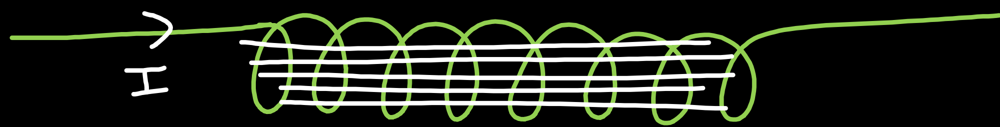
Inductor



$L \rightarrow$ depends on configuration

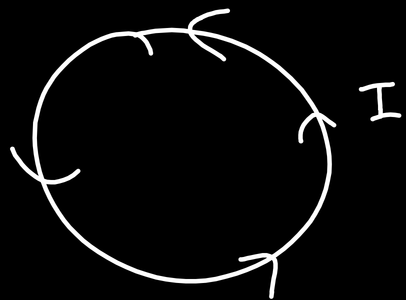
$\rightarrow$ depends on medium

$\phi = (B)(\text{Area})N$

magnetic

$\phi \propto I$

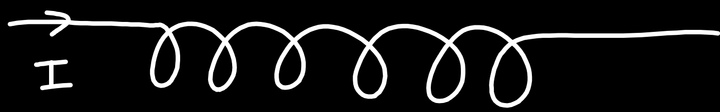
$$\boxed{\phi = LI}$$



$\mu_r \rightarrow$ relative permeability

$\mu_0 \longrightarrow \mu_r \mu_0$

Vaccum medium.



$$\boxed{\phi = LI}$$

Faraday & Lenz law.

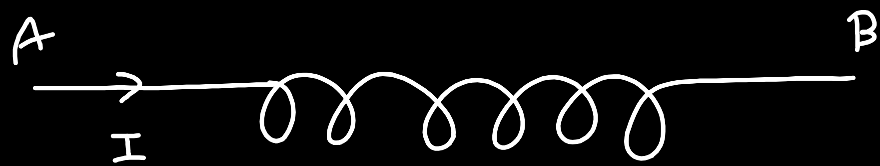
If flux changes

induced Emf ⇒ $\left|\dfrac{d\phi}{dt}\right|$

it oppose the change

$$Emf = \frac{d\phi}{dt}$$

$$\boxed{|Emf| = L\frac{dI}{dt}}$$

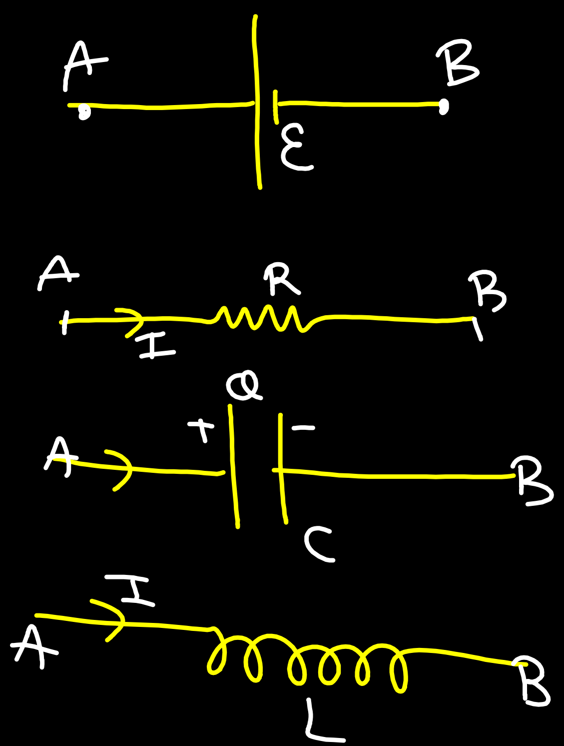


If I constant, ϕ constant, $\text{Emf}_{\text{induced}} = 0$, $V_A = V_B$

If I variable, ϕ change, Emf induce Hoga, $\text{Emf}_{\text{induce}} = L\left(\frac{dI}{dt}\right)$

$$V_A - V_B = L\frac{dI}{dt}$$

Circuit Elements



$$V_A - V_B = \varepsilon$$

$$V_A - V_B = IR$$

$$V_A - V_B = Q/C$$

$$V_A - V_B = L\frac{dI}{dt}$$

$$V_A - V_B = L\frac{dI}{dt}$$

① I constant $\frac{dI}{dt} = 0$ $\quad V_A - V_B = 0 \quad V_A = V_B$

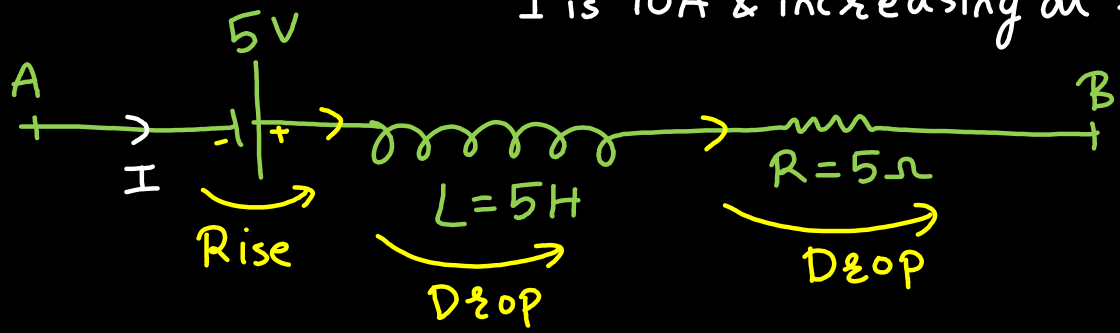
② I increasing $\frac{dI}{dt} = +ve$ $\quad V_A - V_B = +ve \quad V_A > V_B$

③ I decreasing $\frac{dI}{dt} = -ve$ $\quad V_A - V_B = -ve \quad V_A < V_B$

Q// Find $V_A - V_B = ??$

I is 10A & increasing at rate 5A/sec

(#)



a) −45 Volt

b) 50

c) 20

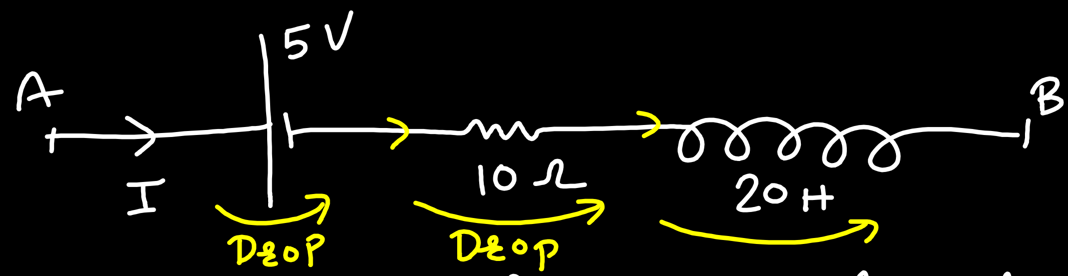
d) 70

$$V_A + 5 - L\frac{dI}{dt} - IR = V_B$$

$$V_A - V_B = L\frac{dI}{dt} + IR - 5$$

$$= (5)(+5) + 10(5) - 5 = \underline{70 \text{ Volt}}$$

#



$I = 10A$ & is decreasing at rate $1A/sec$

a) 85

b) 35

c) 125

d) 110.

