Vector and Scalar Quantities

Physical Quantities

Scalar

➔ Only magnitude no direction

1. Mass ✓
2. Volume ✓
3. Density ✓
4. Speed ✓

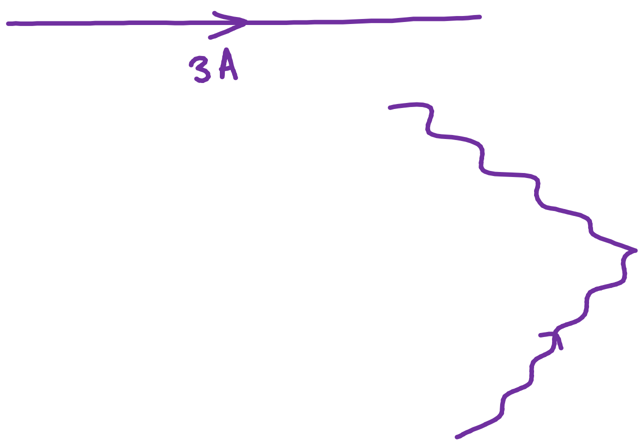
Vector

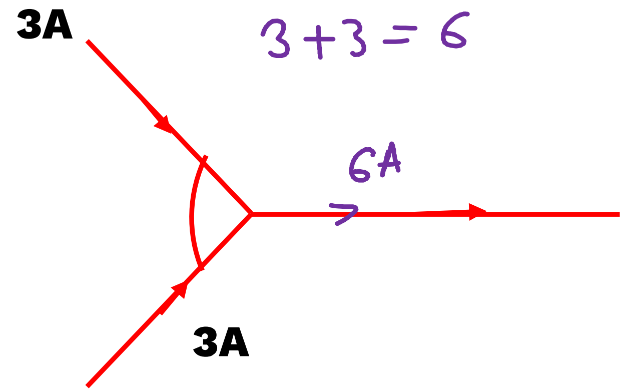
➔ Magnitude as well as direction and OBEYs vector law of algebra

1. Force
2. Velocity
3. Displacement

is current a vector Quantity?

No.



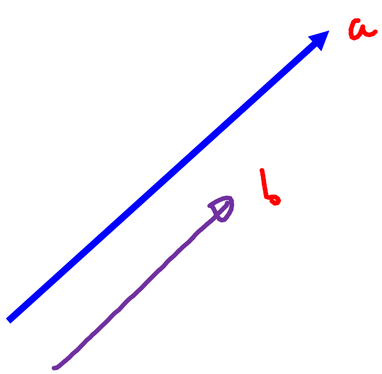


Current

Notation and Representation of Vectors

Vectors are represented by $\vec{a}, \vec{b}, \vec{c}$ and their magnitude (modulus) are represented by a, b, c or $|\vec{a}|, |\vec{b}|, |\vec{c}|$.

Length of arrow ∝ magnitude

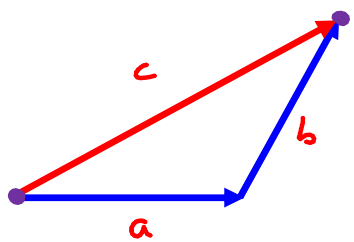


$\vec{a}$ $|\vec{a}|$

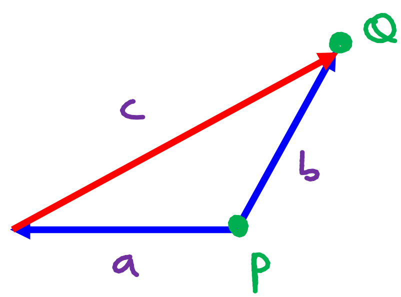
Laws of Vector Addition

Addition of Vectors

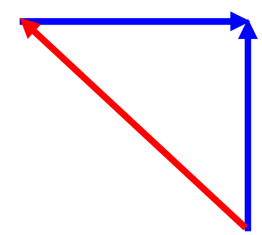
Triangle law of addition



$$\vec{c} = \vec{a} + \vec{b}$$

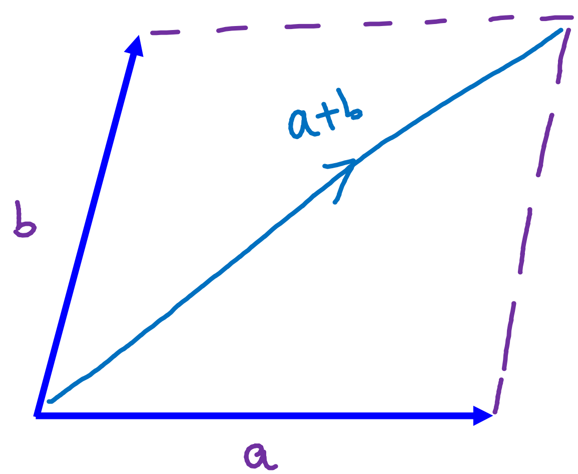


$$\boxed{\vec{b} = \vec{a} + \vec{c}}$$



Addition of Vectors

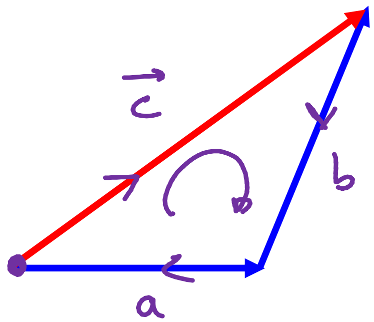
Parallelogram law of addition



Addition of Vectors

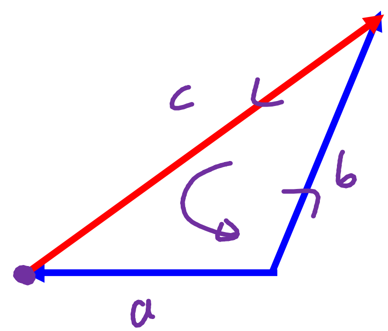
Loop law of addition

#NVStyle $+\vec{a}-\vec{b}+\vec{c}=0$



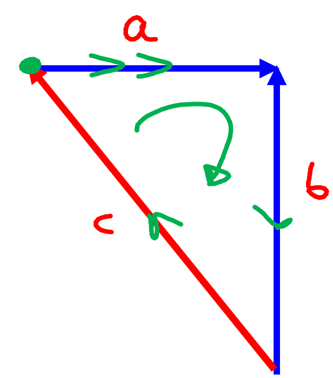
$+\vec{c}-\vec{b}-\vec{a}=0$

$$\boxed{\vec{c}=\vec{a}+\vec{b}}$$



$-a+b-c=0$

$\underline{b=a+c}$



Addition of Vectors

Loop law of addition

A. $\vec{x} = \vec{a} - \vec{b} + \vec{c} - \vec{d}$

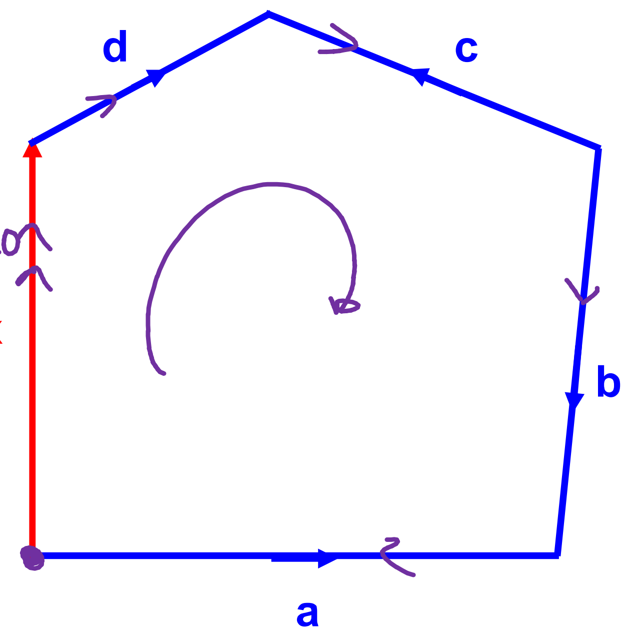
B. $\vec{x} = \vec{a} + \vec{b} + \vec{c} + \vec{d}$

C. $\vec{x} = \vec{a} - \vec{b} - \vec{c} - \vec{d}$

D. $\vec{x} = \vec{a} - \vec{b} + \vec{c} + \vec{d}$

$+\vec{x} + \vec{d} - \vec{c} + \vec{b} - \vec{a} = 0$

$x = a - b + c - d$



Addition of Vectors

Loop law of addition

A. $\vec{x} = \vec{a} + \vec{b} - \vec{c}$

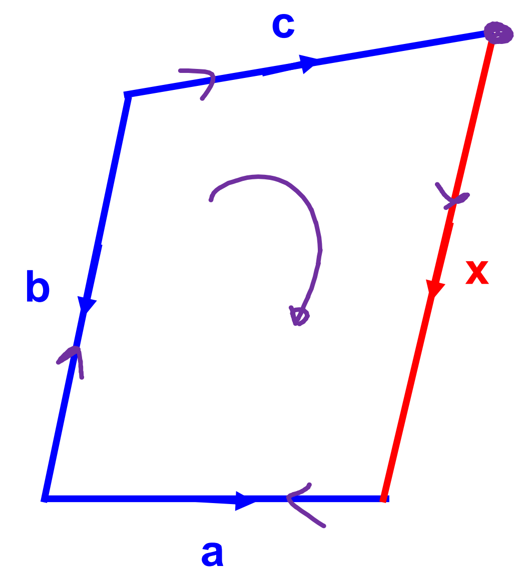
B. $\vec{x} = -\vec{a} + \vec{b} - \vec{c}$

C. $\vec{x} = \vec{a} - \vec{b} - \vec{c}$

D. $\vec{x} = -\vec{a} - \vec{b} + \vec{c}$

$$\vec{x} - \vec{a} - \vec{b} + \vec{c} = 0$$

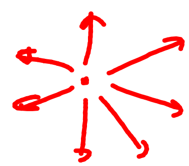
$$\underline{\vec{x} = \vec{a} + \vec{b} - \vec{c}}$$



Types of Vectors

Kinds of Vectors

1 Zero or null vector



Magnitude = 0

Direction = Any arbitrary direction or ~~No direction~~

Kinds of Vectors

2 Unit vector

Magnitude = 1

Direction = ✓

Unit vector in direction of $\vec{a}$ = $\boxed{\hat{a} = \dfrac{\vec{a}}{|\vec{a}|}}$

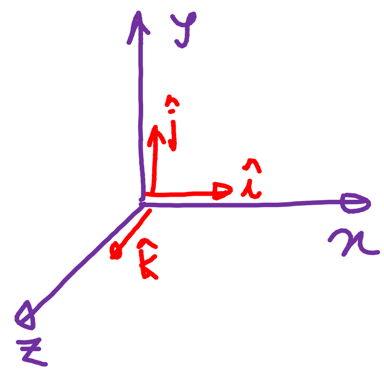
Kinds of Vectors

3 Unit vector

Unit vector in direction of x axis = $\hat{i}$

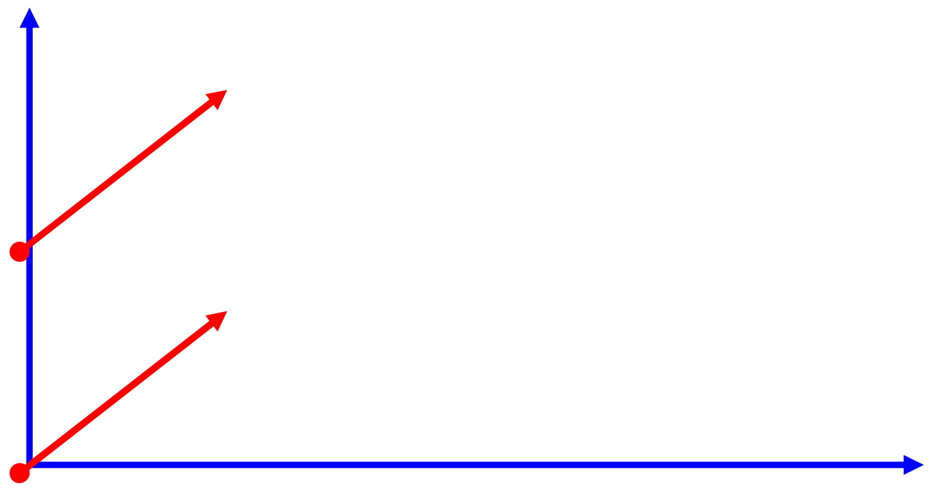
Unit vector in direction of y axis = $\hat{j}$

