Coordinate Geometry

(⊥r distance from x-axis with sign.)

* Basics :-

(⊥r distance from y axis with sign)

→ The address of a point in a plane is called as its coordinates represented as

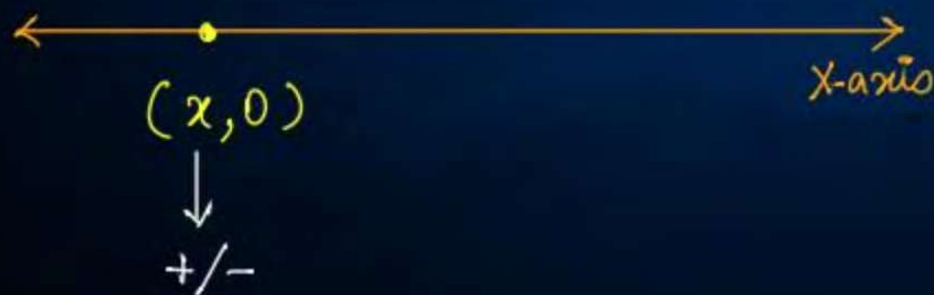
$$\boxed{(x, y)}$$

x-coordinate (abscissa)

y-coordinate (ordinate)

Coordinate Geometry

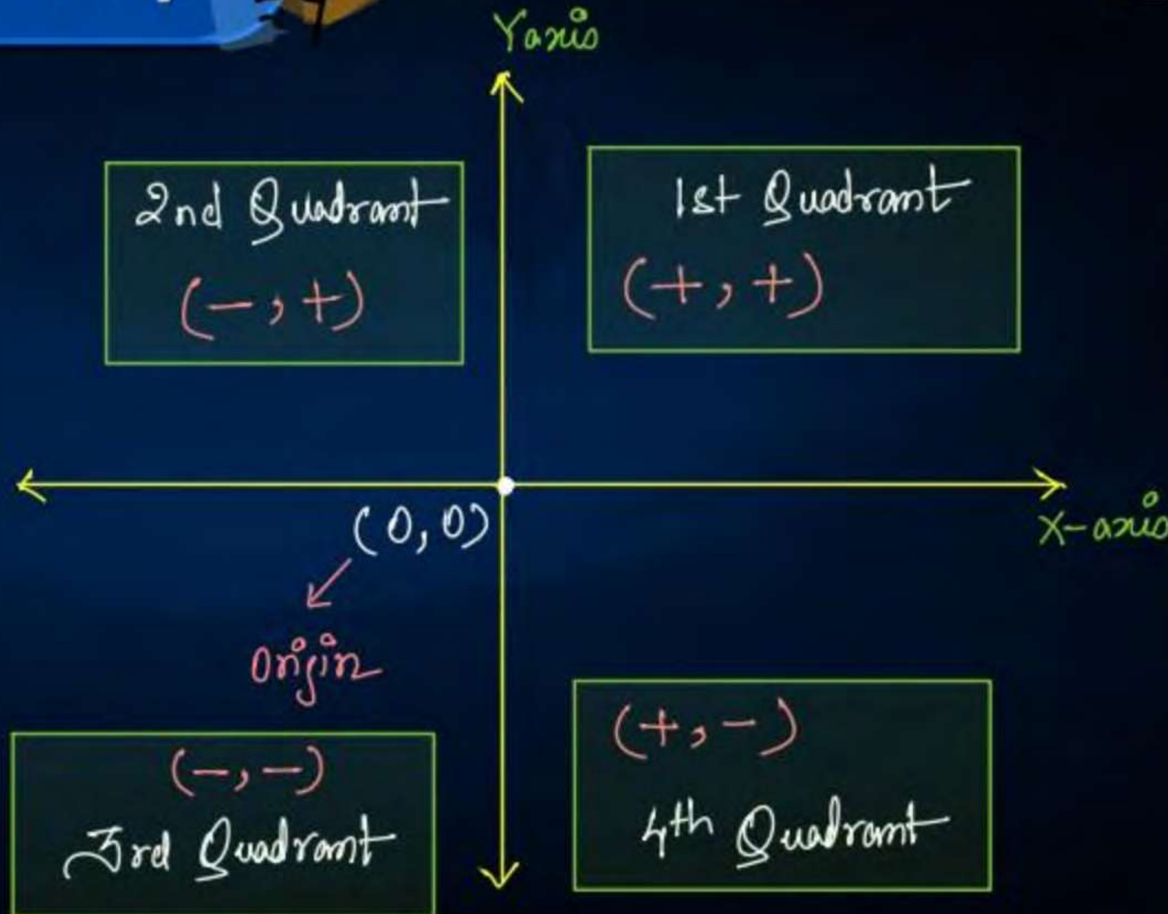
* Basics :-



Coordinate Geometry

* Basics :-

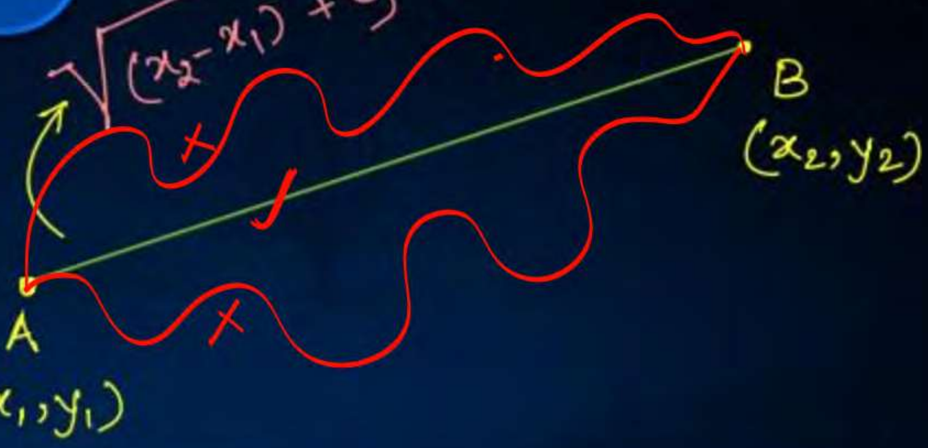
* Quadrants :-



Coordinate Geometry

* **Distance formula** :-

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \text{ units}$$



$$\therefore AB = \sqrt{(\text{difference of abscissae})^2 + (\text{difference of ordinate})^2} \text{ units}$$

To do this, fix one end & call it Master.

