CHAPTER

11

Dual Nature of Matter and Radiation

Planck's Quantum Theory

- According to Planck's quantum theory, light is considered to be made up of small packets (or particles) of energy known as quanta of energy or radiation.

  Energy, $E = h\nu = \frac{hc}{\lambda} = \frac{12400}{\lambda(\text{Å})}$ eV

Photons

- Momentum of one photon is $\frac{h}{\lambda}$.
- When radiation interacts with matter, the radiation behaves as if it is made of particles like photons.
- Einstein proposed that electromagnetic radiation (or simply light) is quantized and exists in elementary amounts (quanta) that we now call photons.
- Photons are not deflected by electric and magnetic fields which shows that they are neutral and do not carry any charge.
- The energy of photon depends upon the frequency of radiation but is independent of the intensity of radiation.

Photoelectric Effect

- When light of suitable frequency illuminates a metal surface, electrons are emitted. This process of ejection of electrons using light is known as photoelectric emission. Photoelectrons ejected from metal have kinetic energies ranging from 0 to K_{max}.
- A certain minimum amount of energy is required for an electron to be pulled out from the surface of a metal. This minimum energy is called the work function (ϕ) of that metal. Work function is minimum for cesium (1.9 eV).
- Einstein equation for photoelectric effect is,

  $$h\nu = \phi + KE_{max} \quad \Rightarrow \frac{hc}{\lambda} = \frac{hc}{\lambda_0} + eV_s$$

- The minimum frequency of the incident light below which photoelectrons are not ejected from the metal surface is known as threshold frequency (ν_0).

  Work function, $\phi = h\nu_0 = \frac{hc}{\lambda_0}$

  λ_0 = threshold wavelength
- The minimum negative potential given to the metal plate with respect to the collector at which the photoelectric current becomes zero is known as stopping potential or cut-off potential. Here $KE_{max} = eV_s$, $V_s =$ stopping potential

  Stopping potential is independent of intensity of light used.
- The number of photoelectrons emitted per second is directly proportional to the intensity of the incident radiation.
- The maximum kinetic energy of the ejected electrons is independent of the intensity of incident radiation but depends upon the frequency of the incident radiation.

Radiation Pressure

Radiation pressure, $P = \frac{I}{c}(1 + r)$. Here I is the intensity of incident radiation, c is the speed of light and r is the reflectivity of the surface.

For 100% reflection, $r = 1$ and for 100% absorption $r = 0$.

de-Broglie Hypothesis

- It says that a wave is associated with a moving material particle. The wavelength associated with a moving particle is given by $\lambda = \frac{h}{mv}$, where m is the mass of the particle moving with v velocity and h is Planck's constant. This wave is called de-Broglie wave.

Key Tips

- de-Broglie wavelength of a material particle, $\lambda = \frac{h}{mv}$
- de-Broglie wavelength in terms of energy of a particle (E), $\lambda = \frac{h}{\sqrt{2mE}}$
- de-Broglie wavelength of an electron accelerated through a potential V volt, $\lambda = \sqrt{\frac{150}{V}}$ Å $= \frac{12.27}{\sqrt{V}}$ Å
- de-Broglie wavelength of a particle in terms of temperature (T), $\lambda = \frac{h}{\sqrt{3mkT}}$

Heisenberg's Uncertainty Principle

According to Heisenberg's Uncertainty Principle, it is not possible to measure exactly both the position and momentum of a microscopic particle (say electron) at the same time. That is,

$$\Delta x \Delta p \geq \frac{\hbar}{2}, \text{ where } \hbar = \frac{h}{2\pi},$$

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12 Atoms

Rutherford Model

- Rutherford concluded that the majority of the space in an atom is empty. The entire positive charge and mass of an atom is concentrated in a small space known as the nucleus. Electrons revolve around the nucleus in circular orbits.
- The perpendicular distance between the initial velocity vector of the alpha-particle from a central line passing through the center of nucleus when the alpha-particle is far away from the nucleus is known as impact parameter.
- Distance of closest approach: Distance of a point from nucleus at which α-particles are nearest to the centre of nucleus.
- The centripetal force required to make an electron move in a circular orbit is provided by the electrostatic force of attraction between the negatively charged electron and the positively charged nucleus.
- Electrons are bound to the nucleus because the total energy of an electron in orbit is negative.
- Rutherford's atomic model could not explain the stability of the atom. It was also not able to explain the line spectra for various elements and the energies of electrons and their distribution around the nucleus.

