



ELECTRO STATICS



Charge



Static

Study of charges at rest

Today's target - Electric Charges and Fields



- Charge and its properties ✓
- Methods of charging. ✓
- Coulomb's Law. ✓
- Electric Field and field Lines ✓
- Field and force due to continuous charge distribution ✓
- Electric Flux and Gauss Law ✓
- Electric Dipole and its Field ✓
- Selected Problems (latest PYQs, Mains and Advanced , NCERT, HCV, IErodov)

ELECTRIC CHARGE

Charge is the property associated with matter due to which it produces and experiences electrical and magnetic effects. The excess or deficiency of electrons in a body gives the concept of charge.

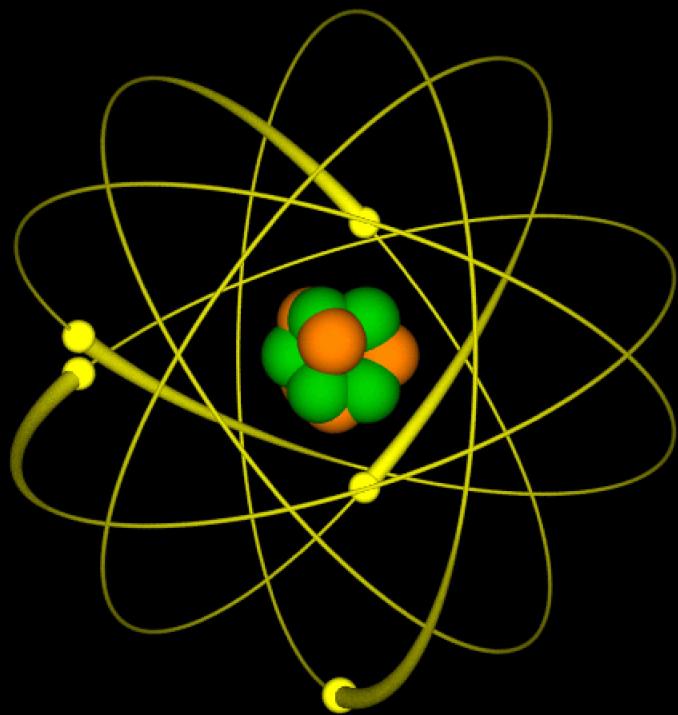
Types of charge :

- (i) Positive charge : It is the deficiency of electrons as compared to proton.
- (ii) Negative charge : It is the excess of electrons as compared to proton.

SI unit of charge : ampere \times second i.e. Coulomb

Dimension : [A T]

$$\left| \begin{array}{l} I = \frac{Q}{t} \\ [I t] - [Q] \\ A \text{ sec} \quad \text{Unit of charge} \\ [AT] \text{ or } [It] \end{array} \right.$$



electron - ve
Proton + ve

$$\frac{e = 1.6 \times 10^{-19} \text{ coulomb}}{\text{charge } p = + e}$$

^{on} +P
^{on} +P
^{on} +P

Nu

Atom neutral
+ve equal -ve

$$m_e = 9.1 \times 10^{-31} \text{ kg}$$

$$m_p = 1.6 \times 10^{-27} \text{ kg}$$

Specific Properties of Charge

Charge is a scalar quantity : ✓

It represents excess or deficiency of electrons. Charges are added like real numbers taking sign into consideration

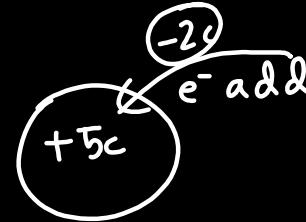
Charge is transferrable :

If a charged body is put in contact with another body, then charge can be transferred to another body.

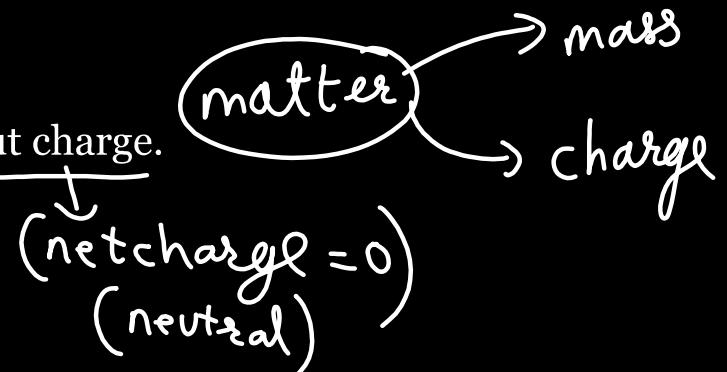
Charge is always associated with mass

Charge cannot exist without mass though mass can exist without charge.

- In charging, the mass of a body changes.
- When body is given positive charge, its mass decreases.
- When body is given negative charge, its mass increases.



$$\text{final} = +5c - 2c = \underline{+3c}$$

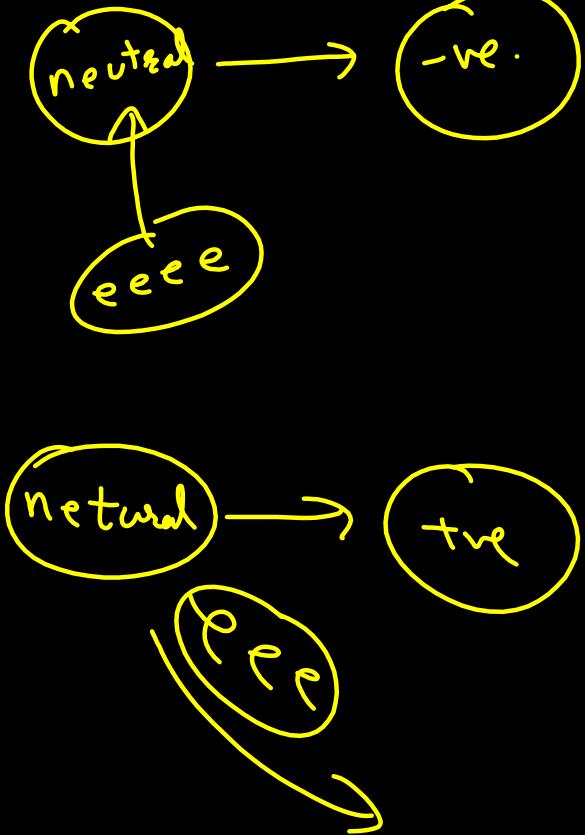


$$1 \text{ esu} = 3.33 \times 10^{-10} \text{ C}$$

$$1 \text{ C} = 3 \times 10^9 \text{ esu} \quad (\text{stat coulomb})$$

1 Faraday = charge of one mole electrons.

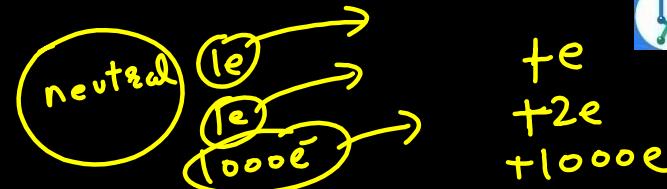
$$= \underline{96500 \text{ C}}$$



Charge is quantized

e

The quantization of electric charge is the property by virtue of which all free charges are integral multiple of a basic unit of charge represented by e.



The quantum of charge is the charge $e = 1.6 \times 10^{-19} C$

$$q = \pm ne$$

$$Q = \pm ne$$

$$I = \frac{Q}{t} = \frac{ne}{t}$$

Charge is conserved

In an isolated system, total charge does not change with time, though individual charge may change

Conservation of charge is also found to hold good in all types of reactions either chemical (atomic) or nuclear.

Q How many Electron in I_c ??

$$Q = ne$$

$$I_c = n(1.6 \times 10^{-19})c$$

$$\frac{1}{1.6 \times 10^{-19}} = n$$

$$6.25 \times 10^{18} = n$$

-

\underline{Q} 10^9 electron are moving out of body in 1 second.

How much time for $Q = 1$ coulomb ??.

$$\underline{6.25 \times 10^{18} \text{ electrons}}$$

$$1 \text{ sec} \rightarrow 10^9$$

$$t \text{ sec} \rightarrow 10^9(t) \text{ no. of } e^-$$

$$(10^9)t = 6.25 \times 10^{18}$$

$$\underline{t = 6.25 \times 10^9 \text{ sec}}$$

Charge is invariant

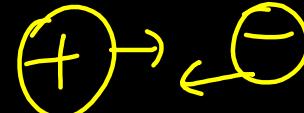
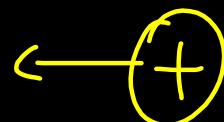
charge on a body does not change whatever be its speed.

Accelerated charge radiates energy

$v = 0$ (i.e. at rest)  Q produces only \vec{E} (electric field)	$v = \text{constant}$  Q produces both \vec{E} and \vec{B} (magnetic field) but no radiation	$v \neq \text{constant}$ (i.e. time varying)  Q produces \vec{E}, \vec{B} and radiates energy
---	--	--

Attraction - Repulsion

Similar charges repel each other while opposite charges attract.



~~60 kg~~ → very fast

$$m' = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Q rest → only E field

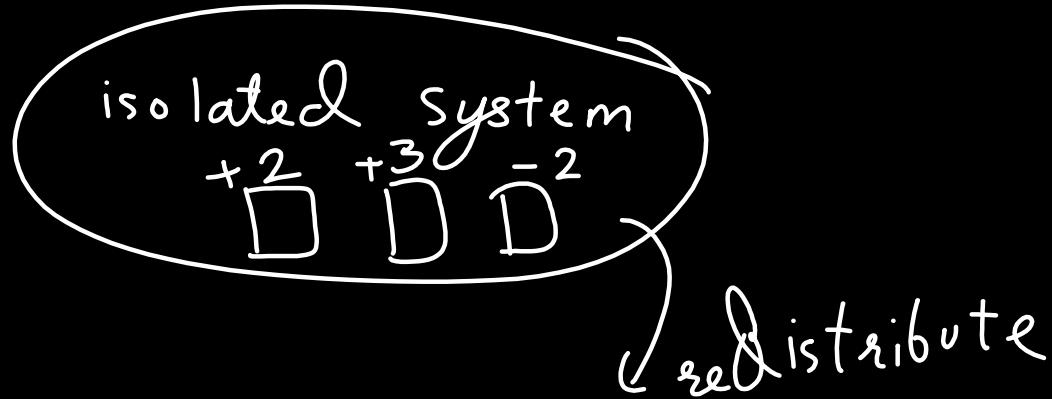
$c \rightarrow 3 \times 10^8 \text{ m/s}$

Q constant vel → Both E & B field

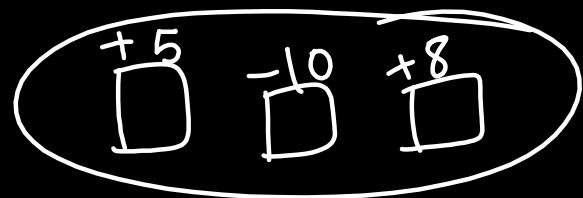
$v \rightarrow \text{speed of particle.}$

Q accelerated → Both E & B field, Electro Magnetic
Radiation.
emissions.

Conservation of Charge



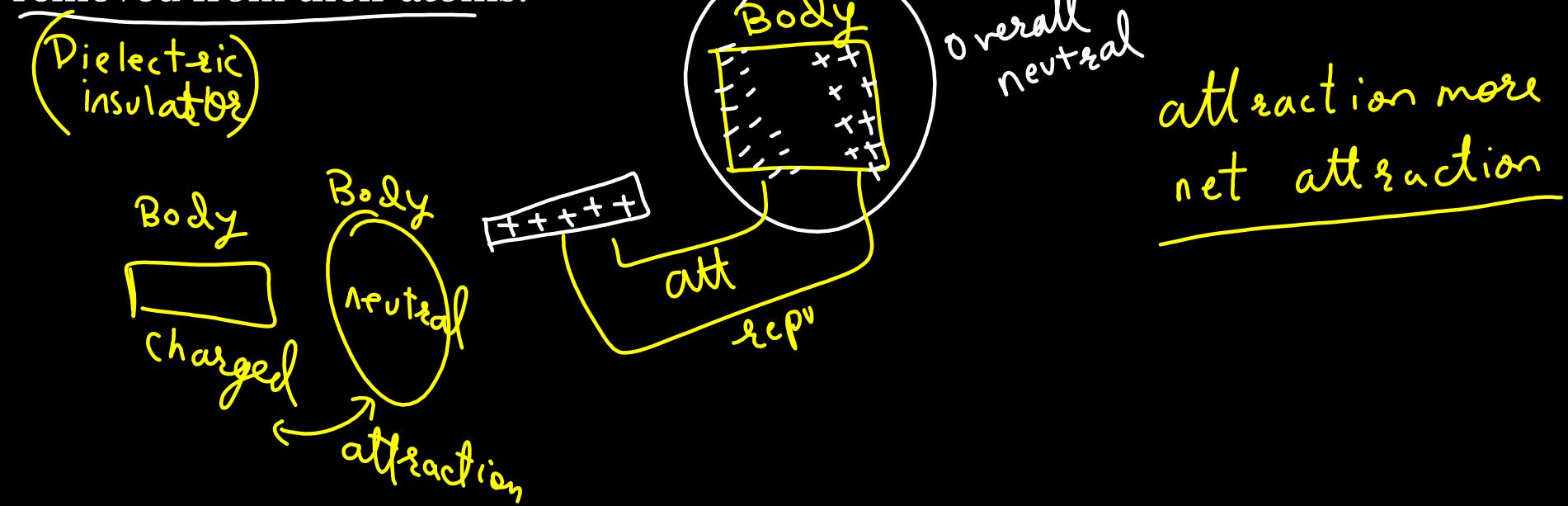
$$Q_{\text{total initial}} = Q_{\text{total final}}$$



CONDUCTORS AND INSULATORS

In conductors, the valence electrons of the atoms can be easily removed from the atoms and moved about in conductor.

In insulators, even the loosest bound electrons are too tightly bond to be easily removed from their atoms.



Repulsion is true test of electrification.

(A B) attract

((C D) repulsion
Both charged)

(a) By Friction

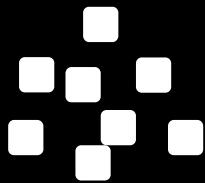


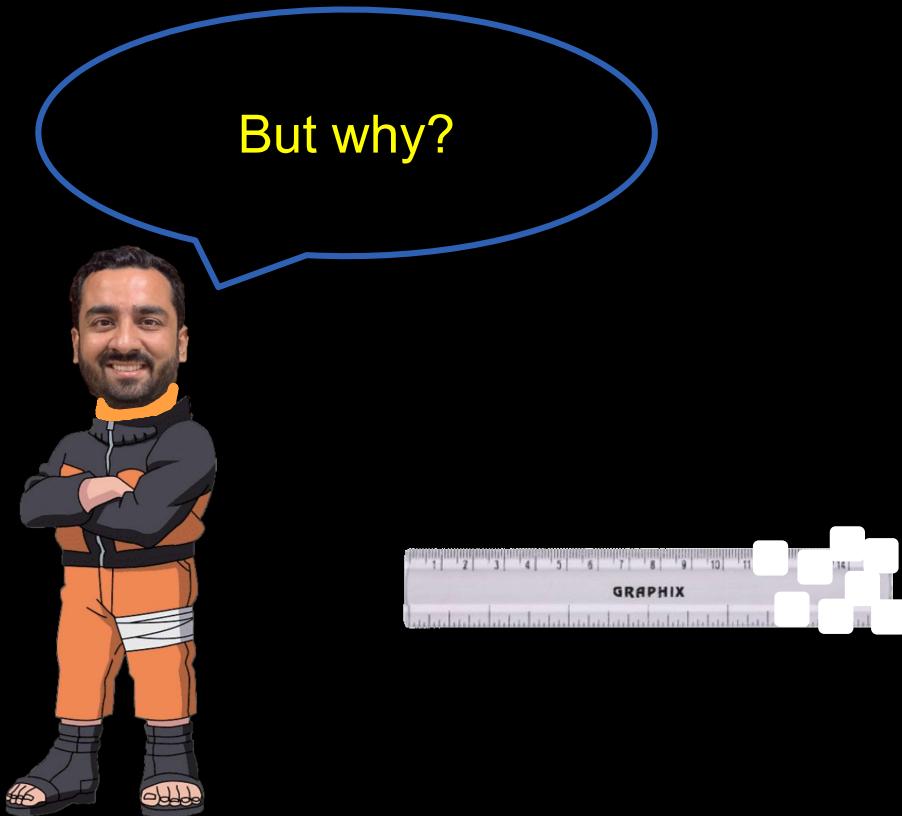
Method of Charging

neutral

neutral

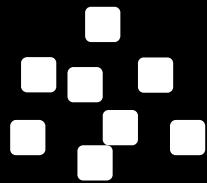
rub
⇒ equal & opposite charge
develop.





But why?





Methods of Charging

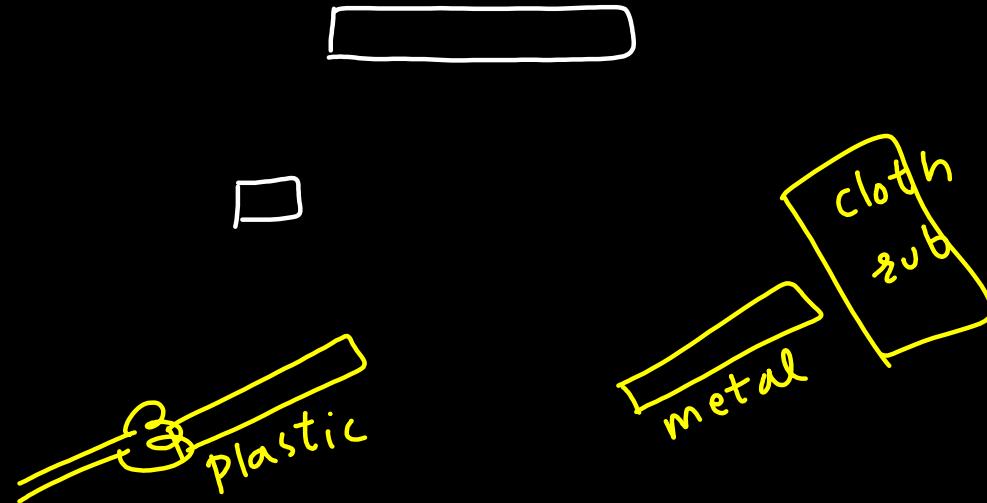
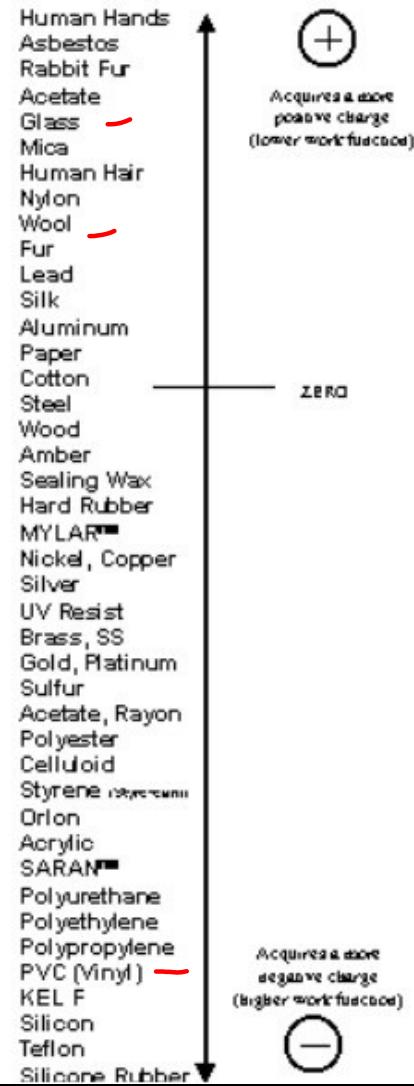
Friction If we rub one body with other body, electrons are transferred from one body to the other. Transfer of electrons takes places from lower work function body to higher work function body.

Positive charge	Negative charge
Glass rod	Silk cloth
Woollen cloth	Rubber shoes, Amber, Plastic objects
Dry hair	Comb
Flannel or cat skin	Ebonite rod
Note : Clouds become charged by friction	



low WF $\Rightarrow e^-$ easily given.

Triboelectric Series

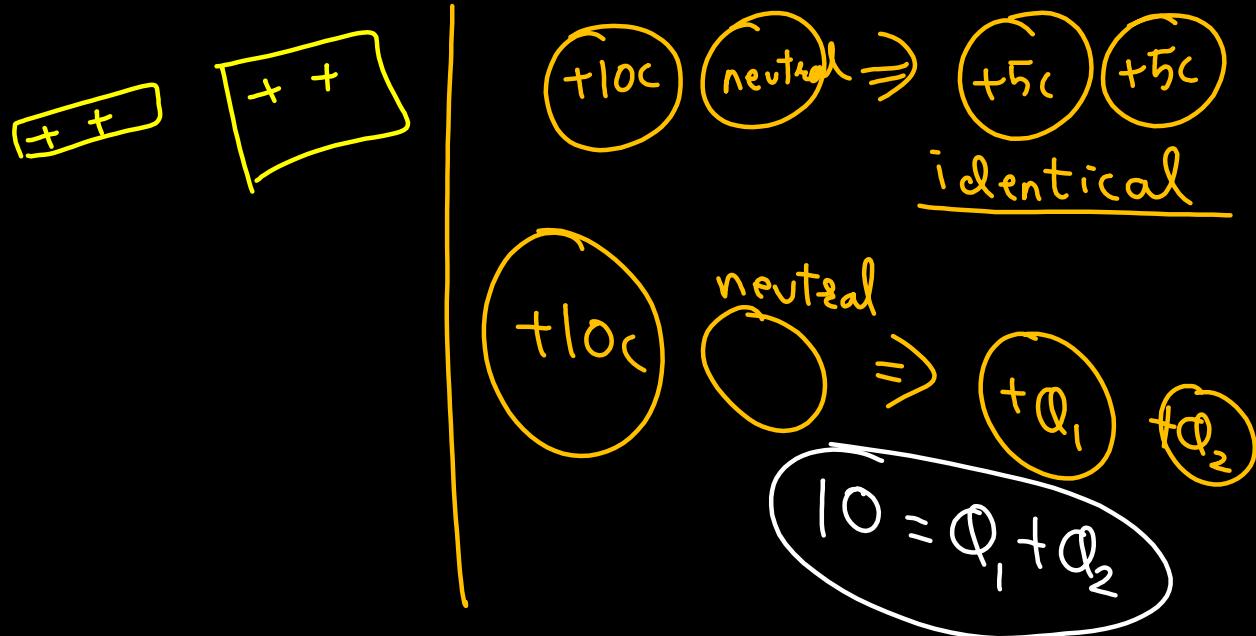
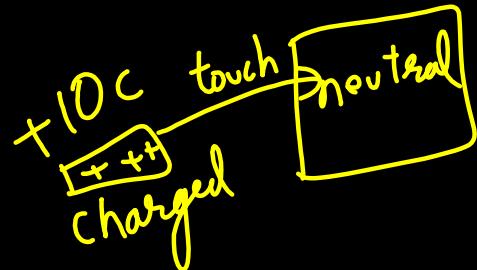


Conduction

(contact)

The process of transfer of charge by contact of two bodies is known as conduction. If a charged body is put in contact with uncharged body, the uncharged body becomes charged due to transfer of electrons from one body to the other.

- The charged body loses some of its charge (which is equal to the charge gained by the uncharged body)
- The charge gained by the uncharged body is always lesser than initial charge present on the charged body.

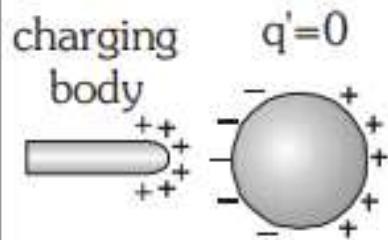


charge flow till potential becomes Same

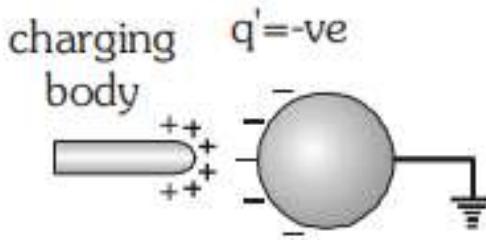
Electrostatic induction (induced)

If a charged body is brought near a metallic neutral body, the charged body will attract opposite charge and repel similar charge present in the neutral body. As a result of this one side of the neutral body becomes negative while the other positive, this process is called 'electrostatic induction'.

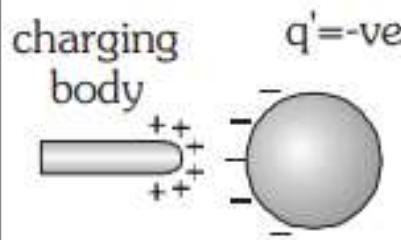
Charging a body by induction (in four successive steps)



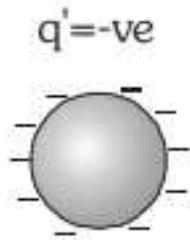
charged body is brought near uncharged body
step-1



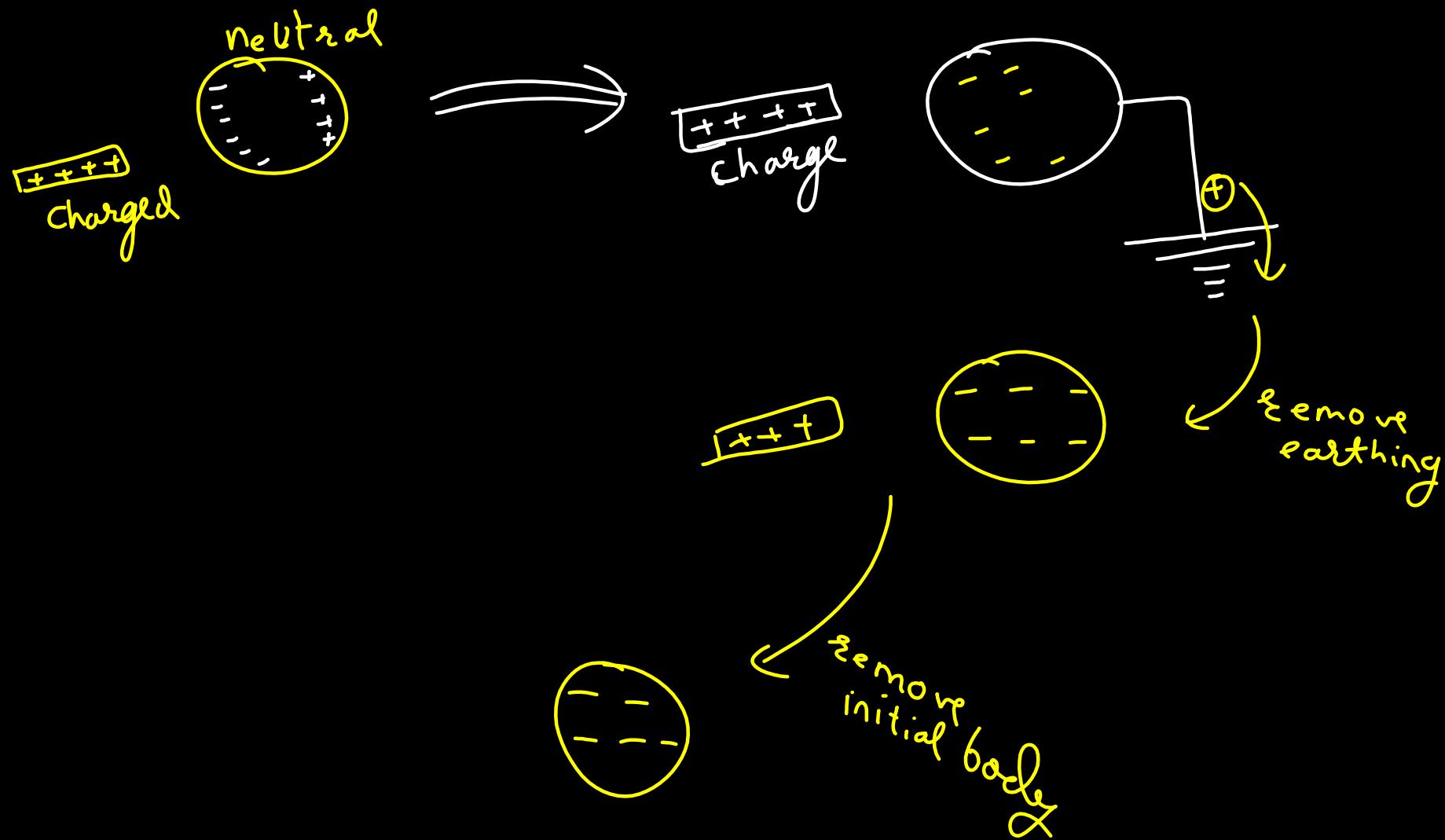
uncharged body is connected to earth
step-2

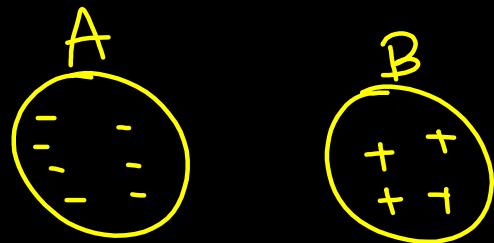
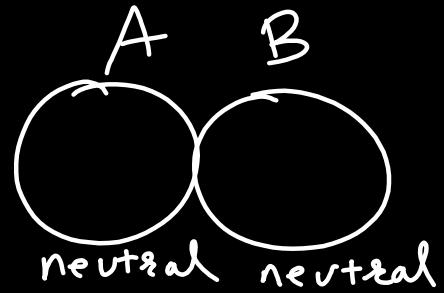


uncharged body is disconnected from the earth
step-3



charging body is removed
step-4





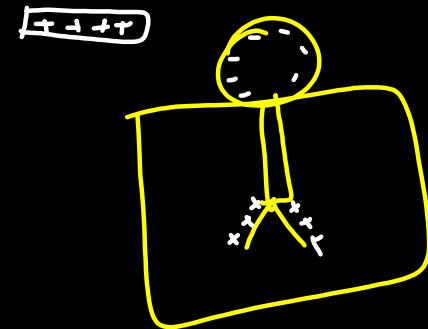
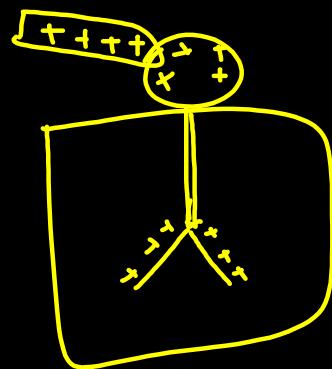
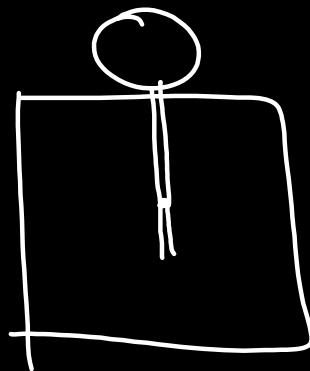
Gold leaf Electroscope

It is used to detect the charge on the body.

It consists of a gold leaf attached to the brass disc which is suspended with the help of a brass rod in a glass jar.

If the gold leaf diverges outward, the body is electrically charged

However if no divergence takes place , then body has no charge.



