TOPICS TO BE COVERED

1. Continuity & Dirichlet functions & IVT
2. Differentiability
3. Function Determination using Differentiability
4. Rolle's Theorem
5. LMVT

CONCEPT HUMARA , CALCULATION (LIMITS) TUMHARA.

TO GET THE ‘BEST’ FROM THIS CLASS

1. Keep a rough copy with you … Don’t rush to write the notes …!
2. Listen to me carefully , have a smile !
3. Keep short notes copy with you & write what I request you to write.
4. Have Infinite Patience …. And enjoy the ride!!

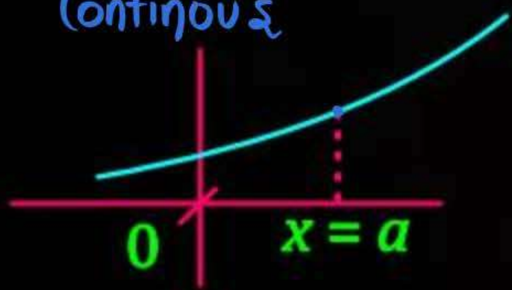
CONTINUITY

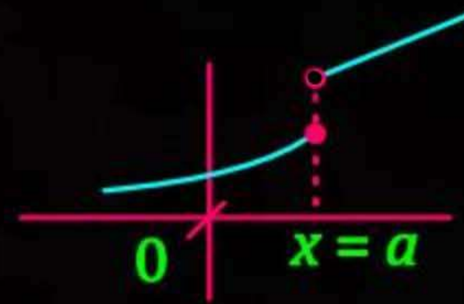
The graph of the function is said to be continuous at $\boxed{x = a}$, if while travelling along the graph form left to right OR right to left and in crossing over the point $x = a$, one does not have to lift the pen.

But if one has to lift the pen, then the function is said to be discontinuous at $x = a$.

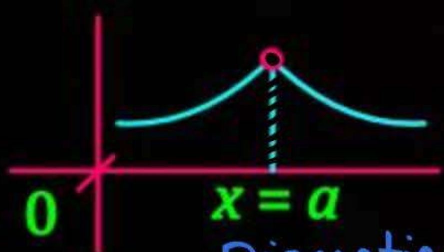
Q. Which of the following Graphs are Continuous at $x = a$?

Continous

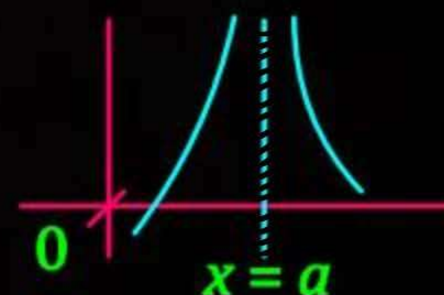




Discontinous



Discontinous



Discontinous

FORMULATIVE DEFINITION OF CONTINUITY

$\boxed{f(x)}$ is $\boxed{\text{Continuous at } x = a}$, if :

$$\underbrace{\boxed{\lim_{x\to a^+} f(x)}}_{R.H.L} = \underbrace{\boxed{\lim_{x\to a^-} f(x)}}_{L.H.L.} = \boxed{f(a)} = \boxed{\text{Finite}}$$

"functional value"

CONTINUITY IN AN $\boxed{\text{INTERVAL}}$

$x \neq a, x \neq b$

(1) Open Interval :

A function is $y = f(x)$ is said to be Continous in $\boxed{(a, b)}$, if it is Continuous at each and every point of this Interval.

(2) Closed Interval :

A function y = f(x) is said to be Continous in $\boxed{[a, b]}$ iff –

$x = a,\ x = b$

(a) It is Continuous in (a, b)

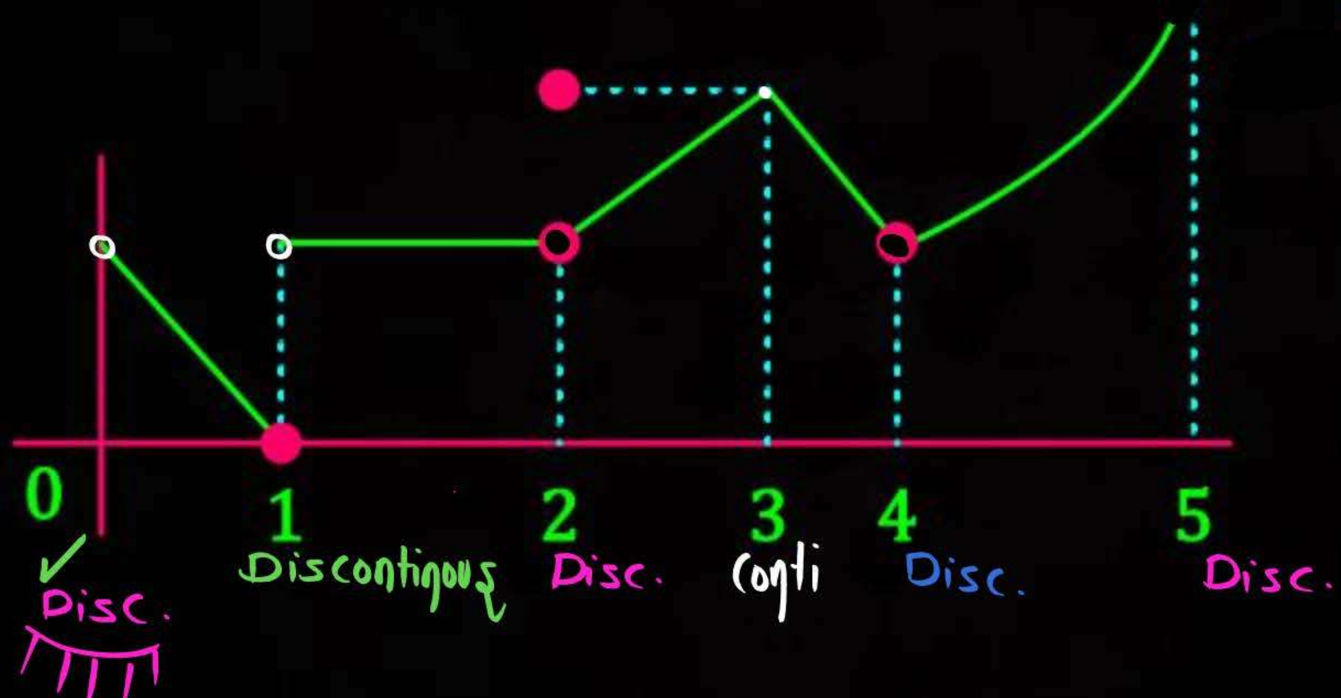
(b) It is right continuous at x = a.
i.e. $\underbrace{\lim_{x \to a^+} f(x)}_{RHL} = f(a) = \text{Finite}$

(c) It is left continuous at x = b
i.e. $\underbrace{\lim_{x \to b^-} f(x)}_{LHL} = f(b) = \text{Finite.}$

$\cap$

Q. Check Continuity of the following graph at **Integral points** :

$x \in$ Integer



NOTE :

All the Polynomial functions, Trigonometric Functions, Exponential Functions and Logarithmic Functions are Continuous in their Domain. ⭐⭐⭐

BKG ★★

Agar Function diya ho but interval (of 'x') nahi diya ho , then by default, the interval , where we have to check continuity will be the domain of the function.

TRUE / FALSE

Domain

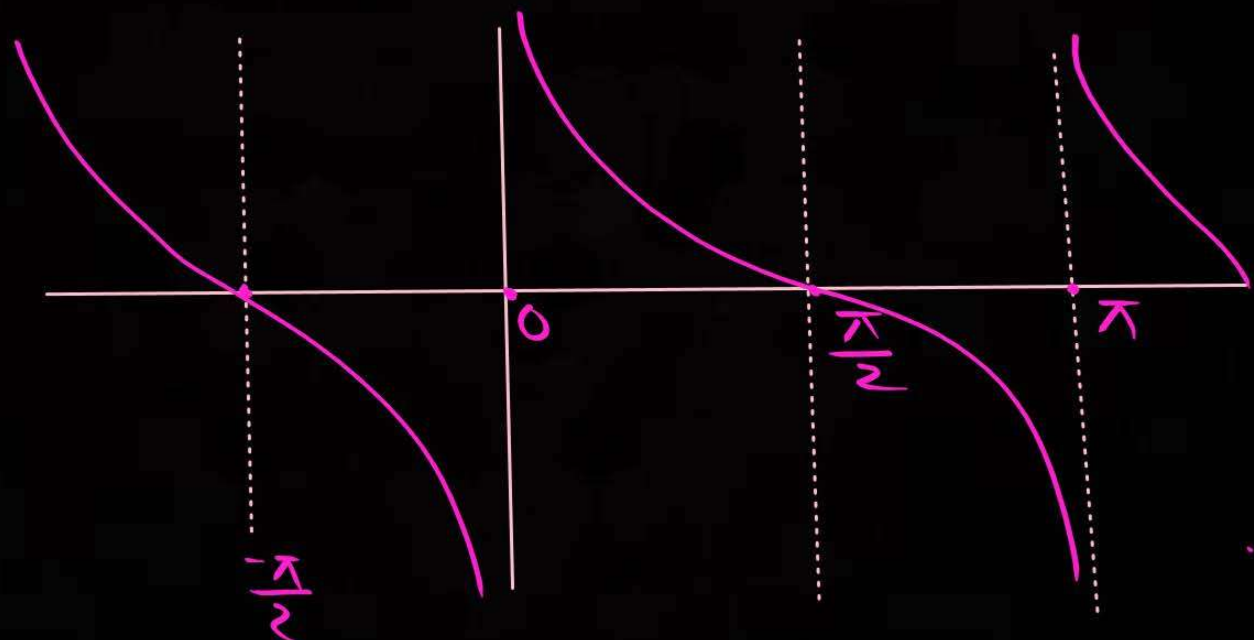
STATEMENT 1 : $\boxed{\text{Cot x}}$ is a $\boxed{\text{Continous Function.}}$ (TRUE)

STATEMENT 2 : $\boxed{\text{Cot x}}$ is a $\boxed{\text{Continous Function}}$ for all $\boxed{x \in R.}$ $\longrightarrow$ (False)

$\boxed{x = n\pi}$

$\downarrow$

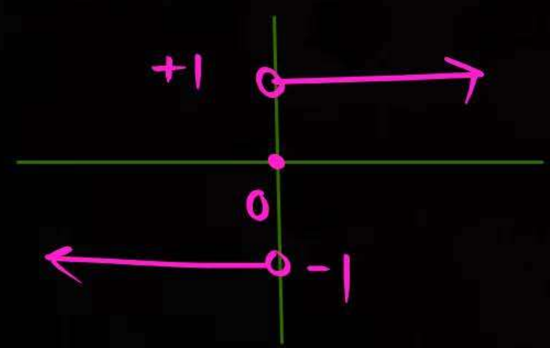
Disc.



IMPORTANT POINTS

1. Sin x , Cos x , e^x , all polynomial functions , $Tan^{-1}x$, $Cot^{-1}x$, $|x|$ are all Continuous for all $x \in R$.
2. $\{x\}$, $[x]$ are continuous for all $x \in R$ (except integers)
3. Tan x , Sec x are continuous for all $x \in R$ (except $\frac{(ODD.\ \pi)}{2}$)
4. Cot x , Cosec x are continuous for all $x \in R$ (except $x = n\pi$)
5. Sgn x is only discontinuous for $x = 0$.

$Sgn(2) = 1$



$$\text{sgn}\,x = \begin{cases} +1 \;; & x>0 \\ 0 \;; & x=0 \\ -1 \;; & x<0 \end{cases}$$

$Sgn(-100)$
$= -1$

BKG

Continuity ko khatara !!!!!!!

Where definition of the function changes.....

"always check at those points"

Q. $f(x) = \begin{cases} \boxed{(1+x^2)}, & \boxed{x > 0} \\ 0, & x = 0 \\ \boxed{-(1+x^2)}, & \boxed{x < 0} \end{cases}$

Check $\boxed{\text{Continuity.}}$

$\boxed{\text{Danger point}} \rightarrow \boxed{x=0}$

$RHL = \lim_{x \to 0^+} (1+x^2) = 1$ ✓

$LHL = \lim_{x \to 0^-} -(1+x^2) = -1$ ✓

$f(0) = 0$ ✓

$\boxed{\text{Ans} \rightarrow \text{Continous } \forall\, x \in R - \{0\}}$

for all

belongs to

Q. Let the function $f(x) = \begin{cases} \dfrac{\log_e(1+5x)-\log_e(1+\alpha x)}{x}; & \text{if } x \neq 0 \\ 10 & ; \text{ if } x = 0 \end{cases}$ be continuous at $x = 0$. The α is equal to :

$\begin{bmatrix} x \to 0 \\ x \neq 0 \end{bmatrix}$

$\boxed{\lim_{x \to 0} f(x) = f(0)}$

[JEE Main 2022 (29 July – Shift 2)]

(A) 10

(B) –10

(C) 5

(D) –5.

$$\lim_{x \to 0} \frac{\ln(1+5x) - \ln(1+\alpha x)}{x} = 10$$

LH RULE

$$\lim_{x \to 0} \frac{\frac{5}{1+5x} - \frac{\alpha}{1+\alpha x}}{1} = 10$$

$\boxed{5 - \alpha = 10}$

$\boxed{\alpha = -5}$ Ans

Q. If $p \neq q \neq 0$, then function $f(x) = \dfrac{\sqrt[7]{p(729 + x)} - 3}{\sqrt[3]{729 + qx} - 9}$ is continuous at $x = 0$, then :

[JEE Main 2022 (27 July – Shift 2)]

$729 = 3^6$

(A) $7pq\, f(0) - 1 = 0$

(B) $63q\, f(0) - p^2 = 0$.

