TOPICS TO BE COVERED

1. Standard Equations of Ellipse
2. Shifted forms of Horizontal and Vertical Ellipse
3. Equations of Tangents, Pair of tangents & Director Circle
4. Equation of Normals to Ellipse
5. Chords & Properties of Ellipse
6. TYQ's- 2022
7. PYQ's- 2021

Definition-Ellipse

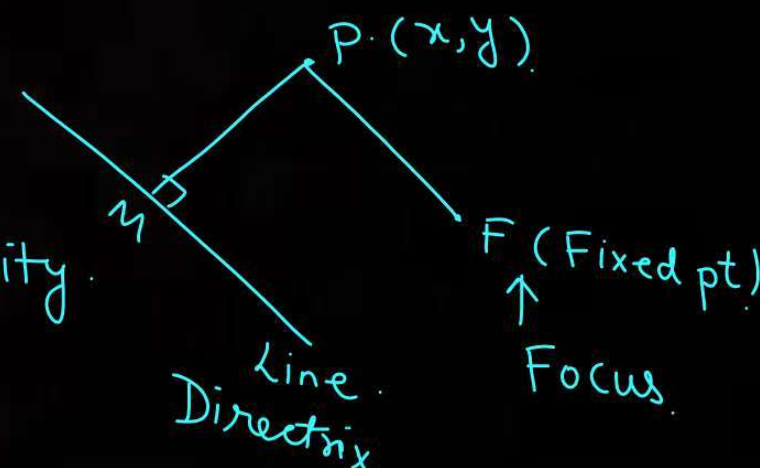
Definition

The locus of a point which moves such that the ratio of its distance from a **fixed point** to its distance from a **fixed straight line** is always a **constant (e) provided $0 < e < 1$.**

Ellipse
$0 < e < 1; \Delta \neq 0$
$h^2 < ab$

$$\frac{PF}{PM} = e$$

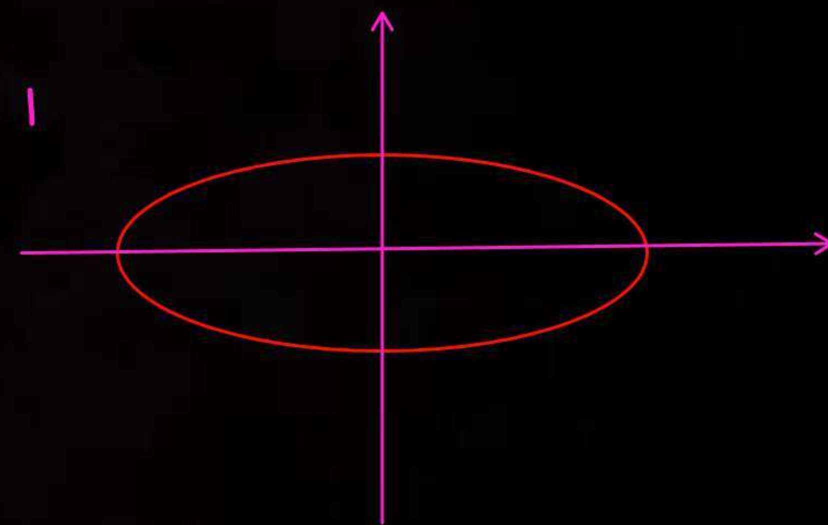
eccentricity.



Standard Equation of Ellipse-1

It is given by $\boxed{\frac{x^2}{a^2}+\frac{y^2}{b^2}=1}$ where $\boxed{b^2=a^2(1-e^2)}$, $(a>b)$

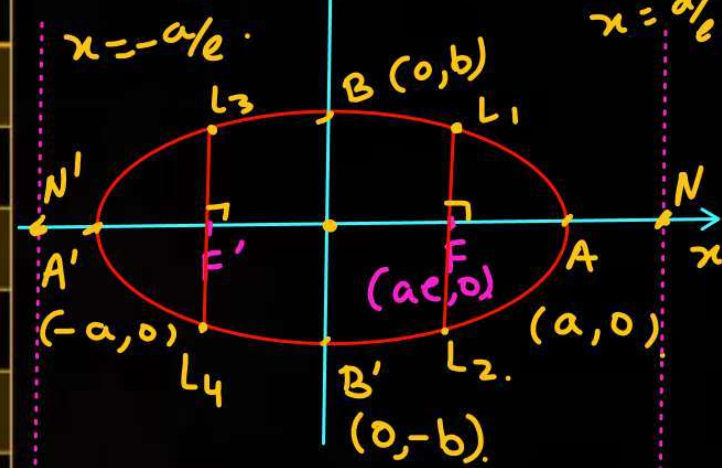
Ex $\rightarrow$ $\frac{x^2}{25}+\frac{y^2}{9}=1$



Standard Equation of Ellipse-1

$(a>b)$. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Equation of Major Axis	$y=0$
Equation of Minor Axis	$x=0$ ✓
Ends of Major Axis/ Vertices	$(a,0)$ & $(-a,0)$
Ends of Minor Axis	$(0,b)$ & $(0,-b)$
Centre →	$(0,0)$
Foci →	$(ae,0)$ & $(-ae,0)$
Directrices →	$x=a/e$ & $x=-a/e$
Feet of Directrices →	$[a/e,0]$ & $[-a/e,0]$
Equation of LR	$x=ae$ or $x=-ae$
Ends of L.R.	$\left[ae,\pm b^2/a\right]$ & $\left[-ae,\pm b^2/a\right]$
Length of LR	$2b^2/a$
Relation (a, b & e)	$b^2=a^2(1-e^2)$
Area of Ellipse	πab



length of Major Axis $=2a$

l (Minor Axis) $=2b$

$\left(\pm\frac{a^2}{b},\ be\right)$

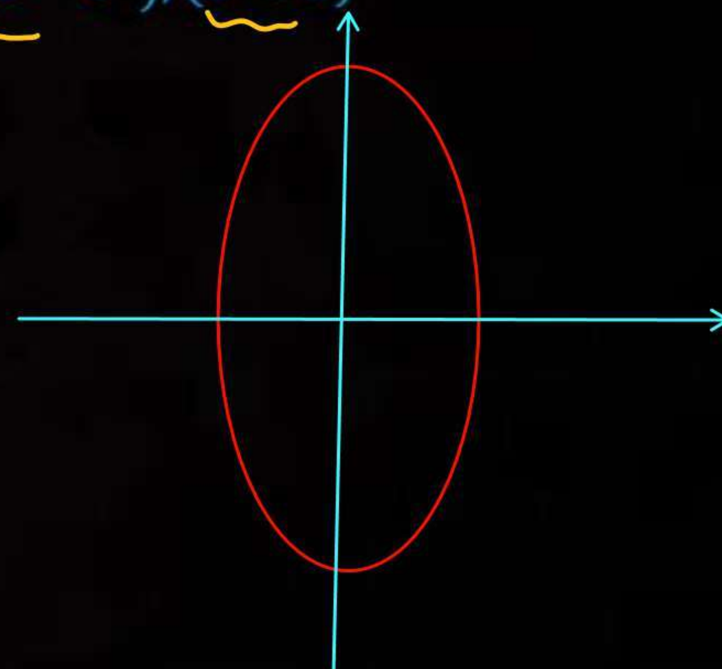
Standard Equation of Ellipse-2

It is given by $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ where $a^2=b^2(1-e^2)$, $(a<b)$

Ex → $\frac{x^2}{9}+\frac{y^2}{10}=1$

$a=3$

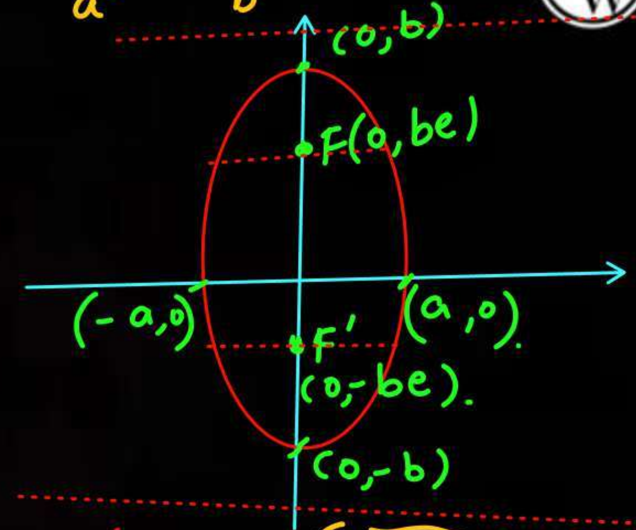
$b=\sqrt{10}$



Standard Equation of Ellipse-2

$a < b$ $\quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Equation of Major Axis	$x = 0$
Equation of Minor Axis	$y = 0$
Ends of Major Axis/ Vertices	$(0, b)$ & $(0, -b)$
Ends of Minor Axis	$(a, 0)$ & $(-a, 0)$
Centre	$(0, 0)$
Foci	$(0, \pm be)$
Directrices	$y = \pm b/e$
Feet of Directrices	$(0, \pm b/e)$
Equation of LR	$y = be$ or $y = -be$
# Ends of L.R.	$\left(\pm \frac{a^2}{b}, be\right)$ & $\left(\pm \frac{a^2}{b}, -be\right)$
Length of LR	$2a^2/b$
Relation (a, b & e)	$a^2 = b^2(1 - e^2)$
Area of Ellipse	πab



$x \leftrightarrow y$

$a \leftrightarrow b$

Q. Find every thing for the Ellipse $4x^2 + 9y^2 - 8x - 36y + 4 = 0$.

Foci

$X = \pm ae, \ Y = 0$

$x - 1 = \pm \cancel{3} \times \sqrt{5}/\cancel{3}$

$x = 1 \pm \sqrt{5}$

$y - 2 = 0 \Rightarrow y = 2.$

Foci $[1 \pm \sqrt{5}, 2]$

Ans

$4(x^2 - 2x) + 9[y^2 - 4y] + 4 = 0.$

$4[(x-1)^2 - 1] + 9[(y-2)^2 - 4] + 4 = 0$

$4(x-1)^2 - \cancel{4} + 9(y-2)^2 - 36 + \cancel{4} = 0.$

$4(x-1)^2 + 9(y-2)^2 = 36.$

$$\boxed{\frac{(x-1)^2}{9} + \frac{(y-2)^2}{4} = 1}$$

$x - 1 = X$ & $y - 2 = Y.$

$$\frac{X^2}{9} + \frac{Y^2}{4} = 1$$

$a = 3, \ b = 2$

$b^2 = a^2(1 - e^2)$

$4 = 9(1 - e^2)$

$1 - e^2 = 4/9.$

$e^2 = 5/9.$

$e = \sqrt{5}/3.$

PYQ-1

$\frac{a^2}{2} = 4a \Rightarrow a = 8.$

$\sqrt{2}b = 8 \Rightarrow b = 4\sqrt{2}.$

Q. Let the length of the latus rectum of an ellipse with its major axis along x-axis and centre at the origin, be 8. If the distance between the foci of this ellipse is equal to the length of its minor axis, then which one of the following points lies on it? $a > b$ [JEE Main-2019]

(A) $(4\sqrt{3}, 2\sqrt{3})$

(B) $(4\sqrt{3}, 2\sqrt{2})$ ✓

(C) $(4\sqrt{2}, 2\sqrt{2})$

(D) $(4\sqrt{2}, 2\sqrt{3})$

$\frac{2b^2}{a} = 8$

✓ $\boxed{b^2 = 4a}$ — (1)

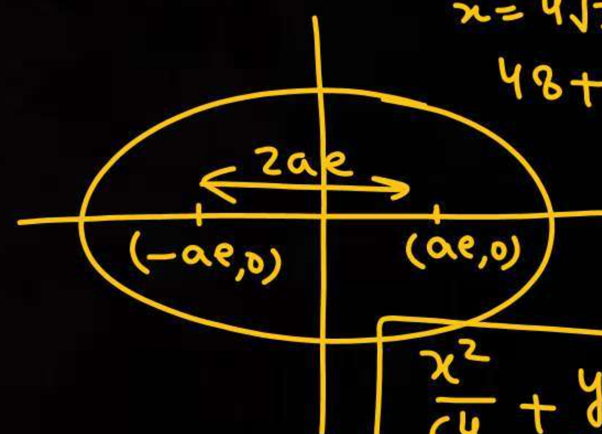
$\cancel{2}ae = \cancel{2}b$

$ae = b$ — (2)

$b^2 = a^2(1-e^2)$ — (3)

$b^2 = a^2 - a^2e^2 \Rightarrow b^2 = a^2 - b^2 \Rightarrow 2b^2 = a^2.$

$\Rightarrow a = \sqrt{2}b.$



$x = 4\sqrt{3} \Rightarrow x^2 = 48$

$48 + 2y^2 = 64$

$2y^2 = 16$

$y^2 = 8.$

$\boxed{\frac{x^2}{64} + \frac{y^2}{32} = 1}$

$\boxed{x^2 + 2y^2 = 64.}$

PYQ-2

Q. An ellipse, with foci at (0, 2) and (0, –2) and minor axis of length 4, passes through which of the following points?

[JEE Main-2019]

