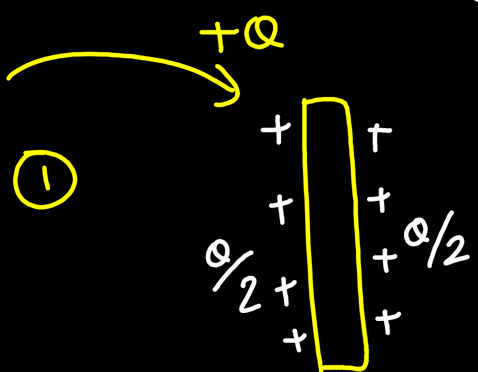
Today's Target

- Charge Distribution in Conducting Plates
- Capacitance
- Types of Capacitors
- Capacitor as Circuit Element
- Redistribution of Charges
- Combination of Capacitors
- Kirchoff's Law & Circuit Solving
- Dielectrics
- R-C circuits

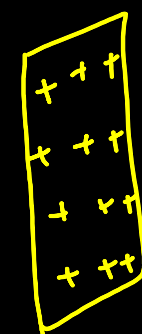
Conducting Plates

①



E due to Non Conducting

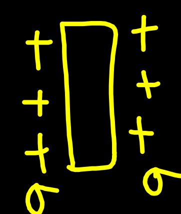
②



$\longrightarrow E = \dfrac{\sigma}{2\varepsilon_0}$

$\sigma = \left(\dfrac{\text{charge}}{\text{area}}\right)$

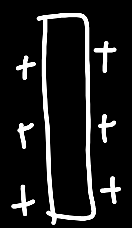
Conducting Plate



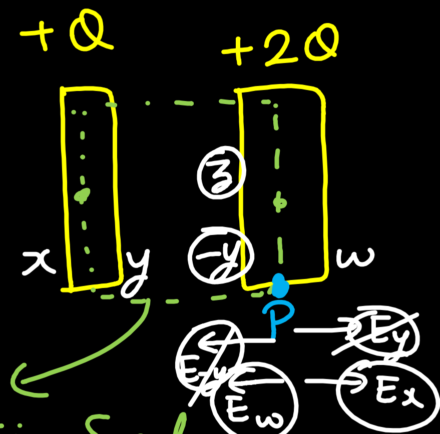
$\longrightarrow \sigma/2\varepsilon_0$

$\longrightarrow \sigma/2\varepsilon_0 \Rightarrow \dfrac{\sigma}{\varepsilon_0}$

Conducting



$E = \frac{\sigma}{\varepsilon_0}$



gaussian Surface

$\phi_{total} = 0$

$Q_{enclosed} = 0$

$y + z = 0$

$z = -y$

$x + y = Q$

$z + w = 2Q$

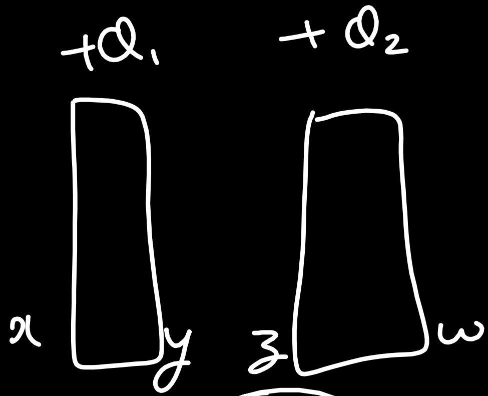
① gauss law

Facing Surfaces Equal & Opp

② E_{net} inside conductor $= 0$

$x = w$

Outer surface charge same.



$x = w$

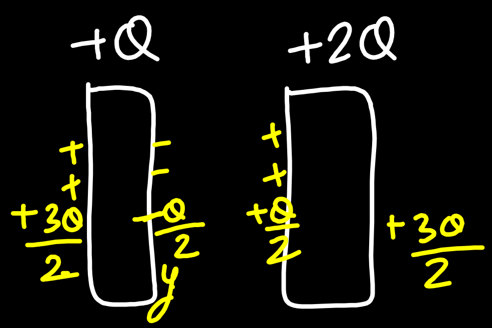
$z = -y$

$$x + \underbrace{\cancel{y+z}}_{0} + w = Q_1 + Q_2$$

$$2x = Q_1 + Q_2$$

$$x = \frac{Q_1 + Q_2}{2}$$

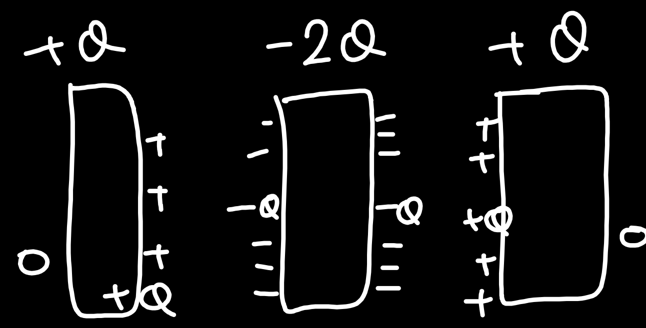
$$\text{Outer} = \frac{\text{total}}{2}$$



$$\text{outer} = \frac{Q + 2Q}{2} = \frac{+3Q}{2}$$

$$\frac{3Q}{2} + y = Q$$

$$y = -Q/2$$



$$\text{outer} = \frac{Q - 2Q + Q}{2} = 0$$

Q

$$|\Delta V| = |E\,d|$$

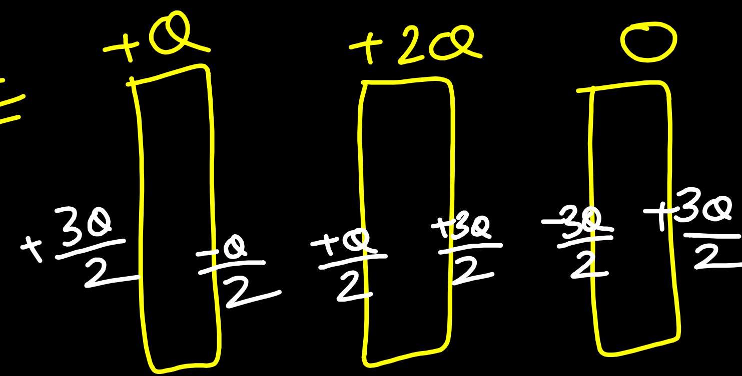
when E is constant

Q

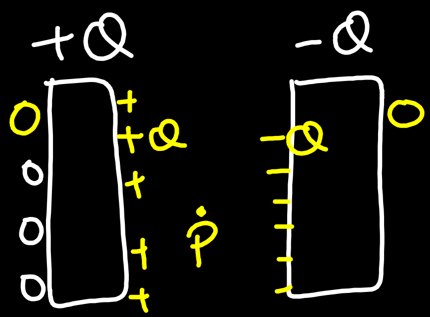
$$|\Delta V| = \left|\int \vec{E}\cdot\vec{dr}\right|$$

when E is variable.

Q



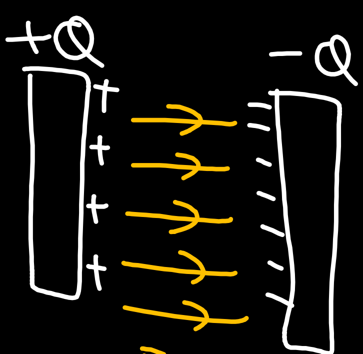
Q



E at P = ?

$+ \quad \longrightarrow E_+ \quad \sigma/2\varepsilon_0$

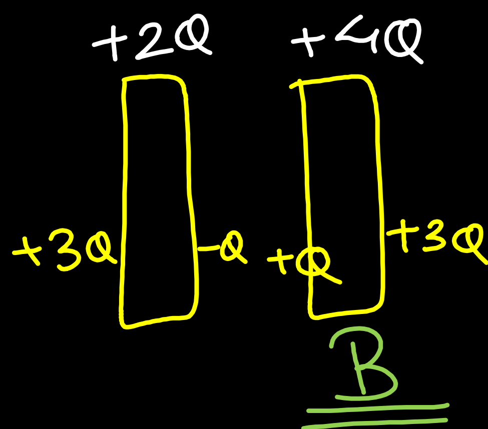
$\longrightarrow E_- \equiv \sigma/2\varepsilon_0 \Rightarrow \sigma/\varepsilon_0$

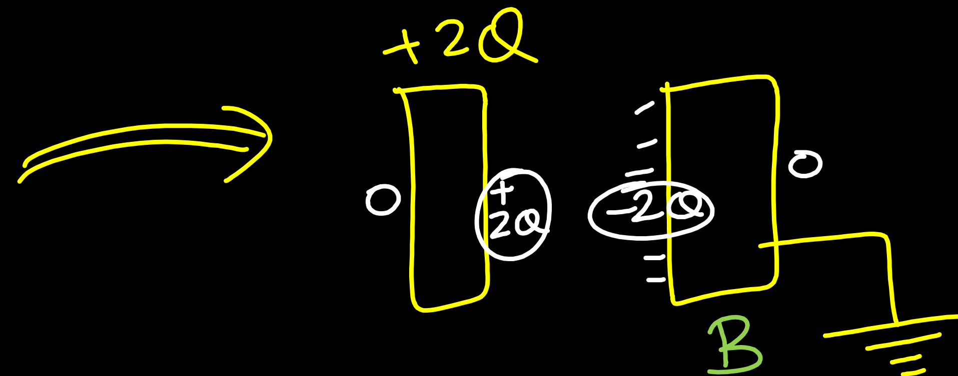


$E = \frac{\sigma}{\varepsilon_0}$

$\sigma = \frac{Q}{A}$

when one plate is Earthed





total charge of system becomes = 0

Charge flow From Plate B to Earth ⇒ +6Q

+4Q initial

−2Q final

B

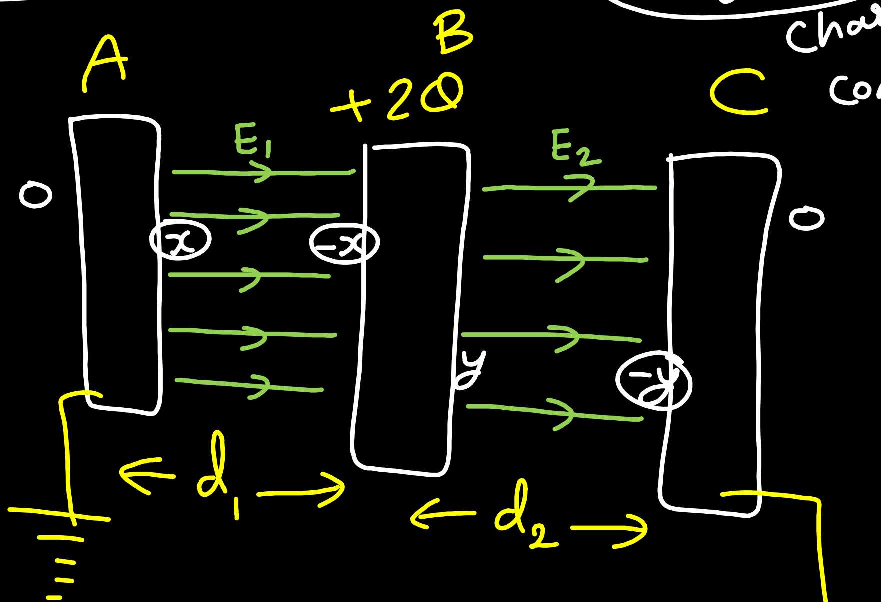
↓

earth

Two plates Earth

$$\boxed{-x+y=2Q}$$
Charge conservation

$V_A = 0 \qquad V_C = 0$



$$E = \frac{\sigma}{\varepsilon_0} \qquad E_2 = \frac{\sigma}{\varepsilon_0}$$

$$E_1 = \frac{x}{A\varepsilon_0} \qquad = \frac{y}{A\varepsilon_0}$$

$$V_A - V_C = 0$$

$$\Delta V_{AC} = 0$$

$$\boxed{E_1 d_1 + E_2 d_2 = 0}$$

$$\frac{x d_1}{A\varepsilon_0} + \frac{y d_2}{A\varepsilon_0} = 0$$

CAPACITANCE

Concept of Capacitance - Capacitance of a conductor is a measure of ability of the conductor to store charge on it. When a conductor is charged then its potential rises. The increase in potential is directly proportional to the charge given to the conductor.

$$Q \propto V \Rightarrow Q = CV$$

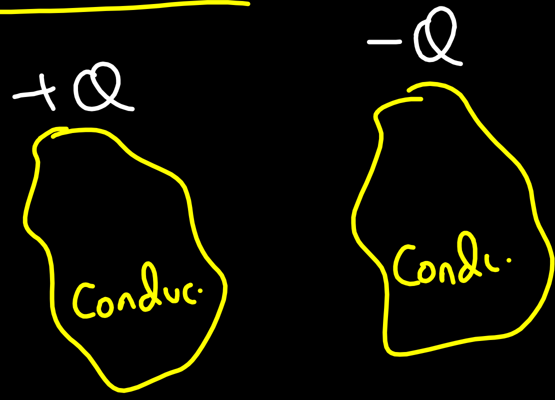
The constant C is known as the capacitance of the conductor.

Capacitance is a scalar quantity with dimension $C = \frac{Q}{V} = \frac{Q^2}{W} = \frac{A^2T^2}{M^1L^2T^{-2}} = M^{-1}L^{-2}T^4A^2$

Unit :- farad, coulomb/volt

The capacity of a conductor is independent of the charge given or its potential raised. It is also independent of nature of material and thickness of the conductor.

Capacitor



$$Q \propto \Delta V$$

$$\boxed{Q = C\,\Delta V}$$

Capacitance.

Potential = $\frac{PE}{\text{charge}}$

$= \frac{[ML^2T^{-2}]}{[AT]}$

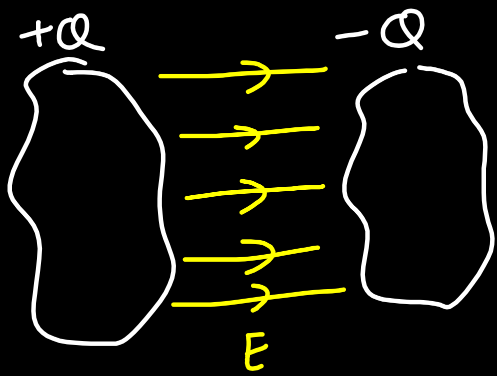
$$C = \frac{Q}{\Delta V}$$

Unit

$\frac{\text{colum}b}{\text{Volt}}$ = Farad.

$$= \frac{[AT]}{[ML^2T^{-2}]/[AT]}$$

$$= [A^2 T^4 M^{-1} L^{-2}]$$



$$C = \frac{Q}{\Delta V}$$

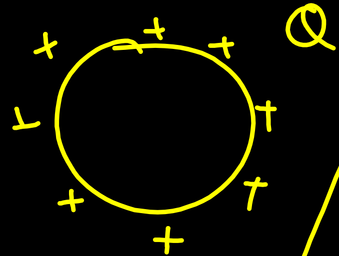
C does not depends Upon

charge given

ΔV

$\infty \; V = 0$

isolated Conductor



$$V = \frac{KQ}{R}$$

$$K \rightarrow \frac{1}{4\pi\varepsilon_0}$$

$$C = \frac{Q}{V}$$

$$C = \frac{Q}{(KQ/R)}$$

$$C = \frac{R}{K}$$

$$\boxed{C = 4\pi\varepsilon_0 R}$$

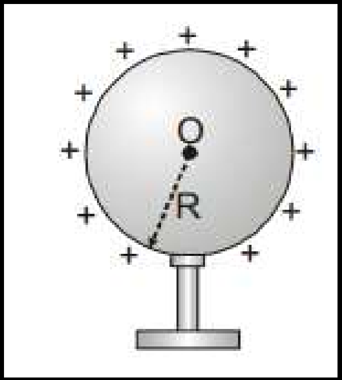
The Capacitance of a Spherical Conductor - When a charge Q is given to a isolated spherical conductor then its potential rises.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} \Rightarrow C = \frac{Q}{V} = \boxed{4\pi\varepsilon_0 R}$$

If conductor is placed in a medium then $C_{\text{medium}} = 4\pi\varepsilon R = 4\pi\varepsilon_0\varepsilon_r R$

Capacitance depends upon :

- Size and Shape of Conductor
- Surrounding medium
- Presence of other conductors nearby

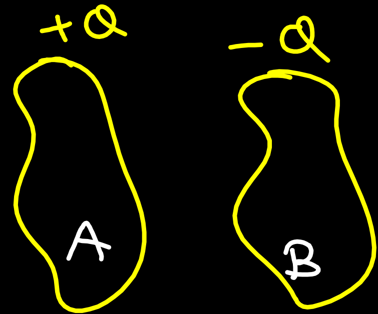


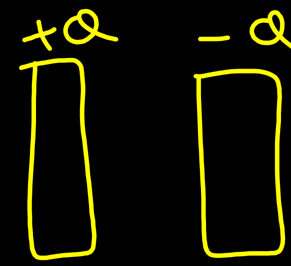
- **Condenser/Capacitor -** The pair of conductor of opposite charges on which sufficient quantity of charge may be accommodated is defined as condenser.

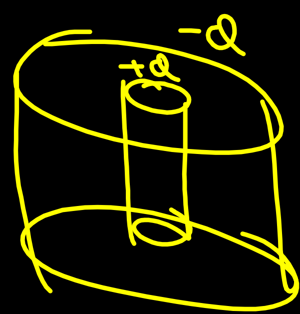
$$\Delta V = V_+ - V_-$$
$$= V_A - V_B$$

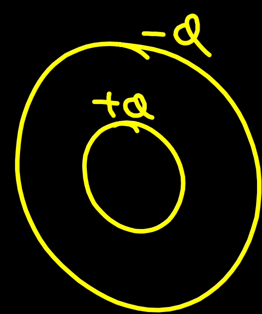
$$\boxed{C = \frac{Q}{\Delta V}}$$

$$\Delta V = |Ed|$$
$$= \left|\int \vec{E}\cdot\vec{dl}\right|$$









Statement 1: It is not possible to make a sphere of capacity 1 farad using a conducting material.
Statement 2: It is possible for earth as its radius is 6.4×10^6 m. **[Online May 26, 2012]**

(a) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation of Statement 1.

(b) Statement 1 is false, Statement 2 is true.

(c) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation of Statement 1.

(d) Statement 1 is true, Statement 2 is false.

$C = 4\pi\varepsilon_0 R = 1$ Farad

$R = 6400$ km

$\varepsilon_0 = 8.85 \times 10^{-12}$

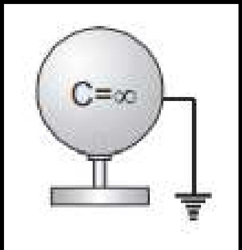
earth

$C = 4\pi\varepsilon_0 R$

$= 4\pi(8.85 \times 10^{-12})(6400 \times 1000)$

$C = 711\,\mu F$

- As the potential of the Earth is assumed to be zero, capacity of earth or a conductor connected to earth will be infinite $C = \frac{q}{V} = \frac{q}{0} = \infty$
- Actual capacity of the Earth $C = 4\pi\varepsilon_0 R = \frac{1}{9\times10^9} \times 64 \times 10^5 = 711\mu F$

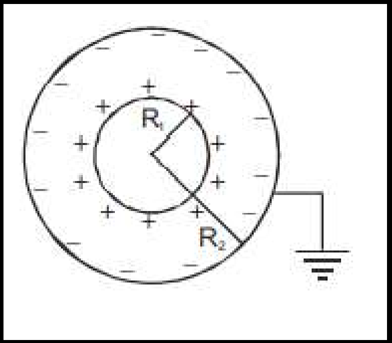


Spherical Capacitor

Outer sphere is earthed - When a charge Q is given to inner sphere it is uniformly distributed on its surface, a charge $-Q$ is induced on inner surface of outer sphere. The charge +Q induced on outer surface of outer sphere flows to earth as it is grounded.

$\mathrm{E} = 0$ for $r < \mathrm{R}_1$ and $\mathrm{E} = 0$ for $r > \mathrm{R}_2$

Potential of inner sphere V_1

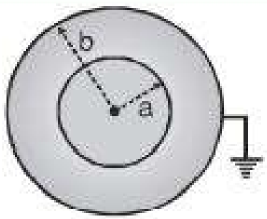
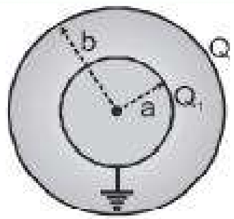
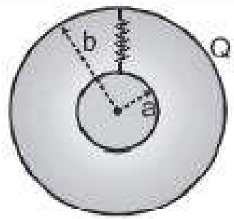
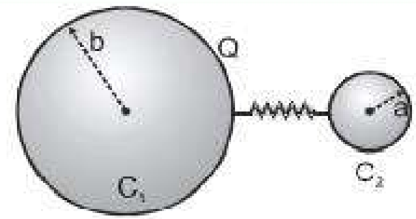


As outer surface is earthed so potential $V_2 = 0$

Potential difference between spheres $\mathrm{V} = \mathrm{V}_1 - \mathrm{V}_2$

So C between both the spheres $= \frac{Q}{V} = 4\pi\varepsilon_0 \frac{R_1 R_2}{R_2 - R_1}$ (in air or vacuum)

In presence of medium between spheres $C = 4\pi\varepsilon_r\varepsilon_0 \frac{R_1 R_2}{R_7 - R_1}$

Spherical capacitor outer is earthed	Inner is earthed and outer is given a charge	Connected and outer is given a charge	Connected spheres
			
$C = \dfrac{4\pi\varepsilon_0 ab}{b-a}$ (b > a)	$C = \dfrac{4\pi\varepsilon_0 b^2}{b-a}$ (b > a)	$C = 4\pi\varepsilon_0 b$	$C = C_1 + C_2$ $C = 4\pi\varepsilon_0(a+b)$

Capacitance of an isolated conducting sphere of radius R_1 becomes n times when it is enclosed by a concentric conducting sphere of radius R_2 connected to earth. The ratio of their radii $\left(\frac{R_2}{R_1}\right)$ is:

(A) $\frac{n}{n-1}$

(B) $\frac{2n}{2n+1}$

(C) $\frac{n+1}{n}$

(D) $\frac{2n+1}{n}$

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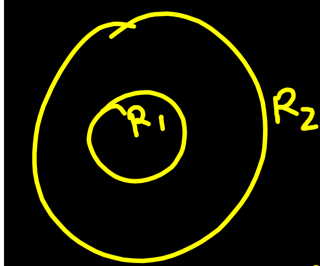
$$C_1 = 4\pi\varepsilon_0 R_1$$



$$C' = nC_1$$

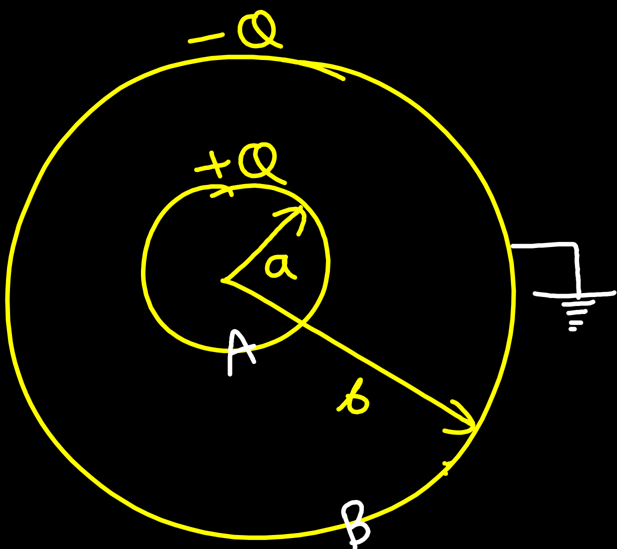
$$\frac{4\pi\varepsilon_0(R_2 R_1)}{(R_2-R_1)} = n(4\pi\varepsilon_0 R_1)$$

$$\frac{R_2}{R_2-R_1} = n$$

$$R_2 = nR_2 - nR_1$$

$$nR_1 = R_2(n-1)$$

$$\frac{R_2}{R_1} = \frac{n}{n-1}$$



$$C = \frac{Q}{\Delta V}$$

$$C = \frac{Q}{KQ\left(\frac{1}{a} - \frac{1}{b}\right)}$$

$$C = \frac{1}{K}\left(\frac{ab}{b-a}\right)$$

$$V_A = \frac{KQ}{a} + \frac{K(-Q)}{b} = KQ\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$V_B = \frac{KQ}{b} + \frac{K(-Q)}{b} = 0$$

$$\boxed{C = \frac{4\pi \varepsilon_0 ab}{b-a}} \star$$

A

+Q surface $\frac{KQ}{\text{radius}}$

-Q inside $\frac{K(-Q)}{\text{radius}}$

B

+Q outside $\frac{KQ}{\text{distance from Center}}$

-Q surface $\frac{K(-Q)}{\text{radius}}$

Cylindrical Capacitor

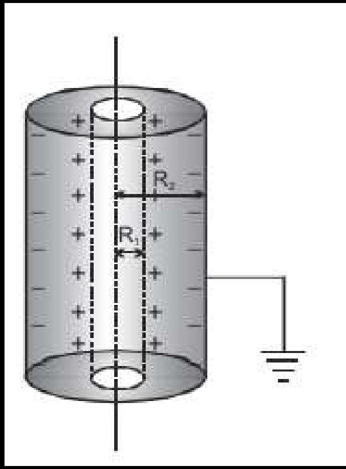
When a charge Q is given to inner cylinder it is uniformly distributed on its surface.

A charge $-\mathrm{Q}$ is induced on inner surface of outer cylinder. The charge $+Q$ induced on outer surface of outer cylinder flows to earth as it is grounded

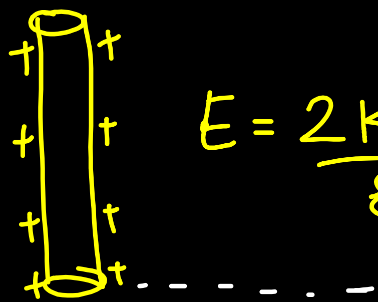
Electrical field between cylinders $E = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{Q/\ell}{2\pi\varepsilon_0 r}$

Potential difference between both cylindrical surfaces

$\mathrm{V} = \int_{\mathrm{R}_1}^{\mathrm{R}_2} \frac{\mathrm{Q}}{2\pi\varepsilon_0 \mathrm{r}\ell} \mathrm{dr} = \frac{\mathrm{Q}}{2\pi\varepsilon_0\ell} \ln\left(\frac{\mathrm{R}_2}{\mathrm{R}_1}\right).$ Capacitance $\mathrm{C} = \frac{\mathrm{Q}}{\mathrm{V}} = \frac{2\pi\varepsilon_0\ell}{\log_e(\mathrm{R}_2/\mathrm{R}_1)}$



In presence of medium $\mathrm{C} = \frac{2\pi\varepsilon_0\varepsilon_{\mathrm{r}}\ell}{\log_e(\mathrm{R}_2/\mathrm{R}_1)}$



$$E = \frac{2k\lambda}{r}$$

$$\lambda = \frac{Q}{\text{length}}$$

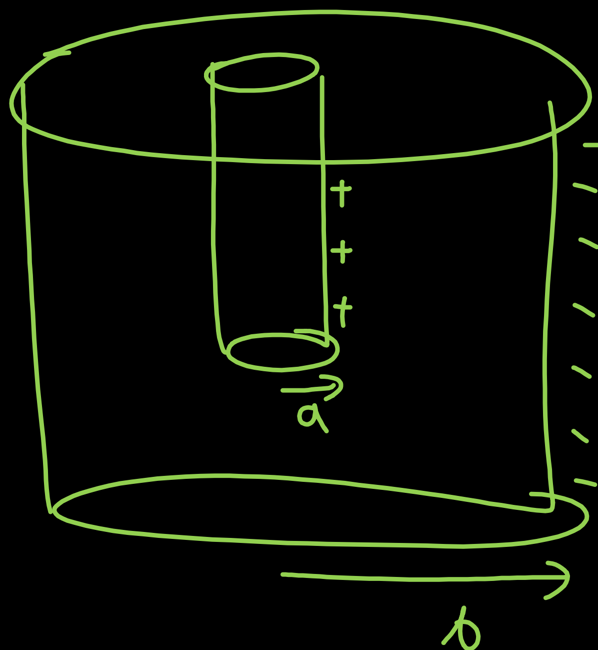
$$\lambda(\text{length}) = Q$$

$$\Delta V = \left|\int \vec{E}\cdot\vec{dr}\right| = \int \frac{2k\lambda}{r}\, dr$$

$$= 2k\lambda \log r \Big]_{r_1}^{r_2}$$

$$= 2k\lambda \log r_2 - \log r_1$$

$$\Delta V = 2k\lambda \ln\left(\frac{r_2}{r_1}\right)$$



$Q = \lambda l \qquad \Delta V = 2k\lambda \ln\left(\frac{b}{a}\right)$

$$C = \frac{Q}{\Delta V} = \frac{\cancel{\lambda} l}{2k\cancel{\lambda}\ln\left(\frac{b}{a}\right)}$$

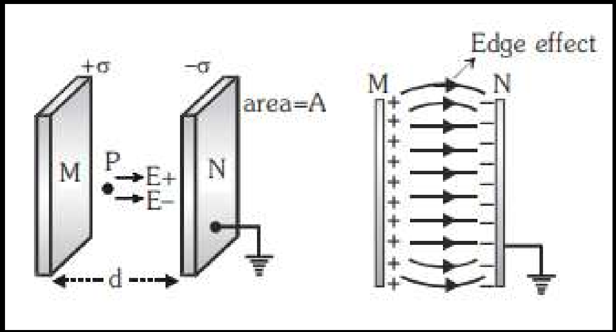
$$= \frac{4\pi\varepsilon_0 l}{2\ln\left(\frac{b}{a}\right)}$$

$$\boxed{C = \frac{2\pi\varepsilon_0 l}{\ln(b/a)}}$$

DIFFERENT TYPES OF CAPACITORS

Parallel Plate Capacitor

(i) Capacitance - It consists of two metallic plates M and N each of area A at separation d. Plate M is positively charged and plate N is earthed. If ε_r is the dielectric constant of the material medium and E is the field at a point P that exists between the two plates, then



I step : Finding electric field

$$E = E_+ + E_- = \frac{\sigma}{2\varepsilon} + \frac{\sigma}{2\varepsilon} = \frac{\sigma}{\varepsilon} = \frac{\sigma}{\varepsilon_0 \varepsilon_r} [\varepsilon = \varepsilon_0 \varepsilon_r]$$

II step : Finding potential difference

$$V = Ed = \frac{\sigma}{\varepsilon_0 \varepsilon_r} d = \frac{qd}{A\varepsilon_0 \varepsilon_r} \left(\because E = \frac{V}{d} \text{ and } \sigma = \frac{q}{A}\right)$$

III step : Finding capacitance $C = \frac{q}{V} = \frac{\varepsilon_r \varepsilon_0 A}{d}$

If medium between the plates is air or vacuum, then $\varepsilon_r = 1 \Rightarrow C_0 = \frac{\varepsilon_0 A}{d}$

so $C = \varepsilon_r C_0 = KC_0$ (where $\varepsilon_r = K =$ dielectric constant)

(ii) Force between the plates - The two plates of capacitor attract each other bec[…] they are oppositely charged.

