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JEE MAINS & ADVANCED

ONE SHOT

* JEE Main (12 Mark)
JEE Adv (8 Mark)

(Basic → Adv)

Hello
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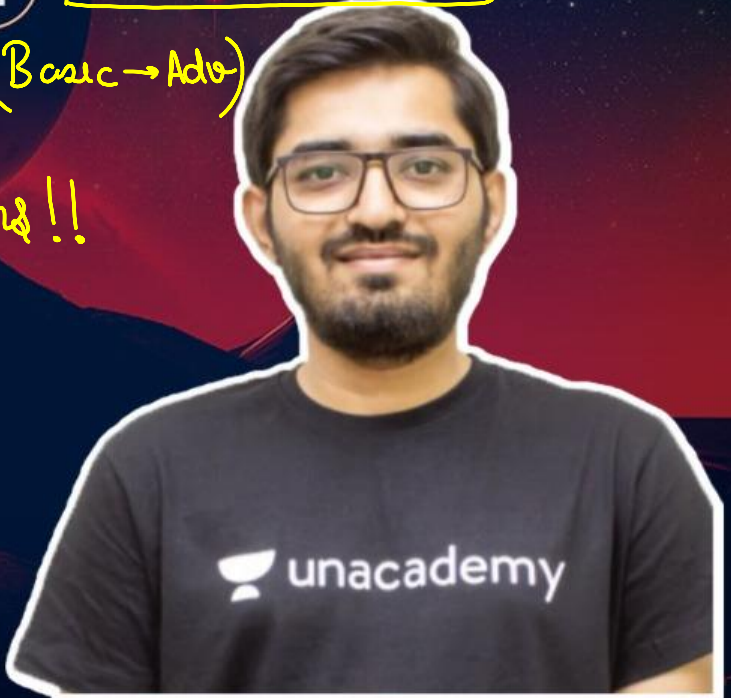
SEQUENCE AND SERIES



NISHANT VORA

Nishant Vora

B.Tech - IIT Patna

- 7+ years Teaching experience
- Mentored 5 lac+ students
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BounceBack

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$\boxed{\text{6-7 hr}}$

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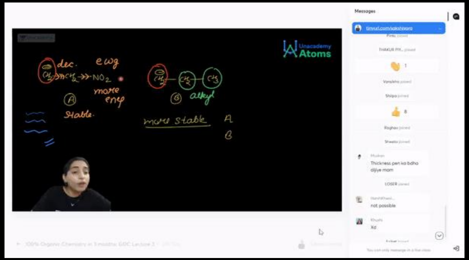
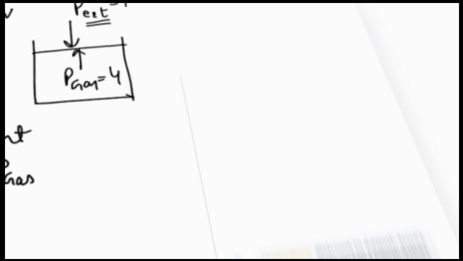
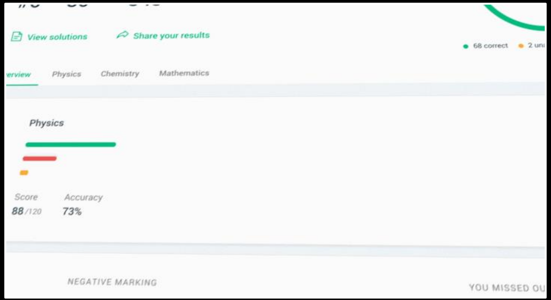




AP
GP
HP
AGP *
Misc *

AM
GM
HM



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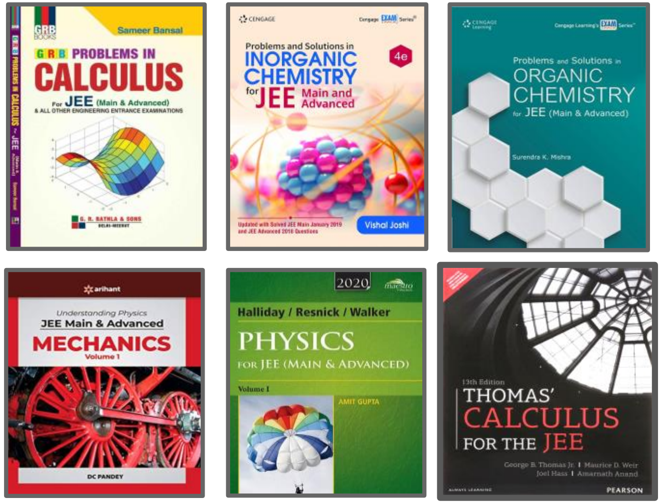

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Sequence and Series

Sequence is arrangement of numbers in some logical pattern

Eg. 1, 2, 4, 8, 16, ? 32

Eg. 1, 4, 9, 16, ? 25

Series is sum of elements of a sequence

Eg. 1 + 2 + 4 + 8 + 16 + ...

Eg. 1 + 4 + 9 + 16 + ...

AP ✓
GP ✓
HP ✓
AGP ✓

Arithmetic Progression

Arithmetic Progression

AP is a sequence whose terms increase or decrease by a fixed number. This fixed number is called the common difference.

1. **E.g:** 1, 4, 7, 10, 13........

+3 +3 +3 +3

$d=3$

$a=1$

2. **E.g:** 4, 2, 0, -2, -4........

-2 -2 -2 -2

$d=-2$

$a=4$

3. **E.g:** 3, 3, 3, 3, 3........

+0 +0 +0

$d=0$

$a=3$

General term/n^{th} term/Last term of G.P. :

If a is the first term and d the common difference, of AP.

$$\boxed{\underbrace{\overset{n^{th}\text{ term}}{T_n}}_{a_n} = \underbrace{a}_{\text{First term}} + (\underbrace{n}_{\text{no of term}}-1)\underbrace{d}_{\text{c.d}}}$$

If 5th & 6th terms of an A.P. are respectively 6 & 5.

Find the 11th term of the A.P.

$T_5 = 6$

$T_6 = 5$

$\boxed{T_{11} = 0}$

$T_{12} = 6+5-12$

$= -1$

$a+4d=6$

$a+5d=5$

$a = ✓$

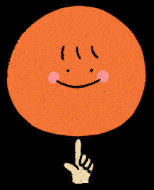
$d = ✓$

$T_{11} = a+10d$

$T_{20} = 6+5-20$

$= 11-20$

$= -9$



If $T_m = n$ and $T_n = m \Rightarrow T_{m+n} = 0$

Shortcut-1

AP

$T_m = n$

$T_n = m$

$\boxed{T_{m+n} = 0}$

Q

In an A.P., if $T_{2000} = 20$ & $T_{20} = 2000$, find $T_{2020} = ?$

$$T_{2000} = 20$$

$$T_{20} = 2000$$

$$T_{2020} = 0 \checkmark$$

If $T_m = n$ and $T_n = m \Rightarrow T_r = m+n-r$

Shortcut-2

$$T_m = n$$

$$T_n = m$$

$$\boxed{T_r = m+n-r}$$

In an A.P., if $T_{64} = 13$ & $T_{13} = 64$, find $T_{100} = ?$

A. -24

B. -23

C. -22

D. -21

$$\left.\begin{aligned} T_{64} &= 13 \\ T_{13} &= 64 \end{aligned}\right\}$$

$$\begin{aligned} T_{100} &= 13 + 64 - 100 \\ &= 77 - 100 \\ &= -23 \end{aligned}$$

If $n\,T_n = m\,T_m \Rightarrow T_{m+n} = 0$

Shortcut-3

$$\boxed{n\,T_n = m\,T_m}$$

$$\boxed{T_{m+n} = 0}$$

Q

If 9 times the 9^{th} term of an AP is equal to 13 times the 13^{th} term, then the 22^{nd} term of the AP is

A. 0 **B.** 22 **C.** 198 **D.** 220

$$9\,T_9 = 13\,T_{13}$$

$$T_{22} = 0$$

If 8^{th} & 15^{th} terms of an A.P. are respectively 37 & 65. Find a and d ?

Shortcut-4

$T_8 = 37$

$T_{15} = 65$

Normal Zindagi

$a + 7d = 37 \quad \text{---} (1)$

$a + 14d = 65 \quad \text{---} (2)$

$-7d = -28$

$\therefore d = \frac{28}{7} = 4$

Mentos Zindagi

$d = \frac{28}{7} = 4$

Q

If $a_1, a_2, a_3 \ldots$ and $b_1, b_2, b_3 \ldots$ are A.P. and $a_1 = 2$, $a_{10} = 3$, $a_1 b_1 = 1 = a_{10} b_{10}$ then $a_4 b_4$ is equal to

A. $\frac{35}{27}$

B. 1

C. $\frac{27}{28}$

D. $\frac{28}{27}$

[JEE M 2022]

$a_1 b_1 = 1$ $\quad a_{10} b_{10} = 1$

$a_1 = 2$

$a_{10} = 3$

$\boxed{d_1 = \frac{1}{9}}$

$b_1 = \frac{1}{2}$

$b_{10} = \frac{1}{3}$

$d_2 = \frac{\frac{1}{3} - \frac{1}{2}}{9} = \frac{-1}{54}$

$a_4 = a_1 + 3d_1$

$= 2 + 3\left(\frac{1}{9}\right) = 2 + \frac{1}{3} = \frac{7}{3}$

$b_4 = b_1 + 3d_2$

$= \frac{1}{2} + 3\left(\frac{-1}{54}\right) = \frac{1}{2} - \frac{1}{18} = \frac{8}{18} = \frac{4}{9}$

$a_4 b_4$

$= \frac{7}{3} \times \frac{4}{9}$

$\Rightarrow \frac{28}{27}$

How many three digit numbers are divisible by 6?

A. 149

B. 150

C. 151

D. 166

Shortcut-5

$$\frac{\cancel{100}}{\cancel{6}} \quad 16 \qquad \frac{\cancel{999}}{\cancel{6}} \quad 166$$

$$166 - 16 = 150$$

Q

How many numbers between 11 and 90 are divisible by 7?

$1 \qquad 12$

$\frac{\cancel{11}}{\cancel{7}} \qquad \frac{\cancel{90}}{\cancel{7}}$

11

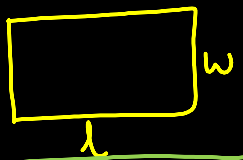
Let $l_1, l_2, \ldots, l_{100}$ be consecutive terms of an arithmetic progression with common difference d_1, and let $w_1, w_2, \ldots, w_{100}$ be consecutive terms of another arithmetic progression with common difference d_2, where $d_1 d_2 = 10$. For each $i = 1, 2, \ldots, 100$, let R_i be a rectangle with length l_i, width w_i and area A_i. If $A_{51} - A_{50} = 1000$, then the value of $A_{100} - A_{90}$ is ________.

10th

[JEE Adv. 2022]

$l_1 \; l_2 \; l_3 \ldots$ AP d_1

$w_1 \; w_2 \; w_3 \ldots$ AP d_2



$\boxed{d_1 d_2 = 10}$ (given)

$\boxed{l_1 d_2 + w_1 d_1 = 10}$

$\boxed{T_n = a + (n-1)d}$

$\boxed{A = l \times b}$

$A_{51} - A_{50} = 1000$

$l_{51} w_{51} - l_{50} w_{50} = 1000$

$(l_1 + 50d_1)(w_1 + 50d_2) - (l_1 + 49d_1)(w_1 + 49d_2) = 1000$

$l_1 d_2 + d_1 w_1 + 50^2 d_1 d_2 - 49^2 d_1 d_2 = 1000$

$$A_{100} - A_{90}$$

$$= l_{100} w_{100} - l_{90} w_{90}$$

$$= (l_1 + 99d_1)(w_1 + 99d_2) - (l_1 + 89d_1)(w_1 + 89d_2)$$

$$= \underline{10 l_1 d_2 + 10 d_1 w_1} + 99^2 d_1 d_2 - 89^2 d_1 d_2$$

$$= 10(10) + d_1 d_2 (99 - 89)(99 + 89)$$

$$= 100 + 10(10)(188)$$

$$= \underline{18900}$$

Sum of n Terms of A.P.

Sum of n terms of an A.P.

