Circles

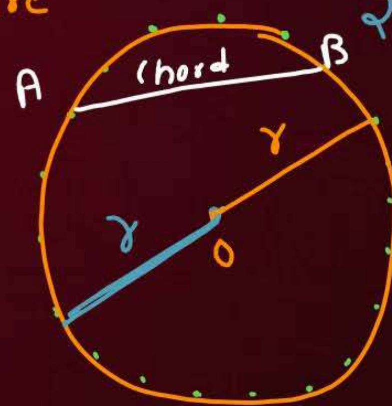
Circle :- A circle is a collection of all points in a plane which are at a constant distance from a fixed point. The constant distance is called the radius and the fixed point is called the centre of the circle.

Fixed point = centre

Constant Distane

↳ 3cm → radius of circle

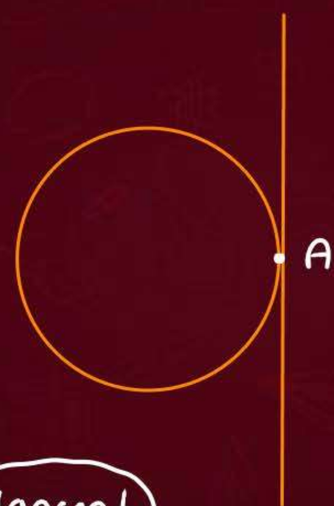
$2r$ = Diameter



Relationship between a Circle and a line

Case 1

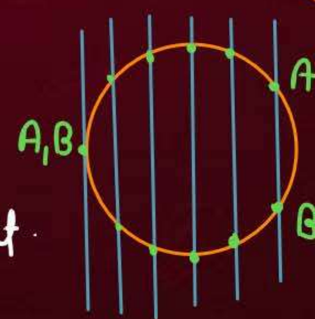
Non-Intersecting line

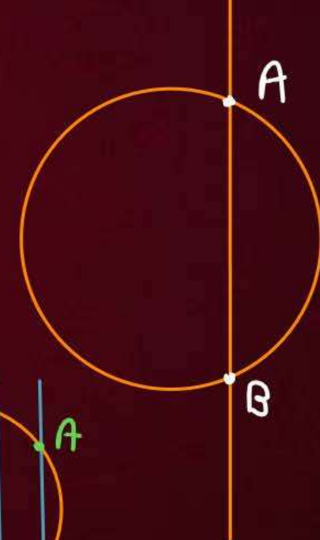


Tangent

• A → point of contact.

Note :→





Intersecting line

Secant

Theorem 1

The tangent at any point of a circle is perpendicular to the radius through the point of contact

$OA = OT$

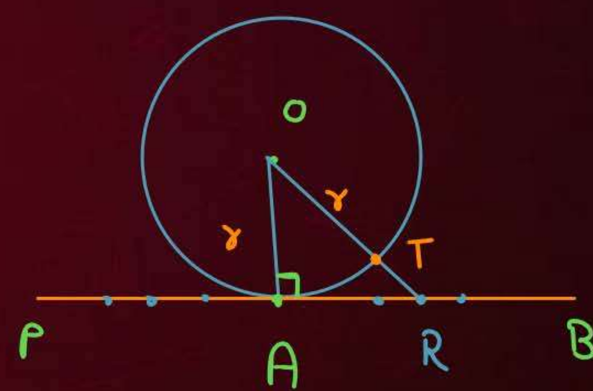
$OA \perp PB$

$OT + TR = OR$

$\boxed{OA < OR}$

OA is shortest

$\boxed{OA \perp BP}$ HP.

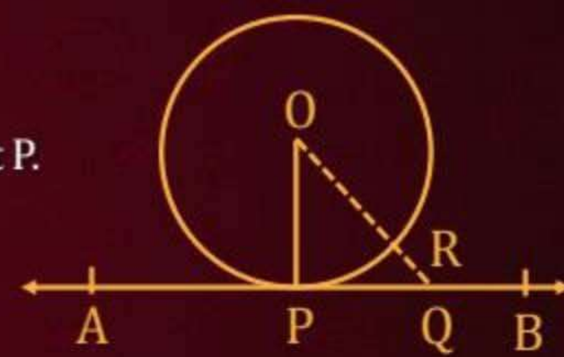


The tangent at any point of a circle is perpendicular to the radius through the point of contact

GIVEN:- A circle with centre O and a tangent AB at a point P of the circle.

TO PROVE :- $OP \perp AB$.

CONSTRUCTION : Take a point Q, other than P, on AB. Join OQ.



PROOF : Q is a point on the tangent AB, other than the point of contact P.

$\therefore$ Q lies outside the circle.

Let OQ intersect the circle at R.

Then, $\boxed{OR < OQ}$ [a part is less than the whole]. ...(i)

But, $\boxed{OP = OR}$ [radii of the same circle] ...(ii)

$\therefore$ $\boxed{OP < OQ}$ [from (i) and (ii)].

Thus, OP is shorter than any other line segment joining O to any point of AB, other than P.

