TOPICS TO BE COVERED

1. Moment of Inertia and torque

2. Angular momentum

3. Rolling Motion

4. Questions

Moment of Inertia

The measure of the property by virtue of which a body revolving about an axis opposes any change in rotational motion is known as moment of inertia.

Point Mass $MoI = m_{body}\, r_\perp^2$

$\Downarrow$

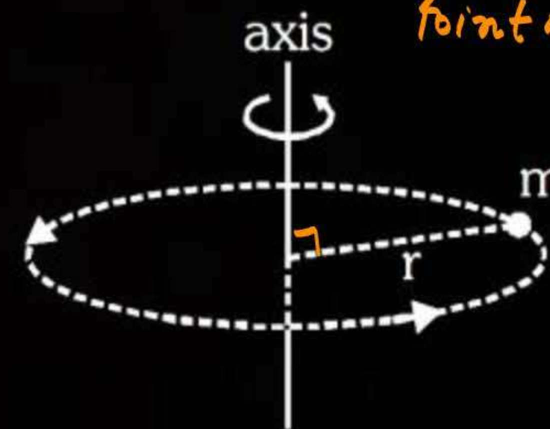
Tensor Qty. "MoI addition/Subtraction"

$\hookrightarrow$ Common axis of Rot.

UNIT : SI : kg–m^2 CGS : g–cm^2

Dimensions : $[M^1L^2T^0]$

Point mass



$r = \perp$ distance from Axis of Rot.

Calculation of MoI

Discrete Masses

point masses (No dimensions)

$$\boxed{I = \sum_{i=1}^{i=n} m_i r_i^2}$$

Subka individual I
nikal kar add karna
hai!

Continous Mass bodies.

disc, Rod, Ring, Cylinder

mass is distributed.

$$I = \int dm\, r^2$$

↳ Mass of Element

1D $dm = \lambda dx$

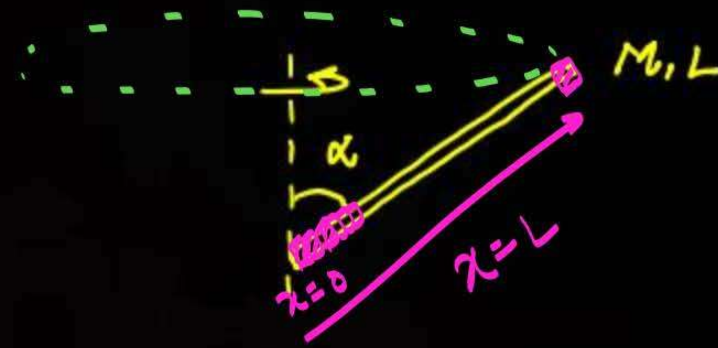
2D $dm = \sigma . dA$

3D $dm = \rho dV$

λ, σ, ρ = Constant (uniform Mass density)

$\lambda = \frac{M}{L}$, $\sigma = \frac{M}{A}$, $\rho = \frac{M}{V}$

Que. Find MoI of Rod about Corner.



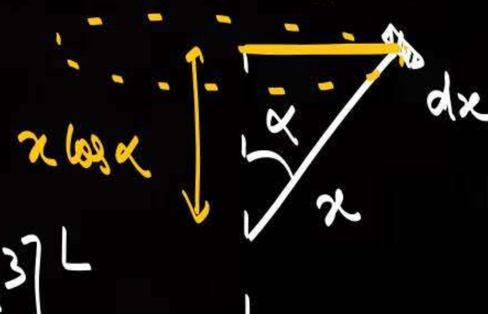
Sol:- uniform mass distribution

$$\lambda = \frac{M}{L}$$

$$dI = dm\, r_{\perp}^2$$

$$r_{\perp} = (x\sin\alpha)$$

$$dI = \frac{M}{L}dx\,(x\sin\alpha)^2$$

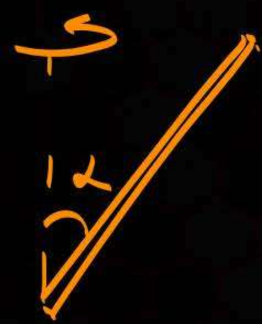


$$dm = \lambda\, dx$$

$$\boxed{dm = \frac{M}{L}dx}$$

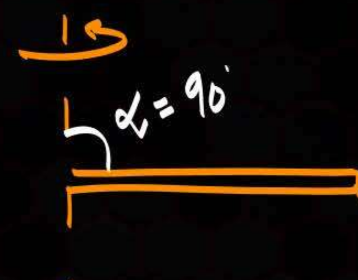
$$I_T = \int dI = \int_0^L \frac{M}{L}\sin^2\alpha\; x^2 dx = \frac{M}{L}\sin^2\alpha\left[\frac{x^3}{3}\right]_0^L = \boxed{\frac{1}{3}ML^2\sin^2\alpha}$$

Analysis



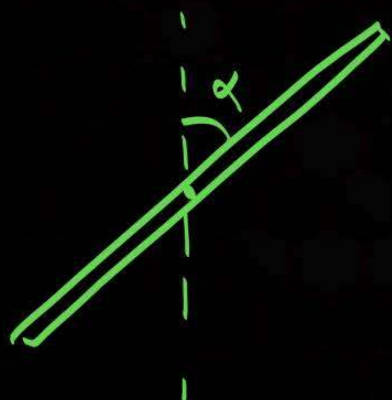
M, L

$$I = \frac{1}{3} M l^2 \sin^2\alpha$$

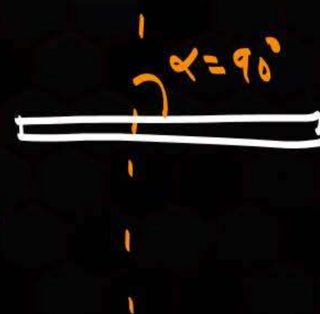


$$I = \frac{1}{3} M l^2$$

$$\boxed{I = 0}$$



$$\boxed{I = \frac{1}{12} m l^2 \sin^2\alpha}$$

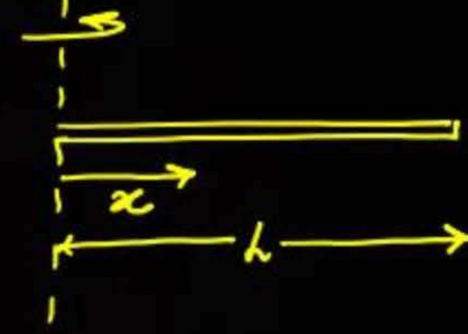


$$\boxed{I = \frac{1}{12} m l^2}$$

Que. Find MoI of Rod whose $\lambda = Cx^2$ about Corner.

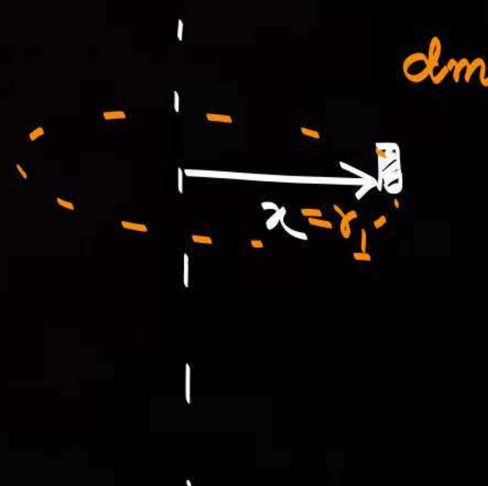
Variable

Sol:- Non-uniform Mass dis. $\lambda = f(x)$



$$dm = \lambda dx$$

$$dm = Cx^2 dx$$



$$dI = dm\, r_1^2$$

$$dI = Cx^2 dx\, x^2$$

$$I_T = \int_0^L Cx^4 dx = \frac{CL^5}{5} \text{ Ans.}$$

Q. The linear mass density of a thin rod AB of length L varies from A to B as $\lambda(x) = \lambda_0\left(1 + \frac{x}{L}\right)$, where x is the distance from A. If M is the mass of the rod then its moment of inertia about an axis passing through A and perpendicular to the rod is:

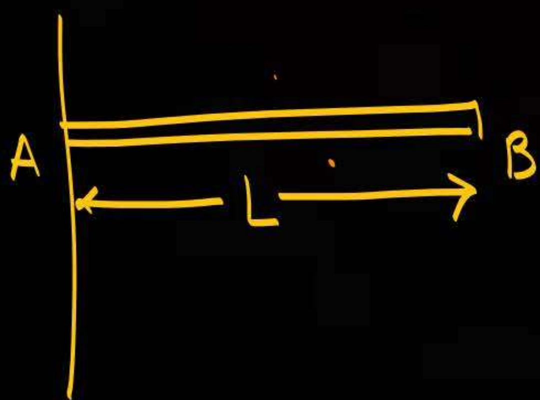
(A) $\frac{5}{12}ML^2$

(B) $\frac{7}{18}ML^2$

(C) $\frac{2}{5}ML^2$

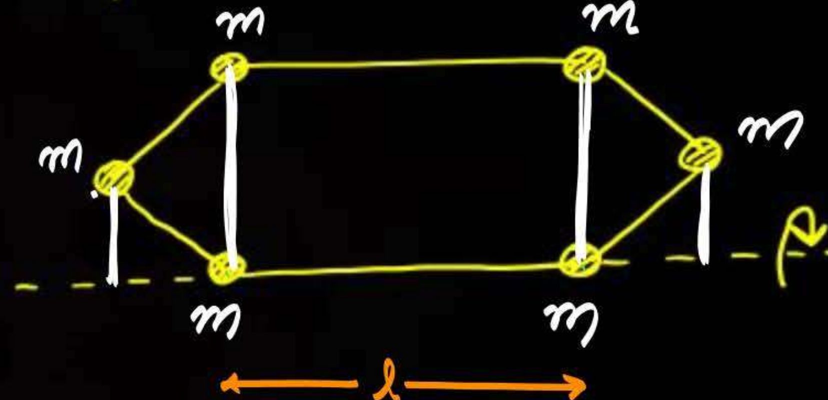
(D) $\frac{3}{7}ML^2$

$\boxed{\text{Approach} \rightarrow \text{Clear}}$ ✓



$\lambda = \lambda_0\left[1 + \frac{x}{L}\right]$

Que. Find MoI of System for point Masses. (Each Mass = m, placed at Corner of Regular hexagon of side l.

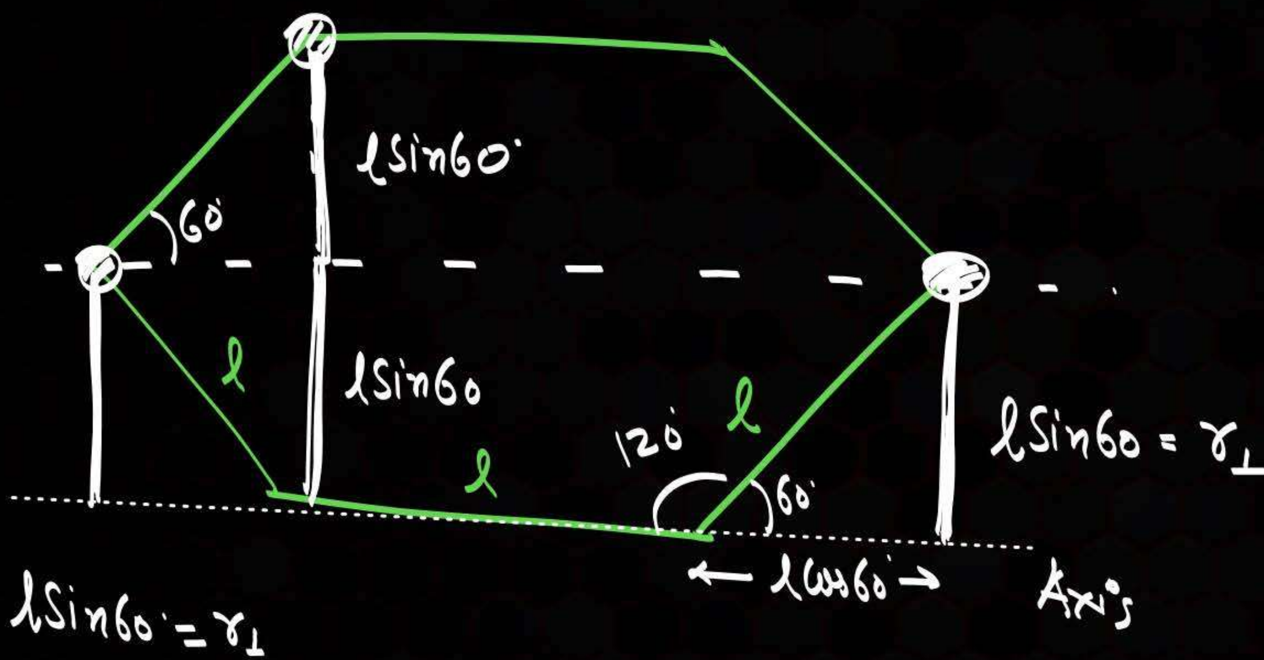


Sol:-

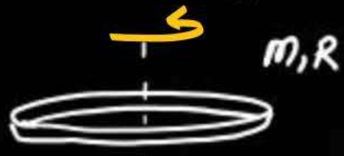
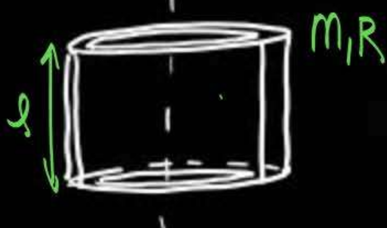
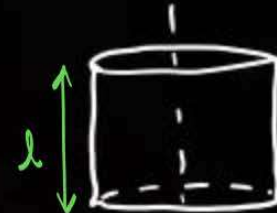
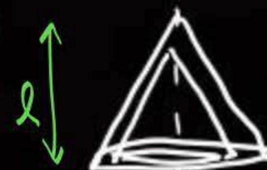
$I_T = \sum m_i r_i^2$ → ⊥ distance from axis of Rot.

Axis = passing through edge of hexagon.

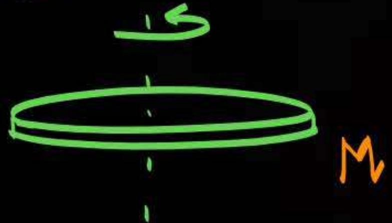
$$\boxed{\begin{aligned} I = m(0)^2 + m(0)^2 + 2m(l\sin 60)^2 \\ + 2m(2l\sin 60)^2 \end{aligned}}$$


lsin60°
60°
l
lsin60
l
120°
l
60°
lsin60 = r⊥
← lcos60° →
Axis
lsin60° = r⊥

List of Important MoI

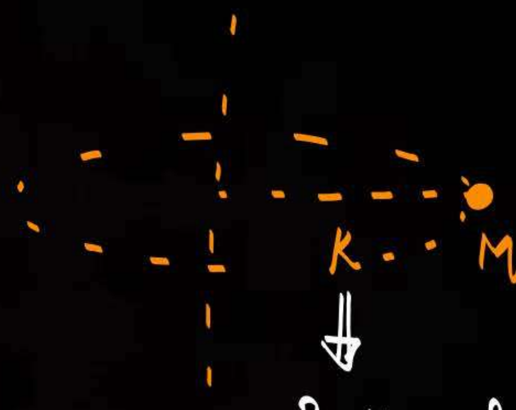
	Ring	MR^2
	disc	$\frac{1}{2}MR^2$
	Rod	$\frac{1}{3}ML^2$
	Rod.	$\frac{1}{12}ML^2$
	hollow cylinder	MR^2
	Solid Cylinder	$\frac{1}{2}MR^2$
	hollow Cone	$\frac{1}{2}MR^2$
	Solid Cone	$\frac{3}{10}MR^2$
	hollow Sphere	$\frac{2}{3}MR^2$
	Solid Sphere.	$\frac{2}{5}MR^2$

Radius of Gyration (K)



$I = \frac{1}{2}MR^2 \implies$

Point Mass



$I = MK^2$

Radius of gyration

Continous Mass dist

$\frac{1}{2}MR^2 = MK^2$

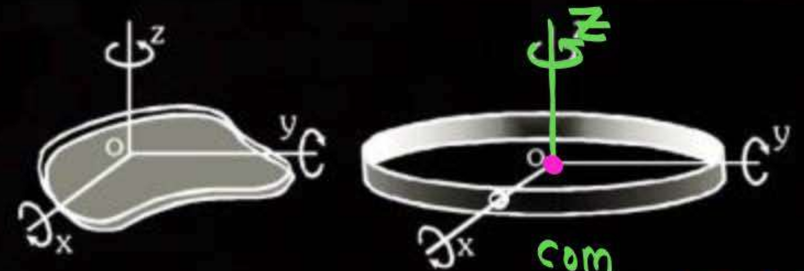
$\boxed{K = \frac{R}{\sqrt{2}}}$

K has no meaning without axis of rotation
K is a scalar quantity.

Theorems of Moment of Inertia

A. Perpendicular Axis Theorem

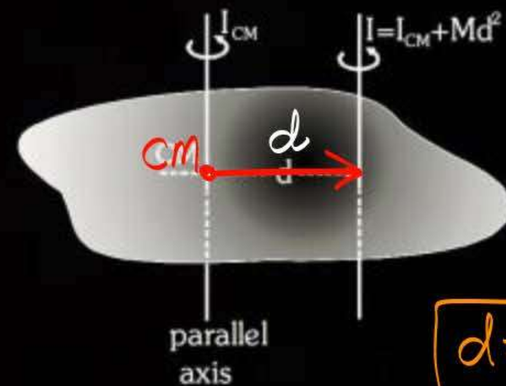
⊛ Planar objects (Ring, disc, Sheets)



$$\boxed{I_z = I_x + I_y}$$

I_x & I_y are two ⊥ axis in Plane of object.

B. Parallel Axis Theorem



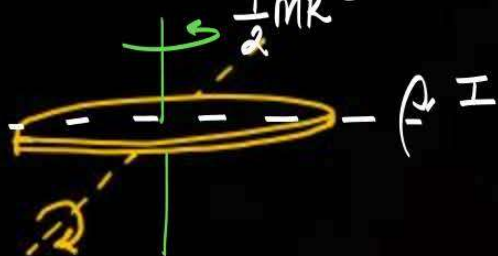
I_{com}

d = Shifted distance

$$\boxed{I_{new} = I_{com} + M_{body}\, d^2}$$

⊛ Application.

a)

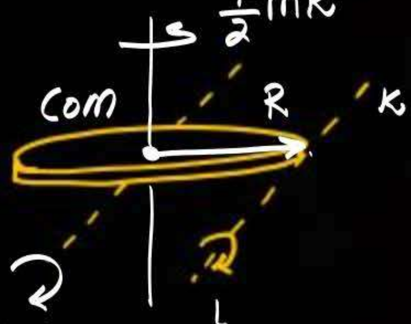


$$I_z = I + I$$

$$\frac{1}{2}MR^2 = 2I$$

$$\boxed{I = \frac{1}{4}MR^2}$$

b)



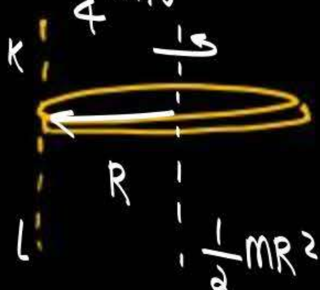
Axis in plane

$$I_{KL} = I_{com} + Mh^2$$

$$= \frac{1}{4}MR^2 + MR^2 = \frac{5}{4}MR^2.$$

$I = \frac{1}{4}MR^2$

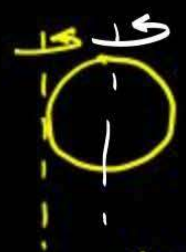
c)



Axis ⊥ to plane

$$I_{KL} = I_{com} + Mh^2 = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2.$$

d)

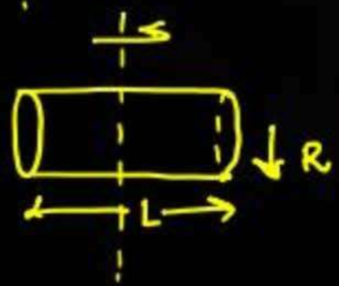


$\frac{2}{5}mR^2$

Solid Sphere

$$I_{new} = I_{com} + MR^2 = \frac{2}{5}MR^2 + MR^2 = \frac{7}{5}MR^2.$$

f)



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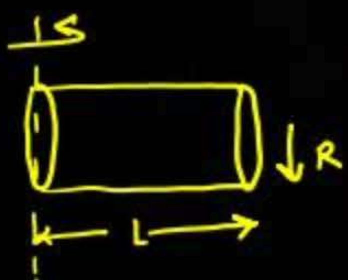
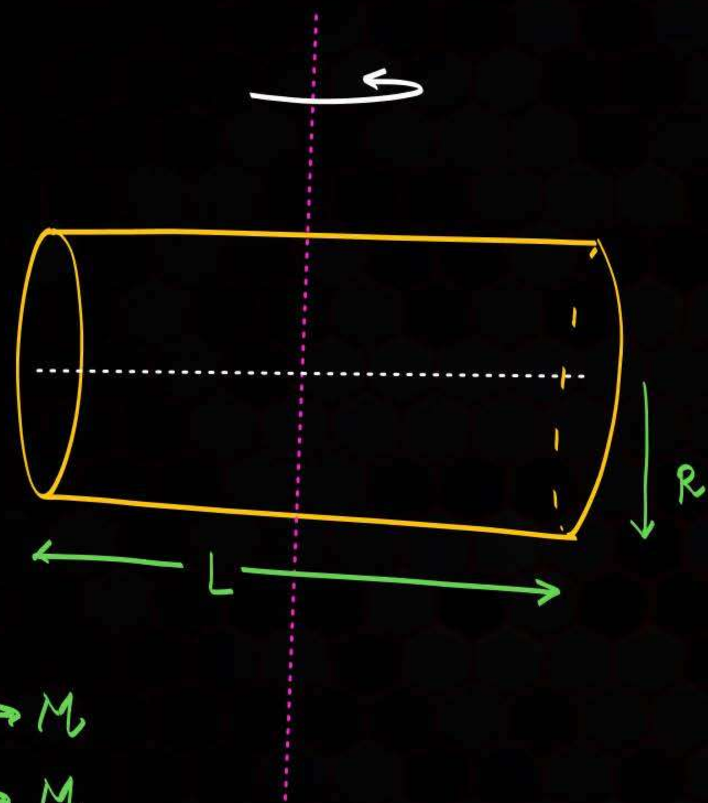
g)



$$\boxed{I_T = \frac{1}{3}mL^2 + \frac{1}{4}mR^2}$$

#

Solid Cylinder.