(A) $(1, 2\sqrt{2})$

(B) $(2, \sqrt{2})$

(C) $(2, 2\sqrt{2})$

(D) $(\sqrt{2}, 2)$ ✓

$be = 2$

$2a = 4$

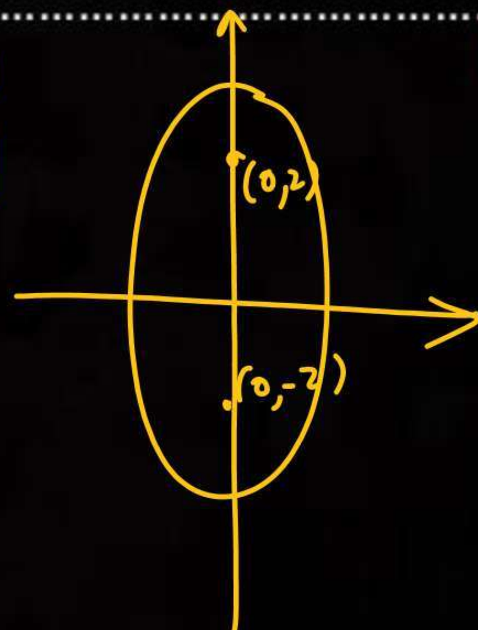
$\boxed{a = 2}$

$a^2 = b^2(1 - e^2)$

$a^2 = b^2 - b^2e^2$

$4 = b^2 - 4$

$\Rightarrow \boxed{b^2 = 8}$



$$\boxed{\frac{x^2}{4} + \frac{y^2}{8} = 1}$$

$y = 2$

$\frac{x^2}{4} + \frac{1}{2} = 1$

$\frac{x^2}{4} = \frac{1}{2}$

$x^2 = 2$

$x = \sqrt{2}$

$(\sqrt{2}, 2)$

PYQ-3

In an ellipse, with centre at the origin, if the difference of the lengths of major axis and minor axis is 10 and one of the foci is at $(0, 5\sqrt{3})$, then the length of its latus rectum is

[2019 Main, 8 April I]

[Ans. A]

(A) 5

(B) 10

(C) 8

(D) 6

$be = 5\sqrt{3} \Rightarrow b^2e^2 = 75.$

$2b - 2a = 10.$

$\Rightarrow \boxed{b - a = 5} \sim (1)$

$b + a = 15 \sim (2)$

$a^2 = b^2(1 - e^2)$

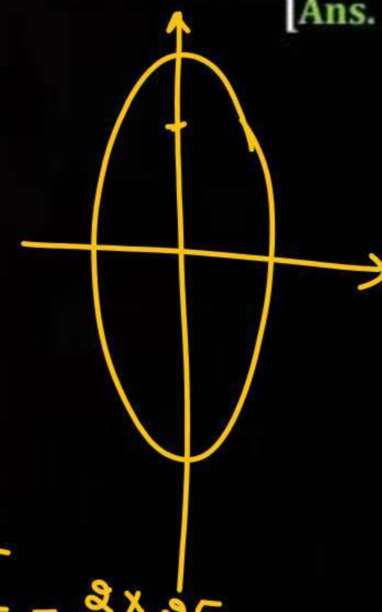
$a^2 = b^2 - b^2e^2$

$b^2 - a^2 = 75.$

$\underbrace{(b-a)}(b+a) = 75$

$(b+a) = 15$

$\left.\begin{array}{l} b = 10 \\ a = 5 \end{array}\right\}$



$L.R. = \frac{2a^2}{b} = \frac{2 \times 25}{10} = 5.$

Shifted forms of Ellipse

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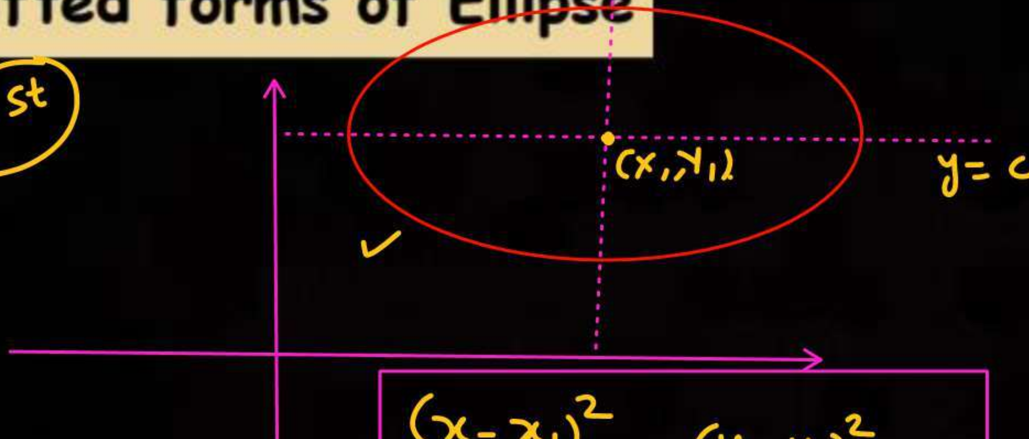
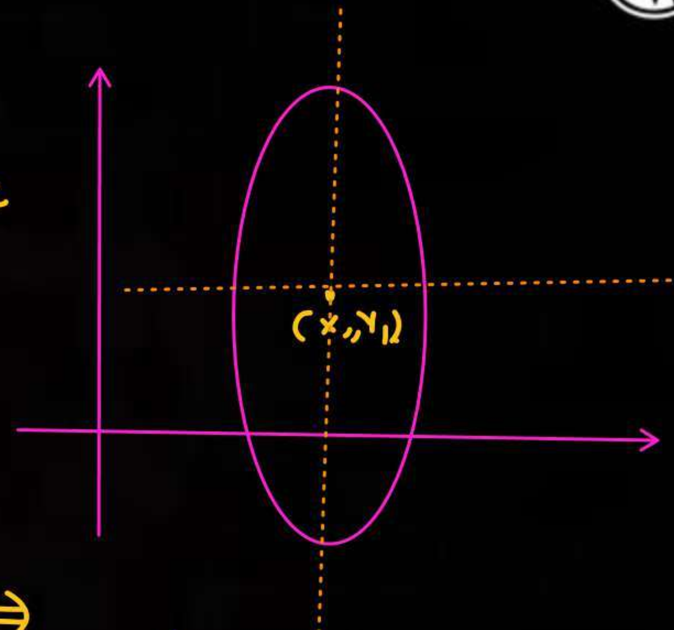


✓ $$\boxed{\frac{(x-x_1)^2}{a^2}+\frac{(y-y_1)^2}{b^2}=1}$$ $\Rightarrow$

$a > b$



$$\frac{(x-x_1)^2}{a^2}+\frac{(y-y_1)^2}{b^2}=1$$

$a < b$

Q. Find the equation of the ellipse its centre at the point (2, –3), one focus at (3, –3) and one vertex at (4, –3).

$$b^2 = a^2 - a^2e^2$$

$$b^2 = 4 - 1$$

$$\boxed{b^2 = 3}$$

$$CA = a = 2.$$

$$CF = ae = 1$$

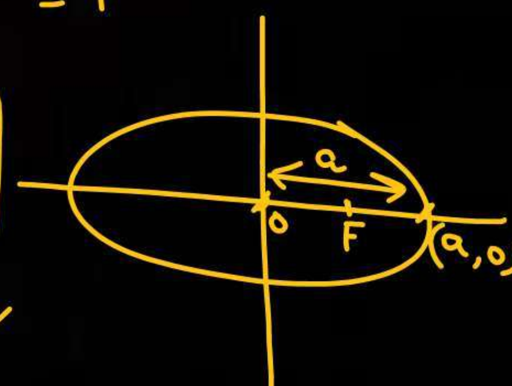
a

$y = -3$

C (2, –3) F (3, –3) A (4, –3)

$$\frac{(x-2)^2}{a^2} + \frac{(y+3)^2}{b^2} = 1$$

$$\boxed{\frac{(x-2)^2}{4} + \frac{(y+3)^2}{3} = 1}$$

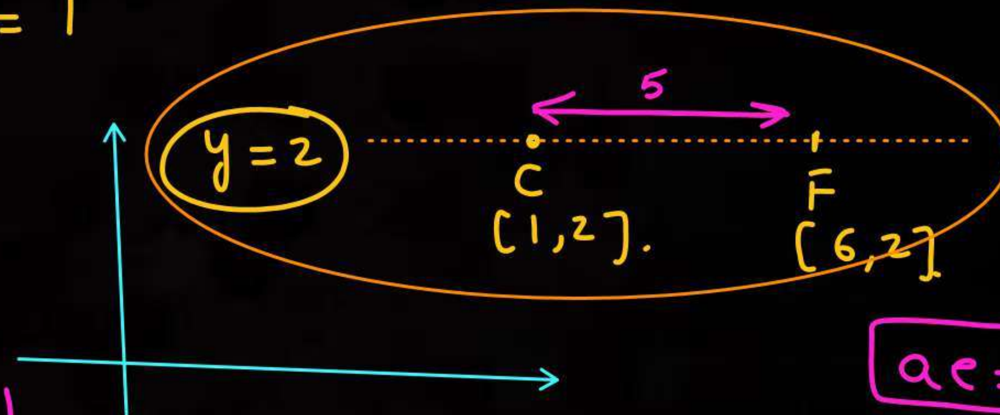


Q. Find the equation of the ellipse having centre at (1,2), one focus at (6,2) and passing through the point (4,6).

$$\frac{(x-1)^2}{45} + \frac{(y-2)^2}{20} = 1$$

Ans.

$$\frac{(x-1)^2}{a^2} + \frac{(y-2)^2}{b^2} = 1$$



$$\boxed{ae = 5}$$

$$b^2 = a^2 - a^2e^2$$

$$b^2 = a^2 - 25$$

$$a^2 = 45 \Rightarrow b^2 = 20.$$

$$\frac{3^2}{a^2} + \frac{4^2}{b^2} = 1$$

$$\frac{9}{a^2} + \frac{16}{a^2-25} = 1$$

$$9[a^2-25] + 16a^2 = a^2(a^2-25).$$

$$25a^2 - 225 = a^4 - 25a^2.$$

$$a^4 - 50a^2 + 225 = 0$$

$$a^2 = 45, \cancel{5}.$$

Note that

The length of latus Rectum of Ellipse is $2b^2/a$

= 2 e (distance between focus and Corresponding Directrix)

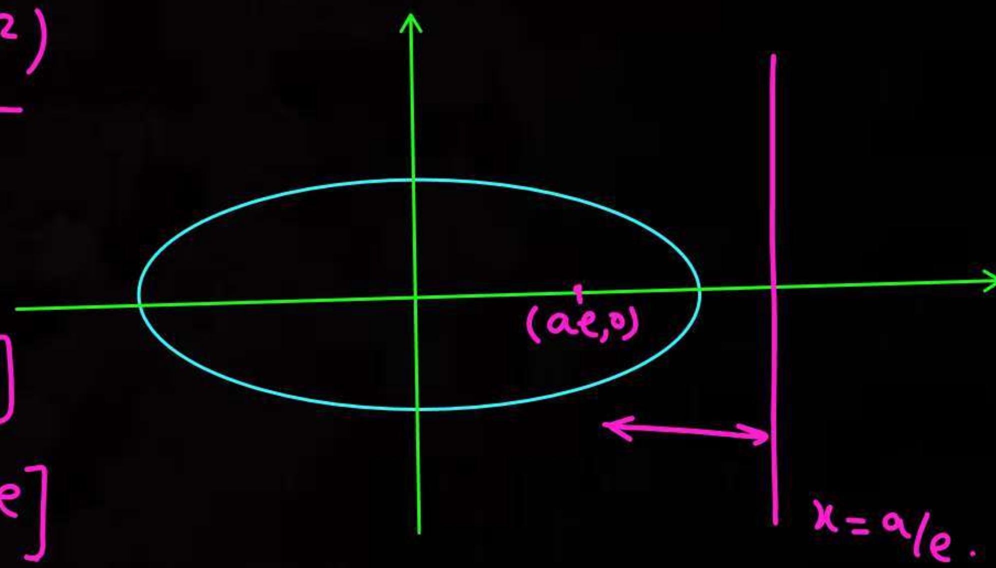
$$L.R. = \frac{2b^2}{a} = \frac{2a^2(1-e^2)}{\cancel{a}}$$

$$= 2a(\underbrace{1-e^2})$$

$$2ae\left[\frac{1}{e} - e\right]$$

$$= 2e\left[\underbrace{a/e - ae}\right]$$

$= 2e \times$ (dis b/w Focus & directrix)



PYQ-4

Q. If the distance between the foci of an ellipse is 6 and the distance between its directrices is 12, then the length of its latus rectum is :

[JEE Main 2020]

(A) $\sqrt{3}$

(B) $2\sqrt{3}$

(C) $3\sqrt{2}$

(D) $\frac{3}{\sqrt{2}}$

$2ae = 6 \Rightarrow ae = 3$

$2a/e = 12 \Rightarrow a/e = 6$

$L.R = ?$

$$\boxed{L.R = 2e\left[a/e - ae\right]}$$

$$= 2 \cdot \frac{1}{\sqrt{2}}(6-3)$$

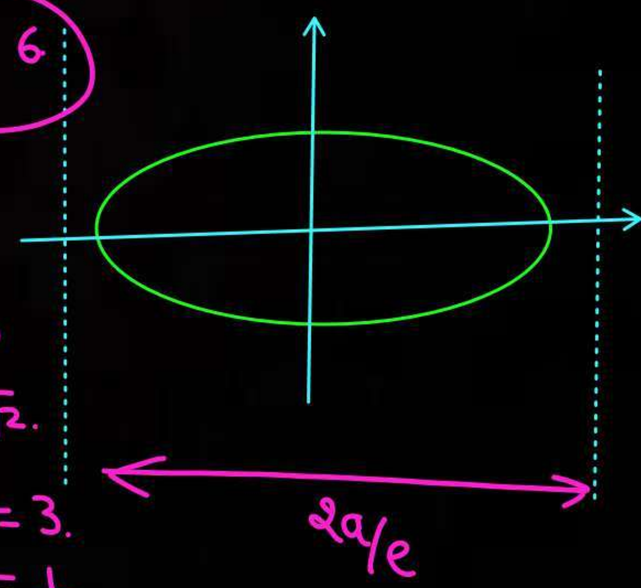
$$= \sqrt{2} \times 3 = 3\sqrt{2}.$$

$a^2 = 18$

$a = 3\sqrt{2}.$

$3\sqrt{2}\,e = 3.$

$e = \frac{1}{\sqrt{2}}.$



PYQ-5 Let S and S′ be the foci of an ellipse and B be any one of the extremities of its minor axis. If $\Delta S'BS$ is a right angled triangle with right angle at B and area $(\Delta S'BS) = 8$ sq. units, then the length of latus rectum of the ellipse is

....

