TOPICS TO BE COVERED

1. COORDINATE GEOMETRY BASICS, INCLINATION & SLOPE
2. EQUATION OF A STRAIGHT LINE, IMAGE FORMULA , PARAMETRIC FORM
3. LOCUS , ANGLE BISECTORS
4. FAMILY OF LINES & PAIR OF STRAIGHT LINES

NOTE *: This chapter is a gateway to Circles/ Parabola / Ellipse / Hyperbola / Complex Numbers*. (Will eventually give approx. **24-28 marks in JEE MAINS** and **5-7 Questions in JEE ADVANCED**)

TO GET THE 'BEST' FROM THIS CLASS

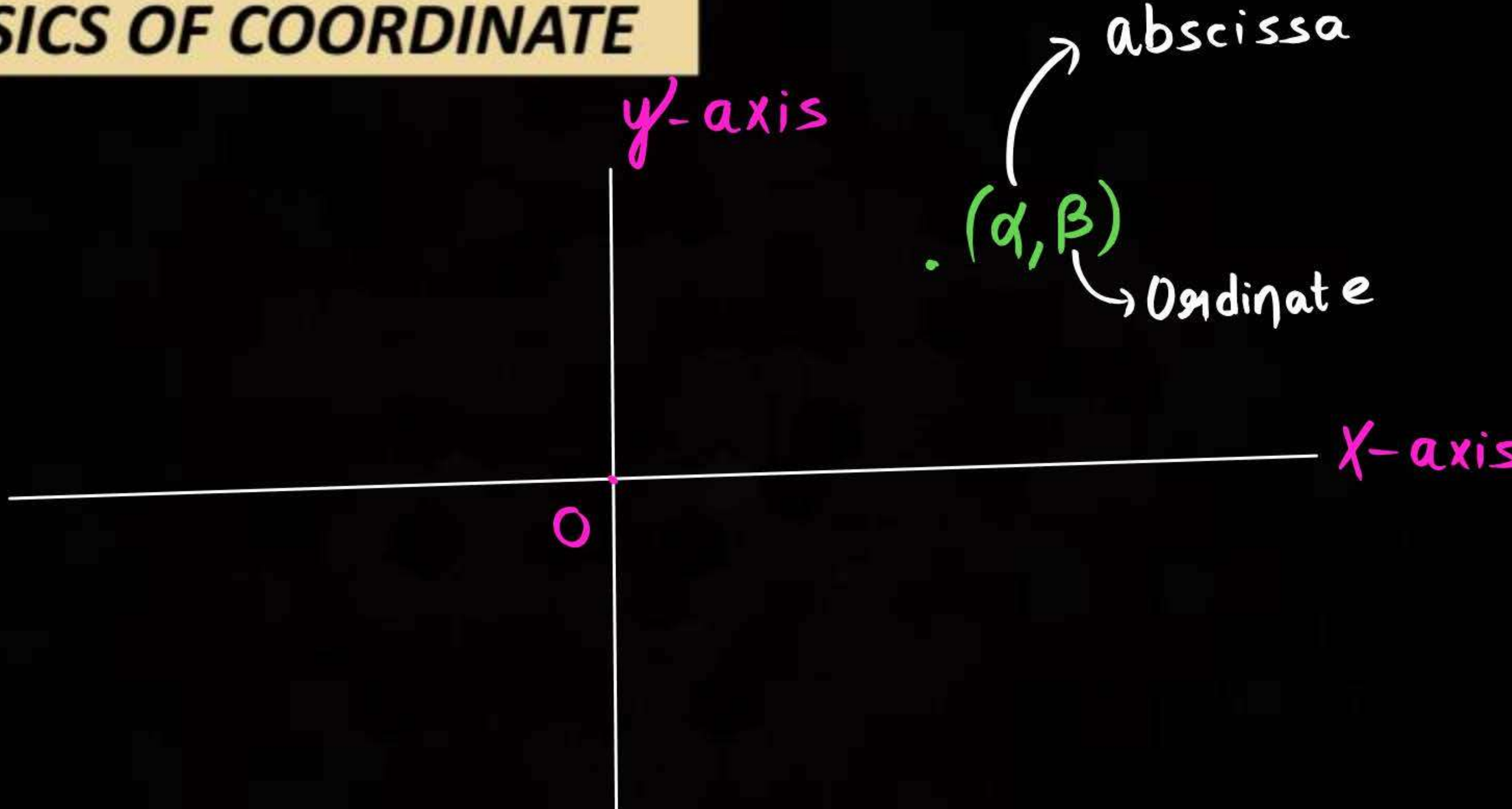
1. Keep a rough copy with you … Don't rush to write the notes …!
2. Listen to me carefully , have a smile !
3. Keep short notes copy with you & write what I request you to write.
4. Have Infinite Patience …. And enjoy the ride!!

Short Notes + Practice sheet

"Concept hum Sikhaenge , Calculation aapki !!

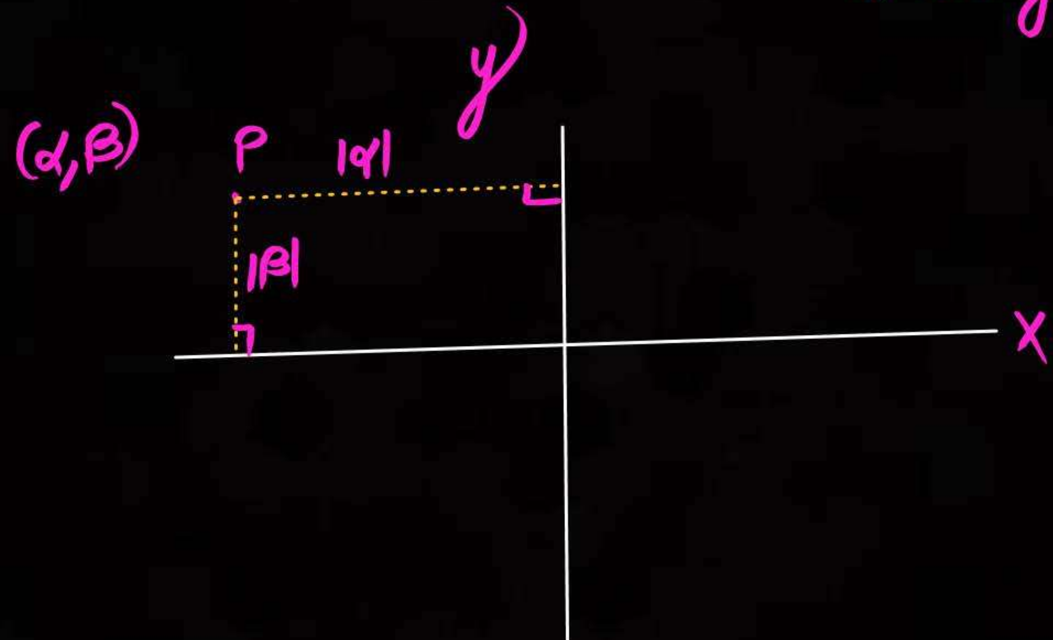
*Khushi hamari aur **Rank aapki** !"*

BASICS OF COORDINATE



DISTANCE OF A POINT FROM COORDINATE AXES

"X and y axis"



MEMORISE ⭐⭐

$P \to (\alpha, \beta)$

$D_{P-x\,axis} = |\beta| = |y\text{-COORDINATE}|$

$D_{P-y\,axis} = |\alpha| = |x\text{-COORDINATE}|$

Q.

Find the distance of following points from the coordinate axes:

(i) (-2 ,-4)

(ii) (-2 , 3)

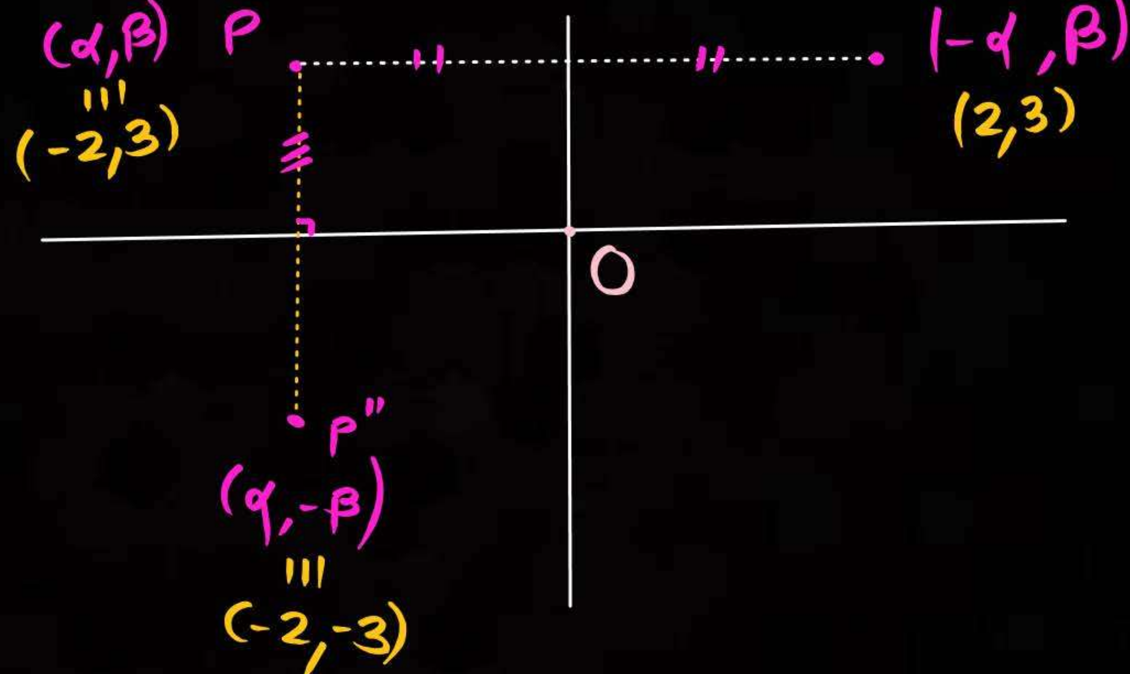
(i) $D_{x\text{-axis}} = |-4| = 4$ units

$D_{y\text{-axis}} = |-2| = 2$ units

(ii) $D_{x\text{-axis}} = |3| = 3$ units

$D_{y\text{-axis}} = |-2| = 2$ units

IMAGE OF A POINT WRT COORDINATE AXES



$\boxed{\text{MEMORISE}}$

$$(\alpha, \beta) \xrightarrow[\text{Image abt x-axis}]{\beta \to -\beta} (\alpha, -\beta)$$

$$(\alpha, \beta) \xrightarrow[\text{Image abt y-axis}]{\alpha \to -\alpha} (-\alpha, \beta)$$

Q.

Find the Image of following points wrt the coordinate axes:

(i) (-2 ,-4)

(ii) (-2 , 3)

(i) $(-2,-4) \xrightarrow{\text{abt x axis}} (-2,4)$ Ans

(i) $(-2,-4) \xrightarrow{\text{abt y-axis}} (2,-4)$ Ans

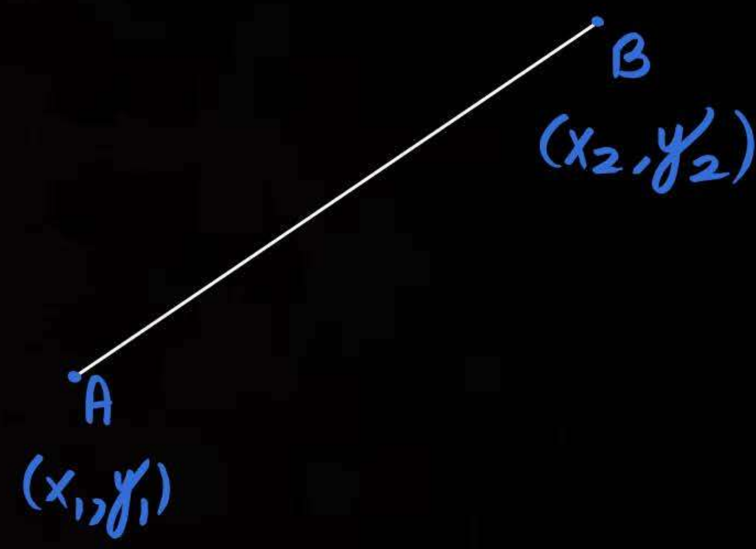
(ii) $(-2,3) \xrightarrow{\text{x axis}} (-2,-3)$ Ans

(ii) $(-2,3) \xrightarrow{\text{y-axis}} (2,3)$ Ans

DISTANCE FORMULA

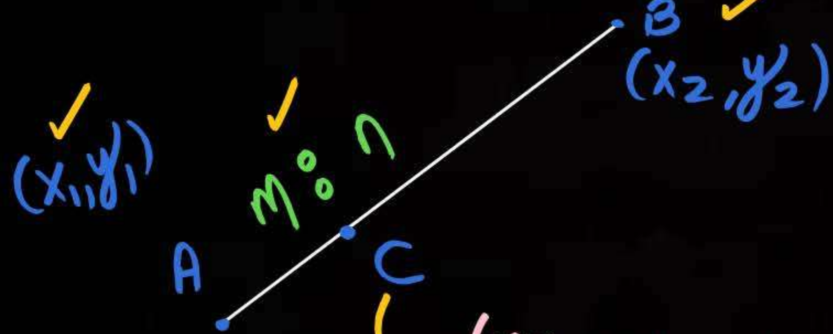
$$AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



SECTION FORMULA

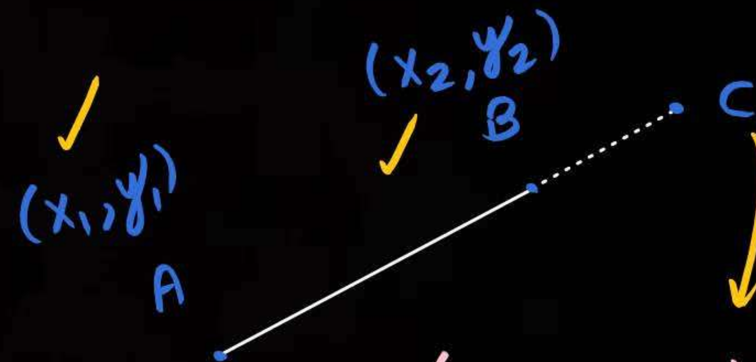
1. Internal Division :



$$\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$

$$\boxed{\frac{AC}{BC}=\frac{m}{n}}$$

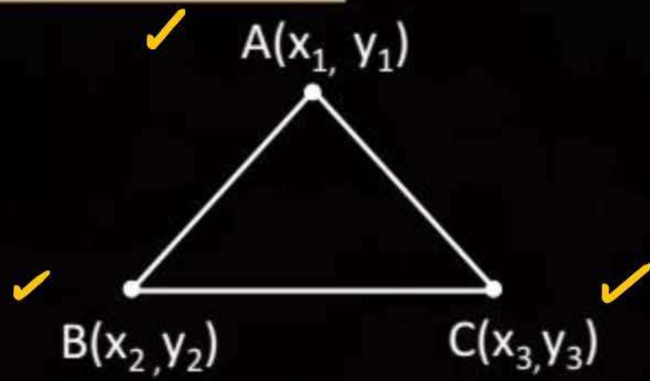
2. External Division :



$$\left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}\right)$$

$$\boxed{\frac{AC}{BC}=\frac{m}{n}}$$

AREA OF A Δ



$$\boxed{\text{Area}(ABC)} = \boxed{\frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|} \text{ square units.}$$

Q. Let $A\left(\frac{3}{\sqrt{a}}, \sqrt{a}\right)$ $a > 0$, be a fixed point in the xy-plane. The image of A in y-axis be B and the image of B in x-axis be C. If $D(3\cos\theta, a\sin\theta)$ is a point in the fourth quadrant such that the maximum area of Δ ACD is 12 square units, then a is equal to ________ .

[JEE Main-2022 (24 June-Shift 1)]

$B \to \left(-\frac{3}{\sqrt{a}}, \sqrt{a}\right)$

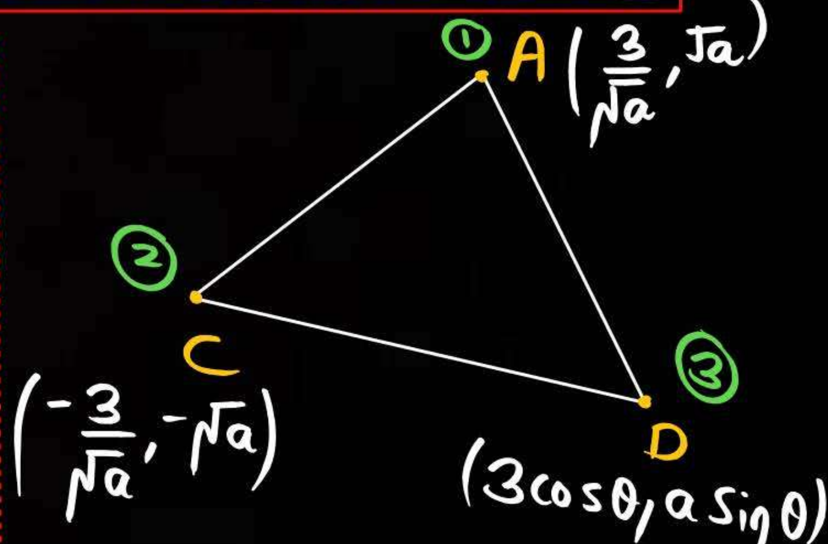
WRT x-axis

$C \to \left(-\frac{3}{\sqrt{a}}, -\sqrt{a}\right)$

area

$$= \frac{1}{2}\left| \frac{3}{\sqrt{a}}\left(-\sqrt{a} - a\sin\theta\right) + \left(-\frac{3}{\sqrt{a}}\right)\left(a\sin\theta - \sqrt{a}\right) + 3\cos\theta\left(\sqrt{a} - (-\sqrt{a})\right) \right|$$

$$= \frac{1}{2}\left| -6\sqrt{a}\sin\theta + 6\sqrt{a}\cos\theta \right|$$



$$= \frac{1}{2} \cdot 6\sqrt{a}\, |\cos\theta - \sin\theta|$$

$$= (3\sqrt{a})\, \underbrace{|\cos\theta - \sin\theta|}_{\max = \sqrt{2}}$$

$$(\text{area})_{max} = \boxed{3\sqrt{2a} = 12}$$

$\boxed{a = 8}$ Ans

$(a\sin\theta + b\cos\theta)$

$\in \left[-\sqrt{a^2+b^2}, \sqrt{a^2+b^2}\right]$

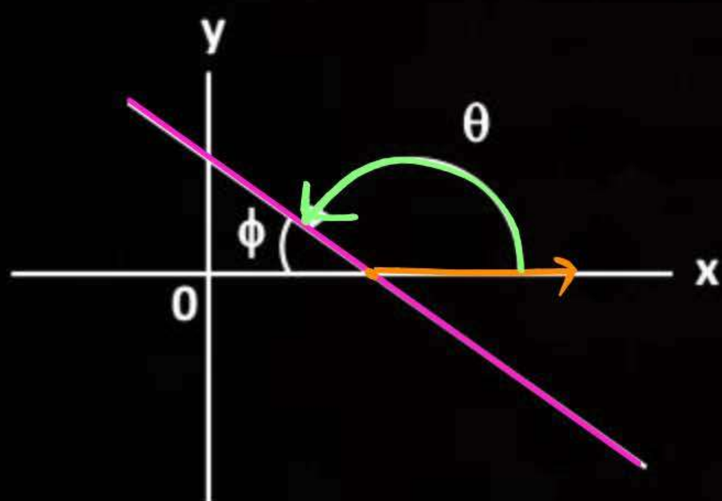
$-\sin\theta + \cos\theta$

$a = -1,\ b = 1$

$[-\sqrt{2}, \sqrt{2}]$

ANGLE OF INCLINATION OF A LINE :

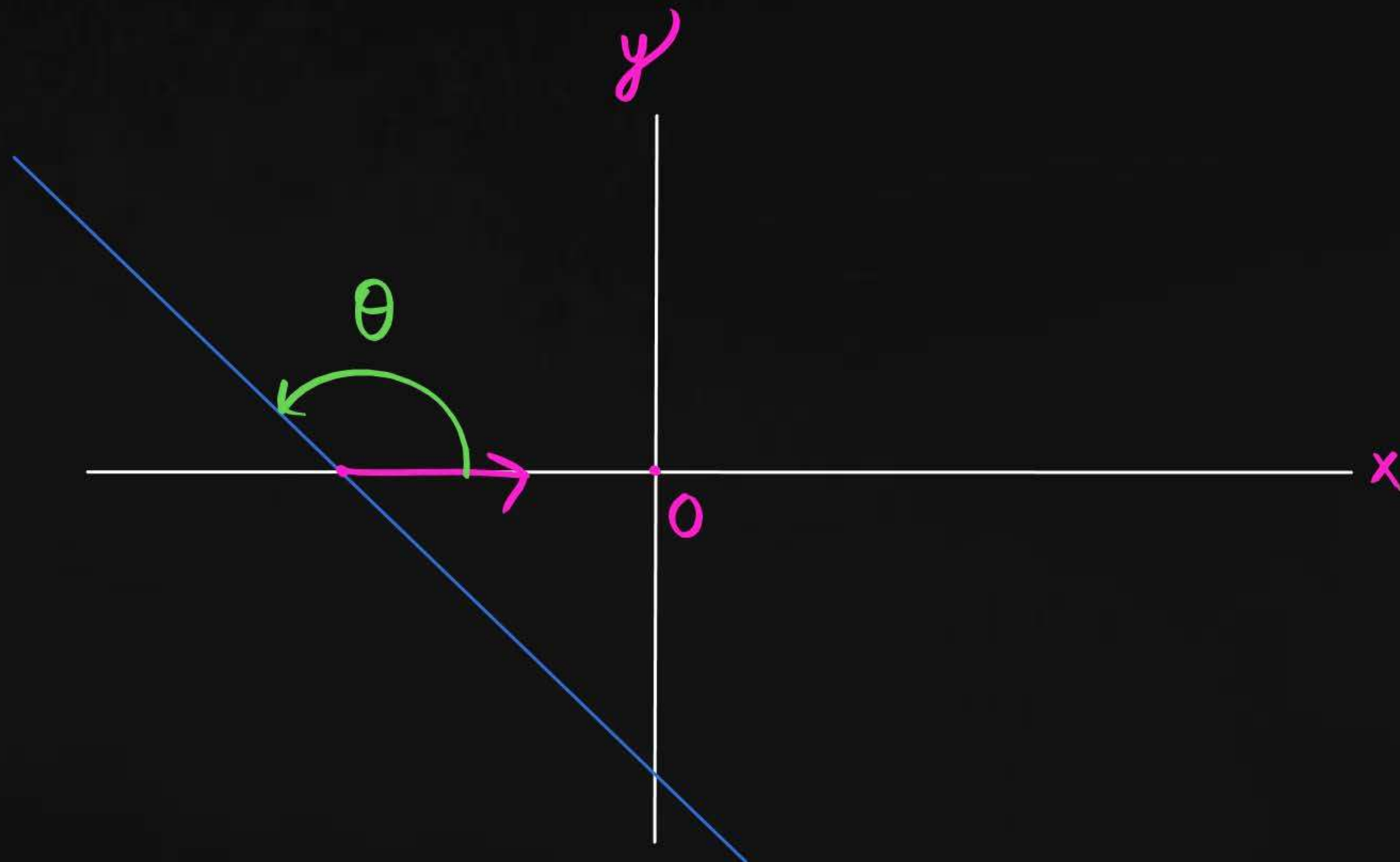
Measure of the angle between the positive direction of the x-axis and the line measured in the anticlockwise direction.



Slope (gradient) $= \tan\theta = m$

$\boxed{\text{Inclination} = \theta}$

$\boxed{\textit{NOTE}: \ 0 \le \theta < \pi}$



y
θ
O
x

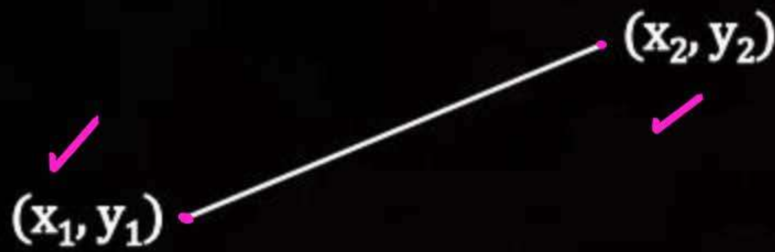
ILLUSTRATION:

Find the inclination of the line with slope $= -\frac{1}{\sqrt{3}}$.

