" CONCEPT HUMARA , CALCULATION TUMHARA."

~ JBK !!

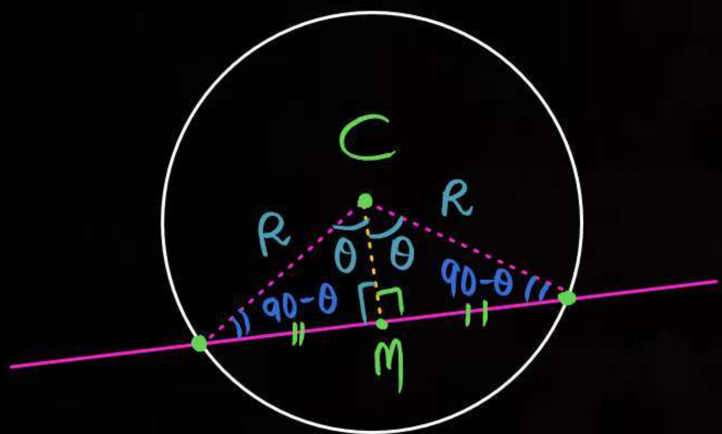
TO GET THE 'BEST' FROM THIS CLASS

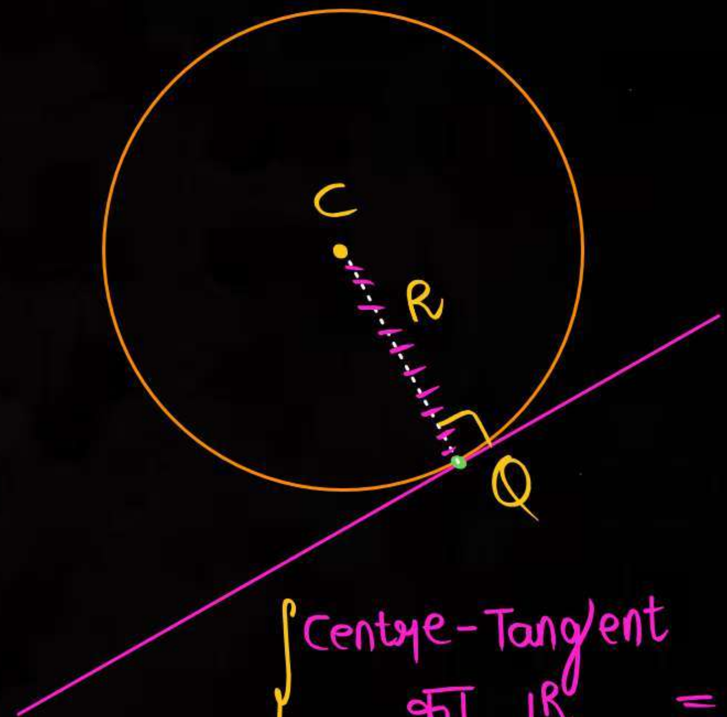
1. Keep a rough copy with you … Don't rush to write the notes …!
2. Listen to me carefully , have a smile !
3. Keep short notes copy with you & write what I request you to write.
4. Have Infinite Patience …. And enjoy the ride!!

TOPICS TO BE COVERED

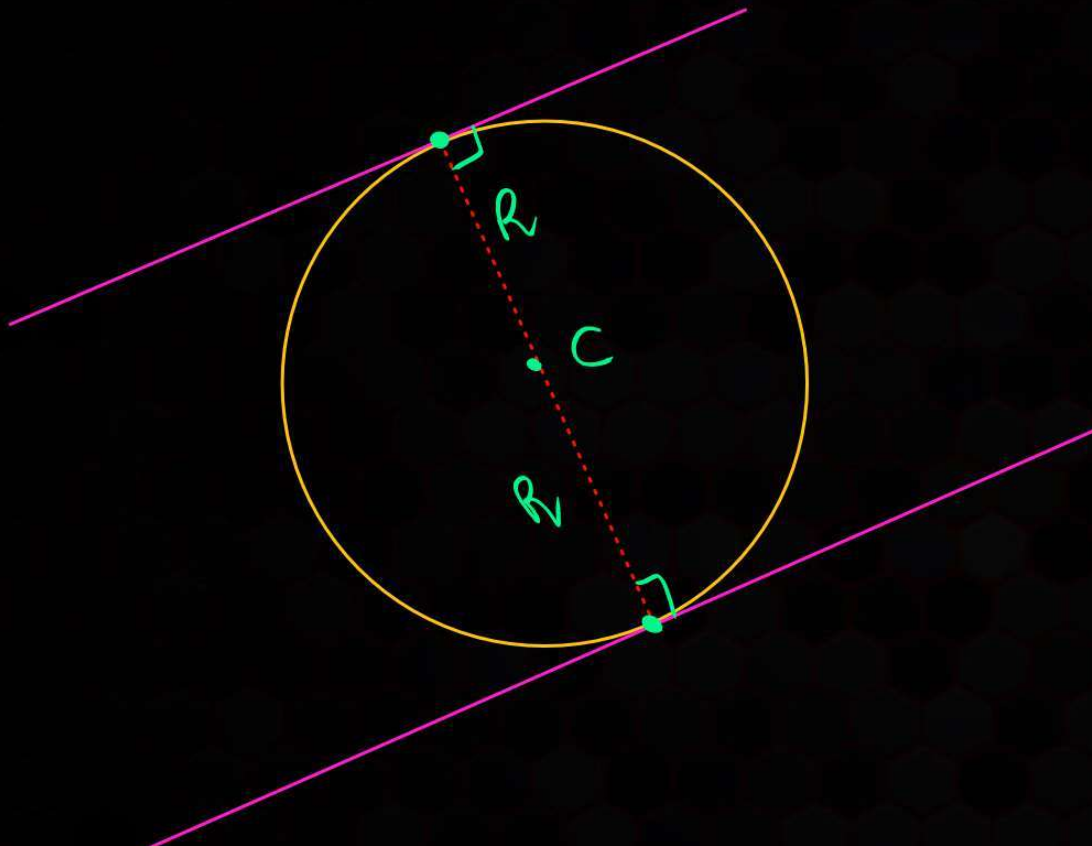
1. DEFINITIONS OF CIRCLE & DIRECTOR CIRCLE ✓
2. TANGENT & NORMALS ✓
3. FAMILY OF CIRCLES ✓
4. RADICAL AXIS & CIRCLES ✓

BACHPAN KI YAADIEN

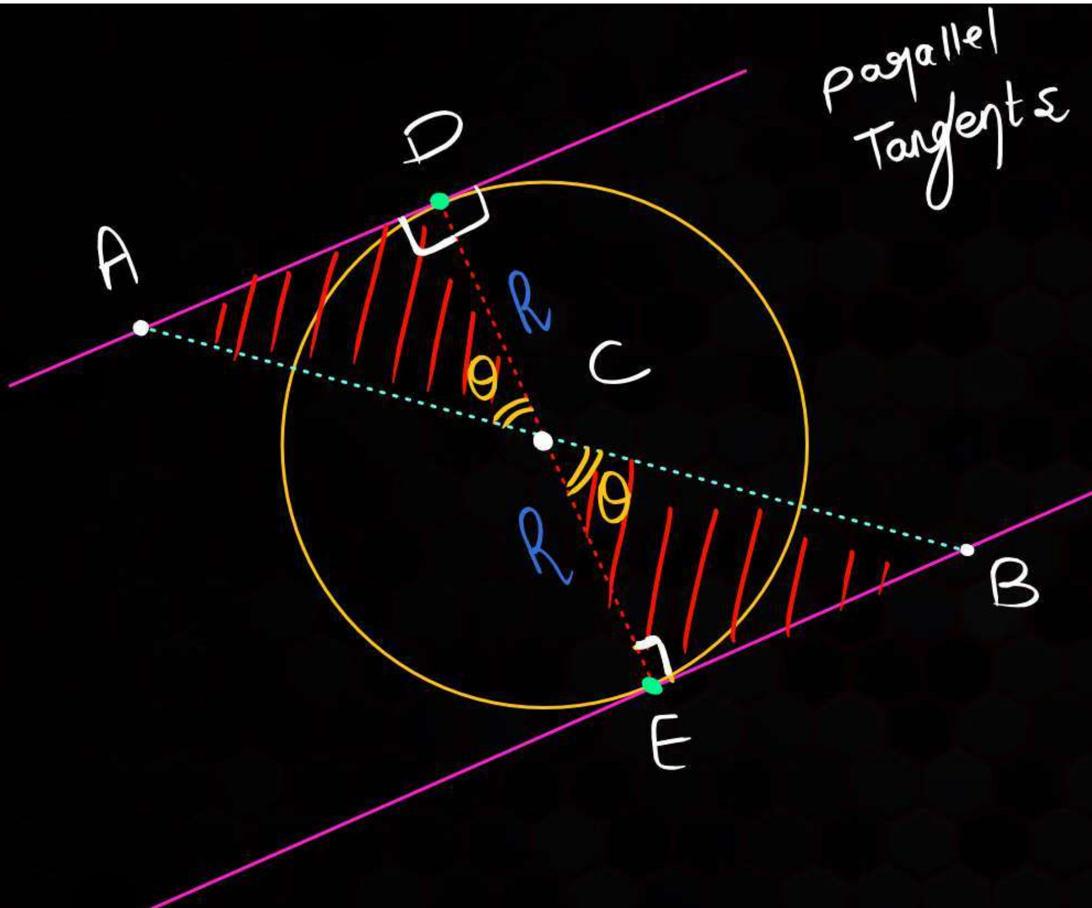




Centre-Tangent का ⊥R Distance $= p = R$
त्रिज्या



Distance between parallel tangents $= 2R$



$\boxed{\Delta ADC \cong \Delta BEC}$

$\boxed{AC = BC}$

OR,

'C' is the mid-point of AB....

DEFINITIONS OF CIRCLES

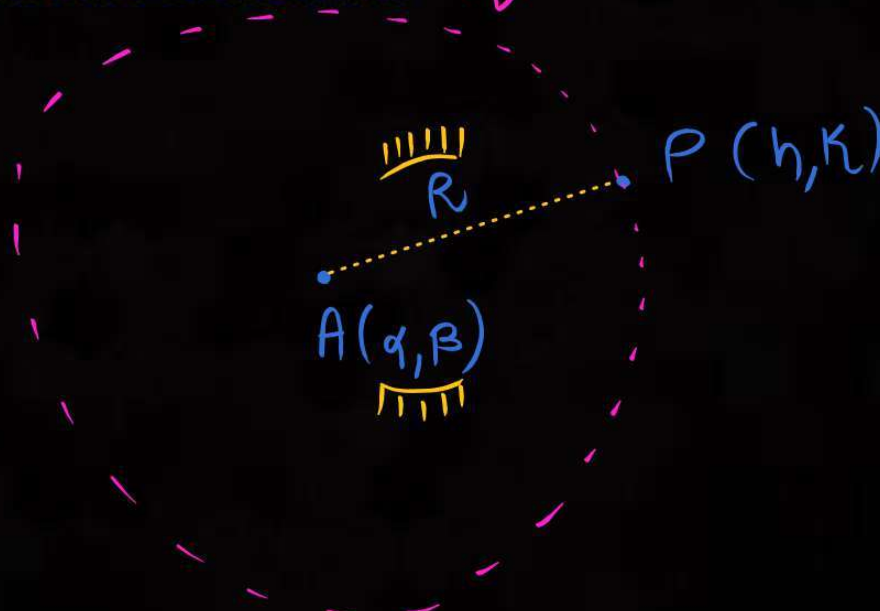
DEFINITION 1

Locus of a **moving point P** $\rightarrow (h, k)$ which moves in such a way that its **distance from a fixed point A** $\rightarrow (\alpha, \beta)$ is always a **Positive Constant.** $= R$

$$AP = R$$

$$(AP)^2 = R^2$$

$$(\underbrace{h}_{x} - \alpha)^2 + (\underbrace{k}_{y} - \beta)^2 = R^2$$



$$\boxed{(x-\alpha)^2+(y-\beta)^2=R^2}$$

α → centre का x-COORDINATE

β → centre का y-COORDINATE

R^2 → $(\text{Radius})^2$

HOW TO WRITE THE **EQUATION** OF A CIRCLE WHOSE **CENTER** & **RADIUS** ARE GIVEN?

✓ Centre $\equiv (-2,3)$

✓ Radius $\equiv$ 9 units

$$(x-(-2))^2+(y-3)^2=9^2$$

$$\boxed{(x+2)^2+(y-3)^2=81}$$

Ans

Q. The equation of the circle whose **two diameters** are the lines $x + y = 4$ and $x - y = 2$ and which passes through $(4,6)$ is

(A) $x^2 + y^2 - 6x - 2y - 16 = 0.$ ✓

(B) $x^2 + y^2 - 6x - 2y = 15$

(C) $x^2 + y^2 = 10$

(D) $5(x^2 + y^2) - 4x = 16$

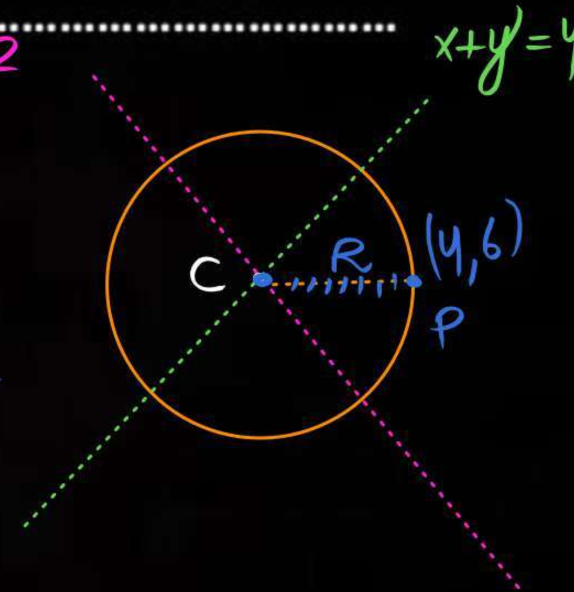
$x - y = 2$

$x = 3$

$y = 1$

$\boxed{C \equiv (3,1)}$

$CP = R = \sqrt{26}$



eqn →

$(x-3)^2 + (y-1)^2 = (\sqrt{26})^2$

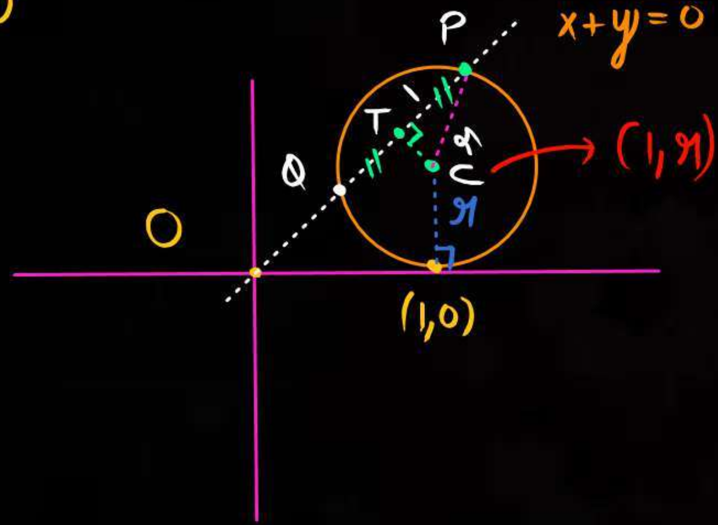
$\boxed{x^2 + y^2 - 6x - 2y - 16 = 0}$ Ans

Q. Let a circle $C : (x-h)^2 + (y-k)^2 = r^2$, $k > 0$, touch the x-axis at $(1, 0)$. If the line $x + y = 0$ intersects the circle C at P and Q such that the length of the chord PQ is 2, then the value of $h + k + r$ is equal to $3+3+1 = 7$ Ans.

$(h, k),\ R = r \qquad x+y=2$

[JEE Main 2022 (24 June – Shift 2)]

$r > 0$



$h = 1,\ k = r$

$$CT = \sqrt{r^2-1} = \left(\frac{|1+r|}{\sqrt{2}}\right)$$

sq

$$r^2 - 1 = \frac{r^2+2r+1}{2}$$

$$2r^2 - 2 = r^2 + 2r + 1$$

$$\boxed{r^2 - 2r - 3 = 0}$$

$r = 3, -1$

$h = 1$
$k = 3$
$r = 3$

Q. Let C be a circle passing through the points A(2, –1) and B(3, 4). The line segment AB is not a diameter of C. If r is the radius of C and its center lies on the circle $(x-5)^2+(y-1)^2 = 13/2$, then r^2 is equal to :

[JEE Main 2022 (26 June – Shift 1)]

$(5,1)$, $R_1 = \sqrt{\frac{13}{2}}$

$P \to (5,1)$

'C', $R = r$

(A) 32

(B) 65/2.

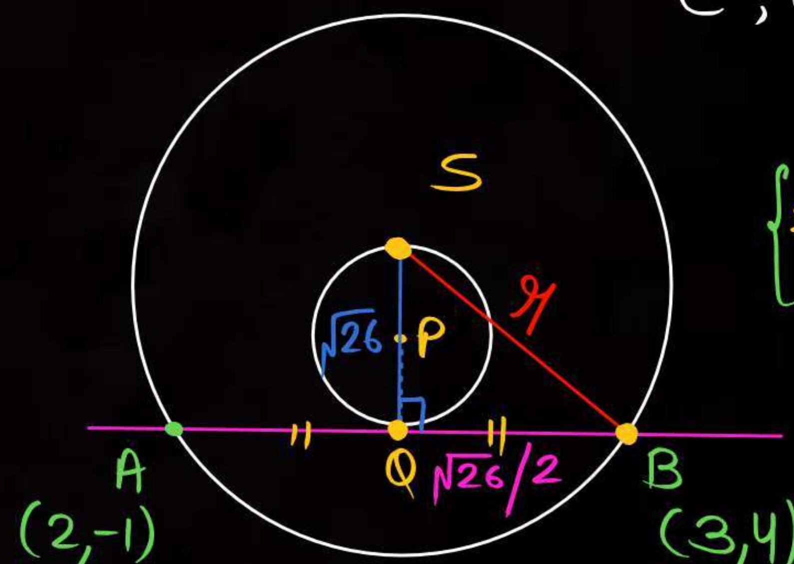
(C) 61/2

(D) 30

$AB = \sqrt{26}$

$PQ = \frac{|1-25+11|}{\sqrt{26}} = \frac{13}{\sqrt{26}} = \sqrt{\frac{13}{2}} = R_1$

$SQ = 2R_1 = 2\sqrt{\frac{13}{2}} = \sqrt{26}$



Using PT in ΔSQB,

$\left\{26 + \frac{26}{4} = r^2\right\}$ Ans

$y - 5x + 11 = 0$

Q. A circle touching the x-axis at $(3, 0)$ and making an intercept of length of 8 on the y-axis passes through the point : **[JEE (Main) 2019]**

$R^2 = 16 + 9$

$\boxed{R = 5}$

(A) ~~(1 , 5)~~

(B) ~~(2 , 3)~~

(C) ~~(3 , 5)~~

(D) (3 , 10). ✓

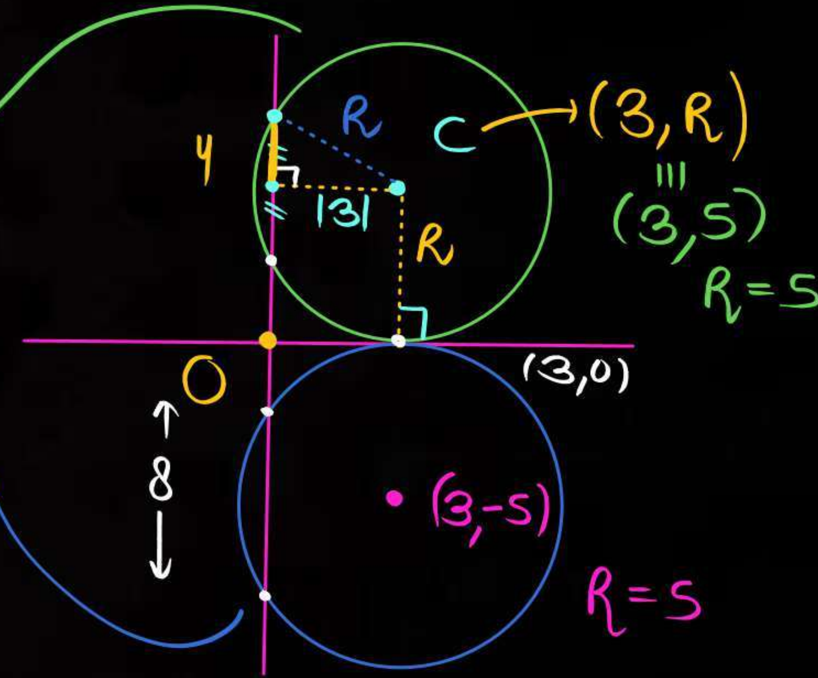
"Two CIRCLE"