Electric field due to positive plate $E = \frac{\sigma}{2\varepsilon_0} = \frac{Q}{2\varepsilon_0\ A}$

Force on negative charge-Q is $F = -QE = -\frac{Q^2}{2\varepsilon_0 A}$

Magnitude of force $F = \frac{Q^2}{2\varepsilon_0\ A} = \frac{1}{2}\varepsilon_0 AE^2$

Force per unit area or energy density or electrostatic pressure $= \frac{F}{A} = u = p = \frac{1}{2} f_0 F^2$

Let C is capacitance of a conductor. On being connected to a battery. It charges to a potential V from zero potential. If q is charge on the conductor at that time then q = CV. Let battery supplies small amount of charge dq to the conductor at constant potential V. Then small amount of work done by the battery against the force exerted by existing charge is

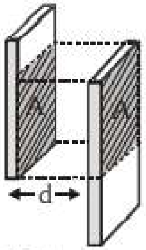
$$dW = Vdq = \frac{q}{C}dq \Rightarrow W = \int_0^Q \frac{q}{C}dq = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q \Rightarrow W = \frac{Q^2}{2C}$$

where Q is the final charge acquired by the conductor. This work done is stored as potential energy, so

$$U = \frac{Q^2}{2C} = \frac{1}{2}\frac{(CV)^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{Q}{V}\right)V^2 = \frac{1}{2}QV$$

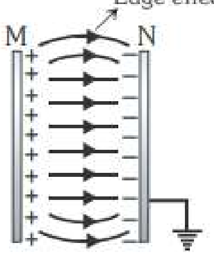
$$\therefore \quad U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

- If one of the plates of parallel plate capacitor slides relatively than C decrease (As overlapping area decreases).



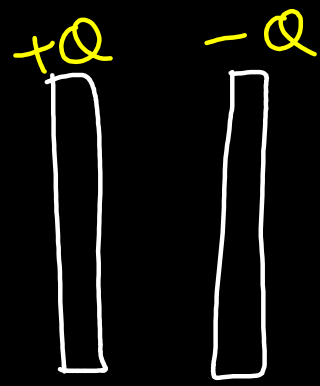
- If both the plates of parallel plate capacitor are touched each other resultant charge and potential became zero.

- Electric field between the plates of a capacitor is shown in figure. Non-uniformity of electric field at the boundaries of the plates is negligible if the distance between the plates is very small as compared to the length of the plates.

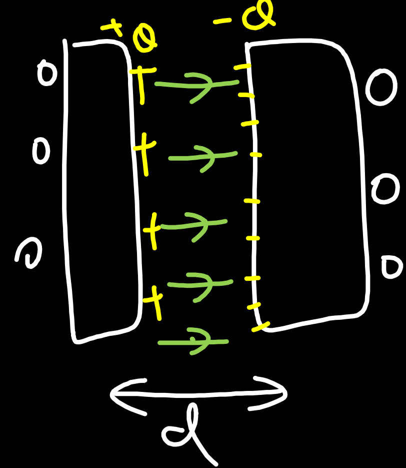


$\vec{E}$ = uniform in the centre

$\vec{E}$ = non-uniform at the edges



$\sigma = \frac{\text{charge}}{\text{area}}$



$$\boxed{E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}}$$

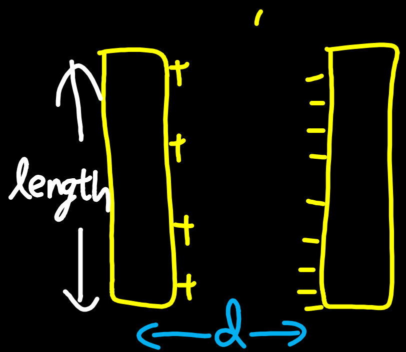
$$\Delta V = Ed = \frac{\sigma}{\varepsilon_0} d = \frac{Qd}{A\varepsilon_0}$$

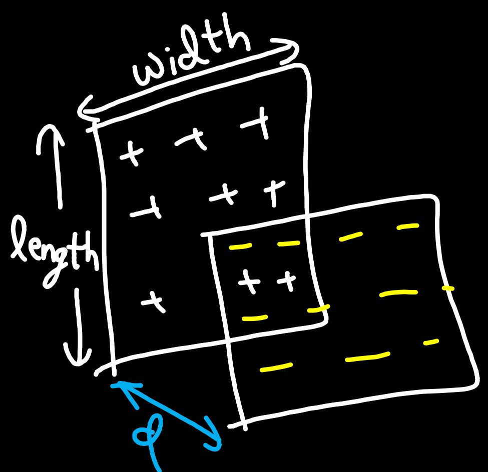
$$C = \frac{Q}{\Delta V} = \frac{\cancel{Q}}{\cancel{Q}d/A\varepsilon_0}$$

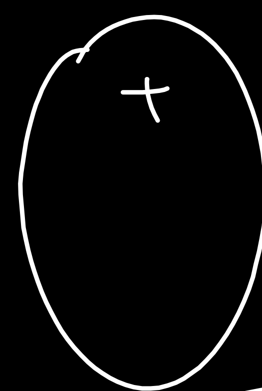
$$C = \frac{A\varepsilon_0}{d}$$

$$\boxed{C = \frac{Q}{\Delta V} = \frac{A\varepsilon_0}{d}}$$

Area of plates = (length) (width)





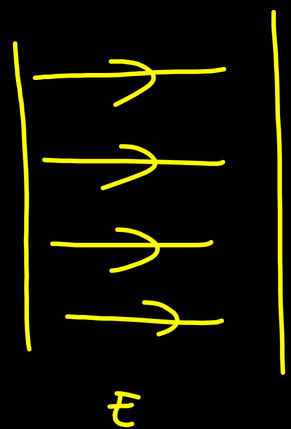


Area $= \pi r^2$

Energy of Capacitor

$$\boxed{\text{energy density} = \frac{\text{energy}}{\text{Vol}} = \frac{1}{2}\varepsilon_0 E^2}$$

Vol = Ad



$$\boxed{\text{energy} = \frac{1}{2}\varepsilon_0 E^2 A d}$$

$$C = \frac{Q}{\Delta V}$$

$$\frac{A\varepsilon_0}{d} = C$$

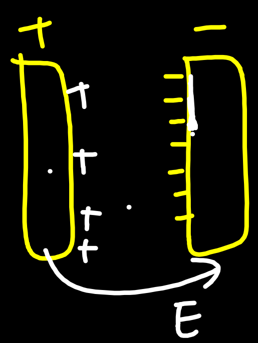
$$A\varepsilon_0 = Cd$$

$$\text{energy} = \frac{1}{2}E^2 d^2 C$$

$$\Delta V = Ed$$

$$U = \boxed{\frac{1}{2}C(\Delta V)^2 = \frac{Q^2}{2C} = \frac{1}{2}Q\Delta V}$$

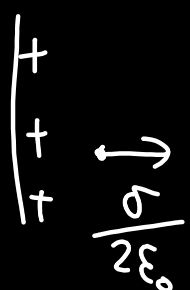
Force b/w Plates



$$\text{force} = (\text{charge})(\text{field})$$

$$= (Q \text{ of plate } 2)\begin{pmatrix}\text{field due to} \\ 1 \text{ over } 2\end{pmatrix}$$

$$\boxed{F = Q\frac{\sigma}{2\varepsilon_0} = \frac{Q^2}{2A\varepsilon_0} = \frac{\sigma^2 A}{2\varepsilon_0}}$$



$$\frac{\text{Force}}{\text{area}} = \text{Pressure} = \frac{\sigma^2}{2\varepsilon_0}$$

If Medium is Changed

vaccum $\rightarrow$ medium

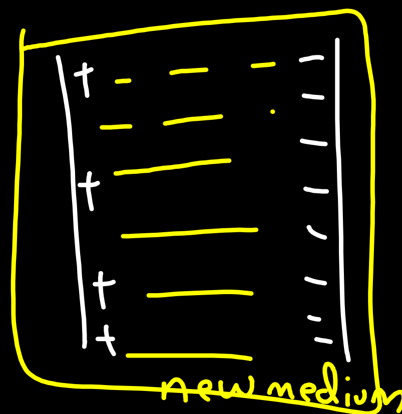
$\varepsilon_0 \longrightarrow \varepsilon_0 \varepsilon_r$ $\rightarrow$ relative permittivity

or

$\varepsilon_0 k$ $\rightarrow$ dielectric constant

$K_{H_2O} = 81$

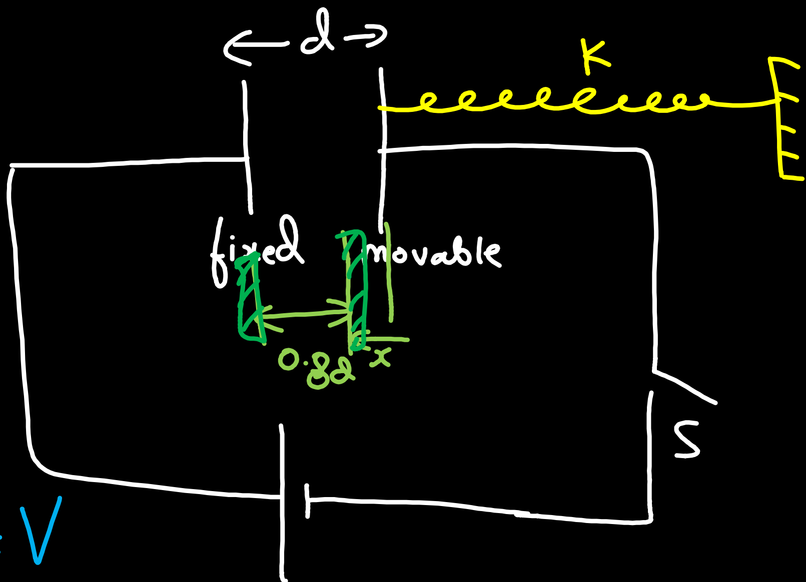
$K_{metal} = \infty$



$$C = \frac{A\varepsilon_0}{d}$$

$$C'_{new} = \frac{A\varepsilon_0\varepsilon_r}{d} = \frac{A\varepsilon_0 k}{d} = kC_{vaccum}$$

Q



initially uncharged & natural

S close

Then distance b/w plates in new equilibrium is (0.8d)

$x = 0.2d$

Extra Info

Battery Voltage $= V$

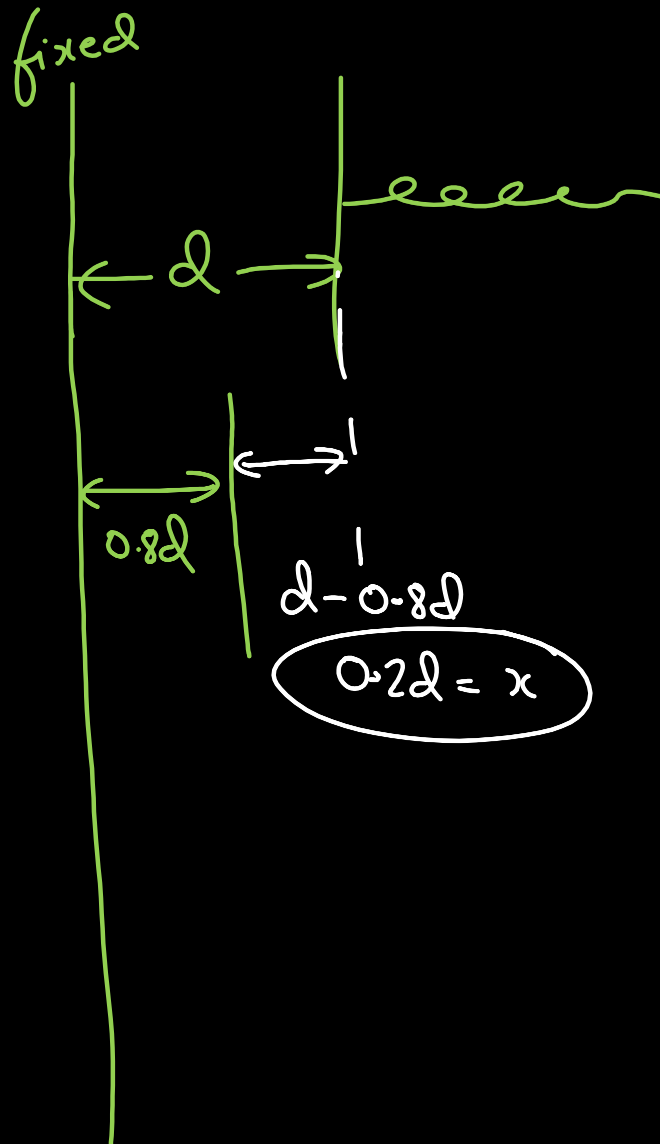
$$C = \frac{Q}{V}$$

$$Q = CV = \left(\frac{A\varepsilon_0}{0.8d}\right)V$$

$$\sigma = \frac{Q}{A}$$

$$K(0.2d) = F$$

$$K(0.2d) = \frac{\sigma^2 A}{2\varepsilon_0}$$



$d - 0.8d$

$0.2d = x$

A parallel plate capacitor with plates of area 1 m^2 each, are at a separation of 0.1 m. If the electric field between the plates is 100 N/C, the magnitude of charge on each plate is :

(Take $\epsilon_0 = 8.85 \times 10^{-12} \dfrac{C^2}{N-M^2}$) [12 Jan. 2019 II]

(a) 7.85×10^{-10} C (b) 6.85×10^{-10} C

(c) 8.85×10^{-10} C (d) 9.85×10^{-10} C

$A = 1 m^2$

$d = 0.1 m$

$E = 100$

$Q = ?$

$E = \dfrac{\sigma}{\varepsilon_0}$

$E = \dfrac{Q}{A\varepsilon_0}$

$Q = E A \varepsilon_0$

$= 100 \times 1 \times 8.85 \times 10^{-12}$

A parallel plate capacitor is made of two circular plates separated by a distance 5 mm and with a dielectric of dialectric constant 2.2 between them. When the electric field in the dielectric is 3×10^4 V/m the charge density of the positive plate will be close to: **[2014]**

(a) $6\times10^{-7}\ C/m^2$ (b) $3\times10^{-7}\ C/m^2$

(c) $3\times10^{4}\ C/m^2$ (d) $6\times10^{4}\ C/m^2$

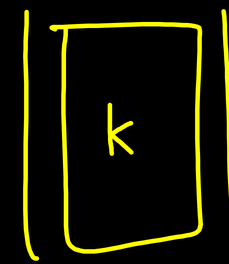
$A = \pi r^2$

$d = 5mm$

$k = 2.2$

$E = 3\times10^4$

$\sigma = ??$

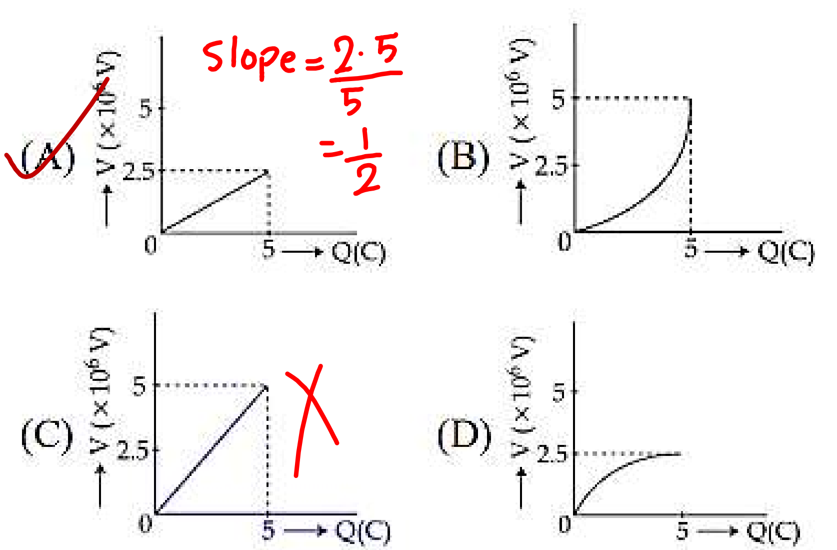


$E = \dfrac{\sigma}{\varepsilon_0}$ Vaccum

$E_{med} = \dfrac{\sigma}{\varepsilon_0 \varepsilon_r} = \dfrac{\sigma}{\varepsilon_0 k}$

$\sigma = (E_{med})\varepsilon_0 k = 3\times10^4 \times 8.85\times10^{-12}\times 2.2$

A condenser of 2 μF capacitance is charged steadily from 0 to 5C. Which of the following graph represents correctly the variation of potential difference (V) across it's plates with respect to the charge (Q) on the condenser ?



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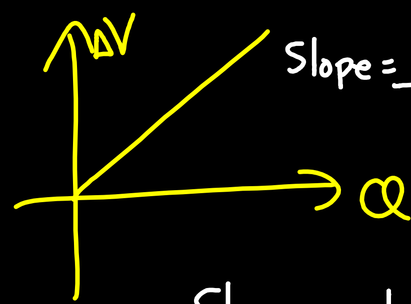
$C = 2\mu F$

$Q = 0$ to $5c$



Slope $= \frac{y}{x}$

Slope $= \frac{1}{C} = \frac{1}{2}$

$Q = C\Delta V$

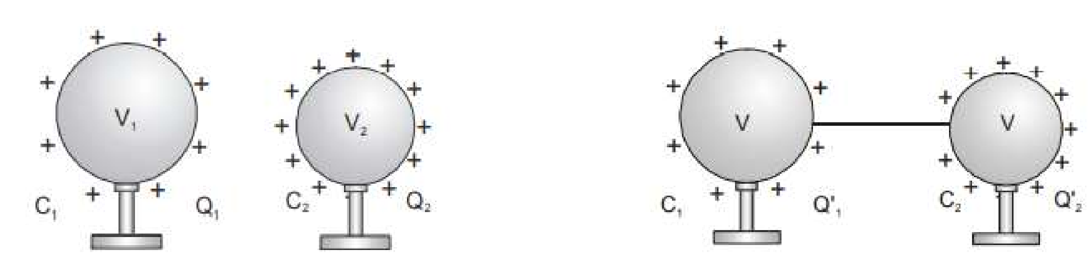
$Q \propto \Delta V$

$\frac{1}{C} = \frac{\Delta V}{Q}$

Redistribution of charges and loss of energy

When two charged conductors are connected by a conducting wire then charge flows from a conductor at higher potential to that at lower potential. This flow of charge stops when the potential of two conductors became equal.

Let the amounts of charges after the conductors are connected are $Q_1{}'$ and $Q_2{}'$ respectively and potential is V then



(Before connection) (After connection)

Common potential - According to law of Conservation of charge

$$Q_{\text{before connection}} = Q_{\text{after connection}} \Rightarrow C_1 V_1 + C_2 V_2 = C_1 V + C_2 V$$

Common potential after connection

$$V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$$

Charges after connection

$$Q_1' = C_1V = C_1\left(\frac{Q_1+Q_2}{C_1+C_2}\right) = \left(\frac{C_1}{C_1+C_2}\right)Q \qquad \text{(Q : Total charge on system)}$$

$$Q_2' = C_2V = C_2\left(\frac{Q_1+Q_2}{C_1+C_2}\right) = \left(\frac{C_2}{C_1+C_2}\right)Q$$

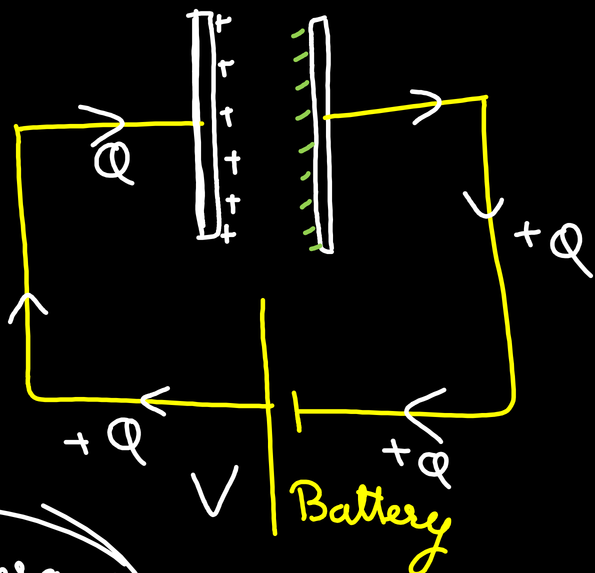
Ratio of the charges after redistribution $\boxed{\frac{Q_1'}{Q_2'} = \frac{C_1V}{C_2V} = \frac{R_1}{R_2} \text{ (in case of spherical conductors)}}$

Loss of energy in redistribution – When charge flows through the conducting wire then **energy is lost mainly on account of Joule effect** , electrical energy is converted into heat energy, so change in energy of this system,

$$\Delta U = U_f - U_i \Rightarrow \left(\frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2\right) - \left(\frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2\right) \Rightarrow \Delta U = -\frac{1}{2}\left(\frac{C_1C_2}{C_1+C_2}\right)(V_1 - V_2)^2$$

Here negative sign indicates that energy of the system decrease in the process

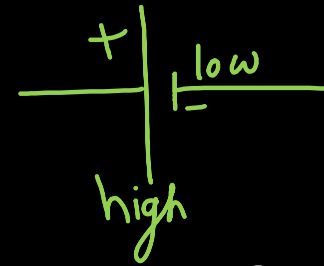
Capacitor as a Circuit Element



charge on Capacitor

$$\boxed{Q = C\Delta V}$$

Battery



Emf = ε

or

Potential diff = V

$$\text{WD by battery} = (Q_{\text{flow}})(\Delta V)$$

$$= (CV)(V)$$

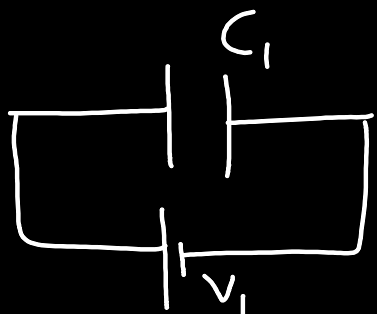
$$= CV^2$$

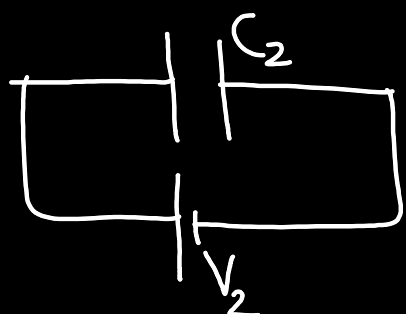
50% energy is lost in heat

energy stored in $C = \frac{1}{2}CV^2$

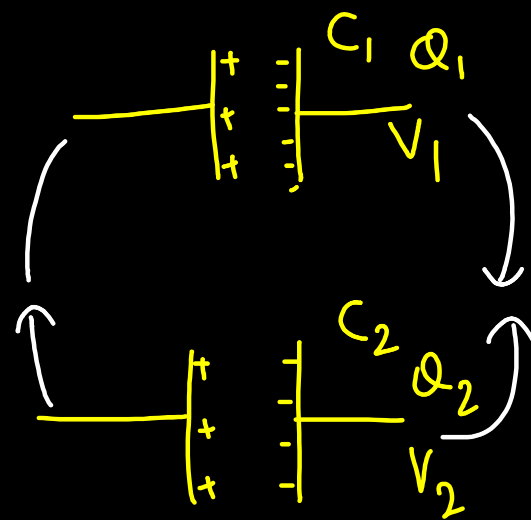
$$WD_{\substack{\text{external}\\\text{battery}}} = \Delta U + \text{heat loss}$$

Redistribution of Charges between Capacitors



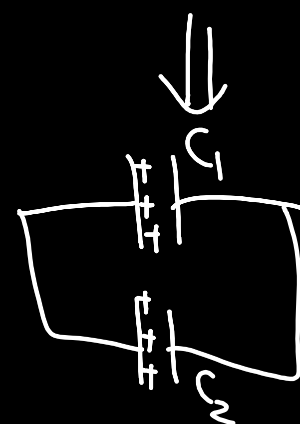


Batteries removed



\# charge conservation

\# V_{final} same



charge flow high V to low V

Jab Tak V_{final} Same of both.

$$C = \frac{Q}{\Delta V}$$

$$Q = C\Delta V$$

$$\frac{Q}{C} = \Delta V$$

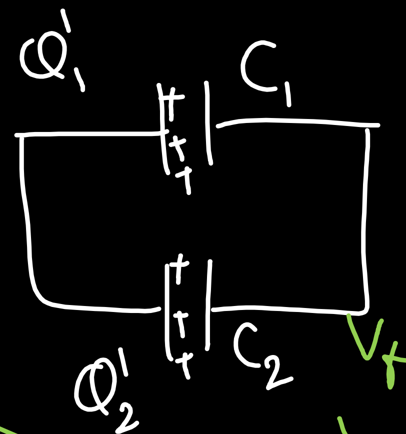
$$Q = C\Delta V$$

$$Q = CV$$

$Q_1 = C_1V_1$

$Q_2 = C_2V_2$

$\Rightarrow$



V_f

$Q_1' = C_1V_f$

$Q_2' = C_2V_f$

$$Q_1' + Q_2' = Q_1 + Q_2$$

$$C_1V_f + C_2V_f = C_1V_1 + C_2V_2$$

$$\boxed{V_f = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}}$$

$$U_{initial} = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2$$

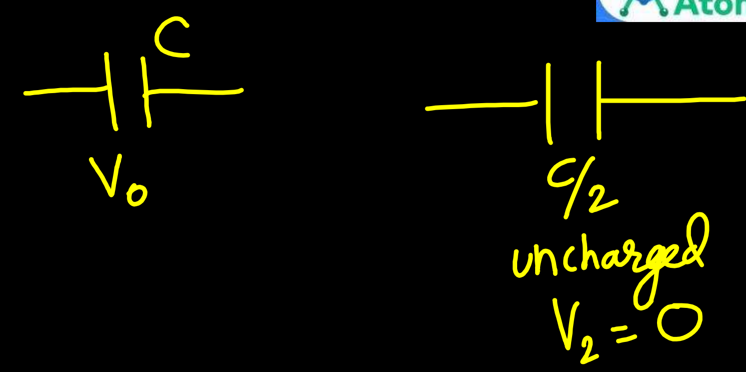
$$U_{final} = \frac{1}{2}C_1V_f^2 + \frac{1}{2}C_2V_f^2$$

$$\text{heat loss} = |\Delta U| = U_{ini} - U_{final} = \frac{1}{2}\frac{C_1C_2}{C_1+C_2}(V_1 - V_2)^2$$

A capacitor C is fully charged with voltage V_0. After disconnecting the voltage source, it is connected in parallel with another uncharged capacitor of capacitance $\frac{C}{2}$. The energy loss in the process after the charge is distributed between the two capacitors is : **[Sep. 04, 2020 (II)]**

(a) $\frac{1}{2}CV_0^2$ (b) $\frac{1}{3}CV_0^2$

(c) $\frac{1}{4}CV_0^2$ (d) $\frac{1}{6}CV_0^2$



$$\frac{1}{2}\frac{C\,C/2}{C+C/2}(V_0-0)^2$$

$$\frac{1}{2}\frac{C\cancel{C}/\cancel{2}}{3\cancel{C}/\cancel{2}} \times V_0^2$$

$$\frac{1}{6}CV_0^2$$

Derivation

Charge Conservation

$$Q_1' + Q_2' = Q_1 + Q_2$$

$$C_1 V_f + C_2 V_f = C_1 V_1 + C_2 V_2$$

$$V_f = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$$

$$Q_1' = C_1 V_f \qquad Q_2' = C_2 V_f$$

$$W_{ini} - W_{final} = \text{heat}$$

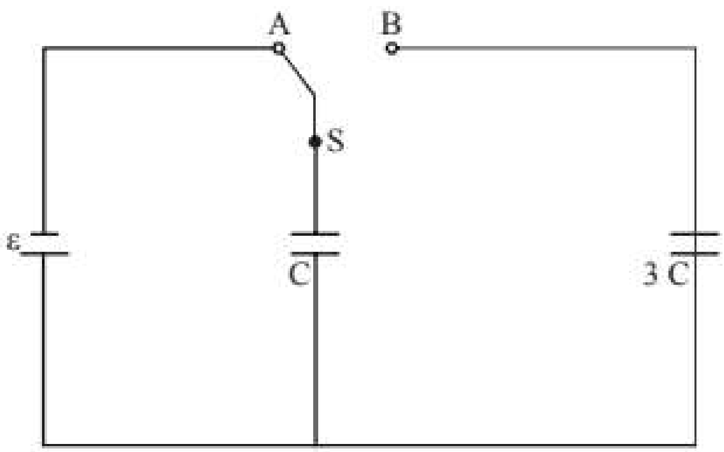
$$\left(\frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_2^2\right) - \left(\frac{1}{2} C_1 V_f^2 + \frac{1}{2} C_2 V_f^2\right)$$

$$\frac{1}{2}(C_1 + C_2) V_f^2$$

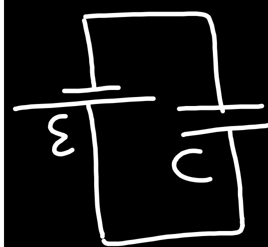
$$\frac{1}{2} \frac{C_1 C_2}{C_1 + C_2} (V_1 - V_2)^2$$

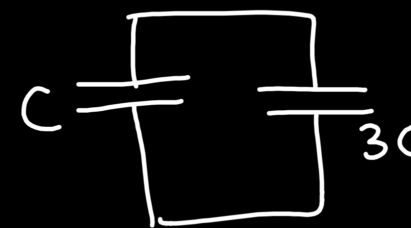
In the figure shown, after the switch 'S' is turned from position 'A' to position 'B', the energy dissipated in the circuit in terms of capacitance 'C' and total charge 'Q' is:

[12 Jan. 2019 I]



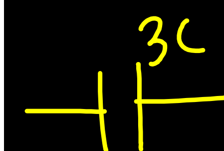
(a) $\frac{1}{8}\frac{Q^2}{C}$ (b) $\frac{3}{8}\frac{Q^2}{C}$ (c) $\frac{5}{8}\frac{Q^2}{C}$ (d) $\frac{3}{4}\frac{Q^2}{C}$





$Q = C\varepsilon$

$\frac{Q}{C} = \varepsilon$



$V_2 = 0$

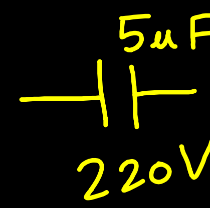
$$\frac{1}{2}\left(\frac{C \times 3C}{C + 3C}\right)(\varepsilon - 0)^2$$

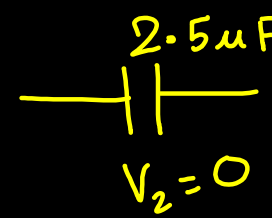
$$= \frac{3}{8} C\varepsilon^2$$

$$\frac{3}{8}\frac{Q^2}{C}$$

A 5 μF capacitor is charged fully by a 220 V supply. It is then disconnected from the supply and is connected in series to another uncharged 2.5 μF capacitor. If the energy change during the charge redistribution is $\frac{X}{100}$ J then value of X to the nearest integer is ________.