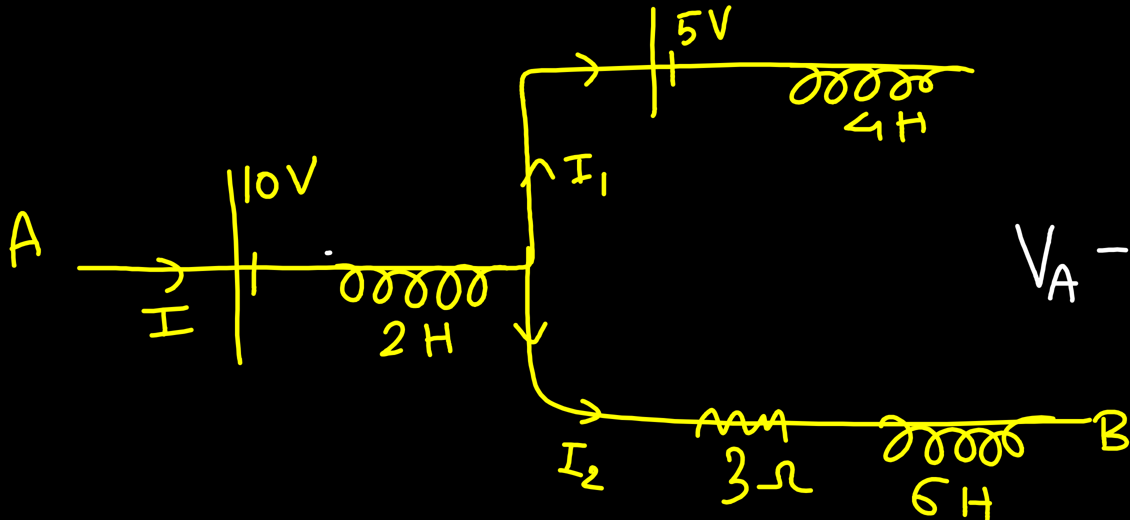
$$V_A - 5 - IR - L\frac{dI}{dt} = V_B$$

$$V_A - V_B = 5 + IR + L\frac{dI}{dt}$$

$$= 5 + 10(10) + 20(-1)$$

$$= 105 - 20$$

$$= 85$$



Find $V_A - V_B = ?$

$$V_A - 10 - L\frac{dI}{dt} - I_2(3) - 6\frac{dI_2}{dt} = V_B$$

$I = 5A$, $I_1 = 2A$

I is increasing at rate $4\,A/s$

I_1 is " " " $6\,A/s$

$$V_A - V_B = 10 + L\frac{dI}{dt} + I_2(3) + 6\frac{dI_2}{dt}$$

$$= 10 + 2(4) + (3)(3) + 6(-2)$$

$$= 15$$

$$I = I_1 + I_2$$

$$5 = 2 + I_2$$

$$3 = I_2$$

$$\frac{dI}{dt} = 4 A/s$$

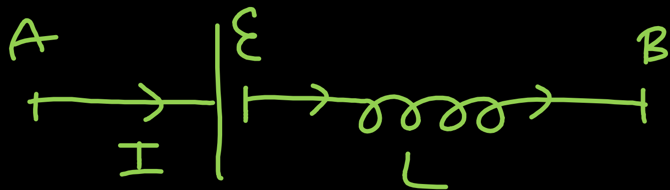
$$I = I_1 + I_2$$

$$\frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt}$$

$$4 = 6 + \frac{dI_2}{dt}$$

$$-2 = \frac{dI_2}{dt}$$

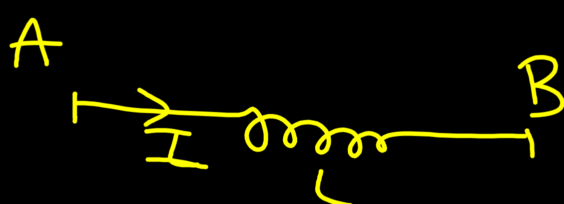
Q



$$V_A - \varepsilon - L\frac{dI}{dt} = V_B$$

$$V_A - V_B = ?$$

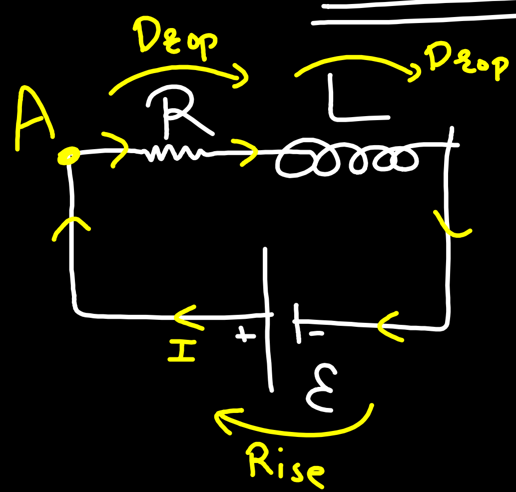
$$V_A - V_B = \varepsilon + L\frac{dI}{dt}$$



$$V_A - L\frac{dI}{dt} = V_B$$

$$V_A - V_B = L\frac{dI}{dt}$$

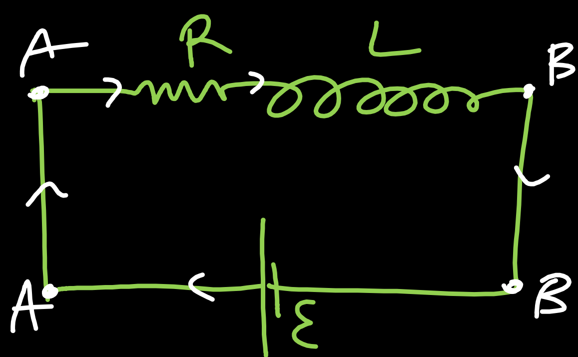
L-R circuits (growth of current)