$\theta \in [0, \pi)$

$\tan\theta = -\frac{1}{\sqrt{3}} \rightarrow \theta = 150^\circ$ $\boxed{\text{Ans}}$

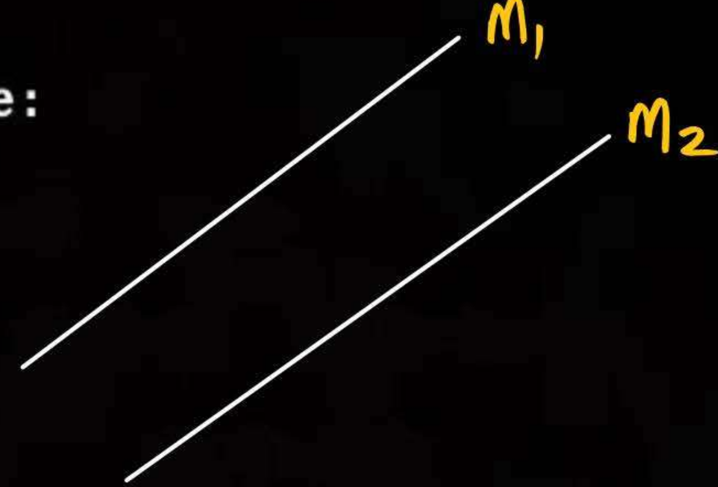
SLOPE OF THE LINE JOINING TWO POINTS



$$\boxed{m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}}$$

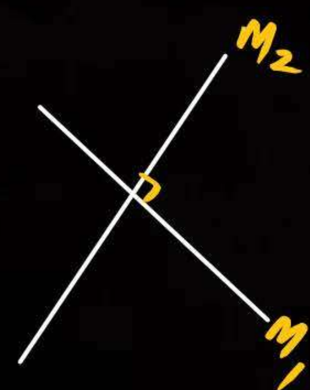
1. Parallel lines have the same slope :

$m_1 = m_2$

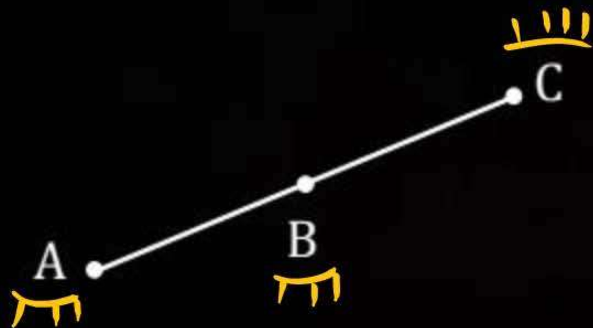


2. Case of Perpendicular lines :

$\boxed{m_1 m_2 = -1}$



COLLINEAR POINTS :



$$\boxed{m_{AB} = m_{BC} = m_{AC}}$$

Q. Let the area of the triangle with vertices A(1, α), B(α, 0) and C(0, α) be 4 sq. units. If the point (α, –α), (–α, α) and (α², β) are collinear, then β is equal to:

D E F

[JEE Main-2022 (24 June-Shift 2)]

(A) 64

(B) –8

(C) –64

(D) 512

$$\frac{1}{2}\left|1(0-\alpha)+\alpha(\alpha-\alpha)+0(\alpha-0)\right|$$

$$= \boxed{\frac{|-\alpha|}{2} = 4}$$

$$\boxed{|\alpha| = |-\alpha| = 8}$$

$$\alpha = \pm 8$$

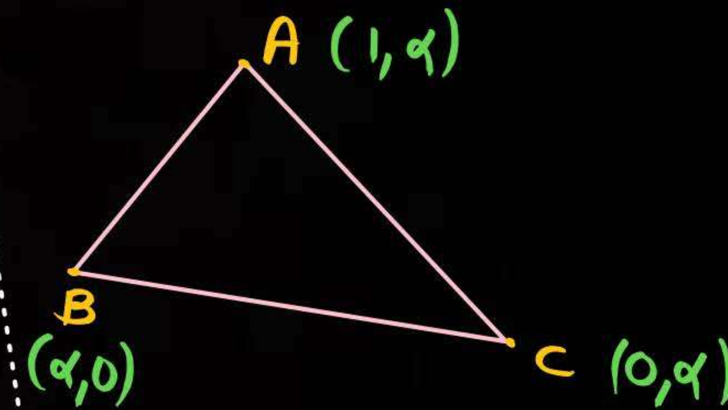
$$\alpha^2 = 64$$

$$m_{DE} = m_{EF} = m_{DF}$$

$$\frac{\alpha-(-\alpha)}{-\alpha-\alpha} = \frac{\beta-\alpha}{\alpha^2-(-\alpha)}$$

$$-1 = \frac{\beta-\alpha}{\alpha^2+\alpha}$$

$$-\alpha^2-\alpha = \beta-\alpha \Rightarrow \boxed{\beta = -\alpha^2}$$



FORMS OF STRAIGHT LINE

Any linear equation in x & y is the equation of a straight line.

Power = 1

$$\boxed{ax + by + c = 0}$$

$a = b = 0$

$c = 2$

$2 = 0$ ✗

$a, b, c \to$ scalars

$a, b, c \in R$

ex $\to 2x - y + 7 = 0$

$2x^1y^1 + 1 = 0$ ✗

'x' and 'y' not multiplied together..

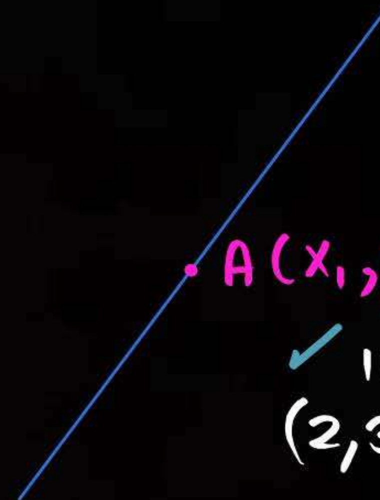
1. POINT-SLOPE FORM

$$\frac{y-y_1}{x-x_1}=m$$

ex) $\dfrac{y-3}{x-2}=-7$

$$y-3=-7x+14$$

$$\boxed{y+7x=17}$$



m = slope

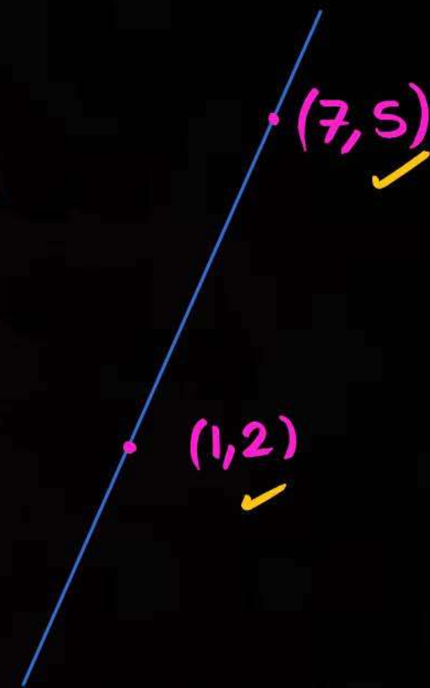
✓ $\equiv -7$

✓ $\parallel$

$(2,3)$

2. TWO -POINT FORM

$$M = \frac{5-2}{7-1} = \frac{1}{2}$$



$$\frac{y-5}{x-7} = \frac{1}{2} \Rightarrow 2y-10 = x-7$$

$$\boxed{2y-x=3}$$

OR

$$\frac{y-2}{x-1} = \frac{1}{2} \Rightarrow 2y-4 = x-1$$

$$\boxed{2y-x=3}$$

Q. If in a parallelogram ABDC, the coordinates of A, B and C are respectively (1, 2), (3, 4) and (2, 5), then the equation of the diagonal AD is:

(JEE MAINS-2019)

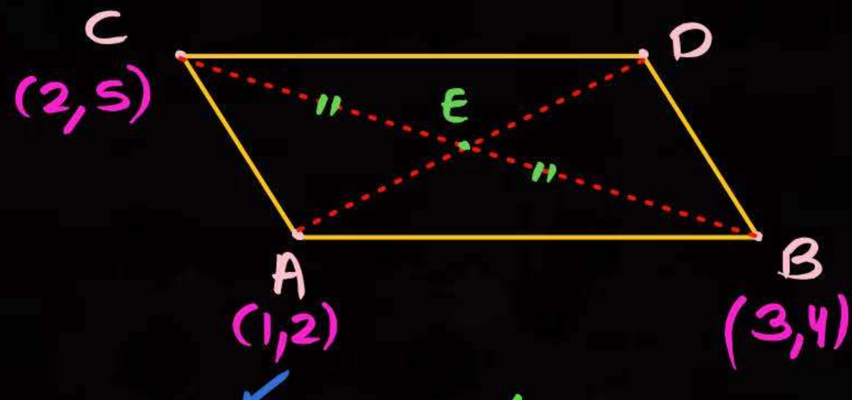
(MAZAAK)

(A) $5x - 3y + 1 = 0$ ✓

(B) $5x + 3y - 11 = 0$

(C) $3x - 5y + 7 = 0$

(D) $3x + 5y - 13 = 0$



$E \equiv \left(\frac{5}{2}, \frac{9}{2}\right)$

$\boxed{AD}$

$$\frac{y-2}{x-1} = \frac{\frac{9}{2} - 2}{\frac{5}{2} - 1}$$

$$\frac{y-2}{x-1} = \frac{5}{3}$$

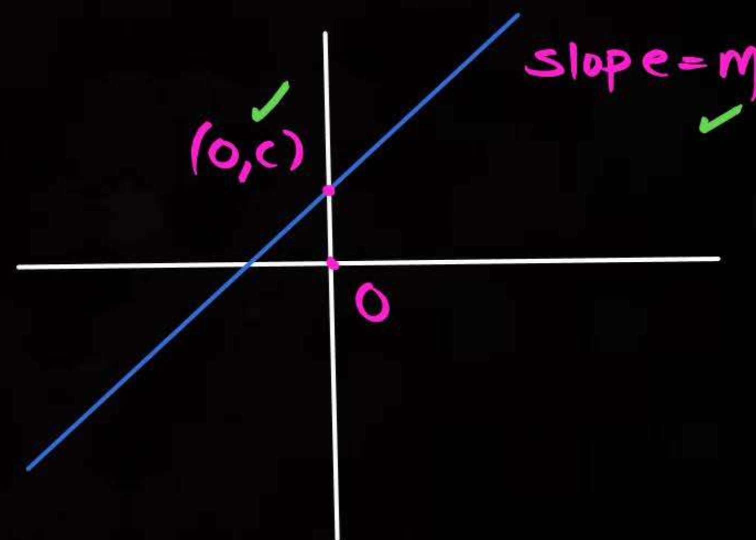
3. SLOPE-INTERCEPT FORM

$$\frac{y-c}{x-0} = m$$

$$y - c = mx$$

$$\boxed{y = mx + c}$$

$$\left\{ \begin{array}{l} y\text{-axis Intercept} = c \\ \text{slope} = m \end{array} \right\}$$



Q→ $c = -2$

$m = 7$,

eqn ≡ ?

$\boxed{y = 7x + (-2)}$ Ans

$\boxed{ax + by + c = 0}$

$by = -ax - c$

$$\boxed{y = \left(-\frac{a}{b}\right)x - \frac{c}{b}}$$

$y = mx + c_1$

$$\boxed{m = -\frac{a}{b} = -\frac{\text{coeff. of } x}{\text{coeff of } y}}$$

$x + 7y = 2$ $\longrightarrow$ $x + 7y - 2 = 0$

$m = ?$

$m = \frac{-1}{7}$ Ans.

Q. If the two lines $x + (a-1)y = 1$ and $2x + a^2y = 1$ $(a \in R - \{0, 1\})$ are perpendicular, then the distance of their point of intersection from the origin is:

(JEE MAINS-2019)

(A) $\sqrt{\frac{2}{5}}$

(B) $\frac{2}{5}$

(C) $\frac{2}{\sqrt{5}}$

(D) $\frac{\sqrt{2}}{5}$

$\boxed{m_1 = \frac{-1}{a-1}}$ $\boxed{m_2 = \frac{-2}{a^2}}$

$m_1 m_2 = -1$

$\frac{+2}{(a-1)a^2} = -1$

$\boxed{a^3 - a^2 + 2 = 0}$ $a = -1$

Long Div. method

$a+1)\, a^3 - a^2 + 2\, (a^2 - 2a + 2$

$a^3 + a^2$

$-2a^2 + 2$

$-2a^2 - 2a$

$2 + 2a$

$2 + 2a$

0

$(a+1)(a^2-2a+2)=0 \longrightarrow \boxed{a=-1}$

$$\begin{cases} x-2y-1=0 \\ 2x+y-1=0 \end{cases}$$

$2x-4y-2=0$

$2x+y-1=0$

$-5y-1=0$

$y=-\frac{1}{5}$ $\boxed{y=-\frac{1}{5}}$

$2x-\frac{1}{5}-1=0$

$\boxed{x=\frac{3}{5}}$

$POI \rightarrow \left(\frac{3}{5}, -\frac{1}{5}\right)$

$(0,0)$

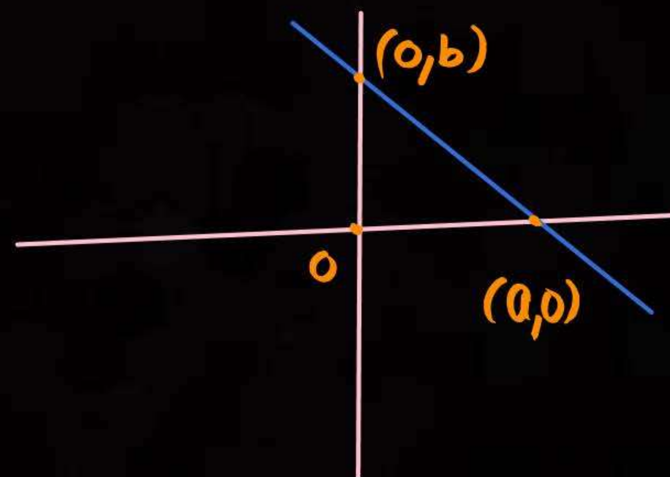
$Ans = \sqrt{\frac{9}{25}+\frac{1}{25}}$

$= \sqrt{\frac{2}{5}}$ units

4. INTERCEPT FORM

x-axis Intercept = a $\rightarrow -2$

y-axis Intercept = b $\rightarrow 3$



$$\boxed{\frac{x}{a} + \frac{y}{b} = 1}$$

Ans $\rightarrow \dfrac{x}{-2} + \dfrac{y}{3} = 1$

GYAAN BHARI BAATEIN

(1) If $a = b$, then $\boxed{m = -1}$ (EQUAL $\boxed{\text{NON ZERO}}$ INTERCEPTS)

$$\frac{x}{a} + \frac{y}{a} = 1 \Rightarrow x + y - a = 0 \longrightarrow m = -\frac{1}{1} = -1$$

(2) If line cuts Equal Distances on coordinate axis, then slope $= \pm 1$.

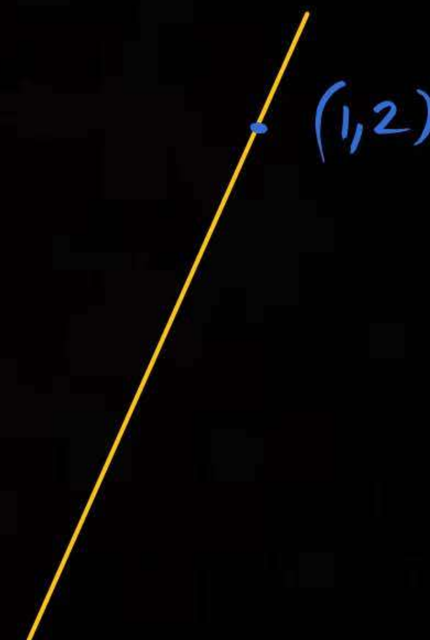
(3) Line is equally inclined with the coordinate axis, then $m = \pm 1$.
(making equal angles with the coordinate axis)

ILLUSTRATION:

$m = -1$

Find the line which passes through the point $(1, 2)$ and cuts equal non zero intercepts on coordinate axis.

$$\frac{y-2}{x-1} = -1 \quad \text{Ans}$$



5. NORMAL FORM OF THE LINE -

Let a line is taken at p distance from origin and perpendicular from the (0, 0) to the line makes α with the positive direction of x axis.

$$x \cos \alpha + y \sin \alpha = p$$

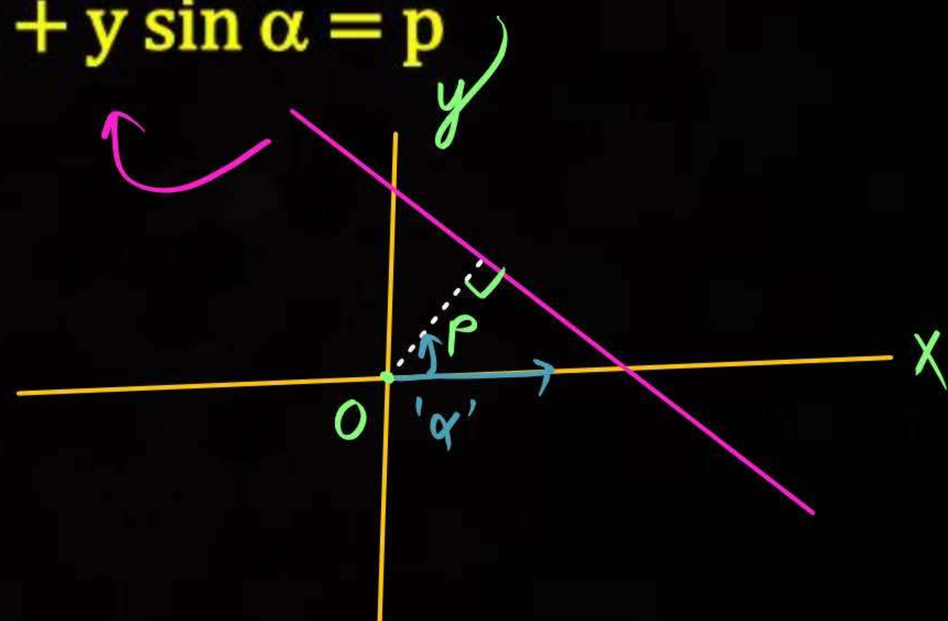


ILLUSTRATION:

Find the equation of the line whose distance from the origin is 2 units and perpendicular to the line from (0, 0) form 60^0 with the positive direction of x-axis.

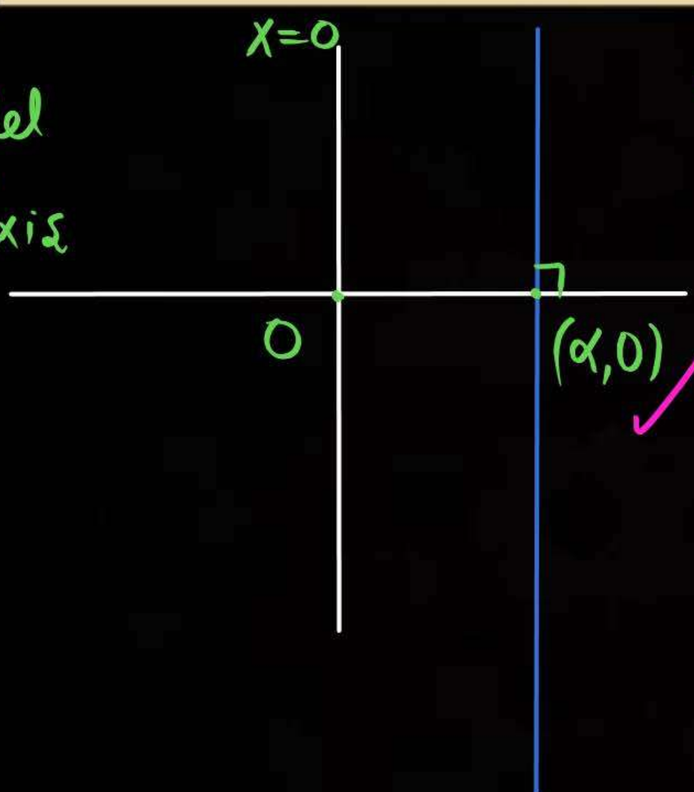
$\boxed{p = 2}$ $\alpha = 60^0$

Ans →

$x\cos 60^\circ + y\sin 60^\circ = 2$

LINES PARALLEL TO COORDINATE AXES

Parallel to y-axis



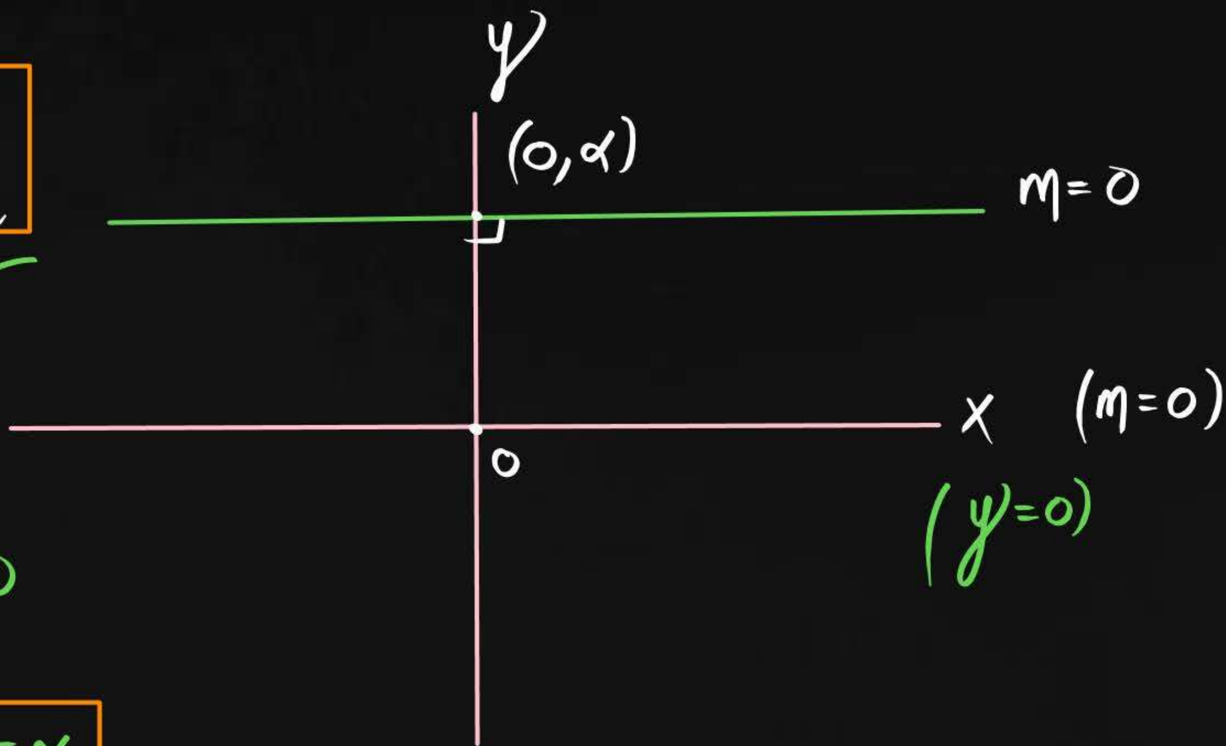
$$\tan 90^\circ = \frac{1}{0} = \frac{\sin 90^\circ}{\cos 90^\circ}$$

$$\frac{y-0}{x-\alpha} = \frac{1}{0} \Rightarrow x-\alpha=0$$

$$\boxed{x=\alpha}$$

parallel to y-axis

Parallel to x-axis



$$\frac{y-\alpha}{x-0} = 0$$

$$\boxed{y = \alpha}$$

Line parallel to x-axis $\longrightarrow$ $y = \alpha$; $(0, \alpha)$

Line parallel to y-axis $\longrightarrow$ $x = \beta$; $(\beta, 0)$

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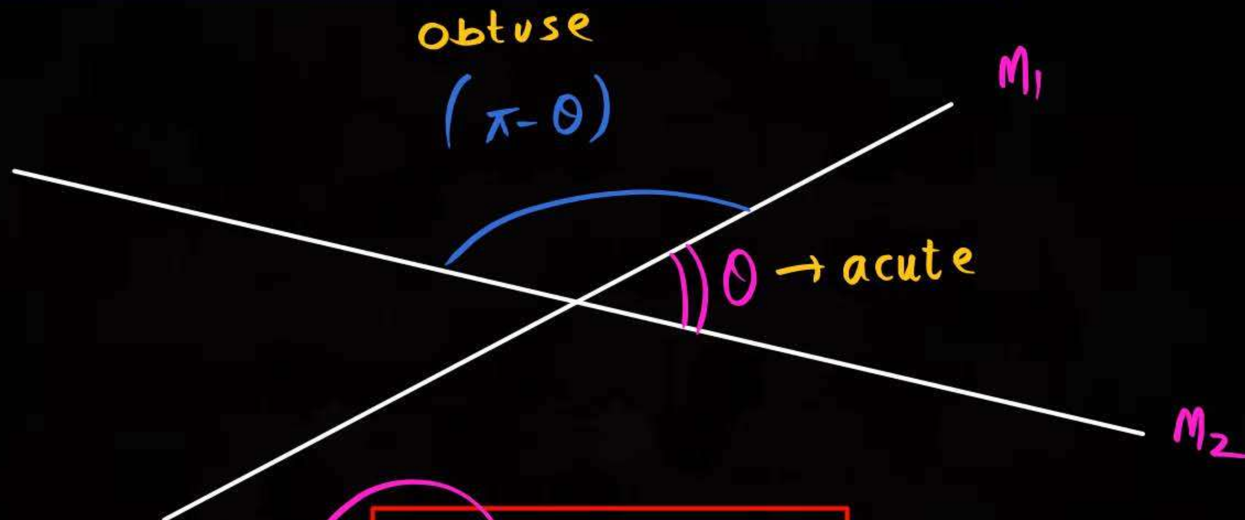
ANGLE BETWEEN TWO LINES



$$\boxed{\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|}$$

acute angle

Q. A straight line L through the point $(3, -2)$ is inclined at an angle 60° to the line $\sqrt{3}x + y = 1$ if L also intersect the x-axis, then the equation of L is:

[JEE 2011 , ADVANCED]

(A) $y + \sqrt{3}x + 2 - 3\sqrt{3} = 0$

(B) $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$

(C) $\sqrt{3}y - x + 3 + 2\sqrt{3} = 0$

(D) $\sqrt{3}y + x - 3 + 2\sqrt{3} = 0$

$\frac{y+2}{x-3} = m \xrightarrow{m=0} \boxed{y=-2}$

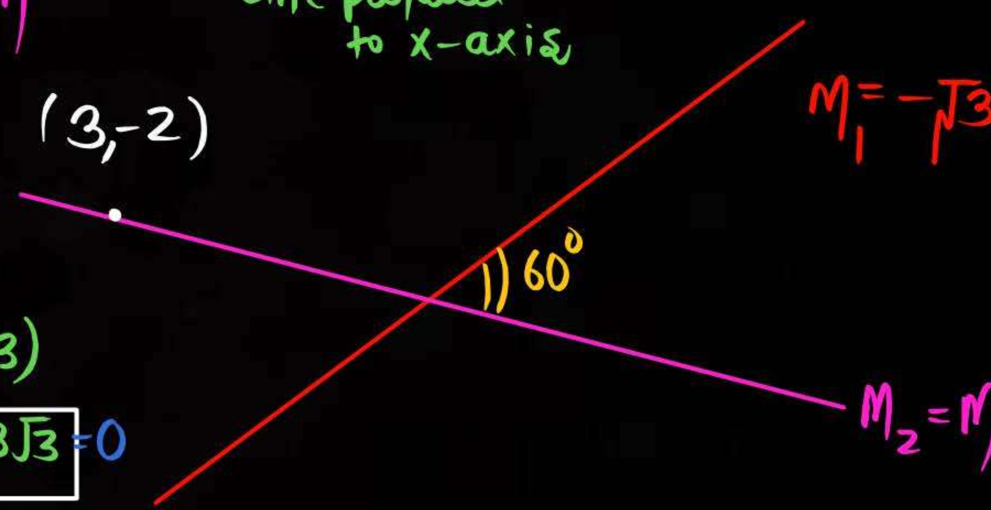
Line parallel to x-axis

$m = \sqrt{3}$

$y + 2 = \sqrt{3}(x-3)$

$\boxed{y - \sqrt{3}x + 2 + 3\sqrt{3}} = 0$

Ans



$$\sqrt{3} = \boxed{\tan 60^\circ = \left|\frac{-\sqrt{3} - m}{1 + (-\sqrt{3})(m)}\right|}$$

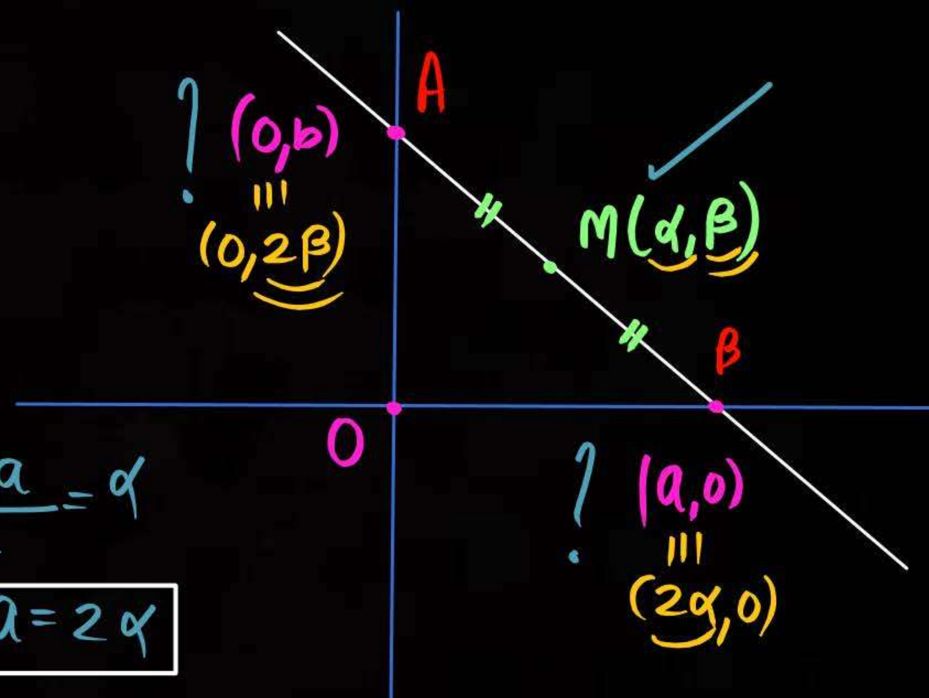
$$\left(\pm\sqrt{3} = \frac{-\sqrt{3} - m}{1 - \sqrt{3}m}\right)$$

$\xrightarrow{(+)}$ $m = \sqrt{3}$

$\xrightarrow{(-)}$ $m = 0$

BKG

(Calculation easy ho Jaegi)



$$\frac{b+0}{2}=\beta$$

$$\boxed{b=2\beta}$$

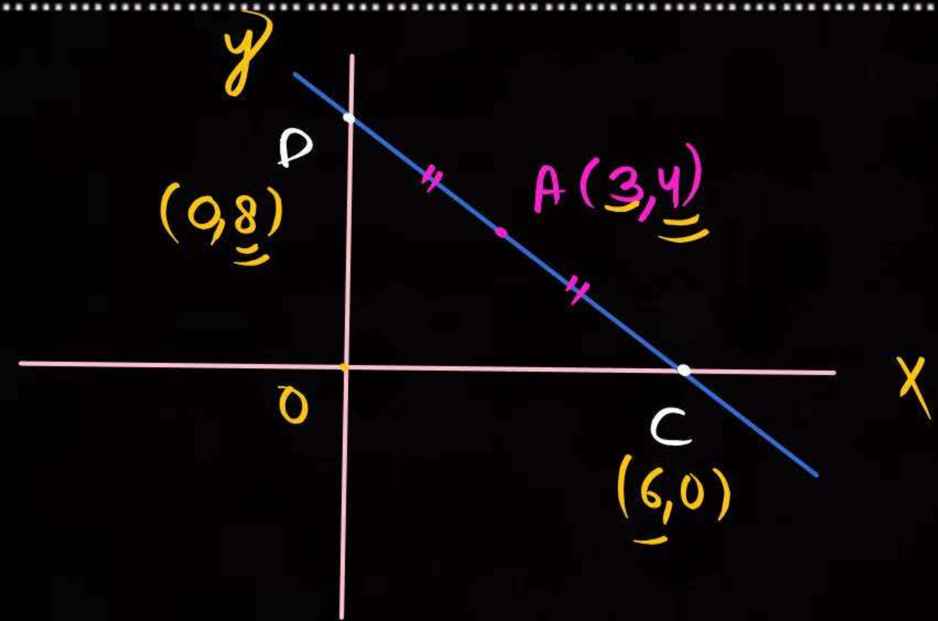
$$\frac{0+a}{2}=\alpha$$

$$\boxed{a=2\alpha}$$

Q. A straight line passing through the point A(3,4) is such that its intercept between the axes is bisected at A. then its equation is-

(CD)

[AIEEE 2006]
(REPEATED IN MAINS 2019)



$$\boxed{\frac{x}{6}+\frac{y}{8}=1}$$

Ans

IMAGE OF A POINT ABOUT A LINE

(JEE mains every year)

$$\boxed{\frac{x_1-\alpha}{a}=\frac{y_1-\beta}{b}=-2\left(\frac{a\alpha+b\beta+c}{a^2+b^2}\right)}$$

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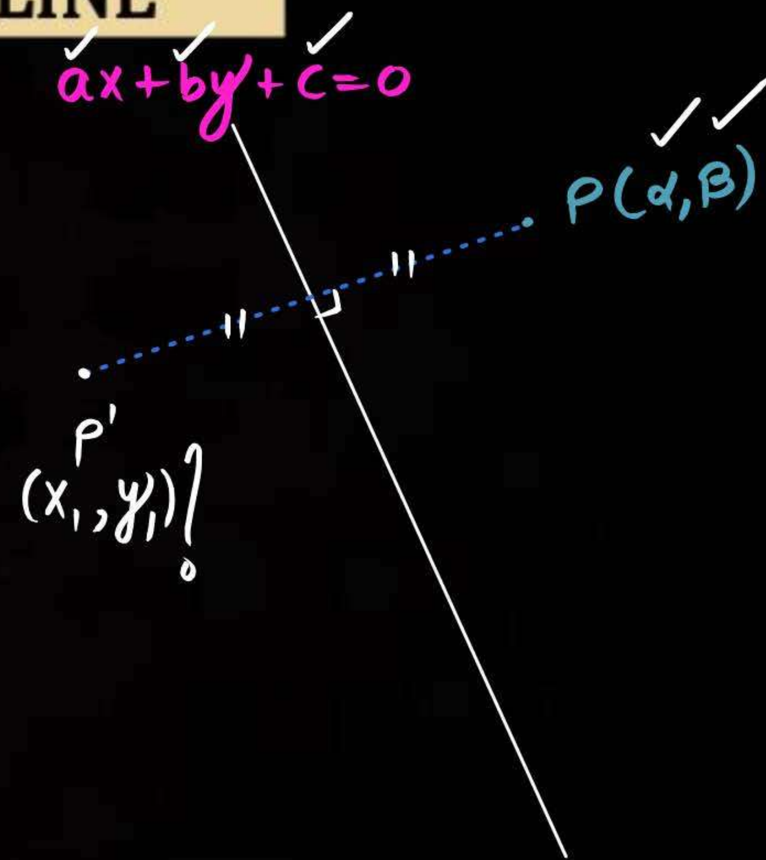


Q. The image of the point $\boxed{(3, 5)}$ in the line $\boxed{x - y + 1 = 0}$, lies on :

$\alpha = 3$
$\beta = 5$

$a = 1$
$b = -1$
$c = 1$

(JEE MAINS-2021)

(A) $(x-2)^2 + (y-4)^2 = 4$

$x = 4,\ y = 4$

(B) $(x-4)^2 + (y+2)^2 = 16$

(C) $(x-4)^2 + (y-4)^2 = 8$

(D) $(x-2)^2 + (y-2)^2 = 12$

$(x_1, y_1) \equiv ?$

$$\frac{x_1 - 3}{1} = \frac{y_1 - 5}{-1} = -\cancel{2}\left(\frac{3 - 5 + 1}{\cancel{2}}\right) = 1$$

$x_1 = 4$

$y_1 = 4$

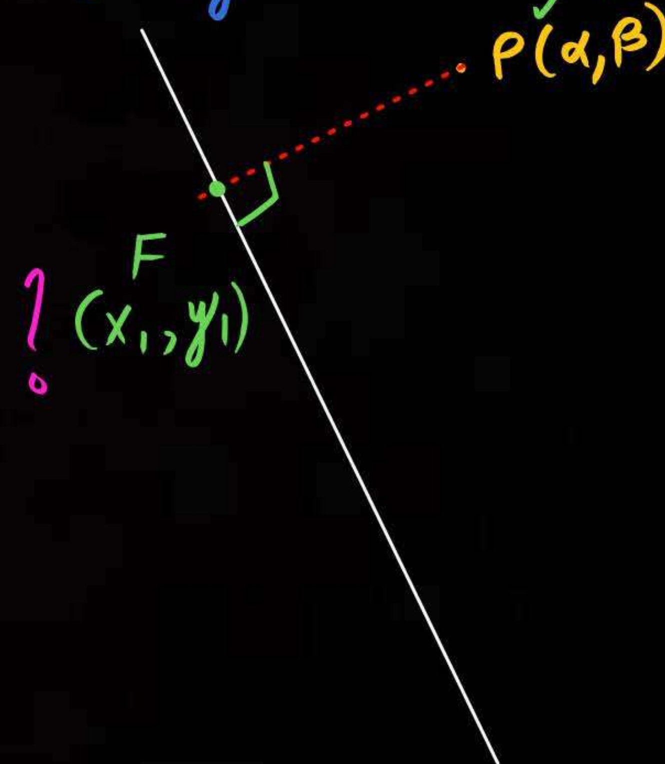
$\boxed{(4, 4)}$

FOOT OF THE PERPENDICULAR

(NEAREST POINT)

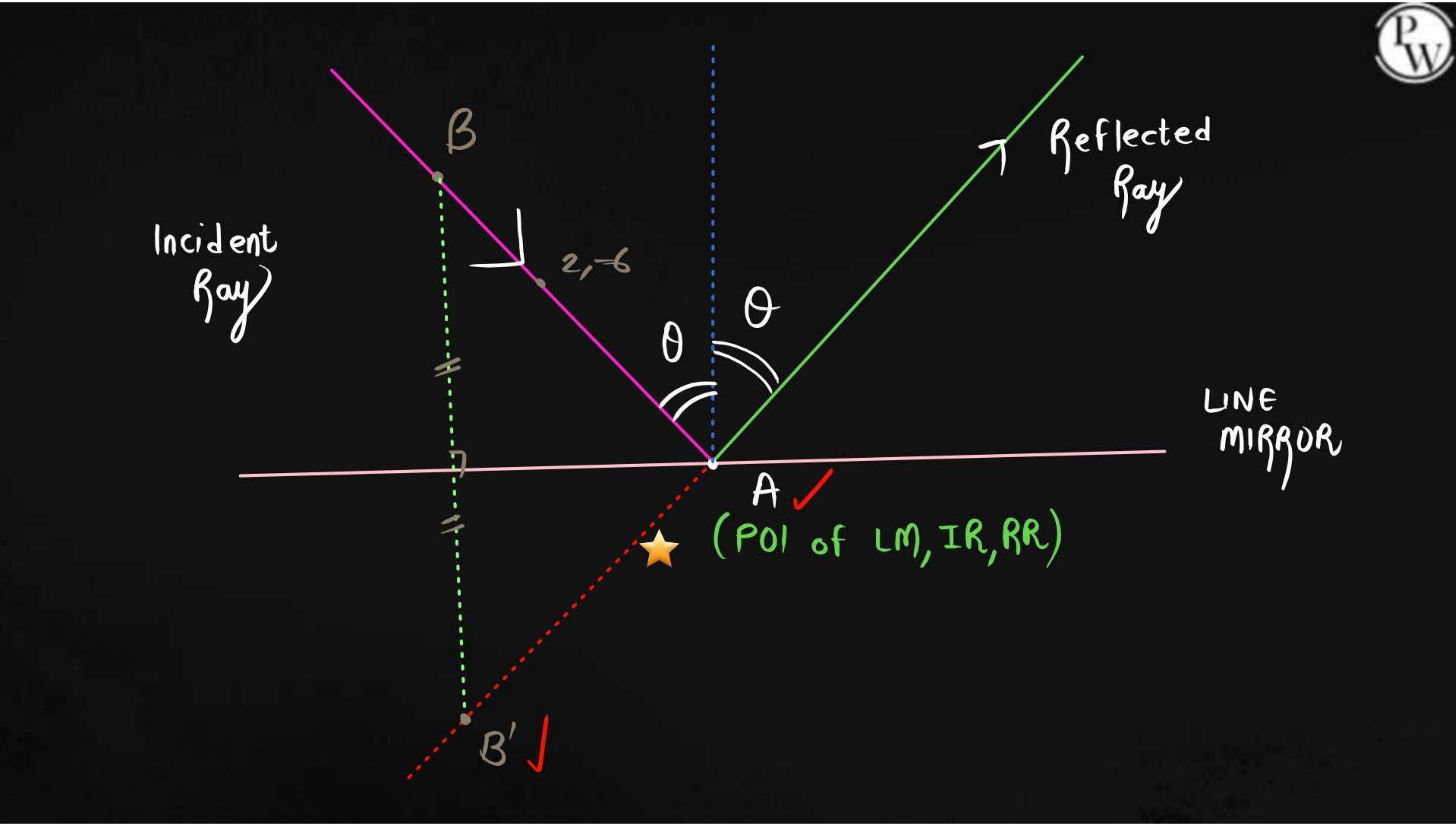
$$\frac{x_1-\alpha}{a}=\frac{y_1-\beta}{b}=-\left(\frac{a\alpha+b\beta+c}{a^2+b^2}\right)$$

$ax+by+c=0$



Every year

OPTICS PROBLEM


B
Incident Ray
2, -6
θ
θ
Reflected Ray
LINE MIRROR
A
(POI of LM, IR, RR)
B'

WORKING RULE

$\boxed{\text{Eqn}_{RR} \equiv ?}$

1. If any point lying on the reflected ray is not given , Find POI with the mirror of Incident Ray. (A)
2. Find any point on the incident ray. (B)
3. Find its reflection about the line mirror (which lies on the reflected ray.) B (B')
4. Use two-point form and get reflected ray equation.

Q. A ray of light along $x + \sqrt{3}y = \sqrt{3}$ gets reflected upon reaching x-axis, the equation of the reflected ray is:

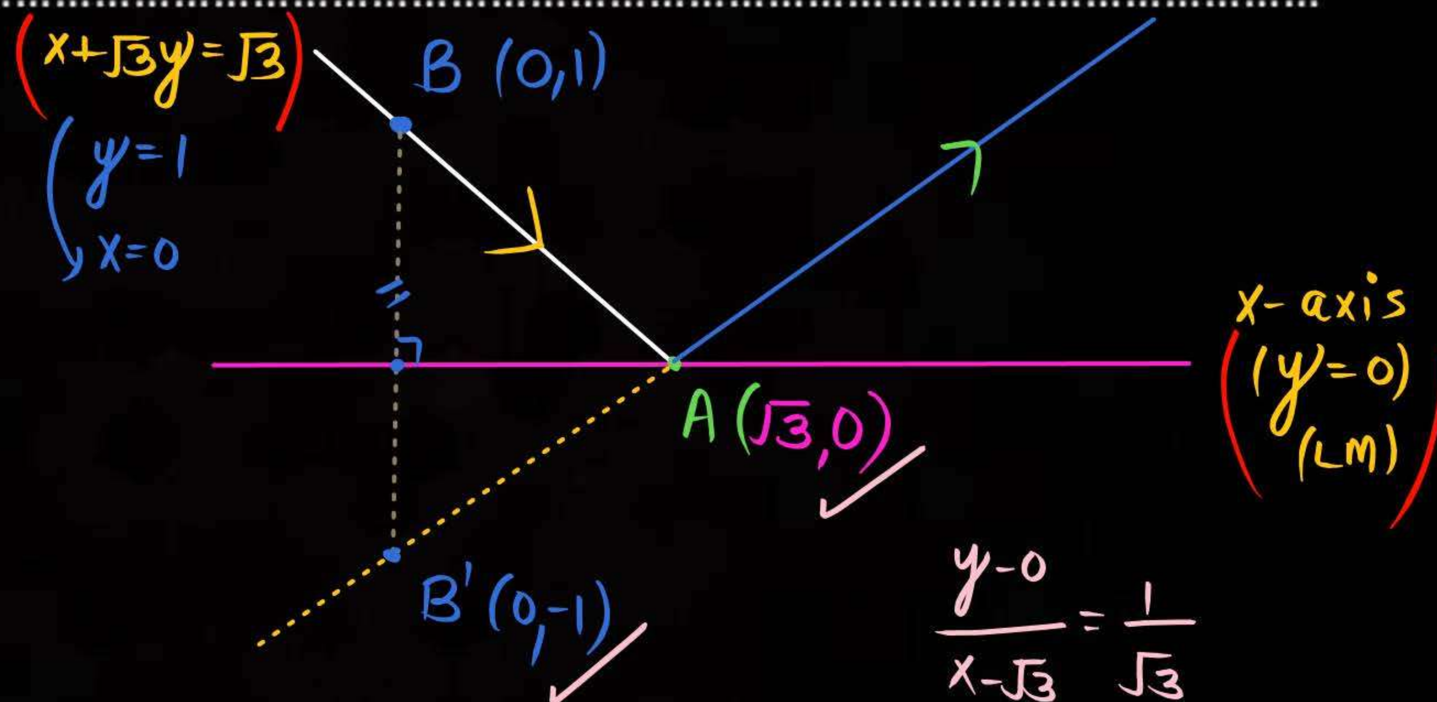
(JEE Main-2013)

(A) $y = x + \sqrt{3}$

(B) $\sqrt{3}y = x - \sqrt{3}$

(C) $y = \sqrt{3}x - \sqrt{3}$

(D) $\sqrt{3}y = x - 1$



$$\frac{y-0}{x-\sqrt{3}} = \frac{1}{\sqrt{3}}$$

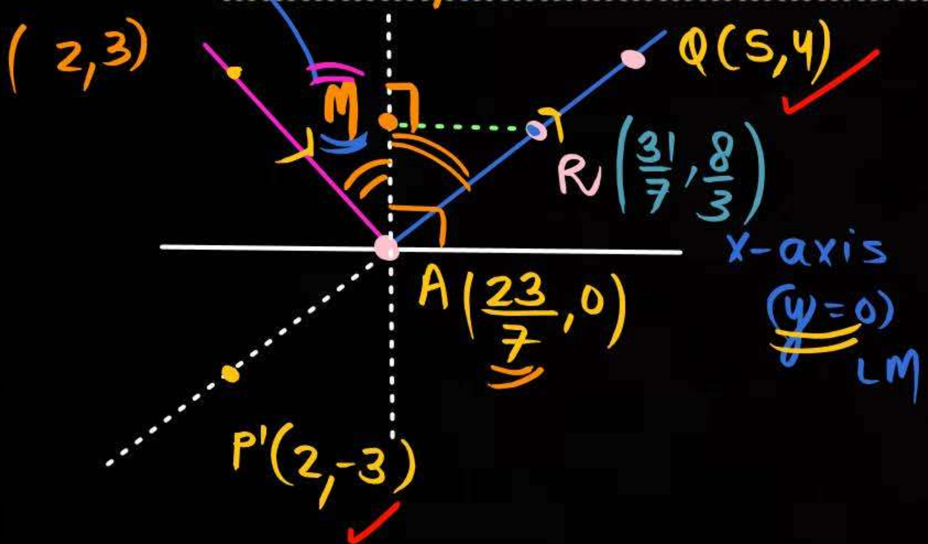
$\boxed{\text{Ans}}$

Q. A ray of light passing through the point P(2, 3) reflects on the x-axis at point A and the reflected ray passes through the point Q(5, 4). Let R be the point that divides the line segment AQ internally into the ratio 2 : 1. Let the co-ordinates of the foot of the perpendicular M from R on the bisector of the angle PAQ be (α, β). Then, the value of $7\alpha + 3\beta$ is equal to 31 Ans.

(α, β)

$\left(\frac{23}{7}, \frac{8}{3}\right)$

$x = 23/7$



Very Calculative But very easy

[JEE Main-2022 (28 June-Shift 1)]

BKG

Calculation Easy

RR

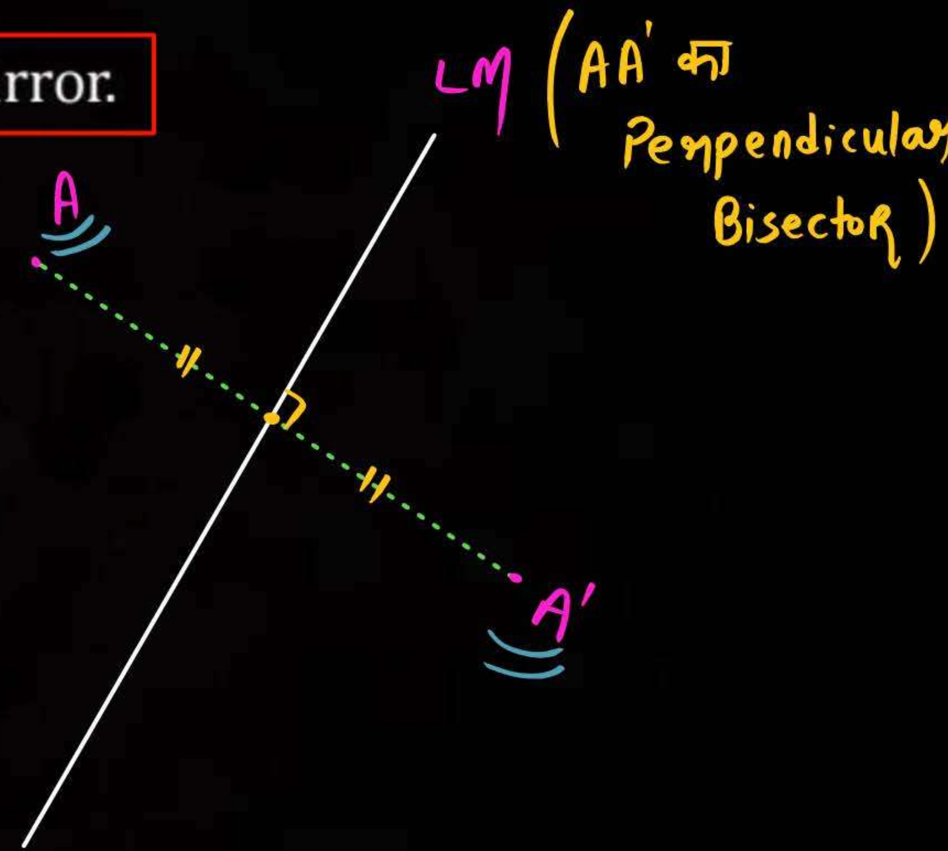
$$\frac{y-4}{x-5} = \frac{7}{3}$$

$$3y - 12 = 7x - 35$$

$$\boxed{7x - 3y = 23}$$

BKG

Perpendicular Bisector acts as Line Mirror.