[NA Sep. 02, 2020 (I)]



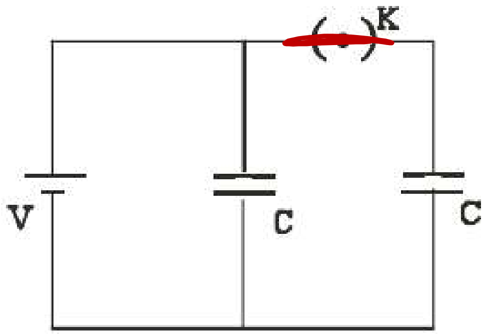


$$\Delta U = \frac{1}{2}\frac{C_1C_2}{C_1+C_2}(V_1 - V_2)^2$$

$$= \frac{1}{2}\left(\frac{5\times 2.5\,\mu\mu}{5+2.5\,\mu}\right)(220-0)^2$$

$X \approx 4$

A source of potential difference V is connected to the combination of two identical capacitors as shown in the figure. When key 'K' is closed, the total energy stored across the combination is E_1. Now key 'K' is opened and dielectric of dielectric constant 5 is introduced between the plates of the capacitors. The total energy stored across the combination is now E_2. The ratio E_1/E_2 will be :



(A) $\frac{1}{10}$ (B) $\frac{2}{5}$

(C) $\frac{5}{13}$ (D) $\frac{5}{26}$

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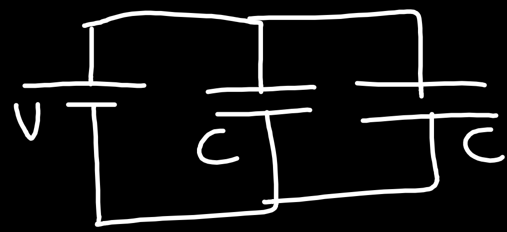
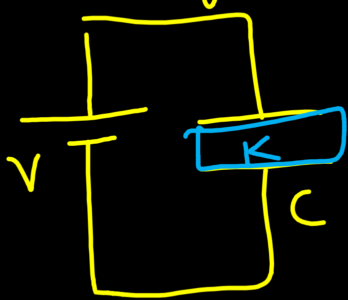
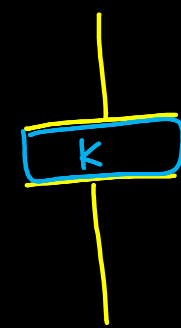


$E_1 \Rightarrow$

Finally

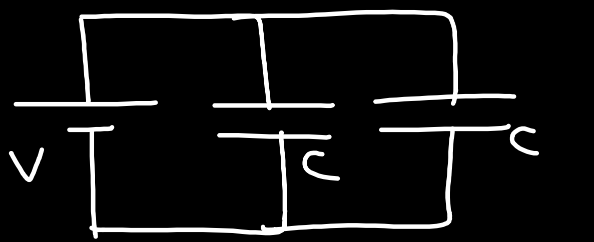




$Q =$

$E_2 =$

$\frac{E_1}{E_2} = ?$



$E_1 = \frac{1}{2}CV^2 + \frac{1}{2}CV^2$

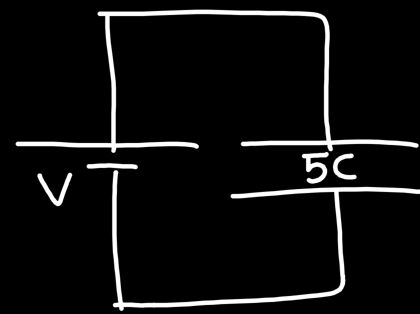
$E_1 = CV^2$

charge on C = CV

final

$C' = \frac{A\varepsilon_0 K}{d}$

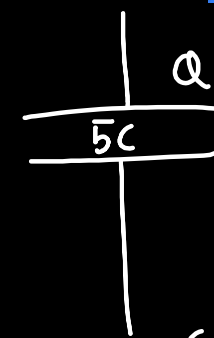
$= KC$



$E_2 = \frac{1}{2}(5C)V^2 + \frac{CV^2}{10}$

$= \frac{26}{10}CV^2$

$E_2 = \frac{13}{5}CV^2$



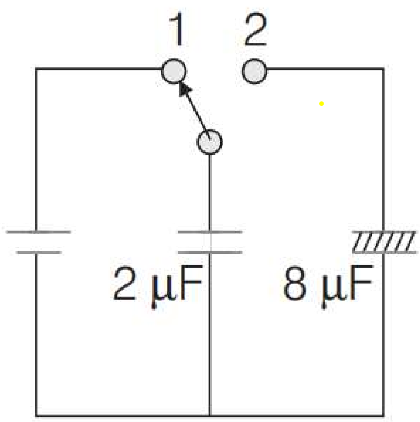
$Q = CV$

$U = \frac{(Q)^2}{2C'}$

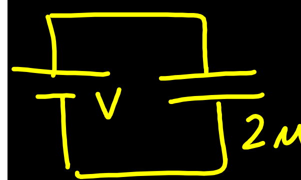
$= \frac{(CV)^2}{2\,5C}$

$= \frac{CV^2}{10}$

A $2\,\mu F$ capacitor is charged as shown in the figure. The percentage of its stored energy dissipated after the switch S is turned to position 2 is **(2011)**



(a) 0%
(b) 20%
(c) 75%
~~(d)~~ 80%



$U_i = \frac{1}{2} C V^2$

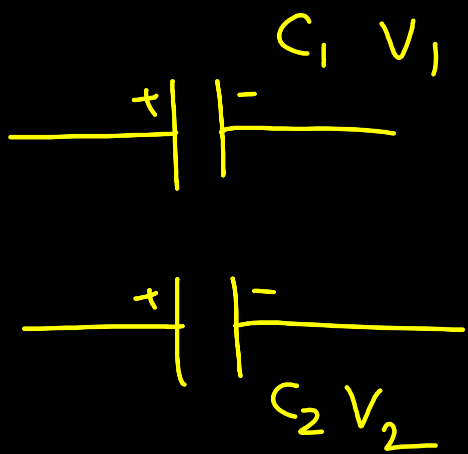
$= \frac{1}{2} 2\mu V^2$

$= V^2$

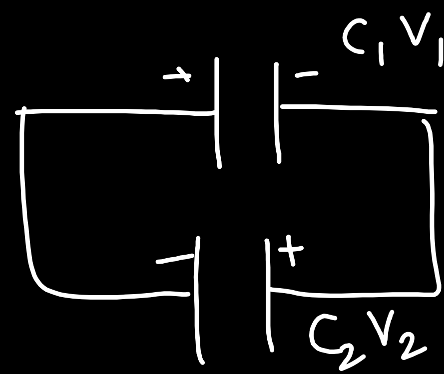
$\Delta U = \frac{1}{\cancel{2}} \frac{\cancel{2} \times 8}{8+2} (V-0)^2$

$\text{loss} = 0.8 V^2$

$\text{lost} = \frac{\text{loss}}{\text{initial}} \times 100$



Reverse Polarity Connection

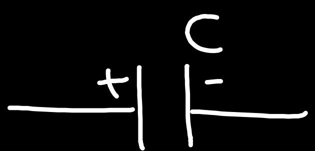


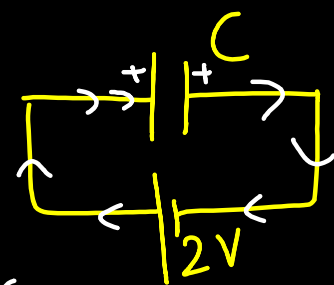
$$V_f = \frac{C_1V_1 + C_2(-V_2)}{C_1 + C_2}$$

$$\text{loss} = \frac{1}{2}\frac{C_1C_2}{C_1+C_2}\left(V_1-(-V_2)\right)^2$$

$$= \frac{1}{2}\frac{C_1C_2}{C_1+C_2}\left(V_1+V_2\right)^2.$$

Charged Capacitor Connected with Battery





$Q_{initial} = CV$

$U_{initial} = \frac{Q^2}{2C}$

$= \frac{(CV)^2}{2C}$

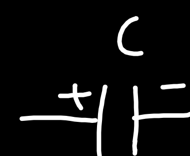
$= \frac{1}{2}CV^2$

$Q_{final} = C(\Delta V_{final})$

$= C\,2V$

$Q_{final} = 2CV$

find heat lost after connecting battery?



$Q_i = CV$

$Q_f = 2CV$

$Q_{\text{flown from battery}} = 2CV - CV$

$= CV$

$WD \text{ by Battery} = (Q_{flow})(\Delta V) = (CV)(2V) = 2CV^2$

$U_{final} = \frac{1}{2}CV_f^2 = \frac{1}{2}C(2V)^2 = \underline{2CV^2}$

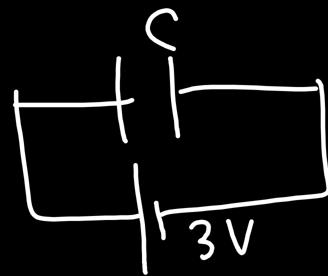
$$\Delta WD_{Batt} = \Delta U + \text{heat loss}$$

$$2CV^2 = (U_f - U_i)$$

$$2CV^2 = 2CV^2 - \frac{1}{2}CV^2 + \text{heat}$$

$$\frac{1}{2}CV^2 = \text{heat}$$

$Q_{ini} = +CV$



Find

① final charge on Capacitor

$Q_{final} = C(\Delta V_{final})$

$= C\,3V$

$= 3CV$

② Q_{flow} from battery.

$\Downarrow$

$3CV - CV$

$= 2CV$

$Q_{ini} = CV$

$Q_{final} = 3CV$

Two capacitors of capacitances C and 2C are charged to potential differences V and 2V, respectively. These are then connected in parallel in such a manner that the positive terminal of one is connected to the negative terminal of the other. The final energy of this configuration is :

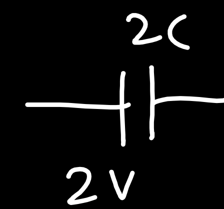
[Sep. 05, 2020 (I)]

(a) $\frac{25}{6}CV^2$

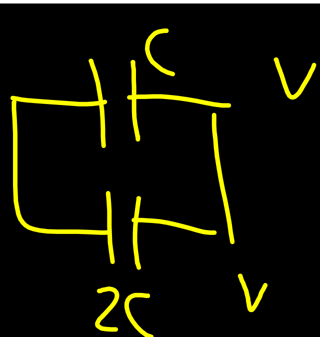
(b) $\frac{3}{2}CV^2$

(c) zero

(d) $\frac{9}{2}CV^2$



$$|V_{final}| = \frac{CV + 2C(-2V)}{C + 2C} = \frac{\cancel{3C}V}{\cancel{3C}} = V$$



$$U_{final} = \frac{1}{2}C(V)^2 + \frac{1}{2}(2C)(V)^2$$

$$= \frac{3CV^2}{2}$$

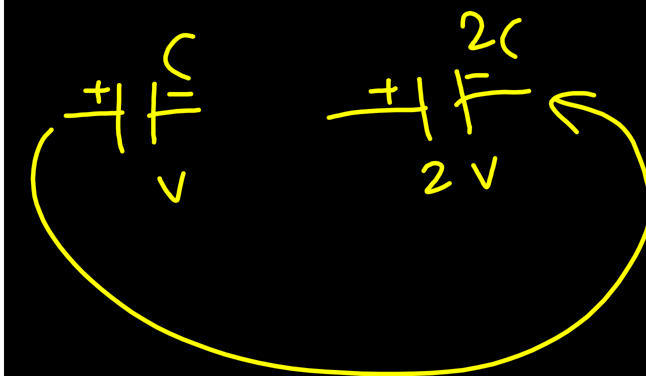
G.O.A.T.

11^{th}

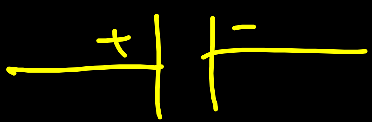
12^{th} Bounce back

A parallel plate capacitor of capacitance C is connected to a battery and is charged to a potential difference V. Another capacitor of capacitance $2C$ is similarly charged to a potential difference $2V$. The charging battery is now disconnected and the capacitors are connected in parallel to each other in such a way that the positive terminal of one is connected to the negative terminal of the other. The final energy of the configuration is **(1995)**

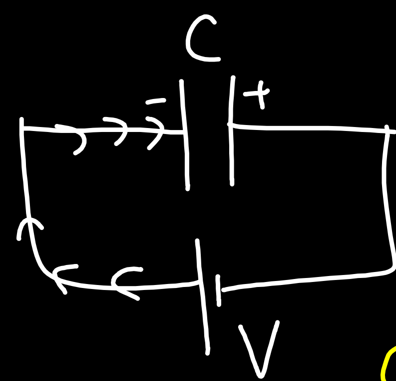
(a) zero (b) $\frac{3}{2}CV^2$ (c) $\frac{25}{6}CV^2$ (d) $\frac{9}{2}CV^2$



Charged C connected with Reverse Polarity



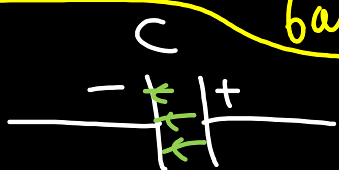
$Q_{init} = CV$



After Connection find heat loss ??

$-ve$ of C connected with $+ve$ of battery

initial



$Q_{ini} = CV$

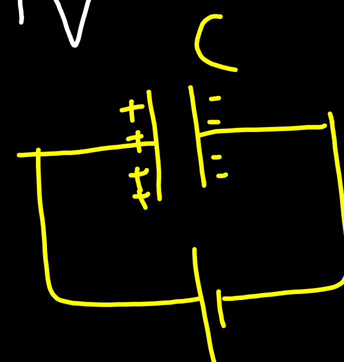
$U_{ini} = \frac{1}{2} C V^2$

finally

$Q_{final} = CV$

$U_{final} = \frac{1}{2} C V^2$

finally



$Q_{final} = CV$

$Q_{\text{flown from battery}} = +2CV$

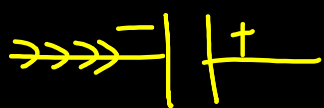
$$WD_{Bat} = \Delta U + heat$$

$$(2cv)(v) = U_f - U_i + heat$$

$$2cv^2 = 0$$

$$2cv^2 = heat$$

Initially



$Q_{ini} = CV$

finally

$Q_{final} = CV$

$Q_{flown} = (\text{final} - \text{initial})$ with polarity

$= 2CV$

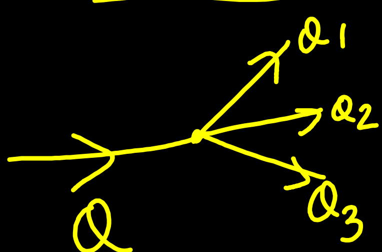
$WD_{Bat} = (Q_{flow})(\text{p.d.})$

$= (2CV)(V) = \underline{\underline{2CV^2}}$

KIRCHHOFF'S LAWS

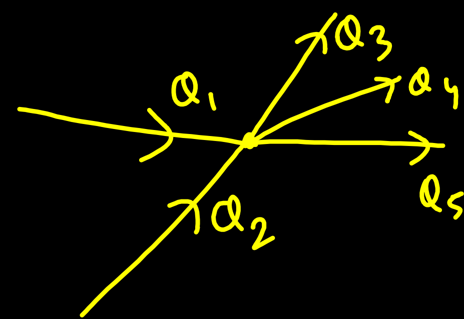
(i) Junction Rule - Sum of charges present on the plates of capacitors connected at a junction is equal to zero (If initially all the capacitors are uncharged) (while adding charges of different plates battery can be neglected as net charge on battery is zero).
(ii) **Loop Rule** - In any closed loop the algebraic sum of potential drops across different elements is equal to zero.

① Junction Rule



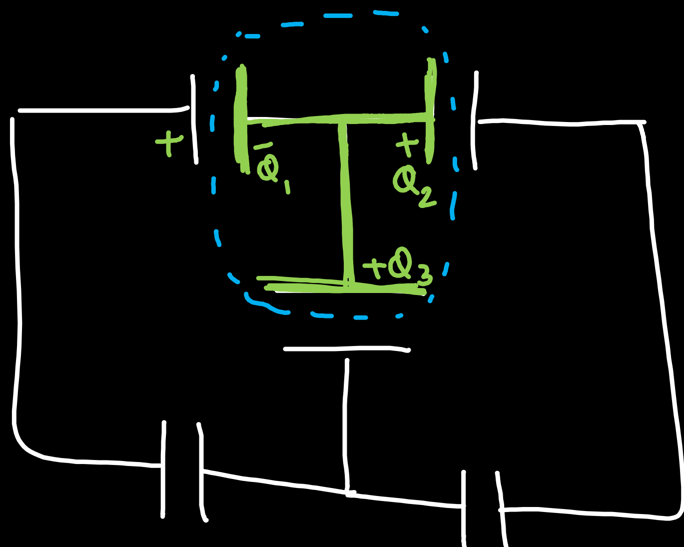
$$Q = Q_1 + Q_2 + Q_3$$

(charge conservation)



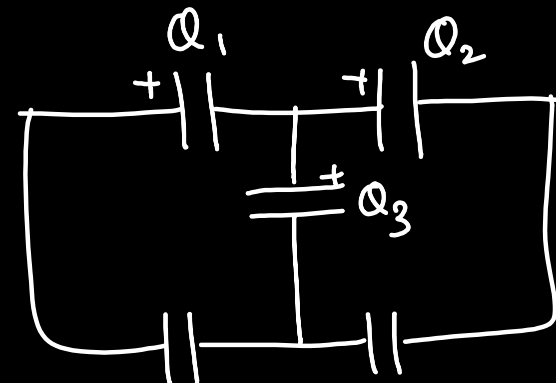
$$Q_1 + Q_2 = Q_3 + Q_4 + Q_5$$

Concept = <u>island</u> (isolated) (initial total charge = final total charge)



initially all No charge

finally

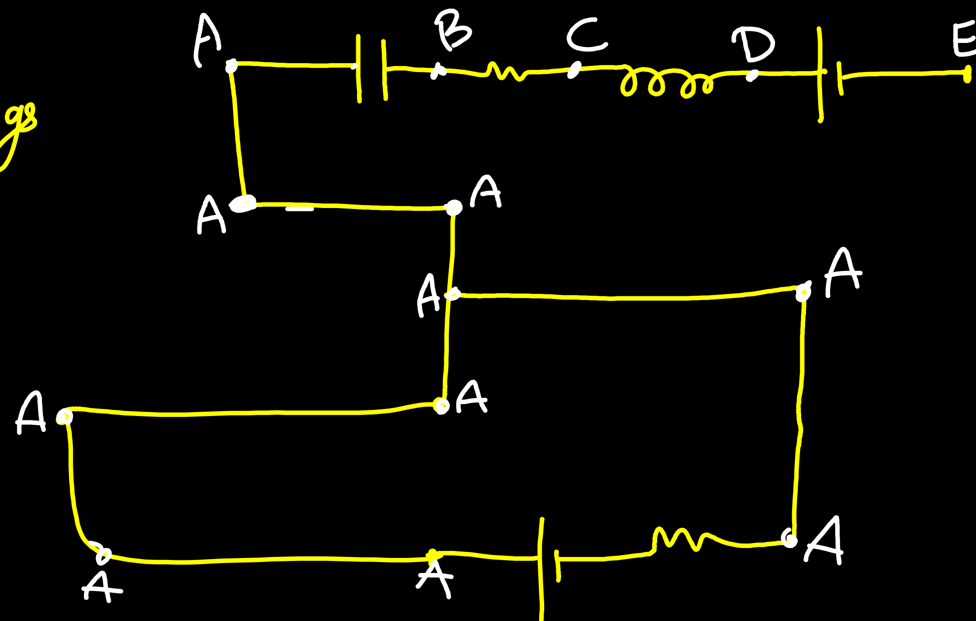


$$0 = -Q_1 + Q_2 + Q_3$$

Loop Law

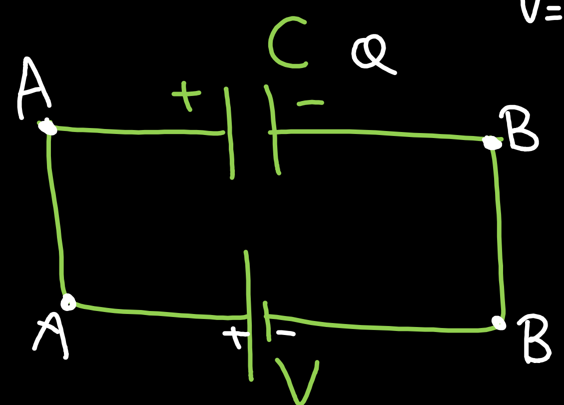
(energy conservation)

Points namings



$Q = CV$

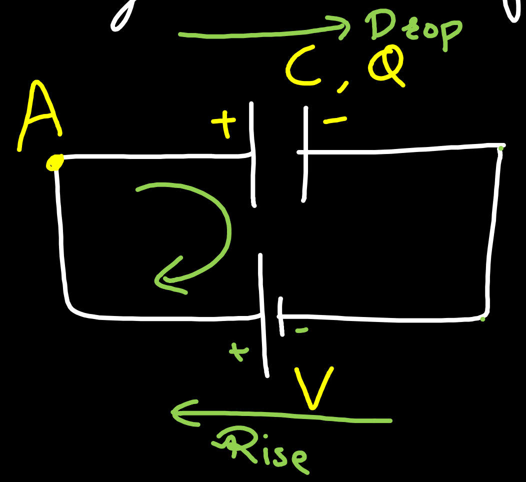
$V = \frac{Q}{C}$



$\Delta V_{AB} = V$

$\Delta V_{AB} = \frac{Q}{C} \Rightarrow \frac{Q}{C} = V$ → eq of loop law.

→ alegrebric sum of all ΔV in loop is 0.



Rise +

Drop -

start

end

$$V_A - \frac{Q}{C} + V = V_A$$

$$0 - \frac{Q}{C} + V = 0$$

$$V = \frac{Q}{C}$$

COMBINATION OF CAPACITORS

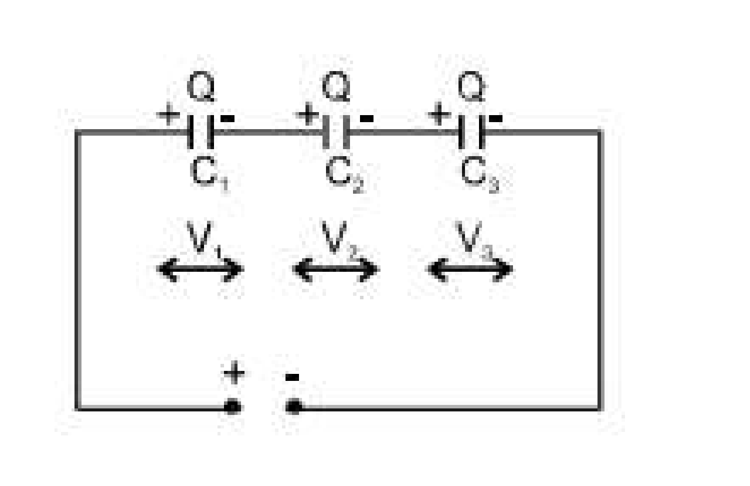
Series Combination (i) When initially uncharged capacitors are connected as shown in the (ii) All capacitors will have same charge but different potential difference across then.

(iii) We can say that

$V_1 = \frac{Q}{C_1}$ $V_1 =$ potential across C_1

$Q =$ charge on positive plate of C_1 $C_1 =$ capacitance of capacitor similarly

$V_2 = \frac{Q}{C_2}$, $V_3 = \frac{Q}{C_3}$ ... .



(iv) $V_1 : V_2 : V_3 = \frac{1}{C_1} : \frac{1}{C_2} : \frac{1}{C_3}$

We can say that potential difference across capacitor is inversely proportional to its capacitance in series combination. $V \propto \frac{1}{C}$

Note : In series combination the smallest capacitor gets maximum potential.

(v) $V_1 = \dfrac{\frac{1}{C_1}}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\ldots\ldots} V$ $\quad V_2 = \dfrac{\frac{1}{C_2}}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\ldots\ldots} V$ $\quad V_3 = \dfrac{\frac{1}{C_3}}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\ldots\ldots} V$

Where $V - V_1 + V_2 + V_3$

(vi) Energy stored in the combination

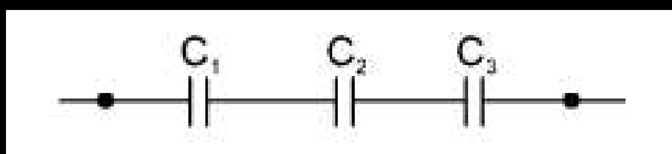
$$U_{\text{combination}} = \frac{Q^2}{2C_1} + \frac{Q^2}{2C_2} + \frac{Q^2}{2C_3} \quad U_{\text{combination}} = \frac{Q^2}{2C_{eq}}$$

Eneryy supplied by the battery in charging the combination

$$U_{battery} = Q \times V = Q \cdot \frac{Q}{C_{eq}} = \frac{Q^2}{C_{eq}} \quad \frac{U_{combination}}{U_{battery}} = \frac{1}{2}$$

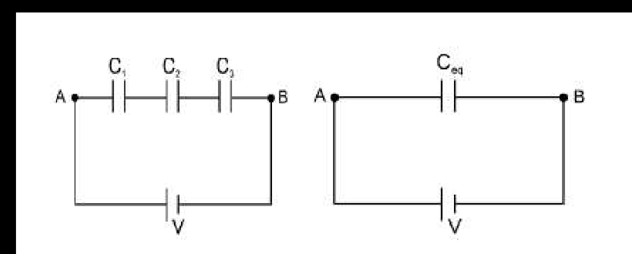
Note : Half of the energy supplied by the battery is stored in form of electrostatic energy and half of the energy is converted into heat through resistance.

<u>**Formulae Derivation for series combination**</u>



meaning of equivalent capacitor

$$C_{eq} = \frac{Q}{V}$$

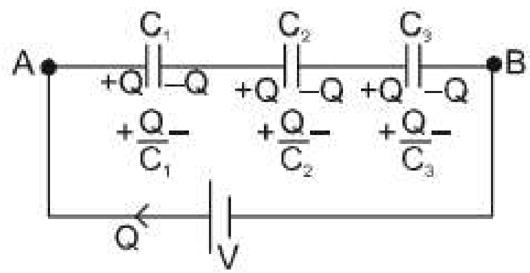


Now, Initially, the capacitor has no charge. Applying Kirchhoff's voltage law

$$\frac{-Q}{C_1} + \frac{-Q}{C_2} + \frac{-Q}{C_3} + V = 0.$$

$$V = Q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right]$$

$$\frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$



$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

in general $$\frac{1}{C_{eq}} = \sum_{n=1}^{n} \frac{1}{C_n}$$

Parallel Combination

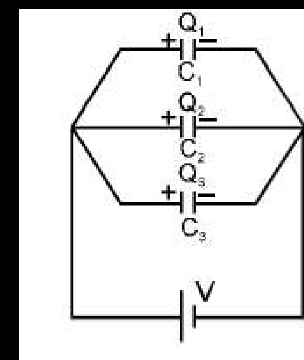
1. When one plate of each capacitors (more than one) is connected together and the other plate of each capacitor is connected together, such combination is called parallel combination.
2. All capacitors have same potential difference but different charges.
3. We can say that :

   $Q_1 = C_1V$

   Q_1 = Charge on capacitor C_1

   C_1 = Capacitance of capacitor C_1

   V = Potential of across capacitor C_1
4. $Q_1 : Q_2 : Q_3 = C_1 : C_2 : C_3$



The charge on the capacitor is proportional to its capacitance $Q \propto C$

5. $Q_1 = \frac{C_1}{C_1+C_2+C_3} Q$; $Q_2 = \frac{C_2}{C_1+C_2+C_3} Q$; $Q_3 = \frac{C_3}{C_1+C_2+C_3} Q$

Where $Q = Q_1 + Q_2 + Q_3 \ldots \ldots$

Note : Maximum charge will flow through the capacitor of largest value.

