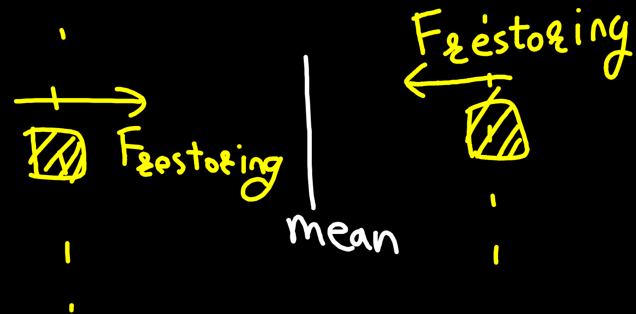
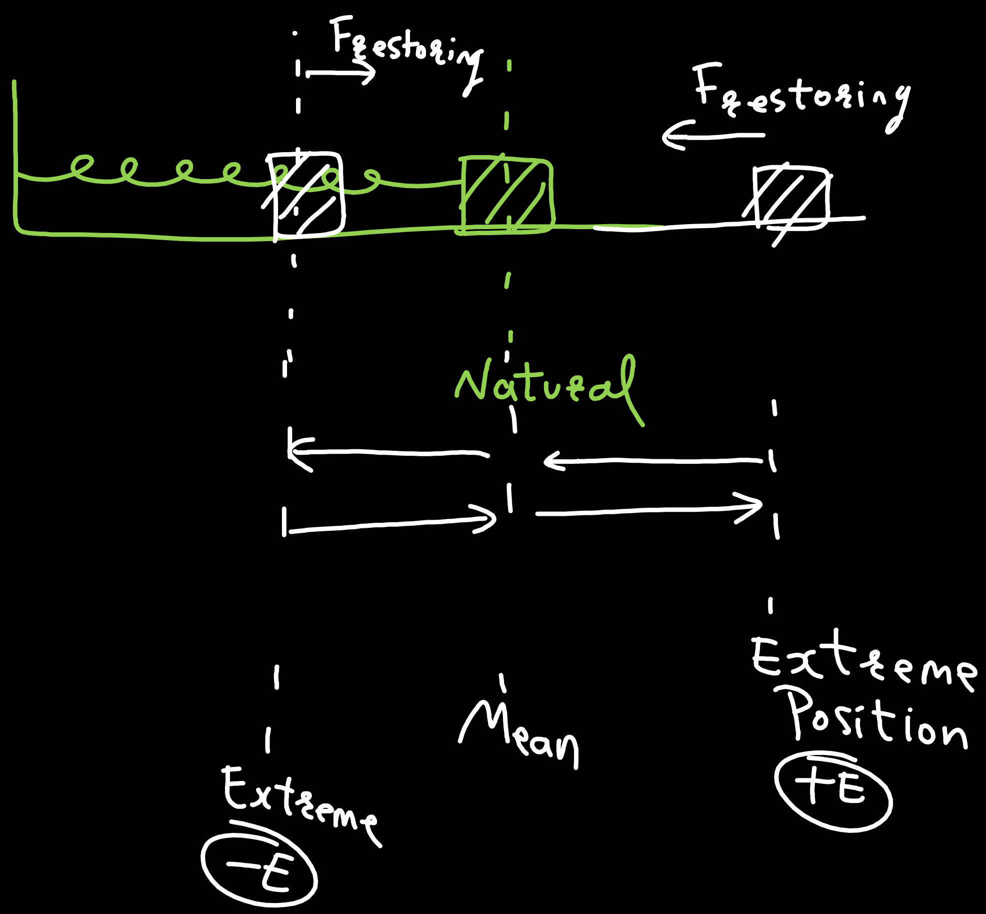
SHM

Oscillation → to & fro motion about mean position

mean/equilibrium net $F = 0$





Cause

$F_{restoring}$

$\downarrow$

direction always towards mean

Suppose

$x = 0$

$\xrightarrow{} F$ ⟵ F

x -ve $\quad$ x +ve

F in +ve $\hat{i}$ $\quad$ mean $\quad$ F in -ve $\hat{i}$

SHM

restoring

$F = -kx$ ✓

$F = -kx^2$ ✗

$F = -kx^3$ ✓

$F = -kx^4$ ✗

$F = -kx^5$ ✓

SHM

$F_{net} \propto (-ve)$ displacement from mean

$\boxed{F = -kx}$

$ma = -kx$

$\boxed{a = -\frac{k}{m} x}$

$\boxed{\frac{d^2x}{dt^2} = \frac{-k}{m} x}$

$\boxed{\frac{d^2x}{dt^2} \propto (-ve) x}$

double diff of x $\propto -(x)$

a

displ

$\downarrow$ diff

vel

$\downarrow$ diff

acc

$a = \frac{d^2x}{dt^2}$

$\sin\theta \xrightarrow{\text{diff}} \cos\theta \xrightarrow{\text{diff}} -\sin\theta$

$\cos\theta \xrightarrow{\text{diff}} -\sin\theta \xrightarrow{\text{diff}} -\cos\theta$

General Eq. of SHM

$\boxed{F = -Kx}$

$$\boxed{\frac{d^2x}{dt^2} \propto (-ve)\,x}$$

Ex

$x = 5\sin(6t)$

$x = 5\sin(6t)$

diff

$vel = 5\cos(6t) \times 6$

$vel = 5(6)\cos(6t)$

diff.

$acc = 5(6) - \sin(6t) \times 6$

$acc = -5(6)^2 \sin(6t)$

$\sin(\omega t)$

$\omega \rightarrow$ angular freq

$x = A\sin(\omega t)$

diff

$v = A\omega\cos(\omega t)$

diff

$acc = -A\omega^2\sin(\omega t)$

Standard EQ.

$\boxed{F = -kx}$

$\boxed{a = -\frac{k}{m}x}$

$\boxed{a = -\omega^2 x}$

$\omega^2 = \frac{k}{m}$

$\omega = \sqrt{\frac{k}{m}}$

$\boxed{x = A \sin(\omega t + \phi)}$

$\boxed{vel = A\omega \cos(\omega t + \phi)}$

$\boxed{acc = -A\omega^2 \sin(\omega t + \phi)}$

$acc = -\omega^2 x$

$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{K}}$

$A \rightarrow$ amplitude

$\omega \rightarrow$ ang. freq.

$\phi \rightarrow$ initial phase

$freq = \frac{1}{T}$

freq = no. of cycles/oscillations in one sec.

Unacademy Atoms

Q

$$F = -kx$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$a = -\omega^2 x$$

$$T = \frac{2\pi}{\omega}$$

Q $F = -5x$

$m = 2\text{kg}$ $T = ?$

$$F = -\underline{5}x$$

$m = 2$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$T = 2\pi\sqrt{\frac{2}{5}}$$

Q $9\frac{d^2x}{dt^2} + 4x = 0$

$m = 2\text{kg}$

$T = ?$

$$9a + 4x = 0$$

$$a = -\frac{4}{9}x$$

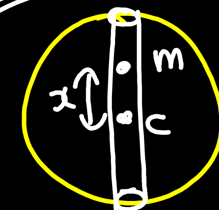
$$F = ma = -\frac{8}{9}x$$

$k = 8/9$ $m = 2$

$$T = 2\pi\sqrt{\frac{2}{8/9}} = 2\pi\sqrt{\frac{9}{4}}$$

$=$

Q



earth M_e R_e

inside solid sphere

$$\text{Force on } (m) = -\frac{GMm}{R^3}x$$

$$T = 2\pi\sqrt{\frac{m}{GMm/R^3}}$$

$$T = 2\pi\sqrt{\frac{R^3}{GM}}$$

$$\boxed{a = -\omega^2 x}$$

$$a = -\frac{4}{9}x$$

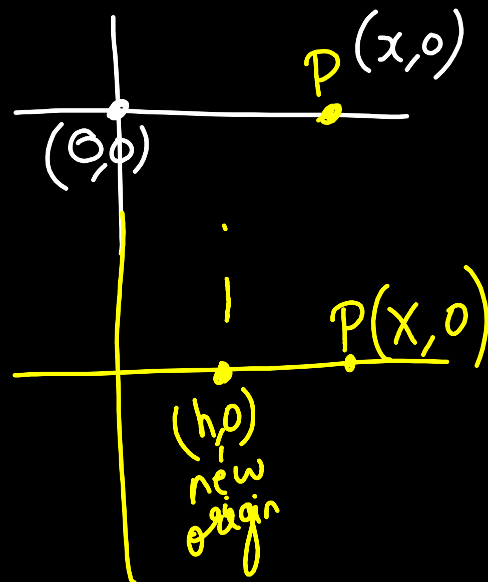
$$\omega^2 = \frac{4}{9}$$

$$\omega = \sqrt{\frac{4}{9}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{9}{4}}$$

<u>change of origin</u> (maths)

shift of origin



$$\boxed{X = x - h}$$

Q

$F = 5 - 8x$

$m = 2\,kg$

Find mean position & Time period.

$F = 0$

$5 - 8x = 0$

$5 = 8x$

$\frac{5}{8} = x$

$F = -kx$

$F = -8\left[\frac{5}{-8} + x\right]$

$F = -8\left[\underbrace{x - \frac{5}{8}}_{x-h}\right]$

$F = -8X$

$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{2}{8}} = 2\pi\sqrt{\frac{1}{4}} = \frac{2\pi}{2} = \pi$

$x = A \sin(\omega t + \phi)$

$x = 5 \sin\left(\pi t + \frac{\pi}{3}\right)$

Phase $\omega t + \phi$

initial Phase ($t=0$) $= \phi$

max x $\Rightarrow 5 =$ Amplitude

max displ from mean

$\omega = \pi$

$T = \frac{2\pi}{\omega} = \frac{2\cancel{\pi}}{\cancel{\pi}} = 2$ sec

initial phase $= \phi = \frac{\pi}{3}$

initial phase $\longrightarrow$ starting position

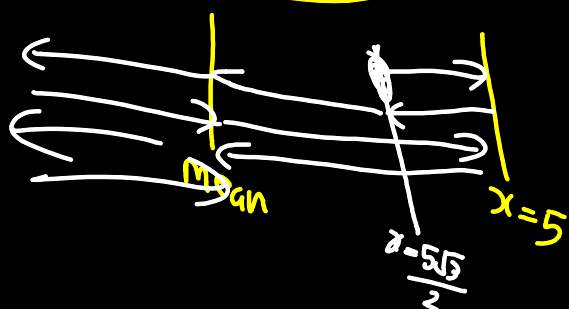
$x = 5\sin\left(\pi t + \frac{\pi}{3}\right)$

$t = 0$

$x = 5\sin\left(\frac{\pi}{3}\right)$

$= 5\sin(60^\circ)$

$x = 5\left(\frac{\sqrt{3}}{2}\right)$



$x = 5\sin(\omega t)$

$t = 0$

$x = 0$

mean

$x = 5\sin(\omega t + 90^\circ)$

$x = 5\cos(\omega t)$

$t = 0$

$x = 5$

extreme.

$$\boxed{x = A\sin(\omega t)}$$

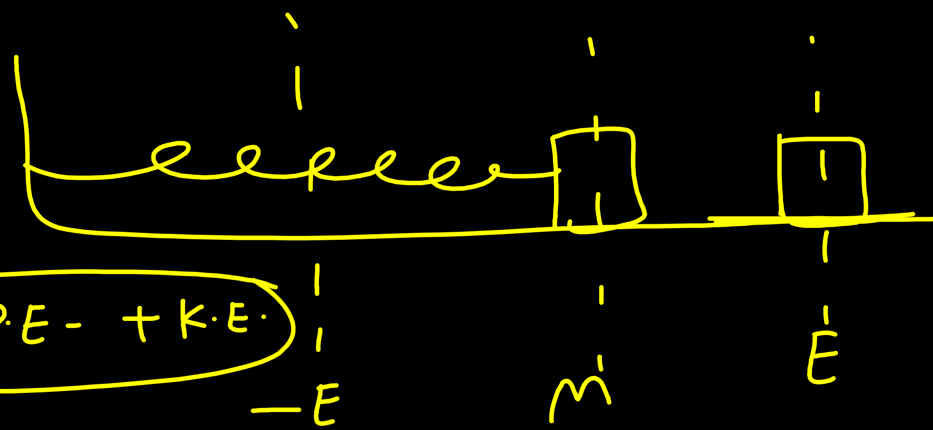
Simplicity $\phi = 0$ start from mean

$$\boxed{v = A\omega\cos(\omega t)}$$

$$\boxed{acc = -A\omega^2\sin(\omega t)}$$

$$\boxed{\text{max Speed} = A\omega}$$

$$\boxed{\text{max acc} = A\omega^2}$$



TE = P.E. + K.E.