$63q\, f(0) - 9 = 0$

$7q\, f(0) - 1 = 0$

(C) $21q\, f(0) - p^2 = 0$

(D) $7pq\, f(0) - 9 = 0$

$$\boxed{\lim_{x \to 0} \frac{(p(3^6 + x))^{1/7} - 3}{(3^6 + qx)^{1/3} - 9} = f(0)}$$

FINITE

$\left[\begin{array}{l} D^R \to 0 \\ N^R \to 0 \end{array}\right]$

$p^{1/7}(3^6)^{1/7} - 3 = 0$

$p^{1/7}\, 3^{6/7} = 3$

$\boxed{p^{1/7} = 3^{1/7}}$

$\boxed{p = 3}$

$\boxed{\text{LH Rule}}$

$$\lim_{x\to 0} \frac{\frac{1}{7}\left(p(3^6+px)\right)^{1/7-1}\cdot p}{\frac{1}{3}(3^6+qx)^{1/3-1}\cdot q} = f(0)$$

$$\frac{\frac{1}{7}\left(3^{\cancel{7}}\right)^{-6/\cancel{7}}\cdot 3}{\frac{1}{3}\cdot(3^{\cancel{6}\,2})^{-2/\cancel{3}}\cdot q} = f(0)$$

$$\frac{\cancel{3}}{7}\cdot\frac{\cancel{3}^{-6}\cdot\cancel{3}}{\cancel{3}^{-4}\cdot q} = f(0)$$

$$1 = 7q\cdot f(0)$$

$$\boxed{0 = 7\cdot q\cdot f(0) - 1}$$

$$\boxed{\text{Continous} \; \frac{\pm}{\times} \; \text{Continous} = \text{Continous}}$$

$$\boxed{\frac{\text{Continous}}{\text{Continous}} = \text{Continous} \qquad D^R \neq 0}$$

Q.

Find the value of a & b so that the function

$$f(x) = \begin{cases} x + a\sqrt{2}.\sin x, & x \in \left(0, \frac{\pi}{4}\right) \\ 2x(\cot x) + b, & x \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right] \\ a(\cos 2x) - b\sin x, & x \in \left(\frac{\pi}{2}, \pi\right) \end{cases}$$

is Continuous in $(0, \pi)$

$$\lim_{x \to \frac{\pi}{2}^+} = -a - b = \lim_{x \to \frac{\pi}{2}^-} = b$$

$= b$

Danger points $\rightarrow$ $\underbrace{\frac{\pi}{4}, \frac{\pi}{2}}$

must be continous

$$\lim_{x \to \frac{\pi}{4}^+} (2x\cot x + b) = \lim_{x \to \frac{\pi}{4}^-} (x + a\sqrt{2}\sin x) = f\left(\frac{\pi}{4}\right)$$

$$\boxed{\frac{\pi}{2} + b = \frac{\pi}{4} + a = \frac{\pi}{2} + b}$$

$$\boxed{a - b = \frac{\pi}{4}}$$ ✓

$$\boxed{-a = 2b}$$ ✓

$$b = -\frac{\pi}{12}$$

$$a = \frac{\pi}{6}$$

Ans

Q. Let $f : R \rightarrow R$ be a function as $f(x) = \begin{cases} 5, & \text{if } x \le 1 \\ a + bx, & \text{if } 1 < x < 3 \\ b + 5x, & \text{if } 3 \le x < 5 \\ 30, & \text{if } x \ge 5 \end{cases}$, then, f is

[JEE Mains-2019]

(A) Continuous if a = –5 and b = 10

(B) Continuous if a = 5 and b = 5

(C) Continuous if a = 0 and b = 5

(D) Not continuous for any values of a and b.

$x = 1$

$\boxed{a + b = 5 = 5}$ ✓

$x = 3$

$a + 3b = b + 15$

$\boxed{a + 2b = 15}$ ✓

$x = 5$

$b + 25 = 30 = 30$

$\boxed{b = 5}$

$b = 10, \quad a = -5$

Q. Let [t] denote the greatest integer ≤ t and $\lim_{x\to 0} x\left[\frac{4}{x}\right] = A$. Then the function $f(x) = [x^2]\sin(\pi.x)$ is Discontinuous, when x is equal to:

[JEE Mains-2020]

$[p] = p - \{p\}$, $\{p\} \in [0,1)$

(A) $\sqrt{A+1}$ $= \sqrt{5}$

(B) $\sqrt{A+5}$ $= 3$

(C) $\sqrt{A+21}$ $= 5$

(D) $\sqrt{A}$ $= 2$

$\lim_{x\to\sqrt{5}^+} = [5^+]\sin(\sqrt{5}\pi)$

$= 5 \cdot \sin\sqrt{5}\pi$

$\lim_{x\to\sqrt{5}^-} = [5^-](\sin\sqrt{5}\pi)$

$= 4\sin\sqrt{5}\pi$

$R \neq L$

$\lim_{x\to 0} x\left(\frac{4}{x} - \left\{\frac{4}{x}\right\}\right)$, $\left\{\frac{4}{x}\right\} \in [0,1)$

$= \lim_{x\to 0} 4 - x\left\{\frac{4}{x}\right\} = \boxed{4 = A}$

BUILT-IN LIMIT

⭐ ***Pehele function ko ' limit' se azaad karo (USING BKG)***

BKG

x^{2n} $\quad$ $n \to \infty$

$(x^2)^n$

$$x^{2n} = (x^2)^n = \begin{cases} \infty & ; x^2 > 1 ; x \in (-\infty,-1) \cup (1,\infty) \\ 1 & ; x^2 = 1 ; x = \pm 1 \\ 0 & ; x^2 < 1 ; x \in (-1,1) \end{cases}$$

$$\boxed{x^{2n} = \begin{cases} \infty & ; x \in (-\infty,-1) \\ 1 & ; x = -1 \\ 0 & ; x \in (-1,1) \\ 1 & ; x = 1 \\ \infty & ; x \in (1,\infty) \end{cases}}$$

Q. If function $f : R \to R$ defined by $f(x) = \lim_{n\to\infty} \dfrac{\cos(2\pi x) - x^{2n}\sin(x-1)}{1 + x^{2n+1} - x^{2n}}$ continuous for all x in :

[JEE Main 2022 (28 July – Shift 2)]

(A) $R - \{-1\}$

(B) $R - \{-1, 1\}$.

(C) $R - \{1\}$

(D) $R - \{0\}$

$$f(x) = \begin{cases} \dfrac{-\sin(x-1)}{(x-1)} & ; x \in (-\infty, -1) \\ -(1+\sin 2) & ; x = -1 \\ \cos 2\pi x & ; x \in (-1, 1) \\ 1 & ; x = 1 \\ \dfrac{-\sin(x-1)}{(x-1)} & ; x \in (1, \infty) \end{cases}$$

$\boxed{x = -1}$ → DC

$R = 1$

$L = \dfrac{-\sin(-2)}{-2} = -\dfrac{\sin 2}{2}$

$f(-1) = -(1+\sin 2)$

$\boxed{x = 1}$ → DC

$R = -1$ $L = 1$ $f(1) = 1$

ALGEBRA OF CONTINUITY

1. When two Continous Functions are added , subtracted , multiplied or divided ; the resultant is always Continous.

2. When one function is continuous and the other is discontinuous ; the function obtained on adding or subtracting will be discontinuous.

3. When one function is continuous and the other is discontinuous ; the function obtained on Multiplying or Dividing can be Discontinuous or Continous.

C $\times$ $\div$ DC = check

4. When two Discontinous Functions are added , subtracted , multiplied or divided ; the resultant can be Continous or Discontinous.

DC $\pm$ $\div$ $\times$ DC = check

$\boxed{\text{Cont}}\ \begin{matrix}\pm\\ \times\\ \div\end{matrix}\ \boxed{\text{Cont}} = \boxed{\text{Cont}}.$

$\boxed{\text{Cont}} \pm \boxed{\text{Disc}} = \boxed{\text{Disc}}$

★★★★ For all Remaining cases; always check...

Q. Discuss Continuity of the following functions at the given point :

(i) $f(x) = \cos x + \{x\}$ for all $x \in R$

(ii) $f(x) = |x-7| + e^{-x+7}$ for all $x \in R$ → (ii) Cont. + cont. = cont. Ans

(iii) $g(x) = x[x]$ at $x=4$ → (iii) (Cont)·(Disc) = check

$F(4^+) = 4(4) = 16$, $F(4^-) = 4(3) = 12$

Disc. at $x=4$ Ans

(iv) $g(x) = x[x]$ at $x=1/2$ → Ans = Cont.

Con. Cont

(i) $\cos x$ → always cont.

$\{x\}$ → always Cont. − {Integer pts}

$x \in Z$: C + DC = DC

$x \notin Z$: Cont + cont = Cont.

Ans

CONTINUITY OF COMPOSITE FUNCTIONS

If g(x) is continuous at x = a & f(x) is continuous at x = g(a) , Then f(g(x)) is continuous at x=a.

a

$\hookrightarrow g(a)$

Conti. at $x=a$.

Always check for all possible combinations. (*PLEASE , I REQUEST YOU !*)

R, L, Func^n value (equal, FINITE)

$\boxed{C(C) = \text{Continous}}$

Q. Discuss Continuity of the $f(g(x))$ at $x=1$, if

(i) $g(x) = x-1$ & $f(x) = \operatorname{sgn} x$

Continous

$f(g(1)) = f(0)$

$g(1) = 0$

$\boxed{\text{Discontinous at } x=1}$ Ans

SUSPICIOUS POINTS FOR CONTINUITY

Most Imp. part

Function kahan-kahan discontinuous ho sakta hai ?

1. Where the definition of the function changes (PIECE-WISE FUNCTIONS)
2. In case of { f(x) } & [f(x)], check for those x where f(x) becomes an Integer.
3. In case of Sgn f(x), check for those x where f(x) =0.
4. The points where the denominator becomes zero.
5. At the end points of given interval.

Q. Discuss Continuity of the $\boxed{f(g(x))}$, if

(i) if $g(x) = \frac{1}{x-1}$ & $f(x) = \frac{1}{x^2+x-2}$

Find those 'x' where $f(g(x))$ is continous..

$= \frac{1}{(x+2)(x-1)}$

Dis. at $x=1$

Disc: $x = -2, 1$

$f(g(x))$

$g(x) = -2, 1$

$\frac{1}{x-1} = -2$

$1 = -2x+2$

$\boxed{x = \frac{1}{2}}$ ✓

$\frac{1}{x-1} = 1$

$\boxed{x=2}$ ✓

$\boxed{x=1}$ ✓

DC at $x \in \left\{\frac{1}{2}, 1, 2\right\}$ And

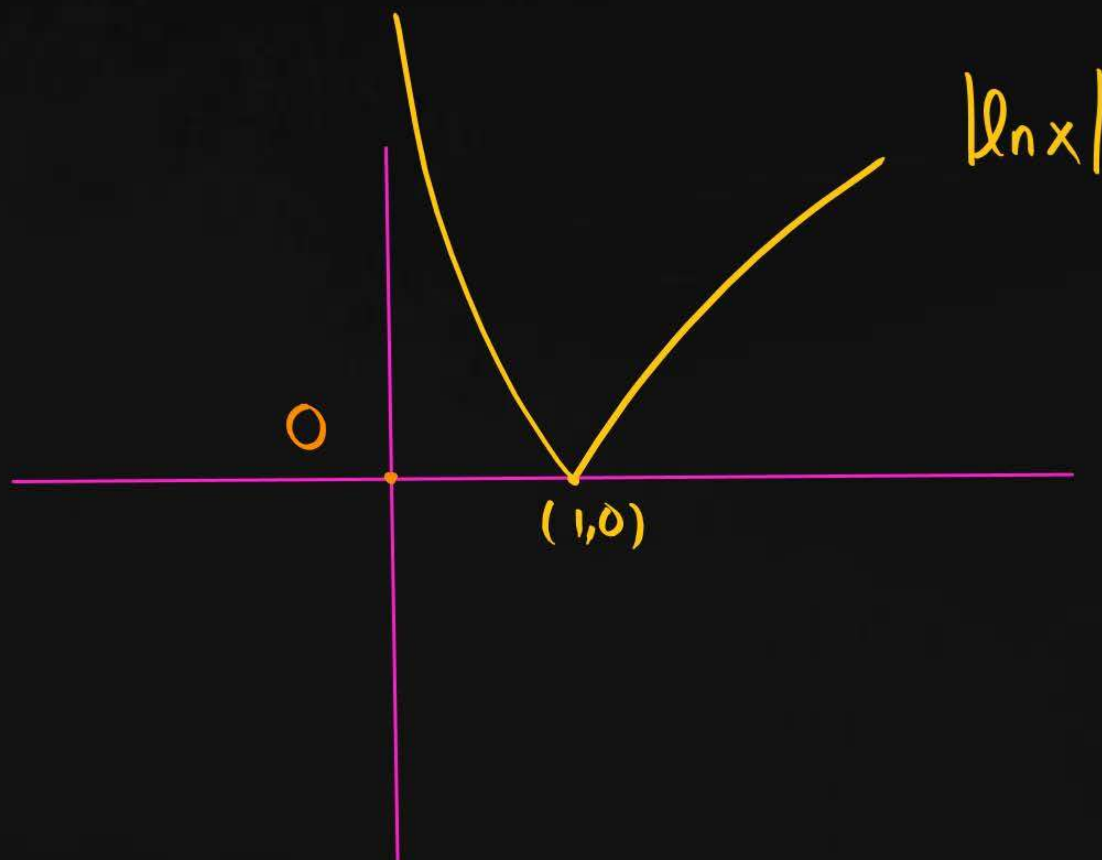
If $\overset{\frown}{f(x)}$ is continous, then $\overset{\frown}{|f(x)|}$ must be continous.