[2019 Main, 12 Jan II]

[Ans. D]

(A) $2\sqrt{2}$

(B) $4\sqrt{2}$

(C) 2

(D) 4

$Ar(\Delta OBS) = 4.$

$\frac{1}{2} \times (ae) \times (b) = 4.$

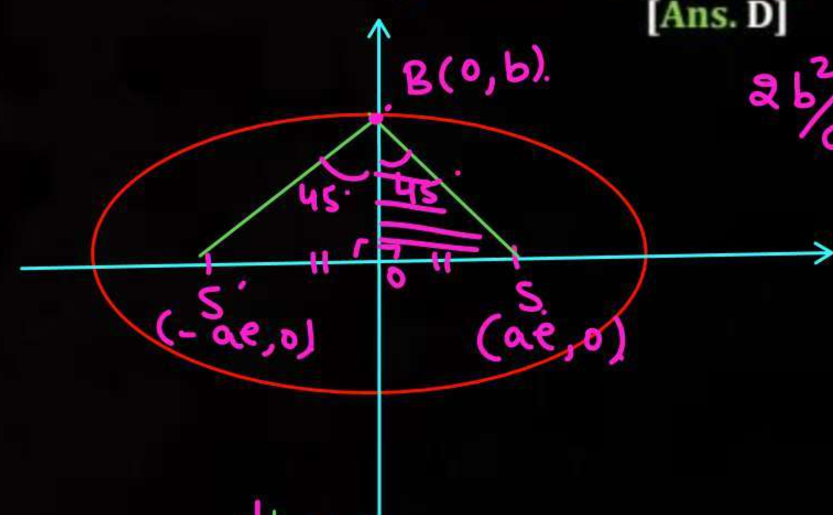
$b^2 = 8.$

$b^2 = a^2 - a^2e^2.$

$b^2 = a^2 - b^2$

$2b^2 = a^2.$

$a^2 = 16 \Rightarrow \boxed{a = 4}$



$2b^2/a$

$\tan 45° = \frac{ae}{b} \Rightarrow \boxed{ae = b}$

LR: $\frac{2b^2}{a} = \frac{2 \times 8}{4} = 4$

PYQ-6

Q.

If the point of intersections of the ellipse $\frac{x^2}{16}+\frac{y^2}{b^2}=1$ and the circle $x^2+y^2=4b, b>4$ lie on the curve $y^2=3x^2$, then b is equal to:

[JEE Main-2021 (March)]

(A) 12

(B) 5

(C) 6

(D) 10

$x^2+3x^2=4b$

$4x^2=4b$

$\boxed{x^2=b}$

$y^2=3b$

$\frac{b}{16}+\frac{3b}{b^2}=1$

$\frac{b}{16}+\frac{3}{b}=1$

$b^2+48=16b$

$b^2-16b+48=0$

$(b-12)(b-4)=0$

$\boxed{b=12}$

PYQ-7

Q. Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $(a > b)$ be a given ellipse, length of whose latus rectum is 10. If its eccentricity is the maximum value of the function, $\phi(t) = \frac{5}{12} + t - t^2$, then $a^2 + b^2$ is equal to:

[JEE Main 2020]

(A) 126

(B) 135

(C) 145

(D) 116

$\frac{2b^2}{a} = 10 \Rightarrow \boxed{b^2 = 5a}$

$\phi'(t) = 0 \Rightarrow 1 - 2t = 0$

$t = \frac{1}{2}$

$\phi_{max} = \frac{5}{12} + \frac{1}{2} - \frac{1}{4}$

$\frac{5}{12} + \frac{1}{4} \times \frac{3}{3}$

$\frac{8}{12} = \frac{2}{3}$

$e = 2/3$

$b^2 = a^2(1 - e^2)$

$5\cancel{a} = a^{\cancel{2}}\left[1 - \frac{4}{9}\right]$

$5 = a \times \frac{5}{9}$

$\boxed{a = 9}$

$b^2 = 45$

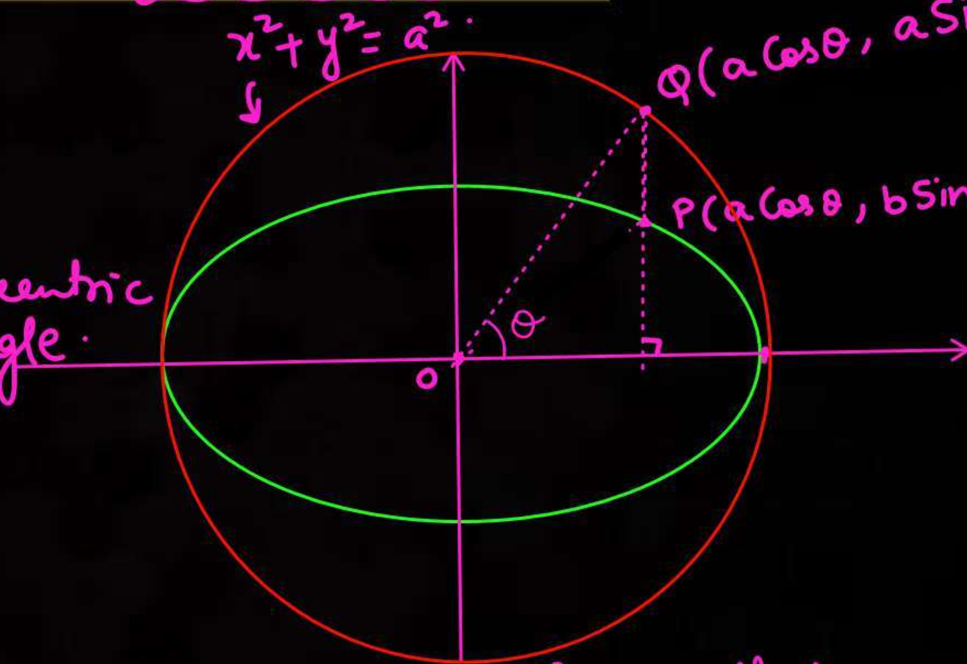
$a^2 + b^2 = 81 + 45 = 126$

Parametric Coordinates & Auxiliary Circle

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

\#. $\left.\begin{array}{l} x = a\cos\theta \\ y = b\sin\theta \end{array}\right\}$ $\theta \in$ Eccentric Angle.

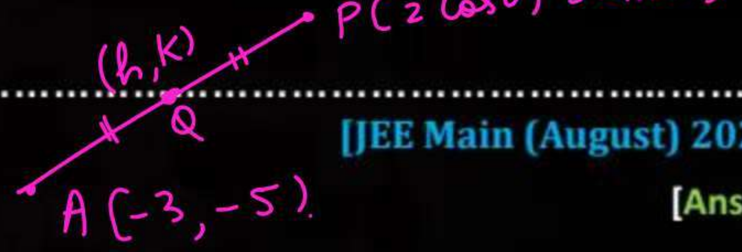
✓



Q is called corresponding pt. of P.

Q. PYQ-8

The locus of mid-points of the line segments joining $(-3, -5)$ and the points on the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$ is :



[JEE Main (August) 2021]

[Ans. C]

(A) $9x^2 + 4y^2 + 18x + 8y + 145 = 0$

(B) $36x^2 + 16y^2 + 90x + 56y + 145 = 0$

(C) $36x^2 + 16y^2 + 108x + 80y + 145 = 0$

(D) $36x^2 + 16y^2 + 72x + 32y + 145 = 0$

$2h = -3 + 2\cos\theta$ & $2k = -5 + 3\sin\theta$

$\frac{2h+3}{2} = \cos\theta$ & $\frac{2k+5}{3} = \sin\theta$.

$\frac{(2h+3)^2}{4} + \frac{(2k+5)^2}{9} = 1$

$9[4h^2+9+12h] + 4[4k^2+25+20k] = 36.$

$36h^2 + 81 + 108h + 16k^2 + 100 + 80k = 36.$

PYQ-9

Q.

If the point P on the curve, $4x^2 + 5y^2 = 20$ is farthest from the point Q(0, – 4), then PQ^2 is equal to:

$\frac{x^2}{5} + \frac{y^2}{4} = 1.$

[JEE Main 2020]

(A) 21

(B) 36

(C) 48

(D) 29

$x = \sqrt{5}\cos\theta$, $y = 2\sin\theta$

$P(\sqrt{5}\cos\theta, 2\sin\theta)$.

$Q[0, -4]$

$(PQ)^2 = (\sqrt{5}\cos\theta)^2 + (2\sin\theta + 4)^2$

$= 5(1 - \sin^2\theta) + 4\sin^2\theta + 16 + 16\sin\theta$

$= 21 - \sin^2\theta + 16\sin\theta$

$= -[\sin^2\theta - 16\sin\theta - 21]$

$= -[(\sin\theta - 8)^2 - 85]$

$$\boxed{(PQ)^2 = 85 - (\underbrace{\sin\theta - 8}_{})^2}$$

$(PQ)^2_{max} = 85 - (1-8)^2$

$= 85 - 49$

$(PQ)^2 = 36$

Focal Distances of any Point P on Ellipse

#.

$$\frac{PF_1}{PM_1} = e$$

$$PF_1 = e\,[\underbrace{PM_1}]$$

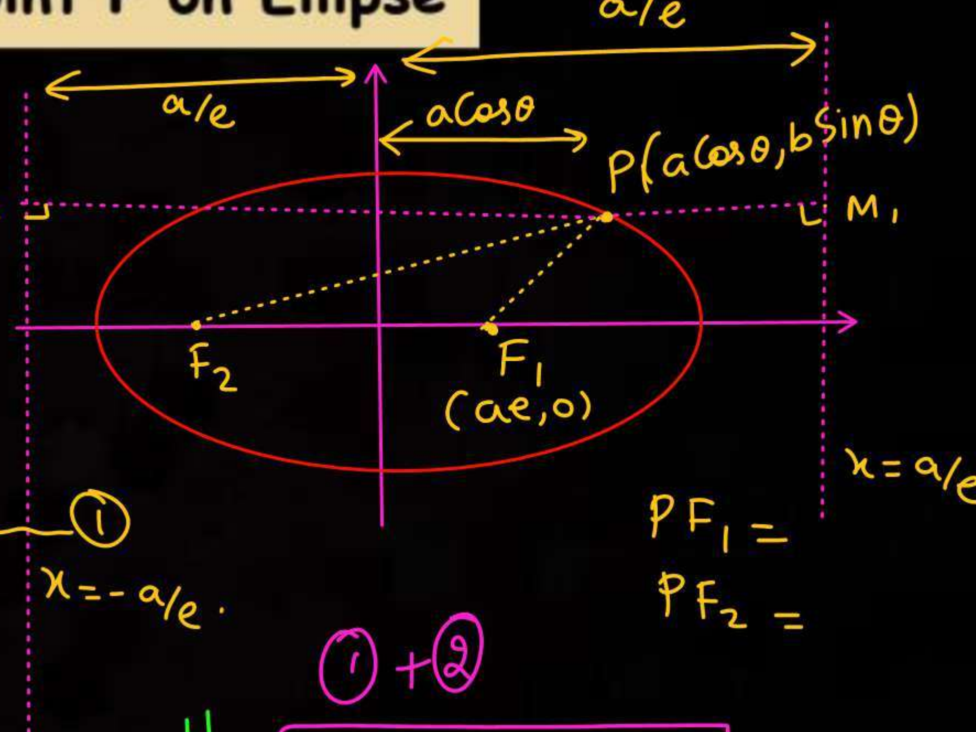
$$PF_1 = e\left[\frac{a}{e} - a\cos\theta\right]$$

$\boxed{PF_1 = a - ae\cos\theta}$ ~ (1)

$$PF_2 = e\,PM_2$$

$$= e\left[\frac{a}{e} + a\cos\theta\right]$$

$\boxed{PF_2 = a + ae\cos\theta}$ ~ (2)



$PF_1 =$

$PF_2 =$

(1) + (2)

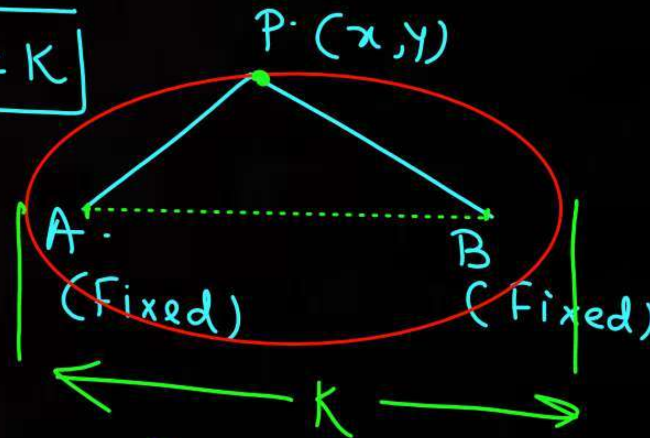
$\boxed{PF_1 + PF_2 = 2a}$

Note that

The **sum of focal distances** of any point P lying on the ellipse is always equal to the **length of Major Axis**.