Unit vector in direction of z axis = $\hat{k}$



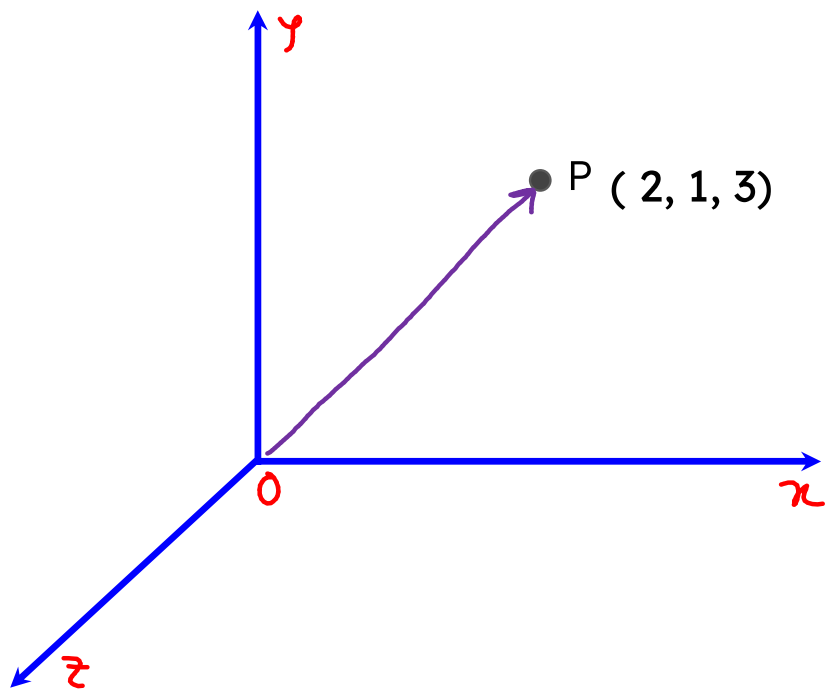
Kinds of Vectors

4 Free vectors



Position Vectors

Position vector of point P is $\overrightarrow{OP} = 2\hat{i} + \hat{j} + 3\hat{k}$



Address of Point

3D

Cartesian System	Vector System (VS)
$P(2,1,3)$	$2\hat{i}+\hat{j}+3\hat{k}$

Position Vectors

P.V of point A = $\vec{OA} = 2\hat{i} + \hat{j} + 0\hat{k}$

P.V of point B = $\vec{OB} = 3\hat{i} + 2\hat{j} + \hat{k}$

$\vec{AB} = \vec{OB} - \vec{OA}$

$\vec{AB}$ = p.v. of B − p.v. of A

$\uparrow \uparrow = \underline{\hat{i} + \hat{j} + \hat{k}}$

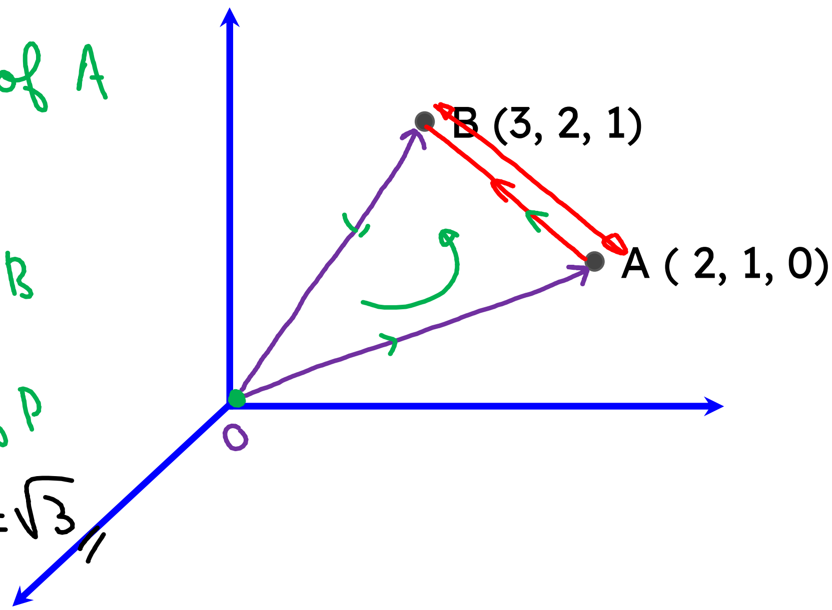
$\vec{BA}$ = p.v. of A − p.v. of B

$\vec{PQ}$ = p.v. of Q − p.v. of P

$|\vec{AB}| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$

$$\boxed{\vec{AB} = \vec{OB} - \vec{OA}}$$

$\vec{OA} + \vec{AB} - \vec{OB} = 0$



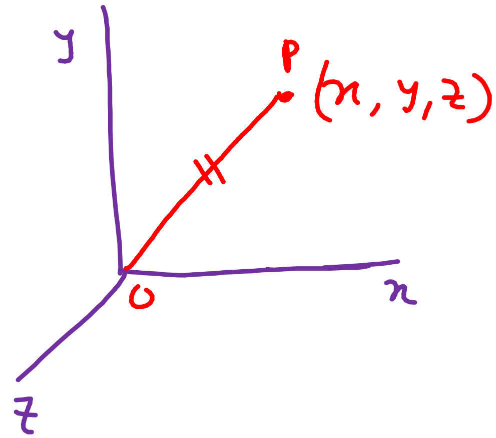
Magnitude of Vectors

distance formula = $|\vec{AB}|$

$$\vec{v} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\boxed{|\vec{v}| = \sqrt{x^2+y^2+z^2}}$$

OP



$$\vec{b} = 2\hat{i} + \hat{j} + 2\hat{k}$$

$$|b| = \sqrt{2^2+1^2+2^2}$$

$$= 3$$

Multiplication of a Vector by Scalar

Multiplication of a Vector by Scalar

If $\boxed{\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}}$

Then $k\vec{a} = (kx)\hat{i} + (ky)\hat{j} + (kz)\hat{k}$

$$2(2\hat{i} + \hat{j} + 3\hat{k}) = \underline{4\hat{i} + 2\hat{j} + 6\hat{k}}$$

Properties of scalar multiplication

1. $k(\vec{a}+\vec{b}) = k\vec{a} + k\vec{b}$

2. $(k+l)\vec{a} = k\vec{a} + l\vec{a}$

$2(\hat{i}+\hat{j}) = 2\hat{i} + 2\hat{j}$

$(2+3)\hat{i} = 2\hat{i} + 3\hat{i}$

$= \underline{\underline{5\hat{i}}}$

① $k(\vec{a}+\vec{b}) = k\vec{a} + k\vec{b}$ ✓

② $(k+l)\vec{a} = k\vec{a} + l\vec{a}$

Collinear Vectors

Angle between two vectors

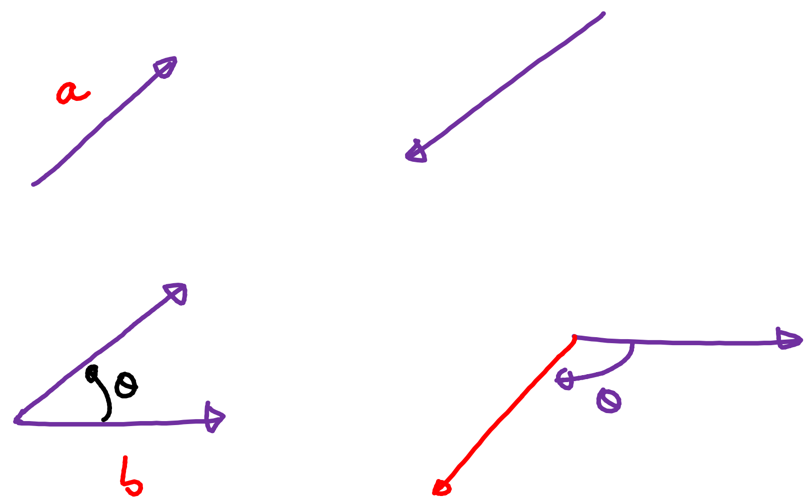
Note: Tail to tail should be connected

$$\boxed{0^\circ \le \theta \le 180^\circ}$$

$\theta \rightarrow$ 1st / 2nd

~~3rd~~ / ~~4th~~

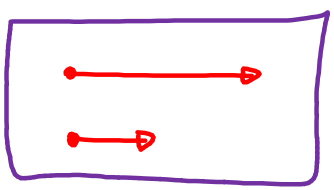
$\sin\theta = \frac{1}{2}$



Collinear Vectors or Parallel Vectors

1 If angle between 2 vectors is either 0° or 180°

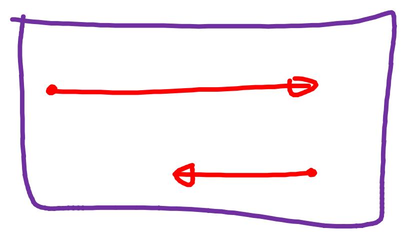
Like



$\theta = 0^\circ$

Ha !

Unlike



$\theta = 180^\circ$

Ha !

Collinear Vectors

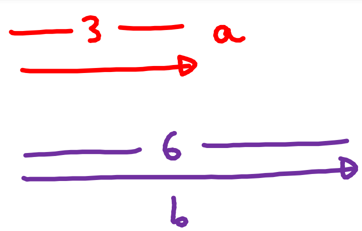
Collinear Vectors

- **Like Vectors** ✓ $\theta = 0^\circ$
- **Unlike Vectors** ✓ $\theta = 180^\circ$

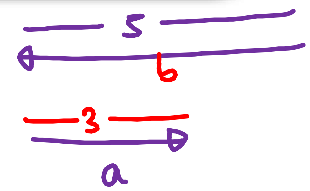
Collinear Vectors or Parallel Vectors

NOTE: If two vectors are parallel they are proportional

NOTE: If two vectors are collinear then a = λ b



$\vec{b} = +2\vec{a}$



$$\boxed{\vec{b} = -\frac{5}{3}\vec{a}}$$

The value of λ when $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$ and $\vec{b} = 8\hat{i} + \lambda\hat{j} + 4\hat{k}$ are parallel is -

A. 4

B. -6

C. -12

D. 1

$\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$

$\vec{b} = 8\hat{i} + \lambda\hat{j} + 4\hat{k}$

$$\boxed{\frac{2}{8} = \frac{-3}{\lambda} = \frac{1}{4}}$$

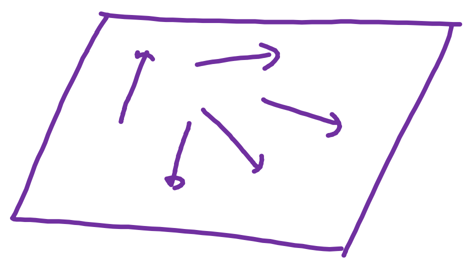
$\vec{a} \parallel \vec{b} \Rightarrow$ proportional

$\Rightarrow$ ~~Cross~~

Coplanar Vectors

Coplanar Vectors

Vectors lying in same plane



Note:

1. Two vectors are always coplanar

2. 3 or more vectors may not be coplanar

Dot Product

Dot Product or Scalar Product

Let $\vec{a}$ and $\vec{b}$ be two non-zero vectors and θ the angle between them then its scalar product is denoted as $\vec{a}\,.\,\vec{b}$ and is defined as

$$\boxed{\vec{a}\,.\,\vec{b} = |\vec{a}|\,|\vec{b}|\cos\theta}$$

$$\boxed{\theta = \vec{a} \wedge \vec{b}}$$

Dot Product or Scalar Product

1. $\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$

2. $\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$

$$\hat{i} \cdot \hat{i} = |\hat{i}||\hat{i}| \cos(0^\circ)$$
$$= 1 \times 1 \times 1$$
$$= 1$$

$$\hat{i} \cdot \hat{j} = |\hat{i}|\,|\hat{j}| \cos 90^\circ$$
$$= 1 \times 1 \times 0$$
$$= 0$$

Dot Product or Scalar Product

If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ then $\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3$

$$\boxed{\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3}$$

If $\vec{a} = 3i + 2j + k$ and $\vec{b} = i - 2j + 5k$ then find $\vec{a}.\vec{b}$.

