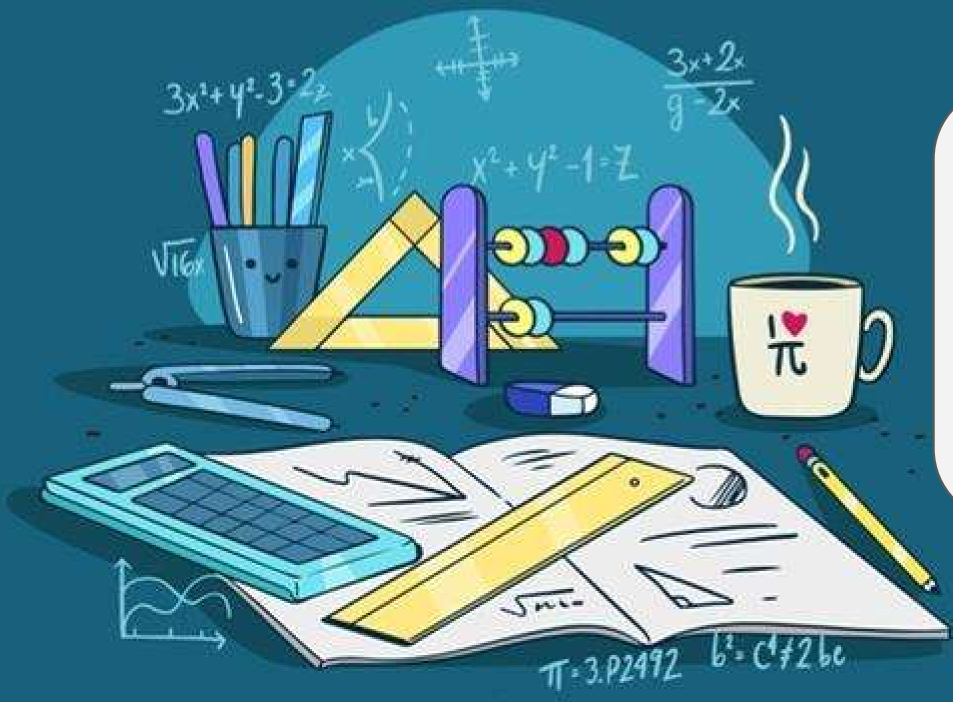


Differential Equations

Differential Equation

Examples

1. $$\left(\frac{d^2y}{dx^2}\right)^{1} + 3\frac{dy}{dx} + 2y = 0$$

O = 2
d = 1

2. $$\left[\frac{d^2y}{dx^2}\right]^{4} = \left[y + \left(\frac{dy}{dx}\right)^{6}\right]$$

O = 2
d = 4

3. $$\boxed{y^2 = x^2 + 2xy\frac{dy}{dx}}$$

O = 1
d = 1

4. $$\left(\frac{d^2y}{dx^2}\right)^{1} = x^2 + 2xy\frac{dy}{dx}$$

O = 2
d = 1

Order and Degree

Order and Degree of Differential Equation

Order

Order is the **highest differential** appearing a differential equation

Degree

Degree is **highest power of highest differential** after converting D.E. to polynomial form (remove fractional powers if any)

e.g. $$\boxed{\left(\frac{d^2y}{dx^2}\right)^1 = x^2 + 2xy\frac{dy}{dx}}$$

$$\boxed{\begin{matrix} O = 2 \\ d = 1 \end{matrix}}$$

Find **order** and **degree** of the following differential equation

$$\left(\frac{d^2y}{dx^2}\right)^4 = \left[y + \left(\frac{dy}{dx}\right)^6\right]^{\cancel{1/4}}$$

$O = 2$

$d = 4$

Find **order** and **degree** of the following differential equation

$$\frac{dy}{dx}+y=\frac{1}{\frac{dy}{dx}}$$

$$\left(\frac{dy}{dx}\right)^{2}+y\left(\frac{dy}{dx}\right)^{1}=1$$

$$O=1$$

$$\boxed{D=2}$$

Find **order** and **degree** of the following differential equation

$$e^{\frac{d^3y}{dx^3}} - x\frac{d^2y}{dx^2} + y = 0$$

$$\left(\frac{d^3y}{dx^3}\right) + \left(\frac{d^2y}{dx^2}\right)$$

$$\boxed{e^{\frac{d^3y}{dx^3}} = x\frac{d^2y}{dx^2} - y}$$

$O = 3$

$d = N.d.$

Find **order** and **degree** of the following differential equation

$$ln\left(\frac{dy}{dx}\right) = ax + by$$

$$\frac{dy}{dx} = \boxed{e^{ax+by}}$$

$$\boxed{\frac{dy}{dx} = f(x, y)}$$

O = 1

d = 1

The order and degree of the differential equation

$$\left(\left(1+3\frac{dy}{dx}\right)^{2/\cancel{3}}\right)^{3} = \left(4\frac{d^3y}{dx^3}\right)^{3}$$ are

A. (1, ⅔)

B. (3, 1)

C. (3,3) ✓

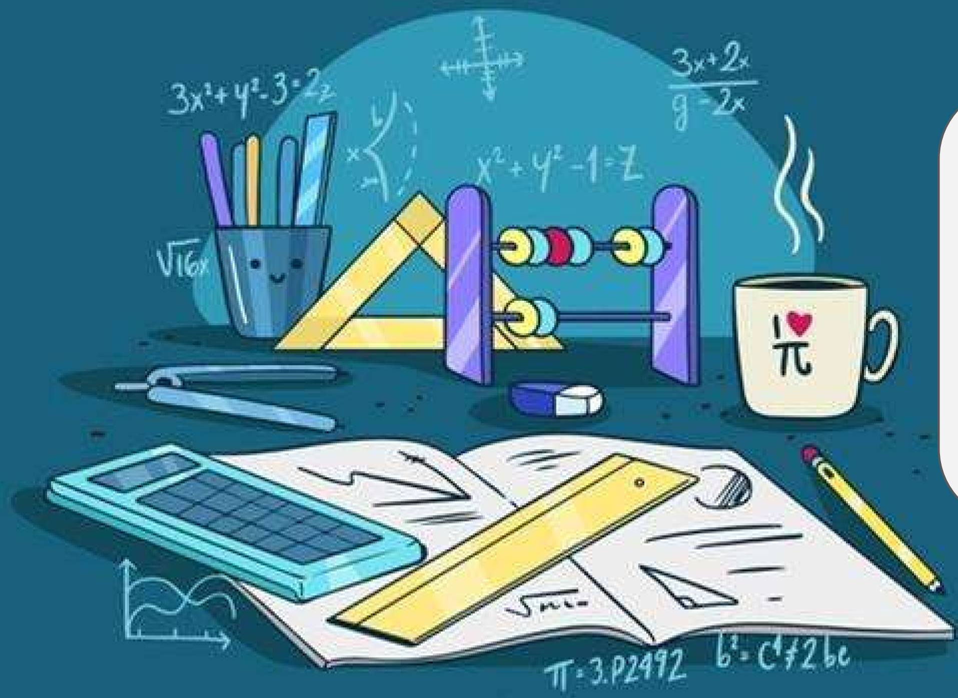
D. (1,2)

2002

$$\left(1+3\frac{dy}{dx}\right)^{2} = 4^{3}\left(\frac{d^3y}{dx^3}\right)^{3}$$

$O = 3$

$d = 3$



Formation of Differential Equations

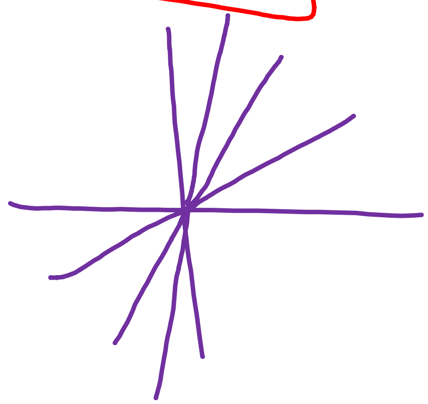
Formation of Differential Equations

Unacademy Atoms

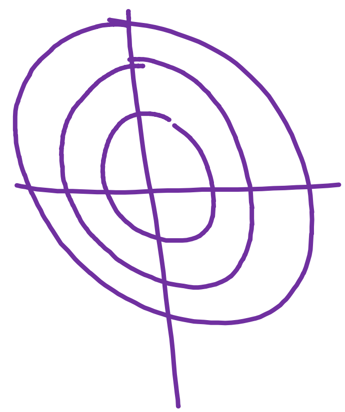
Family of Curves $\xrightarrow{\text{Differentiate}}$ **Differential Equation**

Differential Equation $\xrightarrow{\text{Integrate}}$ **Family of Curves**

$\boxed{y = mx}$



$x^2 + y^2 = r^2$



$y = mx$

$\boxed{\frac{dy}{dx} = m} \longrightarrow$ D.E ✗

$\boxed{\frac{dy}{dx} = \frac{y}{x}} \longrightarrow$ D.E ✓

Formation of Differential Equations

From differential equation for following family of curves

1. $\boxed{y = mx}$ — (1) $m \rightarrow$ parameter. / a.c

\# $\boxed{\dfrac{dy}{dx} = m}$ — (2)

$\boxed{\text{Order} = \text{no. of parameters}}$

\# $\boxed{\dfrac{dy}{dx} = \dfrac{y}{x}}$

$\boxed{O = 1}$

$d = 1$

Formation of Differential Equations

1. Every family of curves has its own differential equation

2. The order of differential equation is always equal to the number of independent arbitrary constant in the equation of family of curves.

3. Final differential equation should not have any arbitrary constant.

The order of the differential equation whose general solution is given by $y = (C_1 + C_2)\cos(x + C_3) - C_4 e^{x+C_5}$, where C_1, C_2, C_3, C_4, C_5, are arbitrary constant is

~~A.~~ 5

B. 4

~~C.~~ 3

D. 2

$$y = c\cos(x + c_3) - \boxed{c_4 e^{c_5}}\, e^{x}$$

$$\boxed{y = c\cos(x + c_3) - k e^{x}}$$

Unacademy Atoms

The differential equation satisfied by the system of parabolas

$y^2 = 4a(x + a)$ is:

A. $y\left(\frac{dy}{dx}\right)^2 - 2x\left(\frac{dy}{dx}\right) - y = 0$

B. $y\left(\frac{dy}{dx}\right)^2 - 2x\left(\frac{dy}{dx}\right) + y = 0$

C. $y\left(\frac{dy}{dx}\right)^2 + 2x\left(\frac{dy}{dx}\right) - y = 0$

D. $\cancel{y\left(\frac{dy}{dx}\right) + 2x\left(\frac{dy}{dx}\right) - y = 0}$

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$y^2 = 4a(x+a)$ —— (1)

$2y\frac{dy}{dx} = 4a$

$\boxed{\frac{yy'}{2} = a}$ —— (2)

$$y^{\cancel{2}} = 4\,\frac{\cancel{y}\,y'}{2}\left(x + \frac{y\,y'}{2}\right)$$

$$y = 2y'\left(x + \frac{y\,y'}{2}\right)$$

$$y = 2xy' + y(y')^2$$

$$y\left(\frac{dy}{dx}\right)^2 + 2x\left(\frac{dy}{dx}\right) - y = 0$$

Unacademy Atoms

The difference between degree and order of differential equation that represents the family of curves given by $y^2 = a\left(x + \frac{\sqrt{a}}{2}\right)$, $a > 0$ is

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$$y^2 = a\left(x + \frac{\sqrt{a}}{2}\right) \quad \text{--- (1)}$$

$$2yy' = a \quad \text{--- (2)}$$

$$y^{\cancel{2}} = 2\cancel{y}y'\left(x + \frac{\sqrt{2yy'}}{2}\right)$$

$$y = 2y'\left(x + \frac{\sqrt{2yy'}}{2}\right)$$

$$y = 2xy' + y'\sqrt{2yy'}$$

$$(y - 2xy')^2 = (y')^2(2yy')$$

$\cancel{y''}$

$$(y - 2xy')^2 = 2y(y')^3$$

$O = 1$

$d = 3$

2

Multi Correct Question

Unacademy Atoms

Consider the family of all circles whose centres lie on the straight line $y = x$. If this family of circle is represented by the differential equation $\boxed{Py'' + Qy' + 1 = 0}$, where P, Q are functions of x, y and y', then which of the following statement is(are) true?