✓ $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

✓ $S_n = \frac{n}{2}\left[F.T + L.T\right] = \frac{n}{2}\left(a_1 + a_n\right)$

Q

Suppose $a_1, a_2, \ldots, a_n, \ldots$ be an arithmetic progression of natural numbers. If the ratio of the sum of the first five terms of the sum of first nine terms of the progression is 5 : 17 and $\boxed{110 < a_{15} < 120}$, then the sum of the first ten terms of the progression is equal to -

[JEE M 2022]

A. 290

B. 380

C. 460

D. 510

① $\boxed{\frac{S_5}{S_9} = \frac{5}{17}}$

② $\boxed{110 < a_{15} < 120}$

$\boxed{S_{10} = ?}$

$$\frac{\frac{\cancel{5}}{\cancel{2}}(2a+4d)}{\frac{9}{\cancel{2}}(2a+8d)} = \frac{\cancel{5}}{17}$$

$$34a + 68d = 18a + 72d$$

$$16a = 4d$$

$$\boxed{4a = d} \quad \therefore \boxed{a = 2} \quad \boxed{d = 8}$$

$$S_{10} = \frac{10}{2}(4 + 9\times 8)$$

$$= 5 \times 76$$

$$= 380$$

$$a_{15} = a + 14d$$

$$= a + 14(4a)$$

$$= 57a$$

$$110 < 57a < 120$$

$$\frac{110}{57} < a < \frac{120}{57}$$

$$\boxed{1.9m < a < 2.1m}$$

Q

If $\{a_i\}_{i=1}^{n}$ where n is an even integer , is an arithmetic progression with $\boxed{\text{common difference } 1}$, and $\boxed{\sum_{i=1}^{n} a_i = 192}$, $\boxed{\sum_{i=1}^{n/2} a_{2i} = 120}$, then n is equal to:

[JEE M 2022]

a_{2i}

A. 48

B. 96

C. 92

D. 104

① $a_1 + a_2 + a_3 + \ldots + a_n = 192$

② $a_2 + a_4 + a_6 + \ldots + a_n = 120$

③ $a_1 + a_3 + a_5 + \ldots + a_{n-1} = 72$

$$a_2 - a_1 = d$$

$$a_4 - a_3 = d$$

$$a_2 + a_4 + a_6 + \dots + a_n = 120$$

$$a_1 + a_3 + a_5 + \dots + a_{n-1} = 72$$

$$d + d + d + \dots d = 48$$

$$d = 1$$

$$d\left(\frac{n}{2}\right) = 48$$

$$\therefore \boxed{n = 96}$$

Let S_1 be the sum of first 2n terms of an arithmetic progression. Let S_2 be the sum of first 4n terms of the same arithmetic progression. If $(S_2 - S_1)$ is 1000, then the sum of the first 6n terms of the arithmetic progression is equal to

[JEE M 2021]

A. 1000

B. 7000

C. 5000

D. 3000

✓ $S_1 = S_{2n} = 2n$

✓ $S_2 = S_{4n} = 4n$

★ $S_{6n} = ?$

✓ $\boxed{S_2 - S_1 = 1000}$

$4n - 2n = 1000$

$2n = 1000$

$\boxed{n = 500}$

#chalaki

AP $\underbrace{1, 1, 1, 1, 1, 1, \ldots}_{2n}$

$S_{6n} = 6n$

$= 6(500)$

$= 3000$

Q

If S_n denotes the sum first 'n' terms of A.P then $\dfrac{S_{3n} - S_{n-1}}{S_{2n} - S_{2n-1}} = ?$

A. 2n-1 = 1

B. 2n = 2

C. 2n+1 = 3

D. n/2 = 1/2

Shortcut-6

$n = 1$

$$\frac{S_3 - S_0}{S_2 - S_1} = \frac{(1+2+3) - (0)}{(1+2) - (1)} = \frac{6}{2} = 3$$

AP $1 + 2 + 3 + 4 + \ldots$

$\boxed{\text{Ninja}}$

n = ✓

Q The sum of first n terms of two A.P.'s are in the ratio $3n + 8 : 7n + 15$, then the ratio of their 12^{th} term is

A. 8 : 7 **B.** 7 : 16 **C.** 74 : 169 **D.** 13 : 47

Shortcut-7

$n \to 2(12) - 1 \to 23$

$n \to 2n - 1$

$$\frac{S_n}{S'_n} \qquad \frac{a_n}{a'_n}$$

$n \to \frac{n+1}{2}$

$n \to 2(12) - 1 \to 23$

$S_n \quad a_1 \; a_2 \; a_3 \; \ldots \; a_n$

$S'_n \quad a'_1 \; a'_2 \; a'_3 \; \ldots \; a'_n$

$$\frac{S_n}{S'_n} = \frac{3n + 8}{7n + 15}$$

$$\frac{a_{12}}{a'_{12}} = \frac{3(23) + 8}{7(23) + 15} = \frac{\cancel{77}^{7}}{\cancel{176}_{16}} = \frac{7}{16}$$

Q

The sum of n terms two A.P.'s are in the ratio of $7n + 1 : 4n + 27$, find the ratio of their 11^{th} terms

A. 8 : 7 **B.** 7 : 16 **C.** 4:3 **D.** 13 : 47

Shortcut-7

$$\frac{S_n}{S_n'} = \frac{7n+1}{4n+27}$$

$n \rightarrow 2(11) - 1$

$n \rightarrow 21$

$$\frac{a_{11}}{a_{11}'} = \frac{7(21)+1}{4(21)+27} = \frac{148}{111} = \frac{4 \times \cancel{37}}{3 \times \cancel{37}} = \frac{4}{3}$$

Q

If the ratio of the n^{th} terms of two APs is (2n + 8) : (5n - 3), then

$$\frac{S_{29}}{S'_{29}} = ?$$

A. 8 : 7 **B.** 7 : 16 **C.** 4:3 **D.** 38 : 72

Shortcut-7

$$\frac{a_n}{a'_n} = \frac{2n+8}{5n-3}$$

$$n \to \frac{29+1}{2}$$

$$\frac{S_{29}}{S'_{29}} = \frac{2(15)+8}{5(15)-3} = \frac{38}{72}$$

Q

Super Imp

The sum of first 20 terms common between the series $3 + 7 + 11 + 15 + \ldots$ and $1 + 6 + 11 + 16 + \ldots$ is

A. 4000 B. 4020 C. 4200 D. 4220

JEE 2014

Shortcut-8

$3 + 7 + 11 + 15 + \ldots \qquad d_1 = 4$

$1 + 6 + 11 + 16 + \ldots \qquad d_2 = 5$

$\mathrm{Lcm}(d_1, d_2)$

Common terms : $11,\ \underline{11+20},\ \underline{11+2(20)},\ \underline{\quad}$ $\longrightarrow$ AP $\boxed{d = 20}$

$$S_{20} = \frac{20}{2}\left(2(11) + 19 \times 20\right)$$

$$= 10(22 + 380)$$

$$= \underline{4020}$$

Q

The number of terms common to two A.P.s
3, 7, 11, ..., 407 & 2, 9, 16, ... , 709 is

A. 12 B. 13 C. 14 D. 15

Shortcut-8

[JEE M 2020]

$3, 7, 11, 15, 19, 23, \ldots, 407$

$2, 9, 16, 23 \ldots, 709$

$d_1 = 4$
$d_2 = 7$
} Lcm

Last common term

Common Terms: $23, 23+28, \ldots\ldots, T_n$

$\boxed{d = 28}$

$T_n \le 407$

$23 + (n-1)(28) \le 407$

$(n-1)28 \le 384$

$n - 1 \le \frac{384}{28}$

$n - 1 \le 13.\text{Sm}$

$\boxed{n \le 14.\text{Sm}}$

$n = 14$

Q

Let 3, 6, 9, 12,... upto 78 terms and 5, 9, 13, 17,... upto 59 terms be two series. Then, the sum of the terms common to both the series is equal to____.

[JEE M 2022]

$T_{59} = 5 + 58(4)$
$= 5 + 232$
$= 237$

$T_{78} = 3 + 77(3)$
$= 3(78)$
$= 234$

3, 6, 9, 12, ..., 234

5, 9, 13, 17, ..., 237

3

4

C.T : $\underbrace{9, 9+12, 9+24, \ldots\ldots, T_n}_{19}$ $\quad \boxed{d = 12}$

$T_n \leq 234$

$9 + (n-1)(12) \leq 234$

$(n-1)(12) \leq 225$

$n - 1 \leq \frac{225}{12}$

$n - 1 \leq 18.\text{sm}$

$n \leq 19.\text{sm}$

$n = 19$

$$S_{19} = \frac{19}{2}\left(2(9) + 18(12)\right)$$

$$= \underline{\qquad\qquad\qquad}$$

$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$

Q

Let X be the set consisting of the first 2018 terms of the arithmetic progression 1, 6, 11, ..., and Y be the set consisting of the first 2018 terms of the arithmetic progression 9, 16, 23, ... Then, the number of elements in the set X ∪ Y is

[JEE Adv. 2018]

$(5, 7)$

$T_{2018} = 1 + 2017(5)$
$= 10086$

$X = \{1, 6, 11, 16, 21, 26, \ldots, \text{chota}\}$

$Y = \{9, 16, 23, \ldots, \text{Bada}\}$

$n(X) = 2018$

$n(Y) = 2018$

#chalaki

$X \cap Y = \{16, 16+35, \ldots, T_n\}$

$T_n \leq 10086$

$16 + (n-1)(35) \leq 10086$

$$\Rightarrow \quad 16+(n-1)35 \le 10086$$

$$\Rightarrow \quad 35n-19 \le 10086$$

$$\Rightarrow \quad n \le \frac{10105}{35} = 288.7$$

$$\therefore \quad n = 288$$

$$\therefore \quad n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$$

$$= 2018 + 2018 - 288 = 3748$$

Q

Different A.P.'s are constructed with the first term 100, the last term 199, And integral common differences. The sum of the common differences of all such, A.P's having at least 3 terms and at most 33 terms is.

[JEE M 2022]

$d = \text{integer}$

$100, \_, \_, \_, \_, \ldots, \underline{199}$

$\Sigma d_i = \_ + \_ + \_$

$a_1 = 100 \quad a_n = 199$

$\boxed{3 \leq n \leq 33}$

$(n-1)d = 99$

$\boxed{2 \leq n-1 \leq 32}$

$\text{int} = d = \frac{99}{n-1}$

$\frac{99}{3} \quad \frac{99}{9} \quad \frac{99}{11}$

$\Rightarrow 33 + 11 + 9$

$\Rightarrow 53$

(Properties)

Highlights of A.P.

Highlights of an A.P.

1. If each of an A.P. is increased, decreased, multiplied or divided by the same non zero number, then the resulting sequence is also an AP.

$\times 2$

$(1 \quad 2 \quad 3 \quad 4 \quad 5 \quad \ldots)$ $+K$

$-K$

$\times K$

$-K$

$3 \quad 4 \quad 5 \quad 6 \quad 7$ ✓

$2 \quad 4 \quad 6 \quad 8 \quad 10$ ✓

Highlight of an A.P.

$a=5$
$d=1$

2. Three numbers in AP can be taken as a-d, a, a+d
   Four numbers in AP can be taken as a-3d, a-d, a+d, a+3d
   Five numbers in AP can be taken as a-2d, a-d, a, a+d, a+2d
   Six terms in AP can be taken as a-5d, a-3d, a-d, a+d, a+3d, a+5d

4, 5, 6 . .

3 $\boxed{a-d, a, a+d}$

4 . $\boxed{a-3d, a-d, a+d, a+3d}$

a-d

$S_{50} = \frac{n}{2}[2a + (n-1)d]$

4

Highlight of an A.P.

3. The common difference can be zero, positive or negative

$$\boxed{d = 0, +, -}$$

Highlight of an A.P.

4. The sum of the two terms of an AP equidistant from the beginning and end is constant and equal to the sum of first and last terms

1 2 3 4 5 6

Highlight of an A.P.

5. For any series, $T_n = S_n - S_{n-1}$. In series if S_n is a quadratic function of n or T_n is a linear function of n, then the series is an A.P.

$$\boxed{T_n = S_n - S_{n-1}}$$

Highlight of an A.P.