$f(x) \longrightarrow |f(x)|$

G.B.K

-ve y wale bhaag, Jao, Jaane se pehele photo Khichao..

$|\ln x|$



Q. The number of points where the function

$$f(x) = \begin{cases} |2x^2 - 3x - 7|, & \text{if } x \le -1 \\ [4x^2 - 1], & \text{if } -1 < x < 1 \\ |x+1| + |x-2|, & \text{if } x \ge 1 \end{cases}$$

Where [t] denotes the greatest integer ≤ t, is Discontinuous is: 7 points Ans

[JEE Mains-2022]

$(0^-)^2 = 0^+$

GKB ♡♡

Check

$x = -1, 1,$ $\underbrace{4x^2-1}_{-1,0,1,2} = \text{Integer}$

$x \in (-1, 1)$

$x^2 \in [0, 1)$

$4x^2 \in [0, 4)$

$\boxed{(4x^2 - 1) \in [-1, 3)}$

Check at

✓ $x = -1$ → R = 2, L = 2, f = 2

✓ $x = 1$ → L = 3, R = 3, f = 2

$x = 0$ Conti. $\begin{cases} -1 = R \\ -1 = L \\ -1 = f \end{cases}$

$x = 1/2$ L ≠ R

$x = -1/2$ L ≠ R

$x = 1/\sqrt{2}$ R, L

$x = -1/\sqrt{2}$ R ≠ L

$x = \sqrt{3}/2$ R ≠ L

$x = -\sqrt{3}/2$ R ≠ L

$4x^2 - 1 = -1 \Rightarrow x = 0$

$4x^2 - 1 = 0 \Rightarrow x = \pm \frac{1}{2}$

$4x^2 - 1 = 1 \Rightarrow x = \pm \frac{1}{\sqrt{2}}$

$4x^2 - 1 = 2 \Rightarrow x = \pm \frac{\sqrt{3}}{2}$

Q. Let $f(x) = [2x^2 + 1]$ and $g(x) = \begin{cases} 2x - 3, & x < 0 \\ 2x + 3, & x \ge 0 \end{cases}$ where $[t]$ is the greatest integer $\le t$. Then, in the open interval $(-1, 1)$, the **number of points** where **fog** is **Discontinuous** is equal to $\underline{31 + 32 = 63}$.

[JEE Main 2022 (25 June – Shift 2)]

$x \in (-1,1)$

$f(x) = [2x^2+1]$, $g(x) = \begin{cases} 2x-3; & x \in (-1,0) \\ 2x+3; & x \in [0,1) \end{cases}$

$$\boxed{f(g(x)) = \begin{cases} [2(2x-3)^2+1]; & x \in (-1,0) \\ [2(2x+3)^2+1]; & x \in [0,1) \end{cases}}$$

$-1 < x < 0 \Rightarrow -2 < 2x < 0$

$-5 < 2x-3 < -3$

$9 < (2x-3)^2 < 25$

$18 < (2x-3)^2 \times 2 < 50$

$\boxed{19 < 2(2x-3)^2+1 < 51}$

$\underbrace{\{20, 21, \dots 50\}}$

31 points ✓

$x \in [0,1)$

$2x \in [0,2)$

$(2x+3) \in [3,5)$

$(2x+3)^2 \in [9,25)$

$2(2x+3)^2 \in [18,50)$

$$\boxed{2(2x+3)^2+1 \in [19,51)}$$

Int.

$$\boxed{\{19, 20, \ldots\ldots 50\}} \rightarrow 32 \text{ pts}$$

DIRICHLET FUNCTION

1. These are piece-wise functions which are separately defined for Rational and Irrational numbers.

2. These function are continuous at those values of 'x' for which both the definitions become equal.

Ex. : $$f(x) = \begin{cases} 1 - x, & x \in Q \\ x, & x \in \overline{Q} \end{cases}$$

$1 - x = x \longrightarrow \boxed{x = \frac{1}{2}}$ Continous at $x = \frac{1}{2}$

2. $f(x) = x^2, x \in Q$

$= 1, \quad x \in \bar{Q}$

$\longrightarrow x^2 = 1 \longrightarrow \boxed{x = \pm 1}$ "Continous"

3. $f(x) = -1 - x, \quad x \in Q$

$= x^2 \quad , \quad x \in \bar{Q}$

$\longrightarrow$ Discontinous at all x.

$-1 - x = x^2$

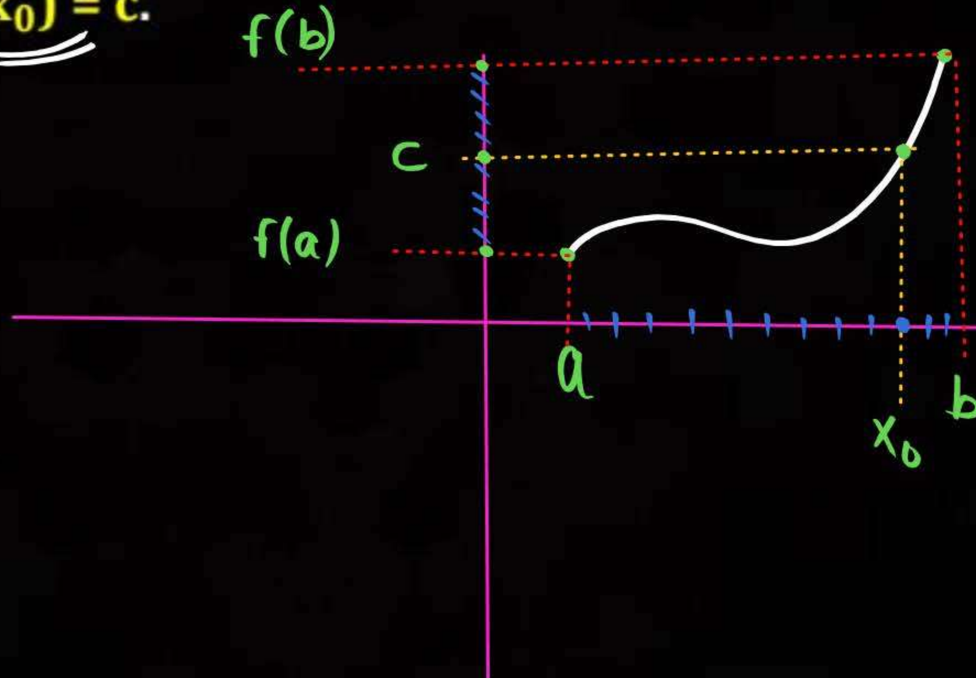
$\boxed{0 = x^2 + x + 1}$

$x \in \phi$

Theorems on Continuity

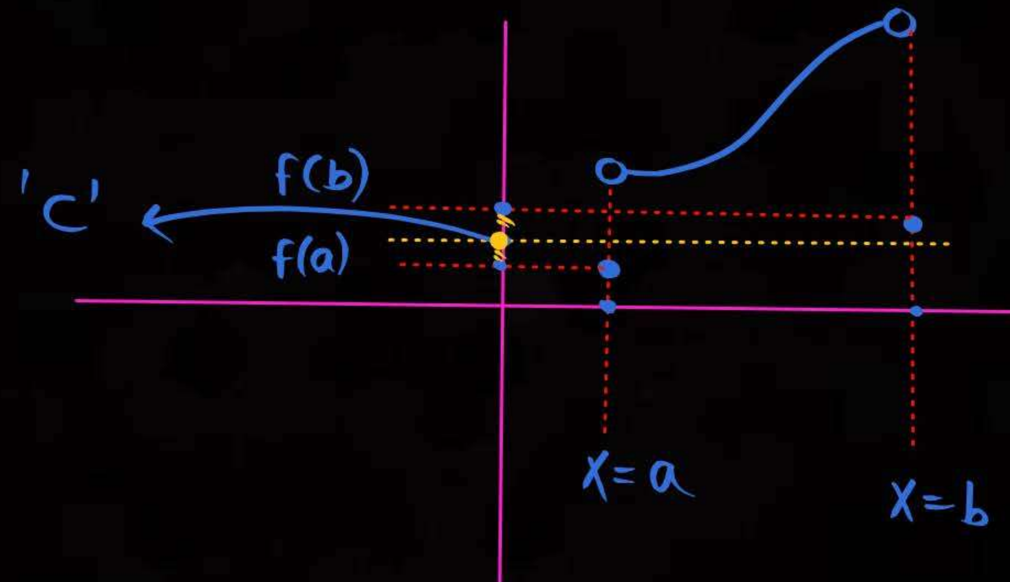
INTERMEDIATE VALUE THEOREM

If f is Continous on [a, b] and $f(a) \neq f(b)$, then for any $c \in [f(a), f(b)]$, there is at-least one x_0 in [a , b] for which $f(x_0) = c$.



NOTE

Continuity in closed interval [a , b] is must for the validity of this theorem.



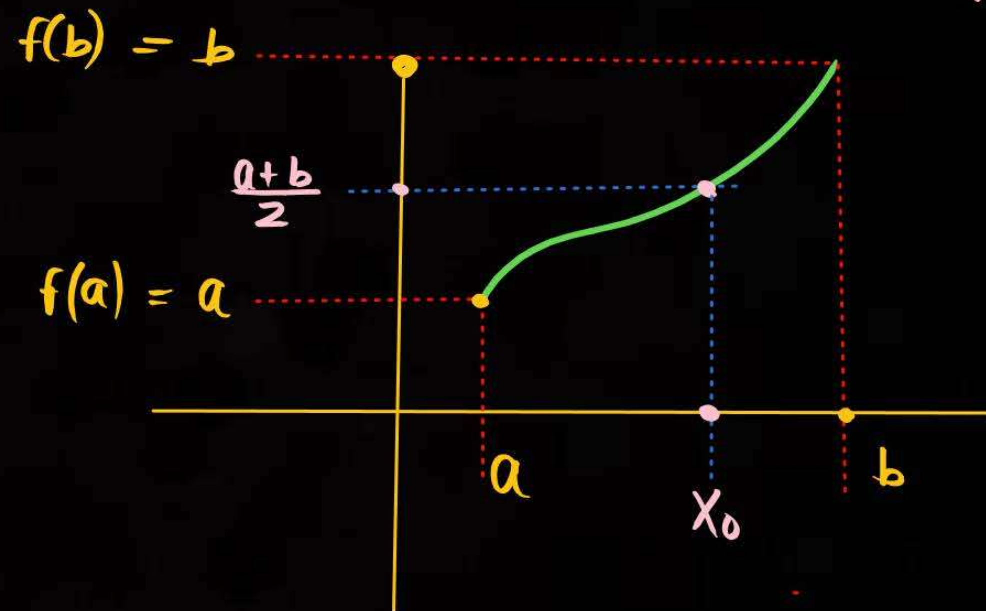
Q. Show that the $\boxed{f(x) = (x-a)^2(x-b)^2 + x}$, takes a value $\boxed{\frac{a+b}{2}}$ for some $x \in [a, b]$.

"4° Degree polynomial"

$\boxed{\text{option}}$ JEE Advanced

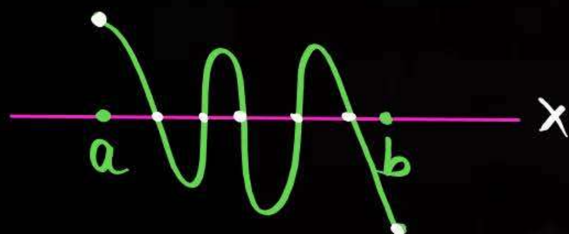
$f(a) = a$

$f(b) = b$



BKG

If a function $f(x)$ is a continous function & $f(a).f(b)<0$ .... Then it has at-least one root (or, odd number of roots) between $x=a$ & $x=b$.