Bohr's Model

- According to Bohr's first postulate, every atom consists of a central core called nucleus in which the entire mass and positive charge of the atom is concentrated. Electrons revolve around the nucleus in circular orbits. The centripetal force required for the circular motion of electrons is provided by the electrostatic force of attraction between the negatively charged electrons and the positively charged nucleus.

$$\frac{(Ze)e}{4\pi\varepsilon_0 r^2} = \frac{mv^2}{r}$$

- According to Bohr's second postulate, an electron can revolve in certain fixed orbits known as stationary orbits. When an electron revolves in a stationary orbit its energy is constant and the angular momentum of the electron is an integral multiple of $h/2\pi$, where h is Planck's constant.

$$mvr = \frac{nh}{2\pi}$$

- According to Bohr's third postulate, when an electron jumps from an inner stationary orbit to an outer stationary orbit it absorbs energy and when it jumps back from an outer stationary orbit to an inner orbit it radiates energy.

$$E_i - E_f = hv = \frac{hc}{\lambda}$$

- **Bohr radius:** First orbit of hydrogen atom is called Bohr radius (a_0).
- **Ground state:** Lowest state of atom, called the ground state, is the state in which electron revolves in the orbit of smallest radius, the Bohr radius, a_0.
- **Ionization energy:** Minimum energy required to free an electron from the ground state of an atom is called the ionization energy.
- The stationary orbits are not equally spaced. The ratio of radii of the first three stationary orbits is $1^2 : 2^2 : 3^2 = 1 : 4 : 9$.

Spectral lines and Hydrogen Like Atoms

- In hydrogen like atoms,

Radius of n^{th} orbit, $r_n = a_0\left(\frac{n^2}{Z}\right)$, $a_0 = 0.529\text{Å}$

Orbital speed, $v_n = v_0\left(\frac{Z}{n}\right)$, $v_0 = 219\times10^6 \, m/s$

Energy in n^{th} orbit, $E_n = E_0\left(\frac{Z^2}{n^2}\right)$, $E_0 = -13.6\text{eV}$

- For central forces like in atoms, $TE = -KE = \frac{PE}{2}$
- Wavelength corresponding to spectral lines is given by the Rydberg's formula,

$$\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

R = Rydberg constant = 1.097×10^7 m^{-1}

- Total number of possible transitions between any two states is $\frac{n(n+1)}{2}$, where n = difference between the two state numbers.
- The lines for hydrogen are said to be grouped into series, according to the level at which upward jump starts and downward jump ends.

- When an electron jumps from a higher energy state n to ground state, $n = 1$, the series of spectral lines emitted is called Lyman series. All of them lie in the ultraviolet region.
- When an electron jumps from a higher energy state n to state 2, the series of spectral lines emitted is called Balmer series. These line in visible region.
- When an electron jumps from a higher energy state n to state 3, the series of spectral lines emitted is called Paschen series. All of them lie in near infrared region.
- When an electron jumps from a higher energy state n to state 4 the series of spectral lines emitted is called Brackett series. All of them lie in infrared region.
- When an electron jumps from a higher energy state n to state 5 the series of spectral lines emitted is called Pfund series. All of them lie in infrared region.

When Effect of motion of Nucleus is Considered,

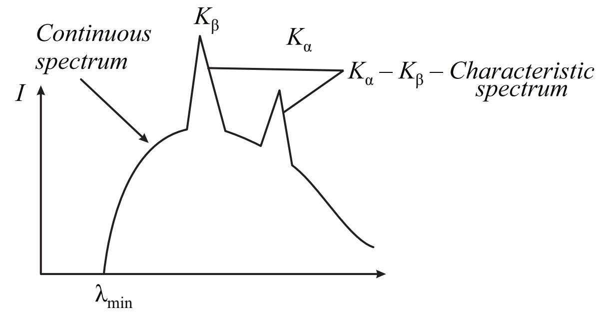
Radius of the orbit, $r_n = (0.529\ \text{Å})\dfrac{n^2}{Z}\cdot\dfrac{m}{\mu}$

Energy of the orbit, $E_n = (-13.6\,\text{eV})\dfrac{Z^2}{n^2}\cdot\dfrac{\mu}{m}$

Here μ is reduced mass, $\mu = \dfrac{Mm}{(M+m)}$,

Where, M = mass of nucleus, m = mass of charge particle

X-Rays

- X-rays are produced by bombarding high speed electrons on a target of high atomic weight and high melting point.
- X-ray are electromagnetic radiations of short wavelength (0.1 Å to 10 Å).
- Are not affected by electric and magnetic fields.
- They cause photoelectric emission.
- 

Characteristics for continuous X-rays, $eV = h\nu_{max} = \dfrac{hc}{\lambda_{min}}$

e = electronic charge;

V = accelerating potential

ν_{max} = maximum frequency of X-Rays

- Intensity of X-rays depends on number of electrons hitting the target.
  - Cut off wavelength or minimum wavelength, $\lambda_{min} \cong \dfrac{12400}{V}$ Å
  - Where V (in volts) is the potential difference applied to the tube
  - Continuous spectrum is due to retardation of electrons.
- Characteristic X-Rays

For $K_\alpha, \lambda = \dfrac{hc}{E_K - E_L}$ For $K_\beta, \lambda = \dfrac{hc}{E_L - E_M}$

- Moseley's Law, $\sqrt{\nu} = a(Z-b)$, here a and b are positive constants for one type of X-rays (independent of Z).