A. 4

B. 5

C. 3

D. -3

$$\vec{a}.\vec{b} = 3 - 4 + 5$$

$$= 4$$

$$\vec{b}.\vec{a} = 3 - 4 + 5 = 4$$

Properties of Dot Product

Properties of Dot Product

1 $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$

$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$

Properties of Dot Product

Unacademy Atoms

2 $\vec{a} \cdot \vec{a} = |\vec{a}|^2$

$$\boxed{\vec{a}^2 = \vec{a} \cdot \vec{a} = |a|^2}$$

$$\vec{a} \cdot \vec{a} = |a|\,|\vec{a}| \cos(0^\circ)$$
$$= \underline{\underline{|a|^2}}$$

a

$$(\vec{a}+\vec{b})^2 = (\vec{a}+\vec{b}) \cdot (\vec{a}+\vec{b})$$
$$|a+b|^2$$

Properties of Dot Product

3 $\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$

$\vec{a} \cdot (\vec{b} \pm \vec{c}) = \vec{a} \cdot \vec{b} \pm \vec{a} \cdot \vec{c}$

Properties of Dot Product

4 $-|\vec{a}|\,|\vec{b}| \le \vec{a}\cdot\vec{b} \le |\vec{a}|\,|\vec{b}|$

$$\vec{a}\cdot\vec{b} = |a|\,|b|\,\underline{\cos\theta}$$

$$(\vec{a}\cdot\vec{b})_{max} = |a||b|$$

$$(\vec{a}\cdot\vec{b})_{min} = -|a|\,|b|$$

Properties of Dot Product

5

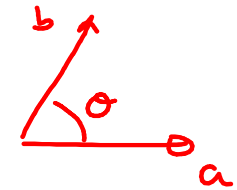
$\theta \rightarrow 1^{st}$

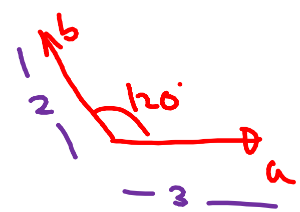
$$\vec{a} \cdot \vec{b} = \begin{cases} + & \text{If } \theta \in [0, \pi/2) \\ - & \text{If } \theta \in (\pi/2, \pi] \\ 0 & \text{If } \theta = \pi/2 \end{cases}$$

$\boxed{0 \le \theta \le \pi}$

$\vec{a} \cdot \vec{b} = |a|\,|b| \cos\theta$

$\oplus \quad \oplus$





$\vec{a} \cdot \vec{b} = |a|\,|b| \cos 120^\circ$

$= 3 \times 2 \times (-\cos 60^\circ)$

Properties of Dot Product

6 $\vec{0} \cdot \vec{a} = 0$

$$\boxed{\vec{0} \cdot \vec{a} = 0}$$

Zero Vector non zero Vector

$\underline{3} \cdot \vec{a} \Rightarrow \times$

Properties of Dot Product

7 Angle between 2 vectors a and b $\cos\theta = \vec{a}.\vec{b}/|\vec{a}||\vec{b}|$

$$\vec{a}.\vec{b} = |a|\,|b| \cos\theta$$

$$\boxed{\cos\theta = \frac{\vec{a}.\vec{b}}{|a||b|}}$$

Properties of Dot Product

8 If $\vec{a} \perp \vec{b}$ then $\vec{a} \cdot \vec{b} = 0$ but if $\vec{a} \cdot \vec{b} = 0$ then either $a = 0$ or $\vec{b} = 0$ or $\theta = 90°$

* $a \perp b \Longrightarrow \vec{a} \cdot \vec{b} = 0$
* $\vec{a} \cdot \vec{b} = 0 \Longrightarrow \vec{a} \perp \vec{b}$

  $\Longrightarrow \vec{a} = \vec{0}$

  $\Longrightarrow \vec{b} = \vec{0}$

Properties of Dot Product

9

If $\vec{b} = \vec{c}$ then $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$ but if $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$ then $\vec{a} = 0$ or $\vec{b} = \vec{c}$ or $\vec{a} \perp (\vec{b} - \vec{c})$

★ $\vec{b} = \vec{c} \implies \vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$

★ $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$

$\vec{a} \cdot \vec{b} - \vec{a} \cdot \vec{c} = 0$

$\vec{a} \cdot (\vec{b} - \vec{c}) = 0 \implies \vec{a} = 0$

$\implies \vec{b} - \vec{c} = 0$

$\implies \vec{a} \perp \vec{b} - \vec{c}$

Properties of Dot Product

10 Identities

a. $(\vec{a} + \vec{b})^2 = |\vec{a}|^2 + 2\,\vec{a} \cdot \vec{b} + |\vec{b}|^2$

b. $(\vec{a} - \vec{b})^2 = |\vec{a}|^2 - 2\,\vec{a} \cdot \vec{b} + |\vec{b}|^2$

c. $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = |\vec{a}|^2 - |\vec{b}|^2$

d. $|\vec{a} + \vec{b}| = |\vec{a}| + |\vec{b}| \Rightarrow \vec{a} \parallel \vec{b}$

e. $|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 \Rightarrow \vec{a} \perp \vec{b}$

f. $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}| \Rightarrow \vec{a} \perp \vec{b}$

$$(\vec{a}+\vec{b})^2 = (\vec{a}+\vec{b}).(\vec{a}+\vec{b})$$

$a.b = b.a$

$$= a.a + a.b + b.a + b.b$$

$$= |a|^2 + 2a.b + |b|^2$$

$$(\vec{a}-\vec{b})^2 = |a|^2 - 2a.b + |b|^2$$

$$(a-b).(a+b) = a.a + \cancel{a.b} - \cancel{b.a} - b.b$$

$$= |a|^2 - |b|^2$$

$$|\vec{a}+\vec{b}|^2 = (|a| + |b|)^2 \qquad \text{T.P} \quad a \parallel b$$

$$\left(|\vec{a}|^2 = \vec{a}\cdot\vec{a} \right)$$

$$\underbrace{(\vec{a}+\vec{b}).(\vec{a}+\vec{b})}$$

$$= |a|^2 + 2|a||b| + |b|^2$$

$$\cancel{|a|^2} + \cancel{2}a.b + \cancel{|b|^2} = \cancel{|a|^2} + \cancel{2}|a||b| + \cancel{|b|^2}$$

$$a.b = |a||b|$$

$$\cancel{|a|}\cancel{|b|}\cos\theta = \cancel{|a|}\,\cancel{|b|}$$

$$\cos\theta = 1$$

$$\boxed{\theta = 0^\circ} \qquad \underline{a \parallel b}$$

Questions on Dot Product

Q

Let $\vec{a}, \vec{b}, \vec{c}$ be three mutually perpendicular vectors of the same magnitude and equally inclined at an angle θ, with the vector $\vec{a} + \vec{b} + \vec{c}$. Then $\boxed{36\cos^2 2\theta}$ is equal to __________.

[JEE Main 2021]

Given :- $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{a} = 0$

$|a| = |b| = |c| = K$

$\boxed{\vec{a} \wedge (\vec{a}+\vec{b}+\vec{c}) = \theta}$

$\vec{b} \wedge (\vec{a}+\vec{b}+\vec{c}) = \theta$

$\vec{c} \wedge (\vec{a}+\vec{b}+\vec{c}) = \theta$

$\cos 2\theta = 2\cos^2\theta - 1$

$= 2\left(\frac{1}{3}\right) - 1$

$\boxed{\cos 2\theta = \frac{-1}{3}}$

$36\cos^2 2\theta = 4$

$$\vec{a}\cdot(\vec{a}+\vec{b}+\vec{c}) = |a|\,|a+b+c|\cos\theta$$

$$\cos\theta = \frac{a\cdot(a+b+c)}{|a|\,|a+b+c|} = \frac{|a|^{\cancel{2}}}{\cancel{|a|}\,|a+b+c|} = \frac{|a|}{|a+b+c|} = \frac{\cancel{k}}{\sqrt{3}\,\cancel{k}}$$

$$= \frac{1}{\sqrt{3}}$$

$$|a+b+c|^2 = |a|^2+|b|^2+|c|^2+2(\cancel{a\cdot b}+\cancel{b\cdot c}+\cancel{c\cdot a})$$

$$|a+b+c|^2 = k^2+k^2+k^2$$

$$\boxed{|a+b+c| = \sqrt{3}\,k}$$

Unacademy Atoms

Q

Let $\vec{a}, \vec{b}$ & $\vec{c}$ be three unit vectors such that $|\vec{a} - \vec{b}|^2 + |\vec{a} - \vec{c}|^2 = 8$. Then $|\vec{a} + 2\vec{b}|^2 + |\vec{a} + 2\vec{c}|^2$ is equal to _________.

[JEE Main 2021]

$$|a|^2 + |b|^2 - 2\,a.b + |a|^2 + |c|^2 - 2\,a.c = 8$$

$$4 - 2(a.b + a.c) = 8$$

$$\Rightarrow \boxed{a.b + a.c = -2}$$

$$|a|^2 + 4|b|^2 + 4\,a.b + |a|^2 + 4|c|^2 + 4\,\vec{a}.\vec{c}$$

$$10 + 4(a.b + a.c)$$

$$10 + 4(-2) = 2$$

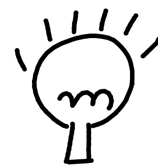
If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors satisfying $|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2 = 9$, then $|2\vec{a} + 5\vec{b} + 5\vec{c}|$ is

[JEE Adv. 2012]

$|a|^2 + |b|^2 - 2\,a\cdot b + |b|^2 + |c|^2 - 2\,b\cdot c + |c|^2 + |a|^2 - 2\,a\cdot c = 9$

$6 - 2(a\cdot b + b\cdot c + c\cdot a) = 9$

$\boxed{a\cdot b + b\cdot c + c\cdot a = -3/2} \Longrightarrow$



chamka

$|a+b+c|^2 = |a|^2 + |b|^2 + |c|^2 + 2(a\cdot b + b\cdot c + c\cdot a)$

$= 3 + 2\left(\frac{-3}{2}\right)$

$= 0$

$$\boxed{\vec{a}+\vec{b}+\vec{c}=\vec{0}}$$

$$\vec{b}+\vec{c}=-\vec{a}$$

$$\left|2\vec{a}+5\vec{b}+5\vec{c}\right|$$

$$\Rightarrow \left|2\vec{a}+5(\vec{b}+\vec{c})\right|$$

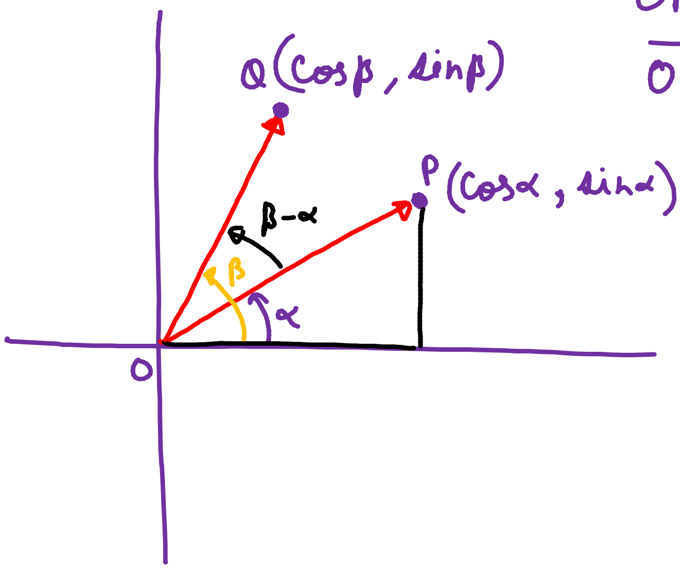
$$\Rightarrow \left|2\vec{a}+5(-\vec{a})\right|$$

$$\Rightarrow \left|-3\vec{a}\right| = 3|a| = 3(1) = 3$$

#Power of Vectors

Prove by vector method the following formula of trigonometry

$\cos(\alpha - \beta) = \cos\alpha \cos\beta + \sin\alpha \sin\beta$



$\vec{OP} = \cos\alpha\,\hat{i} + \sin\alpha\,\hat{j}$

$\vec{OQ} = \cos\beta\,\hat{i} + \sin\beta\,\hat{j}$

$\vec{OP} \cdot \vec{OQ} = |\vec{OP}|\,|\vec{OQ}| \cos(\beta - \alpha)$

$\cos\alpha \cos\beta + \sin\alpha \sin\beta = 1 \times 1 \times \cos(\beta - \alpha)$

Projection of $\vec{a}$ on $\vec{b}$

Projection of $\vec{a}$ on $\vec{b}$

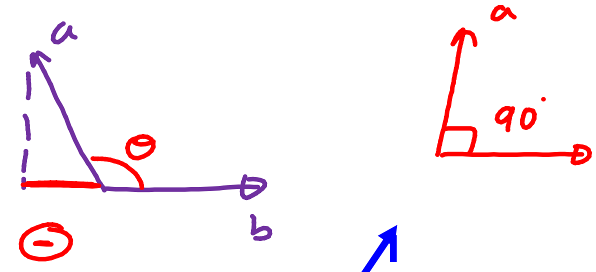
(Scalar)