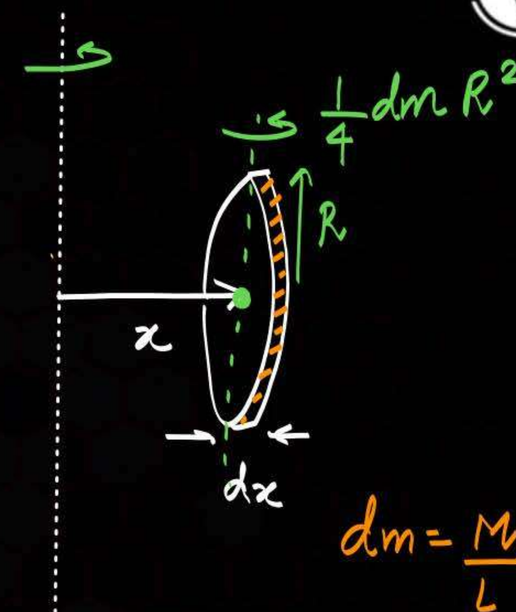
M

$l \longrightarrow M$

$1 \longrightarrow \frac{M}{L}$

$dx \rightarrow \frac{M}{L} dx$

$\Rightarrow$



$dm = \frac{M}{L} dx$

MoI of disc about axis

$dI = I_{com} + Mh^2$

$$\boxed{dI = \frac{1}{4} dm R^2 + dm x^2}$$

$$I_{Total} = \int dI = \int\left(\frac{1}{4}dmR^2 + dmx^2\right)$$

$$= \int_{-L/2}^{+L/2}\left(\frac{1}{4}\frac{m}{L}dx\,R^2 + \frac{m}{L}dx\,x^2\right)$$

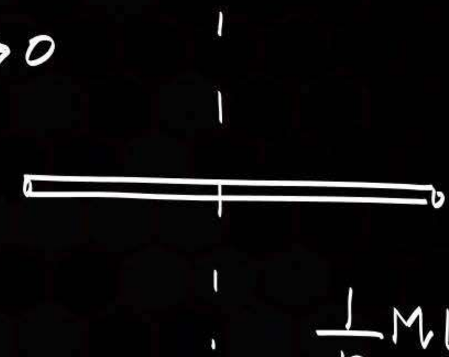
$$= \frac{1}{4}\frac{m}{L}R^2\left[x\right]_{-L/2}^{+L/2} + \frac{m}{L}\left[\frac{x^3}{3}\right]_{-L/2}^{+L/2}$$

$$\boxed{I_T = \frac{1}{4}mR^2 + \frac{1}{12}mL^2}$$

Concept

$$\boxed{I_T = \frac{1}{12}mL^2 + \frac{1}{4}mR^2}$$

$R \to 0$

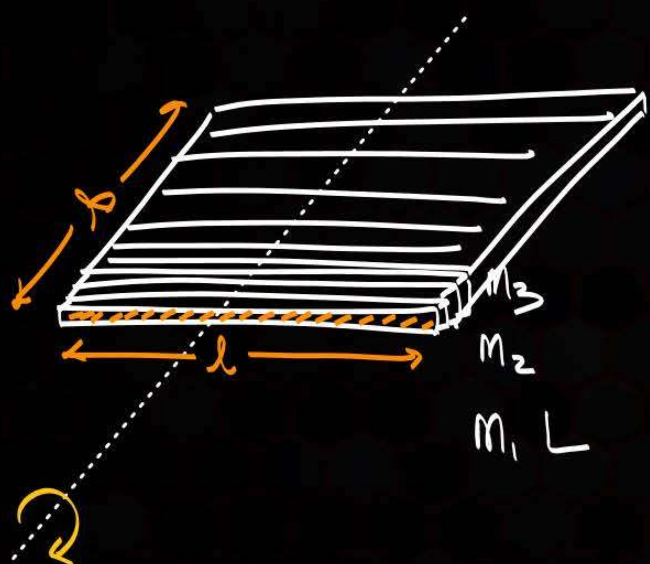


$$\frac{1}{12}ML^2$$

$L \to 0$

$$\frac{1}{4}mR^2$$

Recall the Concept



$$\frac{1}{12}m_1L^2+\frac{1}{12}m_2L^2+\frac{1}{12}m_3L^2$$

$$I=\sum m_i r_i^2$$
$$=\int dm\, r^2$$

↳ ⊥ distance from axis of Rot.

* length ∥ to axis of Rot doesnot Impact MoI.

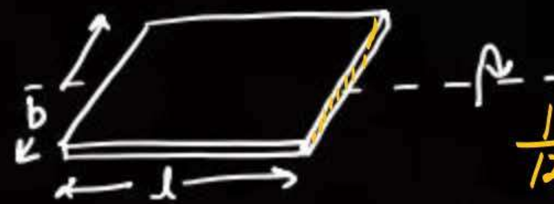
$$I_T=\frac{1}{12}(m_1+m_2+\cdots m_n)L^2$$

$$\boxed{I_T=\frac{1}{12}ML^2}$$

MoI of plates.

Concept :-

$\frac{1}{12}ML^2$



$\frac{1}{12}Mb^2$

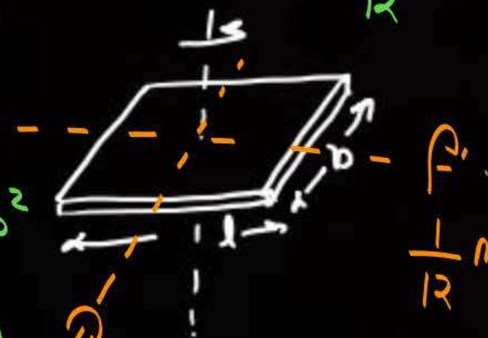
$I = \frac{1}{12}m(l^2+b^2)$

①

$I_z = I_x + I_y$

$= \frac{1}{12}ml^2 + \frac{1}{12}mb^2$

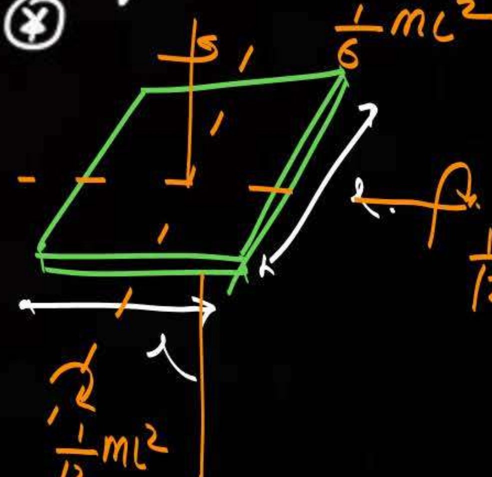
$= \frac{1}{12}m(l^2+b^2)$



$\frac{1}{12}mb^2$

$\frac{1}{12}ml^2$

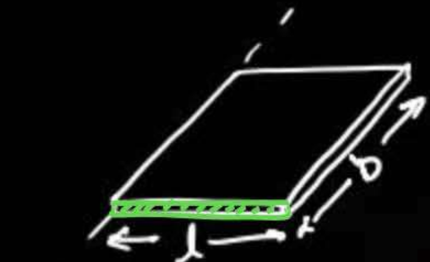
Square Case.



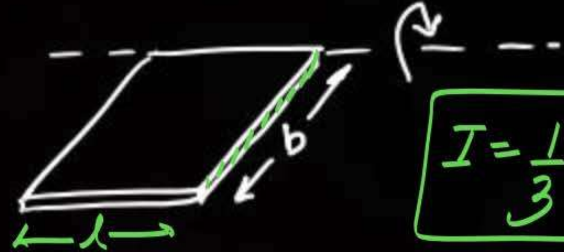
$\frac{1}{6}ml^2$

$\frac{1}{12}ml^2$

$\frac{1}{12}ml^2$

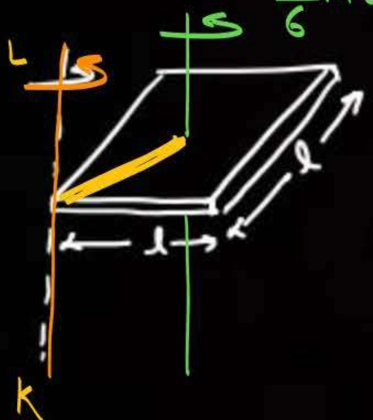


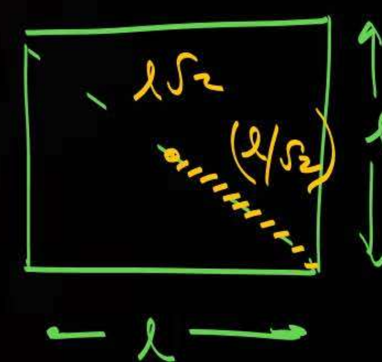
$\boxed{\frac{1}{3}ml^2}$



$\boxed{I = \frac{1}{3}mb^2}$

$\frac{1}{6}ml^2$





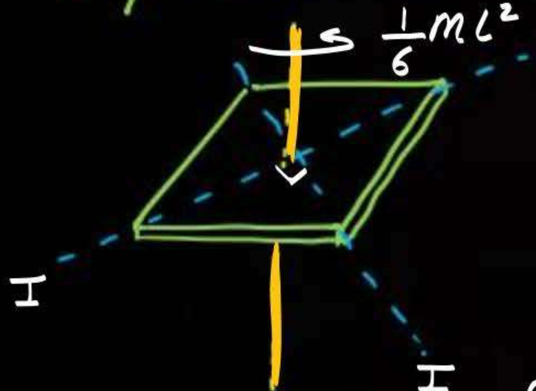
$$I_{KL} = I_{com} + mh^2$$

$$= \frac{1}{6}ml^2 + m\left(\frac{l}{\sqrt{2}}\right)^2 =$$

⊛ Triangular Lamina

$$\boxed{I = \frac{1}{6} m h^2}$$

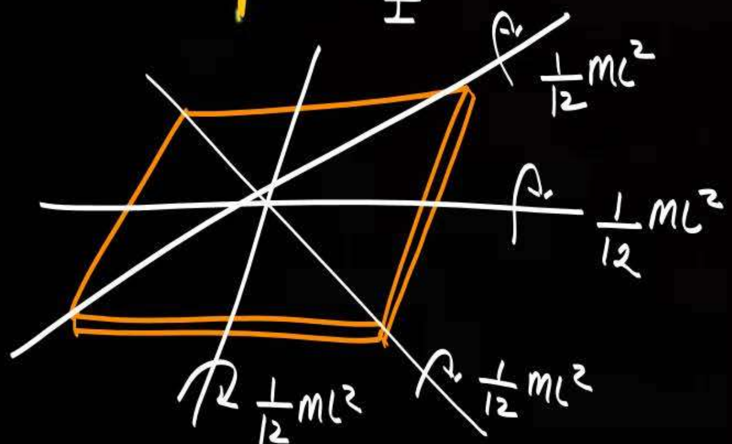
⊛ Important Point for "Square" Lamina

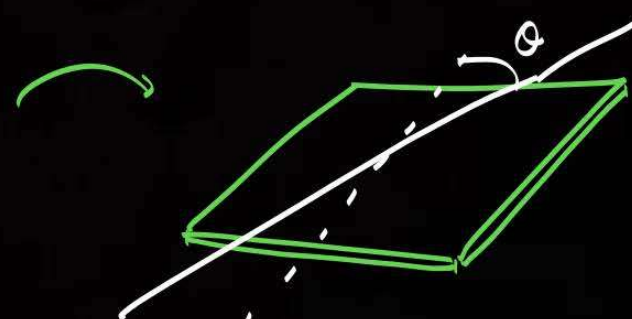


$$2I = \frac{1}{6} m l^2$$

$$\boxed{I = \frac{1}{12} m l^2}$$

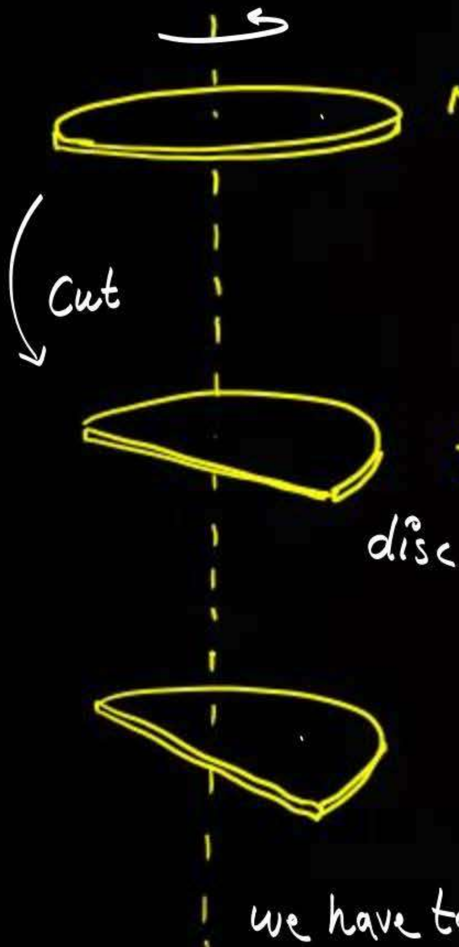
$$\boxed{I_{dia} = \frac{1}{12} m l^2}$$





$$\boxed{I = \frac{1}{12} m l^2}$$ ⊛

* Imp points



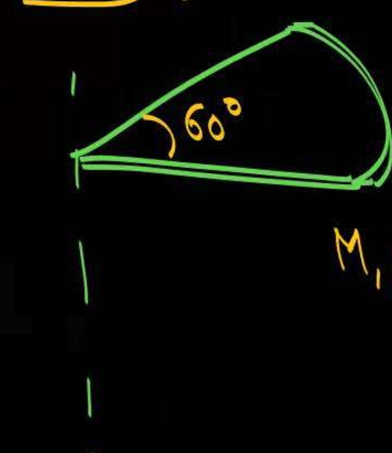
M, R $\qquad \frac{1}{2}MR^2$

$\frac{M}{2}$, R $\qquad I = \frac{1}{2}\left(\frac{M}{2}\right)R^2 = \frac{1}{4}MR^2$

M, R $\qquad \boxed{I = \frac{1}{2}MR^2}$

we have taken a Semi disc of Mass = M

$\frac{1}{2}MR^2$



M, R

Additive and Negative System

Jodna Subtract.

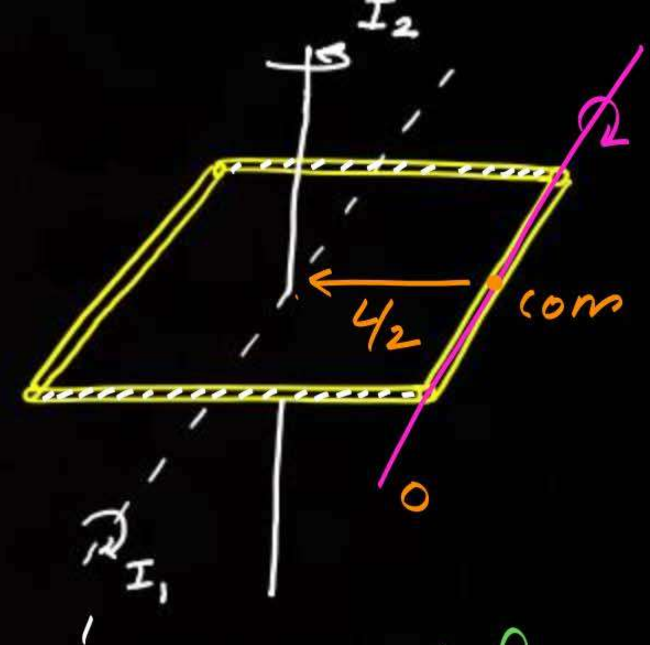
1. Find MoI of Each Segment about Common axis of Rot.
2. Then add/Subtract.

Que. Four Rods are Connected to make a Square frame as Shown Find MoI about Given axis.

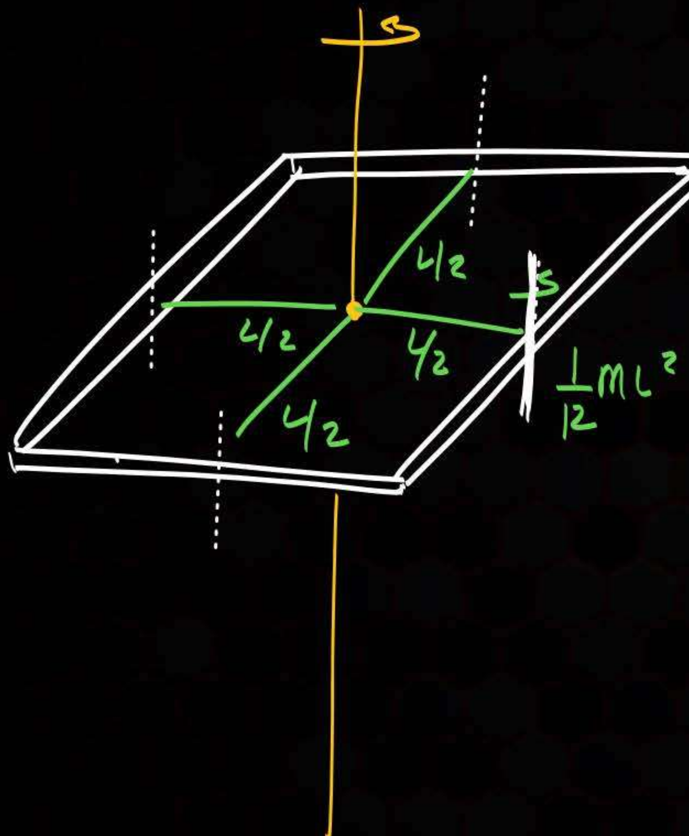
Each Rod of Mass $= m$ & length $= L$.

$$I_1 = 2\left[\frac{1}{12}mL^2\right] + 2\left[I_{com} + mh^2\right]$$

$$= 2\left[\frac{1}{12}mL^2\right] + 2\left[0 + m\left(\frac{L}{2}\right)^2\right]$$



Axis in plane of frame.



$$I_{Total} = 4\left[\frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2\right]$$

Q. A thin disc of mass M and radius R has mass per unit area $\sigma(r) = kr^2$, where r is the distance from its centre. Its moment of inertia about an axis going through its centre of mass and perpendicular to its plane is

[2019 Main] × 5

A $\frac{MR^2}{2}$

B $\frac{MR^2}{6}$

C $\frac{MR^2}{3}$

D $\frac{2MR^2}{3}$ Ans

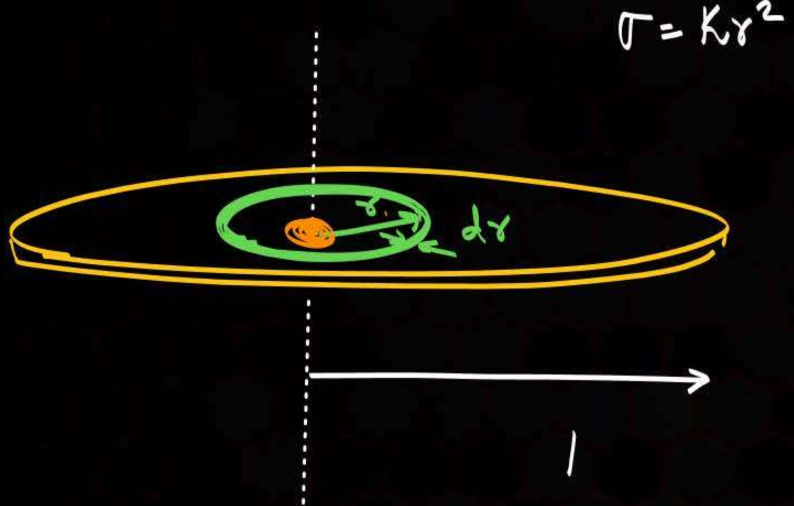
$$M_T = \int dm$$

$$= \int_0^R (kr^2)\, 2\pi r\, dr$$

$$M = 2\pi k \left[\frac{r^4}{4}\right]_0^R$$

$$\boxed{M = \frac{\pi k R^4}{2}}$$

$$\boxed{\pi k = \frac{2M}{R^4}}$$



$dm = \sigma \cdot dA$

$$\boxed{dm = (K r^2 \cdot 2\pi r\, dr)}$$

$dI = dm\, r^2$

$dI = (K r^2\, 2\pi r\, dr)\, r^2$

$$I = \int_0^R 2\pi K\, r^5\, dr \xrightarrow{} \text{variable.}$$

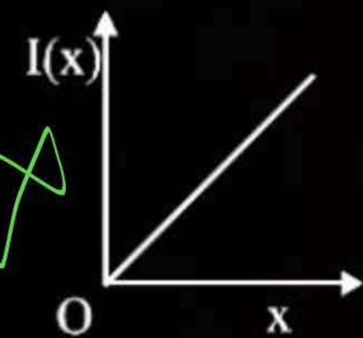
$$I_T = 2\pi K \left[\frac{r^6}{6}\right]_0^R$$

$$= \frac{2\pi K R^6}{6} = \frac{\pi K R^6}{3} = \frac{2M}{R^4}\left(\frac{R^6}{3}\right)$$

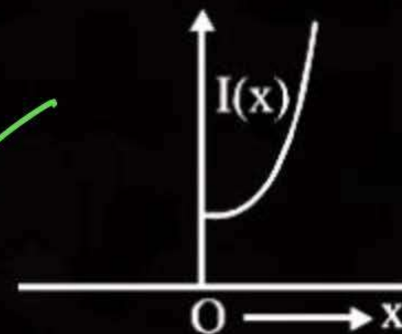
$$\frac{2}{3} m R^2$$

Q. The moment of inertia of a solid sphere, about an axis parallel to its diameter and at a distance of x from it, is 'I(x)'. Which one of the graphs represents the variation of I(x) with x correctly? **[2019 Main]**

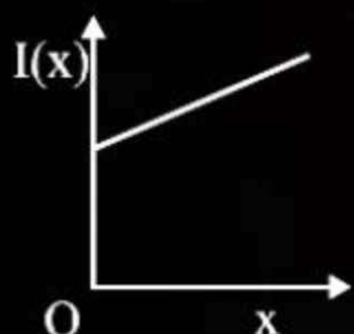
A



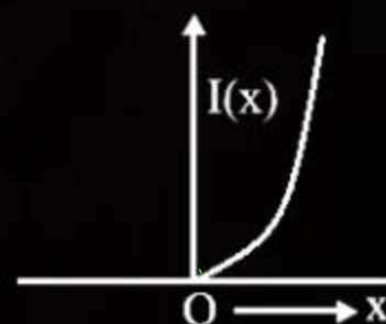
B ✓ Ans

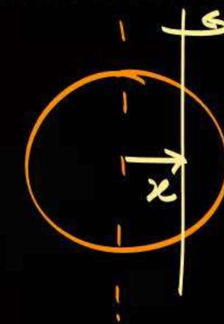


C



D





$I_{new} = I_{com} + Mx^2$

at $x = 0$ $I_{new} = I_{com}$

Que. A Rigid body Can be hinged about any point on x axis. When it is hinged Such that hinge is at x, The Moment of Inertia is Given by

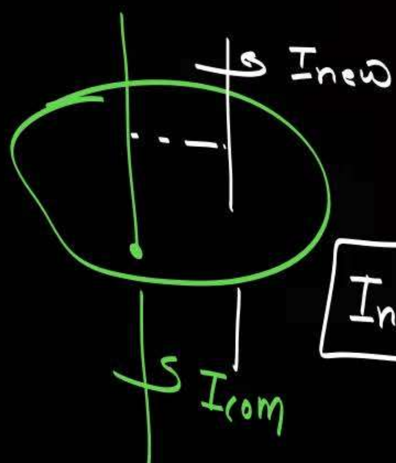
$$I = 2x^2 - 12x + 27$$

Find x Coordinate of COM.

a) $x = 2$ b) $x = 0$ c) $x = 1$ d) $x = 3$. Ans

Sol:- $I = 2x^2 - 12x + 27$.