$(x-3)^2 + (y-5)^2 = 25$

$(x-3)^2 + (y+5)^2 = 25$

Isko satisfy kiya

Satisfy nahi kiya



$P \to (\alpha, \beta)$

D_P - x axis $= |\beta|$

D_P - y axis $= |\alpha|$

Q. Let a circle C touch the lines $L_1 : 4x - 3y + K_1 = 0$ and $L_2 : 4x - 3y + K_2 = 0$, $K_1, K_2 \in R$. If a line passing through the Centre of the circle C intersects L_1 at $(-1, 2)$ and L_2 at $(3, -6)$, then the **equation** of the circle C is :

[JEE Main 2022 (25 June – Shift 1)]

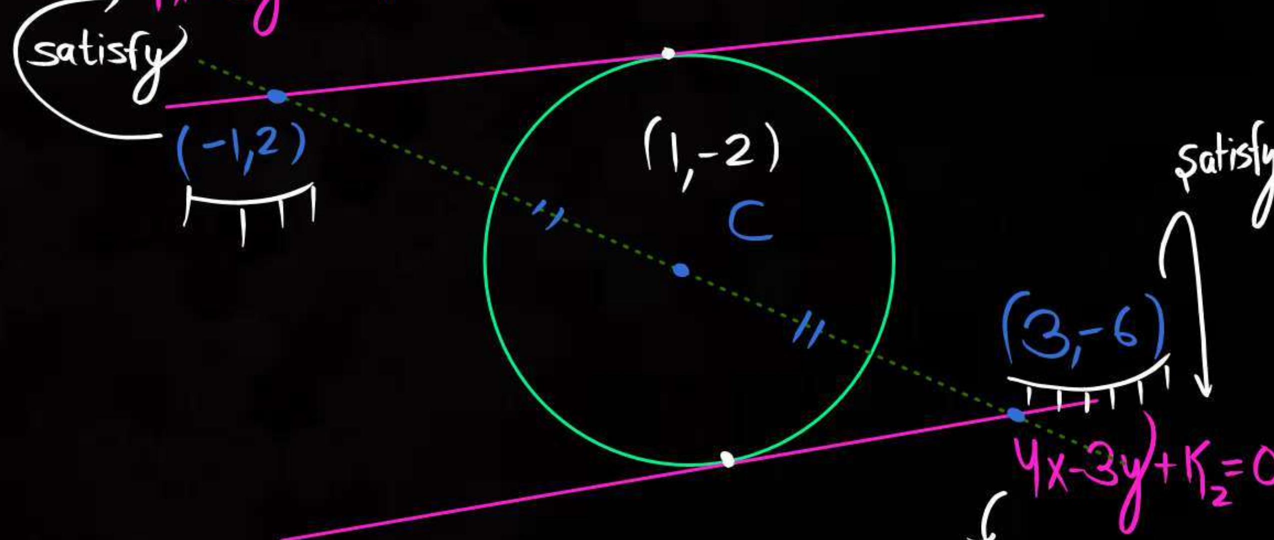
(A) $(x - 1)^2 + (y - 2)^2 = 4$

(B) $(x + 1)^2 + (y - 2)^2 = 4$

(C) $(x - 1)^2 + (y + 2)^2 = 16.$ ✓

(D) $(x - 1)^2 + (y - 2)^2 = 16$

$4x - 3y + K_1 = 0 \rightarrow -4 - 6 + K_1 = 0 \rightarrow \boxed{K_1 = 10}$



$12 + 18 + K_2 = 0$

$\boxed{K_2 = -30}$

$$\begin{cases} 4x-3y+10=0 \\ 4x-3y-30=0 \end{cases}$$

$$2R = \frac{40}{\sqrt{4^2+9}} \Rightarrow \boxed{R=4}$$

$(1,-2)$

Ans $\rightarrow (x-1)^2+(y-(-2))^2 = 4^2$

$x^2+y^2-2x+4y-11=0$

General Eqn of a circle

$$x^2 + y^2 + 2g\,x + 2f\,y + c = 0$$

Make sure coefficients of x^2 and y^2 are 'unity'

Centre $\equiv \left(\dfrac{2g}{-2}, \dfrac{2f}{-2}\right) \equiv (-g, -f)$

$R = \sqrt{g^2 + f^2 - c}$

Circle

$(-g, -f)$

$R = \sqrt{g^2+f^2-c}$

$$(x-(-g))^2+(y-(-f))^2 = g^2+f^2-c$$

$$x^2+\cancel{g^2}+2gx+y^2+\cancel{f^2}+2fy = \cancel{g^2}+\cancel{f^2}-c$$

$$\boxed{x^2+y^2+2gx+2fy+c=0}$$

FINDING CENTER & RADIUS OF A CIRCLE WHEN GENERAL EQUATION IS GIVEN…

① make coeff. of x^2 and $y^2 = 1$

② $\left(\frac{C_x}{-2}, \frac{C_y}{-2}\right) \equiv$ centre

③ $R = (g^2+f^2-c)^{1/2}$

Q → Find centre and Radius of

$$2x^2+2y^2-4x-10y-4=0$$

Sol^n →

$$x^2+y^2-2x-5y-2=0 \quad (c=-2)$$

$$\text{Centre} \equiv \left(\frac{-2}{-2}, \frac{-5}{-2}\right) \equiv \left(1, \frac{5}{2}\right)$$

$$R = \sqrt{1+\frac{25}{4}-(-2)} = \sqrt{3+\frac{25}{4}} = \frac{\sqrt{37}}{2} \text{ Ans}$$

DEFINITION 2

A 2 Degree Curve's General equation represents a circle iff :

$$ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$$

→ POSL ✓

→ Parabola ✓

→ Ellipse ✓

→ Hyperbola ✓

→ $\boxed{\text{CIRCLE}}$

$a = b$

$h = 0$

⭐⭐⭐

DEFINITION 3

Locus of a **moving point P** which moves in such a way that the **ratio** of its distance from two **fixed points A & B** is always **a positive constant** (**not 'unity'**).

$$\left(\frac{PA}{PB}\right) = \text{Positive Constant} (\neq 1)$$

'P' का Locus is a CIRCLE..

PROOF

$A \to (x_1, y_1)$

$B \to (x_2, y_2)$

$P \to (h, k)$

$$\boxed{\frac{PA}{PB} = k_1} \Rightarrow (PA)^2 = k_1^2 (PB)^2$$

$$(h-x_1)^2 + (k-y_1)^2 = k_1^2 \cdot \left((h-x_2)^2 + (k-y_2)^2\right)$$

$$(1-k_1^2)h^2 + (1-k_1^2)k^2 - 2hx_1 - 2ky_1 + x_1^2 + y_1^2$$

$$= k_1^2 x_2^2 + k_1^2 y_2^2 - 2hk_1^2 x_2 - 2k_1^2 k y_2$$

$$\underbrace{(1-k_1^2)}_{\neq 0}\overset{x^2}{h^2} + \underbrace{(1-k_1^2)}_{\neq 0}\underset{y^2}{k^2} - \underset{x}{h}(2x_1 + 2k_1^2 x_2)$$

$$-\underset{y}{k}(2y_1 + 2k_1^2 y_2)$$

$$+ (x_1^2 + y_1^2 - k_1^2 x_2^2 - k_1^2 y_2^2) = 0$$

(i) $C_{x^2} = C_{y^2} = (1-k_1^2)$

(ii) No xy terms

If $\boxed{\dfrac{PA}{PB} = K}$ $\longrightarrow$ 'P' का LOCUS is a CIRCLE..

$\{A, B \rightarrow \text{Fix pts}\}$

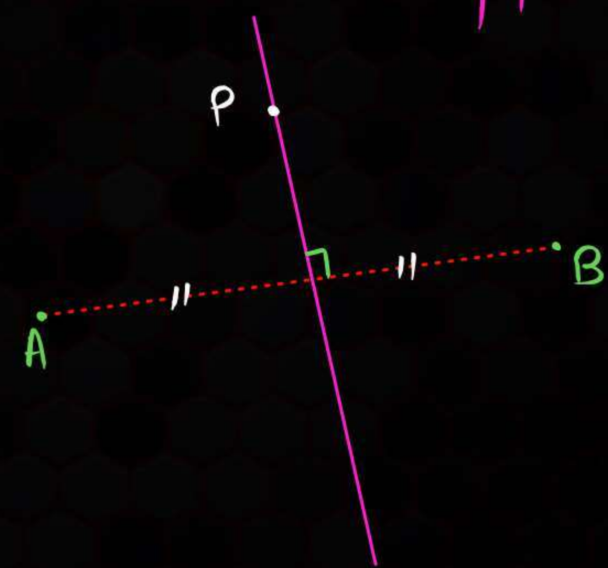
$$\left\{\begin{matrix} K > 0 \\ K \neq 1 \end{matrix}\right\}$$

If $\frac{PA}{PB} = 1 \implies \boxed{PA = PB}$ $\longrightarrow$ 'P' का Locus

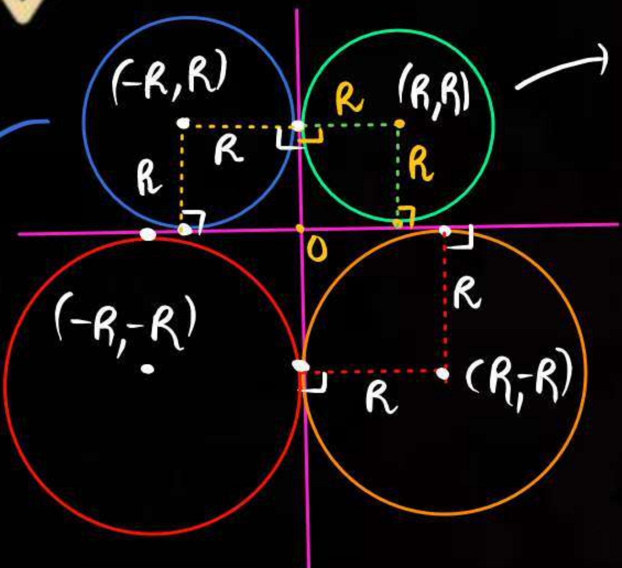
"$\perp^R$ Bisector of Line Segment AB"

$A, B \rightarrow$ Fix pt.

$P \rightarrow$ moving pt.



CIRCLE TOUCHING COORDINATE AXES



$$(x-R)^2+(y-R)^2=R^2$$

$(R>0)$

$$(x+R)^2+(y-R)^2=R^2$$

Q. Find the equation of circle in the first quadrant touching the line $3x + 4y = 12$ and the coordinate axes.

(R, R) ; Radius $= R > 0$

$$p = R$$

$$\frac{|3R + 4R - 12|}{5} = R$$

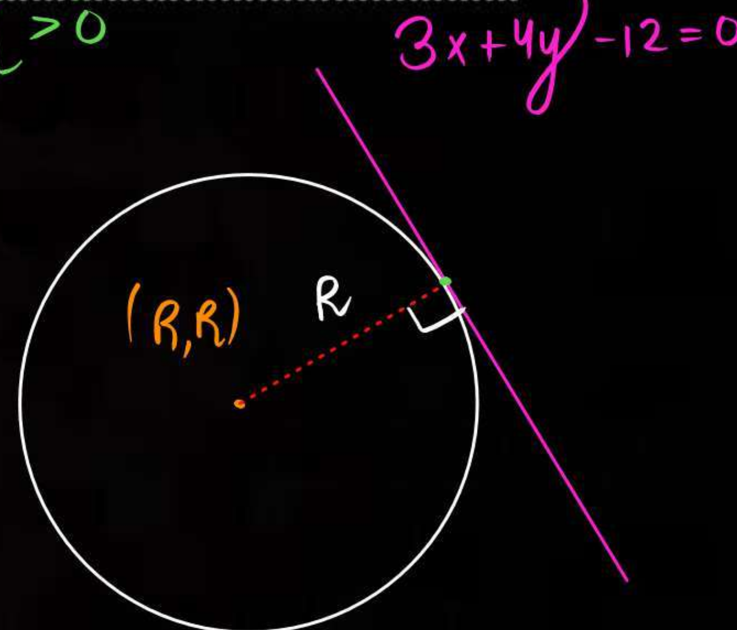
$$\boxed{|7R - 12| = 5R}$$

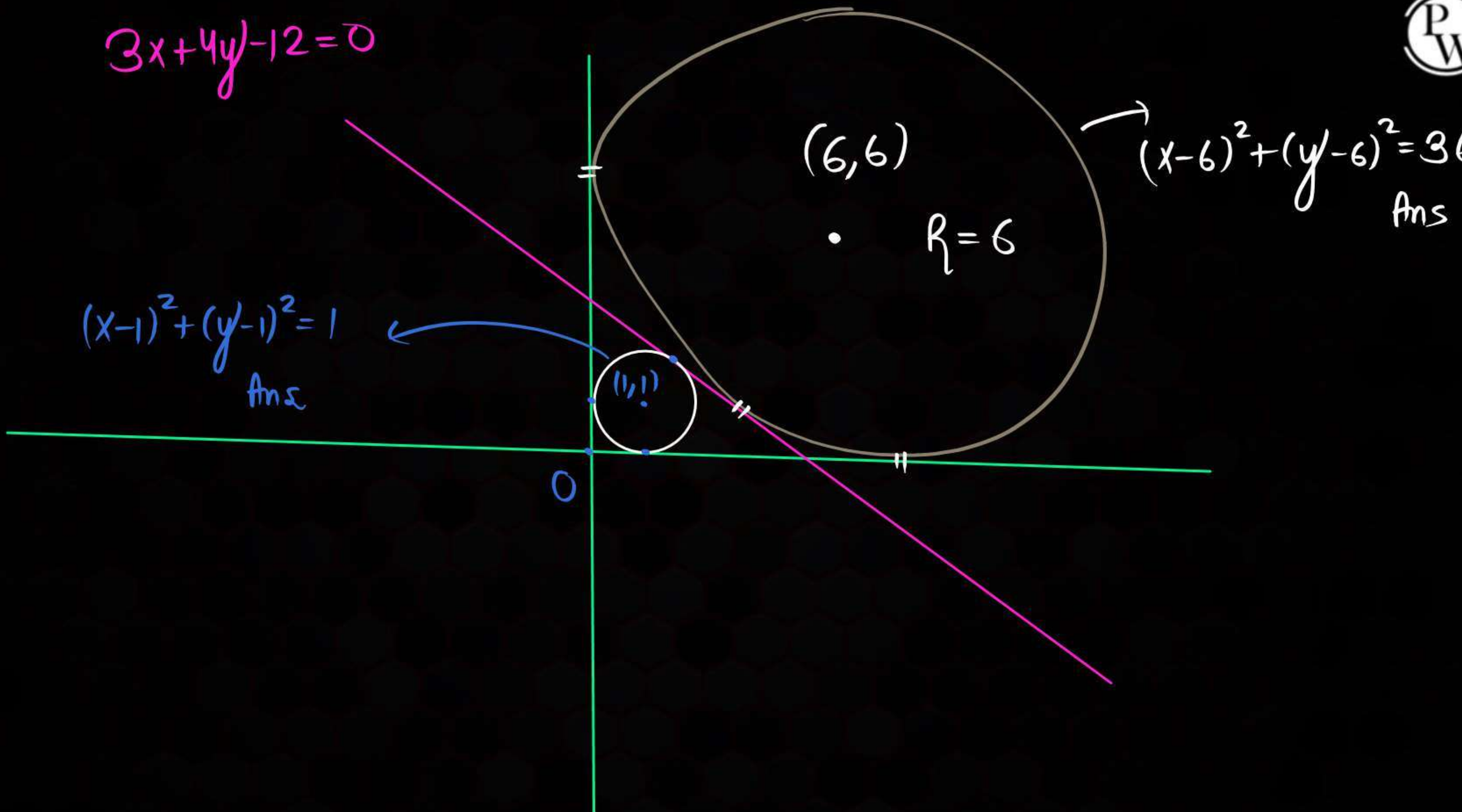
(+) $7R - 12 = 5R$

$$\boxed{R = 6}$$

(−) $7R - 12 = -5R$

$$\boxed{R = 1}$$





$3x+4y-12=0$
$(6,6)$
$R=6$
$(x-6)^2+(y-6)^2=36$
Ans
$(x-1)^2+(y-1)^2=1$
Ans
$(1,1)$
O

POWER OF A POINT

Make sure the coefficients of x^2 and y^2 in the circle's equation = 1

$S: x^2+y^2+2gx+2fy+c=0$

$P:(\alpha,\beta)$

$$\boxed{\text{Power of Point } P = S_1 = \alpha^2+\beta^2+2g\alpha+2f\beta+c}$$

$\boxed{S_1 > 0}$ → Point 'P' lies "outside" the circle. → "Hence 2 tangents can be drawn from 'P' to the circle."