Apply Kirchhoff's Law / KVL in Loop

$$\cancel{V_A} - IR - L\frac{dI}{dt} + \varepsilon = \cancel{V_A}$$

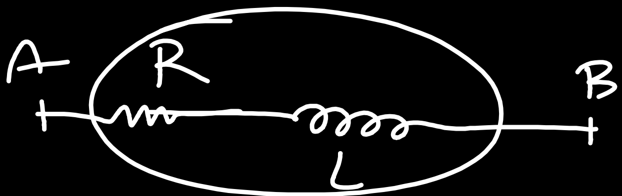
$$\varepsilon = IR + L\frac{dI}{dt}$$



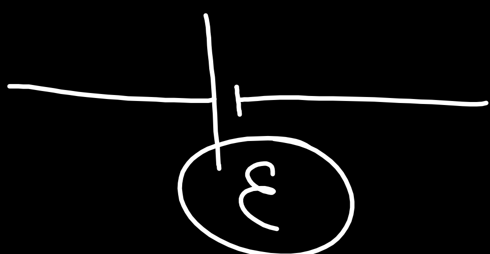
$$V_A - V_B = IR + L\frac{dI}{dt}$$

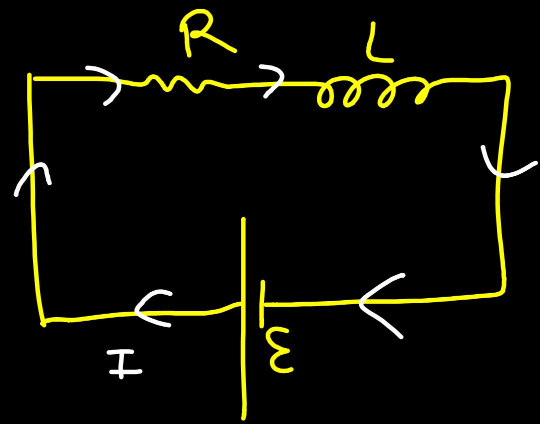
$$V_A - V_B = \varepsilon$$

$$\varepsilon = IR + L\frac{dI}{dt}$$



$$IR + L\frac{dI}{dt}$$





$$\varepsilon = IR + L\frac{dI}{dt}$$

$$\varepsilon - IR = L\frac{dI}{dt}$$

$$dt = \frac{L\,dI}{\varepsilon - IR}$$

$$\frac{dt}{L} = \frac{dI}{\varepsilon - IR}$$

Basic

$$\int \frac{1}{x}\,dx = \log_e x$$

$$\int \frac{1}{ax+b}\,dx = \frac{\log_e(ax+b)}{a}$$

$$\int \frac{dx}{6-7x} = \frac{\log_e(6-7x)}{-7}$$

$$\int \frac{dt}{L} = \int \frac{dI}{\varepsilon - IR}$$

$$\frac{t}{L} = \frac{\log_e(\varepsilon - IR)}{-R}$$

$$\left.\frac{-R}{L}t\right]_0^t = \left.\log_e(\varepsilon - IR)\right]_0^I$$

$$\Rightarrow -\frac{R}{L}t = \log_e(\varepsilon - IR) - \log_e(\varepsilon)$$

$$-\frac{tR}{L} = \log_e\left(\frac{\varepsilon - IR}{\varepsilon}\right)$$

take anti log

$$e^{-\frac{tR}{L}} = \frac{\varepsilon - IR}{\varepsilon}$$

$$\varepsilon e^{-tR/L} = \varepsilon - IR$$

$$IR = \varepsilon - \varepsilon e^{-tR/L}$$

Unacademy Atoms

$$I = \frac{\mathcal{E}}{R}\left(1 - e^{-tR/L}\right)$$

$\tau \Rightarrow$ time constant

$$\boxed{I = I_{steady}\left(1 - e^{-t/\tau}\right)}$$

$$\tau = \frac{L}{R}$$

$I_{steady} = \mathcal{E}/R$

τ unit of time

= seconds.

unit of $\frac{L}{R} \rightarrow$ seconds

Very Special Case / Making it Easy

$$I = I_{steady}\left(1 - e^{-t/\tau}\right)$$

$t = 0$ $e^0 = 1$

$$I = I_{steady}(1-1)$$

$$I = 0$$

$t = \infty$

$$e^{-\infty} = 0$$

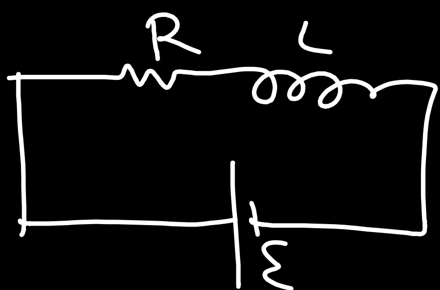
$$\frac{1}{e^{\infty}} = \frac{1}{2.7^{\infty}} = \frac{1}{\infty} = 0$$

$$I = I_{steady}(1-0)$$

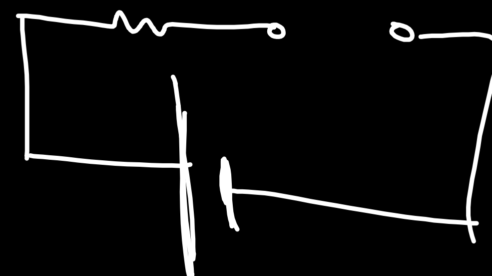
$$= I_{steady}$$

$$I = \frac{\mathcal{E}}{R}$$

$t=0$ behaves as Open Circuit

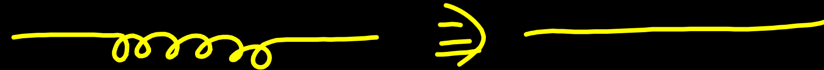


$t=0 \Rightarrow$

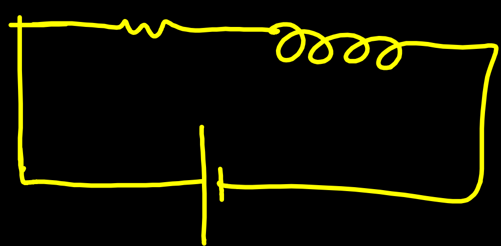


$I = 0$

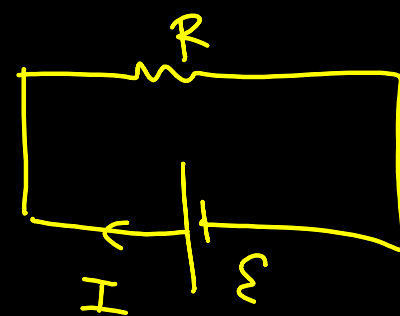
$t = \infty$ behaves as Short Circuit



at $t = \infty$



$t = \infty \Rightarrow$

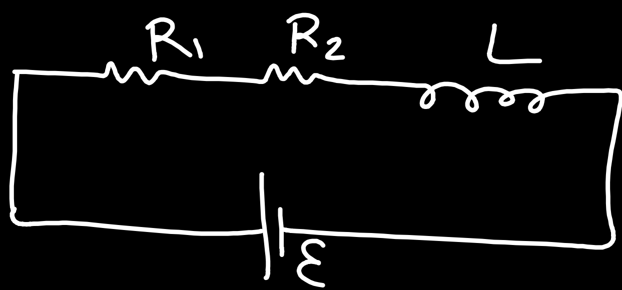


$$I = \frac{\varepsilon}{R}$$

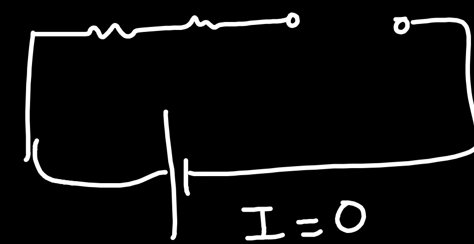
Q Find current in Circuit in each Branch at

(1) $t = 0$ & (2) $t = \infty$.

