 6$

$(x-3)(x+2)$

④ $1x^2 - x - 20$

$(x-5)(x+4)$

Quadratic Equations

$$ax^2+bx+c=0 \left\langle \begin{array}{l} \alpha \\ \beta \end{array} \right.$$

- $\alpha+\beta = \frac{-b}{a}$ —— ①
- $\alpha\beta = \frac{c}{a}$ —— ②

① $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

② $(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta$

③ $\alpha^3+\beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$

④ $\alpha^4+\beta^4 = (\alpha^2+\beta^2)^2 - 2\alpha^2\beta^2$

Quadratic Equations

$$(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta$$

$$= \left(\frac{-b}{a}\right)^2 - 4\left(\frac{c}{a}\right)$$

$$= \frac{b^2}{a^2} - \frac{4ac}{a^2}$$

$$(\alpha-\beta)^2 = \frac{b^2-4ac}{a^2}$$

$$|\alpha-\beta| = \frac{\sqrt{b^2-4ac}}{|a|} = \frac{\sqrt{D}}{|a|}$$

$$\star \boxed{|\alpha-\beta| = \frac{\sqrt{D}}{|a|}}$$

QE $\begin{cases} 5 \\ -2 \end{cases}$

$$\boxed{x^2 - (\text{Sum})x + \text{Product} = 0}$$

$$\underline{x^2 - 3x - 10 = 0}$$

Q

Let f(x) be a quadratic polynomial such that f(-2) + f(3) = 0. If one of the roots of f(x) = 0 is -1, then the sum of the roots of f(x) = 0 is equal to :

(JEE 2022)

#chindi

A. 11/3

B. 7/3

C. 13/3

D. 14/3

① $\boxed{f(-2)+f(3)=0}$

② $f(x) = K(x+1)(x-\alpha)$

$\cancel{K}(-1)(-2-\alpha) + \cancel{K}(4)(3-\alpha) = 0$

$2+\alpha+12-4\alpha = 0$

$\therefore \alpha = \frac{14}{3}$

★ Sum of Roots

$\Rightarrow -1+\alpha$

$\Rightarrow -1+\frac{14}{3}$

$\Rightarrow \frac{11}{3}$

Q

Let α, β be the roots of the equation $\boxed{x^2 - \sqrt{2}x + \sqrt{6} = 0}$ and $\frac{1}{\alpha^2}+1, \frac{1}{\beta^2}+1$ be the roots of the equation $\boxed{x^2 + ax + b = 0.}$ Then the roots of the equation $x^2 - (a+b-2)x + (a+b+2) = 0$ are :

(JEE M 2022)

A. non-real complex numbers

B. real and both negative

C. real and both positive

D. real and exactly one of them is positive

$$x^2 - \sqrt{2}x + \sqrt{6} = 0 \begin{cases} \alpha \\ \beta \end{cases} \qquad \boxed{\alpha\beta = \sqrt{6}}$$

$$x^2 + ax + b = 0 \begin{cases} \frac{1}{\alpha^2} + 1 \\ \frac{1}{\beta^2} + 1 \end{cases}$$

$$x^2 - \left(-\frac{5}{6} - 2\right)x + \left(-\frac{5}{6} + 2\right) = 0 \begin{cases} ? \\ ? \end{cases}$$

$$6x^2 + 17x + 7 = 0 \qquad 7 \times 3 \times 2$$

$$6x^2 + 14x + 3x + 7 = 0$$

$$(2x+1)(3x+7) = 0$$

$$\boxed{x = \frac{-1}{2}, \frac{-7}{3}}$$

$$-\cancel{\frac{1}{\alpha^2}} - \cancel{\frac{1}{\beta^2}} - 2 = a$$

$$\left(\frac{1}{\alpha^2} + 1\right)\left(\frac{1}{\beta^2} + 1\right) = b$$

$$\frac{1}{(\alpha\beta)^2} + 1 - 2 = a + b$$

$$\frac{1}{6} - 1 = a + b$$

$$\therefore \boxed{a + b = \frac{-5}{6}}$$

Q

The minimum value of the sum of the squares of the roots of

$x^2 + (3-a)x + 1 = 2a$ is :

A. 4
B. 5
C. 6
D. 8

(JEE M 2022)

$$x^2 + (3-a)x + (1-2a) = 0 \quad \alpha, \beta$$

$$(\alpha^2+\beta^2)_{mini} = (\alpha+\beta)^2 - 2\alpha\beta$$

$$= (a-3)^2 - 2(1-2a)$$

$$= a^2 - 6a + 9 - 2 + 4a$$

$$= \boxed{a^2 - 2a + 1} + 6$$

$$= (a-1)^2 + 6$$

$\therefore a = 1$

(C for chindi)

Q

If the sum of the squares of the reciprocals of the roots α and β of the equation $3x^2 + \lambda x - 1 = 0$ is 15, then $6(\alpha^3 + \beta^3)^2$ is equal to:

(JEE M 2022)

A. 18
B. 24
C. 36
D. 96

$3x^2 + \lambda x - 1 = 0 \begin{cases} \alpha \\ \beta \end{cases}$

$\alpha^3 + \beta^3 = ?$

$\alpha + \beta = \frac{-\lambda}{3} = (\pm 1)$

$\boxed{\alpha\beta = \frac{-1}{3}}$

$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = 15$

$\frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2} = 15$

$\frac{\frac{\lambda^2}{9} - 2\left(\frac{-1}{3}\right)}{\left(\frac{1}{9}\right)} = 15$

$\rightarrow \frac{\lambda^2}{\cancel{9}} + \frac{6}{\cancel{9}} = \frac{15}{\cancel{9}}$

$\lambda^2 = 9$

$\boxed{\lambda = \pm 3}$

$$\underbrace{\alpha^3 + \beta^3} = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$$

$$= (\pm 1)^3 - \cancel{3}\left(\frac{-1}{\cancel{3}}\right)(\pm 1)$$

$$= (\pm 1) + (\pm 1)$$

$$= 2, 0, -2$$

$$6\left(\alpha^3+\beta^3\right)^2$$

$$= 6(\pm 2)^2$$

$$= 24$$

Q

Suppose a, b denote the distinct real roots of the quadratic polynomial $\boxed{x^2 + 20x - 2020}$ and suppose c, d denote the distinct complex roots of the quadratic polynomial $x^2 - 20x + 2020$. Then the value of $\boxed{ac(a - c) + ad(a - d) + bc(b - c) + bd(b - d)}$ is

[Adv. 2020]

A. 0

B. 8000

C. 8080

D. 16000

$x^2 + 20x - 2020 = 0 \left\langle \begin{matrix} a \\ b \end{matrix} \right.$

$$\boxed{\begin{aligned} a+b &= -20 \\ ab &= -2020 \end{aligned}}$$

$x^2 - 20x + 2020 = 0 \left\langle \begin{matrix} c \\ d \end{matrix} \right.$

$$\boxed{\begin{aligned} c+d &= 20 \\ cd &= 2020 \end{aligned}}$$

Unacademy Atoms

$$\text{Req} \Rightarrow a^2c - ac^2 + a^2d - ad^2 + b^2c - bc^2 + b^2d - bd^2$$

$$\Rightarrow a^2(c+d) + b^2(c+d) - c^2(a+b) - d^2(a+b)$$

$$\Rightarrow a^2(20) + b^2(20) + c^2(20) + d^2(20)$$

$$\Rightarrow 20\left(a^2+b^2+c^2+d^2\right)$$

$$\Rightarrow 20\left((a+b)^2 - 2ab + (c+d)^2 - 2cd\right)$$

$$\Rightarrow 20\left(400 + 2(\cancel{20}20) + 400 - 2(\cancel{20}20)\right)$$

$$\Rightarrow \underline{16000}$$

Newton's Method

Newton's Method: Powers of Roots

Let α and β, are the roots of the quadratic equation $ax^2 + bx + c = 0$, and $S_n = \alpha^n \pm \beta^n$ then $aS_n + bS_{n-1} + cS_{n-2} = 0$

$$\boxed{a\boxed{x^2} + b\boxed{x} + c(1) = 0} \left\langle \begin{matrix} \alpha \\ \beta \end{matrix} \right.$$

$$\boxed{S_n = \alpha^n \pm \beta^n}$$

$$\star \; \boxed{aS_n + bS_{n-1} + cS_{n-2} = 0}$$

$$\boxed{\begin{array}{l} x^2 \to S_n \\ x \to S_{n-1} \\ 1 \to S_{n-2} \end{array}}$$

Unacademy Atoms

Q

Let α and β be the roots of $x^2 - 6x - 2 = 0$, with $\alpha > \beta$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\boxed{\dfrac{a_{10} - 2a_8}{2a_9}}$

JEE Adv. 2011 & JEE Main 2015

(JEE Main 2021)