$\boxed{S_1 = 0}$ → Point 'P' lies "on" the circle → "1 Tangents"

$\boxed{S_1 < 0}$ → Point 'P' lies "INSIDE" the circle. → "0 Tangents"

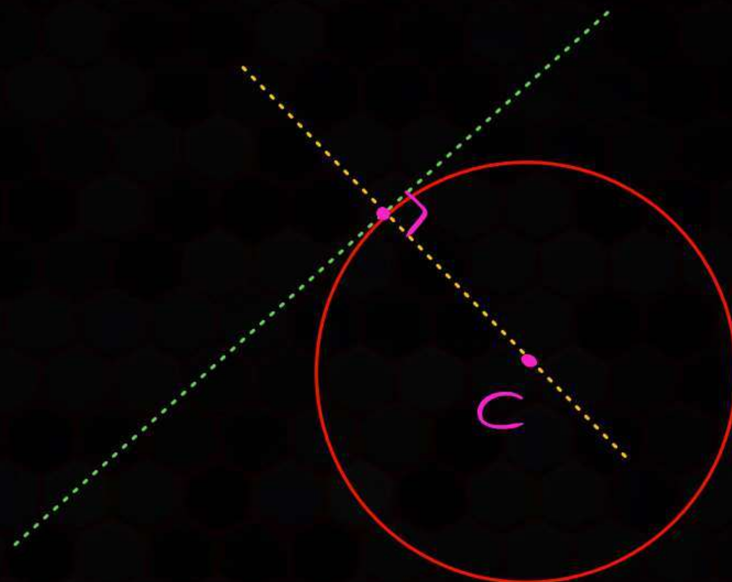
★★★★★★★★★★★★★★

Circle का Normal

"DIAMETER"

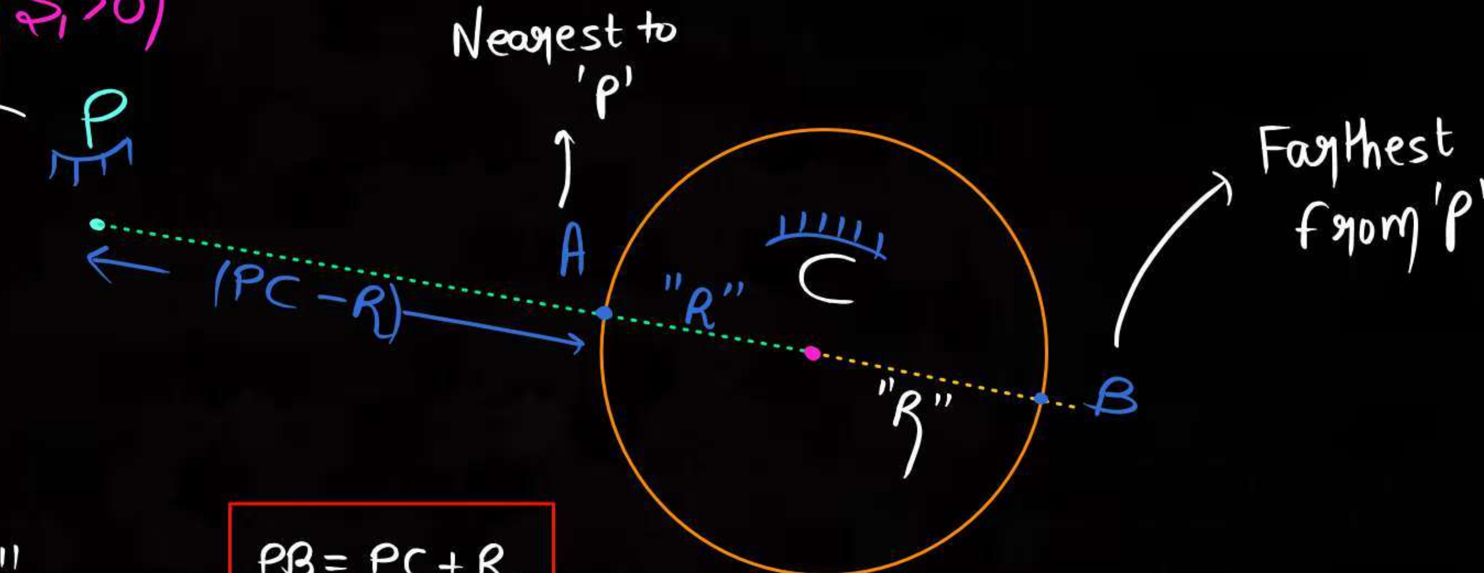
"longest chord"

★★★



POINTS NEAREST/FARTHEST TO A GIVEN POINT

$(S_1 > 0)$



" max^m and min^m Distance always occur along Normal"

$$\boxed{PB = PC + R}$$

Q. The co-ordinates of the point on the circle $\boxed{x^2 + y^2 - 12x + 4y + 30 = 0}$ which is **farthest** from the **origin** are

$(6,-2)$, $R = \sqrt{36+4-30} = \sqrt{10}$

$S_1 = 30 > 0$

(A) (9,-3).

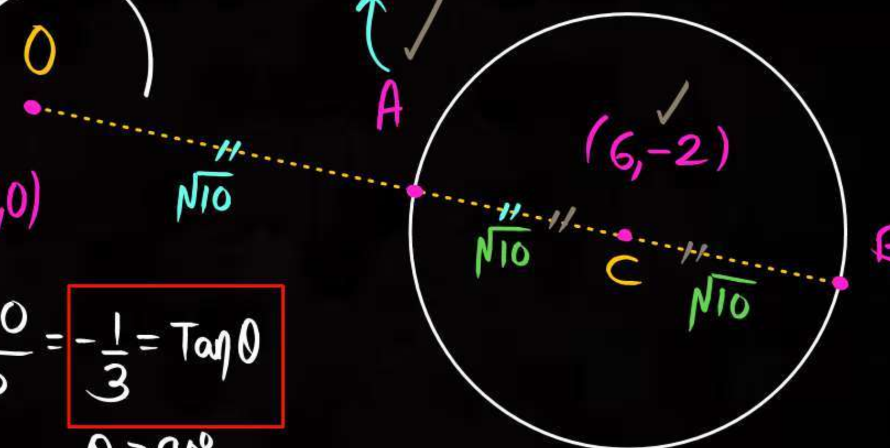
(B) (8,5)

(C) (12,4)

(D) None of These

$\boxed{OC = 2\sqrt{10}}$

$(3,-1)$



$(9,-3)$ Ans

$m = \dfrac{-2-0}{6-0} = \boxed{-\dfrac{1}{3} = \tan\theta}$

$\theta > 90^\circ$

$\sin\theta = 1/\sqrt{10}$

$\cos\theta = -3/\sqrt{10}$

$A, B \equiv \left(6 \pm \cancel{\sqrt{10}} \cdot \left(\dfrac{-3}{\cancel{\sqrt{10}}}\right), -2 \pm \cancel{\sqrt{10}} \left(\dfrac{1}{\cancel{\sqrt{10}}}\right)\right)$

(+) $(3,-1)$

(−) $(9,-3)$

STANDARD CIRCLE

Circle with center at (0,0) and radius = R.

$$(x-0)^2+(y-0)^2=R^2$$

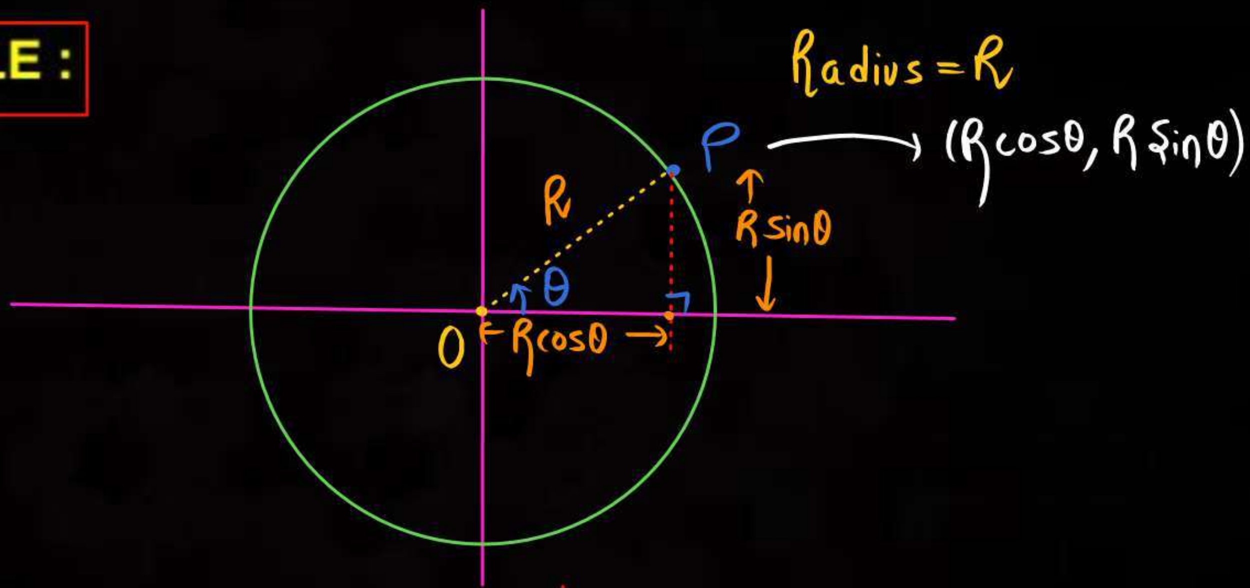
$$\boxed{x^2+y^2=R^2}$$

PARAMETRIC FORM

1. FOR STANDARD CIRCLE :

Any point on the standard CIRCLE $x^2+y^2=R^2$ can be assumed as:

$$(R\cos\theta, R\sin\theta)$$
$$\theta \in [0, 2\pi)$$



2. FOR GENERAL CIRCLE :

Centre $\equiv (-g, -f)$

$R = \sqrt{g^2+f^2-c}$

$$\boxed{x^2+y^2+2gx+2fy+c=0}$$

$$(x+g)^2+(y+f)^2=R^2$$

$$\begin{cases} x+g = X \\ y+f = Y \end{cases}$$

$$X^2+Y^2=R^2$$

$$(X, Y) \equiv (R\cos\theta, R\sin\theta)$$

$$X = R\cos\theta = x + g \longrightarrow x = -g + R\cos\theta$$

$$Y = R\sin\theta = y + f \longrightarrow y = -f + R\sin\theta$$

$\boxed{x^2 + y^2 + 2gx + 2fy + c = 0}$

Any point (x, y) lying on it can be assumed as:

$$(-g + R\cos\theta, -f + R\sin\theta)$$

$$R = (g^2 + f^2 - c)^{1/2}$$

BKG

$x^2+y^2=R^2 \longrightarrow (R\cos\theta, R\sin\theta)$

$(0,0)$

$x^2+y^2+2gx+2fy+c=0 \longrightarrow (-g+R\cos\theta, -f+R\sin\theta)$

$(-g, -f)$

Q. If $x^2 + y^2 - 6x + 4y - 3 = 0$, find the range of $E = 3x - 4y$.

"CIRCLE"

Advanced

$(3, -2)$

$-g \quad -f$

(x, y)

$\equiv (3 + R\cos\theta, -2 + R\sin\theta)$

$R = (9 + 4 + 3)^{1/2} = 4$

$E = 3(3 + R\cos\theta) - 4(-2 + R\sin\theta)$

$$\boxed{E = 9 + 8 + 12\cos\theta - 16\sin\theta}$$

$$E = 17 + 4\underbrace{(3\cos\theta - 4\sin\theta)}_{[-5,5]}$$

$a = 3,\ b = -4$

$[-5, 5]$

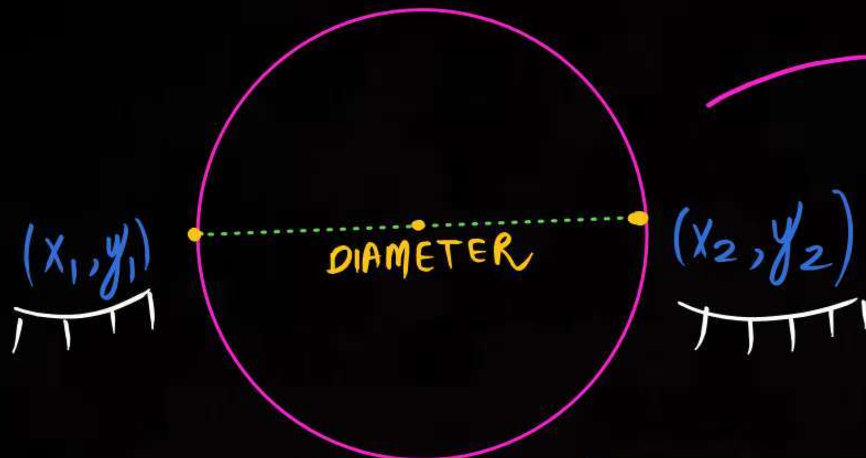
$[-20, 20]$

$$\boxed{E \in [-3, 37]}$$

Ans

$$(a\cos\theta + b\sin\theta)$$

$$\left[-\sqrt{a^2+b^2},\ \sqrt{a^2+b^2}\right]$$



$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$$

CIRCLE'S EQUATION WHEN ENDS OF DIAMETER ARE GIVEN

"JEE का Bahut Poorana Pyaar"

Q. Let the abscissae of the two points P and Q the roots of $2x^2 - rx + p = 0$ $\rightarrow x_1, x_2$ and the ordinates of P and Q be the roots of $x^2 - sx - q = 0$ $\rightarrow y_1, y_2$. If the equation of the circle described on PQ as diameter is $2(x^2 + y^2) - 11x - 14y - 22 = 0$, then $2r + s - 2q + p$ is equal to 7 Ans.

$22 + 7 - 22 = 7$

$2(x^2+y^2) - rx - 2sy + p - 2q = 0$ $\quad (p - 2q = -22)$

[JEE Main 2022 (25 June – Shift 1)]

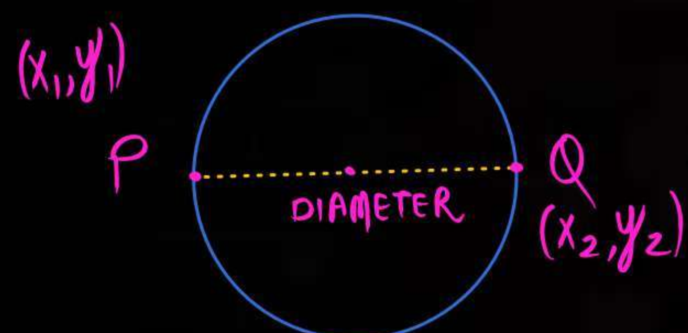
$r = 11,\ s = 7,$

$P \rightarrow (x_1, y_1)$

$Q \rightarrow (x_2, y_2)$

$2x^2 - rx + p = 0 \rightarrow x_1,\ x_2$

$$\boxed{(x - x_1)(x - x_2) = \frac{2x^2 - rx + p}{2}}$$



$y^2 - sy - q = 0 \rightarrow y_1,\ y_2$

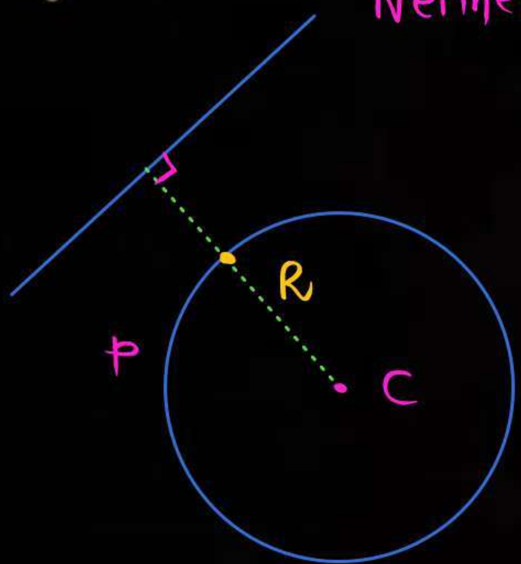
$$\boxed{(y - y_1)(y - y_2) = y^2 - sy - q}$$

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

Ans $$\boxed{\frac{2x^2 - rx + p}{2} + y^2 - sy - q = 0}$$

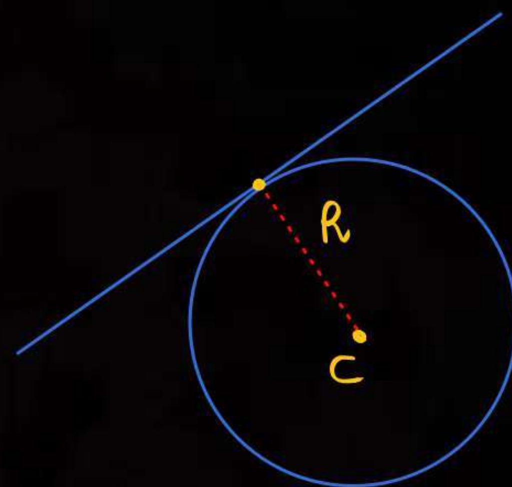
LINE & A CIRCLE

Neither secant nor Tangent



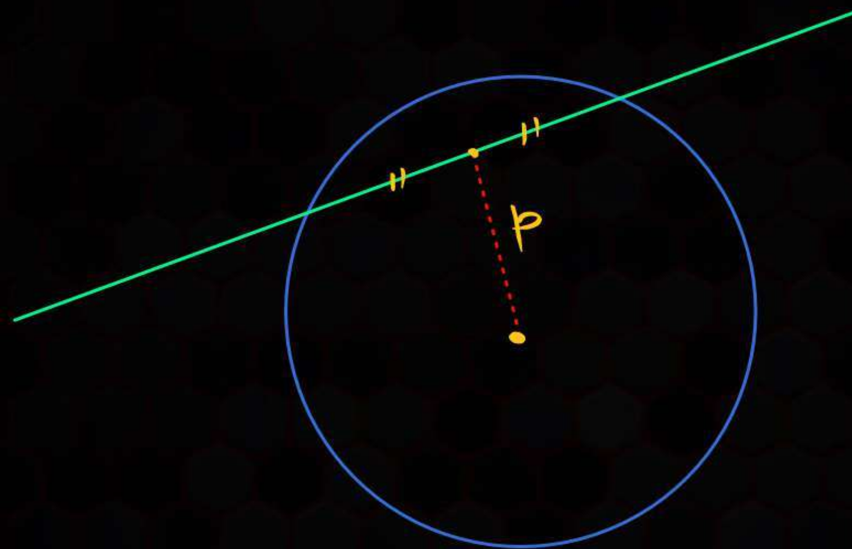
$\boxed{p > R}$

Tangent



$\boxed{p = R}$

$\boxed{p < R}$



secant

Q. Find λ if $\boxed{y = 2x + \lambda}$ line cuts to the circle $\boxed{(x-1)^2 + (y-1)^2 = 1}$

$p < R$

$$\frac{|1-2-\lambda|}{\sqrt{5}} < 1$$

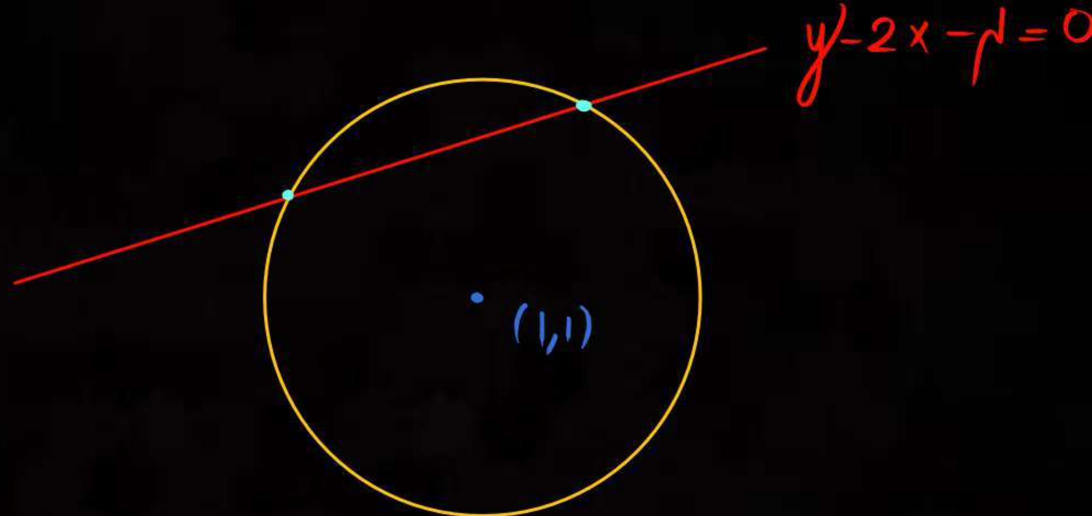
$$\boxed{|-1-\lambda| < \sqrt{5}}$$

$$-\sqrt{5} < -1-\lambda < \sqrt{5}$$

$$\boxed{1-\sqrt{5} < -\lambda < 1+\sqrt{5}}$$

$$\sqrt{5}-1 > \lambda > -(1+\sqrt{5})$$

Ans



Q. For what value of 'm' the line $3x - my + 5 = 0$ is tangent to the circle

$$x^2 + y^2 - 4x + 6y - 3 = 0\ ?$$

$(2, -3)$, $R = (4 + 9 - (-3))^{1/2} = 4$

$p = R$

$$\boxed{\frac{|6 + 3m + 5|}{\sqrt{9 + m^2}} = 4}$$

$\rightarrow (3m + 11)^2 = 16(9 + m^2)$

$9m^2 + 121 + 66m = 144 + 16m^2$

$$\boxed{7m^2 - 66m + 23 = 0}$$

Now $\boxed{K.K}$

Q. A circle touches both the y-axis and the line $x + y = 0$. Then the locus of its center is :

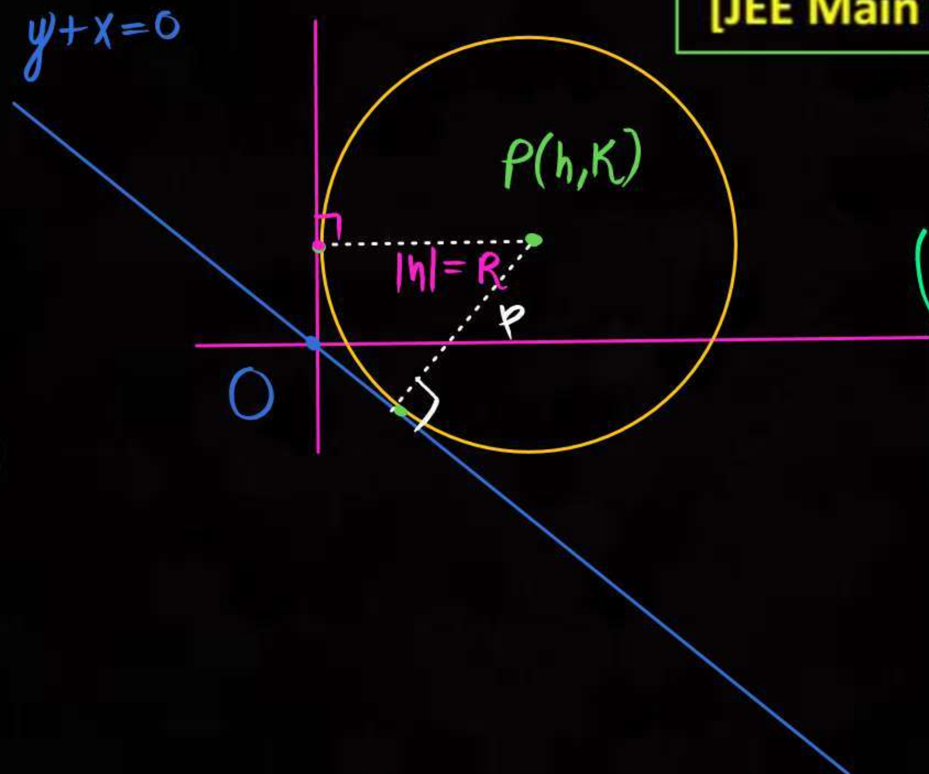
[JEE Main 2022 (25 June – Shift 2)]