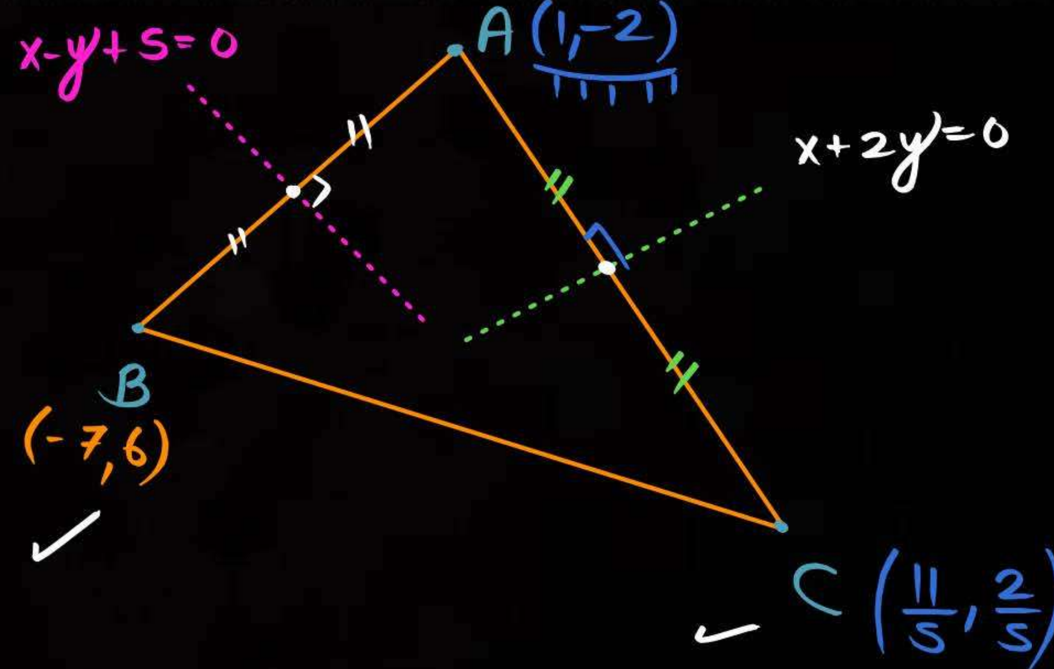
(Line mirror)

Q. Equation of perpendicular bisector of the sides AB and AC of a triangle ABC are $x - y + 5 = 0$ and $x + 2y = 0$. If the vertex A is $(1, -2)$. Find the equation of BC.

FYQ

Ans →

$$\frac{y-6}{x+7} = \frac{\frac{2}{5}-6}{\frac{11}{5}-(-7)}$$



BKG ♡♡
BKG ♡

LINES KI MAYA :

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ (Intersecting)

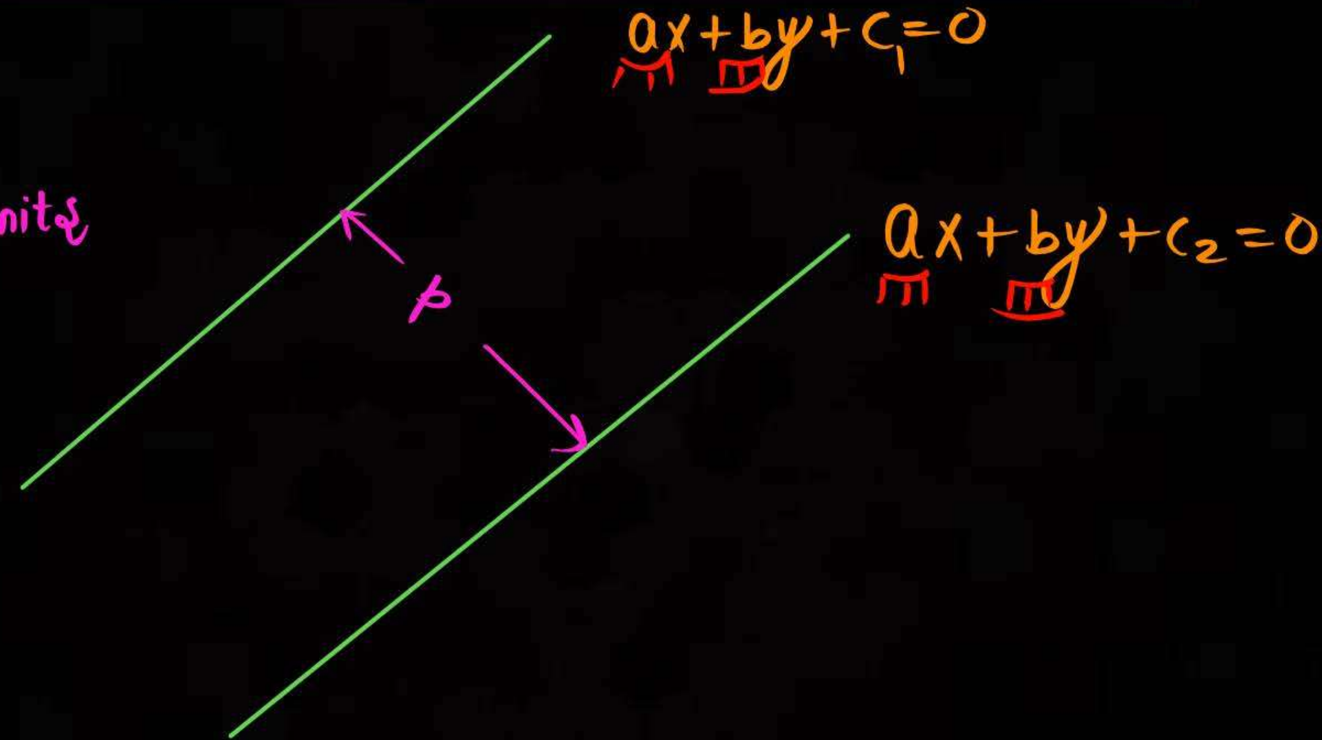
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ (parallel)

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ (Coincident)

DISTANCE BETWEEN PARALLEL LINES

$$p = \frac{|c_1 - c_2|}{\sqrt{a^2+b^2}} \text{ units}$$

⭐⭐



$(1) \rightarrow 2x + 3y - 1 = 0$

$\times 2$

$4x + 6y - 7 = 0$ ✓

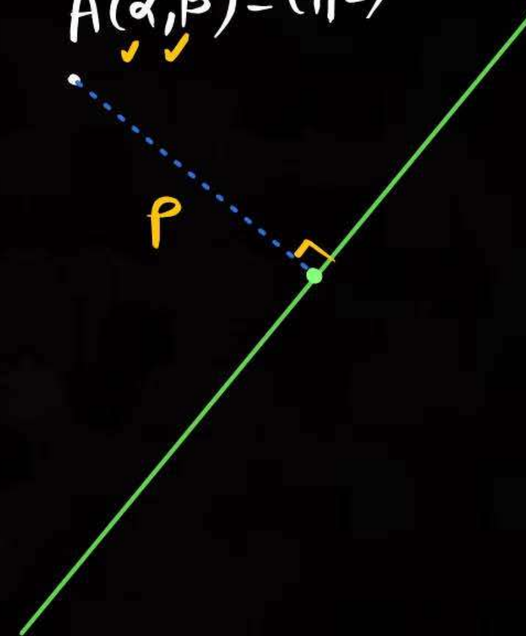
$4x + 6y - 2 = 0$ ✓

$p = ?$

$$p = \frac{|-7-(-2)|}{\sqrt{4^2+6^2}} \text{ units } \boxed{\text{Ans}}$$

DISTANCE BETWEEN A POINT AND A LINE

$$A(\alpha,\beta) \equiv (1,2)$$



$$ax+by+c=0$$

$$x+y+7=0$$

$$\boxed{p = \frac{|a\alpha+b\beta+c|}{\sqrt{a^2+b^2}} \text{ units}}$$

(Learn)

$$p = \frac{|1+2+7|}{\sqrt{1+1}} = \frac{10}{\sqrt{2}} \text{ units..}$$

Q. The line L given by $\frac{x}{5}+\frac{y}{b}=1$ passes through the point $(13, 32)$. The line K is parallel to L and has the equation $\frac{x}{c}+\frac{y}{3}=1$, then the distance between L and K is:

PYQ

(A) $23/\sqrt{15}$

(B) $\sqrt{17}$

(C) $17/\sqrt{15}$

(D) $23/\sqrt{17}$

$\boxed{\frac{13}{5}+\frac{32}{b}=1} \Rightarrow \boxed{b=-20}$ ✓

$(K \parallel L)$ $\frac{\frac{1}{5}}{\frac{1}{c}}=\frac{\frac{1}{b}}{\frac{1}{3}} \longrightarrow \frac{c}{5}=\frac{3}{b} \Rightarrow \boxed{c=\frac{15}{-20}=-\frac{3}{4}}$ ✓

$\boxed{L} \Rightarrow \frac{x}{5}+\frac{y}{-20}=1 \Rightarrow \boxed{4x-y-20=0}$ ✓

$\boxed{K} \Rightarrow \frac{4x}{-3}+\frac{y}{3}=1 \Rightarrow -4x+y=3 \Rightarrow \boxed{4x-y+3=0}$ ✓

Ans →

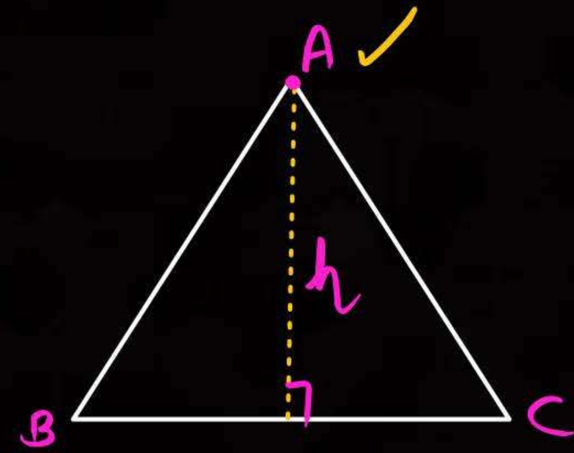
$\frac{|-20-3|}{\sqrt{4^2+1}}$

Ans

BKG

For an Equilateral Triangle, its area is given by $\boxed{\dfrac{(\text{altitude})^2}{\sqrt{3}}}$

$= \left(\dfrac{h^2}{\sqrt{3}}\right)$



Q. Let R be the point (3, 7) and let P and Q be two points on the line $x + y = 5$ such that PQR is an equilateral triangle. Then the area of Δ PQR is:

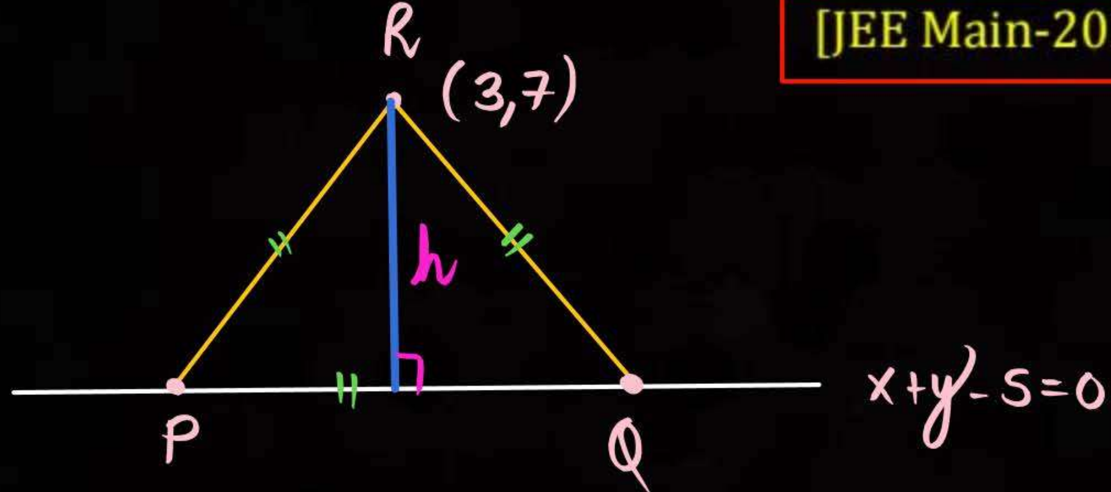
[JEE Main-2022 (26 June-Shift 1)]

(A) $\frac{25}{4\sqrt{3}}$

(B) $\frac{25\sqrt{3}}{2}$

(C) $\frac{25}{\sqrt{3}}$

(D) $\frac{25}{2\sqrt{3}}$ ✓



$$h = \frac{|3+7-5|}{\sqrt{2}} = \frac{5}{\sqrt{2}}$$

$$\text{Ans} = \frac{h^2}{\sqrt{3}} = \frac{25}{2\sqrt{3}} \text{ sq. units}$$

$\boxed{\text{Ans}}$

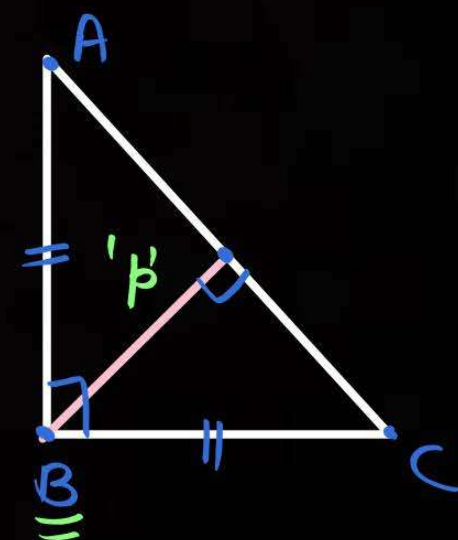
BKG

FYQ

For isosceles Right angled Triangle, its area is given by

(Altitude from Right angled vertex from hypotenuse) ^{2}

$\boxed{p^2}$ sq. units..



Q. Drawn from the origin are two mutually perpendicular lines forming an isosceles triangle together with the straight line $2x + y = a$. Then the area of this triangle is

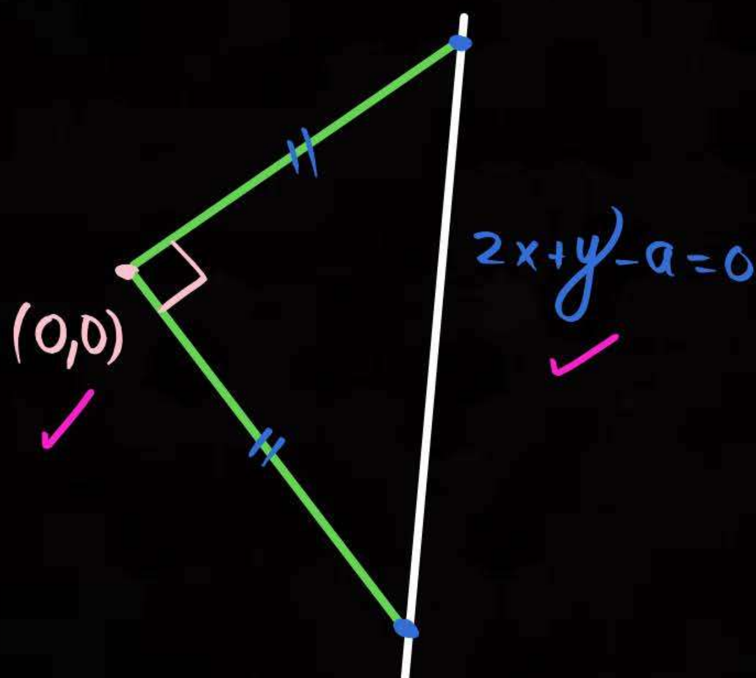
(OP sawaal ♡♡)

A $5a^2$

B $a^2/\sqrt{5}$

C $\sqrt{5}a^2$

D $a^2/5$ ✓



$$p = \frac{|0+0-a|}{\sqrt{5}} = \frac{|-a|}{\sqrt{5}}$$

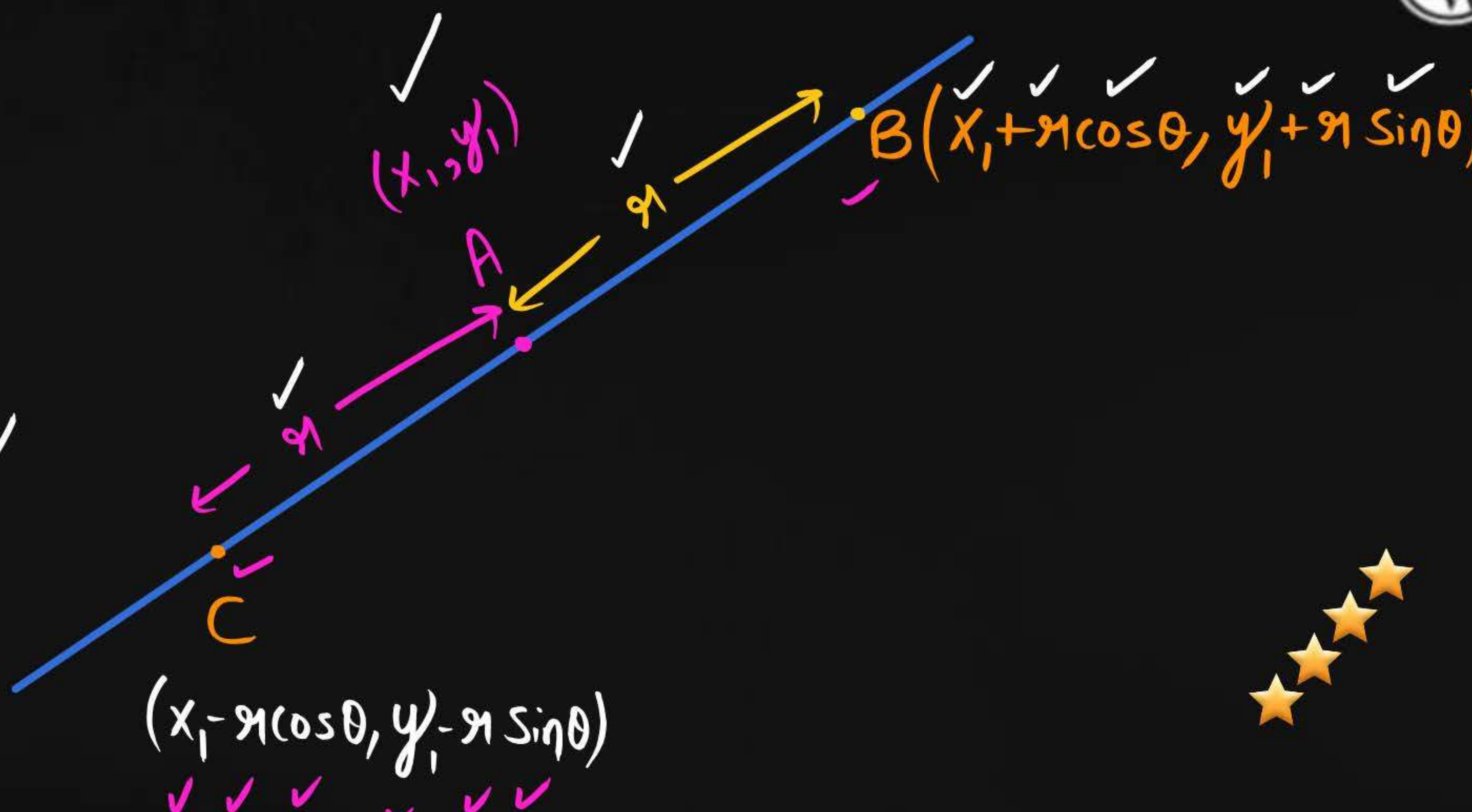
$$\text{area} = p^2 = \frac{|-a|^2}{5} = \boxed{\frac{a^2}{5}}$$

(Very Important for entire Coordinate Geometry)

PARAMETRIC FORM

$m = \tan\theta$

Inclination $= \theta$



TYPE 1

'r' (x_1, y_1) 'θ'

Finding the points at a given distance from the given point on the given line.

ILLUSTRATION :

Find the point at 2 units distance on the line $\underbrace{y = x + 2}_{m = 1 = \tan\theta}$ from (1, 3) point.

(x_1, y_1)

(points) $r = 2$

$$\boxed{\theta = \frac{\pi}{4}}$$

$$\boxed{\begin{array}{c} \text{PW} \heartsuit \heartsuit \\ \text{PW} \heartsuit \end{array}}$$

$(x_1 + r\cos\theta,\ y_1 + r\sin\theta)$

$$\left(1 + 2\cdot\frac{1}{\sqrt{2}},\ 3 + 2\cdot\frac{1}{\sqrt{2}}\right) \text{ Ans}$$

$(x_1 - r\cdot\cos\theta,\ y_1 - r\sin\theta) \rightarrow \left(1 - 2\cdot\frac{1}{\sqrt{2}},\ 3 - 2\cdot\frac{1}{\sqrt{2}}\right)$ Ans

TYPE 2

$r = ?$

BKG ♡♡♡

Distance of a point measured from the given line parallel to another given line.

ILLUSTRATION :

PYQ 2020

Find the distance of (1, 2) point from the line $x + y = 4$ along the line making 60^0 with the positive direction of x– axis in anti-clockwise sense.

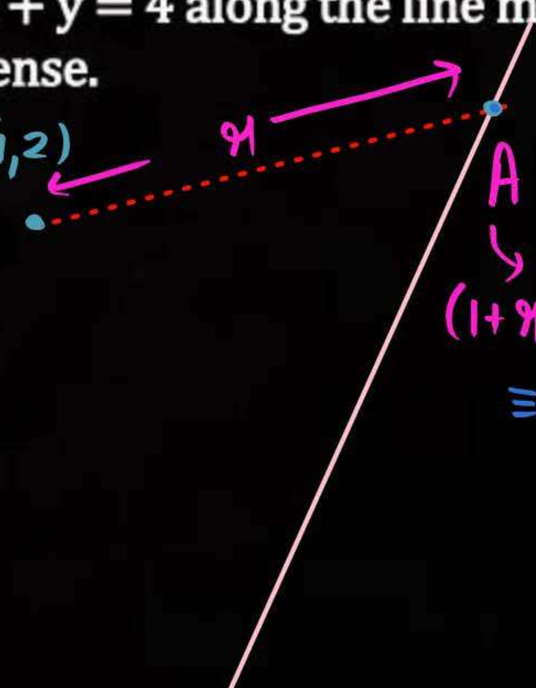
$$\left(1+\frac{r}{2}\right)+\left(2+\frac{r\sqrt{3}}{2}\right)=4$$

$$3+\frac{r}{2}(\sqrt{3}+1)=4$$

$$\boxed{r=\frac{2}{\sqrt{3}+1}\text{ units}}$$

Ans

$\theta = 60^0$



$x+y-4=0$

↑ satisfy ⭐

A ↳ $(1+r\cos 60^\circ,\ 2+r\sin 60^\circ)$

$\equiv \left(1+\frac{r}{2},\ 2+\frac{r\sqrt{3}}{2}\right)$

TYPE 3

To find the slope of the line. $\rightarrow$ $m = ?$

$\tan\theta = ?$

$\theta = ?$

$$\boxed{\theta \in [0, \pi)}$$

ILLUSTRATION :

(Check next page)

$\tan\theta = ?$

Q. Slope of a line passing through $P(2, 3)$ and intersecting the line $x + y = 7$ at a distance of 4 units from P, is:

$r = 4$

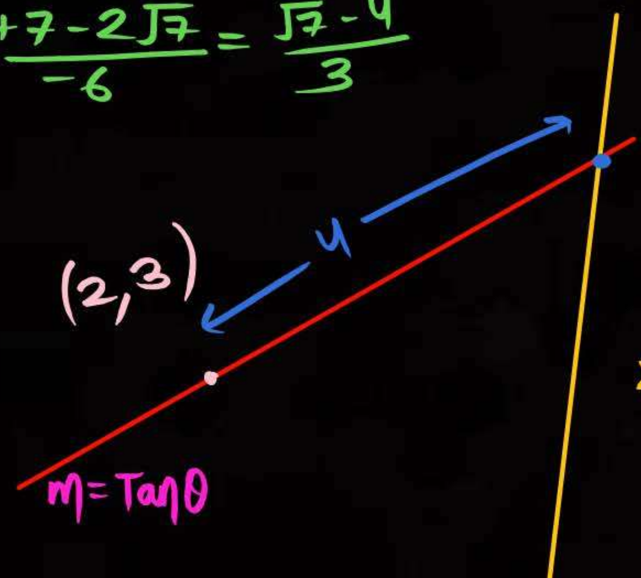
(JEE MAINS-2019)

A $\frac{1-\sqrt{5}}{1+\sqrt{5}}$

B $\frac{1-\sqrt{7}}{1+\sqrt{7}} \times \frac{1-\sqrt{7}}{1-\sqrt{7}} \rightarrow \frac{1+7-2\sqrt{7}}{-6} = \frac{\sqrt{7}-4}{3}$ ✓

C $\frac{\sqrt{7}-1}{\sqrt{7}+1}$

D $\frac{\sqrt{5}-1}{\sqrt{5}+1}$



$\rightarrow (2+4\cos\theta,\ 3+4\sin\theta)$

satisfy

$x+y-7=0$

$$(2+4\cos\theta)+(3+4\sin\theta)=7$$

$$\boxed{\cos\theta+\sin\theta=\frac{1}{2}}$$

"TRIGONOMETRIC EQUATIONS" ⭐⭐⭐

squaring

$$1+\boxed{2\sin\theta\cos\theta}=\frac{1}{4}$$

⭐

⭐ $$\frac{2\tan\theta}{1+\tan^2\theta}=\sin 2\theta=-\frac{3}{4}$$

$\tan\theta=m$

$$\frac{2m}{1+m^2}=-\frac{3}{4}$$

Ans

$$\boxed{m=\frac{-4+\sqrt{7}}{3},\ \frac{-4-\sqrt{7}}{3}}$$

Centroid

H, G, C, I

Orthocentre

Circumcentre

Incentre.