$\boxed{\text{about COM MoI} \longrightarrow \text{Min}}$



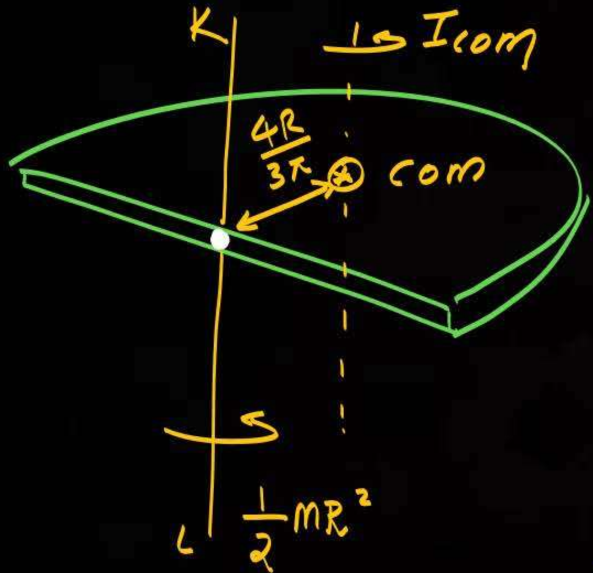
$$\boxed{I_{new} = I_{com} + Mx^2}$$

$$I = 2x^2 - 12x + 27$$

$$\frac{dI}{dx} = 4x - 12$$

$$\boxed{x = 3} \Rightarrow \text{COM}$$

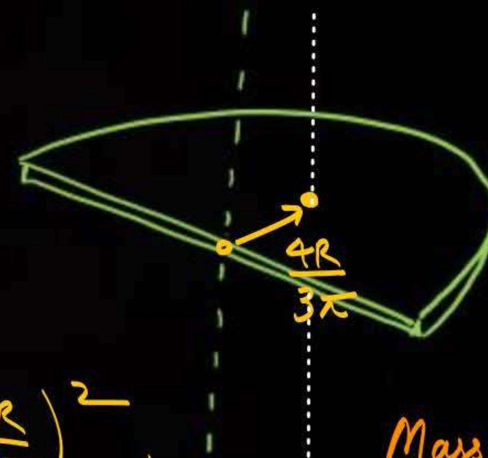
Que. Find MoI of disc of about axis passing through com.
disc mass is M & Radius is R.



$$I_{KL} = I_{com} + Mh^2$$

$$\frac{1}{2}MR^2 = I_{com} + M\left(\frac{4R}{3\pi}\right)^2$$

$$\boxed{\frac{1}{2}MR^2 - M\left(\frac{4R}{3\pi}\right)^2 = I_{com}}$$



Mass = M
Radius = R

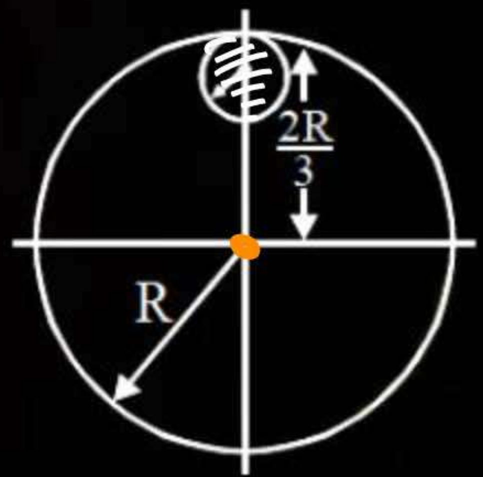
Q. From a uniform circular disc of radius R and mass 9M, a small disc of radius $\frac{R}{3}$ is removed as shown in the figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through centre of disc is **[2018 Main]**

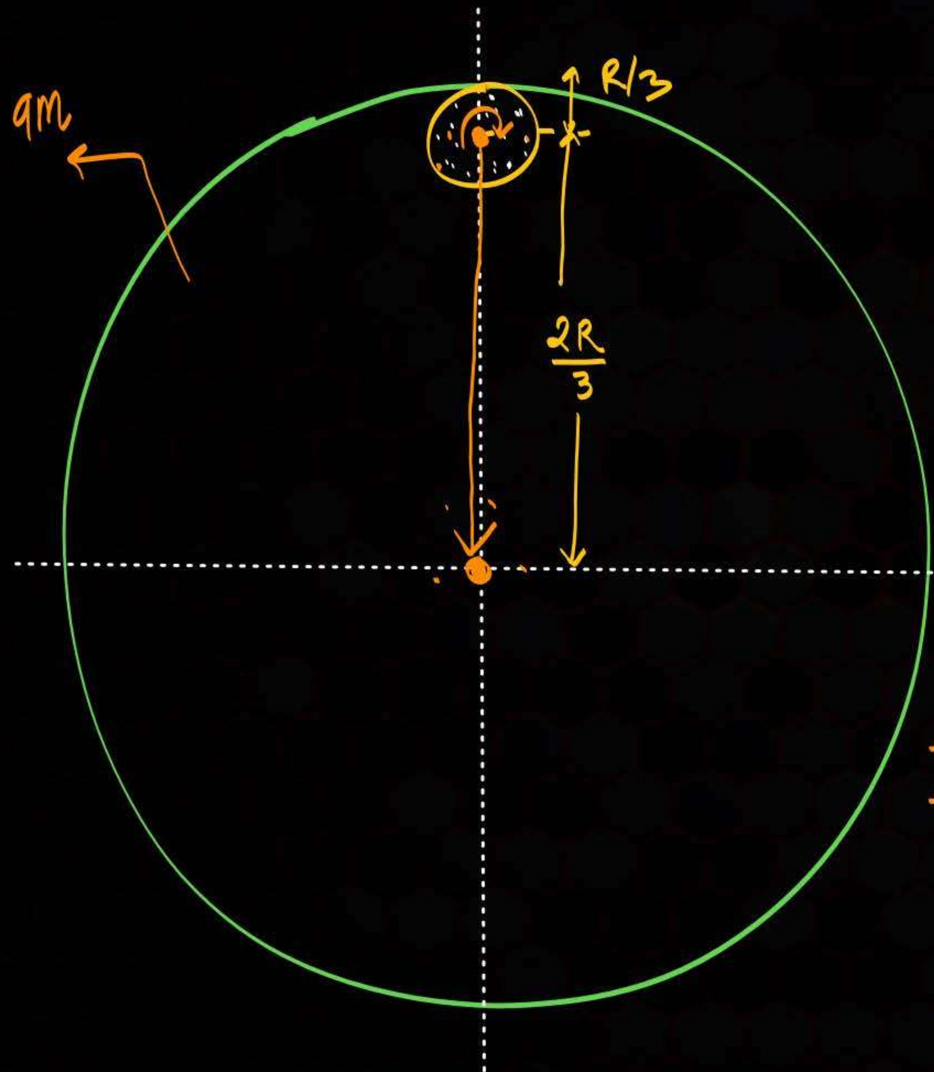
(A) $\frac{37}{9}MR^2$

(B) $4\,MR^2$

(C) $\frac{40}{9}MR^2$

(D) $10\,MR^2$





$$I_{complet\ disc} = \frac{1}{2}(9M)R^2$$

$$I_{smaller\,disc} = \frac{1}{2}(M)\left(\frac{R}{3}\right)^2$$

$$\pi(R)^2 \longrightarrow 9M$$

$$1 \longrightarrow \frac{9M}{\pi R^2}$$

$$\pi\left(\frac{R^2}{9}\right) \rightarrow \frac{9M}{\pi R^2}\cdot\frac{\pi R^2}{9} \Rightarrow M$$

$$I_{final} = I_{bigger} - I_{smaller}\Big|_{common\,axis.}$$

$$= \frac{1}{2}(9M)R^2 - \left[\frac{1}{2}m\left(\frac{R}{3}\right)^2 + M\left(\frac{2R}{3}\right)^2\right]$$

Q. From a solid sphere of mass M and radius R, a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its centre and perpendicular to one of tis faces is **[2015 Main]**

(A) $\dfrac{MR^2}{32\sqrt{2}\,\pi}$

(B) $\dfrac{4MR^2}{9\sqrt{3}\,\pi}$

(C) $\dfrac{MR^2}{16\sqrt{2}\,\pi}$

(D) $\dfrac{4MR^2}{3\sqrt{3}\,\pi}$

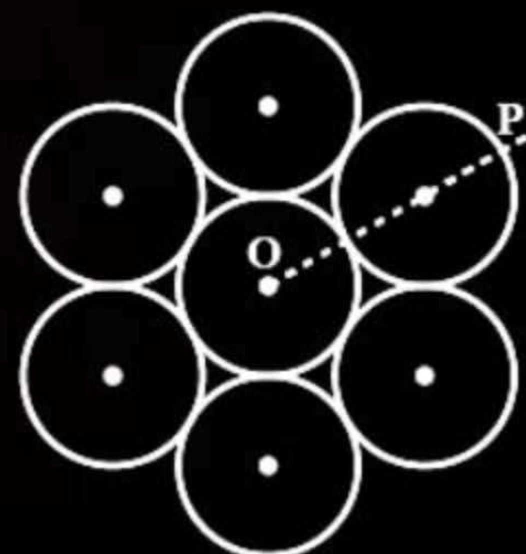
Q. Seven identical circular planer discs, each of mass M and radius R are welded symmetrically as shown in the figure. The moment of inertia of the arrangement about an axis normal to the plane and passing through the point P is **[2018 Main]**

(A) $\frac{181}{2} MR^2$

(B) $\frac{19}{2} MR^2$

(C) $\frac{55}{2} MR^2$

(D) $\frac{73}{2} MR^2$



Torque

Torque is the rotational analogue of force and expresses the tendency of a force applied to an object to cause rotation in it about a given point.

(*) $\boxed{\vec{\tau} = \vec{r} \times \vec{F}}$

(*) $\boxed{\tau = rF\sin\theta}$

θ is angle between $\vec{r}$ & $\vec{F}$.

$\boxed{\tau_{min} = 0}$

$\theta = 0^\circ, \pi$

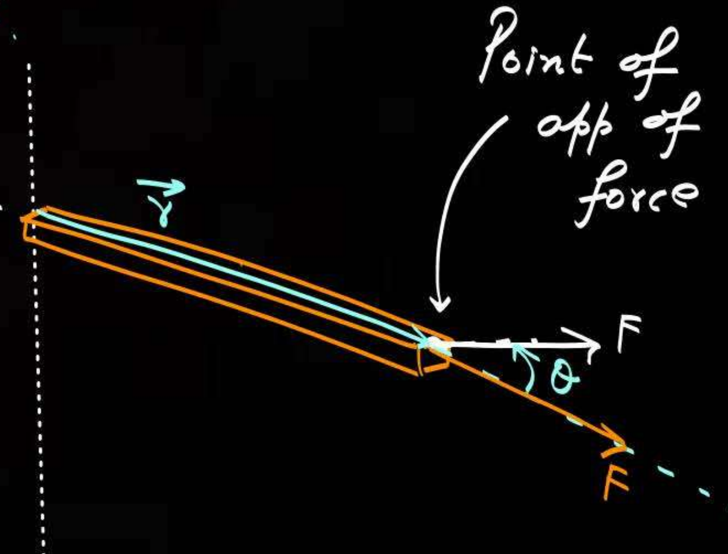
τ_{max}

$\theta = 90^\circ$

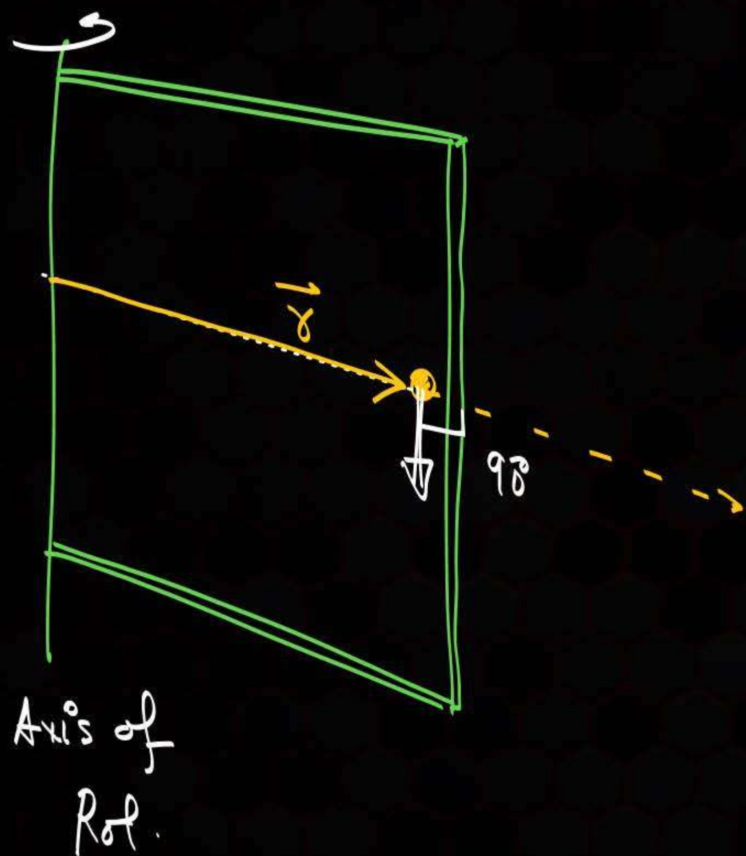
$\boxed{\tau = rF}$

$\boxed{\begin{array}{l} r = 0 \quad \text{force on axis} \\ \tau = 0 \end{array}}$

(*) $\vec{\tau} = \begin{vmatrix} \hat{i} & -\hat{j} & \hat{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$



#



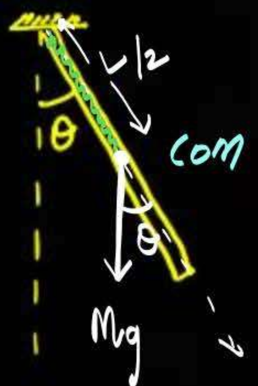
$$\tau = rF\sin 90$$

$$\boxed{\tau = rF}$$

For Rot of a body τ must be alligned with axis of Rot.

Calculation of Torque

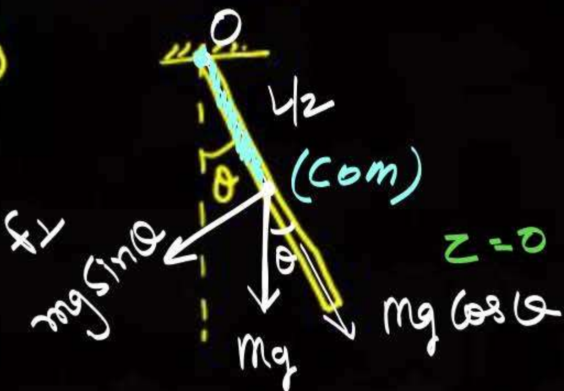
a)



$|\tau| = rF\sin\theta$

$\boxed{|\tau| = \frac{L}{2} mg \sin\theta}$ $(-\hat{k})$

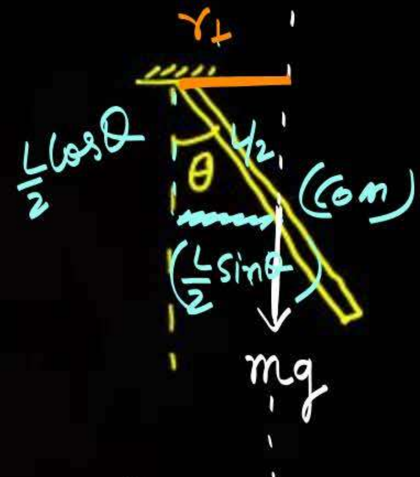
b)



$|\tau| = rF_{\perp}$ → Resolve force ∥ & ⊥ to r.

$\boxed{|\tau| = \frac{L}{2} \times mg\sin\theta}$

c)



$|\tau| = f\, r_{\perp}$ draw Line of Force

→ drop ⊥ on Line of force.

$= mg\left(\frac{L}{2}\sin\theta\right)$

If Multiple forces act on body.

& we have to Calculate Torque about any point.

$$\boxed{\vec{\tau}_{net} = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_3}$$

$\boxed{\text{Vectorilly add}}$

Questions

$\Downarrow$

Rotational Equilibrium

$\Downarrow$

$$\text{System} = \begin{Bmatrix} \text{Rest or} \\ \text{Const velocity} \end{Bmatrix}$$

$$\boxed{\begin{array}{l} \circledast \ \sum F_{net} = 0 \\ \circledast \ \sum \tau_{net} = 0 \end{array}}$$

1. FBD diagram.
2. $\sum F_x = 0$, $\sum F_y = 0$
3. $\sum \vec{\tau}_{any\ point} = 0$

Pure Rotational Motion

$\Downarrow$

accelerated. (α)

1. FBD
2. $\tau_{axis} = I_{axis}\,\alpha$
3. $\alpha = \dfrac{\tau_{axis}}{I_{axis}}$

$\Downarrow$

Const $\rightarrow$

$$\omega_f = \omega_0 + \alpha t$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega_f^2 - \omega_0^2 = 2\alpha\theta.$$

Combinational Problems.

1. FBD
2. $F_{net} = ma$
3. $\tau_{net} = I_{axis}\,\alpha.$

Q. Shown in the figure is rigid and uniform one meter long rod AB held in horizontal position by two strings tied to its ends and attached to the ceiling. The rod is of mass 'm' and has another weight of mass 2 m hung at a distance of 75 cm from A. The tension in the string at A is :

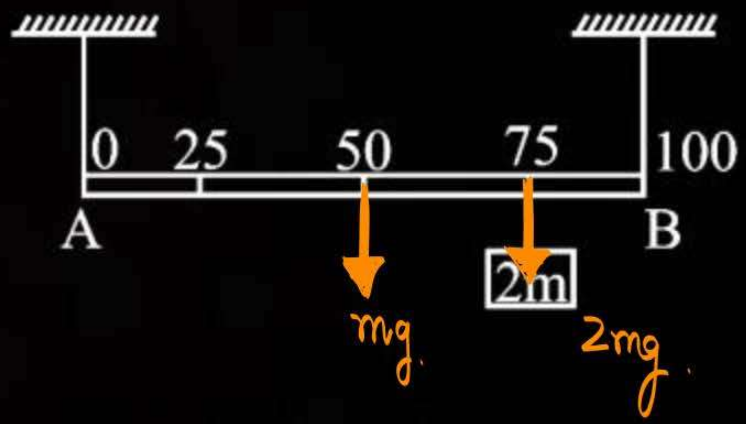
(A) 0.5 mg

(B) 2 mg

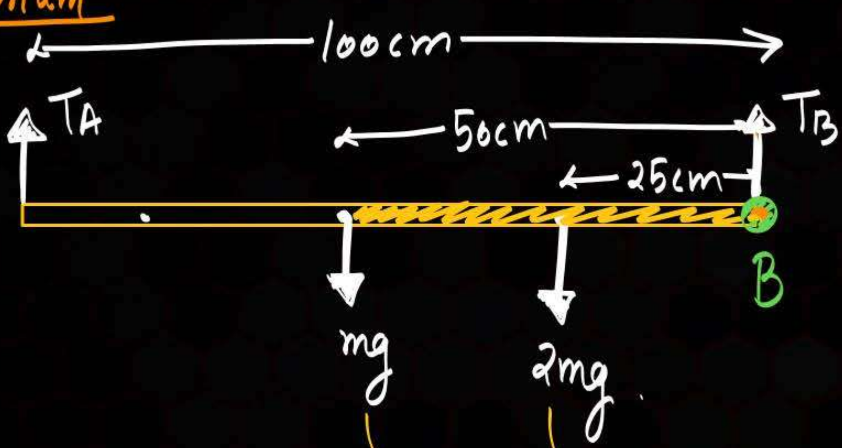
(C) 0.75 mg

(D) 1 mg

Ans



Equilibrium



$\Sigma F_{net} = 0$

$\boxed{T_A + T_B = 3mg}$

$mg + T_B = 3mg$

$\boxed{T_B = 2mg}$

Clockwise = -ve

anticlock = +ve.

Rot Equilibrium

$\tau_{net} = 0$

(any point)

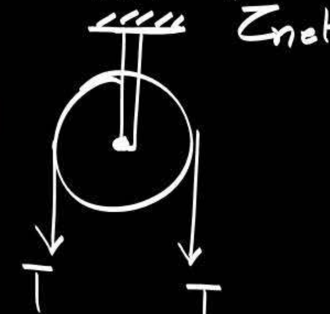
$\tau_{net_B} = T_B(0) + (2mg)\cancel{25} + mg(\overset{2}{\cancel{50}}) - T_A(\overset{4}{\cancel{100}}) = 0$

$2mg + 2mg = 4T_A$

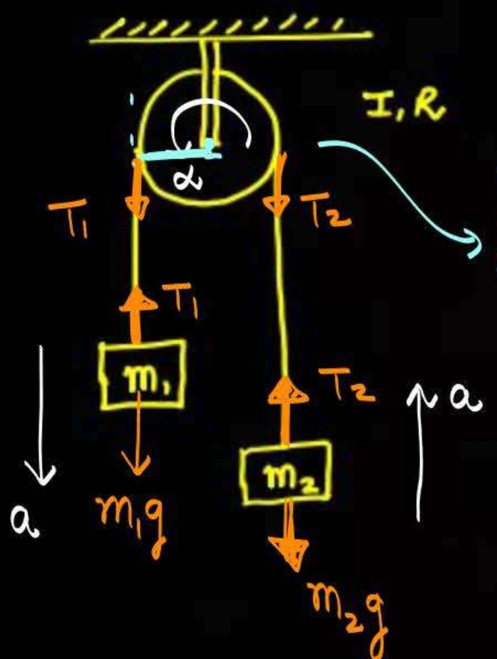
$\boxed{mg = T_A}$

Pulley problems.

In LOM → pulley → massless, frictionless } → do not Rotate



a)



$m_1 > m_2$

→ Pure Rot

$\tau_{net} = I\alpha$

In Rotational problems

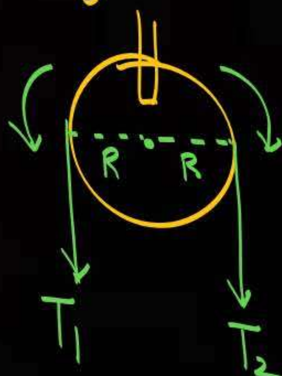
Pulley → MoI = ✓

friction = ✓

M, R

(disc)

Pulley will Rotate.



