LINEAR INEQUALITIES & LINEAR PROGRAMMING PROBLEMS

(1) Types of Inequalities

Numerical Inequalities: Relation between numbers e.g.: $3<5, 4>7$

Literal or Variable Inequalities : Relation between two variables or variable and numbers. e.g.: $y<8$

Double Inequalities: Relationship from two sides e.g. $2<x<5$

Strict Inequalities: An inequality that uses the symbol $<$ or $>$ e.g.: $x<5, 3<5$

Slack Inequality: An inequality that uses the symbols $\le$ or $\ge$. e.g.: $y\le 5$

Linear Inequalities in One Variable: An inequality which involves a linear function in one variable e.g.: $x<5$

Linear Inequalities in two variables: An inequality which involves a linear function in two variables, e.g.: $3x+2y<5$

Quadratic Inequalities: An inequality which involves a quadratic function, eg.: $x^2+2x\le 5$.

(2) Some important results

If a is a positive real number, then

$|x|<a \Leftrightarrow -a<x<a$ i.e. $x\in(-a, a)$.

$|x|\le a \Leftrightarrow -a\le x\le a$ i.e. $x\in[-a,a]$.

$|x|>a \Leftrightarrow x<-a$ or $x>a$ $|x|\ge a \Leftrightarrow x\le -a$ or $x\ge a$

$|x|\ge a \Leftrightarrow x\le -a$ or $x\ge a$

Let r be a positive real number and a be a fixed real number. Then,

$|x-a|<r \Leftrightarrow a-r<x<a+r$ i.e. $x\in(a-r, a+r)$.

$|x-a|\le r \Leftrightarrow a-r\le x\le a+r$ i.e. $x\in[a-r, a+r]$.

$|x-a|>r \Leftrightarrow x<a-r$ or $x>a+r$

$|x-a|\ge r \Leftrightarrow x\le a-r$ or $x\ge a+r$

Let a and b be positive real numbers . Then

$a<|x|<b \Leftrightarrow x\in(-b,-a)\cup(a,b)$

$a\le|x|\le b \Leftrightarrow x\in[-b,-a]\cup[a,b]$

$a\le|x-c|\le b \Leftrightarrow x\in[-b+c,-a+c]\cup[a+c,b+c]$

$a<|x-c|<b \Leftrightarrow x\in(-b+c,-a+c)\cup(a+c,b+c)$

(3) Some Important Terms Related To Lpp

1. Feasible Region: The common region determined by all the constraints including non-negative constraints $x, y\ge 0$ of linear programming problem is known as feasible region (or solution region). If we shade the region according to the given constraints, then the shaded area is the feasible region which is the common area of the regions drawn under the given constraints

2. Feasible Solution: Each point within & on the boundary of the feasible region represents feasible solution of constraints. Note that in the feasible region there are infinitely many points which satisfy the given condition.

3. Optimal Solution: Any point in the feasible region that gives the optimal value (maximum or minimum) of the objective function is called an optimal solution.

(4) Mathematical Form Of Linear Programming Problems

The general mathematical form of a linear programming problem may be written as follow. **Objective Function:** $Z=C_1x+C_2y$ **Subject to constraints are:** $a_1x+b_1y\le d_1$ $a_2x+b_2y\le d_2$ etc and non-negative restrictions are $x\ge 0, y\ge 0$

(1) Objective Function: A linear function $Z=ax+by$, where a & b are constants, which has to be maximized or minimized according to a set of given conditions, is called a linear objective function.

(2) Decision Variables: In the objective function $Z=ax+by$, the variables x, y are said to be decision variables.

(3) Constraints: The restrictions in the form of inequalities on the variables of a linear programming problem are called constraints. The condition $x\ge 0, y\ge 0$ are known as non-negative restrictions. In the constraints given in the general form of a LPP there may be anyone of the 3 signs $\le, =, \ge$

(5) Corner Point Method Of Solving Lpp

Steps Involved:

(1) Find the feasible region of the LPP & determine its corner points (vertices) either by inspection or by solving the two equations of the lines intersecting at that point.

(2) Evaluate the objective function $Z=ax+by$ at each corner point. Let M & m, respectively be the largest & smallest values of these points.

(3) (i) When the feasible region is bounded, M & m are the maximum & minimum values of Z. (ii) In case the feasible region is unbounded, we have:

(a) M is the maximum value of Z, if the open half plane determined by $ax+by>m$ has no point in common with the feasible region. Otherwise, Z has no maximum value.

(b) Similarly, m is the minimum value of Z, if the open half plane determined by $ax+by<m$ has no point in common with the feasible region. Otherwise, Z has no minimum value.