Ispe Bhaukaali, lengthy sawaal aate है

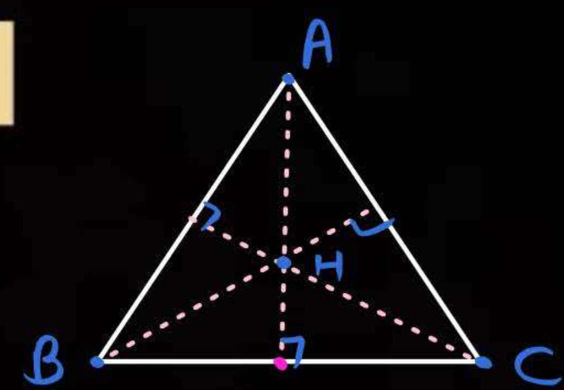
FOUR FRIENDS IN A TRIANGLE

BKG

Calculation will be Easy...

1. ORTHOCENTRE (H)

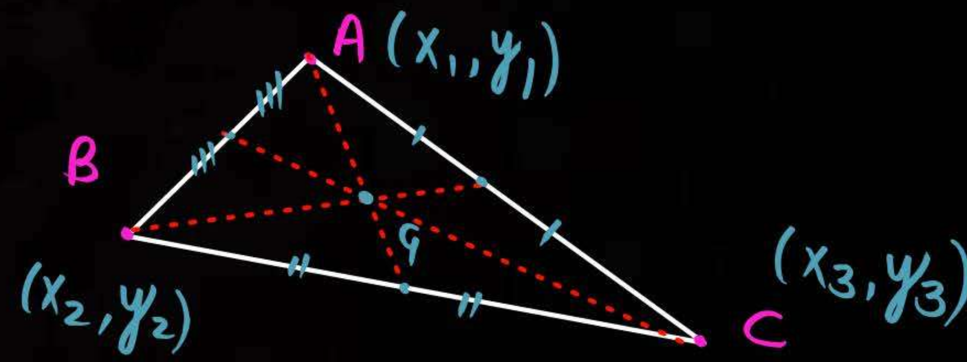
Point of Intersection of altitudes.



2. CENTROID (G)

Point of Intersection of medians.

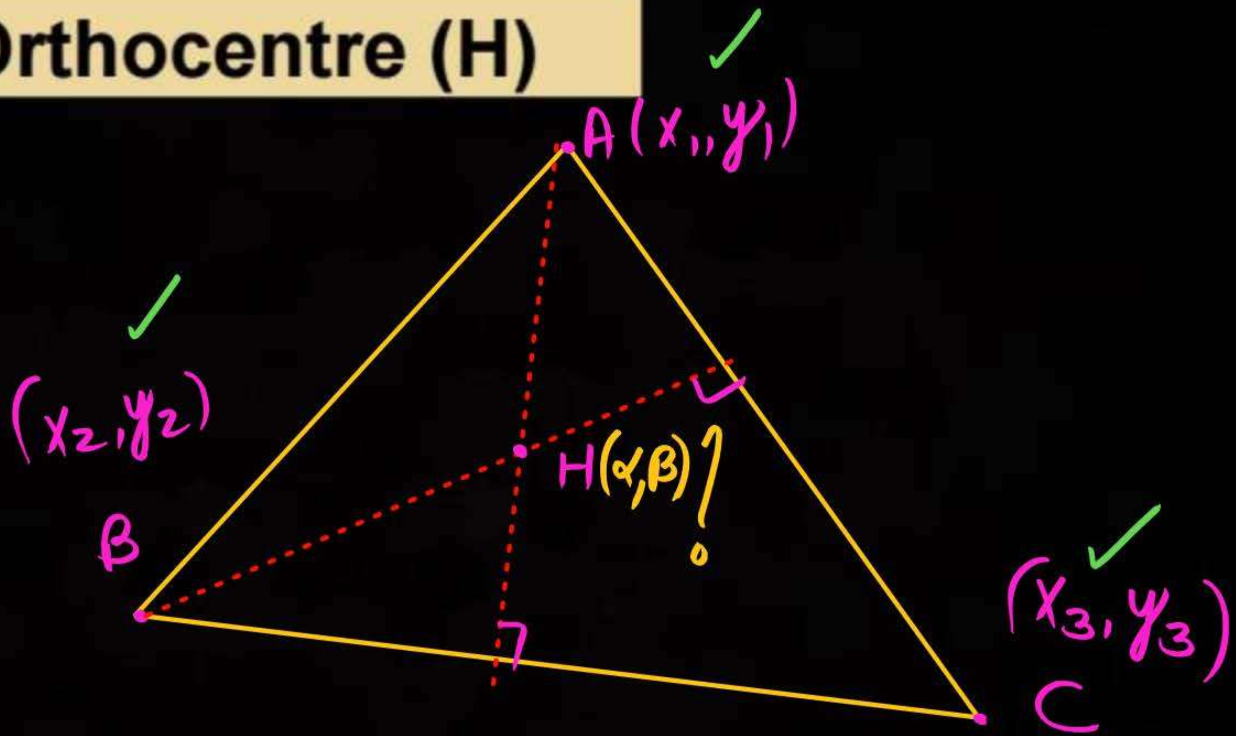
$$G \equiv \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$



Finding Orthocentre (H)

$$\left\{\begin{aligned} m_{BC} \cdot m_{AH} &= -1 \\ m_{AC} \cdot m_{BH} &= -1 \end{aligned}\right\}$$

$\hookrightarrow (\alpha, \beta)$



BKG

1. Line joining the ortho-Centre to any of the vertex is the altitude.

Q. If a ΔABC has vertices $A(-1, 8)$, $B(-7, 1)$ and $C(5, -5)$, then its orthocenter has coordinates

(JEE MAINS-2020)

A $(-3, 3)$

B $\left(-\frac{3}{5}, \frac{5}{5}\right)$

C $\left(-\frac{15}{4}, \frac{5}{2}\right)$

$\alpha \qquad \beta$

D $(-3, -3)$

• $m_{AH} \cdot m_{BC} = -1$

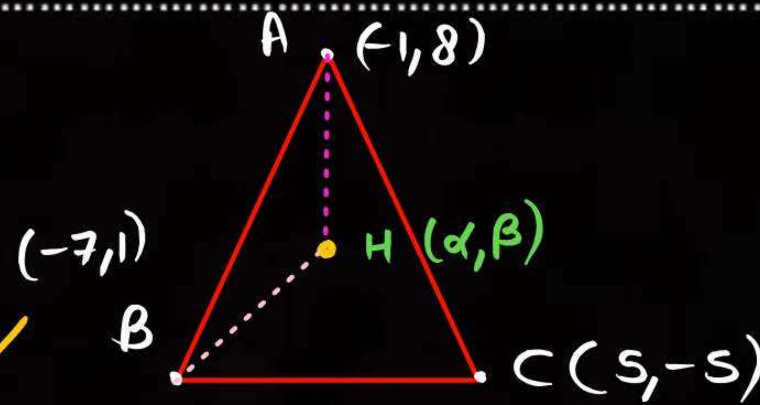
$$\frac{\beta - 8}{\alpha + 1} \cdot \left(\frac{6}{-12}\right) = -1$$

$$\boxed{\beta - 2\alpha = 10}$$

• $m_{BH} \cdot m_{AC} = -1$

$$\frac{\beta - 1}{\alpha + 7} \cdot \frac{13}{-6} = -1$$

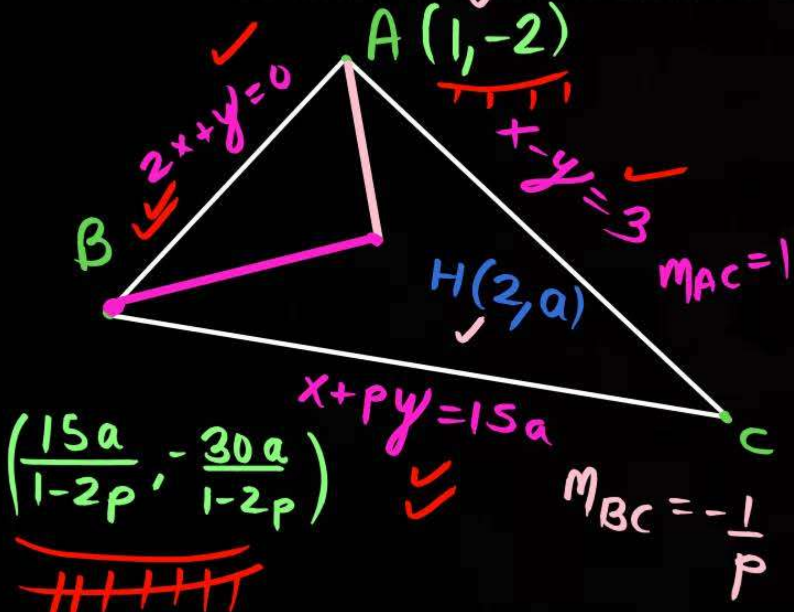
$$\boxed{6\alpha - 13\beta = -55}$$



Q. The equations of the sides AB, BC and CA of a triangle ABC are $2x + y = 0$, $x + py = 15a$ and $x - y = 3$ respectively. If its Orthocentre is $(2, a)$, $-1/2 < a < 2$, then p is equal to :

'H'

[JEE Main-2022 (26 July Shift 1)]



$$\boxed{m_{AH}\cdot m_{BC}=-1}$$

$$\frac{a+2}{2-1}\cdot\left(-\frac{1}{p}\right)=-1$$

$$\boxed{(a+2)=p}$$

Put

$$\boxed{m_{BH}\cdot m_{AC}=-1}$$

$$\frac{\left(a-\left(\frac{-30a}{1-2p}\right)\right)}{\left(2-\frac{15a}{1-2p}\right)}\cdot(1)=-1$$

$$\frac{a(1-2p)+30a}{2(1-2p)-15a}=-1$$

$$\frac{a(1-2(a+2)) + 30a}{2(1-2(a+2)) - 15a} = -1$$

$$a - 2a(a+2) + 30a = (2 - 4(a+2) - 15a)(-1)$$

$$a - 2a^2 - 4a + 30a = -2 + 4(a+2) + 15a$$

$$-2a^2 + 27a = 19a + 6$$

$$-2a^2 + 8a - 6 = 0$$

$$\boxed{a^2 - 4a + 3 = 0}$$

$a = -1, 3$ → Discard

$p = a + 2 = 3$ Ans

$\boxed{\text{Baba ♡ ♡}}$

Q. The distance of the $\boxed{\text{origin from the centroid}}$ of the triangle whose two sides have the equations $x - 2y + 1 = 0$ and $2x - y - 1 = 0$ and whose orthocenter is $\left(\frac{7}{3}, \frac{7}{3}\right)$ is :

$G \rightarrow (2, 2)$ $\quad OG = \sqrt{4+4} = 2\sqrt{2}$ units

[JEE Main-2022 (29 June-Shift 2)]

$\boxed{\textbf{(Bahut Famous Sawaal)}}$

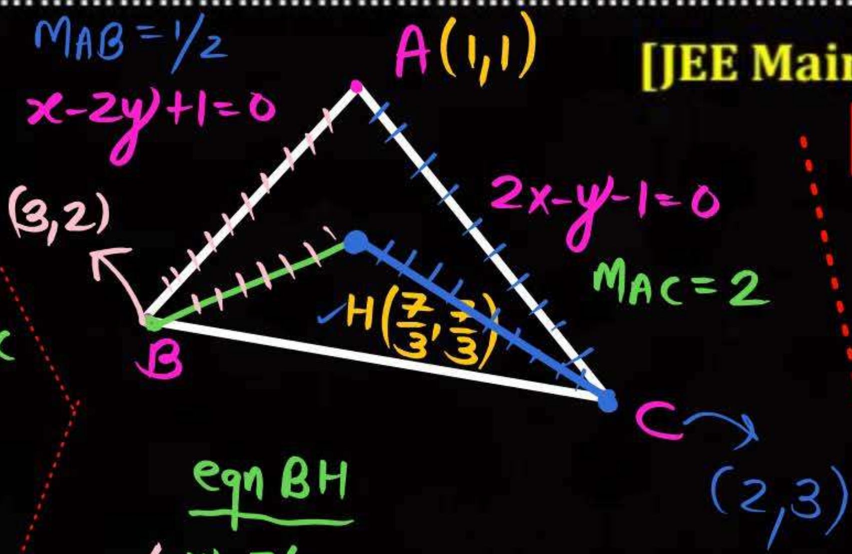
(A) $\sqrt{2}$

(B) 2

(C) $2\sqrt{2}$ ✓

(D) 4

$m_{AB} = 1/2$



$m_{AC} = 2$

$m_{BH} = \frac{-1}{m_{AC}} = -\frac{1}{2}$

eqn BH

$\frac{y - 7/3}{x - 7/3} = -\frac{1}{2}$

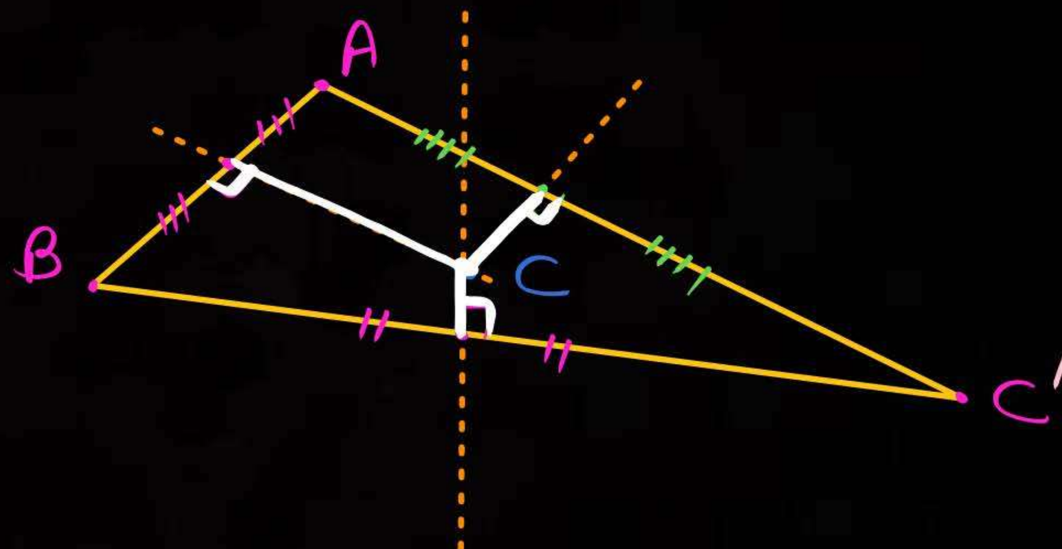
$m_{CH} = \frac{-1}{m_{AB}} = -2$

CH eqn

$\frac{y - 7/3}{x - 7/3} = -2$

3. CIRCUM-CENTRE (C)

Point of Intersection of perpendicular bisectors.



$$\boxed{AC = BC = CC'}$$

$= R$

CIRCUMRADIUS

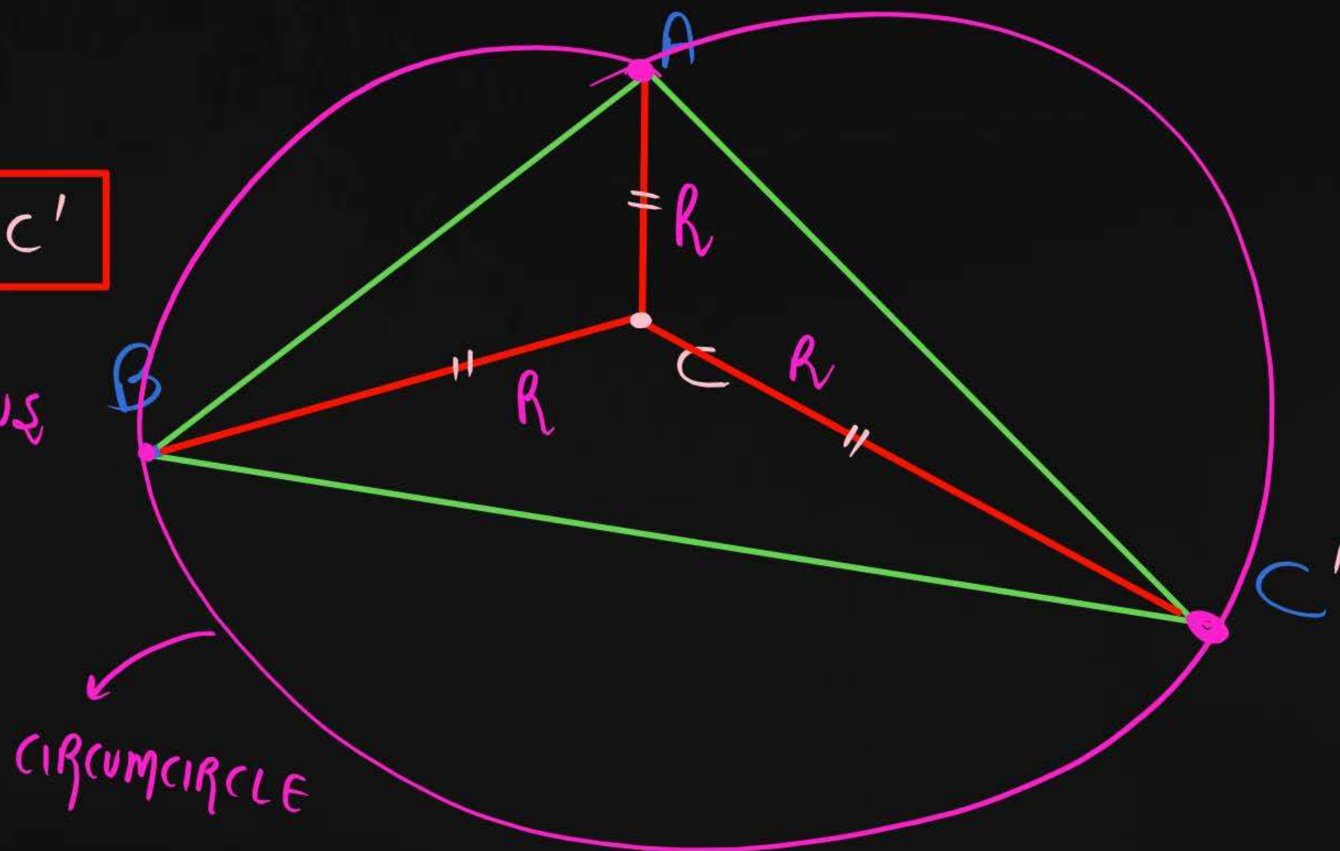


ILLUSTRATION :

$R = AC = 5$ units Ans.

Find the Circum-Centre of a triangle whose vertices are (-2,-3), (-1,0) and (7,-6). Also find its Circumradius.

$$\boxed{AC^2 = BC^2}$$

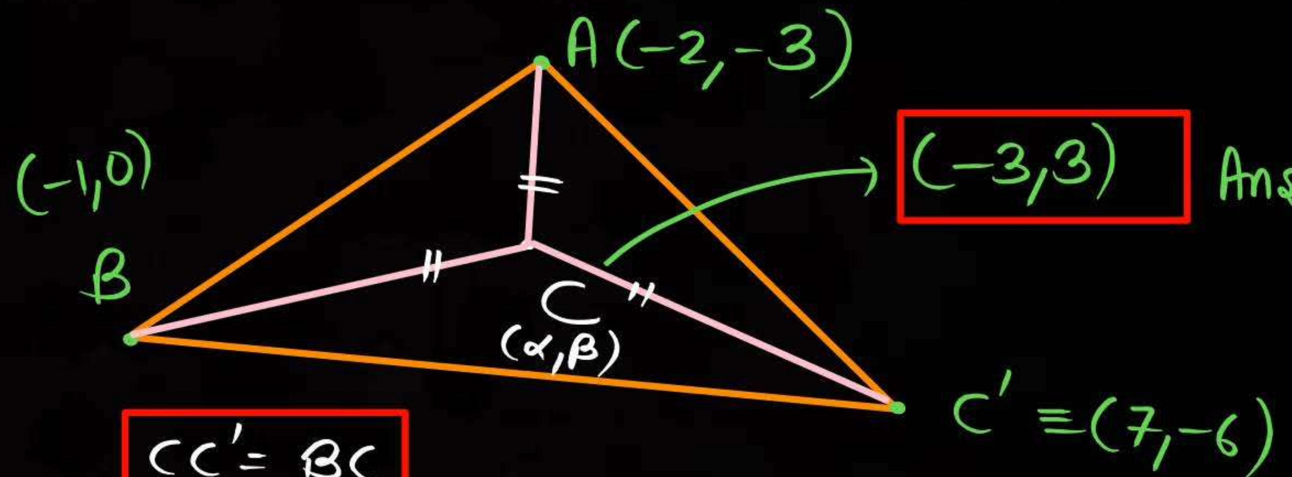
$$(\alpha+2)^2 + (\beta+3)^2 = (\alpha+1)^2 + \beta^2$$

$$\Rightarrow 2\alpha + 12 + 6\beta = 0$$

$$\boxed{\alpha + 3\beta + 6 = 0}$$

$\alpha = -3$

$\beta = +3$



$$\boxed{CC' = BC}$$

$$(\alpha-7)^2 + (\beta+6)^2 = (\alpha+1)^2 + \beta^2$$

$$-16\alpha + 84 + 12\beta = 0$$

$$\boxed{-4\alpha + 3\beta + 21 = 0}$$

BKG

1. Perpendicular from the circum-Centre to any side , bisects it. ⭐⭐
2. Line joining the circum-Centre to any of the vertex is the circumradius. ⭐⭐

Q. Let the Circum-centre of a triangle with vertices A$(a, 3)$, B$(b, 5)$ and C(a, b), $ab > 0$ be P$(1, 1)$. If the line AP intersects the line BC at the point Q(k_1, k_2), then $k_1 + k_2$ is equal to :

[JEE Main-2022 (29 July Shift 1)]

(BAHUT CALCULATIVE SAWAAL)

(A) 2

(B) 4/7

(C) 2/7

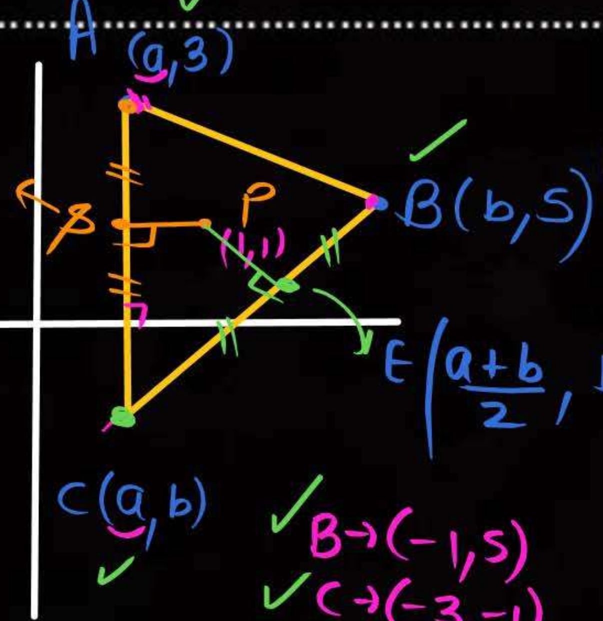
(D) 4

$\left(a, \frac{b+3}{2}\right)$

$(a, 1)$

$\frac{b+3}{2} = 1$

$\boxed{b = -1}$



$E\left(\frac{a+b}{2}, \frac{b+5}{2}\right)$

$B \to (-1, 5)$

$C \to (-3, -1)$

$BC \to \frac{y-5}{x-1} = \frac{6}{-1+3}$

$$\boxed{m_{PE} \cdot m_{BC} = -1 \qquad \frac{\frac{b+5}{2} - 1}{\frac{a+b}{2} - 1} \times \frac{5-b}{b-a} = -1}$$