6. If a, b, c are in A.P. $\Rightarrow$ $2b = a + c$

$$\boxed{2b = a + c}$$

$$\boxed{2(\text{M.T}) = \text{FT} + \text{LT}}$$

Q If $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P, then Prove $\underbrace{\frac{a}{b+c}, \frac{b}{a+c}, \frac{c}{a+b}}$ are in A.P.

$$(a+b+c)\left(\frac{1}{b+c} \quad \frac{1}{c+a} \quad \frac{1}{a+b}\right) \longrightarrow \text{A.P}$$

$$\frac{a+\overline{b+c}}{b+c}, \quad \frac{a+b+c}{c+a}, \quad \frac{a+b+c}{a+b} \longrightarrow \text{AP}$$

$$\frac{a}{b+c}+\cancel{1}, \quad \frac{b}{c+a}+\cancel{1}, \quad \cancel{1}+\frac{c}{a+b} \longrightarrow \text{AP}$$

$$\cancel{-1} \qquad \cancel{-1} \qquad \cancel{-1}$$

$$\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b} \longrightarrow \underline{\text{AP}}$$

Given $a_1, a_2, a_3 \ldots\ldots a_n$ in A.P and

If $$\underbrace{\frac{1}{a_1 a_n}+\frac{1}{a_2 a_{n-1}}+\frac{1}{a_3 a_{n-2}}+\cdots+\frac{1}{a_n a_1}}=\frac{\lambda}{\boxed{a_1+a_n}}\left[\frac{1}{a_1}+\frac{1}{a_2}+\cdots+\frac{1}{a_n}\right]$$

then find the value of λ.

jee

[JEE 2006]

$$LHS=\frac{1}{a_1+a_n}\left(\frac{a_1+a_n}{a_1 a_n}+\frac{a_2+a_{n-1}}{a_2 a_{n-1}}+\frac{a_3+a_{n-2}}{a_3 a_{n-2}}+\cdots+\frac{a_1+a_n}{a_n a_1}\right]$$

$$=\frac{1}{a_1+a_n}\left(\frac{1}{a_n}+\frac{1}{a_1}+\frac{1}{a_{n-1}}+\frac{1}{a_2}+\frac{1}{a_{n-2}}+\frac{1}{a_3}+\cdots+\frac{1}{a_1}+\frac{1}{a_n}\right]$$

$$=\frac{2}{a_1+a_n}\left(\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}+\cdots+\frac{1}{a_n}\right]$$

$$a_1 + a_n = a_2 + a_{n-1} = a_3 + a_{n-2} = \dots = k$$

Q

If the sum of the first 40 terms of the series $3 + 4 + 8 + 9 + 13 + 14 + 18 + 19 + \ldots\ldots\ldots$ is $(102)m$, then m is equal to

A. 20

B. 25

C. 10

D. 5

[JEE M 2021]

$7 + 17 + 27 + 37 + \ldots$ (20 term)

$$= \frac{20}{2}\left(14 + 19 \times 10\right)$$

$$= 10\left(14 + 190\right)$$

$$= 2040 = 102m$$

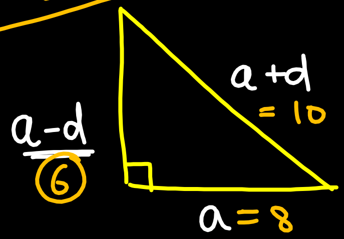
$$\therefore m = \frac{2040}{102} = 20$$

The sides of a right angled triangle are in arithmetic progression. If the triangle has area 24, then what is the length of its smallest side?

[JEE Adv. 2018]

#Chindi

6 – 8 – 10



$$\frac{1}{2}(a)(a-d) = 24 \quad \text{---} (1)$$

$$(a+d)^2 = a^2 + (a-d)^2 \quad \text{---} (2)$$

$$4ad = a^2$$

$$\therefore \boxed{a = 4d}$$

$$\frac{1}{2}(4d)(3d) = 24$$

$$d^2 = 4$$

$$\boxed{d = 2}$$

$$a = 8$$

$$a = 8$$
$$d = 2$$

Smallest side $= a - d$

$= 8 - 2$

$= 6$

Unacademy Atoms

Geometric Progression

Geometric Progression (G.P.) :

If the ratio of two consecutive terms is always constant.
This ratio is called common ratio.

$$1 \xrightarrow{\times 3} 3 \xrightarrow{\times 3} 9 \xrightarrow{\times 3} 27 \xrightarrow{\times 3} 81 \xrightarrow{} $$

$r =$

$$\frac{T_2}{T_1} = \frac{3}{1} = 3$$

$$\frac{T_3}{T_2} = \frac{9}{3} = 3$$

GP

$r = 3$

$a = 1$

General term/n^{th} term/Last term of G.P. :

Let $CR = r \quad FT = a$

$a \xrightarrow{\times r} ar \xrightarrow{\times r} ar^2 \xrightarrow{\times r} ar^3, \ldots$

$a, \; ar, \; ar^2, \; ar^3, \; \ldots$

$T_1 = a$

$T_2 = ar^1$

$T_3 = ar^2$

$T_4 = ar^3$

☆ n^{th} term of GP

$$\boxed{T_n = a r^{n-1}}$$

$T_5 = ar^4$

$T_{20} = ar^{19}$

$T_{19} = ar^{18}$

$1, 3, 9, 27, 81, \ldots \quad T_{20} = ?$

$a = 1 \quad r = 3$

$T_{20} = ar^{19}$

$= (1)(3)^{19} = 3^{19}$

a = first term

r = Common Ratio

n = no of term

Q

If $a_1(>0), a_2, a_3, a_4, a_5$ are in a G.P., $a_2 + a_4 = 2a_3 + 1$ and $3a_2 + a_3 = 2a_4$, then $a_2 + a_4 + 2a_5$ is equal to _____.

[JEE M 2022]

$$a_2 + a_4 = 2a_3 + 1$$

$$ar^1 + ar^3 = 2ar^2 + 1$$

$$ar(1 + r^2 - 2r) = 1$$

$$\boxed{ar(r-1)^2 = 1} \quad \text{--- (1)}$$

$$3a_2 + a_3 = 2a_4$$

$$3\cancel{a}\cancel{r} + \cancel{a}r^2 = 2\cancel{a}r^3$$

$$3 + r = 2r^2$$

$$2r^2 - r - 3 = 0$$

$$2r^2 - 3r + 2r - 3 = 0$$

$$(r+1)(2r-3) = 0$$

$$\boxed{r = -1 \text{ OR } 3/2}$$

$\cancel{r = -1}$ or $r = 3/2$

$\cancel{ar(r-1)^2 = 1}$

$\cancel{a(-1)(4) = 1}$

$\cancel{\boxed{a = \frac{-1}{4}}}$

Rejected

$ar(r-1)^2 = 1$

$a\left(\frac{3}{2}\right)\left(\frac{1}{4}\right) = 1$

$\boxed{a = 8/3}$ $\boxed{r = 3/2}$

$a_2 + a_4 + 2a_5$

$= ar + ar^3 + 2ar^4$

$= ar(1 + r^2 + 2r^3)$

$= \left(\frac{8}{3}\right)\left(\frac{3}{2}\right)\left(1 + \frac{9}{4} + 2\left(\frac{27}{8}\right)\right)$

$= 4\left(1 + \frac{9}{4} + \frac{27}{4}\right)$

$= 40$

Q

Let $A_1, A_2, A_3, \ldots\ldots$ be an increasing geometric progression of positive real numbers. If $A_1A_3A_5A_7 = \frac{1}{1296}$ and $A_2 + A_4 = \frac{7}{36}$, then, the value of $A_6 + A_8 + A_{10}$ is equal to

r>1

A. 33

B. 37

C. 43

D. 47

$r = 1$ Const GP

$r < 1$ dec GP

$r > 1$ inc GP

[JEE M 2022]

$$(a)(ar^2)(ar^4)(ar^6) = \frac{1}{6^4}$$

$$\left(a^4 r^{12}\right)^{\frac{1}{4}} = \left(\frac{1}{6^4}\right)^{\frac{1}{4}}$$

$$A_2 + \frac{1}{6} = \frac{7}{36}$$

$$\therefore \boxed{A_2 = \frac{1}{36}}$$

$ar^3 = \frac{1}{6}$

$\boxed{A_4 = \frac{1}{6}}$

$A_6 + A_8 + A_{10}$

$= 1 + 6 + 36 = 43$

$\_\_, \frac{1}{36}, \_\_, \frac{1}{6}, \_\_, \underline{1}, \_\_, \underline{6}, \_\_, \underline{36}$

$\times 6 \quad \times 6 \quad \times 6 \quad \times 6$

$ar^3 = \frac{1}{6}$

$ar = \frac{1}{36}$

$\boxed{a = \frac{1}{36\sqrt{6}}}$

$r^2 = 6$

$\boxed{r = \sqrt{6}}$

$A_6 + A_8 + A_{10}$

$= ar^5 + ar^7 + ar^9$

Sum of n Terms of G.P.

Sum of n terms of a G.P. :

$$S_n = \frac{a(1-r^n)}{1-r} \quad \text{OR} \quad \frac{a(r^n-1)}{r-1}$$

Sum of n terms of GP

Sum of infinite terms of a G.P. :

$$\boxed{S_\infty = \frac{a}{1-r}}\ ^{*}$$

* if $|r| < 1$

Q

If $2^{10} + 2^9 . 3^1 + 2^8 . 3^2 + \ldots\ldots + 2^1 . 3^9 + 3^{10} = S - 2^{11}$ then S is equal to:

GP

A. $2 . 3^{11}$

B. $3^{11} - 2^{12}$

C. $3^{11}/2 + 2^{10}$

D. 3^{11}

[JEE M 2020]

$a = 2^{10}$

$r = \frac{2^9 . 3^1}{2^{10}} = 3/2$

$n = 11$

D

$$\frac{a(r^n - 1)}{r - 1} = \frac{2^{10}\left(\left(\frac{3}{2}\right)^{11} - 1\right)}{\left(\frac{3}{2} - 1\right)} = \cancel{2^{11}}\left(\frac{3^{11} - 2^{11}}{\cancel{2^{11}}}\right)$$

$= 3^{11} - 2^{11}$

$= S - 2^{11}$

Let $a_1, a_2, a_3, \ldots$ be a sequence of positive integers in arithmetic progression with common difference 2. Also, let $b_1, b_2, b_3, \ldots$ be a sequence of positive integers in geometric progression with common ratio 2. If $a_1 = b_1 = c$, then the number of all possible values of c, for which the equality

$$\boxed{2(\underbrace{a_1 + a_2 + \cdots + a_n}_{AP\ d=2}) = \underbrace{b_1 + b_2 + \cdots + b_n}_{GP\ r=2}} \quad 1$$

$c \rightarrow$ kitni value

holds for some positive integer n, is ______

[JEE Adv. 2020]

$c \quad a_2 \quad a_3 \ldots\ldots$ AP

$c \quad b_2 \quad b_3 \ldots\ldots$ GP

$$\cancel{2} \times \frac{n}{\cancel{2}}\left[2c + (n-1)2\right] = \frac{c(2^n - 1)}{2-1}$$

$$2nc + 2n(n-1) = c(2^n - 1)$$

$$2n(n-1) = c(2^n - 1 - 2n)$$

$$\boxed{c = \frac{2n(n-1)}{2^n - 1 - 2n}}$$

chalaki

$$C = \frac{2n(n-1)}{2^n - 1 - 2n} \geq 1$$

$2n(n-1) \geq 2^n - 1 - 2n$

$2n^2 - \cancel{2n} \geq 2^n - 1 - \cancel{2n}$

$$\underbrace{2n^2 \geq 2^n - 1}$$

$n=1 \quad C = \frac{2(1)(0)}{2^1 - 1 - 2(1)} = 0$ X

$n=2 \quad C = \frac{2(2)(1)}{2^2 - 1 - 2(2)} = -4$ X

$n=3 \quad C = \frac{2(3)(2)}{2^3 - 1 - 2(3)} = \frac{12}{1} = 12$ ✓

$n=4 \quad C = \cdot$ X

$n=5 \quad C = \cdot$ X

$n=6 \quad C = \cdot$ X

$n = \underbrace{\boxed{1, 2, 3, 4, 5, 6}}, \underbrace{7, 8, 9, \ldots}_{X}$

$98 \leq 127$

Q

If $\frac{1}{2 \cdot 3^{10}} + \frac{1}{2^2 \cdot 3^9} + \dots \frac{1}{2^{10} \cdot 3} = \frac{K}{2^{10} \cdot 3^{10}}$, then the remainder when K is divided by 6 is

[JEE M 2022]

A. 1

B. 2

C. 3

D. 5

$a = \frac{1}{2\ 3^{10}} \quad r = \frac{3}{2} \quad n = 10$

$\frac{a(r^n - 1)}{r-1} = \frac{\frac{\cancel{1}}{\cancel{2}\ 3^{10}}\left(\left(\frac{3}{2}\right)^{10} - 1\right)}{\cancel{\frac{1}{2}}}$

$= \frac{3^{10} - 2^{10}}{\cancel{3^{10} 2^{10}}} = \frac{K}{\cancel{2^{10} 3^{10}}}$

$K = 3^{10} - 2^{10}$

$r = ?$

$$\frac{3^{10} - 2^{10}}{6} \Rightarrow 5$$

Kaise ?

Y.T $\boxed{\text{CRACK IN SEC Finding Remainder}}$?

Q

If $\boxed{A=\sum_{n=1}^{\infty}\frac{1}{\left(3+(-1)^n\right)^n}}$ and $B=\sum_{n=1}^{\infty}\frac{(-1)^n}{\left(3+(-1)^n\right)^n}$, then

$\frac{A}{B}$ is equal to :

[JEE M 2022]

A. 11/9

B. 1

C. -11/9

D. -11/3

$\frac{a}{1-r}$

$A=\frac{1}{2}+\frac{1}{4^2}+\frac{1}{2^3}+\frac{1}{4^4}+\ldots\infty$

$=\left(\frac{1}{2}+\frac{1}{2^3}+\ldots\ldots\right)+\left(\frac{1}{4^2}+\frac{1}{4^4}+\ldots\infty\right)$

$=\frac{\frac{1}{2}}{1-\frac{1}{4}}+\frac{\frac{1}{4^2}}{1-\frac{1}{16}}$

$=\frac{2}{3}+\frac{1}{15}=\frac{11}{15}$

$$B = \frac{-1}{2} + \frac{1}{4^2} + \frac{-1}{2^3} + \frac{1}{4^4} \cdots \infty$$

$$= -\left(\frac{1}{2} + \frac{1}{2^3} \cdots\right) + \left(\frac{1}{4^2} + \frac{1}{4^4} \cdots\right)$$

$$= -\frac{2}{3} + \frac{1}{15}$$

$$\boxed{B = \frac{-9}{15}}$$

$$\frac{A}{B} = \frac{11}{-9}$$

(Properties)

Highlights of G.P.

jee

Highlights of G.P :

1. If each term of a GP be **multiplied or divided** by the same non - zero quantity, the resulting sequence is also a GP.