COULOMB'S LAW

Gravitation

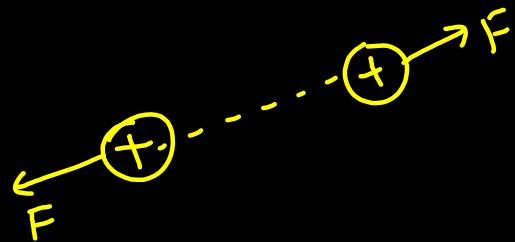


$$F = G \frac{m_1 m_2}{r^2}$$

Q_1 Q_2

$$F = k \frac{Q_1 Q_2}{r^2}$$

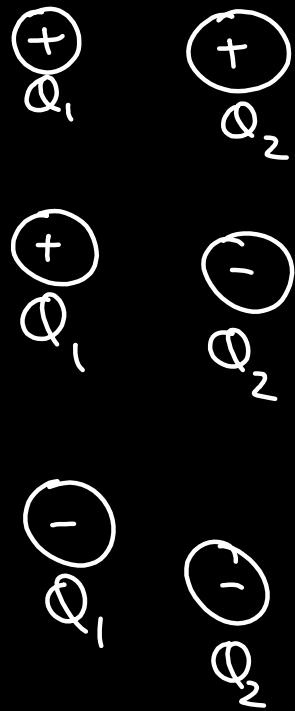
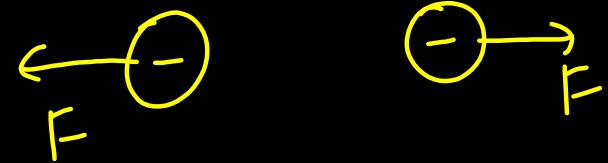
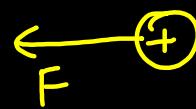
- Always act along the line joining two charges
- It is a field force and not a contact force
- Net force on charge will change in different mediums
- Action – reaction Pair
- it is a conservative force (work done is path independent)
- Formula only for point charges
- It is a central force



$$F = \frac{kQ_1 Q_2}{r^2}$$

$$|F| = \left| \frac{kQ_1 Q_2}{r^2} \right|$$

FBD acche



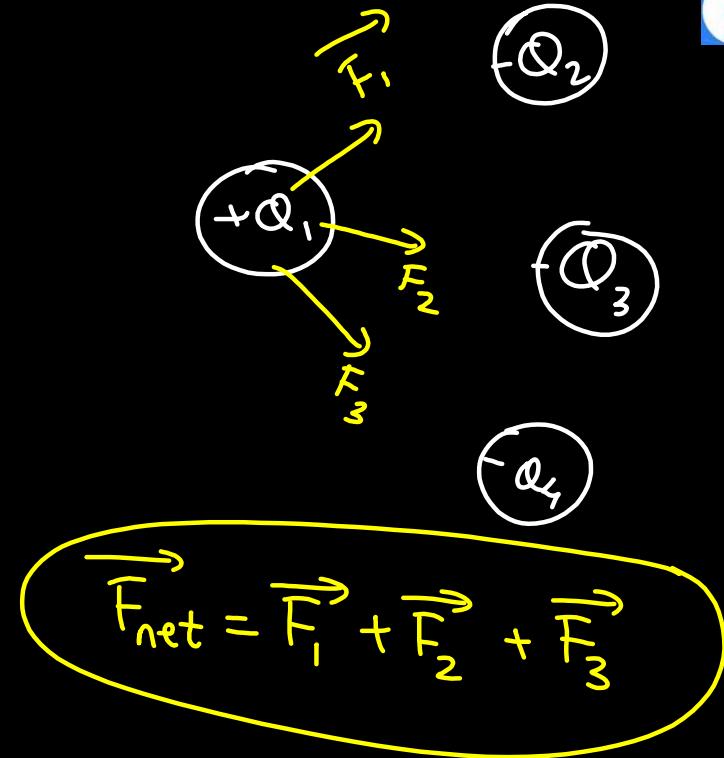
Superposition Theorem

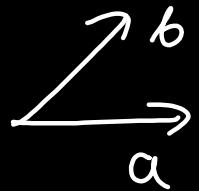
Vector Addition

Resultant

Net force.

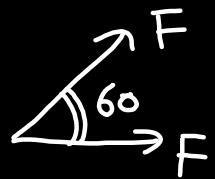
Q_1 & Q_2 ka interaction
Duniya se independent
Hai



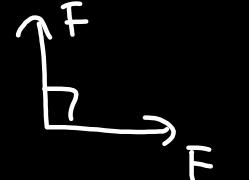


$$\vec{R} = \vec{a} + \vec{b}$$

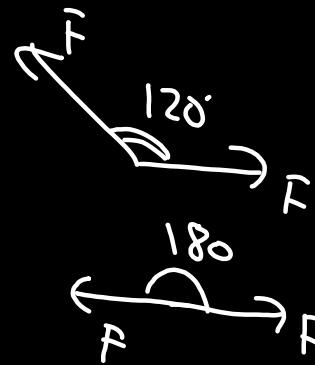
$$|R| = \sqrt{a^2 + b^2 + 2ab \cos\theta}$$



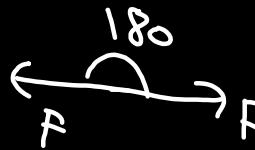
$$R = \sqrt{3} F$$



$$R = F\sqrt{2}$$

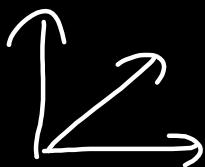


$$R = F$$



$$R = 0$$

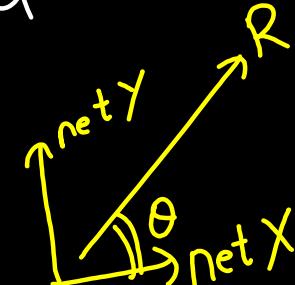
more than 2 vectors



x Component of all
y " " " "

$$\vec{F}_{net} = F_{net,x} \hat{i} + F_{net,y} \hat{j}$$

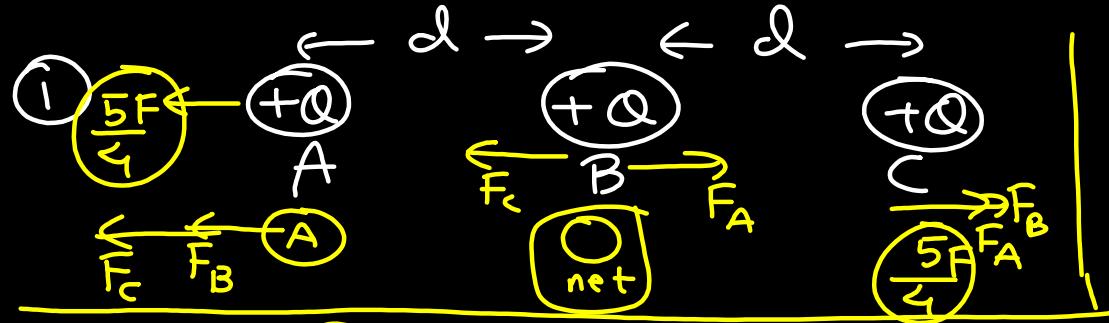
$$F_{net} = \sqrt{x^2 + y^2}$$



$$\tan\theta = \frac{y}{x}$$

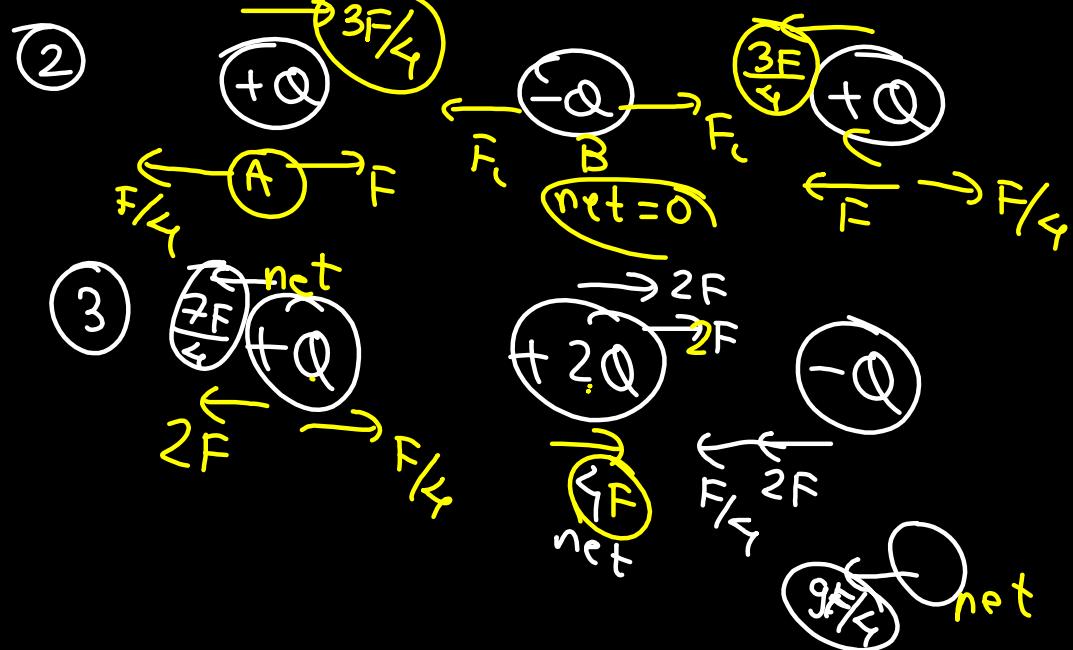
Find net Force

$$F = \frac{KQQ}{d^2} = \frac{KQ^2}{d^2}$$

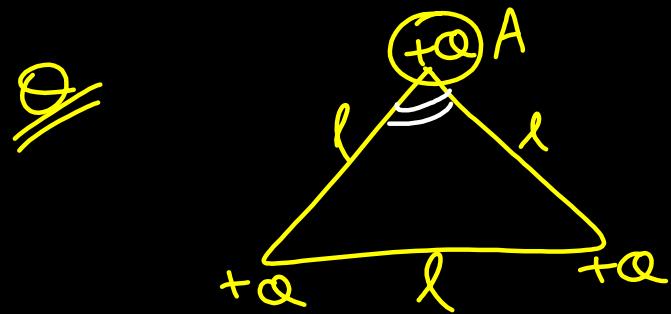


(A) $F_B = \frac{KQ^2}{d^2} = F$

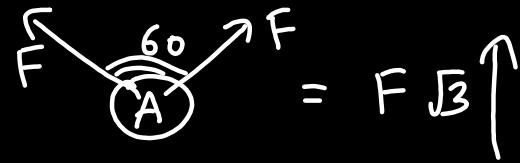
$$F_C = \frac{KQ^2}{(2d)^2} = \frac{KQ^2}{4d^2} = \frac{F}{4}$$



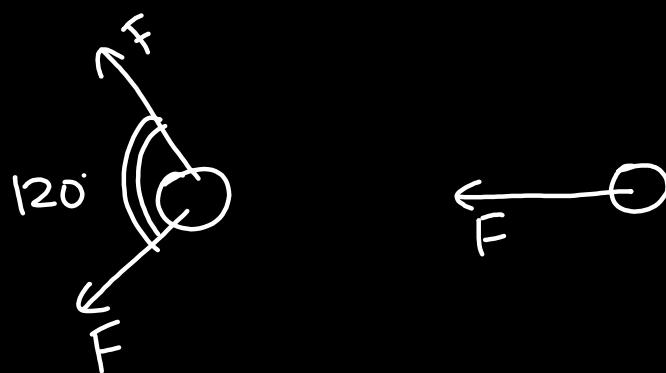
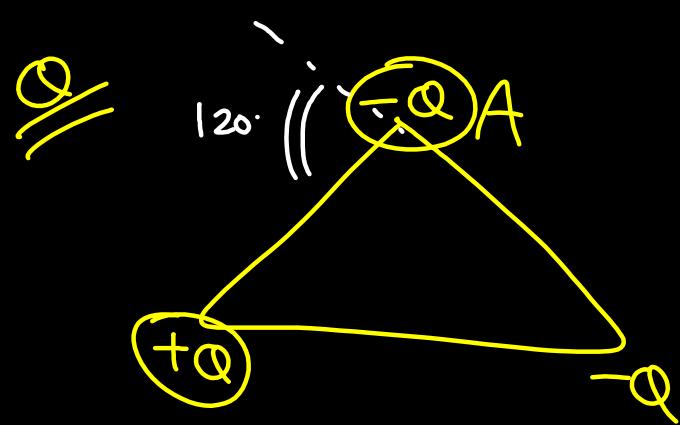
$$F = \frac{kQ^2}{l^2}$$



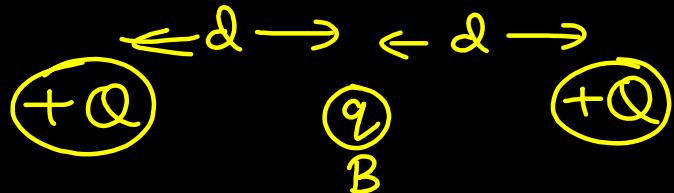
At vertex A:



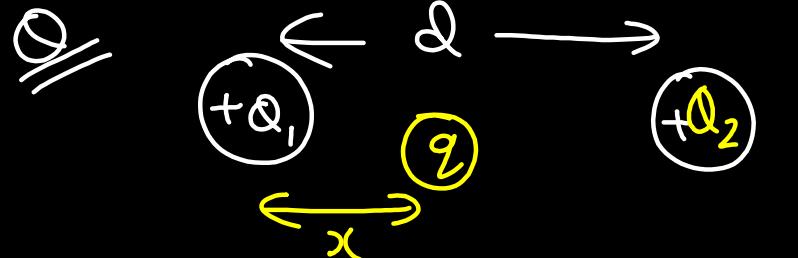
$$F = F \sqrt{3} \uparrow$$



Equilibrium of Charges



$$F_{\text{net}} = 0$$



F_{net} on q is 0. Find x ??.

$$\frac{KQ_1 q}{x^2} = \frac{KQ_2 q}{(d-x)^2}$$

$$\Rightarrow x = \frac{\sqrt{Q_1} d}{\sqrt{Q_1} + \sqrt{Q_2}}$$

$$Q_1 (d-x)^2 = Q_2 x^2$$

$$\sqrt{Q_1} (d-x) = \sqrt{Q_2} x$$

$$\frac{KQ_2 q}{(d-x)^2} \quad \frac{KQ_1 q}{x^2}$$

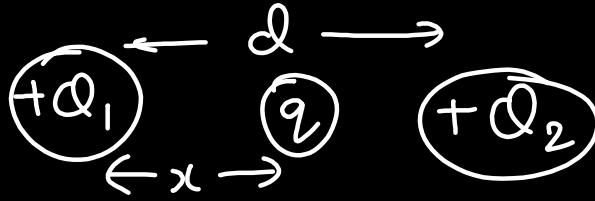
$$x = \frac{(Q) \sqrt{Q_1}}{\sqrt{Q_1} \left(1 + \sqrt{\frac{Q_2}{Q_1}} \right)}$$

$$x = \frac{d}{1 + \sqrt{n}}$$

distance from Q_1

$$\sqrt{n} = \sqrt{\frac{Q_2}{Q_1}}$$

$$n = \frac{Q_2}{Q_1}$$



$$\frac{k Q_1 Q_2}{d^2} = \frac{k Q_1 q}{x^2}$$

$$\frac{x^2}{d^2} Q_2 = q$$

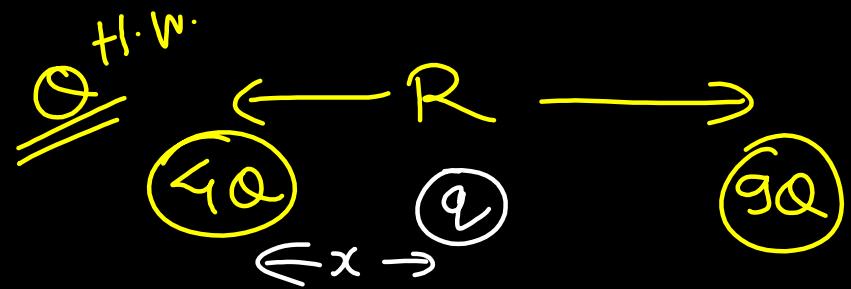
$$\frac{Q_1 \cancel{d^2}}{\cancel{d^2}} Q_2 = q$$

$$(\sqrt{Q_1} + \sqrt{Q_2})^2 = q$$

Find values of \underline{q} & $\underline{Q_2}$
So that all three charges in equilibrium?

q should be
-ve

$$q = -\frac{Q_1 Q_2}{(\sqrt{Q_1} + \sqrt{Q_2})^2}$$

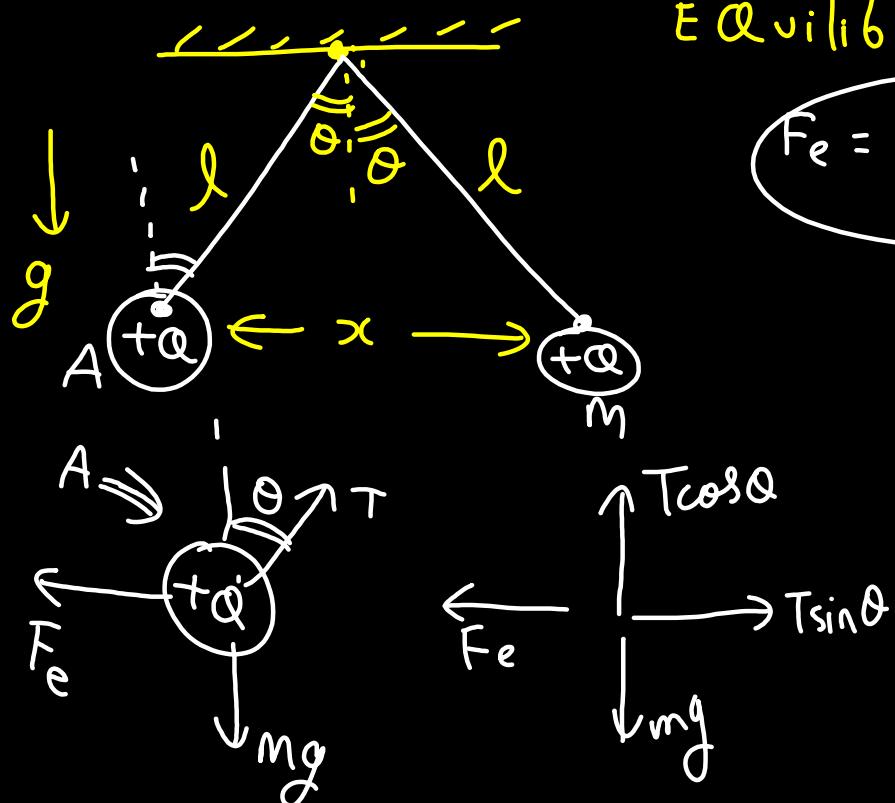


Ans → Comment in Comments

t.me/ajitlulla

Find Q & x for all in equilibrium??

Suspended with String



Equilibrium

$$F_e = \frac{kQ^2}{x^2}$$

geometry



$$\sin \theta = \frac{x}{2l}$$

$$T \sin \theta = F_e$$

$$T \cos \theta = mg$$

$$\tan \theta = \frac{F_e}{mg}$$

square & add

$$T^2 = (F_e)^2 + (mg)^2$$

$$T = \sqrt{(F_e)^2 + (mg)^2}$$

Suppose x small

θ small

$$\sin \theta \approx \theta \approx \tan \theta = \frac{x}{2l}$$

$$\tan \theta = \frac{F_e}{mg} = \frac{kQ^2}{x^2 mg}$$

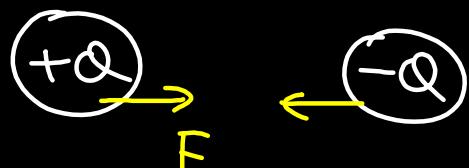
$$\frac{kQ^2}{x^2 mg} = \frac{x}{2l}$$

\Rightarrow

x & l relation
 x & Q relation
 Q & l relation.

Two charges equal in magnitude and opposite in polarity are placed at a certain distance apart and force acting between them is F . If 75% charge of one is transferred to another, then the force between the charges becomes

- (A) $F/16$
- (B) $9F/16$
- (C) F
- (D) $15F/16$



$$F = \frac{kQ^2}{d^2}$$

75% of Q

$$\frac{75}{100} \times Q = \frac{3}{4}Q$$

Mains

$$Q - \frac{3Q}{4}$$

$$-\frac{Q}{4} + \frac{3Q}{4}$$

$$0\%$$

$$-0\%$$

$$F' = \frac{kQ \cdot \frac{Q}{4}}{d^2} = \frac{kQ^2}{16d^2} = \frac{F}{16}$$

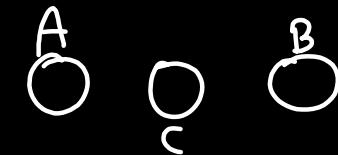
Two identical metallic spheres A and B when placed at certain distance in air repel each other with a force of F. Another identical uncharged sphere C is first placed in contact with A and then in contact with B and finally placed at midpoint between spheres A and B. The force experienced by sphere C will be :

- (A) $3F/2$
- (B) $3F/4$
- (C) F
- (D) $2F$

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$$A + \alpha \quad B + \alpha$$

$$F = \frac{K\alpha^2}{d^2}$$



$A + \alpha$ $C + \alpha/2$ $A + \alpha/2$ $C + \alpha/2$	$B + \alpha$ $\frac{\alpha + \alpha/2}{2} = \frac{3\alpha}{4}$ $\frac{3\alpha}{4}$ $B + \frac{3\alpha}{4}$
--	---

$$A + \alpha/2 \quad \xleftarrow{F_B} \quad \xrightarrow{F_A} \quad B + \frac{3\alpha}{4}$$

$$F_{AC} = \frac{K \left(\frac{Q}{2}\right) \left(\frac{3Q}{4}\right)}{\left(\frac{d}{2}\right)^2}$$

$$= \frac{3KQ^2}{8^2 d^2}$$

$$= \frac{3KQ^2}{2d^2}$$

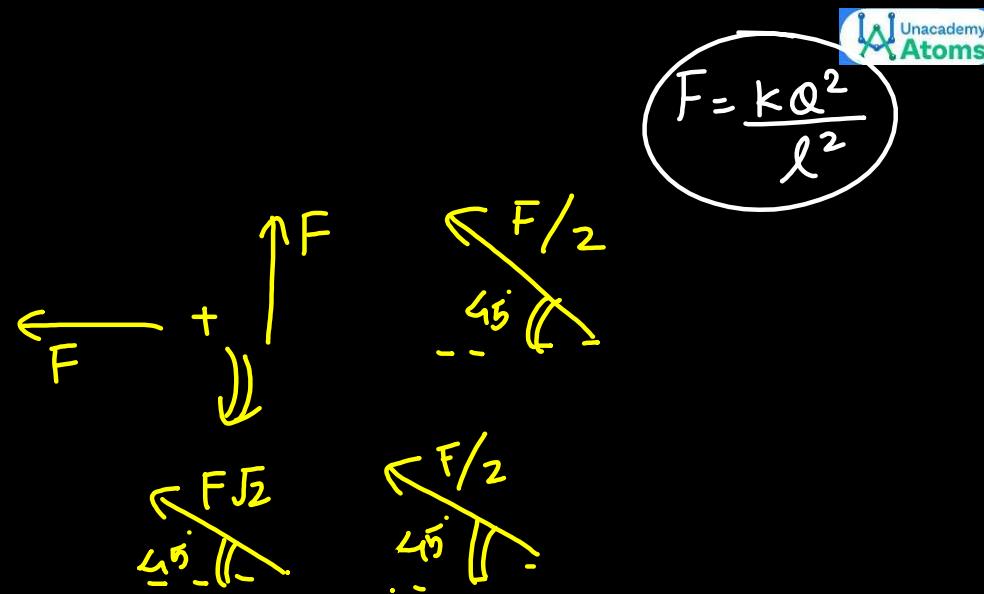
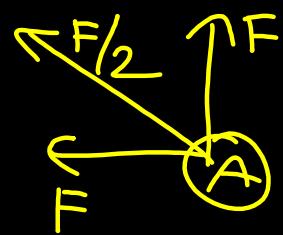
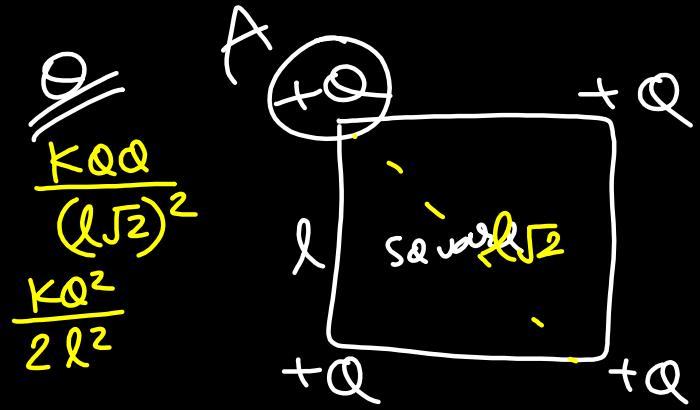
$$F_{BC} = \frac{K \left(\frac{3Q}{4}\right) \left(\frac{3Q}{4}\right)}{\left(\frac{d}{2}\right)^2}$$

$$= \frac{9KQ^2}{4d^2}$$

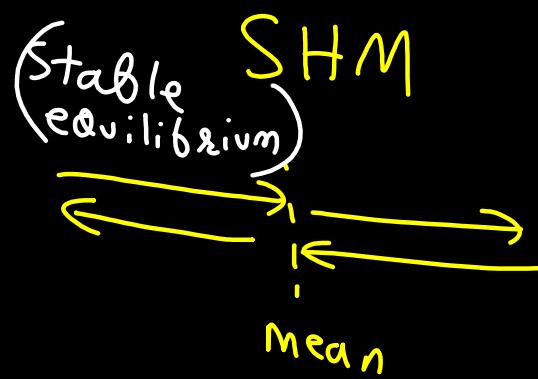
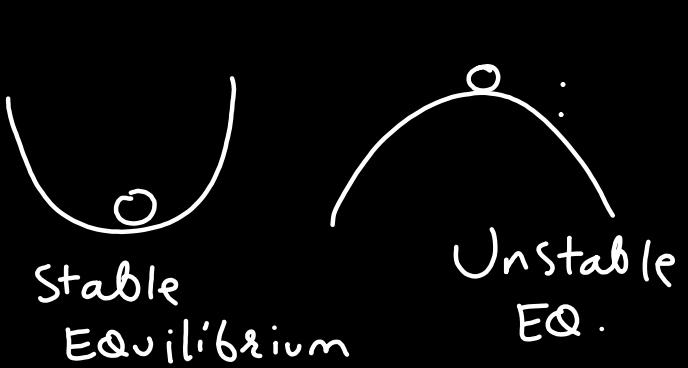
$$F_{\text{net}} = \frac{g}{4} - \frac{3}{2}$$

$$= \frac{3}{4} \frac{KQ^2}{d^2}$$

\$\frac{3}{4} F\$
 ← left (-i)



$$\begin{aligned}
 F\sqrt{2} + \frac{F}{2} &= F\left(\sqrt{2} + \frac{1}{2}\right) \\
 &= \frac{KQ^2}{l^2}\left(\sqrt{2} + \frac{1}{2}\right)
 \end{aligned}$$



$$F = -Kx$$

$$T = 2\pi \sqrt{\frac{m}{k}}$$

$$F = -10x$$

$$T = 2\pi \sqrt{\frac{m}{10}}$$

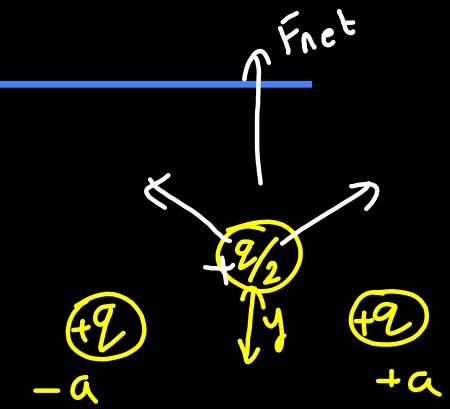
$$F = -\frac{\alpha}{\beta}x$$

$$T = 2\pi \sqrt{\frac{m}{\alpha/\beta}}$$



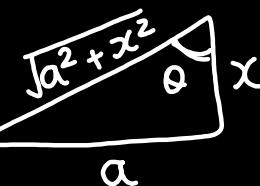
Two charges, each equal to q , are kept at $x = -a$ and $x = a$ on the x-axis. A particle of mass m and charge $q_0 = \frac{q}{2}$ is placed at the origin. If charge q_0 is given a small displacement ($y \ll a$) along the y-axis, the net force acting on the particle is proportional to [2013]

- (a) y (b) $-y$ (c) $\frac{1}{y}$ (d) $-\frac{1}{y}$



Prove motion of $-q$ is SHM
& find its Time Period ??.

$$x \lll a$$



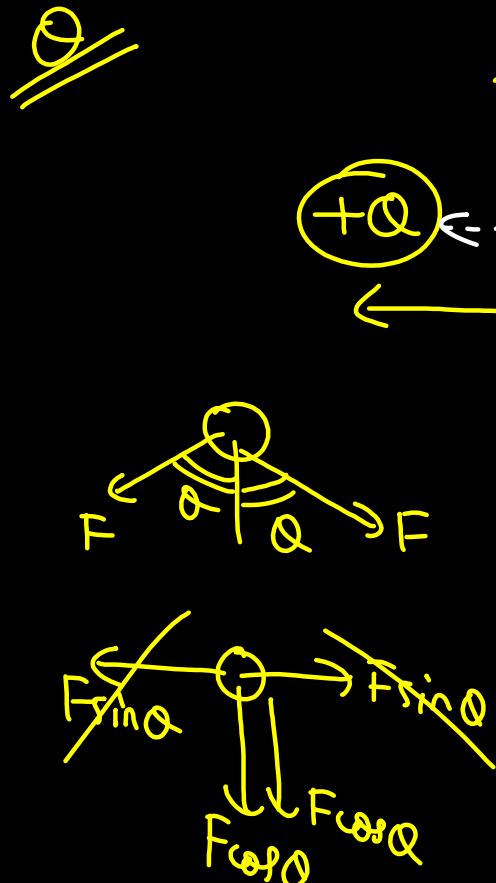
$$F = \frac{KQq}{(Ja^2+x^2)^2}$$

$$H^2 = P^2 + B^2$$

$$\cos \theta = \frac{x}{\sqrt{a^2+x^2}}$$

$$2 \frac{KQq}{(a^2+x^2)} \frac{x}{\sqrt{a^2+x^2}} = \frac{2 KQq x}{a^3}$$

$$x \lll a$$



$$2F \cos \theta \Rightarrow$$

$$F = -\frac{2kQq}{a^3} x$$

$$T = 2\pi \sqrt{\frac{m}{2kQq/a^3}}$$

$$= 2\pi \sqrt{\frac{ma^3}{2kQq}}$$

$$\boxed{F = -kx}$$

$$\boxed{T = 2\pi \sqrt{\frac{m}{k}}}$$

Two identical positive charges Q each are fixed at a distance of '2a' apart from each other. Another point charge q_0 with mass 'm' is placed at midpoint between two fixed charges. For a small displacement along the line joining the fixed charges, the charge q_0 executes SHM. The time period of oscillation of charge q_0 will be :