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13 Nuclei

Characteristics of Nucleus

- The nucleus of an atom is its center. Most of the mass of an atom is concentrated in the nucleus.
- One atomic mass unit is defined as $(1/12)^{th}$ of the mass of the ${}^{12}_{6}C$ isotope. It is represented by the symbol u and it is the average mass of a nucleon.
- The stability of any nucleus depends on the number of protons and neutrons. For small nuclei to be stable, the number of protons must be roughly equal to the number of neutrons. As the number of protons increases, however, more neutrons are needed to maintain stability.
- Nuclei of different elements have different sizes because the mass number (A) for different elements is different. Density of nucleus is the almost same for all elements.
- Average radius of nucleus may be written as,

  $R = R_0 A^{1/3}$, $R_0 = 1.1 \times 10^{-15}$ m

Mass Defect

The sum of the individual masses of the separated protons and neutrons exceeds the mass of the stable nucleus by an amount ΔM. This difference in mass is known as the **mass defect** of the nucleus.

Mass-energy Relation

According to mass-energy relation, the mass of a body is a measure of its energy content.

Equivalence of mass and energy, $E = mc^2$

Note: 1 u = 1.66×10^{-27} kg $\equiv$ 931.5 MeV

Binding Energy

- The binding energy of a nucleus is the energy that would have to be provided to split a nucleus into its individual nucleons. Binding energy of nucleus ${}_Z X^A$ of mass M, is given by $BE = (Zm_p + Nm_n - M)c^2$ and the mass defect, $\Delta m = Zm_p + Nm_n - M$.
- The average energy required to remove a nucleon from the nucleus to infinite distance is known as the **binding energy per nucleon**.
- Nuclei with high binding energies per nucleon are very stable as it takes a lot of energy to split them. Nuclei with lower binding energies per nucleon are easier to split.
- In order to become more stable, unstable nuclei tend to release some of their energy. Releasing this energy would decrease the amount of energy they contained, and therefore increase the amount of energy that must be added to them to split them apart.

Q Value

- Energy released in any nuclear reaction or collision is called Q value of the reaction.
- For nuclear reaction, $A + B \rightarrow C + Q$ (Energy)

  The energy of reaction Q is given by,

  $Q = (m_A + m_B - m_C)c^2 = \text{BE}_C - \text{BE}_A - \text{BE}_B$
- If Q is positive, energy is released and products are more stable in comparison to reactants.

  If Q is negative, energy is absorbed and products are less stable in comparison to reactants.

Nuclear Force

- The strong nuclear force is the force that holds nucleons together in the nucleus of an atom. It acts only over very short distances.
- Inside a nucleus, the nucleons are very close. The pull of the strong nuclear force is much greater than the push of the protons repelling each other, and therefore the nucleus remains intact.
- Adjacent neutrons experience no electrostatic repulsion between each other. There is only the strong force of attraction) between them.
- Two protons do repel each other when they are brought together, but in the nucleus they are so close to each other that the force of repulsion is overcame by the even stronger nuclear force.

Nuclear Fission

- By bombarding a particle on a heavy nucleus ($A > 230$), it splits into two or more light nuclei. In this process certain mass disappears which is obtained in the form of energy (enormous amount)

  $A + p \rightarrow B + C + Q$

- The energy released in a fission reaction comes from the difference between the mass of the original nucleus and the combined mass of the fission fragments.
- A chain reaction occurs when neutron, emitted from the decay of one atom, initiate fission in surrounding nuclei.
- An uncontrolled chain reaction occurs when every free neutron goes on to produce another fission reaction. It occurs in nuclear bombs and releases an enormous amount of energy.
- A controlled chain reaction occurs when only some of the emitted neutrons are able to induce further fission reactions, and the remaining neutrons are absorbed by a material that is not fissionable. In this situation energy can be released at a constant rate.
- Enrichment is the process of increasing the percentage of ^{235}U in a sample of uranium. Enrichment is important because naturally occurring uranium does not have a high enough percentage of ^{235}U to sustain a chain reaction.

Nuclear Fusion

- It is the phenomenon of fusing two or more light nuclei to form a single heavy nucleus.