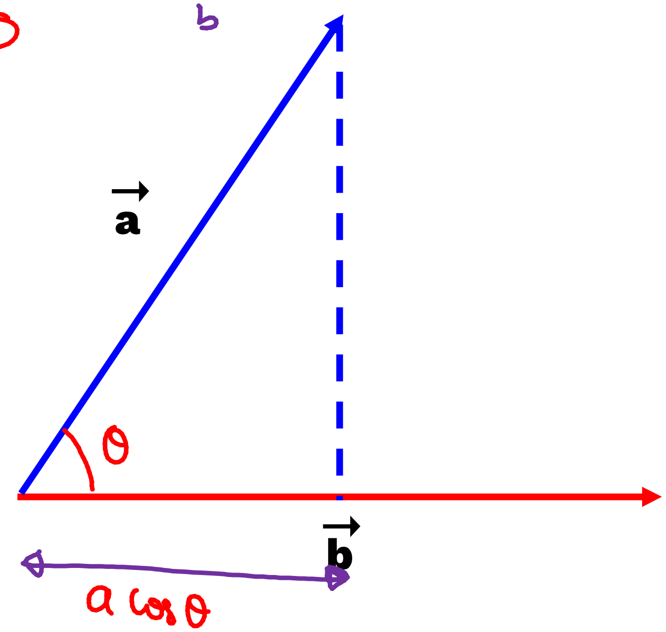
$$\text{Projection of } \vec{a} \text{ on } \boxed{\vec{b}} = \frac{\vec{a}.\vec{b}}{|\vec{b}|}$$

proj. of $\vec{a}$ on $\vec{b}$ $= |a| \cos\theta$

$= \cancel{|a|} \dfrac{\vec{a}.\vec{b}}{\cancel{|a|}\,|b|}$

$= \dfrac{\vec{a}.\vec{b}}{|b|}$





Projection of $\vec{a}$ on $\vec{b}$

$$\text{proj of } \vec{x} \text{ on } \boxed{\vec{y}} = \frac{\vec{x} \cdot \vec{y}}{|y|}$$

$$\text{Proj of } \vec{a}+\vec{b} \text{ on } \boxed{\vec{c}-\vec{d}} = \frac{(a+b)\cdot(c-d)}{|c-d|}$$

The projection of vector $\underbrace{\hat{i} + \hat{j} + \hat{k}}_{a}$ on the vector $\underbrace{\hat{i} - \hat{j} + \hat{k}}_{b}$ is-

A. $\sqrt{3}$

B. $1/\sqrt{3}$

C. $2/\sqrt{3}$

D. $2\sqrt{3}$

$$\frac{(-1, 1, 1) \cdot (1, -1, 1)}{\sqrt{3}} = \frac{-1 \cancel{-1} \cancel{+1}}{\sqrt{3}}$$

$$= \frac{-1}{\sqrt{3}}$$

$$\text{proj of a ON b} = \frac{\vec{a} \cdot \vec{b}}{|b|}$$

$$= \frac{1 - 1 + 1}{\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}}$$

Angle between two vectors

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|a|\,|b|}$$

Find the angle between the vectors $4\hat{i} + \hat{j} + 3\hat{k}$ and $2\hat{i} - 2\hat{j} - \hat{k}$.

$$\cos\theta = \frac{(4, 1, 3) \cdot (2, -2, -1)}{\sqrt{16+1+9}\,\sqrt{4+4+1}}$$

$$\cos\theta = \frac{8-2-3}{\sqrt{26}\,(3)} = \frac{1}{\sqrt{26}}$$

$$\boxed{\theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)}$$

Q

Let the vectors $\vec{a}, \vec{b}, \vec{c}$ be such that $|\vec{a}|=2, |\vec{b}|=4$ and $|\vec{c}|=4$. If the projection of $\vec{b}$ on $\vec{a}$ is equal to the projection of $\vec{c}$ on $\vec{a}$ and $\vec{b}$ is perpendicular to $\vec{c}$, then the value of $|\vec{a}+\vec{b}-\vec{c}|$ is ________.

[JEE Main 2020]

$$* \quad \frac{\vec{b}\cdot\vec{a}}{\cancel{|a|}} = \frac{\vec{c}\cdot\vec{a}}{\cancel{|a|}}$$

$$\Rightarrow \boxed{\vec{b}\cdot\vec{a} = \vec{c}\cdot\vec{a}}$$

$$\Rightarrow \boxed{\vec{b}\cdot\vec{c} = 0}$$

$$|a+b-c|^2 = |a|^2+|b|^2+|c|^2+2(\cancel{a\cdot b}-\cancel{b\cdot c}-\cancel{a\cdot c})$$

$$= 2^2+4^2+4^2$$

$$= 4+16+16$$

$$= 36$$

$$\boxed{|a+b-c| = 6}$$

Unacademy Atoms

$\vec{v} = \underline{x}\,\hat{i} + \underline{y}\,\hat{j} + \underline{z}\,\hat{k}$

$\underline{2\hat{i} + 3\hat{j} + 5\hat{k}}$

Components of Vector

Q

A vector a has components 3p and 1 with respect to a rectangular cartesian system. This system is rotated through a certain angle about the origin in the counter clockwise sense. If, with respect to new system, a has components p + 1 and $\sqrt{10}$, then a value of p is equal to:

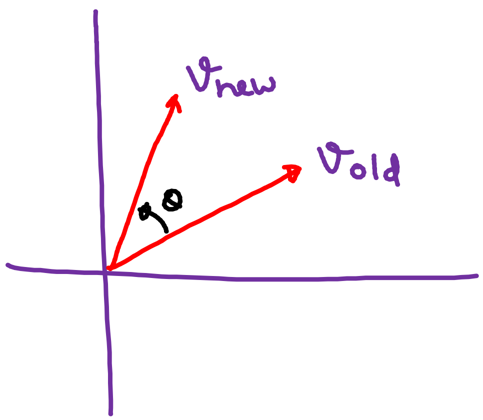
[JEE Main 2021]

(1) 1

(2) $-\frac{5}{4}$

(3) $\frac{4}{5}$

(4) -1 ✓



$$\vec{v}_{old} = 3p\,\hat{i} + 1\,\hat{j} + 0\hat{k}$$

$$\vec{v}_{new} = (p+1)\,\hat{i} + \sqrt{10}\,\hat{j} + 0\hat{k}$$

$$|v_{old}|^2 = |v_{new}|^2$$

$$(3p)^2 + 1^2 = (p+1)^2 + 10$$

$$9p^2 + 1 = p^2 + 2p + 11$$

$$8p^2 - 2p - 10 = 0$$

$$4p^2 - p - 5 = 0$$

$$4p^2 - 5p + 4p - 5 = 0$$

$$p(4p-5) + (4p-5) = 0$$

$$\underline{p = -1 \mid 5/4}$$

Q

Suppose that $\vec{p}, \vec{q}$ and $\vec{r}$ are three non-coplanar vectors in $\mathbb{R}^3$. Let the components of a vector $\vec{s}$ along $\vec{p}, \vec{q}$ and $\vec{r}$ be 4, 3 and 5, respectively. If the components of this vector $\vec{s}$ along $(-\vec{p}+\vec{q}+\vec{r}), (\vec{p}-\vec{q}+\vec{r})$ and $(-\vec{p}-\vec{q}+\vec{r})$ are x, y and z, respectively, then the value of $\boxed{2x+y+z}$ is

[JEE Adv. 2015]

$$\vec{S} = 4\vec{p} + 3\vec{q} + 5\vec{r}$$

$$\vec{S} = x(-p+q+r) + y(p-q+r) + z(-p-q+r)$$

$$= (-x+y-z)p + (x-y-z)q + (x+y+z)\vec{r}$$

$\boxed{x=4}$

$-x+y-z=4$

$x-y-z=3$

$x+y+z=5$

$y-z=8$

$y+z=1$

$y=\frac{9}{2}$

$z=-7/2$

$$2x + y + z = 2(4) + \frac{9}{2} - \frac{7}{2}$$

$$= 8 + 1$$

$$= 9$$

Unacademy Atoms

Linear combination of vectors

Linear combination of vectors

A vector $\vec{r}$ is said to be a linear combination of the vectors $\vec{a}, \vec{b}, \vec{c}$ ....
If ∃ scalars x, y, z, ... Such that $\vec{r} = x\vec{a} + y\vec{b} + z\vec{c} + \ldots\ldots$

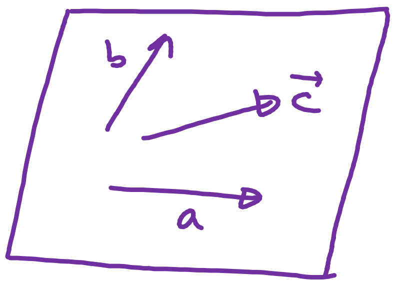
$$\underline{\vec{r} = x\vec{a} + y\vec{b} + z\vec{c} +} \;\ldots\ldots$$

$$\underline{3\vec{a} - 2\vec{b} + 5\vec{c}}$$

Theorem in Plane

Theorem in plane

If three non-zero, non-collinear vectors are **lying in same plane** then any **one vector** can be represented as **linear combination of other two**



$$\boxed{\vec{c} = x\vec{a} + y\vec{b}}$$

$$\boxed{\vec{a} = \lambda\vec{b} + \mu\vec{c}}$$

Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$ be there vectors. A vector $\vec{v}$ in the plane of a and b, whose projection on c is $1/\sqrt{3}$, is given by

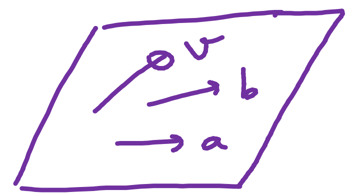
[JEE 2011]

~~A.~~ $1\hat{i} - 3\hat{j} + 3\hat{k}$

~~B.~~ $-3\hat{i} - 3\hat{j} - \hat{k}$

C. $3\hat{i} - \hat{j} + 3\hat{k}$

~~D.~~ $\hat{i} + \hat{j} - 3\hat{k}$



$$\vec{v} = \lambda \vec{a} + \mu \vec{b}$$

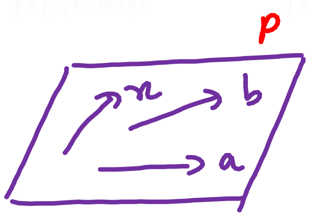
$$\vec{v} = \lambda(1, 1, 1) + \mu(1, -1, 1)$$

$$\vec{v} = (\lambda + \mu, \lambda - \mu, \lambda + \mu)$$

Q

Let $\vec{x}$ be a vector in the plane containing vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$. If the vector $\vec{x}$ is perpendicular to $(3\hat{i} + 2\hat{j} - \hat{k})$ and its projection on $\vec{a}$ is $\frac{17\sqrt{6}}{2}$, then the value of $|\vec{x}|^2$ is equal to ____ .

[JEE Main 2021]



$\vec{x}$

$\hookrightarrow 13^2 + 14^2 + 11^2$

$= \boxed{486}$

$\vec{x} = \lambda \vec{a} + \mu \vec{b}$

$\vec{x} = \lambda(2,-1,1) + \mu(1,2,-1)$

$\vec{a} = (2,-1,1)$

$\vec{x} = (2\overset{8}{\lambda} + \overset{-3}{\mu}, -\overset{8}{\lambda} + 2\overset{-3}{\mu}, \overset{8}{\lambda} - \overset{-3}{\mu}) = (13, -14, 11)$

$|a| = \sqrt{4+1+1}$

$= \sqrt{6}$

$(3, 2, -1)$

$3(2\lambda + \mu) + 2(-\lambda + 2\mu) - 1(\lambda - \mu) = 0$

$\boxed{3\lambda + 8\mu = 0}$ —— (1)

proj of $\vec{x}$ on $\boxed{\vec{a}} = \dfrac{\vec{x}\cdot\vec{a}}{|a|} = \dfrac{17\sqrt{6}}{2}$

$$\frac{2(2\lambda+\mu)-1(-\lambda+2\mu)+1(\lambda-\mu)}{\sqrt{6}} = \frac{17\sqrt{6}}{2}$$