$b = -1$

$a = 5$, $a = -3$

(Discard)

AP $\longrightarrow$ $A \to (-3, 3)$

$P \to (1, 1)$

eqn $\to$ $\dfrac{y-1}{x-1} = \dfrac{3-1}{-3-1}$

POI of AP and BC

$Q \to \left(-\dfrac{13}{7}, \dfrac{17}{7}\right)$

$\underbrace{K_1}$ $\underbrace{K_2}$

$K_1 + K_2 = \dfrac{4}{7}$ Ans

B.C.S ♡♡♡

4. IN-CENTRE (I)

Point of Intersection of internal angle bisectors.

$$I \equiv \left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c} \right)$$

LEARN

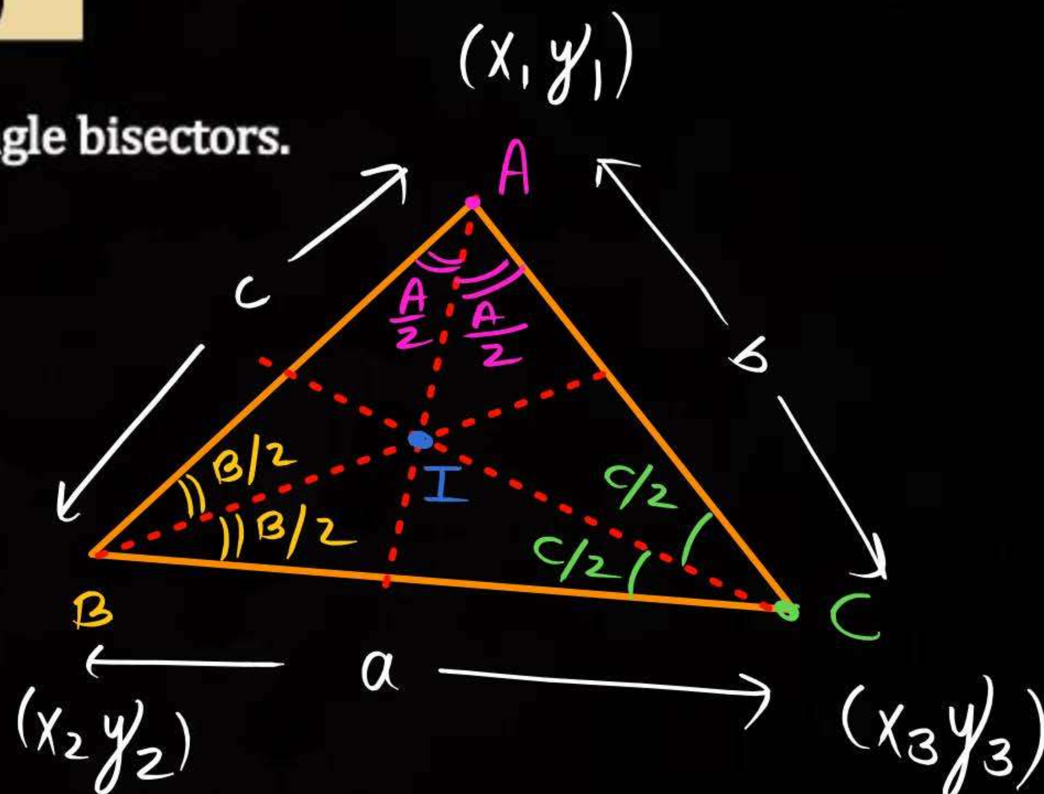
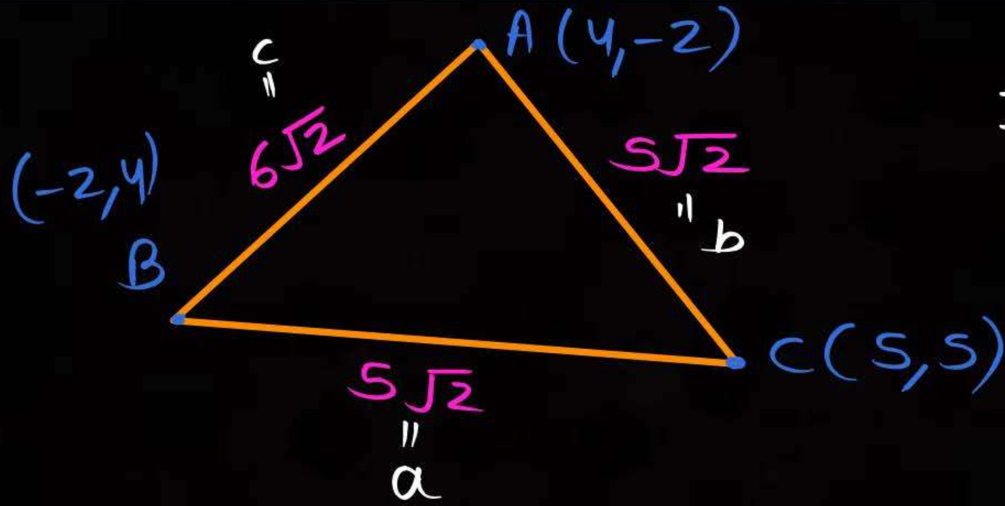


ILLUSTRATION :

Find the coordinates of incentre of a triangle whose vertices are(4,-2) , (-2,4) and (5,5).



$a+b+c$

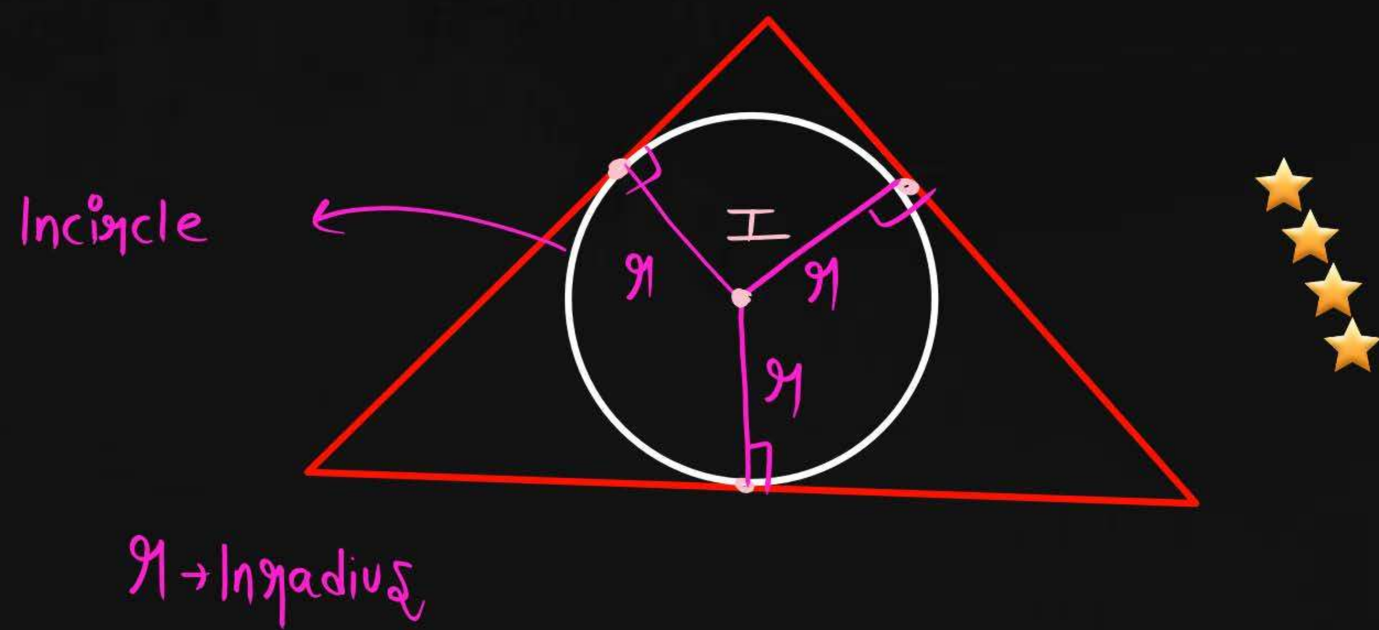
$= 16\sqrt{2}$

$$I_x = \left(\frac{20\sqrt{2}-10\sqrt{2}+30\sqrt{2}}{16\sqrt{2}}\right)$$

$$= \frac{5}{2}$$

$$I_y = \frac{5}{2}$$

$\left(\frac{5}{2}, \frac{5}{2}\right)$ Ans



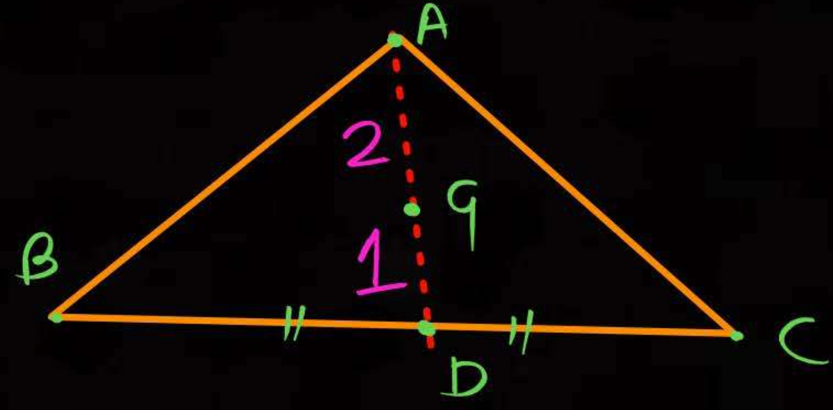
Incircle
I
r
r
r
r → Inradius

BKG

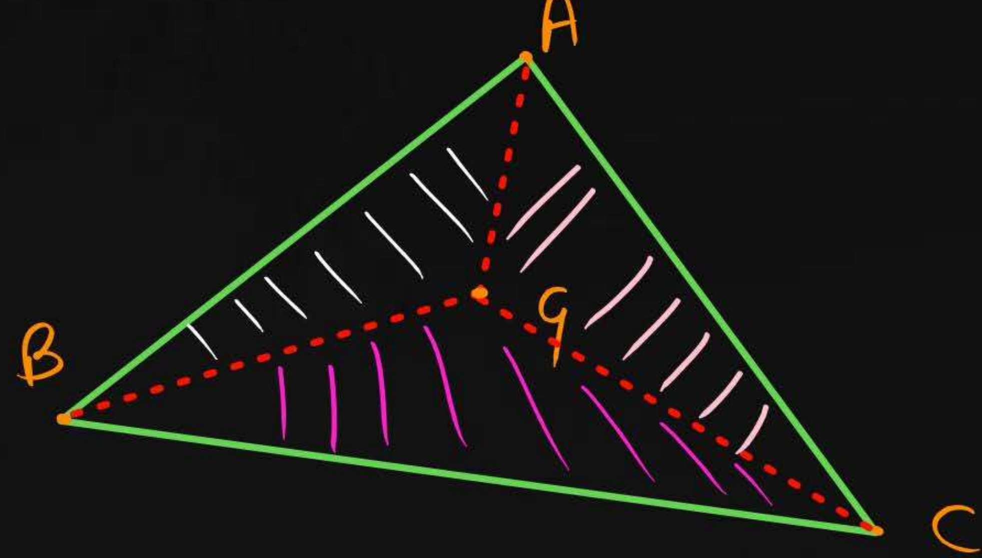
1. Perpendicular from the incentre to any side is the inradius. ⭐⭐⭐
2. Line joining the incentre to any of the vertex is the angle- bisector. ⭐⭐

H,G,C,I KA PARAM-GYAAN

1. Centroid divides each median in the ratio 2:1 measured from the vertex.



2. Centroid is the point which divides the triangle into 3 equal areas.


A
B
G
C

Q. Let O(0,0), P(3,4), Q(6,0) be the vertices of the triangle OPQ. The point R inside the triangle OPQ is such that the triangle OPR, PQR, OQR are of equal area. The coordinates of R are.

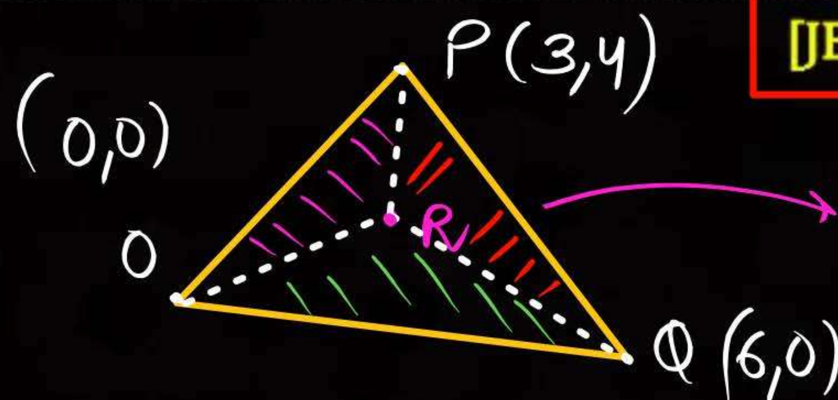
[JEE (Advanced)-2007]

(A) $\left(\frac{4}{3}, 3\right)$

(B) $(3, 2/3)$

(C) $(3, 4/3)$

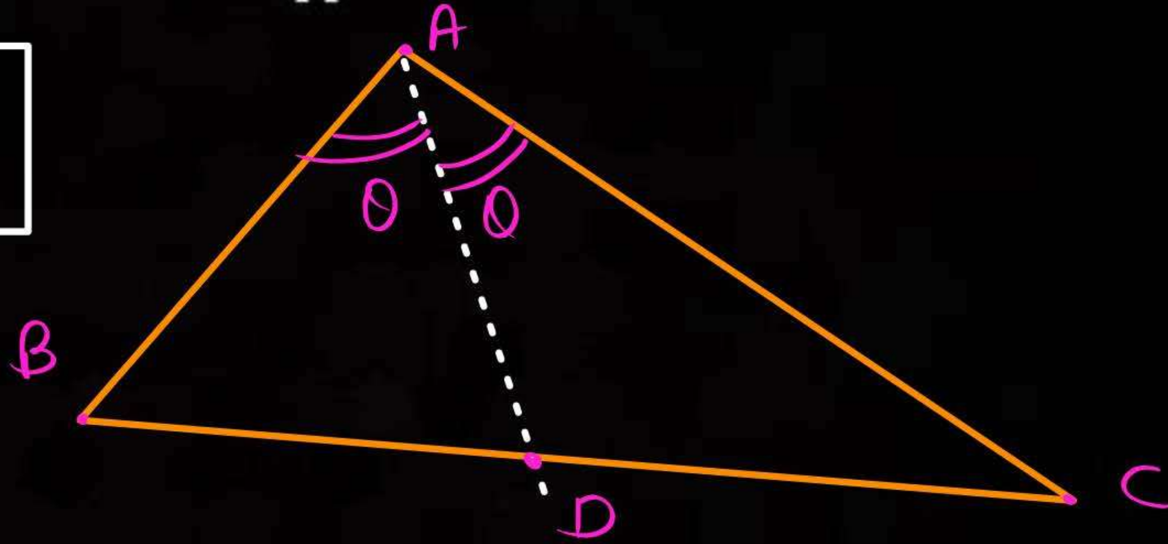
(D) $\left(\frac{4}{3}, 2/3\right)$



centroid

3. Interior angle Bisectors always divide the opposite side in the ratio of the sides containing the angle.

$$\boxed{\frac{AB}{AC} = \frac{BD}{DC}}$$

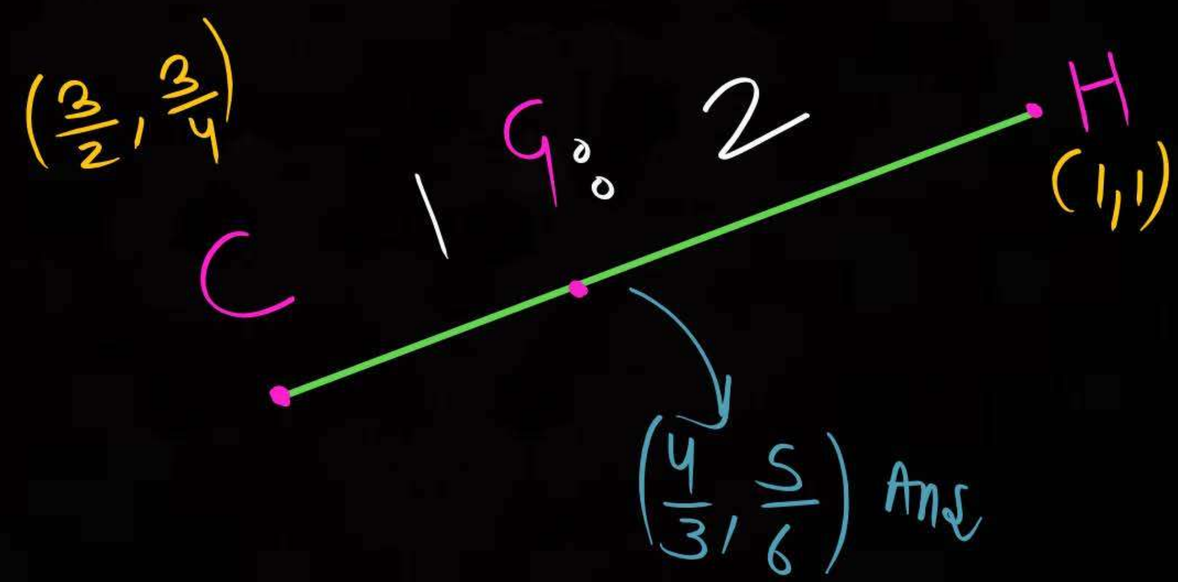


Euler's Line

4. In any triangle ; C ,G ,H are collinear and G divides the Line segment CH in the ratio 1:2.

ILLUSTRATION :

If a Triangle has its Orthocenter at (1,1) and Circumcenter at (1.5, 0.75), then Find G.



5. In case of Isosceles Triangle; $\boxed{\text{H,G,C,I are collinear.}}$

6. $\boxed{\text{In case of Equilateral Triangle; H,G,C,I are coincident.}}$ ⭐⭐

same points

Q. Let the $\boxed{\text{centroid}}$ of an $\boxed{\text{equilateral triangle ABC}}$ be at the $\boxed{\text{origin.}}$ Let one of the sides of the equilateral triangle be along the straight line $x + y = 3$. If R and r be the radius of circumcircle and incircle respectively of ΔABC, then $(R + r)$ is equal to : $\frac{3}{\sqrt{2}} + \frac{6}{\sqrt{2}} = \frac{9}{\sqrt{2}}$ units

(JEE MAINS-2021)

(A) $\frac{9}{\sqrt{2}}$ ✓

(B) $7\sqrt{2}$

(C) $3\sqrt{2}$

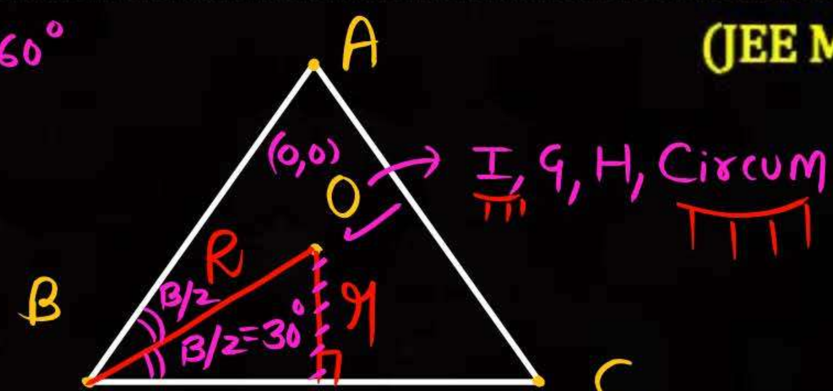
(D) $2\sqrt{2}$

$B = 60^\circ$

$r = \frac{3}{\sqrt{2}}$

$\sin 30^\circ = \frac{r}{R}$

$\left\{ 2r = R = \frac{6}{\sqrt{2}} \right\}$



$x + y - 3 = 0$ ✓

BKG

1. In any right angle triangle ;

(i) Ortho-Centre is 'B'

(90° wallahi vertex)

(ii) Circum-Centre is

'D'

(Hypotenuse का mid-pt)

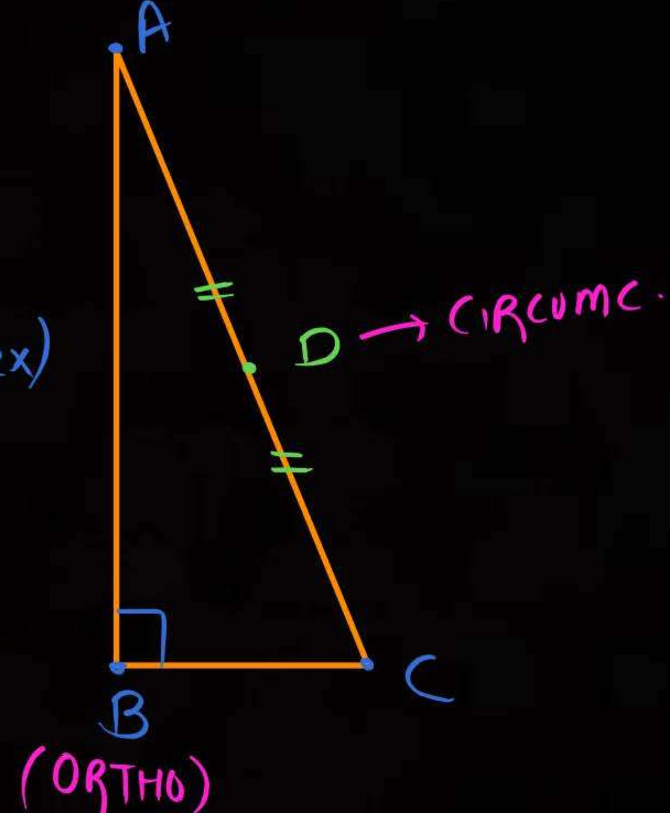
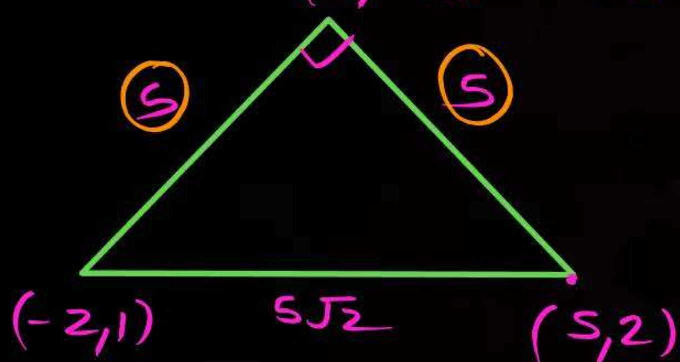


ILLUSTRATION :

Vertices of a triangle are (2,-2) , (-2,1) , (5,2). Find the distance between its Ortho-Centre and Centroid.

$(2,-2) \longrightarrow$ Orthocentre



$\left(G \equiv \frac{5}{3}, \frac{1}{3}\right)$ ✓

Ans =

$\sqrt{\left(\frac{5}{3}-2\right)^2+\left(\frac{1}{3}+2\right)^2}$ units

LOCUS

$\rightarrow$ Graph / eqn

Locus of any moving point P is path traced out by it under some given geometric condition.

STEPS TO FIND LOCUS

(1) Let the variable point (moving point) be (h, k)

(2)Formula lagao.

(3) Eliminate the unknown parameters/ assumed variables / x & y .
(After step 3 , the equation must contain only 'h' and 'k' and constants and known values)

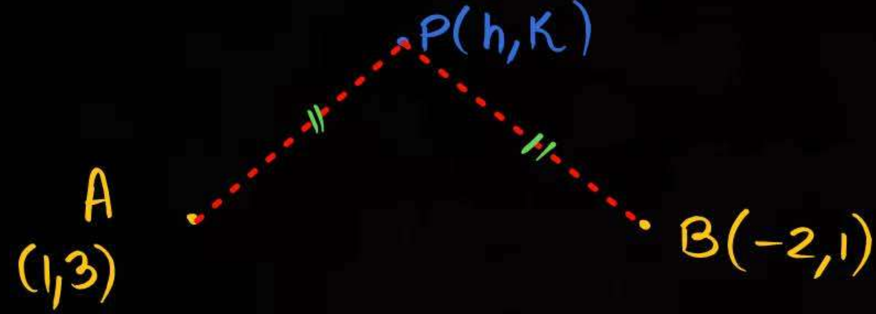
(4) Generalize the locus by putting h = x and k = y.