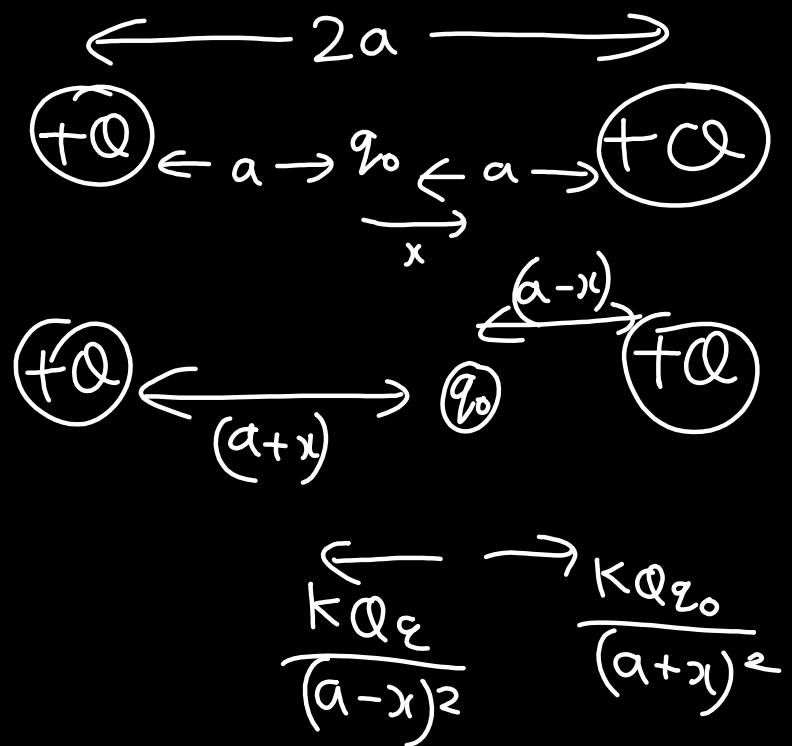
(A) $\sqrt{\frac{4\pi^3 \epsilon_0 m a^3}{q_0 Q}}$

(B) $\sqrt{\frac{q_0 Q}{4\pi^3 \epsilon_0 m a^3}}$

(C) $\sqrt{\frac{2\pi^2 \epsilon_0 m a^3}{q_0 Q}}$

(D) $\sqrt{\frac{8\pi^3 \epsilon_0 m a^3}{q_0 Q}}$

$$K \rightarrow \frac{1}{4\pi \epsilon_0}$$



$$F = \frac{KQq}{(a-x)^2} - \frac{KQq}{(a+x)^2}$$

$$= KQq \left[\frac{a^2 + x^2 + 2ax - a^2 - x^2 + 2ax}{(a-x)^2(a+x)^2} \right]$$

$$= KQq \frac{4ax}{a^4}$$

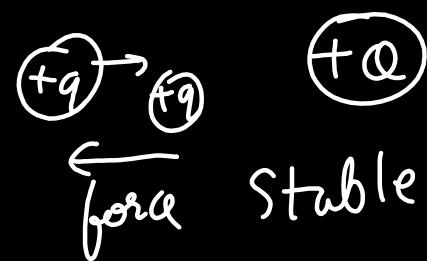
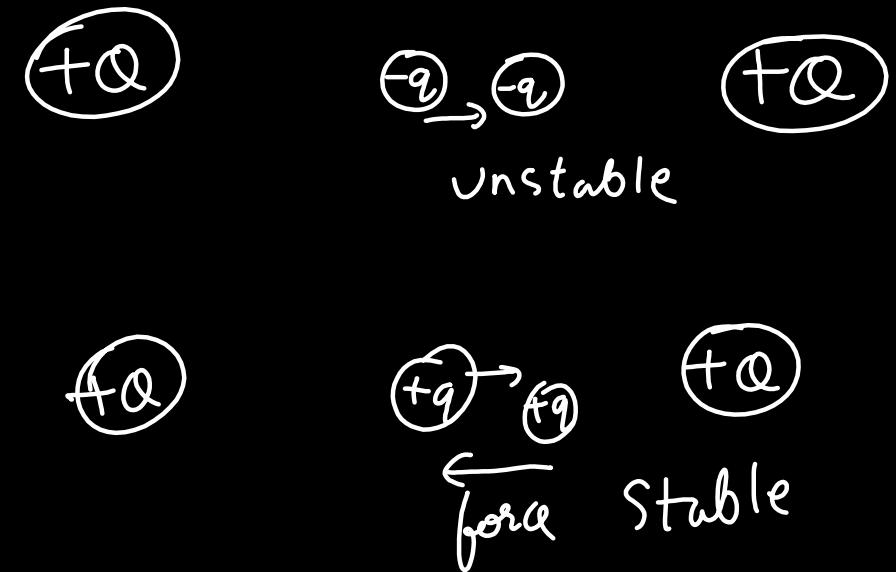
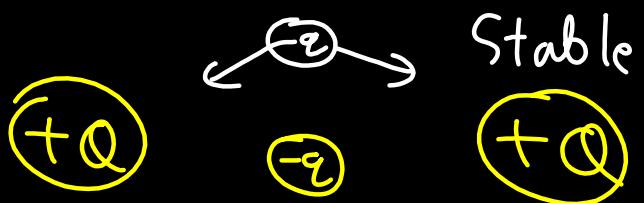
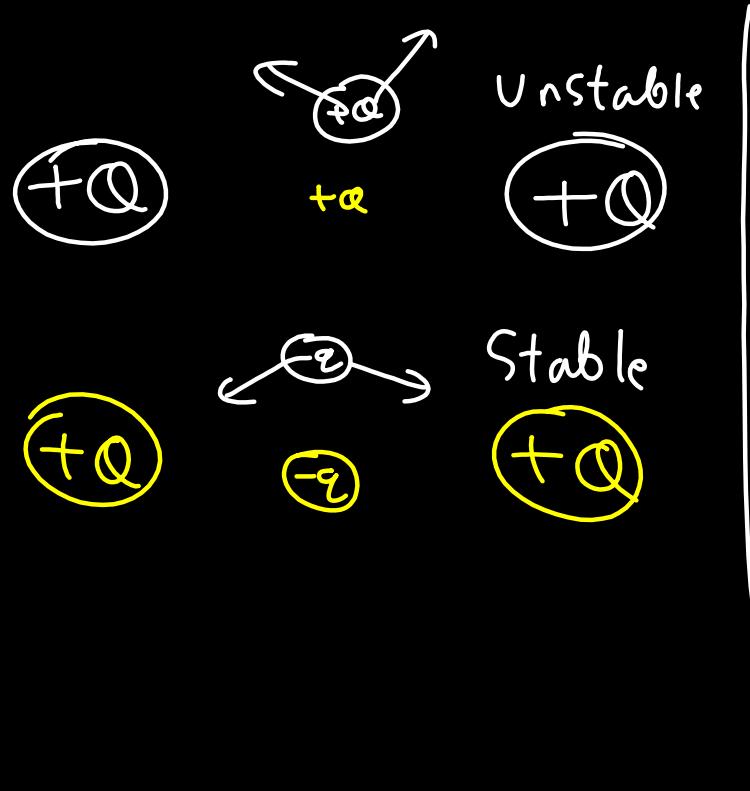
$x \lll a$

$F = -\frac{4KQq}{a^3}$

$$T = 2\pi \sqrt{\frac{m}{4KQq}} a^3$$

$$= 2\pi \sqrt{\frac{ma^2}{4KQq}} \sqrt{\pi \epsilon_0}$$

$$= \sqrt{\frac{4\pi^3 ma^3 \epsilon_0}{Qq}}$$



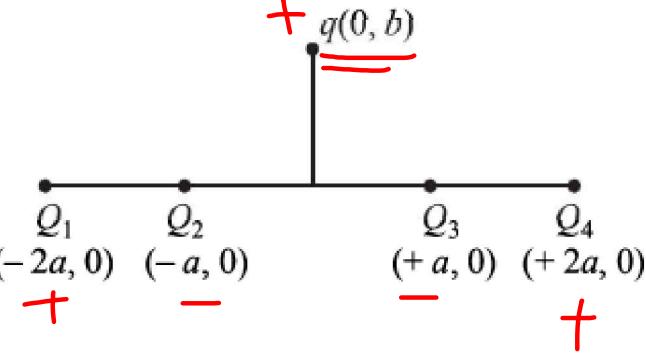
Four charges Q_1, Q_2, Q_3 and Q_4 of same magnitude are fixed along the x axis at $x = -2a, -a, +a$ and $+2a$, respectively. A positive charge q is placed on the positive y axis at a distance $b > 0$. Four options of the signs of these charges are given in List-I. The direction of the forces on the charge q is given in List-II. Match List-I with List-II and select the correct answer using the code given below the lists.

(JEEAdv. 2014)

List - I

- P. Q_1, Q_2, Q_3, Q_4 all positive → 3
- Q. Q_1, Q_2 positive; Q_3, Q_4 negative
- R. Q_1, Q_4 positive; Q_2, Q_3 negative → 4
- S. Q_1, Q_3 positive; Q_2, Q_4 negative

List - II

- 1. $+x$
 - 2. $-x$
 - 3. $+y$
 - 4. $-y$
- 

Codes:

- (a) P-3, Q-1, R-4, S-2
- (b) P-4, Q-2, R-3, S-1
- (c) P-3, Q-1, R-2, S-4
- (d) P-4, Q-2, R-1, S-3



Two balls of same mass and carrying equal charge are hung from a fixed support of length l . At electrostatic equilibrium, assuming that angles made by each thread is small, the separation, x between the balls is proportional to :

- (a) l (b) l^2 (c) $l^{2/3}$ (d) $l^{1/3}$

[Online April 9, 2013]

$$\frac{KQ^2}{x^2 mg} = \frac{x}{2l}$$

x small

$$x^3 = 2 \frac{KQ^2 l}{mg}$$

$$x^3 \propto l$$
$$x \propto l^{1/3}$$



Two identical charged spheres suspended from a common point by two massless strings of length l are initially a distance d ($d \ll l$) apart because of their mutual repulsion. The charge begins to leak from both the spheres at a constant rate. As a result charges approach each other with a velocity v . Then as a function of distance x between them,

- (a) $v \propto x^{-1}$ (b) $v \propto x^{1/2}$ (c) $v \propto x$ (d) $v \propto x^{-1/2}$

[2011]

$$\frac{dq}{dt} = \text{constant}$$

$$v = \frac{dx}{dt}$$

x small

$$\frac{kq^2}{x^2 mg} = \frac{x}{2l} \Rightarrow x^3 = \frac{2lkq^2}{mg}$$

$$x^3 \propto Q^2$$

$$x \propto Q^{2/3} \quad \text{--- (1)}$$

$$x^{3/2} \propto Q$$

$$x \propto Q^{2/3}$$

$$\frac{dx}{dt} \propto \frac{2}{3} Q^{\frac{2}{3}-1} \left(\frac{dQ}{dt} \right)$$

constant

$$v \propto \frac{2}{3} Q^{-1/3}$$

$$v \propto \frac{2}{3} (x^{3/2})^{-1/3}$$

$$v \propto \frac{2}{3} x^{-1/2}$$



3.3. Two small equally charged spheres, each of mass m , are suspended from the same point by silk threads of length l . The distance between the spheres $x \ll l$. Find the rate dq/dt with which

the charge leaks off each sphere if their approach velocity varies as $v = a/\sqrt{x}$, where a is a constant.

I Erodov

$$v = \frac{a}{\sqrt{x}}$$

Preserve.



A charge Q is placed at each of the opposite corners of a square. A charge q is placed at each of the other two corners. If the net electrical force on Q is zero, then Q/q equals: **[2009]**

- (a) -1
- (b) 1
- (c) $-\frac{1}{\sqrt{2}}$
- (d) $-2\sqrt{2}$

H.W.

If a charge q is placed at the centre of the line joining two equal charges Q such that the system is in equilibrium then the value of q is

- (a) $Q/2$ (b) $-Q/2$ (c) $Q/4$ (d) $-Q/4$

[2002]



H.W.

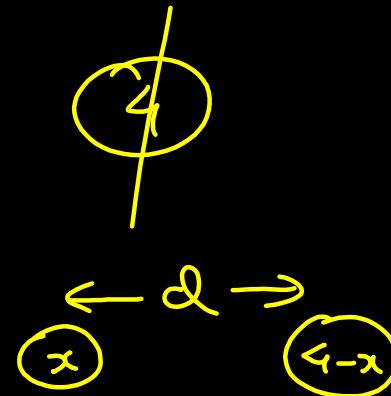
A charge of $4 \mu\text{C}$ is to be divided into two. The distance between the two divided charges is constant. The magnitude of the divided charges so that the force between them is maximum, will be:

- (A) $1 \mu\text{C}$ and $3 \mu\text{C}$ ~~(B) $2 \mu\text{C}$ and $2 \mu\text{C}$~~
 (C) 0 and $4 \mu\text{C}$ (D) $1.5 \mu\text{C}$ and $2.5 \mu\text{C}$

Jee Mains 2022

$$F = \frac{k(x)(4-x)}{d^2}$$

$$F = \frac{k}{d^2} \underbrace{(4x - x^2)}$$



F maxima
1st differentiation

$$\frac{dF}{dx} = \frac{k}{d^2} (4-2x) = 0$$

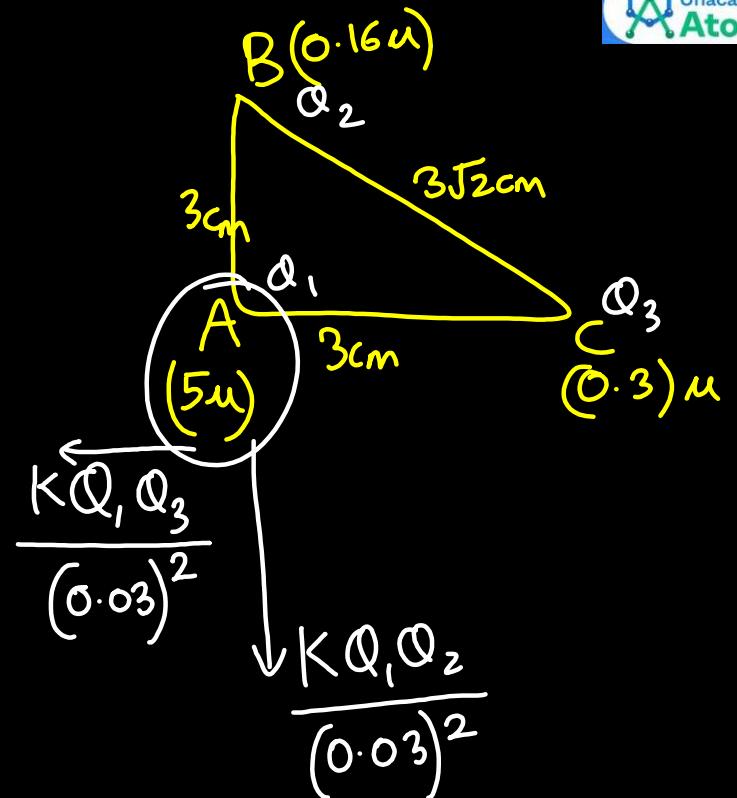
$$4-2x=0 \\ 2=x$$

Three point charges of magnitude $5\mu C$, $0.16\mu C$ and $0.3\mu C$ are located at the vertices A, B, C of a right angled triangle whose sides are $AB = 3\text{cm}$, $BC = 3\sqrt{2}\text{ cm}$ and $CA=3\text{ cm}$ and point A is the right angle corner. Charge at point A experiences _____ N of electrostatic force due to the other two charges.

Jee Mains 2022

$$\begin{array}{c}
 \leftarrow \\
 x \\
 \downarrow \\
 y
 \end{array}$$

$$R = \sqrt{x^2 + y^2}$$



$$\sqrt{\left(\frac{KQ_1Q_3}{(0.03)^2}\right)^2 + \left(\frac{KQ_1Q_2}{(0.03)^2}\right)^2}$$

$$\Rightarrow \frac{KQ_1}{(0.03)^2} (0.34\mu)$$

$$\frac{KQ_1}{(0.03)^2} \sqrt{(0.3\mu)^2 + (0.16\mu)^2}$$

$$\sqrt{0.09 + 0.0256}$$

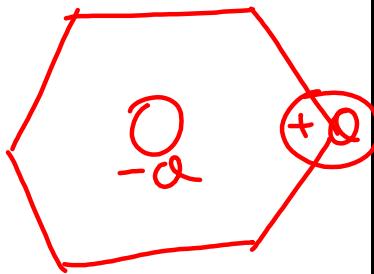
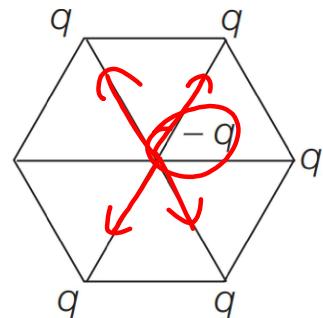
$$\underline{\underline{\sqrt{0.1156}}}$$

$$= \frac{9 \times 10^9 \times 5 \times 10^{-6} \times 0.34 \times 10^{-6}}{9 \times 10^{-4}}$$

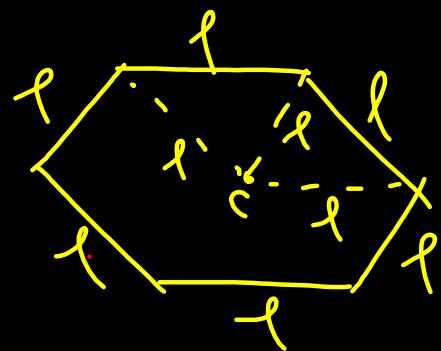
$$= (17)$$

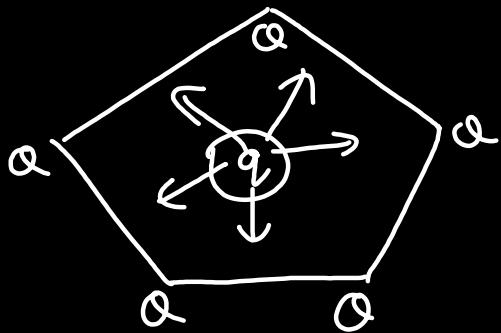
Five point charges, each of value $+q$ coulomb, are placed on five vertices of a regular hexagon of side L metre. The magnitude of the force on the point charge of value $-q$ coulomb placed at the centre of the hexagon isnewton.

(1992)

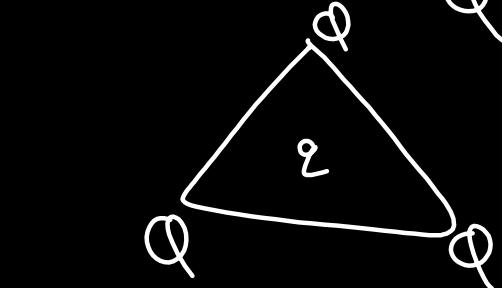
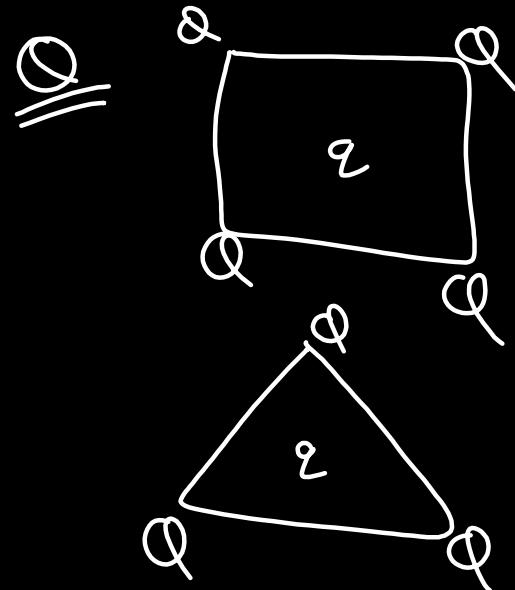
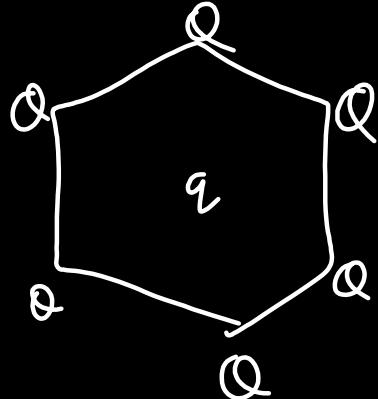


$$\frac{Kq^2}{L^2}$$

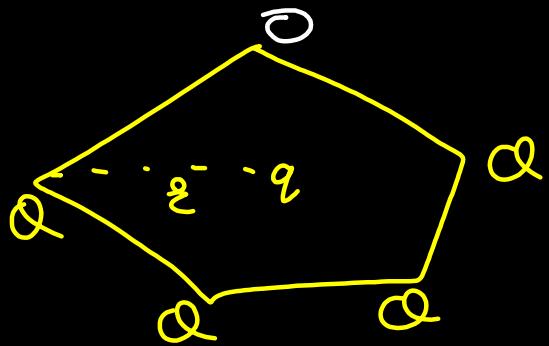




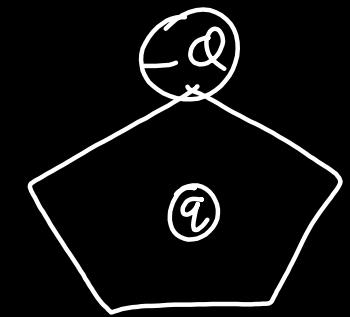
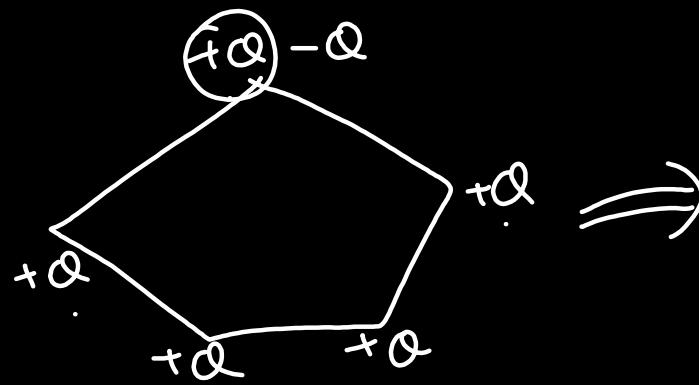
$F_{\text{net}} \text{ on } q = ?$



equal force at
equal angle ($\theta = \frac{360}{n}$)
Resultant = 0



F_{net} on q ??



$$\uparrow \frac{kQq}{r^2}$$

Medium Dependency

$$F = \frac{k \alpha_1 \alpha_2}{r^2}$$

$$N = \frac{(k)(c)(c)}{m^2}$$

$$c \rightarrow [AT]$$

$$m \rightarrow L$$

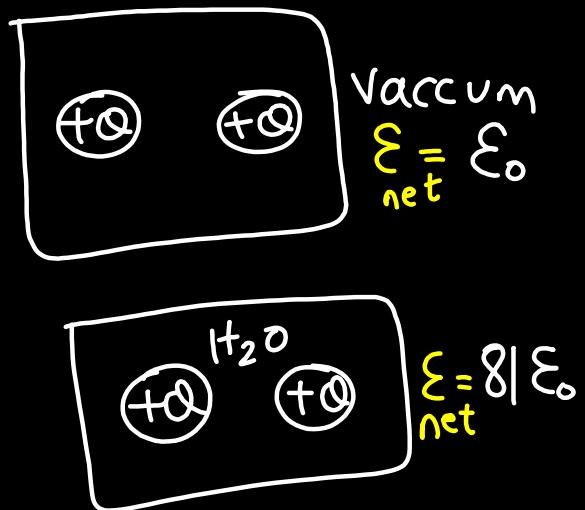
$$N \rightarrow k \frac{m}{s^2} [MLT^{-2}]$$

unit & dimension of $K = 9 \times 10^9 \frac{Nm^2}{C^2} \left[\frac{ML^{-2}L^2}{A^2 T^2} \right] \left[M L^3 T^{-4} A^{-2} \right]$

$$K = \frac{1}{4\pi \epsilon_0}$$

$\epsilon_0 \rightarrow \underline{\text{permittivity}}$ of free space

$$\epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{Nm^2} \left[M^{-1} L^3 A^2 T^{-4} \right]$$

ϵ


Conductor $\epsilon_\infty = \infty$

ϵ_0 vacuum
 medium = $\epsilon_\infty \epsilon_0$
 relative permittivity

$$F_{\text{vacuum}} = \frac{k Q_1 Q_2}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{r^2}$$

$$F_{\text{net medium}} = \frac{1}{4\pi\epsilon_\infty\epsilon_0} \frac{Q_1 Q_2}{r^2} = \frac{F_{\text{vacuum}}}{\epsilon_\infty}$$

$+Q$ $-Q$

medium

$+Q$

$-Q$

$+Q$

$+Q$

δ^-

δ_x

δ_x

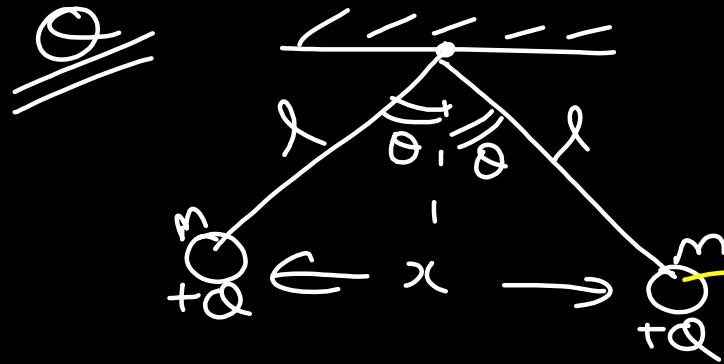
$-Q$

$F_{attraction}$

F_{medium}
in ϵ_0 & c_0

$$F_{net} = F_{charge} - F_{induced \ medium}$$

$$F_{net} = \frac{F_{vacuum}}{\epsilon_r}$$



$$\tan \theta = \frac{F_e}{mg} = \frac{kQ^2}{x^2 mg}$$

(β_{solid})

Now this complete Set Up in liquid of ϵ_r & β_{liquid}

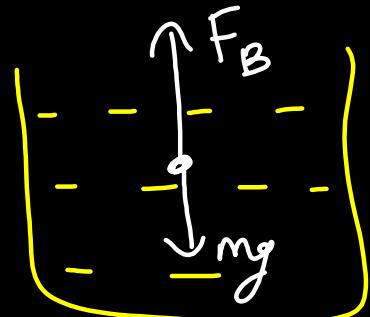
$$F_e = \frac{kQ^2}{x^2}$$

$$F'_e = \frac{kQ^2}{x^2 \epsilon_r}$$

$$\begin{matrix} mg \\ \downarrow \\ mg_{\text{eff}} \end{matrix}$$

$$\tan \theta' = \frac{kQ^2}{x^2 \epsilon_r mg_{\text{eff}}}.$$

$$\tan \theta' = \frac{kQ^2}{x^2 \epsilon_r mg \left(1 - \frac{\beta_L}{\beta_S}\right)}$$



11th fluid statics

$$m g_{\text{eff}} = m g - F_B$$

$$= m g - V \rho_L g$$

$$= m g \left(1 - \frac{V \rho_L}{m} \right)$$

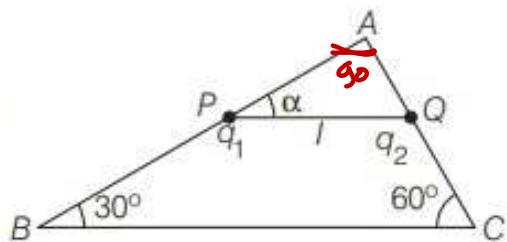
$$= m g \left(1 - \frac{\rho_{\text{liquid}}}{\rho_{\text{solid}}} \right)$$

$$\frac{m}{V} = \rho_{\text{solid}}$$

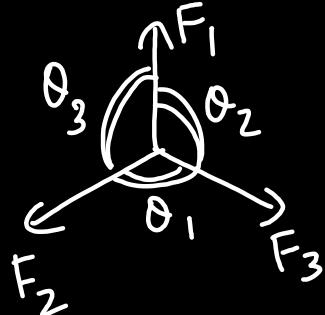
$$m = (\rho_{\text{solid}}) V$$

A rigid insulated wire frame in the form of a right angled triangle ABC , is set in a vertical plane as shown in figure. Two beads of equal masses m each and carrying charges q_1 and q_2 are connected by a cord of length l and can slide without friction on the wires.

Considering the case when the beads are stationary determine
The angle $\alpha = ?$

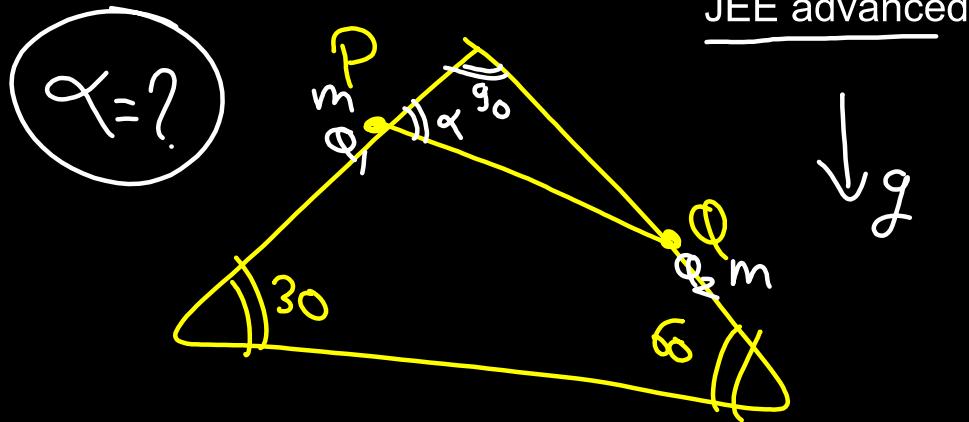


E lectro + NLM + Lami's Theorem



$$\frac{F_1}{\sin \theta_1} = \frac{F_2}{\sin \theta_2} = \frac{F_3}{\sin \theta_3}$$

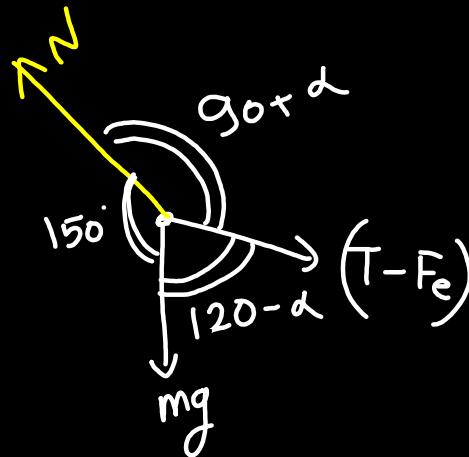
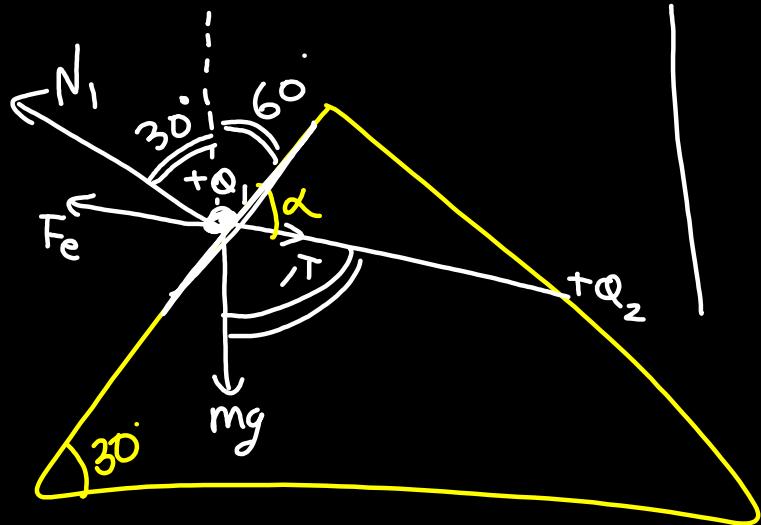
equilibrium



JEE advanced

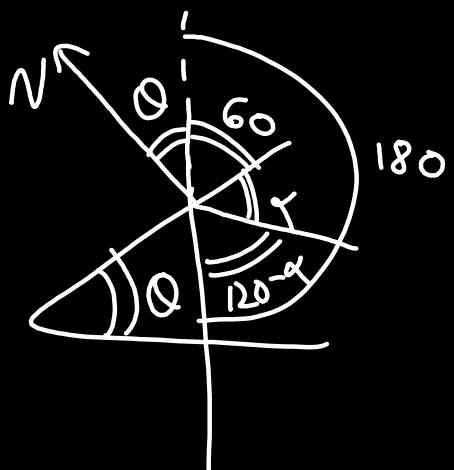
(Challenge)

t.me/ajitlulla



$$360 - (90 + \alpha) - (120 - \alpha) = 150$$

$$\frac{N}{\sin(120 - \alpha)} = \frac{mg}{\sin(90 + \alpha)} = \frac{T - F_e}{\sin(150)}$$



ELECTRIC FIELD

In order to explain 'action at a distance', i.e., 'force without contact' between charges it is assumed that a charge or charge distribution produces a field in space surrounding it.