(A) $y = \sqrt{2}x$

(B) $x = \sqrt{2}y$

(C) $y^2 - x^2 = 2xy.$

(D) $x^2 - y^2 = 2xy$ ✓



$p = R$

$$\frac{|h+K|}{\sqrt{2}} = |h|$$

$$h^2+K^2+2hK = 2h^2$$

$$2hK = h^2 - K^2$$

$$\boxed{2xy = x^2 - y^2}$$

Ans

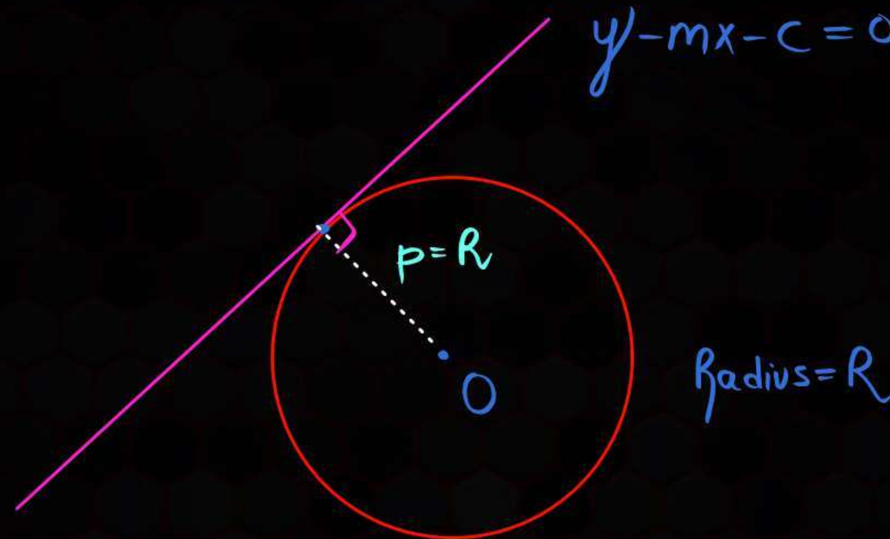
CONDITION OF TANGENCY

$\boxed{y = mx + c}$ 'c' की kis value k liye ; $y = mx + c$ is a Tangent to $x^2 + y^2 = R^2$?

$$\frac{|-c|}{\sqrt{1+m^2}} = R$$

$$c^2 = R^2(1+m^2)$$

$$\boxed{c = \pm R\sqrt{1+m^2}}$$



$$\boxed{x^2+y^2=R^2}$$

Tangent of slope 'm'

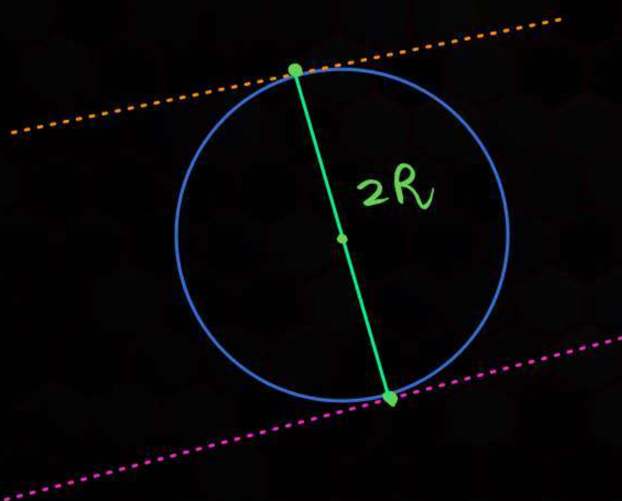
$$\boxed{y = mx \pm R\sqrt{1+m^2}}$$

(+)

$$y = mx + R\sqrt{1+m^2}$$

(−)

$$y = mx - R\sqrt{1+m^2}$$



Q → Find a tangent of slope $m=5$ for the $x^2+y^2=16$

$R=4$

Ans: $y = 5x \pm 4\sqrt{1+25}$

$\boxed{\text{Ans}}$

Q. Find the tangent of slope 2 for the circle $m=2$

(a) $x^2 + y^2 = 16$

(b) $(x-1)^2 + (y-1)^2 = 16$

(a) $y = 2x \pm 4\sqrt{1+4}$

$$\boxed{y = 2x \pm 4\sqrt{5}}$$ Ans

(b) $x-1 = X,\ y-1 = Y$

$$\boxed{X^2 + Y^2 = 16}$$

$Y = mX \pm 4\sqrt{1+m^2}$

$$\boxed{Y = 2X \pm 4\sqrt{5}}$$

$Y \to y-1,\ X \to x-1$

$$\boxed{(y-1) = 2(x-1) \pm 4\sqrt{5}}$$ Ans

Q. Find λ if $y = 2x + \lambda$ line cuts an intercept of length 2 units by the circle $x^2 + y^2 = 16$.

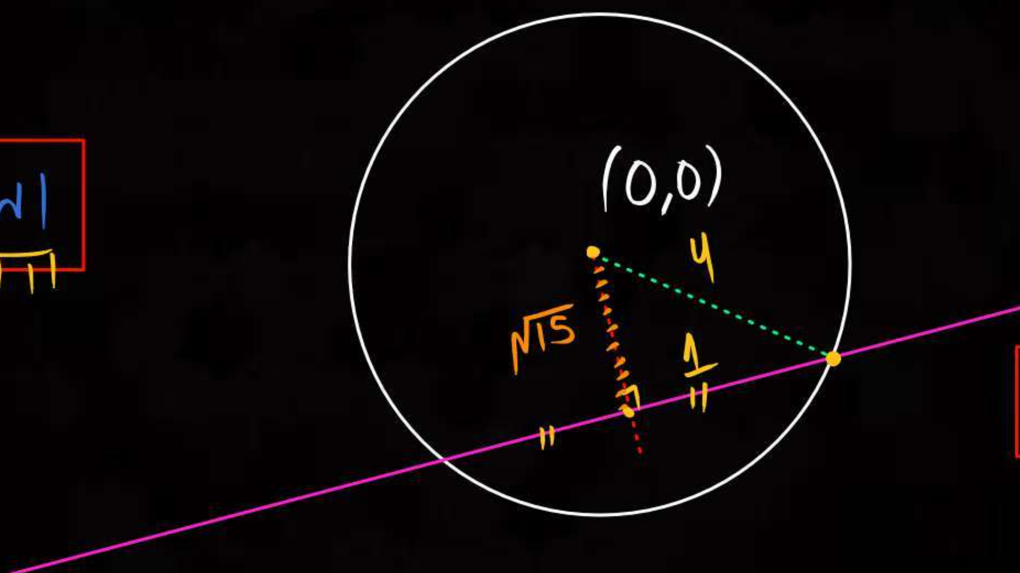
"Famous"

$$\sqrt{15} = \frac{|-\lambda|}{\sqrt{5}}$$

$$\boxed{5\sqrt{3} = |-\lambda| = |\lambda|}$$

$$\lambda = \pm 5\sqrt{3}$$

Ans



$y = 2x + \lambda$

$$\boxed{y - 2x - \lambda = 0}$$

(Diameter)

Q. Let the lines $\boxed{y + 2x = \sqrt{11} + 7\sqrt{7}}$ and $\boxed{2y + x = 2\sqrt{11} + 6\sqrt{7}}$ be **normal** to a circle C : $(x-h)^2 + (y-k)^2 = r^2$. If the line $\sqrt{11}y - 3x = \frac{5\sqrt{77}}{3} + 11$ is tangent to the circle C , then the value of $\boxed{(5h-8k)^2 + 5r^2}$ is equal to 816 Ans.

[JEE Main 2022 (28 June – Shift 1)]

(h, k) ; $R = r$

POI

$$\begin{cases} y + 2x = \sqrt{11} + 7\sqrt{7} \\ (2y + x = 2\sqrt{11} + 6\sqrt{7}) \end{cases}$$

$$4y + \cancel{2x} = 4\sqrt{11} + 12\sqrt{7}$$

"Subtract" $$y + \cancel{2x} = \sqrt{11} + 7\sqrt{7}$$

$$3y = 3\sqrt{11} + 5\sqrt{7}$$

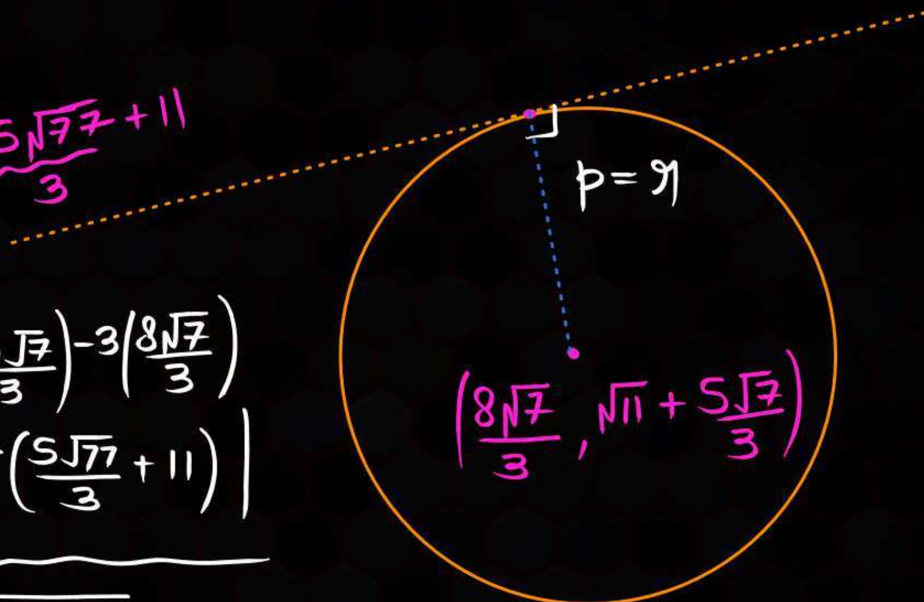
$$y = \boxed{\sqrt{11} + \frac{5}{3}\sqrt{7} = k}$$

$$x = \boxed{\frac{8\sqrt{7}}{3} = h}$$

Ans:

$$\boxed{\left(\frac{40\sqrt{7}}{\cancel{3}} - 8\sqrt{11} - \frac{40\sqrt{7}}{\cancel{3}}\right)^2 + 112 = 816 \text{ Ans}}$$

$$\sqrt{11}\,y - 3x = \frac{5\sqrt{77}}{3} + 11$$



$$p = r = \frac{\left|\sqrt{11}\left(\sqrt{11} + \frac{5\sqrt{7}}{3}\right) - 3\left(\frac{8\sqrt{7}}{3}\right) - \left(\frac{5\sqrt{77}}{3} + 11\right)\right|}{\sqrt{11+9}}$$

$$= \frac{4\sqrt{7}}{\sqrt{5}}$$

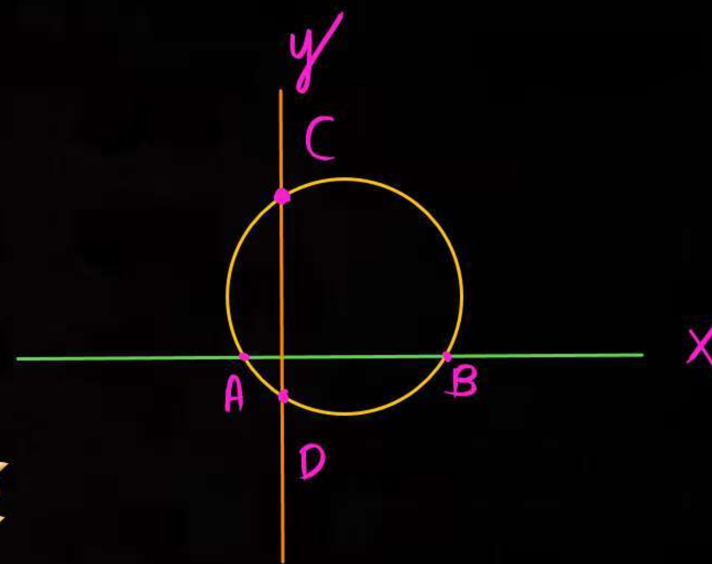
$$r^2 = \frac{16 \times 7}{5} = \frac{112}{5}$$

INTERCEPT MADE BY A CIRCLE ON AXES

$x^2+y^2+2gx+2fy+c=0$

AB = Intercept of x-axis $= 2\sqrt{g^2-c}$

CD = Intercept of y-axis $= 2\sqrt{f^2-c}$



Q. If the circle $\boxed{x^2 + y^2 - 2gx + 6y - 19c = 0}$, g, c ∈ R passes through the point $\boxed{(6, 1)}$ and its center lies on the line $\boxed{x - 2cy = 8}$, then the length of **intercept** made by the circle on **x-axis** is :

$+2gx \quad 2fy \quad + c$

$(2\sqrt{g^2 - c}) \Rightarrow 2\sqrt{g^2 - (-19c)} = 2\sqrt{g^2 + 19c} = 2\sqrt{4+19}$

$= 2\sqrt{23}$

Ans

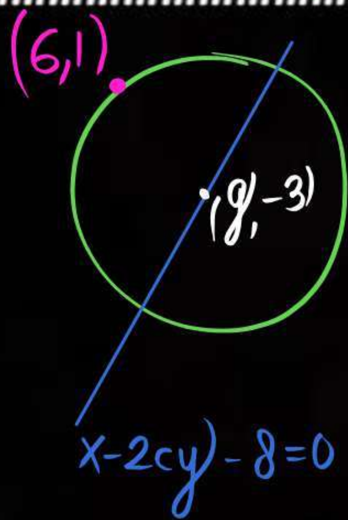
[JEE Main 2022 (27 July – Shift 1)]

(A) $\sqrt{11}$

(B) 4

(C) 3

(D) $2\sqrt{23}$.



$\longrightarrow 36 + 1 - 12g + 6 - 19c = 0$

$\boxed{-12g - 19c + 43 = 0}$

$g = 2, \; c = 1$

$\longrightarrow \boxed{g + 6c - 8 = 0}$

Very Important

Bahut Bhaukaal Sawaal aate hai..

POINT & A CIRCLE

Q. P: (a, b)

S: $(x - x_0)^2 + (y - y_0)^2 = R^2 \longrightarrow (x - x_0)^2 + (y - y_0)^2 - R^2 = 0$

Let PA and PB are 2 tangents from any Point P to the circle, then find :

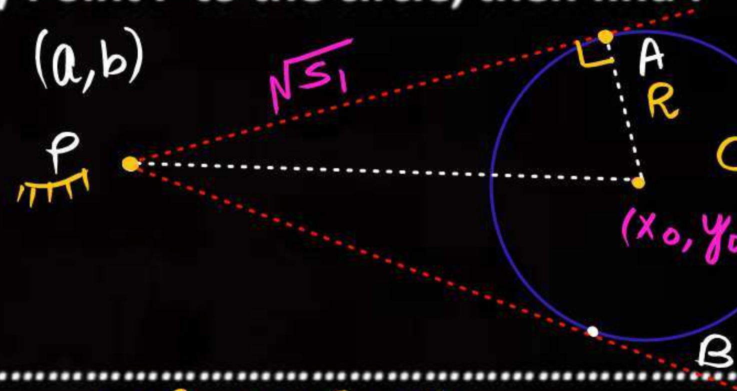
$(PA = PB)$ (1) Length of the tangents

(2) Angle between the tangents.

(3) Length of chord AB

(4) Area of ΔPAB

(5) Circum-Center of ΔPAB



R

$$\boxed{S_1 = (a - x_0)^2 + (b - y_0)^2 - R^2}$$

$$(PA)^2 = (PC)^2 - R^2$$

$$(PB)^2 = \boxed{(PA)^2 = (a - x_0)^2 + (b - y_0)^2 - R^2}$$

$$(PA)^2 = S_1$$

$$\boxed{PA = \sqrt{S_1} = l}$$

★★★

Q. P: (a , b) $S_1 > 0$

S: $(x - x_0)^2 + (y - y_0)^2 = R^2$

Let PA and PB are 2 tangents from any Point P to the circle, then find :

(1) Length of the tangents

(2) Angle between the tangents.

(3) Length of chord AB

(4) Area of ΔPAB

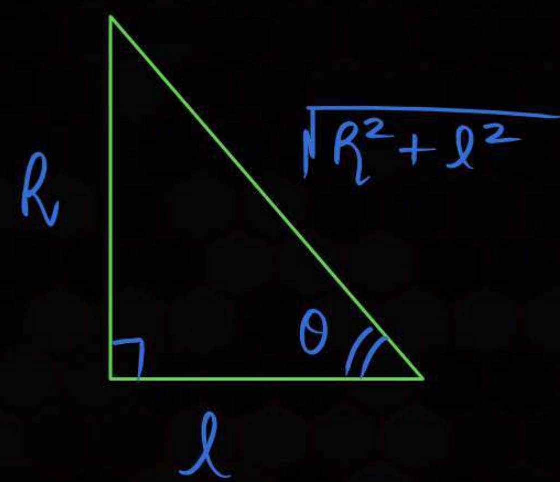
(5) Circum-Center of ΔPAB

$$\boxed{\tan\theta = R/l}$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2R/l}{1-R^2/l^2}$$

$$\boxed{\tan 2\theta = \frac{2Rl}{l^2 - R^2}}$$

$$\boxed{\tan\theta = \frac{R}{l}}$$



$$\boxed{\cos\theta = \frac{l}{\sqrt{R^2+l^2}}}$$

$$\boxed{\sin\theta = \frac{R}{\sqrt{R^2+l^2}}}$$

P: (a , b) $S_1 > 0$

Q. S: $(x - x_0)^2 + (y - y_0)^2 = R^2$

Let PA and PB are 2 tangents from any Point P to the circle, then find :

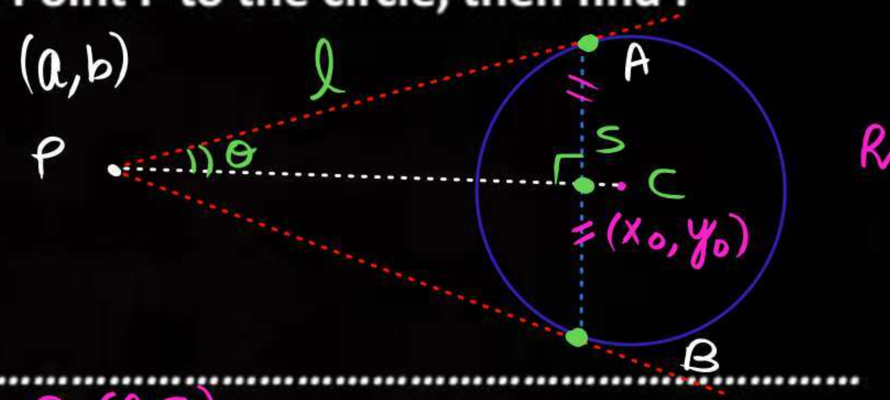
(1) Length of the tangents

(2) Angle between the tangents.