PE = U

TE = U + KE

SHM
TE constant

TE = constant

KE → max mean
KE → 0 extreme

PE → mini mean
PE → max extreme.

extreme speed = 0

mean speed max = $A\omega$

acc mean = 0

acc extreme max = $A\omega^2$

$x = A\sin(\omega t)$

$v = A\omega\cos(\omega t)$

$acc = -A\omega^2\sin(\omega t)$

$KE = \frac{1}{2}mv^2 =$

$$\boxed{KE = \frac{1}{2}m\omega^2A^2\cos^2(\omega t)}$$

$PE = \frac{1}{2}kx^2 + PE_{mean}$

$= \frac{1}{2}kx^2 + PE_0$

$PE = \frac{1}{2}kx^2 + U_0$

$PE = \frac{1}{2}kA^2\sin^2(\omega t) + U_0$

$$PE = \frac{1}{2}m\omega^2A^2\sin^2(\omega t) + U_0$$

Unacademy Atoms

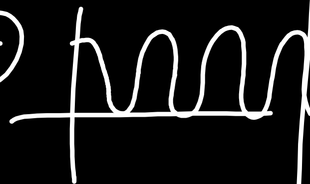
$a = -\frac{k}{m}x$

$a = -\omega^2 x$

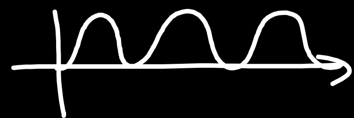
$\omega^2 = \frac{k}{m}$

$\boxed{m\omega^2 = k}$

$KE = \frac{1}{2} m\omega^2 A^2 (\cos^2 \omega t)$

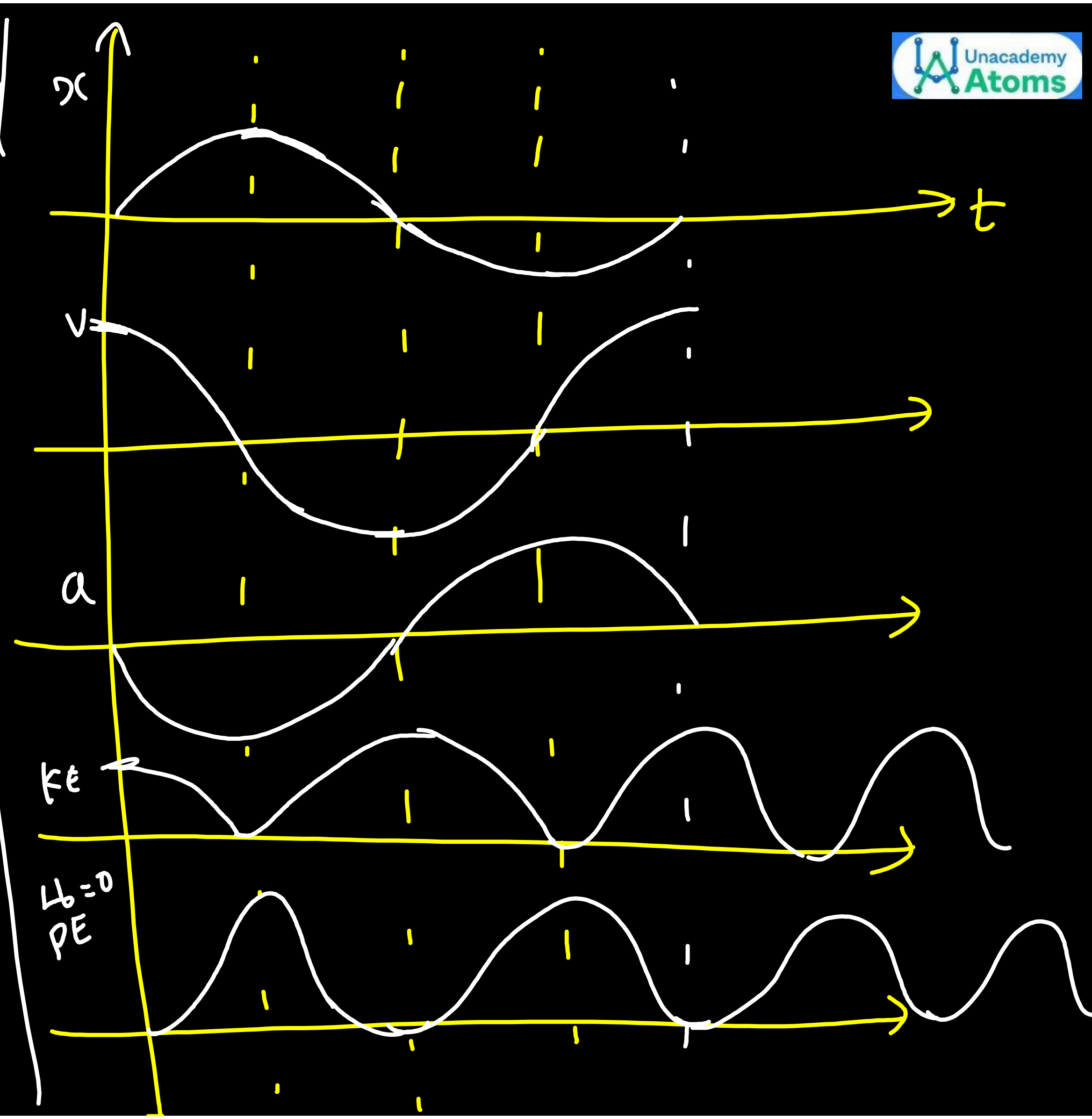


$PE = \frac{1}{2} m\omega^2 A^2 \sin^2(\omega t) + U_0$



$TE = KE + PE$

$$\boxed{TE = \frac{1}{2} m\omega^2 A^2 + U_0}$$



Q $m = 2\,kg$

$TE = 9$ Joule

PE at mean $= 5$ Joule

$A = 0.01\,m$

Find $T = ?$

a) $\frac{\pi}{10}$

b) $\frac{\pi}{20}$

c) $\pi/50$

d) $\pi/100$

$$TE = \frac{1}{2} m A^2 \omega^2 + U_0$$

$$9 = \frac{1}{2}(2)(0.01)^2 \omega^2 + 5$$

$$4 = (0.01)^2 \omega^2$$

$$4 = \frac{1}{100 \cdot 100} \omega^2$$

$$40000 = \omega^2$$

$$200 = \omega$$

$$T = \frac{2\pi}{\omega}$$

$$= \frac{2\pi}{200}$$

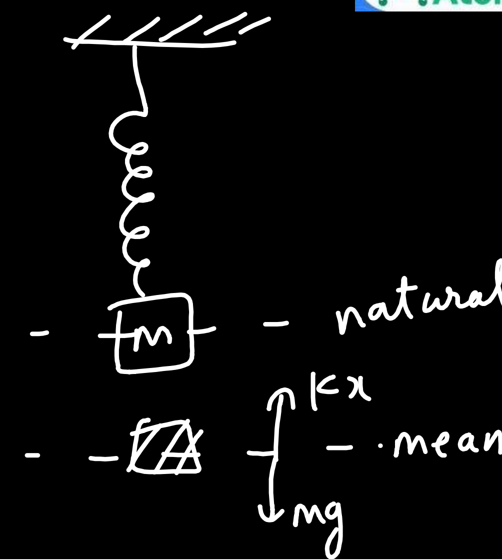
$$T = \frac{\pi}{100}$$

Basic Ex

Natural
mean

PE = 0
mean

$U_0 = 0$



$U_0 \neq 0$

$$\boxed{TE = KE + PE}$$

$$\boxed{TE = KE_{max} + PE_{mini}}$$

$$\boxed{TE = KE_{mini} + PE_{max}}$$

$KE_{mini} = 0$

Q// suppose

$$\underset{(PE)}{U} = U_0 \sin^2(\omega t)$$

Find TE

a) U_0 b) $\frac{U_0}{2}$

c) $\frac{U_0}{3}$ d) $\frac{U_0}{4}$

$PE_{max} = U_0$

$KE_{mini} = 0$

$TE = PE_{max} + KE_{mini}$

$TE = U_0$

Relationship with x

$\boxed{a = -\omega^2 x}$

$\boxed{F = -kx}$