  $A + B \rightarrow C + Q$ (Energy)
- The product (C) is more stable than reactants (A and B) and $m_C < (m_A + m_B)$.
- Mass defect, $\Delta m = [(m_A + m_B) - m_C]$ amu
- Energy released is $E = (\Delta m) \times 931$ MeV

Radioactivity

A nuclear phenomenon in which an unstable nucleus undergoes a decay is known as radioactivity. A decay equation is a representation of a decay reaction. It shows the changes occurring in nuclei undergoing decay and lists the products of the decay. The nucleus remaining after an atom undergoes radioactive decay is called a daughter nucleus. The daughter nucleus is more stable than the original nucleus. Generally, there are three types of radioactive decays,

(i) α decay

(ii) β^- and β^+ decay

(iii) γ decay

α decay

- An α-particle is a relatively slow-moving decay product consisting of two protons and two neutrons. It is the nucleus of helium and so can be written as ${}^4_2He$. α-particles carry positive charge.
- α-decay process: ${}^A_ZX \longrightarrow {}^{A-4}_{Z-2}Y + {}^4_2He$
- Q-value is, $Q = [m({}^A_ZX) - m({}^{A-4}_{Z-2}Y) - m({}^4_2He)]c^2$
- For the above shown decay kinetic energy of α-particle will be given by, $K_\alpha = \frac{m_Y}{m_X}Q$

β^- and β^+ decay

- A β-particle is a fast-moving electron that is ejected from an unstable nucleus. The electron is produced when a neutron transforms into a proton and an electron.
- In β-decay the resulting daughter nucleus has the same number of nucleons as the parent, but has one less neutron and one more proton.
- β-particles are very light when compared to α particles. They travel at a large range of speeds-from that of an α- particle up to 99% of the speed of light.
- β^-decay

  ${}^A_ZX \longrightarrow {}^{A}_{Z+1}Y + \beta^- + v^-$

  Q – value = $[m({}^A_ZX) - m({}^{A}_{Z+1}Y)]c^2$
- β^+ decay

  ${}^A_ZX \longrightarrow {}^{A}_{Z-1}Y + \beta^+ + v$

  Q – value = $[m({}^A_ZX) - m({}^{A}_{Z-1}Y) - m_e]c^2$

  where $m({}^A_ZX)$, $m({}^{A}_{Z-1}Y)$ are atomic masses
- Electron capture (K-capture): When atomic electron is captured by nucleus, X-rays are emitted.

  ${}^A_ZX + {}^{0}_{-1}e \longrightarrow {}^{A}_{Z-1}Y + v$

  Q – value = $[m({}^A_ZX) - m({}^{A}_{Z-1}Y)]c^2$

  where $m({}^A_ZX) - m({}^{A}_{Z-1}Y)$ are atomic masses.

γ decay

A γ-ray is the packet of electromagnetic energy released when a nucleus is formed in excited state after α or β-decay which releases energy to come down to ground state. γ-rays travel at the speed of light, carry no mass or charge, and are not deflected by electric or magnetic fields.

Law of Radioactive Decay

The rate of disintegration is directly proportional to the number of radioactive atoms present at that time i.e., rate of decay $\propto$ number of nuclei.

Rate of decay = λ (number of active nuclei)

i.e., $\frac{dN}{dt} = -\lambda N$.

where λ is called the decay constant.

If N_0 is the number of parent nuclei at $t = 0$. The number that survives at time t is $N = N_0e^{-\lambda t}$

Probability of a nucleus for survival of time $t = \frac{N}{N_0} = \frac{N_0e^{-\lambda t}}{N_0} = e^{-\lambda t}$

Half-life

- A **half-life** is the time taken for half of a group of unstable nuclei to decay. In other words, it is the time during which the number of undecayed atoms in the sample becomes half the total number of atoms present initially in the sample. Half-life is represented by the symbol $T_{1/2}$.

  Half life: $T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$
- Number of nuclei present after n half lives i.e. after a time $t = nT_{1/2}$, $N = \frac{N_0}{2^n}$

- A radioactive nucleus can decay by two different processes having half lives t_1 and t_2 respectively. Effective half-life of nucleus will be given by $\frac{1}{t}=\frac{1}{t_1}+\frac{1}{t_2}$.

Mean or Average Life

The ratio of the total life time of all the atoms of the element to the total number of atoms present initially in the sample of the element is known as the **mean or average life**. Mean life is represented by the symbol τ. It is equal to the reciprocal of the decay constant (λ) of the element.

Average life: $\tau=\frac{1}{\lambda}=\frac{T_{1/2}}{0.693}$

Activity

- Activity of sample:
  $R=\lambda N=R_0e^{-\lambda t}$
- Unit: 1 Bq = 1 decay/s, 1 curie = 3.7×10^{10} Bq, 1 rutherford = 10^6 Bq
- Activity per unit mass is called specific activity.
- Activity after n half lives: $\frac{A_0}{2^n}$