A. 1

B. 2

C. 3 ✓

D. 4

$x^2 - 6x - 2 = 0 \begin{cases} \alpha \\ \beta \end{cases}$

$\star\ a_n = \alpha^n - \beta^n$ $\boxed{\#\text{Newton}}$

$a_n - 6a_{n-1} - 2a_{n-2} = 0$

$\boxed{n=10}$

$a_{10} - 6a_9 - 2a_8 = 0$

$\boxed{\dfrac{a_{10} - 2a_8}{2a_9}} = \dfrac{6}{2} = 3$

Q

Let α and β be the roots of the equation, $5x^2 + 6x - 2 = 0$. If $S_n = \alpha^n + \beta^n$, n = 1, 2, 3, ..., then :

JEE MAIN 2020

A. $6S_6 + 5S_5 = 2S_4$

B. $6S_6 + 5S_5 + 2S_4 = 0$

C. $5S_6 + 6S_5 = 2S_4$ ✓

D. $5S_6 + 6S_5 + 2S_4 = 0$

$5S_n + 6S_{n-1} - 2S_{n-2} = 0$

$n = 6$

$5S_6 + 6S_5 - 2S_4 = 0$

Q

For a natural number n, let $\boxed{a_n = 19^n - 12^n}$. Then, the value of $\dfrac{\boxed{31\alpha_9 - \alpha_{10}}}{57\alpha_8}$ is

(JEE M 2022)

$a_n = \alpha^n - \beta^n$

$\therefore \alpha = 19$, $\beta = 12$ } Roots

$x^2 - S x + P = 0$

$x^2 - 31x + 228 = 0$ ⟨ 12, 19

$a_{10} - 31a_9 + 228a_8 = 0$

✓ 4 $= \dfrac{228\cancel{a_8}}{57\cancel{a_8}} = \dfrac{31a_9 - a_{10}}{57a_8}$

Q Let α, β (α > β) be the roots of the quadratic equation $x^2 - x - 4 = 0$. If $\boxed{P_n = \alpha^n - \beta^n}$, $n \in \mathbb{N}$, then

$$\Rightarrow \frac{P_{15}P_{16} - P_{14}P_{16} - P_{15}^2 + P_{14}P_{15}}{P_{13}P_{14}}$$ is equal to ______. (2022)

★ $P_n - P_{n-1} - 4P_{n-2} = 0$

$P_{15} - P_{14} = 4P_{13}$

$P_{16} - P_{15} = 4P_{14}$

$$\Rightarrow \frac{P_{16}(P_{15} - P_{14}) - P_{15}(P_{15} - P_{14})}{P_{13}P_{14}}$$

$$\Rightarrow \frac{(P_{15} - P_{14})(P_{16} - P_{15})}{P_{13}P_{14}} \Rightarrow \frac{(4\cancel{P_{13}})(4\cancel{P_{14}})}{\cancel{P_{13}}\cancel{P_{14}}} \Rightarrow 16$$

Identity

Identity :

Let $ax^2 + bx + c = 0$ be a quadratic equation. Now, if this quadratic equation has more than two distinct roots then it becomes an identity and in this case $a = b = c = 0$.

$$(x+1)^2 = x^2 + 2x + 1$$ — ∞ Root

$$\boxed{ax^2 + bx + c = 0} \Longrightarrow \text{identity if } \boxed{a = b = c = 0}$$

$$0x^2 + 0x + 0 = 0$$ — ∞ Root

For what values of p, the equation

$(p + 2)(p - 1)x^2 + (p - 1)(2p + 1)x + p^2 - 1 = 0$ has more than two roots.

QE = 0 — more than 2 Root

$a = b = c = 0$

$$\left.\begin{aligned}(p+2)(p-1) &= 0\\(p-1)(2p+1) &= 0\\p^2-1 &= 0\end{aligned}\right\}\quad p = 1$$

Unacademy Atoms

Nature of Roots

Nature of Roots :

i. If $D > 0 \Rightarrow$ roots are real and distinct.
ii. If $D = 0 \Rightarrow$ roots are equal.
iii. If $D < 0 \Rightarrow$ roots are imaginary.

1. If coefficients of the quadratic equation are rational then its irrational roots always occur in pair. If $p + \sqrt{q}$ is one of the roots then other root will be $p - \sqrt{q}$

2. If coefficients of the quadratic equation are real then its imaginary roots always occur in complex conjugate pair. If $p + iq$ is one of the roots then other root will be $p - iq$

3. If coefficients of the quadratic equation are real and D = perfect square then the roots are rational

Nature of Roots :

$a x^2 + b x + c = 0$

$\frac{-b \pm \sqrt{25}}{2a} = \text{Roots} = \frac{-b \pm 5}{2a}$

☆ $a, b, c \to$ Rational	Irrational $\to$ pair $\quad p+\sqrt{q}, \; p-\sqrt{q}$
☆ $a, b, c \to$ Real	Imag $\to$ pair $\quad p+iq, \; p-iq$
☆ $\boxed{D = \text{p. sq.}}$ $a, b, c \Rightarrow$ Rational	Roots = Rational

Unacademy Atoms

Q

The quadratic equation with rational coefficients whose one root is $2 + \sqrt{3}$ is

A. $x^2 - 4x + 1 = 0$

B. $x^2 + 4x + 1 = 0$

C. $x^2 + 4x - 1 = 0$

D. $x^2 + 2x + 1 = 0$

$\alpha = (2+\sqrt{3})$

$\beta = (2-\sqrt{3})$

Sum = 4

Product = 1

$x^2 - 4x + 1 = 0$

The roots of the quadratic equation $x^2 - 2(a + b)x + 2(a^2 + b^2) = 0$ are

A. Rational and different

B. Rational and ~~equation~~ equal

C. Irrational and different

D. Imaginary and different

Concept

Rational Root $\boxed{D = \text{p. sq}}$

Equal $\boxed{D = 0}$ Real

Imag. $\boxed{D < 0}$

$$D = 4(a+b)^2 - 4(2)(a^2+b^2)$$

$$= 4\{a^2+b^2+2ab-2a^2-2b^2\}$$

$$= 4\{-a^2-b^2+2ab\}$$

$$= -4\{a^2+b^2-2ab\}$$

$$\therefore \boxed{D = -4(a-b)^2}$$

$$D < 0$$

The $\boxed{\text{number}}$ of all possible positive integral values of α for which the roots of the quadratic equation, $6x^2 - 11x + \alpha = 0$ are rational numbers is :

JEE M 2019

A. 3 ✓

B. 2

C. 4

D. 5

$D = \text{p.sq} \Rightarrow \text{Rational}$

$D = 121 - 4(6)(\alpha)$

$\therefore D = 121 - 24\alpha$

p.sq.

$= 121 - 24 = 97$ ✗

$= 121 - 24(2) = 73$ ✗

$= 121 - 24(3) = 49$ ✓

$= 121 - 24(4) = 25$ ✓

$= 121 - 24(5) = 1$ ✓

$= 121 - 24(6) = \ominus$

$(7) = \ominus$

$\boxed{\alpha = 3, 4, 5}$

$\Downarrow$

Rational

$\Downarrow$

$D = \text{p.sq.}$

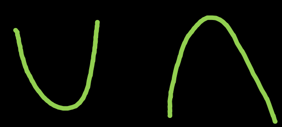
Graph of Quadratic

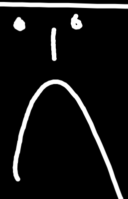
Quadratic Expression and its Graph :

Quad. Exp $\Rightarrow$ parabola

$$\boxed{y = ax^2 + bx + c}$$

$y = f(x)$

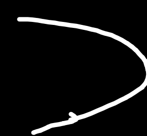


$a > 0$	$a < 0$
	

$$\boxed{x = ay^2 + by + c}$$

* $a > 0$

* $a < 0$



Quadratic Expression and its Graph :

Unacademy Atoms

① $a>0$ $\cup$ $\quad a<0$ $\cap$

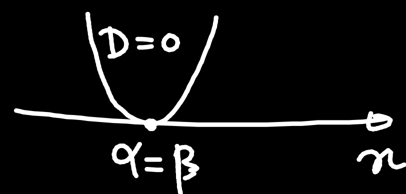
$$y = a\cancel{x^2} + b\cancel{x} + c$$

$x=0 \quad y=c$

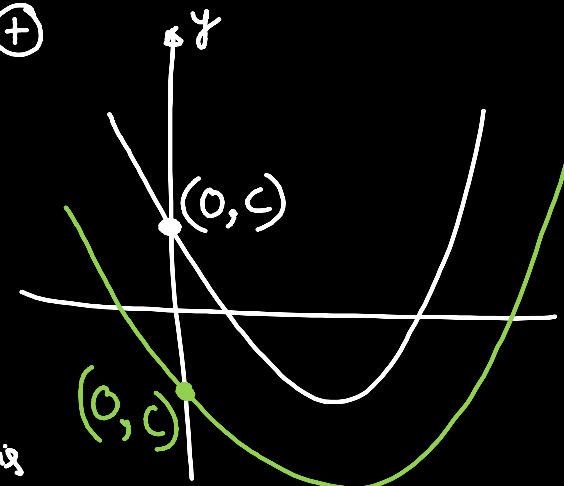
② $D>0$ 2 Real

$c = \oplus$

$D=0$ Real & Equal

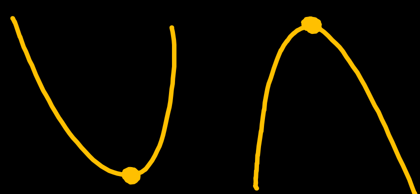


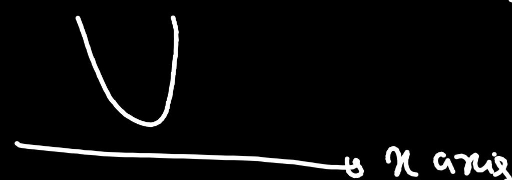
$D<0$ No Real



$c = \ominus$

③





$$\text{Vertex} \equiv \left(\frac{-b}{2a}, \frac{-D}{4a}\right)$$

Quadratic Expression and its Graph :