Coordinate Geometry

* Special case of Distance formula :-

$$\boxed{\sqrt{x^2+y^2} \text{ units}}$$

QUESTION

Find the distance between the following pairs of points:
$\underbrace{(-5, 7), (1, 3)}$

$\underline{\underline{Sol^n}}$ $\sqrt{(-5-1)^2 + (7-3)^2}$ units

$= \sqrt{36+16}$ units

$= \sqrt{52}$ units

$= 2\sqrt{13}$ units

QUESTION

Find the distance between the following pairs of points:
$(a, b), (-a, -b)$

Sol^n Reqd. distance $= \sqrt{(2a)^2 + (2b)^2}$ units

$= \sqrt{4a^2 + 4b^2}$ units

$= \boxed{2\sqrt{a^2 + b^2} \text{ units}}$

QUESTION

15, 36, 39

Find the distance between the points (0, 0) and (36, 15).

Sol^n Reqd. distance $= \sqrt{(36)^2 + (15)^2}$ unit

$= \sqrt{1296 + 225}$ unit

$= \sqrt{1521}$ unit

$= \boxed{39 \text{ units}}$

QUESTION

Determine if the points (1, 5), (2, 3) and (−2,−11) are collinear.

A: (1, 5), B: (2, 3), C: (−2,−11)

$\boxed{\text{Sol}^n}$

$$AB = \sqrt{1+4} = \sqrt{5} \text{ unit}$$

$$BC = \sqrt{16+196} = \sqrt{212} \text{ unit} = 2\sqrt{53} \text{ unit}$$

$$AC = \sqrt{9+256} = \sqrt{265} \text{ unit}$$

2	212
2	106
	53

5	265
	53

Here, we observe that

$$\left.\begin{array}{l} AB+BC > AC \\ BC+AC > AB \\ AB+AC > BC \end{array}\right\} \therefore \text{The 3 pts form a triangle.}$$

QUESTION

Do the points $\overset{P}{(3,2)}$, $\overset{Q}{(-2,-3)}$ and $\overset{R}{(2, 3)}$ form a triangle? If so, name the type of triangle formed.

Sol^n

$PQ = \sqrt{25+25} = 5\sqrt{2}$ units

$QR = \sqrt{16+36} = 2\sqrt{13}$ units

$PR = \sqrt{1+1} = \sqrt{2}$ units

Scalene $\triangle$.

$50 = PQ^2$

$\Rightarrow 52 = QR^2$

$2 = PR^2$

right angled $\triangle$!

$PQ + QR > PR$

$QR + PR > PQ$

$PQ + PR > QR$

Yes, they form a $\triangle$.

QUESTION

Check whether (5, −2), (6, 4) and (7, −2) are the vertices of an isosceles triangle.

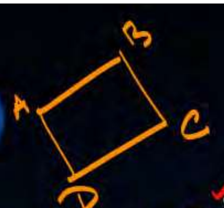
A B C

$\boxed{\text{Sol}^n}$

$$\left.\begin{cases} AB = \sqrt{1+36} = \sqrt{37} \text{ units} \\ BC = \sqrt{1+36} = \sqrt{37} \text{ units} \end{cases}\right\} \longrightarrow$$ Yes, its an isosceles $\triangle$.

$$AC = \sqrt{4+0} = 2 \text{ units}$$

QUESTION



equal ← Side

equal Dia

Show that the points (1, 7), (4, 2), (−1, −1) and (−4, 4) are the vertices of a square.

Sol^n

$AB = \sqrt{9+25} = \sqrt{34}$ unit

$BC = \sqrt{25+9} = \sqrt{34}$ unit

$CD = \sqrt{9+25} = \sqrt{34}$ unit

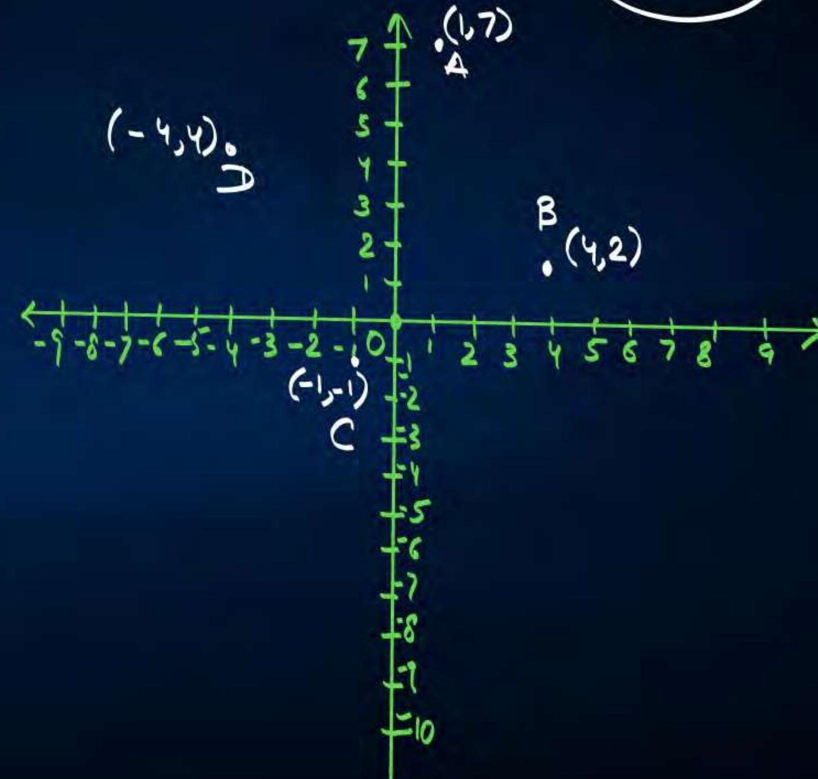
$DA = \sqrt{25+9} = \sqrt{34}$ unit

equal

$AC = \sqrt{4+64} = \sqrt{68}$ unit

$BD = \sqrt{64+4} = \sqrt{68}$ unit

equal



QUESTION

Champa → □

Chameli → $\boxtimes$

In a classroom, 4 friends are seated at the points A, B, C and D as shown in Fig. Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli, "Don't you think ABCD is a square?" Chameli disagrees. Using distance formula, find which of them is correct.