$m_1 g - T_1 = m_1 a$

$T_1 R - T_2 R = I\alpha$

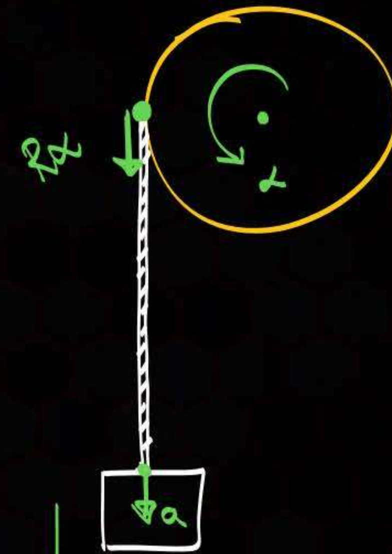
$T_2 - m_2 g = m_2 a$

$$m_1 g - T_1 = m_1 a$$
$$T_1 R - T_2 R = I\alpha$$
$$T_2 - m_2 g = m_2 a$$

$$m_1 g - T_1 = m_1 a$$
$$T_1 - T_2 = I\frac{\alpha}{R}$$
$$T_2 - m_2 g = m_2 a$$

$$m_1 g - \cancel{T_1} = m_1 a$$
$$\cancel{T_1} - \cancel{T_2} = \frac{Ia}{R^2}$$
$$\cancel{T_2} - m_2 g = m_2 a$$

$$\boxed{\frac{m_1 g - m_2 g}{m_1 + m_2 + \frac{I}{R^2}} = a}$$

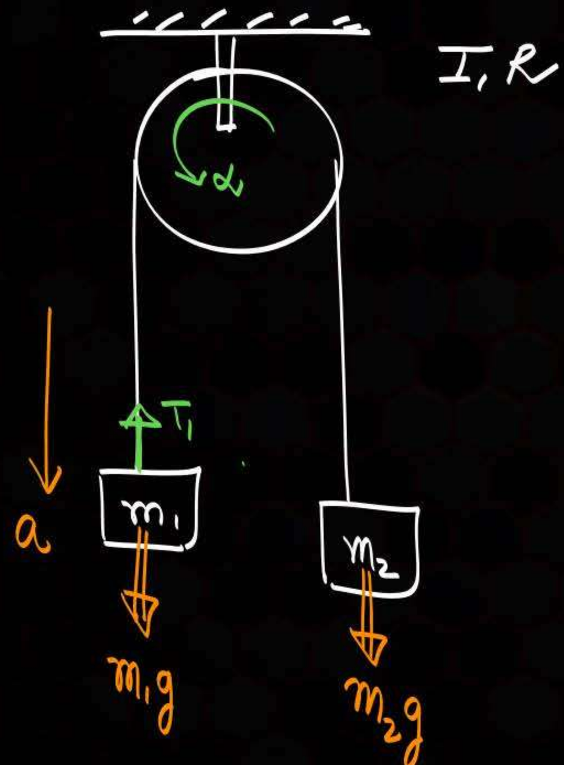


a

$$\boxed{a = R\alpha}$$

$$\alpha = \frac{a}{R}$$

Summary



$$\boxed{a = \frac{m_1 g - m_2 g}{m_1 + m_2 + \frac{I}{R^2}}}$$

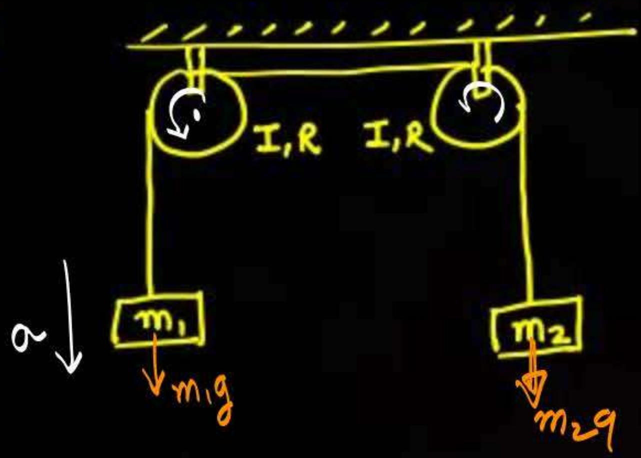
Effective Mass of Pulley in System

$$a = R\alpha$$

$$\alpha = \frac{a}{R}$$

$$m_1 g - T_1 = m_1 a$$

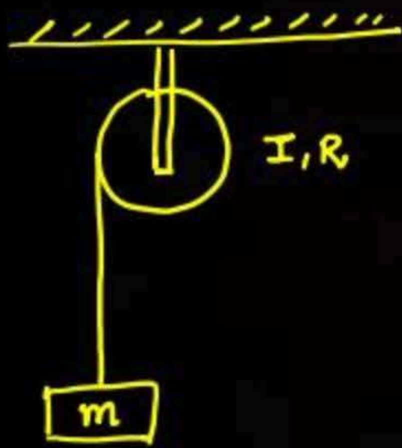
b)



$$\boxed{a = \frac{m_1 g - m_2 g}{m_1 + m_2 + \frac{2I}{R^2}}}$$

$$\alpha = \frac{a}{R}$$

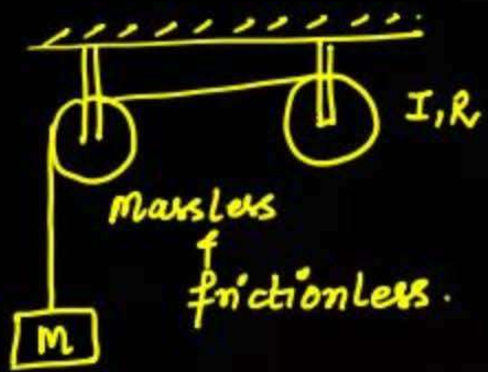
c)



$$\boxed{a = \frac{mg}{m + \frac{I}{R^2}}}$$

$$\alpha = \frac{a}{R}$$

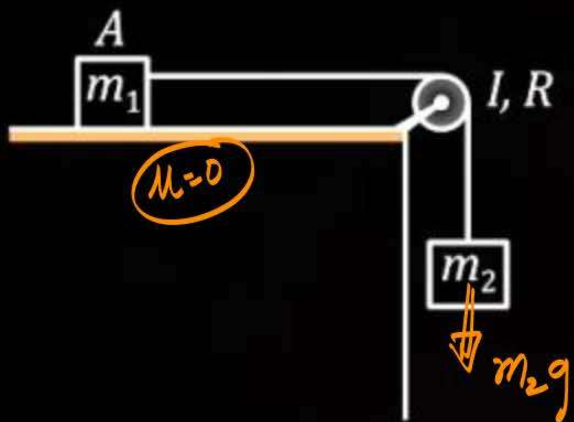
d)



$$\boxed{a = \frac{mg}{m + \frac{I}{R^2}}}$$

$$\boxed{\alpha = \frac{a}{R}}$$

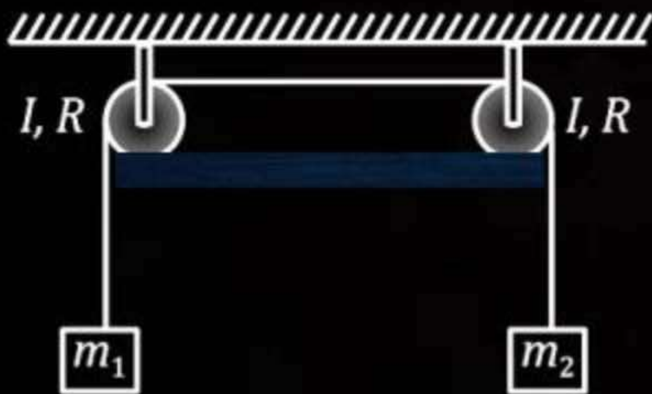
(a)



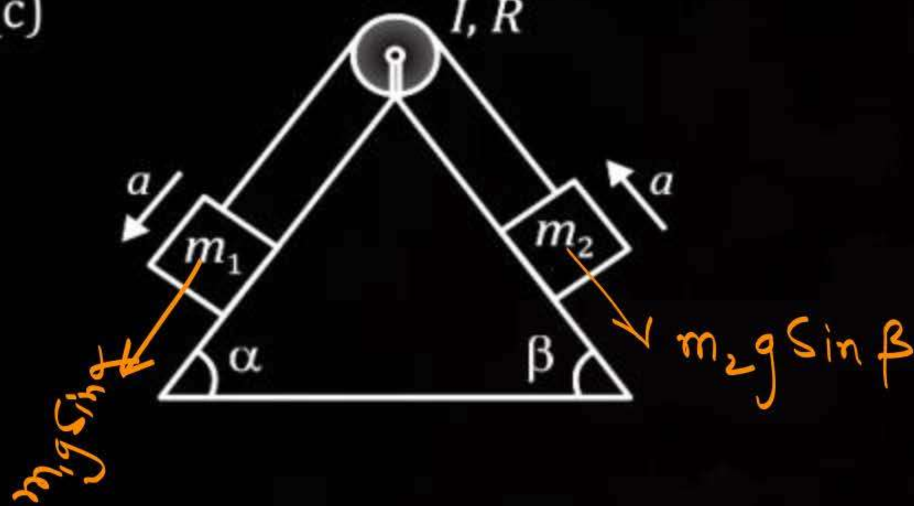
$$\boxed{a = \frac{m_2 g}{m_1 + m_2 + \frac{I}{R^2}}}$$

$$\boxed{\alpha = \frac{a}{R}}$$

(b)



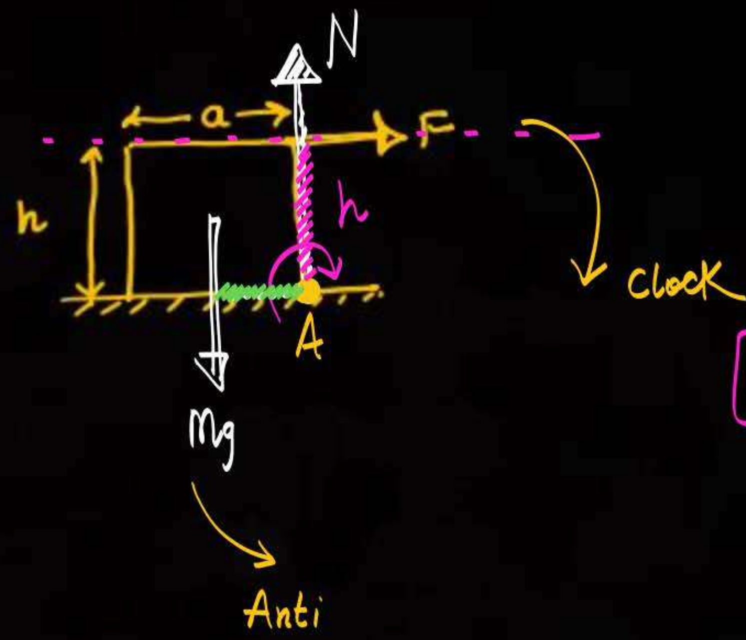
(c)



$$\boxed{a = \frac{m_1 g \sin\alpha - m_2 g \sin\beta}{m_1 + m_2 + \frac{I}{R^2}}}$$

Toppling

Force to topple a body.



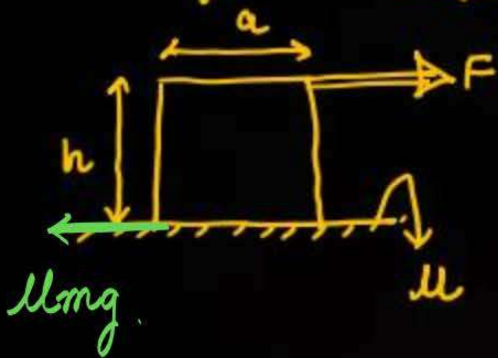
$$\boxed{\tau = F r_\perp}$$

$$Fh = mg\frac{a}{2}$$

$$\circledast \quad \boxed{F = \frac{mga}{2h}}$$

To topple the body.

⊛ Condition of Toppling before slipping



$F_{Topple} < F_{slipping}$

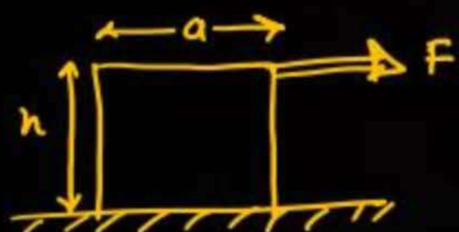
$\frac{\cancel{m}\cancel{g}a}{2h} < \mu_s \cancel{m}\cancel{g}$

$$\boxed{\frac{a}{2h} < \mu_s}$$

$F_{move} = \mu_s mg$

$F_{Topple} = \frac{mga}{2h}$

⊛ Condition of slipping before Toppling



$F_{move} < F_{Toppling}$

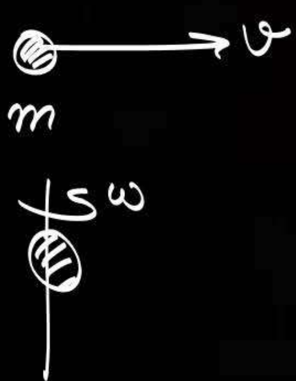
$\mu_s \cancel{m}\cancel{g} < \frac{\cancel{m}\cancel{g}a}{2h}$

$$\boxed{\mu_s < \frac{a}{2h}}$$

Work done by Torque :-

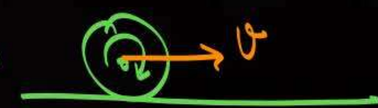
$$\boxed{\text{Work done by Torque} = \vec{\tau} \cdot d\vec{\theta}}$$

Kinetic Energy of Rotating body :-



$$\boxed{KE = \frac{1}{2} m v^2}$$

$$\boxed{KE = \frac{1}{2} I \omega^2}$$



$$KE_{Total} = KE_{Tran} + KE_{Rot}$$

$$\boxed{KE_{Tot} = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2}$$

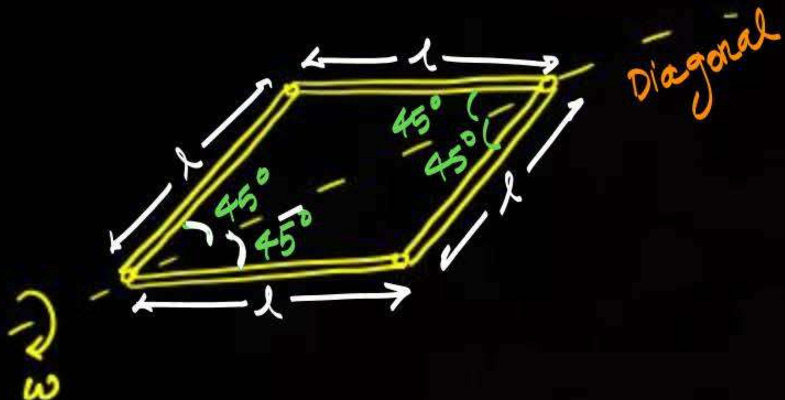
Work done by the torque = Change in kinetic energy of rotation $\boxed{W = \frac{1}{2}I\omega_2^2 - \frac{1}{2}I\omega_1^2}$

$\boxed{W = KE_f - KE_i}$

$\frac{dW}{dt} = \tau\frac{d\theta}{dt}$. Instantaneous rotational power $P_r = \tau\omega$. In general, $P_r = \vec{\tau}.\vec{\omega}$.

$$\boxed{\begin{array}{l} P = \vec{F}\cdot\vec{v} \\ \text{Power} = \vec{\tau}\cdot\vec{\omega} \end{array}}$$

Ex:-

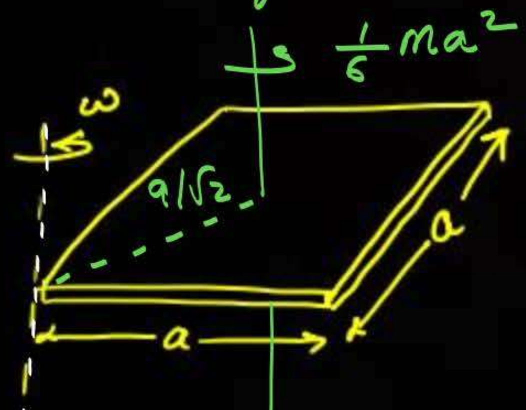


$$KE_{system} = \frac{1}{2}\left(I_{system\,axis}\right)\omega^2$$

$$= \frac{1}{2}\left(4 \times \frac{1}{3} ml^2 \sin^2 45\right)\omega^2$$

Each of mass = m,
Length = L

Ex:-



$$I_{axis} = \frac{1}{6}Ma^2 + M\left(\frac{a}{\sqrt{2}}\right)^2 = \frac{Ma^2}{6} + \frac{Ma^2}{2}$$

$$KE = \frac{1}{2}\left(\frac{Ma^2}{6} + \frac{Ma^2}{2}\right)\omega^2$$

Q. A thin uniform stick of length l and mass m is held horizontally with its end B hinged on the edge of a table. Point A is suddenly released. The acceleration of the centre of mass of the stick at the time of release, is :

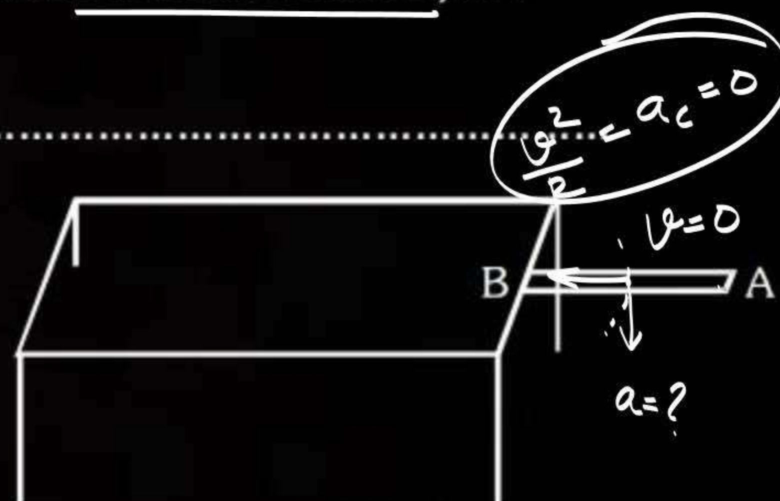
[(Mains) 2007]

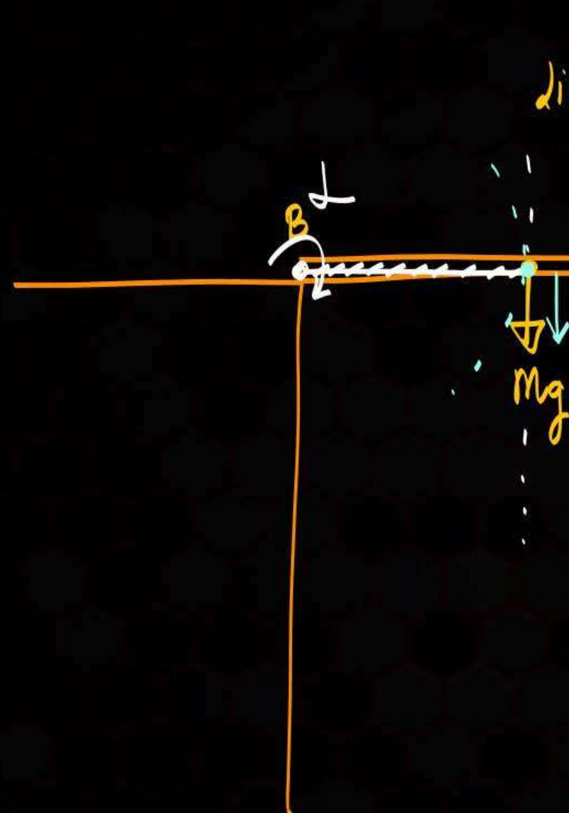
(A) $(3/4)g$ Ans

(B) $(3/7)g$

(C) $(2/7)g$

(D) $(1/7)g$





$$a_t = R\alpha$$

$$= \frac{L}{2}\cdot\frac{3g}{2L}$$

$$\boxed{a = \frac{3g}{4}}$$

$$\tau_B = (mg)\left[\frac{L}{2}\right] = I\alpha$$

$$\frac{\cancel{m}gL}{2} = \frac{1}{3}\cancel{m}L^2\alpha$$

$$\boxed{\frac{3g}{2L} = \alpha}$$

Q. A rigid massless rod of length 3 l has two masses attached at each end as shown in the figure. The rod is pivoted at point P on the horizontal axis (see figure). When released from initial horizontal position, its instantaneous angular acceleration will be: α

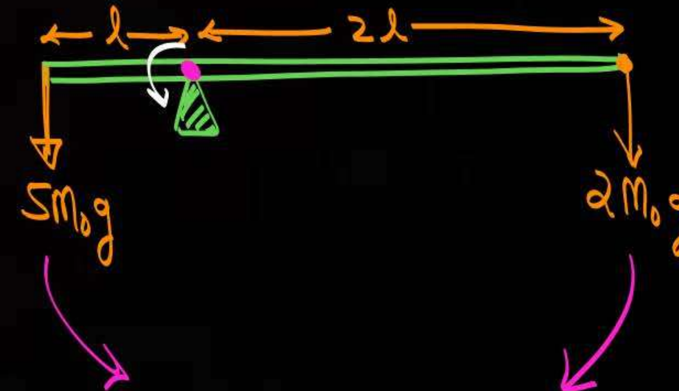
$$I_T = 5m_0 l^2 + 2m_0(2l)^2$$
$$= 5m_0 l^2 + 8m_0 l^2 = 13m_0 l^2$$

(A) $\frac{g}{13l}$ Ans

(B) $\frac{g}{3l}$

(C) $\frac{g}{2l}$

(D) $\frac{7g}{3l}$



$$\tau_{net} = 5m_0 g(l) - (2m_0 g)(2l) = I\alpha$$

$$5m_0 g l - 4m_0 g l = 13m_0 l^2 \alpha$$

$$\boxed{\frac{g}{13l} = \alpha} \quad \frac{m_0 g l}{13m_0 l^2} = \alpha$$

Q. A slab is subjected to two forces $\vec{F}_1$ and $\vec{F}_2$ of same magnitude F as shown in the figure. Force $\vec{F}_2$ is in XY-plane $(\hat{i}, \hat{j})$ while force F_1 acts along axis at the point $(2\vec{i} + 3\vec{j})$. The moment of these forces (Torque) about point O will be :

