CHAPTER

17 Simple Harmonic Motion

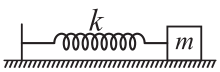
Simple Harmonic Motion

$F = -kx$

General equation of SHM is $x = A\sin(\omega t + \phi)$; $(\omega t + \phi)$ is phase of the motion and ϕ is initial phase of the motion.

$$\omega = \frac{2\pi}{T} = 2\pi f$$

Time period: $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$



Speed : $v = \omega\sqrt{A^2 - x^2}$

Acceleration : $a = -\omega^2 x$

Energy in SHM

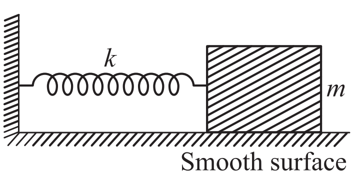
Kinetic Energy: KE = $\frac{1}{2}mv^2 = \frac{1}{2}m\omega^2\left(A^2 - x^2\right) = \frac{1}{2}k\left(A^2 - x^2\right)$

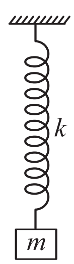
Potential Energy: PE = $\frac{1}{2}kx^2$

Total Mechanical Energy:

$$E = \text{KE} + \text{PE} = \frac{1}{2}k(A^2 - x^2) + \frac{1}{2}Kx^2 = \frac{1}{2}KA^2 = \text{constant}$$

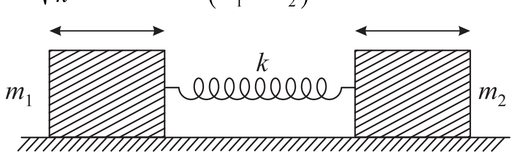
Spring-Mass System

❖ 



Time period, $T = 2\pi\sqrt{\frac{m}{k}}$

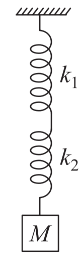
❖ $T = 2\pi\sqrt{\frac{\mu}{k}}$,where $\mu = \frac{m_1 m_2}{(m_1 + m_2)}$ is known as reduced mass.



❖ Series combination

$$k_s = \frac{k_1 k_2}{k_1 + k_2}$$

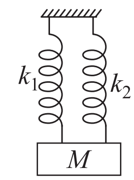
$$T = 2\pi\sqrt{\frac{M}{k_s}}$$



❖ Parallel combination,

$$k_p = k_1 + k_2$$

$$T = 2\pi\sqrt{\frac{M}{k_p}}$$



Simple pendulum

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$= 2\pi\sqrt{\frac{l}{g_{eff.}}}$$ (in accelerating Reference Frame)

where g_{eff} is net acceleration due to psuedo force and gravitational force.

Compound Pendulum/Physical Pendulum

$$T = 2\pi\sqrt{\frac{I}{mgl}}$$

where, $I = I_{cm} + ml^2$; l is distance between point of suspension and centre of mass.

Torsional Pendulum

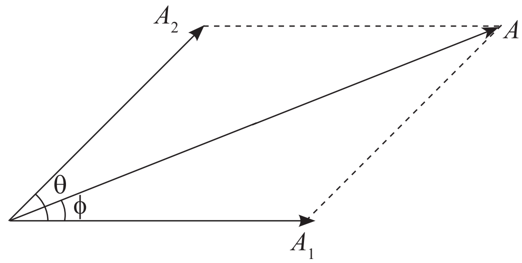
$T = 2\pi\sqrt{\frac{I}{C}}$ where, C = Torsional constant

Superposition of Two SHMs Along the Same Direction

$x_1 = A_1 \sin \omega t$ and $x_2 = A_2 \sin(\omega t + \theta)$

If equation of resultant SHM is taken as $x = A\sin(\omega t + \phi)$, then

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\theta} \quad \text{and} \quad \tan\phi = \frac{A_2\sin\theta}{A_1 + A_2\cos\theta}$$



Small Oscillations

$$\omega = \sqrt{\frac{U''(x_0)}{m}}$$

where $U''(x_0)$ is second derivative of potential at the point x_0 i.e. the point of stable equilibrium

Damped Harmonic Oscillations

If the damping force is given by $\vec{F}_d = -b\vec{v}$, where $\vec{v}$ is the velocity of the oscillator and b is a damping constant, then the displacement of the oscillator is given by

$$x(t) = A_0 e^{-bt/2m} \cos(\omega' t + \phi),$$

where ω' is the angular frequency of the damped oscillator, is given as $\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$

If the damping constant is small $\left(b \ll \sqrt{km}\right)$, then $\omega' \approx \omega$, where ω is the angular frequency of the undamped oscillator.

For small b, the mechanical energy E of the oscillator is given by

$$E(t) = \frac{1}{2} k A_0^2 e^{-bt/m}.$$

Forced Oscillations and Resonance

If an external driving force with angular frequency ω_d acts on an oscillating system with natural angular frequency ω_0, the system oscillates with angular frequency ω_d. The velocity amplitude v_m of the system is greatest when

$$\omega_d = \omega,$$

a condition called **resonance**. The amplitude A_0 of the system is (approximately) greatest under this condition.