Unacademy Atoms

Quadratic Expression and its Graph :

Unacademy Atoms

Q

Draw the graph of $y = x^2 - 7x + 12$

$y = \left(\frac{7}{2}\right)^2 - 7\left(\frac{7}{2}\right) + 12$

$= \frac{-1}{4}$

S-1 $a > 0$ upward

S-2 $D = 49 - 4(12) = 1$

$D > 0$ 2 Root

$x^2 - 7x + 12 = 0$

$(x-3)(x-4) = 0$

$x = 3, 4$

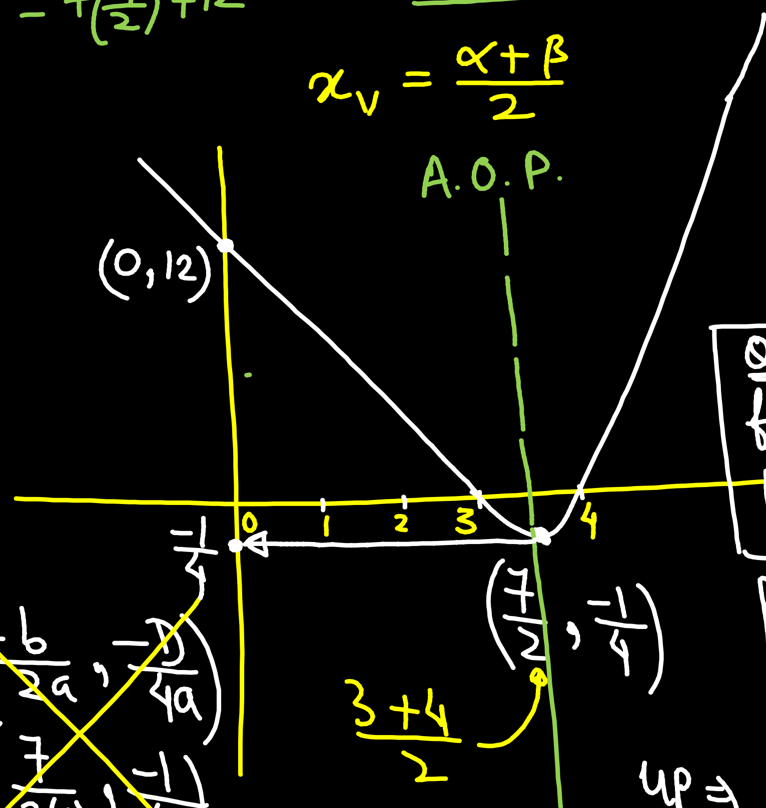
③ $V \equiv \cancel{\left(\frac{-b}{2a}, \frac{-D}{4a}\right)}$

$\equiv \cancel{\left(\frac{7}{2(1)}, \frac{-1}{4}\right)}$

Axis of Sym.

$x_V = \frac{\alpha + \beta}{2}$

A.O.P.



Qns

$f(x) = x^2 - 7x + 12$

Range – ?

$\left[\frac{-1}{4}, \infty\right)$

up $\Rightarrow \left[\frac{-D}{4a}, \infty\right)$

up : Range $\Rightarrow \left[\frac{-D}{4a}, \infty\right)$

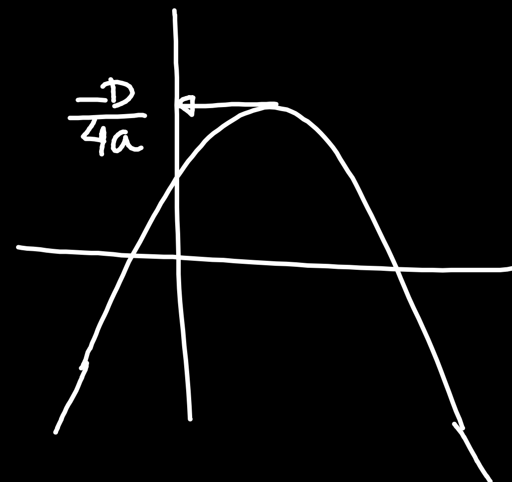
down: Range $\Rightarrow \left(-\infty, \frac{-D}{4a}\right]$

$y_v = ?$

$y = x^2 - 7x + 12$

$\therefore \frac{dy}{dx} = 2x - 7 = 0$

$x_v = 7/2$



Draw the graph of $y = -x^2 + x - 1$

Range : $\left(-\infty, \frac{-3}{4}\right]$

S-1 $a < 0$ $\cap$

S-2 $D = 1^2 - 4(-1)(-1)$

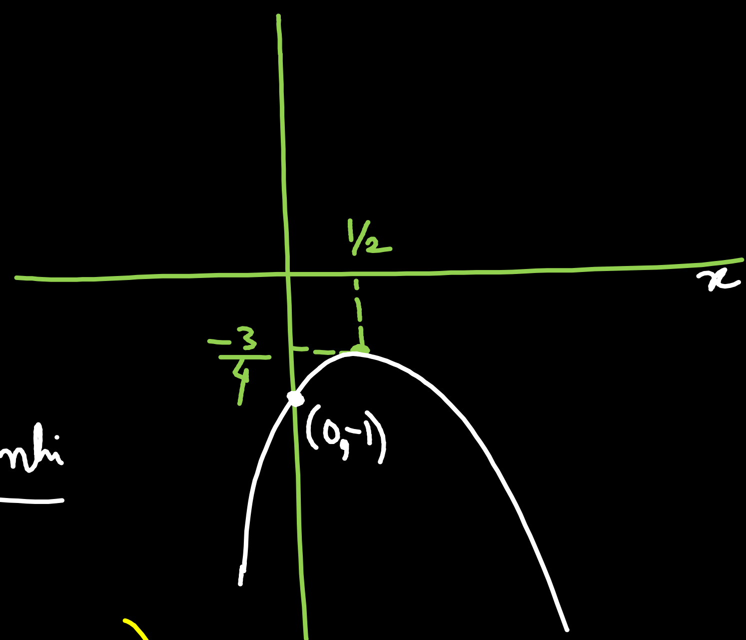
$D = -3$

$D < 0$ No Real Root

$\hookrightarrow$ x-axis Ko cut nhi

S-3 $(0, -1)$

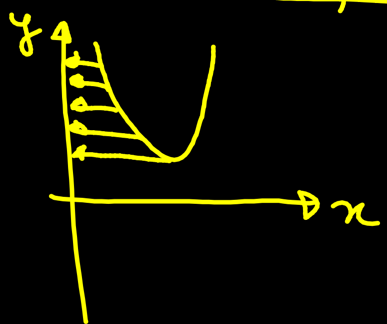
S-4 $V \equiv \left(\frac{-b}{2a}, \frac{-D}{4a}\right) \equiv \left(\frac{-1}{2(-1)}, \frac{3}{4(-1)}\right) \equiv \left(\frac{1}{2}, \frac{-3}{4}\right)$



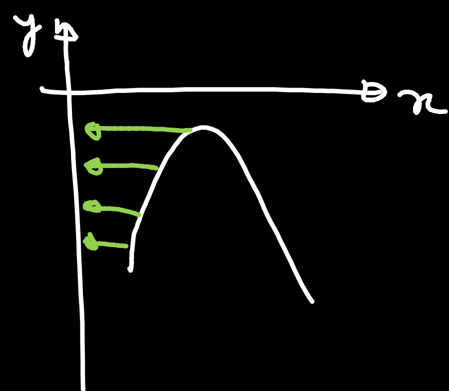
Sign of Quadratic Expression:

- If $a>0$ and $D<0$ then $ax^2 + bx + c$ is always positive
- If $a<0$ and $D<0$ then $ax^2 + bx + c$ is always negative

$\boxed{\text{if } a>0\ ;\ D<0}$



$a<0$ and $D<0$



$y = x^2 + x + 1$

$\boxed{\therefore\ a>0,\ D<0}$

$\boxed{x^2+x+1} \Rightarrow$ Always positive

$(-10)^2 + (-10) + 1 \Rightarrow +$

If the numbers appeared on the two throws of a fair six faced die are α and β, then the probability that $\boxed{x^2 + \alpha x + \beta} > 0$, for all $x \in R$, is :