Q 1

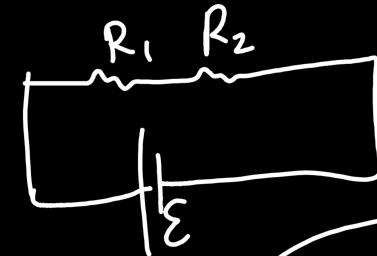


$t = 0$



$I = 0$

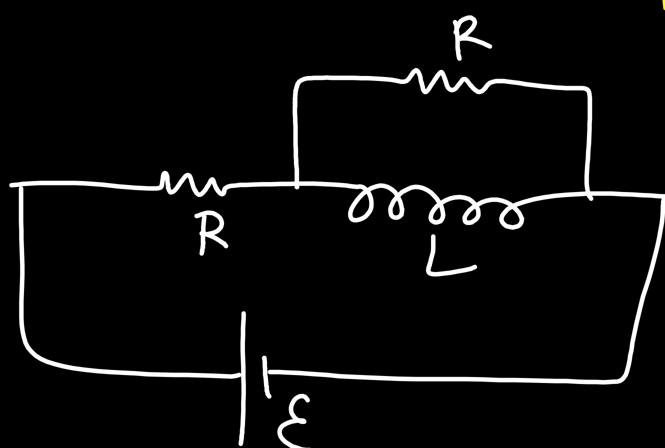
$t = \infty$



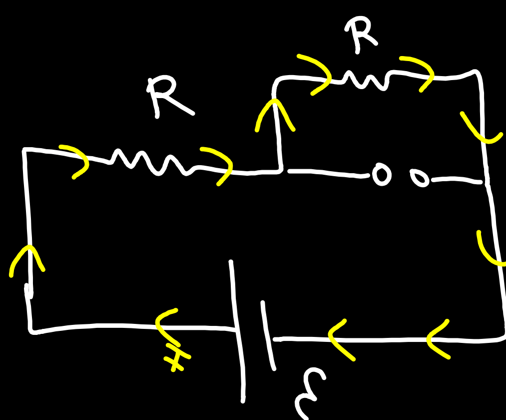
$$I = \frac{\varepsilon}{R_1 + R_2}$$

Q

At $t=0$ & $t=\infty$ Find total Current from battery ??

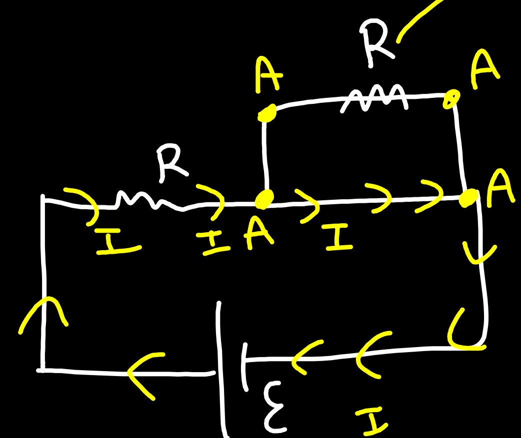


$t=0$



$$I = \frac{\varepsilon}{2R}$$

$t=\infty$



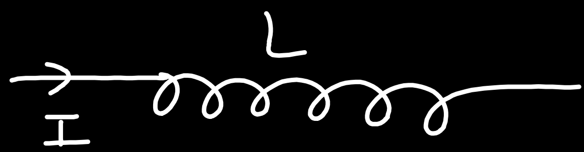
short ho Gya hai

⇓

No current

$$I = \frac{\varepsilon}{R}$$ ✓

Energy Stored in Inductor

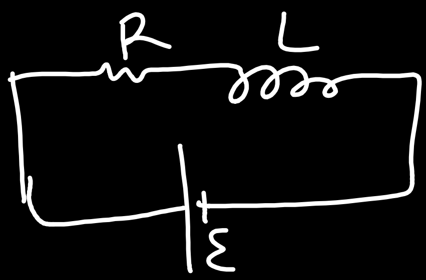
$\#$ 

$$\boxed{\text{Energy stored}_{\text{inside inductor}} = \frac{1}{2} L I^2}$$

$\#$ $I = I_0\left(1 - e^{-t/\tau}\right)$

energy stored in L R circuit

$$\frac{1}{2}L\left[I_0\left(1-e^{-t/\tau}\right)\right]^2$$



$I_0 = \frac{\varepsilon}{R}$

Steady State Current

initial energy $= 0$

maximum energy stored $= \frac{1}{2} L I_0^2$

function of time

Energy stored $= \frac{1}{2} L \left[I_0 \left(1 - e^{-t/\tau}\right) \right]^2$

Power $\longrightarrow$ Rate of Energy Stored in L

$$\text{Eenergy} = \frac{1}{2} L I^2$$

$$\frac{d(\text{Energy})}{dt} = \frac{1}{2} L \left(2 I \frac{dI}{dt} \right)$$

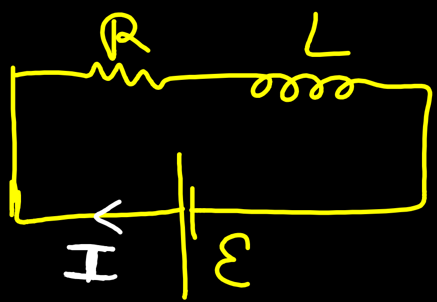
$$\text{Power or rate of energy} = \left(L \frac{dI}{dt} \right) I$$

$$\text{Power} = V I$$

$$= (\mathcal{E}_{mf}) I$$

$$= \left(L \frac{dI}{dt} \right) I$$

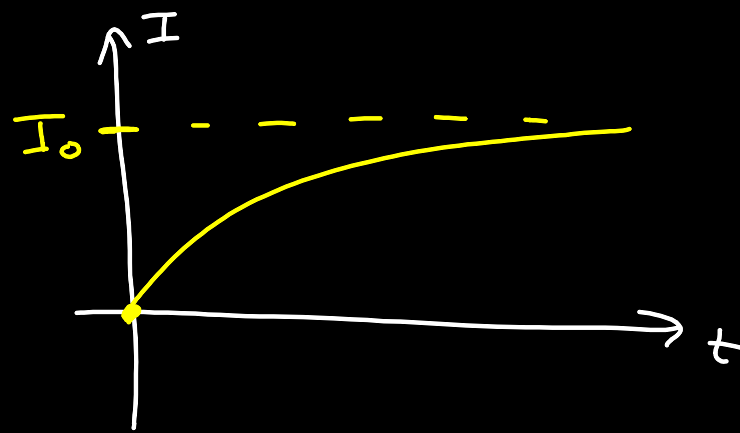
#



$$\boxed{I = I_0\left(1 - e^{-t/\tau}\right)}$$

$$\boxed{I_0 = \frac{\varepsilon}{R}}$$ steady state current

$$\boxed{\tau = \frac{L}{R}}$$



τ = Time Constant $\rightarrow$ time when 63% of steady state current is achieved.