$+2$ $\quad (1 \quad 3 \quad 9 \quad 27 \quad 81)$

$\times \quad 3 \quad 5 \quad 11 \quad 29 \quad 83$ $\quad$ GP

Highlights of G.P :

2. If each term of a G.P. raised the same power then resulting sequence is also a G.P

$(1)^{1/2} \quad (3)^{1/2} \quad (9)^{1/2} \quad (27)^{1/2} \quad (81)^{1/2}$

✓ $1 \quad \sqrt{3} \quad 3 \quad 3\sqrt{3} \quad 3^2$

Highlights of G.P :

3. Any 3 consecutive terms of a GP can be taken as a/r, a, ar
any 4 consecutive terms of a GP can be taken as a/r^3, a/r, ar, ar^3
& so on

$\boxed{\#NVTip}$

3 terms $\underbrace{\frac{a}{\cancel{r}}, a, a\cancel{r}}$

$\boxed{\text{Product} = 30}$

3 terms. a, ar, ar^2

Highlights of G.P :

4. If a finite G.P. the product of the terms equidistant from the beginning and the end are equal $a_1a_n = a_2a_{n-1} = a_3a_{n-2} = ...$

1 3 9 27 81

Highlights of G.P :

5. If a, b, c are in GP $\rightarrow b^2 = ac$

$$\frac{b}{a} = \frac{c}{b}$$

$$\boxed{b^2 = ac}$$

Q

Let $\frac{1}{16}$, a and b be in G.P. and $\frac{1}{a}, \frac{1}{b}, 6$ be in A.P., where $a, b > 0$. Then $72(a+b)$ is equal to ____

[JEE M 2021]

$\boxed{a^2 = b \times \frac{1}{16}}$ — ①

$b = 16a^2$

$\frac{2}{b} = \frac{1}{a} + 6$ — ②

$\frac{2}{16a^2} = \frac{1+6a}{a}$

$\frac{1}{8a} = 1 + 6a \Rightarrow 1 = 8a + 48a^2$

$$48a^2 + 8a - 1 = 0$$

$$48a^2 + 12a - 4a - 1 = 0$$

$$(12a-1)(4a+1) = 0$$

$$\underline{a = \frac{1}{12}}, \cancel{-\frac{1}{4}}$$

$$a = \frac{1}{12}$$

$$b = 16\left(\frac{1}{12}\right)\left(\frac{1}{12}\right) = \frac{1}{9}$$

$$72(a+b)$$

$$= 72\left(\frac{1}{12} + \frac{1}{9}\right)$$

$$= 14$$

Sequence Convertible to G.P.

Type 1: Converting Recurring Decimal to Fraction

$$x = 0.4\overline{23}$$

$x = 0.4232323\ldots$

$x = 0.4 + 0.023 + 0.00023 + \ldots\infty$

$x = \frac{4}{10} + \left(\frac{23}{10^3} + \frac{23}{10^5} + \ldots\infty\right)$

$x = \frac{4}{10} + \dfrac{\frac{23}{1000}}{1 - \frac{1}{100}}$

$x = \frac{4}{10} + \frac{23}{990} = \frac{419}{990}$

$x = 0.4232323\ldots$

$1000x = 423.2323\ldots$

$10x = 4.2323\ldots$

$990x = 419$

$\therefore \boxed{x = \frac{419}{990}}$

Type 2 :

9 ♡

If $9 + 99 + 999 + \ldots\ldots + \text{upto } 49 \text{ terms} = 10\dfrac{(10^{\lambda} - 1)}{\mu} - 49$, where $\lambda, \mu \in N$ then find the value of $\lambda + \mu$

$\Rightarrow 9 + 99 + 999 + 9999 + \ldots\ldots$ upto 49 terms

$\Rightarrow (10 - 1) + (10^2 - 1) + (10^3 - 1) + (10^4 - 1) + \ldots$ 49 terms

$\Rightarrow (10 + 10^2 + 10^3 + \ldots\ldots) - 49$

$\Rightarrow \dfrac{10(10^{49} - 1)}{9} - 49$

$\lambda = 49$

$\mu = 9$

$\lambda + \mu = 58$

$$\frac{10-1}{10} = 1 - \frac{1}{10}$$

The sum of first 20 terms of the sequence 0.7, 0.77, 0.777, ..., is

A. $7/81\,(179 - 10^{-20})$

B. $7/9\,(99 - 10^{-20})$

C. $7/81\,(179 + 10^{-20})$

D. $7/9\,(99 + 10^{-20})$

[JEE 2013]

$$S \Rightarrow 0.7 + 0.77 + 0.777 + \dots \text{(20 terms)}$$

$$\Rightarrow \frac{7}{10} + \frac{77}{100} + \frac{777}{1000} + \dots$$

$$\Rightarrow \frac{7}{9}\left(\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + \dots\right)$$

$$\Rightarrow \frac{7}{9}\left[\left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{10^2}\right) + \left(1 - \frac{1}{10^3}\right) + \dots\right)$$

jee

$$\Rightarrow \frac{7}{9}\left[20-\left(\frac{1}{10}+\frac{1}{10^2}+\frac{1}{10^3}+\cdots\right)\right]$$

$$\Rightarrow \frac{7}{9}\left[20-\frac{\frac{1}{\cancel{10}}\left(1-\left(\frac{1}{10}\right)^{20}\right)}{\frac{9}{\cancel{10}}}\right]$$

$$\Rightarrow \frac{7}{9^2}\left[179+10^{-20}\right]$$

9 m

$5+55+555\ldots$

$$\frac{5}{9}\left(9+99+999+\ldots\right)$$

Type 3 :

$x^3-y^3=(x-y)(x^2+xy+y^2)$

Find the sum S = (x + y) + (x² + xy + y²) + (x³ + x²y + xy² + y³) + … n terms.

[JEE M 2021]

$$S=\frac{x^2-y^2}{x-y}+\frac{x^3-y^3}{x-y}+\frac{x^4-y^4}{x-y}+\dots$$

$$S=\frac{(x^2+x^3+x^4+\dots)-(y^2+y^3+y^4\dots)}{x-y}$$

Q

If $\dfrac{6}{3^{12}}+\left(\dfrac{10}{3^{11}}+\dfrac{20}{3^{10}}+\dfrac{40}{3^{9}}+\ldots+\dfrac{10240}{3}\right)=2^{n}\cdot m$, $= 2^{12}\times 1$

where m is odd, then m.n is equal to ______

[JEE M 2022]

#chalaki

$m=1$

$n=12$

$$\frac{6}{3^{12}}+10\left(\underbrace{\frac{1}{3^{11}}\xrightarrow{\times 6}\frac{2}{3^{10}}\xrightarrow{\times 6}\frac{2^2}{3^9}+\ldots+\frac{2^{10}}{3^1}}_{G.P.}\right)$$

$$\Rightarrow \frac{2}{3^{11}}+\cancel{10}^{\,2}\left(\frac{\frac{1}{3^{11}}\left(6^{11}-1\right)}{\cancel{5}}\right)$$

$n=11$

$a=\dfrac{1}{3^{11}}$

$r=6$

$m\times n$

$=12$

$$\Rightarrow \cancel{\frac{2}{3^{11}}}+\frac{2}{3^{11}}\left(6^{11}-1\right)=\frac{2}{3^{11}}\times 6^{11}=2^{12}$$

Type 4 :

S = 1/2 + 3/4 + 7/8 + 15/16 + ... to n terms.

$$S = \left(1 - \frac{1}{2}\right) + \left(1 - \frac{1}{4}\right) + \left(1 - \frac{1}{8}\right) + \left(1 - \frac{1}{16}\right) + \dots n \text{ term}$$

$$= n - \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots\right)$$

Q

The greatest integer less than or equal to the sum of the first 100 terms of the sequence 1/3, 5/9, 19/27, 65/81, ... is equal to

[JEE M 2022]

$$S = \frac{1}{3} + \frac{5}{9} + \frac{19}{27} + \frac{65}{81} \; \ldots \; \text{100 terms}$$

$$S = \left(1 - \frac{2}{3}\right) + \left(1 - \frac{4}{9}\right) + \left(1 - \frac{8}{27}\right) + \left(1 - \frac{16}{81}\right) + \ldots$$

$$= 100 - \left(\frac{2}{3} + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + \left(\frac{2}{3}\right)^4 + \ldots\right)$$

$$= 100 - \left(\frac{\frac{2}{\cancel{3}}\left(1 - \left(\frac{2}{3}\right)^{100}\right)}{\frac{1}{\cancel{3}}}\right) = 100 - 2\left(1 - \left(\frac{2}{3}\right)^{100}\right)$$

$$2 \times (0.66)^{100} = 0 \text{ sm}$$

$$= \left[98 + \underline{2\left(\frac{2}{3}\right)^{100}}\right]$$

$$= [98 \text{ sm}]$$

$$= 98$$

$$\frac{1}{2} > \left(\frac{1}{2}\right)^{100}$$

Arithmetic – Geometric Progression (A.G.P.)

Sum of n terms and infinite terms of an A.G.P

$$AGP \Rightarrow \underbrace{AP + GP}$$

Q

The sum $\underbrace{1+2\cdot 3+3\cdot 3^2+\ldots\ldots+10\cdot 3^9}$ is equal to

A. $\dfrac{2\cdot 3^{12}+10}{4}$

✓B. $\dfrac{19\cdot 3^{10}+1}{4}$

$2\times 3-1\times 3$

$3(2-1)$

C. $5\cdot 3^{10}-2$

D. $\dfrac{9\cdot 3^{10}+1}{2}$

[JEE M 2022]

AP + GP

$S = 1 + 2\cdot 3 + 3\, 3^2 + 4\, 3^3 + \ldots + 10\, 3^9$

$3S = 1\, 3 + 2\cdot 3^2 + 3\, 3^3 + \ldots + 9\, 3^9 + 10\, 3^{10}$

$-2S = (3^0 + 3 + 3^2 + 3^3 + \ldots + 3^9) - 10\, 3^{10}$

$-2S = \dfrac{1(3^{10}-1)}{2} - 10\, 3^{10} \Rightarrow \boxed{S = \dfrac{19\cdot 3^{10}+1}{4}}$

Q

Let $S = 2 + \frac{6}{7} + \frac{12}{7^2} + \frac{20}{7^3} + \frac{30}{7^4} + \ldots$ then $4S$ is equal to

$\overset{6}{}\quad \overset{8}{}\quad \overset{10}{}$

A. $\left(\frac{7}{3}\right)^2$

B. $\frac{7^3}{3^2}$

C. $\left(\frac{7}{3}\right)^3$ ✓

D. $\frac{7^2}{3^3}$

[JEE M 2022]

$$1S = 2 + \frac{6}{7} + \frac{12}{7^2} + \frac{20}{7^3} + \frac{30}{7^4} + \ldots\ldots$$

$$\frac{1}{7}S = \frac{2}{7} + \frac{6}{7^2} + \frac{12}{7^3} + \frac{20}{7^4} + \ldots\ldots$$

$$\frac{6}{7}S = 2 + \frac{4}{7} + \frac{6}{7^2} + \frac{8}{7^3} + \frac{10}{7^4} + \ldots$$

$$\frac{6S}{7} = \frac{7^2}{3 \times 6}$$

$$\therefore 4S = \frac{7^3 \times \cancel{4}}{3 \times \cancel{6} \times \cancel{6}}$$

$3 \quad 3$

$$4S = \left(\frac{7}{3}\right)^3$$

Let

$$S_1 = 2 + \frac{4}{7} + \frac{6}{7^2} + \frac{8}{7^3} + \cdots$$

$$\frac{1}{7} S_1 = \quad \frac{2}{7} + \frac{4}{7^2} + \frac{6}{7^3} + \cdots$$

$$\frac{6}{7} S_1 = \underbrace{2 + \frac{2}{7} + \frac{2}{7^2} + \frac{2}{7^3} + \cdots}$$

$$\frac{\overset{3}{\cancel{6}}}{7} S_1 = \frac{\cancel{2}}{1 - \frac{1}{7}}$$

$$\frac{3 S_1}{7} = \frac{7}{6} \Rightarrow \boxed{S_1 = \frac{7^2}{3 \times 6}}$$

Q

The sum of the infinite series $1+\frac{5}{6}+\frac{12}{6^2}+\frac{22}{6^3}+\frac{35}{6^4}+\frac{51}{6^5}+\frac{70}{6^6}+\ldots$ is equal to:

A. $\frac{425}{216}$

B. $\frac{429}{216}$

C. $\frac{288}{125}$

D. $\frac{280}{125}$

[JEE M 2022]

DIY

Q

Let $a_1, a_2, a_3, \ldots$ be an A.P. If $\sum_{r=1}^{\infty} \frac{a_r}{2^r} = 4$, then $4a_2$ is equal to $4 \times 4 = 16$ ________.