(JEE 2022)

A. 17/36
B. 4/9
C. 1/2
D. 19/36

$\alpha, \beta \in \{1, 2, 3, 4, 5, 6\}$

$x^2 + \alpha x + \beta \Rightarrow$ Always Positive

$D = \alpha^2 - 4\beta < 0$

$a > 0$

* $D < 0$

$P = \dfrac{\text{fav.}}{\text{Total}}$

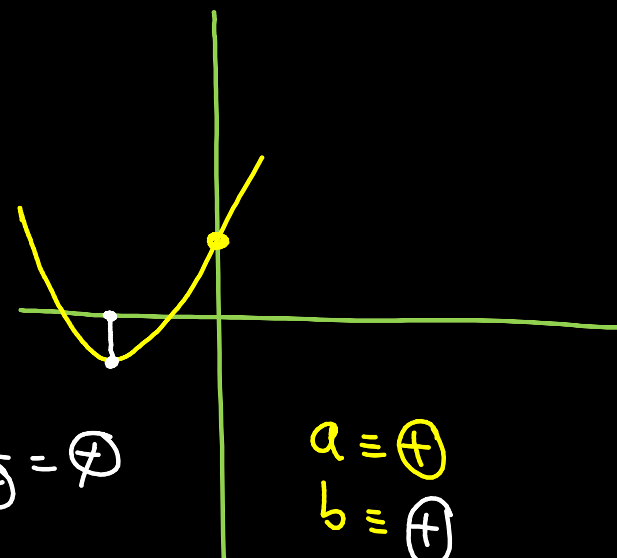
$= \dfrac{17}{6 \times 6}$

Unacademy Atoms

$\alpha^2 < 24$

$$\alpha^2 < 4\beta$$

$\alpha = 1$	$\beta = 1$	$(1,1)$
$\alpha = 1, 2$	$\beta = 2$	$(1,2)$ $(2,2)$
$\alpha = 1, 2, 3$	$\beta = 3$	.
$\alpha = 1, 2, 3$	$\beta = 4$	.
$\alpha = 1, 2, 3, 4$	$\beta = 5$	.
$\alpha = 1, 2, 3, 4$	$\beta = 6$	.



$\frac{-b}{2(+)} = (-)$

$a \equiv (+)$

$b \equiv (+)$

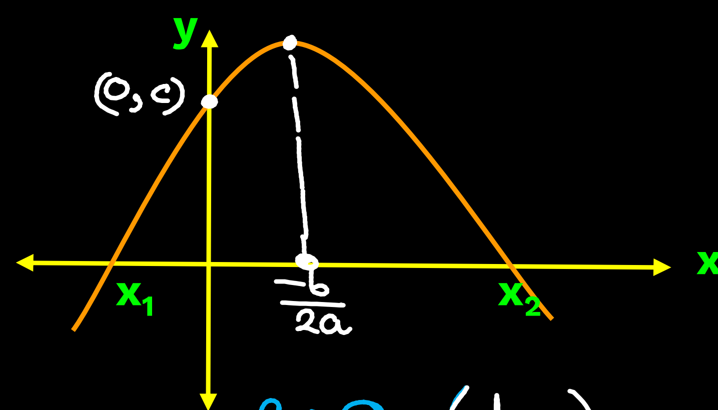
$c \equiv (+)$

Unacademy Atoms

Q

Consider the graph of quadratic trinomial $y = ax^2 + bx + c$ as shown below where x_1 and x_2 are roots of the equation $ax^2 + bx + c = 0$. Which of the following is/are correct?

MCQ



ACD

A. $a - b - c < 0$

~~B.~~ $bc < 0$

C. $b > 0$

D. b and c have the same sign different from a

$\frac{+b}{2\ominus} = \oplus$

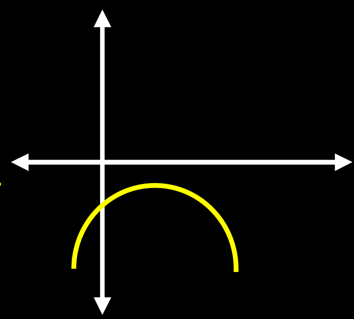
$a = \ominus$ (down)

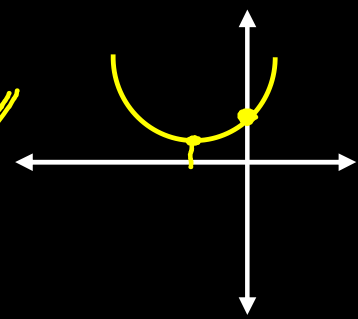
$b = \oplus$

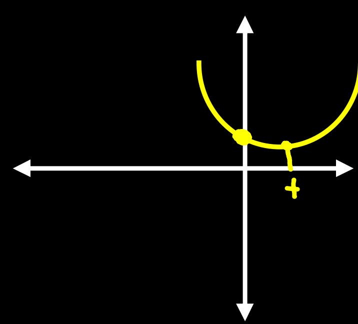
$c = \oplus$ (+y cut)

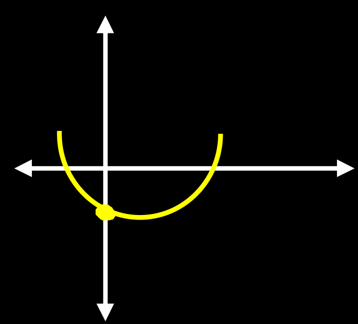
If $ax^2 + bx + c = 0$ has imaginary roots and a, b, c > 0. Then possible graph of $y = ax^2 + bx + c$ is :

$\boxed{a > 0}$
$b > 0$
$\boxed{c > 0}$

~~A.~~


B.


C.


~~D.~~


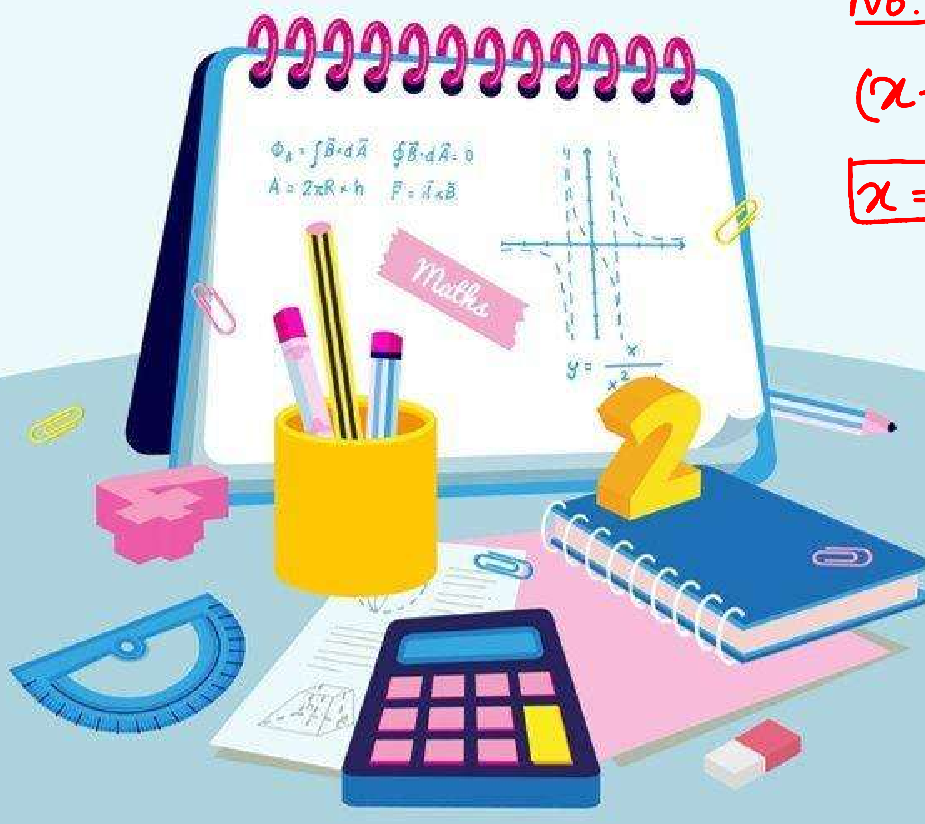
$$\frac{-b}{2a} = \frac{-(+)}{2(+)} = (-)$$

Unacademy Atoms

No. of Root

$(x-1)^2=0$

$\boxed{x=1}$



Symmetric Expressions

Symmetric Expression :

Expressions in α and β, which do not change by interchanging α and β.