JEE Adv. 2015

~~A.~~ $P = y + x$

B. $P = y - x$

C. $P+Q = 1 - x + y + y' + (y')^2$

~~D.~~ $P-Q = x + y - y' - (y')^2$

$C(a, a)$ $\boxed{rad = r}$

$$\boxed{(x-a)^2 + (y-a)^2 = r^2} \quad —— \quad 1$$

$$\cancel{2}(x-a)+\cancel{2}(y-a)y'=0$$

$$\underbrace{x-a}+yy'-\underbrace{ay'}=0$$

$$\boxed{\frac{x+yy'}{1+y'}=a}$$

$$(x-a)+(y-a)y'=0 \quad \text{— } (2)$$

$$\boxed{1+(y-a)\,y''+(y')^2=0} \quad \text{— } (3)$$

$$1+\left(y-\frac{x+yy'}{1+y'}\right)y''+(y')^2=0$$

$$(1+y')\left\{1+\frac{(y-x)\,y''}{1+y'}+(y')^2=0\right\}$$

$$1 + y' + (y - x) y'' + (y')^2 (1 + y') = 0$$

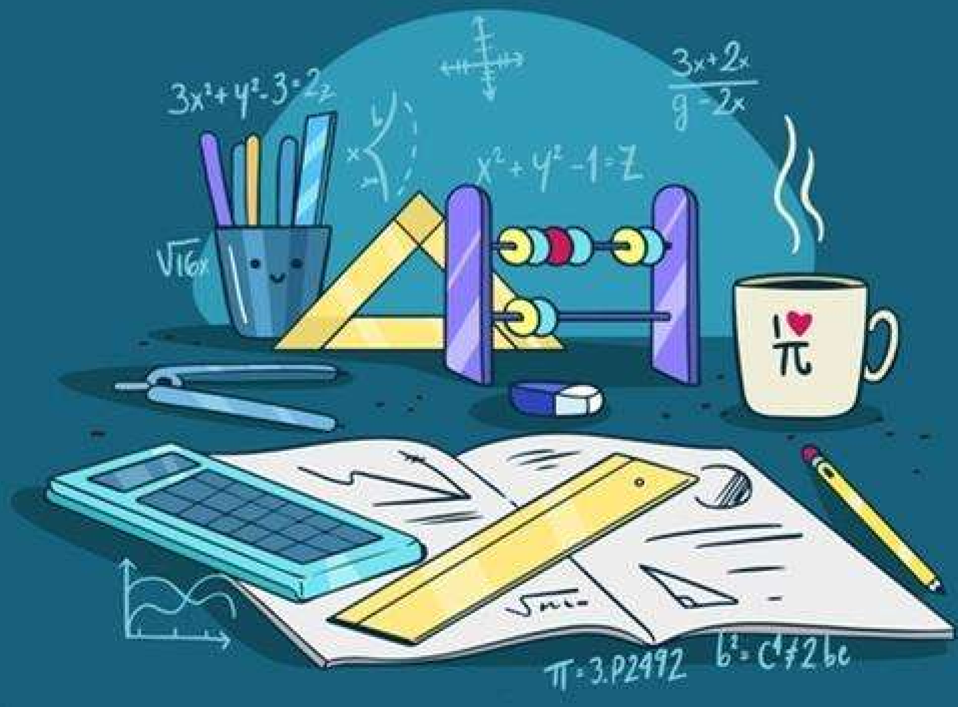
$$(y - x) y'' + \underbrace{y' + (y')^2 + (y')^3} + 1 = 0$$

$$(y - x) y'' + (1 + y' + (y')^2) y' + 1 = 0$$

$$P y'' + Q y' + 1 = 0$$

$$P \equiv y - x$$

$$Q \equiv 1 + y' + (y')^2$$



Solution of Differential Equations

Formation of Differential Equations

Family of Curves $\xrightarrow{\text{Differentiate}}$ Differential Equation

Family of Curves $\xleftarrow[\text{Integrate}]{}$ Differential Equation

Solution of a Differential Equation

Find the solution of differential equation

$O = 1$
$d = 1$

$$\boxed{\left(\frac{dy}{dx}\right)^1 = \frac{y}{x}}$$

$$\int \frac{dy}{y} = \int \frac{dx}{x}$$

$$\ln y = \ln x + c$$

$$\ln\left(\frac{y}{x}\right) = c$$

$$\frac{y}{x} = e^c$$

$$\frac{y}{x} = K$$

$$\boxed{y = Kx}$$

General and Particular Solutions

$\boxed{y = kx}$ G. S

$\Rightarrow (2, 1)$

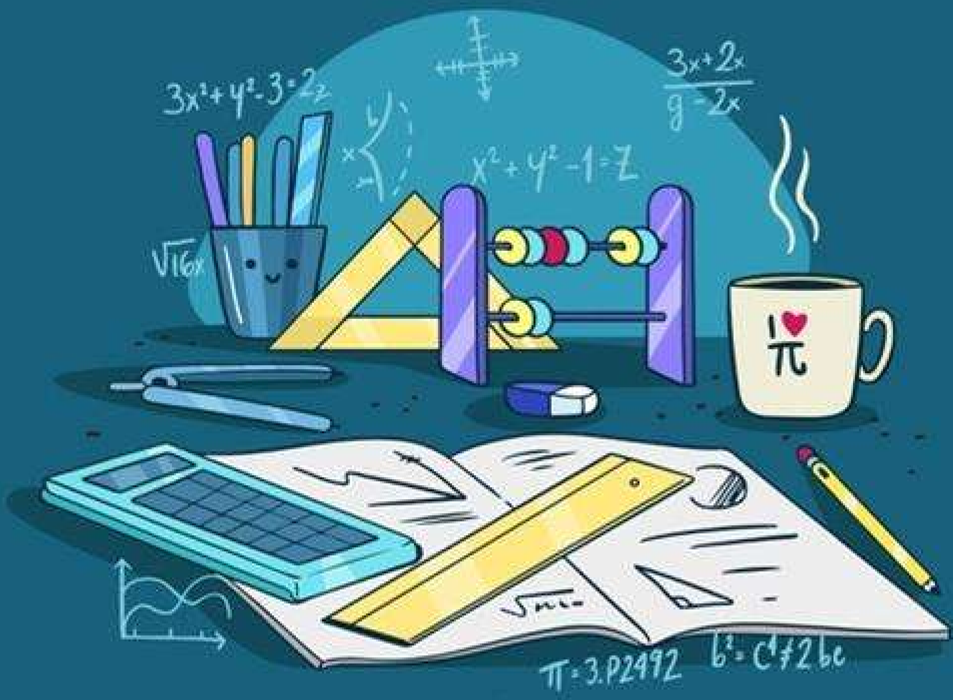
$1 = k(2)$

$\therefore \boxed{k = 1/2}$

$\boxed{y = \frac{1}{2}x}$ P. S

Differential Equation

- Variable Separable ☆☆☆
  - Reducible Variable Separable
    - T-1
    - T-2
    - T-3
- Homogeneous ☆
  - Reducible to Homogeneous ✓
- Linear ☆☆☆
  - Reducible to Linear (Bernoulli's D.E.) ✓



Variable Separable

Variable Separable

$$\frac{dy}{dx} = \frac{y}{x}$$

$$\int \frac{dy}{y} = \int \frac{dx}{x}$$

Unacademy Atoms

Let y = y(x) be solution of the differential equation

$\boxed{\log_e\left(\frac{dy}{dx}\right) = 3x + 4y}$, with $\boxed{y(0) = 0}$

$y(0) = 0$

$\uparrow \quad \uparrow$

$x \quad y$

If $y\left(\underbrace{-\frac{2}{3}\log_e 2}_{x}\right) = \underbrace{\alpha \log_e 2}_{y}$, then the value of α is equal to:

JEE Main 2021

(1) $-\frac{1}{4}$

(2) $\frac{1}{4}$

(3) 2

(4) $-\frac{1}{2}$

$$\frac{dy}{dx} = e^{3x} \cdot e^{4y}$$

$$\int e^{-4y}\, dy = \int e^{3x}\, dx$$

$$-\frac{e^{-4y}}{4} = \frac{e^{3x}}{3} + c$$

$$-\frac{1}{4} = \frac{1}{3} + c$$

$$\therefore \boxed{c = \frac{-7}{12}}$$

$$-\frac{e^{-4y}}{4} = \frac{e^{3x}}{3} - \frac{7}{12}$$

$$-\frac{e^{-4(\alpha \ln 2)}}{4} = \frac{e^{\cancel{3}\left(\frac{-2}{\cancel{3}}\ln 2\right)}}{3} - \frac{7}{12}$$

$$-\frac{2^{-4\alpha}}{4} = \frac{\frac{1}{4}}{3} - \frac{7}{12}$$

$$\cancel{+}\frac{2^{-4\alpha}}{4} = \cancel{+}\frac{1}{2}$$

$$2^{\boxed{-4\alpha}} = 2^{\boxed{1}}$$

$$\alpha = \frac{-1}{4}$$

Unacademy Atoms

Let $y = y(x)$ be the solution of the differential equation $\boxed{e^x\sqrt{1-y^2}\,dx + \left(\frac{y}{x}\right) dy = 0}$, $y(1) = -1$.

Then the value of $\boxed{(y(3))^2}$ is equal to:

(1) $1-4e^3$

(2) $1-4e^6$ ✓

(3) $1+4e^3$

(4) $1+4e^6$

JEE Main 2021

$$\frac{x}{\sqrt{1-y^2}}\left\{e^x\sqrt{1-y^2}\,dx + \frac{y\,dy}{x} = 0\right\}$$

$x = 1$
$y = -1$

$$\int \underbrace{x}_{\text{I}}\,\underbrace{e^x}_{\text{II}}\,dx + \int \frac{\boxed{y\,dy}}{\sqrt{1-y^2}} = 0$$

$1 - y^2 = t^2$

$-\cancel{2}y\,dy = \cancel{2}t\,dt$

$$xe^x - e^x + \int \frac{-\cancel{t}\,dt}{\cancel{t}} = 0$$

$$xe^x - e^x - \sqrt{1-y^2} = C$$

$$xe^x - e^x - \sqrt{1-y^2} = \cancel{C}$$

$x = 1 \quad y = -1$

$y(\overset{x}{3})$

$$0 - 0 = c$$

$$\therefore c = 0$$

$$xe^x - e^x = \sqrt{1-y^2}$$

$$3e^3 - e^3 = \sqrt{1-y^2}$$

$$2e^3 = \sqrt{1-y^2}$$

$$4e^6 = 1 - y^2 \Rightarrow \boxed{y^2 = 1 - 4e^6}$$

If y = y(x) satisfies the differential equation $8\sqrt{x}\left(\sqrt{9+\sqrt{x}}\right)dy = \left(\sqrt{4+\sqrt{9+\sqrt{x}}}\right)^{-1} dx,\ x>0$ and y(0) = √7, then y(256) =

x y

$x = 256$

A. 3

B. 9

C. 16

D. 80

JEE Adv. 2018

$$\int dy = \int \boxed{\frac{\overset{\to\, dt}{1}}{\sqrt{4+\sqrt{9+\sqrt{x}}}} \times \frac{1}{\sqrt{9+\sqrt{x}}} \times \frac{1}{8\sqrt{x}} \cdot dx}$$

$$\sqrt{4+\sqrt{9+\sqrt{x}}} = t$$

$$\frac{1}{2\sqrt{4+\sqrt{9+\sqrt{x}}}} \times \frac{1}{2\sqrt{9+\sqrt{x}}} \times \frac{1}{2\sqrt{x}} \cdot dx = dt$$

$$\int dy = \int dt$$

$$y = t + c$$

$$y = \sqrt{4+\sqrt{9+\sqrt{x}}} + c$$

$x = 0 \quad y = \sqrt{7}$

$$\sqrt{7} = \sqrt{4+3} + c$$

$$\therefore \boxed{c = 0}$$

$$y = \sqrt{4+\sqrt{9+\sqrt{x}}}$$

$$y = \sqrt{4+\sqrt{9+16}}$$

$$y = 3$$

Unacademy Atoms

Let b be a nonzero real number. Suppose $f: \mathbb{R} \to \mathbb{R}$ is a differentiable function such that $f(0) = 1$. If the derivative f' of f satisfies the equation

$$\boxed{f'(x) = \frac{f(x)}{b^2 + x^2}}$$

$x = 0$
$y = 1$

for all $x \in \mathbb{R}$, then which of the following statements is/are TRUE?