6. Energy stored in the combination:

$$V_{\text{combination}} = \frac{1}{2} C_1 V^2 + \frac{1}{2} C_2 V^2 + \cdots = \frac{1}{2}(C_1 + C_2 + C_3 \ldots\ldots)V^2 = \frac{1}{2} C_{eq} V^2$$

$$U_{\text{battery}} = QV = CV^2 = \frac{U_{\text{combination}}}{U_{\text{battery}}} = \frac{1}{2}$$

Note : Half of the energy supplied by the battery is stored in form of electrostatic energy and half of the energy is converted into heat through resistance.

Formulae Derivation for parallel combination :

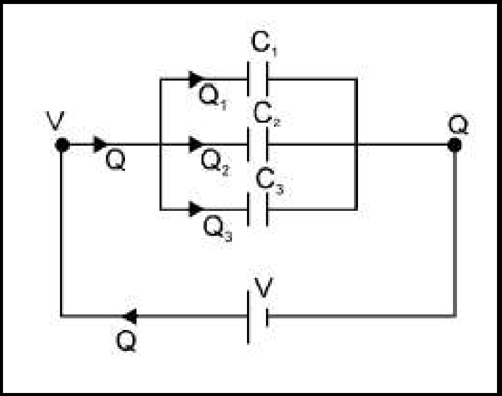
$$Q = Q_1 + Q_2 + Q_3 = C_1V + C_2V + C_3V$$

$$= V(C_1 + C_2 + C_3)$$

$$\frac{Q}{V} = C_1 + C_2 + C_3$$

$$C_{eq} = C_1 + C_2 + C_3$$

In general $C_{eq} = \sum_{n=1}^{n} C_n$



Mixed Combination

The combination which contains mixing of series parallel combinations or other complex combinations fall in mixed category.

There are two types of mixed combinations
(i) Simple (ii) Complex.

Simple Mixed Combination

Combinations which can be easily converted in series parallel combination.

Complex Mixed Combination

The given combination can not be simplified by series or parallel combination such a combination is solved by using Kirchhoff's Law and other techniques. A special case of this combination is Wheatstone bridge.

Wheatstone bridge

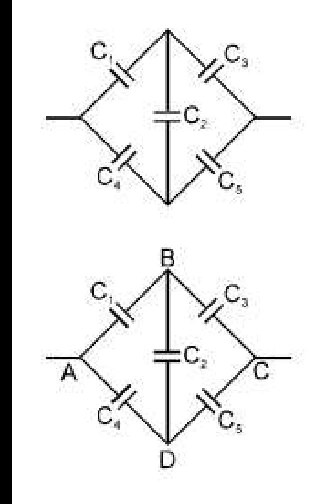
If $\frac{C_1}{C_4} = \frac{C_3}{C_5}$ or $C_1C_5 = C_3C_4$ then

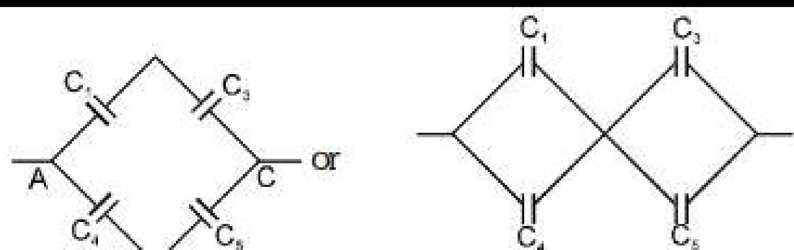
(i) Such combination is called balanced Wheatstone bridge.

(ii) In this case $V_B = V_D$.

(iii) Charge on $C_2 = 0$

(iv) The equivalent can be converted in to given circuits:



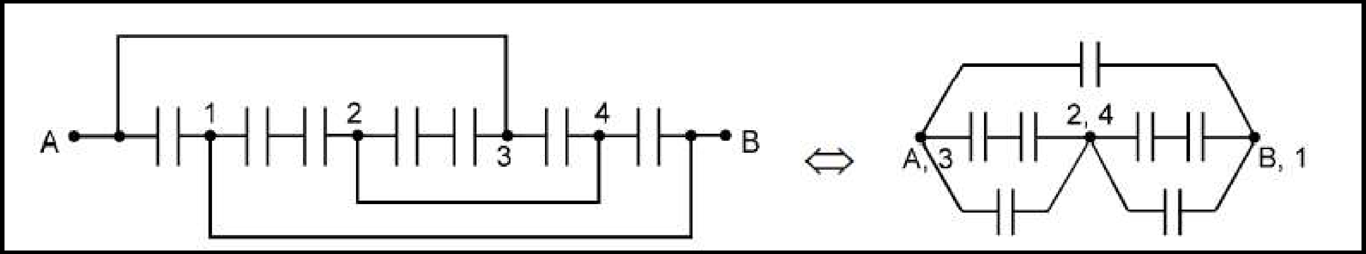


(v) $C_{eq} = \dfrac{(C_1 + C_4)(C_3 + C_5)}{C_1 + C_2 + C_3 + C_5} = \dfrac{C_1C_3}{C_1 + C_3} + \dfrac{C_4C_5}{C_4 + C_5}$

(vi) If $C_1 = C_4 = C_3 = C_5 = C$ then $C_{eq} = C$

Other important circuit solving techniques
(Applicable in both capacitive and resistive networks) .

(A) Equipotential technique – All the junction which are at equal potential (such as junction connected by a connecting wire) can be replaced by a single junction. So redraw the circuit to get it simplified

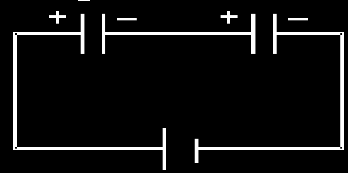


(B) Infinite Circuits - Assume equivalent capacitance / resistance to be Ceq/ R eq of whole network , then add one or more branch (repetitive) in infinite network . Calculate equivalent of this new circuit. It should again be equal to Ceq / Req.

Note : If all the resistance / capacitance of a circuit are made k times then equivalent will also become K-times

GOLDEN POINT

•For a given voltage to store maximum energy capacitors should be connected in parallel.



If N identical capacitors each having breakdown voltage V are joined in

(I) series then the break down voltage of the combination is equal to NV

(ii)parallel then the breakdown voltage of the combination is equal to V.

•Two capacitors are connected in series with a battery. Now battery is removed and loose wires connected together then final charge on each capacitor is zero.

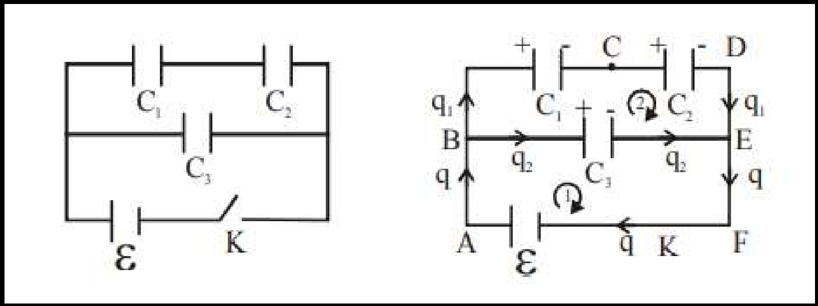
•If N identical capacitors are connected then

Cseries = N   Cparallel= NC

In DC capacitor's offers infinite resistance in steady state, so there will be no current flows through capacitor branch

CIRCUIT SOLVING

For the given circuit we are interested in finding charges and potential difference across, C_1, C_2 and C_2



Now, In loop 1 Using KVI , we get

$$\varepsilon - \frac{q_2}{C_3} = 0 \qquad \Rightarrow q_2 = C_3\varepsilon$$

In loop 2 Using KVL, we get

$$\frac{q_2}{C_3} - \frac{q_1}{C_1} - \frac{q_1}{C_2} = 0 \qquad \Rightarrow \frac{q_2}{C_3} = \frac{q_1}{C_1} + \frac{q_1}{C_2}$$

By solving (i) and (ii)

We get the values of q_1 and q_2

And with the help of q_1, q_2 we can find corresponding potential difference.

Now, we are also interested in calculating $V_F - V_B$ In path BCDEF

By applying K.V.L. we get

$$V_F - V_B = (V_C - V_B) + (V_D - V_C) + (V_E - V_D) + (V_F - V_E)\left(-\frac{q_1}{C_1}\right) + \left(-\frac{q_1}{C_2}\right) + (0) + (0)$$

also from path $BAF = -\varepsilon$

CHARGE FLOW AND HEAT GENERATION

(i) Whenever there is potential difference there will be flow of charge.

(ii) Charge always have tendency to flows from high potential energy to low potential energy when released freely.

(iii) Positive charge always flows from high potential to low potential (if only electric force act on charge)

(iv) Negative charge always flows from low potential to high potential (if only electric force act on charge)

(v) The flow of charge will continue till there is potential difference between the conductors (finally potential difference = 0)

(vi) Formulae related with redistribution of charges.

Unacademy Atoms

Before connecting the conductors		
Parameter	**Conductor I^{st}**	**Conductor II^{nd}**
Capacitance	C_1	C_2
Charge	Q_1	Q_2
Potential	V_1	V_2

After connecting the conductors		
Parameter	**I^{st} Conductor**	**II^{nd} Conductor**
Capacitance	C_1	C_2
Charge	Q_1'	Q_2'
Potential	V	V

$$V = \frac{Q_1'}{C_1} = \frac{Q_2'}{C_2} \Rightarrow \frac{Q_1'}{Q_2'} = \frac{C_1}{C_2}$$ But, $Q_1' + Q_2' = Q_1 + Q_2$

$$\therefore V = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$; $$\therefore Q_1' = \frac{C_1}{C_1 + C_2}(Q_1 + Q_2) \ \& \ Q_2' = \frac{C_2}{C_1 + C_2}(Q_1 + Q_2)$$

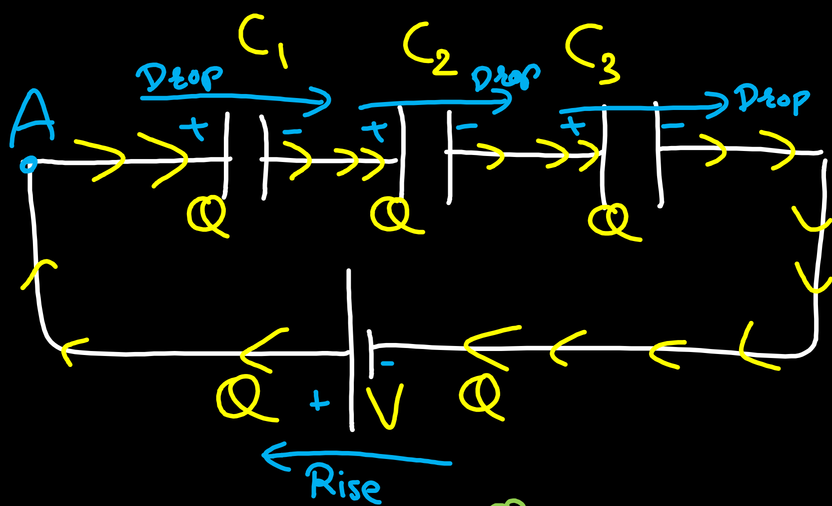
Heat loss during redistribution : $\Delta H = \frac{1}{2}\frac{C_1C_2}{C_1 + C_2}(V_1 - V_2)^2$

The loss of energy is in the form of Joule heating in the wire.

Note : Always put Q_1, Q_2, V_1 and V_2 with sign.

$Q = C\Delta V$

$\frac{Q}{C} = \Delta V$



series Connection

$$-\frac{Q}{C_1} - \frac{Q}{C_2} - \frac{Q}{C_3} + V = 0$$

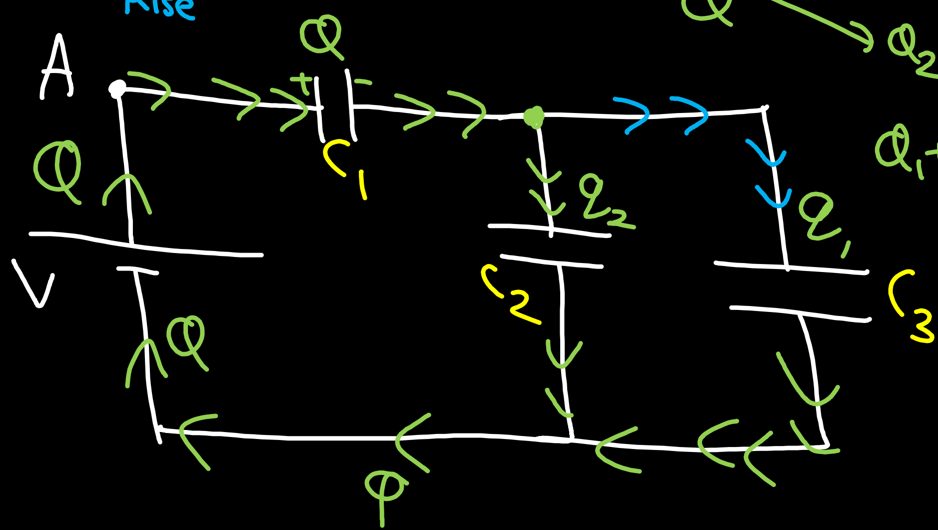
$Q \to Q_1$, $Q \to Q_2$

$Q_1 + Q_2 = Q$

or

$Q \to x$, $Q \to Q - x$.

Q



loop 1

$$-\frac{Q}{C_1} - \frac{q_1}{C_3} + V = 0$$

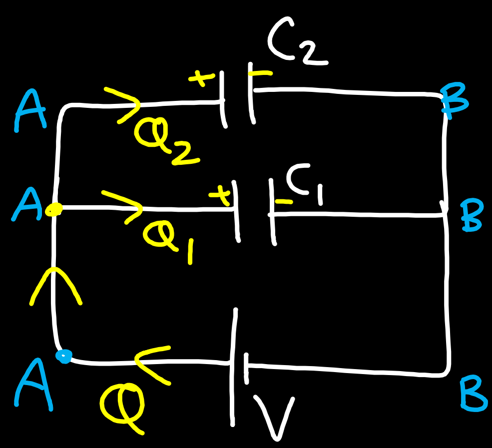
loop 2

$$-\frac{Q}{C_1} - \frac{q_2}{C_2} + V = 0$$

Combination of Capacitors

① Parallel Connection → when connected across same points, (ΔV across is same)

$$\boxed{\frac{Q_2}{C_2} = \frac{Q_1}{C_1} = V}$$



2

$$-\frac{Q_2}{C_2} + V = 0$$

$$\frac{Q_2}{C_2} = V$$

1

$$-\frac{Q_1}{C_1} + V = 0$$

$$\frac{Q_1}{C_1} = V$$

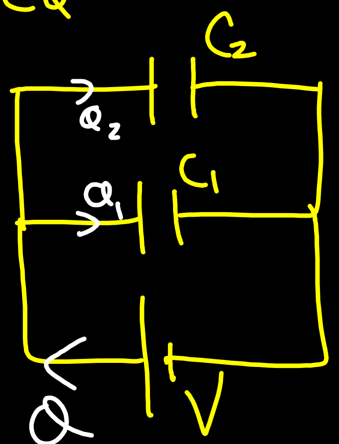
$$\boxed{Q_1 = C_1 V}$$

$$\boxed{Q_2 = C_2 V}$$

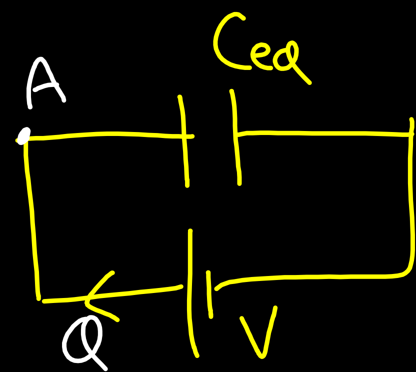
$$\boxed{Q_1 : Q_2 = C_1 : C_2}$$

more C more Q

C_{eq}



$\Rightarrow$



$$-\frac{Q}{C_{eq}} + V = 0$$

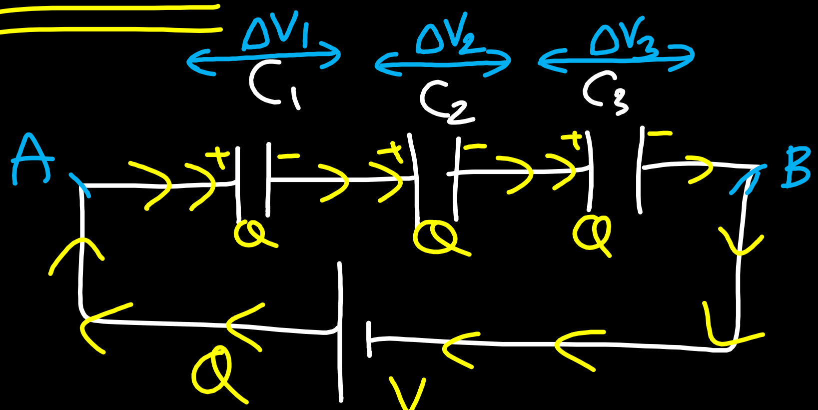
$$\boxed{Q = C_{eq} V}$$

$$Q = Q_1 + Q_2$$

$$C_{eq} V = C_1 V + C_2 V$$

$$\boxed{C_{eq} = C_1 + C_2}$$

Series → charge flow same in Capacitors.



$$\boxed{\Delta V_1 + \Delta V_2 + \Delta V_3 = V}$$

loop $-\frac{Q}{C_1} - \frac{Q}{C_2} - \frac{Q}{C_3} + V = 0$

$$V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

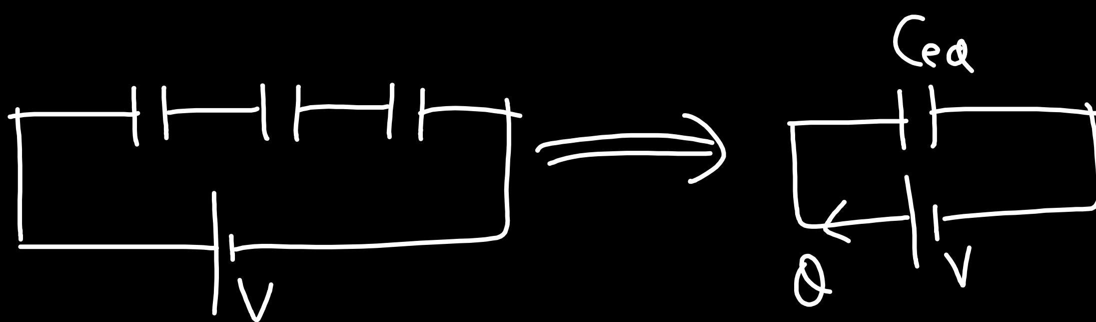
$$\Delta V_1 = \frac{Q}{C_1}$$

$$\Delta V_2 = \frac{Q}{C_2}$$

$$\Delta V_3 = \frac{Q}{C_3}$$

$$\Delta V_1 : \Delta V_2 : \Delta V_3$$

$$= \frac{1}{C_1} : \frac{1}{C_2} : \frac{1}{C_3}$$



$$Q = C_{eq} V$$

$$V = \frac{Q}{C_{eq}}$$

$$V = \frac{\cancel{Q}}{C_1} + \frac{\cancel{Q}}{C_2} + \frac{\cancel{Q}}{C_3}$$

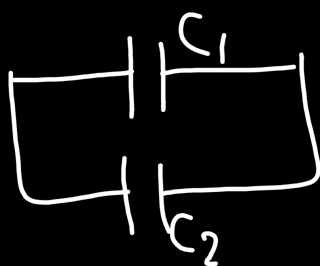
$$\frac{\cancel{Q}}{C_{eq}} =$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$\parallel \quad C_{eq} = C_1 + C_2$

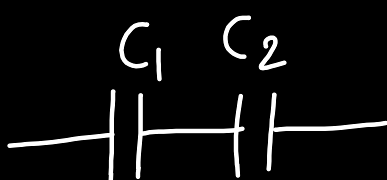
series $\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$

Q1



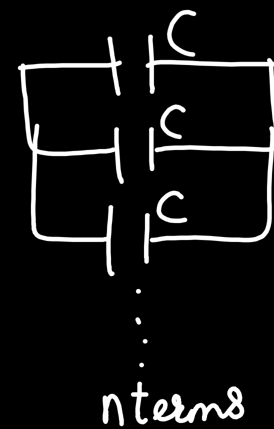
$C_{eq} = C_1 + C_2$

Q



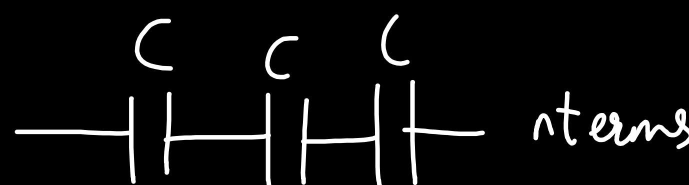
$C_{eq} = \dfrac{C_1 C_2}{C_1 + C_2}$

Q



$\boxed{C_{eq} = nC}$

Q

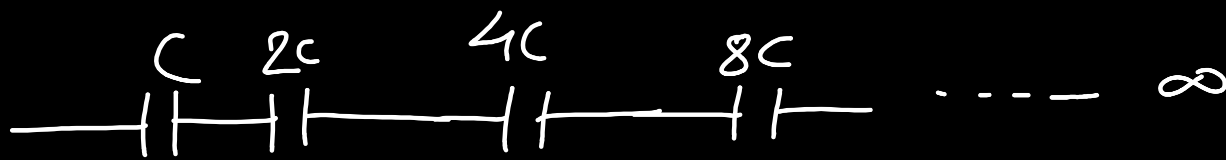


$\dfrac{1}{C_{eq}} = \dfrac{1}{C} + \dfrac{1}{C} \cdots$

$\dfrac{1}{C_{eq}} = \dfrac{n}{C}$

$C_{eq} = \dfrac{C}{n}$

Q//



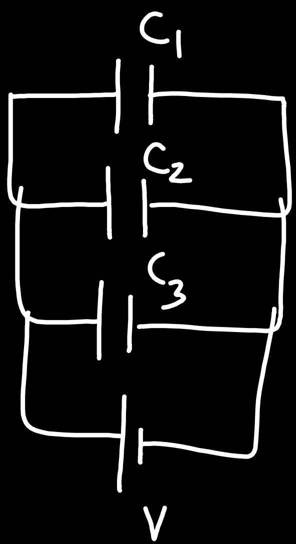
$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{2C} + \frac{1}{4C} + \frac{1}{8C} + \cdots$$

$$= \frac{1}{C}\left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots\right)$$

$$= \frac{1}{C} \frac{a}{1-r}$$

$$\frac{1}{C_{eq}} = \frac{1}{C} \frac{(1)}{(1-1/2)} = \frac{2}{C}$$

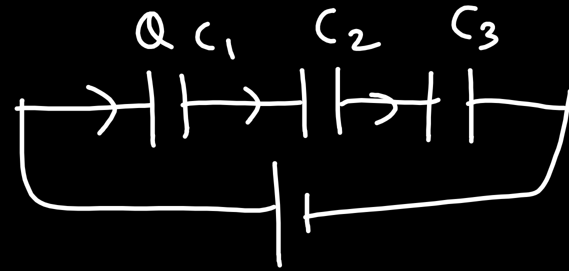
$$C_{eq} = C/2$$



$$U = \frac{1}{2} C_1 V^2$$

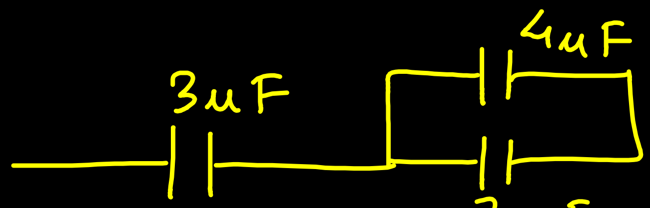
$$U = \frac{1}{2} C_2 V^2$$

$$U = \frac{1}{2} C_3 V^2$$



$$U = \frac{Q^2}{2C_1} + \frac{Q^2}{2C_2} + \frac{Q^2}{2C_3}$$

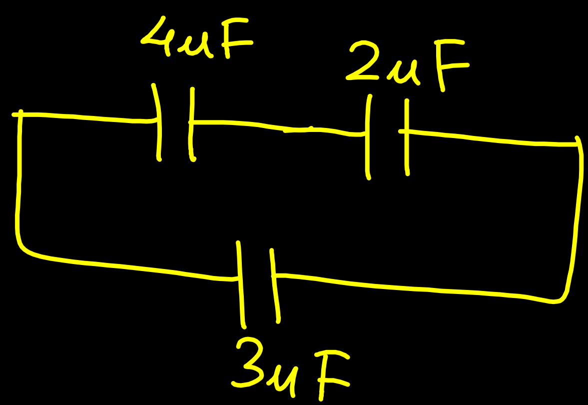
Q



3 6

$$C_{eq} = \frac{3\times 6}{3+6} = \frac{\cancel{3}\times 6}{\cancel{9}\,3} = 2\mu F$$

Q

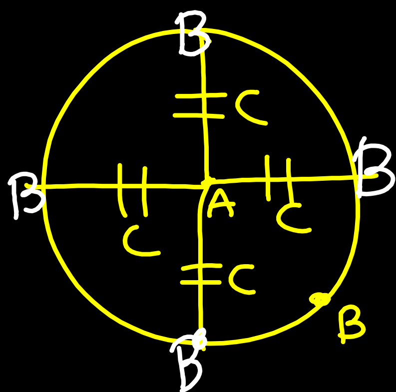


$$\frac{4\times \cancel{2}\,1}{\cancel{6}\,3}$$

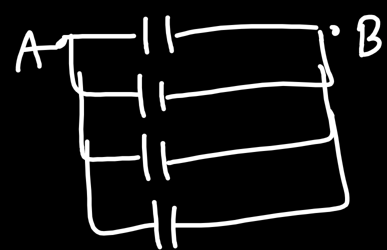
$$\frac{4}{3} + 3 = \frac{13}{3}\mu F$$

Equipotential Point Method

Q

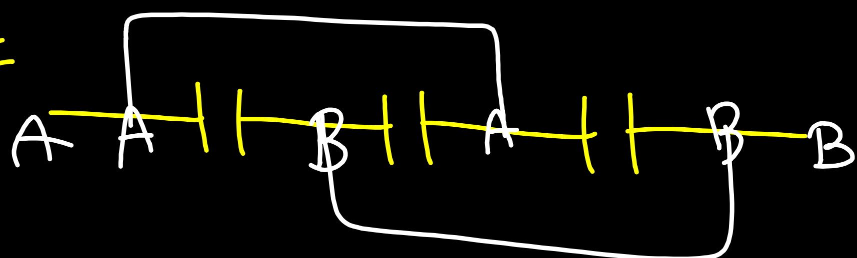


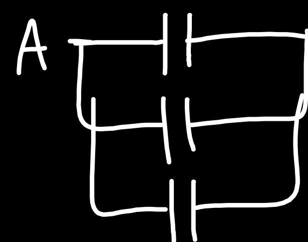
$\Rightarrow$



$\Rightarrow 4C$

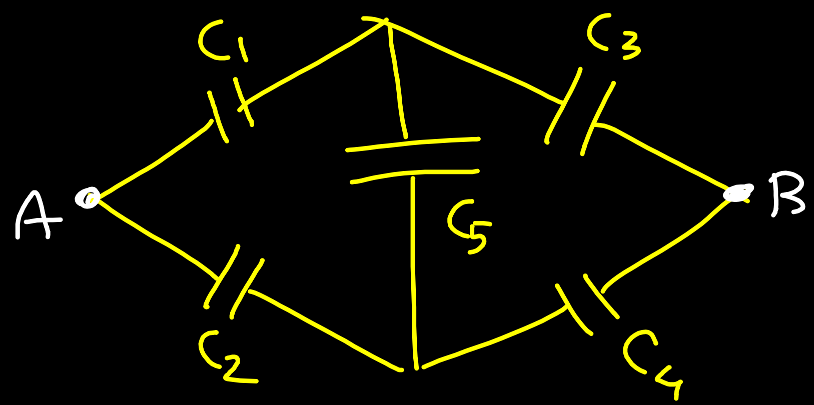
Q





$\Rightarrow 3C$.