(3) Length of chord AB

$\boxed{\text{(4) Area of } \Delta PAB}$

(5) Circum-Center of ΔPAB



$AB = 2(AS)$

$= 2(l \sin\theta) = \dfrac{2lR}{\sqrt{R^2 + l^2}}$ units

Q. P: (a , b) $S_1 > 0$

S: $(x - x_0)^2 + (y - y_0)^2 = R^2$

Let PA and PB are 2 tangents from any Point P to the circle, then find :

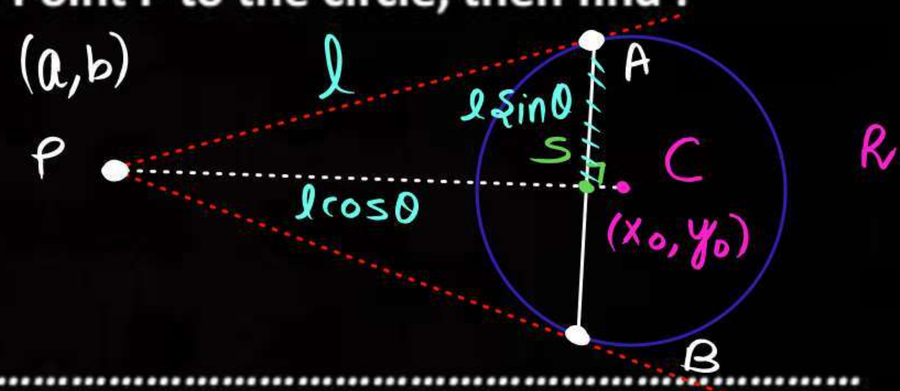
(1) Length of the tangents

(2) Angle between the tangents.

(3) Length of chord AB

★★ $\boxed{\text{(4) Area of } \Delta PAB}$

(5) Circum-Center of ΔPAB



$$\frac{1}{2}(AB)(PS) = \frac{1}{2}(2l\sin\theta)(l\cos\theta)$$

$$= \frac{l^2}{2}\sin 2\theta = \frac{l^2}{\cancel{2}}\left(\frac{\cancel{2}\tan\theta}{1+\tan^2\theta}\right)$$

$$\boxed{\text{area} = \frac{R\,l^3}{R^2+l^2}\ \text{sq units}}$$

★★★★★

Q.

P: (a, b) $S_1 > 0$

S: $(x - x_0)^2 + (y - y_0)^2 = R^2$

Let PA and PB are 2 tangents from any Point P to the circle, then find :

(1) Length of the tangents

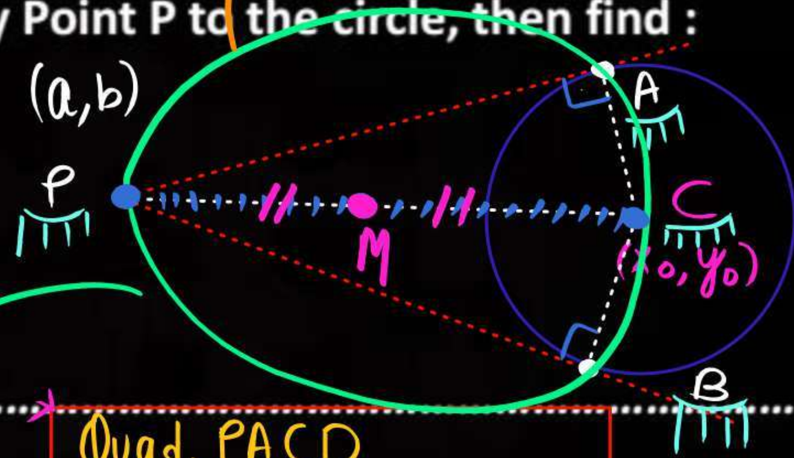
(2) Angle between the tangents.

(3) Length of chord AB

(4) Area of ΔPAB

(5) Circum-Center of ΔPAB

" CIRCUMCIRCLE OF Δ PAB"



PC → Diameter

'M' → center

"mid-pt of PC"

Ans

Quad. PACD

↳ cyclic Quad.

Q. Let the tangents at two points A and B on the circle $x^2 + y^2 - 4x + 3 = 0$ meet at origin O (0, 0). Then the area of the triangle of OAB is:

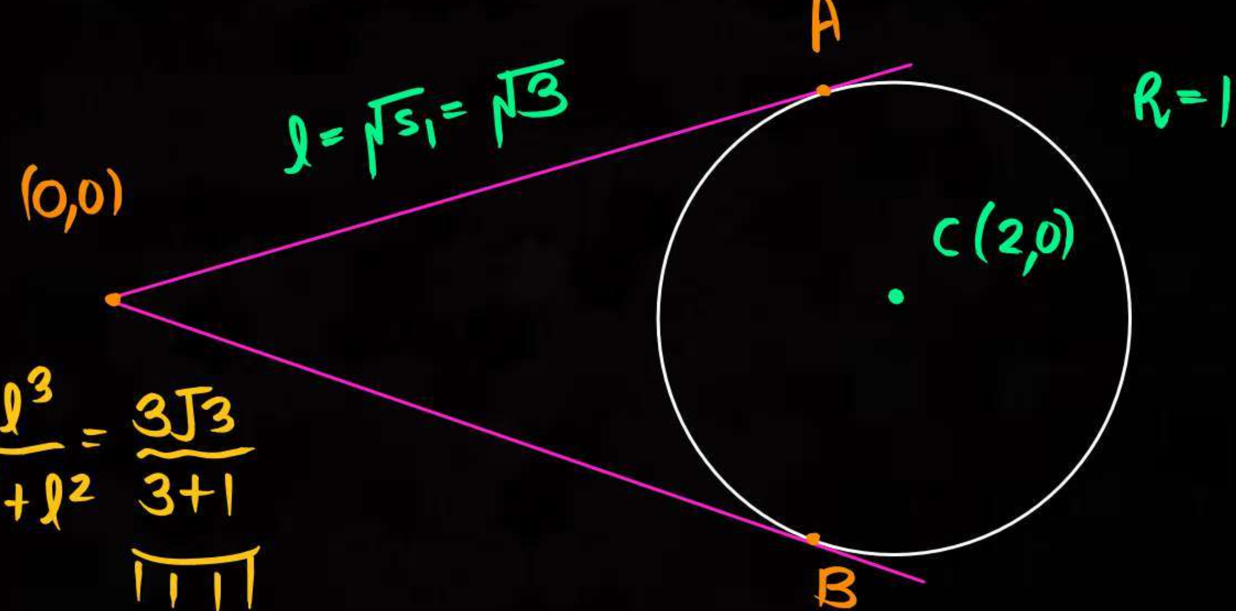
[JEE Main 2022 (28 July – Shift 2)]

(A) $\frac{3\sqrt{3}}{2}$

(B) $\frac{3\sqrt{3}}{4}$ ✓

(C) $\frac{3}{2\sqrt{3}}$

(D) $\frac{3}{4\sqrt{3}}$



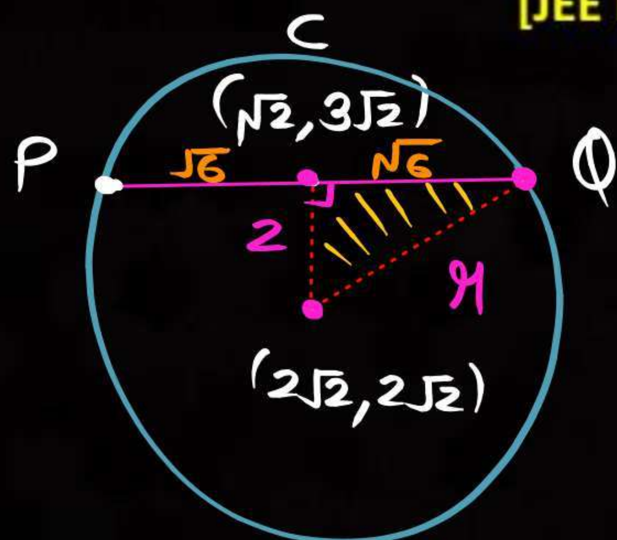
$$\frac{R l^3}{R^2 + l^2} = \frac{3\sqrt{3}}{3+1}$$

Ans

Q. If one of the diameters of the circle $\boxed{x^2 + y^2 - 2\sqrt{2}x - 6\sqrt{2}y + 14 = 0}$ is a chord of the circle $\underbrace{\left(x - 2\sqrt{2}\right)^2 + \left(y - 2\sqrt{2}\right)^2 = r^2}$, then the value of r^2 is equal to 10 _______________.

$R = (2+18-14)^{1/2}$
$= \sqrt{6}$

[JEE Main 2022 (28 June – Shift 2)]



$r^2 = 4 + 6 = 10$

Ans

It is the locus of an external point P(h,K) from where $\perp^{R}$ tangents can be drawn to a given CIRCLE.

DIRECTOR CIRCLE

$\boxed{\text{Concentric with the given CIRCLE.}}$

$\boxed{R_{DC} = \sqrt{2}\, R_{OC}}$

$$\boxed{x^2+y^2+2x+4y-5=0}$$

$(-1,-2)$

$$R_{oc}=\sqrt{1+4+5}$$
$$=\sqrt{10}$$

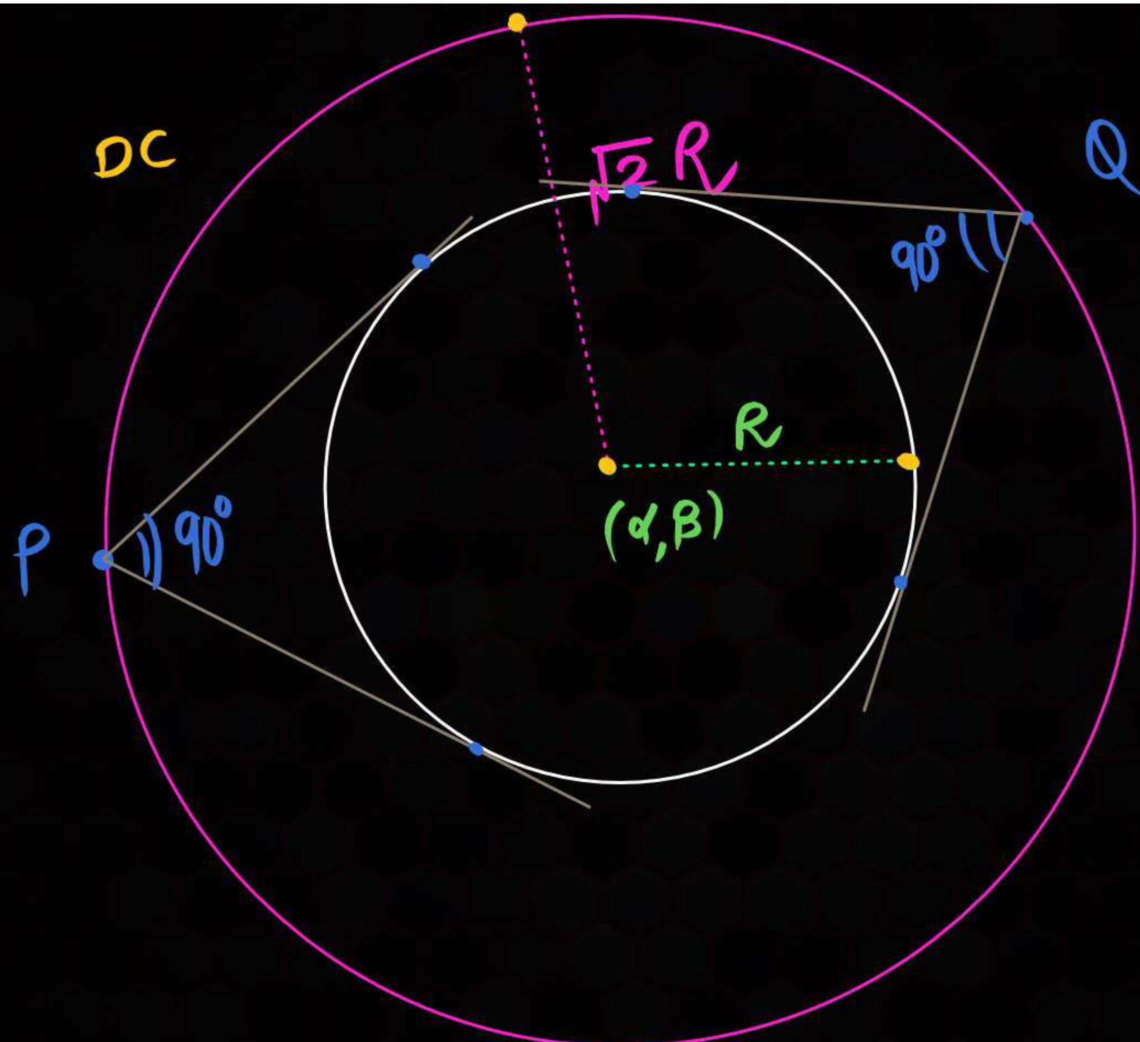
"DIRECTOR CIRCLE"

$(-1,-2)$ ✓

$R_{DC}=\sqrt{20}$ ✓

$$\boxed{(x+1)^2+(y+2)^2=20}$$

Ans



"Take any pt. on the DC, The tangents drawn from this pt to white CIRCLE will always be $\perp^R$."

Q → Find the angle b/w the tangents drawn from $\underbrace{P(1,2)}$ to the CIRCLE $\underbrace{(x-3)^2+(y-5)^2=\frac{13}{2}}$.

JEE advanced

sol^n → $\underbrace{DC}$ → $(3,5)$

$\underbrace{R_{DC}} = \sqrt{2}\cdot\sqrt{\frac{13}{2}} = \sqrt{13}$

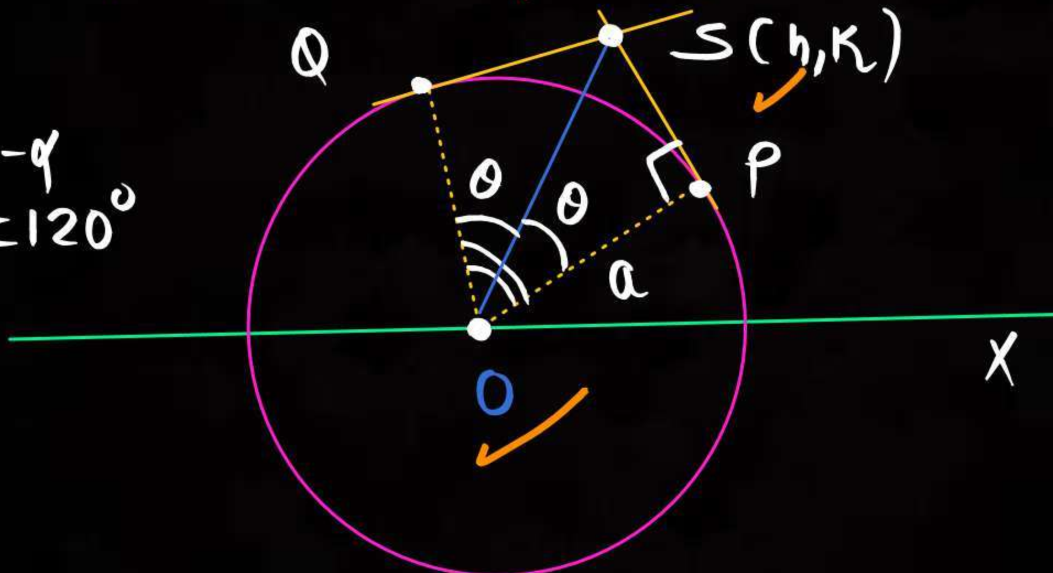
eq^n: $(x-3)^2+(y-5)^2=13$ ← $(1,2)$ lies on it.

$$\boxed{Ans = 90^\circ}$$

Q. Find the locus point of intersection of the pair of tangent drawn to a circle $x^2 + y^2 = a^2$ at $P(\alpha)$ and $Q(\beta)$, where $|\alpha - \beta| = 120°$

$(a\cos\alpha, a\sin\alpha)$ $(a\cos\beta, a\sin\beta)$

$\boxed{\alpha - \beta = \pm 120°}$



$$2\theta = \beta - \alpha = \pm 120°$$

$$\boxed{\theta = \pm 60°}$$

$$\cos\theta = \frac{1}{2}$$

$$\cos\theta = \frac{OP}{OS} = \frac{1}{2}$$

$$\boxed{4(OP)^2 = (OS)^2}$$

$$4a^2 = h^2 + k^2$$

$$\boxed{x^2 + y^2 = 4a^2}$$ Ans

'T'

TECHNICALLY LINEAR

TRICK / SHORTCUT to

WRITE Tangent's eqn, C.O.C.,

Chord whose mid-pt is given,

Pair of Tangent.

$$\boxed{P \to (x_1, y_1)}$$

$$x^2 \longrightarrow x \cdot x_1$$

$$y^2 \longrightarrow y \cdot y_1$$

$$x \longrightarrow \frac{x + x_1}{2}$$

$$y \longrightarrow \frac{y + y_1}{2}$$

$$xy \longrightarrow \frac{x_1 y + x y_1}{2}$$

$$c \longrightarrow c$$

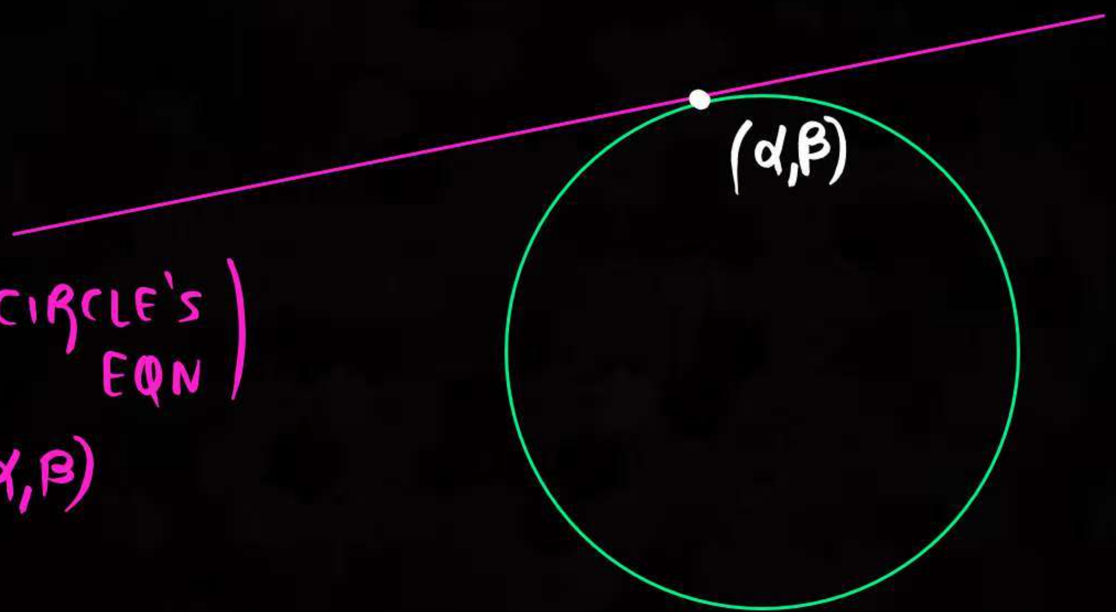
WRITE 'T' FOR $x^2+3y^2+xy+2x-y+7=0$ WRT $(1,2)$