Q. Prove that $f(x) = 2x^3-5x^2-10x+5$ has a root somewhere in $[\overset{a}{-1},\overset{b}{2}]$.

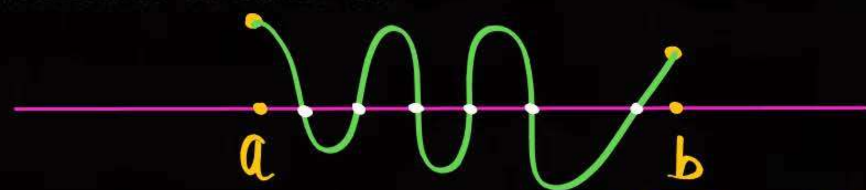
$f(-1)=8$

$f(2)=-19$

$\boxed{f(-1).f(2)<0}$

BKG

If a function **f(x) is a continous function** & **f(a).f(b)>0** .... Then it has either **no root** or, **even number of roots** between x=a & x=b.

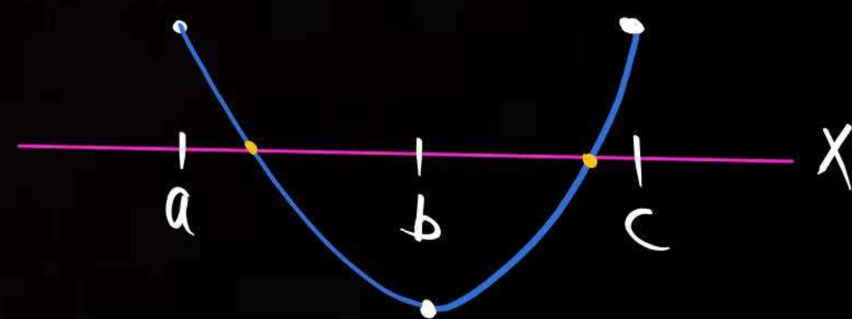


Q. Comment on nature of roots of quadratic $f(x) = (x - a)(x - b) + (x - b)(x - c) + (x - c)(x - a)$ (where $\boxed{a < b < c}$)

$f(a) = (a-b)(a-c) > 0$

$f(b) = (b-c)(b-a) < 0$

$f(c) = (c-a)(c-b) > 0$



Maxm 8 Real Roots

Q. If $\boxed{f(x) = 3x^8 + ax^4 + bx^2 - 7}$ and $\boxed{f(1) = 5}$ and $\boxed{f(-2) = -1}$, then minimum number of real roots of f(x) is ____ 6 Real Roots.

$\boxed{\text{Advanced}}$ ♡♡♡

$= f(-x)$

Even funcn

[graph symm. abt y-axis]

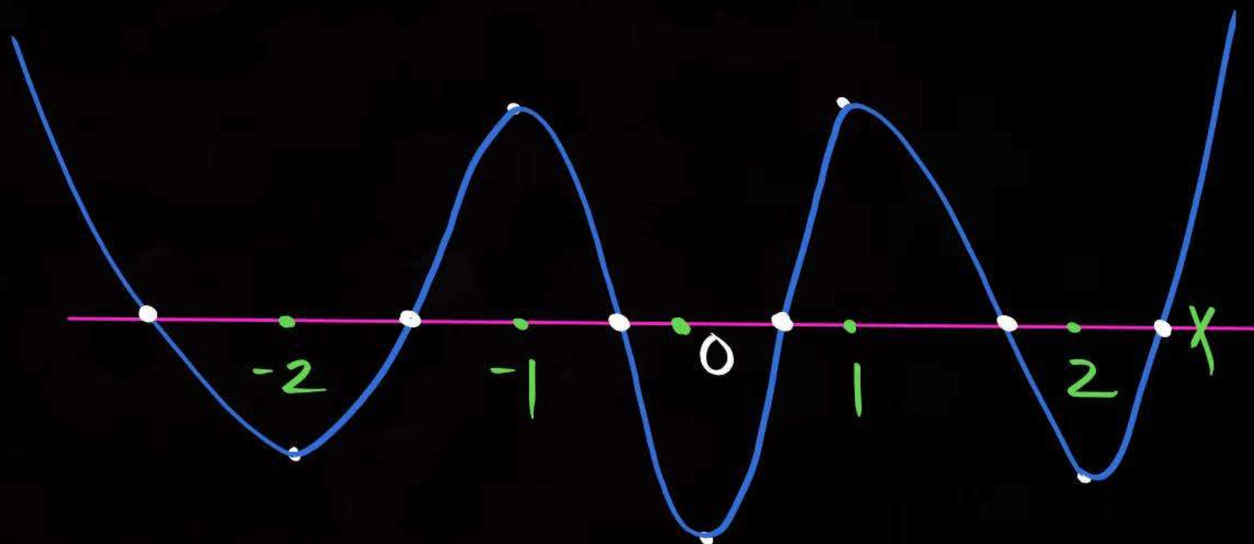
$f(0) = -7$

$f(-1) = 5$

$f(2) = -1$

$f(-\infty) = \infty$

$f(\infty) = \infty$



Gyaan → Always find

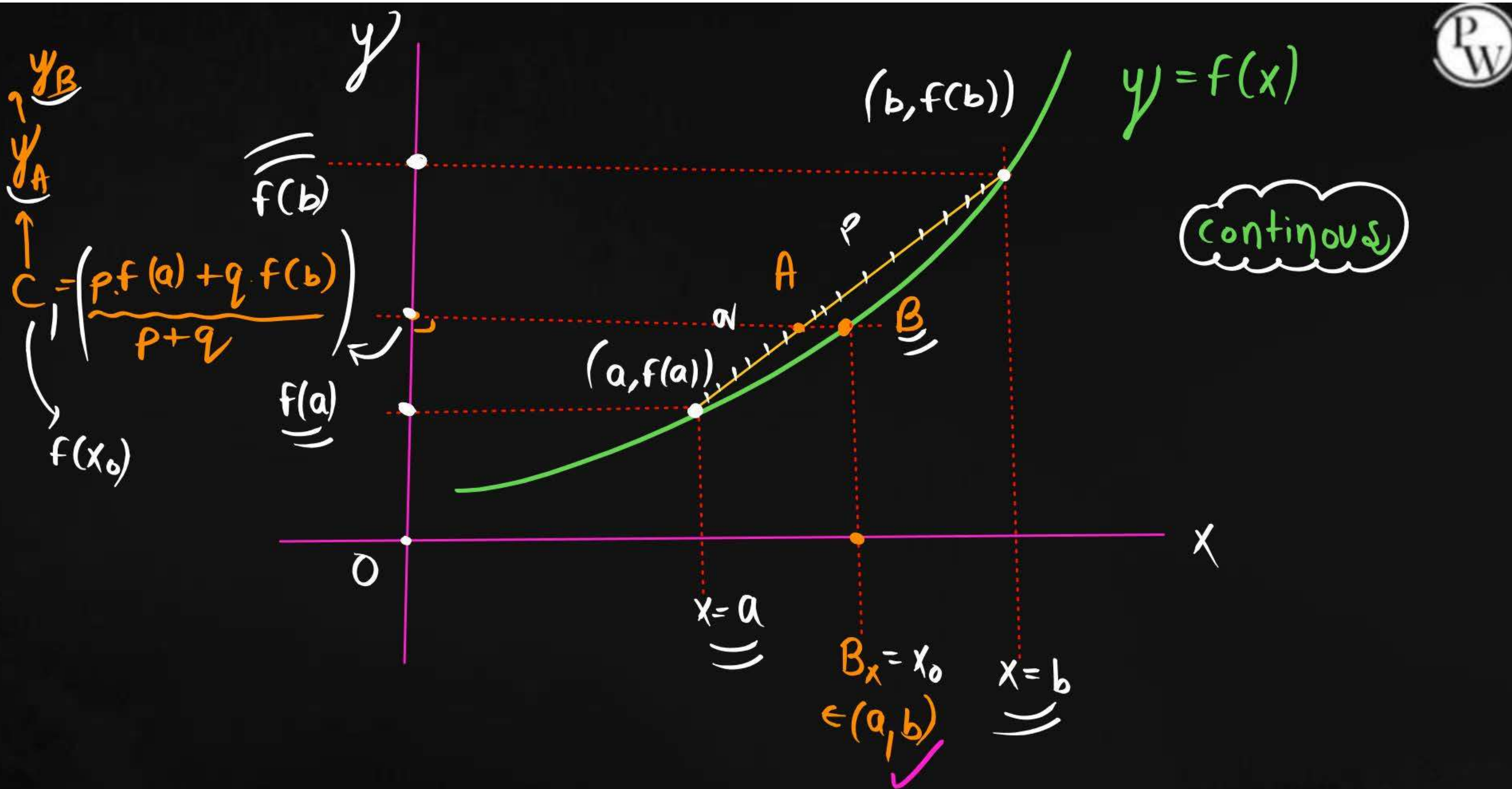
① Even OR odd

② f(0)

③ $f(\infty)$

④ $f(-\infty)$

Advanced ♡♡



y_B
y_A
$C = \left(\frac{p.f(a)+q.f(b)}{p+q}\right)$
$f(x_0)$
y
$f(b)$
$f(a)$
$(b, f(b))$
$y = f(x)$
continous
p
A
q
B
$(a, f(a))$
O
X
$x = a$
$B_x = x_0$
$\in (a, b)$
$x = b$

BKG

Advanced

x_0

(DERIVED FROM SECTION FORMULA)

If f(x) is a continuous function in [a,b] (& $f(a) \neq f(b)$), then there exists some c in (a,b) such that :

$$f(\overset{x_0}{c}) = \frac{p.f(a)+q.f(b)}{p+q}$$, here p & q >0

True / False :

(JEE advanced का option)

If f(x) is continuous in [a, b], (f(a) ≠ f(b)) then for some c ∈ (a, b) 7f(a) + 3f(b) = 10f(c).

$p=7, q=3$

$\boxed{\text{TRUE}}$

DIFFERENTIABILITY

DERIVABILITY

Geometrical Meaning

Here we will talk about the slope of the tangent drawn to the curve at $x = a$ if it exists.

Physical Meaning

Here we talk of the function which are derivable and instantaneous rate of change of functions.

EXISTENCE OF DERIVATIVE

f(x) is Derivable at x = a iff →

$$\left\{\begin{aligned} &LHD = f'(a^-) = \lim_{h\to 0^+} \frac{f(a-h)-f(a)}{-h} \\ &RHD = f'(a^+) = \lim_{h\to 0^+} \frac{f(a+h)-f(a)}{h} \end{aligned}\right\}$$

$$\boxed{\begin{aligned} LHD &= RHD \\ &= \text{Finite} \end{aligned}}$$

Geometrically, if the function is derivable at x = a then it implies that a unique tangent with finite slope exists at this point.

$$f'(a^+) = f'(a^-) = \text{FINITE} = \left.\frac{dy}{dx}\right|_{x=a} = f'(a)$$

unique tangent

"

" Tangent का slope"

(Finite)

$\boxed{\text{Vertical Tangent}}$

$\boxed{LHD = RHD = \infty}$

$\boxed{LHD = RHD = -\infty}$

Func^n is NON-DERIVABLE

But a V_T exists

(Tangent parallel to y-axis)

$\boxed{\begin{array}{l} RHD = \infty \\ LHD = -\infty \end{array}}$ $\boxed{\begin{array}{l} LHD = \infty \\ RHD = -\infty \end{array}}$ → Non-Deri.