JEE Adv. 2020

AC

(A) If $b > 0$, then f is an increasing function

(B) If $b < 0$, then f is a decreasing function

Inc

(C) $f(x)f(-x) = 1$ for all $x \in \mathbb{R}$

(D) $f(x) - f(-x) = 0$ for all $x \in \mathbb{R}$

$$f(x) \neq f(-x)$$

$$\frac{dy}{dx} = \frac{y}{b^2+x^2}$$

$$\int \frac{dy}{y} = \int \frac{dx}{b^2+x^2}$$

$$\ln y = \frac{1}{b}\tan^{-1}\left(\frac{x}{b}\right) + \cancel{c}$$

$$\boxed{y = e^{\frac{1}{b}\tan^{-1}\left(\frac{x}{b}\right)}}$$

$$y = f(x) = e^{\boxed{\frac{1}{b}\tan^{-1}\left(\frac{x}{b}\right)}} = e^{\square} = +$$

$$\frac{dy}{dx} = \frac{\boxed{y}}{\boxed{b^2+x^2}}$$

$y \rightarrow +$, $b^2+x^2 \rightarrow +$

$$\boxed{\frac{dy}{dx} > 0} \quad f \uparrow$$

$$f(x)\, f(-x) = e^{\frac{1}{b}\tan^{-1}\left(\frac{x}{b}\right)} \cdot e^{-\frac{1}{b}\tan^{-1}\left(\frac{x}{b}\right)}$$

$$= e^{0}$$

$$= 1$$

Let $f: \mathbb{R} \longrightarrow \mathbb{R}$ be a differentiable function with $f(0) = 0$. If $y = f(x)$ satisfies the differential equation **$dy/dx = (2 + 5y)(5y - 2)$**, then the value of $\lim_{x \to -\infty} f(x)$ is ____

JEE Adv. 2018

$$\frac{dy}{dx} = (2+5y)(5y-2)$$

$$\Rightarrow \frac{1}{4}\int \frac{(5y+2)-(5y-2)}{(5y+2)(5y-2)}\,dy = \int dx$$

$$\Rightarrow \frac{1}{4}\left[\int \frac{1}{5y-2}\,dy - \int \frac{dy}{5y+2}\right] = x + c$$

$$\Rightarrow \frac{1}{4}\left[\frac{1}{5}\ln|5y-2| - \frac{1}{5}\ln|5y+2|\right] = x + c$$

$$\Rightarrow \frac{1}{20}\ln\left|\frac{5y-2}{5y+2}\right| = x + \cancel{c}$$

$x=0 \quad y=0 \qquad \boxed{c=0}$

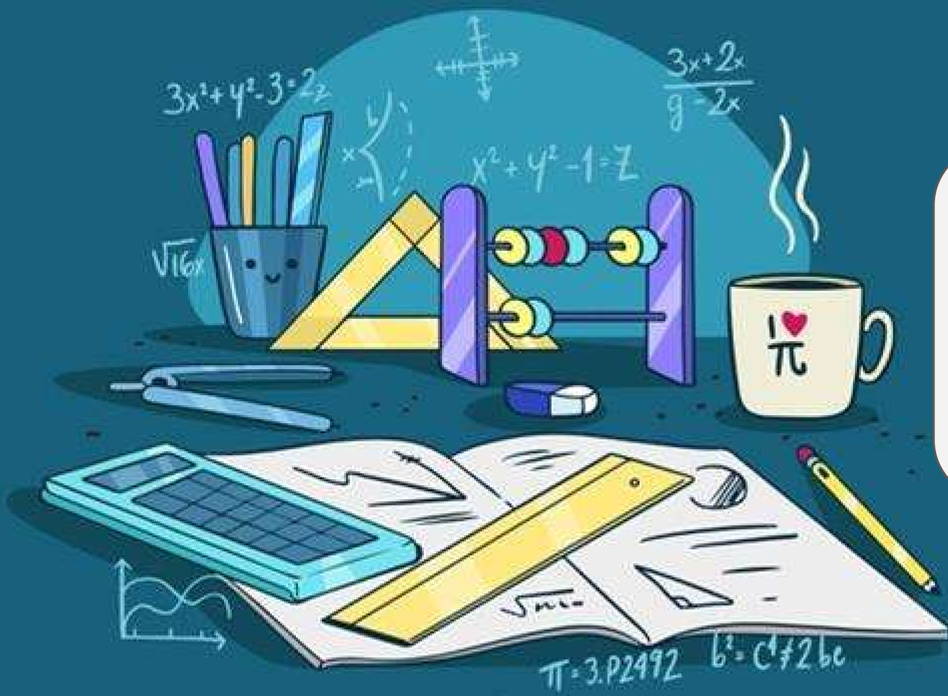
$$\ln\left|\frac{5y-2}{5y+2}\right| = 20x$$

$$\left|\frac{5y-2}{5y+2}\right| = \cancel{e^{20(-\infty)}}$$

$$\left|\frac{5y-2}{5y+2}\right| = 0$$

$$\therefore\ y = 2/5$$

Unacademy Atoms



Reducible Variable Separable: Type 1

Differential Equation

- Variable Separable
  - Reducible Variable Separable
    - T-1
    - T-2
    - T-3
- Homogeneous
  - Reducible to Homogeneous
- Linear
  - Reducible to Linear (Bernoulli's D.E.)

Type 1 : Reducible Variable Separable

Standard Form: $\boxed{\dfrac{dy}{dx} = f(ax + by + c)}$

Method: Put $ax + by + c = t$

The solution of the differential equation, $\boxed{\frac{dy}{dx}} = (x-y)^2$, when $y(1) = 1$, is :

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(a) $\log_e\left|\frac{2-x}{2-y}\right| = x - y$

(b) $-\log_e\left|\frac{1-x+y}{1+x-y}\right| = 2(x-1)$

(c) $-\log_e\left|\frac{1+x-y}{1-x+y}\right| = x+y-2$

(d) $\log_e\left|\frac{2-y}{2-x}\right| = 2(y-1)$

$x - y = t$

$1 - \frac{dy}{dx} = \frac{dt}{dx}$

$\therefore \boxed{\frac{dy}{dx} = 1 - \frac{dt}{dx}}$

$1 - \frac{dt}{dx} = t^2$

$1 - t^2 = \frac{dt}{dx}$

$\int dx = \int \frac{dt}{1^2 - t^2}$

$$x = \frac{1}{2(1)} \ln\left(\frac{1+t}{1-t}\right) + c$$

$$x = \frac{1}{2} \ln\left|\frac{1+x-y}{1-x+y}\right| + c$$

$x=1 \quad y=1$

$$1 = \frac{1}{2}\ln(1) + c$$

$$\therefore \boxed{1 = c}$$

$$x = \frac{1}{2}\ln\left|\frac{1+x-y}{1-x+y}\right| + 1$$

JEE Main 2020

The solution of the differential equation

$$\boxed{\frac{dy}{dx}} - \frac{\boxed{y+3x}}{\log_e \boxed{(y+3x)}} + 3 = 0 \text{ is :}$$

(where C is a constant of integration.)

(1) $x - \frac{1}{2}(\log_e(y+3x))^2 = C$

(2) $x - \log_e(y+3x) = C$

(3) $y + 3x - \frac{1}{2}(\log_e x)^2 = C$

(4) $x - 2\log_e(y+3x) = C$

$y + 3x = t$

$\frac{dy}{dx} + 3 = \frac{dt}{dx}$

$\therefore \boxed{\frac{dy}{dx} = \frac{dt}{dx} - 3}$

$\frac{dt}{dx} - \cancel{3} - \frac{t}{\ln t} + \cancel{3} = 0$

$\frac{dt}{dx} = \frac{t}{\ln t}$

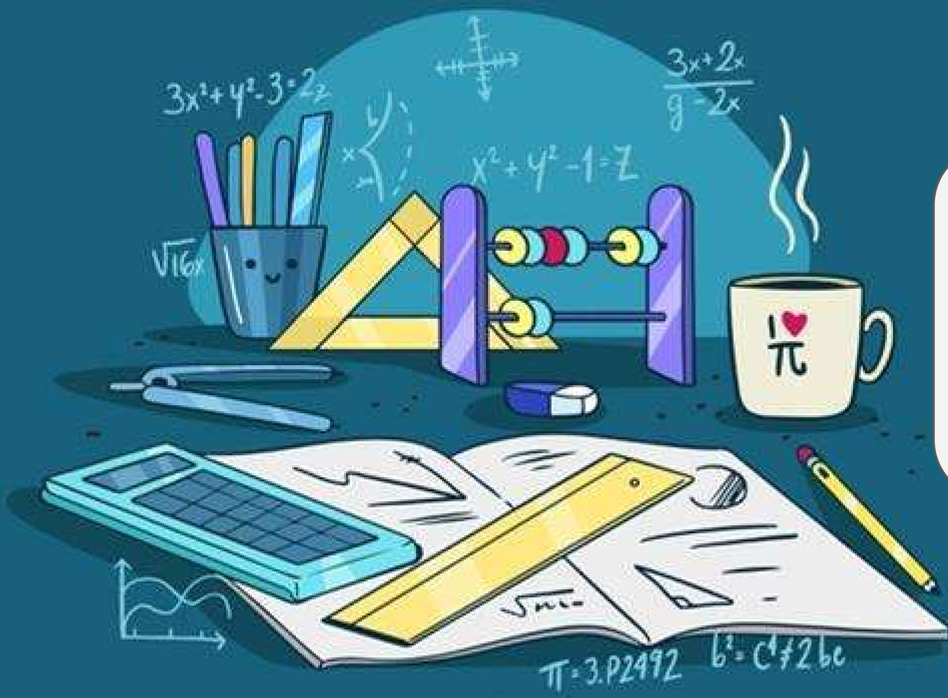
$$\int \frac{\boxed{\ln t}}{t}\, dt = \int dx$$

$\ln t \to z$, $\frac{dt}{t} \to dz$

$$\frac{(\ln t)^2}{2} = x + c$$

$$\frac{\ln^2(y+3x)}{2} = x + c$$

Unacademy Atoms



Reducible Variable Separable: Type 2

Type 2 : Reducible Variable Separable

$$\frac{dy}{dx} = \frac{a_1x + \boxed{b_1}y + c_1}{\boxed{a_2}x + b_2y + c_2}$$ where $\mathbf{b_1 + a_2 = 0}$

Solve the differential equation $\boxed{\frac{dy}{dx} = \frac{x-2y+5}{2x+3y-1}}$

$2 + (-2) = 0$

CROSS MULTIPLICATION

$\int dx = x$

$\int dy = y$

$\int d(xy) = xy$

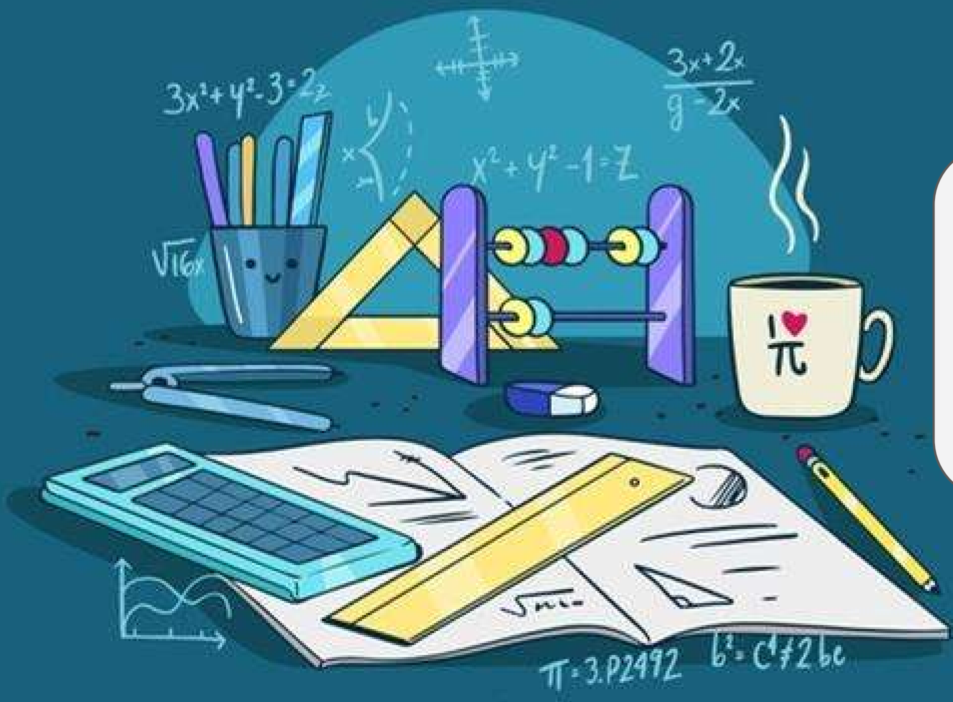
$2x\,dy + 3y\,dy - dy = x\,dx - 2y\,dx + 5\,dx$