$t = 0$ ✓
$t = \infty$ ✓

I at $t = \tau$

$$I = I_0\left(1 - e^{-t/\tau}\right)$$
$$= I_0\left(1 - e^{-\tau/\tau}\right)$$
$$= I_0\left(1 - e^{-1}\right)$$
$$= I_0\left(1 - \frac{1}{e}\right)$$
$$= I_0\left(1 - \frac{1}{2.7}\right)$$
$$= I_0(1 - 0.37) = 0.63 I_0$$

$e = 2.7$

I at $t = (\ln 2)\tau$

Prop $e^{\ln(x)} = x$

$$e^{-\ln 2} = e^{\ln(2)^{-1}} = e^{\ln\left(\frac{1}{2}\right)} = \frac{1}{2}$$

$$I = I_0\left(1 - e^{-t/\tau}\right)$$
$$= I_0\left(1 - e^{-\ln(2)\,\tau/\tau}\right)$$
$$= I_0\left(1 - e^{-\ln 2}\right) = I_0\left(1 - \frac{1}{2}\right)$$
$$= \boxed{I_0\left(\frac{1}{2}\right)}$$

Unacademy Atoms

$$\text{Power (rate of energy)} = \left(L\frac{dI}{dt}\right)I$$

$$I = I_0\left(1 - e^{-t/\tau}\right)$$

$$\frac{dI}{dt} = I_0\left(0 - e^{-t/\tau} \times \frac{-1}{\tau}\right)$$

$$= \frac{I_0}{\tau}e^{-t/\tau}$$

$$\text{Power} = L\left[\frac{I_0}{\tau}e^{-t/\tau}\right]\left(I_0\left(1-e^{-t/\tau}\right)\right)$$

$$= \frac{L I_0 I_0}{\tau}e^{-t/\tau}\left(1-e^{-t/\tau}\right)$$

$$= \frac{L I_0^2}{\tau}e^{-t/\tau}\left(1-e^{-t/\tau}\right)$$

Basic

e^{x}

$\Big\}$ differentiation

e^{x}

e^{-5x}

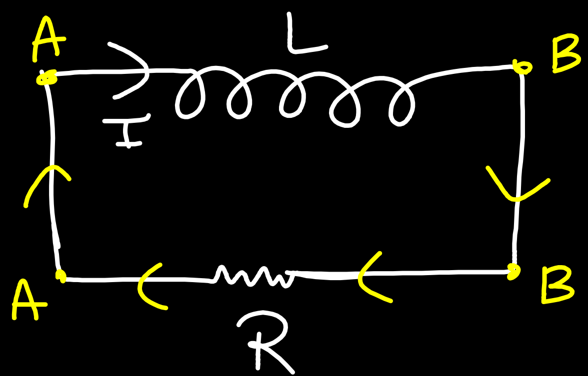
$\Big\}$ diff

$\left(e^{-5x}\right)(-5)$

Decay of Current

(initial current $= I_0$) (I decreasing)

$\frac{dI}{dt}$ -ve.



$$\cancel{V_A} - L\frac{dI}{dt} - IR = \cancel{V_A}$$

$$-L\frac{dI}{dt} = IR$$

$$-L\frac{dI}{dt} = IR$$

$$I = -\frac{L}{R}\frac{dI}{dt}$$

$$-\frac{R}{L}dt = \frac{dI}{I}$$

$$-\frac{R}{L}\int dt = \int \frac{dI}{I}$$

$$-\frac{Rt}{L}\Bigg]_0^t = \log_e I \Bigg]_{I_0}^{I}$$

$$-\frac{R}{L}t = \log_e I_f - \log_e I_0$$

$$-\frac{R}{L}t = \log_e\left(\frac{I_f}{I_0}\right)$$

$\Rightarrow$ anti log

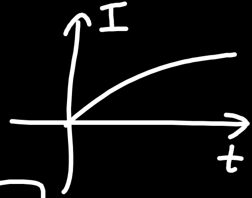
$$e^{-\frac{R}{L}t} = \frac{I_f}{I_0}$$

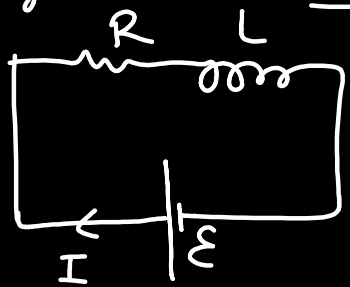
$$I_f = I_0\, e^{-tR/L}$$

$$\boxed{I_f = I_0\, e^{-t/\tau}}$$

$\tau = L/R$

I growth



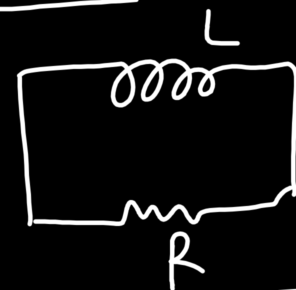


$$I = I_0\left(1 - e^{-t/\tau}\right)$$

Starting $I = 0$

∞ $I = I_0$

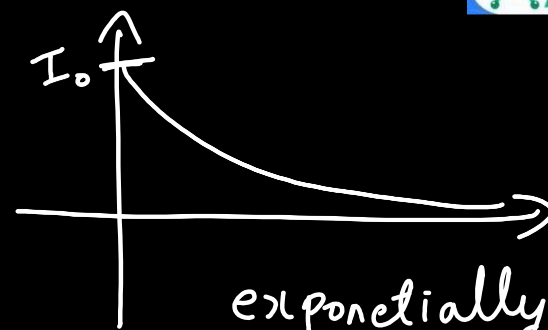
I decay



$$I = I_0\, e^{-t/\tau}$$

$t = 0$ $I = I_0$

∞ $I = 0$



exponetially

decreasing

Starting energy $= \frac{1}{2} L I_0^2$

finally lost by Resistor.

Inductor + Motional Emf + SHM + Circuit Solving + NLM

Basics of SHM

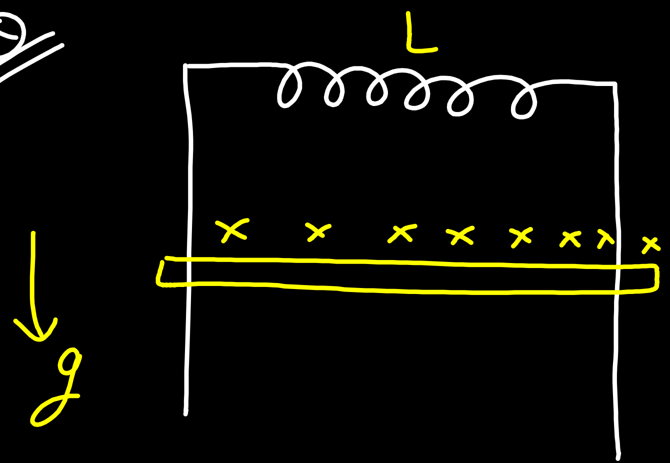
$$\frac{d^2x}{dt^2} = -\omega^2 x$$

$$\boxed{\text{double diff of } x \propto -(x)}$$

$$\text{Time} = \frac{2\pi}{\omega}$$

& $x = A\sin(\omega t)$ if $x=0$ from starting.

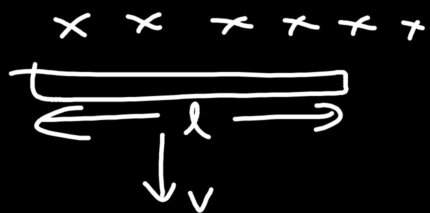
Q

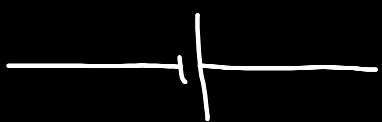


Conducting Rod
movable Rod
mass = m
length = ℓ

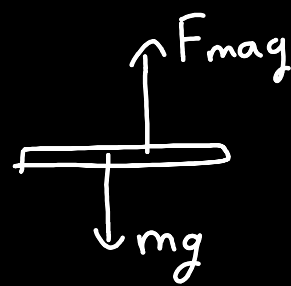
Initial current = 0
Rod released from rest
magnetic field = B
resistance of loop is negligible.