V_T DNE

Check C & D at x = 0, f(x) = |x|. $\longrightarrow f(0^+) = 0 = f(0^-) = f(0)$

Derivability:

$$LHD = f'(0^-) = \lim_{h\to 0^+} \frac{f(-h) - f(0)}{-h}$$

$$= \lim_{h\to 0^+} \frac{|-h|}{-h} = \frac{-(-h)}{-h} = -1$$

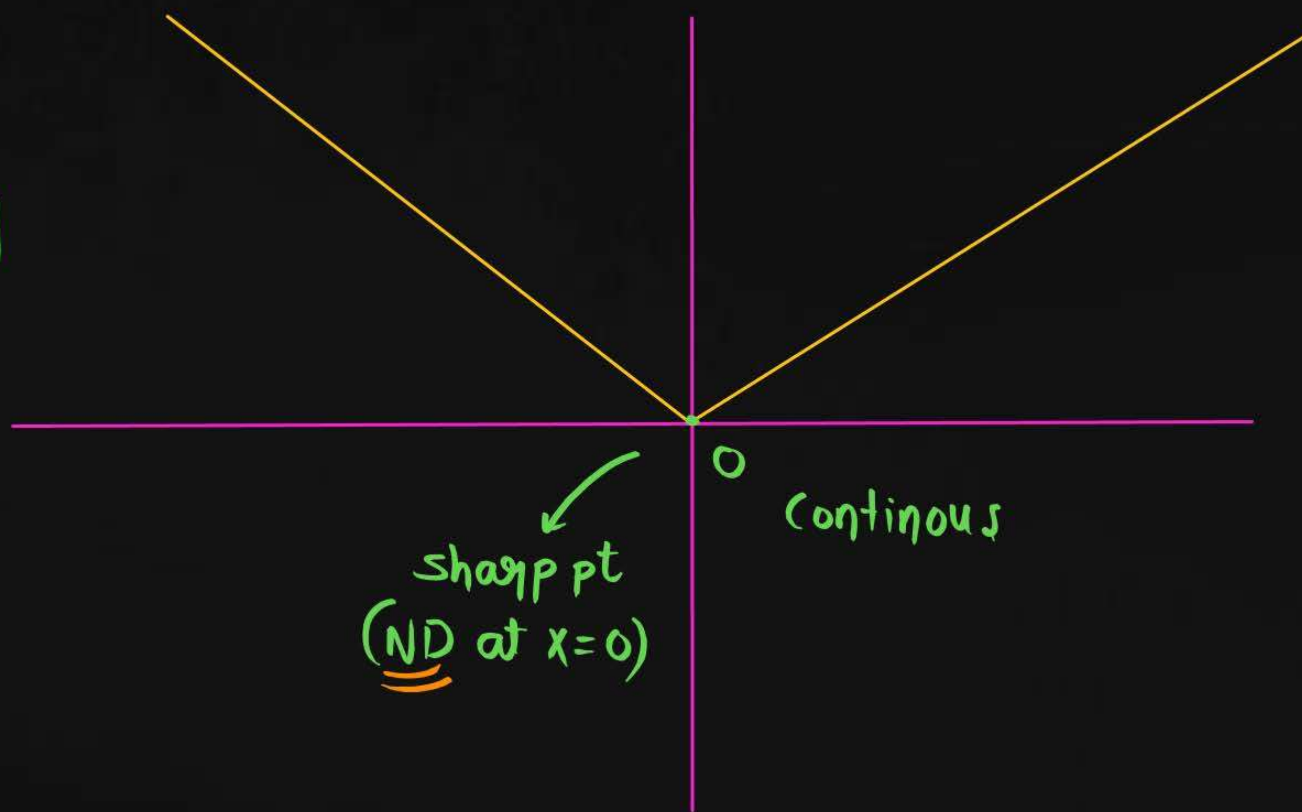
$$RHD = f'(0^+) = \lim_{h\to 0^+} \frac{f(h) - f(0)}{h} = \frac{|h|}{h} = \frac{h}{h} = 1$$

RHD ≠ LHD

$\boxed{\text{Non-Derivable at } x = 0}$

But continous at x = 0.

$|x|$



$\boxed{\text{Sharp edge}} \rightarrow \boxed{\text{Non-Derivable}} \rightarrow \boxed{\text{But continous}}$

Derivability & Continuity

$D \rightarrow C$ ✓

(1) ✓ If a function $y = f(x)$ is derivable at $x = a$, then $f(x)$ is also continuous then at $x = a$.

(2) If $f(x)$ is Derivable at $x = a$ $\Rightarrow$ Continuous at $x = a$. (TRUE)

But

If $f(x)$ is Continuous at $x = a$ $\Rightarrow$ derivable at $x = a$ $\longrightarrow$ (FALSE)

(3) $\boxed{\text{Dis-Continuous at } x = a,}$ $\Rightarrow$ $\boxed{\text{Non differentiable at } x = a}$ TRUE

$\boxed{\text{Non Derivable at } x = a}$ $\Rightarrow$ $\boxed{\text{Dis-Continuous at } x = a}$ (FALSE)

BKMG → Baba Ka Maha-gyaan

If $f'(a^+) = p$

$f'(a^-) = q$

(1) If p and q both are finite and $p = q$, Then f(x) is derivable at $x = a$ and continuous at $x = a$.

D → C

Non-Derivable

(2) If 'p' and 'q' both are finite but $p \neq q$, Then f(x) is ND at $x = a$ and continuous at $x = a$.

(Sharp edge) ★★

(3) If at-least one of p or q approaches ∞ or $-\infty$ or DNE, then the function is Non Derivable at $x = a$.

In this case check continuity separately.

LHD = RHD = FINITE $\longrightarrow$ DER. $\rightarrow$ CON.

LHD $\neq$ RHD $\longrightarrow$ Non Der $\rightarrow$ Cont
(BOTH FINITE) (sharp edge)

any else $\rightarrow$ Non-Deri.
But check continuity

Q. Consider the function $\boxed{f(x) = [x - 1] + |x - 2|}$

Disc + Cont = Disc.

TRUE **Statement –1** : f(x) is Discontinuous at x = 1

TRUE **Statement –2** : f(x) is Non-Derivable at x = 2.

Disc + cont = Disc

DERIVABILITY OVER AN INTERVAL

(1) Open interval :

A function f(x) is said to be derivable in $\boxed{(a, b)}$ if it is derivable at **each and every** point of its interval.

(2) Closed interval :

A function f(x) is said to be derivable in $\boxed{[a, b]}$ if –

(i) It is derivable in $\boxed{(a, b)}$

(ii) Derivable at $x = a^+$ i.e. $f'(a^+)$ must exist finitely

(iii) It must be derivable at $x = b^-$ i.e. $f'(b^-)$ must exist finitely.

Q. (i) $f(x) = |\ln x|$ at $x = 1$

(ii) $f(x) = (\ln^2 x)$ at $x = 1$

Discuss Derivability.

(i) $f'(1^+) = \lim_{h \to 0^+} \frac{f(1+h) - f(1)}{h}$

$= \lim_{h \to 0^+} \frac{|\ln(1+h)|}{h} = \frac{\ln(1+h)}{h} = 1$

$f'(1^-) = \lim_{h \to 0^+} \frac{f(1-h) - f(1)}{-h} = \frac{|\ln(1-h)|}{-h} = \frac{-\ln(1-h)}{-h} = -1$

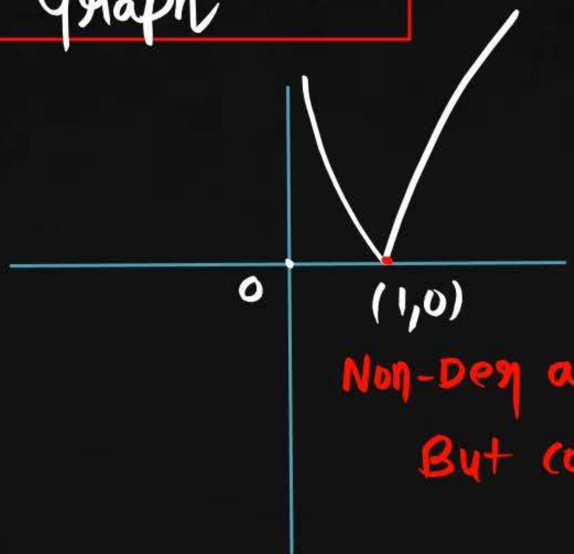
$\boxed{RHD \neq LHD}$

Non-Derivable at $x=1$

(sharp)

Continous at $x=1$

Graph



Non-Deri at $x=1$

But cont. at $x=1$

(ii) $\quad f(x) = (\ln x)^2$

$$f'(1^+) = \lim_{h\to 0^+} \frac{f(1+h) - 0}{h}$$

$$= \lim_{h\to 0^+} \frac{(\ln(1+h))^2}{h} = 0$$

$$f'(1^-) = \lim_{h\to 0^+} \frac{f(1-h) - f(1)}{-h}$$

$$= \lim_{h\to 0^+} \frac{(\ln(1-h))^2}{-h} = 0$$

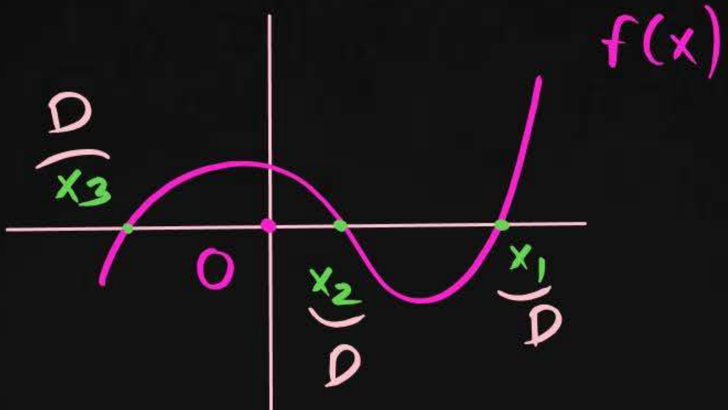
Derivable at $x=1$

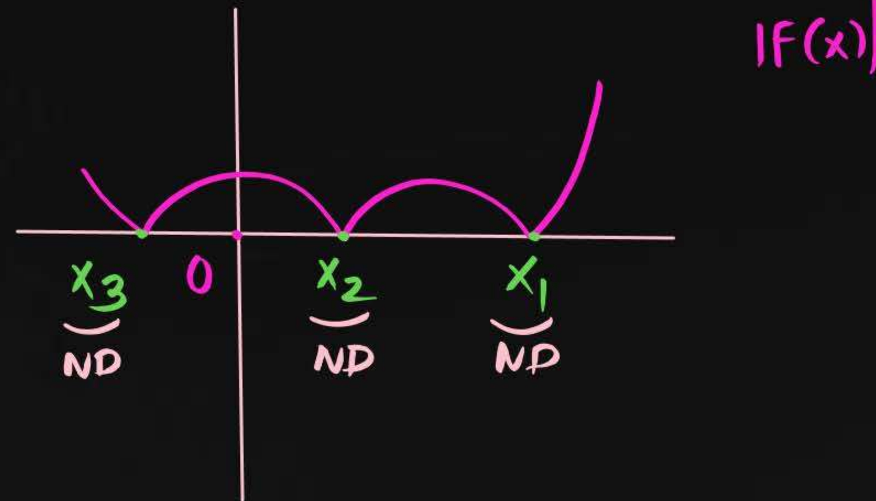
Continous at $x=1$.

GKB

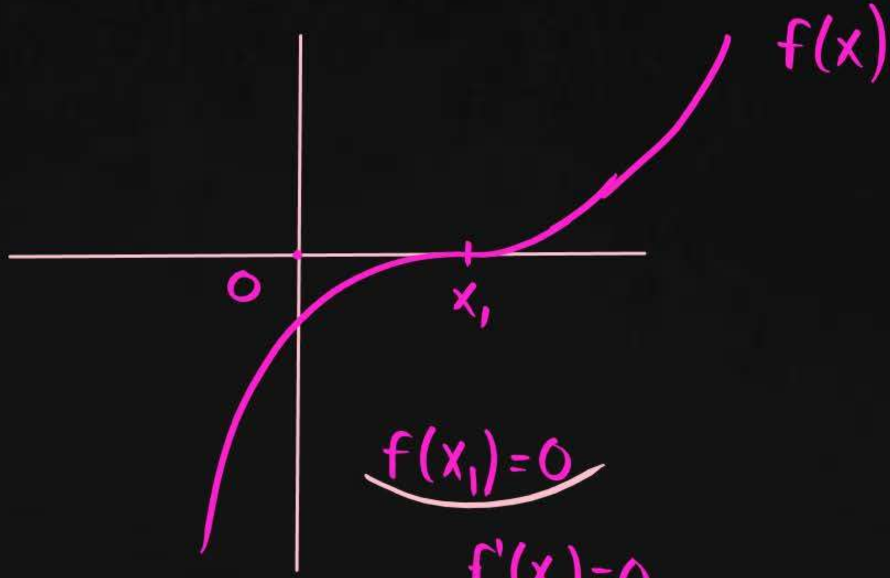
"DERIVABLE"

(1) Consider the graph of a differentiable function f(x). If f(x) is differentiable then |f(x)| will be differentiable at all those points where $f(x) \neq 0$, and those points where $f(x) = 0$, then function |f(x)| may be differentiable or may not be differentiable.