Some examples of symmetric expressions are

i. $\alpha^2 + \beta^2$

ii. $\alpha^2 + \alpha\beta + \beta^2$

iii. $\frac{1}{\alpha} + \frac{1}{\beta}$

iv. $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

v. $\alpha^2\beta + \beta^2\alpha$

vi. $\left(\frac{\alpha}{\beta}\right)^2 + \left(\frac{\beta}{\alpha}\right)^2$

vii. $\alpha^3 + \beta^3$

viii. $\alpha^4 + \beta^4$

Unacademy Atoms

Unacademy Atoms

Q) $ax^2+bx+c=0 \begin{cases}\alpha\\ \beta\end{cases}$

$x \to (x-1)$

$\boxed{\text{New Q.E} = 0} \begin{cases}\alpha+1\\ \beta+1\end{cases}$

$\alpha+1 = x$

$\therefore \alpha = (x-1)$

$$\boxed{a(x-1)^2+b(x-1)+c=0}$$

M-1

$S = (\alpha+1)+(\beta+1)$

$P = (\alpha+1)(\beta+1)$

$\boxed{x^2 - Sx + P = 0}$ ✓

Ex. $\boxed{2x^2+5x+7=0} \begin{cases}\alpha\\ \beta\end{cases}$

$x \to (x+3)$

$\boxed{\text{New Q.E} = 0} \begin{cases}\alpha-3\\ \beta-3\end{cases}$

$\alpha-3 = x$

$\therefore \alpha = x+3$

✓ $\boxed{2(x+3)^2+5(x+3)+7=0}$

$\boxed{2x^2+17x+40=0}$

$\boxed{x^2+\frac{17}{2}x+20=0}$

$P = (\alpha-3)(\beta-3)$

$= \alpha\beta - 3(\alpha+\beta)+9$

$= \frac{7}{2} - 3\left(\frac{-5}{2}\right)+9$

$= \boxed{20}$

$S = \alpha-3+\beta-3$

$= \alpha+\beta-6$

$= \frac{-5}{2} - 6 = -17/2$

$ax^2+bx+c=0 \left< \begin{matrix}\alpha \\ \beta\end{matrix}\right.$ New QE $\left< \begin{matrix}? \\ ?\end{matrix}\right.$

New Roots	Replace.
$-\alpha, -\beta$	$x \longrightarrow (-x)$
$\alpha+K, \beta+K$	$x \longrightarrow (x-K)$
$\alpha-K, \beta-K$	$x \longrightarrow (x+K)$
$K\alpha, K\beta$	$x \rightarrow \frac{x}{K}$
$\frac{\alpha}{K}, \frac{\beta}{K}$	$x \rightarrow Kx$
α^n, β^n	$x \rightarrow x^{1/n}$

$\cancel{\alpha+2, \beta+3}$

$-\alpha = x$

$\therefore \alpha = -x$

$\alpha + K = x$

$\therefore \alpha = x - K$

$\frac{\alpha}{K} = x$

$\alpha = Kx$

$\alpha^n = x$

$\alpha = x^{1/n}$

Unacademy Atoms

If α and β are roots of $2x^2 - 7x + 6 = 0$, then the quadratic equation whose roots are $-2/\alpha$, $-2/\beta$ is

A. $3x^2 + 7x + 4 = 0$

B. $3x^2 - 7x + 4 = 0$

C. $6x^2 + 7x + 2 = 0$

D. $6x^2 - 7x + 2 = 0$

New QE = 0 $-2/\alpha$, $-2/\beta$

$\frac{-2}{\alpha} = x$

$\therefore \alpha = \frac{-2}{x}$

$x \rightarrow \frac{-2}{x}$

$$2\left(\frac{-2}{x}\right)^2 - 7\left(\frac{-2}{x}\right) + 6 = 0$$

$$x^2\left(\frac{8}{x^2} + \frac{14}{x} + 6 = 0\right)$$

$$\frac{1}{2}\left(8 + 14x + 6x^2 = 0\right)$$

$$3x^2 + 7x + 4 = 0$$

$\boxed{deg = 4}$

Bi Quadratic

* Cubic $\boxed{deg = 3}$

Bi Quadratic Equation

$$ax^4 + bx^3 + cx^2 + bx + a = 0$$

* <u>Sym</u>

divide by x^2 :-

$$ax^2 + bx + c + \frac{b}{x} + \frac{a}{x^2} = 0$$

$$\boxed{x + \frac{1}{x} = t}$$

$$a\left(x^2 + \frac{1}{x^2}\right) + b\left(x + \frac{1}{x}\right) + c = 0$$

$$a\left(\left(x + \frac{1}{x}\right)^2 - 2\right) + b\left(x + \frac{1}{x}\right) + c = 0$$

Unacademy Atoms

The sum of the cube of all the roots of the equation $x^4 - 3x^3 - 2x^2 + 3x + 1 = 0$ is_.

(JEE Main 2022)

$$x^4 - 3x^3 - 2x^2 + 3x + 1 = 0 \quad \alpha, \beta, \gamma, \delta$$

$$x^2 - 3x - 2 + \frac{3}{x} + \frac{1}{x^2} = 0$$

$$\left(x - \frac{1}{x}\right)^2 + \cancel{2} - 3\left(x - \frac{1}{x}\right) - \cancel{2} = 0$$

$$t^2 - 3t = 0$$

$$t(t-3) = 0$$

$$t = 0 \text{ OR } 3$$

Req

$$\Rightarrow \alpha^3 + \beta^3 + \gamma^3 + \delta^3$$

$$\Rightarrow \cancel{1^3} + \cancel{(-1)^3} + (\gamma+\delta)^3 - 3\gamma\delta(\gamma+\delta)$$

$$\Rightarrow 0 + 27 + 3(3)$$

$$\Rightarrow 36$$

α, β

$$x - \frac{1}{x} = 0$$

$$x^2 = 1$$

$$\underline{x = \pm 1}$$

$$\left.\begin{aligned} \alpha &= 1 \\ \beta &= -1 \end{aligned}\right\}$$

γ, δ

$$x - \frac{1}{x} = 3$$

$$x^2 - 3x - 1 = 0 \begin{cases} \gamma \\ \delta \end{cases}$$

$$\underline{\gamma + \delta = 3}$$

$$\underline{\gamma\delta = -1}$$

$$\cancel{\boxed{\gamma, \delta \Rightarrow \frac{3 \pm \sqrt{13}}{2}}}$$

Unacademy Atoms

The number of real solutions of the equation $e^{4x} + 4e^{3x} - 58e^{2x} + 4e^{x} + 1 = 0$ is_

(2022)

$\boxed{2 \text{ Sol}^n}$

Let, $e^x = t$

$$t^4 + 4t^3 - 58t^2 + 4t + 1 = 0$$

$$t^2 + 4t - 58 + \frac{4}{t} + \frac{1}{t^2} = 0$$

$$\left(t + \frac{1}{t}\right)^2 - 2 + 4\left(t + \frac{1}{t}\right) - 58 = 0$$

$$t^2 + \frac{1}{t^2} = \left(t + \frac{1}{t}\right)^2 - 2$$

\# Baspan

$\boxed{\# \text{CUTE}}$

$$z^2 + 4z - 60 = 0$$

$$(z + 10)(z - 6) = 0$$

$$z = -10 \mid 6$$

$$t + \frac{1}{t} = -10$$

$$e^x + \frac{1}{e^x} = -10$$

$e^x \longrightarrow$ positive

No soln

$$t + \frac{1}{t} = 6$$

$$t^2 - 6t + 1 = 0 \left< \begin{matrix} t_1 \\ \\ t_2 \end{matrix} \right.$$

$D > 0$

$$t = e^x$$

2 Soln

The sum of all the real roots of the equation $(e^{2x} - 4)(6e^{2x} - 5e^{x} + 1) = 0$ is

(2022)

A. $\log_e 3$

B. $-\log_e 3$

C. $\log_e 6$

D. $-\log_e 6$

Piro (#chindi)

$e^x = t$

$(t^2-4)(6t^2-5t+1)$

$(t-2)(t+2)(3t-1)(2t-1)$

$e^x = 2$

$x = \ln 2$

$(e^x-2)(e^x+2)(3e^x-1)(2e^x-1) = 0$

$e^x = 2, \cancel{-2}, \frac{1}{3}, \frac{1}{2}$

$\therefore x = \ln 2, \ln\frac{1}{3}, \ln\frac{1}{2}$

$\sum x = \cancel{\ln 2} + \ln\frac{1}{3} + \cancel{\ln\frac{1}{2}}$

$= \ln 3^{-1}$

$= -\ln 3$

Unacademy Atoms

QE1 $\langle \alpha, \beta$

QE2 $\langle \alpha, \gamma$

Condition for Common Roots

Condition of Common Roots :

Condition for both the common roots

$$a_1x^2 + b_1x + c_1 = 0 \begin{cases} \alpha \\ \beta \end{cases}$$

$$a_2x^2 + b_2x + c_2 = 0 \begin{cases} \alpha \\ \beta \end{cases}$$

$$\boxed{\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}}$$

then both Roots are common.

$$1x^2 - 3x + 2 = 0 \begin{cases} 1 \\ 2 \end{cases}$$

$$2x^2 - 6x + 4 = 0 \begin{cases} 1 \\ 2 \end{cases}$$

$$\boxed{\frac{1}{2} = \frac{-3}{-6} = \frac{2}{4} = k}$$

Condition of Common Roots :

Condition for one common roots :

$$a_1x^2 + b_1x + c_1 = 0 \quad \begin{cases} \alpha = \checkmark \\ \beta \end{cases} \qquad (\beta \neq \gamma)$$

$$a_2x^2 + b_2x + c_2 = 0 \quad \begin{cases} \alpha = \checkmark \\ \gamma \end{cases}$$

$$\alpha\beta = \frac{c_1}{a_1}$$

$$\boxed{\beta = \frac{c_1}{a_1\alpha}}$$

$$(\alpha)(\gamma) = \checkmark$$

?