$2(3\lambda+8\mu=0)$ $\quad \boxed{6\lambda-\mu=51}$ — 2

$6\lambda-\mu=51$

$6\lambda+16\mu=0$

$-\quad-\quad-$

$-17\mu=51$

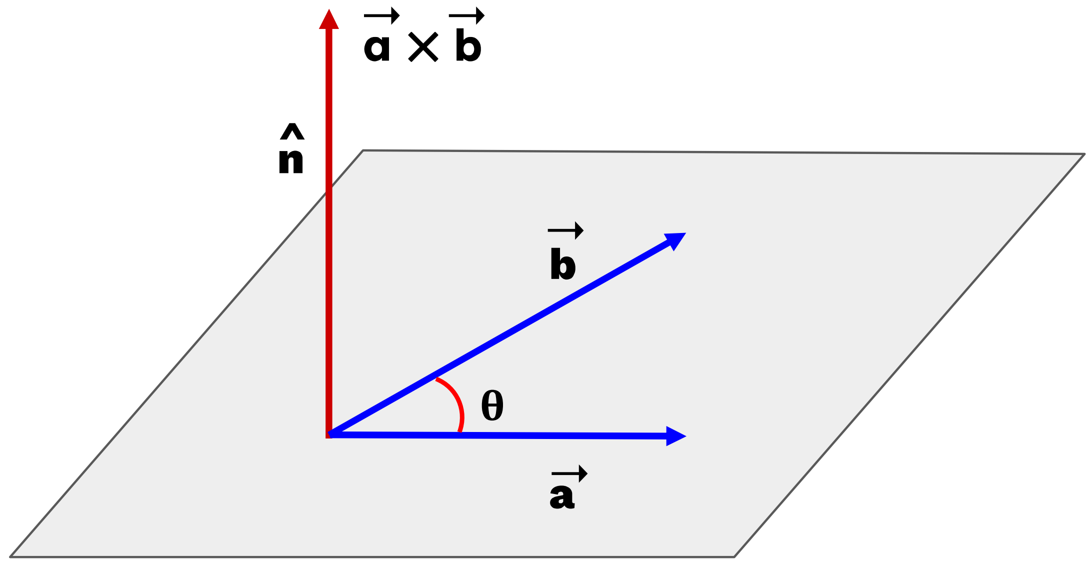
$\boxed{\mu=-3}$ $\quad \lambda=8$

Cross Product

Cross product or Vector product

The vector product or cross product of two vectors $\vec{a}$ and $\vec{b}$ is defined as a vector, written as $\vec{a} \times \vec{b}$ and is defined as

$$\boxed{\vec{a} \times \vec{b} = |\vec{a}|\,|\vec{b}|\sin\theta\,\hat{n}}$$



Properties of Cross Product

Properties of Vector product

1 In general, $\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$. In fact $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$

$\vec{b} \times \vec{a} \neq \vec{a} \times \vec{b}$

$\boxed{\vec{b} \times \vec{a} = -(\vec{a} \times \vec{b})}$

$\underline{\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}}$

Properties of Vector product

2 **For scalar m, $m\vec{a} \times \vec{b} = m(\vec{a} \times \vec{b}) = \vec{a} \times m\vec{b}$.**

$$m(\vec{a} \times \vec{b}) = (m\vec{a}) \times \vec{b} = \vec{a} \times (m\vec{b})$$

Properties of Vector product

3 $\vec{a} \times (\vec{b} \pm \vec{c}) = \vec{a} \times \vec{b} \pm \vec{a} \times \vec{c}$

$$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$$

$$(\vec{a} + \vec{b}) \times \vec{c} = \vec{a} \times \vec{c} + \vec{b} \times \vec{c}$$

Properties of Vector product

4 If $\vec{a} \parallel \vec{b}$ then $\theta = 0$ or $\pi \Rightarrow \vec{a} \times \vec{b} = 0$ (but $\vec{a} \times \vec{b} = 0 \Rightarrow \vec{a} = 0$ or $\vec{b} = 0$ or $\vec{a} \parallel \vec{b}$). In particular $\vec{a} \times \vec{a} = 0$.

$$\vec{a} \parallel \vec{b} \implies \underline{\theta = 0^\circ / 180^\circ} \implies \vec{a} \times \vec{b} = 0$$

$$\vec{a} \times \vec{b} = 0 \implies \vec{a} = \vec{0}$$
$$\implies \vec{b} = \vec{0}$$
$$\implies \vec{a} \parallel \vec{b}$$

Properties of Vector product

5 If $\vec{a} \perp \vec{b}$ then $\vec{a} \times \vec{b} = |\vec{a}|\ |\vec{b}|\hat{n}$ (or $|\vec{a} \times \vec{b}| = |\vec{a}|\ |\vec{b}|$)

$$\vec{a} \perp \vec{b} \implies \theta = 90^\circ$$

$$|\vec{a} \times \vec{b}| = |a|\ |b| \sin 90^\circ$$

$$= \underline{\underline{|a|\ |b|}}$$

Properties of Vector product

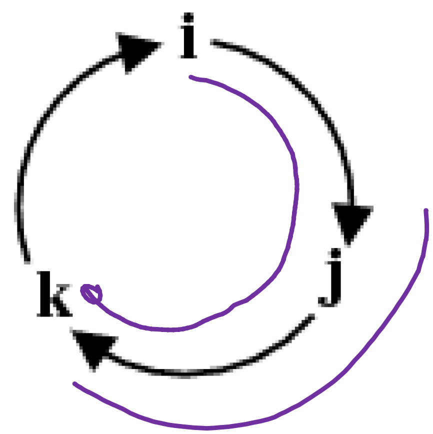
6 $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$ and $\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{k} = \hat{i}$ and $\hat{k} \times \hat{i} = \hat{j}$ (use cyclic system)

$$i \times j = k$$

$$\boxed{\begin{array}{l} ACW = + \\ CW = - \end{array}}$$

$$\hat{k} \times \hat{j} = -\hat{i}$$

$$j \times i = -\hat{k}$$



Properties of Vector product

7 Vector perpendicular to a and b is given by ± (a × b)

$\vec{v} \perp \vec{a}$

$\vec{v} \perp \vec{b}$

$$\boxed{\vec{v} = \pm(\vec{a} \times \vec{b})}$$

Properties of Vector product

8 If θ is angle between $\vec{a}$ and $\vec{b}$ then $\sin\theta = |\vec{a} \times \vec{b}| / |\vec{a}||\vec{b}|$

$$\sin\theta = \frac{|\vec{a}\times\vec{b}|}{|a|\,|b|}$$

Properties of Vector product

9 $|\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2 = |\vec{a}|^2 \, |\vec{b}|^2$ (Lagrange's identity)

$$= \left(|a|\,|b|\sin\theta\right)^2 + \left(|a|\,|b|\cos\theta\right)^2$$

$$= |a|^2\,|b|^2\left(\sin^2\theta + \cos^2\theta\right)$$

$$= \underline{\underline{|a|^2\,|b|^2}}$$

Properties of Vector product

10 If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ then

$$\boxed{\vec{a}} \times \boxed{\vec{b}} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = (a_2a_3 - a_3b_2)\hat{i} + (a_3b_1 - a_1b_3)\hat{j} + (a_1b_2 - a_2b_1)\hat{k}$$

$\vec{a} \times \vec{b}$

If $\vec{a} = 2\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b} = 6\hat{i} - 3\hat{j} + 2\hat{k}$ then $\vec{a} \times \vec{b}$ equals

A. $2\hat{i} - 2\hat{j} - \hat{k}$

B. $\hat{i} - 10\hat{j} - 18\hat{k}$ ✓

C. $\hat{i} + \hat{j} + \hat{k}$

D. $6\hat{i} - 3\hat{j} + 2\hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 2 & 2 & -1 \\ 6 & -3 & 2 \end{vmatrix} = i(1) - j(10) + k(-18)$$

If $|\vec{a}| = 2$, $|\vec{b}| = 5$ and $|\vec{a} \times \vec{b}| = 8$, then $|\vec{a} \cdot \vec{b}|$ is equal to:

Unacademy Atoms

(1) 6

(2) 4

(3) 3

(4) 5

[JEE Main 2021]

$$|a \times b|^2 + (\vec{a} \cdot \vec{b})^2 = |a|^2 |b|^2$$

$$64 + (a \cdot b)^2 = 4 \times 25$$

$$(a \cdot b)^2 = 100 - 64$$

$$\boxed{a \cdot b = 6}$$

Q

Let $\vec{a} = -\hat{i} - \hat{k}, \vec{b} = -\hat{i} + \hat{j}$ and $\vec{c} = \hat{i} + 2\hat{j} + 3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r}.\vec{a} = 0$, then the value of $\boxed{\vec{r}.\vec{b}}$ is 9

[JEE Adv. 2011]

$\vec{a} = (-1, 0, -1)$

$\vec{b} = (-1, 1, 0)$

$\vec{c} = (1, 2, 3)$

$\vec{r}.\vec{a} = 0$

$-1(1-\lambda) - 1(3) = 0$

$-1 + \lambda - 3 = 0$

$\boxed{\lambda = 4}$

$\vec{r} \times \vec{b} - \vec{c} \times \vec{b} = 0$

$(\vec{r} - \vec{c}) \times \vec{b} = 0$

$\vec{r} - \vec{c} \parallel \vec{b}$

$\vec{r} - \vec{c} = \lambda \vec{b}$

$\therefore \vec{r} = \vec{c} + \lambda \vec{b}$

$\therefore \vec{r} = (1, 2, 3) + \lambda(-1, 1, 0)$

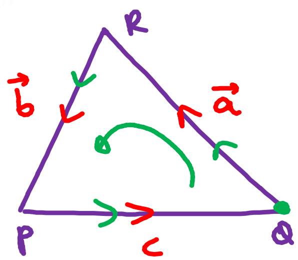
$\therefore \vec{r} = (1-\lambda, 2+\lambda, 3) \equiv (-3, 6, 3)$

In a triangle PQR, let $\vec{a} = \overrightarrow{QR}$, $\vec{b} = \overrightarrow{RP}$ and $\vec{c} = \overrightarrow{PQ}$. If

$$|\vec{a}| = 3, \qquad |\vec{b}| = 4 \quad \text{and} \quad \boxed{\frac{\vec{a}\cdot(\vec{c}-\vec{b})}{\vec{c}\cdot(\vec{a}-\vec{b})} = \frac{|\vec{a}|}{|\vec{a}|+|\vec{b}|}}$$

then the value of $|\vec{a}\times\vec{b}|^2$ is ______

[JEE Adv. 2020]



#NVStyle

$$\vec{a}+\vec{b}+\vec{c}=0$$

$$\therefore \boxed{\vec{c} = -\vec{a}-\vec{b}}$$

$$\boxed{(a+b)\cdot(a-b) = |a|^2-|b|^2}$$

$$\frac{\vec{a}\cdot(-a-b-b)}{(-a-b)\cdot(a-b)} = \frac{3}{7}$$

$$\frac{a\cdot(+a+2b)}{(+a+b)\cdot(a-b)} = \frac{3}{7}$$

$$\boxed{\frac{|a|^2+2\,a\cdot b}{|a|^2-|b|^2} = \frac{3}{7}}$$

$$\frac{9 + 2\, a \cdot b}{9 - 16} = \frac{3}{7}$$

$$9 + 2\, a \cdot b = -3$$

$$\boxed{a \cdot b = -6}$$

$$|\vec{a} \times \vec{b}|^2 + (a \cdot b)^2 = |a|^2 |b|^2$$

$$|a \times b|^2 + 36 = 9 \times 16$$

$$|a \times b|^2 = 144 - 36$$

$$= 108$$

Let △PQr be a triangle. Let $\vec{a} = \overline{QR}$, $\vec{b} = \overline{RP}$ and $c = \overline{PQ}$. if $|\vec{a}| = 12$, $|\vec{b}| = 4\sqrt{3}$, $\vec{b}.\vec{c} = 24$, then which of the following is are) true?

ACD

$24 - 12$

A. $\frac{|\vec{c}|^2}{2} - |\vec{a}| = 12$

$|a| = 12$
$|b| = 4\sqrt{3}$
$\boxed{b.c = 24}$

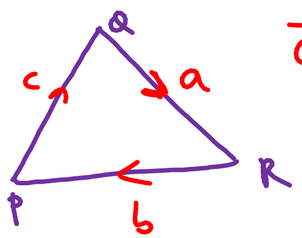
$24 + 12$

B. $\frac{|\vec{c}|^2}{2} + |\vec{a}| = 30$

C. $|\vec{a} \times \vec{b} + \vec{c} \times \vec{a}| = 48\sqrt{3}$

D. $\vec{a} . \vec{b} = -72$

[JEE Adv. 2015]