Conversely, if a point P moves such that sum of its distances from two fixed points is always a constant **(k)** then locus of P is as ellipse provided **k > AB**. ✓

Given $\boxed{PA + PB = K}$



Locus is ellipse only if $\boxed{K > AB}$

$PA + PB > AB$

$\underbrace{PA + PB}$

$\boxed{K > AB}$

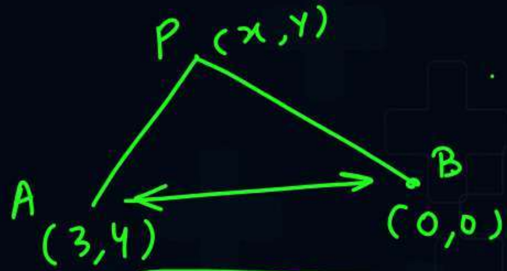
If $K = AB$.

Locus is line segment AB.

If $K < AB \Rightarrow$ no locus.

$$\boxed{PA + PB = 6}$$

Q→ $\sqrt{(x-3)^2+(y-4)^2} + \sqrt{x^2+y^2} = 6$



$$\boxed{AB = 5}$$

$K = 6$

$\because K > AB$

$\Rightarrow$ locus is ellipse.

l(Major Axis) $= 6$.

$2ae = 5$.

$K = AB$

$PA + PB = K$.

$$\boxed{PA + PB = AB}$$

Q. An ellipse having foci at (3,3) and (−4,4) and passing through the origin has eccentricity equal to

(A) $\frac{3}{7}$

(B) $\frac{2}{7}$

(C) $\frac{5}{7}$

(D) $\frac{3}{5}$

$$OF + OF' = 2a$$

$$4\sqrt{2} + 3\sqrt{2} = 2a.$$

$$7\sqrt{2} = 2a.$$

$$a = \frac{7}{\sqrt{2}}.$$

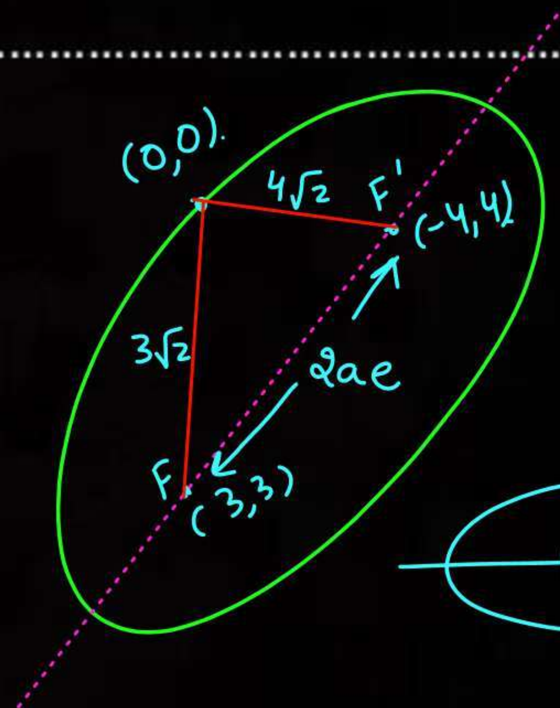
$$FF' = 2ae.$$

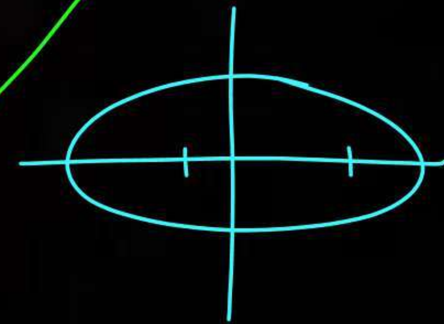
$$\sqrt{7^2 + 1^2} = 2ae.$$

$$\sqrt{50} = 2 \times \frac{7}{\sqrt{2}} e.$$

$$5\cancel{\sqrt{2}} = \cancel{\sqrt{2}} \times 7e$$

$$e = 5/7.$$





PYQ-10

Q. If the co-ordinates of two points A and B are $(\sqrt{7}, 0)$ and $(-\sqrt{7}, 0)$ respectively and P is any point on the conic, $9x^2 + 16y^2 = 144$, then PA + PB is equal to : $= 2a = 8.$

$\frac{x^2}{16} + \frac{y^2}{9} = 1$

[JEE Main 2020]

(A) 8

(B) 6

(C) 16

(D) 9

$F[\sqrt{7}, 0]$

$\& \; F'(-\sqrt{7}, 0)$

$\underbrace{a = 4}, \; b = 3$

$9 = 16(1 - e^2)$

$9/16 = 1 - e^2$

$e^2 = 7/16$

$e = \sqrt{7}/4.$

$ae = 4 \times \sqrt{7}/4 = \sqrt{7}.$

Various forms of Tangents

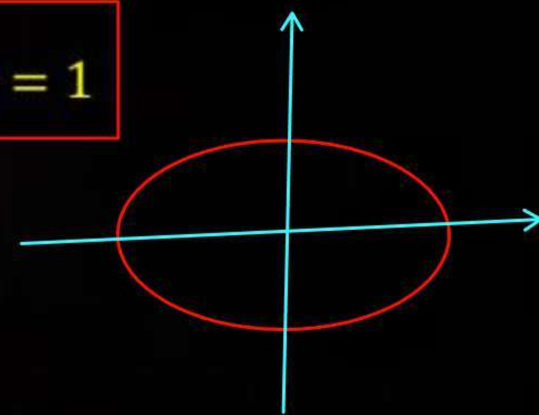
(a) Cartesian Form

Equation of the tangent at (x_1, y_1) is given by $\boxed{\dfrac{xx_1}{a^2} + \dfrac{yy_1}{b^2} = 1}$

(b) Slope Form

The equation of the tangent with slope m is given by

$$\boxed{y = mx \pm \sqrt{a^2m^2 + b^2}}$$



(c) Parametric Form

Equation of the tangent $P(a\cos\theta, b\sin\theta)$, where θ is the eccentric angle of the point P, is $\boxed{\dfrac{x\cos\theta}{a} + \dfrac{y\sin\theta}{b} = 1}$.

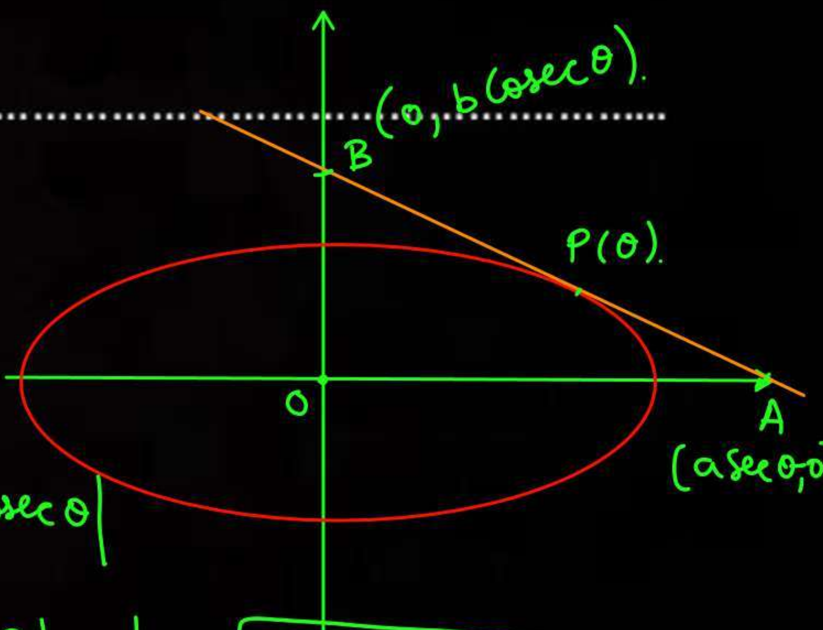
Note →

Q. A variable tangent is drawn to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meeting the coordinate axes at points A & B respectively. Find

(a) The minimum area of Triangle OAB $= ab$.

(b) The minimum length of AB $= a+b$.

HW.



Proof →

(a). $\boxed{\frac{x\cos\theta}{a}+\frac{y\sin\theta}{b}=1}$

$y=0$ for A.

A $[a\sec\theta, 0]$

$$\text{Area of } \triangle OAB = \left|\frac{1}{2}\cdot a\sec\theta\cdot b\,\mathrm{cosec}\,\theta\right|$$

$$= \left|\frac{ab}{2\sin\theta\cos\theta}\right| = \left|\frac{ab}{\sin 2\theta}\right| \Rightarrow \boxed{\text{Area}_{min} = ab}.$$

PYQ-11

Q. If $3x + 4y = 12\sqrt{2}$ is a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{9} = 1$ for some $a \in R$, then the distance between the foci of the ellipse is :

[JEE Main 2020]

(A) 4

(B) $2\sqrt{7}$

(C) $2\sqrt{5}$

(D) $2\sqrt{2}$

$4y = -3x + 12\sqrt{2}$.

$y = -\frac{3x}{4} + 3\sqrt{2}$. —— (1)

$y = mx \pm \sqrt{a^2m^2+b^2}$ —— (2)

Compare (1) & (2)

$m = -3/4$.

$3\sqrt{2} = \pm\sqrt{a^2m^2+b^2}$.

$18 = a^2\left(\frac{9}{16}\right) + 9$

$9 = \frac{9a^2}{16} \Rightarrow a^2 = 16$.

$\boxed{a = 4}$

$2ae$ = dis b/w Foci.

$b^2 = a^2(1-e^2)$.

$b^2 = a^2 - a^2e^2$.

$9 = 16 - a^2e^2$

$\boxed{a^2e^2 = 7}$

$ae = \sqrt{7}$.

PYQ-12

$$\frac{x^2}{2} + \frac{y^2}{1} = 1$$

Q. If tangents are drawn to the ellipse $x^2 + 2y^2 = 2$ at all points on the ellipse other than its four vertices then the mid points of the tangents intercepted between the coordinate axes lie on the curve: $\boxed{a = \sqrt{2},\ b = 1}$

[JEE Main-2019]

(A) $\dfrac{x^2}{2} + \dfrac{y^2}{4} = 1$

(B) $\dfrac{x^2}{4} + \dfrac{y^2}{2} = 1$

(C) $\dfrac{1}{2x^2} + \dfrac{1}{4y^2} = 1$ ✓

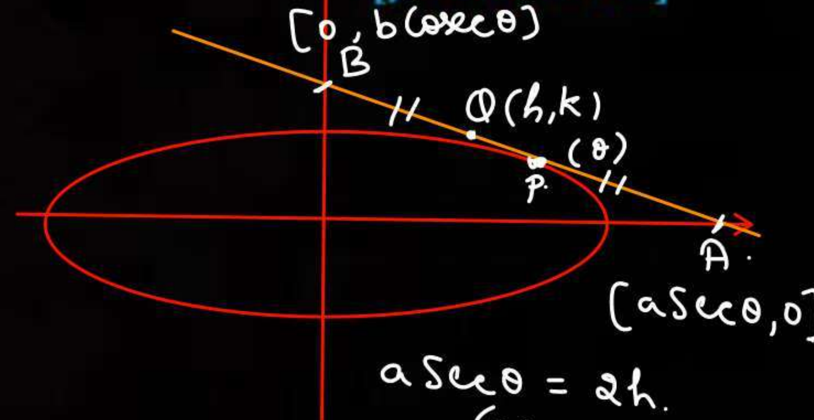
(D) $\dfrac{1}{4x^2} + \dfrac{1}{2y^2} = 1$

$$\boxed{\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1}$$

$$\frac{a^2}{4h^2} + \frac{b^2}{4k^2} = 1$$

$(h, k) \rightarrow (x, y)$

$$\frac{2}{4x^2} + \frac{1}{4y^2} = 1$$



$a\sec\theta = 2h$

$\cos\theta = a/2h$

$b\,\mathrm{cosec}\,\theta = 2k$

$\sin\theta = b/2k$

PYQ-13

$$\frac{x^2}{2}+\frac{y^2}{8}=1 \qquad \frac{xa}{2}+\frac{yb}{8}=1$$

Q. If the tangents on the ellipse $4x^2 + y^2 = 8$ at the points (1, 2) and (a, b) are perpendicular to each other, than a^2 is equal to:

$x_1 = 1,\ y_1 = 2$

[JEE Main-2019]

(A) 64/17

(B) 2/17

(C) 128/17

(D) 4/17

$$4xx_1 + yy_1 = 8.$$

$$4x + 2y = 8.$$

$$T_1:\ \boxed{2x + y = 4}$$

$$\boxed{m = -2}$$

$$\frac{a}{\cancel{2}} = \frac{1}{\cancel{2}\sqrt{17/2}}$$

$$a^2 = \frac{1}{17/2} = \frac{2}{17}.$$

$$T_2:\ m = \tfrac{1}{2}.$$

$$y = mx \pm \sqrt{a^2m^2 + b^2}.$$

$$y = \tfrac{1}{2}x \pm \sqrt{2 \times \tfrac{1}{4} + 8}.$$

$$y = x/2 \pm \sqrt{8 + \tfrac{1}{2}}.$$

$$y = x/2 \pm \sqrt{17/2}.$$

$$\frac{y}{\pm\sqrt{17/2}} - \frac{x}{\pm 2\sqrt{17/2}} = 1$$

PYQ-14

Q. If the line $\boxed{x - 2y = 12}$ is tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point $\left(3, \frac{-9}{2}\right)$, then the length of the latus rectum of the ellipse is:

[JEE Main-2019]

(A) 9

(B) $8\sqrt{3}$

(C) $12\sqrt{2}$

(D) 5

$\frac{x}{12} - \frac{y}{6} = 1$

$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$

$\frac{3x}{a^2} - \frac{9y}{2b^2} = 1$ ~ (1)

$\frac{x}{12} - \frac{y}{6} = 1$ ~ (2)

$\frac{3}{a^2} = \frac{1}{12}$ & $\frac{9}{2b^2} = \frac{1}{6}$

$\Rightarrow a^2 = 36.$ $b^2 = 27$

$LR = \frac{2 \times 27}{6}$

$= 9.$

PYQ-15

Q. The length of the minor axis (along y-axis) of an ellipse in the standard form is $\frac{4}{\sqrt{3}}$. If this ellipse touches the line, $x + 6y = 8$; then its eccentricity is:

HW

[JEE Main 2020]

(A) $\sqrt{\frac{5}{6}}$

(B) $\frac{1}{2}\sqrt{\frac{11}{3}}$

(C) $\frac{1}{3}\sqrt{\frac{11}{3}}$

(D) $\frac{1}{2}\sqrt{\frac{5}{3}}$

Note that

Tangents drawn at the 4 ends of Latera Recta of an ellipse enclose a Rhombus whose area is $2a^2/e$. ✓

$$\frac{x(\cancel{a}e)}{a^{\cancel{2}}} + \frac{y\cdot(\cancel{b^2}/a)}{\cancel{b^2}} = 1$$

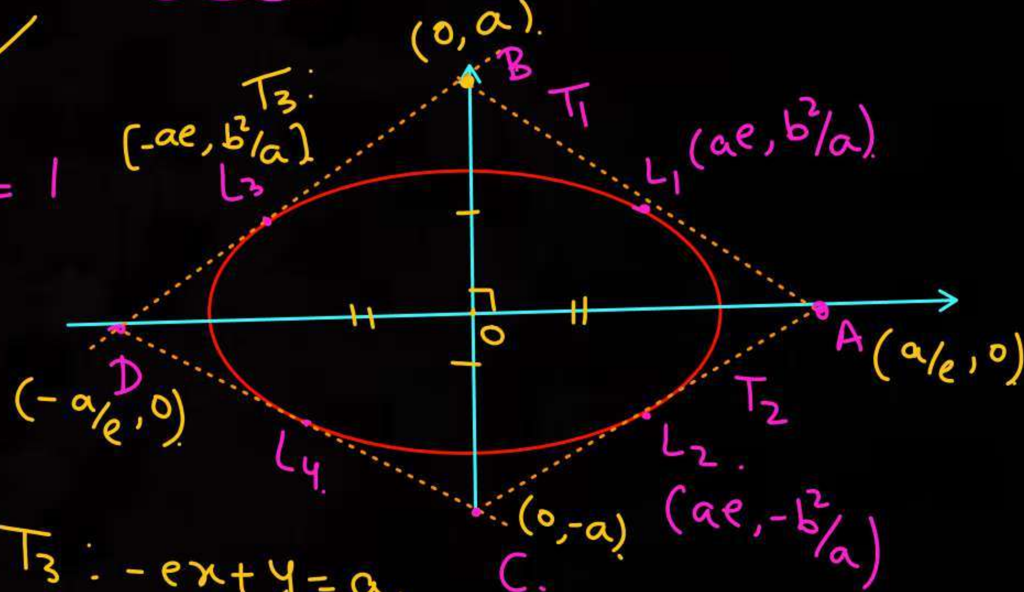
$$\frac{ex}{a} + \frac{y}{a} = 1$$

$$T_1 : \boxed{ex + y = a}$$

$$T_2 : ex - y = a$$

$$2ex = 2a$$

$$x = a/e$$



$$T_3 : -ex + y = a$$

$$T_4 : -ex - y = a$$

Area $= \frac{1}{2} d_1 d_2$

$$\frac{1}{\cancel{2}} \cdot \cancel{2}a/e \cdot 2a = 2a^2/e$$

PYQ-16

Let L be a common tangent line to the curves $4x^2 + 9y^2 = 36$ and $(2x)^2 + (2y)^2 = 31$. Then the square of the slope of the line L is

[JEE-Main-2021 (February)]

Ans. 3

$$\boxed{x^2 + y^2 = \frac{31}{4}}$$

$r = \frac{\sqrt{31}}{2}$

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

$$y = mx \pm \sqrt{9m^2+4}$$

$$\left|\frac{\sqrt{9m^2+4}}{\sqrt{1+m^2}}\right| = \frac{\sqrt{31}}{2}.$$

$$4(9m^2+4) = 31(1+m^2)$$

$$5m^2 = 31-16.$$

$$5m^2 = 15.$$

$$\boxed{m^2 = 3}$$

PYQ-17

Let L be a tangent line to the parabola $y^2 = 4x - 20$ at $(6, 2)$. If L is also a tangent to the ellipse $\frac{x^2}{2} + \frac{y^2}{b} = 1$, then the value of b is equal to

[JEE-Main-2021 (March)]

[Ans. B]

(A) 11

(B) 14

(C) 16

(D) 20

$$y = mx \pm \sqrt{2m^2 + b}$$

$$y = x - 4$$

$$m = 1 \quad \& \quad -4 = -\sqrt{2m^2 + b}$$

S.B.S.

$$16 = 2m^2 + b$$

$$16 - 2 = b$$

$$b = \underline{\underline{14}}$$

$$2y\frac{dy}{dx} = 4$$

$$\frac{dy}{dx} = \frac{2}{y} = \frac{2}{2} = 1$$

$$m_T = 1$$

$$y - 2 = 1(x - 6)$$

$$y - 2 = x - 6$$

$$\boxed{y = x - 4}$$

Director Circle

The locus of the point of intersection of the perpendicular tangents to the ellipse is called the director circle of the ellipse.

Equation of the director circle of the ellipse is $x^2 + y^2 = a^2 + b^2$

Proof → Given $m_1 m_2 = -1$

$$y = mx \pm \sqrt{a^2m^2 + b^2}$$

(h, k) satisfies

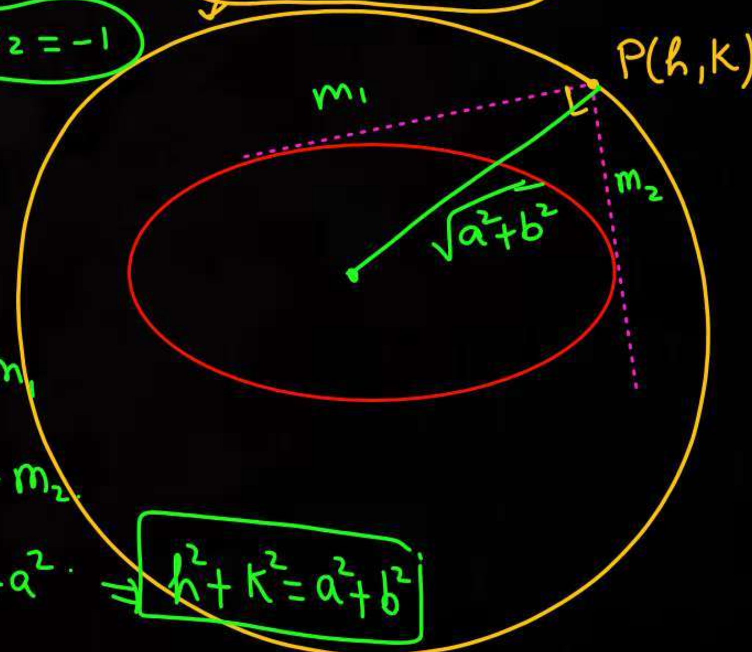
$$k - mh = \pm\sqrt{a^2m^2 + b^2}$$

$$(k - mh)^2 = a^2m^2 + b^2$$

$$m^2(h^2 - a^2) - 2hkm + k^2 - b^2 = 0 \quad \begin{cases} m_1 \\ m_2 \end{cases}$$

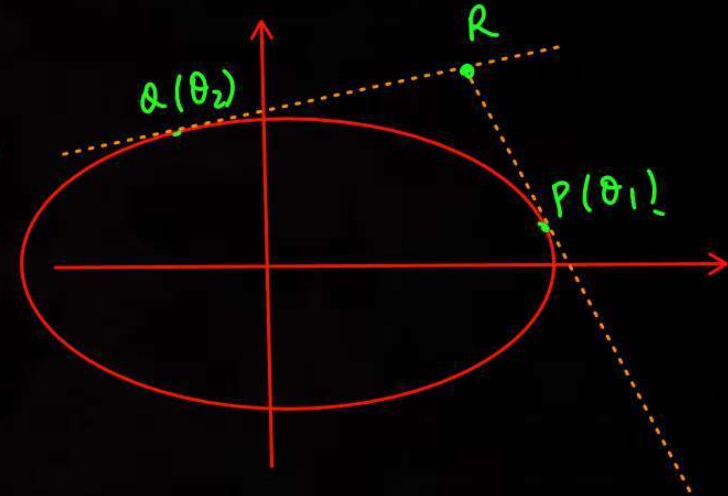
$$m_1 m_2 = -1$$

$$\frac{k^2 - b^2}{h^2 - a^2} = -1 \Rightarrow k^2 - b^2 = -h^2 + a^2 \Rightarrow \boxed{h^2 + k^2 = a^2 + b^2}$$



Point of Intersection of Tangents

$$R\left\{ x = a\,\frac{\cos\left(\frac{\theta_1+\theta_2}{2}\right)}{\cos\left(\frac{\theta_1-\theta_2}{2}\right)}, \quad y = b\,\frac{\sin\left(\frac{\theta_1+\theta_2}{2}\right)}{\cos\left(\frac{\theta_1-\theta_2}{2}\right)} \right\}$$



Equation of Normals

(a) Cartesian Form

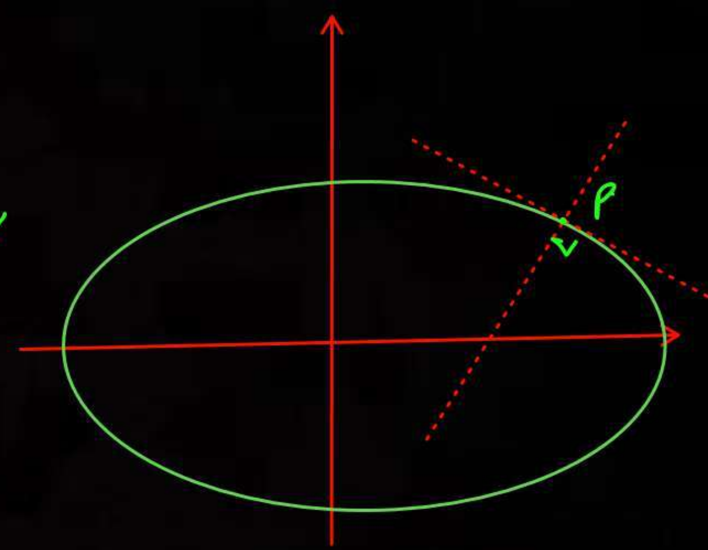
$$\boxed{\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2 - b^2 = a^2e^2}$$

(b) Slope Form

$$\boxed{y = mx \mp \frac{(a^2-b^2)m}{\sqrt{a^2+b^2m^2}}}$$

(c) Parametric Form

$$\boxed{\frac{ax}{\cos\theta} - \frac{by}{\sin\theta} = a^2 - b^2 = a^2e^2}$$



PYQ-18

If the normal at an end of a latus rectum of an ellipse passes through an extremity of the minor axis, then the eccentricity e of the ellipse satisfies:

[JEE Main-2020]