$2(x\,dy + y\,dx) + 3y\,dy - dy = x\,dx + 5\,dx$

$2\int d(xy) + \int 3y\,dy - \int dy = \int x\,dx + \int 5\,dx$

$$\boxed{2(xy) + \frac{3y^2}{2} - y = \frac{x^2}{2} + 5x + c}$$

Unacademy Atoms



Reducible Variable Separable: Type 3

Type 3 : Polar Coordinates

1 i. $\boxed{x\,dx + y\,dy = r\,dr}$ ii. $\boxed{x\,dy - y\,dx = r^2\,d\theta}$

$$\boxed{\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \end{aligned}}$$

$$\boxed{\begin{aligned} &① \quad r = \sqrt{x^2+y^2} \\ &② \quad \theta = \tan^{-1}\left(\frac{y}{x}\right) \end{aligned}}$$

① $x^2 + y^2 = r^2$

$$\boxed{\cancel{2}x\,dx + \cancel{2}y\,dy = \cancel{2}r\,dr}$$

② $\tan\theta = \dfrac{y}{x}$

$$x^2 \sec^2\theta \cdot d\theta = x\,dy - y\,dx$$

$$r^2 \cancel{\cos^2}\theta\ \cancel{\sec^2}\theta\ d\theta = x\,dy - y\,dx$$

Type 3 : Polar Coordinates

2 i. $\boxed{x\,dx - y\,dy = r\,dr}$ ii. $\boxed{x\,dy - y\,dx = r^2 \sec\theta\, d\theta}$

$$\boxed{\begin{aligned} x &= r\sec\theta \\ y &= r\tan\theta \end{aligned}}$$

1. $x^2 - y^2 = r^2$

$\cancel{2}x\,dx - \cancel{2}y\,dy = \cancel{2}r\,dr$

2. $\sin\theta = \frac{y}{x}$

$\cos\theta\, d\theta = \frac{x\,dy - y\,dx}{x^2}$

$r^2 \sec^2\theta \cos\theta\, d\theta = x\,dy - y\,dx$

Solve the following differential Equation

$$xdx + ydy = x(xdy - ydx)$$

A

$$\cancel{r}\,dr = \cancel{r}\cos\theta\,(r^2\,d\theta)$$

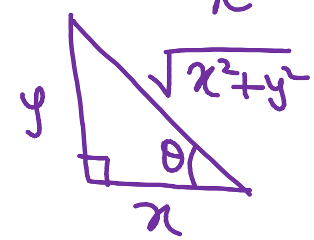
$$\int \frac{dr}{r^2} = \int \cos\theta\, d\theta$$

$$\frac{-1}{r} = \sin\theta + c$$

$$\boxed{\frac{-1}{\sqrt{x^2+y^2}} = \frac{y}{\sqrt{x^2+y^2}} + c}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$\tan\theta = \frac{y}{x}$$



Solve the following differential Equation

$$\frac{x + y\frac{dy}{dx}}{x\frac{dy}{dx} - y} = \sqrt{\frac{1 - x^2 - y^2}{x^2 + y^2}}$$

$$\frac{\boxed{x\,dx + y\,dy}}{x\,dy - y\,dx} = \sqrt{\frac{1 - (x^2+y^2)}{x^2+y^2}}$$

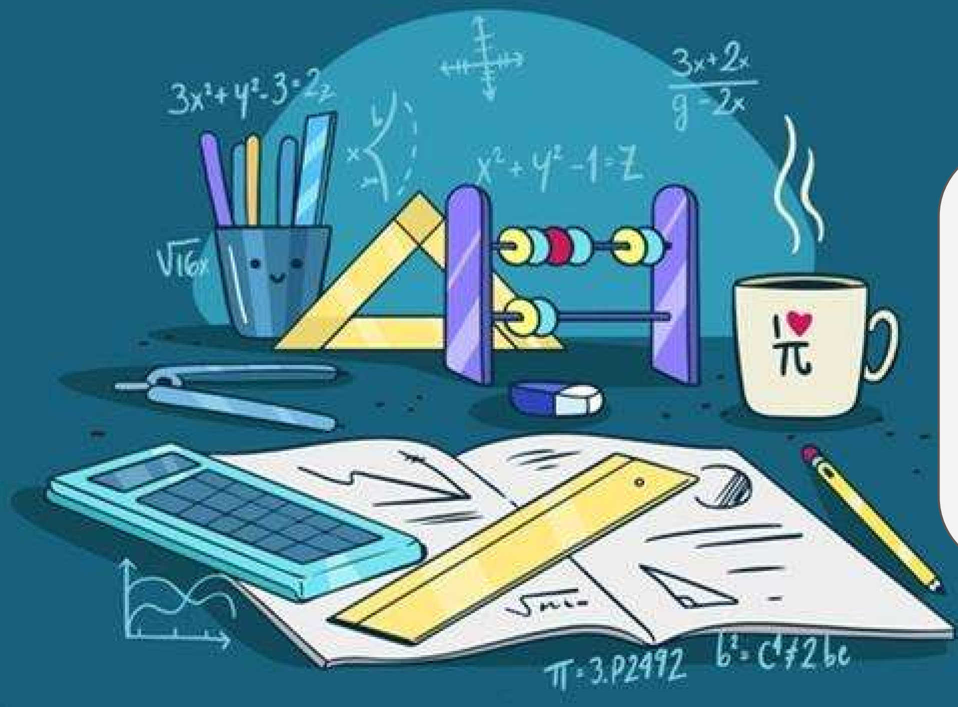
$$\frac{\cancel{r}\,dr}{\cancel{r^2}\,d\theta} = \sqrt{\frac{1 - r^2}{\cancel{r^2}}}$$

$$\frac{dr}{d\theta} = \sqrt{1 - r^2}$$

$$\int \frac{dr}{\sqrt{1-r^2}} = \int d\theta$$

$$\sin^{-1}(r) = \theta + c$$

$$\boxed{\sin^{-1}\left(\sqrt{x^2+y^2}\right) = \tan^{-1}\left(\frac{y}{x}\right) + c}$$



Homogeneous Differential Equation

Homogeneous Equations

The function f(x, y) is said to be a homogeneous function of degree n if for any real number t(≠0), we have $\boxed{f(\lambda x, \lambda y) = \lambda^n(x, y)}$

$$\boxed{f(\lambda x, \lambda y) = \lambda^n f(x, y)}$$

<u>For example</u>

$f(x, y) = x^{2/3} + 2x^{1/3}y^{1/3} + 5y^{2/3}$

$$f(\lambda x, \lambda y) = (\lambda x)^{2/3} + 2(\lambda x)^{1/3}(\lambda y)^{1/3} + 5(\lambda y)^{2/3}$$

$$\boxed{f(\lambda x, \lambda y) = \lambda^{2/3}\left(f(x, y)\right)}$$

D.E.

Unacademy Atoms

Shortcut

JEE Main 2020

If a curve $y = f(x)$, passing through the point $(1, 2)$, is the solution of the differential equation,

$\boxed{2x^2 dy = (2xy + y^2)\,dx}$, then $\boxed{f\left(\frac{1}{2}\right)}$ is equal to :

$x = \frac{1}{2} \quad y = ?$

(1) $\frac{1}{1+\log_e 2}$ ✓

(2) $\frac{1}{1-\log_e 2}$

(3) $1+\log_e 2$

(4) $\frac{-1}{1+\log_e 2}$

$$\boxed{\frac{dy}{dx}} = \frac{2xy + y^2}{2x^2}$$

Variable

Put $\boxed{y = vx}$

$v = \frac{y}{x}$

$$\boxed{\frac{dy}{dx} = v + x\frac{dv}{dx}}$$

Unacademy Atoms

$$v + x\frac{dv}{dx} = \frac{2\cancel{x}(v\cancel{x}) + v^2\cancel{x^2}}{2\cancel{x^2}}$$

$$\cancel{v} + x\frac{dv}{dx} = \cancel{v} + \frac{v^2}{2}$$

$$x\frac{dv}{dx} = \frac{v^2}{2}$$

$$\int\frac{dv}{v^2} = \frac{1}{2}\int\frac{dx}{x}$$

$$-\frac{1}{v} = \frac{1}{2}\ln x + c$$

$$-\frac{x}{y} = \frac{1}{2}\ln x + c$$

$x = 1 \quad y = 2$

$$\boxed{-\frac{1}{2} = c}$$

$$-\frac{x}{y} = \frac{1}{2}\ln x - \frac{1}{2}$$

$$-\frac{1}{\cancel{2}y} = \frac{1}{\cancel{2}}\ln\left(\frac{1}{2}\right) - \frac{1}{\cancel{2}}$$

$$+\frac{1}{y} = +\ln 2 + 1$$

$$\therefore \boxed{y = \frac{1}{\ln 2 + 1}}$$

$$\frac{dy}{dx} = \frac{2xy + y^2}{2x^2}$$

S.C

$$\boxed{\begin{array}{l} \dfrac{dy}{dx} \to v + x\dfrac{dv}{dx} \\ y \to v \\ x \to 1 \end{array}}$$

$$\cancel{v} + x\frac{dv}{dx} = \cancel{\frac{2v}{2}} + \frac{v^2}{2}$$

$$\underline{x\frac{dv}{dx} = \frac{v^2}{2}}$$

Unacademy Atoms

If $\boxed{\dfrac{dy}{dx} = \dfrac{xy}{x^2 + y^2}}$; $y(1) = 1$; then a value of x satisfying $y(x) = e$ is:

$x = ?\quad y = e$

(1) $\frac{1}{2}\sqrt{3}\,e$

(2) $\frac{e}{\sqrt{2}}$

$\boxed{\begin{array}{l} y \to v \\ x \to 1 \end{array}}$

(3) $\sqrt{2}\,e$

~~(4)~~ $\sqrt{3}\,e$

Shortcut

JEE Main 2020

$$x\frac{dv}{dx} = \frac{v}{1+v^2} - v$$

$$\frac{x\,dv}{dx} = \frac{-v^3}{1+v^2}$$

$$\frac{v^{-3+1}}{-3+1}$$

$$\int \frac{(1+v^2)}{v^3}\,dv = \int -\frac{dx}{x}$$

$$v = y/x$$

$$\int \left(\frac{1}{v^3} + \frac{1}{v}\right) dv = -\int \frac{dx}{x}$$

$$\frac{-1}{2v^2} + \underline{\ln v} = \underline{-\ln x} + c$$

$$-\frac{x^2}{2y^2} + \ln y = c$$

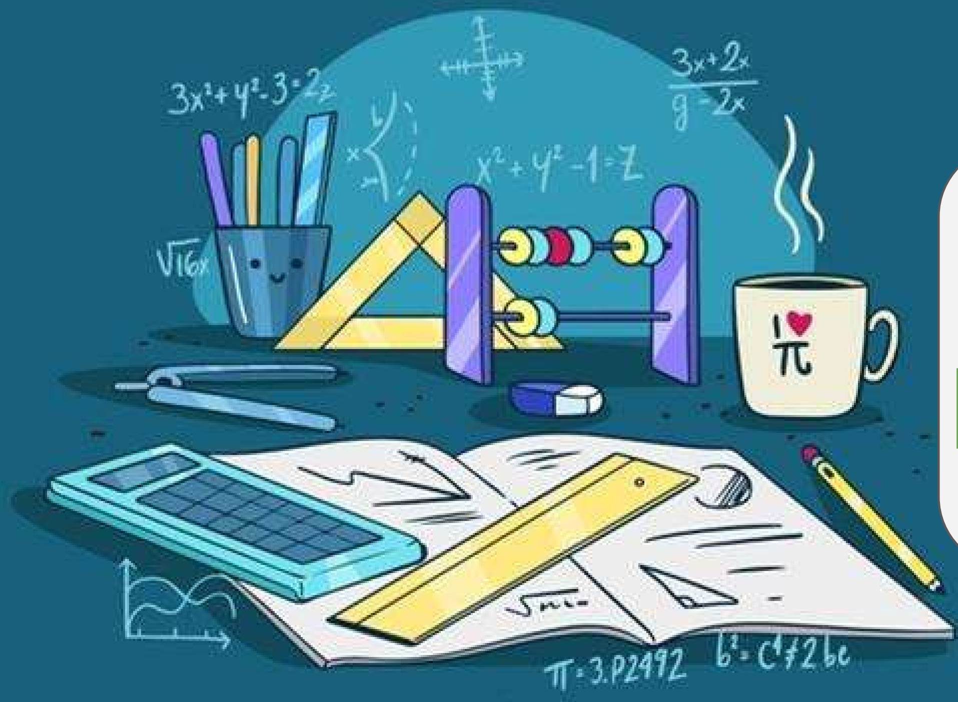
$$\boxed{-\frac{1}{2} + 0 = c}$$

$$-\frac{x^2}{2y^2} + \ln y = \frac{-1}{2}$$

$$-\frac{x^2}{2e^2} + 1 = \frac{-1}{2}$$

$$+\frac{x^2}{\cancel{2}e^2} = \frac{+3}{\cancel{2}}$$

$$\boxed{x = \sqrt{3}e}$$



Reducible to Homogeneous D.E.