A $(3\hat{i} - 2\hat{j} + 3\hat{k})F$

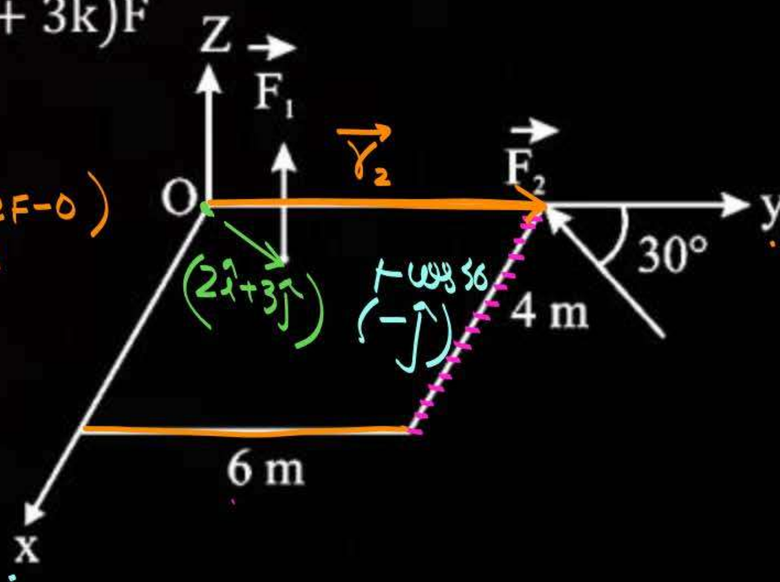
B $(3\hat{i} - 2\hat{j} - 3\hat{k})F$

C $(3\hat{i} + 2\hat{j} - 3\hat{k})F$

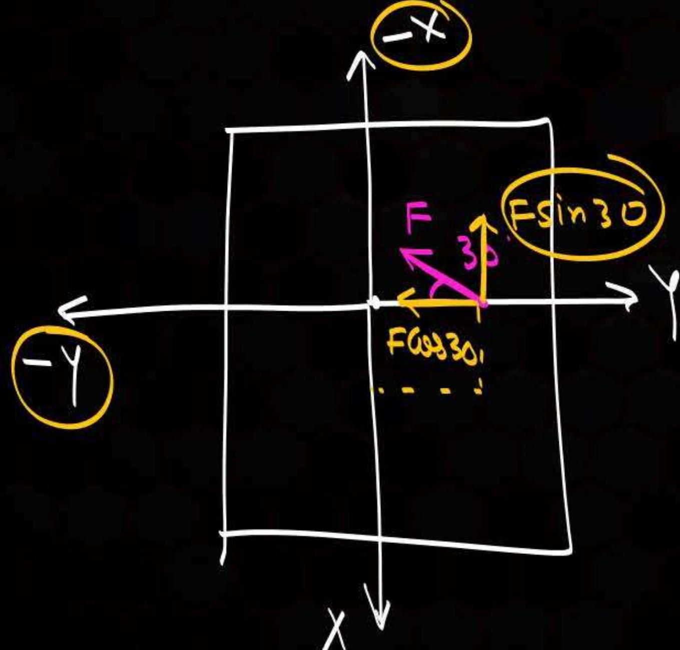
D $(3\hat{i} + 2\hat{j} + 3\hat{k})F$

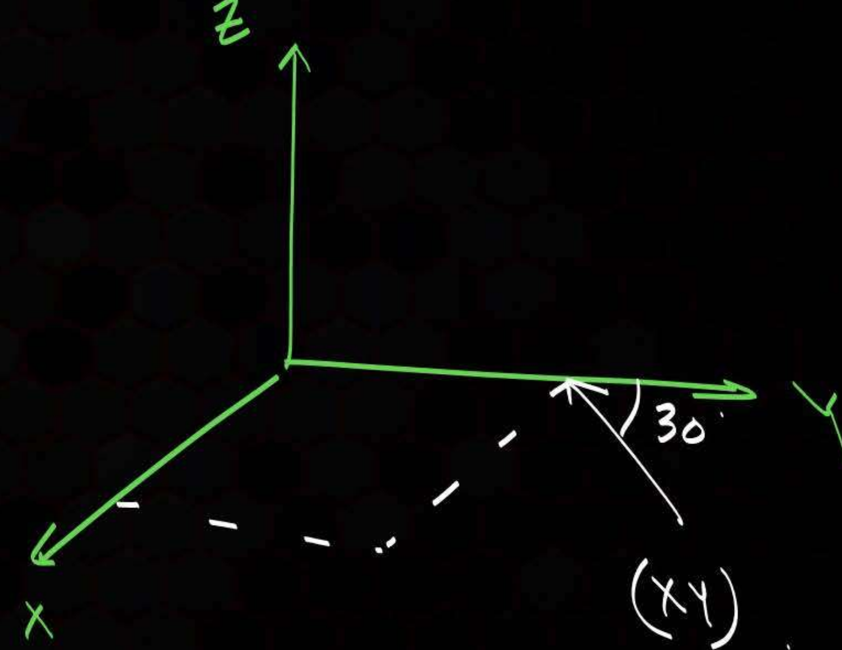
$$\vec{\tau}_{net} = \vec{\tau}_1 + \vec{\tau}_2$$

$$\vec{\tau}_1 = \vec{r}_1 \times \vec{F}_1 = \begin{vmatrix} \hat{i} & -\hat{j} & k \\ 2 & 3 & 0 \\ 0 & 0 & F \end{vmatrix} = \hat{i}(3F - 0) - \hat{j}(2F - 0) + \hat{k}(0 - 0)$$



$$\vec{\tau_2} = \vec{r_2} \times \vec{F} = \begin{vmatrix} \hat{i} & -\hat{j} & \hat{k} \\ 0 & 6 & 0 \\ -F\sin 30 & -F\cos 30 & 0 \end{vmatrix}$$





Q. A cord is wound round the circumference of wheel of radius r. The axis of the wheel is horizontal and moment of inertia about it is I. A weight mg is attached to the end of the cord and falls from the rest. After falling through a distance h, the angular velocity of the wheel will be

(A) $\sqrt{\dfrac{2gh}{I + mr}}$

(B) $\left[\dfrac{2mgh}{I + mr^2}\right]^{1/2}$

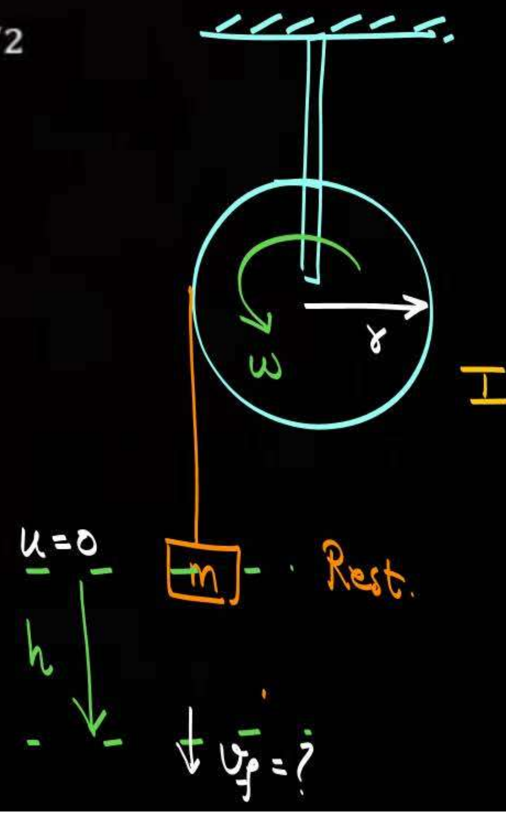
(C) $\left[\dfrac{2mgh}{I + 2mr^2}\right]^{1/2}$

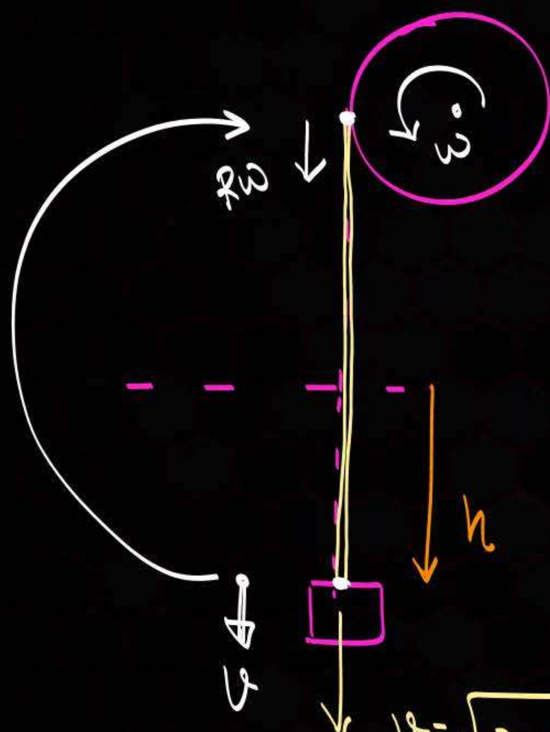
(D) $\sqrt{2gh}$

a) $a = \dfrac{mg}{m + \frac{I}{R^2}}$ Const.

$v_f^2 - \cancel{u^2} = 2ah$

$\boxed{v_f = \sqrt{2ah}}$





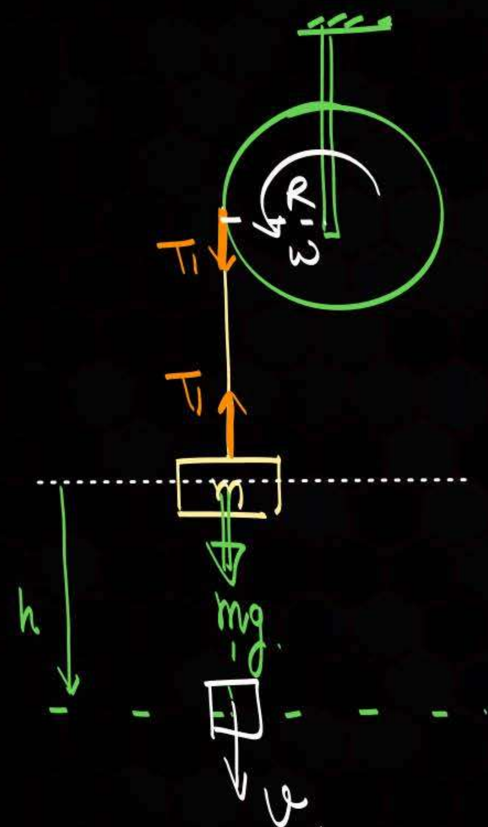
$$v = \sqrt{2ah} = \sqrt{2\left(\frac{mg}{m+\frac{I}{R^2}}\right)h}$$

$$v = R\omega$$

$$\frac{v}{R} = \omega_{Pulley}.$$

$$\boxed{\frac{\sqrt{2\left(\frac{mg}{m+\frac{I}{R^2}}\right)h}}{R} = \omega_{Pulley}}$$

Energy



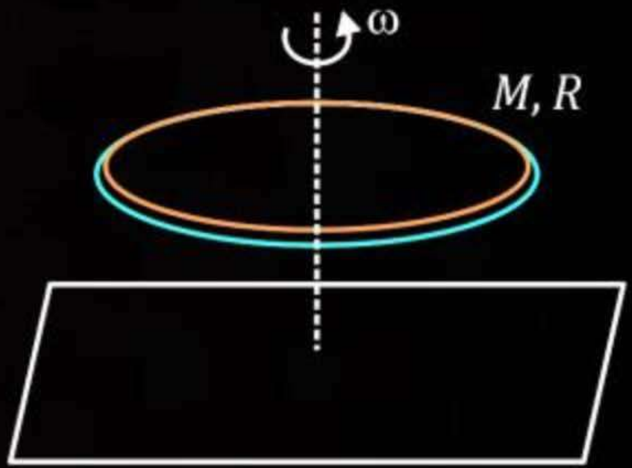
$$+mgh = K_f - K_i$$

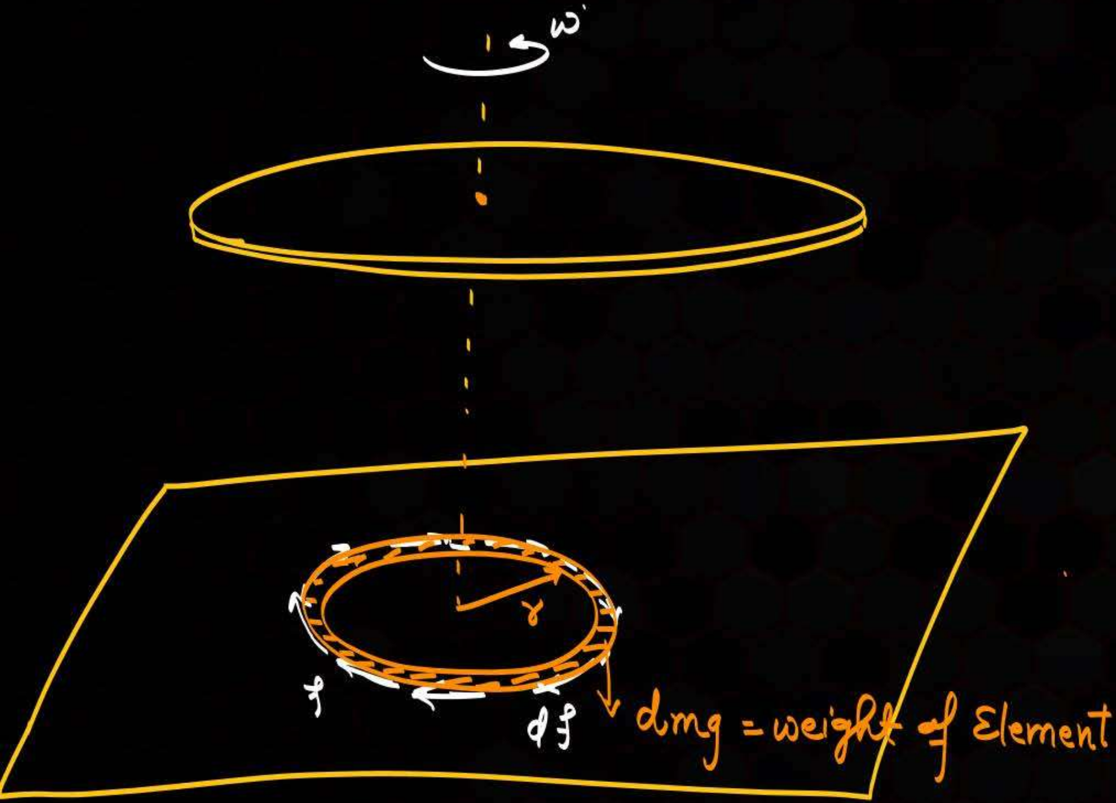
$$mgh = \left(\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2\right) - 0$$

$$v = R\omega$$

$$\omega = \_\_\_\_\_\_$$

Q. A uniform disc of radius R is first spined about its axis with angular speed 'ω' and then carefully placed on rough surface with coefficient of friction = μ. Find stopping time.





$$dm = \sigma \cdot 2\pi r\,dr$$

$$dm = \frac{M}{\cancel{\pi} R^2}\, 2\cancel{\pi} r\,dr$$

$$dm = \frac{2M r\,dr}{R^2}$$

$$df = \mu \overset{N}{dm\,g} = \frac{\mu 2 M r\,dr}{R^2}\, g$$

$$d\tau = df\, r_\perp = \left(\frac{2\mu M g}{R^2}\, r\,dr\right) r$$

$$d\tau = \frac{2\mu M g}{R^2}\, r^2\,dr$$

$$f = \mu N = \mu(dmg)$$

$$\tau_{Total} = \int d\tau$$

$$= \int_0^R \frac{2\mu M g}{R^2} r^2 dr$$

$$= \frac{2\mu M g}{R^2}\left[\frac{R^3}{3}\right]$$

$$\boxed{\tau_{net} = \frac{2\mu M g R}{3}}$$

$$\tau = I\alpha$$

$$\frac{2\mu \cancel{M} g R}{3} = \frac{1}{2}\cancel{M}R^2\alpha$$

$$\alpha = \frac{4\mu g}{3R}$$

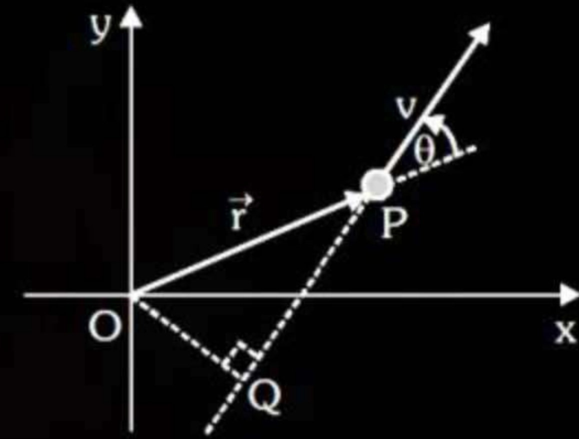
$$\cancel{\omega_f}^{\,0} = \omega_0 + \alpha t$$

$$\omega_0 = \frac{4\mu g}{3R}t$$

$$\frac{3R\omega_0}{4\mu g} = t$$

Angular momentum of a particle

Angular momentum $\vec{L}_o$ about the origin O of a particle of mass m moving with velocity $\vec{v}$ is defined as the moment of its linear momentum $\vec{p} = m\vec{v}$ about the point O.



$$\boxed{\vec{L}_o = \vec{r} \times (m\vec{v})} \qquad \boxed{\vec{L} = \vec{r} \times \vec{p}}$$

⊗ Analysis.

$$\vec{L} = rp\sin\theta.$$

$L = 0$ ← → L_{max}

$\theta = 0, \pi$ $\qquad$ $\theta = 90^\circ$

$$\boxed{L = rp}$$

⊗ Vector form.

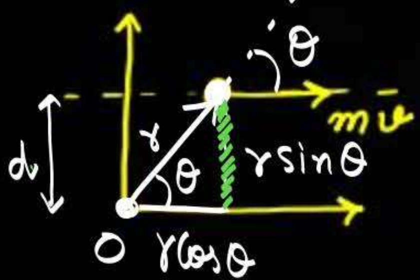
$$\vec{r} = r_x\hat{i} + r_y\hat{j} + r_z\hat{k}$$

$$\vec{p} = p_x\hat{i} + p_y\hat{j} + p_z\hat{k}$$

$$\vec{L} = \begin{vmatrix} \hat{i} & -\hat{j} & \hat{k} \\ r_x & r_y & r_z \\ p_x & p_y & p_z \end{vmatrix}.$$

Calculation of angular momentum

a) Point mass moving with Constant velocity.



$L_0 = r p \sin\theta$

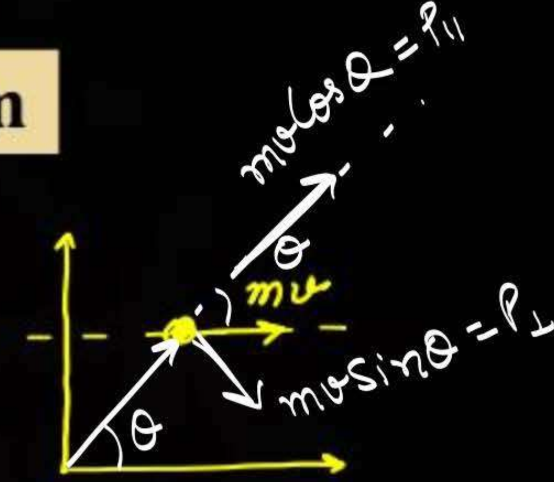
$L_0 = r p \sin\theta$

$\boxed{L_0 = p d}$

#

$L = p\, r_\perp$

$\boxed{L = p d}$



$L = r\, P_\perp$

$L = r m v \sin\theta$

$\boxed{L = m v d = p d}$

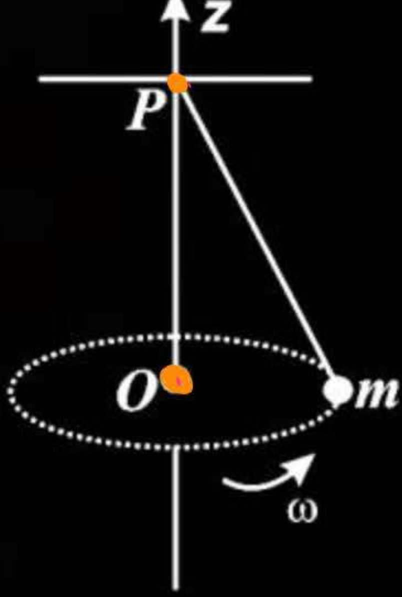
Q. A small mass m is attached to a massless string whose other end is fixed at P as shown in the figure. The mass is undergoing circular motion in the x–y plane with centre at O and constant angular speed ω. If the angular momentum of the system, calculated about O and P are denoted by $\vec{L}_0$ and $\vec{L}_p$ respectively, then

(A) $\vec{L}_0$ and $\vec{L}_p$ do not vary with time

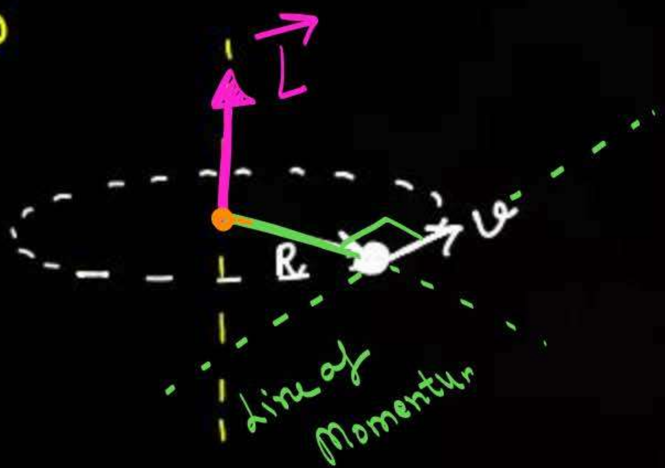
(B) $\vec{L}_0$ varies with time while $\vec{L}_p$ remains constant

(C) $\vec{L}_0$ remains constant while $\vec{L}_p$ varies with time Ans

(D) $\vec{L}_0$ and $\vec{L}_p$ both vary with time



Calculation of Angular Momentum.



about Centre.

$L = p r_\perp$

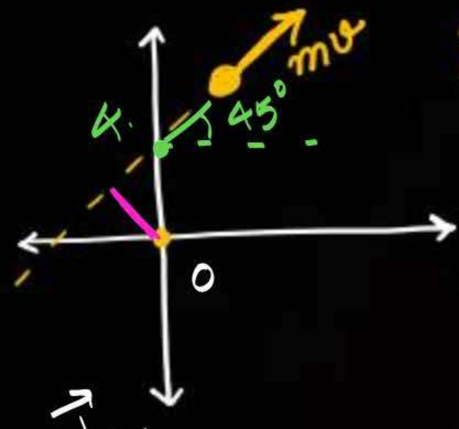
$|L| = mvR$

Constant → mag = Const
→ dir = Const

about point O.

$|L| = m v r_\perp$ = Const.