[JEE M 2022]

$$\sum_{r=1}^{\infty} \frac{a_r}{2^r} = 4$$

$$\Rightarrow \underbrace{\frac{a_1}{2^1} + \frac{a_2}{2^2} + \frac{a_3}{2^3} + \cdots}_{} = 4$$

$$a_2 = 4$$

$$S = \frac{a_1}{2} + \frac{a_2}{2^2} + \frac{a_3}{2^3} + \frac{a_4}{2^4} + \cdots\cdots$$

$$\frac{1}{2}S = \frac{a_1}{2^2} + \frac{a_2}{2^3} + \frac{a_3}{2^4} + \cdots\cdots$$

$$\frac{S}{2} = \frac{a_1}{2} + \left(\frac{d}{2^2} + \frac{d}{2^3} + \frac{d}{2^4} + \cdots\cdots\right)$$

$$\frac{S}{2} = \frac{a_1}{2} + \frac{\frac{d}{2^2}}{1 - \frac{1}{2}}$$

$$\frac{S}{\cancel{2}} = \frac{a_1}{\cancel{2}} + \frac{d}{\cancel{2}} = a_1 + d$$

$$S = a_1 + d$$

$$\underline{S = a_2}$$

Miscellaneous Series

jee

Type 1 :

Sequence dealing with $\sum n ; \sum n^2 ; \sum n^3$

$$\sum n = \boxed{\frac{n(n+1)}{2} = 1+2+3+\dots+n}$$

$$\sum n^2 = \boxed{\frac{n(n+1)(2n+1)}{6}} = 1^2+2^2+3^2+\dots+n^2$$

$$\sum n^3 = \left\{\frac{n(n+1)}{2}\right\}^2 = 1^3+2^3+3^3+\dots+n^3$$

$$\sum n = \frac{n(n+1)}{2}$$

$$\sum n^3 = \left(\frac{n(n+1)}{2}\right)^2$$

$$\sum n^2 = \frac{n(n+1)(2n+1)}{6}$$

jee

Q

If the sum of the first ten terms of the series

$$\left(1\frac{3}{5}\right)^2+\left(2\frac{2}{5}\right)^2+\left(3\frac{1}{5}\right)^2+4^2+\left(4\frac{4}{5}\right)^2+\text{.......}, \text{ is } \frac{16}{5}m,$$

A. 100 B. 99 C. 102 D. 101

[JEE 2016]

$$S=\left(\frac{8}{5}\right)^2+\left(\frac{12}{5}\right)^2+\left(\frac{16}{5}\right)^2+\left(\frac{20}{5}\right)^2+\left(\frac{24}{5}\right)^2+\ldots \quad \text{10 terms}$$

$$=\left(\frac{4}{5}\right)^2\left\{\underbrace{1^2+2^2+3^2+4^2+\ldots\ldots+11^2}-1^2\right\}$$

$$=\frac{16}{25}\left[\frac{11(12)(23)}{6}-1\right]$$

$$=\frac{16}{25}\left\{22\times23-1\right\}=\frac{\cancel{16}}{\cancel{25}_{5}}\times 505=\frac{\cancel{16}}{\cancel{5}}m$$

$$\therefore \boxed{m=101}$$

Type 2 :

Continued Product

S= 1.2 + 2.3 + 3.4 + ... + n(n+1)

Method 1: Sigma Method

$$S = 1\times2 + 2\times3 + 3\times4 + 4\times5 + \dots + n(n+1)$$

1

$$S = \sum_{r=1}^{n} r(r+1)$$

$$= \sum r^2 + r = \sum_{r=1}^{n} r^2 + \sum_{r=1}^{n} r$$

$$= \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} = \frac{n(n+1)}{2}\left\{\frac{2n+1}{3} + 1\right\}$$

$$= \boxed{\frac{n(n+1)(n+2)}{3}}$$

Type 2 :

Continued Product

$S = 1.2 + 2.3 + 3.4 + \ldots\ldots + n(n+1)$

Method 2: Method of Difference (# Baarish)

① $T_r = \dfrac{r(r+1)\left((r+2)-(r-1)\right)}{3}$ # chalaki

$T_r = \frac{1}{3}\left[r(r+1)(r+2) - (r-1)(r)(r+1) \right]$

$$T_1 = \frac{1}{3}\left[1\cdot 2\cdot 3 - 0\cdot 1\cdot 2\right]$$

$$T_2 = \frac{1}{3}\left[2\cdot 3\cdot 4 - 1\cdot 2\cdot 3\right]$$

$$T_3 = \frac{1}{3}\left[3\cdot 4\cdot 5 - 2\cdot 3\cdot 4\right]$$

$$T_n = \frac{1}{3}\left[n(n+1)(n+2) - (n-1)n(n+1)\right]$$

$$S_n = \frac{1}{3}\left[n(n+1)(n+2)\right]$$

Type 2 :

Continued Product

$S = 1.2.3 + 2.3.4 + 3.4.5 + \ldots\ldots + n(n+1)(n+2)$

$$T_r = \frac{r(r+1)(r+2)\big((r+3)-(r-1)\big)}{4}$$

$$T_r = \frac{1}{4}\Big[\, r(r+1)(r+2)(r+3) - (r-1)\,r\,(r+1)(r+2) \Big]$$

$$T_1 = \frac{1}{4}\left[1\cdot2\cdot3\cdot4 - 0\cdot1\cdot2\cdot3\right]$$

$$T_2 = \frac{1}{4}\left[2\cdot3\cdot4\cdot5 - 1\cdot2\cdot3\cdot4\right]$$

$$\vdots \qquad\qquad \vdots$$

$$T_n = \frac{1}{4}\left[n(n+1)(n+2)(n+3) - \boxed{}\right]$$

$$S_n = \frac{1}{4}\,n(n+1)(n+2)(n+3)$$

Type 2 :

Continued Product

$S = 1.2.3 + 2.3.4 + 3.4.5 + \ldots\ldots + n(n+1)(n+2)$

Method 3: Shortcut

Shortcut-9

① $1\times 2 + 2\times 3 + 3\times 4 + \ldots + n(n+1) = \frac{n(n+1)(n+2)}{3}$

② $1\times 2\times 3 + 2\times 3\times 4 + \ldots\ldots + n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}$

③ $1\times 2\times 3\times 4 + 2\times 3\times 4\times 5 + \ldots\ldots + n(n+1)(n+2)(n+3) = \frac{n(n+1)(n+2)(n+3)(n+4)}{5}$

Type-3 (Using method of difference) :

Shortcut-10

Find the sum of series .

i. 6 + 13 + 22 + 33 + ... n terms

#NVExclusive

$$S = 6 + 13 + 22 + 33 + \dots + T_n$$

1^{st} diff $\Rightarrow$ 7 9 11

2^{nd} diff $\Rightarrow$ 2 2

3^{rd} diff $\Rightarrow$ 0

$$S = \overbrace{6 + \underline{13 + 22 + 33 + \dots + T_n}}^{n}$$

$$S = \quad 6 + 13 + 22 + \dots + T_{n-1} + T_n$$

$$0 = 6 + \Big(\underbrace{7 + 9 + 11 + \dots}_{(n-1)\text{ terms}}\Big) - T_n$$

$$T_n = 6 + \frac{n-1}{2}\left[14 + (n-2)2\right]$$

$$T_n = 6 + (n-1)(7+n-2)$$

$$T_n = 6 + n^2 + 4n - 5$$

$$\boxed{T_n = n^2 + 4n + 1} \qquad \boxed{S_n = \Sigma n^2 + 4\Sigma n + \Sigma 1}$$

6 13 22 33 . . . .

7 9 11

2 2

0

$$T_n = 6 + 7\frac{(n-1)}{1!} + \cancel{2}\frac{(n-1)(n-2)}{\cancel{2}!} + \cancel{0}$$

$$= 6 + 7n - 7 + n^2 - 3n + 2$$

$$= \underline{n^2 + 4n + 1}$$

Q

$S = 2 + 5 + 10 + 17 + 26 + 37 + \dots\dots\dots\dots + T_n$

jee

$b \Rightarrow$ 2 5 10 17 26 37

3 5 7 9 11

2 2 2 2

0 0 0

$T_1 = 1^2+1 = 2$

$T_2 = 2^2+1 = 5$

$T_3 = 3^2+1 = 10$

$T_n = 2 + 3(n-1) + \cancel{2}\dfrac{(n-1)(n-2)}{\cancel{2!}}$

$= 2 + \cancel{3n} - 3 + n^2 - \cancel{3n} + 2$

$\boxed{T_n = n^2+1}$

$S_n = \Sigma T_n$

$= \Sigma n^2 + 1$

$= \Sigma n^2 + \Sigma 1$

$\boxed{S_n = \dfrac{n(n+1)(2n+1)}{6} + n}$

Shortcut-10

Q

Let $a_1 = b_1 = 1$, $a_n = a_{n-1} + 2$ and $b_n = a_n + b_{n-1}$ for every natural number $n \geq 2$. Then $\sum_{n=1}^{15} a_n \cdot b_n$ is equal to ________.

[JEE M 2022]

$a_1 = b_1 = 1$

$a_n - a_{n-1} = 2 = d$

$b_n - b_{n-1} = a_n$

$a_n \rightarrow$ A P.

$b_n \rightarrow$

$\Sigma (2n-1) n^2$

$a : \boxed{1, 3, 5, 7, 9, 11}$

$a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5 \quad a_6$

$T_n = a_n = 1 + (n-1)2$

$= 2n - 1$

$b_n - b_{n-1} = a_n$

$n=2 \quad b_2 - b_1 = 3 \Rightarrow b_2 = 4$

$n=3 \quad b_3 - b_2 = 5 \Rightarrow b_3 = 9$

$n=4$

b.

1		4		9		16
	3		5		7	
		2		2		
			0			

$$T_n = 1 + 3(n-1) + \cancel{2}\frac{(n-1)(n-2)}{\cancel{2}}$$

$$= \cancel{3n} - \cancel{2} + n^2 - \cancel{3n} + \cancel{2}$$

$$= n^2 \checkmark$$

$$\Rightarrow \sum 2n^3 - n^2$$

$$\Rightarrow 2\sum n^3 - \sum n^2$$

$$\Rightarrow 2\left(\frac{15 \quad 16}{2}\right)^2 - \frac{15 \quad 16 \quad 31}{6}$$

$$\Rightarrow \underline{\qquad\qquad\qquad}$$

Q

Let $a_1, a_2, a_3, \ldots$ be an arithmetic progression with $a_1 = 7$ and common difference 8. Let $T_1, T_2, T_3, \ldots$ be such that $T_1 = 3$ and $T_{n+1} - T_n = a_n$ for $n \geq 1$. Then, which of the following is/are TRUE ?