Field
Concept

Field

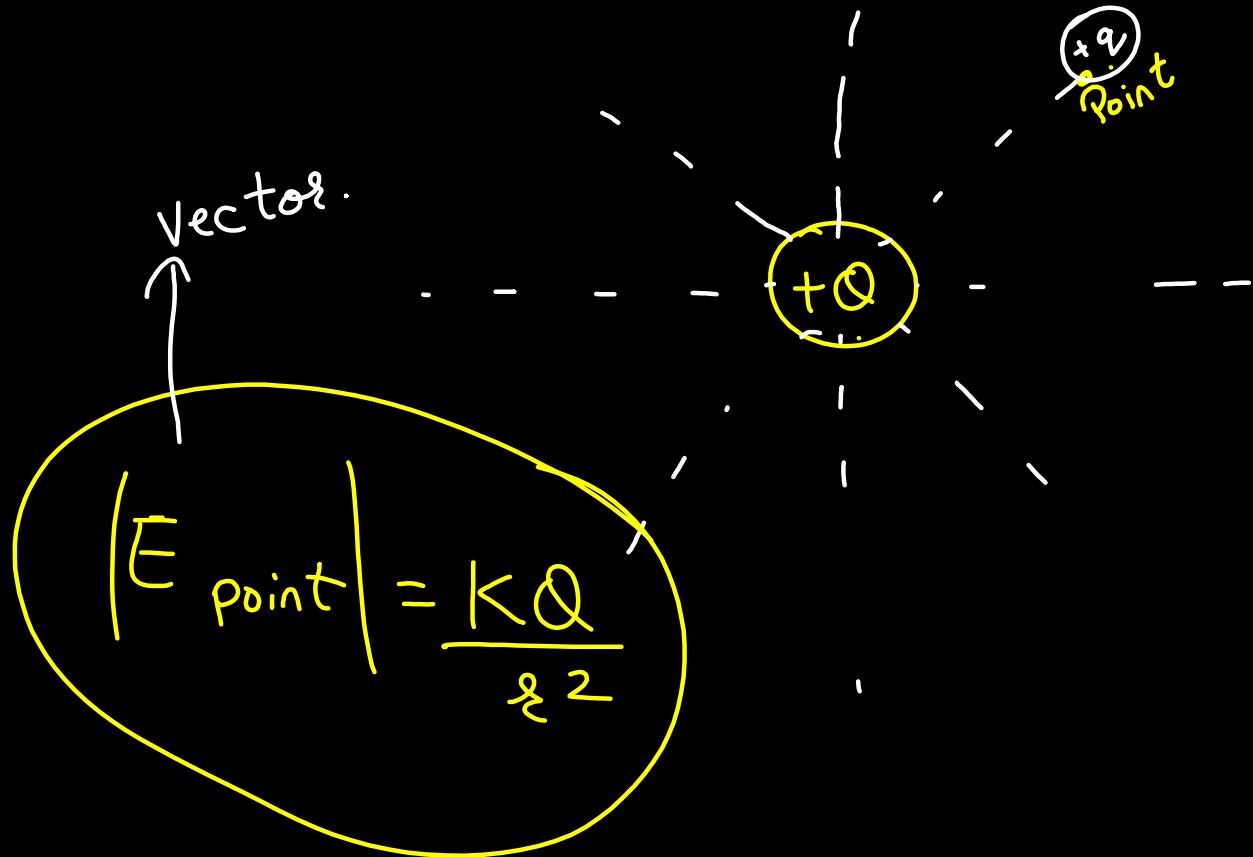
$$E = \frac{\text{force}}{\text{charge}} = \frac{\text{force}}{\text{test charge}}$$

+q_{test}

+q

field

$$\lim_{q_{\text{test}} \rightarrow 0} \frac{KQ q_{\text{test}}}{r^2} = \frac{KQ}{r^2}$$



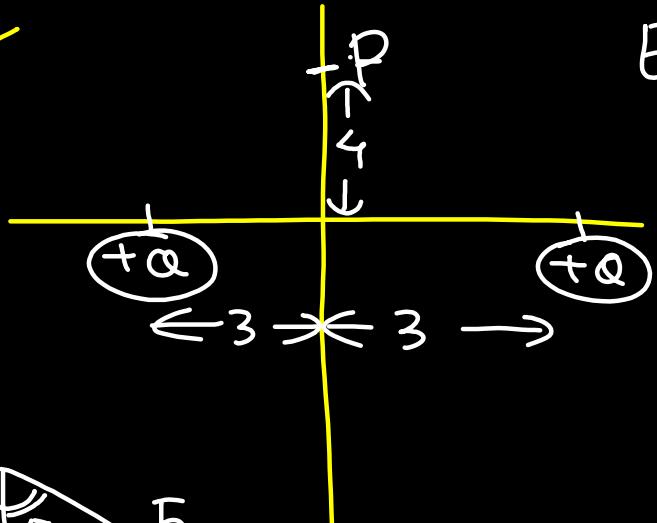
influence
effect
force ability
interaction

$+Q$

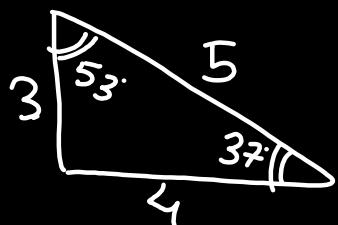
Point
 q_c

$E \rightarrow$ Force की unit charge
experience करेगा at point
 P due to $(+Q)$

Q



E_{net} at P = ?

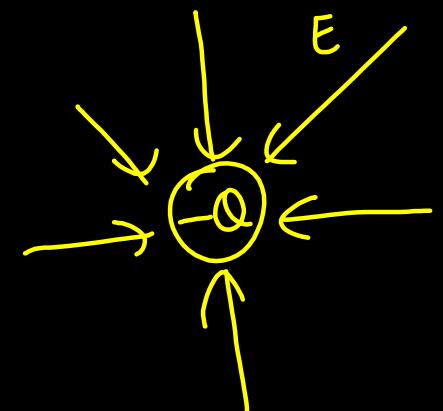
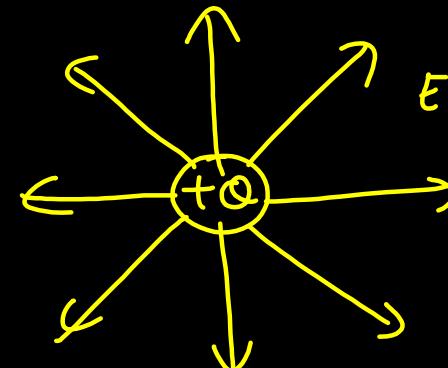


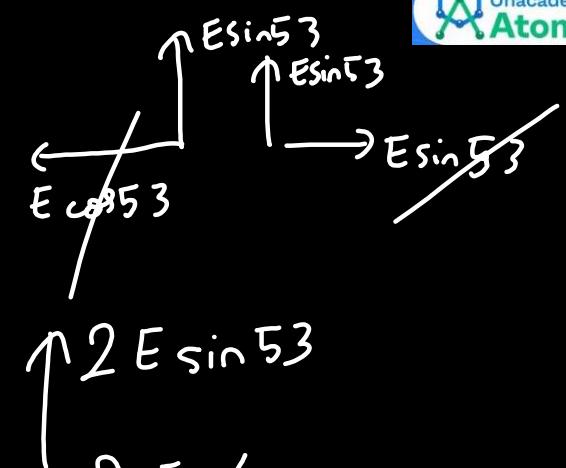
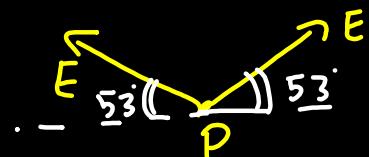
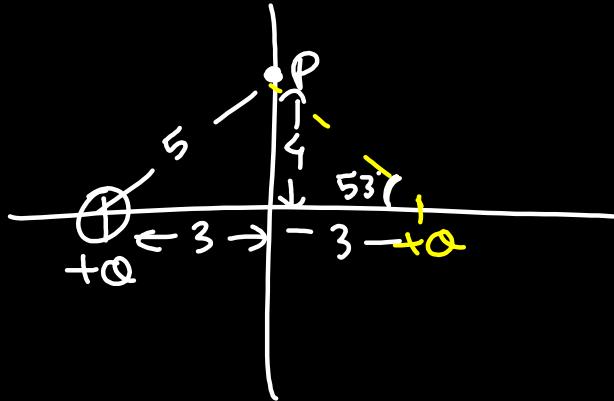
$$\sin 37^\circ = \frac{3}{5}$$

$$\cos 37^\circ = \frac{4}{5}$$

$$\sin 53^\circ = \frac{4}{5}$$

$$\cos 53^\circ = \frac{3}{5}$$





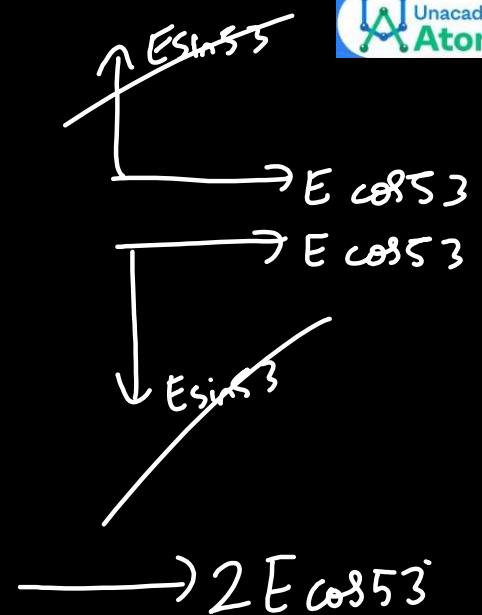
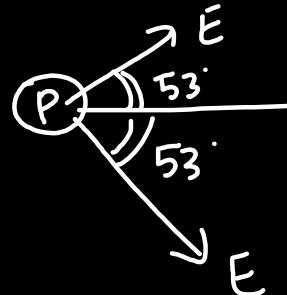
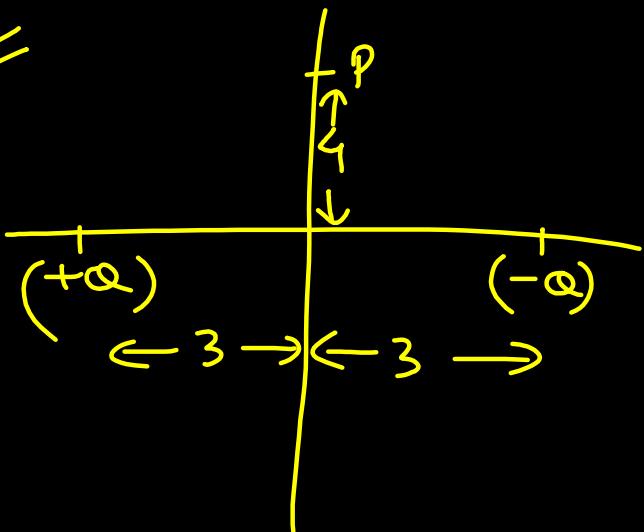
$$E = \frac{kQ}{r^2}$$

$$= \frac{kQ}{5^2}$$

$$= \frac{kQ}{25}$$

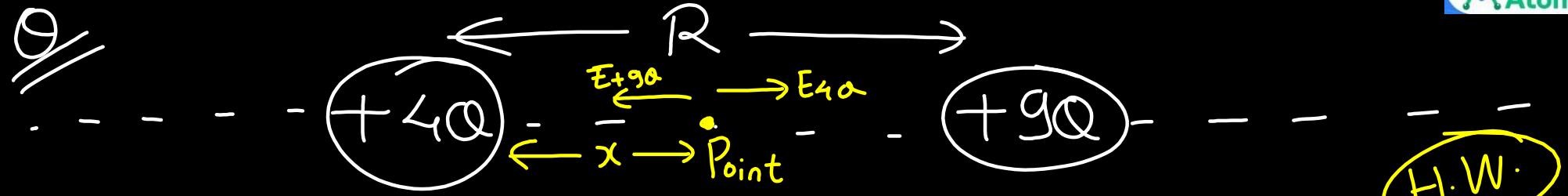
$$2 E \frac{4}{5}$$

$$\frac{8E}{5} = \frac{8kQ}{125}$$



$$= \frac{6 k_0 Q}{125} ?$$

$$2 \frac{k_0}{25} \frac{3}{5}$$



Point on X axis is field = 0 (null point) ??.

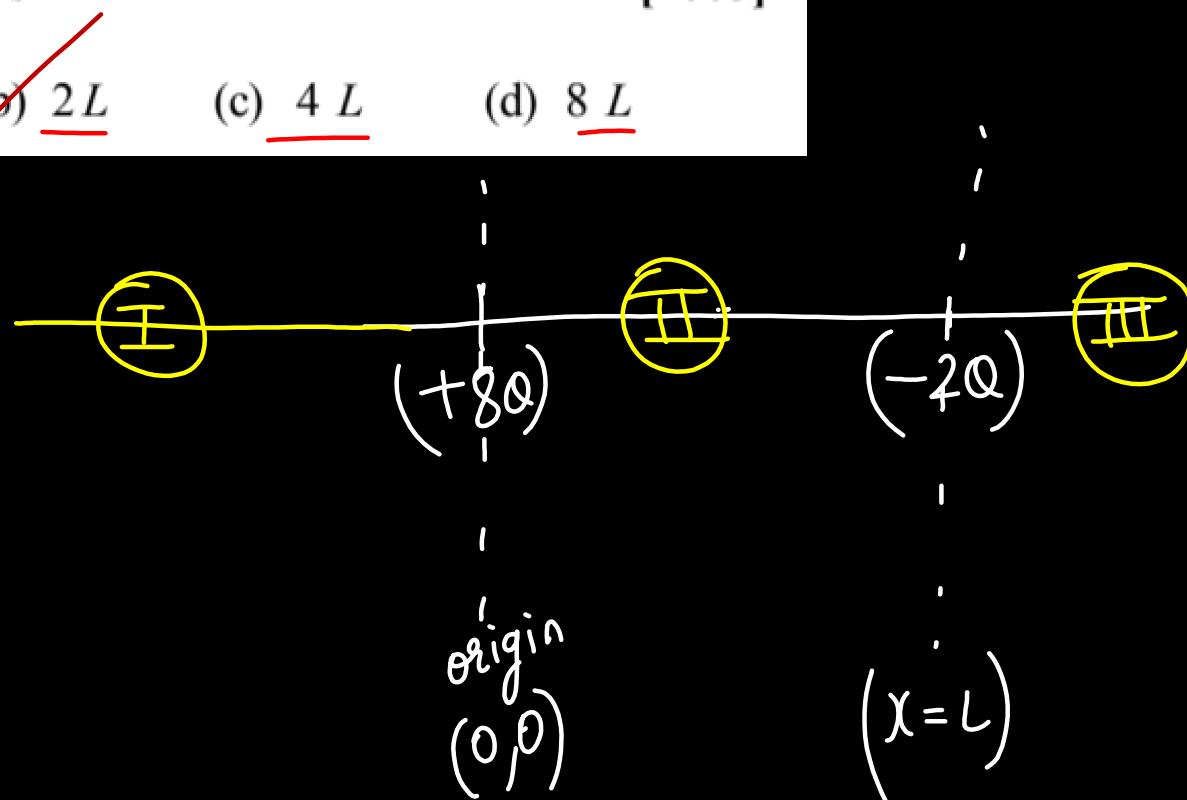
Solve

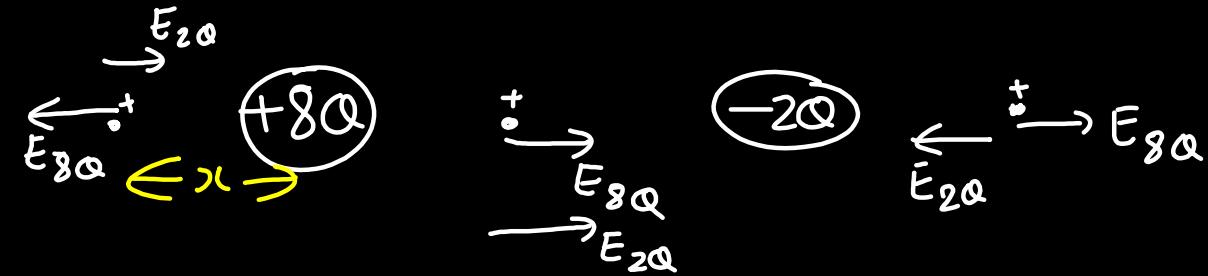
$$\frac{k4Q}{x^2} = \frac{k9Q}{(R-x)^2}$$

Two point charges $+8q$ and $-2q$ are located at $x = 0$ and $x = L$ respectively. The location of a point on the x axis at which the net electric field due to these two point charges is zero is [2005]

- (a) $\frac{L}{4}$
- (b) $2L$
- (c) $4L$
- (d) $8L$

Coordinate nullpoint





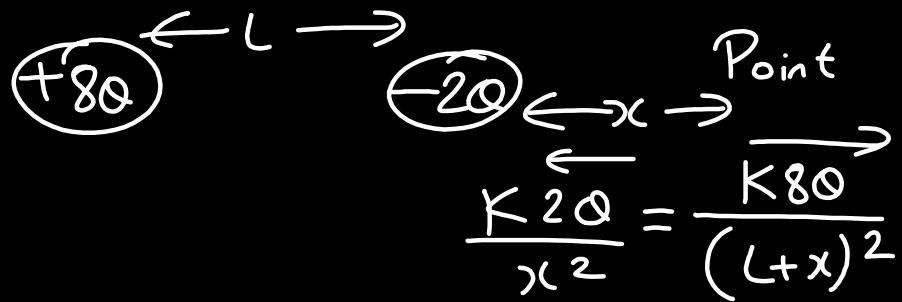
Null Point
Chote Ke Karib

$$\frac{N_u}{Deno}$$

N_u more
 $Deno$ less

$$E = \frac{K8Q}{(R+x)^2}$$

$$E_{20} = \frac{2KQ}{x^2}$$

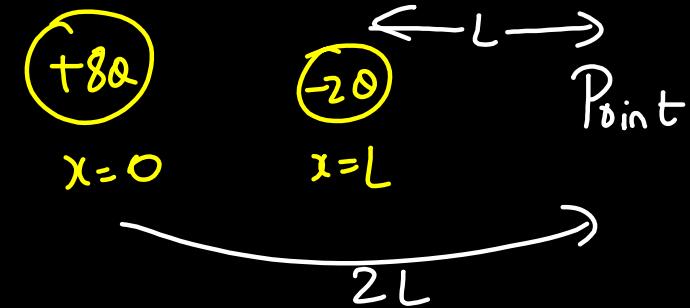


$$\frac{1}{x^2} = \frac{2}{(L+x)^2}$$

$$\frac{1}{x} = \frac{2}{L+x}$$

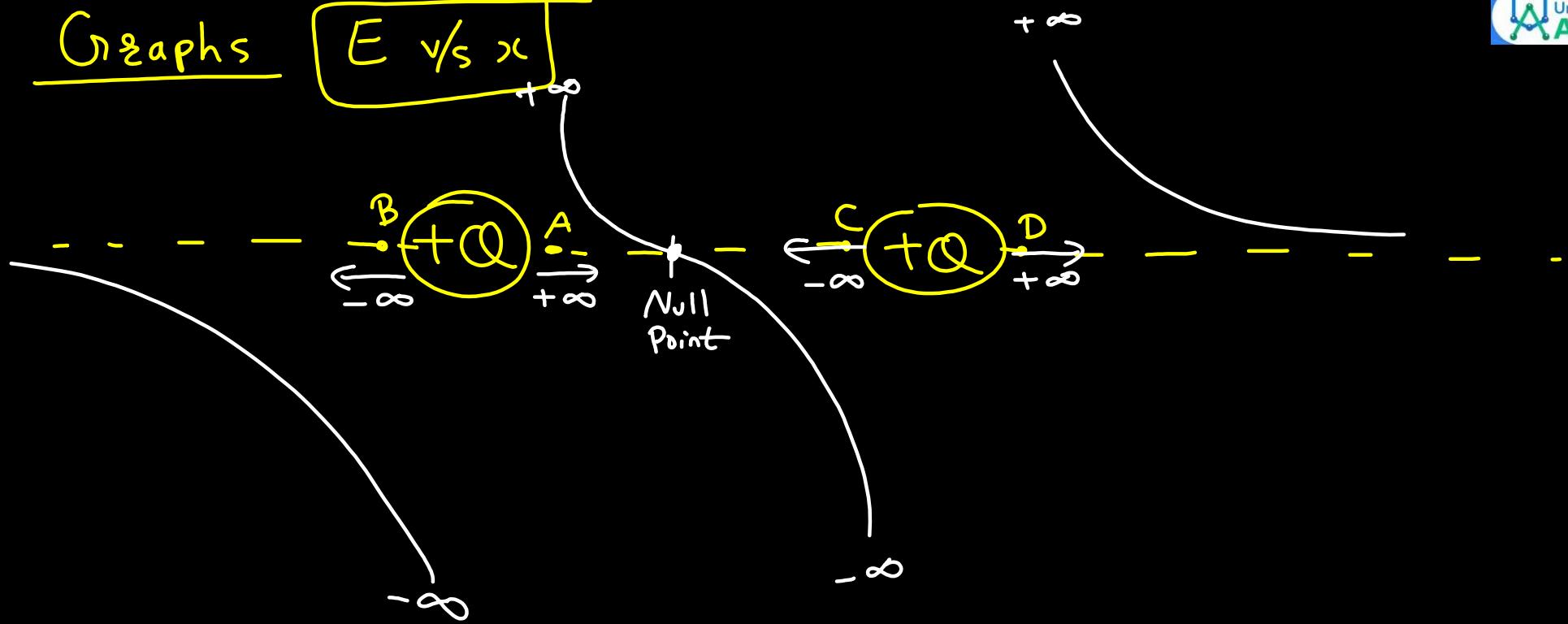
$$L+x = 2x$$

$$L = x$$



Graphs

$E \text{ v/s } x$



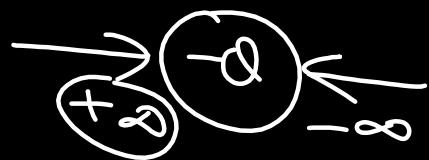
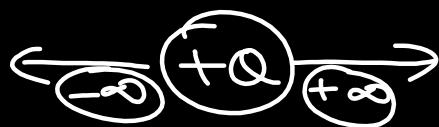
Steps

① charge ke Agal Bgal field Dekho $(+\infty \text{ or } -\infty)$



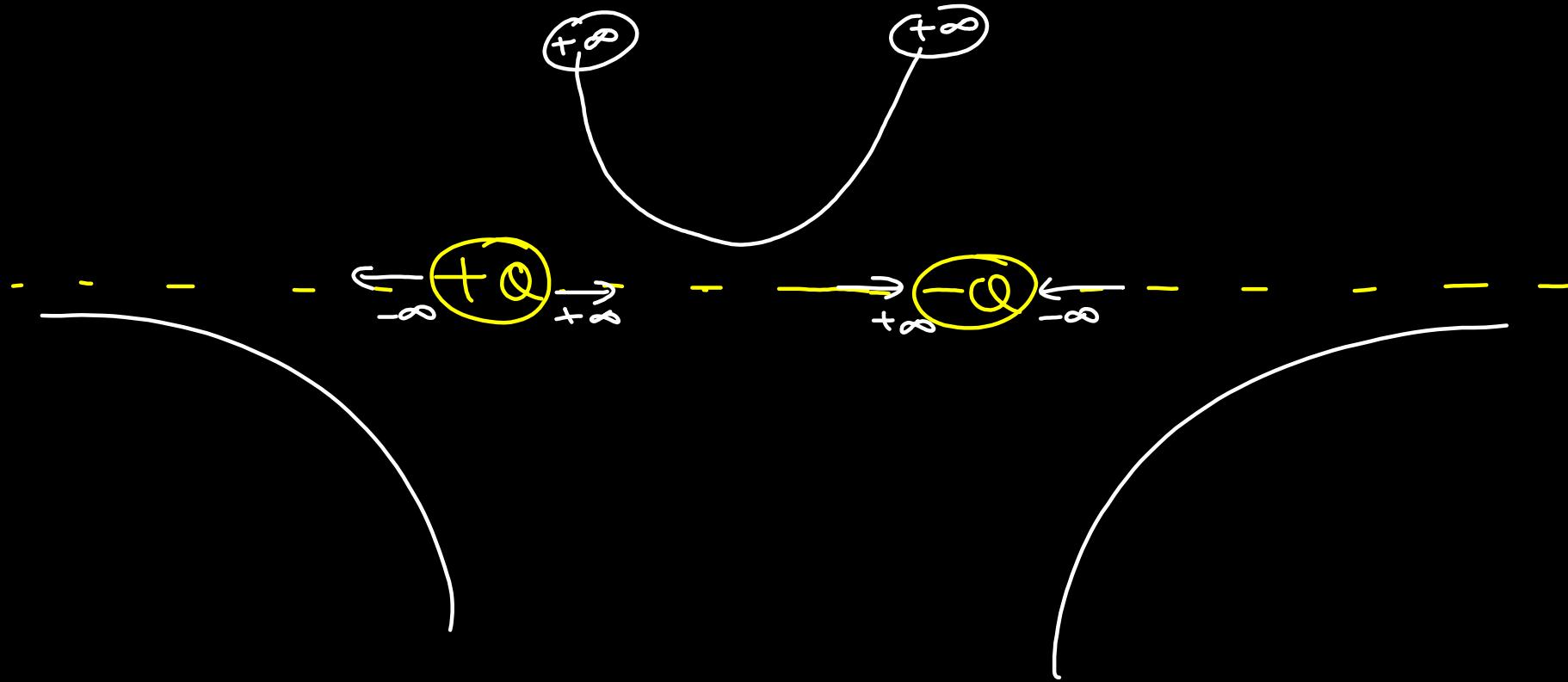
$$E = \frac{kQ}{r^2} \quad r \rightarrow 0$$

$$E \rightarrow \infty$$

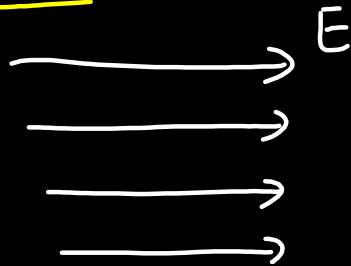


② Find E at ∞ distance (LOL)

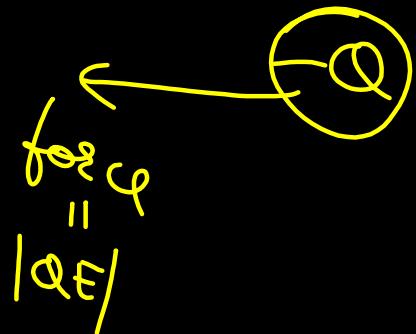
③ Find Null point if Any



Uniform E field



$(+Q) \rightarrow$ force $= QE$
 (constant)



$$E = \frac{\text{force}}{\text{charge}}$$

$$\vec{F} = Q \vec{E}$$



smooth table

Basic



rest

$$U = 0$$

$$a = \frac{QE}{m}$$

$$V = U + at$$

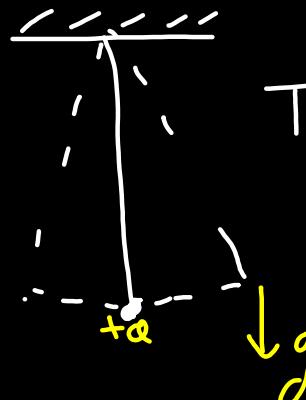
$$S = U t + \frac{1}{2} a t^2$$

$$V^2 = V^2 + 2as$$

$$\rightarrow E$$

$$F = QE$$

$$\downarrow E$$



$$T = 2\pi \sqrt{\frac{l}{g}}$$

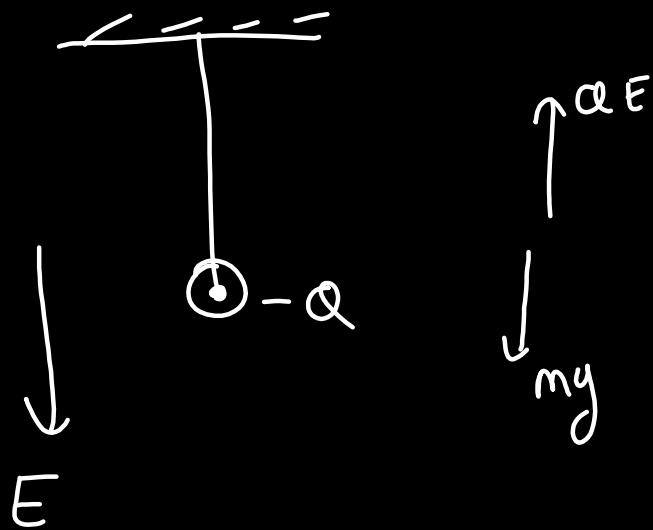
$$\text{new } T' = ?$$

$g \rightarrow g_{\text{effective}}$

$$\downarrow mg + QE$$

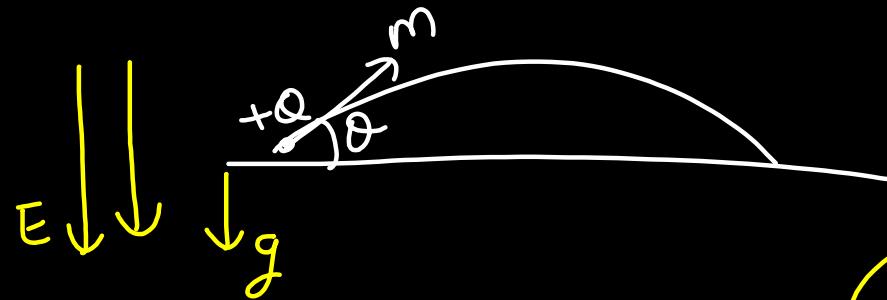
$$\text{net acc} = \frac{mg + QE}{m} = g + \frac{QE}{m}$$

$$T' = 2\pi \sqrt{\frac{l}{g + \frac{QE}{m}}}$$



$$a_{\text{eff}} = \frac{mg - QE}{m} = g - \left(\frac{QE}{m}\right) = g_{\text{eff}}$$

$$T' = 2\pi \sqrt{\frac{l}{g - \frac{QE}{m}}}$$

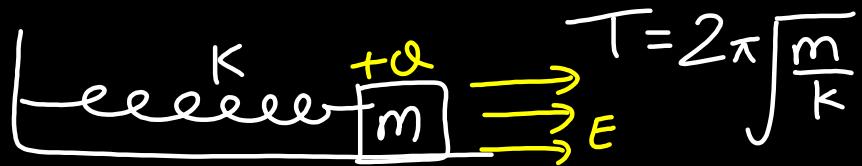


$$g \rightarrow g_{\text{eff}} = g + \left(\frac{QE}{m} \right)$$

$$T = \frac{2v \sin \theta}{g}$$

$$H_{\max} = \frac{v^2 \sin^2 \theta}{2g}$$

$$R = \frac{v^2 \sin 2\theta}{g}$$



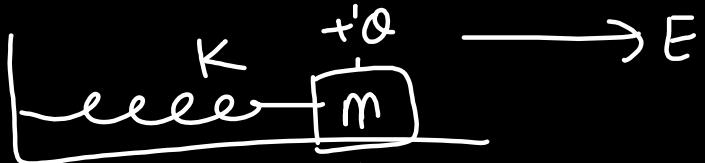
$$T = 2\pi \sqrt{\frac{m}{k}}$$

$$\text{new } T' = 2\pi \sqrt{\frac{m}{k}}$$

after

No effect of constant force F in Spring block.