(A) $e^2 + 2e - 1 = 0$

(B) $e^2 + e - 1 = 0$

(C) $e^4 + 2e^2 - 1 = 0$

(D) $e^4 + e^2 - 1 = 0$ ✓

$$\frac{a^2 x}{ae} - \frac{b^2 y}{b^2/a} = a^2 - b^2 = a^2e^2$$

$$\frac{ax}{e} - ay = a^2e^2$$

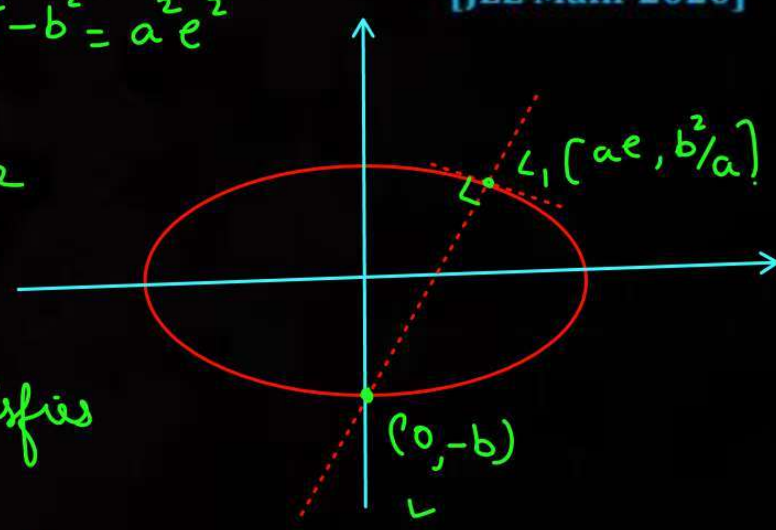
$$\frac{x}{e} - y = ae^2$$

$(0, -b)$ satisfies

$$b = ae^2$$

$$b/a = e^2 \Rightarrow b^2/a^2 = 1 - e^2$$

$$b^2/a^2 = e^4 \Rightarrow \boxed{e^4 = 1 - e^2}$$



PYQ-19 The tangent and normal to the ellipse $\underbrace{3x^2}$ + $5y^2 = 32$ at the point P(2, 2) meets the x-axis at Q and R, respectively. Then, the area (in sq. units) of the ΔPQR is

[2019 Main, 10 April II]

..........[Ans. D]

(A) 16/3

(B) 14/3

(C) 34/15

(D) 68/15

T: $3x(x_1) + 5yy_1 = 32.$

$3x(2) + 5y(2) = 32$

$\boxed{3x + 5y = 16}$

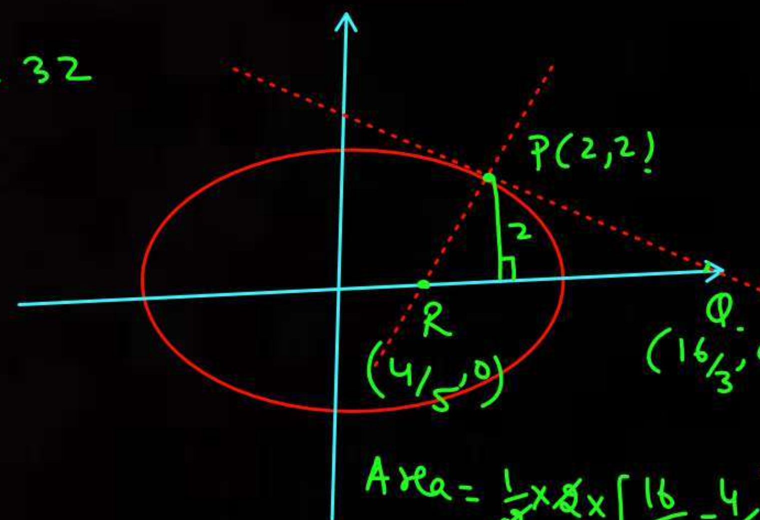
$m_T = -3/5$

$m_N = 5/3.$

N: $y - 2 = 5/3(x-2).$

$3y - 6 = 5x - 10.$

$\boxed{3y = 5x - 4}$



Area $= \frac{1}{\cancel{2}} \times \cancel{2} \times \left[\frac{16}{3} - \frac{4}{5}\right]$

$= \frac{80-12}{15} = \frac{68}{15}$

PYQ-20 the normal to the ellipse $3x^2 + 4y^2 = 12$ at a point P on it is parallel to the line, $2x + y = 4$ and the tangent to the ellipse at P passes through Q(4, 4) then PQ is equal to

$\frac{x^2}{4} + \frac{y^2}{3} = 1$

[2019 Main, 12 April I]

[Ans. A]

(A) $\frac{5\sqrt{5}}{2}$

(B) $\frac{\sqrt{221}}{2}$

(C) $\frac{\sqrt{61}}{2}$

(D) $\frac{\sqrt{157}}{2}$

$m_N = -2$

$\Rightarrow m_T = \frac{1}{2}$

$y = mx \pm \sqrt{a^2m^2 + b^2}$

$y = \frac{1}{2}x \pm \sqrt{4 \times \frac{1}{4} + 3}$

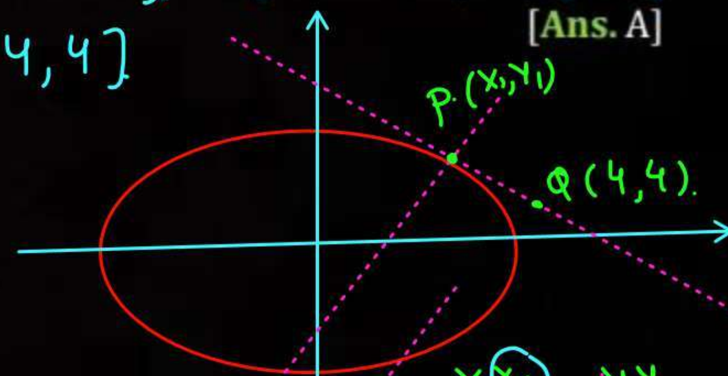
$\boxed{y = x/2 \pm 2}$

$\boxed{y = x/2 + 2}$ as (4,4) satisfies

$y - x/2 = 2 \Rightarrow y/2 - x/4 = 1$

$P[-1, 3/2]$

$Q[4, 4]$



$\frac{x x_1}{4} + \frac{y y_1}{3} = 1$

$-x/4 + y/2 = 1$

$x_1 = -1$ & $\frac{y_1}{3} = \frac{1}{2}$

$y_1 = 3/2$.

$(PQ)^2 = 25 + (4 - 3/2)^2$.

$25 + 25/4$.

$= 25 \times 5/4$.

$PQ = \frac{5\sqrt{5}}{2}$.

PYQ-21

Q. Let x = 4 be a directrix to an ellipse whose centre is at the origin and its eccentricity is $\frac{1}{2}$. If P (1, β), β > 0 is a point on this ellipse, then the equation of the normal to it at P is :-

[JEE Main 2020]

(A) $7x - 4y = 1$

(B) $4x - 2y = 1$

(C) $4x - 3y = 2$

(D) $8x - 2y = 5$

$\boxed{a/e = 4}$ $e = 1/2$.

$\boxed{a = 2}$

$b^2 = a^2(1 - e^2)$

$b^2 = 4\left[1 - \frac{1}{4}\right]$.

$b^2 = 3$.

$\boxed{\frac{x^2}{4} + y^2/3 = 1}$

$\frac{1}{4} + \beta^2/3 = 1$

$\beta^2/3 = 3/4$ $(1, 3/2)$.

$\boxed{\beta = 3/2}$

$\frac{a^2 x}{x_1} - \frac{b^2 y}{y_1} = a^2 - b^2$.

$\frac{4x}{1} - \frac{\cancel{3}y}{\cancel{3}/2} = 1$

$\boxed{4x - 2y = 1}$.

PYQ-22

Q. Let the line $y = mx$ and the ellipse $2x^2 + y^2 = 1$ intersect at a point P in the first quadrant. If the normal to this ellipse at P meets the co-ordinate axes at $\left(-\frac{1}{3\sqrt{2}}, 0\right)$ and $(0, \beta)$, then β is equal to

HW ✓

[JEE Main 2020]

(A) $\frac{2}{\sqrt{3}}$

(B) $\frac{2\sqrt{2}}{3}$

(C) $\frac{2}{3}$

(D) $\frac{\sqrt{2}}{3}$

Chords of Ellipse

Chord of Contact

$$\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$$

Chord with a Given Mid Point T = S_1

$$T=\frac{xx_1}{a^2}+\frac{yy_1}{b^2}-1, S_1=\frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}-1$$

Chord joining P (θ_1) & Q (θ_2)

$$\frac{x}{a}\cos\left(\frac{\theta_1+\theta_2}{2}\right)+\frac{y}{b}\sin\left(\frac{\theta_1+\theta_2}{2}\right)=\cos\left(\frac{\theta_1-\theta_2}{2}\right)$$

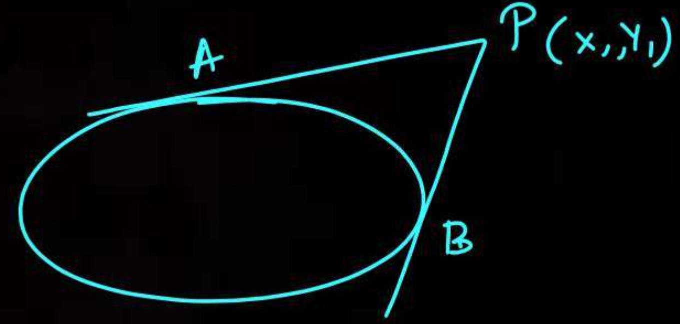
Equation of Pair of Tangents

Equation of pair of tangents of PA and PB is

✓ $SS_1 = T^2$ where

$S = \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} - 1,\qquad S_1 = \dfrac{x_1^2}{a^2} + \dfrac{y_1^2}{b^2} - 1,$

$T = \dfrac{xx_1}{a^2} + \dfrac{yy_1}{b^2} - 1$



✓ $$\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1\right)\left(\frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} - 1\right) = \left(\frac{xx_1}{a^2} + \frac{yy_1}{b^2} - 1\right)^2$$

Properties of Ellipse

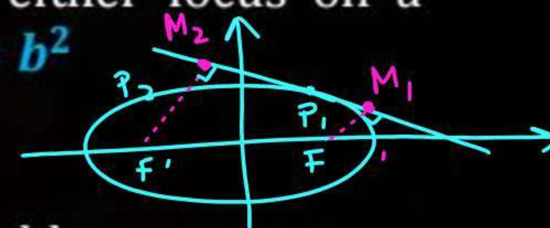
Property -1

The **sum of focal distances** of any point P lying on the ellipse is always equal to the **length of Major Axis**. $PF_1 + PF_2 = 2a.$

Property -2

Product of the length's of the perpendiculars from either focus on a variable tangent to an **Ellipse = (semi minor axis)^2 = b^2**

$\boxed{P_1 P_2 = b^2}$



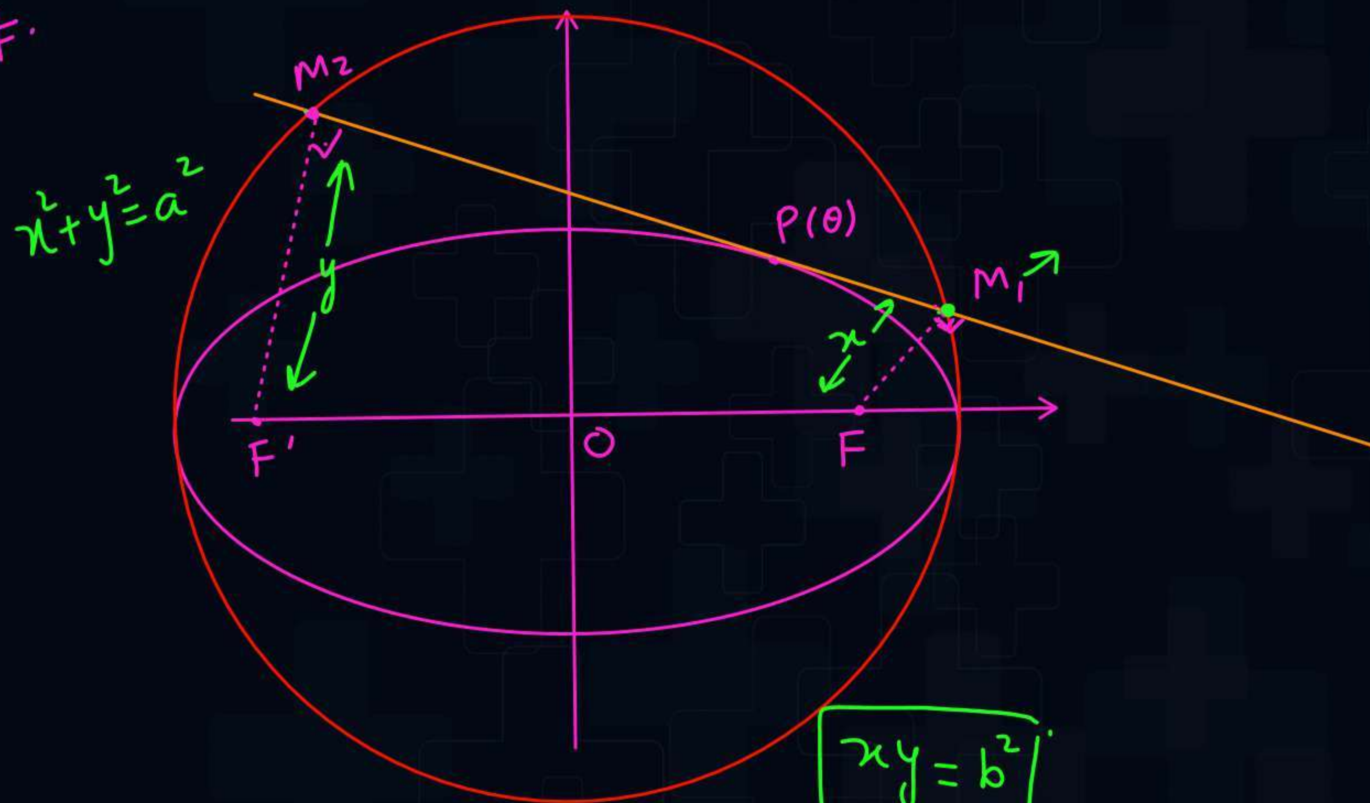
Property -3

Feet of the perpendiculars from either foci on a variable tangent to an **ellipse/hyperbola** lies on its auxiliary circle.