$\boxed{\text{Sol}^n}$

$AB = \sqrt{9+9} = \sqrt{18}$ units

$BC = \sqrt{9+9} = \sqrt{18}$ units

$CD = \sqrt{9+9} = \sqrt{18}$ units

$DA = \sqrt{9+9} = \sqrt{18}$ units

sides equal

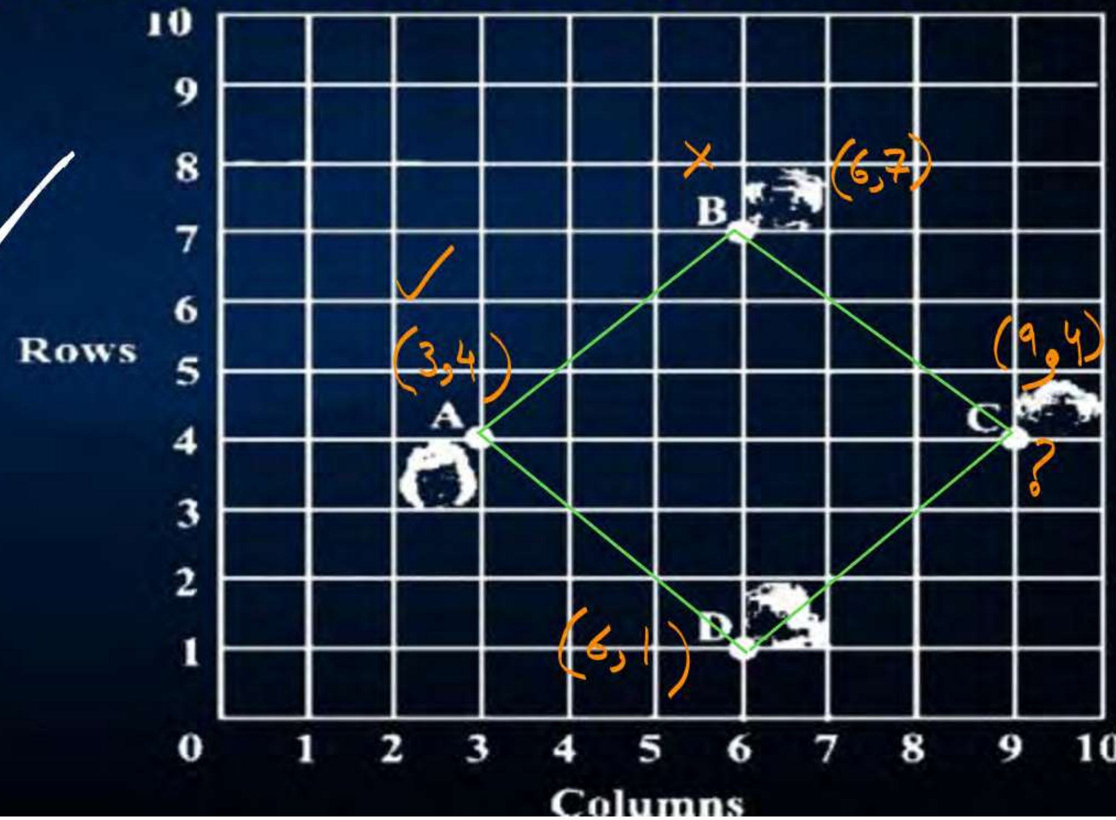
$AC = \sqrt{36+0} = 6$ units

$BD = \sqrt{0+36} = 6$ units

dia equal

→ sq. ✓

* Champa was right.



QUESTION

Name the type of quadrilateral formed, if any, by the following points, and give reasons for your answer:

(−1,−2), (1, 0), (1,2), (−3,0)

A B C D

S×

Soln

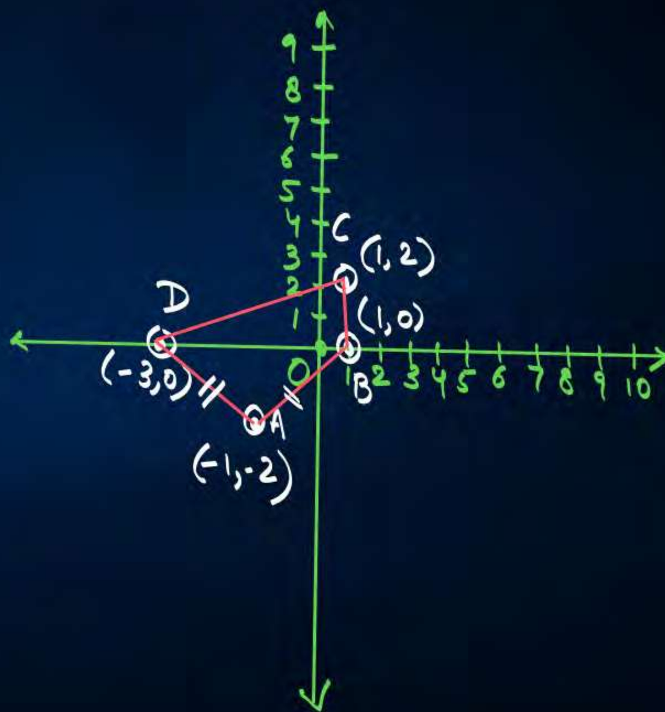
$AB = \sqrt{4+4} = \sqrt{8}$ unit $\longrightarrow$ $2\sqrt{2}$ unit

$BC = \sqrt{0+4} = 2$ unit

$CD = \sqrt{16+4} = \sqrt{20}$ unit $\longrightarrow$ $2\sqrt{5}$ unit

$DA = \sqrt{4+4} = \sqrt{8}$ unit $\longrightarrow$ $2\sqrt{2}$ unit

* Its a quadrilateral.