$\vec{a} + \vec{b} + \vec{c} = 0$

$|b + \vec{c}|^2 = |+\vec{a}|^2$

$|b|^2 + |c|^2 + 2\, b.c = |a|^2$

$48 + |c|^2 + 2(24) = 144$

$\Rightarrow \boxed{|c|^2 = 48}$

$\boxed{|c| = 4\sqrt{3}}$

$$|a+b|^2 = |-c|^2$$

$$|a|^2 + |b|^2 + 2\,a.b = |c|^2$$

$$12^2 + \cancel{48} + 2\,a.b = \cancel{48}$$

$$a.b = \frac{-144}{2}$$

$$a.b = -72$$

$$|a\times b|^2 + (a.b)^2 = |a|^2|b|^2$$

$$|a\times b|^2 + (72)^2 = 144\times 48$$

$$\boxed{|a\times b| = 24\sqrt{3}}$$

☆☆

if $\vec{a}+\vec{b}+\vec{c}=0$

then $a\times b = b\times c = c\times a$

(c) $2|a\times b|$

$\Rightarrow 2\times 24\sqrt{3}$

$\Rightarrow 48\sqrt{3}$

Geometrical Interpretation of Cross Product

Area of Parallelogram

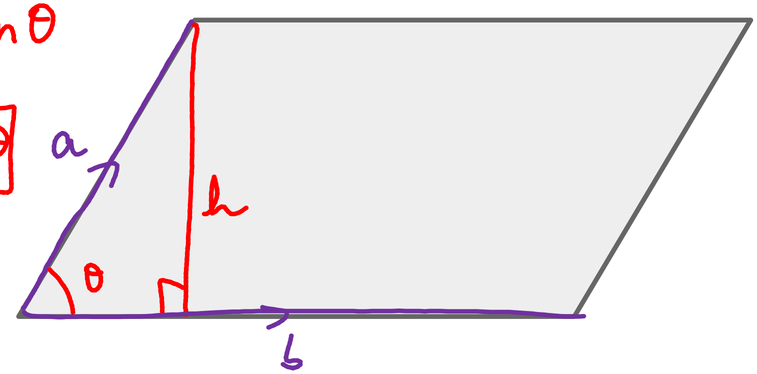
CASE 1 If two adjacent sides are given

$\vec{a}$ = sides
$\vec{b}$ = sides

$$\frac{h}{a} = \sin\theta$$

$$\boxed{h = a\sin\theta}$$

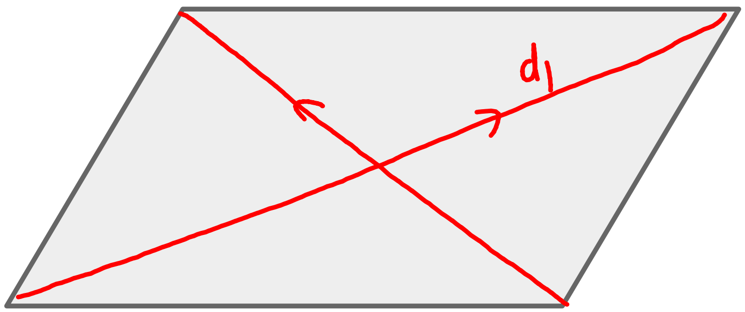
$$A = b \times h$$
$$= b(a\sin\theta)$$
$$= ab\sin\theta$$
$$= |\vec{a} \times \vec{b}|$$



Area of Parallelogram

CASE 2 If diagonal vectors are given

$$\boxed{A = \frac{1}{2}\left|d_1 \times d_2\right|}$$



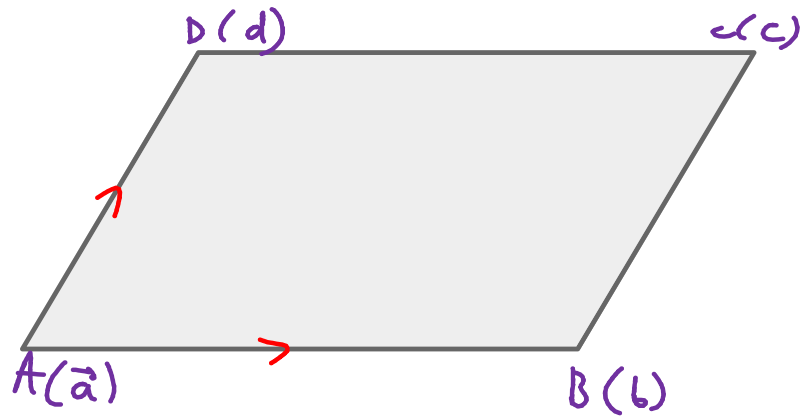
Area of Parallelogram

CASE 3 If position vectors of vertex are given

$\vec{AB} = \vec{b} - \vec{a}$

$\vec{AD} = \vec{d} - \vec{a}$

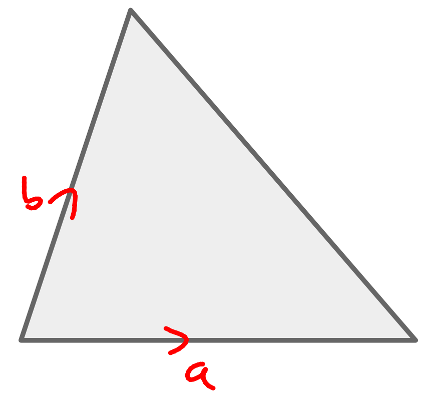
$$\boxed{\text{Area} = |\vec{AB} \times \vec{AD}|}$$



Area of Triangle

CASE 1 If any two adjacent sides are given

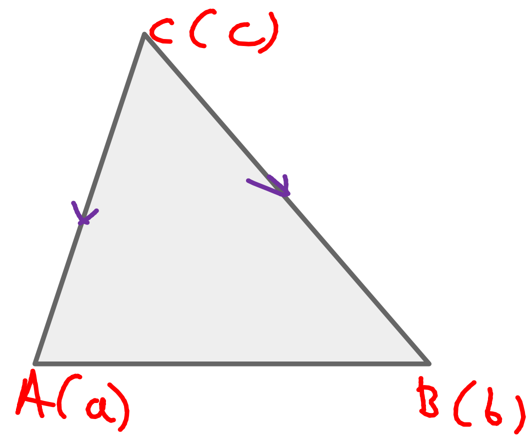
$$A = \frac{1}{2}|\vec{a} \times \vec{b}|$$



Area of Triangle

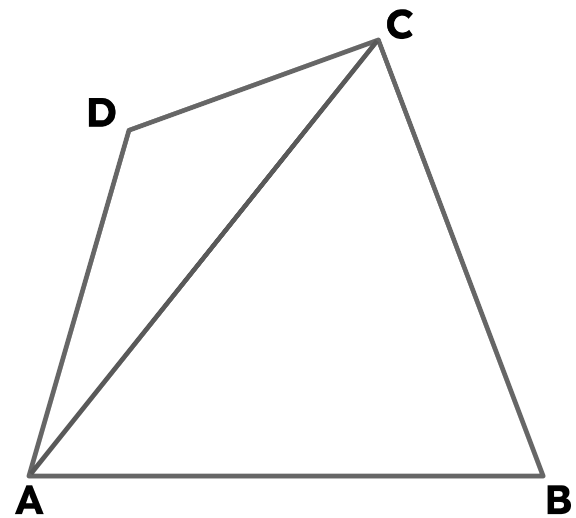
CASE 2 If position vectors of vertex are given

$$\text{Area} = \frac{1}{2} \left| \overrightarrow{CA} \times \overrightarrow{CB} \right|$$



Area of Quadrilateral

$$\boxed{A = \frac{1}{2}\left|d_1 \times d_2\right|}$$



Find the area of parallelogram whose two adjacent **sides** are represented by $\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $b = 2\hat{i} - 2\hat{j} + 4\hat{k}$.

A. $8\sqrt{3}$ ✓

B. $2\sqrt{3}$

C. $\sqrt{3}$

D. $4\sqrt{3}$

$$A = |\vec{a} \times \vec{b}| = \sqrt{8^2+8^2+8^2} = \underline{\underline{8\sqrt{3}}}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 3 & 1 & 2 \\ 2 & -2 & 4 \end{vmatrix} = i(8) - j(+8) + k(-8)$$

$$= (8, -8, -8)$$

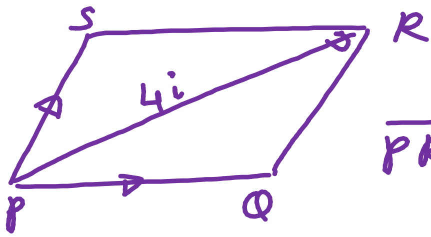
Unacademy Atoms

Q

Let a and b be positive real numbers. Suppose $\overrightarrow{PQ} = a\hat{i} + b\hat{j}$ $(a, b, 0)$ and $\overrightarrow{PS} = a\hat{i} - b\hat{j}$ $(a, -b, 0)$ are adjacent sides of a parallelogram $PQRS$. Let $\vec{u}$ and $\vec{v}$ be the projection vectors of $\vec{w} = \hat{i} + \hat{j}$ along $\overrightarrow{PQ}$ and $\overrightarrow{PS}$, respectively. If $|\vec{u}| + |\vec{v}| = |\vec{w}|$ and if the area of the parallelogram $PQRS$ is 8, then which of the following statements is/are TRUE?

$|w| = \sqrt{1^2+1^2}$

[JEE Adv. 2020]



(A) $a + b = 4$

(B) $a - b = 2$

$\overrightarrow{PR} = ai + \cancel{bj} + ai - \cancel{bj}$

$= 2ai = 4i$

(C) The length of the diagonal PR of the parallelogram $PQRS$ is 4

u = proj of $\vec{w}$ on $\overrightarrow{PQ}$

v = '' '' $\vec{w}$ on $\overrightarrow{PS}$

$\boxed{i + j}$ $\boxed{2i+2j}$ $2i-2j$ $2(i+j)$

(D) $\vec{w}$ is an angle bisector of the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PS}$

AC

$$\frac{\vec{w}\cdot\overrightarrow{PQ}}{|\overrightarrow{PQ}|} + \frac{\vec{w}\cdot\overrightarrow{PS}}{|\overrightarrow{PS}|} = \sqrt{2}$$

$$\frac{(1,1)\cdot(a,b)}{\sqrt{a^2+b^2}} + \frac{(1,1)\cdot(a,-b)}{\sqrt{a^2+b^2}} = \sqrt{2}$$

$$\frac{a+\cancel{b}}{\sqrt{a^2+b^2}}+\frac{a-\cancel{b}}{\sqrt{a^2+b^2}}=\sqrt{2}$$

$$2a=\sqrt{2}\sqrt{a^2+b^2}$$

$$4a^2=2(a^2+b^2)$$

$$4a^2=2a^2+2b^2$$

$$2a^2=2b^2$$

$$a=b=+$$

$$PQ\times PS=\begin{vmatrix} i & j & k \\ a & b & 0 \\ a & -b & 0 \end{vmatrix}$$

$$PQ\times PS=i(0)-j(0)+k(-2ab)$$

$$|PQ\times PS|=\sqrt{0^2+0^2+(2ab)^2}$$

$$8=2ab$$

$$4=ab$$

$$a=2$$
$$b=2$$

PQ || QR

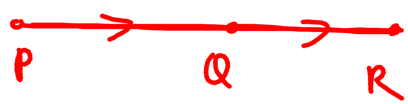
P Q R

$\Delta = 0$ ✗

Collinearity of 3 Points

Collinearity of three points

Three points are collinear if **ar(ABC)=0** or **$\overrightarrow{AB} \parallel \overrightarrow{BC} \parallel \overrightarrow{CA}$**



$\overrightarrow{PQ} \parallel \overrightarrow{QR} \parallel \overrightarrow{PR}$

If $A \equiv (2\hat{i} + 3\hat{j})$, $B \equiv (p\hat{i} + 9\hat{j})$ and $C \equiv (\hat{i} - \hat{j})$ are collinear, then the value of p is-

A. 1/2

B. 3/2

C. 7/2

D. 5/2

$\vec{CA} \parallel \vec{AB}$

A B C

$A \equiv (2, 3)$

$B \equiv (p, 9)$

$C \equiv (1, -1)$

$\vec{AB} = (p-2, 6)$

$\vec{AC} = (-1, -4)$

$\vec{AB} \parallel \vec{AC}$

$$\frac{p-2}{-1} = \frac{6}{-4} \Rightarrow p = 2 + \frac{3}{2} = 7/2$$

STP/Box

Scalar Triple Product / Box Product

Scalar Triple Product or Box Product

$$\underline{\vec{a}}.\underline{(\vec{b} \times \vec{c})} = [\vec{a}\ \ \vec{b}\ \ \vec{c}]$$

Formula for Scalar Triple Product

If $\vec{a} = a_1 i + a_2 j + a_3 k, \vec{b} = b_1 i + b_2 j + b_3 k$ and $\vec{c} = c_1 i + c_2 j + c_3 k$,

then $[\vec{a}\,\vec{b}\,\vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

$[\vec{a} \quad \vec{b} \quad \vec{c}]$

Three vectors are given by, **a** = $\hat{i} - \hat{j} + \hat{k}$, **b** = $2\hat{i} + \hat{j} + \hat{k}$,and **c** = $\hat{i} + \hat{j} - 2\hat{k}$.