Equation Reducible to Homogeneous Form

Equation of the form $\boxed{\dfrac{dy}{dx} = \dfrac{ax + \boxed{b}y + c}{\boxed{A}x + By + C}}$ can be reduced to a homogeneous by substituting $\boxed{x = X + h}$ and $\boxed{y = Y + k.}$

$\boxed{\dfrac{dy}{dx} = \dfrac{L}{L}}$

$A + b = 0$ Reducible V.S. Ty-2

$\boxed{x = \underbrace{X}_{\text{Var}} + \underbrace{h}_{\text{Const.}}}$ $\boxed{y = \underbrace{Y}_{\text{Var}} + \underbrace{k}_{\text{const}}}$

Solve : $\boxed{\dfrac{dy}{dx} = \dfrac{2x - y + 1}{x - 2y - 1}}$

$x = X + h \qquad y = Y + k$

$dx = dX \qquad dy = dY$

$$\boxed{\frac{dy}{dx} = \frac{dY}{dX}}$$

$$\frac{dY}{dX} = \frac{2(X+h) - (Y+k) + 1}{X + h - 2(Y+k) - 1}$$

$$\frac{dY}{dX} = \frac{2X - Y + (2h - k + 1)}{X - 2Y + (h - 2k - 1)}$$

$x = X - 1 \qquad y = Y - 1$

$\boxed{X = x + 1 \qquad Y = y + 1}$

$$\boxed{\frac{dY}{dX} = \frac{2X - Y}{X - 2Y}}$$

$\boxed{Y = VX} \qquad \boxed{V = \frac{Y}{X}}$

$$V + X\frac{dV}{dX} = \frac{2 - V}{1 - 2V}$$

$$X\frac{dV}{dX} = \frac{2 - V}{1 - 2V} - V$$

Unacademy Atoms

Unacademy Atoms

$2h - k + 1 = 0$

$h - 2k - 1 = 0$

$h = -1 \quad k = -1$

$2 - 2v + 2v^2 = t$

$(-2 + 4v)\,dv = dt$

$-2\underbrace{(1 - 2v)\,dv}_{} = dt$

$$x\frac{dv}{dx} = \frac{2 - v - v + 2v^2}{1 - 2v}$$

$$\int \frac{\boxed{(1-2v)\,dv}}{\boxed{2 - 2v + 2v^2}} = \int \frac{dx}{x}$$

$$-\frac{1}{2}\int \frac{dt}{t} = \int \frac{dx}{x}$$

$$-\frac{1}{2}\ln t = \ln x + c$$

$$-\frac{1}{2}\ln(2 - 2v + 2v^2) = \ln x + c$$

$$-2\left(-\frac{1}{2}\ln\left(2-\frac{2Y}{X}+\frac{2Y^2}{X^2}\right)=\ln X+c\right)$$

$$\ln\left(2-\frac{2Y}{X}+2\frac{Y^2}{X^2}\right)=-2\ln X - 2c$$

$$\ln\left(\frac{2X^2-2XY+2Y^2}{X^2}\right)+\ln X^2=k$$

$$\ln\left(2X^2-2XY+2Y^2\right)=k$$

$$2X^2-2XY+2Y^2=e^k=c'$$

$$2(x+1)^2-2(x+1)(y+1)+2(y+1)^2=c'$$

Solve : $\dfrac{dy}{dx} = \dfrac{2x - y + 1}{x - 2y - 1}$

$$\boxed{\dfrac{dY}{dX} = \dfrac{2X - Y}{X - 2Y}}$$

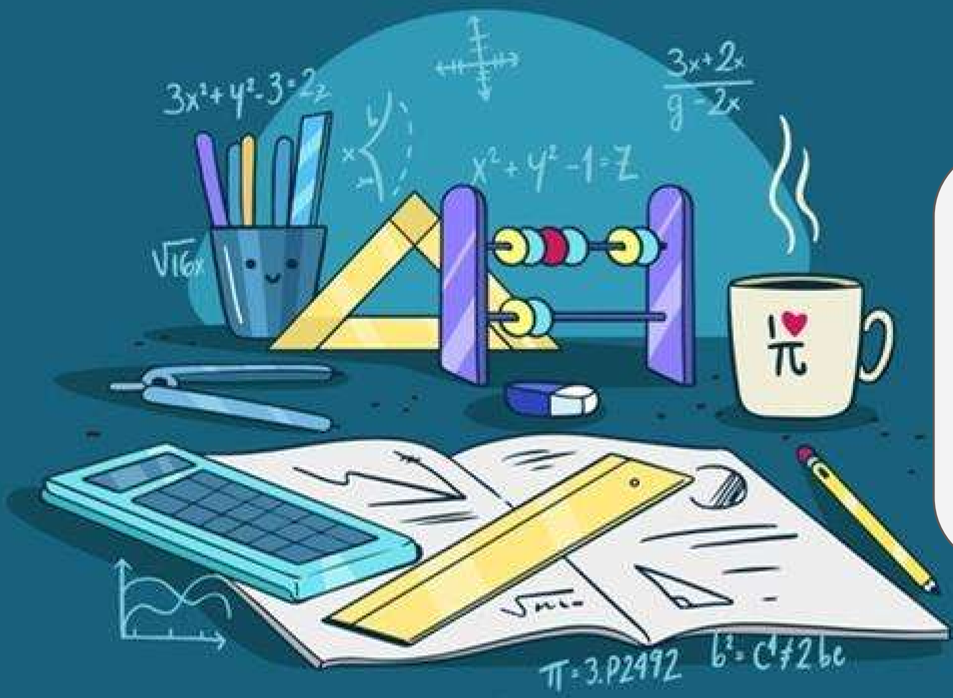
$$\boxed{\begin{array}{l} 2h - k + 1 = 0 \\ h - 2k - 1 = 0 \end{array}} \quad \begin{array}{l} h = -1 \\ k = -1 \end{array}$$

🔥 Shortcut

$$V + X\frac{dV}{dX} = \frac{2 - V}{1 - 2V}$$

$$x = X - 1$$

$$y = Y - 1$$



Linear Differential Equation

Linear Differential Equation

$y = f(x)$

A differential equation is said to be linear if the $\boxed{\text{dependent}}$ variable and all its differentials coefficients occur in **degree one only** and are **never multiplied together**.

$y \cdot \frac{dy}{dx} \to \times$

The nth order linear differential equation if of the form.

$$\boxed{a_0(x)\left(\frac{d^n y}{dx^n}\right)^1 + a_1(x)\left(\frac{d^{n-1}y}{dx^{n-1}}\right)^1 + \cdots + a_n(x).y = \phi(x).}$$

$\left(\frac{dy}{dx}\right)^1$ $\quad x\frac{dy}{dx} \to ✓$

Where $a_0(x)$, $a_1(x)$ …. $a_n(x)$ are the coefficients of the differential equation

Examples : LDE

1. $y\dfrac{dy}{dx} = x^2y + x^3$ ✗

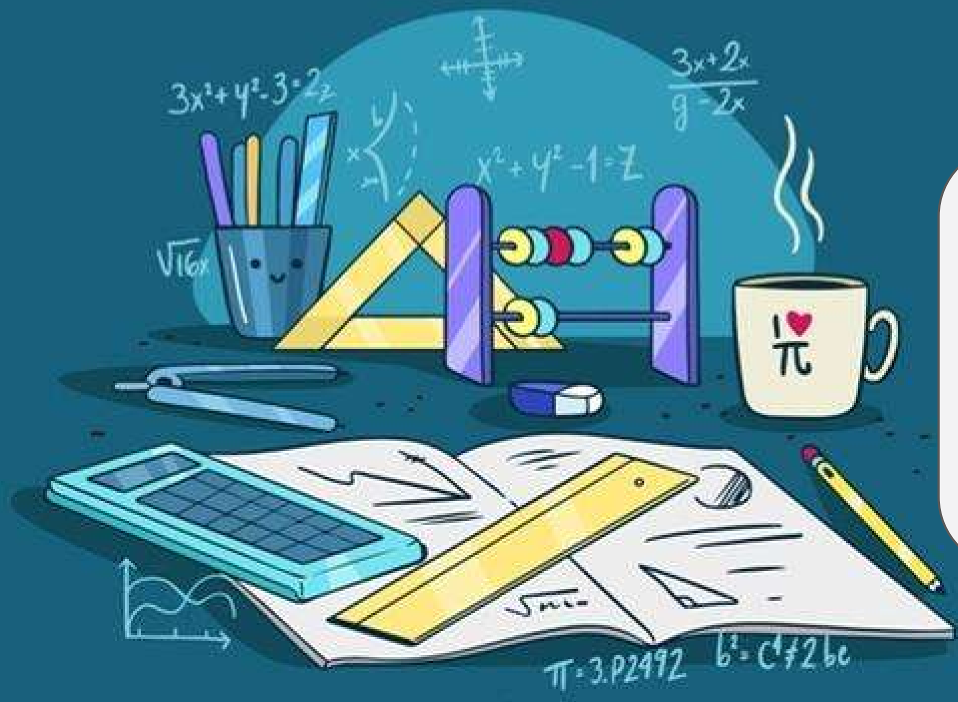
2. $x^4\dfrac{dy}{dx} = xy + 5$ ✓ → LDE O = 1

3. $x\dfrac{d^2y}{dx^2} + x^2\dfrac{dy}{dx} = 5y + 7$ ✓ ⇒ LDE ✓ O = 2, D = 1

4. $y\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} = 5$ ✗

$y^2\dfrac{d^3y}{dx^3}$ → LDE ✗

Unacademy Atoms



Linear Differential Equation of First Order

Linear Differential Equations of First Order

Standard form of L.D.E of first order

$$\boxed{\frac{dy}{dx} + P(x)\,y = Q(x)}$$

L.D.E of O = 1

$$\frac{dy}{dx} + P(x)\,y = Q(x)$$

Method to Solve L.D.E of First Order

$$\boxed{\frac{dy}{dx} + P(x)\,y = Q(x)}$$

$$\boxed{I.f = e^{\int P(x)\,dx}}$$

$$\boxed{e^{\int P(x)\,dx}} \cdot \boxed{\frac{dy}{dx}} + \boxed{P(x)\,e^{\int P(x)\,dx}} \cdot \boxed{y} = Q(x) \cdot e^{\int P(x)\,dx}$$

$$\int \cancel{d}\left(y \cdot e^{\int P(x)\,dx}\right) = \int Q(x)\, e^{\int P(x)\,dx}\, dx$$

Method to Solve L.D.E of First Order

$$y \cdot e^{\int P(x)\,dx} = \int \left(Q(x) \cdot e^{\int P(x)\,dx} \right) dx$$

Solⁿ :- of LDE of O=1

★★★

$$\boxed{y \cdot (I.F.) = \int \left(Q(x) \cdot IF \right) dx}$$

Solve the following differential equations

$$x^2\frac{dy}{dx}+y=1 \qquad \frac{dy}{dx}+P(x)\,y=Q(x)$$

$$\boxed{\frac{dy}{dx}+\left(\frac{1}{x^2}\right)y=\frac{1}{x^2}}$$

$P(x)$ $\quad$ $Q(x)$

$$\text{I.F}=e^{\int\frac{1}{x^2}dx}=e^{-1/x}$$

$$\frac{-1}{x}=t \Rightarrow \frac{1}{x^2}dx=dt$$

$$\text{Sol}^n:-\quad y\times e^{-1/x}=\int e^{-1/x}\times\frac{1}{x^2}dx$$

$$\boxed{y\times e^{-1/x}=e^{-1/x}+c}$$

Unacademy Atoms

Let y = y(x) be the solution of the differential equation $\operatorname{cosec}^2 x\,dy + 2\,dx = (1 + y\cos 2x)\operatorname{cosec}^2 x\,dx$, with $y\left(\frac{\pi}{4}\right) = 0$. Then, the value of $(y(0)+1)^2$ is equal to:

$(y+1)^2$

(1) $e^{1/2}$

(2) $e^{-1/2}$

(3) e^{-1}

(4) e

JEE Main 2021

$$\left[\operatorname{cosec}^2 x \frac{dy}{dx} + 2 = (1 + y\cos 2x)\operatorname{cosec}^2 x\right] \times \sin^2 x$$

$$\frac{dy}{dx} + 2\sin^2 x = 1 + y\cos 2x$$

$$\boxed{\frac{dy}{dx} + (-\cos 2x)\,y = \cos 2x}$$

$P(x)$ $\quad$ $Q(x)$

$$\frac{dy}{dx} = y\cos 2x + \cos 2x$$

$$\int \frac{dy}{y+1} = \int \cos 2x\, dx$$

$$I.f = e^{\int -\cos 2x\, dx} = e^{-\frac{\sin 2x}{2}}$$

$$-\frac{\sin 2x}{2} = t \Rightarrow \boxed{-\cos 2x\, dx = dt}$$

Solⁿ :- $y \times e^{-\frac{\sin 2x}{2}} = \int \underline{\cos 2x} \cdot e^{\boxed{-\frac{\sin 2x}{2}}} \underline{dx}$

$$= -\int e^{t}\, dt$$

$$\boxed{\begin{aligned} x &= \frac{\pi}{4} \\ y &= 0 \end{aligned}}$$

$$y \times e^{-\frac{\sin 2x}{2}} = -e^{-\frac{\sin 2x}{2}} + e^{-1/2}$$

$$y \times 1 = -1 + e^{-1/2}$$

$$(y+1)^2 = \left(e^{-1/2}\right)^2 = \underline{e^{-1}}$$

Let y(x) be a solution of the differential equation **$(1 + e^x)\, y' + ye^x = 1$. If y(0) = 2**, then which of the following statement is (are) true ?

$x=0 \quad y=2$

A. y(-4) = 0

B. y(-2) = 0

$\frac{dy}{dx}=0$

C. y(x) has $\boxed{\text{a}}$ critical point in the interval (-1, 0)

D. y(x) has $\boxed{\text{no}}$ critical point in the interval (-1, 0)

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AC

$$(1+e^x)\frac{dy}{dx} + ye^x = 1$$

$$\frac{dy}{dx} + \left(\frac{e^x}{1+e^x}\right)y = \frac{1}{1+e^x}$$

$$IF = e^{\int \frac{e^x}{1+e^x} dx}$$

$1+e^x = t$

$e^x dx = dt$

$$= e^{\ln(1+e^x)}$$

$$= 1+e^x$$

Sol:- $y(1+e^x) = \int \cancel{(1+e^x)} \times \frac{1}{\cancel{(1+e^x)}} . dx$

$y(1+e^x) = x + c$

$2(1+1) = c$

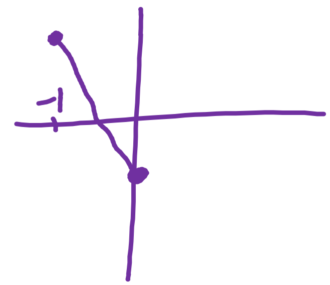
$\boxed{c = 4}$

$y(1+e^x) = x+4$

$$\boxed{y = \frac{x+4}{1+e^x}}$$

$$y = f(x) = \frac{x+4}{1+e^x}$$

f'



$y=0 \quad x=-4$

$1-\frac{2}{e}$

$1-x-4$

$$\frac{dy}{dx} = \frac{1(1+e^x) - e^x(x+4)}{(1+e^x)^2}$$

$$\frac{dy}{dx} = \frac{1+e^x(-x-3)}{\boxed{(1+e^x)^2}}$$

$$\left.\frac{dy}{dx}\right|_{-1} = \frac{1+e^{-1}(-2)}{+} = +$$

$$\left.\frac{dy}{dx}\right|_{x=0} = -$$

Alternate Form of L.D.E.

Standard Form: $\boxed{\dfrac{dy}{dx} + P(x)\,y = Q(x)}$

Solution : $y \times e^{\int P(x)\,dx} = \int \left(Q(x) \times e^{\int P(x)\,dx}\right) dx$

Standard Form: $\boxed{\dfrac{dx}{dy} + P(y)\,x = Q(y)}$

Solution : $x \times e^{\int P(y)\,dy} = \int \left(Q(y) \times e^{\int P(y)\,dy}\right) dy$

Let $y = y(x)$ be the solution curve of the differential equation, $\boxed{\left(y^2 - x\right)\frac{dy}{dx} = 1}$, satisfying $y(0) = 1$. This curve intersects the x-axis at a point whose abscissa is:

(1) $2 - e$ (2) $-e$

(3) 2 (4) $2 + e$

[JEE]

$y^2 - x = \frac{dx}{dy}$

$$\boxed{\frac{dx}{dy} + 1x = y^2}$$

P(y) Q(y)

I.f $= e^{\int 1\, dy} = e^y$

Sol^n:- $x \times e^y = \int y^2 e^y\, dy$

$$x e^y = \int y^2 e^y \, dy$$

	D	I
$+$	y^2	e^y
$-$	$2y$	e^y
$+$	2	e^y
$-$	0	e^y

$$x e^y = y^2 e^y - 2y e^y + 2e^y + c$$

$$0 = e - 2\cancel{e} + 2\cancel{e} + c$$

$$\underline{c = -e}$$

$$x e^y = \cancel{y^2} e^y - 2\cancel{y} e^y + 2e^y - e$$

$y = 0 \quad x = ?$

$$\boxed{x = 2 - e}$$

Let f : (0, ∞) ⟶ R be a differentiable function such that

$\boxed{f'(x) = 2 - \frac{f(x)}{x}}$ for all x ∈ (0, ∞) and f(1) ≠ 1. Then

Multi-correct

JEE Adv. 2016

A. $\lim_{x \to 0+} f'\left(\frac{1}{x}\right) = 1$

B. $\lim_{x \to 0+} xf\left(\frac{1}{x}\right) = 2$

C. $\lim_{x \to 0+} x^2 f'(x) = 0$

D. $|f(x)| \le 2$ for all x ∈ (0, 2)

$$\frac{dy}{dx} = 2 - \frac{y}{x}$$

$$\boxed{\frac{dy}{dx} + \frac{y}{x} = 2}$$

$P(x) = \frac{1}{x}$ $Q(x) = 2$

$I.F = e^{\int \frac{1}{x} dx} = e^{\ln x} = x$

Sol^n:- $y \times x = \int x \times 2 \, dx$

$xy = x^2 + c$

$$\boxed{\begin{array}{c} y = \dfrac{x^2 + c}{x} \\ 1 \neq 1 + c \\ \boxed{c \neq 0} \end{array}}$$

$$\boxed{f(x) = x + \frac{c}{x}} \Rightarrow \boxed{f'(x) = 1 - \frac{c}{x^2}}$$

$$f\left(\frac{1}{x}\right) = \frac{1}{x} + cx$$

$$\lim_{x \to 0^+} x\, f\left(\frac{1}{x}\right) = 1 + c\cancel{x^2}$$

$$= 1$$

$$f(x) = x + \frac{c}{x}$$

$$x \in (0, 2)$$

$$f(0^+) = 0 + \frac{c}{0^+}$$

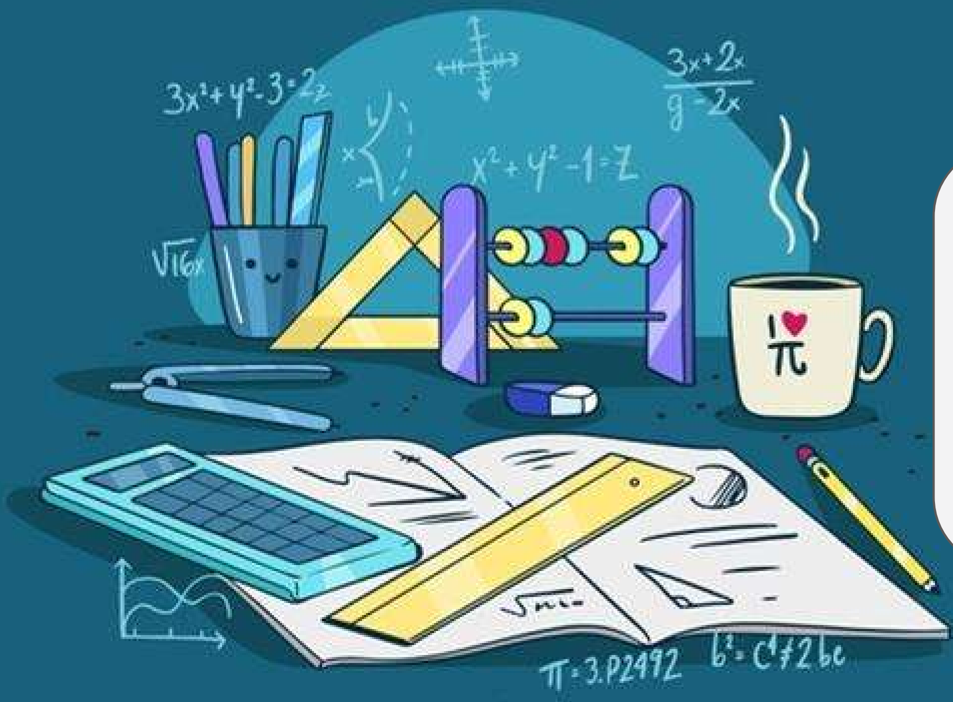
$$= \underline{\pm\infty}$$

$$f'(x) = 1 - \frac{c}{x^2} \Longrightarrow f'\left(\frac{1}{x}\right) = 1 - c\cancel{x^2}$$

$$x^2 f'(x) = \cancel{x^2} - c$$

$$\lim_{x \to 0^+} f'\left(\frac{1}{x}\right) = 1$$

$$\lim_{x \to 0^+} x^2 f'(x) = -c \neq 0$$



Reducible Linear Differential Equation

Reducible Linear Differential Equation

#Standard form of Reducible L.D.E

$$\boxed{\frac{dy}{dx} + P(x)\,y = Q(x)\,y^n}$$

#Method to solve Reducible L.D.E

Solve the following differential equations

$$\boxed{\frac{dy}{dx} = xy + x^3y^2}$$

$P(x)$ $\qquad Q(x)$

$$\frac{dy}{dx} + (-x)\,y = x^3\,\underline{y^2}$$

$$\boxed{\frac{1}{y^2}\frac{dy}{dx}} + (-x)\,\boxed{y^{-1}} = x^3$$

$$y^{-1} = t$$

$$-\frac{1}{y^2}\cdot\frac{dy}{dx} = \frac{dt}{dx}$$

$$\boxed{\frac{dt}{dx} + (x)\,t = -x^3}$$

$$If = e^{\int x\,dx} = e^{x^2/2}$$

$$Sol^n :- \; t\,e^{x^2/2} = \int -x^3 \cdot e^{x^2/2}\,dx$$

$$t \cdot e^{x^2/2} = \int -x^2 \, e^{x^2/2} \, \boxed{x\,dx}$$

$$\frac{x^2}{2} = z$$

$$x\,dx = dz$$

$$= -\int 2z \, e^{z} \, dz$$

$$t \cdot e^{x^2/2} = -2\left(z e^{z} - e^{z}\right) + c$$

$$\boxed{y^{-1} e^{x^2/2} = -2\left(\frac{x^2}{2} e^{x^2/2} - e^{x^2/2}\right) + c}$$