$x = A\sin(\omega t)$

$V = A\omega\cos(\omega t)$

$\sin^2 + \cos^2 = 1$

$\boxed{\dfrac{x^2}{A^2} + \dfrac{v^2}{A^2\omega^2} = 1}$ ellipse

$\boxed{V = \omega\sqrt{A^2 - x^2}}$

$KE = \frac{1}{2}mv^2$

$\boxed{KE = \frac{1}{2}m\omega^2(A^2 - x^2)}$

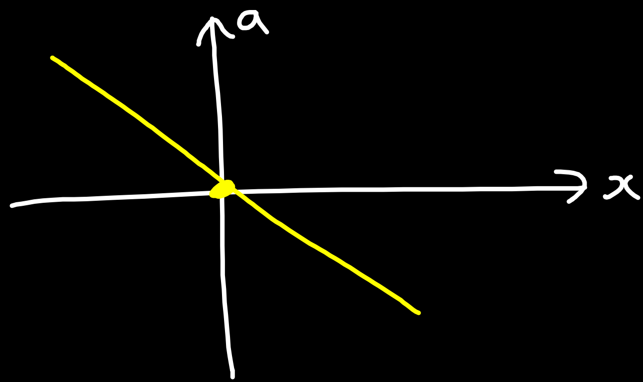
$PE = \frac{1}{2}kx^2 + U_0$

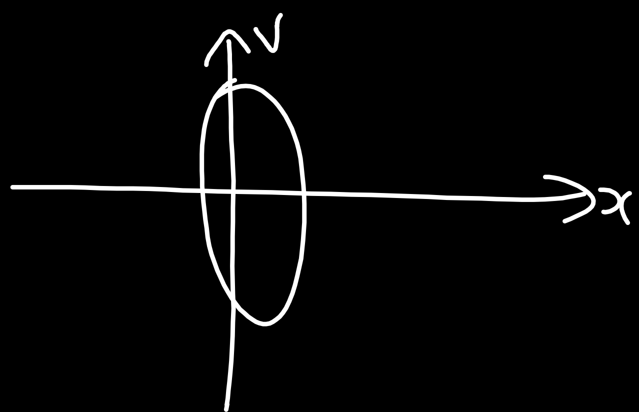
$k = m\omega^2$

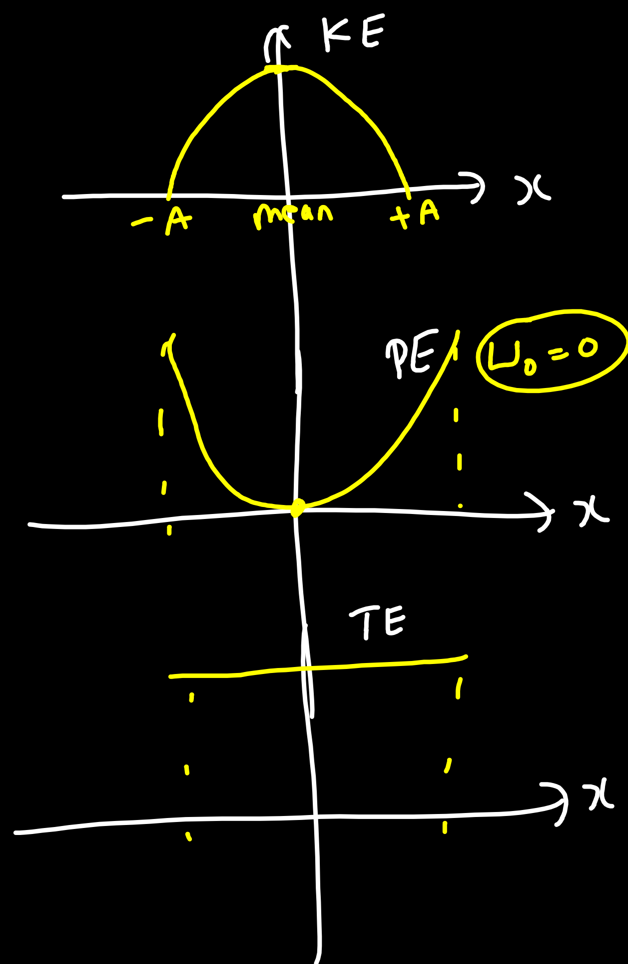
$\boxed{PE = \frac{1}{2}m\omega^2 x^2 + U_0}$

$TE = KE + PE$

$\boxed{TE = \frac{1}{2}m\omega^2 A^2}$


a
x


v
x


KE
x
-A
mean
+A
PE
$U_0 = 0$
x
TE
x

Q// Find t when $PE = KE$

Q// Find position where $PE = KE$

① $PE = KE$

$$\frac{1}{2}mA^2\omega^2\sin^2\omega t = \frac{1}{2}mA^2\omega^2\cos^2\omega t$$

$\sin^2 = \cos^2$

$\sin = \cos$

$\theta = 45^\circ = \pi/4$

$\theta = \omega t$

$\frac{\pi}{4} = \omega t$

$T = \frac{2\pi}{\omega}$

$\omega = \frac{2\pi}{T}$

$\frac{\cancel{\pi}}{4} = \frac{2\cancel{\pi}}{T} t$

$$\boxed{\frac{T}{8} = t}$$

given

starts from $t=0$ at $x=0$

$PE_{mean} = 0$

time $= T$

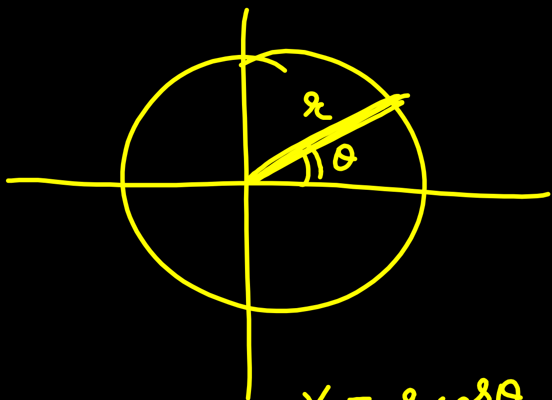
amplitude $= A$.

② $\cancel{\frac{1}{2}}\cancel{m}\cancel{\omega^2}x^2 = \cancel{\frac{1}{2}}\cancel{m}\cancel{\omega^2}(A^2 - x^2)$

$2x^2 = A^2$

$x = A/\sqrt{2}$

Phasor Diagram



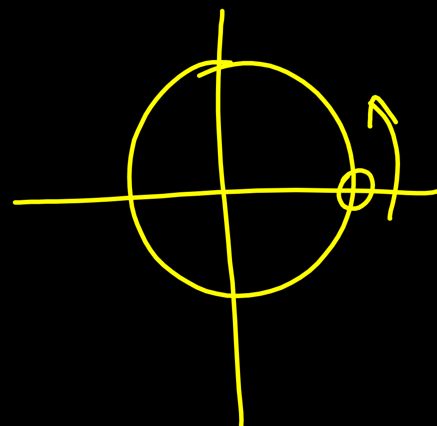
$X = r\cos\theta$

$Y = r\sin\theta$

circular

$s = ut + \frac{1}{2}at^2$

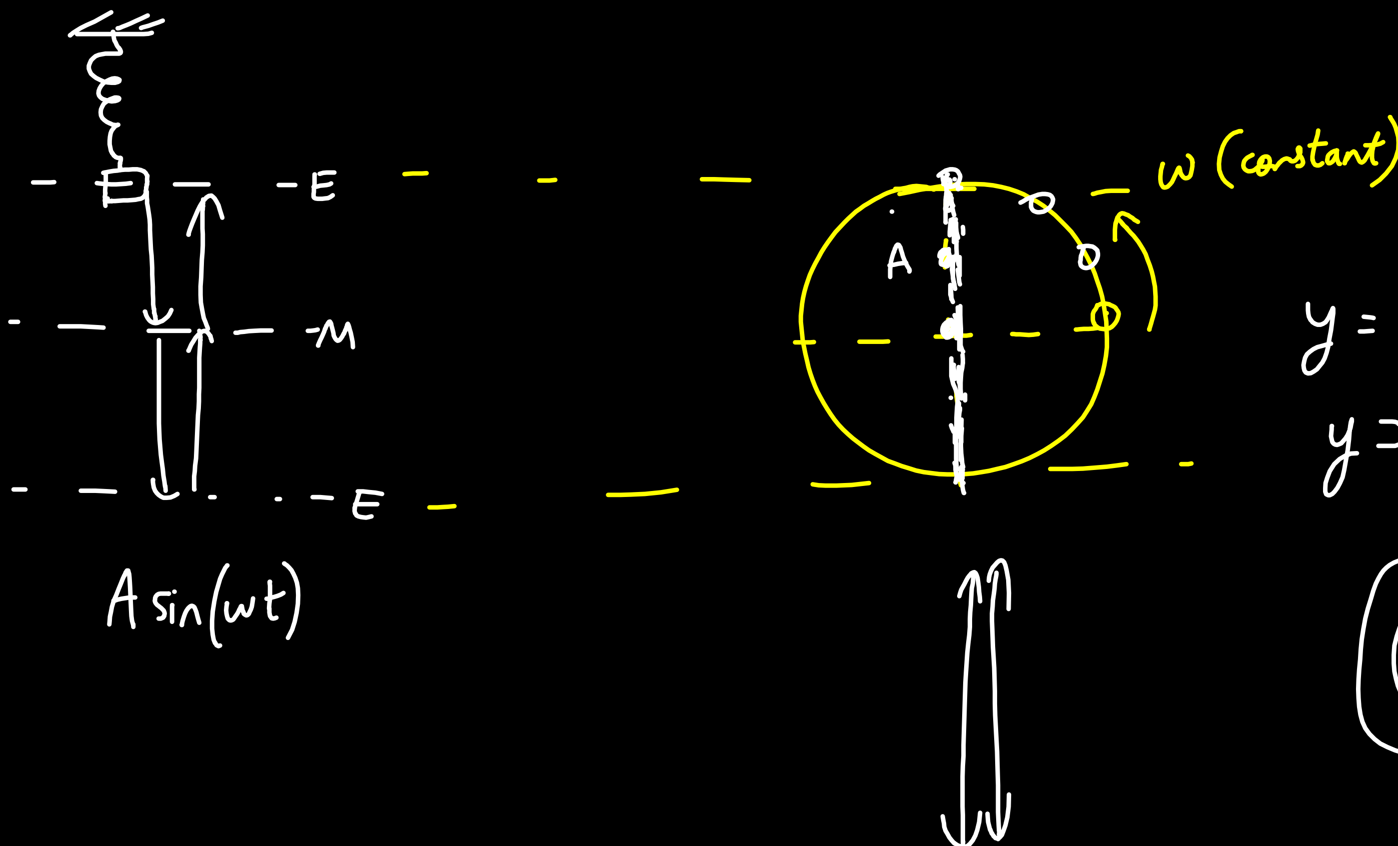
$\theta = \omega t + \frac{1}{2}\alpha t^2$



Uniform circular motion

$\alpha = 0$

$\boxed{\theta = \omega t}$

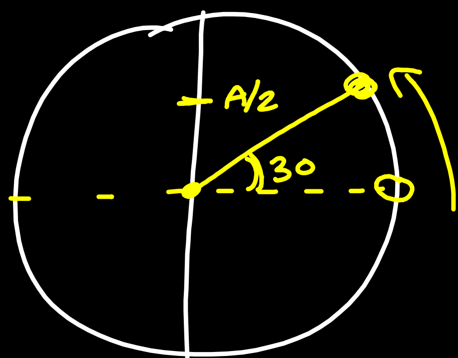


$$y = A\sin\theta$$

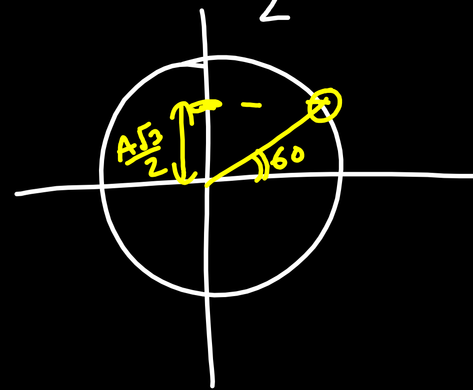
$$y = A\sin(\omega t)$$

$$\boxed{\theta_{rotated} = \omega t}$$

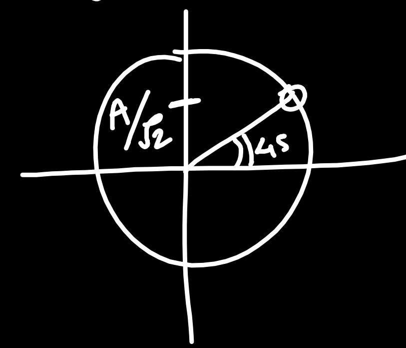
$$x = A\sin(\omega t)$$



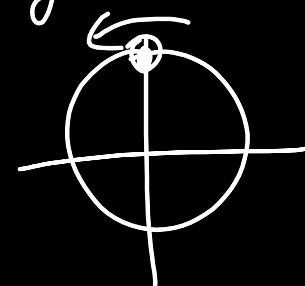
$$x = \frac{A\sqrt{3}}{2}$$



$$y = A/\sqrt{2}$$



$$y = A$$



Find time taken from (direct)