Or



equilibrium

Fnet balance

Maximum

Elongation

Work energy theorem

$$WD_E = (F)(\text{displ}) \quad (\text{constant})$$

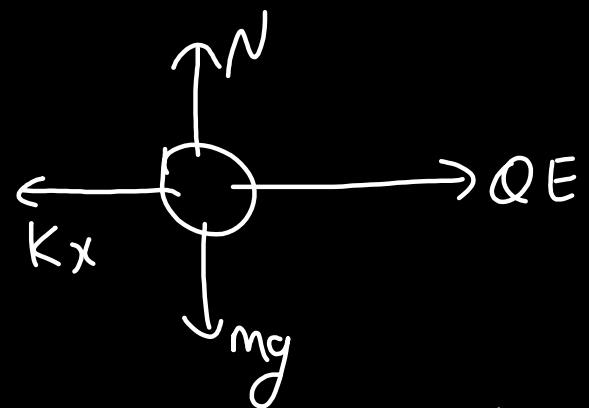
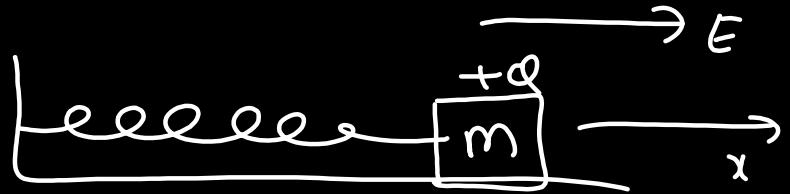
$$WD_{SP} = -\frac{1}{2}Kx^2 \quad (\text{elong or comp})$$

Find

① displ in mean position

② max-possible elongation.

rest natural



$$Kx = QE$$

$$\textcircled{1} \quad x = \frac{QE}{K}$$

$$\begin{aligned} \max WD &= (\text{force})(\text{displ}) \\ &= QEx \end{aligned}$$

$$WD_{sp} = -\frac{1}{2} Kx^2$$

$$WD_{\text{total}} = KE_f - KE_i$$

$$QE_x - \frac{1}{2} Kx^2 = \text{O} - \text{O}$$

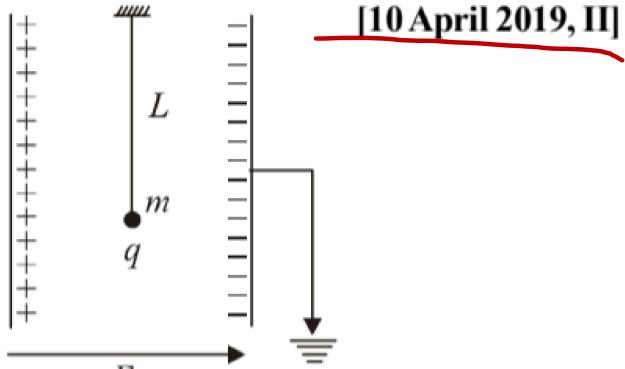
for max elong

$$QE x = \frac{1}{2} Kx^2$$

$$x = \frac{2E_Q}{K}$$

A simple pendulum of length L is placed between the plates of a parallel plate capacitor having electric field E, as shown in figure. Its bob has mass m and charge q. The time period of the pendulum is given by :

[10 April 2019, II]



$$(a) \quad 2\pi \sqrt{\frac{L}{\left(g + \frac{qE}{m}\right)}}$$

$$(b) \quad 2\pi \sqrt{\frac{L}{\sqrt{g^2 - \frac{q^2 E^2}{m^2}}}}$$

$$(c) \quad 2\pi \sqrt{\frac{L}{\left(g - \frac{qE}{m}\right)}}$$

$$\checkmark (d) \quad 2\pi \sqrt{\frac{L}{\sqrt{g^2 + \left(\frac{qE}{m}\right)^2}}}$$

$$+q \longrightarrow qE$$

$$\downarrow mg$$

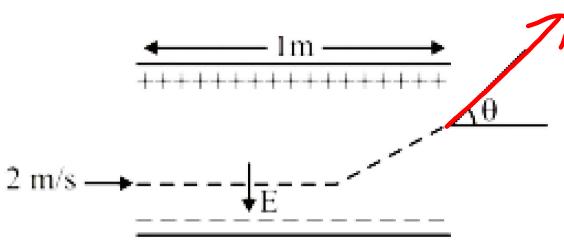
$$\text{net force} = \sqrt{(mg)^2 + (qE)^2}$$

$$= m \sqrt{g^2 + \left(\frac{qE}{m}\right)^2}$$

$$T' = 2\pi \sqrt{\frac{l}{g_{\text{eff}}}}$$

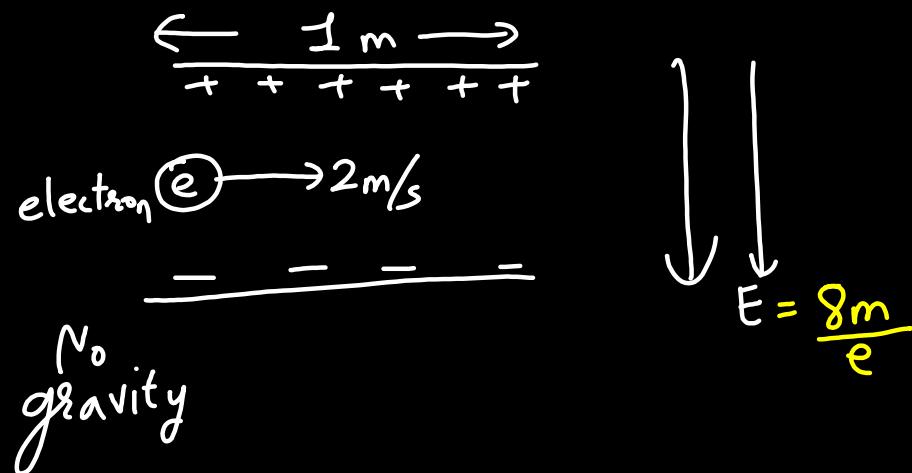
$$g_{\text{eff}} = \frac{\text{net F}}{m} = \sqrt{g^2 + \left(\frac{qE}{m}\right)^2}$$

A uniform electric field $E = (8m/e) \text{ V/m}$ is created between two parallel plates of length 1m as shown in figure, (where m = mass of electron and e = charge of electron). An electron enters the field symmetrically between the plates with a speed of 2m/s. The angle of the deviation (θ) of the path of the electron as it comes out of the field will be



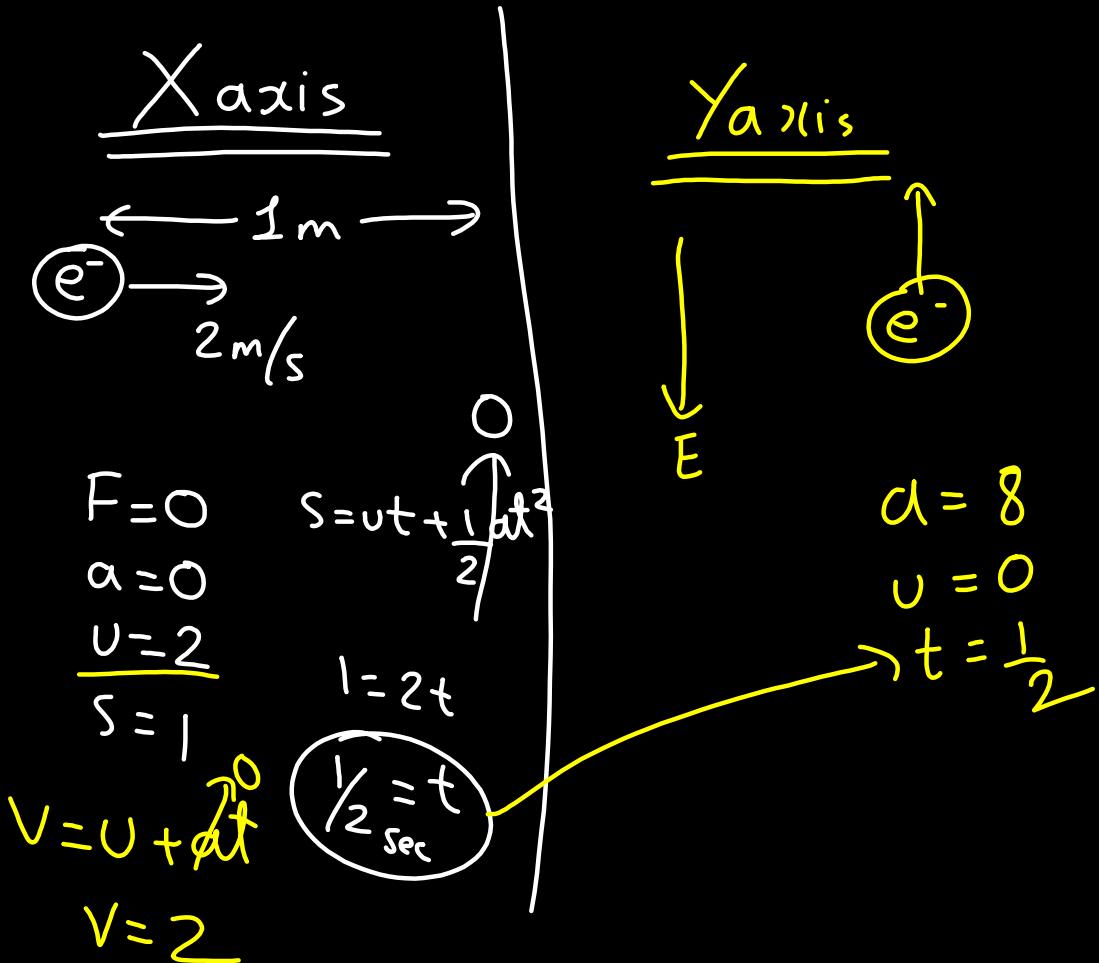
- (A) $\tan^{-1} (4)$ (B) $\tan^{-1} (2)$
 (C) $\tan^{-1} \left(\frac{1}{3} \right)$ (D) $\tan^{-1} (3)$

JEE Mains 2022



$$\begin{aligned} V_f &= V_x \hat{i} + V_y \hat{j} \\ &= 2 \hat{i} + 4 \hat{j} \end{aligned}$$

$$\begin{aligned} \tan \theta &= \frac{Y}{X} \\ &= \frac{4}{2} \\ \theta &= \tan^{-1}(2) \end{aligned}$$



$$F = QE = (e) \left(\frac{8m}{e} \right) = 8m$$

$$acc = \frac{F}{m} = 8$$

$$a = 8$$

$$U = 0$$

$$t = \frac{1}{2}$$

$$V = U + at$$

$$V_y = 4$$

Figure 1.33 shows tracks of three charged particles in a uniform electrostatic field. Give the signs of the three charges. Which particle has the highest charge to mass ratio?

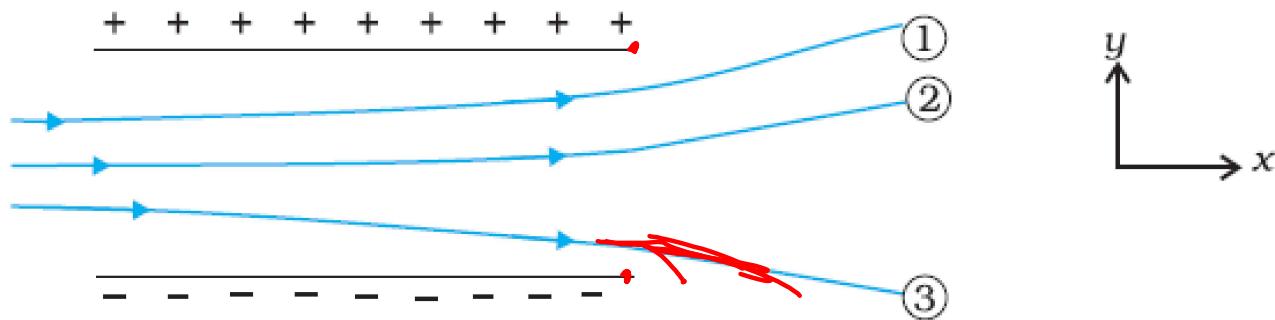
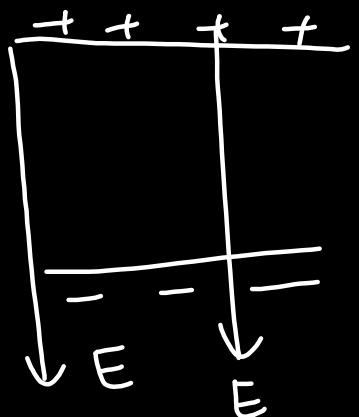
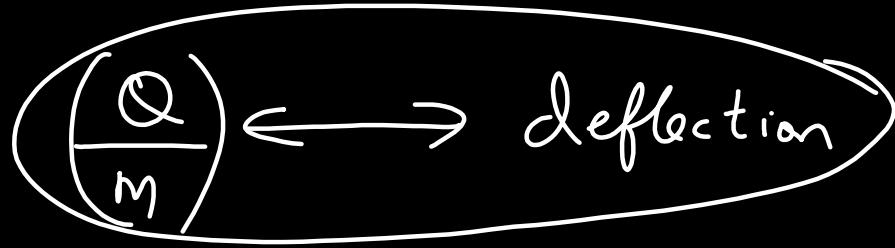


FIGURE 1.33



highest $\left| \frac{Q}{m} \right|$ ratio ^{specific charge}

③ more

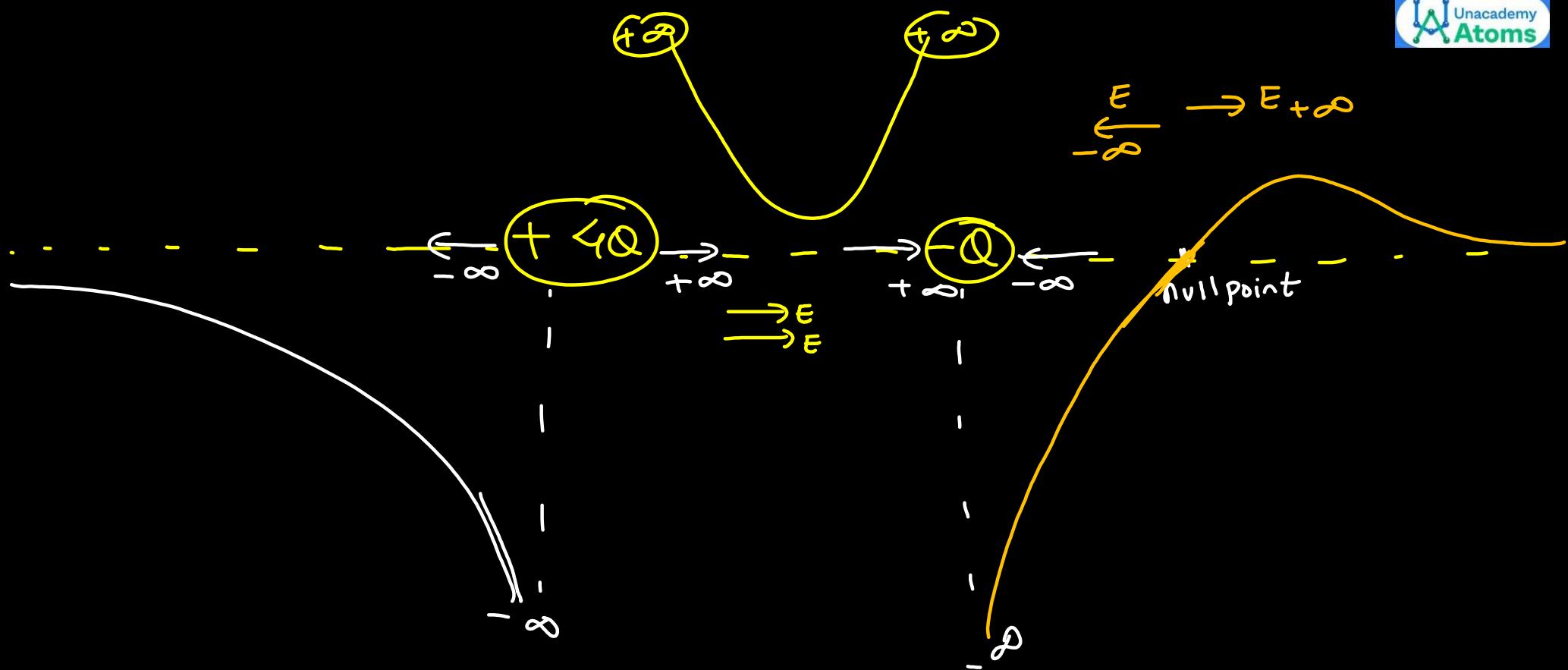


$Q \uparrow \uparrow$ force $\uparrow \uparrow$
deflection more.

$$\frac{N_u}{D_{en}}$$

mass $\uparrow \uparrow$ deflection less

$N_u \uparrow \uparrow D_{en} \downarrow \downarrow$ ratio value more



A particle of charge q and mass m is subjected to an electric field $E = E_0(1 - ax^2)$ in the x -direction, where a and E_0 are constants. Initially the particle was at rest at $x = 0$. Other than the initial position the kinetic energy of the particle becomes zero when the distance of the particle from the origin is :

[Sep. 04, 2020 (II)]

- (a) a (b) $\sqrt{\frac{2}{a}}$ (c) $\sqrt{\frac{3}{a}}$ (d) $\sqrt{\frac{1}{a}}$

$$F = qE(-ax^2)$$

$$acc = \frac{qE}{m}(-ax^2)$$

$$x=0 \quad v=0$$

$$v_f = 0$$

Variable force
Variable acc

$$a = \frac{dv}{dt} = \frac{v dv}{ds}$$

$$v = \frac{dx}{dt}$$

$$acc = \frac{QE}{m} (1 - \alpha x^2)$$

$$x = \frac{\alpha x^3}{3}$$

$$\sqrt{\frac{dv}{dx}} = \dots$$

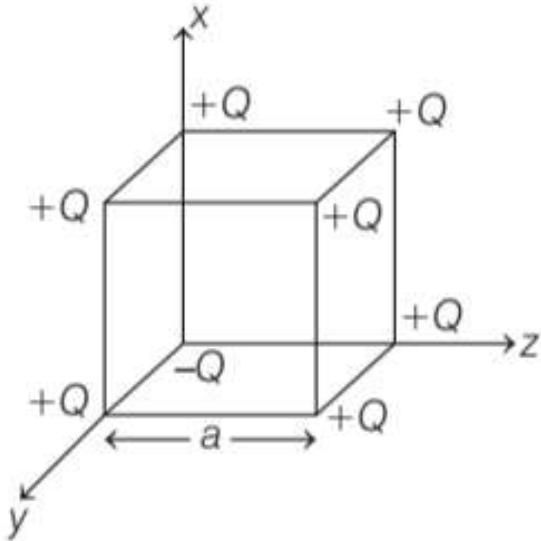
$$\int v dv = \frac{QE}{m} \left(\int dx - \frac{\alpha x^2}{3} dx \right)$$

$$\begin{aligned} \frac{v^2}{2} &= \frac{QE}{m} \left(x - \frac{\alpha x^3}{3} \right) \\ \sqrt{\frac{v^2}{2}} &= \sqrt{x - \frac{\alpha x^3}{3}} \end{aligned}$$

$$\sqrt{\frac{v^2}{2}} = \sqrt{x - \frac{\alpha x^3}{3}}$$

A cube of side a has point charges $+Q$ located at each of its vertices except at the origin, where the charge is $-Q$. The electric field at the centre of cube is

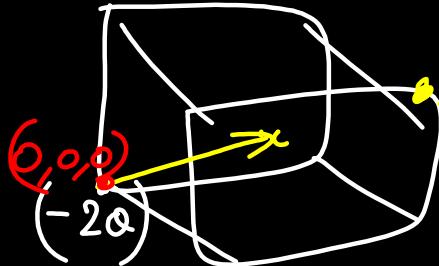
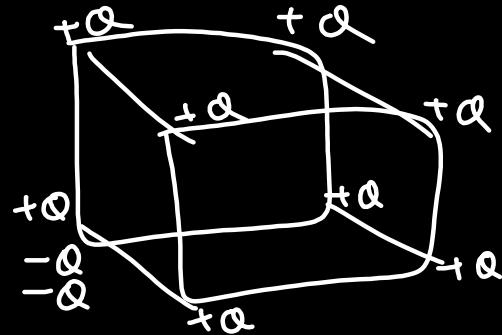
- a. $\frac{-Q}{3\sqrt{3}\pi\epsilon_0 a^2} (\hat{x} + \hat{y} + \hat{z})$
- b. $\frac{Q}{3\sqrt{3}\pi\epsilon_0 a^2} (\hat{x} + \hat{y} + \hat{z})$
- c. $\frac{-2Q}{3\sqrt{3}\pi\epsilon_0 a^2} (\hat{x} + \hat{y} + \hat{z})$
- d. $\frac{2Q}{3\sqrt{3}\pi\epsilon_0 a^2} (\hat{x} + \hat{y} + \hat{z})$



2021
Mains.

$$|\vec{E}| = \frac{kQ}{r^2}$$

$\vec{E} = \frac{kQ}{r^3} \hat{r}$
 on Point \vec{E} Point w.r.t. Charge

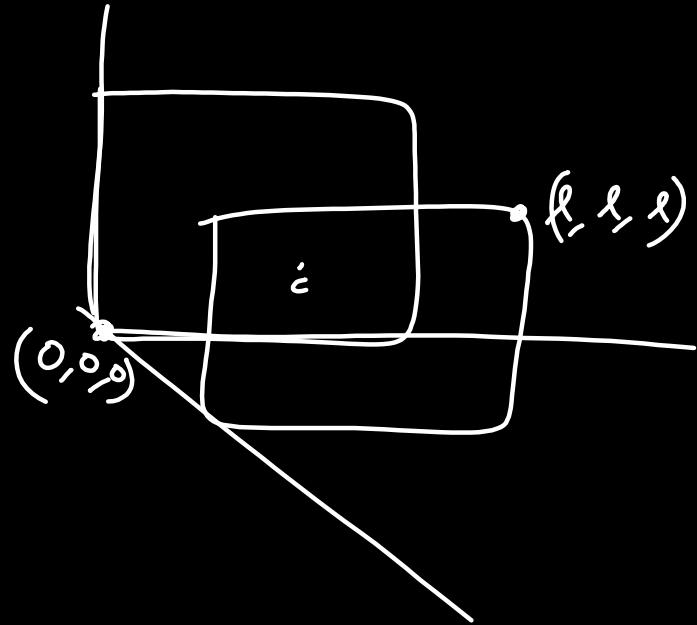


$$\vec{E}_{atC} = \frac{kQ}{\epsilon^3} \hat{e}_{center} \text{charge}$$

$$= \frac{k(-2Q)}{\left(\frac{l\sqrt{3}}{2}\right)^3} \left(\frac{\hat{i} + \hat{j} + \hat{k}}{2} \right)$$

$$= \frac{1}{4\pi\epsilon_0} \frac{-2Q}{3\sqrt{3}l^3} \hat{e}^2 \left(\frac{l}{2} \right) \left(\hat{i} + \hat{j} + \hat{k} \right)$$

$$|\epsilon| = \frac{l\sqrt{3}}{2}$$

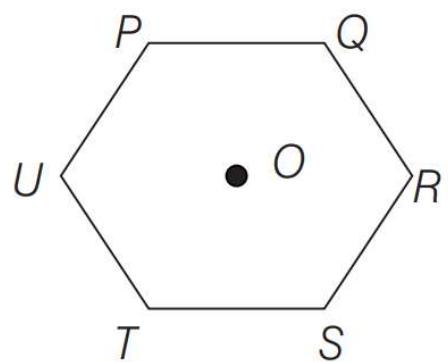


$$C = \left(\frac{l}{2}, \frac{l}{2}, \frac{l}{2} \right)$$

$$\vec{r}_{\text{center charge}} = \vec{r}_{\text{center}} - \vec{r}_{\text{charge}}$$

Six charges, three positive and three negative of equal magnitude are to be placed at the vertices of a regular hexagon such that the electric field at O is double the electric field when only one positive charge of same magnitude is placed at R . Which of the following arrangements of charge is possible for, P , Q , R , S , T and U respectively? (2004)

- | | |
|----------------------|----------------------|
| (a) +, -, +, -, -, + | (b) +, -, +, -, +, - |
| (c) +, +, -, +, -, - | (d) -, +, +, -, +, - |



Vector form

$$F = \frac{k Q_1 Q_2}{r^2}$$

$\vec{F} = \frac{k Q_1 Q_2}{r^2} \hat{r}$ Position of 1 w.r.t 2
 on charge 1 due to 2

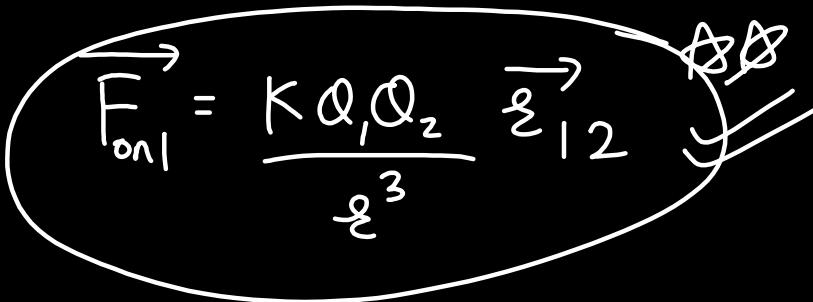
$$\vec{F}_{\text{on } 1} = \frac{k Q_1 Q_2}{r^2} \hat{r} \text{ of 1 w.r.t 2}$$

vector = (magnitude) (unit vector)

$$\hat{\vec{r}} = \frac{\vec{r}}{|\vec{r}|}$$

$$\hat{\vec{r}} = \frac{\vec{r}}{|\vec{r}|}$$

$$F = \frac{k q_1 q_2}{r^2} \hat{\vec{r}}$$

$$\vec{F}_{on1} = \frac{k q_1 q_2}{r^3} \hat{\vec{r}}_{12}$$


$\text{Q} \equiv$ Force vector on $Q(1, -1, 2)$ by $\frac{Q}{2}(0, 0, -2)$

$$\vec{F}_{\text{on } A} = \frac{k Q_1 Q_2}{r^3} \vec{\epsilon}_{AB}$$

$$\vec{\epsilon}_{AB} = \vec{\epsilon}_A - \vec{\epsilon}_B$$

$$= \frac{k Q \left(\frac{Q}{2}\right) (1\hat{i} - 1\hat{j} + 2\hat{k})}{(\sqrt{18})^3}$$

$$\vec{\epsilon}_{AB} = (1\hat{i} - 1\hat{j} + 2\hat{k}) - (-2\hat{k})$$

$$= 1\hat{i} - 1\hat{j} + 4\hat{k}$$

$$|\vec{\epsilon}| = \sqrt{1^2 + 1^2 + 4^2}$$

t.me/ajitlulla



Break Time | 10 min Break

Resume at
9:25 pm

Gauss Law Continue

Continuous Charge Distribution

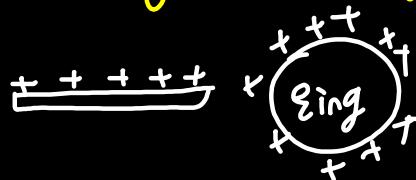
Basics



$\lambda \rightarrow$ linear charge density $= \frac{Q}{l} = \frac{\text{charge}}{\text{length}}$.