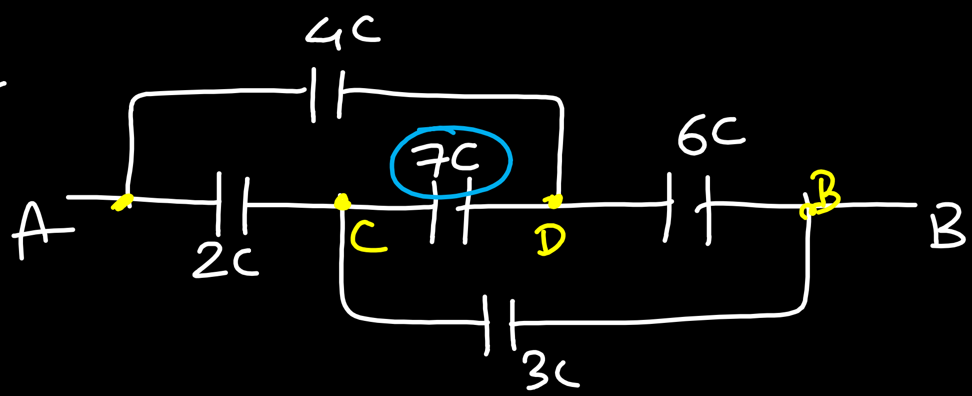
Wheat Stone Bridge



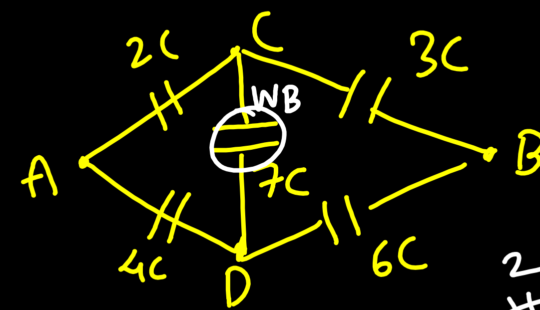
If $\frac{C_1}{C_2} = \frac{C_3}{C_4}$ than charge on $C_5 = 0$

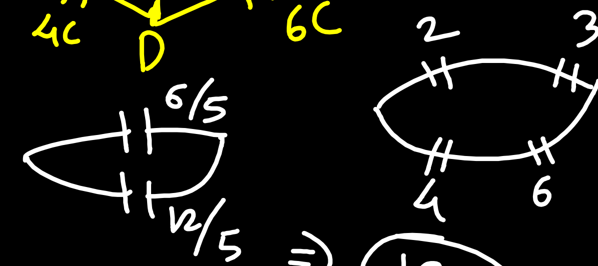
(we can remove it)

Q



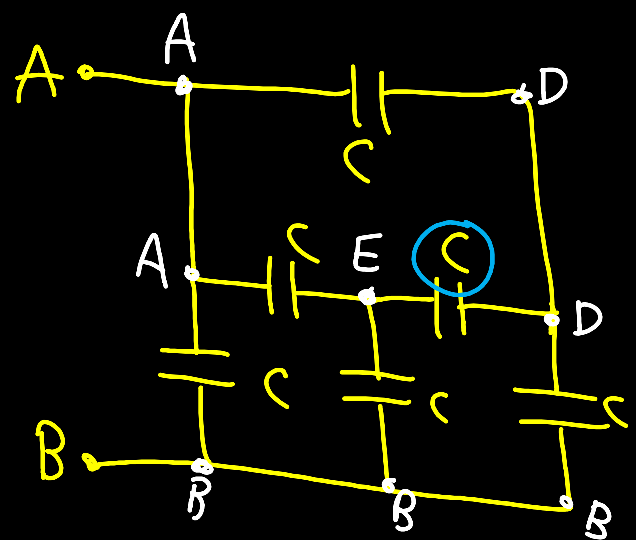
⇒

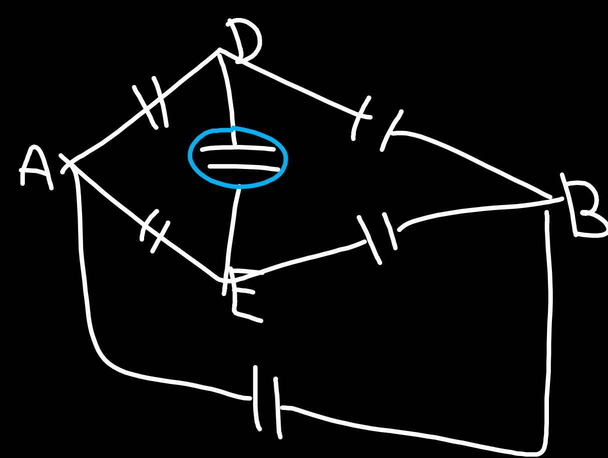




$$\Rightarrow \frac{18C}{5}$$

Q

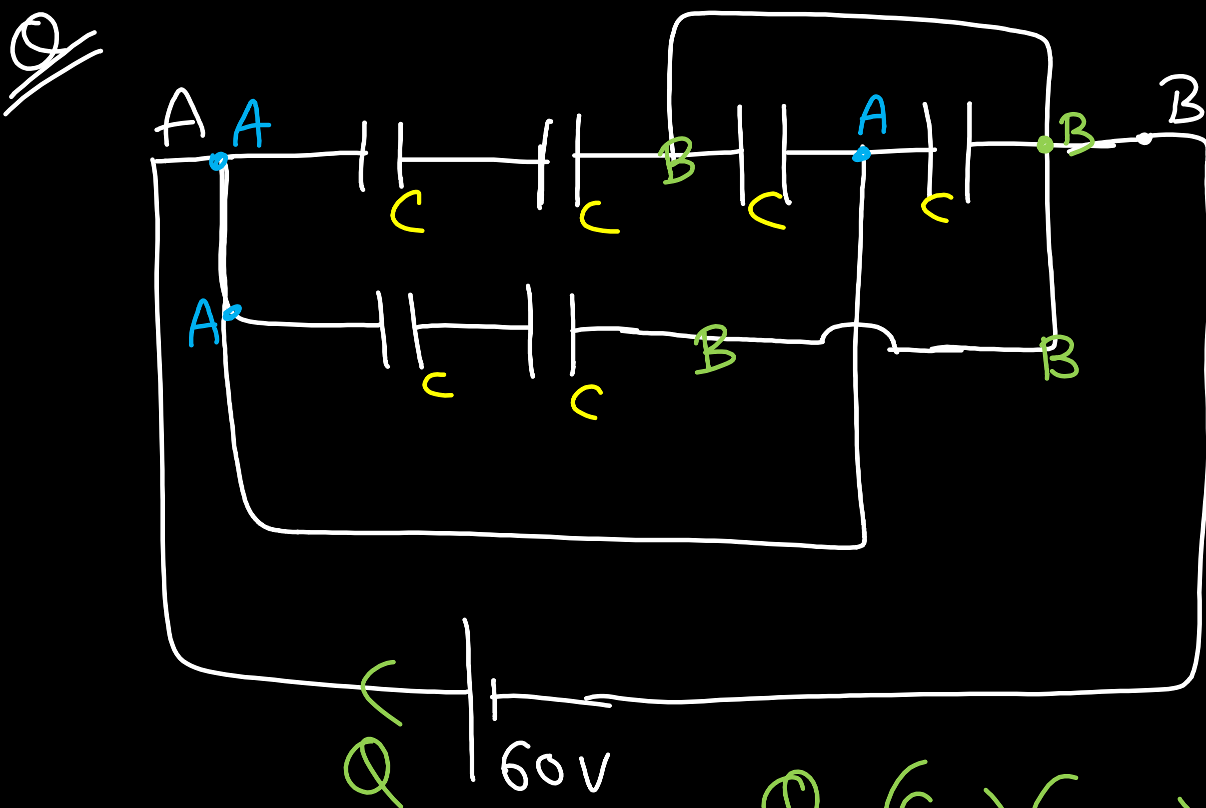




$$\frac{C}{2} + \frac{C}{2} + C$$

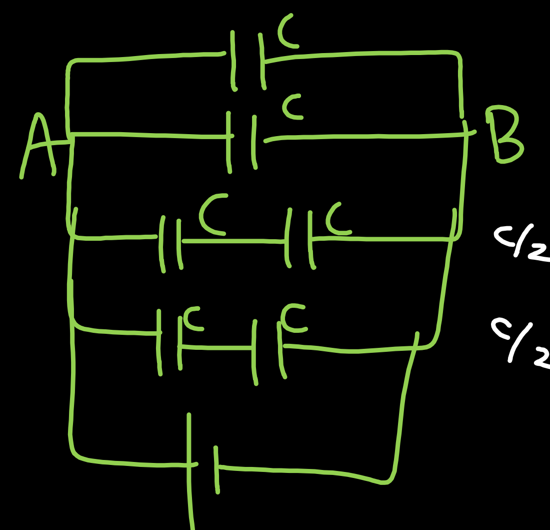
$$= 2C$$

Q



Find net charge from battery??

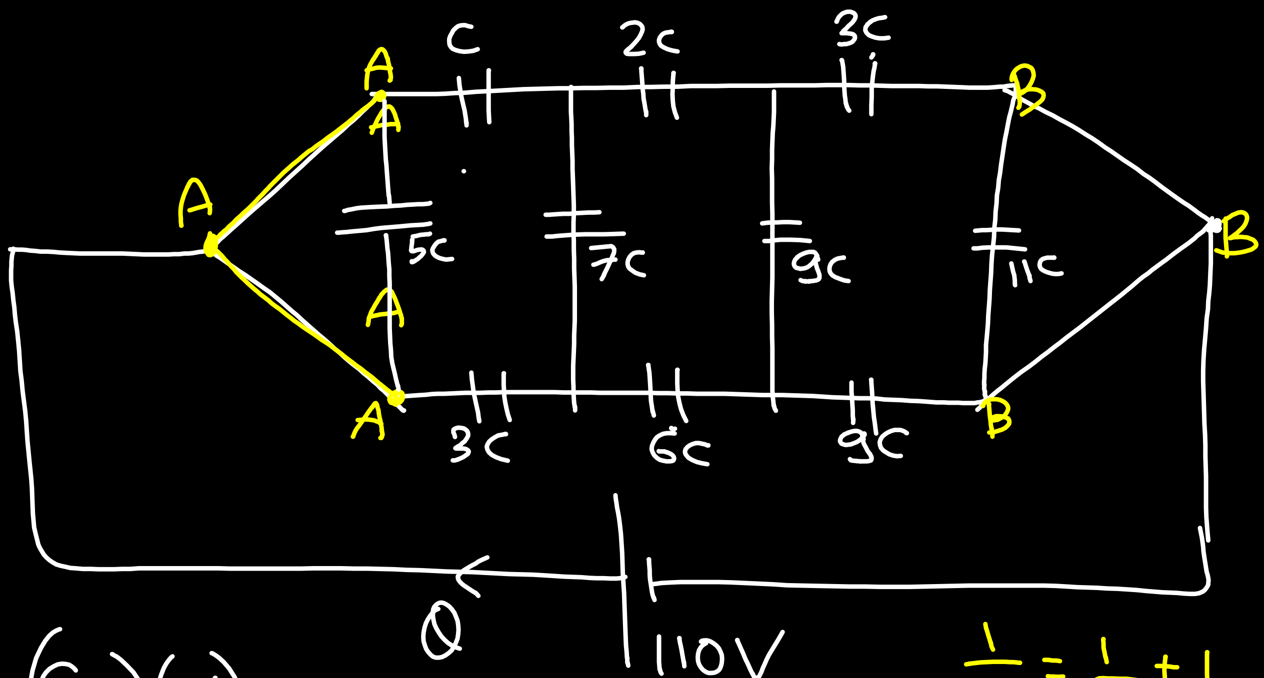
$Q = (C_{eq})(V)$



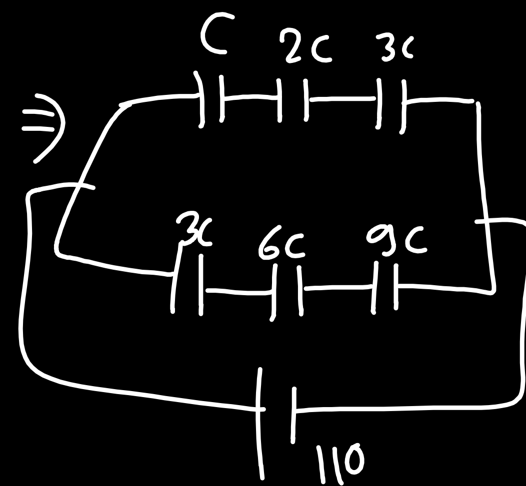
$Q = (3C)(60) = \underline{180\,C}$

Q

$Q = ?$



$\Rightarrow$



$Q = (C_{eq})(V)$

$= \frac{24C}{11} \times 110$

$Q = 240C$

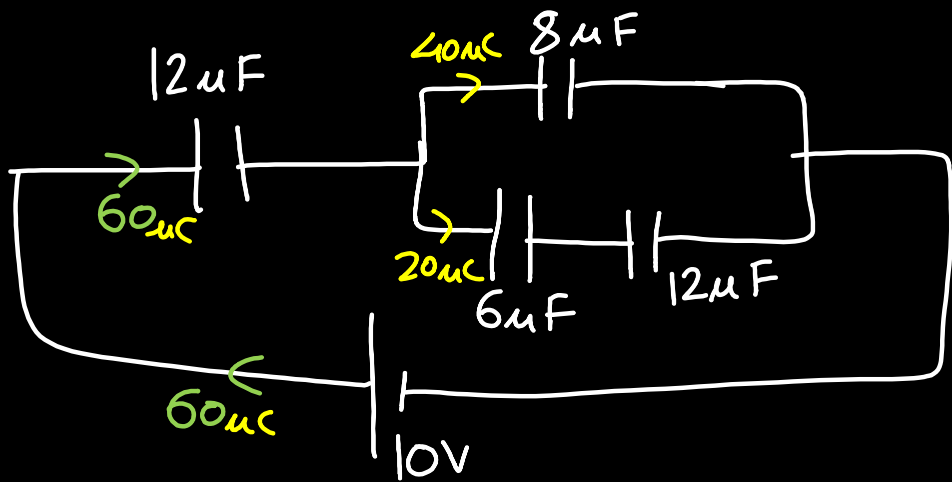
$C_{eq} = \frac{24C}{11}$

$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{2C} + \frac{1}{3C} = \frac{11}{6C}$

$C_{eq} = \frac{6C}{11}$

$\frac{1}{C_{eq}} = \frac{1}{3}\left(\frac{1}{C} + \frac{1}{2C} + \frac{1}{3C}\right) = \frac{1}{3}\frac{11}{6C} \Rightarrow C_{eq} = \frac{18C}{11}$

Q

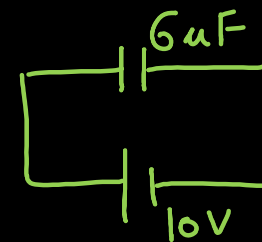


Find ① charge on each C
② U stored in each C
③ Q total from battery.

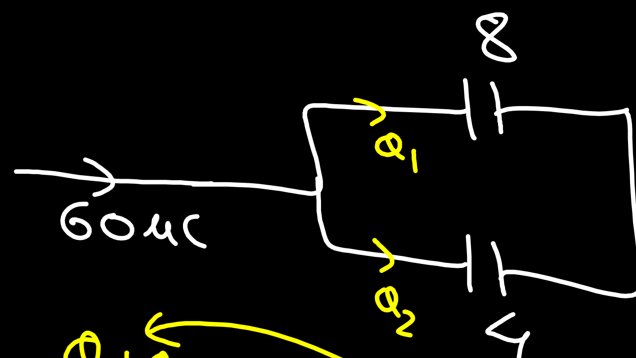
$\frac{6\times 12}{18\;3} = 4$

$4+8=12$

$\Rightarrow$



$Q = C_{eq} V$
$= 6\mu \; 10$
$Q = 60\mu C$



$\frac{Q_1}{8} = \frac{Q_2}{4}$

$Q_1 = 2Q_2$

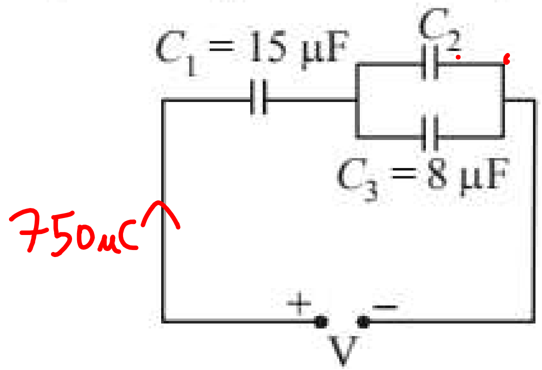
$Q_1 + Q_2 = 60$

$3Q_2 = 60$

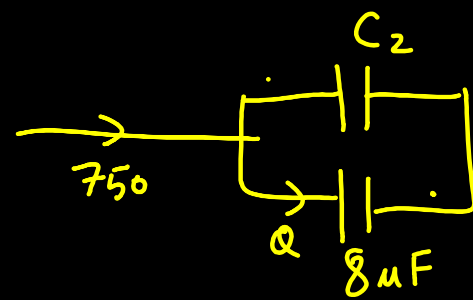
$Q_2 = 20$

$$U = \frac{Q^2}{2C}$$

In the circuit shown in the figure, the total charge is 750 μC and the voltage across capacitor C_2 is 20 V. Then the charge on capacitor C_2 is : **[Sep. 03, 2020 (I)]**



(a) 450 μC
(b) 590 μC
(c) 160 μC
(d) 650 μC



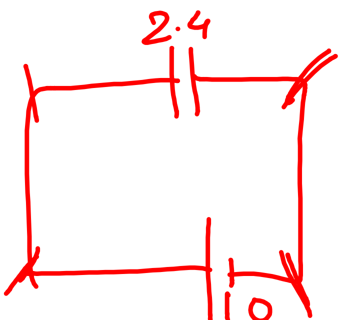
$\frac{Q}{8} = 20$

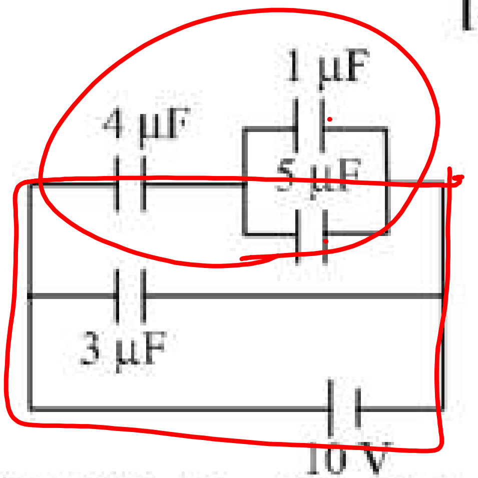
$Q = 160$

$750 - 160$

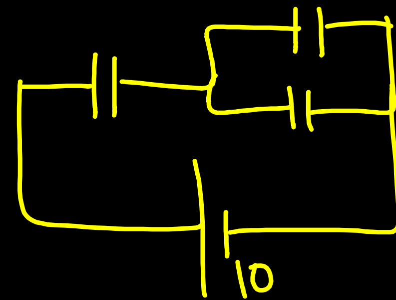
$= 590$

In the given circuit, the charge on 4 µF capacitor will be : **[12 April 2019 II]**





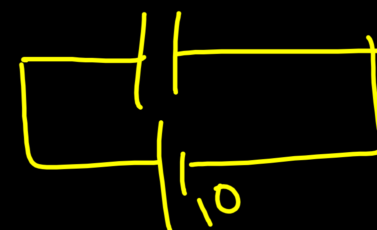
$\frac{6 \times \cancel{4}^{2}}{\cancel{10}\ 5} = 2.4$



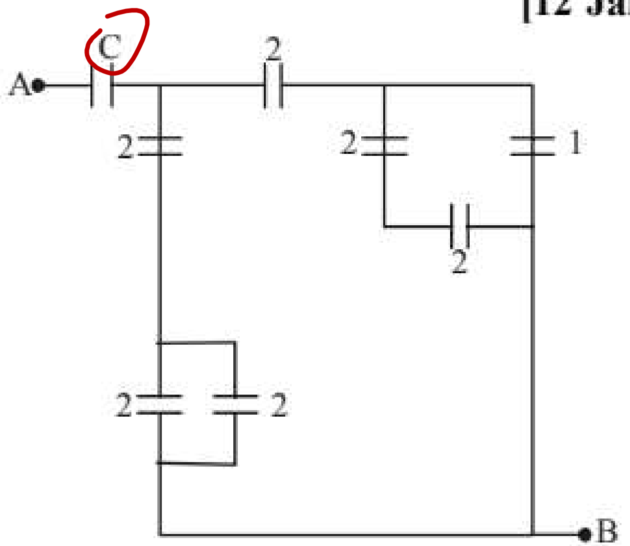
$Q = (C_{eq})(10)$

(a) 5.4 µC (b) 9.6 µC (c) 13.4 µC (d) 24 µC ✓

$= 24$

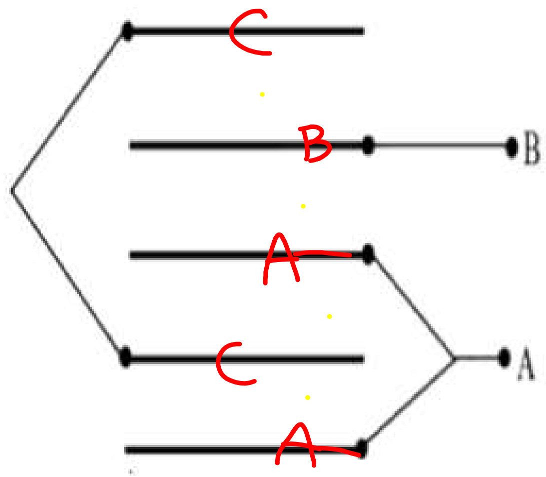


In the circuit shown, find C if the effective capacitance of the whole circuit is to be 0.5 μF. All values in the circuit are in μF. **[12 Jan. 2019 II]**



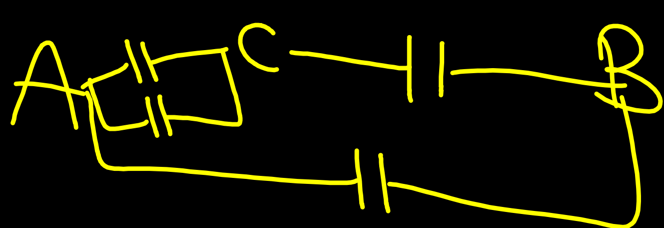
(a) $\frac{7}{11}$ μF (b) $\frac{6}{5}$ μF (c) 4 μF (d) $\frac{7}{10}$ μF

Five identical plates of equal area A are placed parallel to each other and at equal distance d from each other as shown in fig. The effective capacity of the system between the terminals A and B id

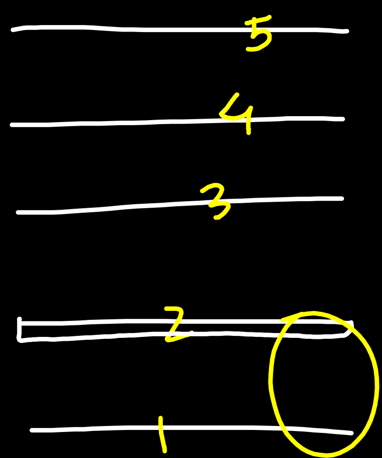


(A) $\frac{3}{5}\frac{\varepsilon_0 A}{d}$ (B) $\frac{5}{4}\frac{\varepsilon_0 A}{d}$

(C) $\frac{5}{3}\frac{\varepsilon_0 A}{d}$. (D) $\frac{4}{5}\frac{\varepsilon_0 A}{d}$



$\Rightarrow \frac{5C}{3} = \frac{5}{3}\frac{A\varepsilon_0}{d}$



$$C = \frac{A\varepsilon_0}{d}$$

$\Rightarrow$

The total charge on the system of capacitance $C_1 = 1\mu F$, $C_2 = 2\mu F$, $C_3 = 4\mu F$ and $C_4 = 3\mu F$ connected in parallel is

(Assume a battery of 20V is connected to the combination)

(A) $200\mu C$ (B) $200 C$

(C) $10\mu C$ (D) $10 C$

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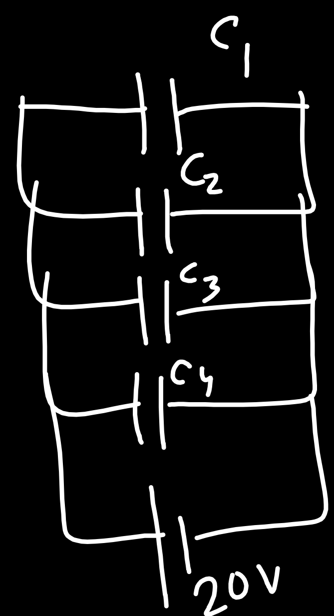
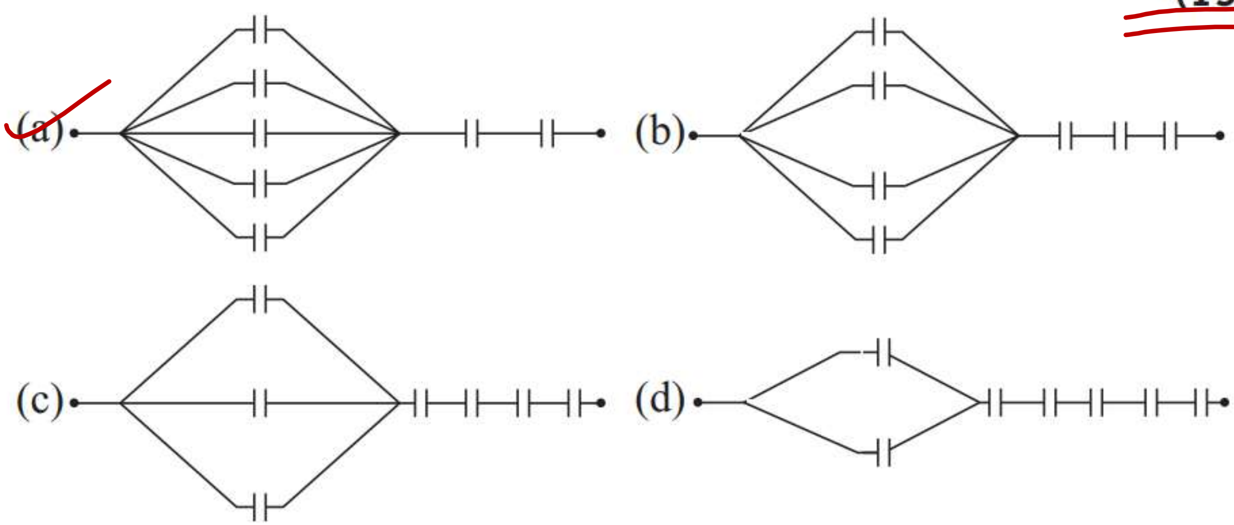


Seven capacitors each of capacitance 2μF are connected in a configuration to obtain an effective capacitance $\frac{10}{11}$μF. Which of the following combination will achieve the desired result ?

(1990)



5C C C

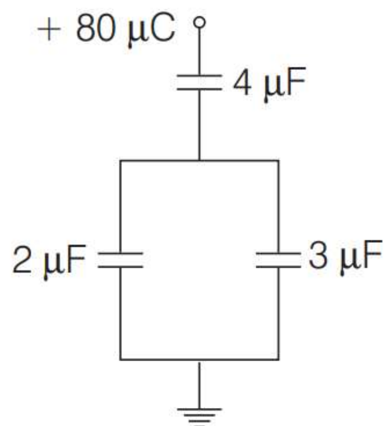
$$\frac{1}{5C} + \frac{1}{C} + \frac{1}{C} = \frac{1}{C_{eq}}$$

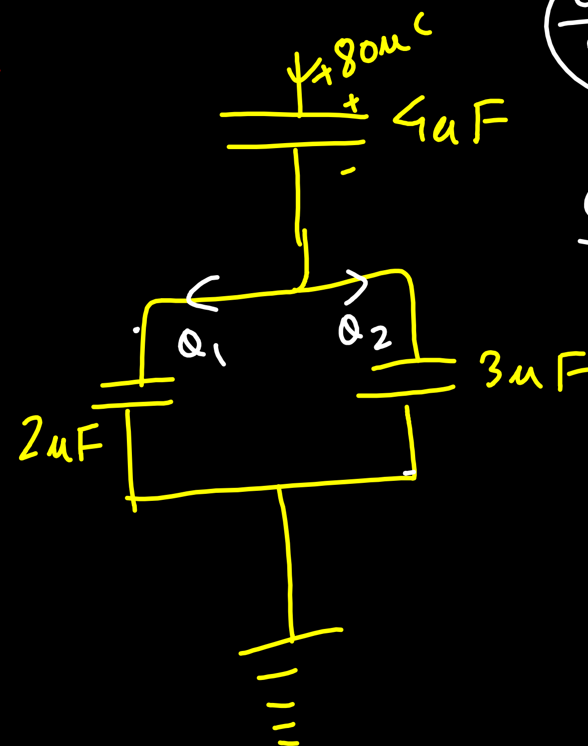
$$\frac{1+5+5}{5} = \frac{1}{C_{eq}}$$

$$C_{eq} = \frac{5}{11}(2)$$

In the given circuit, a charge of +80 μC is given to the upper plate of the 4 μF capacitor. Then in the steady state, the charge on the upper plate of the 3 μF capacitor is **(2012)**

H.W.