T

$\omega = \frac{2\pi}{T}$

$\theta_{rot} = \omega t$

① $x = 0$ to $x = A$

90° $\theta = \omega t$

$\frac{\pi}{2} = \frac{2\pi}{T} t$

$t = T/4$

② $x = 0$ to $x = A/2$

30°

$\theta = \omega t$

$\frac{\pi}{6} = \frac{2\pi}{T} t$

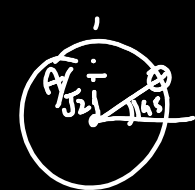
$t = T/12$

③ $x = A/2$ to $x = A$

60°

$\frac{\pi}{3} = \frac{2\pi}{T} t \Rightarrow t = T/6$

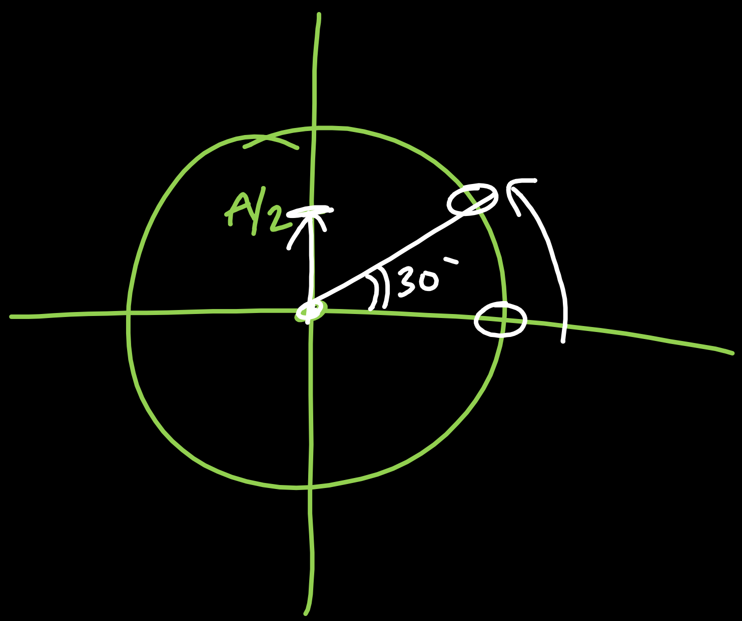
④ $x = 0$ to $x = A/\sqrt{2}$

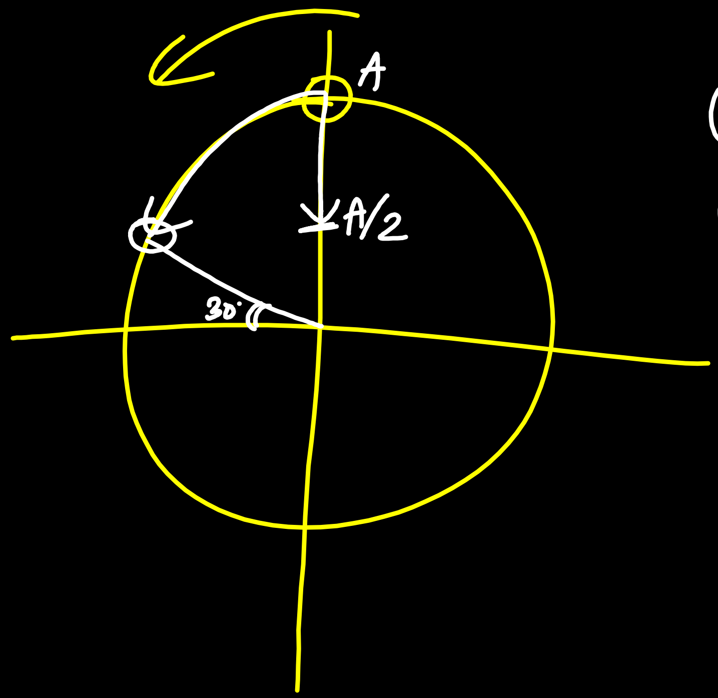


$\frac{\pi}{4} = \frac{2\pi}{T} t$

$t = T/8$

⑤ $x = A$ to $x = A/2$


A/2
30°



$\theta_{rot} = \omega t$

$60°$

$\frac{\pi}{3} = \frac{2\pi}{T} t$

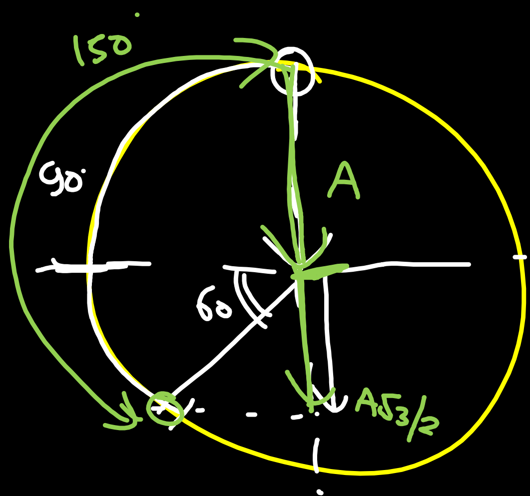
$t = T/6$

Q. Distance covered by particle executing SHM

A T

Particle starts from extreme

Find distance travelled in $\frac{5T}{12}$ sec



$\theta = \omega t$

$= \left(\frac{2\pi}{T}\right)\left(\frac{5T}{12}\right)$

$= \frac{\pi 10}{12} = \frac{10 \times \cancel{180}^{\cancel{30}\,15}}{\cancel{12}\,8}$

$= 150°$

$A_{ns} = A + \frac{A\sqrt{3}}{2}$

Break 10min

Resume 9:15pm

After Break

→ Spring

→ Simple

→ Time Calculation

→ Superposition / SHM.

Calculation of Time Period

Find mean position $\longleftrightarrow$

Displace by small amount x

Find F_{net} & acc at this position

Compare with

$$F = -kx \qquad T = 2\pi\sqrt{\frac{m}{k}}$$

$$a = -\omega^2 x$$

#

$x \to \theta$

$F \to \tau$

$m \to MoI$

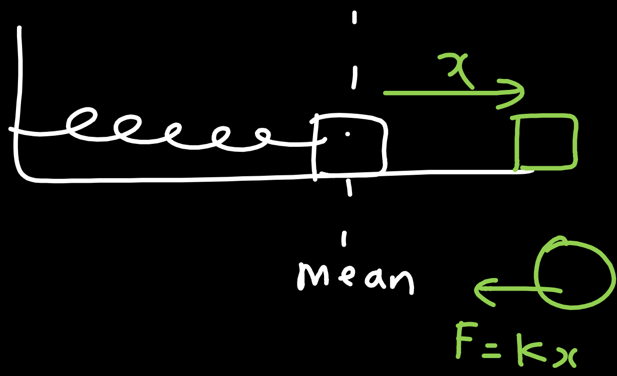
$a \to \alpha$

$$\tau = -k\theta$$

$$T = 2\pi\sqrt{\frac{I}{k}}$$

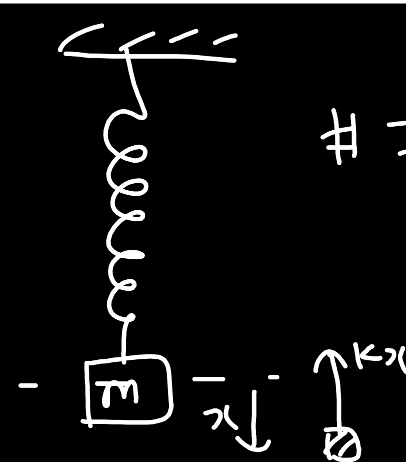
$$\alpha = -\omega^2\theta.$$

Q



$$F_{spring} = -kx$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

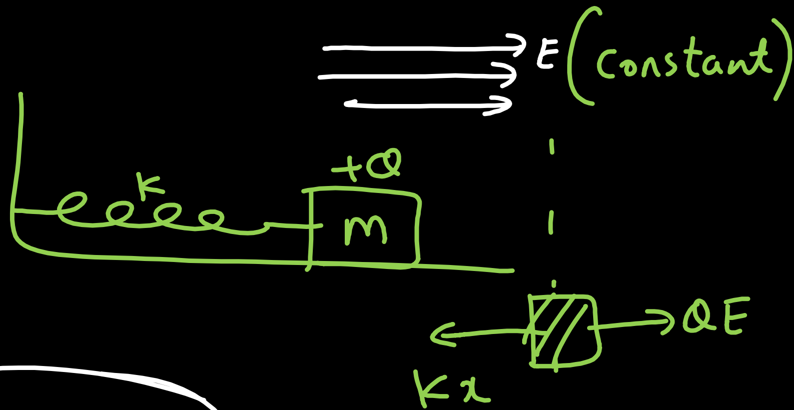


In spring block system for calculation of T we can ignore any constant forces (like mg)

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$F_{net} = -kx$$

Q



$\boxed{\text{force} = QE}$

Find ① mean position = ?

② Time period = ?