NOTE: 1. *Parameters are unknowns.*

2. *If anything(value) is assumed in the question, then it is also an unknown parameter.*

ILLUSTRATIONS :

1. Find the equation to the locus of a point equidistant from the points A(1, 3) and B(–2, 1)



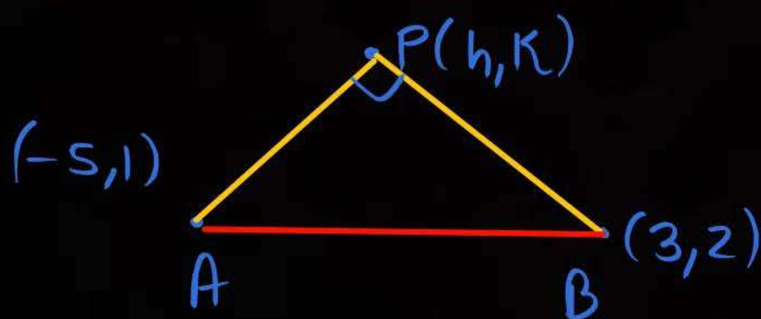
$$\boxed{PA^2 = PB^2}$$

$$\left(\sqrt{(h-1)^2+(K-3)^2}\right)^2 = \left(\sqrt{(h+2)^2+(K-1)^2}\right)^2$$

$$\boxed{-6h-4K+5=0} \quad h \to x,\ K \to y$$

$$-6x-4y+5=0 \quad \boxed{\text{Ans}}$$

2. Locus of a point, so that the join of (–5, 1) and (3, 2) subtend a right angle at the moving point.



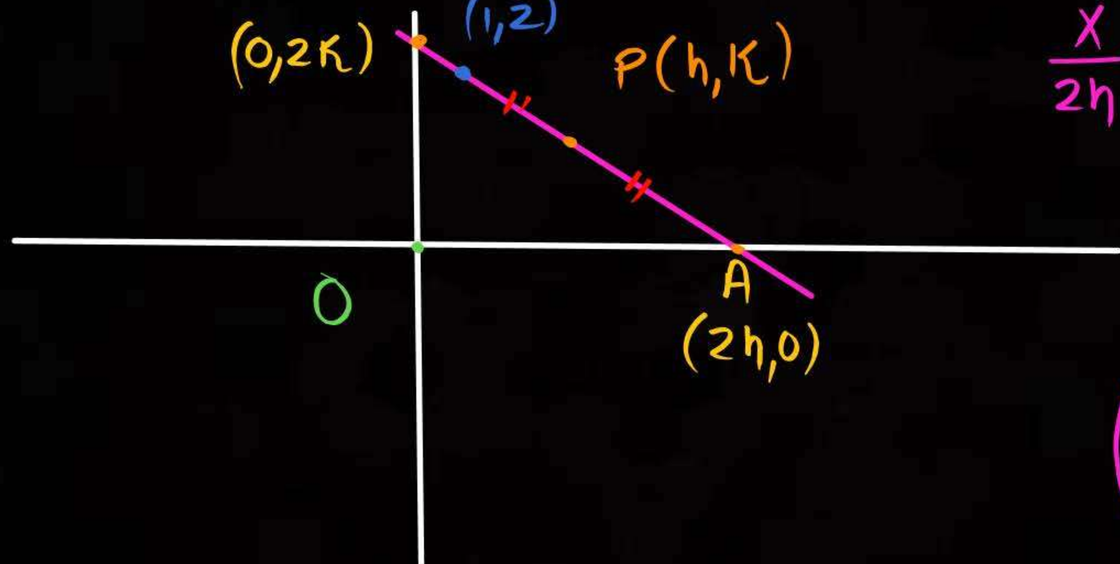
$$m_{PA} \cdot m_{PB} = -1$$

$$\boxed{\frac{K-1}{h+5} \cdot \frac{K-2}{h-3} = -1}$$

$K \to y,\ h \to x$

$\boxed{\text{Ans}}$

3. Find the locus of the midpoint of segment intercepted between the coordinate axis passing through (1, 2).



$$\frac{x}{2h} + \frac{y}{2K} = 1 \quad (1,2)$$

$$\frac{1}{2h} + \frac{\cancel{2}}{\cancel{2}K} = 1$$

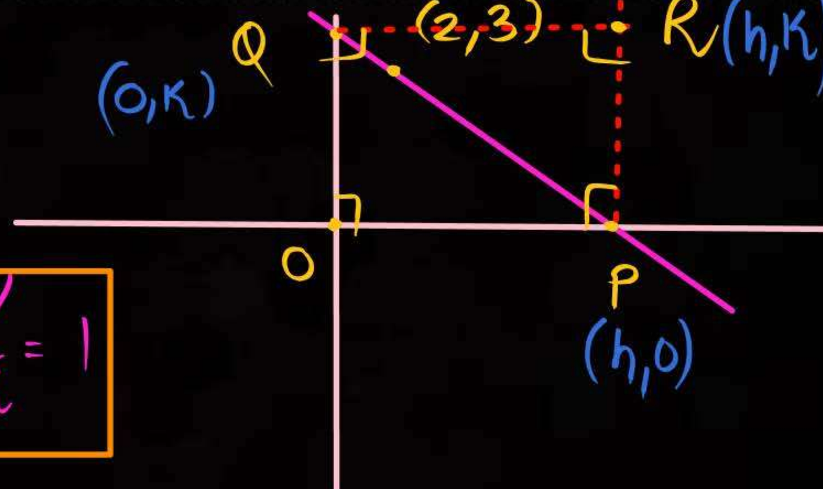
$h \to x$

$K \to y$

$$\boxed{\frac{1}{2x} + \frac{1}{y} = 1}$$ Ans

Q. A straight line through a fixed point (2, 3) intersects the coordinate axes at distinct points P and Q. If O is the origin and the rectangle OPRQ is completed, then the locus of R is

(JEE Main-2018)



(A) $2x + 3y = xy$

(B) $3x + 2y = xy$

(C) $3x + 2y = 6xy$

(D) $3x + 2y = 6$

$$\boxed{\frac{x}{h} + \frac{y}{k} = 1}$$

(2,3)

$$\frac{2}{h} + \frac{3}{k} = 1$$

$h \to x$

$k \to y$

$$\frac{2}{x} + \frac{3}{y} = 1 \quad \boxed{\text{Ans}}$$

Q. A point P moves so that the sum of squares of its distances from the points $(1, 2)$ and $(-2, 1)$ is 14. Let $f(x, y) = 0$ be the locus of P, which intersects the x-axis at the points A, B and the y-axis at the point C, D. Then the area of the quadrilateral ACBD is equal to :

[JEE Main-2022 (26 July Shift 1)]

$\boxed{\text{TRY By yourself}}$

(A) $9/2$

(B) $\frac{3\sqrt{17}}{2}$

(C) $\frac{3\sqrt{17}}{4}$

(D) 9

POWER OF A POINT

$L: ax+by+c=0$

$A: (\alpha,\beta)$

$\boxed{P_A = a\alpha+b\beta+c}$

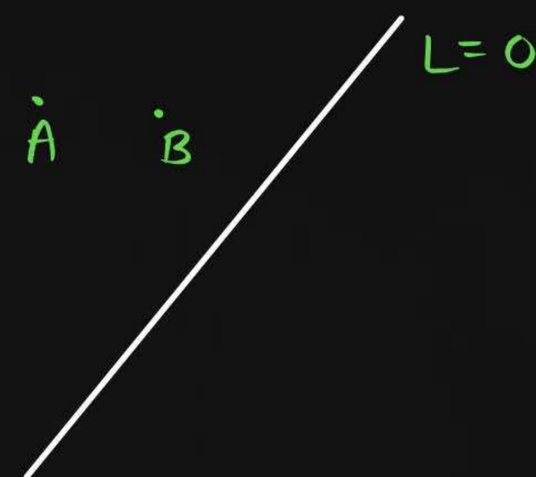
$L: ax+by+c=0$

$A \to (\alpha_1, \beta_1)$

$B \to (\alpha_2, \beta_2)$

$$\boxed{P_A \cdot P_B > 0}$$

'A' and 'B' lie on the same side.



$L : ax + by + c = 0$

$A \to (\alpha_1, \beta_1)$

$B \to (\alpha_2, \beta_2)$

$$\boxed{P_A \cdot P_B < 0}$$

'A' and 'B' lie on the opposite side

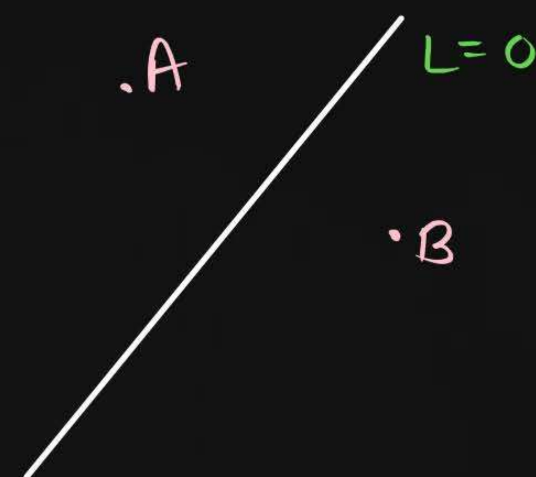


ILLUSTRATION:

Find the range of α if (α, α) and $(1, 1)$ points lie on same side of the line $3x + 4y - 5 = 0$ and opposite side of the line $x + 2y - 7 = 0$.

A $\cdot (\alpha, \alpha)$

B $\cdot (1, 1)$

$3x + 4y - 5 = 0$

$P_A \cdot P_B > 0$

$(7\alpha - 5)(2) > 0$

$\boxed{\alpha > \frac{5}{7}}$

(α, α)

$\cdot A$

$x + 2y - 7 = 0$

$\cdot (1, 1)$

$P_A \cdot P_B < 0$

$(3\alpha - 7)(3 - 7) < 0$

$\boxed{\alpha > \frac{7}{3}}$

$\cap$

$a \in \left(\frac{7}{3}, \infty\right)$ $\boxed{\text{Ans}}$

Q. Let the point P(α, β) be a unit distance from each of the two lines $L_1 : 3x - 4y + 12 = 0$ and $L_2 : 8x + 6y + 11 = 0$. If P lies below L_1 and above L_2, then $100(\alpha + \beta)$ is equal to :

[JEE Main-2022 (25 July Shift 2)]

(A) –14

(B) 42

(C) –22

(D) 14

$$\frac{|3\alpha - 4\beta + 12|}{5} = 1 \longrightarrow +(3\alpha - 4\beta + 12) = 5$$

$$\boxed{3\alpha - 4\beta + 7 = 0}$$

$$\frac{|8\alpha + 6\beta + 11|}{10} = 1 \longrightarrow \boxed{8\alpha + 6\beta + 1 = 0}$$

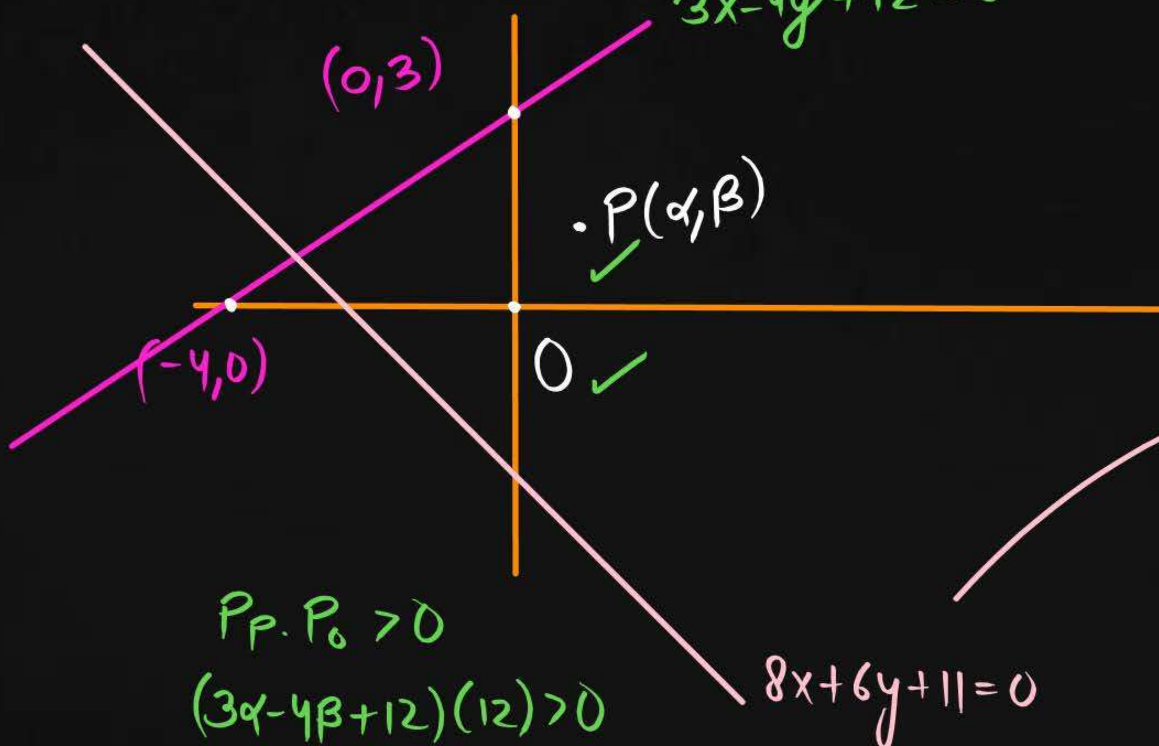
(α, β) ✓

$|f(x)| = +f(x)$

$f(x) > 0$

$|f(x)| = -f(x)$

$f(x) < 0$



$P_P \cdot P_O > 0$

$(3\alpha-4\beta+12)(12)>0$

$$\boxed{3\alpha-4\beta+12>0}$$

L_2

$P_P \cdot P_O > 0$

$\rightarrow (8\alpha+6\beta+11)(11)>0$

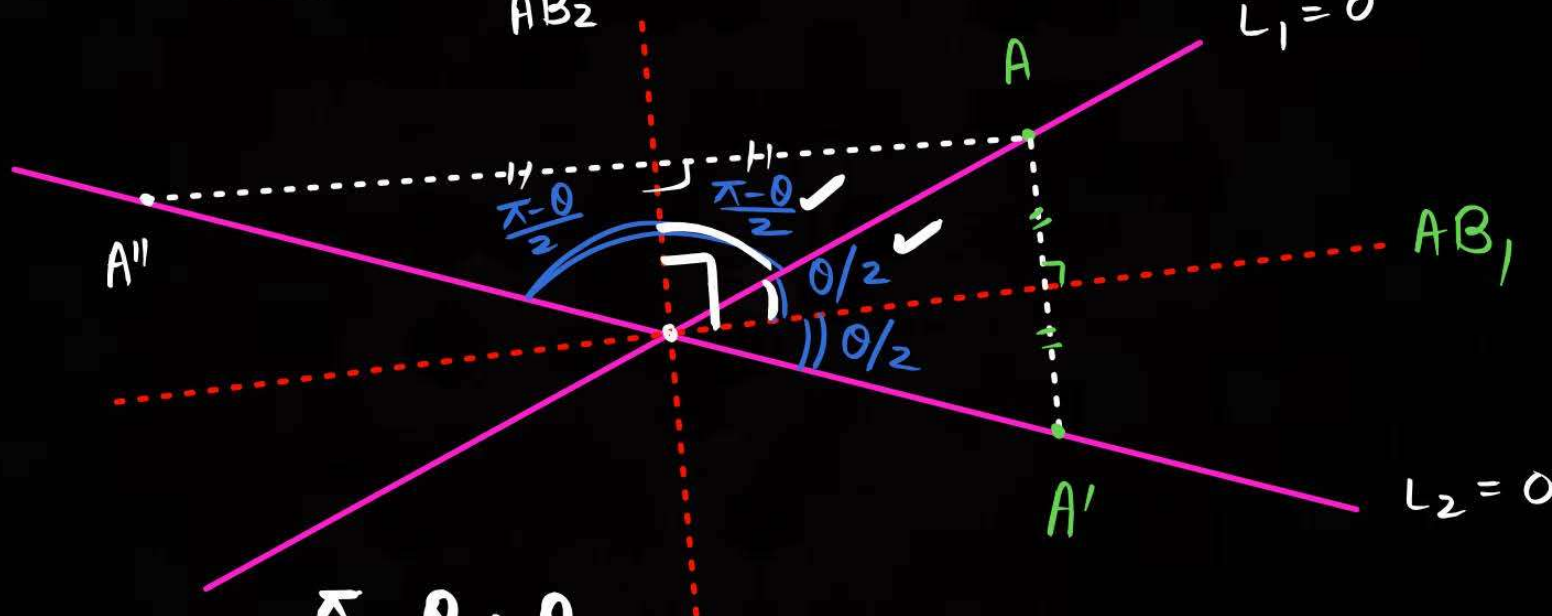
$$\boxed{8\alpha+6\beta+11>0}$$

ANGLE BISECTORS

$$\left.\begin{aligned} a_1x+b_1y+c_1=0 \\ a_2x+b_2y+c_2=0 \end{aligned}\right\} \longrightarrow \boxed{\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}} = \pm\left(\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}\right)}$$

Note:

1. Image of any point on the first line L_1 about bisector line on the second line L_2.
2. Angle Bisectors are perpendicular to each other.



$$\frac{\pi}{2}-\frac{\theta}{2}+\frac{\theta}{2}=\frac{\pi}{2}$$

ACUTE & OBTUSE ANGLE BISECTORS

For two lines

$a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$,

If c_1 and c_2 are of same sign.

Then check the value of

$$\left(a_1a_2 + b_1b_2\right)$$

> 0	< 0
(+) obtuse	(+) acute
(–) acute	(–) obtuse

ILLUSTRATION:

Find the obtuse ∠ bisector of the line 3x + 4y = 5, 2x –7y = - 6.

$3x+4y-5=0, \quad 2x-7y+6=0$

$\boxed{-3x-4y+5=0}$ $\quad$ $\boxed{2x-7y+6=0}$

$a_1a_2+b_1b_2$

$= -6+28 > 0$

$$\frac{-3x-4y+5}{5} = \pm\left(\frac{2x-7y+6}{\sqrt{53}}\right)$$

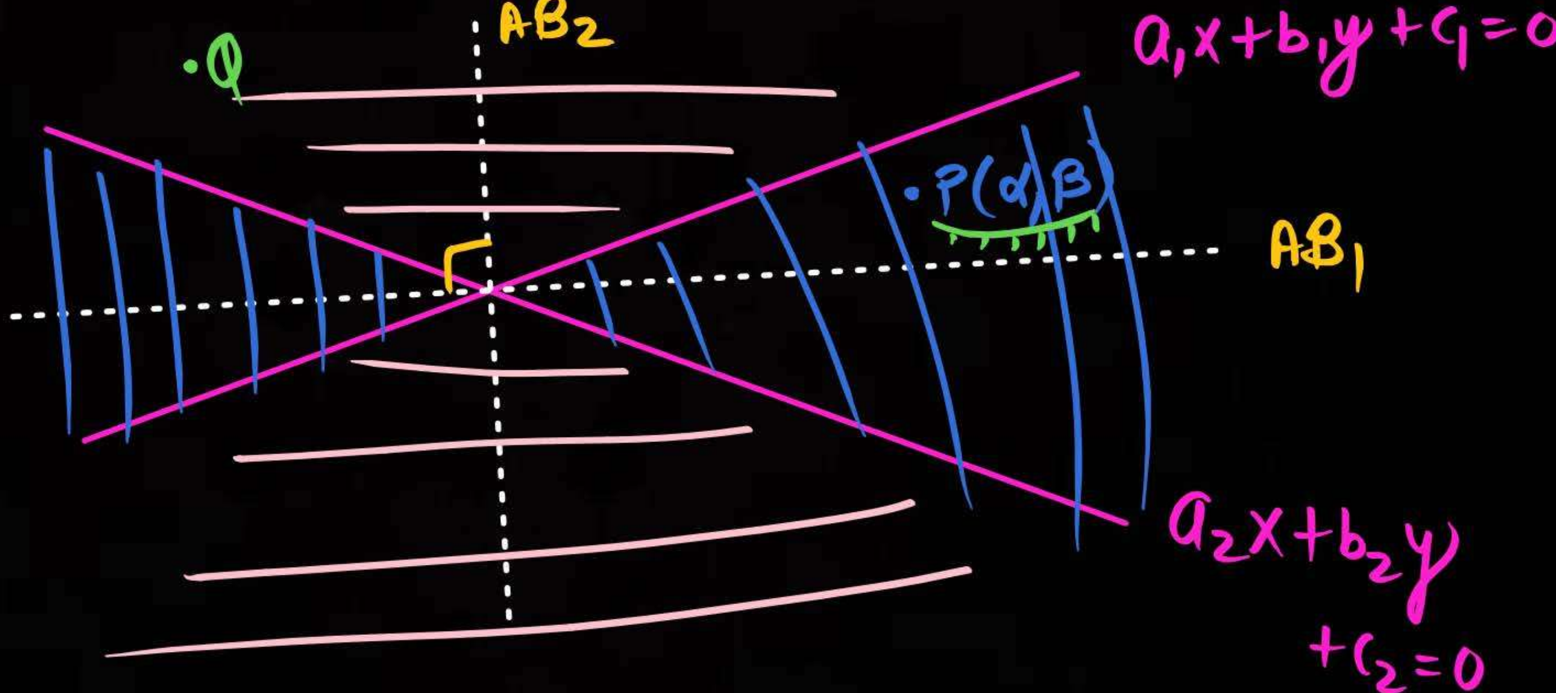
Obtuse ← (+)

$\boxed{\text{Ans}}$

acute ← (−)

IMPORTANT TYPE :

To find the bisector of the angle b/w given lines in which a given point lies.



WORKING method

⭐⭐⭐⭐

① Find Power of point of given pt WRT BOTH LINES.

② If same sign ⟶ (+) wallah

opposite sign ⟶ (−) wallah

ILLUSTRATION :

Find the bisector of that $\angle$ b/w the lines $\underbrace{x + 2y = 3}_{}$ & $\underbrace{3x + 2y = 1}_{}$ in which

L1 L2

(i) (2,3) lies

(ii) (0,1) lies

(iii) (0,0) lies

(i) $P_{L_1} = 2+6-3 = 5$

$P_{L_2} = 6+6-1 = 11$

(+) wallah

$$\frac{x+2y-3}{\sqrt{5}} = +\left(\frac{3x+2y-1}{\sqrt{13}}\right) \text{ Ans}$$

$x+2y-3=0$; $3x+2y-1=0$

$$\frac{x+2y-3}{\sqrt{5}} = \pm\left(\frac{3x+2y-1}{\sqrt{13}}\right)$$

(ii) $P_{L_1} = -1$, $P_{L_2} = 1$

(−) wallah

$$\frac{x+2y-3}{\sqrt{5}} = -\left(\frac{3x+2y-1}{\sqrt{13}}\right) \text{ Ans}$$

(iii) $P_{L_1} = -3$

$P_{L_2} = -1$

(+) wallah

$$\frac{x+2y-3}{\sqrt{5}} = +\left(\frac{3x+2y-1}{\sqrt{13}}\right)$$

$\boxed{\text{Ans}}$

Q. Find the range of α if $(0, 0)$ lies in the bisector of acute $\angle$ b/w the lines $(\alpha^2 + 3)x + 4y = 3$ and $x + \alpha y = 1$.

(ADVANCED)

$(\alpha^2+3)x+4y-3=0$ (L_1)

$a_1=\alpha^2+3$

$b_1=4$

$P_{L_1}=-3$

(L_2)

$x+\alpha y-1=0$

$a_2=1$

$b_2=\alpha$

$P_{L_2}=-1$

(+) wallah $\Rightarrow$ acute angle Bis.

(−) wallah $\Rightarrow$ obtuse angle Bis.

★★ $\boxed{a_1a_2+b_1b_2<0}$ $\alpha^2+3+4\alpha<0 \Rightarrow \boxed{\alpha\in(-3,-1)}$ Ans

FAMILY OF LINES

$L_1 + \lambda L_2 = 0, \lambda \in R$

represents to the family of lines passing through a fix point P, where given two lines L_1 & L_2 intersects.

∞ many lines can pass through 'A'

$\boxed{L_1 + \lambda L_2 = 0, \ \lambda \in R}$

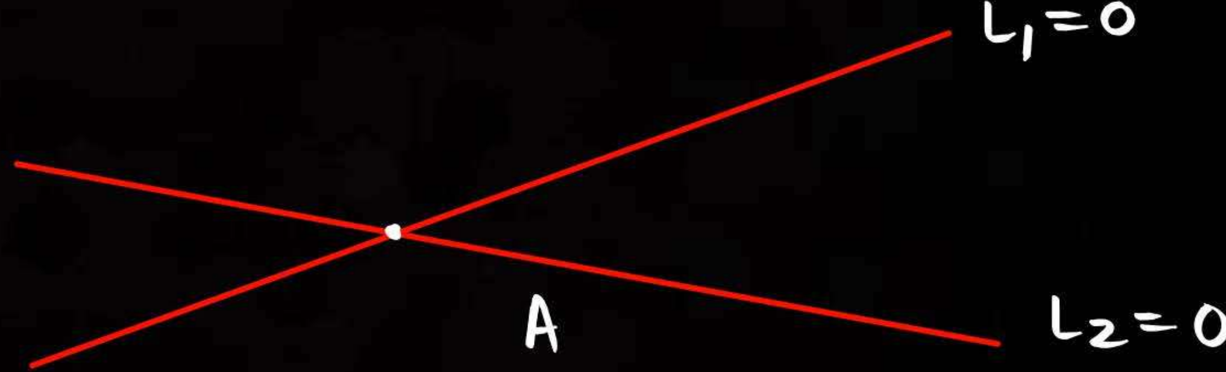


ILLUSTRATION :

Equation of the straight line which passes through the point (2, -3) and the POI of $x + y + 4 = 0$ & $3x - y - 8 = 0$.