$(1,2) \to x_1, y_1$

Ans

$$x(1)+3(y)(2)+\frac{1(y)+2(x)}{2}+2\left(\frac{x+1}{2}\right)-\left(\frac{y+2}{2}\right)+7=0$$

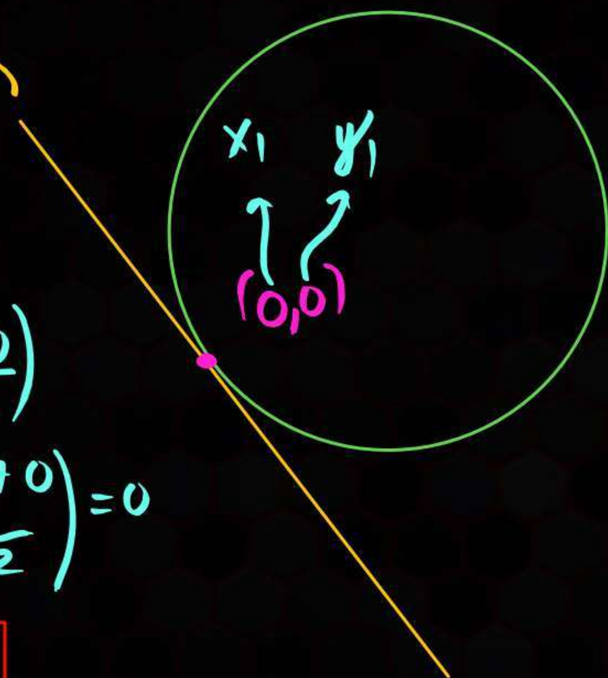
EQUATION OF TANGENT



$T=0$ (CIRCLE'S EQN WRT (α,β))

Ex →

$$x^2+y^2+7x+9y=0$$



T=0

$$0x+0y+7\left(\frac{x+0}{2}\right)+9\left(\frac{y+0}{2}\right)=0$$

$$\boxed{\frac{7x}{2}+\frac{9y}{2}=0}$$

Ans

Q. If the tangents drawn at the point $O(0, 0)$ and $P\left(1+\sqrt{5}, 2\right)$ on the circle $x^2 + y^2 - 2x - 4y = 0$ intersect at the point Q, then the area of the triangle OPQ is equal to :

$R = \sqrt{1+4} = \sqrt{5}$

$\dfrac{Rl^3}{R^2+l^2}$

[JEE Main 2022 (28 June – Shift 1)]

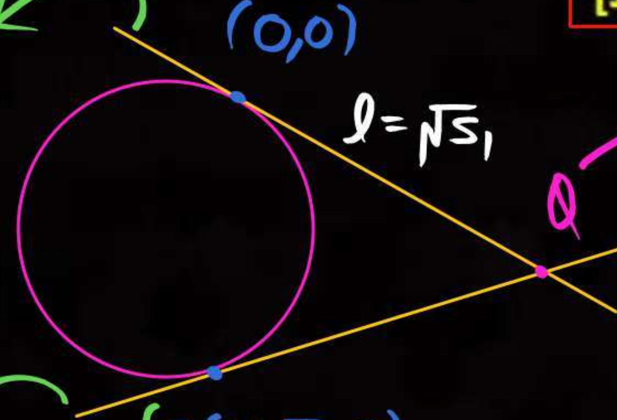
(A) $\frac{3+\sqrt{5}}{2}$

(B) $\frac{4+2\sqrt{5}}{2}$

(C) $\frac{5+3\sqrt{5}}{2}$

(D) $\frac{7++\sqrt{5}}{2}$

$x+2y=0$ ✓



$\sqrt{5}x-5-\sqrt{5}=0$

$\boxed{x-\sqrt{5}-1=0}$ ✓

$Q \rightarrow \left(\sqrt{5}+1, -\left(\frac{\sqrt{5}+1}{2}\right)\right)$

$\sqrt{S_1} = l = \sqrt{(\sqrt{5}+1)^2+\left(\frac{\sqrt{5}+1}{2}\right)^2 - 2(\cancel{\sqrt{5}}+1) + 2(\cancel{\sqrt{5}}+1)}$

$= \left(\frac{5+\sqrt{5}}{2}\right)$ ✓

$$\text{Ans} = \frac{(\sqrt{5})\left(\frac{5+\sqrt{5}}{2}\right)^3}{5+\left(\frac{5+\sqrt{5}}{2}\right)^2}$$

$\boxed{\text{Ans}}$

$\boxed{\text{Calculation ♡ ♡}}$

Q. Let the tangent to the circle $C_1 : x^2 + y^2 = 2$ at the point $M(-1, 1)$ intersect the circle $C_2 : (x - 3)^2 + (y - 2)^2 = 5$, at two distinct points A and B. If the tangents to C_2 at the points A and B intersect at N, then the area of the triangle ANB is equal to :

[JEE Main 2022 (29 June – Shift 1)]

$\boxed{\text{K.K}}$

(A) 1/2

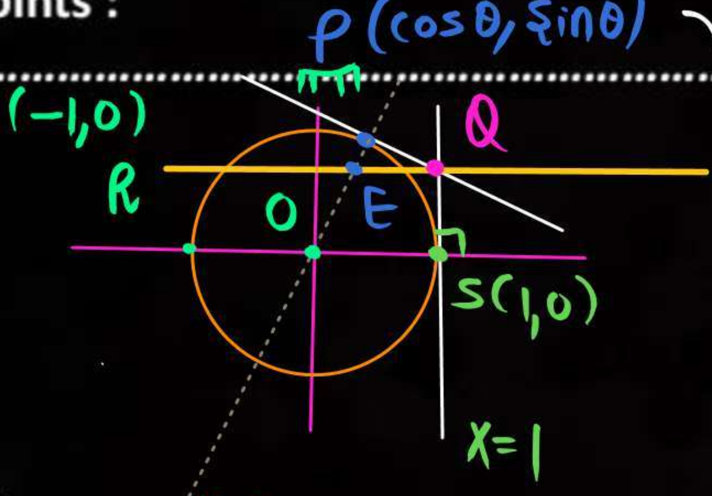
(B) 2/3

(C) 1/6.

(D) 5/3

Q. Let RS be the diameter of the circle $x^2 + y^2 = 1$, where S is the point $(1, 0)$. Let P be a variable point other than R and S on the circle and tangents to the circle at S and P meet at the point Q. The normal to the circle at P intersects a line drawn through Q parallel to RS at point E. Then the locus of E passes through the points :

[JEE Advanced 2016]



(A) $\left(\frac{1}{3}, \frac{1}{\sqrt{3}}\right)$.

(B) $\left(\frac{1}{4}, \frac{1}{2}\right)$

(C) $\left(\frac{1}{3}, -\frac{1}{\sqrt{3}}\right)$.

(D) $\left(\frac{1}{4}, \frac{-1}{.2}\right)$

Tangent at P $\Rightarrow T = 0$

$$\boxed{x\cos\theta + y\sin\theta - 1 = 0}$$

$x = 1$

$Q \rightarrow \left(1, \tan\frac{\theta}{2}\right)$

$$y = \frac{1-\cos\theta}{\sin\theta} = \tan\frac{\theta}{2}$$

$$\frac{y-0}{x-0} = \frac{\sin\theta}{\cos\theta}$$

$N_P \rightarrow \boxed{y = (\tan\theta)\,x}$

$$\frac{y - \tan\frac{\theta}{2}}{x-1} = 0$$

$$\boxed{y = \tan\frac{\theta}{2}}$$

line ∥ to RS and passing thru 'Q'

$$y = (\tan\theta)\,x \quad , \quad y = \tan\frac{\theta}{2}$$

$$\text{POI} \equiv E \equiv (h, k)$$

$$k = (\tan\theta)\,h$$

$$k = \left(\frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}\right)h$$

$$k = \tan\frac{\theta}{2}$$

PUT

$$k = \left(\frac{2\cancel{k}}{1-k^2}\right)h$$

$$\boxed{k=0}$$

$$y = 0 \quad \text{Ans}$$

$$1-k^2 = 2h$$

$$\boxed{1-y^2 = 2x}$$

Ans

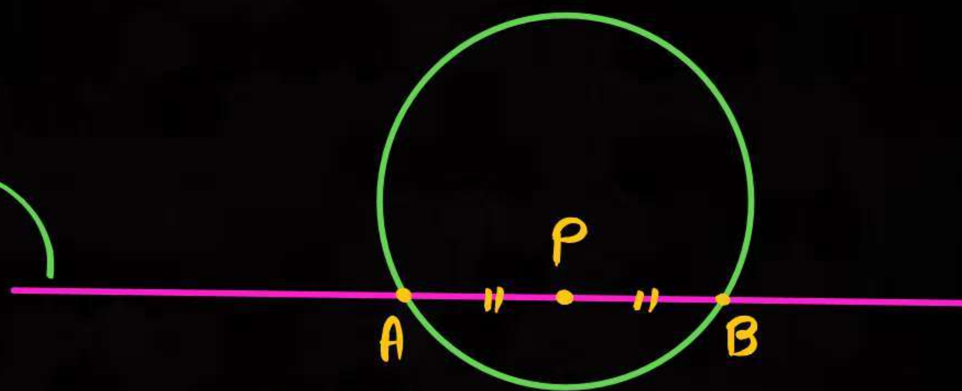
$\boxed{BKG ♡ ♡}$

EQUATION OF CHORD WHEN MID-POINT IS GIVEN

$S_1 < 0$

$P \rightarrow$ given

$P \longrightarrow \boxed{T = S_1}$



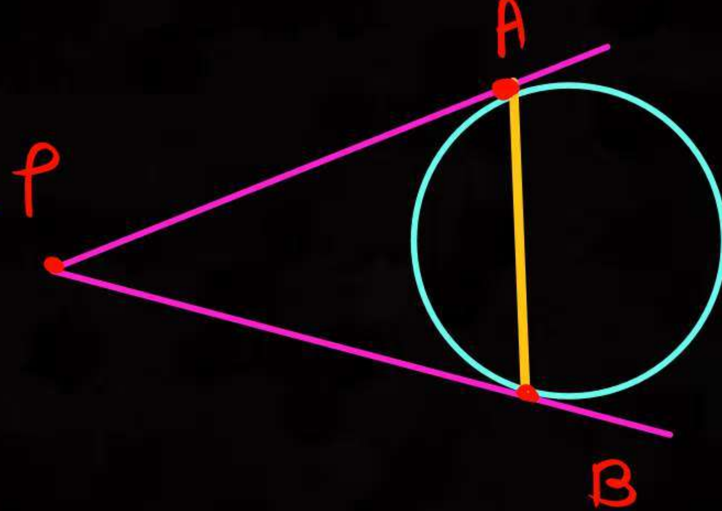
⭐⭐

EQUATION OF C.O.C

" CHORD OF CONTACT "

AB eqn

$\boxed{T=0}$



AB → C.O.C of Tangents from 'P'

EQUATION OF PAIR OF TANGENTS

Joint eqn of Pair of Tangents:

PA and PB...

P

$$\boxed{\underbrace{S}_{\text{CIRCLE'S EQN}} \cdot \underbrace{S_1}_{\text{'P' का POWER OF PT}} = T^2}$$

★★★★

Q → WRITE J.E of P.O.T. drawn from $(1,2)$ to the CIRCLE $x^2+y^2+2x+6y-1=0$

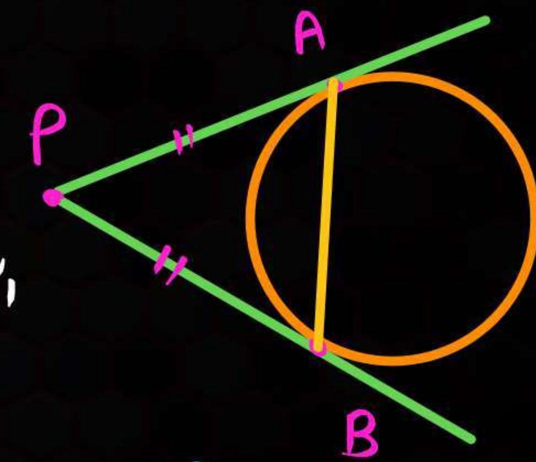
$S_1>0$

$SS_1 = T^2$

$$(x^2+y^2+2x+6y-1)(18) = \left(1.x+2y+2\left(\frac{x+1}{2}\right)+6\left(\frac{y+2}{2}\right)-1\right)^2$$

Ans

$(1,2)$ → x_1, y_1



AB eqn : $T=0$

$$1.x+2.y+(x+1)+3(y+2)-1=0$$

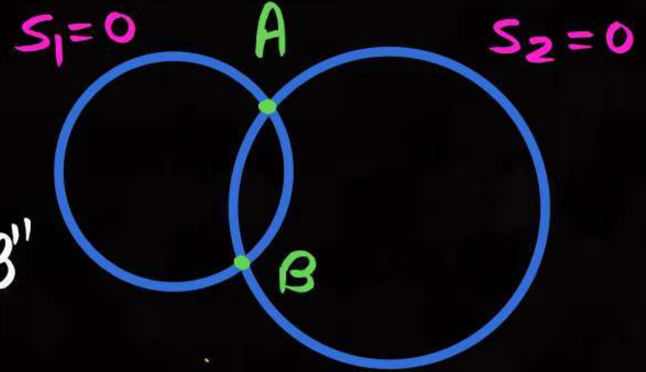
$\boxed{\text{Ans}}$

FAMILY OF CIRCLES

$x^2+y^2+2g_1x+2f_1y+c_1=0$

TYPE 1 :

Equation of family of circles passing through Points of Intersection of Circles $S_1 = 0$ and $S_2 = 0$ is given by:



$x^2+y^2+2g_2x+2f_2y+c_2=0$

"∞ many circles may pass through A and B"

$\boxed{S_1+\lambda S_2=0}$

$\lambda \neq -1$

$\lambda \in R$

$\boxed{\text{Eqn of common chord}} \Rightarrow \boxed{S_1 - S_2 = 0}$ ⭐⭐⭐⭐

Q. Find the equation of a circle passing through the POI of the circles $x^2 + y^2 - 6x = 0$ and $x^2 + y^2 - 4y = 0$ having its center on the line $2x - 3y + 12 = 0$.

(JEE MAINS 2020)

$S_1 = 0$ $\qquad$ $S_2 = 0$

$$\boxed{S_1 + \lambda S_2 = 0}$$

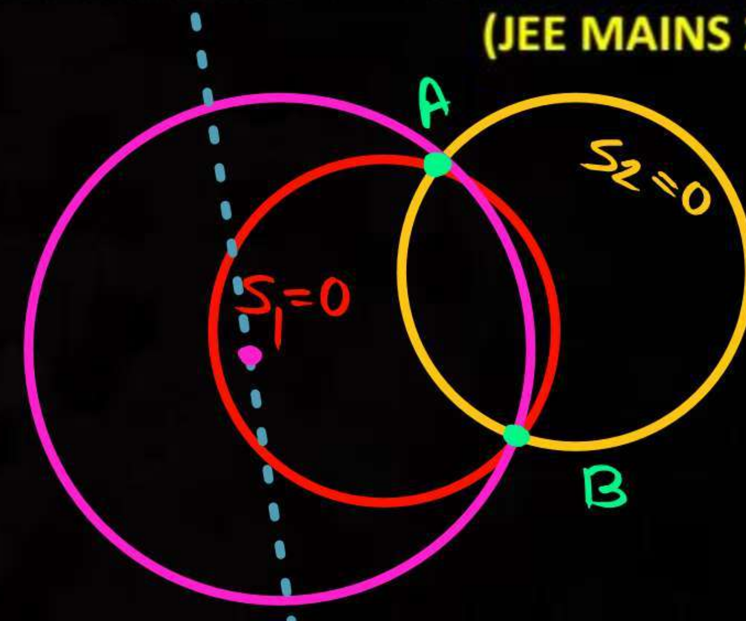
$$(x^2+y^2-6x) + \lambda(x^2+y^2-4y) = 0$$

$$(1+\lambda)x^2 + (1+\lambda)y^2 - 6x - 4\lambda y = 0$$

$$\boxed{x^2+y^2-\frac{6}{1+\lambda}x - \frac{4\lambda}{\lambda+1}y = 0}$$

$$\left(\frac{3}{\lambda+1}, \frac{2\lambda}{\lambda+1}\right)$$

Satisfy



$$2x - 3y + 12 = 0$$

$$2\left(\frac{3}{r+1}\right)-3\left(\frac{2r}{r+1}\right)+12=0 \longrightarrow \boxed{r=-3}$$

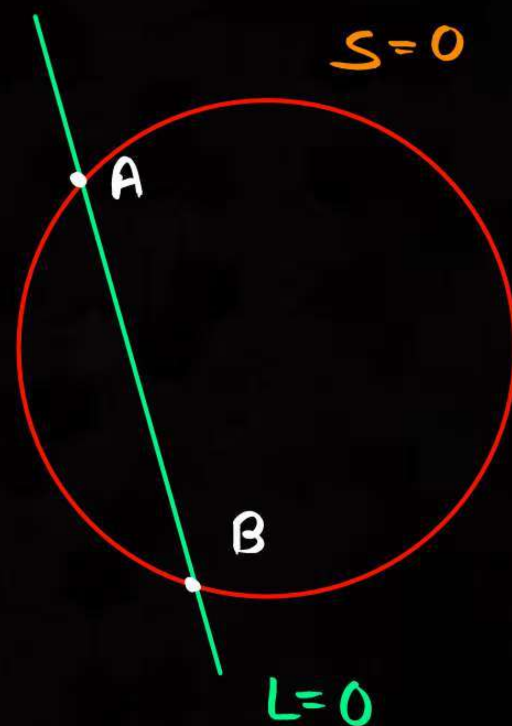
$$\text{Ans} \rightarrow \boxed{x^2+y^2+3x-6y=0}$$

TYPE 2:

Equation of family of circles passing through the Points of intersection of Circle $S = 0$ and a line L = 0 is given by:

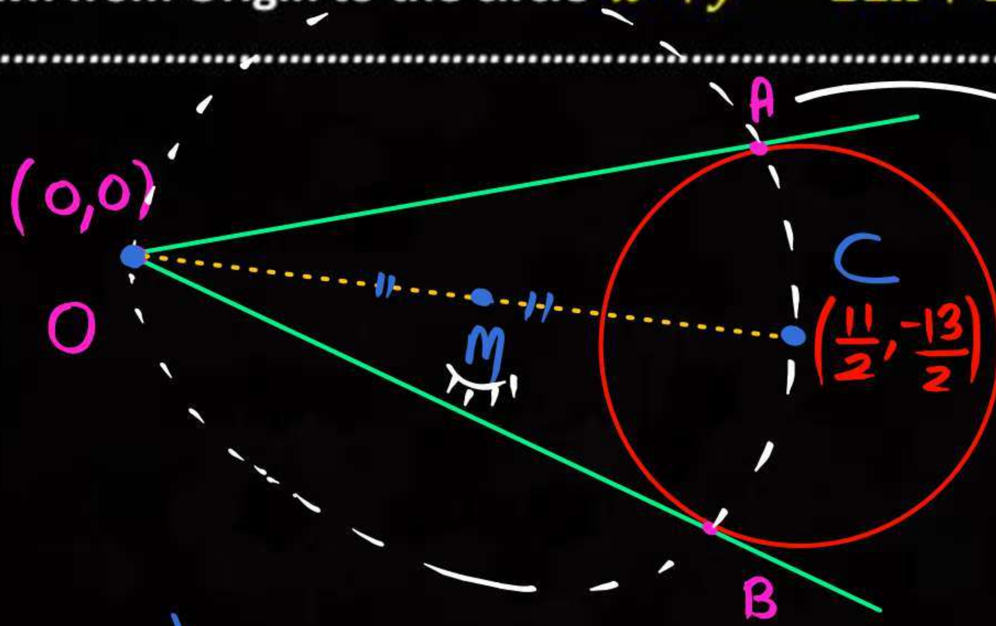
(∞ many CIRCLE)

$$\boxed{\begin{gathered} S + \lambda L = 0 \\ \lambda \in R \end{gathered}}$$



Q. Find the equation of a circle which pass through origin and the POC of the tangents drawn from Origin to ~~the circle~~ $x^2+y^2-11x+13y+17=0$.