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$Kx = QE$$

$$x = \frac{QE}{K} \Rightarrow \text{mean position.}$$

Combination of Springs

$$T = 2\pi\sqrt{\frac{m}{k_{eq}}}$$

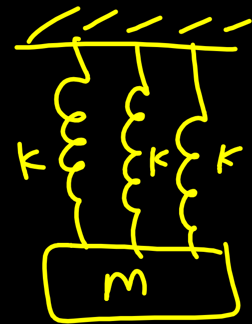
Parallel

$$k_{eq} = k_1 + k_2$$

Series

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$$

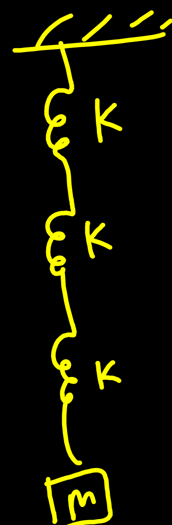
Q//



$k_{eq} = 3k$

$$2\pi\sqrt{\frac{m}{3k}}$$

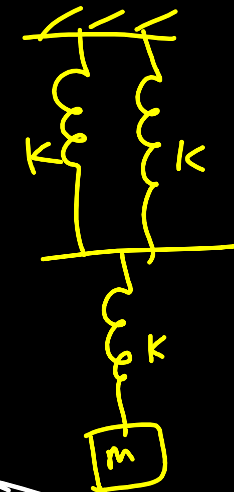
Q//



$k_{eq} = k/3$

$$T = 2\pi\sqrt{\frac{m}{k/3}}$$

$$= 2\pi\sqrt{\frac{3m}{k}}$$



$2k$

k

$$\frac{1}{k_{eq}} = \frac{1}{k} + \frac{1}{2k}$$

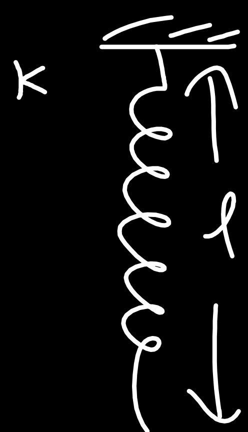
$$\frac{1}{k_{eq}} = \frac{3}{2k}$$

$$k_{eq} = \frac{2k}{3}$$

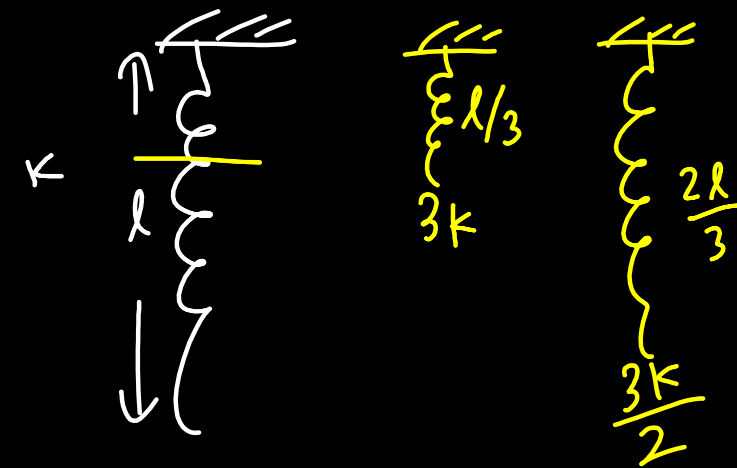
$$T = 2\pi\sqrt{\frac{3m}{2k}}$$

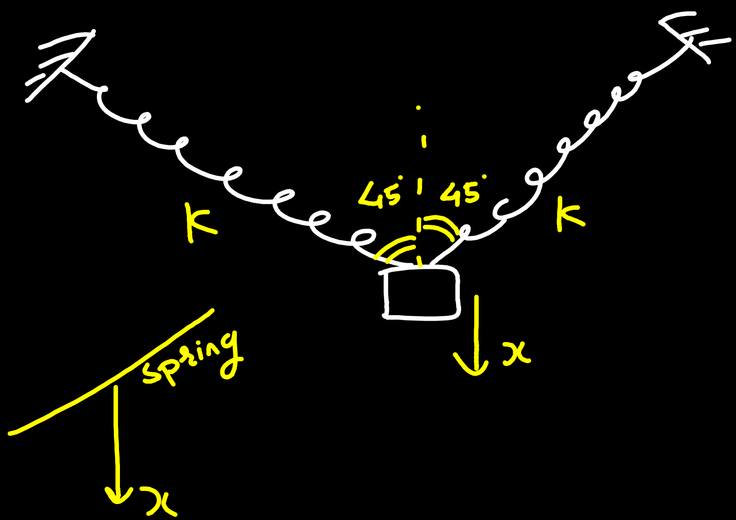
Cutting of Springs

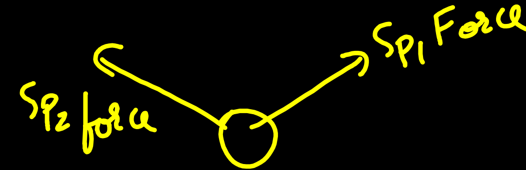
$$K \propto \frac{1}{\text{natural length}}$$



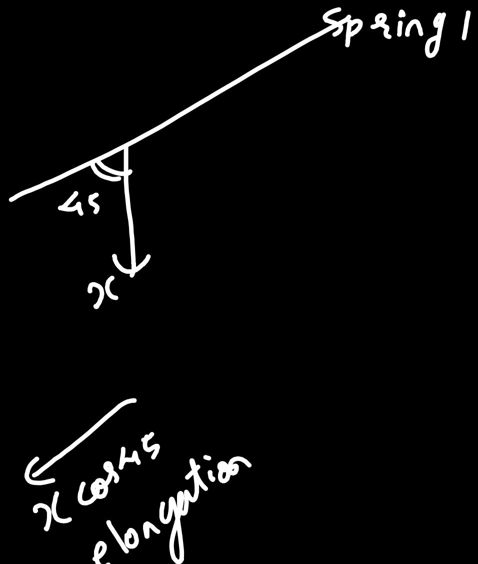
$$\boxed{K l = k_1 l_1 = k_2 l_2}$$



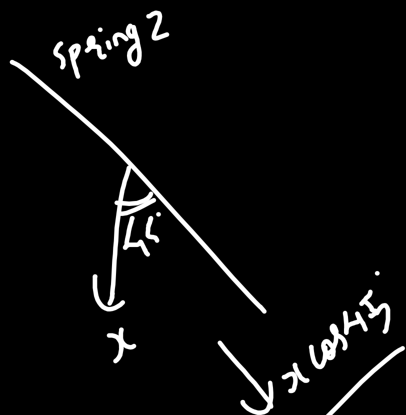


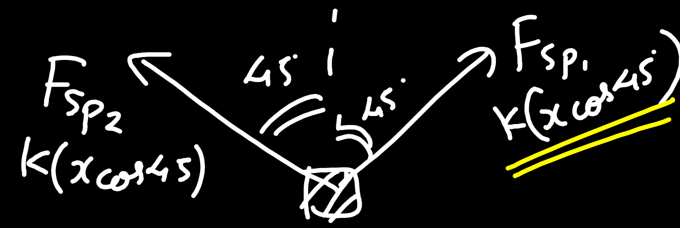


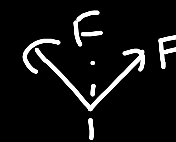
Spring की elongation $\equiv$ x component

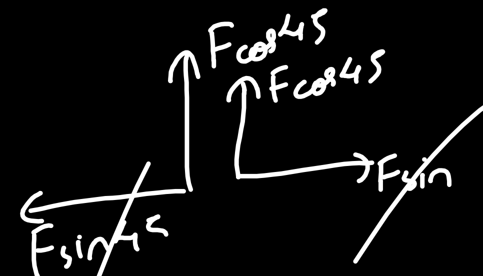


t.me/ajitlulla









$$\uparrow 2F\cos 45$$

$$2(Kx\cos 45)\cos 45$$

$$2Kx\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} \Rightarrow Kx$$

$$T = 2\pi\sqrt{\frac{m}{K}}$$

$\Rightarrow$

$$T = 2\pi\sqrt{\frac{\mu}{k}}$$

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

reduced mass

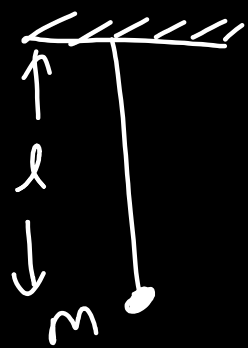
Simple Pendulum

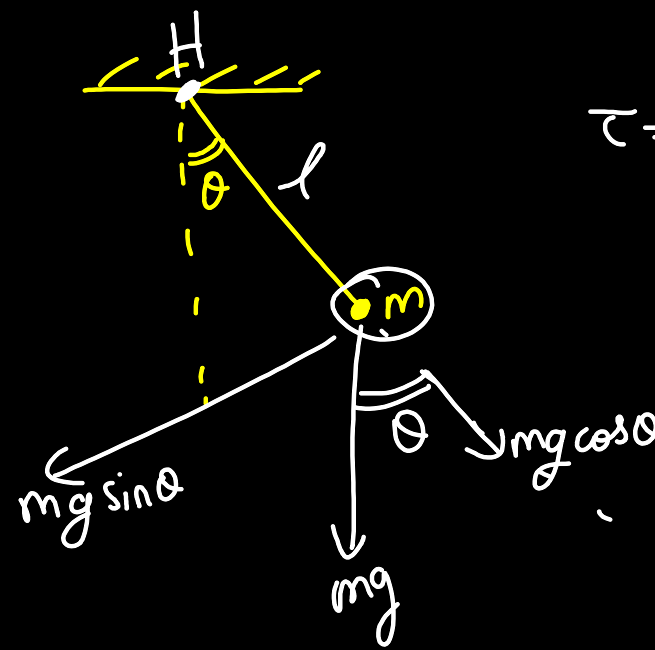
$F = ma$

τ

$\tau = I\alpha$

$\alpha = -\omega^2 \theta$





$\tau = (\text{force})(\perp\text{'ur dist})$

$\tau = -(mg\sin\theta)\,l$

for small angle

$\sin\theta \approx \theta$

$\tau = -mgl\theta$

$I\alpha = -mgl\theta$

$\alpha = \frac{-mgl}{I}\theta$

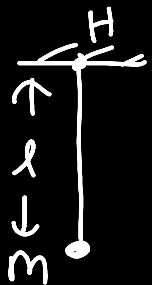
$\alpha = -\omega^2\theta$

$\omega = \sqrt{\frac{mgl}{I}}$

$T = 2\pi/\omega$

$T = 2\pi\sqrt{\frac{I}{mgl}}$

Point mass MoI $= ml^2$
(I)



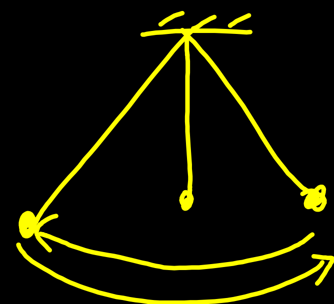
$$T = 2\pi\sqrt{\frac{I}{mgl}}$$

$$= 2\pi\sqrt{\frac{ml^2}{mgl}}$$

$$\boxed{T = 2\pi\sqrt{\frac{l}{g}}}$$

Second's pendulum

$T = 2\,\text{sec}$



$$T = 2\pi\sqrt{\frac{l}{g}}$$

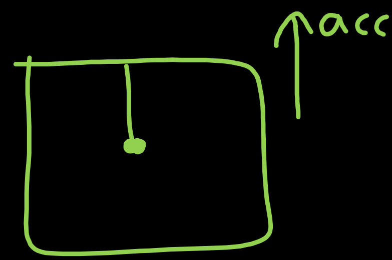
$$2 = 2\pi\sqrt{\frac{l}{g}}$$

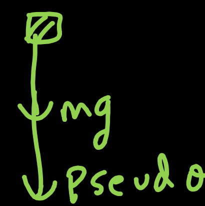
$\pi = \sqrt{10}$

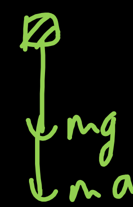
$l \approx 1\text{m}$

$$T = 2\pi\sqrt{\frac{l}{g_{effective}}}$$

Pseudo force



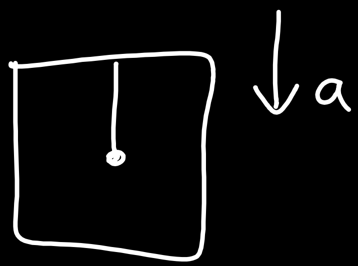


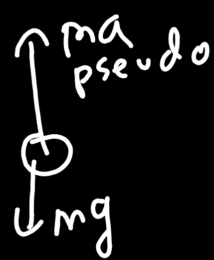


$$T = 2\pi\sqrt{\frac{l}{g+a}}$$

$$g_{eff} = g + a$$

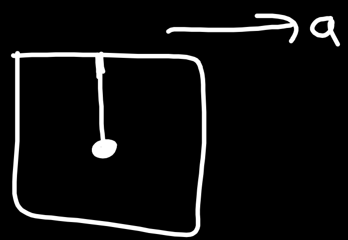
Q

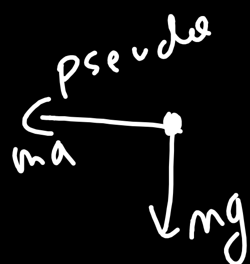


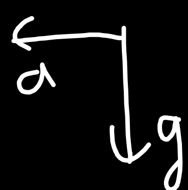


$$g_{eff} = g - a \qquad T = 2\pi\sqrt{\frac{l}{g-a}}$$

Q



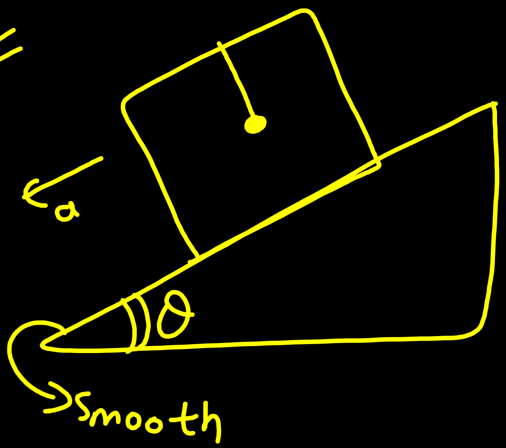


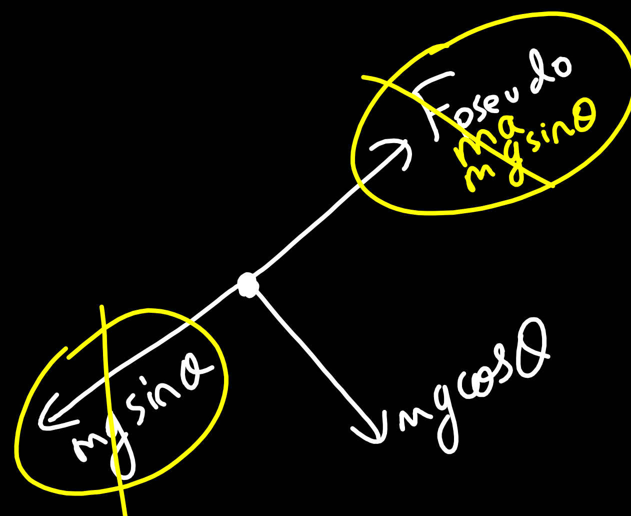


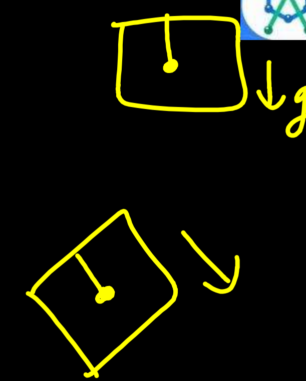
$$\sqrt{a^2+g^2} = g_{eff}$$

$$\boxed{T = 2\pi\sqrt{\frac{l}{\sqrt{g^2+a^2}}}}$$

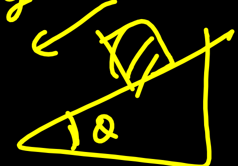
Q.



