CHAPTER

14 Mechanical Properties of Fluids

Properties of Fluid

Density $= \dfrac{\text{mass}}{\text{volume}}$ (kg m^{-3})

Specific weight $= \dfrac{\text{weight}}{\text{volume}} = \rho g$ (kg m^{-2} s^{-2})

Relative density $= \dfrac{\text{density of given liquid}}{\text{density of pure water at } 4^{\circ}\text{C}}$ (Unitless)

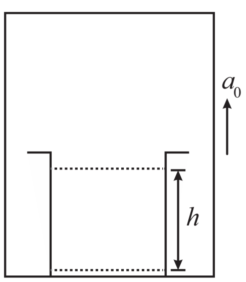
Pressure $= \dfrac{\text{normal force}}{\text{area}} = \dfrac{\text{thrust}}{\text{Area}}$ (Nm^{-2})

Variation of pressure with depth *h*

$$\Delta P = h\rho g$$

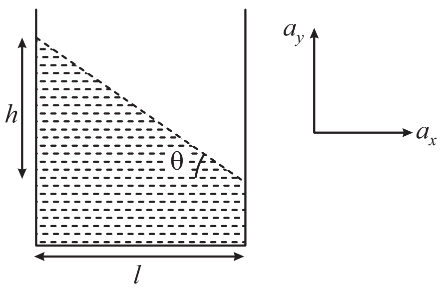
Pressure in case of accelerating fluid

Liquid placed in elevator: When elevator accelerates upward with acceleration a_0 then pressure in the fluid, at depth h may be given by, $P = h\rho[g + a_0]$



Free surface of liquid in accelerating container

Container accelerating in horizontal as well as in vertical direction.



$\tan\theta = \dfrac{h}{l} = \dfrac{a_x}{g + a_y}$ (goint up i.e. $a_y = +$)

$\tan\theta = \dfrac{h}{l} = \dfrac{a_x}{g - a_y}$ (going down i.e. $a_y = -$)

Buoyant force = weight of displaced liquid

$= \rho_l \cdot v_l \cdot g$

ρ_l = density of liquid, v_l = volume of displaced liquid

Steady and Unsteady Flow : The fluid characteristics like velocity, pressure and density at a point do not change with time in steady flow, whereas if any change in fluid characteristics then the flow is unsteady.

Streamline Flow : All the particles passing through a given point follow the same path and hence a unique line of flow. This line or path is called a streamline.

Laminar and Turbulent Flow : In Laminar flow fluid particles move along well-defined streamlines which are straight and parallel, whereas the motion of fluid particles is random in turbulent flow.

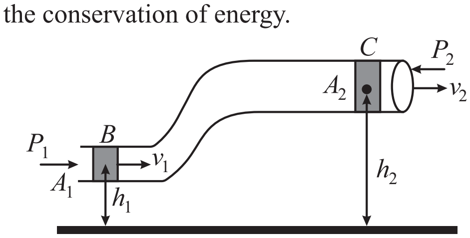
Compressible and Incompressible Flow : In compressible flow the density is not constant for the fluid whereas in incompressible flow the density of the fluid remains constant throughout.

Rotational and Irrotational Flow : The fluid particles while flowing along path-lines also rotate about their own axis, In irrotational flow particles do not rotate about their axis.

Equation of continuity $A_1v_1 = A_2v_2$ (Based on conservation of mass.)

Bernoulli's theorem : $\boxed{P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}}$

Based on the conservation of energy.



$$P_1 + \rho g h_1 + \frac{\rho v_1^2}{2} = P_2 + \rho g h_2 + \frac{\rho v_2^2}{2}$$

Kinetic Energy

Kinetic energy per unit volume

$$= \frac{\text{Kinetic Energy}}{\text{volume}} = \frac{1}{2}\frac{m}{V}v^2 = \frac{1}{2}\rho v^2$$

Potential Energy

Potential energy per unit volume

$$= \frac{\text{Potential Energy}}{\text{volume}} = \frac{m}{V}gh = \rho gh$$

Pressure Energy

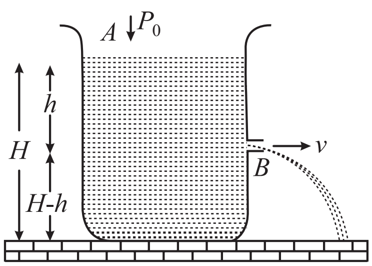
Pressure energy per unit volume $= \frac{\text{Pressure energy}}{\text{volume}} = P$

Rate of flow

Volume of water flowing per second $Q = A_1v_1 = A_2v_2$

Velocity of efflux $V = \sqrt{2gh}$

Horizontal range $R = 2\sqrt{h(H-h)}$



Surface Tension

$$\text{Surface Tension}(T) = \frac{\text{Total force on either sides of imaginary line}(F)}{\text{Length of line}(l)}$$

Surface Energy (U) = Surface Tension (T) × Exposed Area (A)

For liquid drop or water bubble $A = 4\pi r^2$

For soap bubble $A = 8\pi r^2$

Splitting of bigger drop into smaller droplets $R = n^{1/3}\, r$

Where, R = Radius of bigger drop

r = Radius of smaller drops

Work done = Change in surface energy $= 4\pi R^2 T\,(n^{1/3} - 1)$

Excess pressure $P_{ex} = P_{in} - P_{out}$

In liquid drop $P_{ex} = \frac{2T}{R}$

In soap bubble $P_{ex} = \frac{4T}{R}$

Angle of Contact (θ_C)

The angle enclosed between the tangent plane at the liquid surface and the tangent plane at the solid surface at the point of contact inside the liquid is defined as the angle of contact.

Angle of contact $\theta < 90^\circ \Rightarrow$ concave shape, Liquid rise up in capillary

Angle of contact $\theta > 90^\circ \Rightarrow$ convex shape, Liquid falls down in capillary

Angle of contact $\theta = 90^\circ \Rightarrow$ plane shape, Liquid neither rise nor falls

Capillary rise $h = \frac{2T\cos\theta}{r\rho g}$

When two soap bubbles are in contact then, radius of curvature of the common surface.

$$r = \frac{r_1r_2}{r_1 - r_2}\,(r_1 > r_2)$$

Viscosity

Newton's law of viscosity $F = \eta A\frac{\Delta V_x}{\Delta y}$

SI Units of η : $\frac{N\times s}{m^2}$

CGS Units : dyne–s/cm^2 or poise (1 decapoise = 10 poise)

Poiseuille's formula $Q = \frac{dV}{dt} = \frac{\pi pr^4}{8\eta L}$

Stoke's Law $F_V = 6\pi\eta rv$

Terminal velocity $V_T = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta} \Rightarrow V_T \propto r^2$

Reynolds number $R_e = \frac{\rho Vd}{\eta}$

R_e < 1000, then laminar flow, If R_e > 2000, than turbulent flow