$$[\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & -2 \end{vmatrix}$$

$$= 1(-2-1) + 1(-5) + 1(1)$$

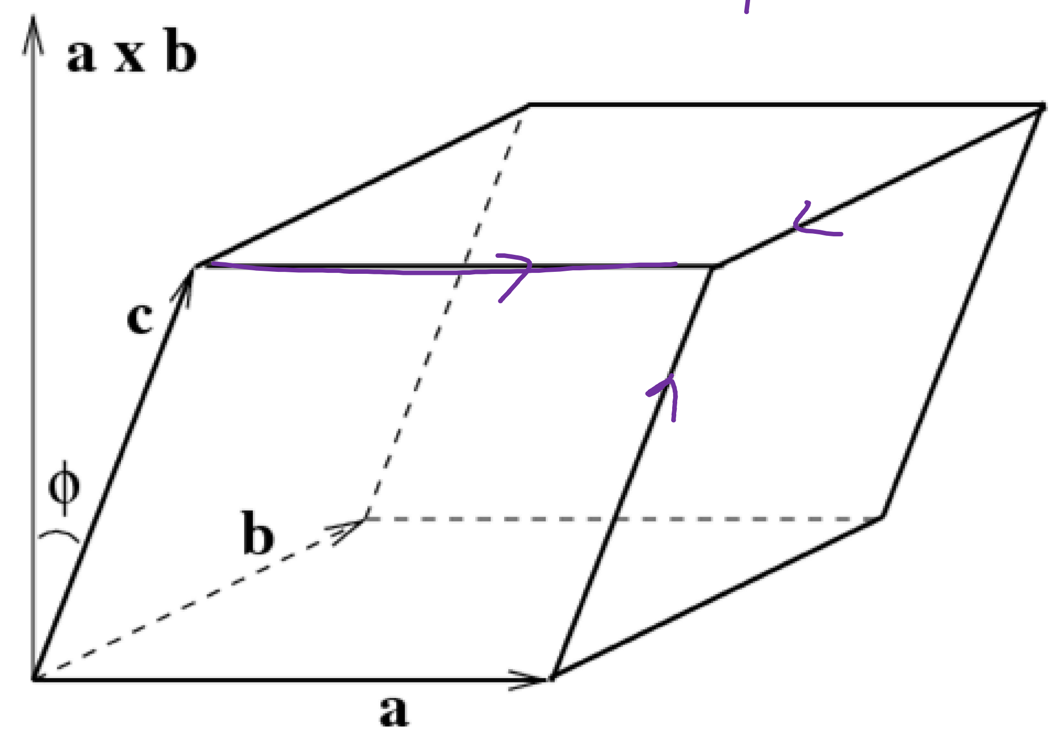
$$= -3 - 5 + 1$$

$$= -7$$

Geometrical Interpretation of Box Product

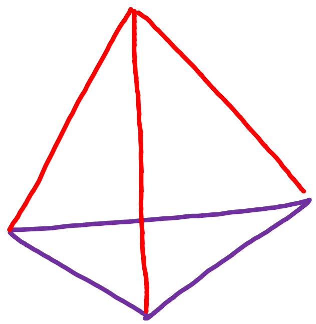
Volume of Parallelepiped

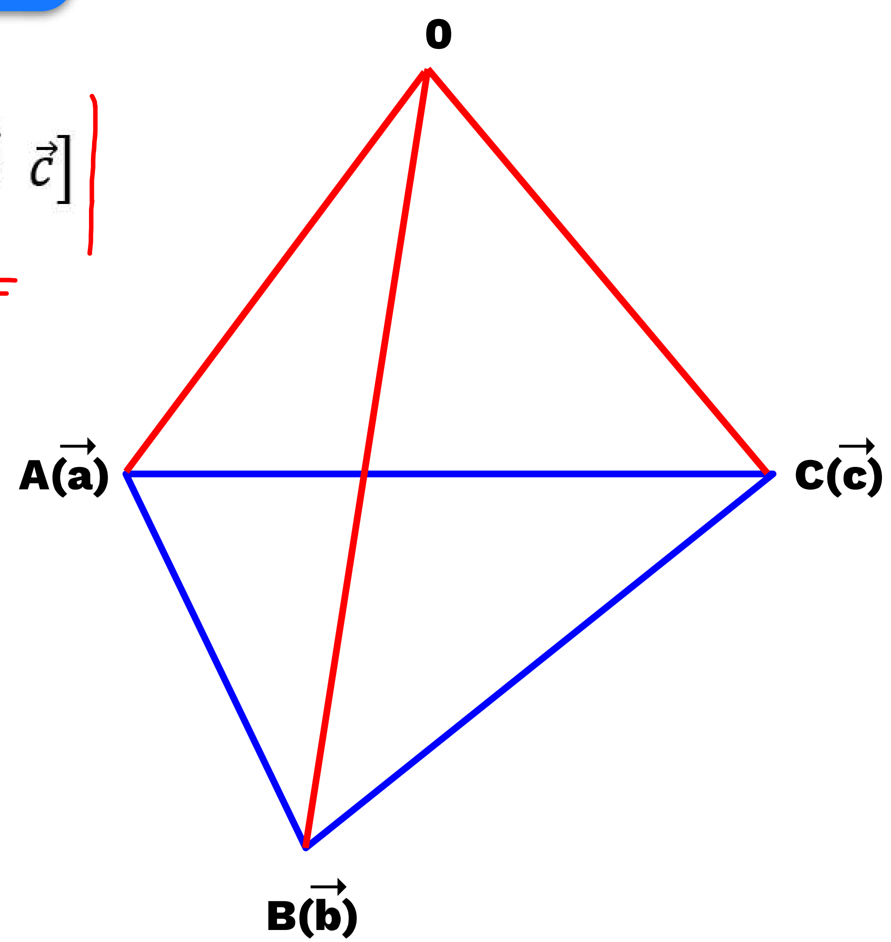
$$Volume\ of\ Parallelepiped = \left|[\vec{a}\ \ \vec{b}\ \ \vec{c}]\right|$$



Volume of Tetrahedron

$$Volume\ of\ Tetrahedron = \frac{1}{6}\left[\vec{a}\ \vec{b}\ \vec{c}\right]$$





If $\vec{a} = 2\hat{i} - 3\hat{j}$, $\vec{b} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{c} = 3\hat{i} - \hat{k}$ represent three coterminous edges of a parallelepiped then the volume of that parallelepiped is -

A. 2

B. 4

C. 6

D. 10

$$V = \left|[\vec{a}\ \vec{b}\ \vec{c}]\right| = \begin{vmatrix} 2 & -3 & 0 \\ 1 & 1 & -1 \\ 3 & 0 & -1 \end{vmatrix}$$

$$= 2(-1) + 3(2) + 0$$

$$= -2 + 6$$

$$= 4$$

Properties of Box Product

Properties of Box Product (STP)

1 **The position of (.) and (×) can be interchanged.**
i.e. $\vec{a}.(\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}).\vec{c}$

$\cancel{1}$ $\vec{a}.(\vec{b} \times \vec{c}) = (a \times b) . c$

Properties of Box Product (STP)

2 $[\vec{a}\ \vec{b}\ \vec{c}] = -[\vec{a}\ \vec{c}\ \vec{b}]$

$$[\vec{a}\ \vec{b}\ \vec{c}] = -[\vec{a}\ \vec{c}\ \vec{b}]$$

Properties of Box Product (STP)

3 You can rotate $\vec{a}, \vec{b}, \vec{c}$ in cyclic order

$$[a \; b \; c] = [c \; a \; b] = [\vec{b} \; \vec{c} \; \vec{a}]$$

Properties of Box Product (STP)

4 $[\vec{a}\ \vec{b}\ \vec{b}] = [\vec{a}\ \vec{b}\ \vec{a}] = 0$

$[a\ \ b\ \ b] = 0$

$[\vec{x}\ \ \vec{y}\ \ \vec{x}] = 0$

Properties of Box Product (STP)

5 The **scalar triple product** of three mutually perpendicular unit vectors is ± 1. Thus $[\hat{i}\ \hat{j}\ \hat{k}] = 1$, $[\hat{i}\ \hat{j}\ \hat{k}] = -1$

$$[i\ k\ j] = -1$$

$$[i\ j\ k] = 1$$

$$[a\ b\ c] = \vec{a}\cdot(\vec{b}\times\vec{c})$$

$$[i\ j\ k] = \hat{i}\cdot(j \times k)$$

$$= \hat{i}\cdot\hat{i}$$

$$= 1$$

Properties of Box Product (STP)

6 If two of the three vectors a, b, c are parallel then $[\vec{a}\ \vec{b}\ \vec{c}] = 0$

$$[\vec{a}\ \vec{b}\ \vec{c}] = 0 \qquad b \parallel c \Rightarrow \boxed{\vec{b} = \lambda \vec{c}}$$

$$\Rightarrow [a\ \ \lambda\vec{c}\ \ \vec{c}]$$

$$\Rightarrow \lambda[\vec{a}\ \ \vec{c}\ \ \vec{c}]$$

$$\Rightarrow 0$$

Q) $[a\ b\ c] = 2$

find $[3\vec{a}\ \ 2\vec{b}\ \ \vec{c}]$

$$= 3 \times 2 \times [a\ b\ c]$$

$$= 6 \times 2$$

$$= 12$$

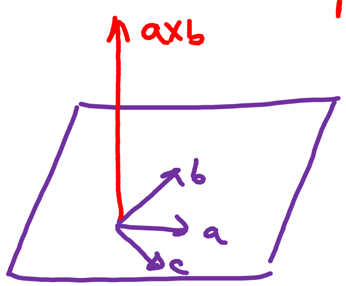
Properties of Box Product (STP)

★★★ max

7 Three non-zero non-collinear vectors are coplanar if $[\vec{a}\ \vec{b}\ \vec{c}] = 0$

$$\text{Coplanar} \Longrightarrow [a\ b\ c] = 0$$

$$\not\Longleftarrow$$



$$\vec{c} \perp \vec{a} \times \vec{b}$$

$$c \cdot (\vec{a} \times \vec{b}) = 0$$

$$[c\ a\ b] = [a\ b\ c] = 0$$

Properties of Box Product (STP)

8 $[\vec{a}+\vec{b}\ \ \vec{c}\ \ \vec{d}] = [\vec{a}\ \ \vec{c}\ \ \vec{d}] + [\vec{b}\ \ \vec{c}\ \ \vec{d}]$

$$[\boxed{a}\ \ \boxed{c}\ \ \boxed{d}] + [\boxed{b}\ \ \boxed{c}\ \ \boxed{d}] = [\vec{a}+\vec{b}\ \ \vec{c}\ \ \vec{d}]$$

$$[a\ \ b\ \ c] + [\vec{a}\ \ \vec{x}\ \ \vec{c}] = [\vec{a}\ \ b+x\ \ \vec{c}]$$

Properties of Box Product (STP)

9. $[\vec{a}+\vec{b} \quad \vec{b}+\vec{c} \quad \vec{c}+\vec{a}] = 2[\vec{a} \; \vec{b} \; \vec{c}]$

10. $[\vec{a}-\vec{b} \quad \vec{b}-\vec{c} \quad \vec{c}-\vec{a}] = 0$

11. $[\vec{a}\times\vec{b} \quad \vec{b}\times\vec{c} \quad \vec{c}\times\vec{a}] = [\vec{a} \; \vec{b} \; \vec{c}]^2$

Properties of Box Product (STP)

$$[a+b \quad b+c \quad c+a]$$

$$\Rightarrow [a \quad b+c \quad c+a] + [b \quad b+c \quad c+a]$$

$$\Rightarrow [a \quad b \quad c+a] + [a \quad c \quad c+a] + [b \quad \cancel{b} \quad c+a] + [b \quad c \quad c+a]$$

$$\Rightarrow [a \quad b \quad c] + [a \quad \cancel{b} \quad \cancel{a}] + [a \quad \cancel{c} \quad c] + [a \quad \cancel{c} \quad \cancel{a}] + [b \quad \cancel{c} \quad c] + [b \quad c \quad a]$$

$$\Rightarrow [a \quad b \quad c] + [b \quad c \quad a]$$

$$\Rightarrow [a \quad b \quad c] + [a \quad b \quad c]$$

$$\Rightarrow 2[a \quad b \quad c]$$

For any three vectors $\vec{a}, \vec{b}, \vec{c}$ $[\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}]$ equals

A. $[\vec{a}\ \vec{b}\ \vec{c}]$

B. $2\,[\vec{a}\ \vec{b}\ \vec{c}]$

C. $[\vec{a}\ \vec{b}\ \vec{c}]^2$

D. 0

(JEE)

Unacademy Atoms

Q

Let a, b and c be distinct positive numbers. If the vectors $a\hat{i} + a\hat{j} + c\hat{k}$, $\hat{i} + \hat{k}$ and $c\hat{i} + c\hat{j} + b\hat{k}$ are co-planar, then c is equal to

[JEE Main 2021]

A. $\dfrac{2}{\frac{1}{a}+\frac{1}{b}}$

B. $\dfrac{1}{a} + \dfrac{1}{b}$

C. $\dfrac{a+b}{2}$

D. $\sqrt{ab}$

$$\begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0$$

$$a(-c) - a(b-c) + c(c) = 0$$

$$-\cancel{ac} - ab + \cancel{ac} + c^2 = 0$$

$$c^2 = ab$$

Unacademy Atoms

Linearly Dependent vs Independent

Linearly Dependent v/s Independent Vectors

For 2 non zero Vectors

If $\boxed{\vec{a} \parallel \vec{b}}$ ⇒ Linearly Dependent

If $\vec{a} \nparallel \vec{b}$ ⇒ Linearly Independent

$\vec{a} \parallel \vec{b} \Rightarrow$ L.D. $\quad \boxed{\vec{a} = \lambda \vec{b}}$

$a \nparallel b \Rightarrow$ L.I.