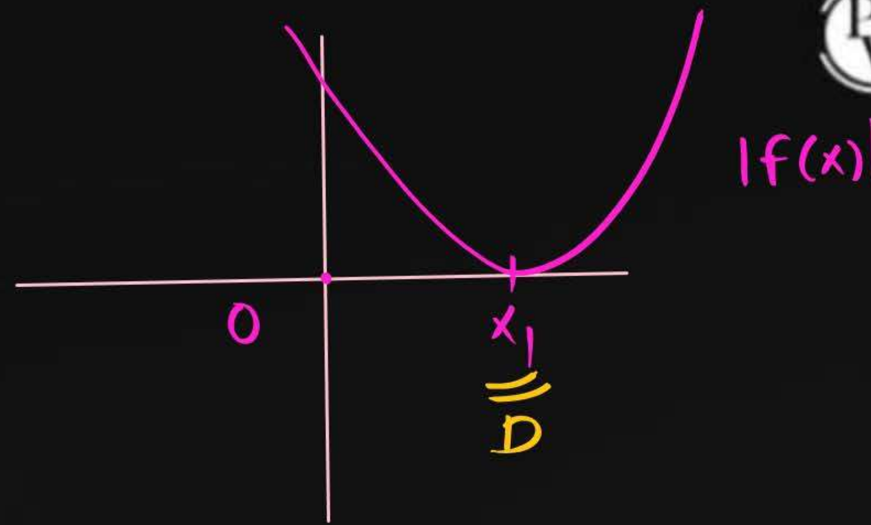


$$\begin{Bmatrix} f(x_1)=0 \\ f(x_2)=0 \\ f(x_3)=0 \end{Bmatrix} \qquad \begin{matrix} f'(x_1)\neq 0 \\ f'(x_2)\neq 0 \\ f'(x_3)\neq 0 \end{matrix}$$



$f(x_1)=0$

$f'(x_1)=0$



(2) If $\boxed{f(x) \text{ is 'D' at } x = x_0}$, then $\boxed{|f(x)| \text{ is also 'D' at } x = x_0}$ iff $\boxed{f(x_0) = 0 = f'(x_0)}$.

($\boxed{\text{This statement is only for those pts where } f(x) = 0}$ and if $\boxed{f(x_0) = 0 \neq f'(x_0)}$, then $\boxed{|f(x)| \text{ is not differentiable}}$ at that point.)

$\boxed{\text{BKG}}$

$f(x)$ → 'Derivable'

$f(x) \rightarrow f(x) = 0 \rightarrow |f(x)|$
- $\rightarrow$ 'D' ✓ $\rightarrow f'(x) = 0$
- $\rightarrow$ 'ND' ✓ $\rightarrow f'(x) \neq 0$

$f(x) \rightarrow f(x) \neq 0 \rightarrow |f(x)|$
- $\rightarrow$ 'D'
- $\rightarrow$ ~~'ND'~~

Q. Let $f(x) = \begin{cases} |14x^2 - 8x + 5|, & \text{if } 8x^2 - 6x + 1 \geq 0 \\ [4x^2 - 8x + 5], & \text{if } 8x^2 - 6x + 1 < 0 \end{cases}$, where $[\alpha]$ denotes the greatest integer less than or equal to α. Then the number of points in R where f is not differentiable is ________ .

[JEE Main 2022 (25 July – Shift 1)]

THEOREMS ON DERIVABILITY

"Same as Continuity"

1. If f(x) and g(x) both are derivable at x = a, then $(f(x) +/-/\times/\div g(x))$ will always be derivable at x = a.

However, $h(x) = f(x)/g(x)$ will be derivable at x = a, if $g(a) \neq 0$.

$$\boxed{\begin{array}{l} D \times D = D \\ D + D = D \\ D - D = D \end{array}}$$

$$\boxed{\frac{D}{D} = D}$$

$D \pm ND = ND$

(2) If $\left.\begin{array}{l} f(x) \longrightarrow D \\ g(x) \longrightarrow ND \end{array}\right\}$ at $x = a$. Then $f(x) \pm g(x)$ is ND at $x = a$. but nothing definite can be said about product and quotient.

ex)- $\underset{D}{\cos|x|} \pm \underset{ND}{|x|} \longrightarrow$ ND at $x = 0$.

(3) If $\left.\begin{array}{l} f(x) \longrightarrow ND \\ g(x) \longrightarrow ND \end{array}\right\}$ at $x = a$.

Then nothing definite can be said about sum, difference, product and quotient.

$$D \begin{matrix} + \\ - \\ \times \\ \div \end{matrix} D = D$$

$$D \pm ND = ND$$

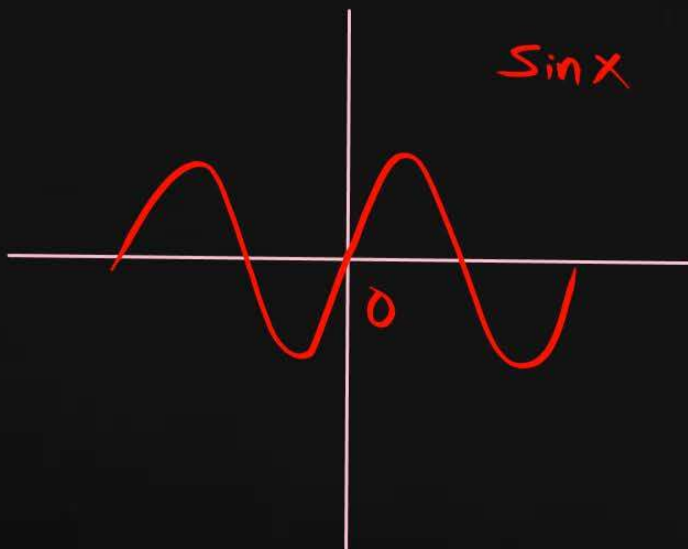
Always check for other cases.

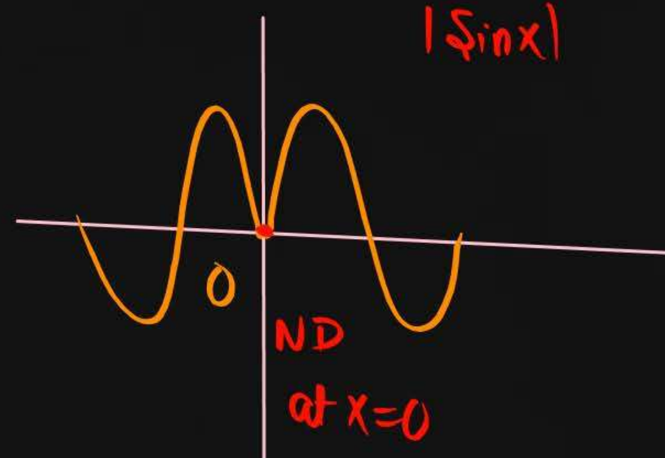
$f(x) \rightarrow$ given

$\xrightarrow{x \rightarrow |x|}$

$f(|x|) \rightarrow -ve\ x$ Bhaag Bacche hue photo Khichao..

" y MIRROR "





Q. Which of the following are derivable at x=0 ?

(JEE 2001)

(A) $\cos|x| + |x|$ $= ND$

D ND

(B) $\cos|x| - |x|$ $= ND$

D ND

(C) $\sin|x| + |x|$

ND ND

(D) $\sin|x| - |x|$

ND ND

check

$f(x) = \sin|x| + |x|$

$f'(0^+) = \lim_{h\to 0^+} \frac{f(h)-f(0)}{h} = \frac{\sin|h|+|h|}{h}$

$= 2$

$f'(0^-) = \lim_{h\to 0^+} \frac{f(-h)-f(0)}{-h} = \frac{\sin|-h|+|-h|}{-h}$

$= -2$

check

$f(x) = \sin|x| - |x|$

$f'(0^+) = \lim_{h\to 0^+} \frac{f(h)}{h} = 0$ ✓

$f'(0^-) = \lim_{h\to 0^+} \frac{f(-h)}{-h} = 0$ ✓

" Inko lagae
PYQ के
answers turant
baaye"

"GREAT GOD BABA"

BHOKAALI RESULTS

RESULT 1

If a function f(x) is derivable at x=a & f(a)=0, & other function g(x) is continuous at x = a; then f(x).g(x) will always be derivable at x=a.

Converse not True

Example :

$f(x) = x|x|$.
Check its derivability at x=0.

Aam Jindagi :
LHD,
RHD

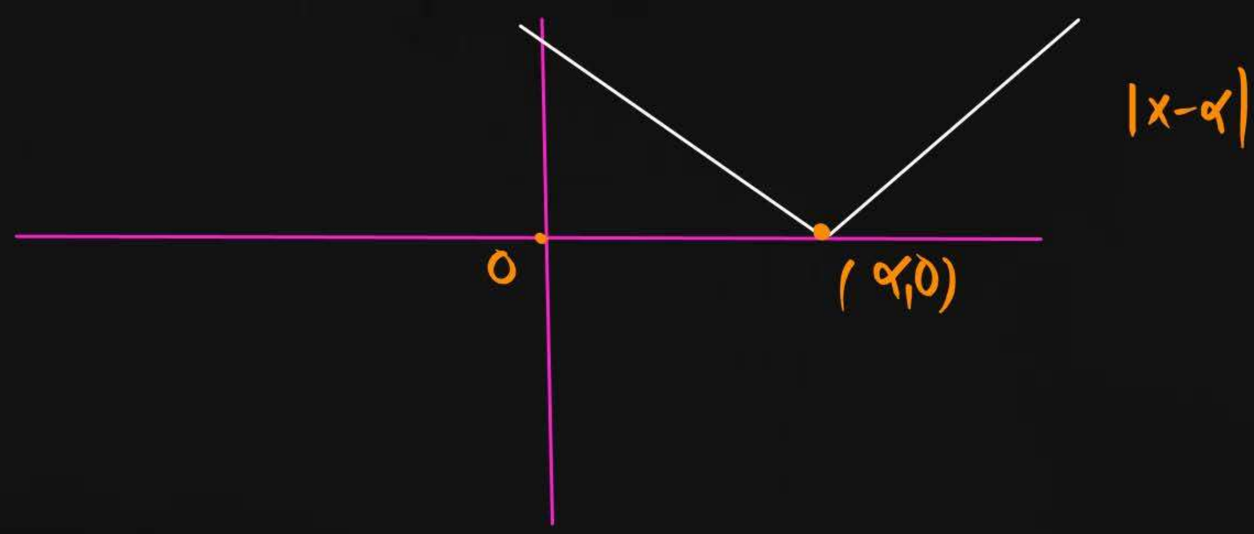
OR

Baba Jindagi:
$x \quad |x|$
Deri. at x=0
x=0
conti
at
x=0
will be derivable
at x=0.

$f(x) . g(x) = H(x)$
'D' at x=a
f(a)=0
Continous
at x=a
must be
derivable
at x=a

⭐⭐⭐⭐⭐⭐⭐

$|x-\alpha|$ always ND but continous at $x=\alpha$



Q. (i) $f(x) = (\ln x).\,|x-1|$ at $x = 1$

(ii) $f(x) = (\sin \pi x)\,|(x-1)(x-2)(x-3)|$ at $x = 1, 2, 3$

(i) $\ln x \rightarrow$ 'D' at $x=1$

$\ln 1 = 0$

$|x-1| \rightarrow$ 'C' at $x=1$

$(\ln x)\,|x-1|$ will be Derivable at $x=1$.

(ii) $\sin \pi x$

$x=1,2,3$

Derivable

$x=1 \rightarrow 0$

$x=2 \rightarrow 0$

$x=3 \rightarrow 0$

$|(x-1)(x-2)(x-3)|$

conti. at

$x=1,2,3$

Product $\rightarrow$ 'D' at $x=1,2,3$

Ans

Result 2

If a function f(x) has a repeated root at x = a ; then f(x) is derivable at x=a.

Q. (i) Check derivability of $f(x) = \underbrace{|(x-3)^8|}_{'D'\ \forall x \in R} + \underbrace{|(x-4)^7|}_{'D'\ \forall x \in R} + \underbrace{|(x-5)|}_{D\ \forall x \in R - \{5\}}$ for x belonging to R.

$f(x) = (x-3)^8$

$x = 3$ 'D'

$f(3) = 0$

$f'(3) = 0$

$|(x-3)^8| \rightarrow$ 'D' at $x = 3$

$\hookrightarrow$ 'D' at $x \in R$

$x = 5$: D+D+ND = ND

$x \neq 5$: D+D+D = D

Ans

Q. Let $S = \{ t \in R : f(x) = |x-\pi|.(e^{|x|}-1).\text{Sin}|x|$ is ND at $x = t\}$, Then the set S is

ND at $x=\pi$ | ND at $x=0$ | ND at $x=0$

(MAINS 2018)

Doubtful points

(A) Phi.

(B) $\{0\}$

(C) $\{\pi\}$

(D) $\{0, \pi\}$

$\boxed{x=0}$ "RR" "Derivable"

$\boxed{x=\pi}$ "RR" "Derivable"

"mod" may be NON-DER at all those pts where अंदर wallah is ZERO

Q. Let the function $f : R \to R$ be defined by $f(x) = x^3 - x^2 + (x-1)\sin x$ and $g : R \to R$ be an arbitrary function. Let $fg : R \to R$ be the product function defined by $(f\,g)\,(x) = f(x)\,g(x)$. Then which of the following statements is/are True? **[JEE (Advanced)-2020 (Paper-I)]**

"D"

✓ 'D' at $x=1$ (conti $x=1$)

✓ $f(1)=0$

(A) If g is continuous at x = 1, then (f g) (x) is differentiable at x = 1. (conti.)