$r_\perp = \sqrt{R^2 + x^2}$



$y = x + 4$

Slope $= 1$

$\tan\theta = 1$

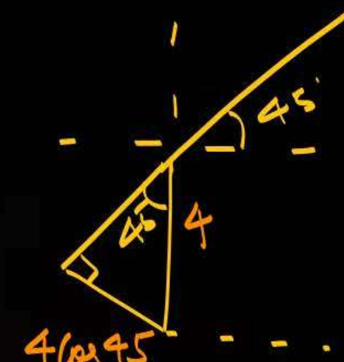
$\theta = 45^\circ$

Intercept

$c = 4$

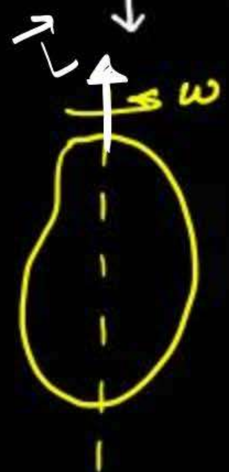
$\vec{L}_o = p\,r_\perp$

$= (mv)(2\sqrt{2})$



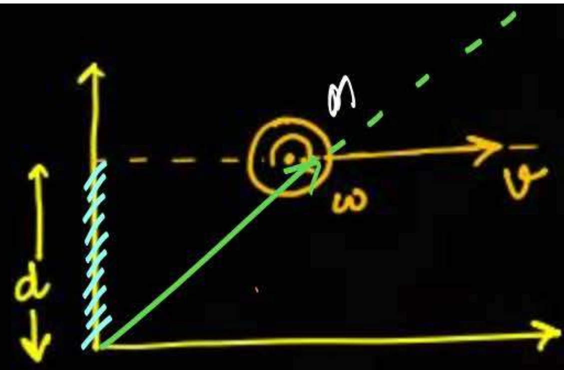
$4\cos 45$

$\frac{4}{\sqrt{2}} = (2\sqrt{2})$



$$\boxed{\vec{L}_{axis} = I_{body\ axis}\,\vec{\omega}}$$

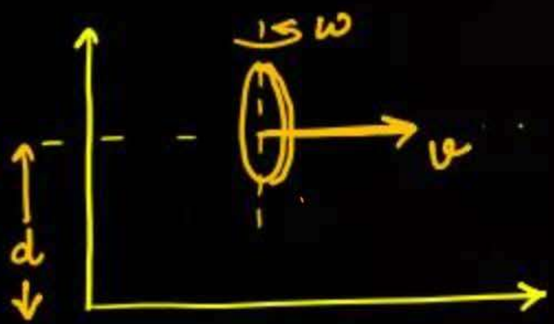
Continous body.



disc of mass m & Radius = R

$$\vec{L}_{Total\,O} = \vec{r} \times \vec{p} + I_{axis}\vec{\omega}$$

$$= (mvd)(-\hat{k}) + \left(\frac{1}{2}MR^2\omega\right)(-\hat{k})$$



disc of mass m, Radius R

$$\boxed{L_{Total} = mvd(-\hat{k}) + \left(\frac{1}{4}MR^2\omega\right)\hat{j}}$$

Q. Four point masses, each of mass m, are fixed at the corners of a square of side l. The square is rotating with angular frequency ω, about and axis passing through one of the corners of the square and parallel to its diagonal, as shown in the figure. The angular momentum of the square about this axis is :

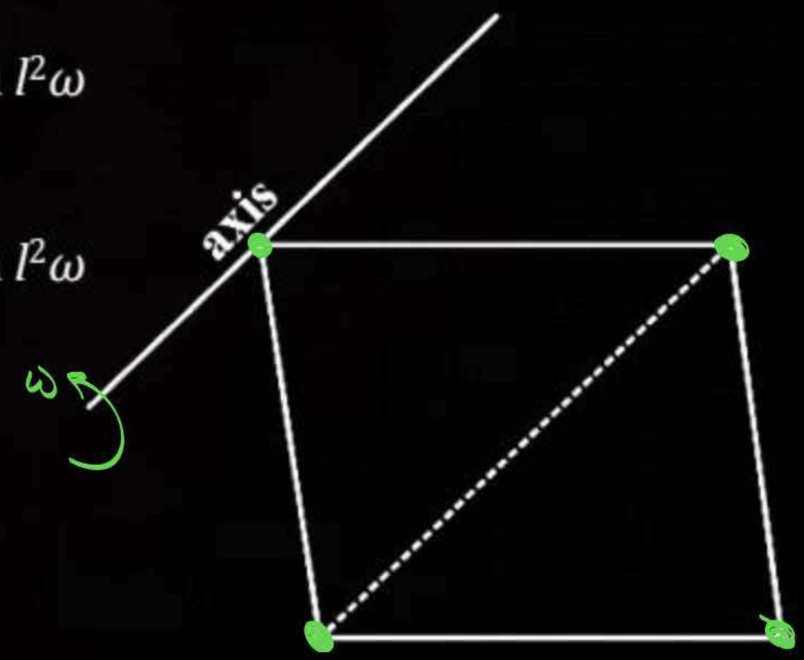
(Main Sep. 06, 2020)

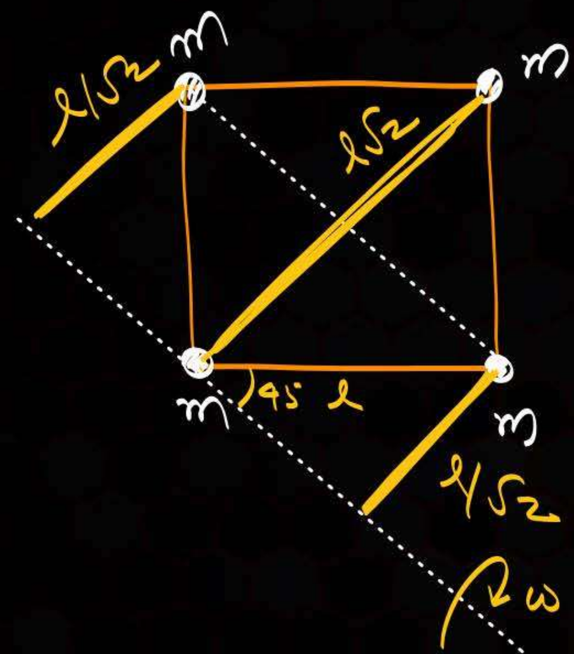
(A) $m l^2 \omega$

(B) $4 m l^2 \omega$

(C) $3m l^2 \omega$ Ans

(D) $2 m l^2 \omega$





$$L_{system} = (I_{axis})\,\omega$$

$$\boxed{L_{sym} = 3ml^2\omega}$$

$$I_{axis} = m(0)^2 + 2m\left(\frac{l}{\sqrt{2}}\right)^2 + m(l\sqrt{2})^2$$

$$I_{axis} = \frac{2ml^2}{2} + 2ml^2$$

$$= 3ml^2$$

Relation between torque & angular momentum

Translational Motion

Newton's Second Law

$$f_{ext} = \frac{dP}{dt}$$

$f_{ext} = 0$

$$\boxed{\vec{P} = \text{Conserved}}$$

$$\boxed{\vec{P_i} = \vec{P_f}}$$

Rotation.

$$\boxed{\vec{\tau}_{ext} = \frac{d\vec{L}}{dt}}$$

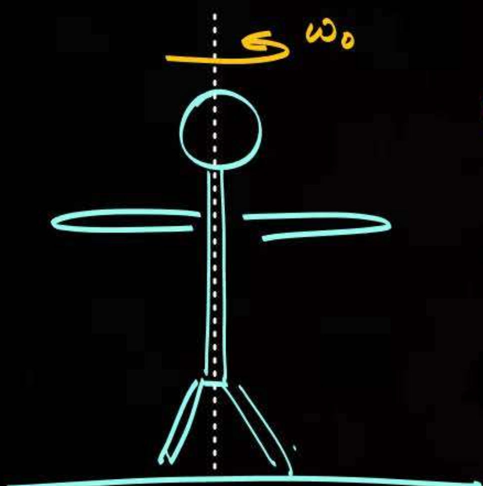
If $\tau_{ext} = 0$

$$0 = \frac{d\vec{L}}{dt}$$ → Constant

$\vec{L}$ = Conserved

$$\boxed{\vec{L_i} = \vec{L_f}}$$

Spinning Ice Skater



$\tau_{ext} = 0$

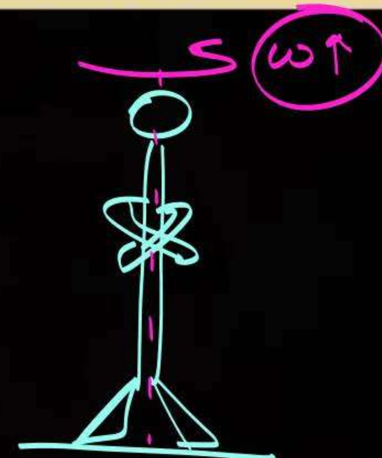
$\vec{L_i} = \vec{L_f}$

$I_1 \omega_1 = I_2 \omega_2$

$I\omega = \text{Constant}$

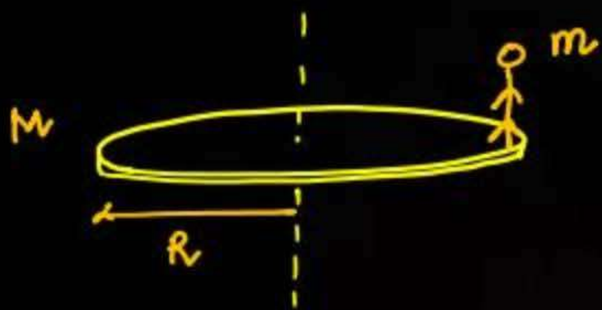
$\downarrow \ \uparrow$

Student on rotating turntable



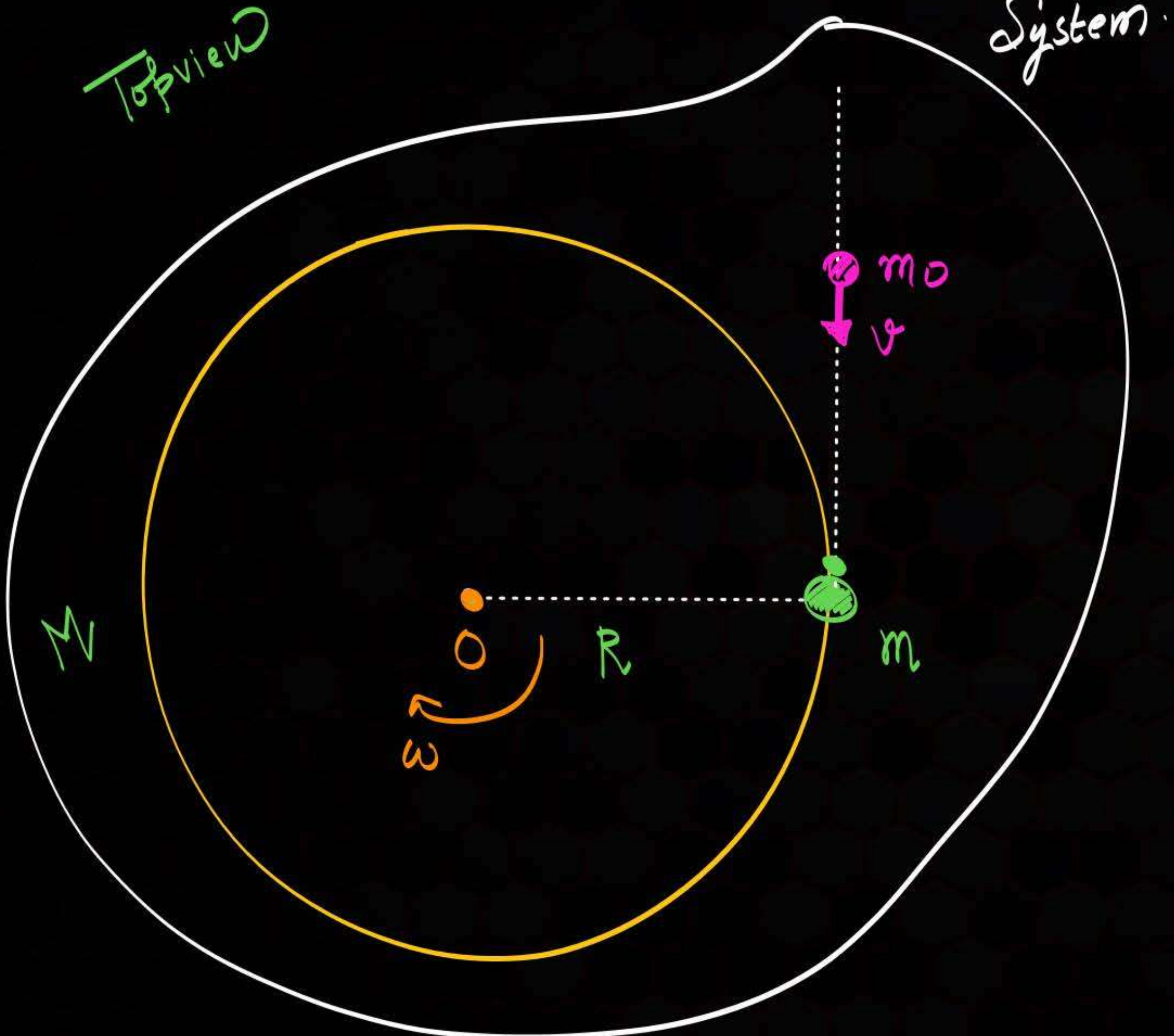
$I \downarrow$

#



Catches a ball of mass $= m_0$ moving with velocity v_0, tangentially to disc.

Find final ω of system.



$$L_i = m_0 v R$$

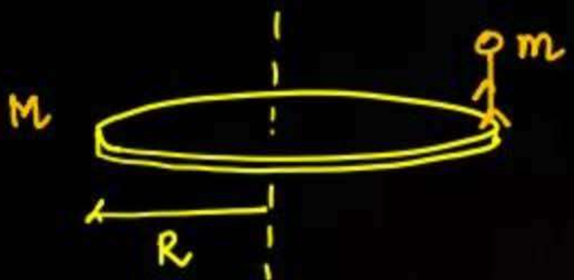
$$L_f = \left(I_{\text{system axis}}\right)\omega$$

$$= \left(\frac{1}{2}MR^2 + mR^2 + m_0R^2\right)\omega_f$$

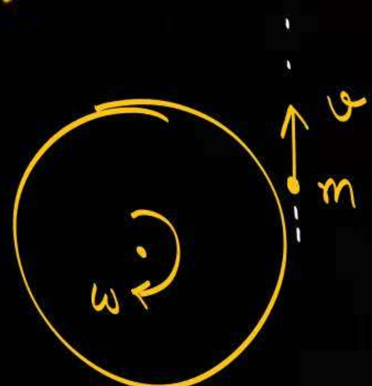
$$\vec{L}_i = \vec{L}_f$$

$$m_0 v R = \left(\frac{1}{2}MR^2 + mR^2 + m_0R^2\right)\omega_f$$

#



a) Person jumps with v wrt ground.



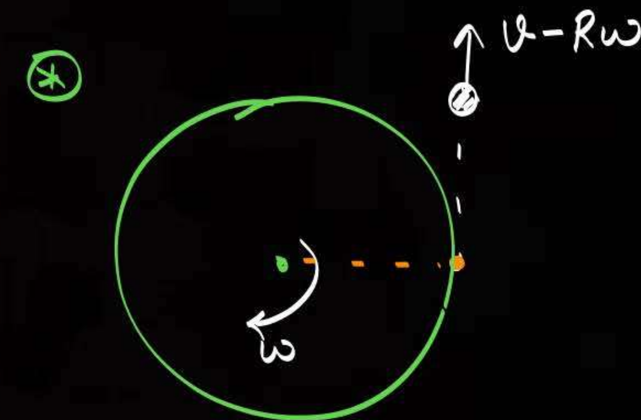
$$L_i = 0$$

$$L_f = mvR - I\omega$$

$$0 = mvR - I\omega$$

$$\boxed{\omega = \frac{mvR}{I}}$$

b) Person jumps with v relative to disc.



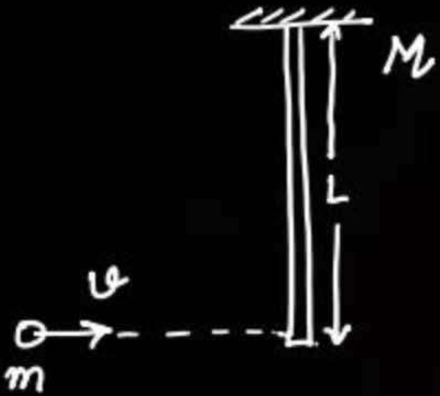
$$L_i = 0$$

$$L_f = m(v - R\omega)R - I\omega$$

$$mvR = mR^2\omega + I\omega$$

$$\boxed{\frac{mvR}{mR^2 + I} = \omega}$$

#



a) ball sticks to Rod find angular velocity of Rod.

b) If $e = 1$, Find final ω of Rod.

A solid sphere is rotating about a diameter at an angular velocity ω. If it cools so that its radius reduces to $\frac{1}{n}$ of its original value, its angular velocity becomes

(A) $\frac{\omega}{n}$

(B) $\frac{\omega}{n^2}$

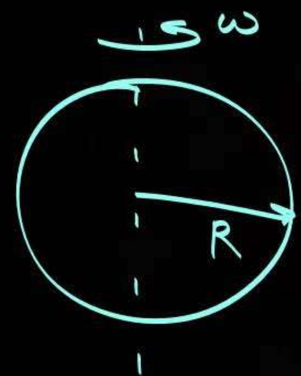
(C) $n\,\omega$

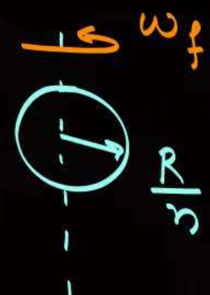
(D) $n^2\omega$ Ans

$\tau_{ext} = 0$

$$\frac{\cancel{2}}{\cancel{5}}\cancel{m}R^2\omega = \frac{\cancel{2}\cancel{m}\cancel{R^2}}{\cancel{5}\,n^2}\omega_f$$

$$\boxed{n^2\omega = \omega_f}$$





$$L_i = \frac{2}{5}mR^2\omega \quad = \quad L_f = \frac{2}{5}m\left(\frac{R}{n}\right)^2\omega_f$$

Q. A force $\vec{F} = \alpha\,\hat{i} + 3\,\hat{j} + 6\,\hat{k}$ is acting at a point $\vec{r} = 2\,\hat{i} - 6\,\hat{j} - 12\,\hat{k}$. The value of α for which angular momentum about origin is conserved is

[2015]

(A) 2

(B) Zero

(C) 1

(D) −1

$\alpha = ?$

$\vec{L_0}$ = Conserved.

$\vec{r} \times \vec{F} = 0$

$\vec{F} = \alpha\hat{i} + 3\hat{j} + 6\hat{k}$

$\vec{r} = 2\hat{i} - 6\hat{j} - 12\hat{k}$.

$$\frac{\alpha}{2} = \frac{3}{-6} = \frac{6}{-12} = \frac{-1}{2}$$

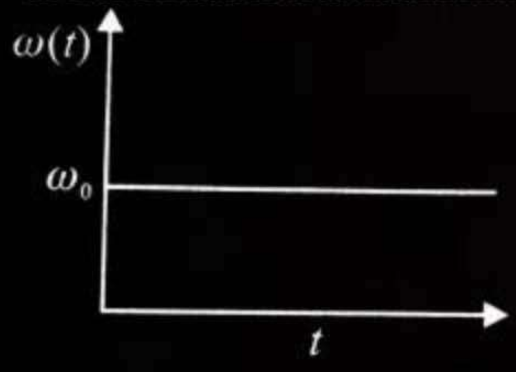
Ans

$$\frac{\alpha}{2} = \frac{-1}{2}$$

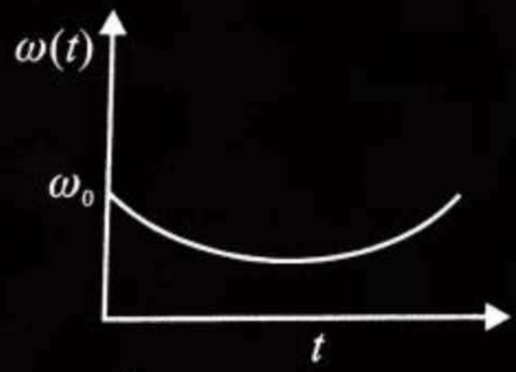
$\alpha = -1$

Q. A circular platform is free to rotate in a horizontal plane about a vertical axis passing through its centre. A tortoise is sitting at the edge of the platform. Now, the platform is given an angular velocity ω_0. When the tortoise move along a chord of the platform with a constant velocity (with respect to the platform). The angular velocity of the platfrom $\omega(t)$ will vary with time t as **[2002]**

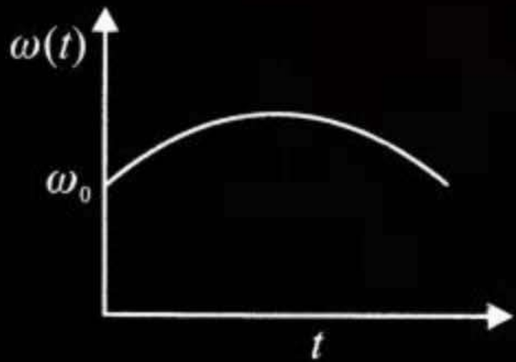
A



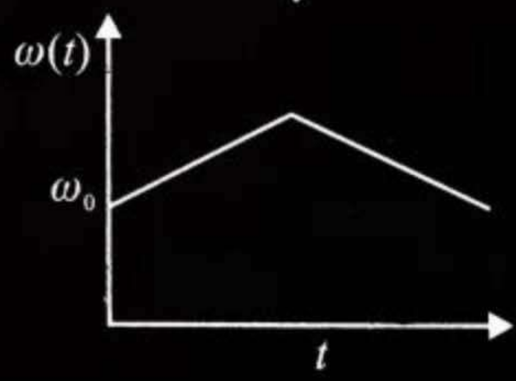
B



C

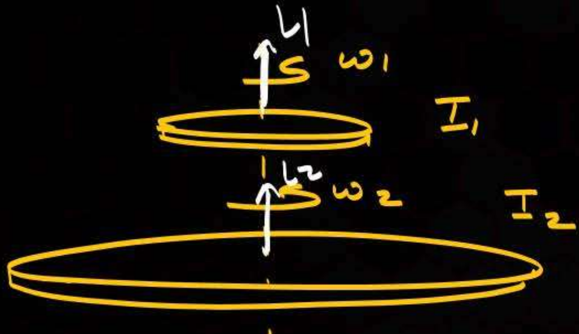


D



Combination of disc

Similar to Perfectly Inelastic Collission

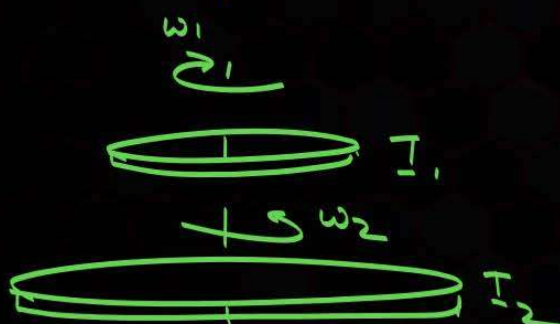


$$L_i = I_1\omega_1 + I_2\omega_2$$

$$L_f = (I_1 + I_2)\,\omega_f$$

$$\omega_f = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$$

$$\text{Loss} = \frac{1}{2}\frac{I_1 I_2}{I_1 + I_2}(\omega_1 - \omega_2)^2$$



$$\omega_f = \frac{I_2\omega_2 - I_1\omega_1}{I_1 + I_2}$$

$$\text{Loss} = \frac{1}{2}\frac{I_1 I_2}{I_1 + I_2}(\omega_1 + \omega_2)^2$$

Considering $I_2\omega_2 > I_1\omega_1$

Q. Two coaxial discs, having moments of inertia I_1 and $\frac{I_1}{2}$ are rotating with respective angular velocities ω_1 and $\frac{\omega_1}{2}$, about their common axis. They are brought in contact with each other and thereafter they rotate with a common angular velocity. If E_f and E_i are the final and initial total energies, then $(F_f - E_i)$ is **[2019 Main]**

HW

(A) $-\frac{I_1\omega_1^2}{24}$

(B) $-\frac{I_1\omega_1^2}{12}$

(C) $\frac{3}{8} I_1\omega_1^2$

(D) $\frac{I_1\omega_1^2}{6}$

Q. A horizontal circular plate is rotating about a vertical axis passing through its centre with an angular velocity ω_0. A man sitting at the centre having two blocks in his hands stretches out his hands so that the moment of inertia of the system doubles. If the kinetic energy of the system is K initially, its final kinetic energy will be

....