Linearly Dependent v/s Independent Vectors

For 3 non zero Vectors

If $[\vec{a}\ \vec{b}\ \vec{c}] \neq 0 \Rightarrow$ Linearly Independent

If $[\vec{a}\ \vec{b}\ \vec{c}] = 0 \Rightarrow$ Linearly Dependent

$[a\ b\ c] = 0 \Longrightarrow$ L.D

$[a\ b\ c] \neq 0 \Rightarrow$ L.I.

if $[a\ b\ c] = 0$

$\Downarrow$

a, b, c Coplanar

$\Downarrow$

$\vec{c} = x\vec{a} + y\vec{b}$

Linearly Dependent v/s Independent Vectors

4 or more non zero Vectors

Four or more vectors are always $\boxed{\text{Linearly Dependent}}$

Check whether $\hat{i} - 3\hat{j} + 2\hat{k}$, $2\hat{i} - 4\hat{j} - \hat{k}$ and $3\hat{i} + 2\hat{j} - \hat{k}$ are linearly independent or dependent?

$$[a\ b\ c] = \begin{vmatrix} 1 & -3 & 2 \\ 2 & -4 & -1 \\ 3 & 2 & -1 \end{vmatrix} \neq 0$$

$\underline{\underline{3}}$

L.I.

Vector Triple Product (VTP)

VTP - Definition

$$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a}\,.\,\vec{c})\vec{b} - (\vec{a}\,.\,\vec{b})\vec{c}$$

$$a \times (b \times c) = (a \cdot c)\, b - (a \cdot b)\, c$$

Vector Triple Product (VTP) - Properties

1. Vector triple product is a vector quantity

Vector Triple Product (VTP) - Properties

2 $\vec{a} \times (\vec{b} \times \vec{c}) \neq (\vec{a} \times \vec{b}) \times \vec{c}$

$a \times (b \times c) \neq \underline{(a \times b) \times c}$

Vector Triple Product (VTP) - Properties

$\vec{y} \times \vec{x} = -(\vec{x} \times \vec{y})$

3 $(\vec{a} \times \vec{b}) \times \vec{c} =$

$(a \times b) \boxed{\times} c$

$= -\{c \times (\vec{a} \times \vec{b})\}$

$= -\{(c \cdot b)a - (a \cdot c)b\}$

$= -(c \cdot b)\vec{a} + (a \cdot c)\vec{b}$

Vector Triple Product (VTP) - Properties

4

$[\vec{a} \times \vec{b} \quad \vec{b} \times \vec{c} \quad \vec{c} \times \vec{a}] = [\vec{a} \ \vec{b} \ \vec{c}]^2$

$|a|=3$

Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. If $\vec{c}$ is a vector such that $\vec{a} \cdot \vec{c} = |\vec{c}|, |\vec{c} - \vec{a}| = 2\sqrt{2}$ and the angle between $(\vec{a} \times \vec{b})$ and $\vec{c}$ is $\frac{\pi}{6}$, then the value of $|(\vec{a} \times \vec{b}) \times \vec{c}|$ is :

[JEE Main 2021]

(1) $\frac{2}{3}$

(2) 4

(3) 3

(4) $\frac{3}{2}$

$$|(a \times b) \times c|$$

$$= |a \times b|\,|c| \sin\frac{\pi}{6}$$

$$= 3 \times |c| \times \frac{1}{2}$$

$$= \boxed{3 \times 1 \times \frac{1}{2}}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 2 & 1 & -2 \\ 1 & 1 & 0 \end{vmatrix}$$

$$= i(2) - j(2) + k(1)$$

$$= (2, -2, 1)$$

$$|a \times b| = \sqrt{2^2 + 2^2 + 1^2} = 3$$

$$a \cdot c = |c|$$

$$|c - a|^2 = (2\sqrt{2})^2$$

$$|c|^2 + |a|^2 - 2\,\underline{\underline{a \cdot c}} = 8$$

$$|c|^2 + 9 - 2|c| = 8$$

$$|c|^2 - 2|c| + 1 = 0$$

$$(|c| - 1)^2 = 0$$

$$\underline{|c| = 1}$$

Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that

$\vec{a} = \vec{b} \times (\vec{b} \times \vec{c})$. If magnitudes of the vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are $\sqrt{2}$, 1 and 2 respectively and the angle between $\vec{b}$ and $\vec{c}$ is $\theta \left(0 < \theta < \frac{\pi}{2}\right)$, then the value of $1 + \tan\theta$ is equal to:

[JEE Main 2021]

(1) $\sqrt{3} + 1$

(2) 2

(3) 1

(4) $\frac{\sqrt{3}+1}{\sqrt{3}}$

$\vec{a} = b \times (b \times c)$

$\vec{a} = (b \cdot c)\, b - (b \cdot b)\, \vec{c}$

$|\vec{a}|^2 = |(2\cos\theta)\vec{b} - \vec{c}|^2$

$2 = 4\cos^2\theta\, |b|^2 + |c|^2 - 4\cos\theta\, (b \cdot c)$

$2 = 4\cos^2\theta\, (1) + 4 - 4\cos\theta\, (2\cos\theta)$

$\vec{b} \wedge \vec{c} = \theta$

$\vec{b} \cdot \vec{c} = |b|\,|c| \cos\theta$

$= 2\cos\theta$

$$2 = 4\cos^2\theta + 4 - 8\cos^2\theta$$

$$\therefore \quad 4\cos^2\theta = 2$$

$$\cos^2\theta = \frac{1}{2}$$

$$\boxed{\theta = \pi/4}$$

Q

If $\vec{a}$ and $\vec{b}$ are vectors in space given by $\vec{a} = \frac{\hat{i} - 2\hat{j}}{\sqrt{5}}$ and $\vec{b} = \frac{2\hat{i} + \hat{j} + 3\hat{k}}{\sqrt{14}}$, then find the value of $(2\vec{a}+\vec{b}).[(\vec{a}\times\vec{b})\times(\vec{a}-2\vec{b})]$

[JEE Adv. 2010]

$$\frac{(1,-2,0)}{\sqrt{5}} \cdot \frac{(2,1,3)}{\sqrt{14}}$$

$$a \cdot a = |a|^2 = 1$$

$$b \cdot a = 0$$

$$b \cdot b = |b|^2 = 1$$

$$(2a+b) \cdot (2a+b)$$

$$= 4a \cdot a + 0 + 0 + b \cdot b$$

$$= 4(1) + 1$$

$$= 5$$

$$= (a \times b) \times (a - 2b)$$

$$= -\{(a - 2b) \times (a \times b)\}$$

$$= -\{((a-2b)\cdot b)\vec{a} - ((a-2b)\cdot a)\, b\}$$

$$= -\{(-2)\vec{a} - (1)\, b\}$$

$$= -\{-2\vec{a} - \vec{b}\}$$

$$= 2\vec{a} + \vec{b}$$

$$(a-2b)\cdot b$$
$$= a\cdot b - 2b\cdot b$$
$$= 0 - 2(1)$$

$$(a-2b)\cdot a$$
$$= a\cdot a - 2\cancel{a\cdot b}$$
$$= 1$$

Scalar Product of Four Vectors

$$(\vec{a} \times \vec{b}).(\vec{c} \times \vec{d}) = \begin{vmatrix} \vec{a}.\vec{c} & \vec{a}.\vec{d} \\ \vec{b}.\vec{c} & \vec{b}.\vec{d} \end{vmatrix}$$

$$(a \times b).(c \times d) = \begin{vmatrix} a.c & b.c \\ a.d & b.d \end{vmatrix}$$

Vector Product of Four Vectors

$$\vec{V} = (\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a}\ \vec{b}\ \vec{d}]\vec{c} - [\vec{a}\ \vec{b}\ \vec{c}]\vec{d}$$

$$(a \times b) \times (c \times d)$$

$$= [a\ b\ d]\vec{c} - [a\ b\ c]\,d$$

Q

If $\vec{a}, \vec{b}, \vec{c}$ and d are unit vectors such that $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = 1$ and $\vec{a} \cdot \vec{c} = 1/2$, then

[JEE Adv. 2009]

~~A.~~ $\vec{a}, \vec{b}, \vec{c}$ are non - coplanar

~~B.~~ $\vec{b}, \vec{c}, \vec{d}$ are non - coplanar

C. $\vec{b}, \vec{d}$ are non - parallel

~~D.~~ $\vec{a}, \vec{d}$ are parallel and b, c are parallel

$$(a \times b) \cdot (c \times d) = 1$$

$$|a \times b|\,|c \times d| \cos\theta = 1$$

$$|a|\,|b| \sin\alpha\,|c|\,|d| \sin\beta \cos\theta = 1$$

$$\sin\alpha \sin\beta \cos\theta = 1$$

$$(a \times b) \wedge (c \times d) = \theta = 0^\circ$$

$$\vec{a} \wedge \vec{b} = \alpha = 90^\circ$$

$$\vec{c} \wedge \vec{d} = \beta = 90^\circ$$

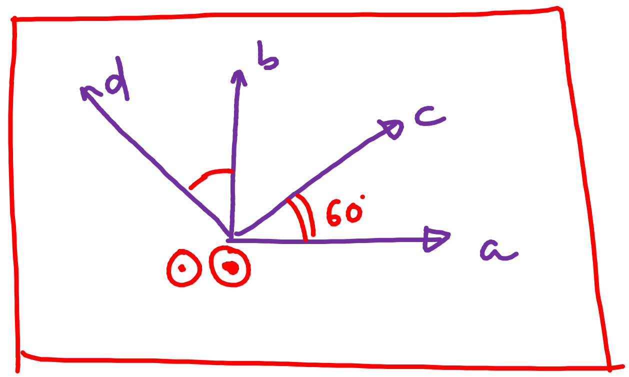
$$\vec{a} \wedge \vec{c} = 60^\circ$$

$a \cdot c = \frac{1}{2}$ $\quad \underline{a \wedge c = \gamma}$

$|a||c| \cos\gamma = \frac{1}{2}$

$\cos\gamma = 1/2$

$\boxed{\gamma = 60^\circ}$

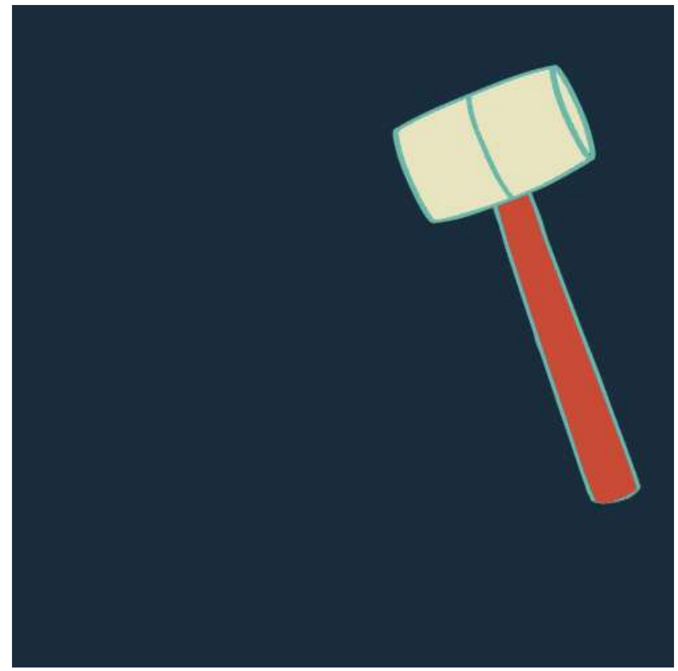


$a \not\parallel d$

Isolating Unknown Vectors

#Hathoda concept

V.T.P



Find vector $\vec{r}$ if $\vec{r}.\vec{a} = m$ and $\vec{r} \times \vec{b} = \vec{c}$ where $\vec{a}.\vec{b} \neq 0$

$\vec{r} = ?$

- $\vec{r}.\vec{a} = m$
- $\vec{r} \times \vec{b} = c$

V.T.P.

$$\vec{a} \times (\vec{r} \times \vec{b}) = \vec{a} \times \vec{c}$$

$$(a.b)\vec{r} - (a.r)\,b = a \times c$$

$$(a.b)\vec{r} = a \times c + mb$$

$$\boxed{\vec{r} = \frac{a \times c + mb}{(a.b)}}$$