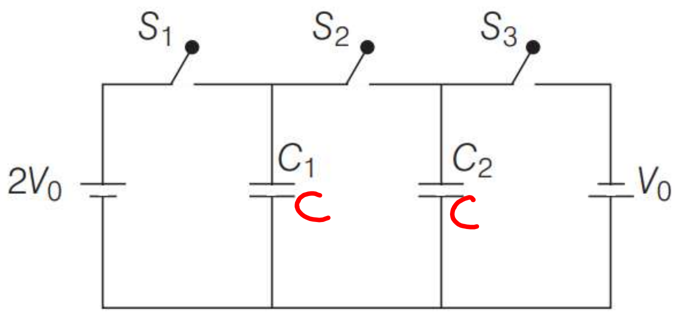


$$\frac{Q_1}{2} = \frac{Q_2}{3}$$

$$Q_1 + Q_2 = 80$$

(a) + 32 μC
(b) + 40 μC
(c) + 48 μC
(d) + 80 μC

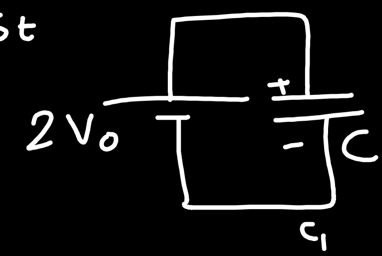
In the circuit shown in the figure, there are two parallel plate capacitors each of capacitance C. The switch S_1 is pressed first to fully charge the capacitor C_1 and then released. The switch S_2 is then pressed to charge the capacitor C_2. After some time, S_2 is released and then S_3 is pressed. After some time **(2013 Adv.)**



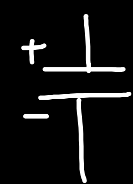
More than one correct

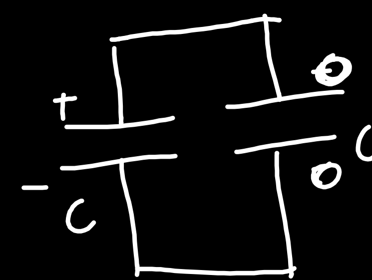
(a) the charge on the upper plate of C_1 is $\cancel{2CV_0}$
(b) the charge on the upper plate of C_1 is CV_0
(c) the charge on the upper plate of C_2 is $\cancel{0}$
(d) the charge on the upper plate of C_2 is $-CV_0$

1st

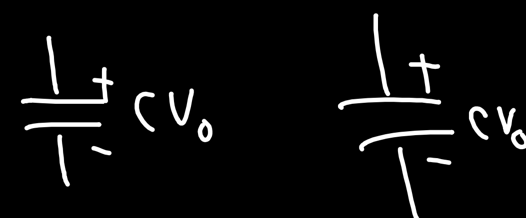


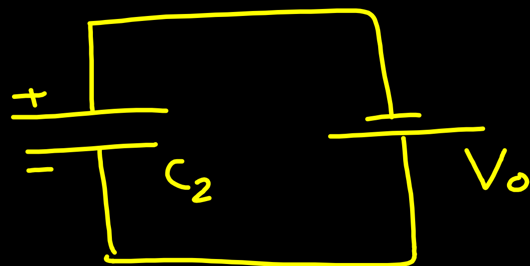
$Q = (C)(\Delta V)$
$= C\,2V_0$

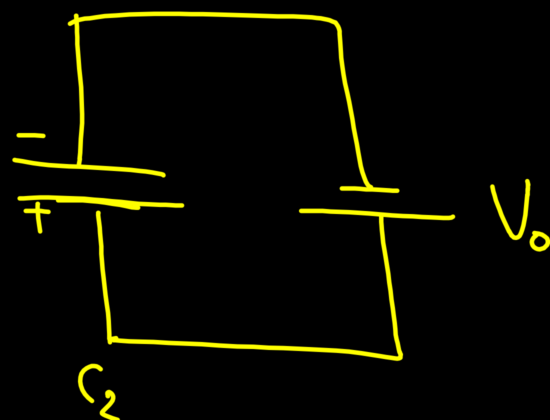




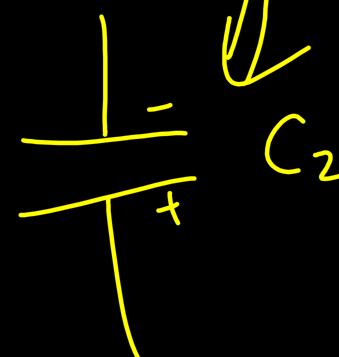
$\Rightarrow$



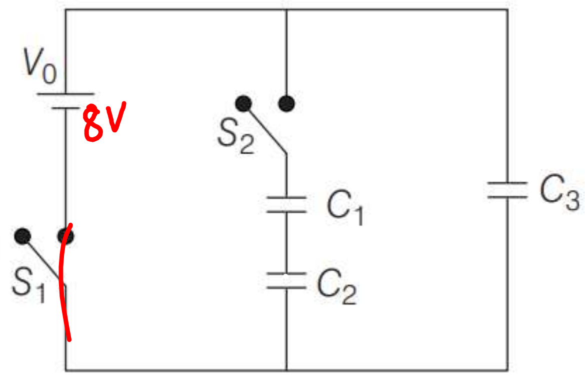




$$Q = (C_2)(V_0)$$



Three identical capacitors C_1, C_2 and C_3 have a capacitance of $1.0 \mu F$ each and they are uncharged initially. They are connected in a circuit as shown in the figure and C_1 is then filled completely with a dielectric material of relative permittivity ε_r. The cell electromotive force (emf) $V_0 = 8V$. First the switch S_1 is closed while the switch S_2 is kept open. When the capacitor C_3 is fully charged, S_1 is opened and S_2 is closed simultaneously. When all the capacitors reach equilibrium, the charge on C_3 is found to be $5 \mu C$. The value of $\varepsilon_r = ...$



(2018 Adv.)

$C_1 = C_2 = C_3 = 1 \mu F$

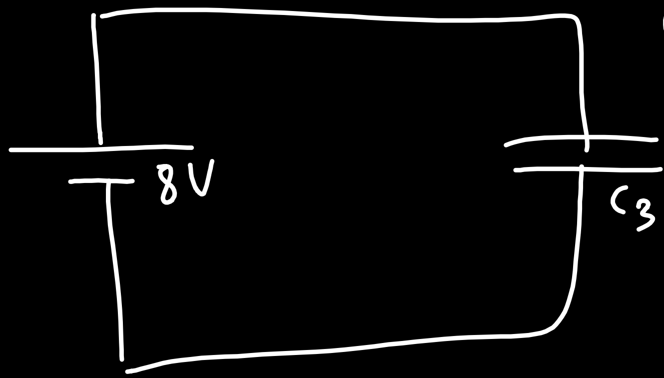
$C = \frac{A\varepsilon_0}{d}$

$C' = \frac{A\varepsilon_0}{d} \varepsilon_r$

$C' = \varepsilon_r C$

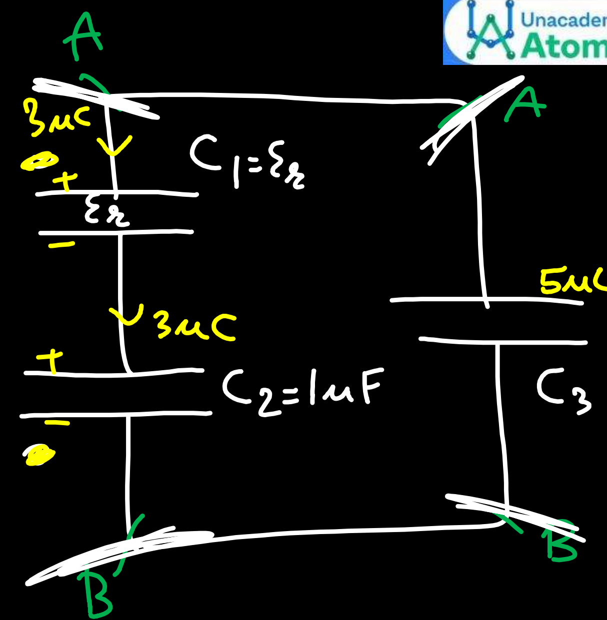
ε_r C_1

$C_1' = \varepsilon_r \ \mu F$



$$Q = C_3(8) = 8\mu C$$

$\Rightarrow$



$$\frac{3}{\varepsilon_r} = 2$$

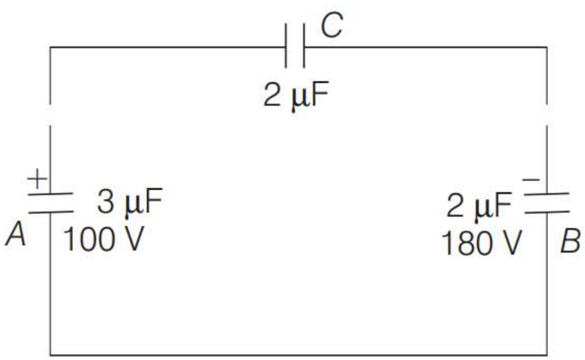
$$\frac{3}{2} = \varepsilon_r$$

$$1.5 = \varepsilon_r$$

$$\frac{3\mu C}{\varepsilon_r} + \frac{3\mu C}{1\mu F} = \frac{5\mu C}{1\mu F}$$

$$\frac{3}{\varepsilon_r} + 3 = 5$$

Two capacitors A and B with capacities $3\mu F$ and $2\mu F$ are charged to a potential difference of 100 V and 180 V respectively. The plates of the capacitors are connected as shown in the figure with one wire of each capacitor free. The upper plate of A is positive and that of B is negative. An uncharged $2\ \mu F$ capacitor C with lead wires falls on the free ends to complete the circuit. Calculate **(1997)**

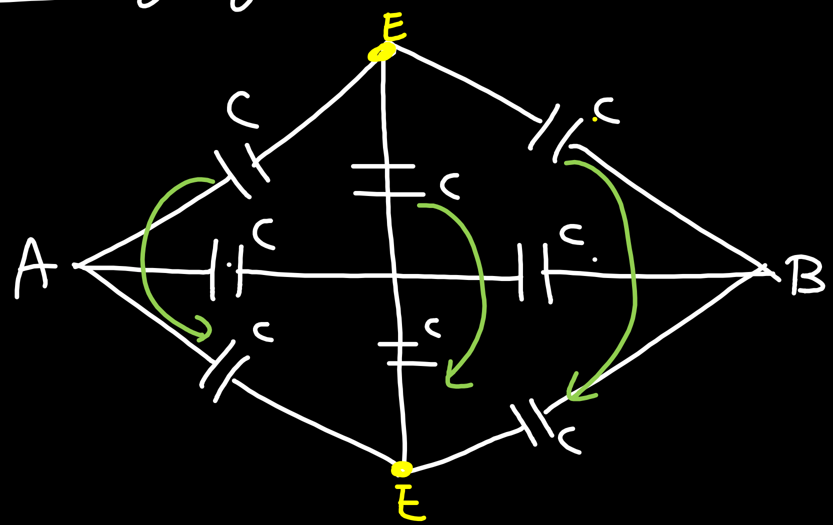


(a) the final charge on the three capacitors and

(b) the amount of electrostatic energy stored in the system before and after completion of the circuit.

Folding Symmetry

Q

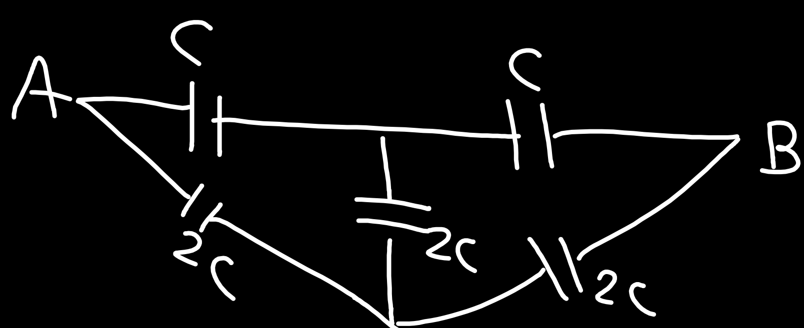


Find $C_{eq} = ?$ b/w AB

AB itself is line of symmetry.

→ can be folded about AB

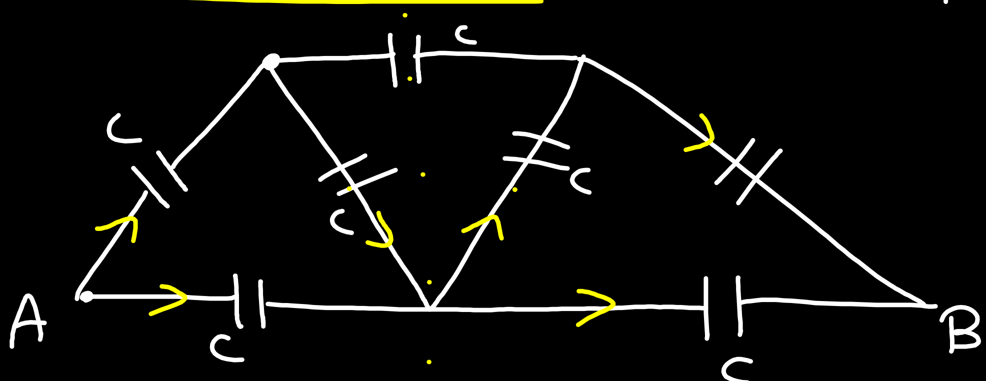
→ Mirror images Points are equipotential.



$c/2$

c $= \frac{3c}{2}$

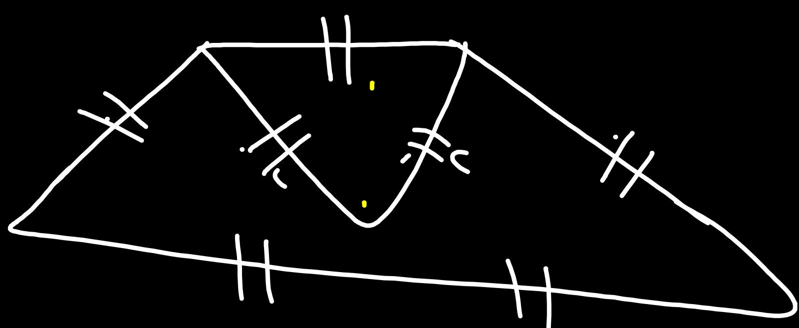
Connection Removal

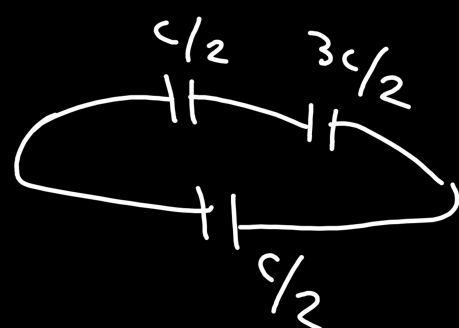


\# line ⊥'er to AB is line of symmetry.

- → all points lying one this ⊥'er line is equipotential
- → mirror image C & H

Same charge

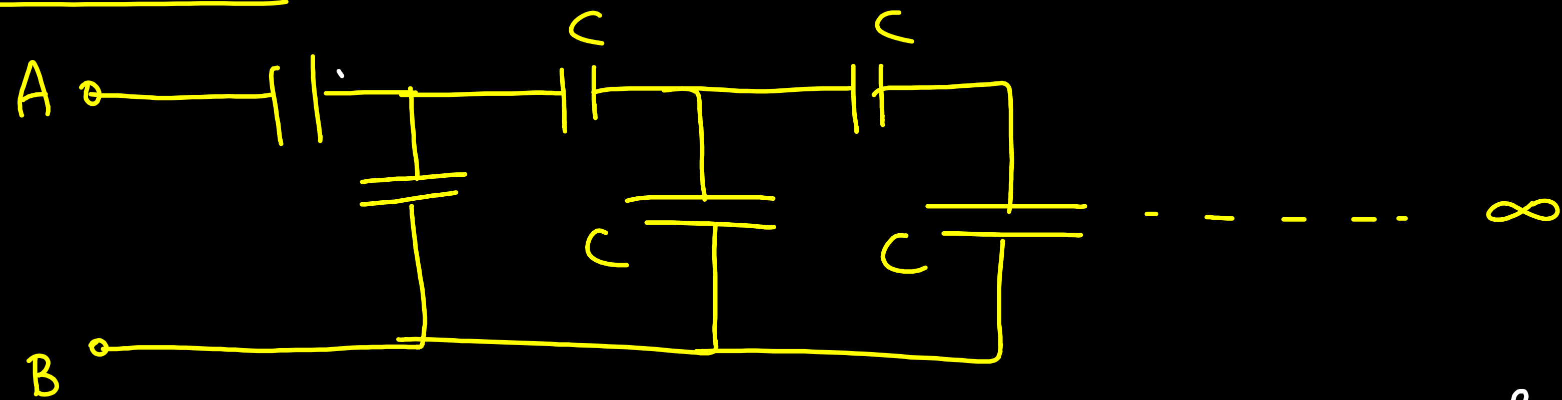


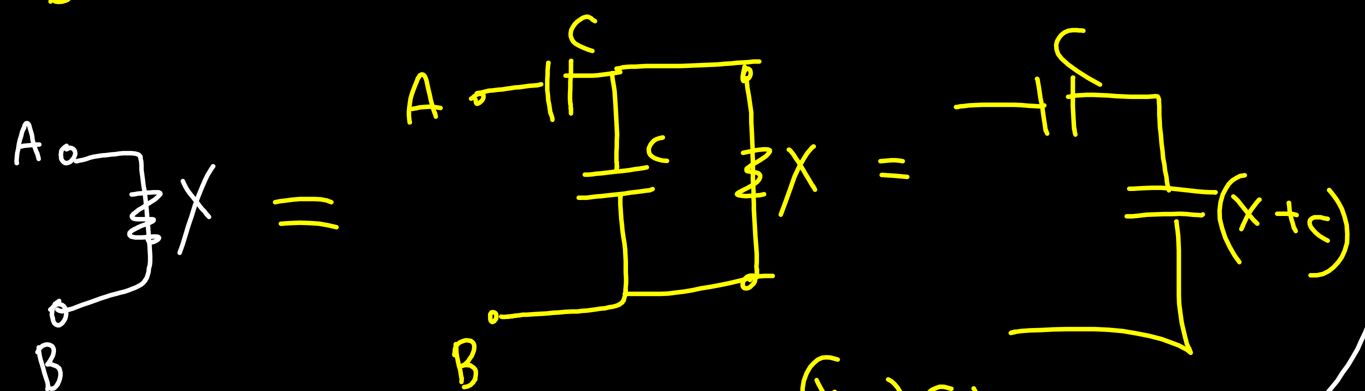


$$\frac{\frac{c}{2}\cdot\frac{3c}{2}}{\frac{4c}{2}} = \frac{3c}{8}$$

$$\frac{3c}{8} + \frac{c}{2} = \frac{7c}{8}$$

∞ ladder





$$X = \frac{(X+C)(C)}{X+C+C}$$

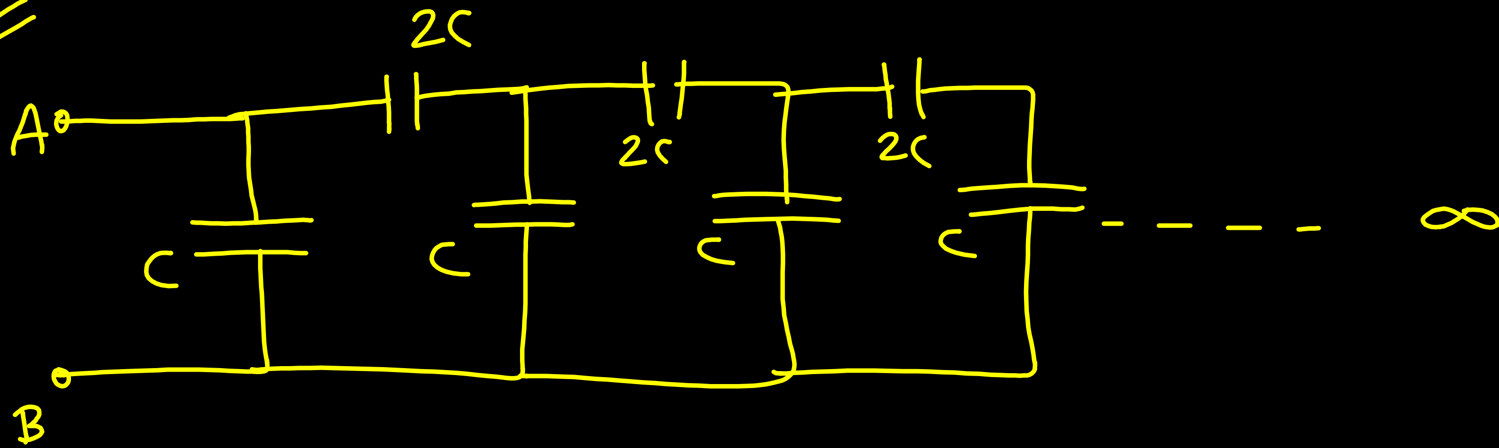
$$\Rightarrow X^2 + 2XC = XC + C^2$$

$$X^2 + XC - C^2 = 0$$

$$X = \frac{-C + \sqrt{5C^2}}{2}$$

$$X = \left(\frac{\sqrt{5} - 1}{2}\right) C$$

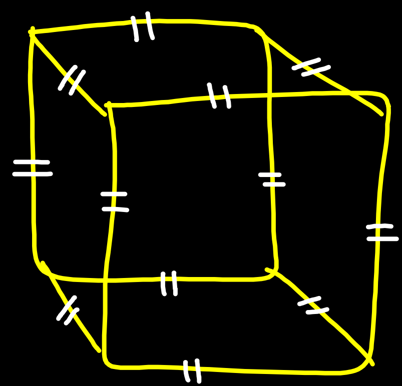
Q H.W.



Check if Ans

$X = 2C$

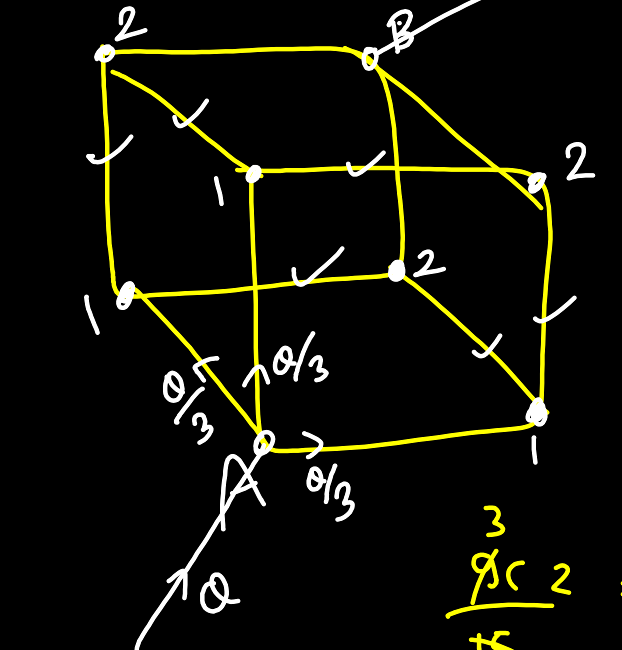
Cube Standard Result

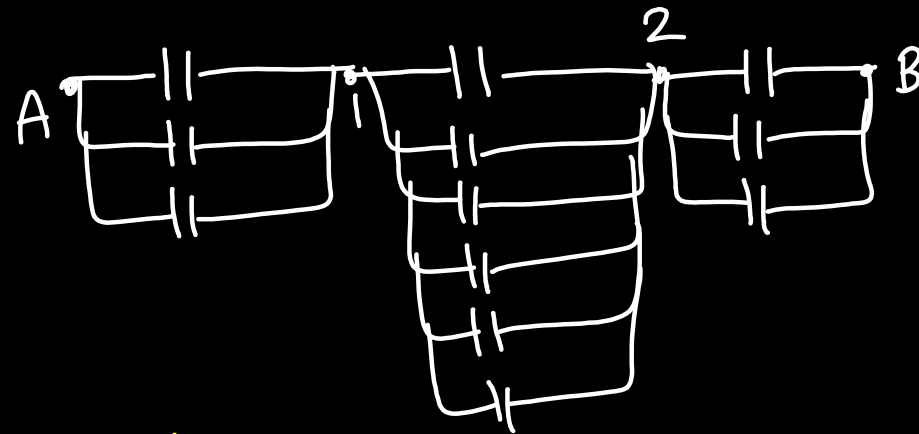


Find eq capacitance

① about Body diagonal
② " Face "
③ about edge

Body Diagonal



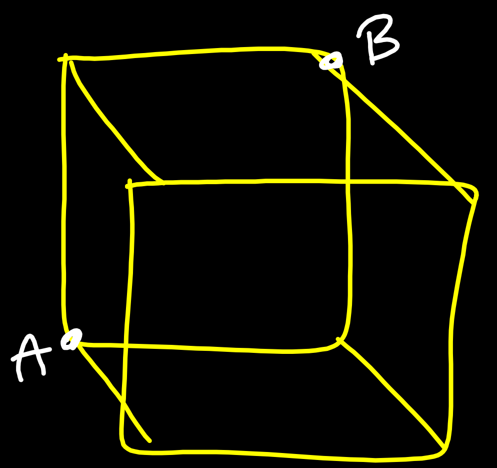


$$\frac{\overset{3}{\cancel{9}}C\;2}{\underset{5}{\cancel{15}}} = \frac{\frac{3C}{2}\times 6C}{\frac{3C}{2}+6C}$$

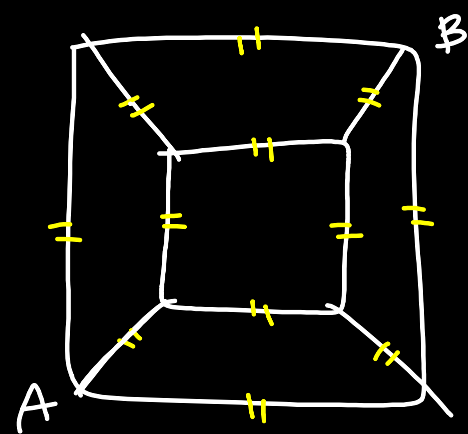
$$\frac{1}{3C}+\frac{1}{6C}+\frac{1}{3C}=\frac{1}{C_{eq}}$$

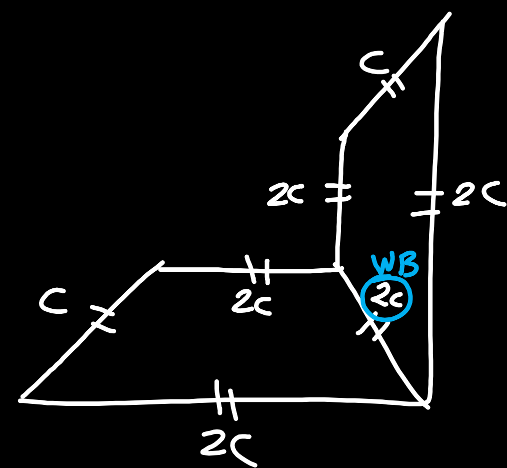
$$\frac{6C}{5}$$

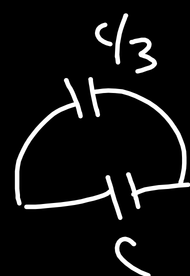
Face Diagonal



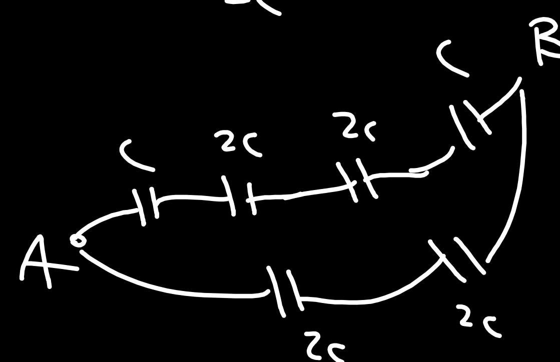
Fold



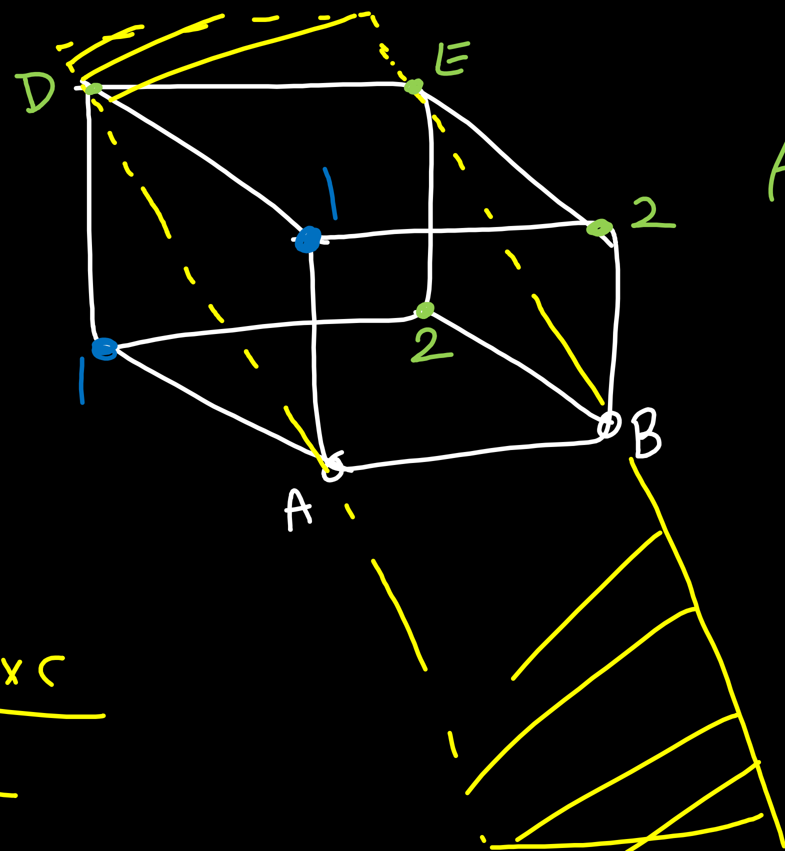


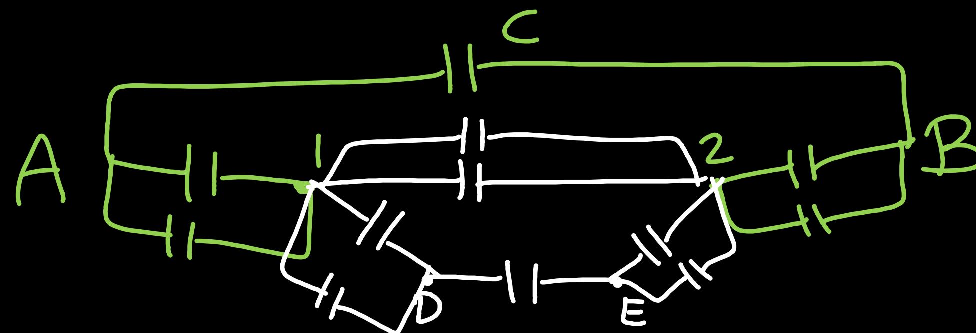


$$= \frac{4C}{3}$$



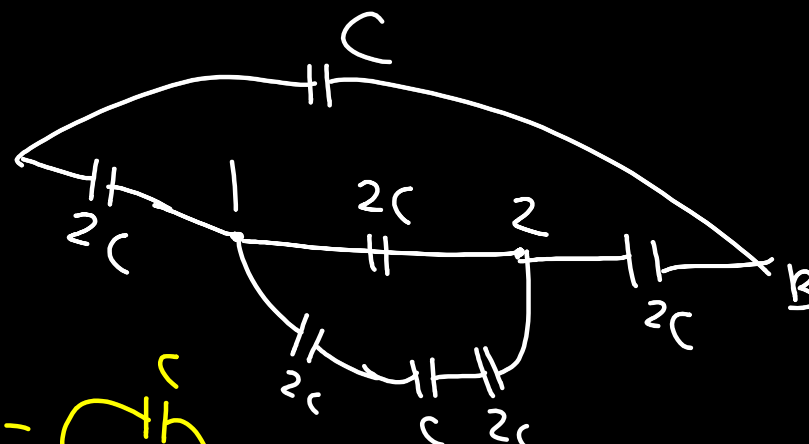
Edge (plane of symmetry) $\Rightarrow$ (mirror image same potential)



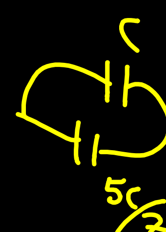


$\frac{C}{2}+2C$

$\frac{\frac{5C}{2}\times C}{\frac{7C}{2}}$



$=$



$= \frac{12C}{7}$

EFFECT OF DIELECTRIC

- The insulators in which microscopic local displacement of charges takes place in presence of electric field are known as dielectrics.
- Dielectrics are non conductors up to certain value of field depending on its nature. If the field exceeds this limiting value called dielectric strength they lose their insulating property and begin to conduct.
- Dielectric strength is defined as the maximum value of electric field that a dielectric can tolerate without breakdown. Unit is volt/meter. Dimensions $M^1 L^1 T^{-3} A^{-1}$

<u>Polar dielectrics</u>

- In absence of external field the centers of positive and negative charge do not coincide-due to asymmetric shape of molecules.
- Each molecule has permanent dipole moment.
- The dipole are randomly oriented so average dipole moment per unit volume of polar dielectric in absence of external field is nearly zero.
- In presence of external field dipoles tends to align in direction of field.
  Ex. Water, Alcohol, $CO_2, HC\ell, NH_3$

Non polar dielectrics

- In absence of external field the center of positive and negative charge coincides in these atoms or molecules because they are symmetric.
- The dipole moment is zero in normal state.
- In presence of external field they acquire induced dipole moment.
  Ex. Nitrogen, Oxygen, Benzene, Methane

Polarisation

The alignment of dipole moments of permanent or induced dipoles in the direction applied electric field is called polarization .