QUESTION

Name the type of quadrilateral formed, if any, by the following points, and give reasons for your answer:

(4,5), (7,6), (4,3), (1,2)

A B C D

//gm, ~~Rect~~

$\boxed{\text{Sol}^n}$

$AB = \sqrt{9+1} = \sqrt{10}$ units

$BC = \sqrt{9+9} = \sqrt{18}$ units

$CD = \sqrt{9+1} = \sqrt{10}$ units

$DA = \sqrt{9+9} = \sqrt{18}$ units

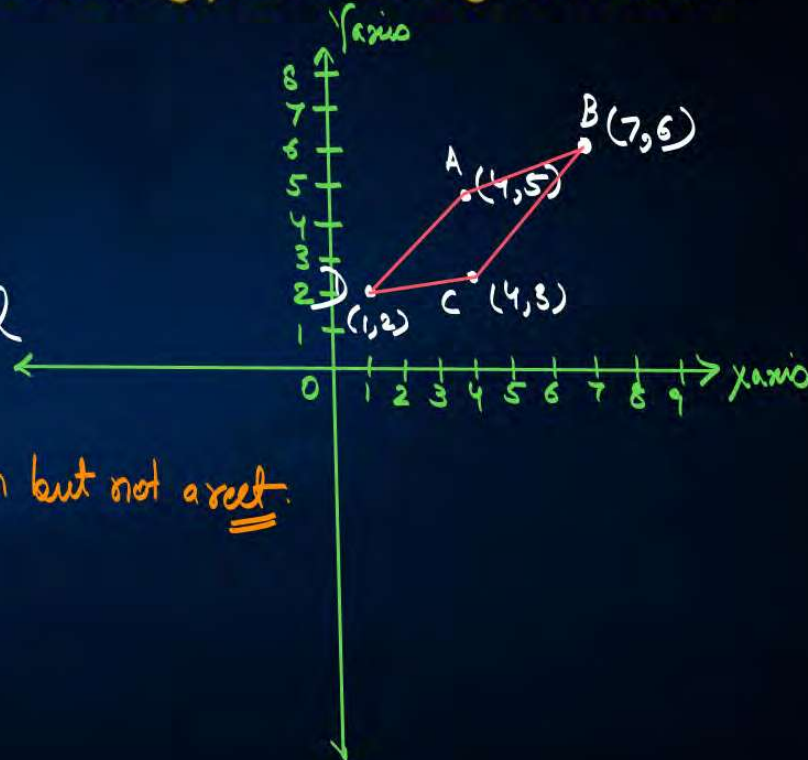
opp side equal

→ Its a //gm but not a rect.

$AC = \sqrt{0+4} = 2$ units

$BD = \sqrt{36+16} = \sqrt{52}$ units

≠ diag



QUESTION

$$\begin{array}{r} 81 \\ -\,25 \\ \hline 56 \end{array} \qquad (x, 0)$$

Find the point on the x-axis which is equidistant from (2, -5) and (-2, 9).

$\boxed{\text{Sol}^n}$

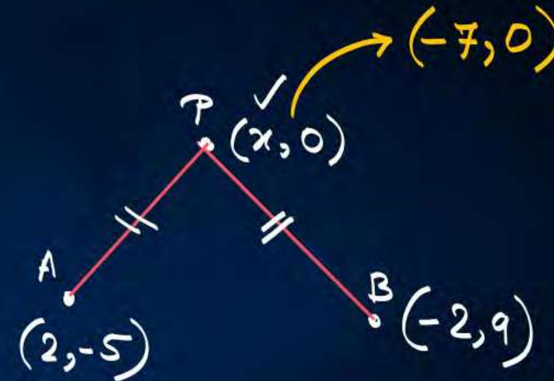
$$\Rightarrow PA^2 = PB^2 \text{ (Given)}$$

$$\Rightarrow (x-2)^2 + 25 = (x+2)^2 + 81$$

$$\Rightarrow (x+2)^2 - (x-2)^2 = 25 - 81$$

$$\Rightarrow \cancel{4} \times x \times \cancel{2} = -\cancel{56}\ \cancel{28}\ 7$$

$$\Rightarrow x = -7$$



QUESTION

Find the values of y for which the distance between the points P(2,−3) and Q(10, y) is 10 units.

$(\ )^2 = \square$

$(\ ) = \pm\sqrt{\square}$

$(3+y)^2 = 36$

$3+y = \pm\sqrt{36}$

$3+y = \pm 6$

$3+y = 6$ | $3+y = -6$

$y = 3$ | $y = -9$

Sol^n

$PQ = 10$ unit

$PQ^2 = 100$

$\Rightarrow 64 + (-3-y)^2 = 100$

$\Rightarrow (-3-y)^2 = 36$

$\Rightarrow \boxed{(3+y)^2 = 36}$

$\Rightarrow 9 + y^2 + 6y - 36 = 0$

$\Rightarrow \boxed{y^2 + 6y - 27 = 0}$

$y^2 + 9y - 3y - 27 = 0$

$y(y+9) - 3(y+9) = 0$

$(y+9)(y-3) = 0$

$y = -9$ | $y = 3$

P (2, −3) —10 unit— Q (10, y): 3, −9

$\sqrt{64+36} = 10$

$\sqrt{64+36} = 10$

QUESTION

$\sqrt{81+81}$ $\sqrt{162}$ $\sqrt{1+81}$

If Q(0, 1) is equidistant from P(5, -3) and R(x, 6), find the values of x. Also find the distances QR and PR.

$\boxed{\text{Sol}^n}$

$QP^2 = QR^2$

$\Rightarrow \cancel{25} + 16 = x^2 + \cancel{25}$

$\Rightarrow x^2 = 16$

$\Rightarrow x = \pm\sqrt{16}$

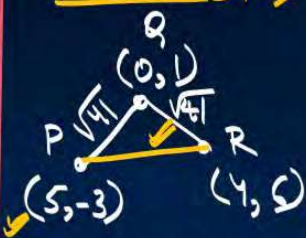
$\Rightarrow x$ → 4, -4

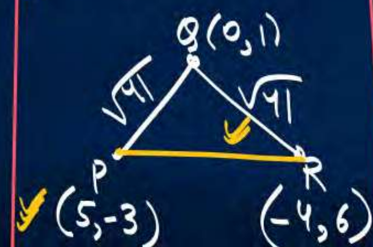
For $x = 4$: $QR = \sqrt{41}$ unit, $PR = \sqrt{82}$ unit