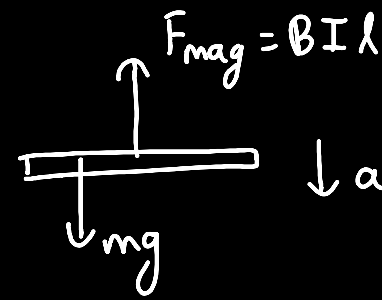
Find
① Draw FBD of rod
② vel as function of time





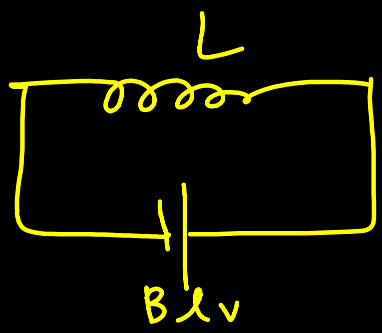
$$\mathcal{E}_{mf} = Bl v$$





$$F_{net} = ma$$

$$\boxed{mg - BIl = ma}$$



$$L\frac{dI}{dt} = Bl v$$

$$\boxed{\frac{dI}{dt} = \frac{Blv}{L}} \text{ — } (2)$$

$$\boxed{mg - BIl = ma} \text{ — } (2)$$

$$mg - ma = BIl$$

$$\Downarrow \text{ diff}$$

$$0 - m\frac{da}{dt} = Bl\frac{dI}{dt}$$

$$-\frac{m}{Bl}\frac{da}{dt} = \frac{dI}{dt}$$

$$\frac{dI}{dt} = \frac{Blv}{L} = \frac{-m}{Bl}\frac{da}{dt}$$

$$-\frac{B^2l^2}{mL}v = \frac{da}{dt}$$

$$a = \frac{dv}{dt}$$

$$\frac{da}{dt} = \frac{d^2v}{dt}$$

$$\frac{d^2v}{dt^2} = -\left(\frac{B^2l^2}{mL}\right)v \Longrightarrow$$ Differential equation

SHM

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

$$x = A\sin(\omega t)$$

if $t=0$ पर $x=0$

$$\frac{d^2v}{dt^2} = -\left(\frac{B^2l^2}{mL}\right)v$$

$$\omega^2 = \frac{B^2l^2}{mL}$$

$$\omega = \frac{Bl}{\sqrt{mL}}$$

$$\boxed{v = A\sin(\omega t)}$$

If displ required

$$\int_0^t v\,dt = x \Rightarrow \int_0^t A\sin(\omega t) = \frac{A\left[-\cos(\omega t)\right]_0^t}{\omega}$$

$$\text{Displ} = \frac{A}{\omega}\left[1-\cos\omega t\right]$$

$V = A \sin(\omega t)$

How to find A

$t = 0 \qquad acc = g$

$$a = \frac{dv}{dt} = A \cos(\omega t) \times \omega$$

$$acc = A\omega \cos(\omega t)$$

$t = 0$

$$A\omega = g$$

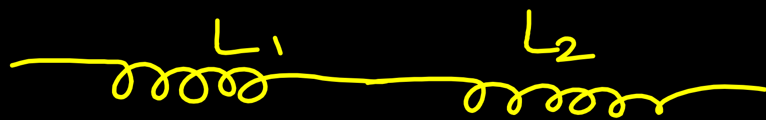
Combination of Inductors :

(without considering mutual inductance)