M-1



$M \equiv \left(\frac{11}{4}, \frac{-13}{4}\right)$

$R = \frac{OC}{2}$

$= \left(\frac{\sqrt{\frac{121}{4}+\frac{169}{4}}}{2}\right)$

Ans:

$\left(x-\frac{11}{4}\right)^2+\left(y+\frac{13}{4}\right)^2 = R^2$

Q. Find the equation of a circle which pass through origin and the POC of the tangents drawn from Origin to the circle $x^2+y^2-11x+13y+17=0$. $S=0$

$\boxed{M-2}$

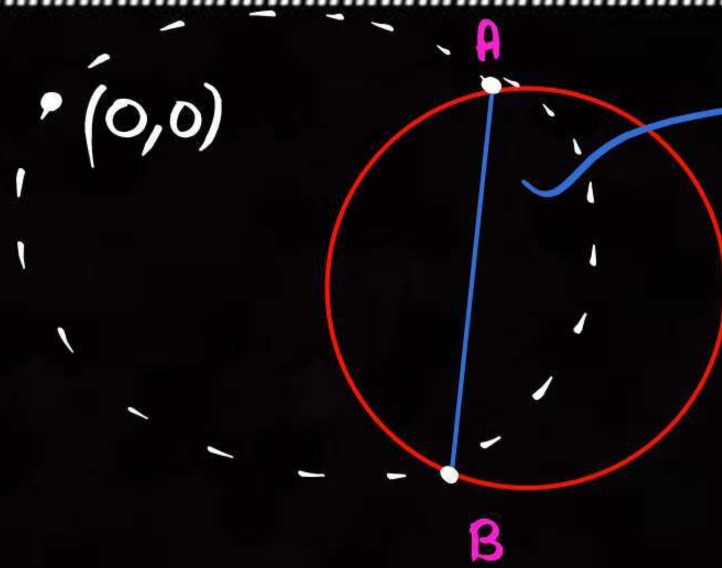
∞ many CIRCLES passing thru AB

$S+\lambda L=0$

$x^2+y^2-11x+13y+17$
$+\lambda(-11x+13y+34)=0$

$(0,0)$

$\lambda=-\frac{1}{2}$



$T=0$

$0x+0y-\frac{11}{2}(x+0)+\frac{13}{2}(y+0)+17=0$

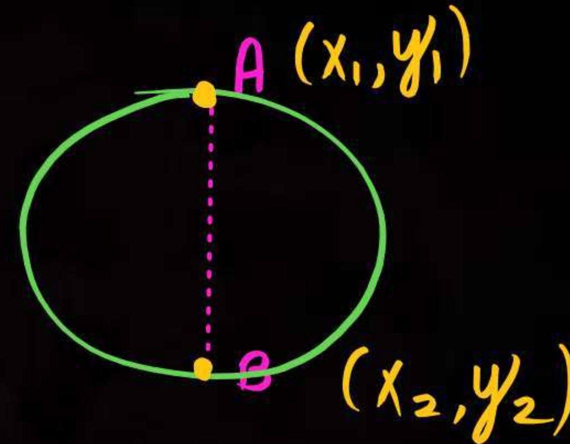
$\boxed{-11x+13y+34=0}$

Ans: $x^2+y^2-11x+13y+17$
$-\frac{1}{2}(-11x+13y+34)=0$

TYPE 3:

Equation of the family of circles passing through two given Points A (x_1, y_1) and B (x_2, y_2) is :

$$S: (x-x_1)(x-x_2) + (y-y_1)(y-y_2) = 0$$



$$AB \equiv L: \frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$$

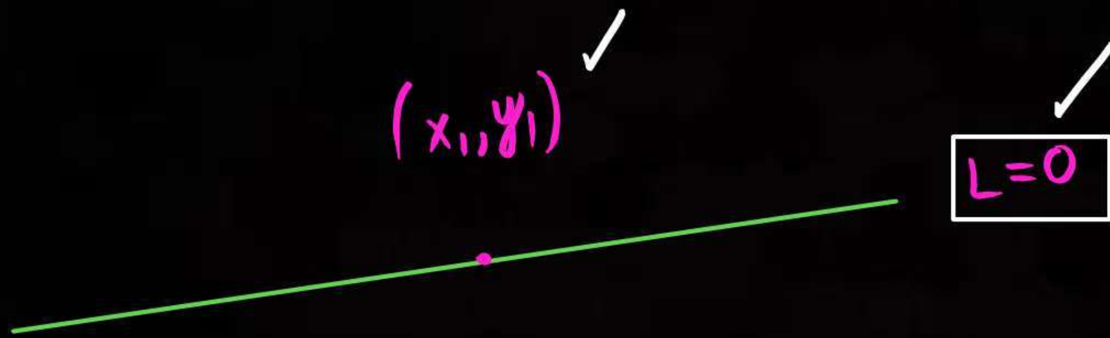
$$(y-y_1)(x_2-x_1) - (x-x_1)(y_2-y_1) = 0$$

(∞ many CIRCLES)

$$\boxed{S + \lambda L = 0}$$

TYPE 4:

Equation of the family of circles touching a line at its fixed Point (x_1, y_1) is:



$\boxed{\text{Point CIRCLE}}$ $R=0$

$S: (x-x_1)^2+(y-y_1)^2=0$

Eqn of all the CIRCLES Touching $L=0$ at (x_1, y_1) is given

$$\boxed{S+\lambda L=0}$$

$\lambda \in R$

Q → (i) Find the eqn of all the circles touching the line $x+y-2=0$ at $(1,1)$ pt.

(ii) also passes $(3,1)$

Soln →

$S: (x-1)^2+(y-1)^2=0$

$x+y-2=0$

$L=0$

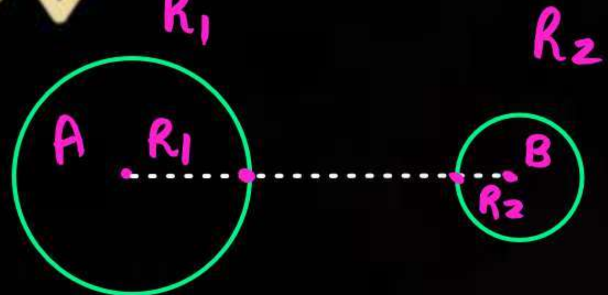
(i) $S+\lambda L=0$

$$\boxed{(x-1)^2+(y-1)^2+\lambda(x+y-2)=0}$$

$(3,1) \rightarrow 4+0+\lambda(4-2)=0 \Rightarrow \boxed{\lambda=-2}$

(ii) $(x-1)^2+(y-1)^2-2(x+y-2)=0$ Ans

SYSTEM OF 2 CIRCLES



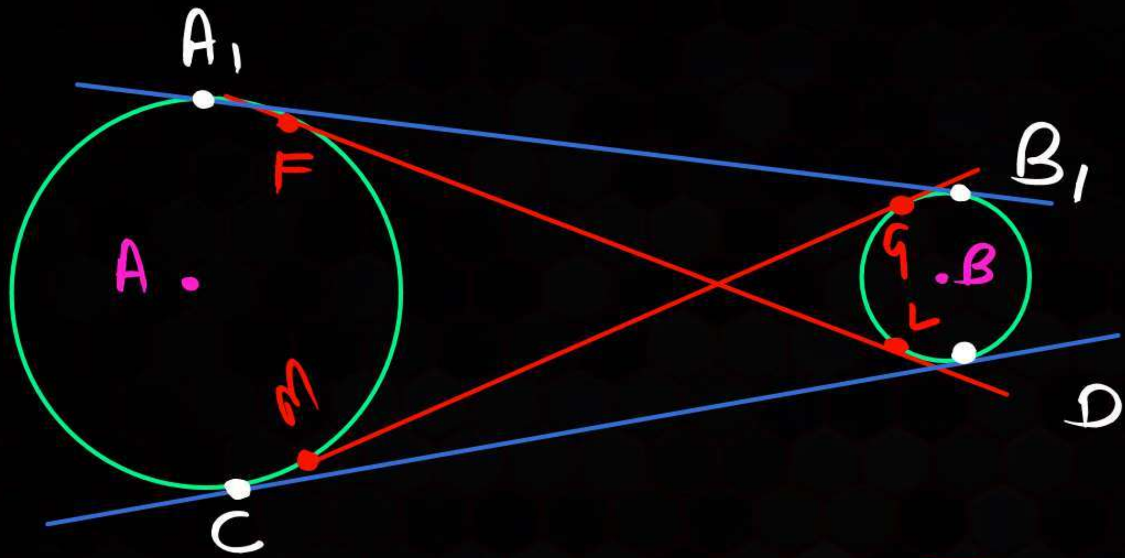
$$\boxed{AB > R_1 + R_2}$$

Condn:

⭐⭐⭐⭐

Neither Touch nor INTERSECT Each other

$\boxed{\text{Case 1}}$



4 common Tangents

TRANSVERSE COMMON TANGENTS

DIRECT COMMON TANGENTS

$$L_{DCT} = \sqrt{(AB)^2 - (R_1 - R_2)^2} \rightarrow A_1B_1 = CD$$

$$L_{TCT} = \sqrt{(AB)^2 - (R_1 + R_2)^2} \rightarrow FL = MG$$

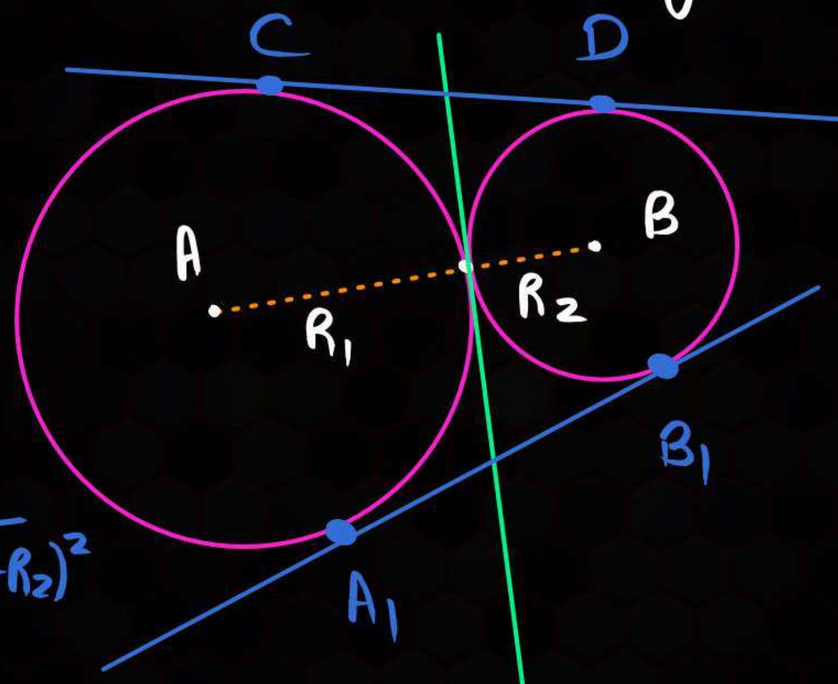
Case 2

External touching

3 common Tangents

$\boxed{AB = R_1 + R_2}$

Cond^n:



2 DCTs

1 more Tangent

$L_{DCT} = \sqrt{(R_1+R_2)^2 - (R_1-R_2)^2}$

$CD = A_1B_1 \quad = 2\sqrt{R_1R_2}$

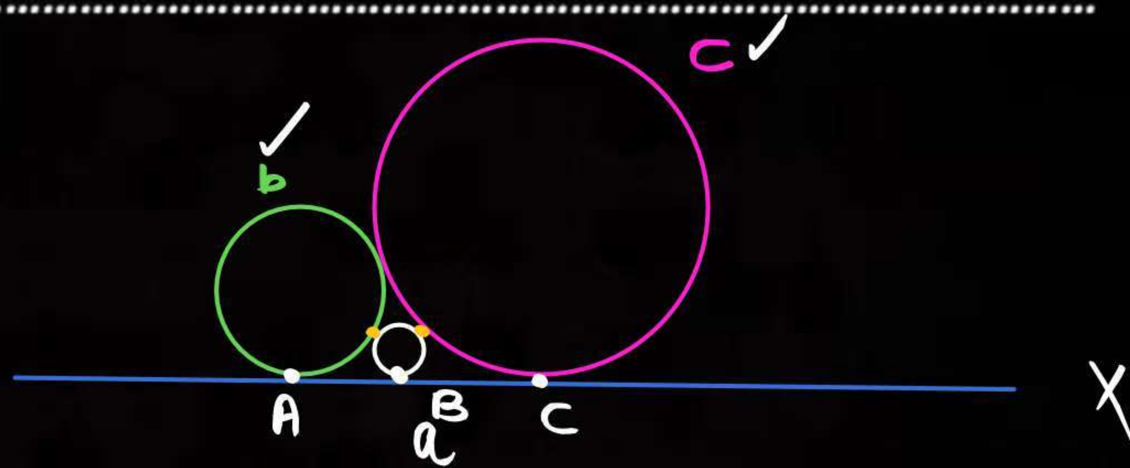
Q. Three circles of radii a , b, c (a < b < c) touch each other **externally**. If they have x-axis as a common tangent, then: [JEE Main 2019]

(A) $\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{b}} + \frac{1}{\sqrt{c}}$.

(B) $\sqrt{a}, \sqrt{b}, \sqrt{c}$ are in A. P.

(C) a , b , c are in A.P

(D) $\frac{1}{\sqrt{b}} = \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{c}}$



$$\boxed{AB + BC = AC}$$

$$\cancel{2}\sqrt{ab} + \cancel{2}\sqrt{ac} = \cancel{2}\sqrt{bc}$$

$\sqrt{abc}$

$$\frac{1}{\sqrt{c}} + \frac{1}{\sqrt{b}} = \frac{1}{\sqrt{a}}$$

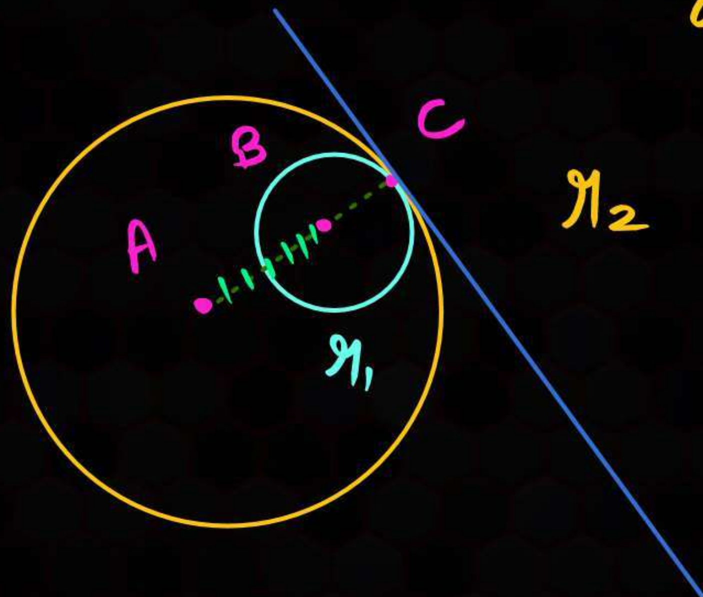
Case 3

"Case of internal touching"

$AC = AB + BC$

$AB = AC - BC$

$\boxed{AB = (r_2 - r_1)}$

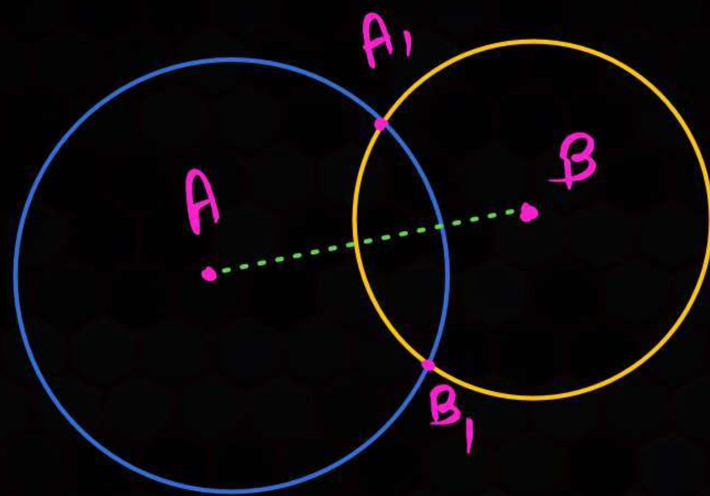


$AB = |r_2 - r_1|$

$\boxed{N_{CT} = 1}$

Case 4

"CIRCLES INTERSECT EACH OTHER"



2 common Tangents

$$\boxed{|r_1 - r_2| < AB < (r_1 + r_2)}$$

Q. If the circles $x^2 + y^2 - 16x - 20y + \overbrace{164 - r^2}^{=c} = 0$ and $(x-4)^2 + (y-7)^2 = 36$ intersect at two distinct points, then

$B \equiv (4,7)$, $R_2 = 6$

$A \equiv (8,10)$
$R_1 = r$

$|R_1 - R_2| < AB < R_1 + R_2$

$$\boxed{|r-6| < 5 < 6+r}$$

$|r-6| < 5$

$-5 < r-6 < 5$

$\boxed{1 < r < 11}$

$5 < 6 + r$

$\boxed{r > -1}$

$\cap$

(A) $r > 11$

(B) $r = 11$

(C) $0 < r < 1$

(D) $1 < r < 11$.

ORTHOGONAL INTERSECN

Two circles intersecting each other at 90°

$$x^2+y^2+2g_1x+2f_1y+c_1=0$$
$$x^2+y^2+2g_2x+2f_2y+c_2=0$$

Condn for ORTHOGONALITY:

$$\boxed{2g_1g_2+2f_1f_2=c_1+c_2}$$

⭐⭐⭐⭐

Q → Find 'K' such that:

$$x^2+y^2+4x+8y+2=0$$

$$2x^2+2y^2+28x+32y+2K=0 \quad \text{intersect orthogonally.}$$

$$x^2+y^2+14x+16y+K=0$$

$g_1=2$

$f_1=4$

$c_1=2$

$g_2=+7, \; f_2=8, \; c_2=K$

$$\# \; 2(2)(7)+2(4)(8)=2+K$$

$$\boxed{K=90}$$

Ans

Pair of CIRCLES

RADICAL AXIS

"line"

$$\boxed{\underbrace{S_1}_{} - \underbrace{S_2}_{} = 0}$$

CIRCLE$_1$ की eqn

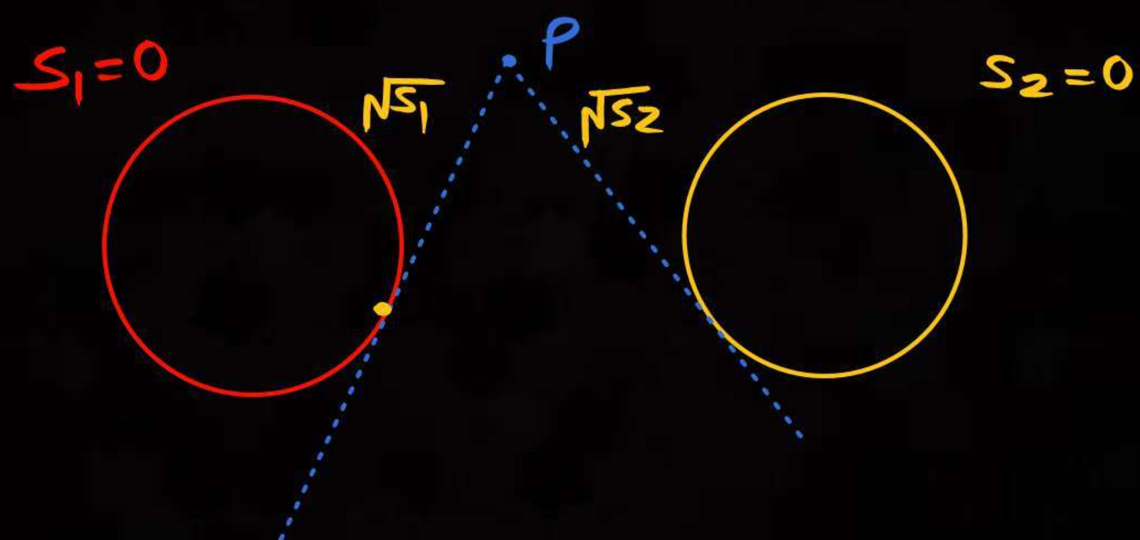
CIRCLE$_2$ की eqn

Radical axis of 2 circles is the locus of a point whose powers wrt the two circles are equal.