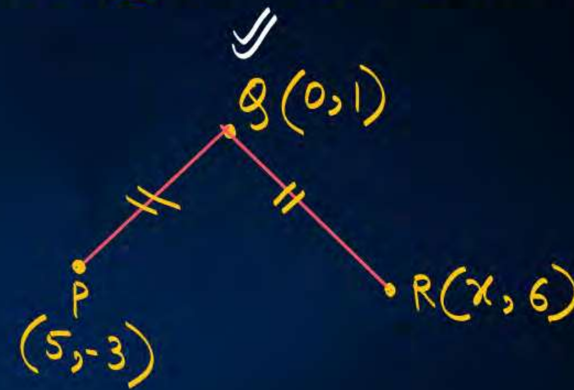
For $x = -4$: $QR = \sqrt{41}$ unit, $PR = \sqrt{162}$ unit

$\sqrt{16+25}$

Check (Ans)

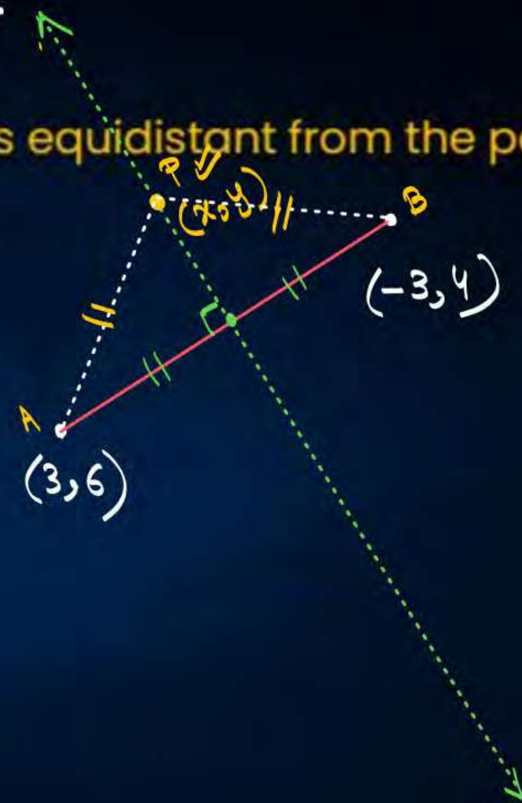






QUESTION

Find a relation between x and y such that the point (x, y) is equidistant from the point (3,6) and (−3,4).



$\boxed{\text{Sol}^n}$ $PA^2 = PB^2$

$$\Rightarrow (x-3)^2 + (y-6)^2 = (x+3)^2 + (y-4)^2$$

$$\Rightarrow (x+3)^2 - (x-3)^2 = (y-6)^2 - (y-4)^2$$

$$\Rightarrow \cancel{4} \times x \times 3 = \cancel{2}(y-5) \times (-\cancel{2})$$

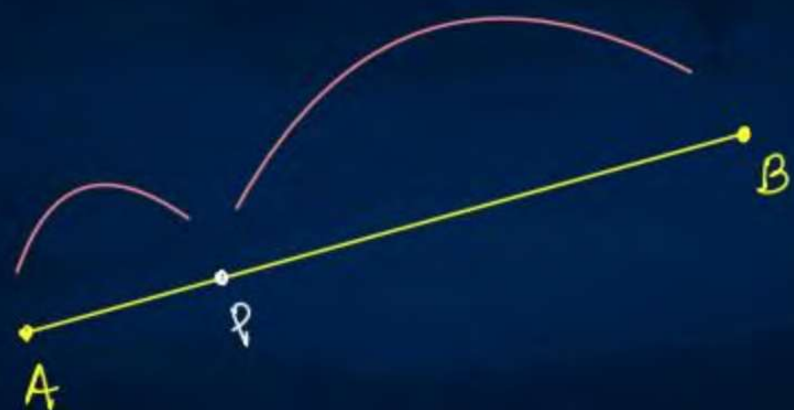
$$\Rightarrow 3x = -(y-5)$$

$$\Rightarrow 3x = -y+5$$

$$\Rightarrow \boxed{3x+y-5=0}$$

Coordinate Geometry

* Section Formula :- (Internal Division)