Property -4 (REFLECTION PROPERTY) ✓

If incident ray passes through one focus after getting reflected from the surface of the ellipse passes through other focus.

#.



$$\boxed{xy = b^2}$$

Reflection property :→.

Properties of Ellipse

Property -5

The portion of the tangent to an ellipse between the point of contact & the directrix subtends a right angle at the corresponding focus.

Property -6

The circle on any focal distance as diameter touches the auxiliary circle.

Property -7

Perpendiculars from the centre upon all chords which join the ends of any perpendicular diameters of the ellipse are of constant length.

PYQ-23 Which of the following points lies on the locus of the foot of perpendicula[r] drawn upon any tangent to the ellipse, $\frac{x^2}{4}+\frac{y^2}{2}=1$ from any of its foci?

Locus is aux. circle.

$x^2+y^2=a^2$

$\boxed{x^2+y^2=4}$ ✓

[JEE Main-2020(September)]

(A) $(1,2)$

(B) $(-2,\sqrt{3})$

(C) $(-1,\sqrt{3})$ ✓

(D) $(-1,\sqrt{2})$

TYQ-1

[JEE Main June 2022]

Q. The locus of the mid point of the line segment joining the point (4,3) and the points on the ellipse $x^2 + 2y^2 = 4$ is an ellipse with eccentricity:

(A) $\frac{\sqrt{3}}{2}$

(B) $\frac{1}{2\sqrt{2}}$

(C) $\frac{1}{\sqrt{2}}$ ✓

(D) $\frac{1}{2}$

$\frac{x^2}{4} + \frac{y^2}{2} = 1$

$P(2\cos\theta, \sqrt{2}\sin\theta)$

$Q[4, 3]$.

$2h = 2\cos\theta + 4 \Rightarrow \cos\theta = h-2$.

$2K = \sqrt{2}\sin\theta + 3 \Rightarrow \sin\theta = \frac{2K-3}{\sqrt{2}}$

$\frac{(x-2)^2}{1} + \frac{(2y-3)^2}{2} = 1$

$\frac{(x-2)^2}{1} + \frac{(y-3/2)^2}{1/2} = 1$ $\quad \frac{X^2}{1} + \frac{Y^2}{1/2} = 1$

P ---- (h, k) ---- Q

$a^2 = 1, \ b^2 = 1/2$

$1/2 = 1 - e^2$

$e^2 = 1/2$

$e = \frac{1}{\sqrt{2}}$

TYQ-2

[JEE Main June 2022]

Q. Let the eccentricity of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $a>b$, be $\frac{1}{4}$. If this ellipse passes through the point $\left(-4\sqrt{\frac{2}{5}}, 3\right)$, then a^2+b^2 is equal to :

$e = 1/4.$

$b^2 = a^2\left(1-\frac{1}{16}\right)$

$b^2 = \frac{15a^2}{16}.$

$\frac{a^2}{16} = \frac{b^2}{15}.$

$a^2 = 16.$

$\frac{16\times 2/5}{a^2} + \frac{9}{b^2} = 1$

$\cancel{16}\times\frac{2}{5\cdot\cancel{16}\cdot b^2}\cdot 15 + \frac{9}{b^2} = 1$

$\frac{6}{b^2}+\frac{9}{b^2} = 1$

$\boxed{b^2 = 15}$

(A) 29

(B) 31 ✓

(C) 32

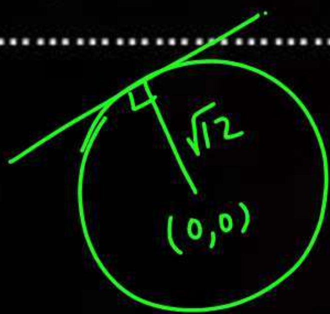
(D) 34

TYQ-3

[JEE Main June 2022]

Q. If m is the slope of a common tangent to the curves $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and $x^2 + y^2 = 12$, then $12m^2$ is equal to:

$y = mx \pm \sqrt{16m^2 + 9}$



$$\left| \frac{\sqrt{16m^2+9}}{\sqrt{1+m^2}} \right| = \sqrt{12}$$

$16m^2 + 9 = 12(1+m^2)$

$4m^2 = 3$

$\boxed{12m^2 = 9}$

(A) 6

(B) 9 ✓

(C) 10

(D) 12

TYQ-4 [JEE Main June 2022]

Q. The line $y = x + 1$ meets the ellipse $\frac{x^2}{4} + \frac{y^2}{2} = 1$ at two points P and Q. If r is the radius of the circle with PQ as diameter then $(3r)^2$ is equal to

$x = y - 1$

$r = \sqrt{4/9 + 1/9 + 5/3 \times 3/3}$ $\sqrt{20/9}$

$r = \frac{\sqrt{20}}{3}$

$3r = \sqrt{20}$

A) 20

B) 12

C) 11

D) 8

#. $\underbrace{(x-x_1)(x-x_2)}_{\text{Quad in } x} + \underbrace{(y-y_1)(y-y_2)}_{\text{Quad in } y} = 0$

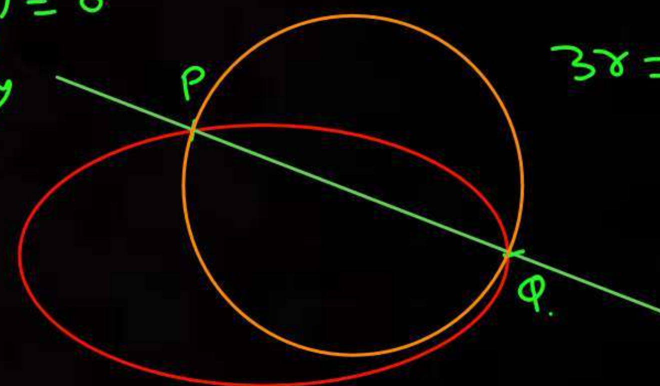
$\frac{x^2}{4} + \frac{(x+1)^2}{2} = 1$

$x^2 + 2(x+1)^2 = 4.$

$\boxed{3x^2 + 4x - 2 = 0} \longrightarrow (1)$

$\frac{(y-1)^2}{4} + \frac{y^2}{2} = 1 \Rightarrow (y-1)^2 + 2y^2 = 4.$

$\Rightarrow \boxed{3y^2 - 2y - 3 = 0} \longrightarrow (2)$



$3x^2 + 3y^2 + 4x - 2y - 5 = 0.$

$\boxed{x^2 + y^2 + 4x/3 - 2y/3 - 5/3 = 0}$

$g = 2/3,\ f = -1/3,\ c = -5/3$

TYQ-5

Q.

[JEE Main July 2022]

If the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meets the line $\frac{x}{7}+\frac{y}{2\sqrt{6}}=1$ on the x-axis and the line $\frac{x}{7}-\frac{y}{2\sqrt{6}}=1$ on the y-axis, then the eccentricity of the ellipse is

$(0, -2\sqrt{6})$ $\qquad$ $(7, 0)$

(A) $\frac{5}{7}$

(B) $\frac{2\sqrt{6}}{7}$

(C) $\frac{3}{7}$

(D) $\frac{2\sqrt{5}}{7}$

$a = 7$

$b = 2\sqrt{6}.$

$b^2 = a^2(1-e^2)$

$24 = 49(1-e^2)$

$\frac{24}{49} = 1-e^2$

$e^2 = 25/49$

$e = 5/7.$

TYQ-6

$\frac{3\sqrt{3}a}{2} = 6\sqrt{3} \Rightarrow \boxed{a = 4}$ ✓

[JEE Main June 2022]

Q. Let the maximum area of the triangle that can be inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{4} = 1, a > 2$, having one of its vertices at one end of the major axis of the ellipse and one of its sides parallel to the y-axis, be $6\sqrt{3}$. Then the eccentricity of the ellipse is:

$4 = 16[1 - e^2] \Rightarrow 1 - e^2 = 1/4$

$e^2 = 3/4$.

....

(A) $\frac{\sqrt{3}}{2}$ ✓

(B) $\frac{1}{2}$

(C) $\frac{1}{\sqrt{2}}$

(D) $\frac{\sqrt{3}}{4}$

$\frac{1}{2} \cdot BC \cdot AM = \frac{1}{2} \cdot 4\sin\theta \cdot (a - a\cos\theta)$

$A = a\,2\sin\theta\,(1 - \cos\theta)$

$dA/d\theta = 0$

$2a[\cos\theta(1 - \cos\theta) + \sin\theta(\sin\theta)] = 0$

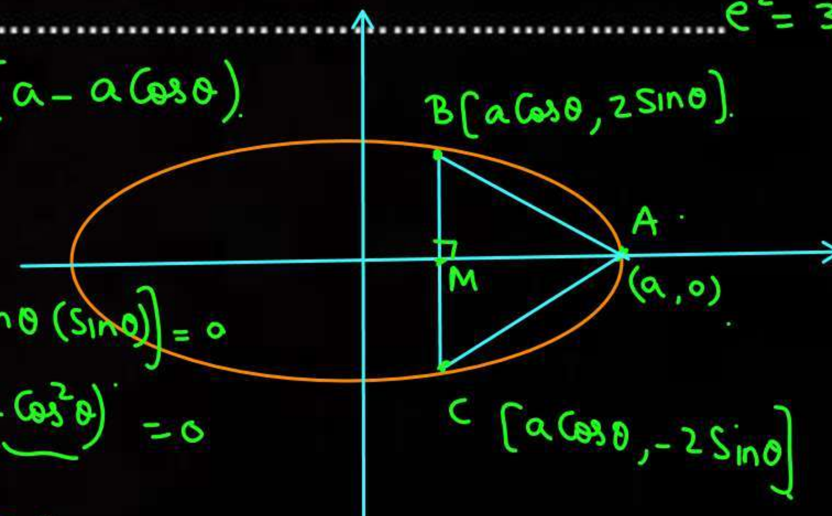
$\cos\theta(1 - \cos\theta) + (1 - \cos^2\theta) = 0$

$\cos\theta + 1 + \cos\theta = 0$

$\boxed{\cos\theta = -\frac{1}{2}} \Rightarrow \theta = 2\pi/3$

$A_{max} = 2a \times \sqrt{3}/2 \cdot 3/2$

$= \frac{3\sqrt{3}a}{2}$



TYQ-7

$x^2+y^2=9/4.$

[JEE Main June 2022]

Q. Let the common tangents to the curves $4(x^2+y^2)=9$ and $y^2=4x$ intersect at the point Q. Let an ellipse, centered at the origin O, has lengths of semi-minor and semi-major axes equal to OQ and 6, respectively. If e and l respectively denote the eccentricity and the length of the latus rectum of this ellipse, then $\frac{l}{e^2}$ is equal to $\underline{\frac{3}{3/4}=4}$.