T_n

$n=1$

$n=2$

[JEE Adv. 2022]

(A) $T_{20} = 1604$

(B) $\sum_{k=1}^{20} T_k = 10510$

(C) $T_{30} = 3454$

(D) $\sum_{k=1}^{30} T_k = 35610$

a_1, a_2, a_3

7, 15, 23, ... (AP) $d=8$

3, 10, 25 ... (Seq)

7, 15, 23

8, 8

0

$T_2 - T_1 = a_1 \Rightarrow T_2 = 3 + 7 = 10$

$T_3 - T_2 = a_2 \Rightarrow T_3 = T_2 + a_2$

$= 10 + 15$

$= 25$

$$T_n = 3 + 7(n-1) + \frac{8(n-1)(n-2)}{2}$$

$$= 7n - 4 + 4(n^2 - 3n + 2)$$

$$\boxed{T_n = 4n^2 - 5n + 4}$$

$$T_{20} = 4(20)^2 - 5(20) + 4 = 1504$$

$$T_{30} = 4(30)^2 - 5(30) + 4 = 3454$$

$$S_n = 4 \sum n^2 - 5 \sum n + 4 \sum 1$$

$$\boxed{S_n = 4\,\frac{n(n+1)(2n+1)}{6} - 5\,\frac{(n)(n+1)}{2} + 4n}$$

S_{20}

S_{30}

Type -4 (Splitting the nth term as a difference of two) :

$$S = \frac{1}{1\times 2} + \frac{1}{2\times 3} + \frac{1}{3\times 4} + \cdots$$

$$S = \frac{1}{1\times 2} + \frac{1}{2\times 3} + \frac{1}{3\times 4} + \cdots + \frac{1}{n(n+1)}$$

① $T_r = \dfrac{1}{r(r+1)}$

② #chalaki $T_r = \dfrac{(r+1)-(r)}{r(r+1)}$

$$\boxed{T_r = \frac{1}{r} - \frac{1}{r+1}}$$

③

$$T_1 = \frac{1}{1} - \cancel{\frac{1}{2}}$$

$$T_2 = \cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}$$

$$T_3 = \cancel{\frac{1}{3}} - \cancel{\frac{1}{4}}$$

$$\vdots$$

$$T_n = \cancel{\frac{1}{n}} - \frac{1}{n+1}$$

$$S_n = 1 - \frac{1}{n+1}$$

$$S_\infty = 1 - \cancel{\frac{1}{\infty}}$$

$$\boxed{S_\infty = 1}$$

$$S_\infty \rightarrow \underline{S_n \ (n=\infty)}$$

Q

Find the sum of n terms of the series and also find S_∞.

$$\frac{1}{1\times2\times3}+\frac{1}{2\times3\times4}+\frac{1}{3\times4\times5}+\cdots$$

$$T_r=\frac{1}{2}\left[\frac{(r+2)-(r)}{r(r+1)(r+2)}\right]$$

Boda - chota

$$T_r=\frac{1}{2}\left[\frac{1}{r(r+1)}-\frac{1}{(r+1)(r+2)}\right]$$

$$T_1=\frac{1}{2}\left[\frac{1}{1\times2}-\cancel{\frac{1}{2\times3}}\right]$$

$$T_2=\frac{1}{2}\left[\cancel{\frac{1}{2\times3}}-\cancel{\frac{1}{3\times4}}\right]$$

$\vdots$

$$T_n=\frac{1}{2}\left[\cancel{\frac{1}{n(n+1)}}-\frac{1}{(n+1)(n+2)}\right]$$

$$S_n=\frac{1}{2}\left[\frac{1}{2}-\frac{1}{(n+1)(n+2)}\right]$$

$$S_n=\frac{1}{2}\left[\frac{1}{2}-\frac{1}{(n+1)(n+2)}\right]$$

$$\boxed{S_\infty=\frac{1}{4}}$$

Q

Find the sum of n terms of the series and also fins S_∞.

$$\frac{1}{1\times2\times3}+\frac{1}{2\times3\times4}+\frac{1}{3\times4\times5}+\cdots$$

Shortcut-11

① $\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+\cdots+\frac{1}{n(n+1)}=\frac{1}{1}\left(\frac{1}{1}-\frac{1}{n+1}\right)=1-\frac{1}{n+1}$

② $\frac{1}{1\times2\times3}+\frac{1}{2\times3\times4}+\cdots+\frac{1}{n(n+1)(n+2)}=\frac{1}{2}\left(\frac{1}{1\times2}-\frac{1}{(n+1)(n+2)}\right)$

③ $\frac{1}{1\times2\times3\times4}+\frac{1}{2\times3\times4\times5}+\cdots+\frac{1}{n(n+1)(n+2)(n+3)}=\frac{1}{3}\left(\frac{1}{1\times2\times3}-\frac{1}{(n+1)(n+2)(n+3)}\right)$

$S_\infty=\frac{1}{18}$

Q

Find the sum of n terms of the series and also fins S_∞.

$$\frac{1}{1\cdot2\cdot3\cdot4}+\frac{1}{2\cdot3\cdot4\cdot5}+\frac{1}{3\cdot4\cdot5\cdot6}+\ldots\ldots$$

Shortcut-11

Q If $\frac{1}{2\times3\times4}+\frac{1}{3\times4\times5}+\frac{1}{4\times5\times6}+\ldots+\frac{1}{100\times101\times102}=\frac{k}{101}$, then 34 k is equal to _____.

[JEE M 2022]

Q) $\frac{1}{2\times3\times\cancel{4}}+\frac{1}{3\times4\times5}+\frac{1}{4\times5\times6}+\quad+\frac{1}{\cancel{100}\times101\times102}=\frac{k}{101}$

$\frac{1}{2}\left(\frac{1}{6}-\frac{1}{101\times6\times17}\right)=\frac{k}{101}$

$\frac{1}{2}\left(\frac{101\times17-1}{\cancel{101}\times6\times17}\right)=\frac{k}{\cancel{101}}$

$$\cancel{34}^{\cancel{2}} \times \frac{1}{\cancel{2}}\left(\frac{1716}{6 \times \cancel{17}}\right) = 34K$$

$$34\cancel{K} = \frac{1716}{6}$$

$$= \boxed{286}$$

If the sum of the first ten terms of the series

$$\frac{1}{5}+\frac{2}{65}+\frac{3}{325}+\frac{4}{1025}+\frac{5}{2501}+\ldots \text{ is } \frac{m}{n}, \text{ where}$$

m and n are co-prime numbers, then m + n is equal to ________.

[JEE M 2022]

$$S = \frac{1}{4+1} + \frac{2}{64+1} + \frac{3}{324+1} + \frac{4}{1024+1} + \frac{5}{2500+1} + \ldots$$

$$\boxed{T_r = \frac{r}{4r^4+1}}$$

4 64 324 1024 2500

4(1 16 81 256 625)

$4(1^4\ 2^4\ 3^4\ 4^4\ 5^4)$

$$T_r = \frac{r}{(2r^2)^2 + 1^2} = \frac{r}{(2r^2+1)^2 - (2r)^2} = \frac{1}{4}\left[\frac{4r}{(2r^2+2r+1)(2r^2-2r+1)}\right]$$

$$= \frac{1}{4}\left[\frac{(2r^2+2r+1)-(2r^2-2r+1)}{(2r^2+2r+1)(2r^2-2r+1)}\right]$$

$$\boxed{T_r = \frac{1}{4}\left[\frac{1}{2r^2-2r+1} - \frac{1}{2r^2+2r+1}\right]}$$

$$T_1 = \frac{1}{4}\left[\frac{1}{1} - \cancel{\frac{1}{5}}\right]$$

$$T_2 = \frac{1}{4}\left[\cancel{\frac{1}{5}} - \cancel{\frac{1}{13}}\right]$$

$$T_{10} = \frac{1}{4}\left[\cancel{\bullet} - \frac{1}{221}\right]$$

$$S_{10} = \frac{1}{4}\left[1 - \frac{1}{221}\right] = \frac{1}{4}\left[\frac{220}{221}\right] = \frac{55}{221} = \frac{m}{n}$$

$$m+n = 55$$

$$\begin{array}{r} 55 \\ +221 \\ \hline 276 \\ \hline \end{array}$$

Q

Let for n = 1, 2, ..., 50, S_n be the sum of the infinite geometric progression whose first term is n^2 and whose common ratio is $\frac{1}{(n+1)^2}$. Then the value of $\frac{1}{26}+\sum_{n=1}^{50}\left(S_n+\frac{2}{n+1}-n-1\right)$ is equal to

[JEE M 2022]

DIY

Q

The sum $\sum_{n=1}^{21} \frac{3}{\underbrace{(4n-1)}\underbrace{(4n+3)}}$ is equal to

A. $\frac{7}{87}$

B. $\frac{7}{29}$

B

C. $\frac{14}{87}$

D. $\frac{21}{29}$

[JEE M 2022]

$$\frac{3}{4}\sum_{n=1}^{21}\frac{(4n+3)-(4n-1)}{(4n-1)(4n+3)}$$

$$\frac{3}{4}\sum_{n=1}^{21}\left(\frac{1}{4n-1}-\frac{1}{4n+3}\right)$$

$$\frac{3}{4}\left(\frac{1}{3}-\cancel{\frac{1}{7}}\right)$$

$$\frac{3}{4}\left(\cancel{\frac{1}{7}}-\cancel{\frac{1}{11}}\right)$$

$$\frac{3}{4}\left(\cancel{\frac{1}{83}}-\frac{1}{87}\right)$$

$$\text{Sum}=\frac{3}{4}\left(\frac{1}{3}-\frac{1}{87}\right)$$

$$=\frac{3}{4}\left(\frac{1}{3}-\frac{1}{3\times 29}\right)$$

$$=\frac{\cancel{3}}{4}\left(\frac{28}{\cancel{3}\times 29}\right)=\frac{7}{29}$$

Q

If $\sum_{k=1}^{10} \frac{k}{k^4+k^2+1} = \frac{m}{n}$, where m and n are co-prime, then m + n is equal to

[JEE M 2022]

DIY

$$\frac{1}{2} \sum_{K=1}^{10} \frac{2K}{\underbrace{(K^2+K+1)}\underbrace{(K^2-K+1)}}$$

$$\frac{1}{2} \sum \frac{(K^2+K+1)-(K^2-K+1)}{(K^2+K+1)(K^2-K+1)}$$

Q

If $\frac{1}{(20-a)(40-a)} + \frac{1}{(40-a)(60-a)} + \ldots\ldots + \frac{1}{(180-a)(200-a)} = \frac{1}{256}$, then the maximum value of a is :

[JEE M 2022]

A. 198

B. 202

C. 212

D. 218

$$\underbrace{\frac{(40-a)-(20-a)}{(20-a)(40-a)}} + \underbrace{\frac{(60-a)-(40-a)}{(40-a)(60-a)}} + \cdots + \frac{(200-a)-(180-a)}{(180-a)(200-a)} = \frac{20}{256}$$

$$\frac{1}{20-a} - \cancel{\frac{1}{40-a}}$$

$$\cancel{\frac{1}{40-a}} - \cancel{\frac{1}{60-a}}$$

$$\vdots$$

$$\cancel{\frac{1}{180-a}} - \frac{1}{200-a}$$

$$\boxed{\frac{1}{20-a} - \frac{1}{200-a} = \frac{20}{256}}$$

$a \to \phi E \quad a \to$

$H_1 \quad H_2 \quad H_3 \quad H_4 \quad \ldots \quad HP$

$\frac{1}{H_1} \quad \frac{1}{H_2} \quad \frac{1}{H_3} \quad \frac{1}{H_4} \longrightarrow AP$

Harmonic Progression

Harmonic Progression (H.P.) :

$a \quad b \quad c \rightarrow HP$

$$\underbrace{\frac{1}{a} \quad \frac{1}{b} \quad \frac{1}{c}} \rightarrow AP$$

Illustration

If the 3rd, 6th and last term of a H.P are 1/3, 1/5, 3/203, find the number of terms.

H P

$$T_3 = \frac{1}{3}$$
$$T_6 = \frac{1}{5}$$
$$T_n = \frac{3}{203}$$

A P.

$$T_3 = 3$$
$$T_6 = 5$$
$$T_n = \frac{203}{3}$$

$$d = \frac{2}{3}$$

$$a + 2d = 3$$
$$a + 2\left(\frac{2}{3}\right) = 3$$
$$a = 3 - \frac{4}{3} = \frac{5}{3}$$

$$\frac{5}{\cancel{3}} + (n-1)\left(\frac{2}{\cancel{3}}\right) = \frac{203}{\cancel{3}}$$
$$(n-1)(2) = 198$$
$$\boxed{n = 100}$$

Important Result

If m^{th} term of an H.P is n, and n^{th} term is equal to m then $(m + n)^{th}$ term is $\frac{mn}{m+n}$

Shortcut-12

A.P.

$T_m = n$

$T_n = m$

$T_{m+n} = 0$

H P

$T_m = n$

$T_n = m$

$$\boxed{T_{m+n} = \frac{mn}{m+n}}$$

Q

If the roots of the equation $x^3 - 11x^2 + 36x - 36 = 0$ are in H.P. Find the middle root.

$$x^3 - 11x^2 + 36x - 36 = 0 \quad \alpha,\ \beta = 3,\ \gamma$$

$\alpha, \beta, \gamma \rightarrow$ H.P

$\frac{1}{\alpha}\ \frac{1}{\beta}\ \frac{1}{\gamma} \rightarrow$ A.P

$$x \rightarrow \frac{1}{x}$$

$$-x^3\left(\frac{1}{x^3} - \frac{11}{x^2} + \frac{36}{x} - 36 = 0\right) \quad \frac{1}{\alpha},\ \frac{1}{\beta},\ \frac{1}{\gamma}$$

$$36x^3 - 36x^2 + 11x - 1 = 0$$

$\frac{1}{\alpha} = A - D$

$\frac{1}{\beta} = A = \frac{1}{3}$

$\frac{1}{\gamma} = A + D$

$3A = 1$

$\therefore A = \frac{1}{3}$

Ans. 3

Q

Let the positive numbers a, b, c, d be in A.P. The abc, abd, acd, bcd are

A. Not in A.P./G.P/H.P

B. In A.P.

C. In G.P.

D. In H.P. ✓

$1, 2, 3, 4 \longrightarrow AP$

$4, 3, 2, 1 \longrightarrow AP$

$$\frac{a}{abcd}, \frac{b}{abcd}, \frac{c}{abcd}, \frac{d}{abcd} \Longrightarrow AP$$

$$\frac{1}{bcd}, \frac{1}{acd}, \frac{1}{abd}, \frac{1}{abc} \Rightarrow A.P$$

$$\Rightarrow \frac{1}{abc}, \frac{1}{abd}, \frac{1}{acd}, \frac{1}{bcd} \Rightarrow AP$$

$$HP \Rightarrow abc, abd, acd, bcd \Rightarrow HP$$

Q

If $x=\sum_{n=0}^{\infty} a^n, y=\sum_{n=0}^{\infty} b^n, z=\sum_{n=0}^{\infty} c^n$, where a, b, c are in A.P. and $|a|<1, |b|<1, |c|<1, abc \neq 0$, then

[JEE M 2022]

A. x, y, z are in A.P.

B. x, y, z are in G.P.

C. $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in A.P.

D. $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1-(a+b+c)$

$x=\sum_{n=0}^{\infty} a^n$

$x = a^0+a^1+a^2+a^3+\ldots\infty$

$\boxed{x=\frac{1}{1-a}}$ $\boxed{y=\frac{1}{1-b}}$ $\boxed{z=\frac{1}{1-c}}$

$\frac{1}{x}=1-a$ $\frac{1}{y}=1-b$ $\frac{1}{z}=1-c$

$a\ ,\ b\ ,\ c \Longrightarrow A.P.$

$1-a\ ,\ 1-b\ ,\ 1-c \Rightarrow AP\ \checkmark$

$\frac{1}{x}\ ,\ \frac{1}{y}\ ,\ \frac{1}{z} \rightarrow AP$

$\frac{1}{1-a}\quad \frac{1}{1-b}\quad \frac{1}{1-c} \Rightarrow HP$

$x\quad y\quad z \rightarrow HP.$

$\frac{1}{x}\quad \frac{1}{y}\quad \frac{1}{z} \rightarrow AP$

Arithmetic Mean (A.M.)