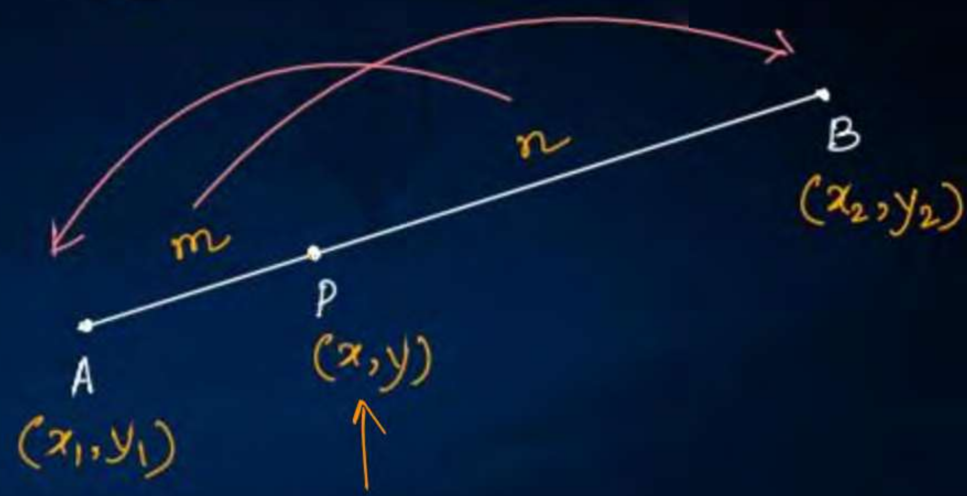


Coordinate Geometry

$$x = \frac{mx_2 + nx_1}{m+n}$$

$$y = \frac{my_2 + ny_1}{m+n}$$

parts sum

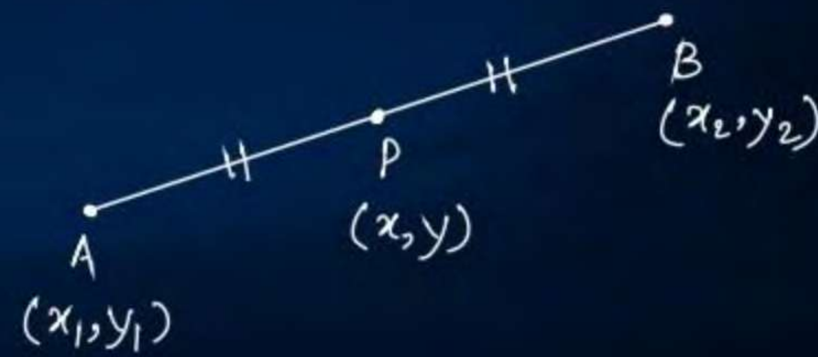


Coordinate Geometry

* **Special case of Section Formula (Midpt Formula) :-**

→ If 'P' is the midpoint of AB, then

$$\boxed{\begin{aligned} x &= \frac{x_1 + x_2}{2} \\ y &= \frac{y_1 + y_2}{2} \end{aligned}}$$



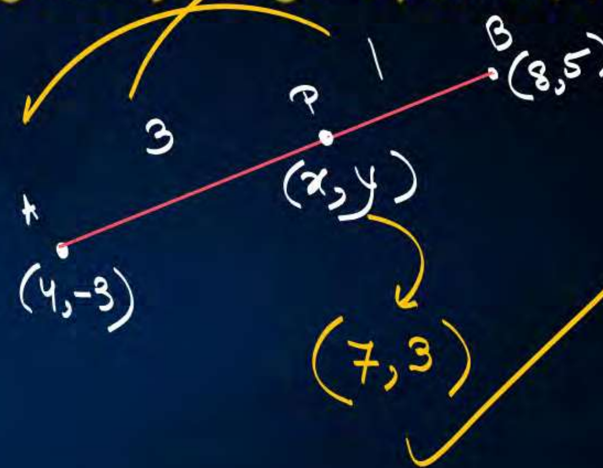
QUESTION

Find the coordinates of the point which divides the line segment joining the points (4,–3) and (8,5) in the ratio 3 : 1 internally.

$\boxed{Sol^n}$

$$x = \frac{24+4}{4} = \frac{\cancel{28}}{\cancel{4}} = 7$$

$$y = \frac{15+(-3)}{4} = \frac{\cancel{12}}{\cancel{4}} = 3$$



QUESTION

3 : 1

5 : 1

k : 1

In what ratio does the point (−4, 6) divide the line segment joining the points A(−6, 10) and B(3,−8)?

Soln

$$-4 = \frac{3k + (-6)}{k+1}$$

$$-4 = \frac{3k-6}{k+1}$$

$$\Rightarrow 3k - 6 = -4k - 4$$

$$\Rightarrow 7k = 2$$

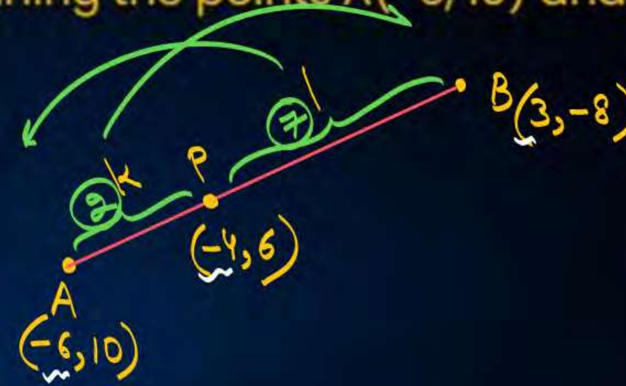
$$\Rightarrow \boxed{k = \frac{2}{7}}$$

$$6 = \frac{-8k+10}{k+1}$$

$$k : 1$$

$$\frac{2}{7} : 1$$

$$k \leftarrow \boxed{2 : 7} \rightarrow 1$$



QUESTION

Find the coordinates of the points of trisection (i.e., points dividing in three equal parts) of the line segment joining the points A(2,-2) and B(-7, 4).

$\boxed{Sol^n}$

$$x_1 = \frac{-7+4}{3} = \frac{\cancel{-3}^{1}}{\cancel{3}} = -1$$

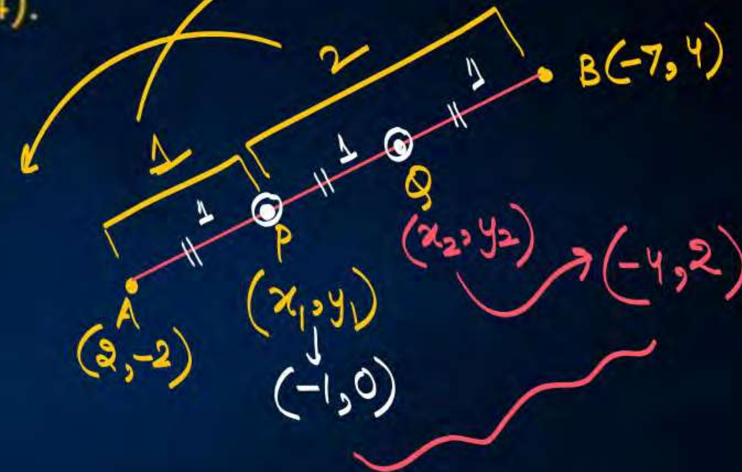
$$y_1 = \frac{4+(-4)}{3} = 0$$

$$x_2 = \frac{(-7)+(-1)}{2}$$

$$x_2 = -4$$

$$y_2 = \frac{0+4}{2} = 2$$

$$y_2 = 2$$



QUESTION

Find the ratio in which the y-axis divides the line segment joining the points (5,-6) and (-1,-4). Also find the point of intersection.

ratio / point
(1) (2)