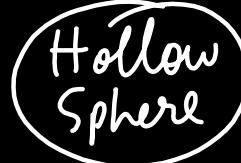
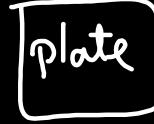
1D \rightarrow length

$$\lambda = \frac{Q}{l}$$



2D \rightarrow area

$$\sigma = \frac{Q}{\text{area}}$$



3D \rightarrow volume

$$\rho = \frac{Q}{\text{Vol}}$$



Differential Charge

$$\lambda = \frac{dQ}{dx}$$

$$dQ = \lambda dx$$

$$\sigma = \frac{dQ}{dA}$$

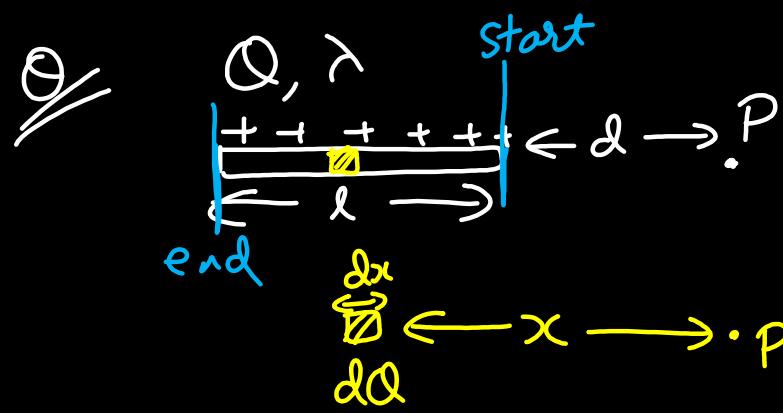
$$dQ = \sigma dA$$

$$S = \frac{dQ}{dV}$$

$$dQ = S dV$$

$$\text{area} = (\text{length})(\text{thickness})$$

$$\text{Volume} = (\text{area})(\text{thick})$$



$$\lambda = \frac{Q}{l}$$

$$= K\lambda \left(\frac{x^{-1}}{-1} \right)^{d+l}$$

$$= -K\lambda \left[\frac{1}{x} \right]_d^{d+l}$$

$$dQ = \lambda dx$$

$$E = \frac{K(dQ)}{x^2}$$

$$= \int \frac{K\lambda dx}{x^2}$$

$$= K\lambda \int x^{-2} dx$$

$$= -K\lambda \left[\frac{1}{d+l} - \frac{1}{d} \right]$$

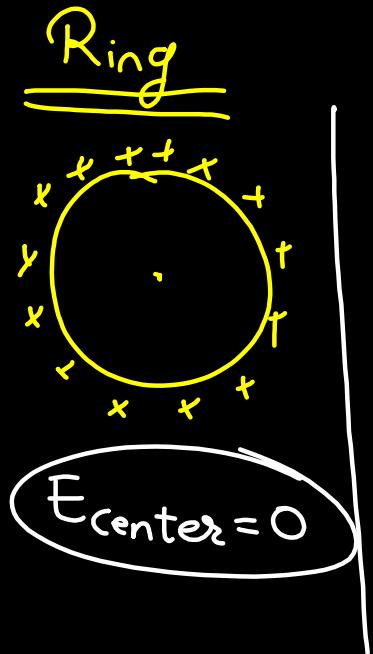
$$= -K\lambda \left[\frac{d-d-l}{d(d+l)} \right]$$

$$= \frac{K\lambda l}{d(d+l)}$$

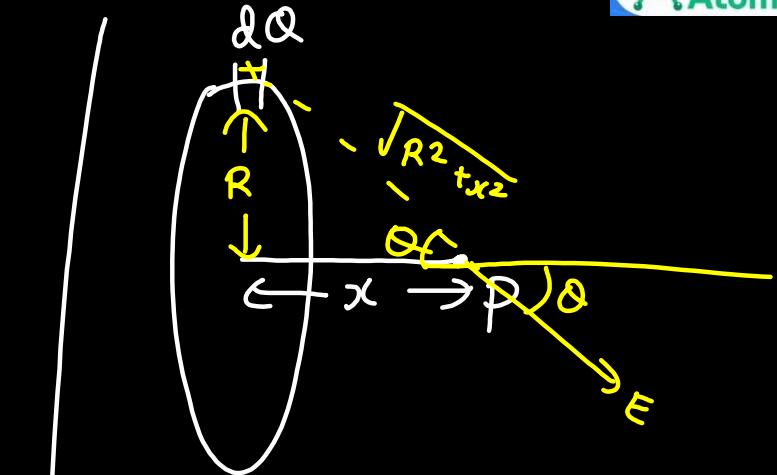
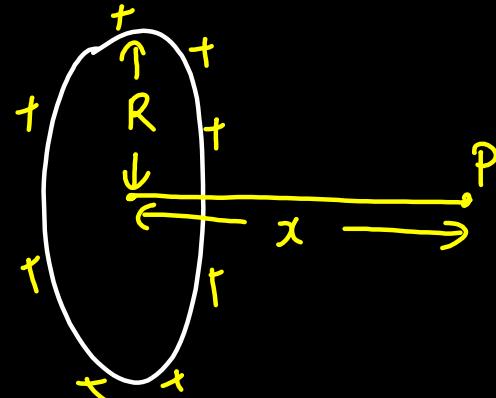


?

$$\begin{aligned}\text{Force} &= (q) \text{ field} \\ &= q \frac{k \lambda l}{d(d+l)}\end{aligned}$$



On Axis of Ring

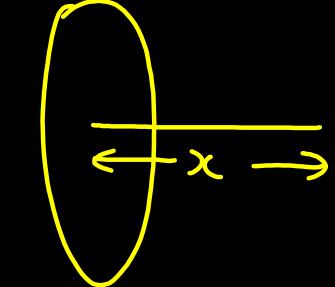


$$E_x = E \cos \theta$$

$$= K \frac{(Q dQ)}{\left(\sqrt{R^2 + x^2}\right)^2} \frac{x}{\sqrt{R^2 + x^2}}$$

$$= \frac{K x}{(x^2 + R^2)^{3/2}} \int dQ$$

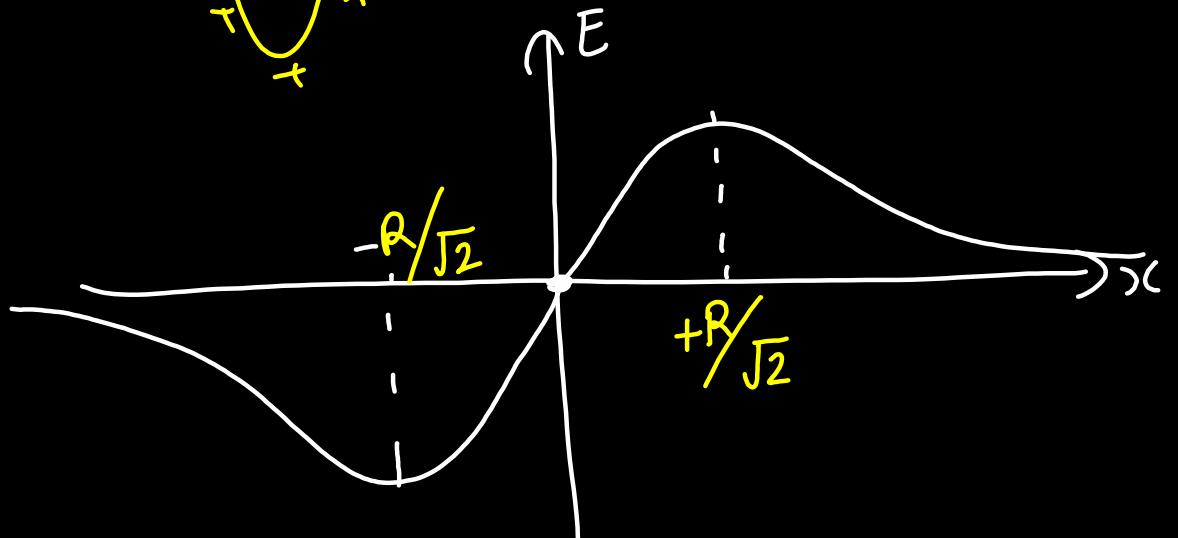
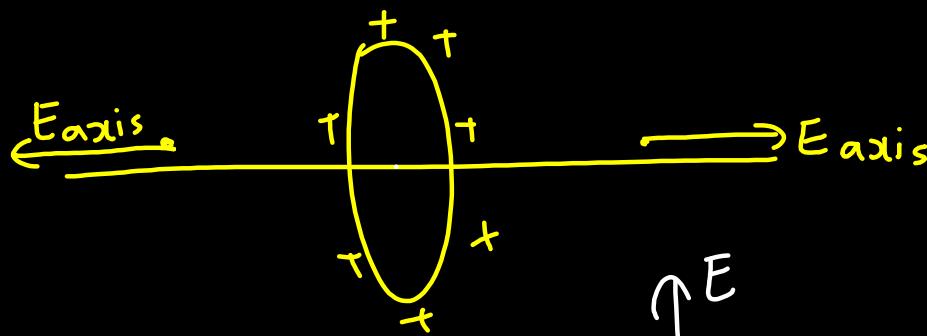
$$= \frac{K Q x}{(x^2 + R^2)^{3/2}}$$



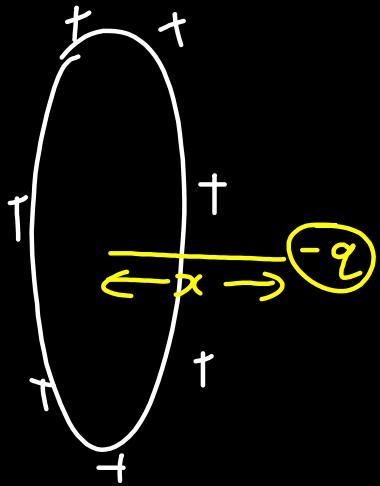
$$E = \frac{kQx}{(x^2 + R^2)^{3/2}}$$

max

$$\frac{dE}{dx} = 0$$



SHM



$x \ll \ll R$

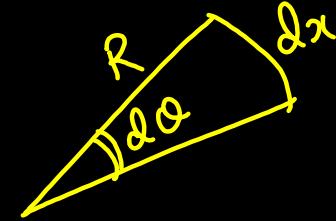
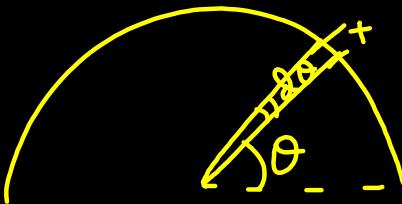
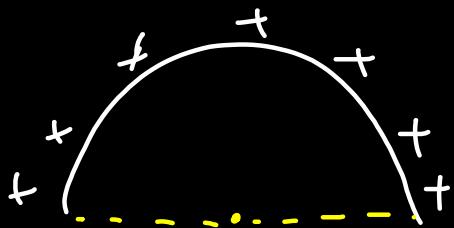
$$\text{force} = (q) E$$

$$= \frac{q K Q x}{(x^2 + R^2)^{3/2}}$$

$$F = -\frac{K Q q}{R^3} x$$

$$T = 2\pi \sqrt{\frac{m R^3}{K Q q}}$$

E due to Arc of Ring at Center



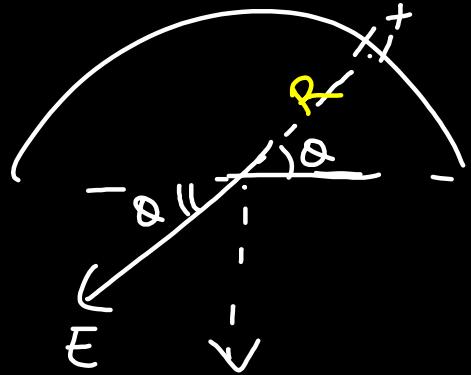
$$\text{angle} = \frac{\text{arc length}}{\text{radius}}$$

$$d\theta = \frac{d\alpha}{R}$$

$$dx = R d\theta$$

$$d\alpha = \lambda dx$$

$$d\alpha = \lambda R d\theta$$



$$E_y = E \sin \theta$$

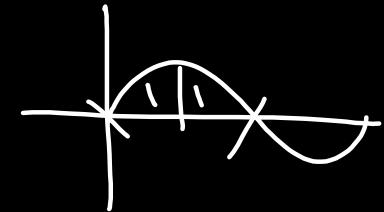
$$= \frac{K (\partial\alpha)}{R^2} \sin \theta$$

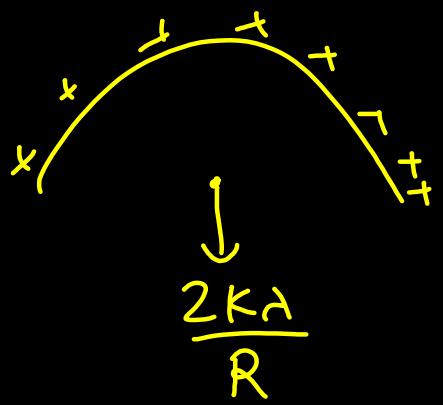
$$= \frac{K \lambda}{R^2} \sin \theta d\alpha$$

$$= \frac{K \lambda}{R^2} R d\theta \sin \theta$$

$$= \frac{K \lambda}{R} \int_0^\pi \sin \theta d\theta$$

$$= \frac{K \lambda}{R} (2) = \frac{2 K \lambda}{R}$$



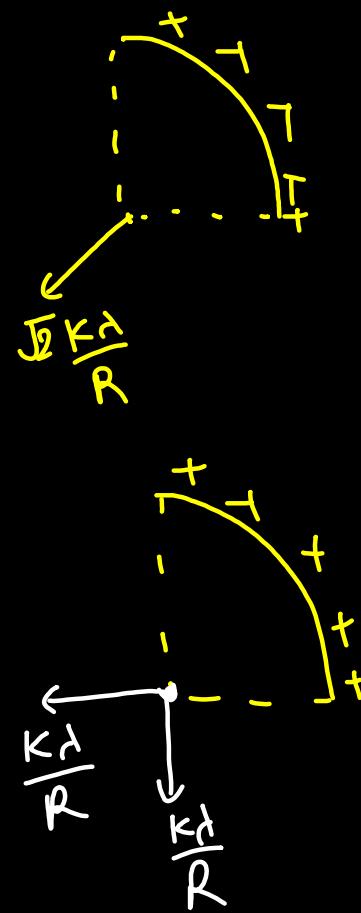


General



along
angle
bisector.

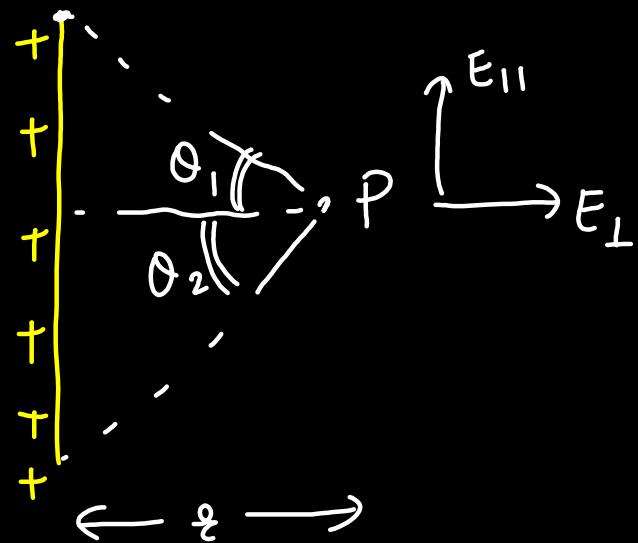
$$E = \frac{2k\lambda}{R} \sin(\theta/2)$$



$$\theta = 90^\circ$$

$$E = \frac{2k\lambda}{R} \sin 45^\circ = \sqrt{2} \frac{k\lambda}{R}$$

Linear Charge



$$E_{\perp} = \frac{K\lambda}{\epsilon} (\sin\theta_1 + \sin\theta_2)$$

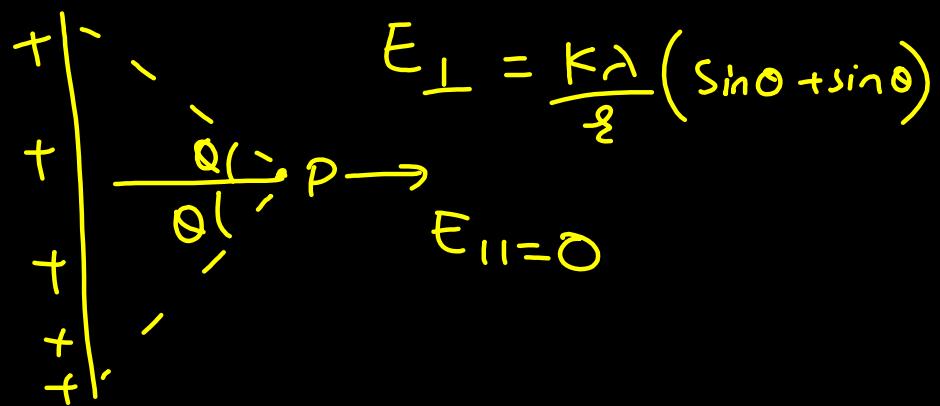
$$E_{||} = \frac{K\lambda}{\epsilon} (\cos\theta_1 - \cos\theta_2)$$

Special Case

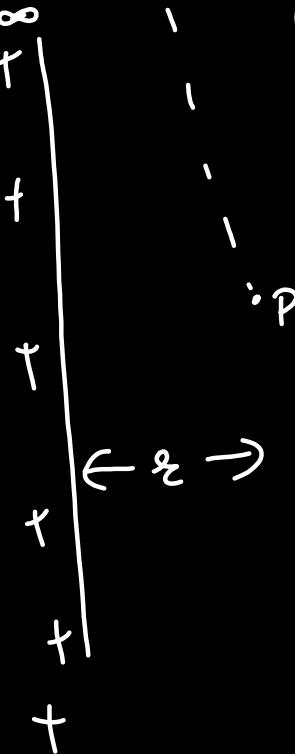
1'ur bisector

$$E_{\perp} = \frac{K\lambda}{2} (\sin\theta + \sin\theta)$$

$\rightarrow E_{||} = 0$

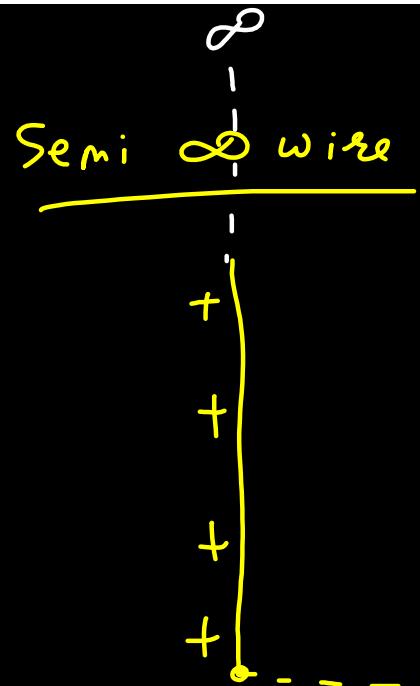


∞ wire



$$\theta_1 = 90^\circ = \theta_2$$

$$E = 2 \frac{K\lambda}{2}$$

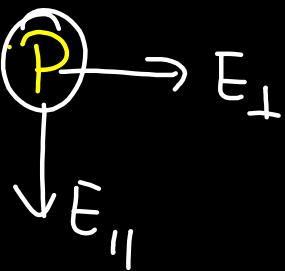


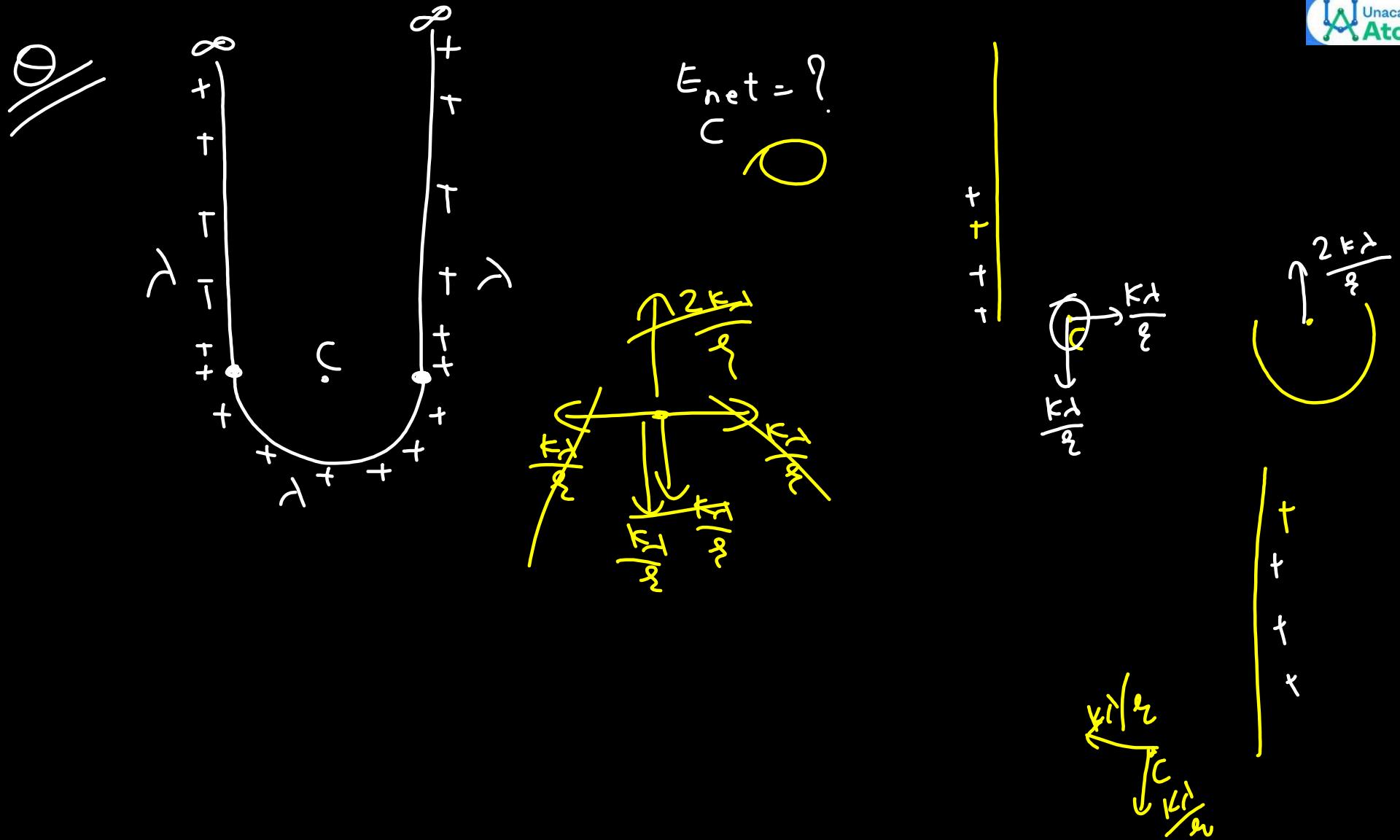
$$\theta_1 = 90^\circ$$

$$\theta_2 = 0$$

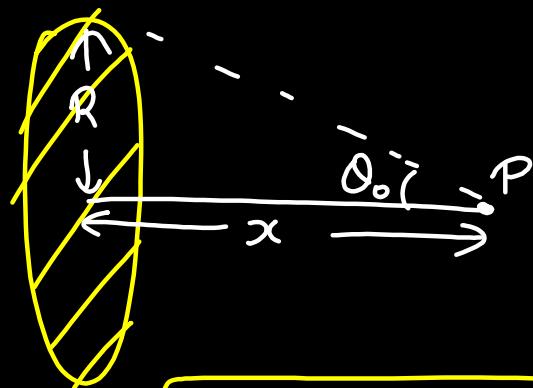
$$E_{\perp} = \frac{k\lambda}{z}$$

$$E_{||} = \frac{k\lambda}{z}$$

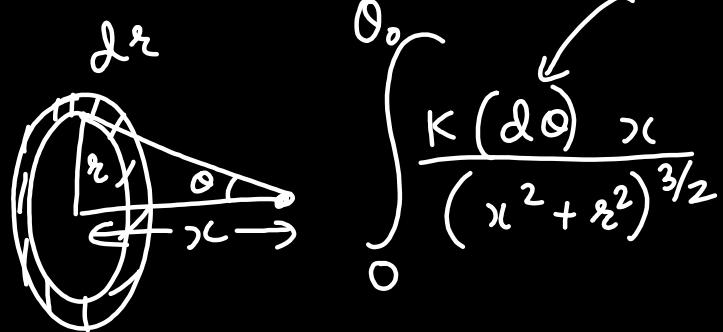




E due to Disc on its Axis



$$E = \frac{\sigma}{2\epsilon_0} (1 - \cos\theta)$$

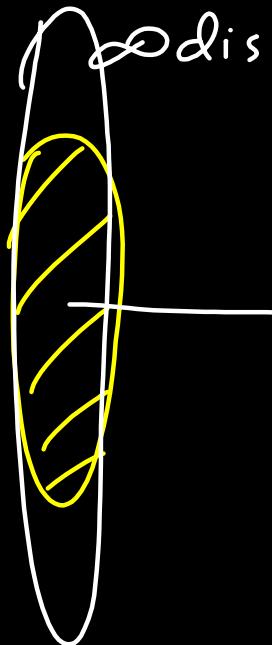


$$dQ = \sigma dA = \sigma 2\pi x dz$$

$$\frac{z}{x} = \tan\theta$$

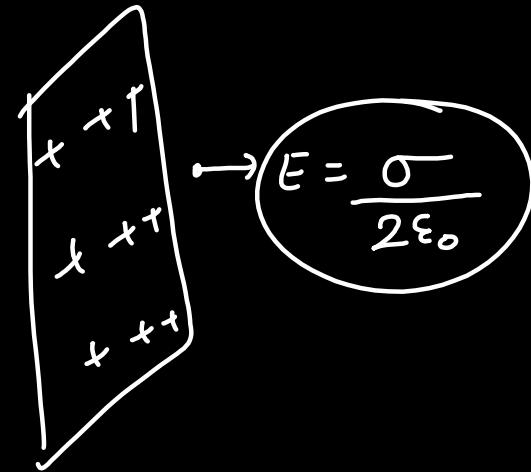
$$z = x \tan\theta$$

$$dz = x \sec^2\theta d\theta$$

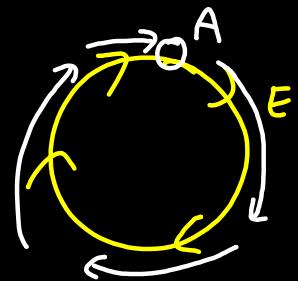


$$\theta = 90^\circ$$

$$E = \frac{\sigma}{2\epsilon_0}$$



Conservative WD in closed loop = 0

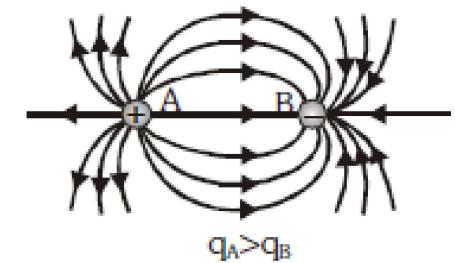


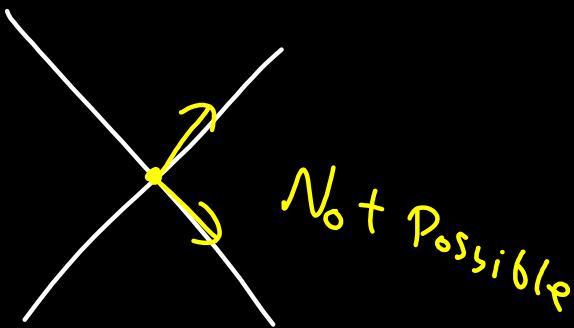
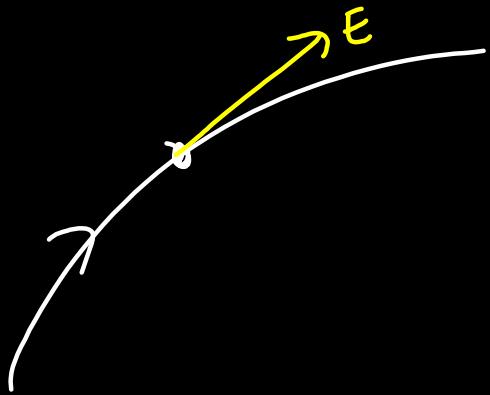
$WD + ve$

Electric lines of force

Electric lines of electrostatic field have following properties

- (i) Imaginary
- (ii) Can never cross each other
- (iii) Can never be closed loops
- (iv) The number of lines originating or terminating on a charge is proportional to the magnitude of charge.
- (v) Lines of force ends or starts normally at the surface of a conductor.
- (vi) If there is no electric field there will be no lines of force.
- (vii) Lines of force per unit area normal to the area at a point represents magnitude of intensity , crowded lines represent strong field while distant lines weak field.
- (viii) Tangent to the line of force at a point in an electric field gives the direction of intensity.



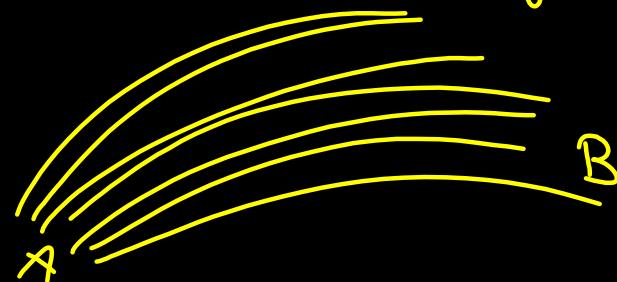


E is \perp to metals & equipotential surface

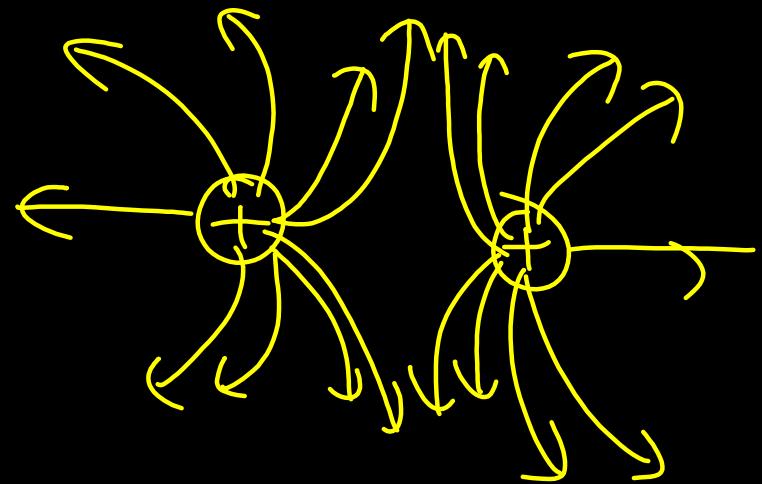
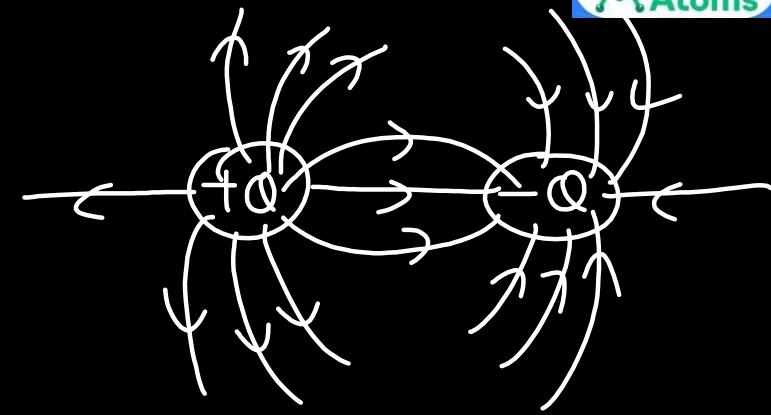
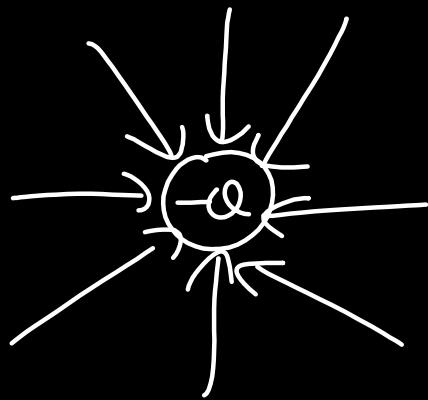
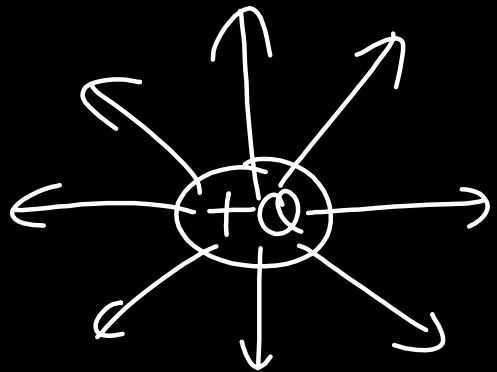
E flow from high potential to low potential.

E is \circ inside metal volume.

density $\propto E$ field intensity

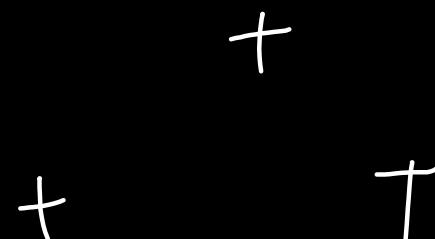
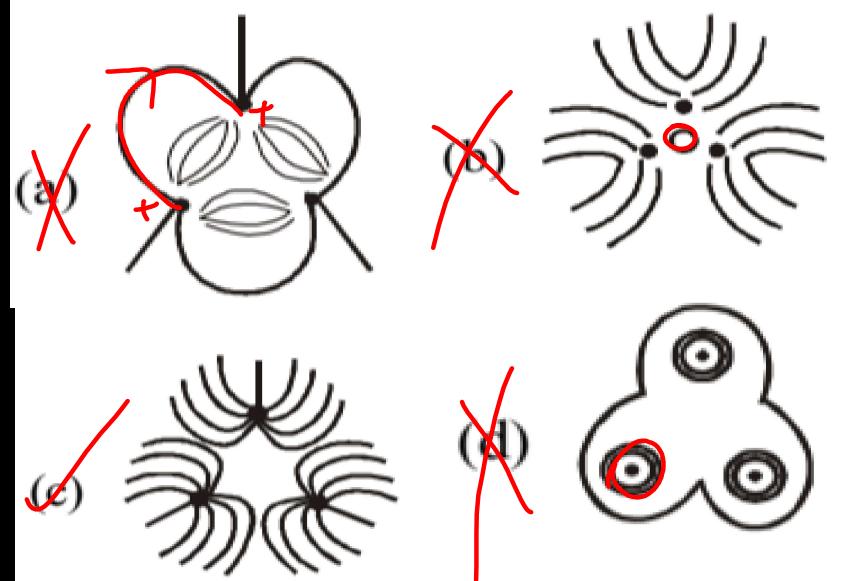


$$E_A > E_B$$



No. of field lines \propto charge magnitude

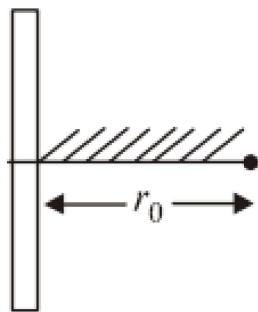
Three positive charges of equal value q are placed at vertices of an equilateral triangle. The resulting lines of force should be sketched as in [Online May 26, 2012]



E originates +ve or ∞
 E terminate -ve or ∞

A positive point charge is released from rest at a distance r_0 from a positive line charge with uniform density. The speed (v) of the point charge, as a function of instantaneous distance r from line charge, is proportional to :

[8 April 2019 III]

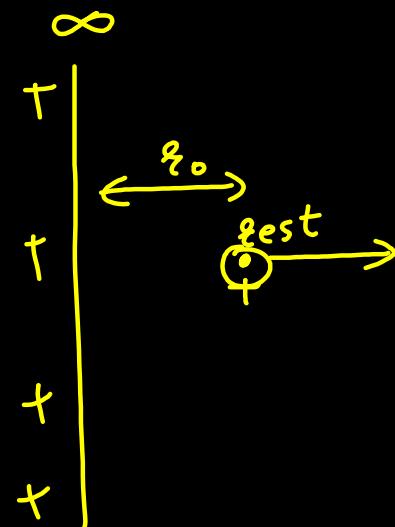


(a) $v \propto e^{+r/r_0}$

(b) $\checkmark v \propto \sqrt{\ln\left(\frac{r}{r_0}\right)}$

(c) $v \propto \ln\left(\frac{r}{r_0}\right)$

(d) $v \propto \left(\frac{r}{r_0}\right)$



$$\text{force} = q E \\ = q \frac{2k\lambda}{\epsilon}$$

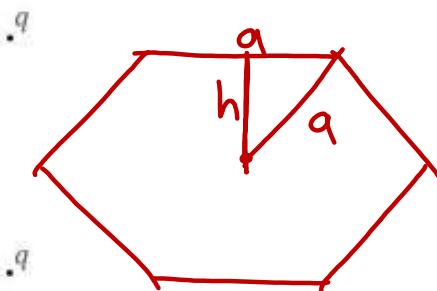
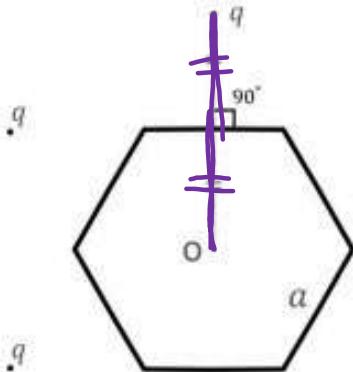
$$\text{acc} = \frac{2k\lambda q}{\epsilon m}$$

$$\frac{\sqrt{dv}}{dr} = \frac{2k\lambda q}{\epsilon m}$$

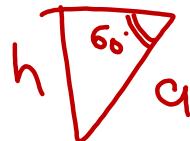
$$\frac{v^2}{2} = \frac{2k\lambda q}{m} \frac{dr}{r}$$

$$v \propto \sqrt{\log\left(\frac{r_0}{r}\right)}$$

Six charges are placed around a regular hexagon of side length a as shown in the figure. Five of them have charge q , and the remaining one has charge x . The perpendicular from each charge to the nearest hexagon side passes through the center O of the hexagon and is bisected by the side.