(A) 2K

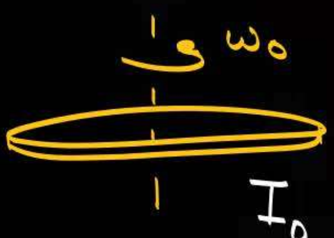
(B) K/2 Ans

(C) K

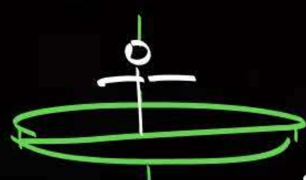
(D) K/4

$$KE_f = \frac{1}{2} I_f \omega_f^2$$

$$= \frac{1}{2}(2I_0)\frac{\omega_0^2}{4}$$



$K = \frac{1}{2} I_0 \omega_0^2$



KE_f

$$\cancel{I_0}\,\omega_0 = 2\cancel{I_0}\,\omega_f$$

$$\boxed{\frac{\omega_0}{2} = \omega_f}$$

$$\boxed{KE_f = \frac{K}{2}}$$

Q. A round disc of moment of inertia I_2 about its axis perpendicular to its plane and passing through its centre is placed over another disc of moment of inertia I_1 rotating with an angular velocity ω about the same axis. The final angular velocity other combination of discs is **[2004]**

H/W

(A) $\dfrac{I_2\omega}{I_1 + I_2}$

(B) ω

(C) $\dfrac{I_1\omega}{I_1 + I_2}$

(D) $\dfrac{(I_1+I_2)\omega}{I_1}$

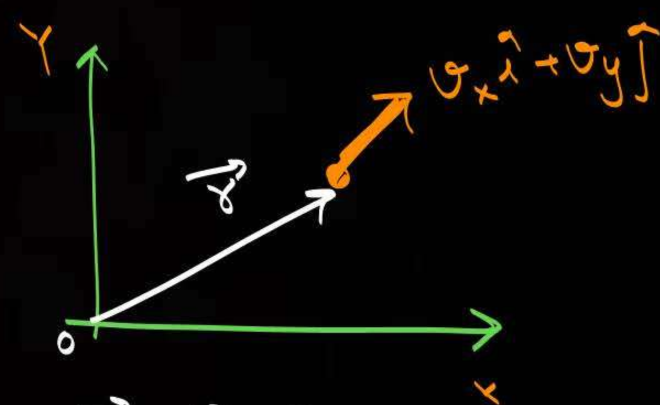
Q. The time dependence of the position of a particle of mass $m = 2$ is given by $\vec{r}\,(t) = 2\,t\,\hat{\imath} - 3t^2\,\hat{\jmath}$. Its angular momentum, with respect to the origin, at time $t = 2$ is : **[Main 10 Apr. 2019 II]**

(A) $48(\hat{\imath} + \hat{\jmath})$

(B) $36\hat{k}$

(C) $-34(\hat{k} - \hat{\imath})$

(D) $-48\,\hat{k}$



$t=2 \quad 4\hat{i} - 12\hat{j}$

$\vec{r} = 2t\,\hat{i} - 3t^2\hat{j}$ $\qquad t = 2s. \qquad \vec{L}_0$

$\vec{v} = \dfrac{d\vec{r}}{dt} = 2\hat{i} - 6t\hat{j}$

at $t=2$ $\vec{v} = 2\hat{i} - 12\hat{j}$

$\vec{L}_0 = \vec{r} \times \vec{p}$

$\vec{L}_0 = m(\vec{r} \times \vec{v})$

$= 2\begin{vmatrix} \hat{i} & -\hat{j} & \hat{k} \\ 4 & -12 & 0 \\ 2 & -12 & 0 \end{vmatrix} =$

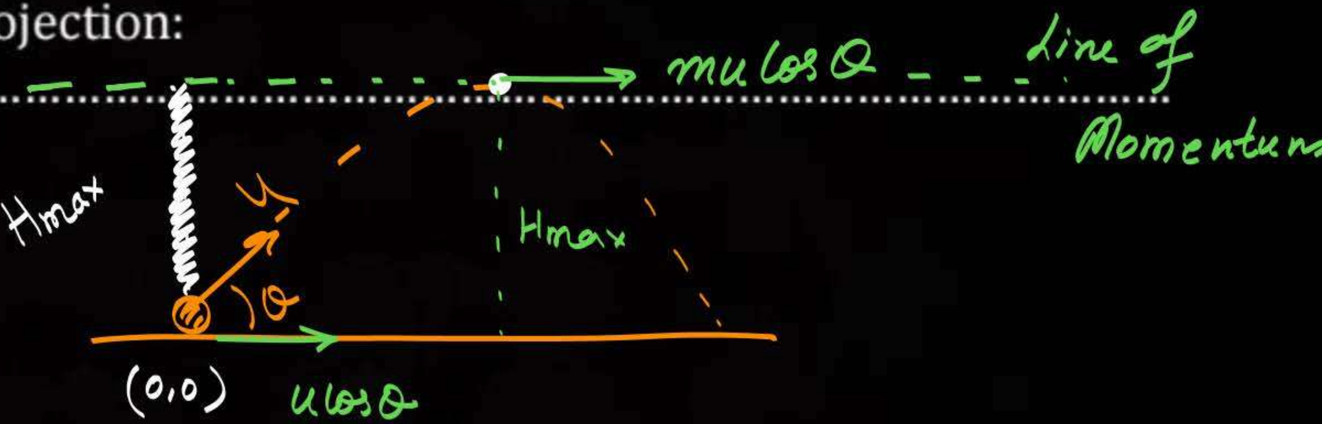
Q. A ball of mass 160 g is thrown up at angle 60° to the horizontal at speed 10 m/s. the angular momentum of ball at highest point of trajectory with respect to point of projection:

(A) 1.73

(B) 3.0

(C) 3.46

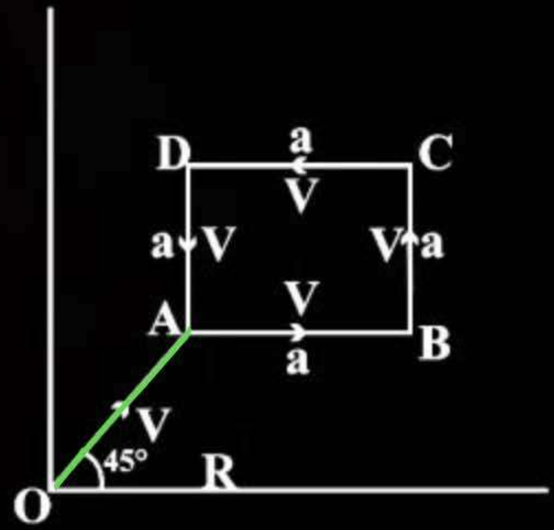
(D) 6.0



$$L_{Projection} = p\,r_{\perp}$$

$$= mu\cos\theta\left(\frac{u^2\sin^2\theta}{2g}\right).$$

Q. A particle of mass m is moving along the side of a square of side 'a', with a uniform speed v in x–y plane as shown in the figure. Which of the following statements is false for the angular momentum $\vec{L}$ about the origin?

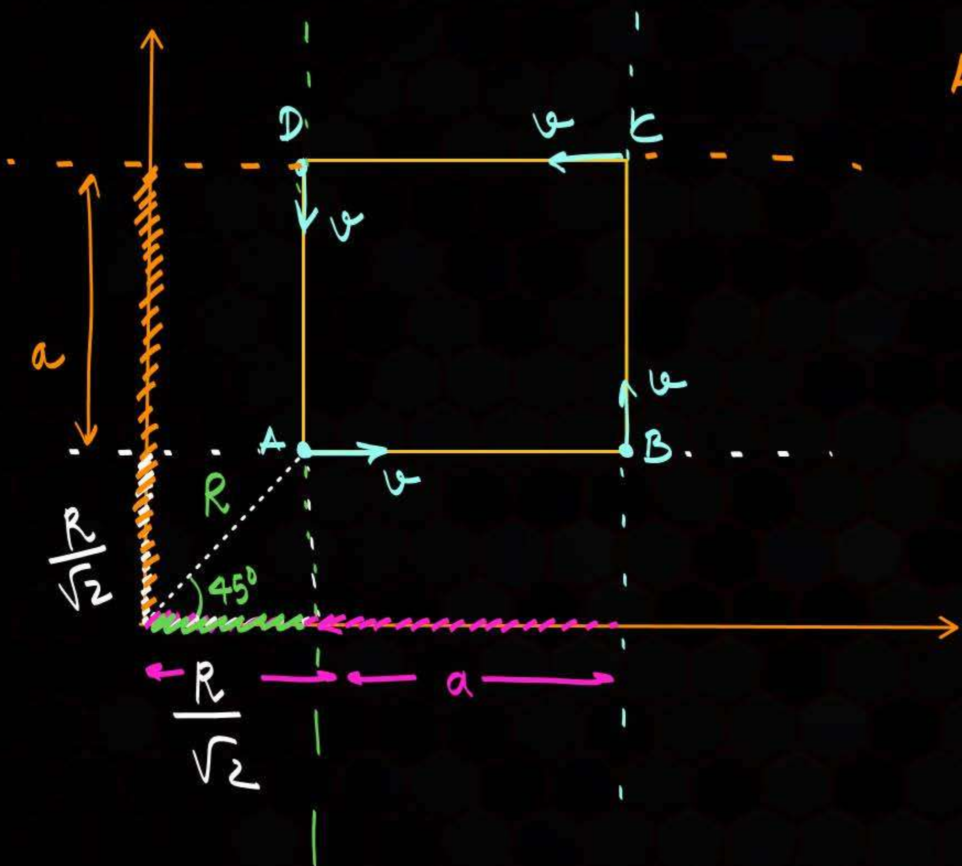


(A) $\vec{L} = \text{mv}\left[\frac{R}{\sqrt{2}} + a\right]\vec{k}$ when the particle is moving from B to C.

(B) $\vec{L} = \frac{mv}{\sqrt{2}} \text{R}\,\vec{k}$ when the particle is moving from D to A.

(C) $\vec{L} = -\frac{mv}{\sqrt{2}} \text{R}\,\vec{k}$ when the particle is moving from A to B.

(D) $\vec{L} = \text{mv}\left[\frac{R}{\sqrt{2}} - a\right]\vec{k}$ when the particle is moving from C to D.



$A \to B$

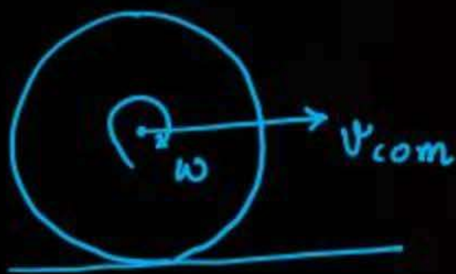
$$L_{A\to B} = \frac{mvR}{\sqrt{2}}(-\hat{k})$$

$$L_{B\to C} = mv\left(\frac{R}{\sqrt{2}} + a\right)(\hat{k})$$

$$L_{C\to D} = mv\left(\frac{R}{\sqrt{2}} + a\right)(\hat{k})$$

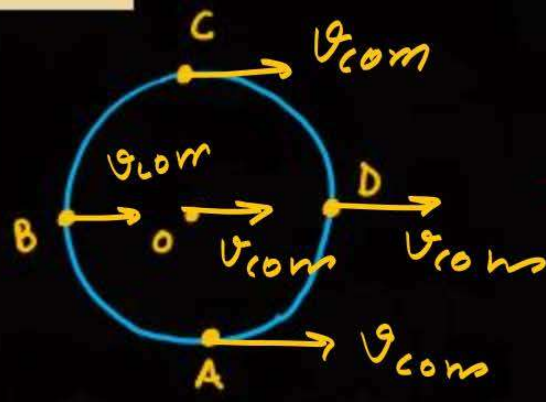
$$L_{D\to A} = mv\left(\frac{R}{\sqrt{2}}\right)(-\hat{k})$$

ROLLING MOTION



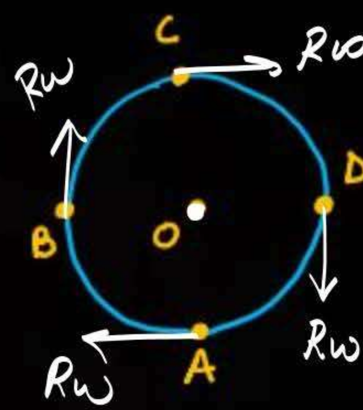
Rolling

=



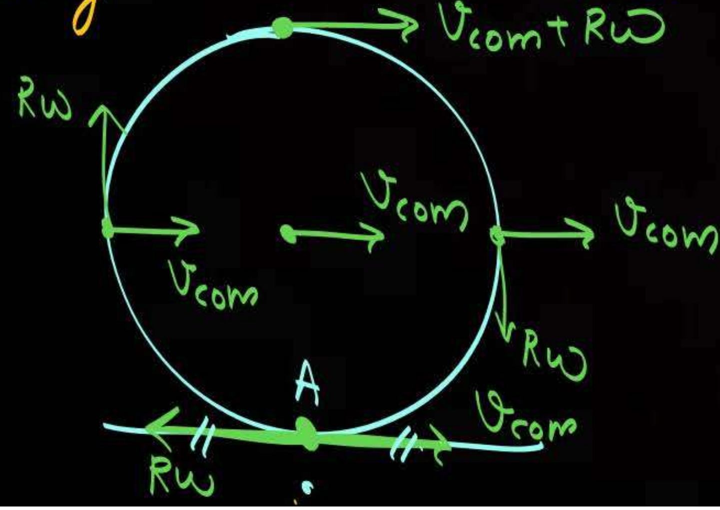
Pure Trans

+



Pure Rot.

Break of 5mins

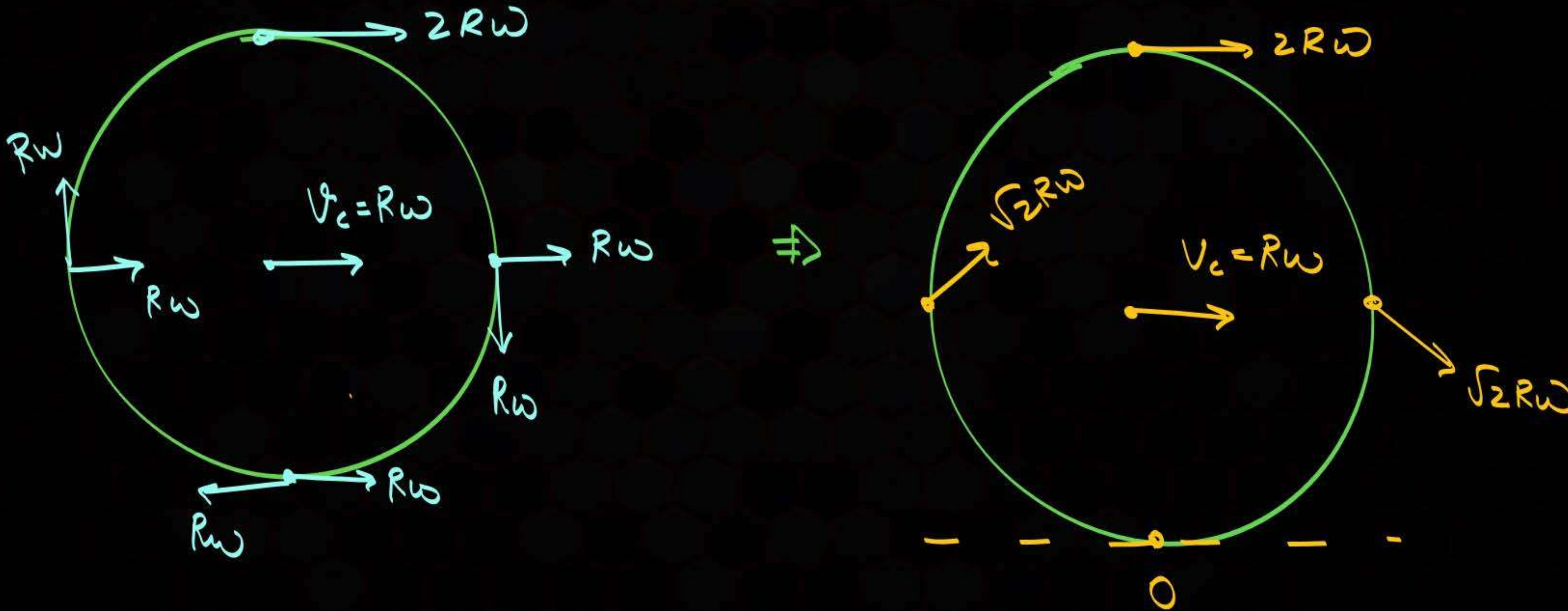


Rolling without Slip.

$\boxed{\text{Relative velocity of Point A wrt gr} = 0}$

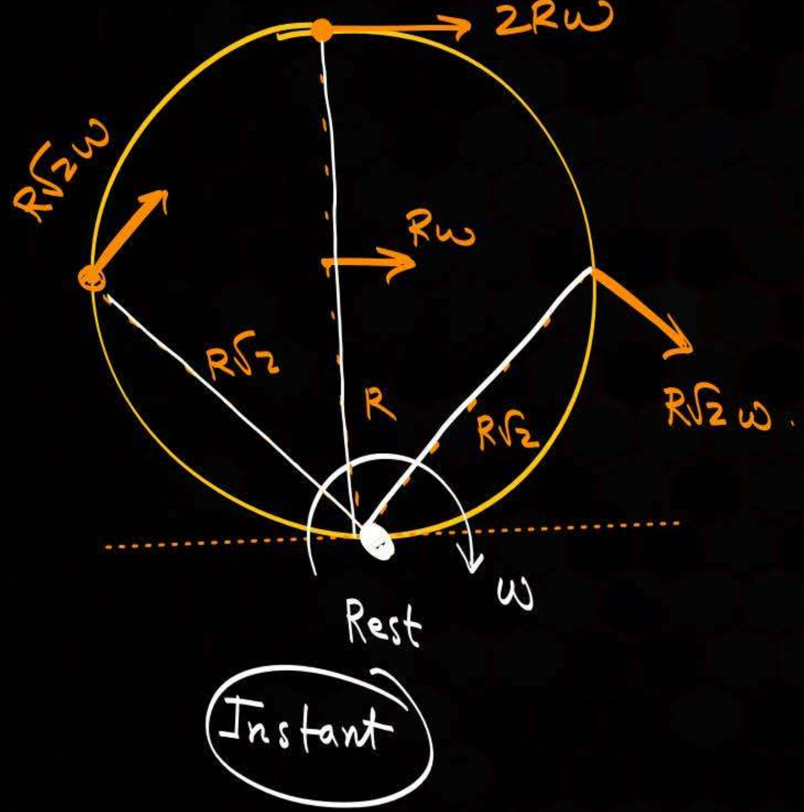
Condition of Pure Rolling

$$\boxed{v_{com} = R\omega}$$



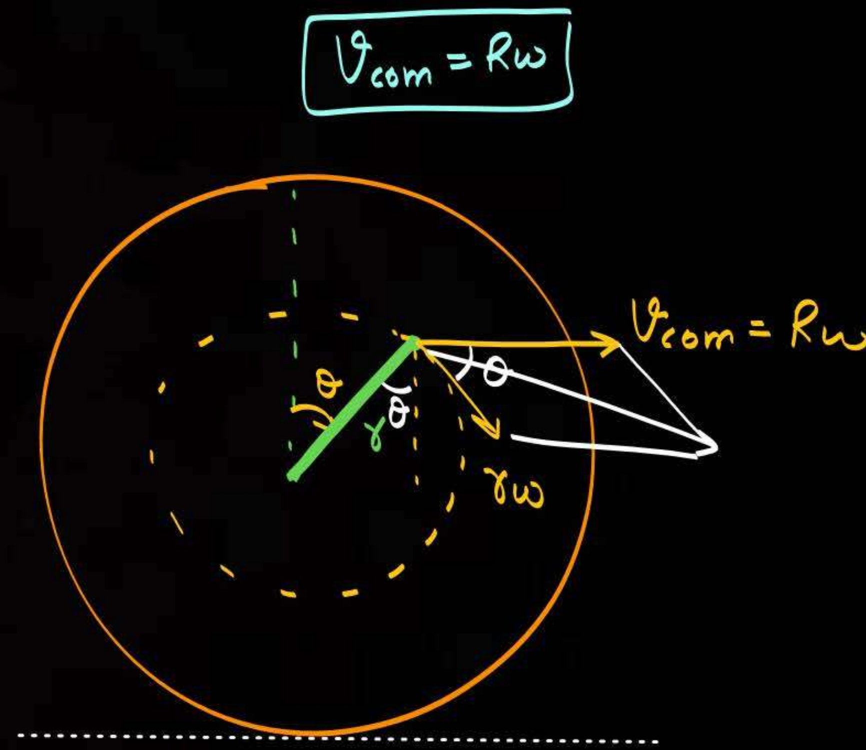
Instantaneous axis of Rot

$\boxed{v = r\omega}$



Velocity at a point on the rim of a rolling sphere

$$\boxed{v_{net} = \sqrt{(r\omega)^2 + (R\omega)^2 + 2Rr\omega^2\cos\theta}}$$



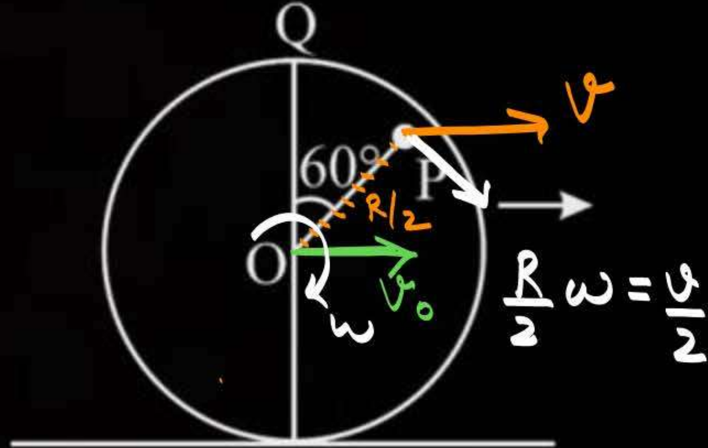
Q. A disc of radius r rolls without slipping on a rough horizontal floor. If velocity of its center of mass is v_0, then velocity of point P, as shown in the figure (OP = r/2), is

(A) v_0

(B) $\frac{v_0}{2}$

(C) $\frac{v_0}{2}\sqrt{7}$ ✓

(D) $\frac{v_0}{2}\sqrt{3}$



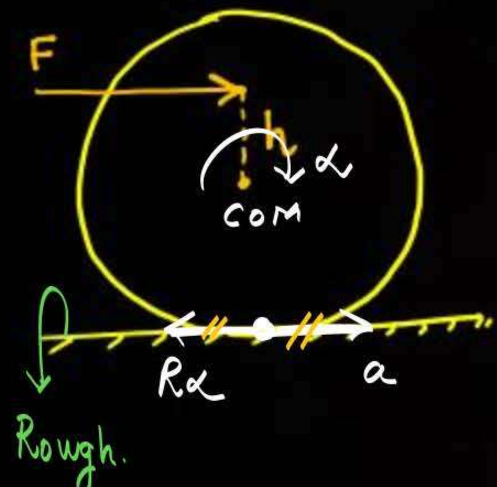
Ans

$$v_{net} = \sqrt{v^2 + \left(\frac{v}{2}\right)^2 + 2 \cdot v \cdot \frac{v}{2}\cos 60}$$

$$= \sqrt{v^2 + \frac{v^2}{4} + \frac{v^2}{2}} = \sqrt{\frac{4v^2 + v^2 + 2v^2}{4}} = \sqrt{\frac{7v^2}{4}}$$

$v = R\omega$

Rolling without slipping (Pure rolling)



Let h be the height were $f = 0$.