$\boxed{\text{Sol}^n}$

$$0 = \frac{-k+5}{k+1}$$

$$\Rightarrow -k+5 = 0$$

$$\Rightarrow \boxed{k=5}$$

$$\boxed{k : 1}$$

$$\boxed{5 : 1}$$

k 1

$$y = \frac{-20+(-6)}{6}$$

$$= \frac{-20-6}{6}$$

$$= \frac{-\cancel{26}\,13}{\cancel{6}\,3}$$

$$= -\frac{13}{3}$$

Check

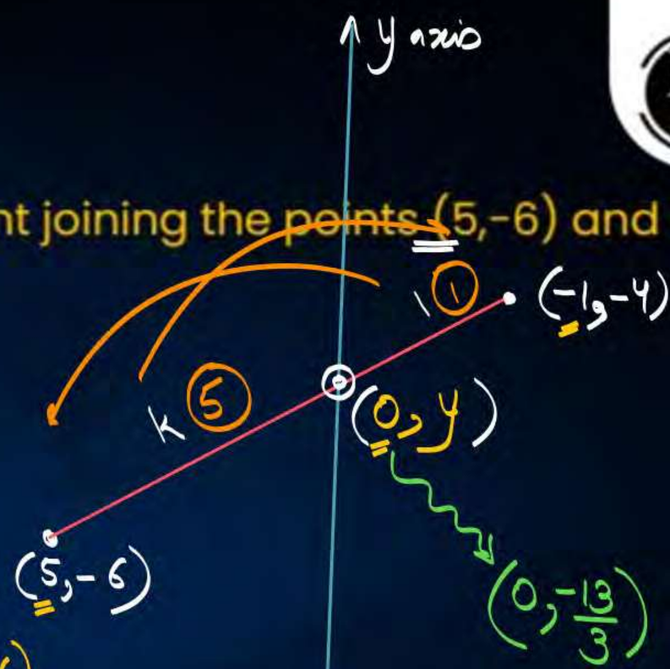
$$-\frac{13}{3} = \frac{-4k+(-6)}{k+1}$$

$$-\frac{13}{3} = \frac{-4k-6}{k+1}$$

$$\frac{13}{3} = \frac{4k+6}{k+1}$$

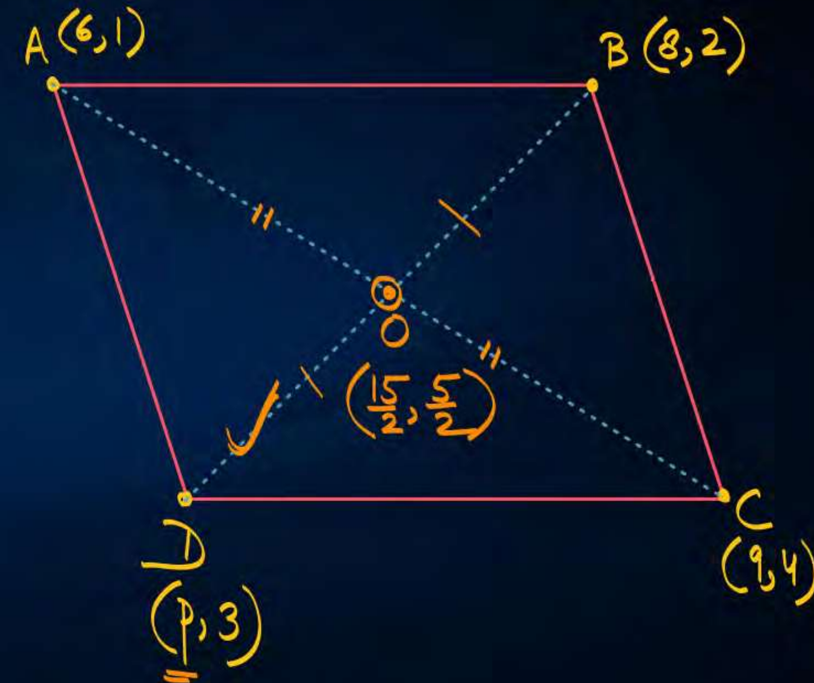
$$13k+13 = 12k+18$$

$$k=5$$



QUESTION

If the points A(6, 1), B(8, 2), C(9, 4) and D(p, 3) are the vertices of a parallelogram, taken in order, find the value of p.



Soln

$$\frac{15}{\cancel{2}} = \frac{p+8}{\cancel{2}}$$

$$p + 8 = 15$$

$$p = 7$$

QUESTION

If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y.

$\boxed{\text{Sol}^n}$

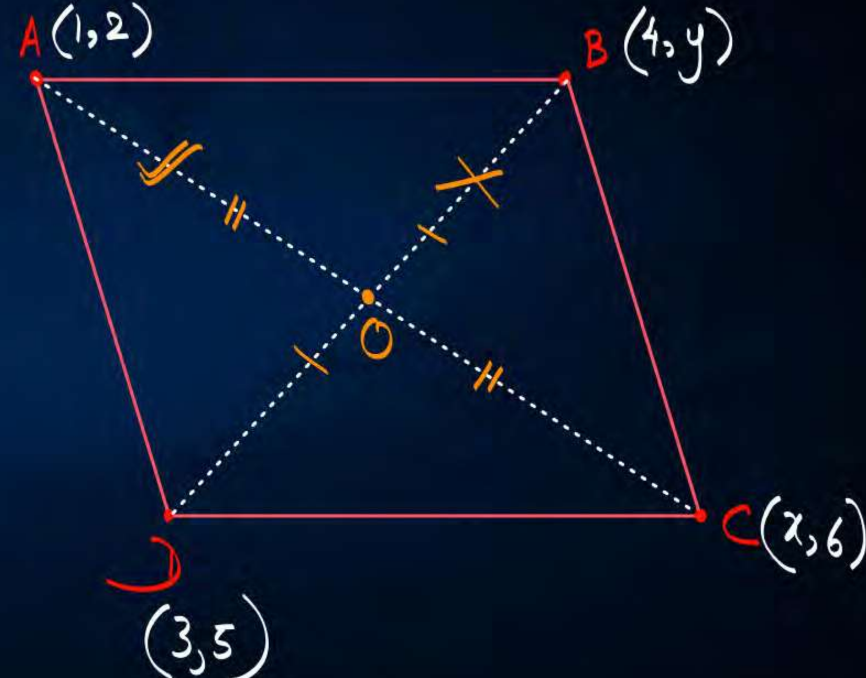
$$AC \longrightarrow \left(\frac{1+x}{2}, 4\right)$$