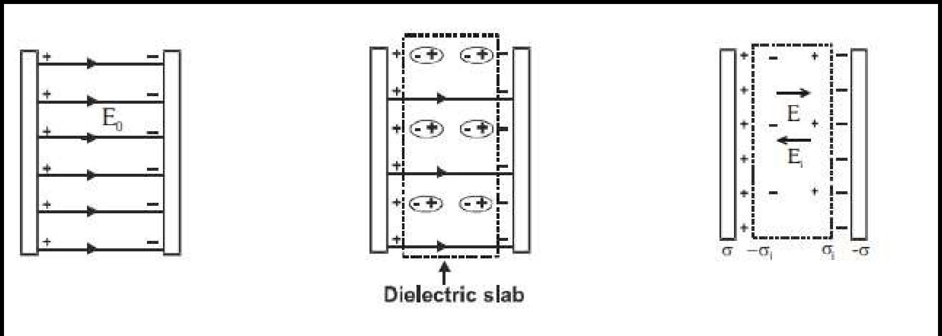
Polarization vector $\vec{\mathbf{P}}$ - This is a vector quantity which describes the extent to which molecules of dielectric become polarized by an electric field or oriented in direction of field.

$\vec{P}$ = the dipole moment per unit volume of dielectric = $n\vec{p}$

where n is number of atoms per unit volume of dielectric and $\vec{p}$ is dipole moment of an atom or molecule.

$\vec{P} = n\vec{p} = \frac{q_i d}{Ad} = \left(\frac{q_i}{A}\right) = \sigma_i$ = induced surface charge density.

Unit of $\vec{P}$ is C/m^2 Dimension is $L^{-2}\ T^1\ A^1$



Let E_0, V_0, C_0 be electric field, potential difference and capacitance in absence of dielectric. Let E, V, C are electric field, potential difference and capacitance in presence of dielectric respectively.

Electric field in absence of dielectric $E_0 = \frac{V_0}{d} = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$

Electric field in presence of dielectric $E = E_0 - E_i = \frac{\sigma - \sigma_i}{\varepsilon_0} = \frac{Q - Q_i}{\varepsilon_0} = \frac{V}{d}$

Capacitance in absence of dielectric $C_0 = \frac{Q}{V_0}$

Capacitance in Presence of dielectric $C = \frac{Q - Q_i}{V}$

The dielectric constant or relative permittivity K or $\varepsilon_r = \frac{F_0}{E} = \frac{V_0}{V} = \frac{C}{C_0} = \frac{Q}{Q - Q_i} = \frac{\sigma}{\sigma - \sigma_i} = \frac{\varepsilon}{\varepsilon_0}$

From $K = \frac{Q}{Q - Q_i} \Rightarrow Q_i = Q\left(1 - \frac{1}{K}\right)$ and $K = \frac{\sigma}{\sigma - \sigma_i} \Rightarrow \sigma_i = \sigma\left(1 - \frac{1}{K}\right)$

Capacity of Different Configurations

In case of parallel plate capacitor $C = \frac{\varepsilon_0 A}{d}$

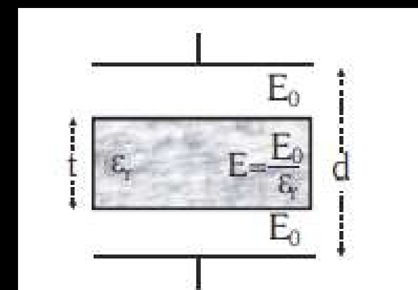
If capacitor is partially filled with dielectric

When the dielectric is filed partially between plates, the thickness of dielectric slab is $t(t < d)$.

If no slab is introduced between the plates of the capacitor, then a field E_0 given by $E_0 = \frac{\sigma}{\varepsilon_0}$, exists in a space d. On inserting the slab of thickness t and a field $E = \frac{E_0}{\varepsilon_r}$ exists inside the slab of thickness t and a field E_0 exists in remaining space $(d - t)$. If V is total potential then $V = E_0(d - t) + Et \Rightarrow$

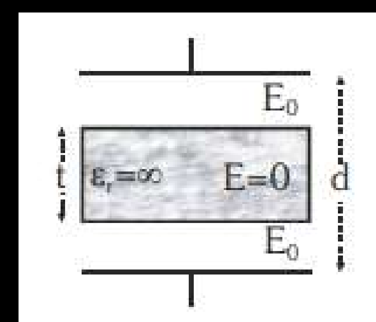
$$V = E_0\left[d - t + \left(\frac{E}{E_0}\right)t\right] \because \frac{E_0}{E} = \varepsilon_r = \text{Dielectric constant}$$

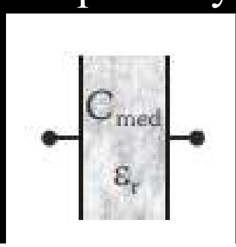
$$\Rightarrow V = \frac{\sigma}{\varepsilon_0}\left[d - t + \frac{t}{\varepsilon_r}\right] = \frac{q}{A\varepsilon_0}\left[d - t + \frac{t}{\varepsilon_r}\right] \Rightarrow C = \frac{q}{V} = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{\varepsilon_r}\right)}$$



If medium is fully present between the space. $\because$ t = d Now from equation (i) $C_{medum} = \frac{\varepsilon_0 \varepsilon_r A}{d}$

If capacitor is partially filled by a conducting slab of thickness ($t < d$).





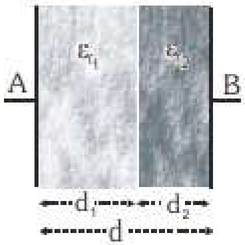
8.3 Distance and area division by dielectric

Distance Division

(i) Distance is divided and area remains same.

(ii) Capacitors are in series.

(iii) Individual capacitances are $C_1 = \frac{\varepsilon_0 \varepsilon_{r_1} A}{d_1}$, $C_2 = \frac{\varepsilon_0 \varepsilon_{r_2} A}{d_2}$



These two in series $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow \frac{1}{C} = \frac{d_1}{\varepsilon_0 \varepsilon_{r_1} A} + \frac{d_2}{\varepsilon_0 \varepsilon_{r_2} A}$

$$\Rightarrow \quad \frac{1}{C} = \frac{1}{\varepsilon_0 A}\left[\frac{d_1 \varepsilon_{r_2} + d_2 \varepsilon_{r_1}}{\varepsilon_{r_1} \varepsilon_{r_2}}\right] \Rightarrow C = \varepsilon_0 A\left[\frac{\varepsilon_{r_1} \varepsilon_{r_2}}{d_1 \varepsilon_{r_2} + d_2 \varepsilon_{r_1}}\right]$$

These two in series $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow \frac{1}{C} = \frac{d_1}{\varepsilon_0 \varepsilon_5 A} + \frac{d_2}{\varepsilon_0 \varepsilon_{\varepsilon_2} A}$

$$\Rightarrow \frac{1}{\mathrm{C}} = \frac{1}{\varepsilon_0\ \mathrm{A}}\left[\frac{\mathrm{d}_1 \varepsilon_{r_2} + \mathrm{d}_2 \varepsilon_{r_1}}{\varepsilon_{r_1} \varepsilon_{r_2}}\right] \Rightarrow \mathrm{C} = \varepsilon_0\ \mathrm{A}\left[\frac{\varepsilon_{r_1} \varepsilon_{r_2}}{\mathrm{d}_1 \varepsilon_{r_2} + \mathrm{d}_2 \varepsilon_{r_1}}\right]$$

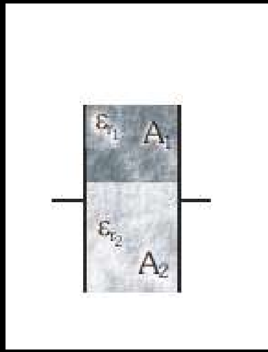
Special case - If $\mathrm{d}_1 = \mathrm{d}_2 = \frac{\mathrm{d}}{2} \Rightarrow \mathrm{C} = \frac{\varepsilon_0\ \mathrm{A}}{\mathrm{d}}\left[\frac{2\varepsilon_{\mathrm{r}_1} \varepsilon_{\mathrm{r}_2}}{\varepsilon_{r_1} + \varepsilon_{\tau_2}}\right]$

Area Division

(i) Area is divided and distance remains same.

(ii) Capacitors are in parallel.

(iii) Individual capacitances are $C_1 = \frac{\varepsilon_0 \varepsilon_{r_1} A_1}{d}\ C_2 = \frac{\varepsilon_0 \varepsilon_{r_2} A_2}{d}$



These two in parallel so $C = C_1 + C_2 = \frac{\varepsilon_0 \varepsilon_{r_1} A_1}{d} + \frac{\varepsilon_0 \varepsilon_{r_2} A_2}{d} = \frac{\varepsilon_0}{d}(\varepsilon_n A_1 + \varepsilon_{r2} A_2)$

Special case - If $A_1 = A_2 = \frac{A}{2}$ Then $C = \frac{\varepsilon_0 A}{d}\left(\frac{\varepsilon_{r_1} + \varepsilon_{t2}}{2}\right)$

Variable Dielectric Constant –

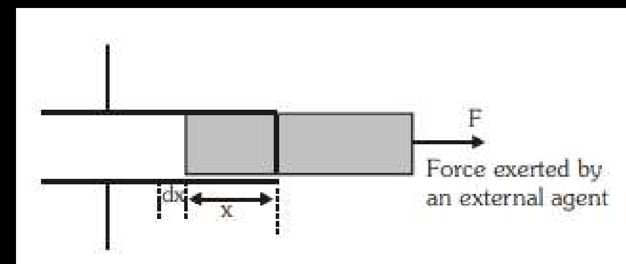
If the dielectric constant is variable, then equivalent capacitance can be obtained by selecting an element as per the given condition and then integrating.

(i) If different elements are in parallel, then $\mathrm{C} - \int \mathrm{dC}$, where dC - capacitance of selected differential element.

(ii) If different element are in series, then $\frac{1}{C} = \int d\left(\frac{1}{C}\right)$ is solved to get equivalent capacitance C.

<u>Force on a Dielectric in a capacitor</u>

Consider a differential displacement dx of the dielectric as shown in figure always keeping the net force on it zero so that the dielectric moves slowly without acceleration. Then, $W_{\text{Ekctrostaxic}} + W_F = 0$, where W_F denotes the work done by external agent in displacement dx



$$W_F = -W_{\text{Evedrostatic}}\ W_F = \Delta U$$

$$\Rightarrow\ -F \cdot dx = \frac{Q^2}{2}\, d\left[\frac{1}{C}\right]\left[W = \frac{Q^2}{2C}\right]$$

$$\Rightarrow\ -F \cdot dx = \frac{-Q^2}{2C^2} dC \Rightarrow F = \frac{Q^2}{2C^2}\left(\frac{dC}{dx}\right)$$

This is also true for the force between the plates of the capacitor. If the capacitor has battery connected to it, then as the p . d. V across the plates is maintained constant. $F = \frac{1}{2} V^2 \frac{dC}{dx}$.

Dielectric without battery

As shown in figure, a battery with a potential difference $|\Delta V_0|$ across its terminals is first connected to a capacitor C_0, which holds a charge $Q_0 = C_0|\Delta V_0|$. We then disconnect the battery, leaving $Q_0 =$ constant.

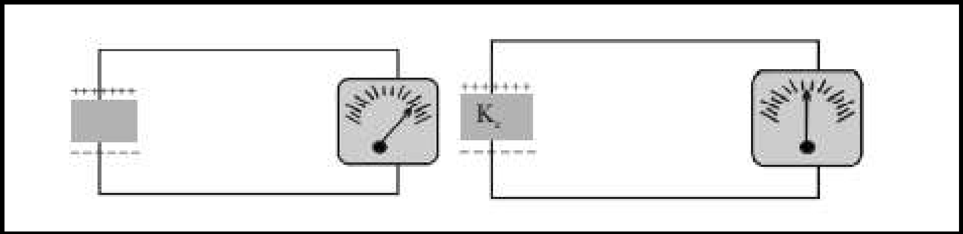


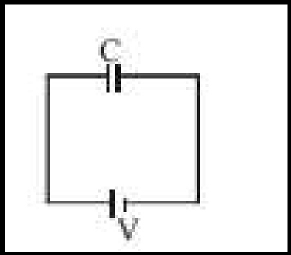
Figure : Inserting a dielectric material between the capacitor plates while keep point the charge $\mathbf{Q}_0$ constant

If we then insert a dielectric between the plates, while keeping the charge constant, experimentally it is found that the potential difference decreases by a factor of K_e. $|\Delta V_0| = \frac{|\Delta V_0|}{K_e}$

This implies that the capacitance is changed to $C = \frac{Q}{|\Delta V_0|} = \frac{Q}{|\Delta V_0|/K_e} = K_e \frac{Q}{|\Delta V_0|} = K_e C_0$ Thus, we see that the capacitance has increased by a factor of K_e, The electric field within the dielectric is now

$$E = \frac{|\Delta V|}{d} = \frac{|\Delta V_0|/K_e}{d} = \frac{1}{K_e}\left(\frac{|\Delta V_0|}{d}\right) = \frac{E_0}{K_e}$$

We see that in the presence of a dielectric, the electric field decreases by a factor of K_e.



EFFECT ON :	BEFORE	AFTER
Capacitance	C	KC
Charge	CV	CV
P.D	V	V/K
Energy	$\frac{1}{2}CV^2$	$\frac{1}{2}\frac{(CV)^2}{KC}=\frac{1}{2}\frac{CV^2}{K}$
Electric field	$E \times d = V$ $E = \frac{V}{d}$	$E' \times d = \frac{V}{K}$ $E' = \frac{V}{Kd}$

Dielectric with battery

Consider a second case where a battery supplying a potential difference remains connected as the dielectric is inserted. Experimentally, it is found (first by Faraday) that the change on the plates is increased by a factor K_e

$$Q = K_e Q_0$$

Where Q_0 is the charge on the plates in the absence of any dielectric

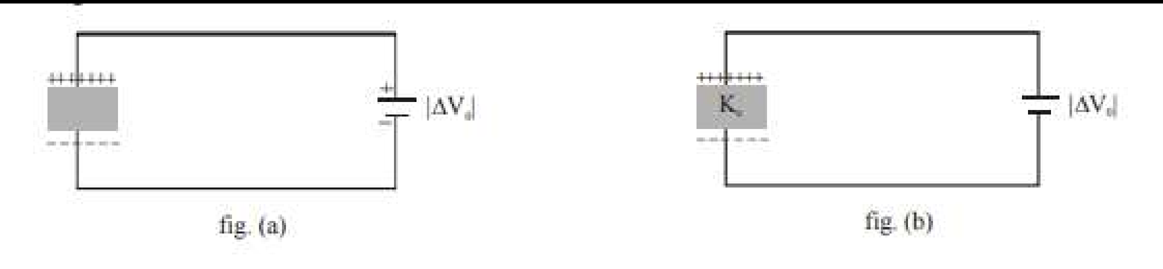
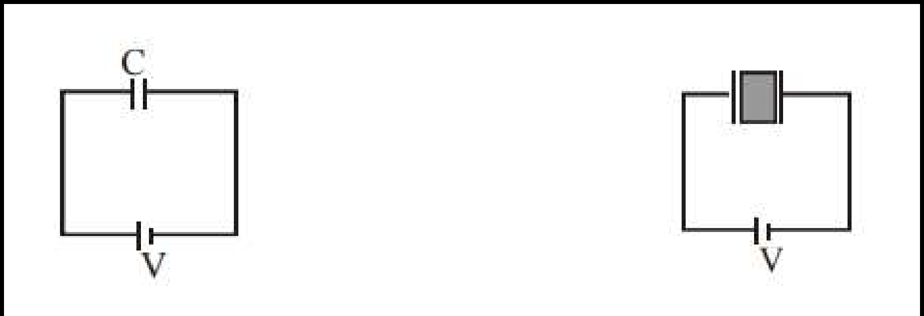


Figure: inserting a dielectric material between the capacitor plates while maintaining a constant potential difference $|V_0|$

The capacitance becomes

$$C = \frac{Q}{|\Delta V_0|} = \frac{K_e Q_0}{|\Delta V_0|} = K_e C_0$$

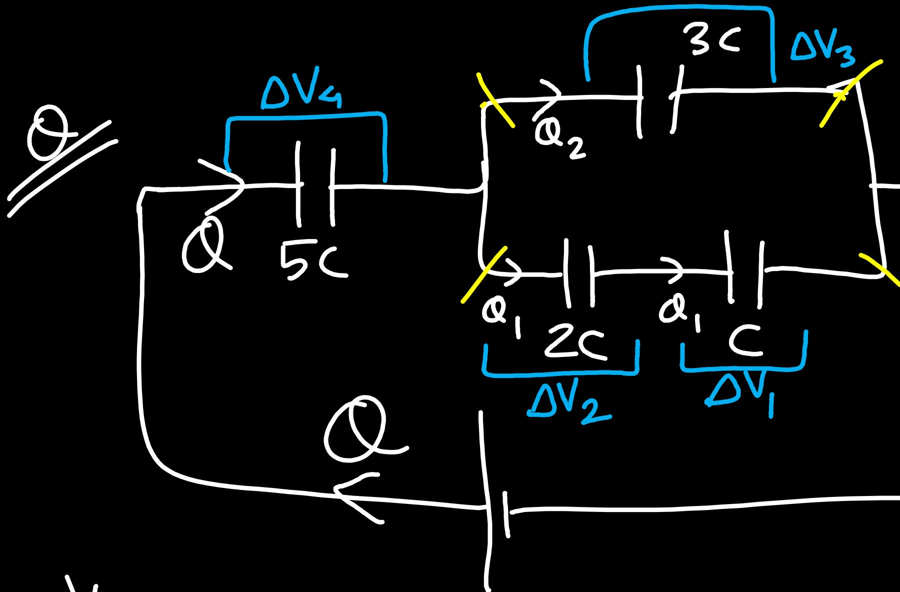
Which is the same as the first case where the charge Q_0 is kept constant, but now the charge has increased.



EFFECT ON :	BEFORE	AFTER
Charge	CV	KCV
P.D.	V	V
Capacitance	C	KC
Energy	$\frac{1}{2}CV^2$	$\frac{1}{2}KCV^2$
Electric Field	$E = \frac{V}{d}$	$E = \frac{V}{d}$

Break 20 min

Resume at 8:15 min



Voltage across C is 10 Volts

Find Emf of battery?

$\Delta V = \frac{Q_1}{C} = 10$

$Q_1 = 10C$

$\Delta V_1 = 10$

$\Delta V_2 = \frac{Q_1}{2C} = \frac{10C}{2C} = 5V$

$\Delta V_1 + \Delta V_2 = \Delta V_3$

$10 + 5 = \Delta V_3$

$\Delta V_3 = \frac{Q_2}{3C} = 15$

$Q_2 = 45C$

$Q = Q_1 + Q_2$

$= 10C + 45C$

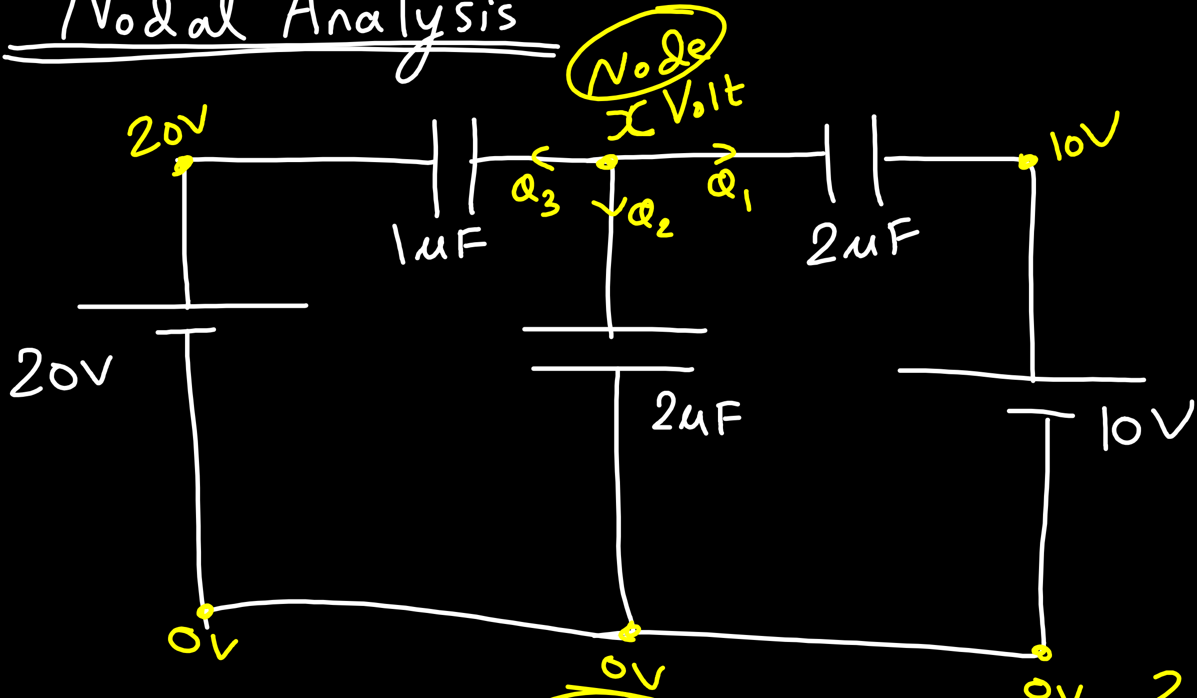
$Q = 55C$

$\Delta V_4 = \frac{Q}{5C} = \frac{55C}{5C}$

$= 11$

$Emf = (\Delta V_4) + (\Delta V_3) = 26$ Volt

Nodal Analysis



Find charge on each capacitor??

Node Se Har Jagah Charge Exit Karo

$$Q_1 + Q_2 + Q_3 = 0$$

$$2(x-10) + 2(x-0) + 1(x-20) = 0$$

$$5x = 40$$

$$x = 8$$

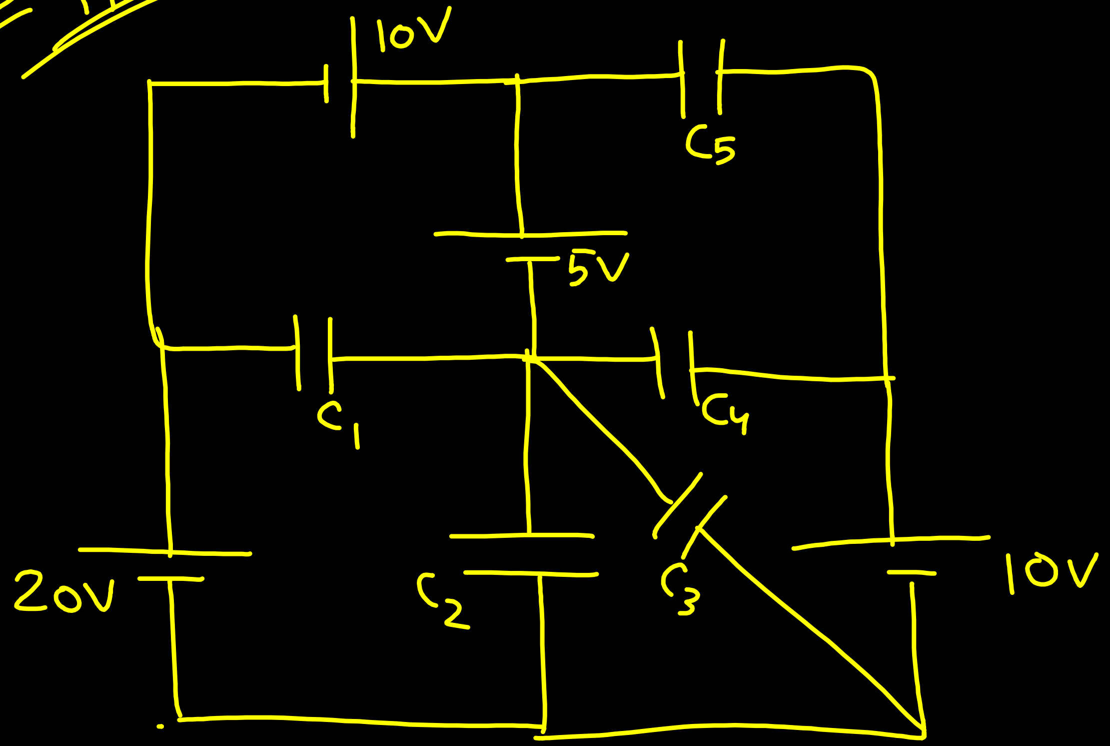
$$Q = C(\Delta V)$$

$$Q_2 = 16\,\mu C$$

$$|Q_1| = 4\,\mu C$$

$$|Q_3| = 12\,\mu C$$

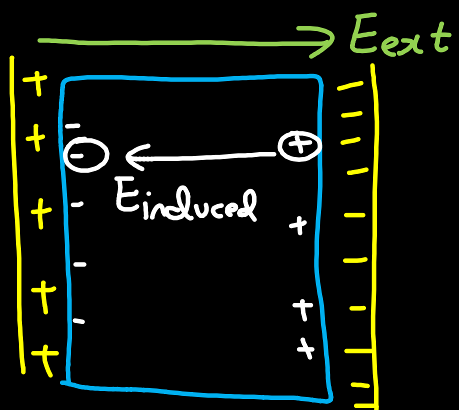
Q. H.W.



all $C = 100\mu F$

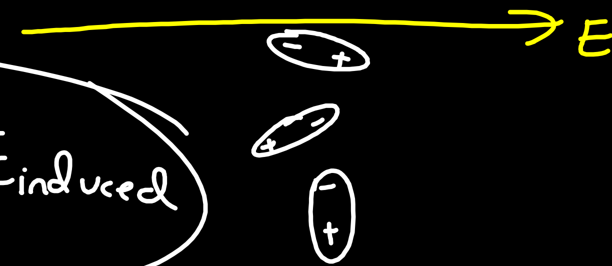
Find charge on each capacitor??

Dielectric

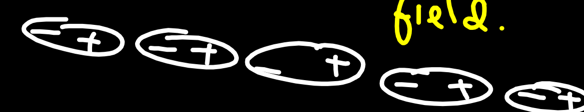


$$E_{\substack{net\\inside}} = E_{ext} - E_{induced}$$

insulator



dipole wants to align along the field.