Resis
Induct

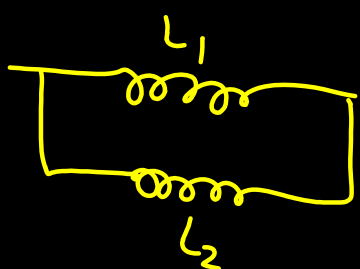
Capacitor
Spring

① Series



$\Rightarrow \quad L_{eq} = L_1 + L_2$

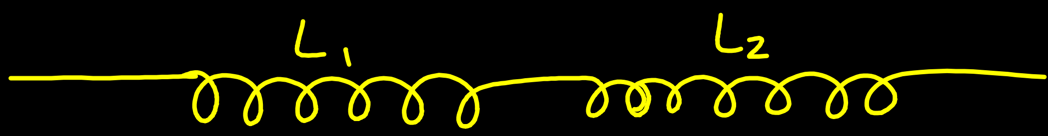
② Parallel



$$\Rightarrow \quad \frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$$

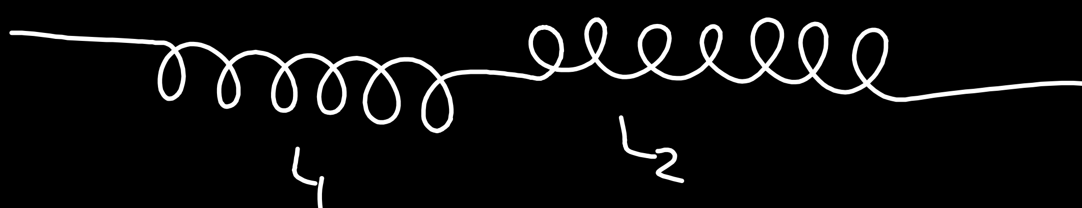
If Mutual inductance considered

Both Sense



$L_1\frac{dI}{dt}$ $M\frac{dI}{dt}$ $L_2\frac{dI}{dt}$ $M\frac{dI}{dt}$

$$\boxed{L_{eq} = L_1 + L_2 + 2M}$$

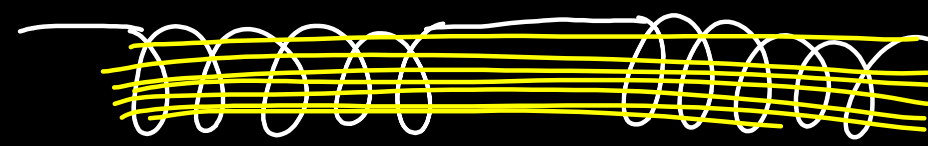


$$L_{eq} = L_1 + L_2 - 2M$$

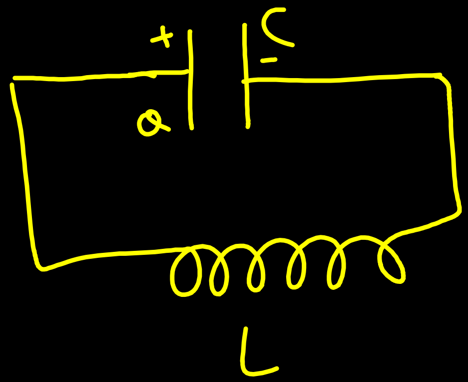
$$M = k\sqrt{L_1 L_2}$$

$k = 1$ for ideal coupling

$$M = \sqrt{L_1 L_2}$$



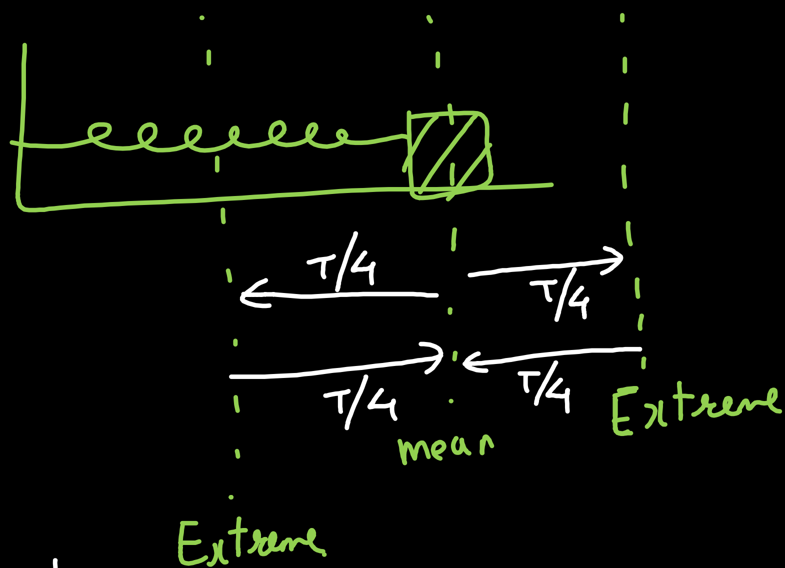
L-C oscillation



oscillation matches with SHM

SHM Basics

T = time period



$\lvert displ \rvert \Rightarrow$	max	0	maximum
$\lvert speed \rvert \Rightarrow$	0	maximum	0

Standard Equations

$\dfrac{d^2x}{dt^2} = -\omega^2 x$

$\dfrac{d^2x}{dt^2} \propto -x$

$T = \dfrac{2\pi}{\omega}$

$x = A\sin(\omega t)$ $t=0$ पर $x=0$

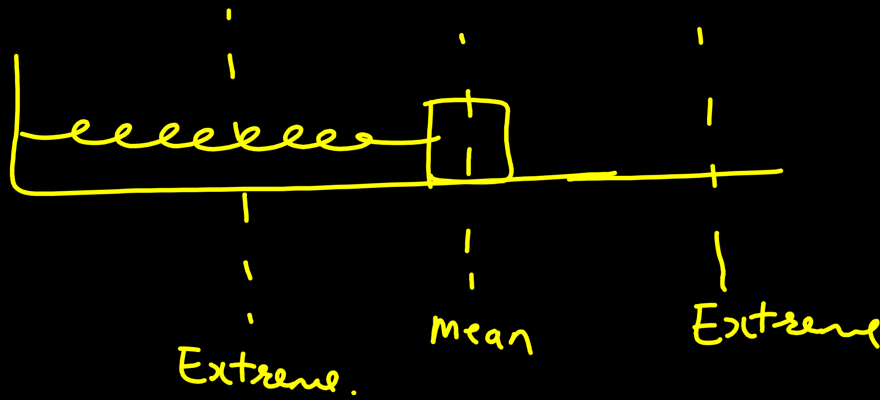
$x = A\cos(\omega t)$ $t=0$ पर $x = \text{maximum}$

Energy

$TE = PE + KE$

$TE = \text{constant}$

at mean PE = 0 here



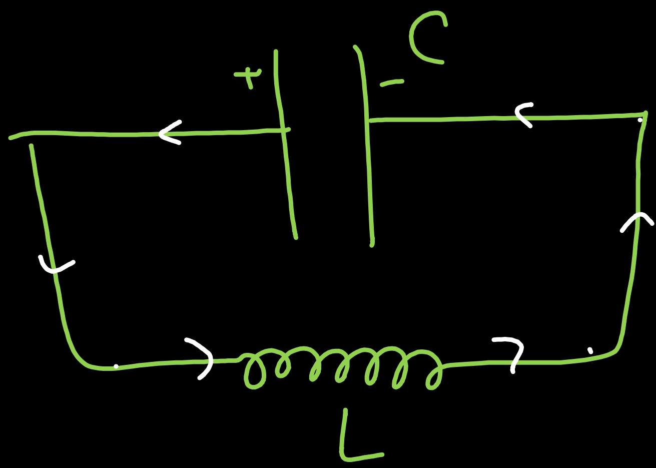
$PE = \frac{1}{2}kx^2$

$KE = \frac{1}{2}mv^2$

PE ⇒	maximum	0	maximum
KE ⇒	0	maximum	0

$$\boxed{\begin{aligned} TE &= PE + KE \\ &= PE_{max} + 0 \\ &= 0 + KE_{max} \end{aligned}}$$

LC oscillation



Suppose initially charge on capacitor $= Q_0$

current $= 0$

$$\left|\frac{Q}{C}\right| = \left|L\frac{dI}{dt}\right|$$

$$\frac{1}{LC}Q = \frac{dI}{dt}$$

$$\frac{1}{LC}Q = -\frac{d^2Q}{dt^2}$$

$$I = \frac{dQ}{dt}$$

But Q is decreasing

dQ -ve

$$I = -\frac{dQ}{dt}$$

$$\frac{dI}{dt} = -\frac{d^2Q}{dt^2}$$

$$\boxed{\frac{d^2Q}{dt^2} = \frac{-1}{LC}\,Q}$$

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

double diff of $Q \propto -Q$

SHM

Q is performing SHM

$$\Rightarrow \quad \omega^2 = \frac{1}{LC}$$

$$\boxed{\begin{aligned} \omega &= \frac{1}{\sqrt{LC}} \\ T &= \frac{2\pi}{\omega} = 2\pi\sqrt{LC} \\ f &= \frac{1}{T} = \frac{1}{2\pi\sqrt{LC}} \end{aligned}}$$

☆☆☆.

$$\boxed{\begin{array}{l} Q = Q_0 \sin(\omega t) \quad \text{if } t=0 \quad Q=0 \\ Q = Q_0 \cos(\omega t) \quad \text{if } t=0 \quad Q = Q_0 \text{ max} \end{array}}$$

if initially Capacitor charged

$$\boxed{Q = Q_0 \cos(\omega t)}$$

$$\frac{dQ}{dt} = -Q_0 \sin(\omega t) \times \omega$$

$$\boxed{I = -Q_0 \omega \sin(\omega t)}$$

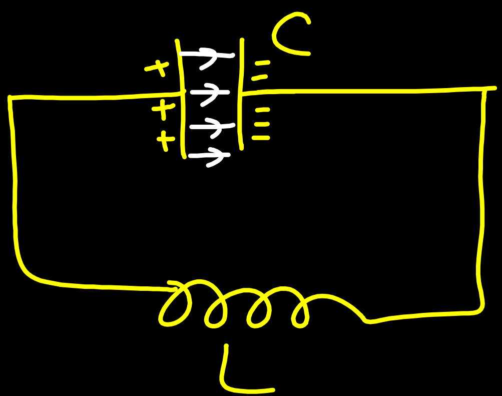
$$E \text{ of capacitor} = \frac{Q^2}{2C}$$