Tr $\quad F = Ma$

$a = \frac{F}{M}$

Rot $\quad Fh = I\alpha$

$Fh = ZMR^2\alpha$

$Fh = ZMR(R\alpha)$

$\frac{\cancel{F}h}{ZMR} = R\alpha = a_t = a = \frac{\cancel{F}}{M}$

$\boxed{h = ZR}$

$I = ZMR^2$

$\Downarrow$

Ring $\quad Z = 1$

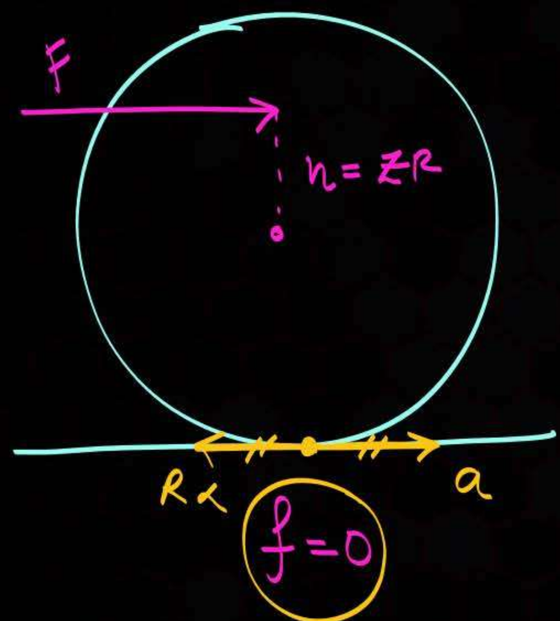
Disc $\quad Z = 1/2$

hollow sphere $\quad = 2/3$

Solid Sphere $\quad = 2/5$

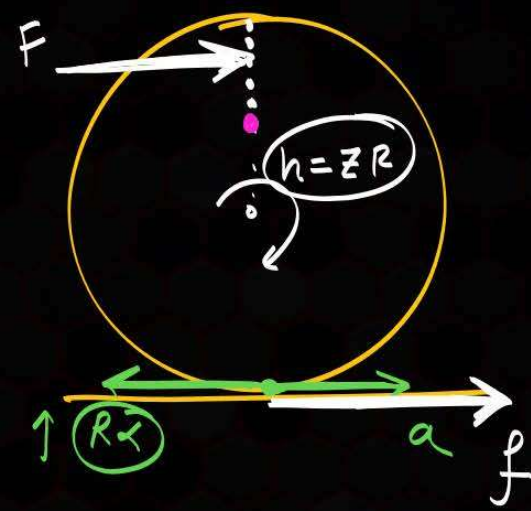
friction direction:-

1

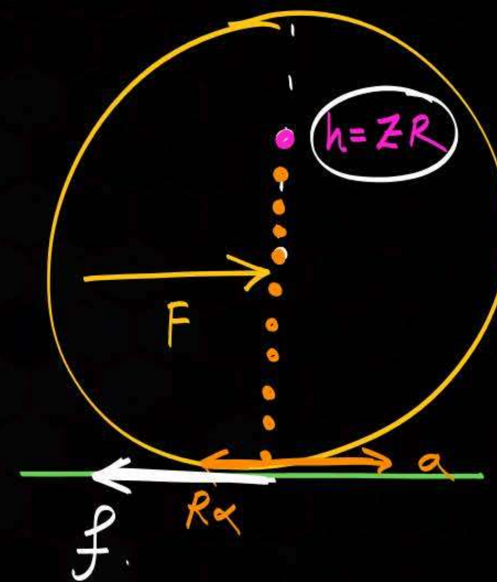


2

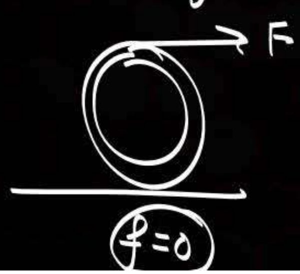
h > 2R



3



Ex :- Ring 2 = 1

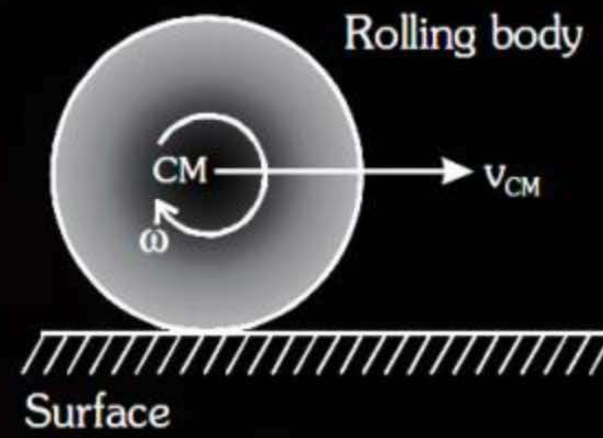


Rolling Kinetic Energy

Rolling Kinetic Energy $E = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \Rightarrow \frac{1}{2}mv^2 + \frac{1}{2}mK^2\left(\frac{v^2}{R^2}\right)$

Rolling Kinetic Energy $E = \frac{1}{2}mv^2\left(1 + \frac{K^2}{R^2}\right)$

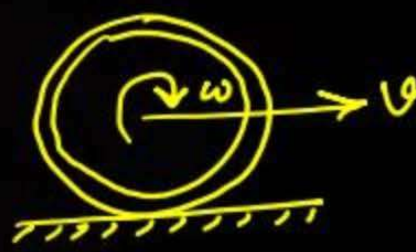
$$E_{translation} : E_{rotation} : E_{Total} = 1 : \frac{K^2}{R^2} : 1 + \frac{K^2}{R^2}$$



Total energy in rolling = Translatory kinetic energy + Rotatory kinetic energy.

Body	$\frac{K^2}{R^2}$	$\frac{E_{trans}}{E_{rotation}} = \frac{1}{\left(\frac{K^2}{R^2}\right)}$	$\frac{E_{trans}}{E_{total}} = \frac{1}{\left(1+\frac{K^2}{R^2}\right)}$	$\frac{E_{rotation}}{E_{total}} = \frac{K^2/R^2}{\left(1+\frac{K^2}{R^2}\right)}$
Ring	1	1	$\frac{1}{2}$	$\frac{1}{2}$
Disc	$\frac{1}{2}$	2	$\frac{2}{3}$	$\frac{1}{3}$
Solid sphere	$\frac{2}{5}$	$\frac{5}{2}$	$\frac{5}{7}$	$\frac{2}{7}$
Spherical shell	$\frac{2}{3}$	$\frac{3}{2}$	$\frac{3}{5}$	$\frac{2}{5}$
Solid cylinder	$\frac{1}{2}$	2	$\frac{2}{3}$	$\frac{1}{3}$
Hollow cylinder	1	1	$\frac{1}{2}$	$\frac{1}{2}$

Q. A ring of mass m is rolling without slipping with linear velocity v as shown in figure. A rod of identical mass is fixed along one of its diameter. The total kinetic energy of system is:



(A) $(7/5)mv^2$

(B) $(2/3)mv^2$

(C) $(4/3)mv^2$

(D) $(5/3)mv^2$

ROLLING MOTION ON AN INCLINED PLANE

a) Case-1 $\mu = 0$

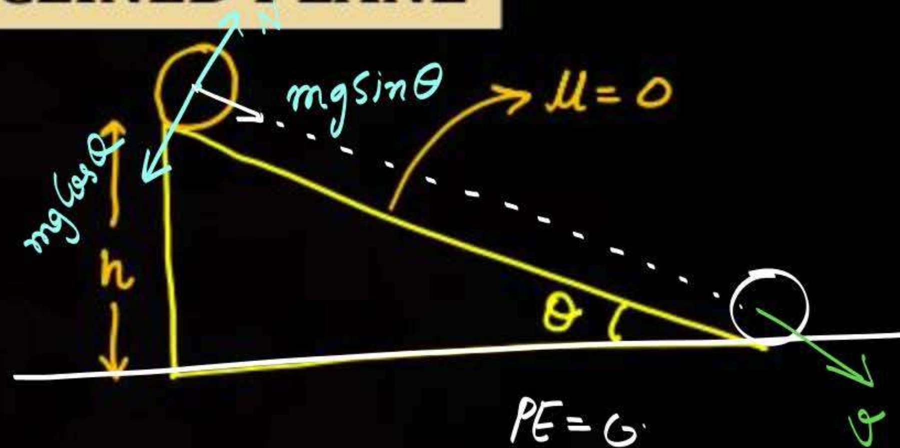
Body will not Roll down!

It will slip.

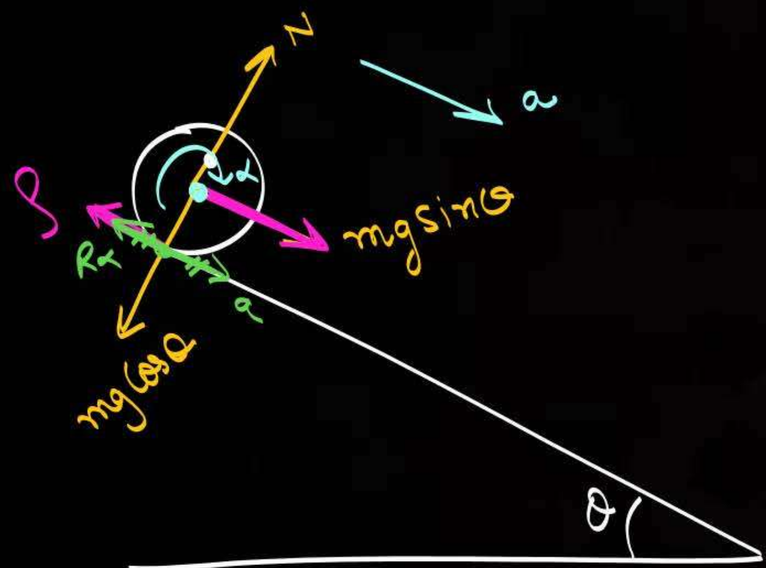
$$E_i = E_f$$

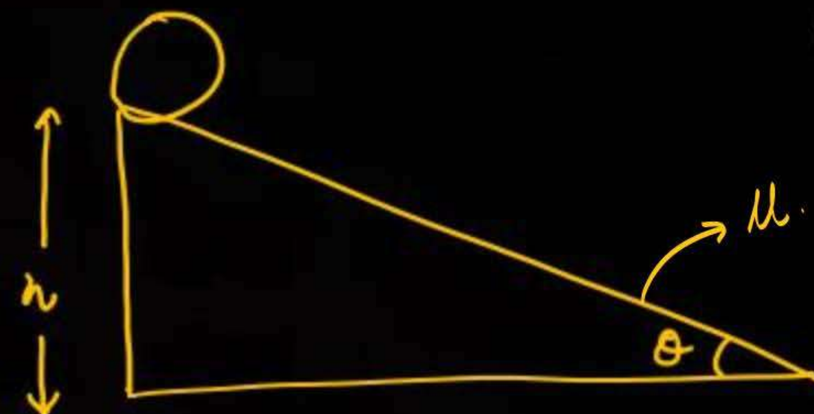
$$\cancel{m}gh = \frac{1}{2}\cancel{m}v^2$$

$$\boxed{\sqrt{2gh} = v}$$



when friction is present.





$$T_r = mg\sin\theta - f = ma$$
$$R_r = fR = I\alpha$$

$$mg\sin\theta - f = ma$$
$$fR = mK^2 \frac{a}{R}$$

$$mg\sin\theta - f = ma$$
$$f = \frac{Ia}{R^2}$$

$$\boxed{\frac{mg\sin\theta}{m + \frac{I}{R^2}} = a}$$

$$a = \frac{mg\sin\theta}{m + \frac{mK^2}{R^2}}$$

$$\boxed{a = \frac{g\sin\theta}{1 + \frac{K^2}{R^2}}}$$

for Same Incline θ = Same.

K = Radius of Gyration

more is $\frac{K^2}{R^2}$ $a \downarrow$

more time $\uparrow$

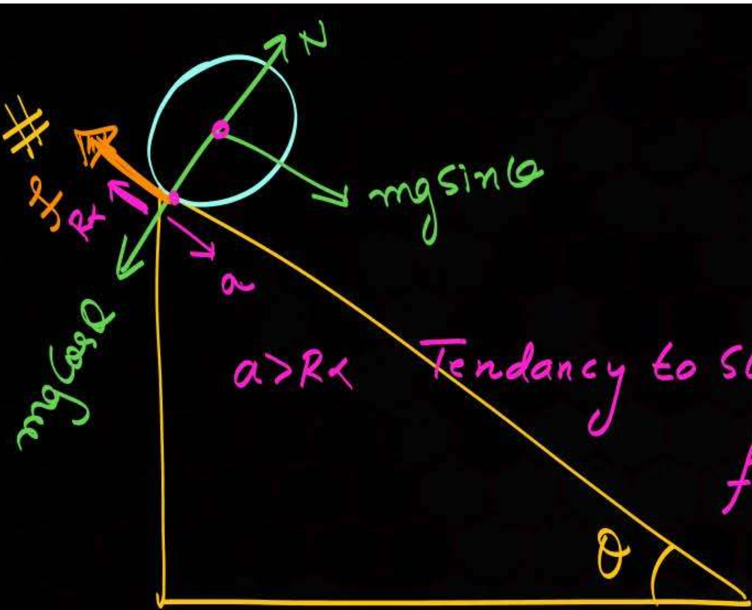
Hollow Cylinder, Ring $K = R$

Solid Cylinder, disc $K = \frac{R}{\sqrt{2}}$

hollow sphere $K = \sqrt{\frac{2}{3}}R$

Solid sphere $K = \sqrt{\frac{2}{5}}R$

#



$a > R\alpha$ Tendancy to slip forward.

friction → backward

$\boxed{a = R\alpha}$

There is no slipping

f → static

$\boxed{W_f = 0}$

$E_i = E_f$

$$\boxed{mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}mK^2\frac{v^2}{R^2}$$

$$2gh = v^2 + \frac{K^2}{R^2}v^2$$

$$\boxed{v = \sqrt{\frac{2gh}{1+\frac{K^2}{R^2}}}}$$

$$KE_{Total} = \frac{1}{2}mv^2\left[1+\frac{K^2}{R^2}\right]$$

$$KE_{Tran} = \frac{1}{2}mv^2$$

$$KE_{Rot} = \frac{1}{2}mv^2\frac{K^2}{R^2}$$

$$\frac{KE_{Total}}{KE_{Tran}} = \left[1+\frac{K^2}{R^2}\right]$$

$$\frac{KE_{Trans}}{KE_{Rot}} = \frac{R^2}{K^2}$$

$$\boxed{\frac{KE_{Total}}{KE_{Rot}} = \frac{1+\frac{K^2}{R^2}}{\frac{K^2}{R^2}}}$$

Acceleration of the body

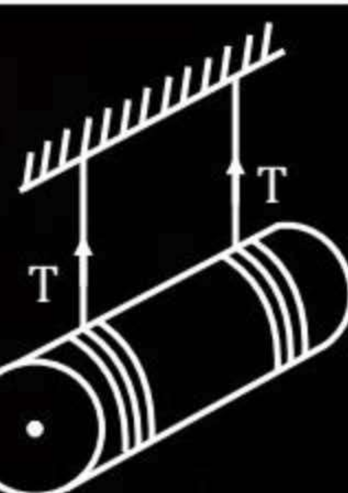
$$\boxed{a_{rolling} = \frac{g\sin\theta}{\left(1+\frac{K^2}{R^2}\right)}}$$

$$\boxed{v_{Rolling} = \sqrt{\frac{2gh}{1+\frac{K^2}{R^2}}} = \sqrt{\frac{2gs\sin\theta}{1+\frac{K^2}{R^2}}}}$$

$$\boxed{t_{rolling} = \sqrt{\frac{2s}{g\sin\theta}\left(1+\frac{K^2}{R^2}\right)} = \sqrt{\frac{2h}{g\sin^2\theta}\left(1+\frac{K^2}{R^2}\right)} = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}\left(1+\frac{K^2}{R^2}\right)}}$$

Q. A cylinder of mass – m suspended by two strings wrapped around cylinder one near each end. The free ends of string are attached to ceiling such that cylinder is horizontal. The cylinder is released from rest and allowed to roll down.

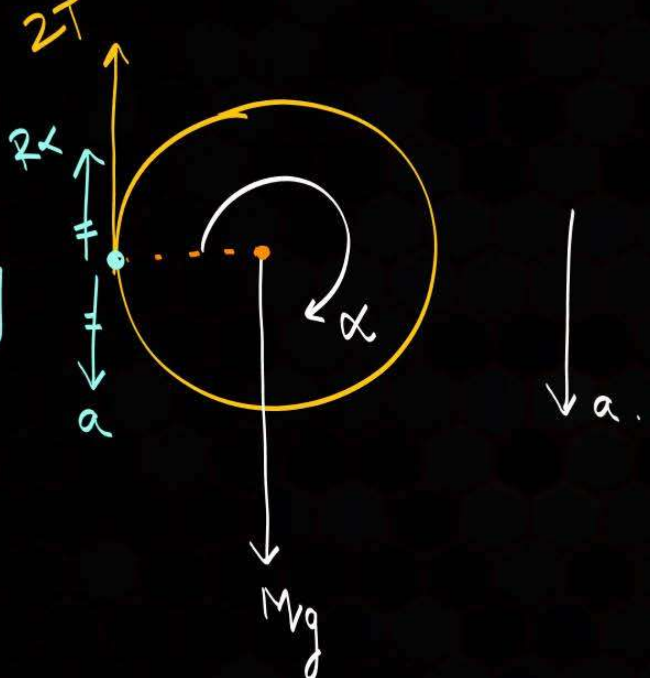
(a) Find tension in strings (b) Acceleration of cylinder



Solid Cylinder $I = \frac{1}{2}MR^2$

$\boxed{a = R\alpha}$

$\alpha = \frac{a}{R}$



$$\boxed{Tr = \quad Mg - 2T = Ma}$$

$$2T(R) = I_{axis}\,\alpha$$

$$Mg - 2T = Ma$$

$$2T\cancel{R} = \frac{MR^{\cancel{2}}}{2}\left(\frac{a}{\cancel{R}}\right)$$

$$Mg - 2T = Ma$$

$$2T = \frac{Ma}{2}$$

$$\cancel{M}g = \frac{3\cancel{M}a}{2}$$

$$\frac{2g}{3} = a$$

A ball rolls without slipping. The radius of gyration of the ball about an axis passing through its centre of mass is K. If radius of the ball be R, then the fraction of total energy associated with its rotational energy will be

(A) $\frac{K^2}{R^2}$

(B) $\frac{K^2}{K^2+R^2}$ Ans

(C) $\frac{R^2}{K^2+R^2}$

(D) $\frac{K^2+R^2}{R^2}$

$$\frac{KE_{Rot}}{KE_{Total}} = \frac{\frac{K^2}{R^2}}{1+\frac{K^2}{R^2}}$$

$$= \frac{K^2}{R^2+K^2}$$

Q. A small object of uniform density rolls up a curved surface with an initial velocity v. It reached up to a maximum height of $\frac{3v^2}{4g}$ with respect to the initial position. The object is **[2007]**

(A) Ring

(B) Solid sphere

(C) Hollow sphere

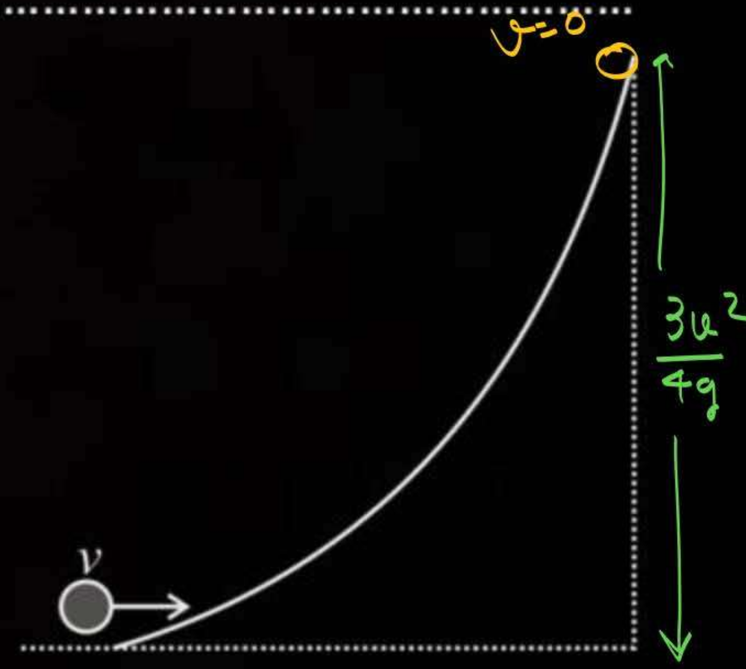
(D) Disc

$$v = \sqrt{\frac{2gh}{1+\frac{K^2}{R^2}}}$$

$$v = \sqrt{\frac{2g\left(\frac{3u^2}{4g}\right)}{1+\frac{K^2}{R^2}}}$$

$$v^2 = \frac{2g\,3u^2}{4g\left(1+\frac{K^2}{R^2}\right)}$$

$K = \_\_\_\_$



Q. A disk and a sphere of same radius but different masses roll off on two inclined planes of the same altitude and length. Which one of the two objects gets to the bottom of the plane first

....

(A) Disk

(B) Sphere

(C) Both reach at the same time

(D) Depends on their masses

Q. A ball rolls without slipping. The radius of gyration of the ball about an axis passing through its centre of mass is K. If radius of the ball be R, then the fraction of total energy associated with its rotational energy will be

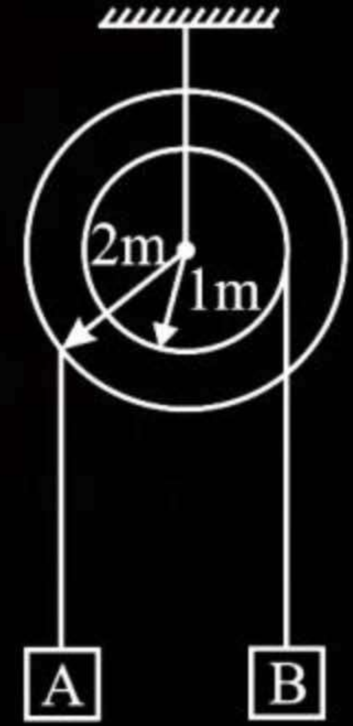
(A) $\frac{K^2}{R^2}$

(B) $\frac{K^2}{K^2+R^2}$

(C) $\frac{R^2}{K^2+R^2}$

(D) $\frac{K^2+R^2}{K^2}$

Q. In the pulley system shown, if radii of the bigger and smaller pulley are 2m and 1 m respectively and the acceleration of block A is 5 m/s^2 in the downward direction, then the acceleration of block B will be



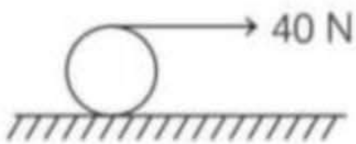
(A) 0 m/s^2

(B) 5 m/s^2

(C) 10 m/s^2

(D) 5/2 m/s^2

A string is wound around a hollow cylinder of mass 5 kg and radius 0.5 m. If the string is now pulled with a horizontal force of 40 N and the cylinder is rolling without slipping on a horizontal surface (see figure), then the angular acceleration of the cylinder will be (Neglect the mass and thickness of the string)



(2019 Main, 11 Jan II)

a) 10 rad/ s^2 b) 16 rad/ s^2

c) 20 rad/ s^2 d) 12 rad/ s^2

A homogeneous solid cylindrical roller of radius R and mass m is pulled on a cricket pitch by a horizontal force. Assuming rolling without slipping, angular acceleration of the cylinder is

(2019 Main, 10 Jan I)

a) $\frac{F}{2mR}$ b) $\frac{2F}{3mR}$ c) $\frac{3F}{2mR}$ d) $\frac{F}{3mR}$

A solid sphere and solid cylinder of identical radii approach an incline with the same linear velocity (see figure). Both roll without slipping all throughout. The two climb maximum heights h_{sph} and h_{cyl} on the incline. The ratio $\frac{h_{sph}}{h_{cyl}}$ is given by

(2019 Main, 8 Ap[ril II)

a) $\frac{2}{\sqrt{5}}$ b) $\frac{14}{15}$ c) 1 d) $\frac{4}{5}$

A cylinder rolls up an inclined plane, reaches some height and then rolls down (without slipping throughout these motions). The directions of the frictional force acting on the cylinder are (2002)

a) up the incline while ascending and down the incline while descending
b) up the incline while ascending as well as descending
c) down the incline while ascending and up the incline while descending
d) down the incline while ascending as well as descending