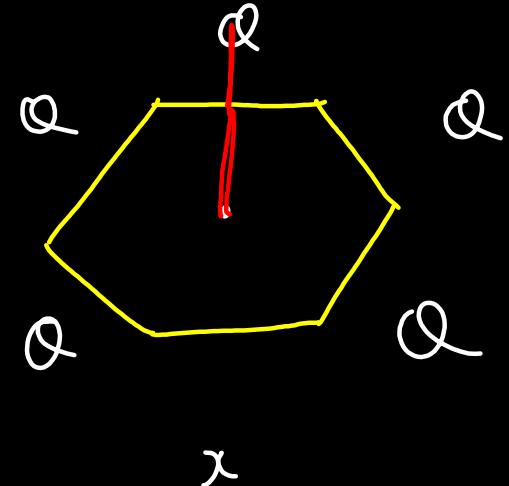
$\bullet x$

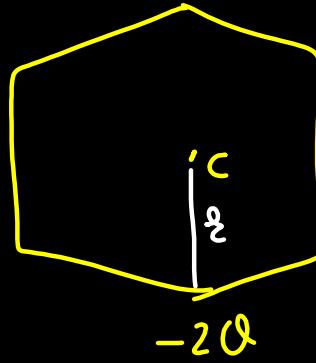
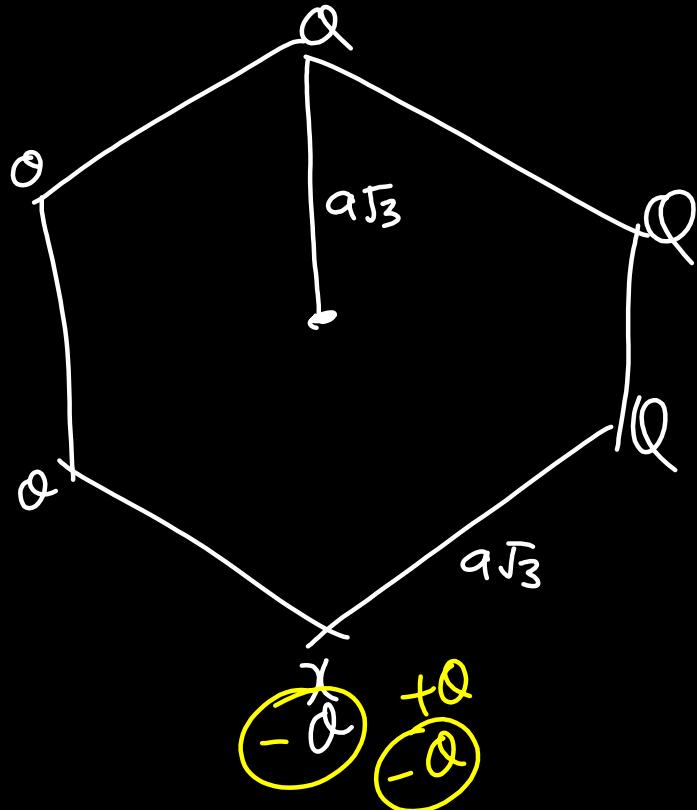


Which of the following statement(s) is(are) correct in SI units?

- (A) When $x = q$, the magnitude of the electric field at O is zero.
- (B) When $x = -q$, the magnitude of the electric field at O is $\frac{q}{6\pi\epsilon_0 a^2}$.
- (C) When $x = 2q$, the potential at O is $\frac{7q}{4\sqrt{3}\pi\epsilon_0 a}$.
- (D) When $x = -3q$, the potential at O is $-\frac{3q}{4\sqrt{3}\pi\epsilon_0 a}$.

$$\frac{q}{6\pi\epsilon_0 a^2} \quad h = q \sin 60^\circ \quad = \frac{qa\sqrt{3}}{2}$$





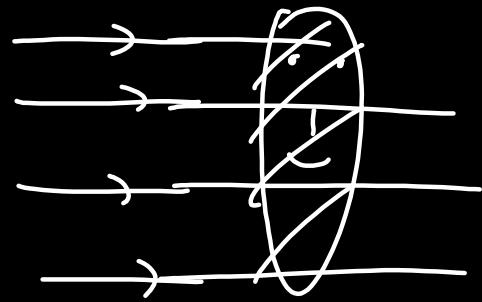
$$\begin{aligned}
 E &= \frac{k(2Q)}{\epsilon^2} \\
 &= \frac{k \cdot 2Q}{(a\sqrt{3})^2} \\
 &= \frac{1}{4\pi\epsilon_0} \frac{2Q}{3a^2} \\
 &= \frac{1}{6\pi\epsilon_0} \frac{Q}{a^2}
 \end{aligned}$$

ELECTRIC FLUX

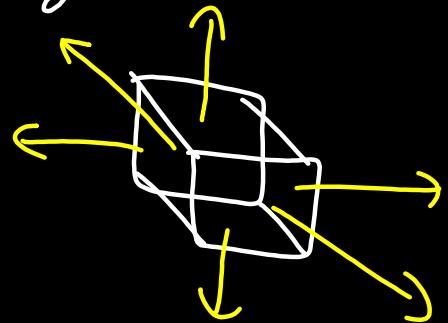
Physical Meaning :

The electric flux through a surface inside an electric field represents the total number of electric lines of force crossing the surface in a direction normal the surface. It is a property of electric field

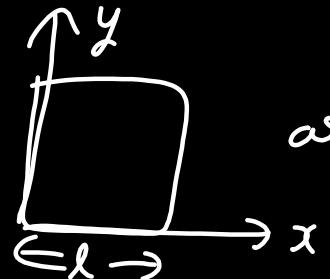
$$\Phi = \vec{E} \cdot \text{area} \quad [\text{if } E \text{ constant}]$$
$$\Phi = \int \vec{E} \cdot d\vec{A} \quad \text{or} \quad \int \vec{E} \cdot d\vec{s}$$



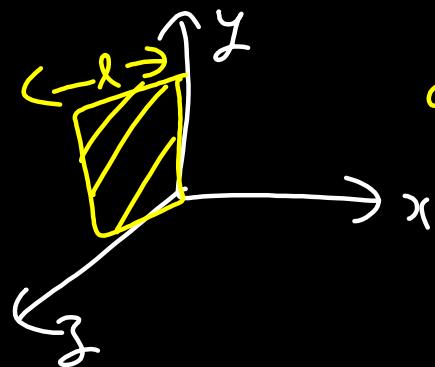
area of closed surface
always outwards



$$\vec{area} = (\text{magnitude}) (\text{unit vector } \perp \text{ to surface})$$



$$area = l^2 (+\hat{k}) \text{ or } l^2 (-\hat{k})$$

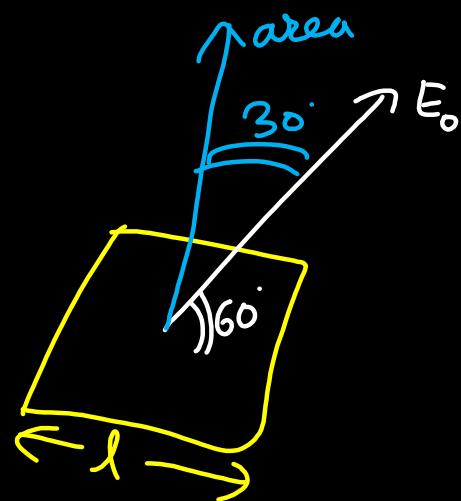


$$area = l^2 (\hat{i})$$

$$\phi = \vec{E} \cdot \vec{Area}$$

$$\phi = E A \cos \theta$$

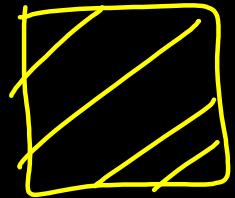
angle b/w E & area



ϕ
passing
through
Surface

$$= E A \cos 30$$

$$= \frac{EA\sqrt{3}}{2}$$



ϕ
 $E \perp$ wr to surface

$$\phi = EA \cos \theta$$

$2 \rightarrow T$

$4 \rightarrow F$

angle with
area
2 & 4

T/F

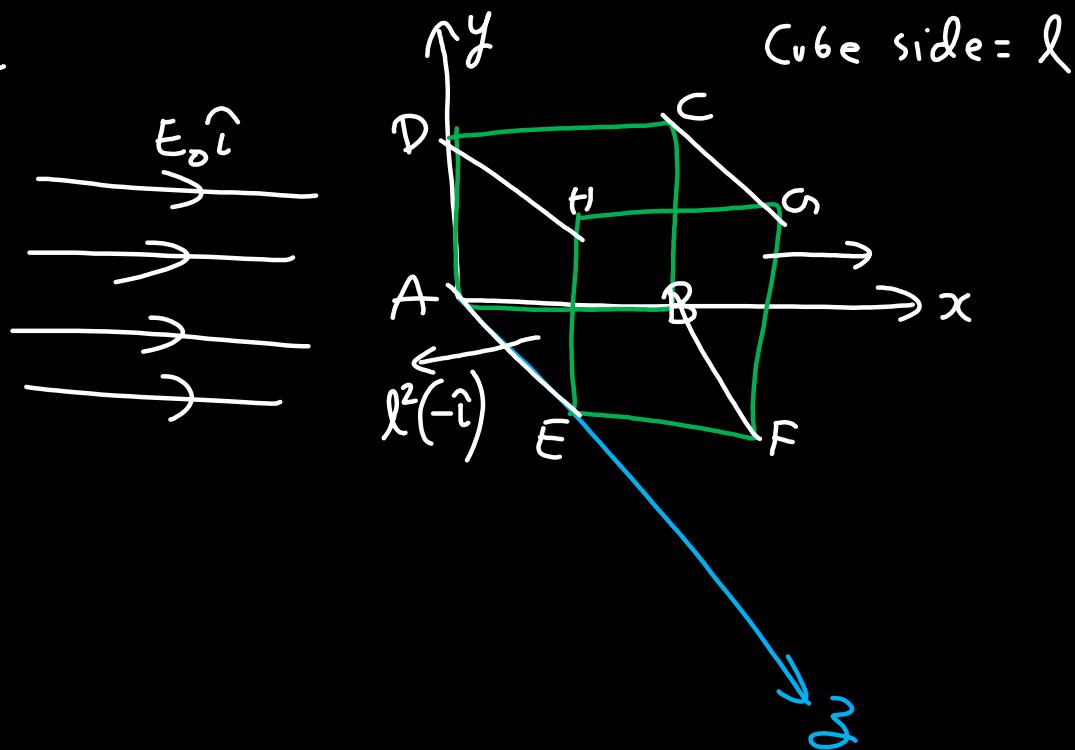
① ϕ is 0 when $E \perp$ wr to surface F

② ϕ is 0 " $E \perp$ wr to area T

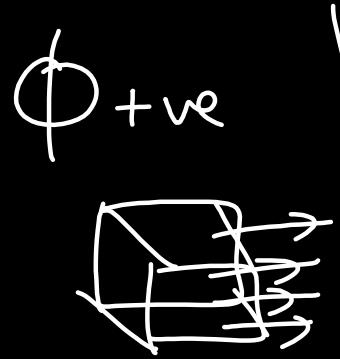
③ ϕ is 0 " " || to surface T

④ ϕ is 0 " " || to area F

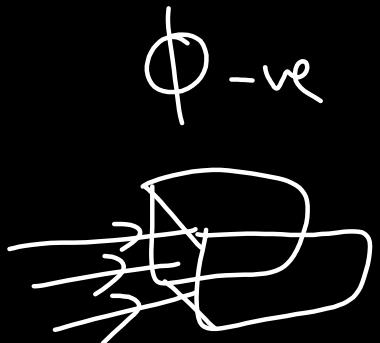
\odot



ϕ	area
$AEHD = -E_0 l^2$	$-l^2 \hat{i}$
$BFGC = +E_0 l^2$	$+l^2 \hat{i}$
$DHGC = 0$	$\hat{l}^2 \hat{j}$
$AEBF = 0$	$-\hat{l}^2 \hat{j}$
$ABCD = 0$	$-\hat{l}^2 \hat{k}$
$EFGH = 0$	$+\hat{l}^2 \hat{k}$
<hr/>	
$\phi_{\text{total}} =$	\odot



E_{exist}
from
surface

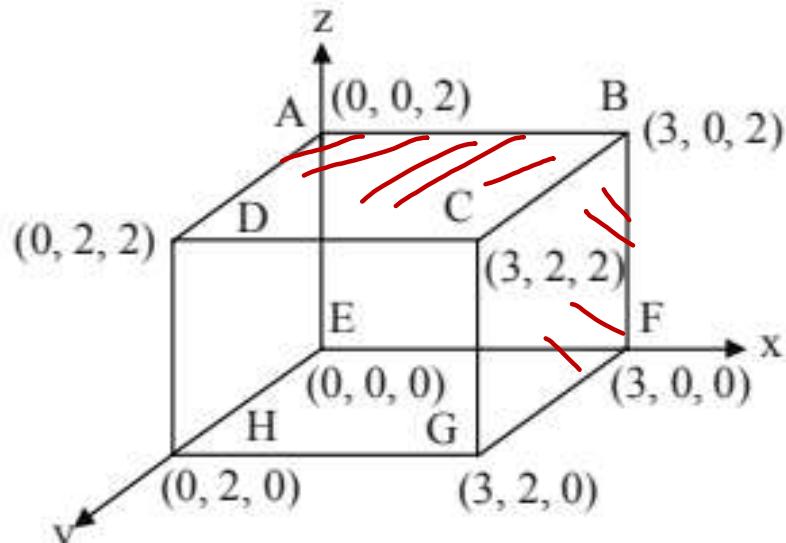


$\phi -ve$
mean
 E_{entry}

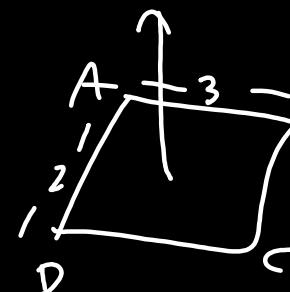
$$\phi_{total} = 0$$

$E_{entry} = E_{exist}$

An electric field $\vec{E} = 4x\hat{i} - (y^2 + 1)\hat{j}$ N/C passes through the box shown in figure. The flux of the electric field through surfaces ABCD and BCGF are marked as ϕ_1 and ϕ_{11} respectively. The difference between $(\phi_1 - \phi_{11})$ is (in Nm²/C) 48. [9 Jan 2020, II]

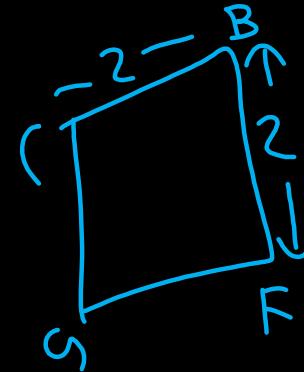


$$\vec{E} = 4x\hat{i} - (y^2 + 1)\hat{j}$$



$$\text{area} = 6(\hat{i} + \hat{k})$$

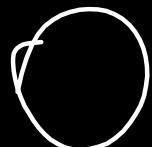
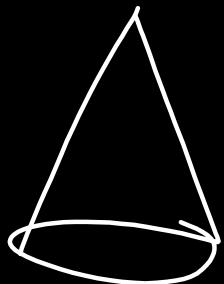
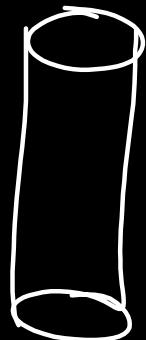
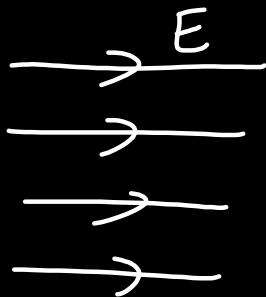
$$\begin{aligned}\phi_1 &= \vec{E} \cdot \text{area} \\ &= 0\end{aligned}$$



$$\text{Area} = 4\hat{i}$$

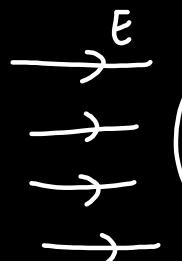
$$\begin{aligned}\phi_{11} &= 16x \\ &= 48\end{aligned}$$

Uniform field



$$\phi_{\text{total}} = 0$$

ϕ_{entering}



$$= (\text{field})(\text{Projected area})$$

$$= -E \pi R^2$$

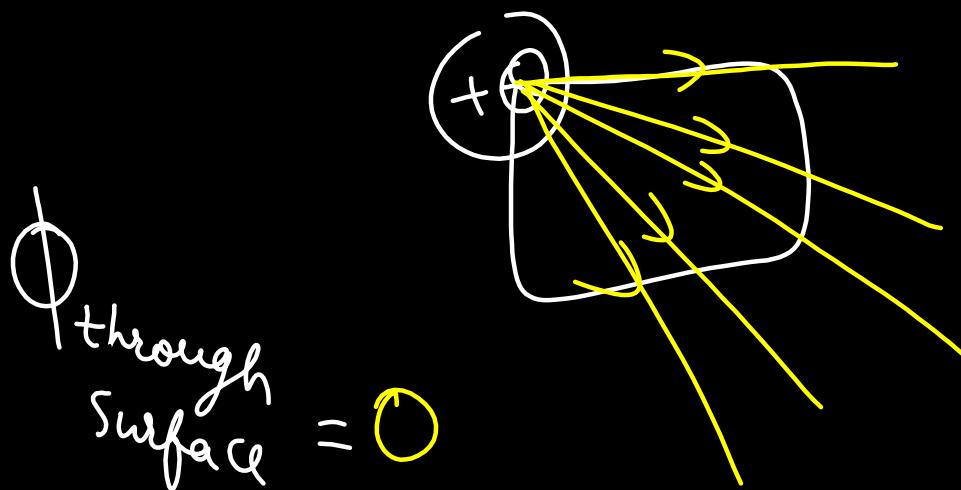
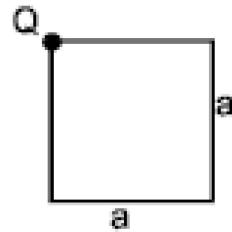


$$= -E_0 (\xi) (l)$$



$$= -E_0 \left(\frac{1}{2} (R_\xi) h \right)$$

A point charge Q is placed at the corner of a square of side a , then find the flux through the square.



The volume charge density of a sphere of radius 6 m is $2 \mu\text{C cm}^{-3}$. The number of lines of force per unit surface area coming out from the surface of the sphere is _____ $\times 10^{10} \text{ NC}^{-1}$.

[Given : Permittivity of vacuum

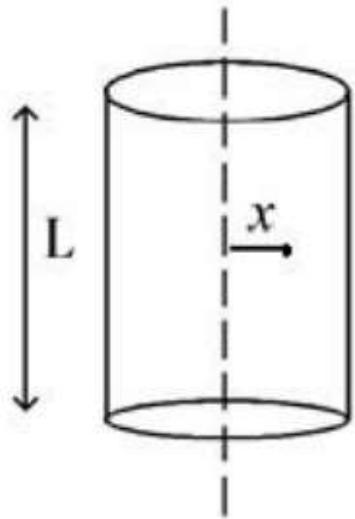
$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

Jee Mains 2022

A long cylindrical volume contains a uniformly distributed charge of density $\rho \text{ Cm}^{-3}$. The electric field

inside the cylindrical volume at a distance $x = \frac{2\epsilon_0}{\rho} \text{ m}$

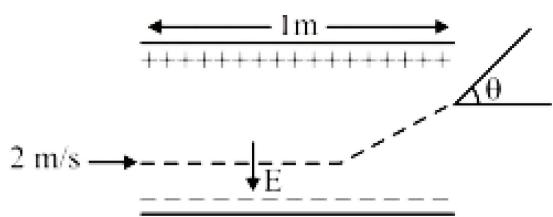
from its axis is _____ Vm^{-1}



Jee Mains 2022

A uniform electric field $E = (8m/e) \text{ V/m}$ is created between two parallel plates of length 1m as shown in figure, (where m = mass of electron and e = charge of electron). An electron enters the field symmetrically between the plates with a speed of 2m/s. The angle of the deviation (θ) of the path of the electron as it comes out of the field will be

.....



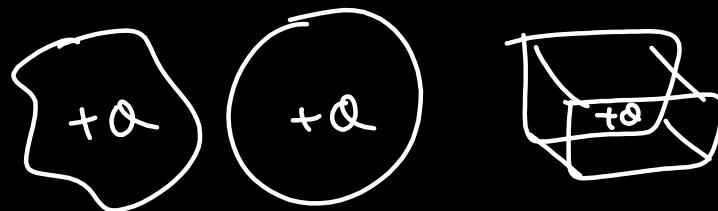
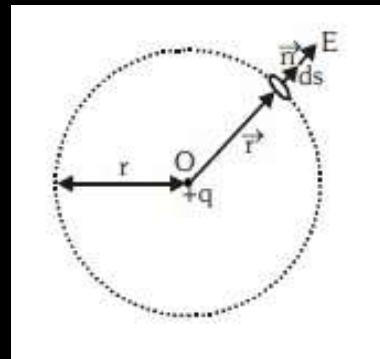
- (A) $\tan^{-1} (4)$ (B) $\tan^{-1} (2)$
(C) $\tan^{-1}\left(\frac{1}{3}\right)$ (D) $\tan^{-1} (3)$

JEE Mains 2022

GAUSS'S LAW

It relates with the total flux of an electric field through a closed surface to the net charge enclosed by that surface and according to it, the total flux linked with a closed surface is $(1/\epsilon_0)$ times the charge enclosed by the closed surface i.e.,

$$\int_S \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0}$$



Regarding Gauss's law it is worth noting that

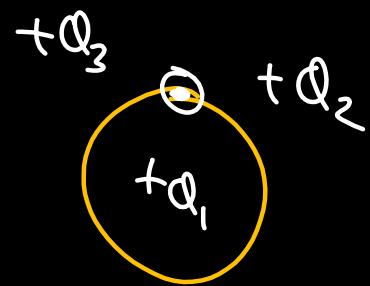
- (i) Flux through gaussian surface is independent of its shape.
- (ii) Flux through gaussian surface depends only on charges present inside gaussian surface.
- (iii) Flux through gaussian surface is independent of position of charges inside gaussian surface.
- (iv) Electric field intensity at the gaussian surface is due to all the charges present (inside as well as out side)
- (v) In a close surface incoming flux is taken negative while outgoing flux is taken positive.
- (vi) In a gaussian surface $\phi = 0$ does not imply $E = 0$ but $E = 0$ at all the points of the surface implies $\phi = 0$.

GAUSS'S LAW & COULOMB'S LAW are equivalent.

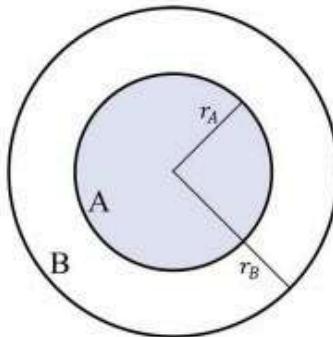
Gauss law is a powerful tool for calculating electric intensity in case of
symmetrical charge distribution.

$$\oint \vec{E} \cdot d\vec{s} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

due to all charges
over 3D surface



In the figure, the inner (shaded) region A represents a sphere of radius $r_A = 1$, within which the electrostatic charge density varies with the radial distance r from the center as $\rho_A = kr$, where k is positive. In the spherical shell B of outer radius r_B , the electrostatic charge density varies as $\rho_B = \frac{2k}{r}$. Assume that dimensions are taken care of. All physical quantities are in their SI units.

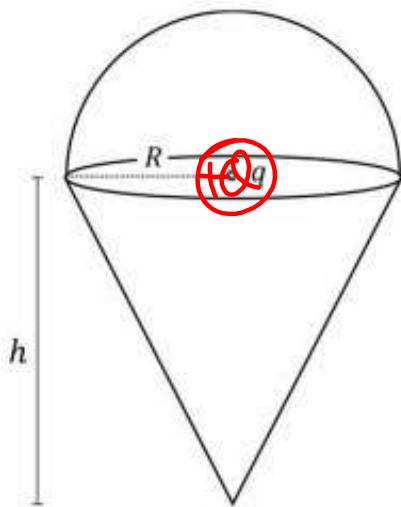


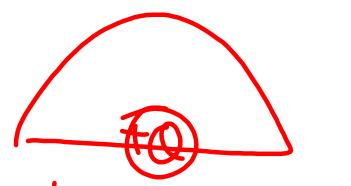
Which of the following statement(s) is(are) correct?

- (A) If $r_B = \sqrt{\frac{3}{2}}$, then the electric field is zero everywhere outside B.
- (B) If $r_B = \frac{3}{2}$, then the electric potential just outside B is $\frac{k}{\epsilon_0}$.
- (C) If $r_B = 2$, then the total charge of the configuration is $15\pi k$.
- (D) If $r_B = \frac{5}{2}$, then the magnitude of the electric field just outside B is $\frac{13\pi k}{\epsilon_0}$.

A charge q is surrounded by a closed surface consisting of an inverted cone of height h and base radius R , and a hemisphere of radius R as shown in the figure. The electric flux through the conical surface is $\frac{nq}{6\epsilon_0}$ (in SI units). The value of n is _____.
advanced
2022

$$n=3$$





$$\Phi_{\text{hemis}} = \frac{Q}{2\epsilon_0}$$

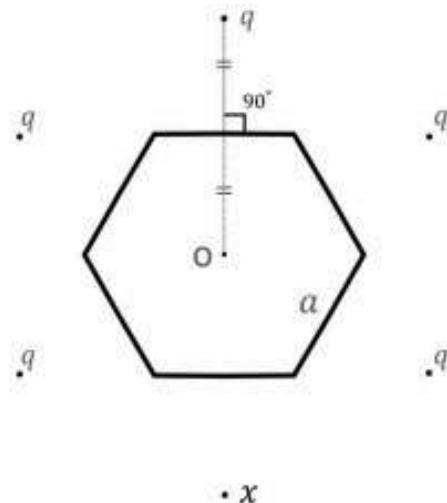
$$\Phi_{\text{cone}} = \frac{nQ}{6\epsilon_0} ?$$

$$\Phi_{\text{total}} = \frac{Q}{\epsilon_0}$$

$$\Phi_{\text{cone}} + \Phi_{\text{hemis}} = \frac{Q}{\epsilon_0}$$

$$\Phi_{\text{cone}} = \frac{Q}{\epsilon_0} - \frac{Q}{2\epsilon_0} \quad \left(\frac{Q}{2\epsilon_0} \right)$$

Six charges are placed around a regular hexagon of side length a as shown in the figure. Five of them have charge q , and the remaining one has charge x . The perpendicular from each charge to the nearest hexagon side passes through the center O of the hexagon and is bisected by the side.



Which of the following statement(s) is(are) correct in SI units?

- (A) When $x = q$, the magnitude of the electric field at O is zero.
- (B) When $x = -q$, the magnitude of the electric field at O is $\frac{q}{6\pi\epsilon_0 a^2}$.
- (C) When $x = 2q$, the potential at O is $\frac{7q}{4\sqrt{3}\pi\epsilon_0 a}$.
- (D) When $x = -3q$, the potential at O is $-\frac{3q}{4\sqrt{3}\pi\epsilon_0 a}$.