The equation of radical axis of two circles $S_1 = 0$ and $S_2 = 0$ is given by

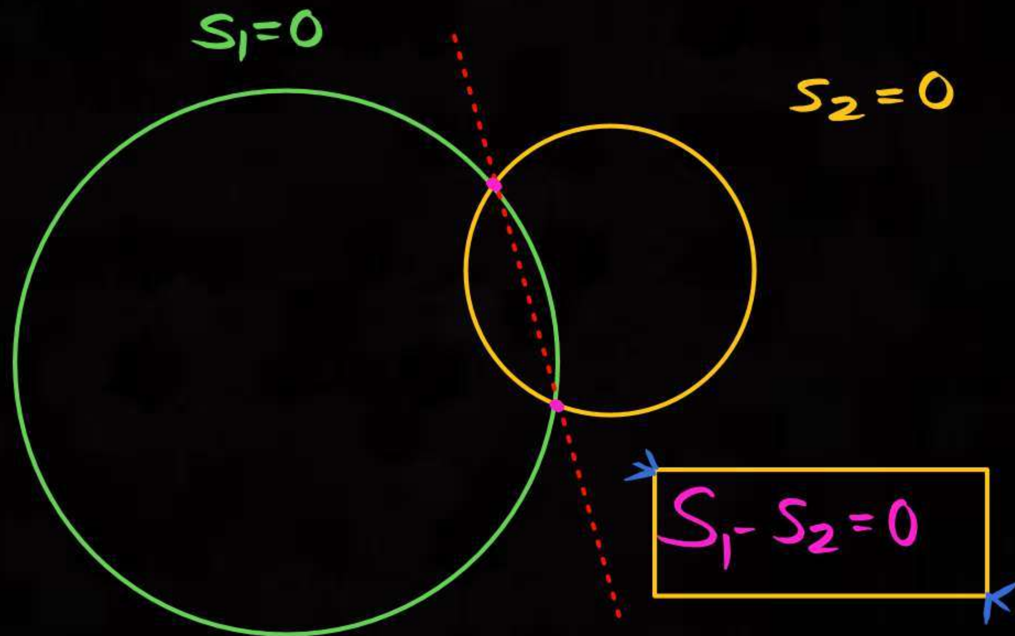
$S_1 = S_2$

$\boxed{S_1 - S_2 = 0}$

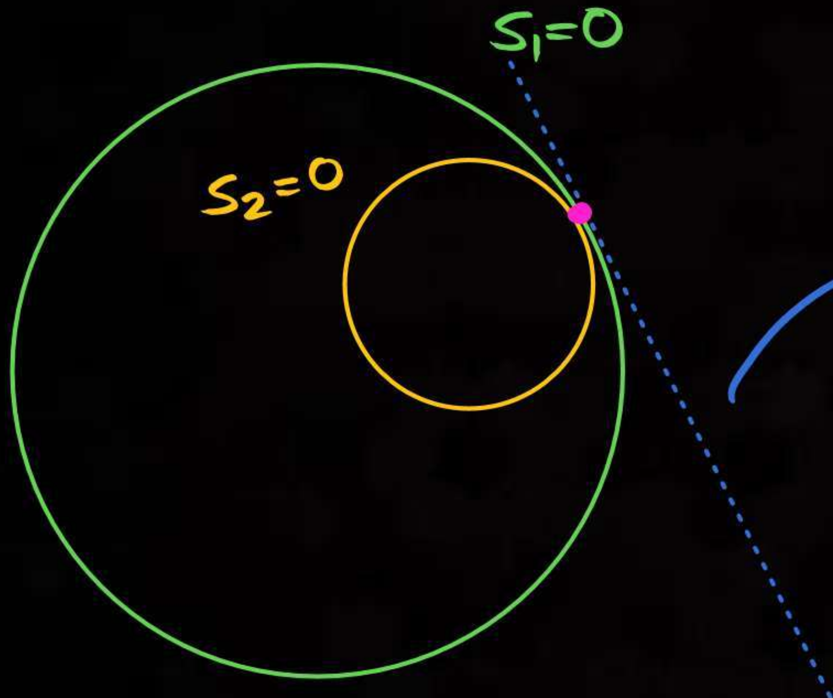


REMEMBER THESE POINTS

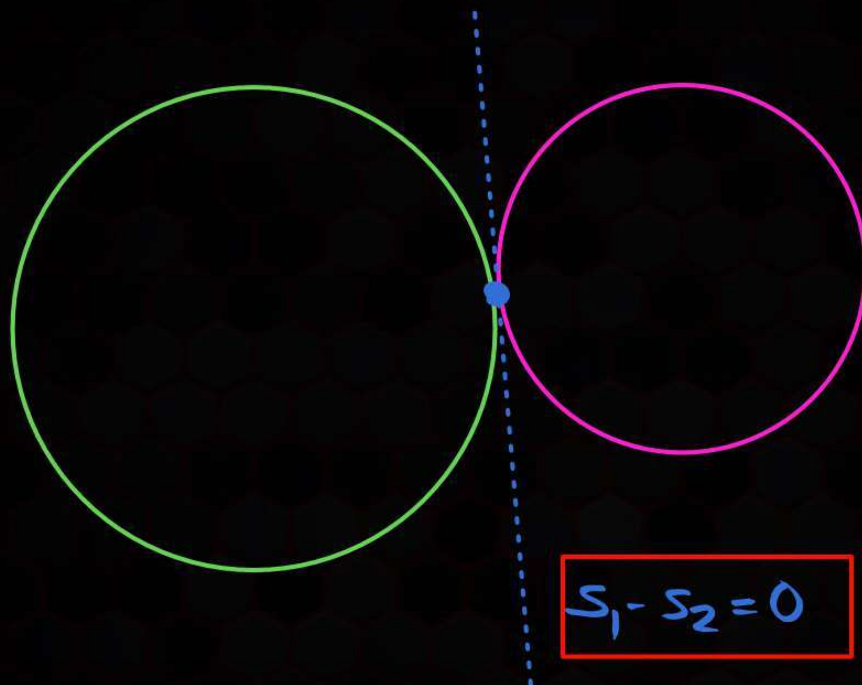
1. If two circles intersect, then the radical axis is the common chord of the two circles.



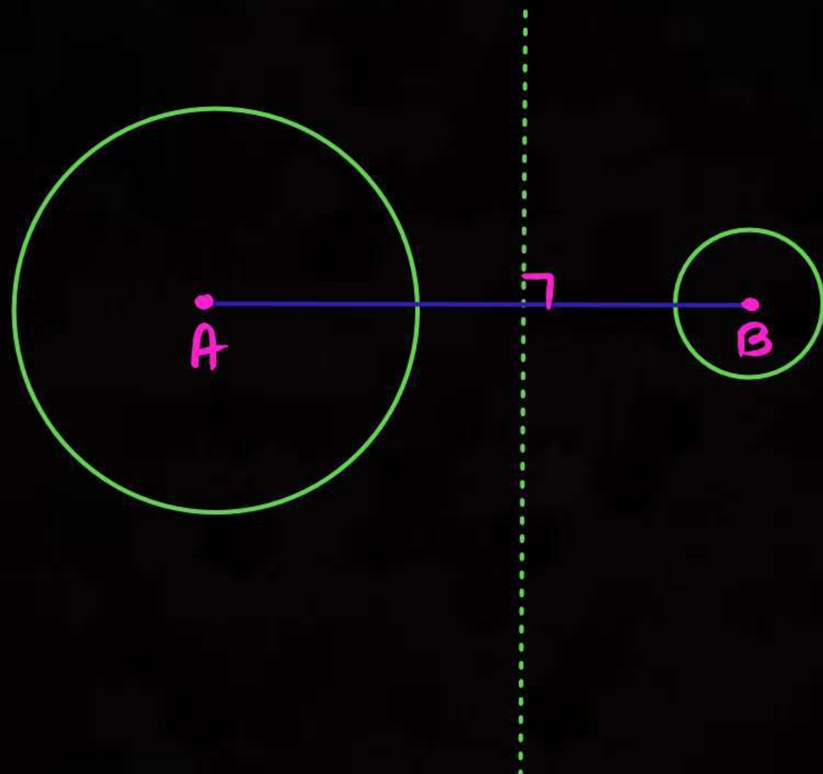
2. If two circles touch each other then the radical axis is the common tangent of the two circles at the common point of contact.



$$\boxed{S_1 - S_2 = 0}$$

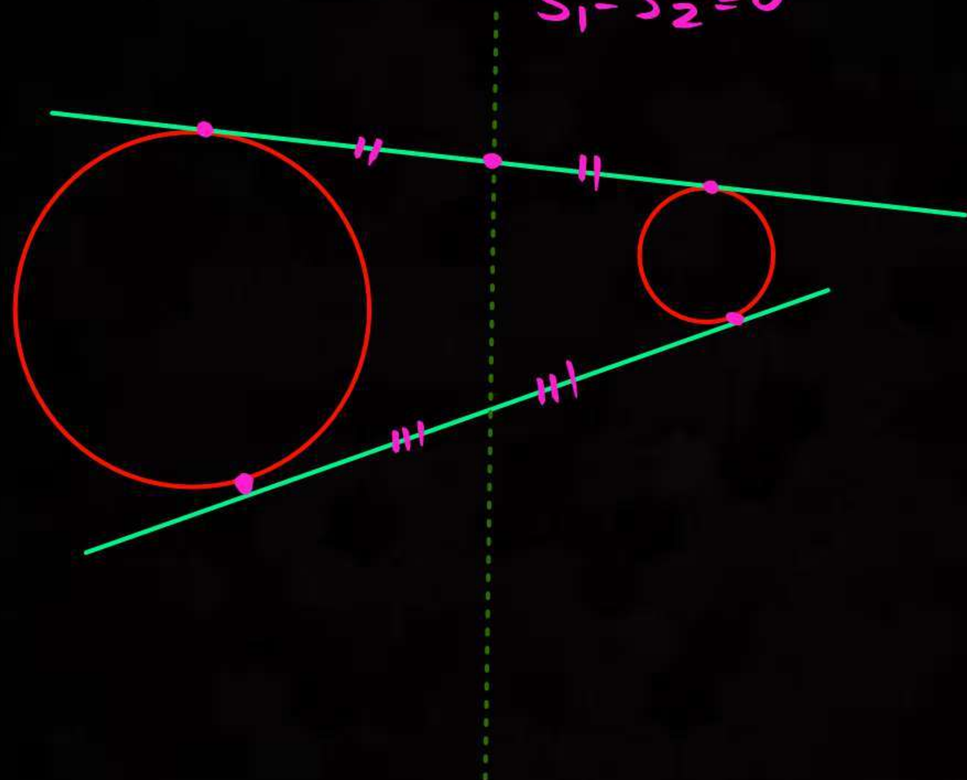

$S_1 - S_2 = 0$

3. Radical axis is always perpendicular to the line joining the centers of the two circles.



5. Radical axis bisects a common tangent between the two circles.

$S_1 - S_2 = 0$



"DCT and TCT Both"

6. Pairs of circles which do not have radical axis are concentric.

$$S_1 = x^2 + y^2 + 2gx + 2fy + c_1 = 0$$

$$S_2 = x^2 + y^2 + 2gx + 2fy + c_2 = 0$$

$x^2+y^2+6x+2y+1=0$

$x^2+y^2+6x+2y+\frac{1}{2}=0$

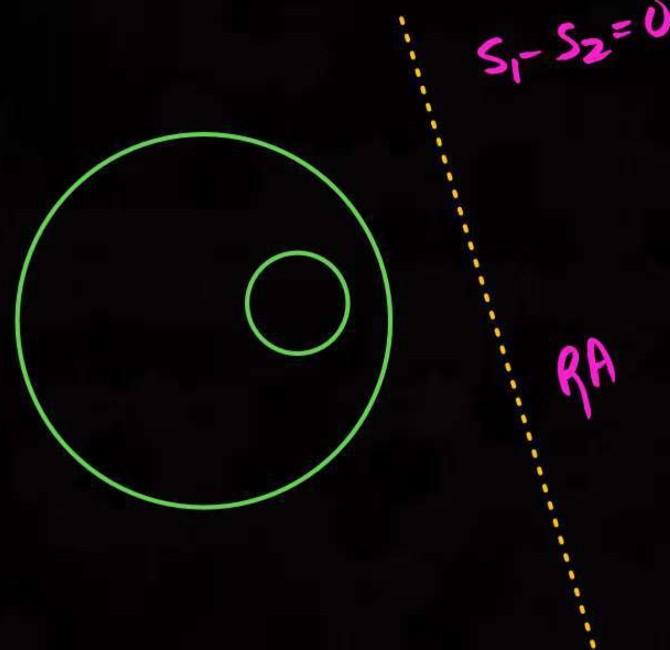
RA

$\boxed{\frac{1}{2}=0}$

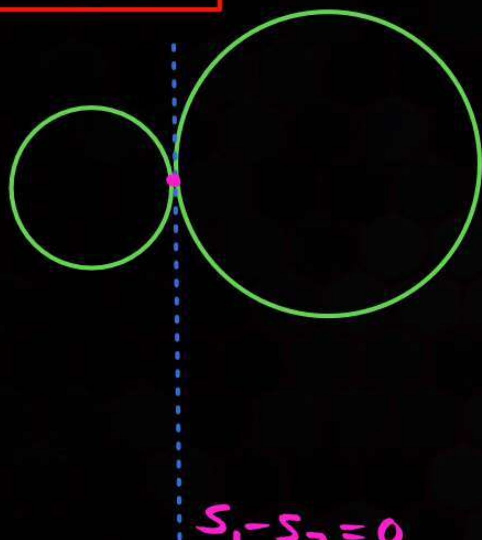
Completely Inside

7. If one circles is contained in another circle when radical axis passes outside to both the circles.

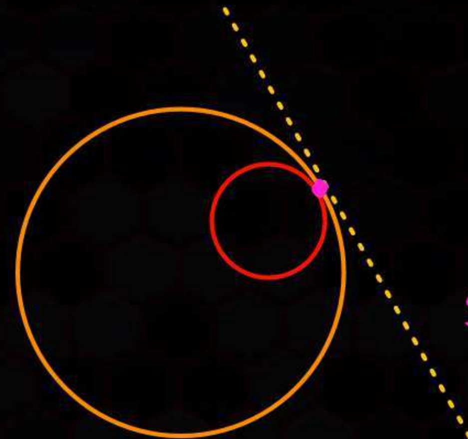
Not Concentric



Summary



$S_1 - S_2 = 0$

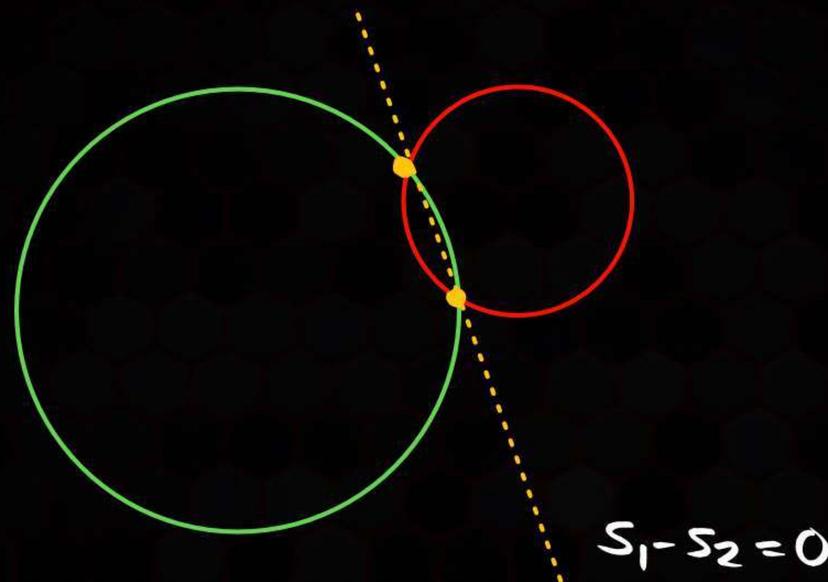


$S_1 - S_2 = 0$

Make SURE $(x^2 = (y^2 = 1$

in $S_1 = 0$

$S_2 = 0$


$S_1 - S_2 = 0$

RADICAL CENTER

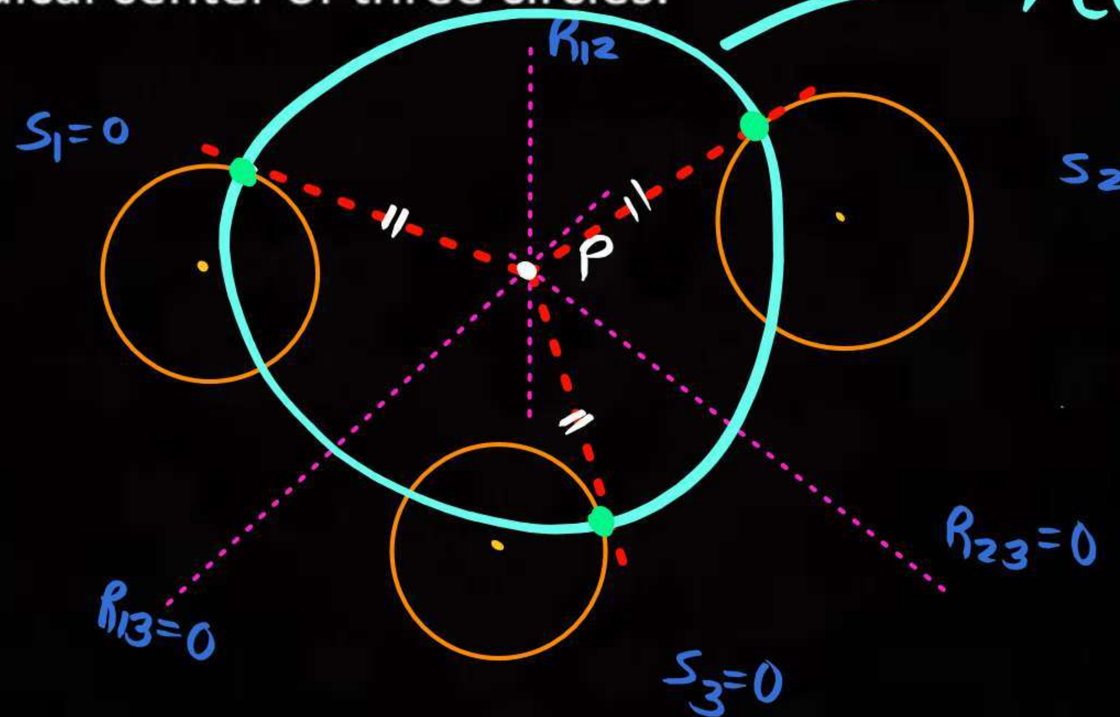
with Non-collinear centres

The common point of intersection of the radical axis of 3 circles taken 2 at a time called the Radical center of three circles.

centRE $\equiv$ P

$R = \sqrt{S_1} = \sqrt{S_2} = \sqrt{S_3}$

P → Radical centre



Radical CIRCLE →

eqn ?

Radical Centre = centre ≡ P

Radical Radius = $\sqrt{S_1} = \sqrt{S_2} = \sqrt{S_3}$

(FROM P)

"Radical CIRCLE is the only CIRCLE which intersect 3 given circles $S_1 = 0$, $S_2 = 0$ and $S_3 = 0$ ORThogonally"

★★★★★★★★★★★