(B) If (fg) (x) is differentiable at x = 1, then g is continuous at x = 1 (converse)

(C) If g is differentiable at x = 1, then (f g) (x) is continuous at x = 1

continous at $x=1$ → Derivable

(D) If (fg) (x) is differentiable at x = 1, then g is differentiable at x = 1.

Tending to G.O.A.T ♡

" Oscillatory limit "

$$\lim_{x\to 0} \sin\left(\frac{1}{x}\right)$$

$$R = \left(\lim_{x\to 0^+} \sin \infty\right) \in [-1,1]$$

$$L = \left(\lim_{x\to 0^-} \sin(-\infty)\right) \in [-1,1]$$

$$\boxed{DNE}$$

Methods to $\boxed{\text{Check Derivability}}$ → (Summary)

1. USING STANDARD RESULTS. → "GREAT GOD BABA"
2. USING LHD & RHD
3. $\boxed{\text{DIFFERENTIATE THE GIVEN FUNCTION}}$ & $\boxed{\text{CHECK ITS CONTINUITY.}}$ → f'(x)

(BUT TC OF :

1. BEFORE APPLYING ; THE GIVEN FUNCTION MUST BE CONTINOUS. f(x)
2. AFTER DIFFERENTIATION ; OSCILLATORY LIMIT , ND , ± ∞ MUSTN'T APPEAR.

ND: Not Defined

↳ If it appears, then: Go back to $\boxed{\text{M-2}}$

$1 \longrightarrow 3 \longrightarrow 2$

ex)- $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$,

Check derivability at x=0.

Standard Result X

$f(x) \to$ 'D' at $x=0$

$f'(x) \to$ 'C' at $x=0$

$$f'(x) = \begin{cases} \underbrace{\cancel{x^2}\left(-\frac{1}{\cancel{x^2}}\right)\cos\frac{1}{x}}_{\text{O.L.}} + \left(\sin\frac{1}{x}\right)2x \; ; & x \neq 0 \\ 0 \; , & x = 0 \end{cases}$$

$\boxed{\text{Method 3 Fails}}$

$\lim_{x \to 0} f'(x) = \text{DNE}$

$f'(0) = 0$

$f'(x) \to$ DC

Method 2

$$\underbrace{RHD}_{} = f'(0^+) = \lim_{h \to 0^+} \frac{f(h) - f(0)}{h} = \frac{h^2 \cdot \sin\frac{1}{h}}{h} = \underbrace{h}_{0} \cdot \underbrace{\left(\sin\frac{1}{h}\right)}_{[-1,1]} = 0$$

$$\underbrace{LHD}_{} = f'(0^-) = \lim_{h \to 0^+} \frac{f(-h) - 0}{-h} = +\underbrace{h}_{0} \cdot \underbrace{\sin\frac{1}{h}}_{[-1,1]} = 0$$

Hence Derivable at $x = 0$.

Result 3

Derivative function of a continuous and derivable function need not to be continuous.

ex)- $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$, $\longrightarrow f(0^+) = 0 \cdot ([-1,1]) = 0 = f(0^-) = f(0)$

f(x) is Differentiable at x = 0 and Continuous at x = 0.

Q. Find the value of $\boxed{\text{'m' and 'n'}}$ if the function $\boxed{f(x) \text{ is derivable } \forall\ x \in R}$,

$$f(x) = \begin{cases} \sin x, & x < \pi \\ \underline{mx + n}, & x \geq \pi \end{cases}$$

✓ $f(x)$ Continous $\forall x \in R$

Standard Result ✗

M3

$$f(\pi^+) = f(\pi^-) = f(\pi)$$

$$m\pi + n = \boxed{0 = m\pi + n}$$

$m = -1$

$\boxed{n = \pi}$ Ans

$$f'(x) = \begin{cases} \cos x ; & x < \pi \\ m ; & x \geq \pi \end{cases}$$

$\cos\pi = \boxed{m = -1}$

Ans

DETERMINATION OF FUNCTION

Determination of function which are differentiable & satisfying the given functional rule –

$\boxed{\text{Method 1}}$ $\rightarrow$ $\boxed{\text{Using LHD-RHD}}$

OR

$\rightarrow$ $\boxed{\text{Partial Diff'n}}$ ✓

① PD wRT 'x'

② PD wrt 'y'

③ Divide both the Relation (ek tareeke se)

④ Enjoy

Using Partial Derivative Method

Q. Let f(x) be a derivable function satisfying $f(x/y) = f(x) - f(y)$ $\forall$ x, y $\geq$ 0. If $f'(1) = 1$, then find f(x).

$x=y=1 \Rightarrow f(1) = f(1) - f(1) = 0$

$f\left(\frac{x}{y}\right) = f(x) - f(y)$

PD WRT 'x'

$$\boxed{f'\left(\frac{x}{y}\right) \cdot \frac{1}{y} = f'(x) - 0}$$

PD WRT y

$$\boxed{f'\left(\frac{x}{y}\right) \times \left(-\frac{1}{y^2}\right) = 0 - f'(y)}$$

DIVIDING

$$\frac{\frac{1}{y}}{\frac{-x}{y^2}} = \frac{f'(x)}{-f'(y)} \Rightarrow \boxed{\frac{y}{x} = \frac{f'(x)}{f'(y)}}$$

"VS"

$$\boxed{y \cdot f'(y) = x \cdot f'(x) = K}$$

$$x \cdot f'(x) = K \xrightarrow{x=1} 1 \cdot f'(1) = K \Rightarrow \boxed{K=1}$$

$$\boxed{x \cdot f'(x) = 1}$$

$$\int f'(x)\,dx = \int \frac{1}{x}\,dx$$

$$\boxed{f(x) = \ln|x| + K}$$

$|x| = x$

$$f(x) = \ln x + K$$

$x = 1$

$$f(1) = 0 + K = 0$$

$$\boxed{f(x) = \ln x}$$

Ans

Q. Suppose f(x) is derivable function that satisfies the equation $f(x+y) = f(x) + f(y) + x^2y + xy^2$ $\forall$ x, y $\in$ R and $\lim_{x\to 0} \frac{f(x)}{x} = 1$, then find

$\xrightarrow{LH} \lim_{x\to 0^+} f'(x) = 1$

$\boxed{f'(0) = 1}$

$x=0, y=0$

(i) f(0) $\boxed{f(0)=0}$ (ii) f'(0) (iii) f'(x) (iv) f(3).

$0 =$

$f'(x+y) = f'(x) + y(2x) + y^2$

$f'(x+y) = f'(y) + x^2 + 2xy$

DIVIDE:

$$1 = \frac{f'(x) + 2xy + y^2}{f'(y) + x^2 + 2xy}$$

"VS"

$\boxed{f'(x) - x^2 = f'(y) - y^2} = K$

$\boxed{f'(x) = K + x^2}$

$x=0$: $1 = K$

$\boxed{f'(x) = x^2 + 1}$ Ans

$x=0$: $f'(0) = 1$ Ans

$f'(x) = x^2 + 1$

$$\boxed{f(x) = \frac{x^3}{3} + x + K} \xrightarrow{x=0} 0 = K$$

$$\boxed{f(x) = \frac{x^3}{3} + x}$$ Ans

$x = 3$

$f(3) = 12$ Ans

PD ♡ ♡ ♡

ROLLE’S THEOREM

Let $f(x)$ be a function subjected to the following conditions-

(i) $f(x)$ is continuous in $[a, b]$
(ii) $f(x)$ is derivable in (a, b)
(iii) $f(a) = f(b)$.

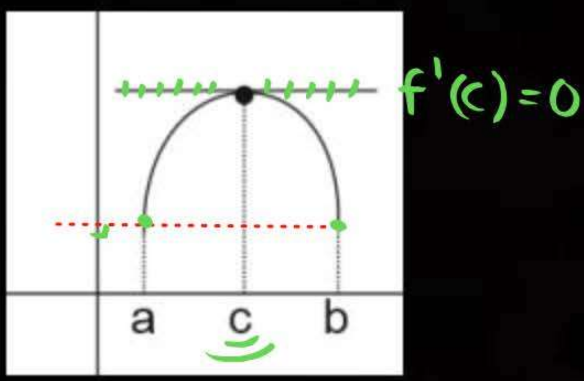
Then there exists at-least one point $x = c$ in (a, b) such that $f'(c) = 0$

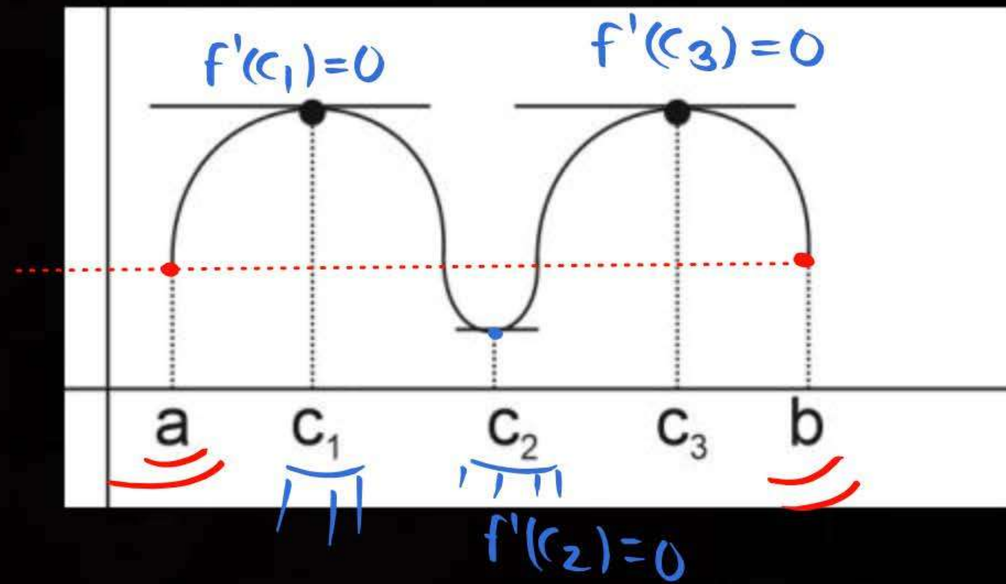
OR

there exists atleast one Root of $f'(x) = 0 \cdots$

$x = c$

$c \in (a, b)$


f'(c)=0
a
c
b


f'(c1)=0
f'(c3)=0
a
c1
c2
c3
b
f'(c2)=0

NOTE :

(i) Geometrical interpretation-

Geometrically Rolle's theorem says in (a , b) the curve has at-least one tangent parallel to x-axis.

(ii) Alternative statement of Rolle's theorem-

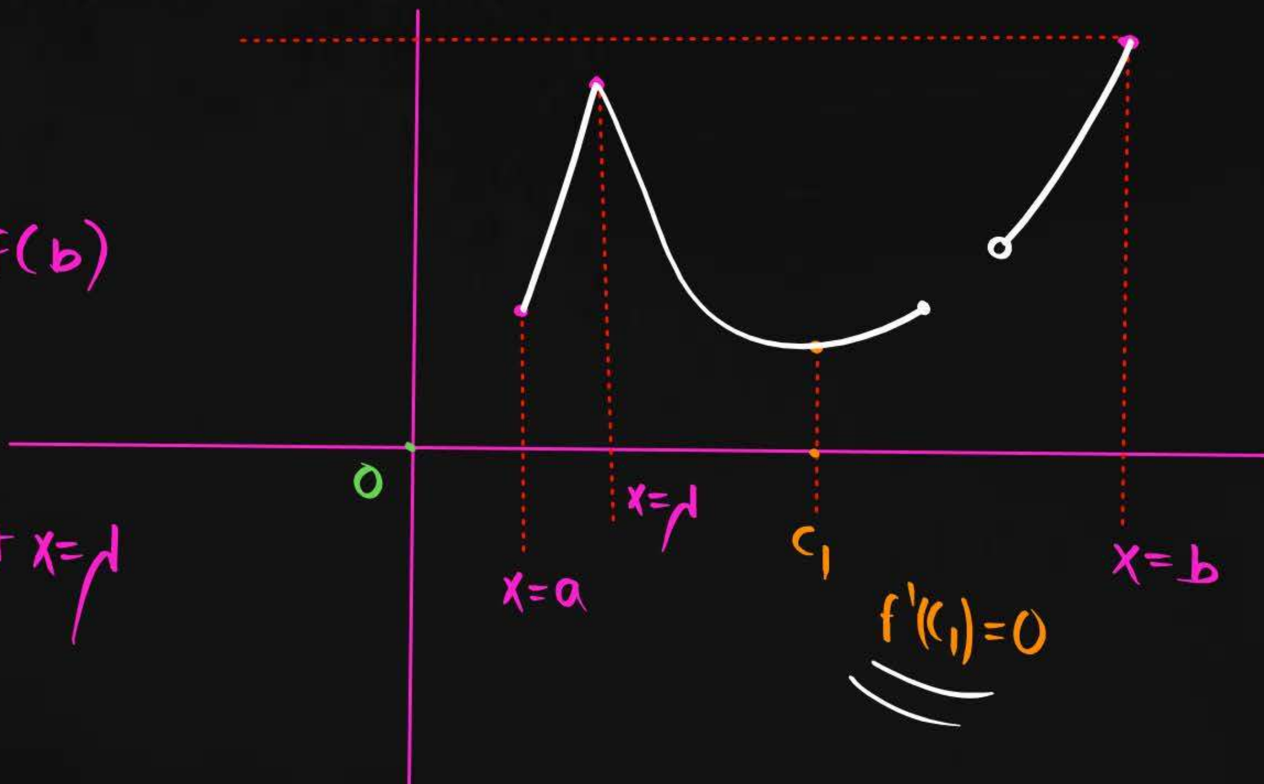
If f(x) satisfies all the three conditions then the equation $f'(x) = 0$ has at least one root in (a, b).