$$BD \longrightarrow \left(\frac{7}{2}, \frac{y+5}{2}\right)$$

$$\frac{1+x}{\cancel{2}} = \frac{7}{\cancel{2}} \qquad\qquad 4 = \frac{y+5}{2}$$

$$x+1 = 7 \qquad\qquad y+5 = 8$$

$$x = 6 \qquad\qquad y = 3$$



QUESTION

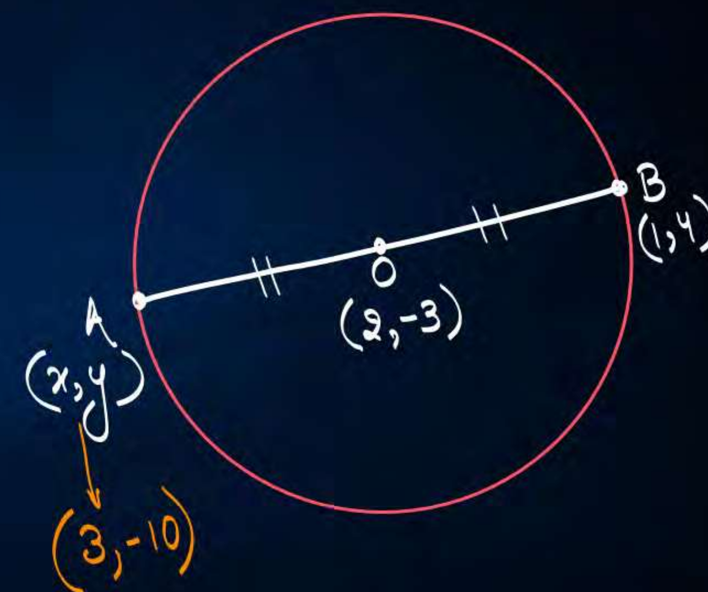
Find the coordinates of a point A, where AB is the diameter of a circle whose centre is (2,-3) and B is (1,4).

Sol^n

$$2 = \frac{x+1}{2} \qquad\Big|\qquad -3 = \frac{y+4}{2}$$

$$\Rightarrow x+1 = 4 \qquad\Big|\qquad y+4 = -6$$

$$\Rightarrow x = 3 \qquad\Big|\qquad y = -10$$



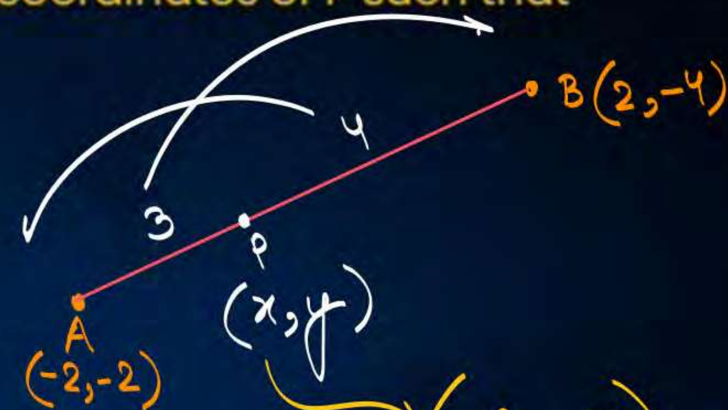
QUESTION

If A and B are $(-2,-2)$ and $(2,-4)$, respectively, find the coordinates of P such that $AP = 3AB/7$ and P lies on the line segment AB.

$\boxed{Sol^n}$

$$x = \frac{6 + (-8)}{7} = -\frac{2}{7}$$

$$y = \frac{-12 + (-8)}{7} = -\frac{20}{7}$$



$$\left(-\frac{2}{7}, -\frac{20}{7}\right)$$

QUESTION

Find the coordinates of the points which divide the line segment joining A(−2, 2) and B(2,8) into four equal parts.

H.W

QUESTION

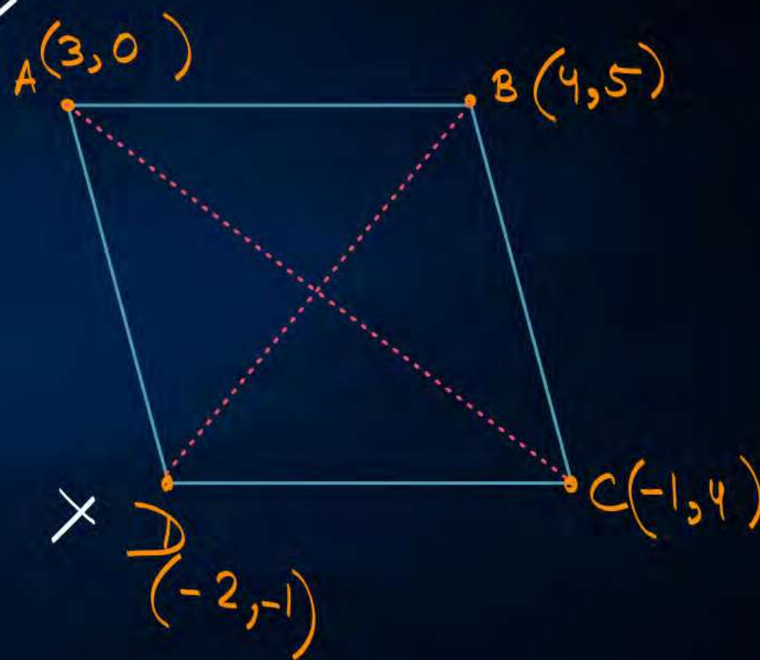
$\frac{1}{2} \times$ prod. of diag.

Find the area of a rhombus if its vertices are $\overset{A}{(3,0)}$, $\overset{B}{(4,5)}$, $\overset{C}{(-1,4)}$ and $\overset{D}{(-2,-1)}$ taken in order.

$\boxed{Sol^n}$

$AC = \sqrt{16+16} = \sqrt{32}$ units $\rightarrow 4\sqrt{2}$

$BD = \sqrt{36+36} = \sqrt{72}$ units $\rightarrow 6\sqrt{2}$



$Ar = \frac{1}{2} \times$ (prod of diag)

$= \frac{1}{2} \times AC \times BD$

$= \frac{1}{\cancel{2}} \times 4\cancel{\sqrt{2}} \times 6\cancel{\sqrt{2}}$

$= 24$ sq. units