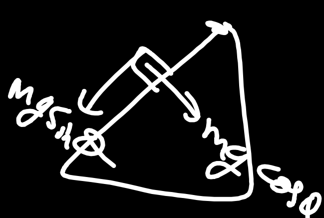


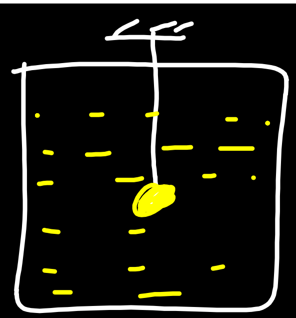


$g\sin\theta = acc$



$$\boxed{T = 2\pi\sqrt{\frac{l}{g\cos\theta}}}$$





$\rho_L = \rho_{Liquid}$

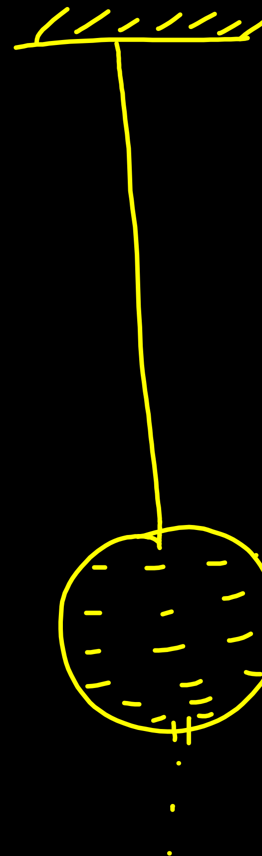
$\rho_s = \rho_{solid}$

$$mg \longrightarrow mg_{effective} = mg\left(1 - \frac{\rho_L}{\rho_s}\right)$$

$$T = 2\pi\sqrt{\frac{l}{g\left(1 - \frac{\rho_L}{\rho_s}\right)}}$$

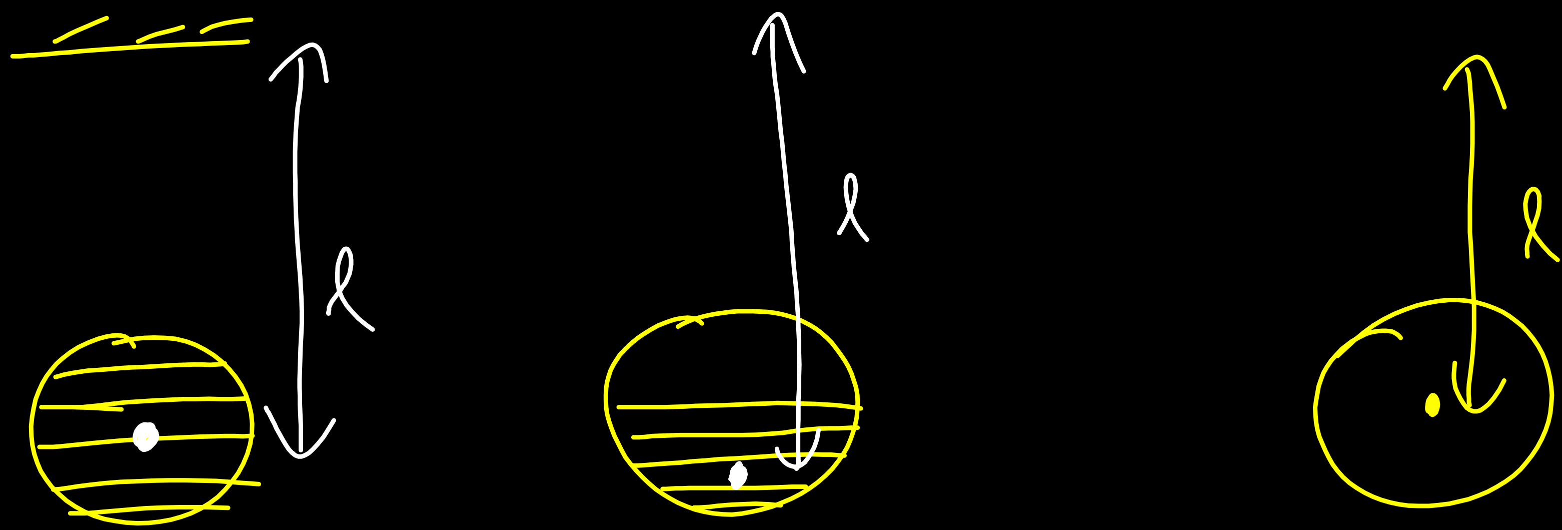
$$T = 2\pi\sqrt{\frac{l}{g}}$$

$l \rightarrow$ distance of COM from hinge



leakage

Time changes ?



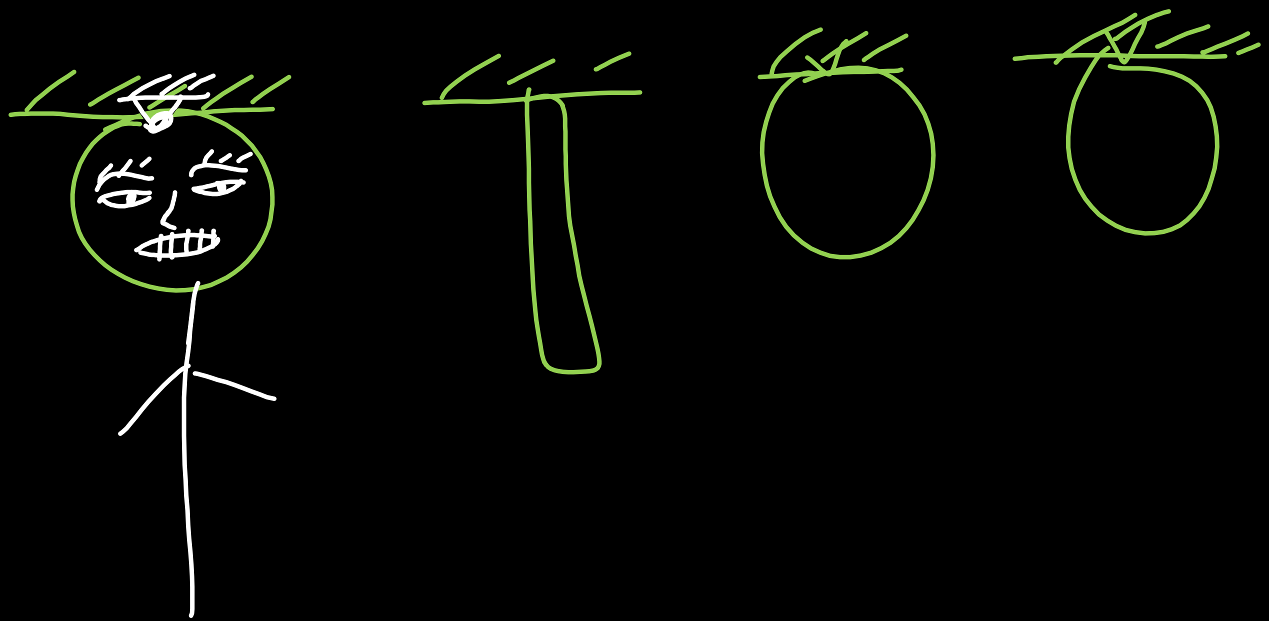
l increase than decrease & init l = final l.

T 1st increases than decrease

& $T_{ini} = T_{final}$

Physical Pendulum

Compound Pendulum

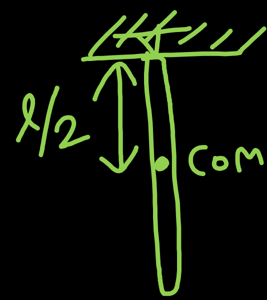


$$\boxed{T = 2\pi\sqrt{\frac{I}{mg\,d}}}$$

$I \rightarrow$ moment of intertia of body about hige

$d \rightarrow$ distance of com from hinge.

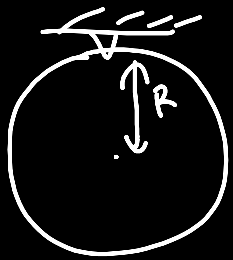
$m \rightarrow$ mass of system.



$$I = \frac{ml^2}{3}$$

$$T = 2\pi\sqrt{\frac{ml^2/3}{mg(l/2)}}$$

$$T = 2\pi\sqrt{\frac{2l}{3g}}$$



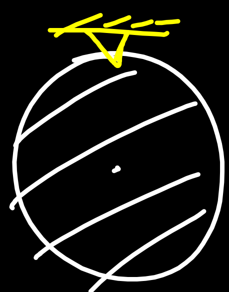
Ring
(m, R)

$$I_{com} + Md^2$$

$$I = Mr^2 + Mr^2$$

$$I = 2Mr^2$$

$$T = 2\pi\sqrt{\frac{2Mr^2}{mg\,r}} = 2\pi\sqrt{\frac{2r}{g}}$$



Disc
m, R

$$MOI = \frac{mr^2}{2} + mr^2$$

$$= \frac{3}{2}mr^2$$

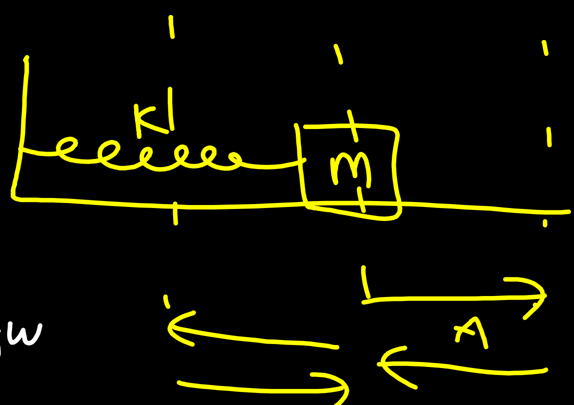
$$T = 2\pi\sqrt{\frac{\frac{3}{2}m r^{\cancel{2}}}{mg\,\cancel{r}}} = 2\pi\sqrt{\frac{3m}{2g}}$$