If the curve $y = y(x)$ represented by the solution of the differential equation $\boxed{(2xy^2 - y)\,dx + x\,dy = 0}$ passes through the intersection of the lines, $2x - 3y = 1$ and $3x + 2y = 8$, then $|y(1)|$ is equal to 1

$$\frac{(2xy^2 - y)\cancel{dx}}{\cancel{dx}} + \frac{x\,dy}{dx} = 0$$

$$2xy^2 - y + x\frac{dy}{dx} = 0$$

$$x\frac{dy}{dx} = y - 2xy^2$$

$$\frac{dy}{dx} = \frac{y}{x} - 2y^2$$

$$\boxed{\begin{aligned} 2x - 3y &= 1 \\ 3x + 2y &= 8 \end{aligned}}$$

$(2, 1)$

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Unacademy Atoms

$$\frac{dy}{dx} - \frac{y^1}{x} = -2y^2$$

$$\boxed{\frac{1}{y^2}\frac{dy}{dx}} - \frac{y^{-1}}{x} = -2$$

$$+\frac{dt}{dx} + \frac{t}{x} = +2$$

$$\boxed{\frac{dt}{dx} + \frac{t}{x} = 2}$$

$$If = e^{\int \frac{1}{x} dx} = e^{\ln x} = x$$

Sol^n :- $t.x = \int 2x\, dx$

$$y^{-1} = t$$

$$\underline{-\frac{1}{y^2}\frac{dy}{dx} = \frac{dt}{dx}}$$

$$\boxed{\frac{1}{y} \times x = x^2 + c}$$

$$\frac{2}{1} = 4 + c$$

$$\underline{c = -2}$$

$$(2xy^2 - y)\,dx + x\,dy = 0$$

$$2xy^2\,dx - y\,dx + x\,dy = 0$$

$$\frac{2x\cancel{y^2}\,dx}{\cancel{y^2}} = \boxed{\frac{y\,dx - x\,dy}{y^2}}$$

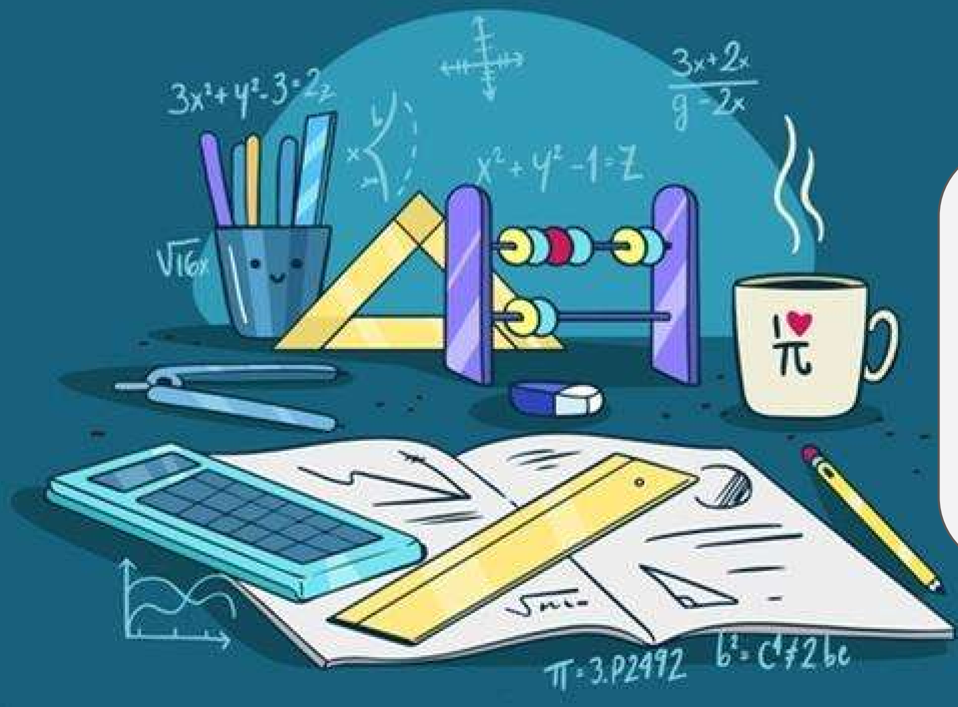
$$\int 2x\,dx = \int d\left(\frac{x}{y}\right)$$

$$x^2 = \frac{x}{y} + c$$

$$\frac{x}{y} = x^2 - 2$$

$$\frac{1}{y} = 1 - 2$$

$$|y(1)| = |-1| = 1$$



Exact Differentials

Exact Differentials

i $\boxed{x\,dy + y\,dx = d(xy)}$

ii $\boxed{\dfrac{x\,dy - y\,dx}{x^2} = d\left(\dfrac{y}{x}\right)}$

iii $\boxed{\dfrac{y\,dx - x\,dy}{y^2} = d\left(\dfrac{x}{y}\right)}$

iv $\dfrac{x\,dy + y\,dx}{xy} = d(\ln xy)$

v $\dfrac{dx + dy}{x + y} = d(ln\,(x + y))$

vi $\dfrac{x\,dy - y\,dx}{xy} = d\left(\ln \dfrac{y}{x}\right)$

vii $\dfrac{ydx - xdy}{xy} = d\left(ln\,\dfrac{x}{y}\right)$

viii $\boxed{\dfrac{x\,dy - y\,dx}{x^2 + y^2} = d\left(\tan^{-1}\dfrac{y}{x}\right)}$

ix $\boxed{\dfrac{ydx - xdy}{x^2 + y^2} = d\left(\tan^{-1}\dfrac{x}{y}\right)}$

x $\dfrac{x\,dx + y\,dy}{x^2 + y^2} = d\left[\ln\sqrt{x^2 + y^2}\right]$

xi $d\left(-\dfrac{1}{xy}\right) = \dfrac{x\,dy + y\,dx}{x^2y^2}$

xii $d\left(\dfrac{e^x}{y}\right) = \dfrac{y\,e^y\,dx - e^x\,dy}{y^2}$

xiii $d\left(\dfrac{e^y}{x}\right) = \dfrac{xe^y\,dy + e^y\,dx}{x^2}$

Solutions of the differential equation ydx + (x + x^{2}y)dy = 0 is

A. log y = C

B. $-\frac{1}{xy} + \log y = C$ ✓

C. $\frac{1}{xy} + \log y = C$

D. $-\frac{1}{xy} = C$

2004

$$\underbrace{y\,dx + x\,dy} + x^2 y\,dy = 0$$

$$\boxed{\int \frac{dt}{t^2} = \frac{-1}{t}}$$

$$\frac{d(xy)}{x^2y^2} + \frac{\cancel{x^2}\,y\,dy}{\cancel{x^2}\,y^2} = 0$$

$$\int \frac{d(xy)}{(xy)^2} + \int \frac{dy}{y} = 0$$

$$\boxed{\frac{-1}{xy} + \ln y = c}$$

$$\int t^{-2}\,dt$$

$$\Rightarrow \frac{t^{-2+1}}{-2+1}$$

$$\Rightarrow \frac{-1}{t}$$

$$y\frac{dx}{dy} + \frac{(x+x^2y)\cancel{dy}}{\cancel{dy}} = 0$$

$$\frac{dx}{dy} + \frac{x}{y} = -\boxed{x^2}$$

$$\frac{1}{x^2}\frac{dx}{dy} + \frac{x^{-1}}{y} = -1$$

$$x^{-1} = t$$

$$\boxed{-\frac{1}{x^2}\frac{dx}{dy} = \frac{dt}{dy}}$$

Unacademy Atoms

If a curve y = f(x) passes through the point (1, -1) and satisfies the differential equation y(1 + xy) dx = x dy, then f(-1/2) is equal to:

A. 2/5

B. 4/5

C. -2/5

D. -4/5

JEE M 2016

$$y\,dx + xy^2\,dx = x\,dy$$

$$\frac{x\cancel{y^2}\,dx}{\cancel{y^2}} = \frac{x\,dy - y\,dx}{y^2}$$

$$x\,dx = -\left[\frac{y\,dx - x\,dy}{y^2}\right]$$

$$\int x\,dx = -\int d\left(\frac{x}{y}\right)$$

$$\frac{x^2}{2} = -\frac{x}{y} + c$$

$x = 1 \quad y = -1$

$$\frac{1}{2} = -\frac{1}{-1} + c$$

$$\therefore c = \frac{1}{2} - 1 = -\frac{1}{2}$$

$$x = -\frac{1}{2}$$

$$\frac{x^2}{2} = \frac{-x}{y} - \frac{1}{2}$$

$$2\left(\frac{1}{8} = \frac{+1}{2y} - \frac{1}{2}\right)$$

$$\frac{1}{4} = +\frac{1}{y} - 1$$

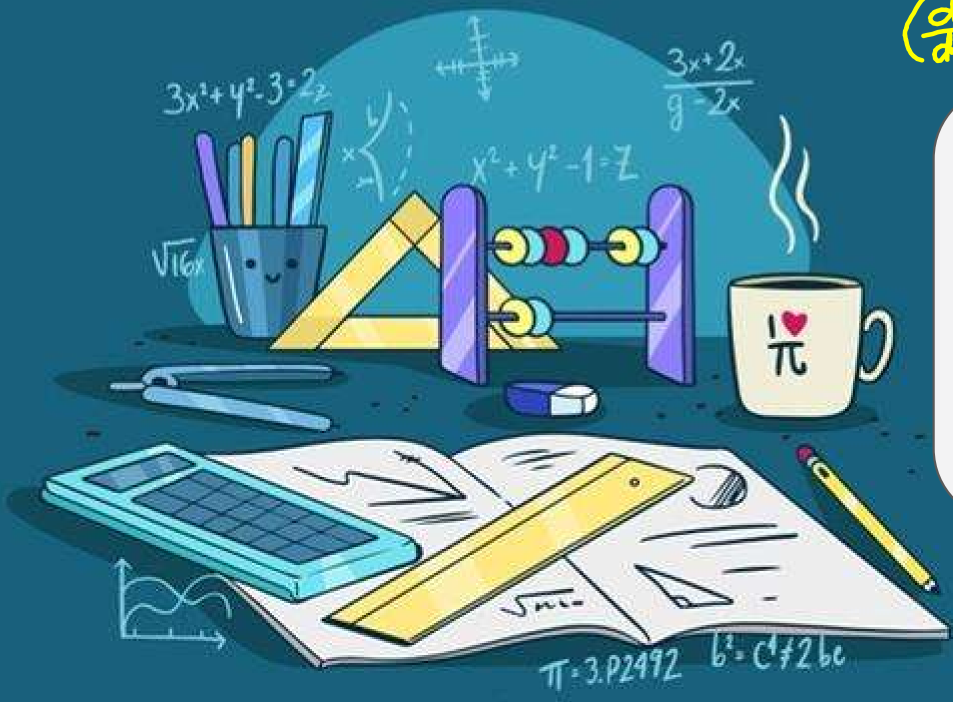
$$\frac{5}{4} = +\frac{1}{y}$$

$$y = 4/5$$

$$\boxed{\frac{dy}{dx} = m_T} \qquad \boxed{-\frac{dx}{dy} = m_N}$$

$$m_N = \frac{2y}{x}$$

$$\frac{-1}{\left(\frac{dy}{dx}\right)} = \frac{2y}{x} \qquad -\frac{dx}{dy} = \frac{2y}{x}$$



Geometrical Application

Let a curve $y = f(x)$ pass through the point $\left(2, (\log_e 2)^2\right)$ and have slope $\frac{2y}{x \log_e x}$ for all positive real value of x. Then the value of f(e) is equal to ___

Unacademy Atoms

JEE Main 2021

$P\left(2, (\ln 2)^2\right)$

$$\boxed{\frac{dy}{dx} = \frac{2y}{x \ln x}}$$

$$\int \frac{dy}{y} = \int \frac{2\,dx}{x \ln x}$$

$$\ln y = 2\int \frac{dt}{t}$$

$$\ln y = 2\ln(\ln x) + c$$

$$\ln(\ln 2)^2 = 2\ln(\ln 2) + c$$

$$\therefore \boxed{c = 0}$$

$$\ln y = 2\ln(\ln x)$$

$$\boxed{y = (\ln x)^2}$$

$$f(e) = 1$$

Unacademy Atoms

If a curve passes through the origin and the slope of the tangent to it at any point (x, y) is $\frac{x^2-4x+y+8}{x-2}$, then this curve also passes through the point :

(1) (4,5)

(2) (5,4)

(3) (4,4)

(4) (5,5)