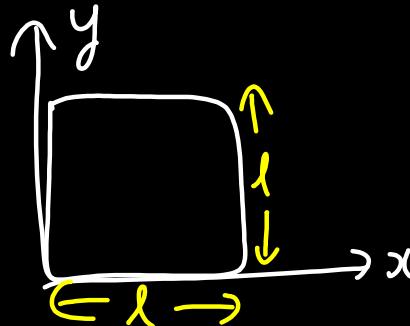
$$\phi = \int \vec{E} \cdot \vec{\text{area}}$$

E variable

or

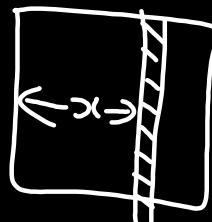
$$E A \cos \theta$$

any one changing



$$E = E_0 \propto (\hat{k})$$

$\phi_{\text{through surface}} = ?$



$$\int l \, d\text{area} = l \, dx \, \hat{k}$$

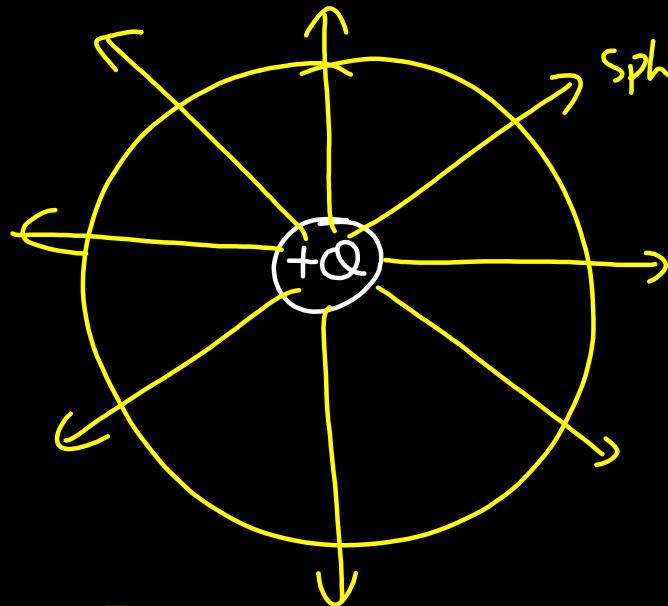
$$\phi = \int (E_0 \propto \hat{k}) \cdot (l \, dx) \hat{k}$$

$$= E_0 l \int_0^l x \, dx$$

$$= E_0 l \left(\frac{l^2}{2} \right)$$

$$= \boxed{E_0 l^3 / 2}$$

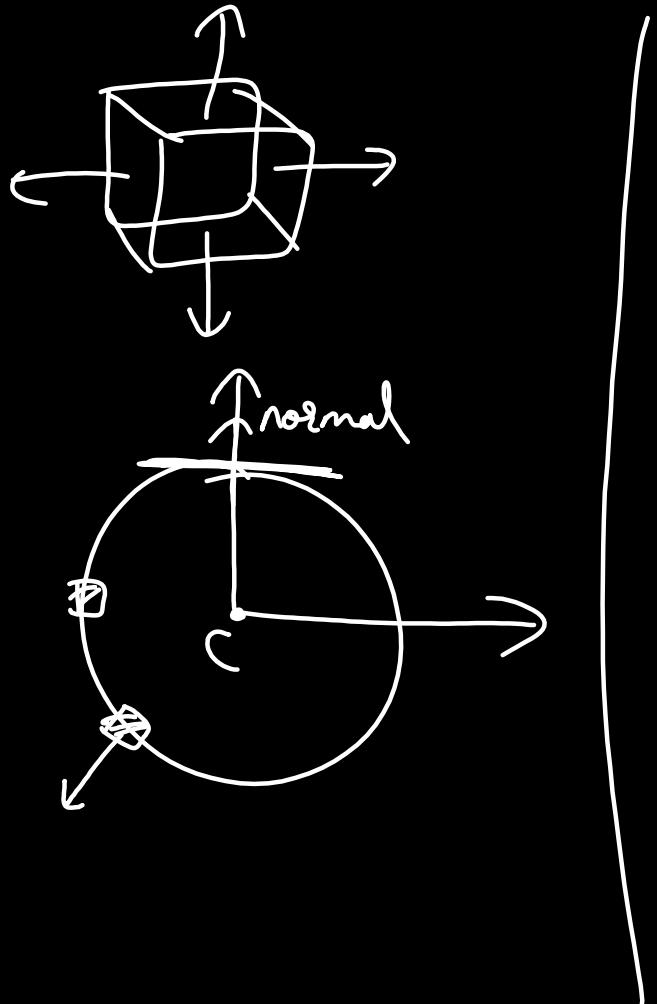
Imp



Spherical Surface.

$$\phi_{\text{total}} = \frac{Q}{\epsilon_0}$$

$$\begin{aligned}
 \phi_{\text{surface}} &= \int \vec{E} \cdot d\vec{A} \\
 &= \int |E| dA \cos\theta \\
 &= |E| \cos\theta \int (dA) \\
 &= \frac{kQ}{r^2} \times \pi r^2 \\
 &= k \epsilon_0 Q \\
 &= \frac{1}{4\pi k \epsilon_0} \oint \phi
 \end{aligned}$$



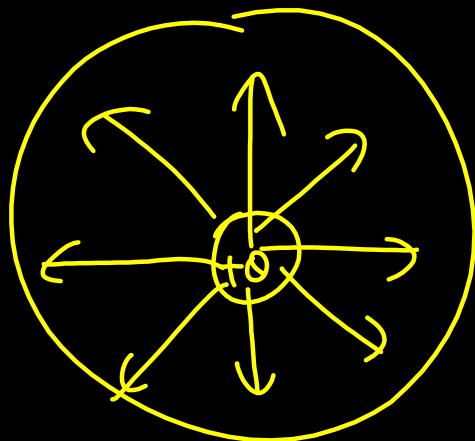
$$\int \vec{E} \cdot d\vec{A}$$

$$\int |E| |dA| |\cos \theta|$$

$$E \int |dA|$$

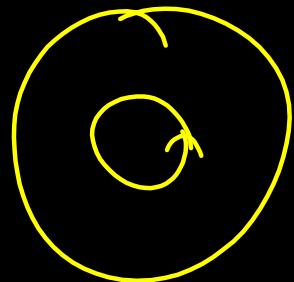
$$E A = \frac{kQ}{\epsilon^2} 4\pi r^2 = \frac{kQ 4\pi}{4\pi \epsilon_0 Q / f_f} = \frac{1}{f_f \epsilon_0} = \frac{Q}{\epsilon_0}$$

$$\Phi_{\text{total}} = \frac{Q}{\epsilon_0}$$

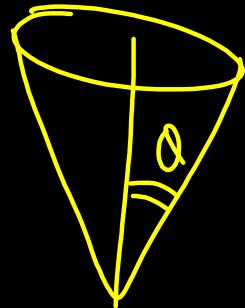


$\frac{Q}{\epsilon_0}$ total flux is
equally distributed
over the complete
solid angle

$$\frac{Q}{\epsilon_0}$$

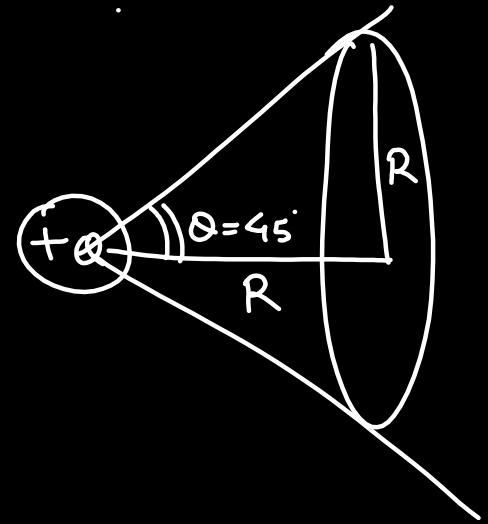


$$\text{total solid angle} = 4\pi$$



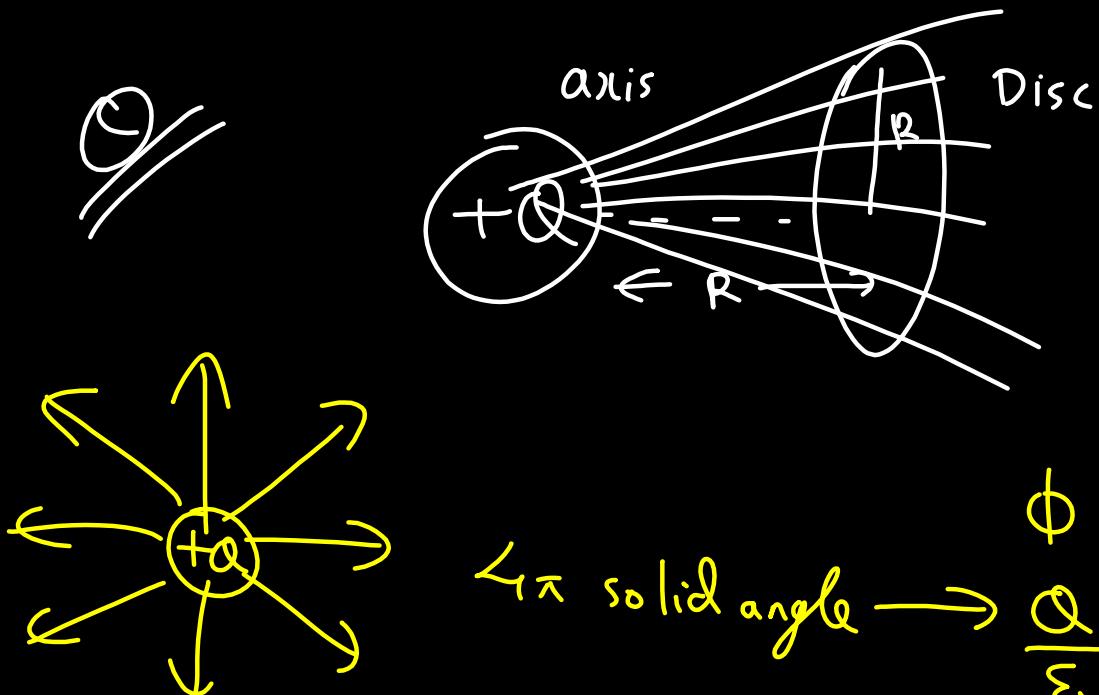
$$\text{Solid angle} = 2\pi(1 - \cos\theta)$$

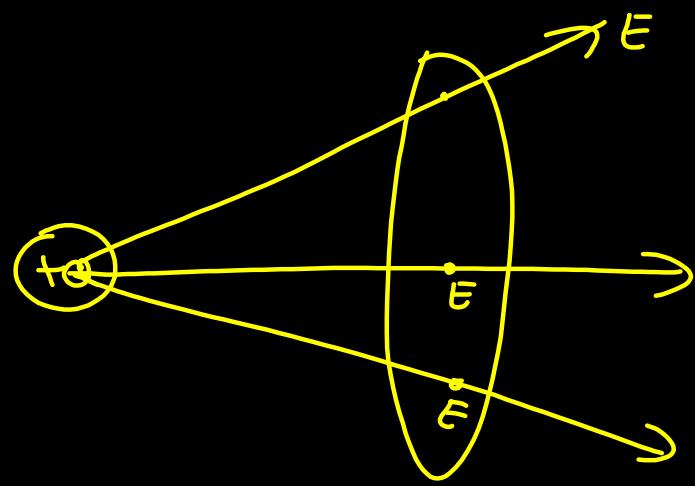
Find flux passing through Disc?



$$4\pi \text{ solid angle} \rightarrow \frac{Q}{\epsilon_0}$$

$$2\pi(1 - \cos\theta) \rightarrow \frac{Q}{\epsilon_0} 2\pi(1 - \cos 45^\circ) \sim \frac{Q}{2\epsilon_0} \left(1 - \frac{1}{\sqrt{2}}\right)$$



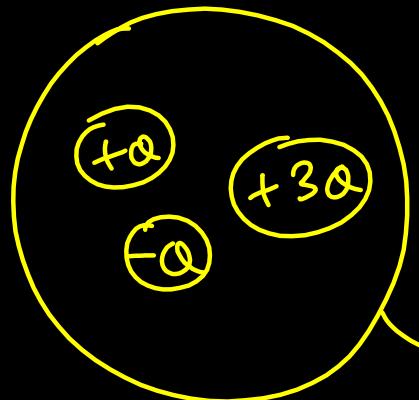


Gauss law

$\oint \vec{E} \cdot d\vec{S}$ = $\frac{Q_{\text{enclosed}}}{\epsilon_0}$

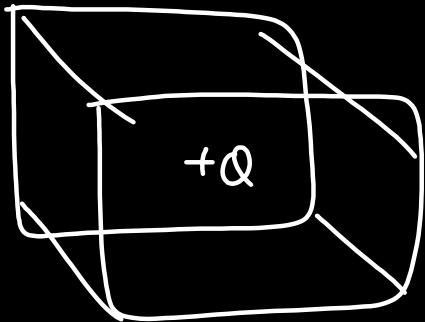
 through a closed 3D surface

$$\oint \vec{E} \cdot d\vec{S} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$



$$\phi = \frac{0}{\epsilon_0} - \frac{Q}{\epsilon_0} + \frac{3Q}{\epsilon_0} = \frac{3Q}{\epsilon_0} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

ϕ



$+Q$ at Center

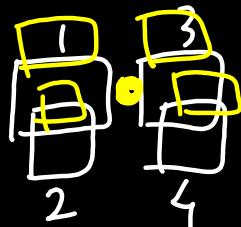
$$\phi_{\text{total cube}} = Q/\epsilon_0$$

$$\phi_{\text{through each face}} = \left(\frac{Q}{\epsilon_0}\right) \frac{1}{6} = \frac{Q}{6\epsilon_0}$$



Corner how many = 8

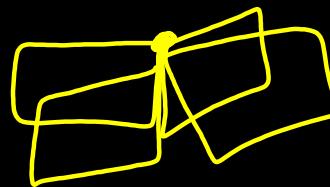
Structure

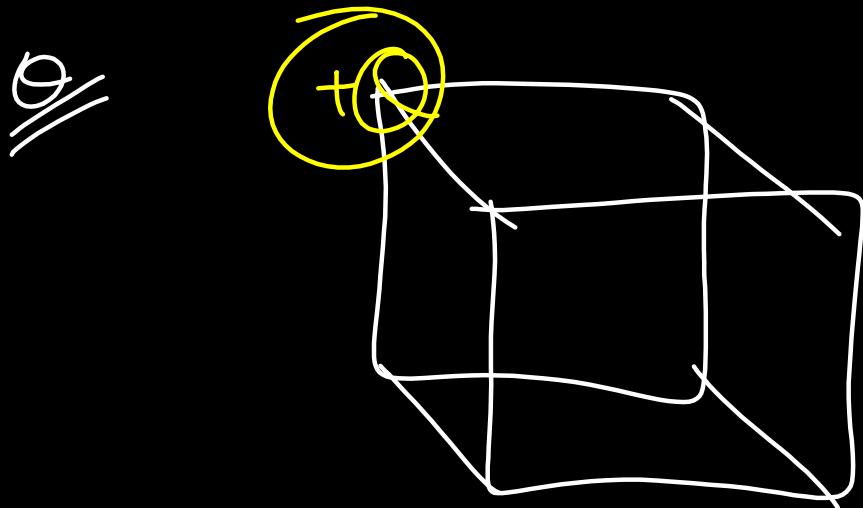


↳ Box on ground floor

↳ More Box over them on 1st floor

Corner Shared in 8 Cubes



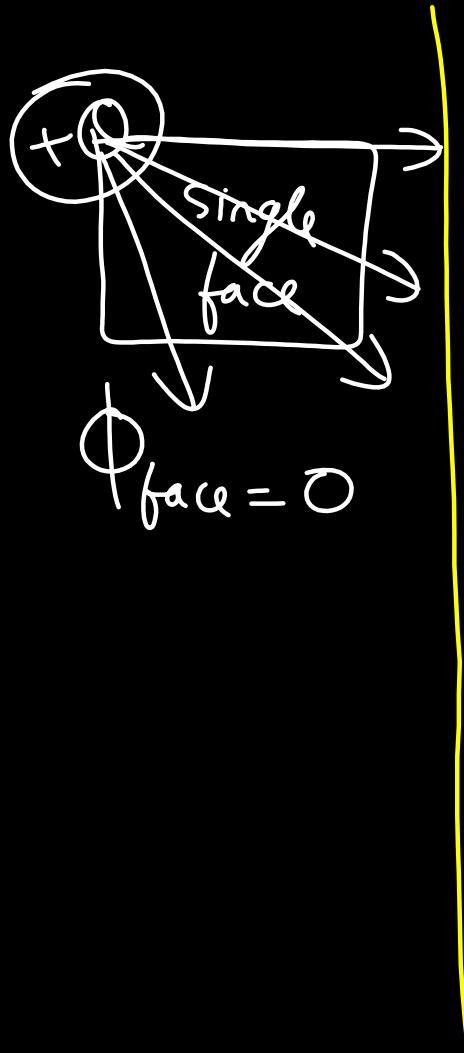


$$\text{Cube } \Phi_{\text{total}} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

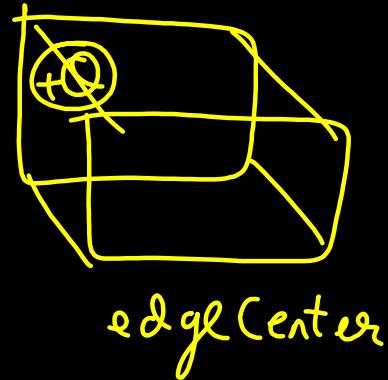
$$= \frac{Q/8}{\epsilon_0} = \frac{Q}{8\epsilon_0}$$

3 face in touch with charge $\cup_n \leq \phi = 0$

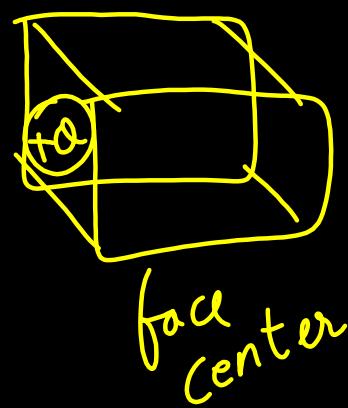
3 face not in " " " $\cup_n \leq \phi = \left(\frac{Q}{8\epsilon_0}\right) \frac{1}{3}$ individually



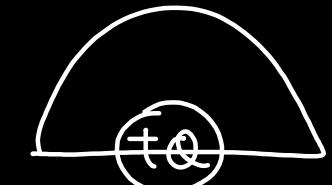
$$\phi_{\text{face}} = 0$$



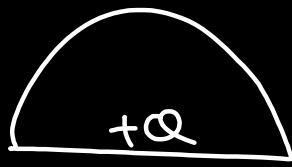
$$\phi_{\text{cube}} = \frac{Q}{4\epsilon_0}$$



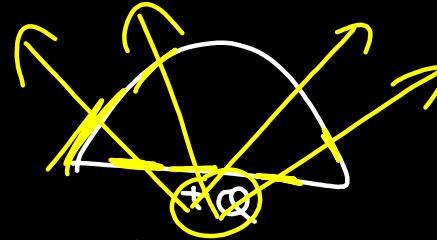
$$\phi_{\text{cube}} = \frac{Q}{2\epsilon_0}$$



$$\phi = \frac{Q}{2\epsilon_0}$$



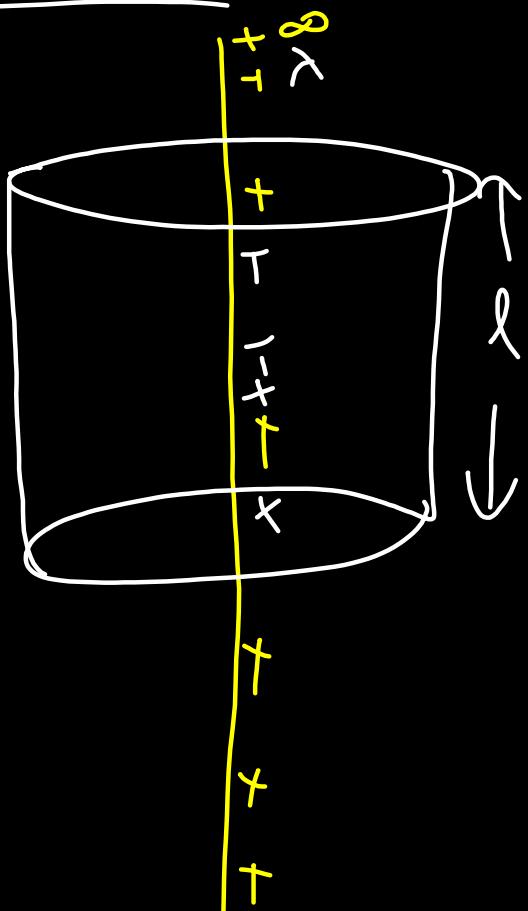
$$\phi = \frac{Q}{\epsilon_0}$$



$$\phi = 0$$

E derivation

$$\lambda = \frac{Q}{l}$$

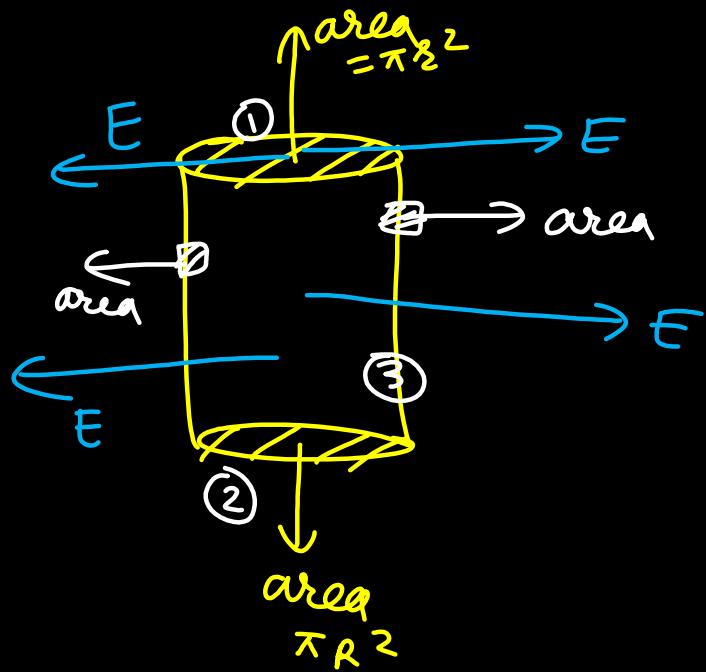


$$Q_{\text{enclosed}} = \lambda l$$

$$\oint \vec{E} \cdot d\vec{s} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

$$E 2\pi r l = \frac{\lambda l}{\epsilon_0}$$

$$E = \frac{\lambda}{2\pi\epsilon_0 r} \times \frac{2}{2} = \frac{2\lambda}{(4\pi\epsilon_0)r} = \frac{2k\lambda}{r}$$



$$\phi_1 = E A \cos 90^\circ = 0$$

$$\phi_2 = 0$$

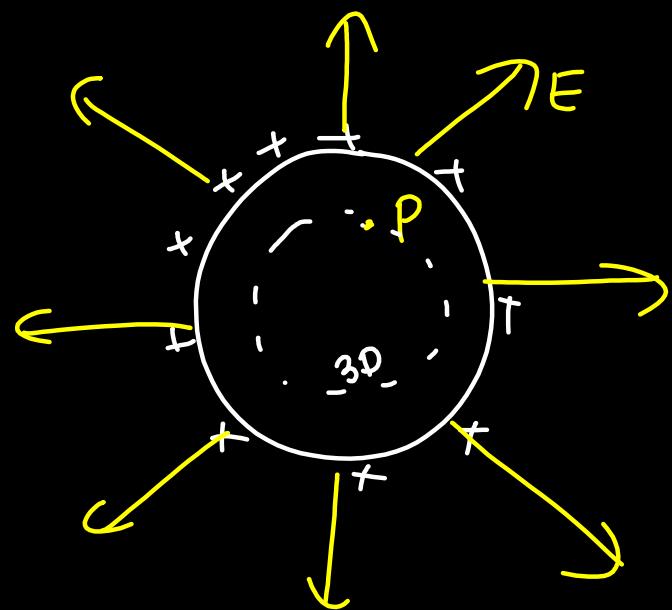
$$\begin{aligned}\phi_3 &= \vec{E} \cdot \vec{A} \\ &= EA \cos 0^\circ \\ &= E 2\pi r l\end{aligned}$$

$$\oint \vec{E} \cdot d\vec{s}$$

↓

$$\begin{aligned}&1 + 2 + 3 \\ &= \underline{E 2\pi r l}\end{aligned}$$

E hollow Sphere

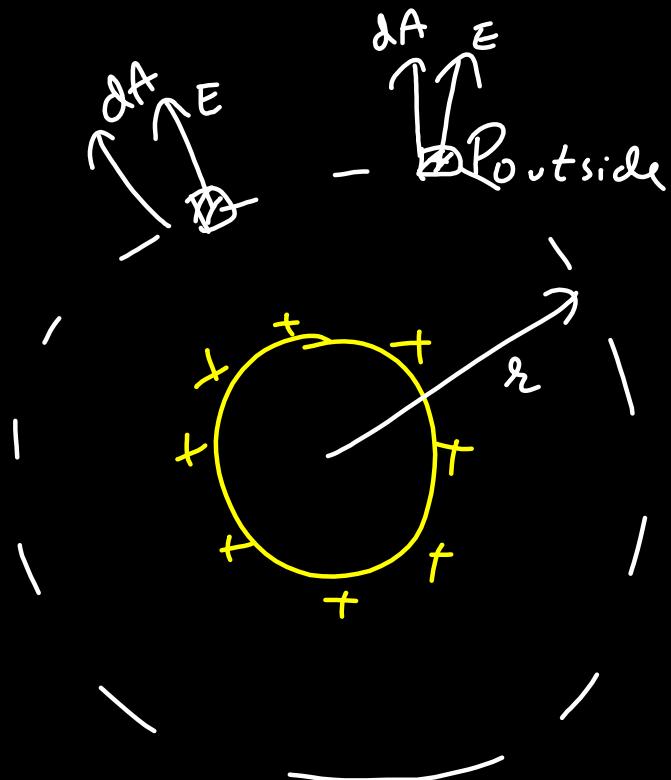


E_{inside}

$$\oint \vec{E} \cdot d\vec{s} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

$$E \cdot 4\pi r^2 = 0$$

$$E_{\text{inside}} = 0$$



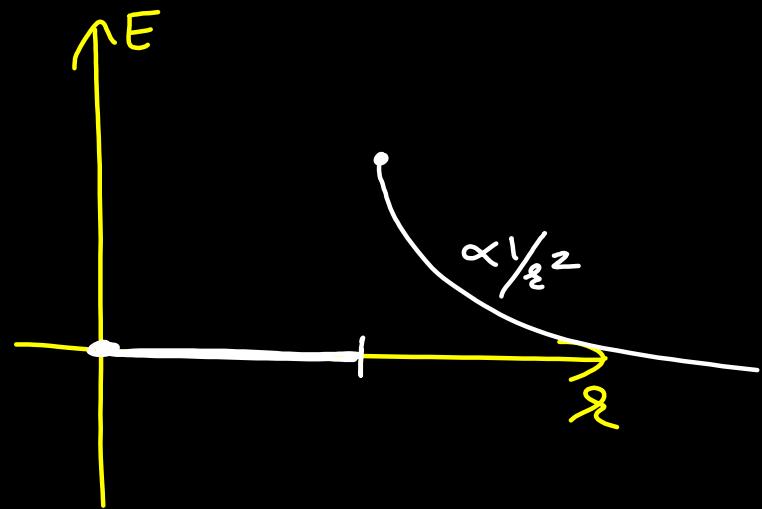
$$\oint \vec{E} \cdot d\vec{s} = \frac{Q_{en}}{\epsilon_0}$$

$$E = \frac{K Q_{enclosed}}{r^2}$$

$$\int |E| |dA| \cos 0 \\ E \int |dA|$$

$$E 4\pi r^2 = \frac{Q_{en}}{\epsilon_0}$$

$$E = \frac{1}{4\pi\epsilon_0 r^2} Q_{en}$$



Solid Sphere



$$\oint \vec{E} \cdot d\vec{s} = \frac{Q}{\epsilon_0}$$

$$E 4\pi r^2 = \frac{Q}{\epsilon_0}$$

$$E_{\text{outside}} = \frac{kQ}{r^2}$$

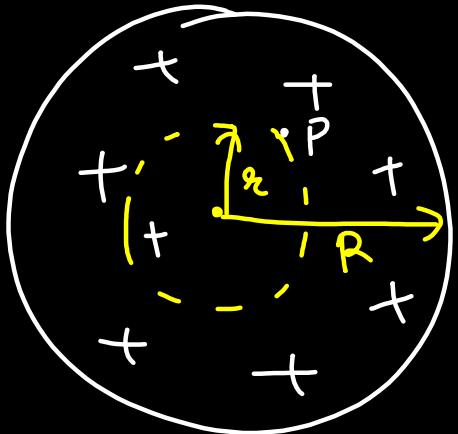
$$\frac{4}{3}\pi R^3 \rightarrow Q$$

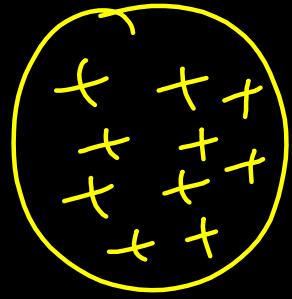
$$\frac{4}{3}\pi r^3 \rightarrow \frac{Q}{\frac{4}{3}\pi R^3} \quad \frac{4}{3}\pi r^3 = \frac{Q r^3}{R^3}$$

$$\int \vec{E} \cdot \vec{ds} = \frac{Q_{en}}{\epsilon_0}$$

$$E \propto \pi r^2 = \frac{Q r^3}{R^3 \epsilon_0}$$

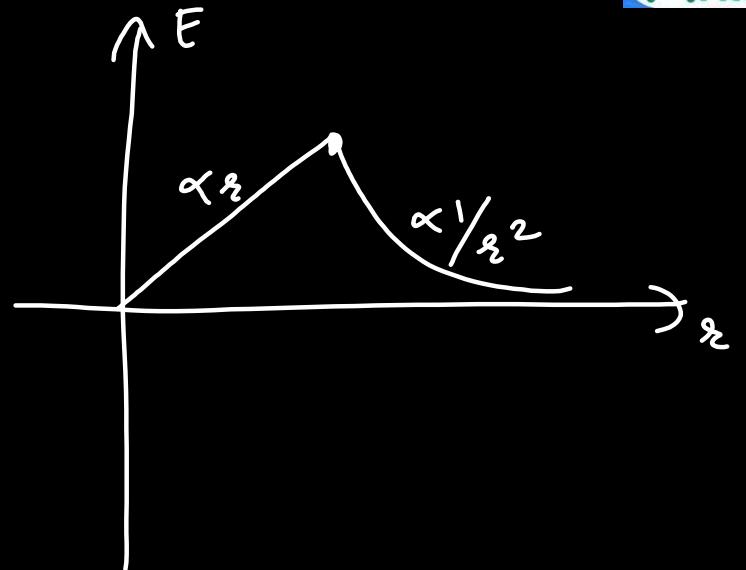
$$E_{\text{inside}} = \frac{Q r}{\frac{4}{3}\pi \epsilon_0 R^3} = \frac{K Q r}{R^3}$$

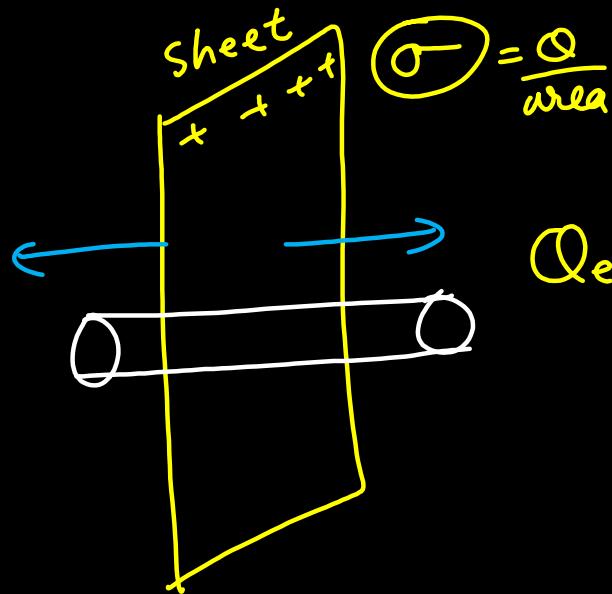




$$E_{\text{insi}} = \frac{kQ}{R^3} r$$

$$E_{\text{out}} = \frac{kQ}{r^2}$$





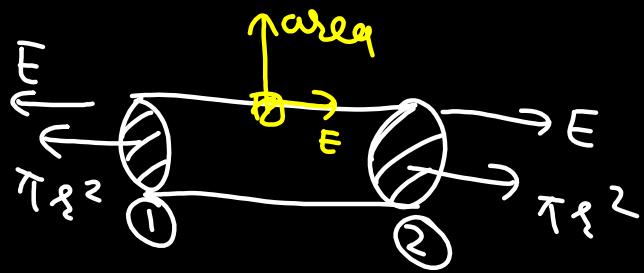
$$Q_{\text{enclosed}} = \sigma \pi r^2$$

$$\oint \vec{E} \cdot d\vec{s}$$

$$\int_1 \vec{E} \cdot d\vec{s} = E \pi r^2$$

$$\int_2 \vec{E} \cdot d\vec{s} = E \pi r^2$$

$$\int \vec{E} \cdot d\vec{s} = 0$$



$$2E \pi r^2 = \underline{\underline{\sigma \pi r^2}}$$

$$E = \frac{\sigma}{2\epsilon_0}$$

$$\oint \vec{E} \cdot d\vec{s} = 2E \pi r^2$$

total