Mechanics Use

$$V = \omega\sqrt{A^2 - x^2}$$

$$V = A\omega$$

Q

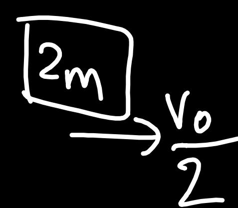


$V_0 = A\omega$

when it was crossing mean position, another block ~~of~~ of mass m is dropped over it & it sticks to body. Find new Amplitude?

initial

final



$P_{ini} = P_{fin}$

$mV_0 = 2mV_f$

$\frac{V_0}{2} = V_f$

initial final

V_0 $\frac{V_0}{2}$

$A\omega$ $A'\omega'$

$V_0 = A\omega$

$\omega = \sqrt{\frac{k}{m}}$

$\omega' = \sqrt{\frac{k}{2m}}$

$V_{final} = \frac{V_0}{2}$

$A'\omega' = \frac{A\omega}{2}$

$A'\sqrt{\frac{\cancel{k}}{2\cancel{m}}} = \frac{A}{2}\sqrt{\frac{\cancel{k}}{\cancel{m}}}$

$A' = A/\sqrt{2}$

Unacademy Atoms

Rotational

τ method

$\tau = I\alpha$

$\alpha = -\omega^2\theta$

Energy method

$TE = \text{constant}$

$\Big\downarrow$ diff

0

Find mean

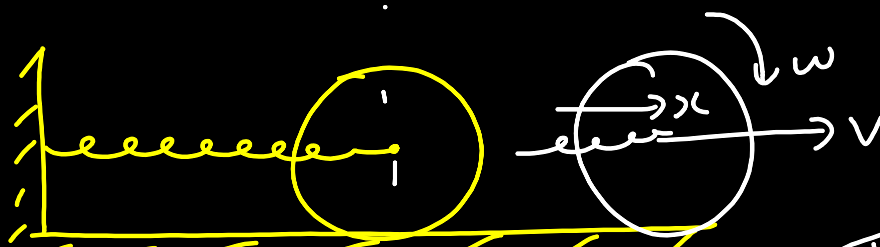
Displace by small amount x

Find $TE = KE + PE$

$\dfrac{d(TE)}{dx} = 0$

or $\dfrac{d(TE)}{dt} = 0.$

Unacademy Atoms

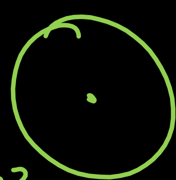


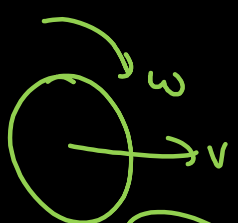
Disc

Pure rolling

SHM

Disc





$I = \frac{mr^2}{2}$

$v = r\omega$

$\omega = v/r$

$$TE = \frac{1}{2}kx^2 + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2.$$

$$TE = \frac{1}{2}kx^2 + \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{mr^2}{2}\right)\left(\frac{v}{r}\right)^2$$

$$\frac{1}{2}kx^2 + \frac{1}{2}mv^2 + \frac{1}{2}\,\frac{m\cancel{r^2}}{2}\,\frac{v^2}{\cancel{r^2}}$$

$$\frac{1}{2}kx^2 + \frac{1}{2}mv^2 + \frac{1}{4}mv^2$$

$$TE = \frac{1}{2}kx^2 + \frac{3}{4}mv^2$$

$$TE = \frac{1}{2}kx^2 + \frac{3}{4}mv^2$$

$$\frac{d(TE)}{dx} = \frac{1}{2}k(2x) + \frac{3}{4}m\left(2v\frac{dv}{dx}\right)$$

$$= kx + \frac{3}{2}m\,v\frac{dv}{dx}$$

$$0 = kx + \frac{3ma}{2}$$

$$a = -\omega^2 x$$

$$a = -\frac{2k}{3m}x$$

$$\omega = \sqrt{\frac{2k}{3m}}$$

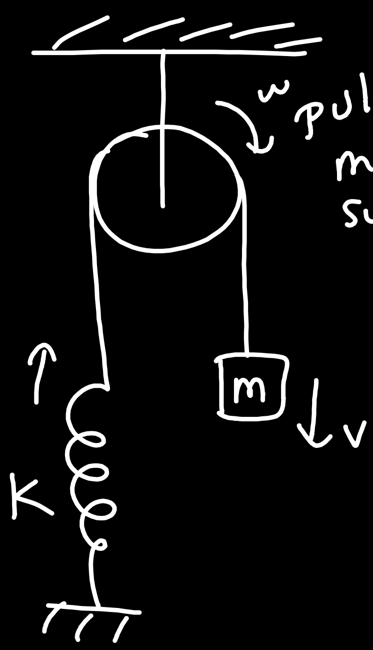
$$T = 2\pi\sqrt{\frac{3m}{2k}}$$

v^2

diff

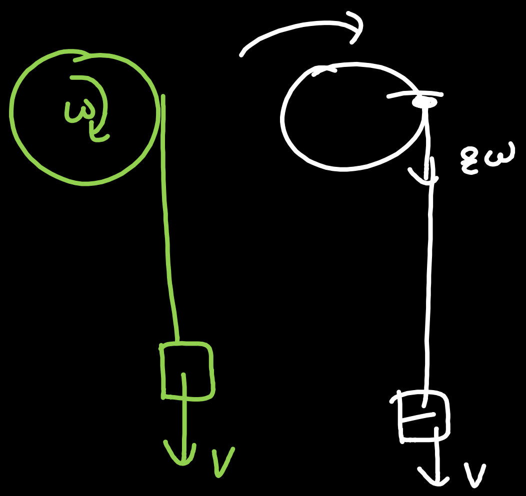
$$2v\frac{dv}{dx}$$

Q



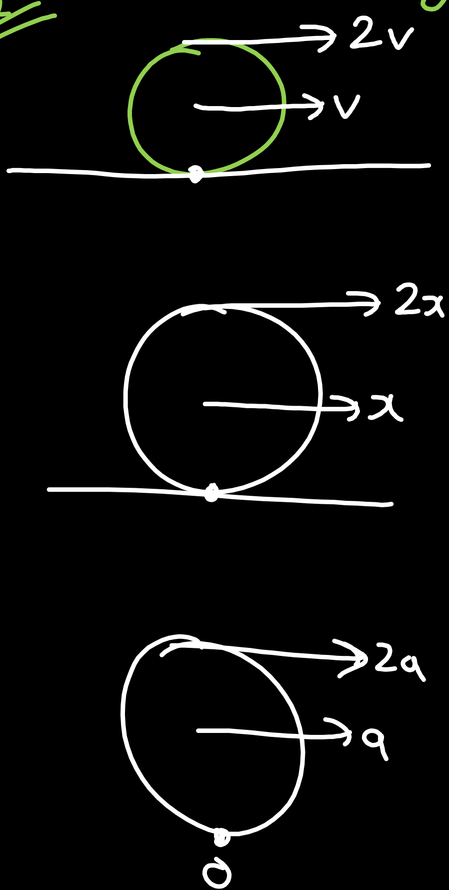
$$TE = \underbrace{\frac{1}{2}kx^2}_{\text{spring}} + \underbrace{\frac{1}{2}mv^2}_{\text{Block}} + \underbrace{\frac{1}{2}I\omega^2}_{\text{pulley}}$$

$$T = 2\pi\sqrt{\frac{3m}{2k}}$$

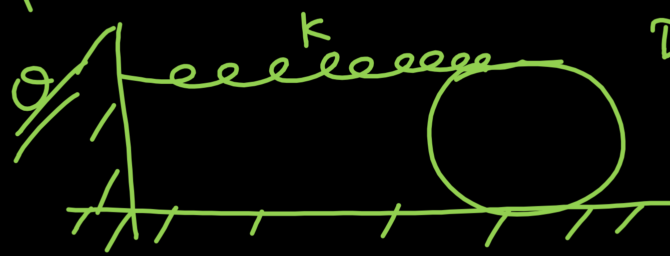


$$V = r\omega$$

Q. Rolling on ground

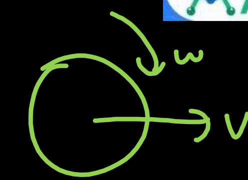


Q.



Disc
m, R

$I = \frac{mR^2}{2}$



$v = R\omega$ $\quad \omega = v/R$

$$TE = \underbrace{\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2}_{} + \frac{1}{2}k(2x)^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}\frac{mR^2}{2}\frac{v^2}{R^2} + \frac{1}{2}k4x^2$$

$$\left(\frac{1}{2} + \frac{1}{4}\right)mv^2$$

$$TE = \frac{3}{4}mv^2 + 2kx^2$$

$$TE = \frac{3}{4}mv^2 + 2kx^2$$

$$\frac{d(TE)}{dx} = \frac{3}{4}m\left(2v\frac{dv}{dx}\right) + 2K(2x)$$

$$0 = \frac{3}{2}ma + 4kx$$

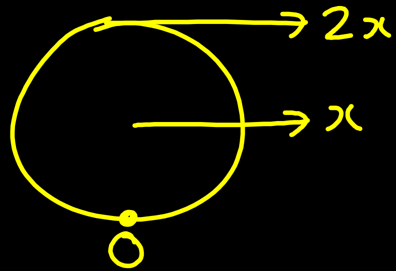
$$a = -\frac{8K}{3m}x \qquad = -\omega^2 x$$

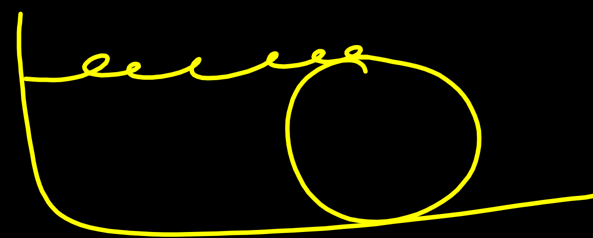
$$\omega = \sqrt{\quad}$$

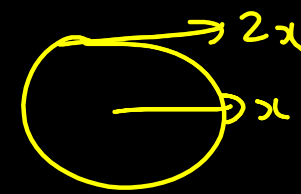
$$T = \frac{2\pi}{\omega}$$

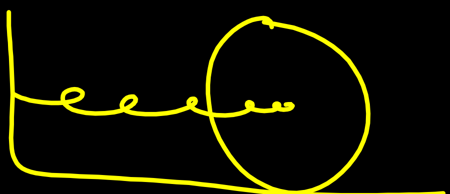
$$T = 2\pi\sqrt{\frac{3m}{8K}}$$

Pure Rolling









$\frac{1}{2} k x^2$

Superposition of SHM's along same axis

$y_1 = A_1 \sin(\omega t)$

$y_2 = A_2 \sin(\omega t + \phi)$

Same freq

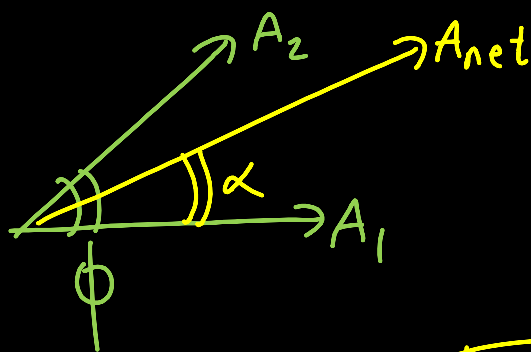
Same axis

Result motion $\Rightarrow$ SHM of same freq of A_{new}.