$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$$

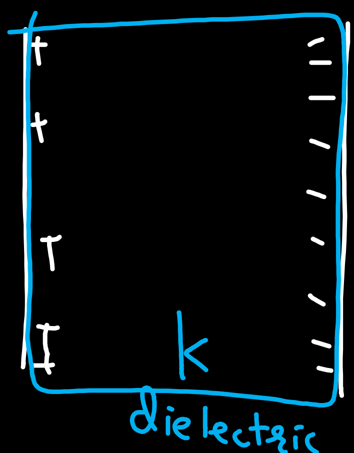
$$\Delta V = Ed = \frac{\sigma d}{\varepsilon_0}$$

$$C = \frac{Q}{\Delta V}$$

$$C = \frac{A\varepsilon_0}{d}$$

$$U = \frac{1}{2}\varepsilon_0 E^2 A d$$
$$= \frac{Q^2}{2C}$$
$$= \frac{1}{2}CV^2$$

full Space filled with dielectric



$$E_{net} = E_0 - E_{induced}$$

$$\frac{E_0}{k} = E_0 - E_{induced}$$

$$E_{induced} = E_0 - \frac{E_0}{k}$$

$$\boxed{E_{induced} = E_0\left(1 - \frac{1}{k}\right)}$$

$$C_{new} = \frac{A\varepsilon_0}{d}k = kC \rightarrow k \text{ times.}$$

$$E_{new} = \frac{\sigma}{\varepsilon_0\varepsilon_r} = \frac{Q}{A\varepsilon_0\varepsilon_r} = \frac{Q}{A\varepsilon_0 k} \quad [Q \text{ fixed}]$$

$$E_{net\ inside} = \frac{E_{vaccum}}{k}$$

$$E = \frac{\sigma}{\varepsilon_0}$$

$$E_{ind} = \frac{\sigma_{ind}}{\varepsilon_0}$$

$$E_{ind} = E_0\left(1 - \frac{1}{K}\right)$$

$$\sigma_{ind} = \sigma\left(1 - \frac{1}{K}\right)$$

Q

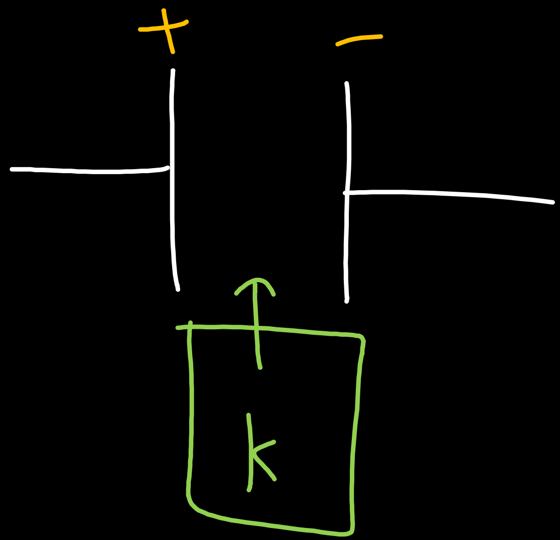
$$U = \frac{1}{2} C V^2$$

V constant prefered

$$U = \frac{Q^2}{2C}$$

Q constant prefered.

Q. C is charged battery disconnected / not connected. Dielectric is inserted. Change in following Quantities.



$C \longrightarrow kC \longrightarrow k$ times

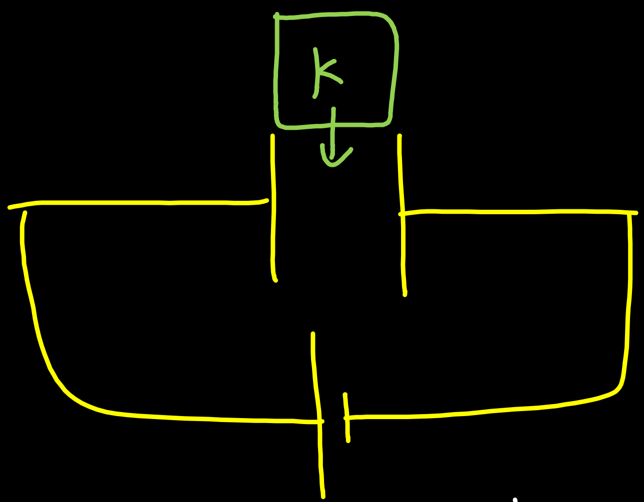
$E \longrightarrow \dfrac{\sigma}{\varepsilon_0} \quad \dfrac{Q}{A\varepsilon_0\varepsilon_r} \longrightarrow \dfrac{1}{k}$ times.

$\Delta V \longrightarrow Ed \longrightarrow \dfrac{1}{k}$ times.

$U \longrightarrow \dfrac{Q^2}{2C} \longrightarrow \dfrac{1}{k}$ times.

$Q \longrightarrow$ constant

Q/ C is kept connected with battery   Repeat



$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0} = \boxed{\frac{\Delta V}{d}}$$

$$Q = C\Delta V$$

$C \longrightarrow \qquad \longrightarrow$ K times.

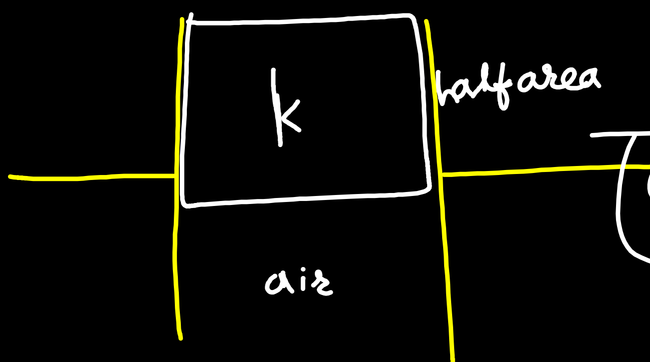
$E \longrightarrow \frac{\Delta V}{d} \longrightarrow$ constant

$\Delta V \longrightarrow$ constant   constant

$U \longrightarrow \frac{1}{2}CV^2 \longrightarrow$ K times.

$Q \longrightarrow Q = C\Delta V \longrightarrow$ K times.

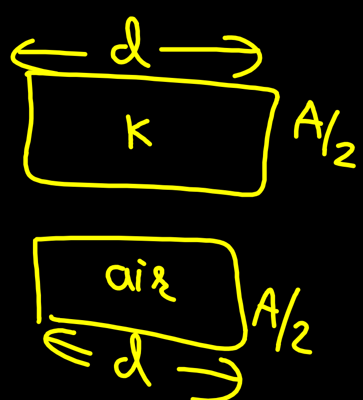
Q: Find C_{eq}



$$C_{eq} = C_1 + C_2$$

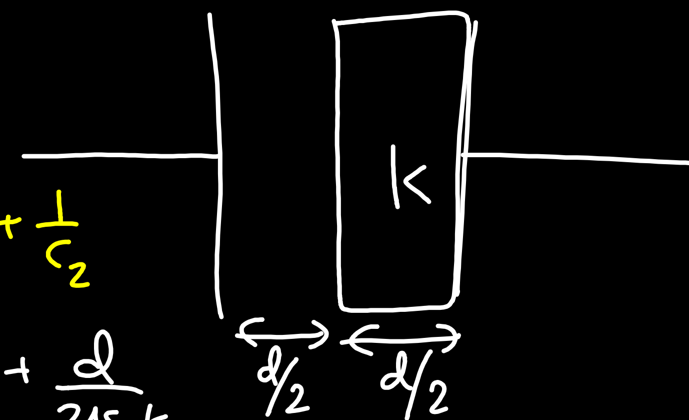
$$= \frac{A\varepsilon_0 k}{2d} + \frac{A\varepsilon_0}{2d}$$

$$C_{eq} = \frac{A\varepsilon_0(1+k)}{2d}$$



$$C_1 = \frac{(A/2)(\varepsilon_0)k}{d}$$

$$C_2 = \frac{(A/2)(\varepsilon_0)}{d}$$

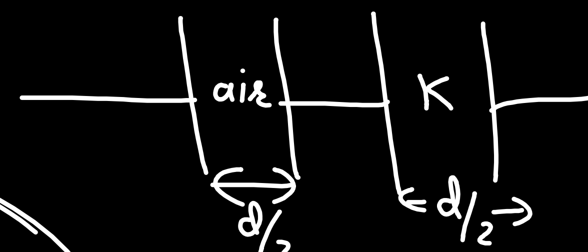


$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$= \frac{d}{2A\varepsilon_0} + \frac{d}{2A\varepsilon_0 K}$$

$$\frac{1}{C_{eq}} = \frac{d(k+1)}{2A\varepsilon_0 K}$$

$$C_{eq} = \frac{2A\varepsilon_0 K}{d(k+1)}$$



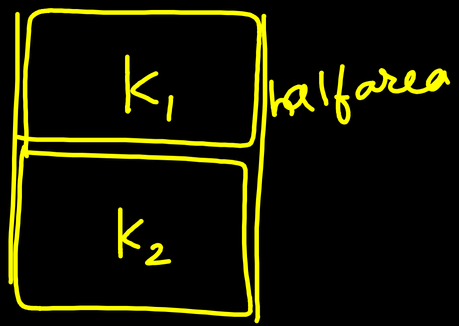
$$C_1 = \frac{A\varepsilon_0}{d/2} = \frac{2A\varepsilon_0}{d}$$

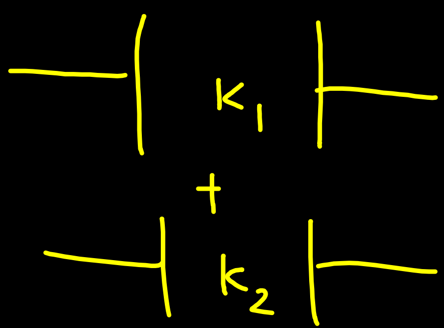
$$C_2 = \frac{A\varepsilon_0 K}{d/2}$$

$$C_2 = \frac{2A\varepsilon_0 K}{d}$$

Q.

Find equivalent dielectric constant which can replace both when filled completely





$$C_1 = \frac{(A/2)(\varepsilon_0)K_1}{d}$$

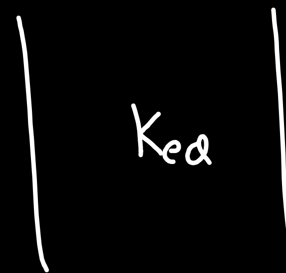
$$C_2 = \frac{(A/2)\varepsilon_0 K_2}{d}$$

$$C_{eq} = C_1 + C_2 = \frac{A\varepsilon_0}{2d}(K_1 + K_2)$$

$$C_{eq} = C_{eq}$$

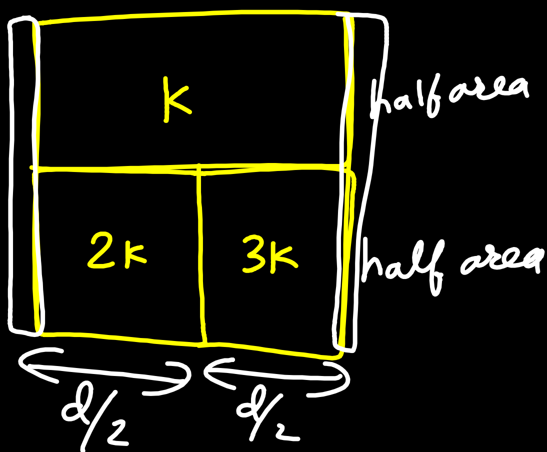
$$\frac{A\varepsilon_0}{2d}(K_1+K_2) = \frac{A\varepsilon_0 K_{eq}}{d}$$

$$\frac{K_1+K_2}{2} = K_{eq}$$



$$C_{eq} = \frac{A\varepsilon_0 K_{eq}}{d}$$

Q



Find $C_{eq} = ?$

$=$

$\boxed{K}$

$$C_1 = \frac{(A/2)(\varepsilon_0)K}{(d)}$$

$$C_1 = \frac{A\varepsilon_0 K}{2d}.$$

$$\boxed{C_{eq} = \left(\frac{C_2 C_3}{C_2 + C_3}\right) + C_1}$$

$\boxed{2K}$ $\boxed{3K}$

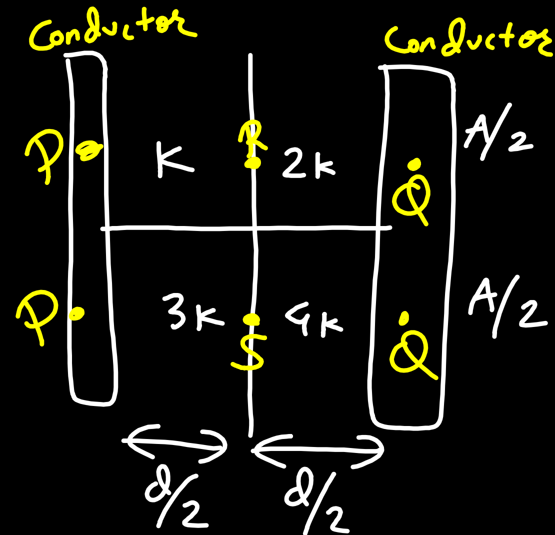
$$C_2 = \frac{(A/2)(\varepsilon_0)2K}{(d/2)} = \frac{2KA\varepsilon_0}{d}.$$

$$C_3 = \frac{(A/2)(\varepsilon_0)3K}{(d/2)}$$

$$C_2 = \frac{3A\varepsilon_0 K}{d}$$

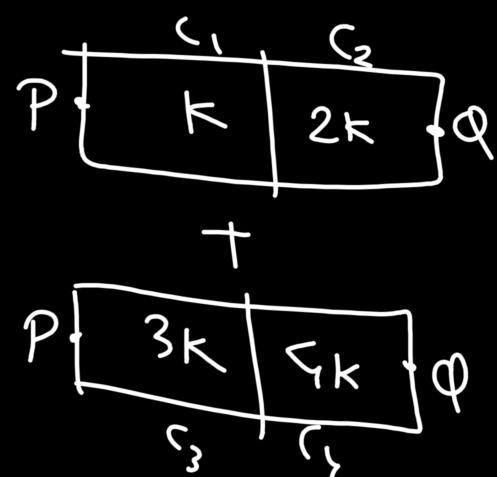
Q

H.W.

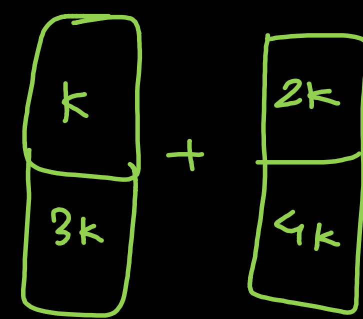


$$C_{eq} = \frac{C_1C_2}{C_1+C_2} + \frac{C_3C_4}{C_3+C_4}$$

Correct Setting



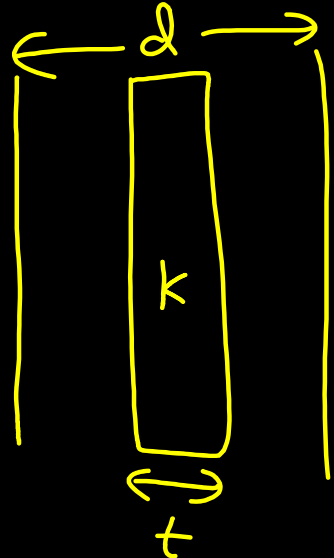
This Setting is Wrong



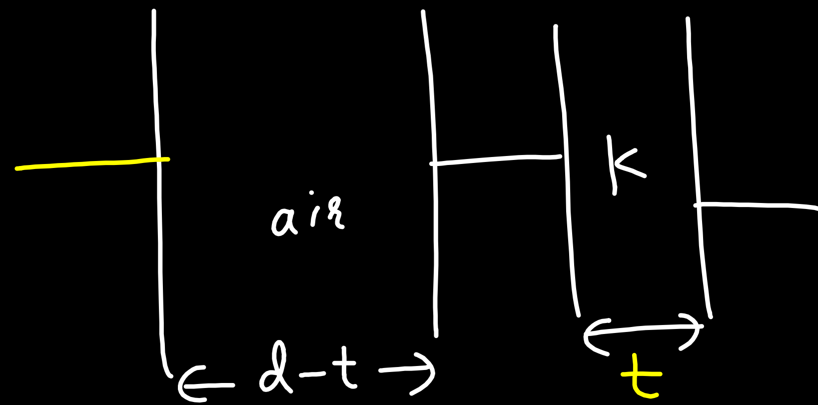
K & 3K Ko Parallel Nahi Man

2K & 4K " " " "

Q



$\Rightarrow$



$$C_1 = \frac{A\varepsilon_0}{d-t} \qquad C_2 = \frac{A\varepsilon_0 k}{t}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

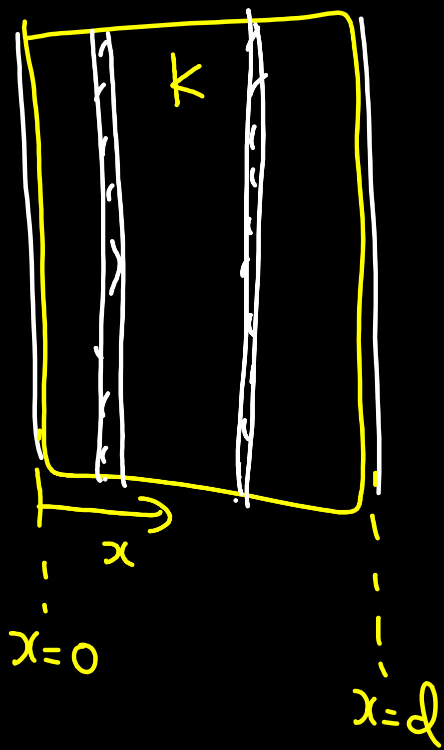
$$= \frac{d-t}{A\varepsilon_0} + \frac{t}{A\varepsilon_0 k} = \frac{dk - tk + t}{A\varepsilon_0 k}$$

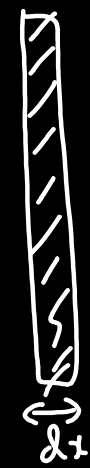
$$\Rightarrow \boxed{C_{eq} = \frac{A\varepsilon_0}{d + t\left(\frac{1}{k} - 1\right)}}$$

Variable K

Q

$$K = \alpha x + \beta$$





$$C = \frac{(A)(\varepsilon_0)(K)}{(dx)}$$

$$C = \frac{A\varepsilon_0(\alpha x + \beta)}{dx}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots \infty$$

$$\frac{1}{C_{eq}} = \int \frac{1}{C}$$

Unacademy Atoms

$$\frac{1}{C_{eq}} = \int \frac{1}{C}$$

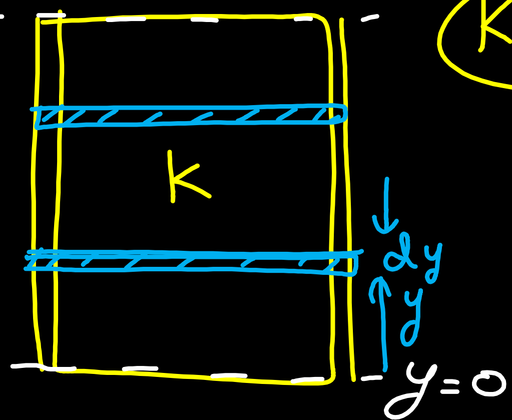
$$= \int_{x=0}^{x=d} \frac{dx}{A\varepsilon_0 (\alpha x + \beta)}$$

$$= \frac{1}{A\varepsilon_0} \left. \frac{\log_e(\alpha x + \beta)}{\alpha} \right]_0^d$$

$$\frac{1}{C_{eq}} = \frac{1}{\alpha A \varepsilon_0}\left[\log(\alpha d + \beta) - \log\beta\right]$$

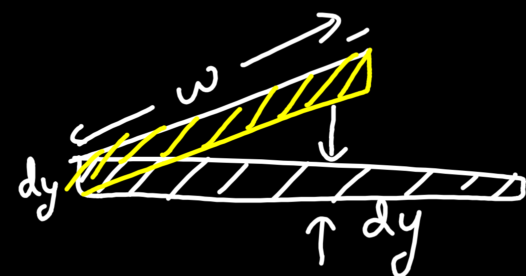
$$\boxed{C_{eq} = \frac{\alpha A \varepsilon_0}{\log\left(\frac{\alpha d + \beta}{\beta}\right)}}$$

Q



$K = \alpha y + \beta$

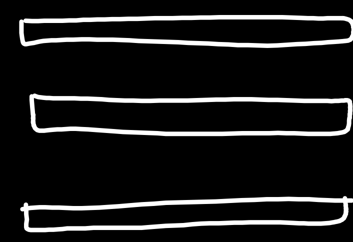
width of plate = ω (into the plane)



Find C_{eq}.

area = $(dy)\,\omega$

$$C = \frac{(\omega\, dy)(\varepsilon_0)K}{(d)} = \frac{\omega (dy)\varepsilon_0(\alpha y + \beta)}{d}$$



$$C_{eq} = C_1 + C_2 + C_3 + \cdots$$

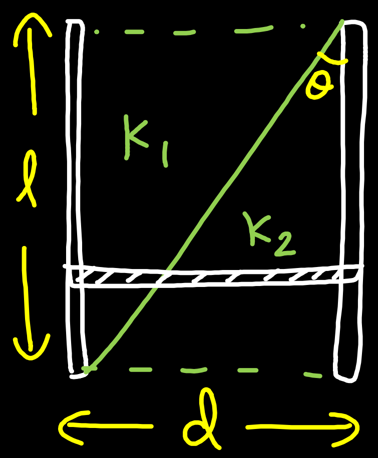
$$= \int C$$

$$= \int \frac{\omega \varepsilon_0 (\alpha y + \beta)\, dy}{d}$$

$$= \frac{\omega \varepsilon_0}{d} \left(\frac{\alpha y^2}{2} + \beta y \right) \Bigg]_0^{l}$$

$$C_{eq} = \frac{\omega \varepsilon_0}{d} \left(\frac{\alpha l^2}{2} + \beta l \right)$$

Q



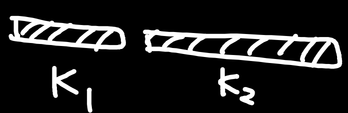
K_1 & K_2 constant

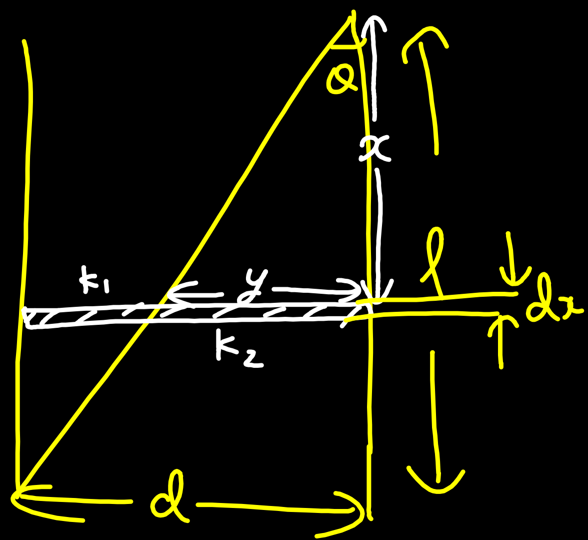
Find C_{eq}

Square plates

area $= l^2$

width $= l$





area

$$\text{area} = l(dx)$$

$$C_1 = \frac{(l\,dx)(\varepsilon_0)k_1}{(d-y)}$$

$$C_2 = \frac{(l\,dx)(\varepsilon_0)k_2}{y}$$

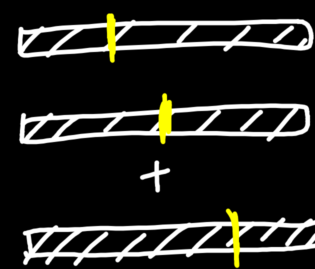
$$\tan\theta = \frac{d}{l} = \frac{y}{x}$$

$$\frac{xd}{l} = y$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$= \frac{d-y}{k_1\varepsilon_0 l(dx)} + \frac{y}{\varepsilon_0 k_2 l\,dx}$$

$$\frac{1}{C_{eq}} = \frac{K_2(d-y) + K_1 y}{K_1 K_2 \varepsilon_0 \ell \, dx}$$



$$C_{eq} = \frac{K_1 K_2 \varepsilon_0 \ell \quad dx}{K_2 d + y(K_1 - K_2)}$$

$$y = \frac{xd}{\ell}$$

$$C_{eq} = \int C_{eq} = k_1 k_2 \varepsilon_0 l \int \frac{dx}{k_2 d + \frac{xd}{l}(k_1 - k_2)}$$

$$= \frac{k_1 k_2 \varepsilon_0 l}{\frac{d}{l}(k_1 - k_2)} \log_e \left(k_2 d + \frac{xd}{l}(k_1 - k_2) \right) \Bigg]_0^l$$

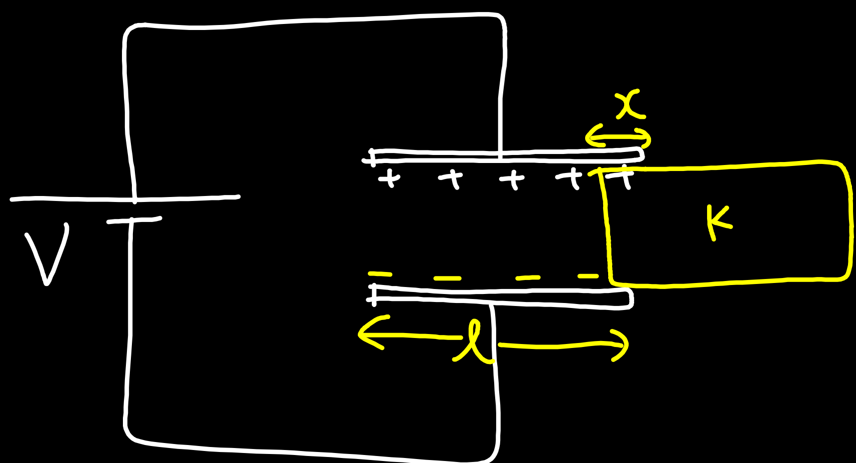
l^2 = area of plate

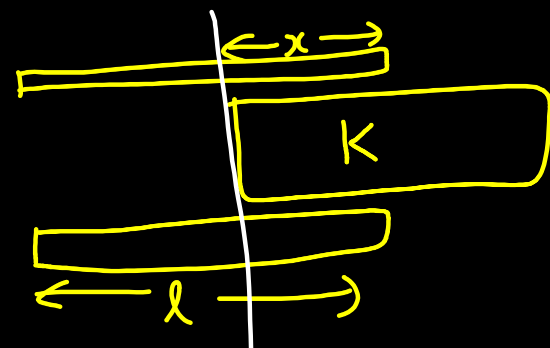
$$\boxed{C_{eq} = \frac{k_1 k_2 \varepsilon_0 l^2}{d(k_1 - k_2)} \log_e \left(\frac{k_1}{k_2} \right)}$$

Force on Dielectric

$$\boxed{F = -\frac{dU}{dx}}$$

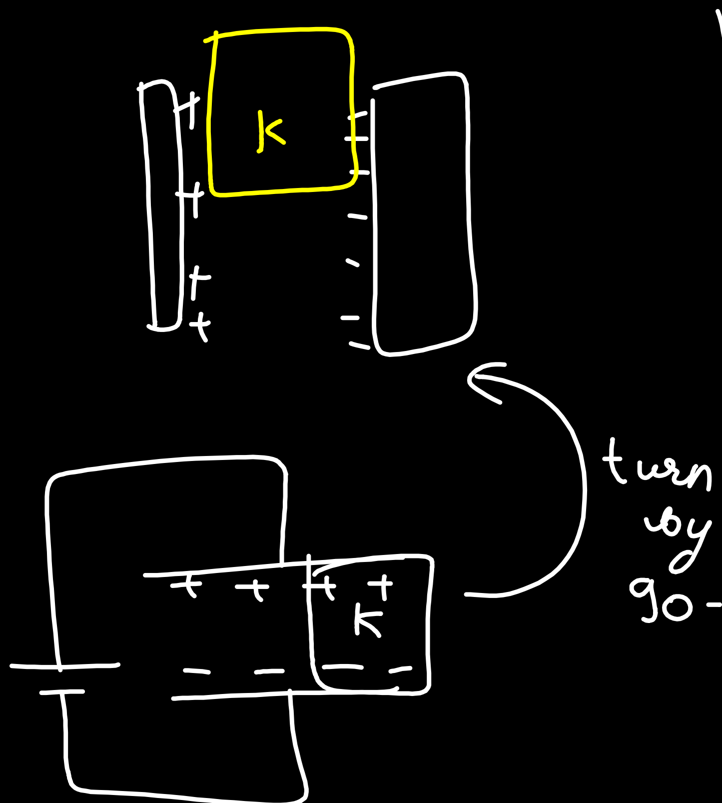
width $= \omega$





$$C_1 = \frac{(\omega)(l-x)\,\varepsilon_0}{d}$$

$$C_2 = \frac{(\omega x)\,\varepsilon_0\,K}{d}$$



$$C_{eq} = C_1 + C_2$$

$$= \frac{w(l-x)\varepsilon_0}{d} + \frac{w x \varepsilon_0 K}{d}$$

$$\boxed{C_{eq} = \frac{w\varepsilon_0}{d}\left[l - x + Kx\right]}$$

$$U = \frac{1}{2} C V^2$$

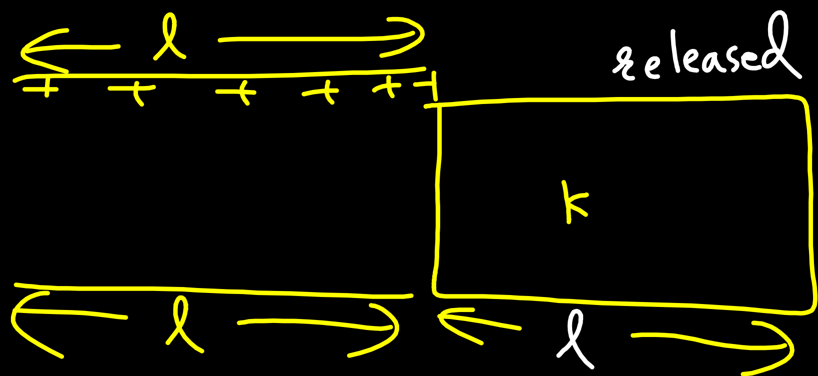
$$U = \frac{1}{2} \frac{\omega \varepsilon_0}{d} (l - x + kx) V^2$$

$$|F| = \left|\frac{\partial U}{\partial x}\right| = \frac{1}{2} \frac{\omega \varepsilon_0}{d} (0 - 1 + k) V^2$$

$$\boxed{F = \frac{\omega \varepsilon_0 V^2}{2d} (k - 1)}$$

Constant

$$acc = \frac{F}{m}$$



$$U = 0$$

$$a = F/M$$

$$s = l$$

$$s = \cancel{ut}^{\,0} + \frac{1}{2} a t^2$$

$$\sqrt{\frac{2s}{a}} = t_0$$

Time Period oscillation $= 4t$