Method :- $a_2(a_1\alpha^2 + b_1\alpha + c_1 = 0)$

S-1 ✓

S-2

$a_1(a_2\alpha^2 + b_2\alpha + c_2 = 0)$

$$\boxed{\alpha = \text{Value}}$$

Note :

\# chalaki

* Given one root is common but one of the QE has D<0 then both roots will be common

✓ $a_1x^2 + b_1x + c_1 = 0$ $\begin{cases} \alpha = p + iq \\ \beta = p - iq \end{cases}$

✓ # $a_2x^2 + b_2x + c_2 = 0$ $\begin{cases} \alpha = p + iq \\ \gamma = p - iq \end{cases}$

D < 0

imag. ⇒ pair

$$\boxed{\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}}$$

Unacademy Atoms

Let $\lambda \neq 0$ be in R. If α and β are roots of the equation, $x^2 - x + 2\lambda = 0$ and α and γ are the roots of the equation, $3x^2 - 10x + 27\lambda = 0$, the $\beta\gamma / \lambda$ is equal to :

A. 27

B. 18

C. 9

D. 36

JEE MAIN 2020

$$\frac{\beta \cdot \gamma}{\lambda} = \frac{\left(\frac{2}{3}\right)(3)}{\left(\frac{1}{9}\right)} = 18$$

$$\boxed{x^2 - x + 2\lambda = 0} \quad \alpha = 3\lambda, \quad \beta = 2/3$$

$$3x^2 - 10x + 27\lambda = 0 \quad \alpha = 3\lambda, \quad \gamma = 3$$

$$3(\alpha^2 - \alpha + 2\lambda = 0)$$

$$\cancel{3\alpha^2} - 10\alpha + 27\lambda = 0$$

$$- \quad + \quad -$$

$$7\alpha - 21\lambda = 0$$

$$\therefore \boxed{\alpha = 3\lambda}$$

$$(\alpha)(\beta) = 2\lambda$$

$$(3\cancel{\lambda})(\beta) = 2\cancel{\lambda}$$

$$\therefore \boxed{\beta = \frac{2}{3}}$$

$$(\alpha)(\gamma) = 9\lambda$$

$$(3\lambda)(\gamma) = 9\lambda$$

$$\therefore \gamma = 3$$

$$(3\lambda)^2 - 3\lambda + 2\lambda = 0$$

$$9\lambda^2 = \lambda$$

$$\boxed{\lambda = \frac{1}{9}}$$

Q

Unacademy Atoms

Let $a, b \in \mathbb{R}$, $a \neq 0$ be such that the equation, $ax^2 - 2bx + 5 = 0$ has a repeated root α, which is also a root of the equation, $x^2 - 2bx - 10 = 0$. If β is the other root of this equation, then $\alpha^2 + \beta^2$ is equal to :

(a) 25 (b) 26 (c) 28 (d) 24

(P.Y.Q)

JEE MAIN 2020

$ax^2 - 2bx + 5 = 0$ — roots α, α

$x^2 - 2bx - 10 = 0$ — roots α, β

$$\frac{b^2}{a^2} - 2b\left(\frac{b}{a}\right) - 10 = 0$$

$$\frac{5a}{a^2} - \frac{2(5\cancel{a})}{\cancel{a}} - 10 = 0$$

$\cancel{2}\alpha = \frac{\cancel{2}b}{a}$

$\therefore \boxed{\alpha = \frac{b}{a}}$

$\frac{5}{a} = 20$

$\therefore \boxed{a = \frac{1}{4}}$

$\alpha^2 = \frac{5}{a}$

$\frac{b^2}{a^2} = \frac{5}{a}$

$\boxed{b^2 = 5a}$

$b^2 = \frac{5}{4}$

$\alpha^2 + \beta^2$

$\Rightarrow (\alpha + \beta)^2 - 2\alpha\beta$

$\Rightarrow 4b^2 + 20$

$\Rightarrow 4\left(\frac{5}{4}\right) + 20$

$\Rightarrow 25$

Unacademy Atoms

Q

Let a, b $\in$ R be such that the equation $ax^2 - 2bx + 15 = 0$ has a repeated root α. If α and β are the roots of the equation $x^2 - 2bx + 21 = 0$, then $\alpha^2 + \beta^2$ is equal to :

(2022)

A. 37
B. 58
C. 68
D. 92

P.Y.Q

If $a, b, c \in R$ and equations $ax^2 + bx + c = 0$ and $x^2 + 2x + 9 = 0$ have a common root, then find $a : b : c$.

ek

$\boxed{1 : 2 : 9}$

$D = 4 - 4(9)$

$D < 0$

#Chalaki

both

JEE 2013

Unacademy Atoms

A-1 Condi Banao
A-2 Condi Solve

~~Ratta~~

Smj

**

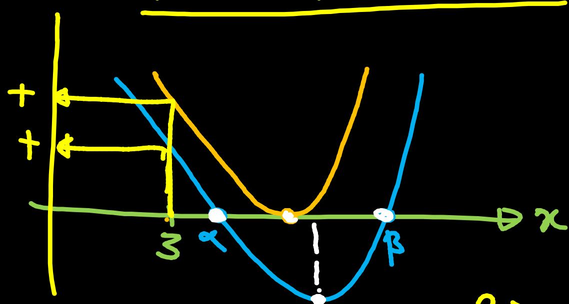
Location of Roots

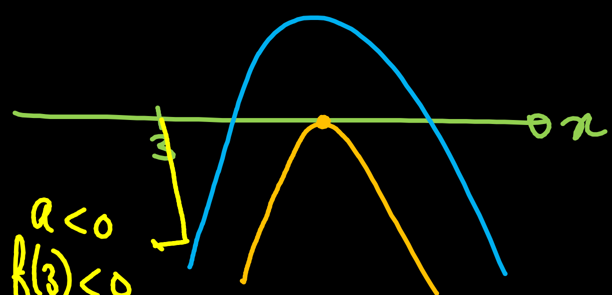
Location of the Roots:

$x_0 = 3$

1. Both the roots are greater than x_0:

Both $\alpha, \beta > 3$





$a > 0$ $\quad$ $a < 0$

$f(3) > 0$ $\quad$ $f(3) < 0$

$a\,f(3) > 0$

Condition

① $D \geq 0$

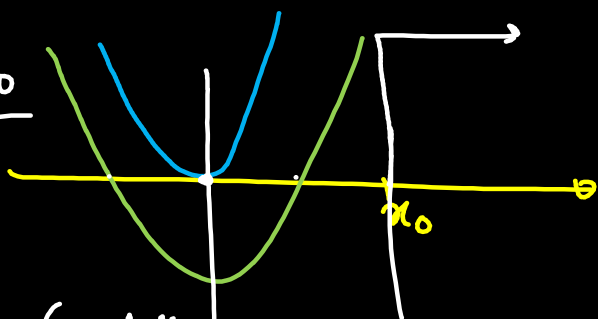
② $\dfrac{-b}{2a} > 3$

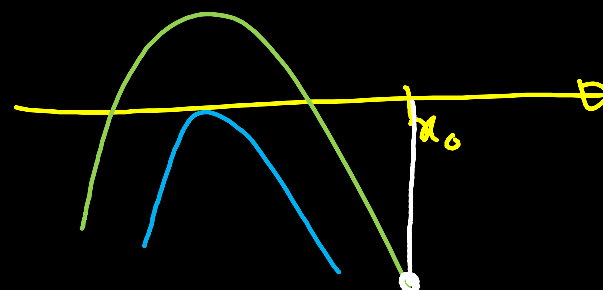
③ $a\,f(3) > 0$

Location of the Roots:

2. Both the roots are less than x_0:

$a > 0$
$f(x_0) > 0$





Conditions :-

i) $D \geq 0$

ii) $\frac{-b}{2a} < x_0$

iii) $a f(x_0) > 0$

Q

If both the roots of quadratic equation $x^2 - 2kx + k^2 + k - 5 = 0$ are less than 5, then k lies in the interval

(up)