JEE Main 2021

$$\frac{dy}{dx} = \frac{\boxed{x^2-4x} + y + \boxed{4} + 4}{x-2}$$

$$\frac{dy}{dx} = \frac{(x-2)^2}{(x-2)} + \frac{y+4}{(x-2)}$$

$$\frac{dy}{dx} = x-2 + \frac{y+4}{x-2}$$

$$\boxed{\frac{dY}{dX} = X + \frac{Y}{X}}$$

$x-2 = X \Rightarrow dx = dX$

$y+4 = Y \Rightarrow dy = dY$

$$\boxed{\frac{dy}{dx} = \frac{dY}{dX}}$$

$$\frac{dY}{dx} - \frac{Y}{X} = X$$

$$e^{\int -\frac{1}{X} dx} = e^{-\ln X} = \frac{1}{X}$$

$$Y \cdot \frac{1}{X} = \int \frac{1}{\cancel{X}} \cdot \cancel{X} \cdot dx$$

$$\boxed{\frac{Y}{X} = X + c}$$

$$\frac{y+4}{x-2} = x - 2 + \cancel{4}$$

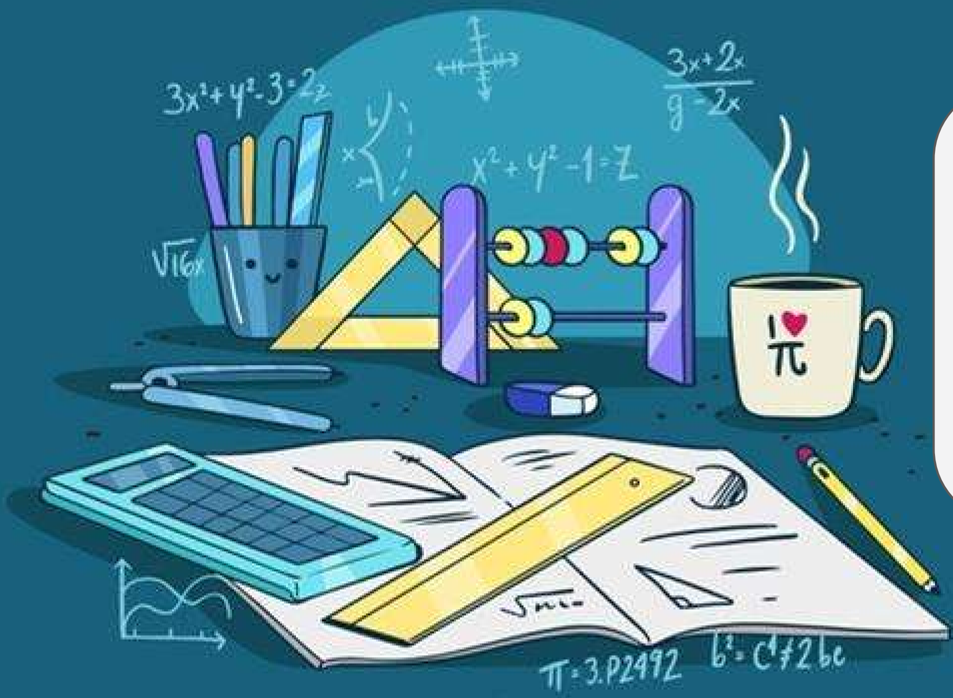
$(0, 0)$

$$\frac{4}{-2} = -2 + c$$

$\therefore \boxed{c = 0}$

$$\boxed{y + 4 = (x-2)^2}$$

Unacademy Atoms



Word Problems

Unacademy Atoms

The population $P = P(t)$ at time $'t'$ of a certain species follows the differential equation $\boxed{\frac{dP}{dt} = 0.5P - 450}$ If $P(0) = 850$, then the time at which population becomes zero is :

(1) $\frac{1}{2}\log_e 18$

(2) $2\log_e 18$

(3) $\log_e 9$

(4) $\log_e 18$

JEE Main 2021

$$\frac{dP}{dt} - 0.5P = -450$$

$$If = e^{-\int 0.5\, dt} = e^{-t/2}$$

Soln :- $P \cdot e^{-t/2} = \int -450 \cdot e^{-t/2}\, dt$

$$P e^{-t/2} = \frac{-450}{\left(\frac{-1}{2}\right)} e^{-t/2} + C$$

$$\left[P \cdot e^{-t/2} = 900\, e^{-t/2} + c\right] \times e^{t/2}$$

$$P = 900 + c\, e^{t/2}$$

$$850 = 900 + c(1)$$

$$\therefore \boxed{c = -50}$$

$$P = 900 - 50\, e^{t/2}$$

$$P = 0 \quad t = ?$$

$$0 = 900 - 50\, e^{+t/2}$$

$$\frac{900}{50} = e^{+t/2}$$

$$\ln 18 = +\frac{t}{2}$$

$$t = 2 \ln 18$$

Unacademy Atoms

The rate of growth of bacteria in a culture is proportional to the number of bacteria present and the bacteria count is 1000 at initial time $t = 0$. The number of bacteria is increased by 20% in 2 hours. If the population of bacteria is 2000 after $\boxed{\dfrac{k}{\log_e\left(\frac{6}{5}\right)}}$ hours, then $\left(\dfrac{k}{\log_e 2}\right)^2$ is equal to

(1) 4

(2) 2

(3) 16

(4) 8

JEE Main 2021

P = no. of bacteria

$$\frac{dP}{dt} \propto P$$

$$\frac{dP}{dt} = \lambda P$$

$$\int \frac{dP}{P} = \int \lambda\, dt$$

$$\ln P = \lambda t + c$$

$$P = e^{\lambda t} \cdot \boxed{e^c}$$

$$\boxed{P = k e^{\lambda t}}$$

$P = \underline{\underline{K}}\, e^{\lambda t}$

$$\boxed{\begin{array}{ll} t=0 & P=1000 \\ t=2 & P=1200 \end{array}}$$

$1000 = K e^{0}$

$\boxed{K=1000}$

$12\cancel{00} = 10\cancel{00}\, e^{\lambda(2)}$

$\frac{6}{5} = e^{\lambda(2)} \Rightarrow \boxed{\frac{1}{2}\ln\left(\frac{6}{5}\right) = \lambda}$

$P = 1000\, e^{\frac{1}{2}\ln\left(\frac{6}{5}\right)\cdot t}$

$P = 2000 \quad t = \frac{K}{\ln\left(\frac{6}{5}\right)}$

$2\cancel{000} = 1\cancel{000}\, e^{\frac{1}{2}\cancel{\ln\left(\frac{6}{5}\right)}\cdot\frac{K}{\cancel{\ln(6/5)}}}$

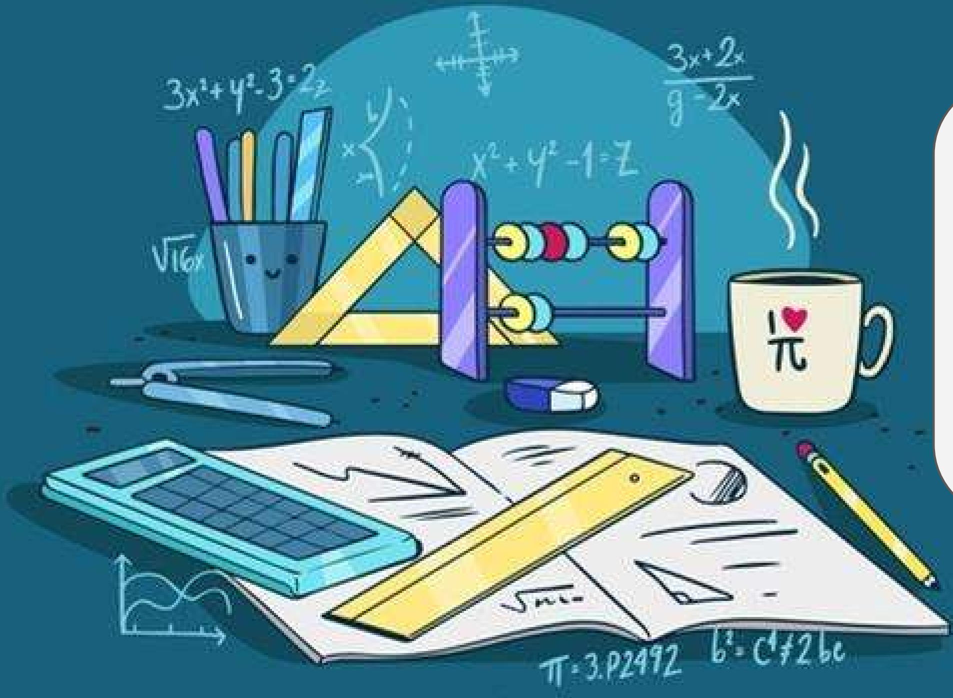
$2 = e^{K/2}$

$\ln 2 = \frac{K}{2}$

$K = 2\ln 2$

$\left(\frac{K}{\ln 2}\right)^2 = 2^2 = 4$

$\boxed{C_1 \perp C_2}$



Orthogonal Trajectories

Orthogonal Trajectories

$m = \frac{dy}{dx}$

$m_1 m_2 = -1$

$m_2 = \frac{-1}{m_1}$

$m_2 = \frac{-1}{\left(\frac{dy}{dx}\right)}$

$m_2 = -\frac{dx}{dy}$

Procedure for finding the orthogonal trajectory

1. Let f(x, y, c) = 0 be the equation of given family, where c is an arbitrary parameter.
2. Find the differential equation.
3. Replace dy/dx by -dx/dy in the equation obtained above
4. Solve the new differential equation to get the answer.

Find the orthogonal trajectory of the following curves

i. $\boxed{x^2 + y^2 = a^2}$ $a = 1/2/3 \ldots$

S-1 $\cancel{2}x\,dx + \cancel{2}y\,dy = 0$

$x + y\frac{dy}{dx} = 0$

S-2 $\boxed{\frac{dy}{dx} = \frac{-x}{y}}$

S-3 $-\frac{1}{\left(\frac{dy}{dx}\right)} = -\frac{x}{y}$

$\boxed{\frac{dx}{dy} = \frac{x}{y}}$ New D.E.

$\int \frac{dx}{x} = \int \frac{dy}{y}$

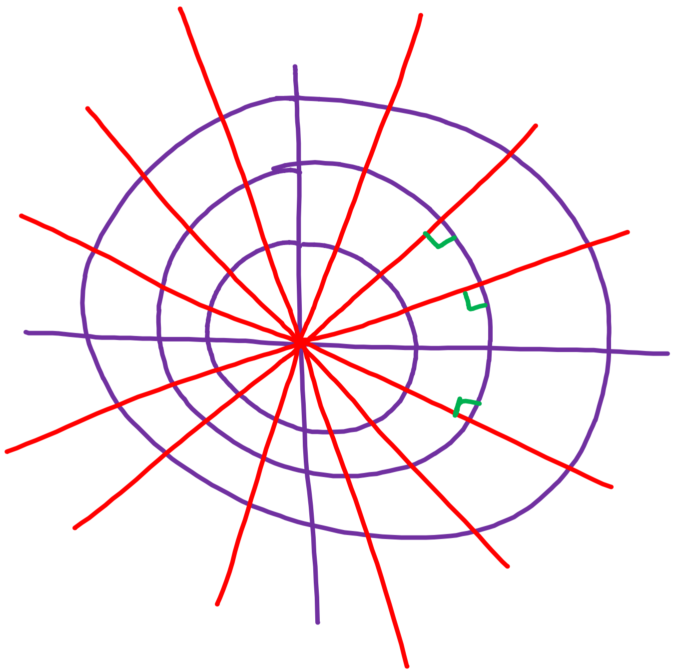
$\ln x = \ln y + c$

$\ln\left(\frac{x}{y}\right) = c$

$\frac{x}{y} = e^c$

$y = e^{-c}x$

$\boxed{y = mx}$



Find the orthogonal trajectory of $y^2 = 4ax$

A. $2x^2 + y^2 = k$

B. $x^2 + 2y^2 = k$

C. $x^2 + 2y = k$

D. $x^2 + 2y^2 = k$

$4a = \frac{y^2}{x}$

$y^2 = \boxed{4a}x$ — 1

S-1 $2y\frac{dy}{dx} = 4a$ — 2

$2\cancel{y}\frac{dy}{dx} = \frac{y^{\cancel{2}}}{x}$

S-2 $\boxed{\frac{dy}{dx} = \frac{y}{2x}}$

New D.E.

$$\boxed{-\frac{dx}{dy} = \frac{y}{2x}}$$

$$-\int 2x\,dx = \int y\,dy$$

$$2\left(-x^2 = \frac{y^2}{2} + c\right)$$

$$-2x^2 = y^2 + 2c$$

$$\boxed{y^2 + 2x^2 = k}$$