Arithmetic Mean (A.M.) :

↳ Average

$$Am(a,b) = \frac{a+b}{2}$$

$$Am(a,b,c) = \frac{a+b+c}{3}$$

$$Am(2,3,4,5) = \frac{2+3+4+5}{4}$$

$$= \frac{14}{4} = \frac{7}{2} = 3.5$$

To insert 'n' AM's between a and b :

$CD = d$

$$a, \overset{+d}{\underline{A_1}}, \overset{+d}{A_2}, \overset{+d}{A_3}, \ldots, \overset{+d}{\underline{A_n}}, b \implies AP$$

$A_1 = a + d$

$A_2 = a + 2d$

$A_3 = a + 3d$

$A_n = a + nd$

$b = a + \underline{nd} + d$

$b = a + (n+1)d$

$$\boxed{d = \frac{b-a}{n+1}}$$ ★★

Sum of 'n' AM's inserted between a and b :

No of inserted AMs

$$\sum A_i = n\left(\frac{a+b}{2}\right)$$

$$\sum A_i =$$

Insert 20 AM's between 4 and 67.

Shortcut-13

$4 \quad A_1 \quad A_2 \quad A_3 \; \ldots\ldots \; A_{20} \quad 67$

$\underbrace{4 \overset{+3}{\curvearrowright} \boxed{7 \overset{+3}{,} 10 \overset{+3}{,} 13, \ldots\ldots, 64,} \; 67}$

$$d = \frac{b-a}{n+1}$$

$$= \frac{67-4}{20+1} = \frac{63}{21} = 3$$

$$A_1 + A_2 + \ldots A_{20} = n\left(\frac{a+b}{2}\right)$$

$$= 20\left(\frac{4+67}{2}\right)$$

$$= \boxed{\quad}$$

Q

If eleven A.M's are inserted between 28 and 10, then find the number of integral A.M's.

$28 \quad A_1 \quad A_2 \quad A_3 \quad \dots\dots \quad A_{11} \quad 10$

$d = \frac{10-28}{11+1}$

$= \frac{-18}{12}$

$\boxed{d = -1.5}$

$28 \quad \boxed{26.5 \quad 25 \quad 23.5 \quad 22 \quad 20.5 \quad 19 \quad 17.5 \quad 16 \quad 14.5 \quad 13 \quad 11.5} \quad 10$

integral AM $\rightarrow$ 5

$\Sigma A_i = n\left(\frac{a+b}{2}\right)$

$= 11\left(\frac{28+10}{2}\right)$

$= \frac{11 \times 38}{2} =$

Q

If n arithmetic means are inserted between 2 and 38, then the sum of the resulting series is obtained as 200. Then find the value of n.

$$\boxed{2 + \underbrace{A_1 + A_2 + A_3 + \cdots + A_n}_{} + 38} = 200 \qquad d = \frac{38-2}{n+1} = \frac{36}{n+1}$$

$$n\left(\frac{a+b}{2}\right) + 40 = 200$$

$$n\left(\frac{2+38}{2}\right) + 40 = 200$$

$$n(20) = 160$$

$$n = 8$$

Let a, b, c be positive integers such that b/a is an integer. If a, b, c are in geometric progression and the arithmetic mean of a, b, c is b + 2, then the value of $\frac{a^2 + a - 14}{a + 1}$ is

Ans. 4

[JEE Adv. 2014]

$a, b, c \rightarrow gp$

$a \quad ar \quad ar^2$

$\frac{+}{+} \quad \frac{b}{a} = r = \text{int}$ positive

$\frac{a+b+c}{3} = b+2$

$\frac{a+ar+ar^2}{3} = ar+2$

$a+ar+ar^2 = 3ar+6$

$\frac{36+6-14}{6+1} = \frac{28}{7} = 4$

jee

$$a - 2ar + ar^2 = 6$$

$$a(1 - 2r + r^2) = 6$$

$$a(r-1)^2 = 6$$

$$a = \frac{6}{(r-1)^2} = \text{positive int.}$$

$r = (+)$ int

$a = (+)$ int

$$a = \frac{6}{1}, \frac{6}{2}$$

$$(r-1)^2 = 1, \cancel{2}, \cancel{3}, \cancel{6}$$

$\cancel{r = 0}$ $r = 2$

$a = 6$ $\boxed{r = 2}$ ✓

Q

If n arithmetic means are inserted between a and 100 such that the ratio of the first mean to the last mean is 1 : 7 and $\boxed{a + n = 33}$, then the value of n is

[JEE M 2022]

A. 21

B. 22

C. 23

D. 24

$a \xrightarrow{+d} A_1 \quad A_2 \quad A_3 \quad \cdot \quad A_n \quad 100$

$$\boxed{d = \frac{100 - a}{n+1}}$$

$$\boxed{\frac{A_1}{A_n} = \frac{1}{7}}$$

$$\boxed{a + n = 33}$$

$$\frac{a+d}{100-d} = \frac{1}{7}$$

$$7a + 7d = 100 - d$$

$$7a + 8d = 100$$

$a = 33 - n$

$$\boxed{7a + \frac{8(100-a)}{n+1} = 100} \qquad \boxed{a + n = 33}$$

$a =$

$n = ?$

Geometric Mean (G.M.)

Geometric Mean (G.M):

$$GM(a, b) = (ab)^{\frac{1}{2}}$$

$$GM(a, b, c) = (abc)^{1/3}$$

$$GM(a, b, c, d) = (abcd)^{1/4}$$

$$GM(2, 3, 5) = (2 \times 3 \times 5)^{1/3}$$

To insert 'n' GM's between a and b:

$$\underbrace{a \xrightarrow{\times r} \underline{G_1} \xrightarrow{\times r} \underline{G_2} \xrightarrow{\times r} \underline{G_3} \ldots\ldots \underline{G_n}}_{} \xrightarrow{\times r} b$$

$\boxed{r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}}$

$\boxed{\prod G_i = \left(\sqrt{ab}\right)^{n}}$

Result

Product of GM's inserted between a and b:

Q

Insert 4 GM's between 5 and 160.

$$5 \quad G_1 \quad G_2 \quad G_3 \quad G_4 \quad 160$$

$$5 \quad \underbrace{10 \quad 20 \quad 40 \quad 80} \quad 160$$

$$r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$$

$$= \left(\frac{160}{5}\right)^{\frac{1}{5}}$$

$$\boxed{r = 2}$$

Shortcut-14

$$G_1 G_2 G_3 G_4 = 10 \times 20 \times 40 \times 80$$

$$= 640000$$

$$G_1 G_2 G_3 G_4 = \left(\sqrt{5 \times 160}\right)^4 = 640000$$

Harmonic Mean (H.M.)

Harmonic Mean (H.M):

$$HM(a,b) = \frac{2}{\frac{1}{a}+\frac{1}{b}}$$

$$HM(a,b,c) = \frac{3}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}$$

$$HM(2,5,15)$$

$$= \frac{3}{\frac{1}{2}+\frac{1}{5}+\frac{1}{15}}$$

$$= \_$$

To insert 'n' HM's between a and b:

$a \quad H_1 \quad H_2 \quad H_3 \quad \ldots \quad H_n \quad b \longrightarrow$ H.P.

$\frac{1}{a} \xrightarrow{+d} \frac{1}{H_1} \xrightarrow{+d} \frac{1}{H_2} \xrightarrow{+d} \frac{1}{H_3} \quad \cdot \quad \frac{1}{H_n} \quad \frac{1}{b} \longrightarrow$ AP

$$d = \frac{\frac{1}{b} - \frac{1}{a}}{n+1}$$

Result $\boxed{\sum \frac{1}{H_i} = \frac{n}{HM(a,b)}}$

Sum of Reciprocals of HM's

Result

✓ AM $\quad d = \frac{b-a}{n+1}$

✓ GM $\quad r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

✓ HM $\quad d = \frac{\frac{1}{b} - \frac{1}{a}}{n+1}$

* $\sum A_i = n\left(\frac{a+b}{2}\right)$

* $\prod G_i = \left(\sqrt{ab}\right)^n$

* $\sum \frac{1}{H_i} = \frac{n}{HM(a,b)}$

Q

If 9 arithmetic and harmonic means be inserted between 2 and 3, prove that $A + 6/H = 5$ where A is any one of the A.M's and H the corresponding H.M.

$2 \quad A_1 \quad A_2 \quad A_3 \ldots A_9 \quad 3 \longrightarrow AP$

$d = \dfrac{3-2}{9+1} = \dfrac{1}{10}$

$2 \quad H_1 \quad H_2 \quad H_3 \ldots H_9 \quad 3 \longrightarrow HP$

$\dfrac{1}{2} \quad \dfrac{1}{H_1} \quad \dfrac{1}{H_2} \quad \dfrac{1}{H_3} \ldots \dfrac{1}{H_9} \quad \dfrac{1}{3} \longrightarrow A.P$

$d = \dfrac{\frac{1}{3} - \frac{1}{2}}{9+1} = \dfrac{-1}{60}$

$A_1 = 2 + \dfrac{1}{10} = \dfrac{21}{10}$

$A_2 = \dfrac{21}{10} + \dfrac{1}{10} = \dfrac{22}{10}$

$\dfrac{1}{H_1} = \dfrac{1}{2} + \left(\dfrac{-1}{60}\right)$

$\boxed{\dfrac{1}{H_1} = \dfrac{29}{60}}$

$\dfrac{1}{H_2} = \dfrac{29}{60} + \dfrac{-1}{60}$

$\dfrac{1}{H_2} = \dfrac{28}{60}$

$$A + \frac{6}{H} = 5$$

$$A_1 + \frac{6}{H_1} = 5$$

$$A_2 + \frac{6}{H_2} = 5$$

$$A_3 + \frac{6}{H_3} = 5$$

$$= \frac{21}{10} + 6\left(\frac{29}{60}\right)$$

$$= \frac{21}{10} + \frac{29}{10}$$

$$= 5$$

$$\frac{22}{10} + 6\left(\frac{28}{60}\right) = 5$$

$$\vdots$$

⭐ Relation between A.M., G.M. and H.M.

jee

Relation between A.M, G.M. and H.M

If a & b are two positive numbers the $A \geq G \geq H$ & A, G, H are in G.P i.e $G^2 = AH$

(1) $\boxed{G^2 = A \times H}$

(2) $\boxed{A \geq G \geq H}$

positive nos

$A = \frac{a+b}{2}$

$H = \frac{2}{\frac{1}{a}+\frac{1}{b}} = \frac{2ab}{a+b}$

$G = \sqrt{ab}$

$A \times H = \left(\frac{\cancel{a+b}}{\cancel{2}}\right)\left(\frac{\cancel{2}ab}{\cancel{a+b}}\right) = \left(\sqrt{ab}\right)^2 = G^2$

Relation between A.M and G.M.

$$A - G = \frac{a+b}{2} - \sqrt{ab}$$

$$A - G = \frac{a+b-2\sqrt{ab}}{2}$$

$a=b$

$$A - G = \frac{(\sqrt{a}-\sqrt{b})^2}{2}$$

$$A - G > 0$$

$$\boxed{A > G}$$

$A \geq G$ ✓

$$\frac{A}{G} \geq 1$$

We know, $G^2 = AH$

$$\frac{G}{H} = \frac{A}{G} \geq 1$$

$$\frac{A}{G} \geq 1$$

$$\frac{G}{H} \geq 1$$

$$G \geq H$$

Relation between A.M, G.M and H.M.