A. (5, 6]

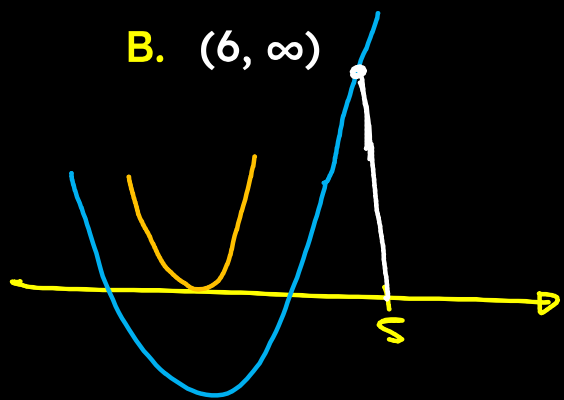
C. $(-\infty, 4)$

B. $(6, \infty)$

D. [4, 5]

JEE MAIN 2005

Unacademy Atoms



① $D \geq 0$

② $\frac{-b}{2a} < 5$

③ $1 \cdot f(5) > 0$

① $D \geq 0$

$\cancel{4K^2} - 4(\cancel{K^2} + K - 5) \geq 0$

$-4(K-5) \geq 0$

$K - 5 \leq 0$

$\boxed{K \leq 5}$

② $\frac{-b}{2a} < 5$

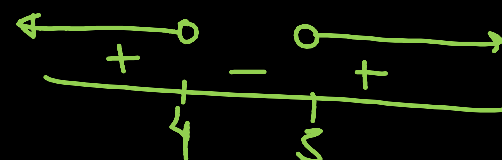
$\Rightarrow \frac{-(-2K)}{2(1)} < 5$

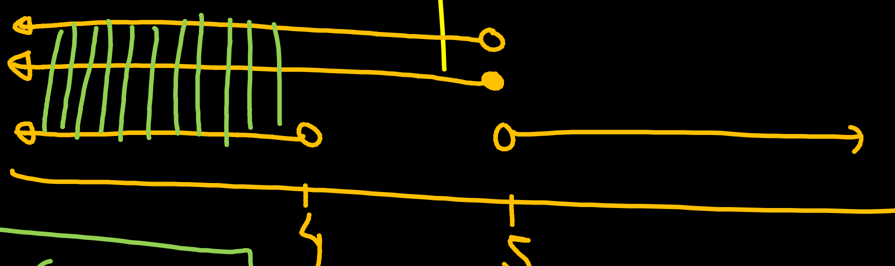
$\Rightarrow \boxed{K < 5}$

$25 - 10K + K^2 + K - 5 > 0$

$K^2 - 9K + 20 > 0$

$(K-4)(K-5) > 0$



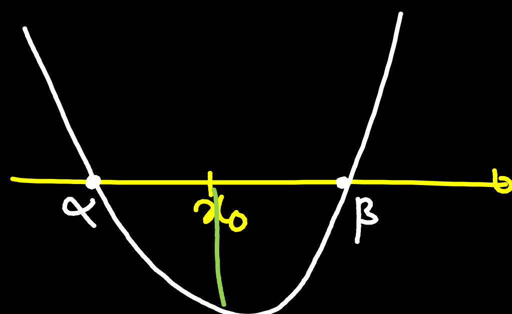


$\boxed{K \in (-\infty, 4)}$

Unacademy Atoms

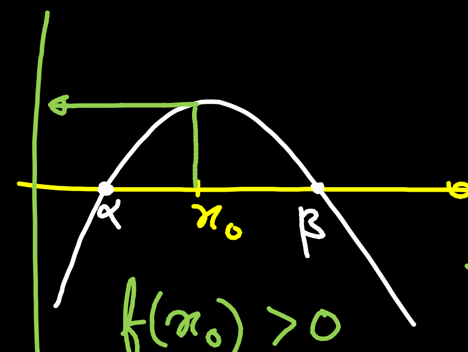
Location of the Roots:

3. One root less than x_0 and other greater than x_0:



$f(x_0) = \ominus$

$a = \oplus$



$f(x_0) > 0$

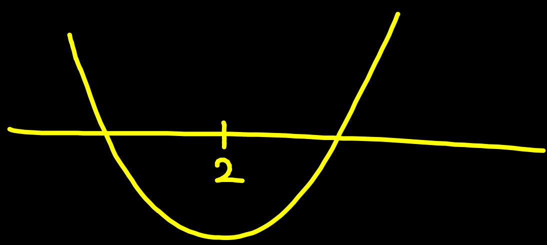
$a = \ominus$

$\cancel{D}$

$\cancel{-\frac{b}{2a}}$

★ $\boxed{a \cdot f(x_0) < 0}$

Find the value of **k** for which one root of the equation of
$x^2 - (k + 1)x + k^2 + k - 8 = 0$ exceed **2** and other is smaller than **2**.



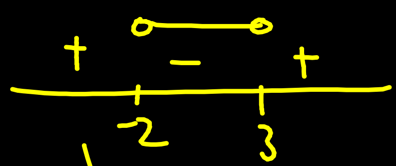
$$a \cdot f(2) < 0$$

$$1\{4 - (k+1)(2) + k^2 + k - 8\} < 0$$

$$4 - 2k - 2 + k^2 + k - 8 < 0$$

$$k^2 - k - 6 < 0$$

$$(k-3)(k+2) < 0$$



$$\boxed{k \in (-2, 3)}$$

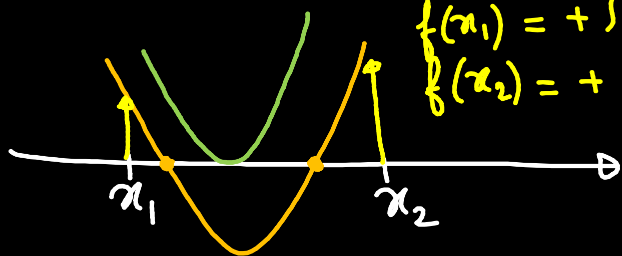
Unacademy Atoms

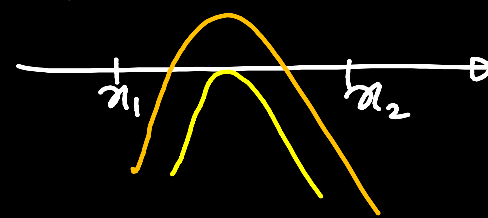
Location of the Roots:

4. Both root between x_1 and x_2: $f(x) = ax^2 + bx + c$

$a = +$
$f(x_1) = +$
$f(x_2) = +$

$a = \ominus$
$f(x_1) = \ominus$
$f(x_2) = \ominus$





① $D \geq 0$

② $x_1 < \dfrac{-b}{2a} < x_2$

③ $a\,f(x_1) > 0$

④ $a \cdot f(x_2) > 0$

Q

If both the roots of quadratic equation $x^2 - mx + 4 = 0$ are real and $\boxed{\text{distinct}}$ and they lie in the interval [1, 5], then m lies in the interval

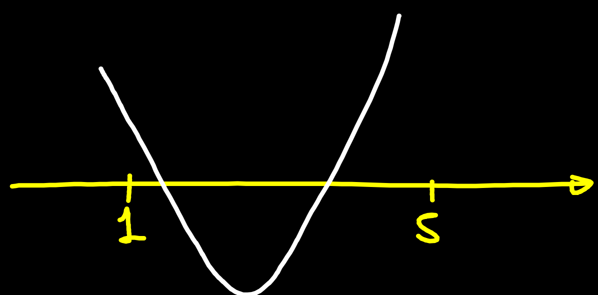
A. (-5, -4) **B.** (4, 5) **C.** (5, 6) **D.** (3, 4)

Unacademy Atoms

JEE MAIN 2019

$1x^2 - mx + 4 = 0$

(up)



Conditions:-

① $D > 0$

② $f(1) > 0$

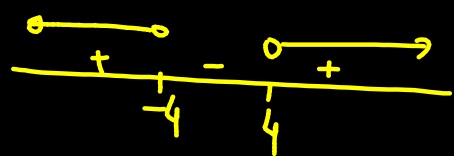
③ $f(5) > 0$

④ $1 < \frac{-b}{2a} < 5$

① $D > 0$

$m^2 - 16 > 0$

$(m-4)(m+4) > 0$



② $1 - m + 4 > 0$

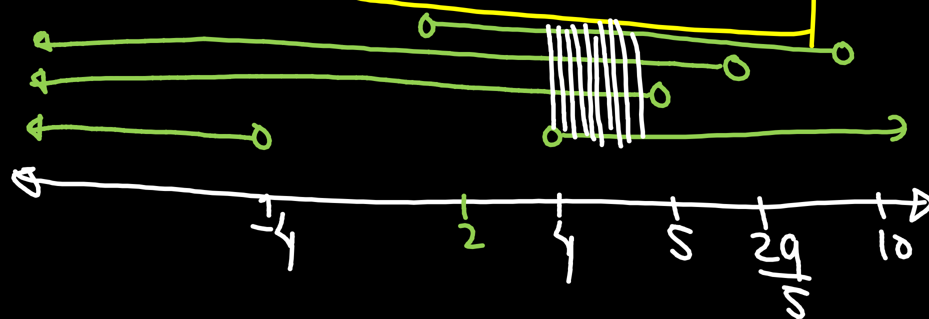
$m < 5$

③ $25 - 5m + 4 > 0$

$\boxed{m < \frac{29}{5}}$

④ $1 < \frac{m}{2} < 5$

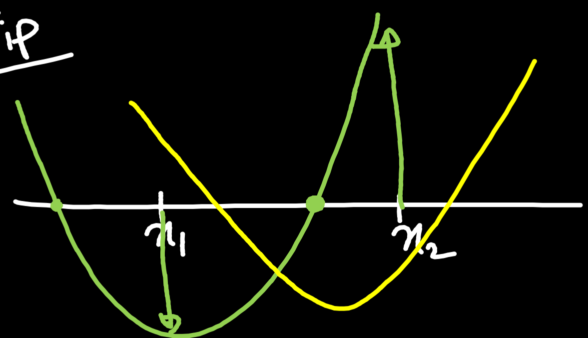
$\boxed{2 < m < 10}$

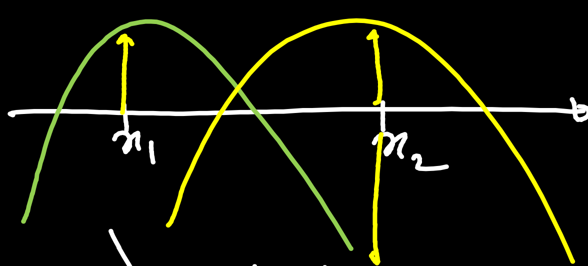


Location of the Roots:

5. Exactly one root between x_1 and x_2:

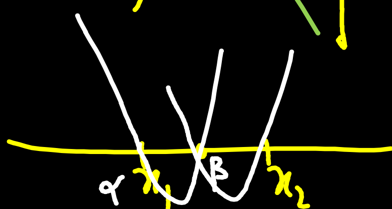
#NVTip





$f(x_1) = \ominus$

$f(x_2) = \oplus$

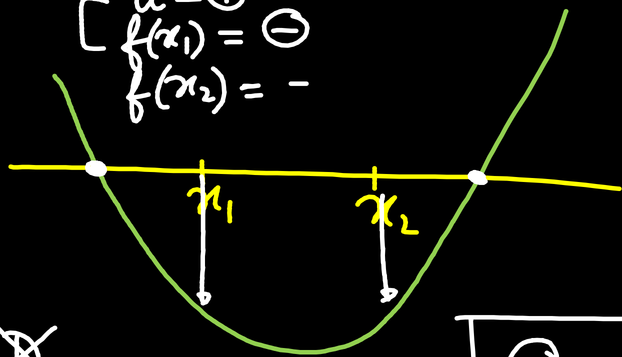


$\boxed{① \ f(x_1) \cdot f(x_2) \leq 0} \Rightarrow \boxed{\text{Solve}}$ $\boxed{\text{End points} \Rightarrow \text{Verify}}$

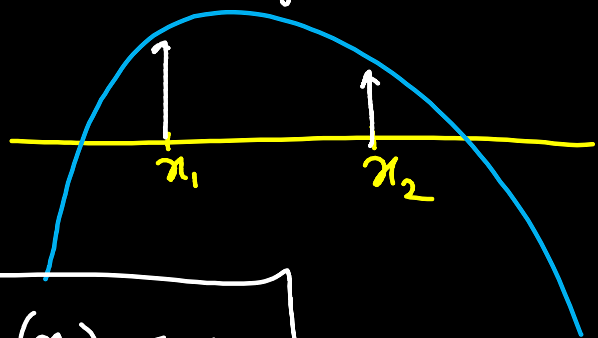
Location of the Roots:

6. Both root outside x_1 and x_2:

$a = \oplus$
$f(x_1) = \ominus$
$f(x_2) = -$



$a = -$
$f(x_1) = +$
$f(x_2) = +$



$\cancel{D}$

$\cancel{-\frac{b}{2a}}$

$\cancel{x_1 < -\frac{b}{2a} < x_2}$

$$\boxed{\begin{array}{l} (1)\ a f(x_1) < 0 \\ (2)\ a f(x_2) < 0 \end{array}}$$

Unacademy Atoms

Q

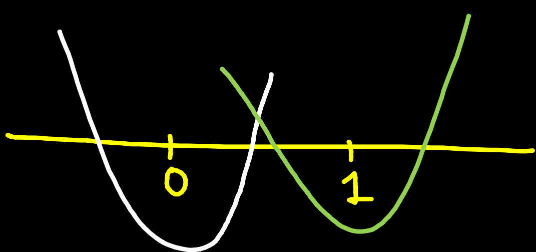
The set of all real values of λ for which the quadratic equations, $\boxed{(\lambda^2 + 1)x^2 - 4\lambda x + 2 = 0}$ always have exactly one root in the interval (0, 1) is :

Ques

A. (0, 2) B. (2, 4) C. (1, 3] D. (-3, -1)

JEE MAIN 2020

$a = \lambda^2 + 1 = \oplus$ (up)



① $f(0) \cdot f(1) \le 0$

$2\{\lambda^2 + 1 - 4\lambda + 2\} \le 0$

$\lambda^2 - 4\lambda + 3 \le 0$

$(\lambda - 1)(\lambda - 3) \le 0$

$\lambda \in (1, 3]$

$\lambda = 1$

(Rej)

$$2x^2 - 4x + 2 = 0$$

$$x^2 - 2x + 1 = 0$$

$$(x-1)^2 = 0$$

$\lambda = 3$

$$10x^2 - 12x + 2 = 0$$

$$5x^2 - 6x + 1 = 0$$

$$5x^2 - 5x - x + 1 = 0$$

$$(5x-1)(x-1) = 0$$

$$\boxed{x = 1, \frac{1}{5}}$$

Unacademy Atoms

Theory of Equations

Theory of Equations:

For Quadratic Equation :

change - not change - change - not change ...

$$ax^2 + bx + c = 0 \quad (\deg = 2)$$

$\div 1$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0 \quad \alpha, \beta$$

$$\alpha + \beta = \frac{-b}{a} \quad \text{——} (1)$$

$$\alpha\beta = \frac{c}{a} \quad \text{——} (2)$$

Theory of Equations:

For Cubic Equation :

#GANNA (∞)

$$ax^3 + bx^2 + cx + d = 0$$

$\div a$

$$x^3 + \frac{b}{a}x^2 + \frac{c}{a}x + \frac{d}{a} = 0 \quad \alpha, \beta, \gamma$$

$\ominus$ $\oplus$ $\ominus$

$$\alpha + \beta + \gamma = \frac{-b}{a}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{+c}{a}$$

$$\alpha\beta\gamma = \frac{-d}{a}$$

Theory of Equations:

Unacademy Atoms

For Bi-quadratic Equation :

$$\ominus \quad \oplus \quad \ominus \quad \oplus$$

$$x^4 + \frac{b}{a}x^3 + \frac{c}{a}x^2 + \frac{d}{a}x + \frac{e}{a} = 0 \quad \alpha, \beta, \gamma, \delta$$

$$\alpha + \beta + \gamma + \delta = \frac{-b}{a}$$

$$\alpha\beta + \beta\gamma + \gamma\delta + \delta\alpha + \alpha\gamma + \delta\beta = \frac{c}{a}$$

$$\alpha\beta\gamma + \beta\gamma\delta + \alpha\gamma\delta + \alpha\beta\delta = \frac{-d}{a}$$

$$\alpha\beta\gamma\delta = \frac{e}{a}$$

Solve the cubic $4x^3 + 16x^2 - 9x - 36 = 0$, the sum of its two roots being equal to zero.

$$4x^3 + 16x^2 - 9x - 36 = 0$$

$$x^3 + 4x^2 - \frac{9}{4}x - 9 = 0$$

$\alpha = 3/2$

$-\alpha = -3/2$

$\beta = -4$

$$\cancel{\alpha} + (-\cancel{\alpha}) + \beta = -4$$

$$(\alpha)(-\alpha)(-4) = 9$$

$$\alpha^2 = \frac{9}{4} \Rightarrow \boxed{\alpha = \pm\frac{3}{2}}$$

$$\boxed{\frac{3}{2}, \frac{-3}{2}, -4}$$

If the sum of all the roots of the equation

$e^{2x} - 11e^{x} - 45e^{-x} + 81/2 = 0$ is $\log_e p$, then p is equal to 45.

(JEE 2022)

$$\boxed{e^{2x} - 11e^{x} - \frac{45}{e^{x}} + \frac{81}{2} = 0}$$

$e^{x} = t$

$x_1 + x_2 + x_3 = ?$

$$2\cancel{t}\left(t^2 - 11t - \frac{45}{\cancel{t}} + \frac{81}{2} = 0\right)$$

$t_1 = e^{x_1}$

$t_2 = e^{x_2}$

$t_3 = e^{x_3}$

$$t^3 - \frac{22}{2}t^2 + \frac{81}{2}t - \frac{90}{2} = 0 \quad t_1, t_2, t_3$$

$$t_1 t_2 t_3 = 45$$

$$e^{x_1} \cdot e^{x_2} \cdot e^{x_3} = 45 \Rightarrow e^{x_1+x_2+x_3} = 45 \Rightarrow \boxed{x_1+x_2+x_3 = \ln 45}$$

If a, b, c are the roots of cubic $x^3 - x^2 + 1 = 0$ then find the value of $a^{-2} + b^{-2} + c^{-2}$.

H.W.