$L_1 = 0$ $\quad$ $L_2 = 0$

$L_1 + \lambda L_2 = 0$

$$\boxed{(x+y+4) + \lambda(3x - y - 8) = 0}$$

$(2, -3)$

$(2 - 3 + 4) + \lambda(6 + 3 - 8) = 0$

$3 + \lambda = 0$

$$\boxed{\lambda = -3}$$

$\lambda = -3$

$-8x + 4y + 28 = 0$

$$\boxed{-2x + y + 7 = 0}$$ Ans

Q. A man is walking on a straight line. The arithmetic mean of the reciprocals of the intercepts of this line on the coordinate axes is 1/4. Three stones A, B and C are placed at the points (1, 1), (2, 2) and (4, 4) respectively. Then which of these stones is/are on the path of the man?

B

(JEE MAINS-2021)

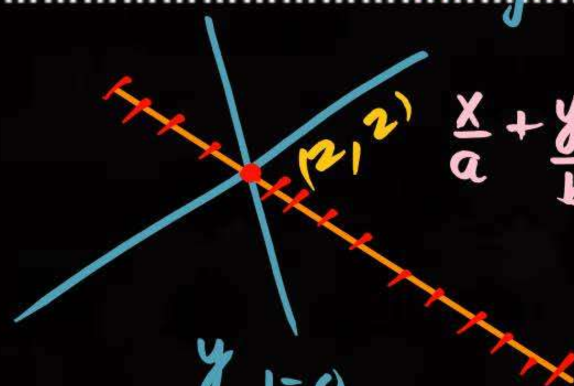
(A) B only ✓

(B) A only

(C) All the three

(D) C only

$x-y=0$



$\frac{y}{2}-1=0$

$$\frac{x}{a}+\frac{y}{b}=1$$

$$\frac{x}{a}+y\left(\frac{1}{2}-\frac{1}{a}\right)=1$$

$$\boxed{\frac{1}{a}(x-y)+\left(\frac{y}{2}-1\right)=0}$$

$$\frac{1}{a},\frac{1}{b}$$

$$AM=\frac{\frac{1}{a}+\frac{1}{b}}{2}=\frac{1}{4}$$

$$\frac{1}{a}+\frac{1}{b}=\frac{1}{2}$$

$$\boxed{\frac{1}{b}=\frac{1}{2}-\frac{1}{a}}$$

$$\boxed{\underbrace{\left(\frac{y}{2}-1\right)}_{L_1}+\underbrace{\left(\frac{1}{a}\right)}_{\lambda}\underbrace{(x-y)}_{L_2}=0}$$

always pass through POI of $L_1=0$ and $L_2=0$.

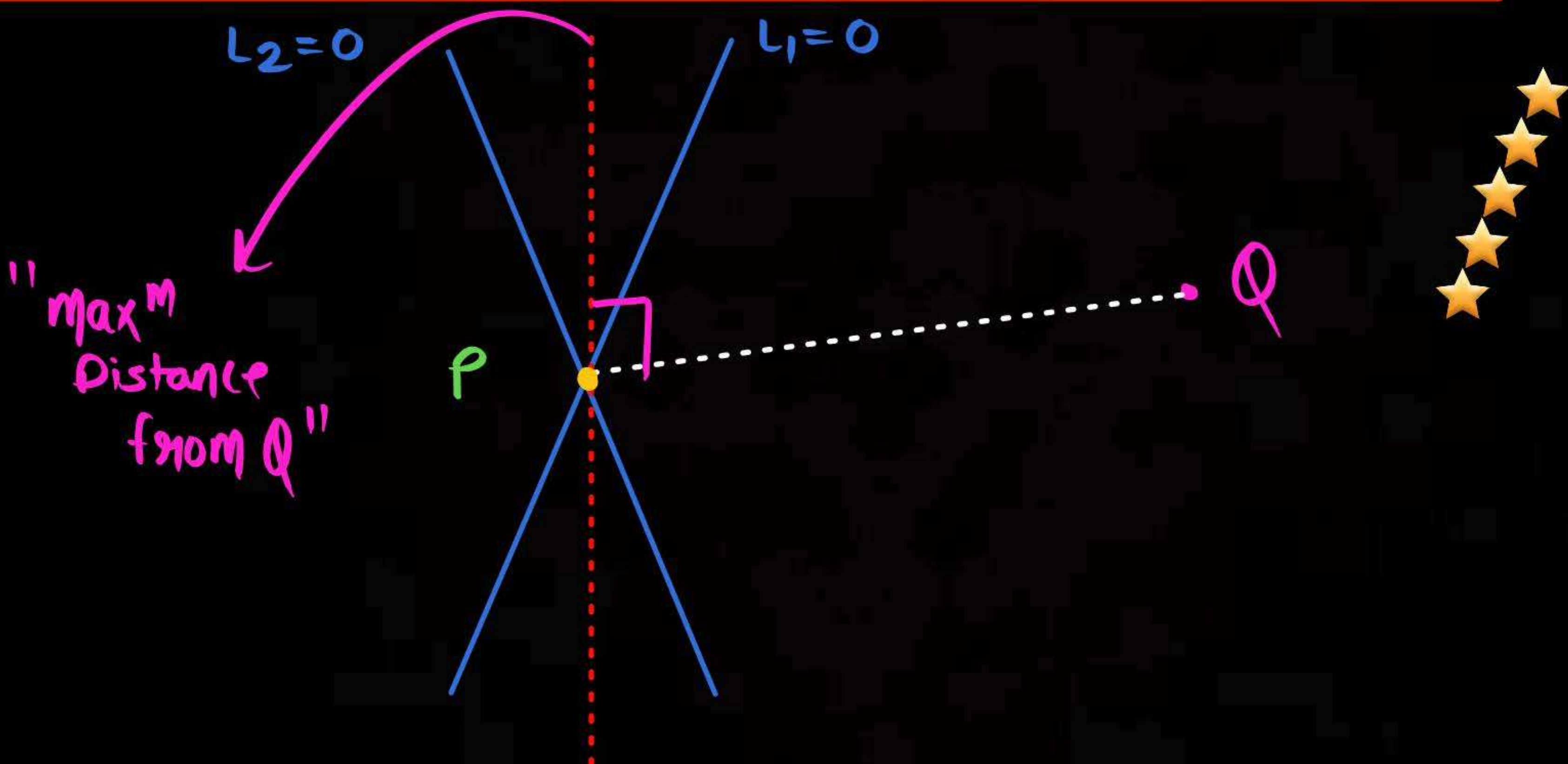
$$\underbrace{\frac{y}{2}-1=0}_{y=2}, \quad x-y=0 \;\Rightarrow\; x=2$$

$(2,2)$

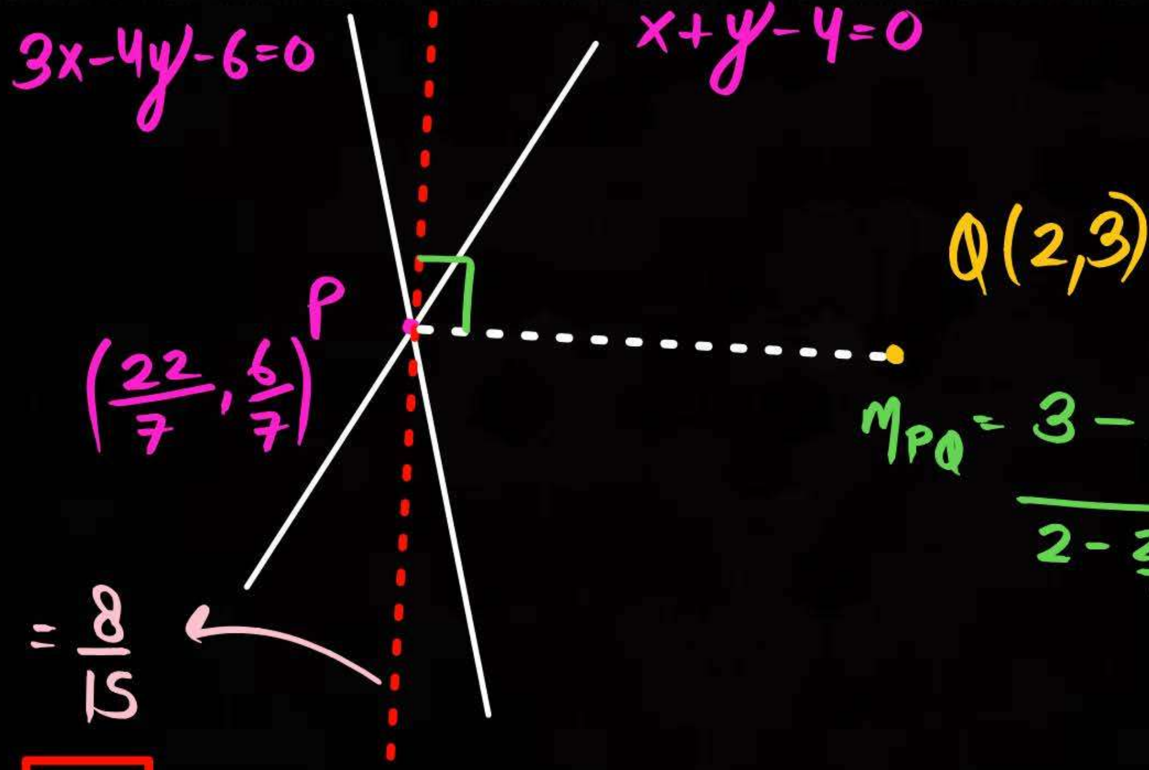
msm ♡♡

NOTE :

Particular member of the family at maximum distance from Q is $\perp^r$ to PQ.



Q. Find the line passing through a fix point P where given two lines $3x - 4y = 6$ and $x + y = 4$ intersect and also at maximum distance from Q (2 ,3)



$$m_{PQ} = \frac{3 - \frac{6}{7}}{2 - \frac{22}{7}} = \frac{-15}{8}$$

$$\frac{y - \frac{6}{7}}{x - \frac{22}{7}} = \frac{8}{15}$$

$\boxed{\text{Ans}}$

TYPE 2

→ Relation b/w a, b, c will be given,

Find that point from where

$ax+by+c=0$ will pass.

$\boxed{\text{Make coefficient of 'c' same in Both expressions}}$

ex) → $a-2b+4c=0$

$ax+by+3c=0 \longrightarrow$ pt ?

$\dfrac{a}{4}-\dfrac{2b}{4}+c=0$

$\dfrac{a}{3}x+\dfrac{by}{3}+c=0$

Compare → $\dfrac{a}{4}=+\dfrac{ax}{3} \to x=\dfrac{3}{4}$

$-\dfrac{2\cancel{b}}{4}=\dfrac{\cancel{b}y}{3} \to y=-\dfrac{3}{2}$

$\left(\dfrac{3}{4},-\dfrac{3}{2}\right)$ $\boxed{\text{Ans}}$

Q. Consider the set of all lines $\boxed{px + qy + r = 0}$ such that $\boxed{3p + 2q + 4r = 0}$. Which one of the following statements is true?

(JEE MAINS-2019)

(Mains ♡♡)

(A) The $\boxed{\text{lines are concurrent}}$ at the point $\left(\frac{3}{4}, \frac{1}{2}\right)$

(B) Each line passes through the origin

(C) The lines passes through the origin

(D) The lines are not concurrent

$\frac{3}{4}p + \frac{1}{2}q + r = 0$

$px + qy + r = 0$

$x = \frac{3}{4}, \quad y = \frac{1}{2}$

PAIR OF STRAIGHT LINES

↳ (Help in Conic sec^{n}s)

$a_1x + b_1y = c_1$

$a_2x + b_2y = c_2$

$a_1x + b_1y - c_1 = 0$

$a_2x + b_2y - c_2 = 0$

$(a_1x + b_1y - c_1)(a_2x + b_2y - c_2) = 0$

$\boxed{2^\circ \text{ Degree eq}^n}$

TWO DEGREE CURVE'S EQUATION

$$ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$$

POSL

CIRCLE

Parabola

Ellipse

hyperbola

a, b, h, g, f, c

scalars

$$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$$

(Learn)

If $\Delta = 0$, Then POSL

ILLUSTRATION:

For what value of λ does the equation

$12x^2 - 10xy + 2y^2 + 11x - 5y + \lambda = 0$ represents a pair of straight lines?

$a = 12, \; b = 2, \; 2h = -10$

$2g = 11, \; 2f = -5, \; c = \lambda$

$\Delta = 0$

$\boxed{\lambda = 2}$ Ans

JOINT EQUATION OF 2 LINES

Q→ $x+y-7=0$

$2x+y-6=0$

$(x+y-7)(2x+y-6)=0$

$\boxed{\text{Ans.}}$

SEPARATE EQUATION OF 2 LINES

Working Method

To find separate equation of two lines when their joint equation is given, first make their RHS equal to 0 and then resolve LHS into two linear factors or use **Shri-Dharacharya Method.**

ILLUSTRATION:

Find the separate equation of lines represents by the equation **$12x^2 - 10xy + 2y^2 + 11x - 5y + 2 = 0$**

$$12x^2 - 10xy + 11x + 2y^2 - 5y + 2 = 0$$

$$\boxed{\underbrace{12}_{A}x^2 + \underbrace{(11-10y)}_{B}x + \underbrace{(2y^2 - 5y + 2)}_{C} = 0}$$

$$x = \frac{-(11-10y) \pm \sqrt{(11-10y)^2 - 4(12)(2y^2 - 5y + 2)}}{24}$$

$(2y+5)^2$

$$24x = -(11-10y) \pm (2y+5)$$

$(+)$

$$24x = -11+10y+2y+5$$

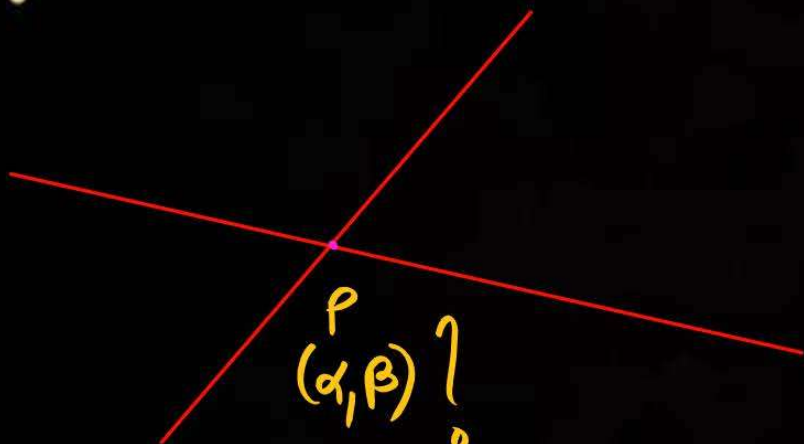
$$\boxed{24x-12y+6=0}$$ Ans

$(-)$

$$24x = -11+10y-2y-5$$

$$\boxed{24x-8y+16=0}$$ Ans

POI OF POSL (Partial Differentiation Method)



$$ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$$

1. PD WRT 'y' → x and y
2. PD WRT 'x' → x and y
3. Solve above 2 eqns
   ↳ get (α, β)

ILLUSTRATION:

For what value of λ does the equation

$\boxed{12x^2 - 10xy + 2y^2 + 11x - 5y + \lambda = 0}$ represents a pair of straight lines? Find their POI.

$\lambda = 2$

PD WRT x (y-constant)

$24x - 10y + 0 + 11 + 0 + 0$

$\boxed{24x - 10y + 11 = 0}$

PD WRT y

($x \to$ constant)

$0 - 10x + 4y + 0 - 5 + 0 = 0$

$\boxed{-10x + 4y - 5 = 0}$

POI

Ans $\to \left(-\frac{3}{2}, -\frac{5}{2}\right)$

HOMEGENOUS EQUATIONS

all terms are of same Degree

$2x^3 + y^3 + 3x^2y$ ✓ (3°)

$\underset{2^\circ}{2x^2} + \underset{1^\circ}{y} + \underset{2^\circ}{xy}$ ✗

HOMEGENOUS EQUATIONS OF 2 DEGREE

$$\boxed{ax^2 + 2hxy + by^2 = 0}$$

$\rightarrow$ Represents 2 line passing through (0,0)

$\boxed{y = m_1 x}$ and $\boxed{y = m_2 x}$

a, h, b $\rightarrow$ constants

(0,0)

$0+0+0=0$ ✓

$y = mx$

$\hookrightarrow$ passing through (0,0)

ANGLE B/W POSL

(Bahut use hoga)

$\boxed{\Delta = 0}$

$$ax^2 + by^2 + 2g(x) + 2fy + 2hxy + c = 0$$

JARH

$$\boxed{\tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a+b}\right|}$$

acute angle

$\theta = \frac{\pi}{2}$

$\Downarrow$

$\boxed{a + b = 0}$

$cx^2 + cy^2 = 0$

Line are $\perp^R$

Q. Find angle between the lines whose joint equation is $12x^2 - 10xy + 2y^2 + 11x - 5y + 2 = 0$

$h = -5$

$a = 12, \ b = 2$

$$\tan\theta = \left|\frac{2\sqrt{25-24}}{12+2}\right| = \frac{1}{7}$$

$$\boxed{\theta = \tan^{-1}\frac{1}{7}}$$ Ans

Q. The angle between the pair of lines whose equation is

$$\boxed{4x^2 + 10xy + my^2 + 5x + 10y = 0 \text{ is}}$$

$a=4,\ b=m,\ h=5$

$\boxed{\Delta=0}$

$m=4$

$\tan\theta = \left|\dfrac{2\sqrt{25-4m}}{m+4}\right|$

$= \left|\dfrac{2}{8}\cdot 3\right| = \dfrac{3}{4}$

(A) $\tan^{-1}\left(\frac{3}{8}\right)$

(B) $\tan^{-1}\left(\frac{4}{3}\right)$

(C) $\tan^{-1}\left(\frac{3}{4}\right)$

(D) $\tan^{-1}\frac{\sqrt{25-4m}}{m+4}$

HOMOGENISATION

Last Topic

(TRICK)

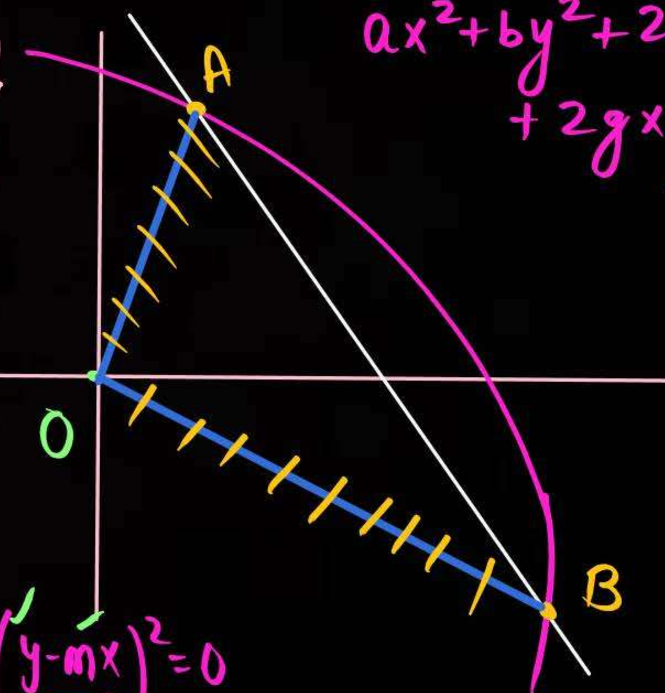
$y = mx + K$

$\left[\dfrac{y - mx}{K} = 1\right]$

OA and OB Ki Joint eqn :

$ax^2 + by^2 + 2hxy$
$+ 2gx(1) + 2fy(1)$
$+ c(1)^2 = 0$

PuT



$ax^2 + by^2 + 2hxy$
$+ 2gx + 2fy$
$+ c = 0$

#

$$ax^2 + by^2 + 2hxy + 2gx\left(\frac{y - mx}{K}\right) + 2fy\left(\frac{y - mx}{K}\right) + c\left(\frac{y - mx}{K}\right)^2 = 0$$

Line ✓
Curve ✓

Joint eqn of POI with (0,0)

$$\frac{y - mx}{c} = 1$$

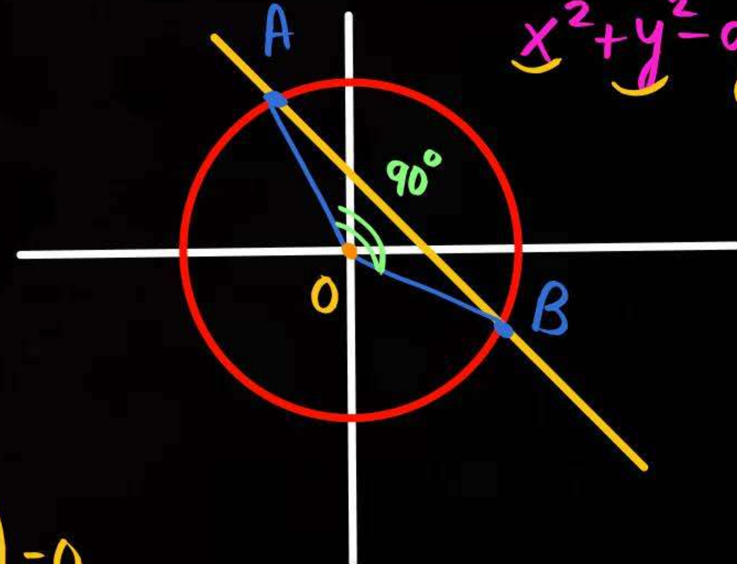
Q. Find the condition that the pair of straight lines joining the origin to the intersection of the line y = mx + c and the circle $x^2 + y^2 = a^2$ may be at right angles.

$\boxed{\text{Joint eqn of OA, OB} \rightarrow}$

$$x^2 + y^2 - a^2 (1)^2 = 0$$

$$\boxed{x^2 + y^2 - a^2 \left(\frac{y - mx}{c}\right)^2 = 0}$$

$$(x^2 + (y^2 = 0 = \left(1 - \frac{a^2 m^2}{c^2}\right) + \left(1 - \frac{a^2}{c^2}\right) = 0$$



$$x^2 + y^2 - a^2 = 0$$

0°

$$2 - \frac{a^2}{c^2}(1+m^2) = 0$$

$$\boxed{2c^2 = a^2(1+m^2)}$$ Ans:

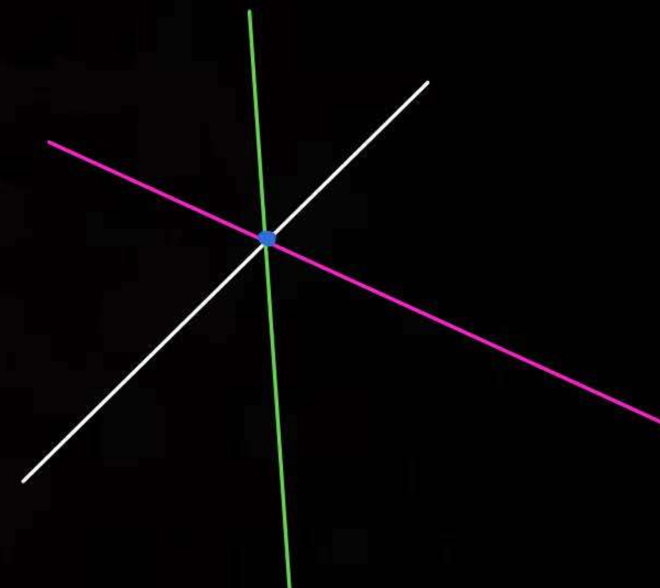
CONDITION OF CONCURRENCY

Intersecting at the same pt.

$$a_1x+b_1y+c_1=0$$

$$a_2x+b_2y+c_2=0$$

$$a_3x+b_3y+c_3=0$$



Condⁿ → $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}=0$

A SPECIAL DETERMINANT

(Circulant)

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 3abc - a^3 - b^3 - c^3$$

Q. If the lines $ax + by + c = 0$, $bx + cy + a = 0$ and $cx + ay + b = 0$ be concurrent, then:

(A) $a^3 + b^3 + c^3 - 3abc = 0$

(B) $a^3 + b^3 + c^3 - abc = 0$

(C) $a^3 + b^3 + c^3 + 3abc = 0$

(D) None of these

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$$

$$3abc - a^3 - b^3 - c^3 = 0$$