① $G^2 = AH$

② $A \geq G \geq H$

$A = \frac{2+2}{2} = 2$

$G = \sqrt{2 \times 2} = 2$

$H = \frac{2(2)(2)}{4} = 2$

a, b, c → distinct positive

$$\boxed{A > G > H}$$

a, b, c → positive

$$\boxed{A \geq G \geq H}$$

Q If a + b + c = 3 and a, b, c are positive then prove that $a^2b^3c^2 \le \dfrac{3^{10}\cdot 2^4}{7^7}$

GM ≤ AM

$$a+b+c=3$$

$$\frac{\boxed{\frac{a}{2}+\frac{a}{2}+\frac{b}{3}+\frac{b}{3}+\frac{b}{3}+\frac{c}{2}+\frac{c}{2}}}{7} \geq \left(\frac{a}{2}\cdot\frac{a}{2}\cdot\frac{b}{3}\cdot\frac{b}{3}\cdot\frac{b}{3}\cdot\frac{c}{2}\cdot\frac{c}{2}\right)^{1/7}$$

$$\left(\frac{3}{7}\right)^7 \geq \left(\frac{a^2b^3c^2}{2^4\,3^3}\right)$$

$$\frac{3^7\cdot 2^4\cdot 3^3}{7^7} \geq a^2b^3c^2 \Rightarrow \boxed{\frac{3^{10}\cdot 2^4}{7^7} \geq a^2b^3c^2}$$

The minimum of $f(x) = a^{a^x} + a^{1-a^x}$, where α, x $\in$ R and $\alpha > 0$, is equal to:

[JEE M 2021]

A. a + 1/a

B. $\alpha + 1$

C. 2a

D. $2\sqrt{a}$

$f(x) = a^{a^x} + a^{1-a^x}$ $\quad a^x = t$

$= a^t + a^{1-t}$

$\frac{a^t + a^{1-t}}{2} \geq (a^t \cdot a^{1-t})^{1/2}$

$\boxed{a^t + a^{1-t} \geq 2\sqrt{a}}$

$f(x) \geq 2\sqrt{a}$

Q

Let f : R ⟶ R be such that for all x ∈ R ($2^{x+1} + 2^{1-x}$), f(x) and ($3^x + 3^{-x}$) are in A.P., then the minimum value of f(x) is:

[JEE M 2020]

A. 1

B. 2

C. 3

D. 4

$$f(x) = \frac{\overbrace{\boxed{2^{x+1} + 2^{1-x}}}^{\geq 4} + \overbrace{\boxed{3^{x} + 3^{-x}}}^{\geq 2}}{2} = \frac{6}{2} = 3$$

$$\frac{2^{x+1} + 2^{1-x}}{2} \geq \sqrt{2^{x+1}\, 2^{1-x}}$$

$$\underbrace{2^{x+1} + 2^{1-x}} \geq 4$$

$$\frac{3^{x} + 3^{-x}}{2} \geq \sqrt{3^{x} \cdot 3^{-x}}$$

$$3^{x} + 3^{-x} \geq 2$$

Q

The minimum value of $2^{\sin x} + 2^{\cos x}$ is:

A. $2^{-1} + \sqrt{2}$

B. $2^{1-\sqrt{2}}$

C. $2^{1-1/\sqrt{2}}$

D. $2^{-1+1/\sqrt{2}}$

[JEE M 2020]

HW

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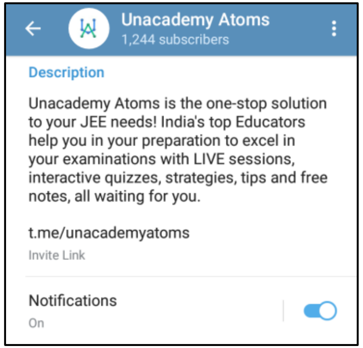

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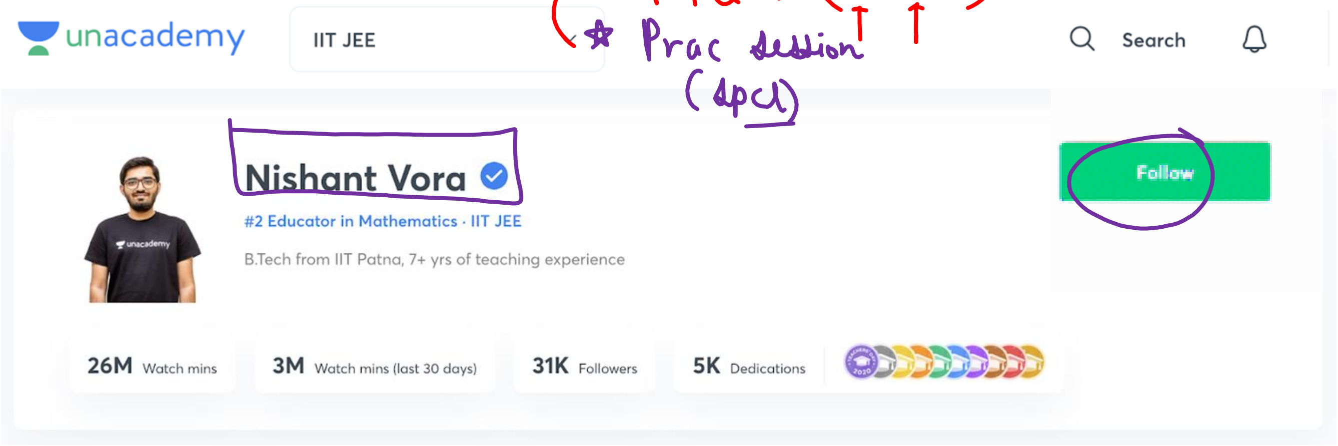

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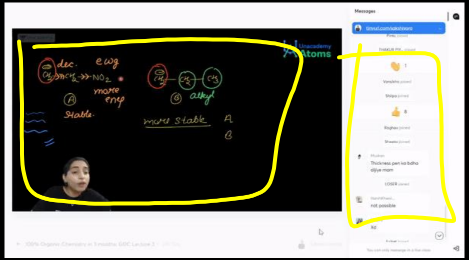

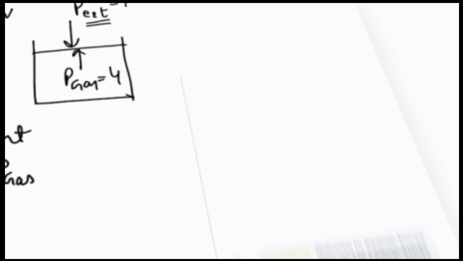

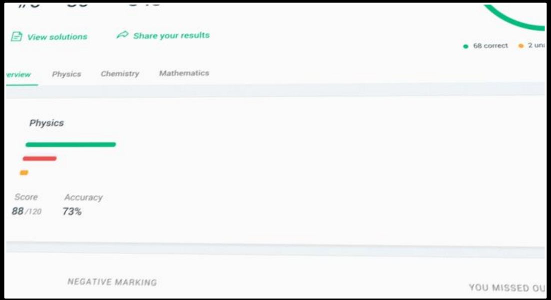

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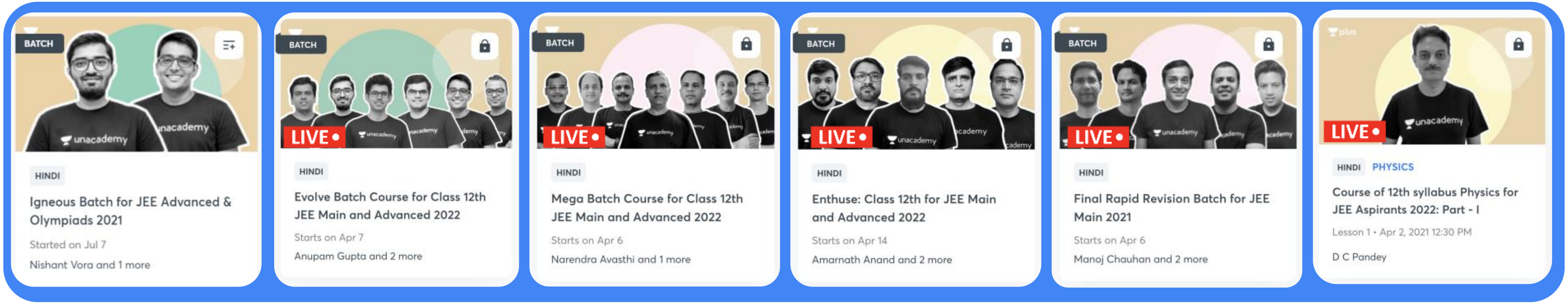

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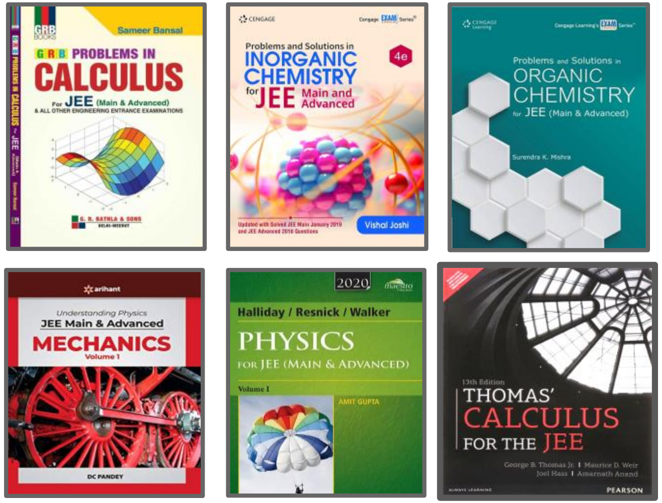

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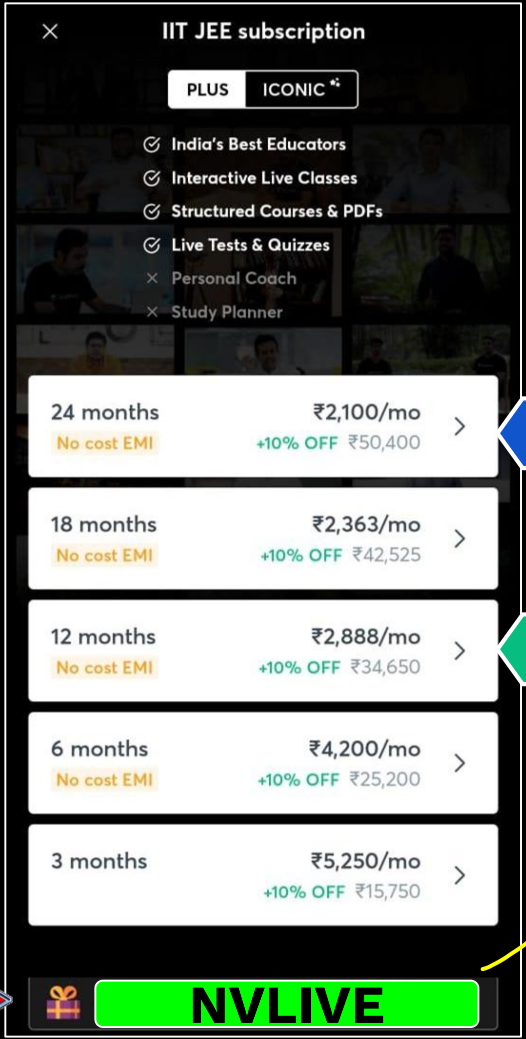

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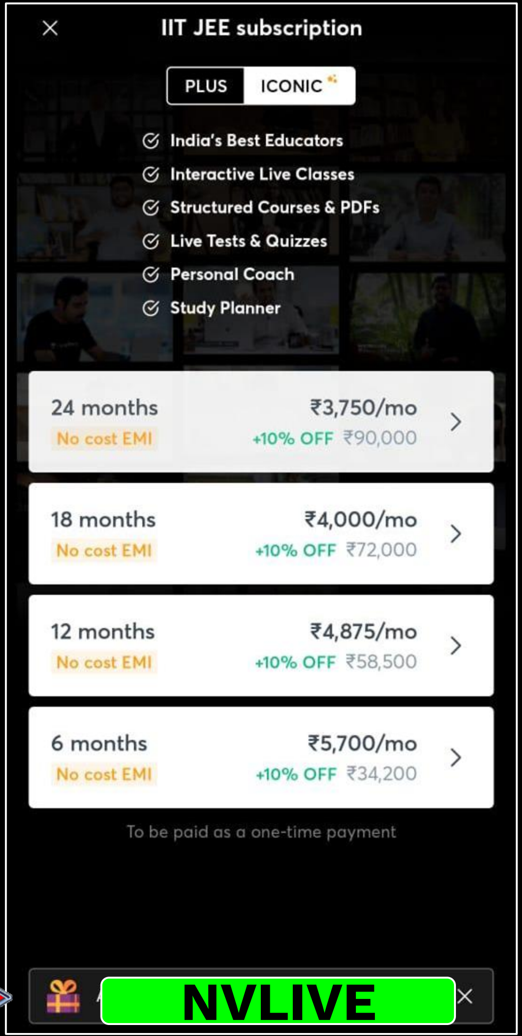

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