$y_{net} = y_1 + y_2$

$= A_1 \sin(\omega t) + A_2 \sin(\omega t + \phi)$

$y_{net} = A_{net} \sin(\omega t + \alpha)$



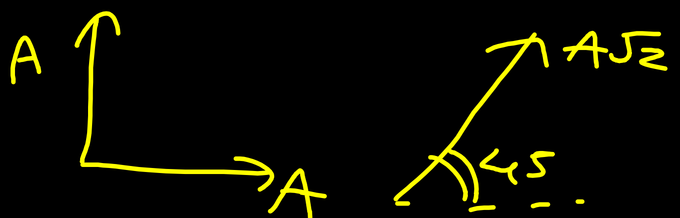
$$A_{net} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$$

$$\tan\alpha = \frac{A_2 \sin\phi}{A_1 + A_2\cos\phi}$$

Q

$$y_1 = A\sin(\omega t)$$

$$y_2 = A\sin(\omega t + 90)$$

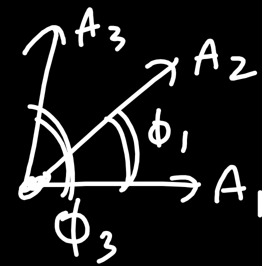


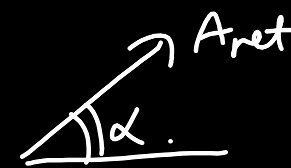
$$y_{net} = (A\sqrt{2})\sin(\omega t + \alpha)$$

$$y_1 = A_1 \sin(\omega t)$$

$$y_2 = A_2 \sin(\omega t + \phi_1)$$

$$y_3 = A_3 \sin(\omega t + \phi_2)$$





$$y_{net} = A_{net} \sin(\omega t + \alpha)$$

Unacademy Atoms

2022

Two massless springs with spring constants 2 K and 9 K , carry 50 g and 100 g masses at their free ends .These two masses oscillate vertically such that their maximum velocities are equal . Then , the ratio of their respective amplitudes will be :

$\omega = \sqrt{\frac{K}{m}}$

$A\omega$

A. 1:2

B. 3:2

C. 3:1

D. 2:3

$A_1 \omega_1 = A_2 \omega_2$

$A_1 \sqrt{\frac{2K}{50}} = A_2 \sqrt{\frac{9K}{100g}}$

$A_1 = A_2 \sqrt{\frac{9}{4}}$

$\frac{A_1}{A_2} = \frac{3}{2}$

2022

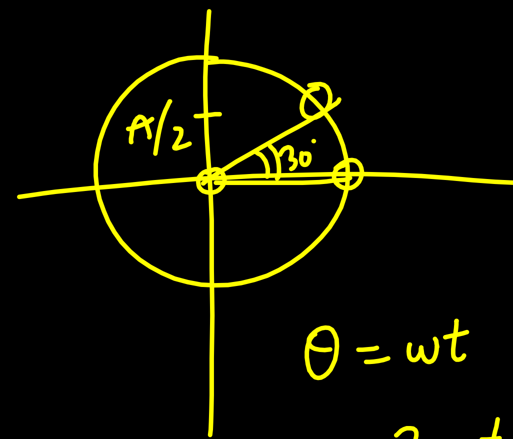
The displacement of simple harmonic oscillator after 3 seconds starting from its Mean position is equal to half of its amplitude . The time period of harmonic motion is :

$x = 0$ to $x = A/2$

$t = 3\ sec$

A. 6 s
B. 8 s
C. 12 s
~~D. 36 s~~

$\frac{T}{12} = 3$



$\theta = \omega t$

$\frac{\pi}{6} = \frac{2\pi}{T} t$

$\frac{T}{12} = t$

Time period of a simple pendulum in a stationary lift is 'T'. If the lift accelerates with $\frac{g}{6}$ vertically upwards then the time period will be :

(Where g=acceleration due to gravity)

A $\sqrt{\frac{6}{5}}\,T$

B $\sqrt{\frac{5}{6}}\,T$

C $\sqrt{\frac{6}{7}}\,T$

D $\sqrt{\frac{7}{6}}\,T$

$\uparrow a$

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$T' = 2\pi\sqrt{\frac{l}{g+g/6}} = \sqrt{\frac{6}{7}}\,2\pi\sqrt{\frac{l}{g}}$$

$$= \sqrt{\frac{6}{7}}\,T$$

2022

The equation of a particle executing simple harmonic motion is given by $x = \sin \pi\left(t+\frac{1}{3}\right) m$. At $t = 1$ s, the speed of particle will be (Given: $\pi = 3.14$) -

A. 0 cm s^{-1}

B. 157 cm s^{-1}

C. 272 cm s^{-1}

D. 314 cm s^{-1}

$x = \sin\left(\pi t + \frac{\pi}{3}\right)$

$v = \cos\left(\pi t + \frac{\pi}{3}\right) \times \pi$

$v = \pi \cos\left(\pi t + \frac{\pi}{3}\right)$

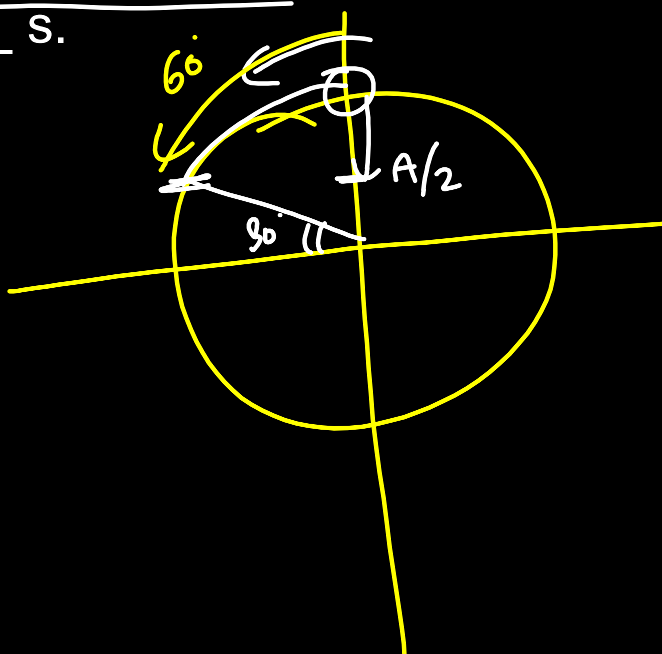
$\pi \cos\left(\pi + \frac{\pi}{3}\right) = -\pi \cos 60 = -\frac{\pi}{2} = -\frac{3.14}{2}$

$\cos(180+60)$

$= -1.57 \, m/s$

2022

A particle executes simple harmonic motion . Its amplitude is 8 cm and time period is 6 s. The time it will take to travel from its position of maximum displacement to the point corresponding to half of its amplitude is ___1 sec___ s.



$$\theta = \omega t$$

$$\frac{\pi}{3} = \frac{2\pi}{T} t$$

$$T/6 = t$$

$$T = 6$$

Motion of a particle in $x - y$ plane is described by a set of following equations $x = 4\sin\left(\frac{\pi}{2} - \omega t\right)$ m and $y = 4\sin(\omega t)$m. The path of the particle will be :

cos θ   sin θ

A. Circular.

B. Helical .

C. Parabolic.

D. Elliptical.

Superposition of two SHM's along x & y axis

Find ~~locus~~ / relation b/w x & y

$\sin\theta = x/A$

① $x = A\sin\omega t$

$y = A\sin\omega t$

$x = y$

St. line

② $x = A\sin(\omega t)$

$y = A\cos(\omega t)$

$\sin^2 + \cos^2 = 1$

$$\frac{x^2}{A^2} + \frac{y^2}{A^2} = 1$$

$x^2 + y^2 = A^2$

circle

③ $x = A\sin(\omega t + \phi)$

$y = A\sin(\omega t)$

ellipse

$x = A_1 \sin(\omega t)$

$y = A_2 \cos(\omega t)$

2022

A body is performing simple harmonic with an amplitude of 10 cm. The velocity of the body was tripled by air Jet when it is at 5 cm from its mean position . The new amplitude of vibration is $\sqrt{x}$ cm . The value of x is ________.

mechanics use

AT24

$A = 10cm$

$\omega = \sqrt{\frac{k}{m}}$

$V = \omega\sqrt{A^2 - x^2}$

$V = \omega\sqrt{100 - 25}$

$V_{initi} = \omega\sqrt{75}$

$V_{final} = \omega\sqrt{A'^2 - x^2}$

$3\cancel{\omega}\sqrt{75} = \cancel{\omega}\sqrt{A'^2 - (5)^2}$

$9 \times 75 = A'^2 - 25$

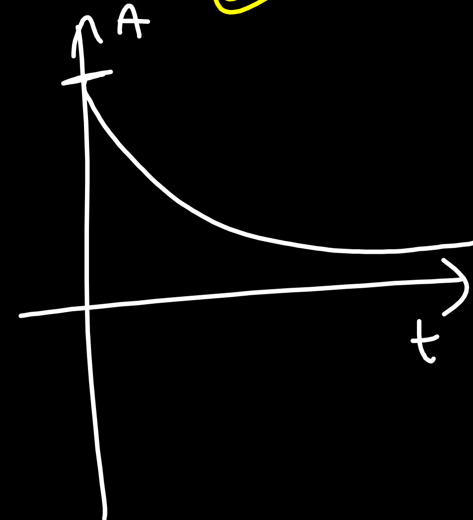
$\sqrt{700} = A'$

A damped harmonic oscillator has a frequency of 5 oscillations per second. The amplitude drops to half its value for every 10 oscillations. The time it will take to drop to $\frac{1}{1000}$ of the original amplitude is close to **(2019 Main)**

(a) 20 s (b) 50 s

(c) 100 s (d) 10 s

$$\boxed{A = A_0 e^{-kt}}$$



$$A \xrightarrow[\text{10 oscillation}]{} \frac{A}{2}$$

$$A \longrightarrow \frac{A}{1000}$$

exponential decrease

Half life $A \xrightarrow{t_{1/2}} \frac{A}{2} \xrightarrow{t_{1/2}} \frac{A}{4} \xrightarrow{t_{1/2}} \frac{A}{8}$

$$A \xrightarrow{t_{1/3}} \frac{A}{3} \xrightarrow{t_{1/3}} \frac{A}{9} \xrightarrow{t_{1/3}} \frac{A}{27}$$

freq = 5

$T = \frac{1}{5}$

1 oscillation $\frac{1}{5}$ sec

10 oscillation 2 sec

$A \longrightarrow \frac{A}{2}$

$t = 2$ sec

$A_f = A_0 e^{-kt}$

$\frac{A}{2} = A e^{-k2}$

$\frac{A}{1000} = A e^{-kt}$

Divide

$\frac{A/2}{A/1000} = \frac{e^{-k2}}{e^{-kt}}$

$500 = e^{k(t-2)}$

$\underline{\underline{\ln(500) = k(t-2)}}$

$$\frac{A}{2} = A e^{-k2}$$

$$\frac{1}{2} = e^{-2k}$$

$$(2)^{-1} = e^{-k2}$$

$$\ln 2 = 2k$$

$$\frac{1}{2}\ln(2) = k$$

$$\frac{A}{1000} = A e^{-kt}$$

$$(10)^{-3} = e^{-kt}$$

$$-3\ln(10) = -kt$$

$$3\ln(10) = \frac{1}{2}\ln(2)\, t$$

$$\frac{6\ln(10)}{\ln(2)} = t$$

$$\approx 20^{sec}$$