$ty=x+t^2$

$\boxed{\begin{aligned}\sqrt{3}y&=x+3.\\ -\sqrt{3}y&=x+3\end{aligned}}$

add $\quad x=-3.$

$y=0.$

$Q\,[-3,0].$

$\boxed{OQ=3}$ $\quad$ $\boxed{\begin{aligned}b&=3\\a&=6\end{aligned}}$

$9=36(1-e^2)$

$\frac{1}{4}=1-e^2\Rightarrow e^2=3/4$

$l=2b^2/a$

$=\frac{2\times9}{6}.$

$=3.$

$\left|\frac{t^2}{\sqrt{1+t^2}}\right|=3/2.$

$t^2=3$

$t=\pm\sqrt{3}.$

$t^4=\frac{9}{4}(1+t^2)$

$4t^4=9+9t^2.$

$4t^4-9t^2-9=0.$

$4t^4-12t^2+3t^2-9=0.$

$(\cancel{4t^2+3})(t^2-3)=0.$

TYQ-8

[JEE Main June 2022]

Q. Let PQ be a focal chord of the parabola $y^2 = 4x$ such that it subtends an angle of $\frac{\pi}{2}$ at the point (3, 0). Let the line segment PQ be also a focal chord of the ellipse E: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a^2 > b^2$. If e is the eccentricity of the ellipse E, then the value of $\frac{1}{e^2}$ is equal to:

$\boxed{t_1^2 + t_2^2 = 2} \longrightarrow (1)$

$\boxed{t_1 t_2 = -1}$ (2)

(A) $1 + \sqrt{2}$

(B) $3 + 2\sqrt{2}$

(C) $1 + 2\sqrt{3}$

(D) $4 + 5\sqrt{3}$

$m_{PA} = \frac{2t_1}{t_1^2 - 3}$

$m_{QA} = \frac{2t_2}{t_2^2 - 3}$

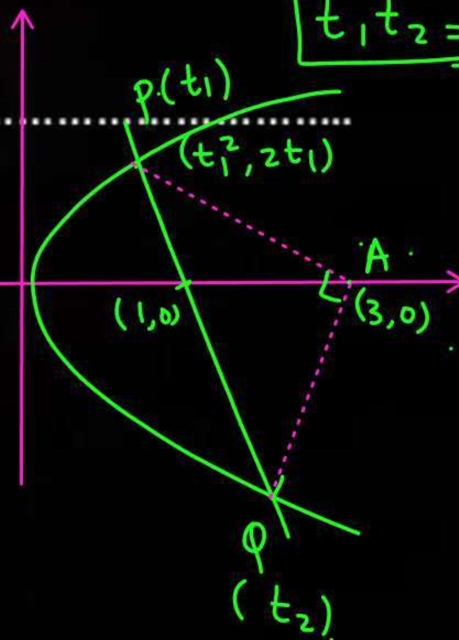
$\frac{4\cancel{t_1 t_2}}{(t_1^2-3)(t_2^2-3)} = -1$

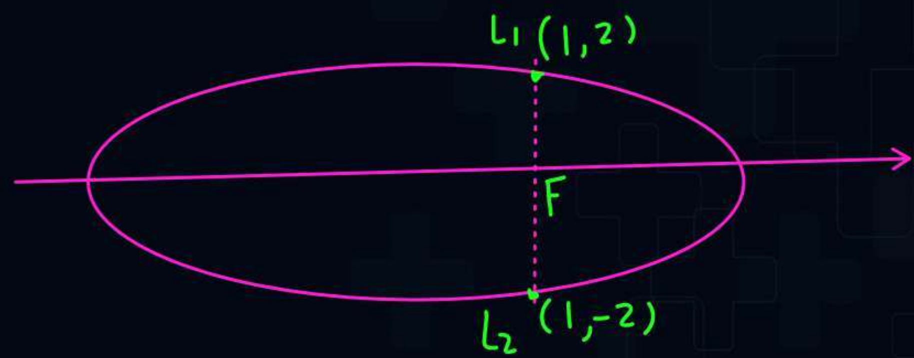
$4 = (t_1^2 - 3)(t_2^2 - 3) \Rightarrow 4 = (t_1 t_2)^2 - 3t_1^2 - 3t_2^2 + 9$

$3 + 3t_1^2 + 3t_2^2 = 9$

$\boxed{\begin{array}{l} t_1 = 1 \\ t_2 = -1 \end{array}}$

$\left.\begin{array}{l} P[1, 2] \\ Q[1, -2] \end{array}\right\}$ L.R.





$$a^2-2a-1=0.$$

$$a=\frac{2\pm\sqrt{4+4}}{2}$$

$$a=\frac{2\pm 2\sqrt{2}}{2}$$

$$a=1\pm\sqrt{2}.$$

$\ominus \rightarrow$ reject

$a>0.$

$$a=1+\sqrt{2}.$$

$$ae=1 \Rightarrow \frac{1}{e}=a$$

$$\frac{b^2}{a}=2$$

$$b^2=2a$$

$$a^2-a^2e^2=2a.$$

$$a^2-1=2a.$$

$$\frac{1}{e^2}=a^2$$

$$=(\sqrt{2}+1)^2.$$

$$=(3+2\sqrt{2})$$

TYQ-9

[JEE Main July 2022]

Q. If the length of the latus rectum of the ellipse $x^2 + 4y^2 + 2x + 8y - \lambda = 0$ is 4 , and l is the length of its major axis, then $\lambda + l$ is equal to

HW

TYQ-10

Consider the ellipse $\frac{x^2}{4}+\frac{y^2}{3}=1$. [JEE Adv. 2022]

Let $H(\alpha,0), 0<\alpha<2$, be a point. A straight line drawn through H parallel to the y-axis crosses the ellipse and its auxiliary circle at points E and F respectively, in the first quadrant. The tangent to the ellipse at the point E intersects the positive x-axis at a point G Suppose the straight line joining F and the origin makes an angle ϕ with the positive x-axis.

$A = \frac{2\sin^3\phi}{\cos\phi}$.

List I		List II
(I)	If $\phi=\frac{\pi}{4}$, then the area of the triangle FGH is $\rightarrow 2\times\frac{1}{2}=1$	(P) $\frac{(\sqrt{3}-1)^4}{8}$
(II)	If $\phi=\frac{\pi}{3}$, then the area of the triangle FGH is $A=\frac{2\times\left(\sqrt{3}/2\right)^3}{1/2}$. $=3\sqrt{3}/2$.	(Q) 1
(III)	If $\phi=\frac{\pi}{6}$, then the area of the triangle FGH is	(R) $\frac{3}{4}$
(IV)	If $\phi=\frac{\pi}{12}$, then the area of the triangle FGH is	(S) $\frac{1}{2\sqrt{3}}$.
		(T) $\frac{3\sqrt{3}}{2}$

The correct option is:

(a) (I) → (R); (II) → (S); (III) → (Q); (IV) → (P)
(b) (I) → (R); (II) → (T); (III) → (S); (IV) → (P)
(c) (I) → (Q); (II) → (T); (III) → (S) ; (IV → (P)
(d) (I) → (Q); (II) → (S); (III) → (Q); (IV → (P)

TYQ-10

Consider the ellipse $\frac{x^2}{4}+\frac{y^2}{3}=1$. [JEE Adv. 2022]

Let $H(\alpha,0), 0<\alpha<2$, be a point. A straight line drawn through H parallel to the y-axis crosses the ellipse and its auxiliary circle at points E and F respectively, in the first quadrant. The tangent to the ellipse at the point E intersects the positive x-axis at a point G Suppose the straight line joining F and the origin makes an angle ϕ with the positive x-axis.

$E\,[a\cos\phi,\ b\sin\phi]$

$$\frac{x\cos\phi}{a}+\frac{y\sin\phi}{b}=1$$

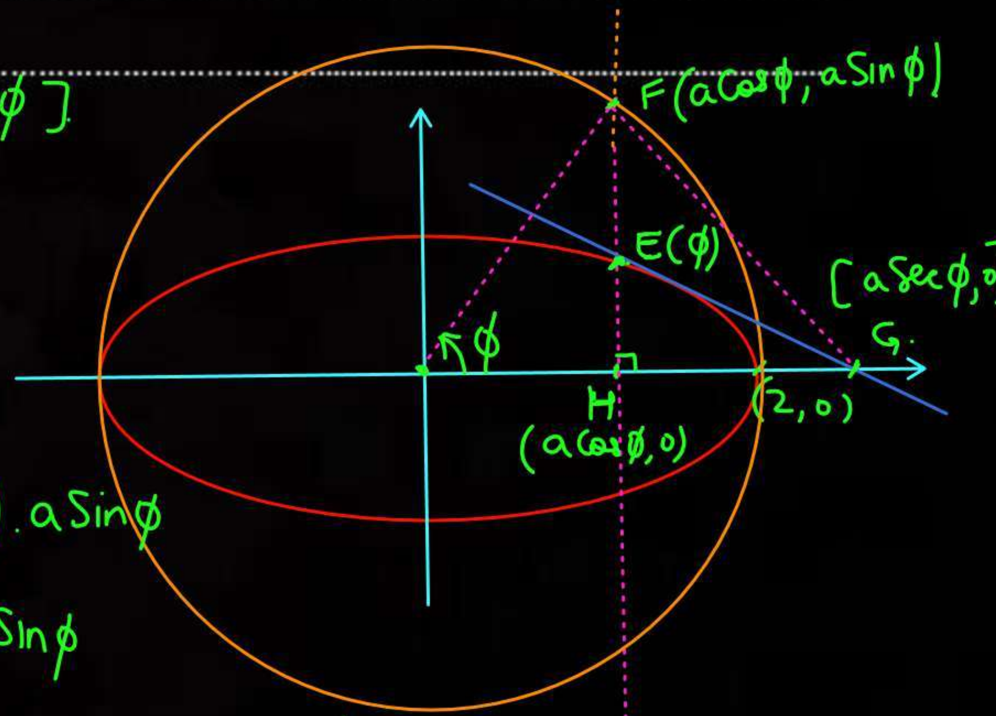
$y=0$ for G.

$x=a\sec\phi$

Area of $\Delta FGH = \frac{1}{2}\cdot(a\sec\phi-a\cos\phi)\cdot a\sin\phi$

$$\frac{a^2}{2}\left[\frac{1}{\cos\phi}-\cos\phi\right]\sin\phi$$

$$\boxed{A=\frac{a^2}{2}\frac{\sin^3\phi}{\cos\phi}}=\frac{2\sin^3\phi}{\cos\phi}.$$



$HG=a\sec\phi-a\cos\phi$

$HF=a\sin\phi.$

PYQs- 2021

(HW)

PYQ-1

Q. The line $12x \cos\theta + 5y \sin\theta = 60$ is tangent to which of the following curves?

[JEE Main (August) 2021]

(A) $x^2 + y^2 = 169$

(B) $144x^2 + 25y^2 = 3600$

(C) $25x^2 + 12y^2 = 3600$

(D) $x^2 + y^2 = 60$

PYQ-2

Q. If the minimum area of the triangle formed by a tangent to the ellipse $\frac{x^2}{b^2}+\frac{y^2}{4a^2}=1$ and the co-ordinate axis is kab, then k is equal to .

....

[JEE Main (August) 2021]

[Ans. 2]

PYQ-3

Q. On the ellipse $\frac{x^2}{8}+\frac{y^2}{4}=1$ let P be a point in the second quadrant such that the tangent at P to the ellipse is perpendicular to the line $x + 2y = 0$. Let S and S′ be the foci of the ellipse and e be its eccentricity. If A is the area of the triangle SPS′ then, the value of $(5 - e^2)$. A is :

[JEE Main (August) 2021]

[Ans. A]

(A) 6

(B) 12

(C) 14

(D) 24

PYQ-4

Q. A ray of light through (2, 1) is reflected at a point P on the y-axis and then passes through the point (5, 3). If this reflected ray is the directrix of an ellipse with eccentricity 1/3 and the distance of the nearer focus from this directrix is $8/\sqrt{53}$,, then the equation of the other directrix can be:

[JEE Main (July) 2021]

(A) $11x + 7y + 8 = 0$ or $11x + 7y - 15 = 0$

(B) $11x - 7y - 8 = 0$ or $11x + 7y + 15 = 0$

(C) $2x - 7y + 29 = 0$ or $2x - 7y - 7 = 0$

(D) $2x - 7y - 39 = 0$ or $2x - 7y - 7 = 0$

PYQ-5

Q.

Let $E_1 : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$. Let E_2 be another ellipse such that it touches the end points of major axis of E_1 and the foci of E_2 are the end points of minor axis of E_1. If E_1 and E_2 have same eccentricities, then its value is :

[JEE Main (July) 2021]

[Ans. A]

(A) $\frac{-1+\sqrt{5}}{2}$

(B) $\frac{-1+\sqrt{8}}{2}$

(C) $\frac{-1+\sqrt{3}}{2}$

(D) $\frac{-1+\sqrt{6}}{2}$

PYQ-6

Q. Let an ellipse $E: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a^2 > b^2$, passes through $\left(\sqrt{\frac{3}{2}}, 1\right)$ and has eccentricity $\frac{1}{\sqrt{3}}$. If a circle, centered at focus $F(\alpha, 0)$, $\alpha > 0$, of E and radius $\frac{2}{\sqrt{3}}$, intersects E at two points P and Q, then PQ^2 is equal to :

[JEE Main (July) 2021]

[Ans. C]

(A) $\frac{8}{3}$

(B) $\frac{4}{3}$

(C) $\frac{16}{3}$

(D) 3

PYQ-7

Q. If a tangent to the ellipse $x^2 + 4y^2 = 4$ meets the tangents at the extremities of its major axis at B and C, then the circle with BC as diameter passes through the point:

[JEE Main (July) 2021]

[Ans. 1]

(A) $(\sqrt{3}, 0)$

(B) $(\sqrt{2}, 0)$

(C) $(1, 1)$

(D) $(-1, 1)$

PYQ-8

Q.

If the curve $x^2 + 2y^2 = 2$ intersects the line $x + y = 1$ at two points P and Q, then the angle subtended by the line segment PQ at the origin is:

[JEE Main (Feb.) 2021]

[Ans. 1]

(A) $\frac{\pi}{2}+\tan^{-1}\left(\frac{1}{4}\right)$

(B) $\frac{\pi}{2}-\tan^{-1}\left(\frac{1}{4}\right)$

(C) $\frac{\pi}{2}+\tan^{-1}\left(\frac{1}{3}\right)$

(D) $\frac{\pi}{2}-\tan^{-1}\left(\frac{1}{3}\right)$

PYQ-9

Q.

Let E be an ellipse whose axes are parallel to the co-ordinates axes, having its center at $(3, -4)$, one focus at $(4, -4)$ and one vertex at $(5, -4)$.
If $mx - y = 4$, $m > 0$ is a tangent to the ellipse E, then the value of $5m^2$ is equal to

....

[JEE Main (July) 2021]

[Ans. 3]