But, the shortest distance between a point and a line is the perpendicular distance. $OP \perp AB$

QUESTION

A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Length PQ is :

(A) 12 cm

(B) 13 cm

(C) 8.5 cm

(D) $\sqrt{119}$ cm

$OQ^2 = OP^2 + PQ^2$

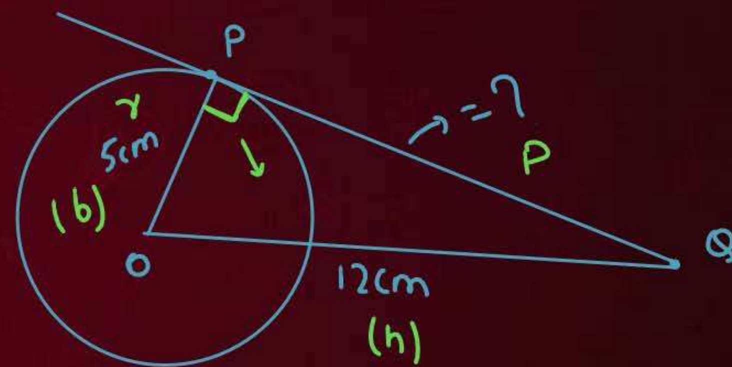
$12^2 = 5^2 + PQ^2$

$144 = 25 + PQ^2$

$144 - 25 = PQ^2$

$119 = PQ^2$

$PQ = \sqrt{119}$ cm



$OP \perp PQ$ {theorem 10.1}

Theorem 2

$\angle O + \angle A + \angle P + \angle B = 360^\circ$

$\angle O + \angle P = 360 - 180$

$\angle O + \angle P = 180^\circ$

The lengths of tangents drawn from an external point to a circle are equal.

Given:→

To prove:→ $AP = BP$

Construction:→ Join OP, OA and OB

Proof:→

$OA \perp AP$ and $OB \perp BP$

In right angled $\triangle AOP$ and $\triangle BOP$

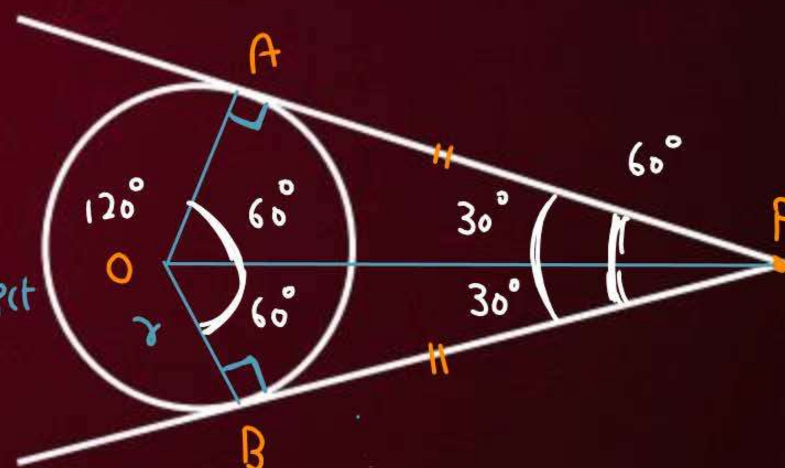
$OP = OP$ → common

$OA = OB$ {radii}

By using RHS

$\triangle AOP \cong \triangle BOP$

$\boxed{AP = BP}$ by cpct



QUESTION

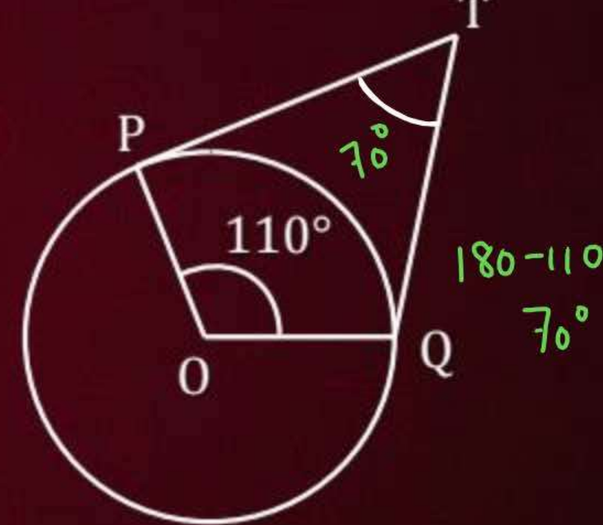
In given fig., if TP and TQ are the two tangents to a circle with centre O so that $\angle POQ = 110°$, then $\angle PTQ$ is equal to

(A) 60°

(B) 70°

(C) 80°

(D) 90°



QUESTION

Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

$AB = ?$

$OP \perp AB$ {10 1}

In ΔOAP

$OA^2 = OP^2 + AP^2$

$5^2 = 3^2 + AP^2$

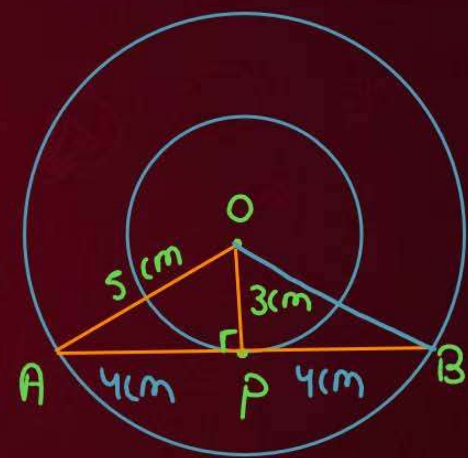
$25 - 9 = AP^2$

$AP^2 = 16$

$AP = \sqrt{16}$

$AP = 4cm$

$AB = 8cm$



QUESTION

A quadrilateral ABCD is drawn to circumscribe a circle (see in given Fig.). Prove that AB + CD = AD + BC

$AB = 10\,cm$

$CD = 5\,cm$

$AD = 9\,cm$

$BC = ?$

$AP = AS \rightarrow (i)$

$BP = BQ - (ii)$

$CR = CQ - (iii)$

$DR = DS - (iv)$

Adding (i), (ii), (iii) and (iv)

$AP + BP + CR + DR = AS + BQ + CQ + DS$

$\boxed{AB + CD = BC + AD}$