(iii) Converse of Rolle's theorem is not true.

$f(a) \neq f(b)$

$x = d$

ND at $x = d$



"DC"

"Converse NOT TRUE"

Q. Verify $\boxed{\text{Rolle's theorem}}$ and find $\boxed{\text{its 'c'}}$ –

(i) $f(x) = x(x+3)e^{-x/2}$ in $[-3, 0]$

(ii) $f(x) = \frac{\sin x}{e^x}$ in $[0,\pi]$

(iii) $x^3 - 3x^2 + 2x + 5$ in $[0, 2]$

(iv) $f(x) = 1 - x^{2/3}$ in $[-1,1]$

$\boxed{\text{DIBY}}$

(i) Continous on $[-3,0]$

Derivable on $(-3,0)$

Rolles' ✓

$f(-3) = 0$

$f(0) = 0$

$$f'(x) = \frac{(x^2+3x)e^{-x/2}}{-2} + e^{-x/2}(2x+3) = 0$$

$$(e^{-x/2})\left(\frac{-x^2-3x}{2} + 2x + 3\right) = 0$$

$\boxed{x = \cancel{3}, -2}$

'c'

Ans

Q. Let $n \in N$, If the value of 'c' in Rolle's Theorem for $f(x) = 2x(x-3)^n$ in $[0,3]$ is ¾, find 'n'

Cont ✓

Der ✓

$f(0)=0=f(3)$ ✓

$f'(x) = 2xn(x-3)^{n-1} + (x-3)^n(2)$

$= (x-3)^{n-1}(2nx + 2(x-3)) = 0$

Put $c = x = \frac{3}{4}$

$\underbrace{\left(\frac{3}{4}-3\right)^{n-1}}_{\text{non-zero}}\left(\frac{3n}{2}-\frac{18}{4}\right)=0$

$\boxed{n=3}$ Ans

Rolle's pe sawaal k type:

1. Given 2 func^{n}s $\to f(x)$, $\to g(x)$

2. Unke Intersection ki Baat hoti है i.e.

$$\boxed{f(x) = g(x)}$$

3. $f(x) - g(x) = 0$ के Root ki Baat ho Rahi है।

4. $H(x) = f(x) - g(x) \longrightarrow$ Rolles' ✓

Q. Let $f: [0,1] \to R$ be a twice differentiate function in $(0, 1)$ such that $f(0) = 3$ and $f(1) = 5$. If the line $y = 2x + 3$ intersects the graph of f at only two distinct points in $(0, 1)$, then the least number of points $x \in (0, 1)$ at which $f''(x) = 0$, is ______. 2

OR $\boxed{H''(x)=0}$

[JEE Main-2022 (28 July - Shift 1)]

$f(x) = 2x+3$

$\boxed{f(x) - (2x+3) = 0}$

Rolles

$\boxed{H(x) = f(x) - (2x+3)}$ → $(0,1)$

C D, C D

$H(0) = 3-3 = 0$

$H(1) = 5-5 = 0$

→ $H'(x)$ atleast one Root in $(0,1)$

$H'(x) = f'(x) - 2$

$H''(x) = f''(x)$

$H'(x) \to$ D, C

$$H(x) \begin{cases} \to x_1 \; ; \quad H(x_1) = 0 \\ \to x_2 \; ; \quad H(x_2) = 0 \end{cases}$$

$H(0)=0$

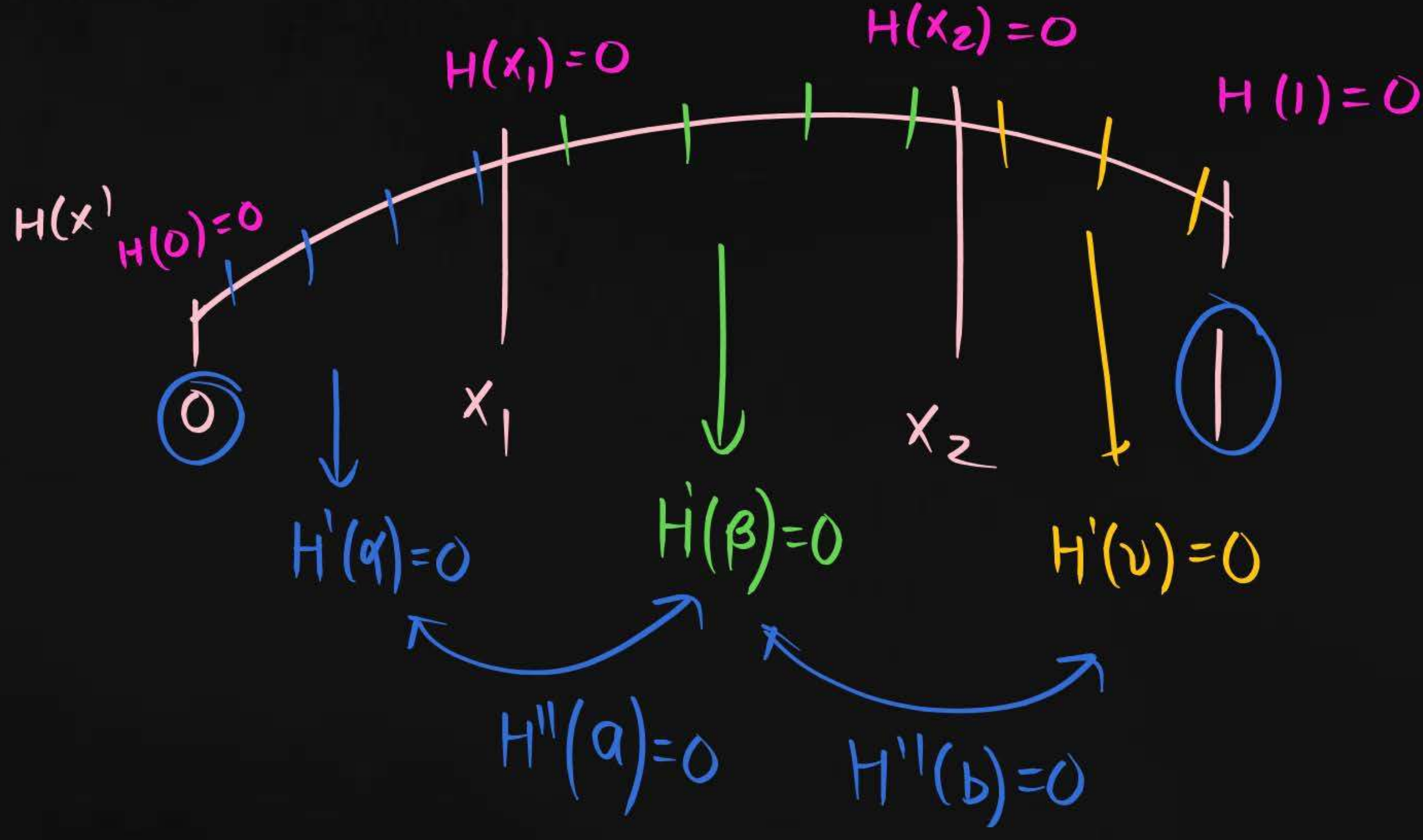
$H(1)=0$

$0 \leftrightarrow x_1 \leftrightarrow x_2 \quad 1$

$H'(x)=0 \qquad H'(x)=0 \qquad H'(x)=0$

$H''(x)=0 \qquad H''(x)=0$

- $f''(x), f'(x)$ K sol^n ya phiR RooTs की Baat ho... Then always use Rolles'..


$H(x)$
$H(0)=0$
$H(x_1)=0$
$H(x_2)=0$
$H(1)=0$
0
x_1
x_2
1
$H'(\alpha)=0$
$H'(\beta)=0$
$H'(\nu)=0$
$H''(a)=0$
$H''(b)=0$

OP ♡ ♡ ♡

OP ♡ ♡

LMVT

Let f(x) is a function such that-

(i) It is continuous in [a,b]

(ii) It is differentiable in (a,b)

Then there exists at-least one point x = c, such that :

$c \in (a,b)$

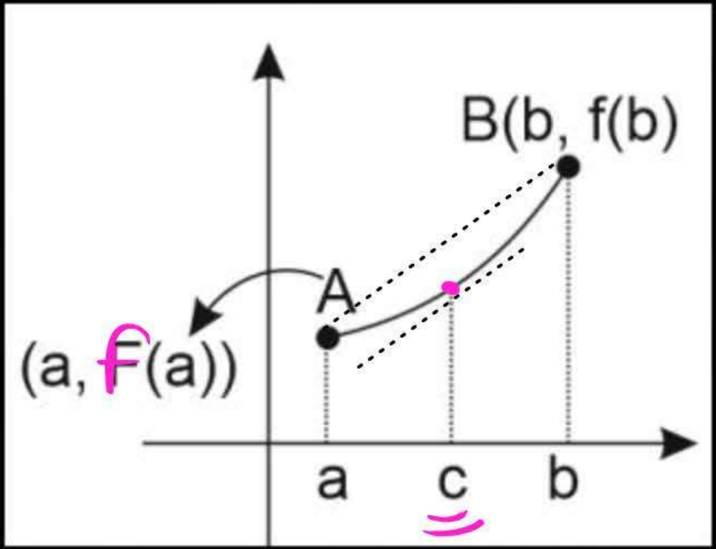
$$\boxed{f'(c) = \frac{f(b)-f(a)}{(b-a)}}$$

final initial

$$M_{AB} = \frac{f(b) - f(a)}{b - a}$$

$$= \left.\frac{dy}{dx}\right|_{x=c}$$

$$= f'(c)$$



1. Geometrically the slope of the secant line joining the curve at $x = a$ and $x = b$ is equal to the slope of the tangent line drawn to the curve at $x = c$

2. Average rate of change of function over some interval is equal to instantaneous rate of change of function at some point of the interval.

3. Rolle's is a special case of LMVT.

Q. Find 'c' of LMVT $f(x) = \sqrt{x-1}, x \in [1, 3]$.

$f'(x) = \frac{1}{2\sqrt{x-1}}$

$\frac{\sqrt{2}-0}{2} = \frac{f(3)-f(1)}{3-1} = f'(c) \longrightarrow f'(c) = \frac{1}{\sqrt{2}} = \frac{1}{2\sqrt{c-1}}$

$\boxed{c = 3/2}$ Ans

Q. Find a point on the curve $f(x) = \sqrt{x-2}$ in $[2, 3]$ where the tangent is parallel to the chord joining the end points.

$a = 2$, $b = 3$

$\frac{1-0}{3-2} = \frac{f(b)-f(a)}{b-a} = f'(c) = \frac{1}{2\sqrt{c-2}} \rightarrow 4(c-2) = 1$

$\boxed{c = \frac{9}{4}}$ Ans

Q. If $a < b$. Then Prove that a real number ' c ' can be found in (a, b) such that $\boxed{3c^2 = a^2 + ab + b^2}$

func^n assume $\rightarrow f(x) = x^3 + K$

C ✓, D ✓

LMVT:

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

$$\frac{(b^3 + \cancel{K}) - (a^3 + \cancel{K})}{b - a} = 3c^2$$

$$\frac{\cancel{(b-a)}(b^2 + a^2 + ab)}{\cancel{b-a}} = 3c^2$$

$$\boxed{a^2 + b^2 + ab = 3c^2}$$

And

"middle se"

DIVIDING THE GIVEN INTERVAL & APPLYING LMVT IN THOSE INTERVALS

Q. If $f(x)$ is continous on $[0, 2]$ and derivable on $(0, 2)$ such that $f(0) = 2$, $f(2) = 8$ and $f'(x) \leqslant 3\ \forall\ x$ in the interval (0,2). Then $f(1)$ has a value :

c_1 LMVT

0 LMVT 1 c_2 2

$$\frac{f(2)-f(1)}{2-1} = f'(c_2) \leq 3$$

$$8 - f(1) \leq 3$$

$$\boxed{f(1) \geq 5}$$

$$\frac{f(1)-f(0)}{1-0} = f'(c_1)$$

$$f(1) - 2 = f'(c_1) \leq 3$$

$$f(1) - 2 \leq 3$$

$$\boxed{f(1) \leq 5}$$

Ans $\rightarrow f(1) = 5$