$$\boxed{E_{capacitor} = \frac{Q_0^2}{2C} \cos^2 \omega t}$$

$$E \text{ of inductor} = \frac{1}{2} L I^2$$

$$\boxed{E_{inductor} = \frac{L Q_0^2 \omega^2}{2} \sin^2(\omega t)}$$

Focus on Energy

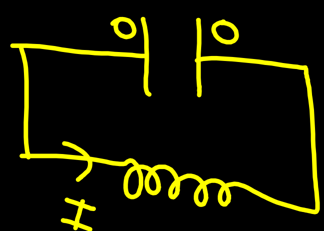


\# $t = 0$ $\quad Q = Q_0 \quad I = 0$

$$TE = E_{capacitor} + E_{inductor}$$

$$TE = \frac{Q_0^2}{2C} + 0$$

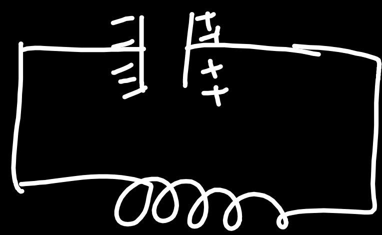
\# $t = T/4$



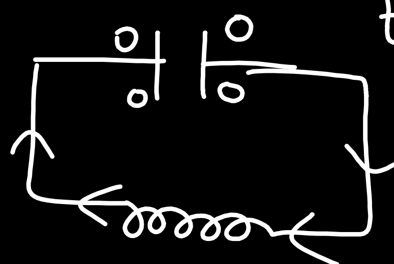
$$Q = 0 \quad I = I_{max} = Q_0 \omega$$

$$TE = E_{capacitor} + E_{inductor}$$

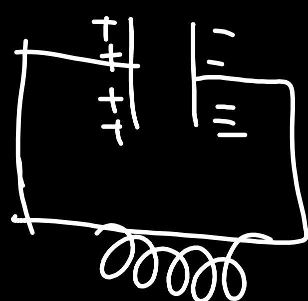
$$= 0 + \frac{1}{2} L (I_{max})^2$$



$t = T/2$ $\quad Q = Q_0$ (max.) $\quad I = 0$



$t = 3T/4$ $\quad Q = 0$ $\quad I = \text{max}$



$t = T$ $\quad Q = Q_0$ (max) $\quad I = 0$

at any general time

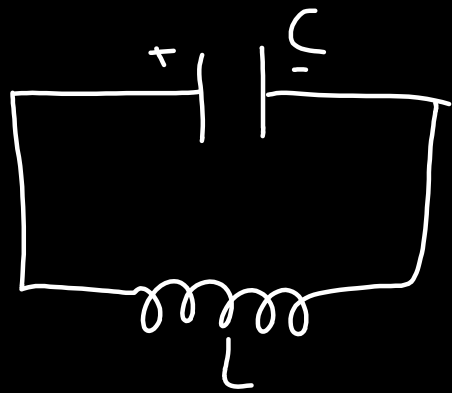
$$Q = Q_0 \cos \omega t$$

$$I = Q_0 \omega \sin(\omega t)$$

$$E_{capacitor} = \frac{Q_0^2}{2C} \cos^2 \omega t$$

$$E_{inductor} = \frac{1}{2} L I_0^2 \sin^2 \omega t$$

Q//



$C = 100\,\mu F$

$L = 40\,mH$

initial $Q_0 = 100\,\mu$ (columb)

Find

① angular freq of oscillation.

② time when energy is equally stored in electric & magnetic form

③ time when all energy is in magnetic form.

① $\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{40\times10^{-3}\times100\times10^{-6}}} = \frac{1}{\sqrt{4\times10^{-6}}} = \frac{1}{2\times10^{-3}} = \frac{1000}{3}$

$= 500$ rad/sec.

② $E_{capcitor} = \frac{Q^2}{2C}$ $\qquad Q = Q_0\cos(\omega t)$

$E_{inductor} = \frac{1}{2}LI^2$ $\qquad I = Q_0\omega\sin(\omega t)$

$E_{capacitor} = \frac{Q_0^2}{2C}\cos^2$

$E_{ind} = \frac{1}{2}LQ_0^2\omega^2\sin^2 = \frac{1}{2}\cancel{L}Q_0^2\frac{1}{\cancel{L}C}\sin^2 = \frac{Q_0^2}{2C}\sin^2$

$E_{cap} = E_{ind}$

$\cos = \sin$

45° or $\pi/4$

$\omega t = \frac{\pi}{4}$

$t = \frac{\pi}{4\omega}$

(2) $t = \frac{\pi}{4(500)} = \frac{\pi}{2000}$ sec

(3) $t = \frac{T}{4} = \left(\frac{2\pi}{\omega}\right)\frac{1}{4} = \frac{\pi}{2\omega} = \frac{\pi}{2(500)} = \frac{\pi}{1000}$ sec.

In above Question

Find (4) max I in circuit

(5) TE of circuit

(4) $I_{max} = Q_0 \omega$

$= (100\mu) 500$

$= 5 \times 10^4 \times 10^{-6}$

$= \underline{0.05} A$

$TE = \dfrac{Q^2}{2C} + \dfrac{1}{2} L I^2$

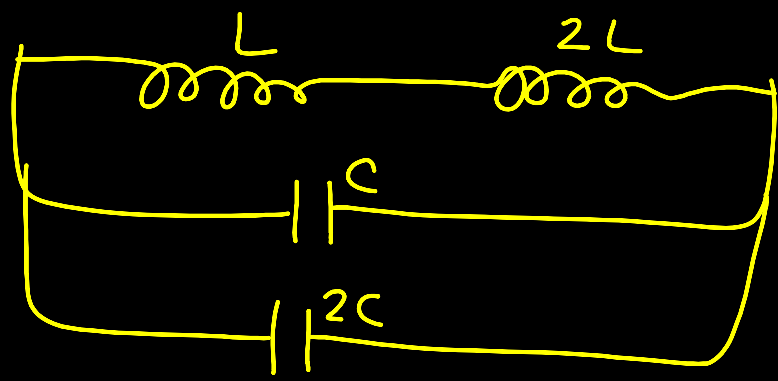
$= \dfrac{Q_{max}^2}{2C} + 0$

$= 0 + \dfrac{1}{2} L I_{max}^2$

(5) $TE = \frac{Q_{max}^2}{2C} = \frac{(100\mu)^2}{2(100\mu)} = \frac{100\mu}{2} = 50\mu$

$= \underline{5 \times 10^{-5} \text{ Joule}}$

Q freq of oscillation of circuit



series

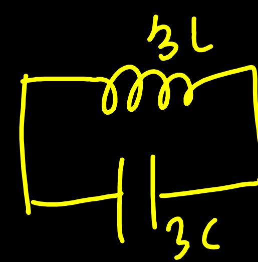
$L = L_1 + L_2$

$L_{eq} = 3L$

parallel

$C_{eq} = C_1 + C_2$

$= 3C$



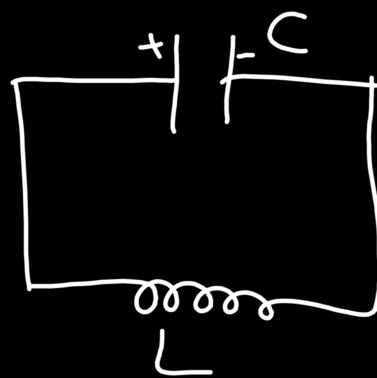
$$f = \frac{1}{2\pi\sqrt{(3L)(3C)}}$$

$$f = \frac{1}{6\pi\sqrt{LC}}$$

Q

initial charge on capacitor $= Q_0$

What will be the charge on capacitor when $E_{electrical}$ is equal to $E_{magnetic}$?



a) $Q_0/2$

b) $Q_0/\sqrt{2}$

c) $Q_0/\sqrt{3}$

d) $\frac{\sqrt{3}}{2} Q_0$

$$Q = Q_0 \cos(\omega t)$$

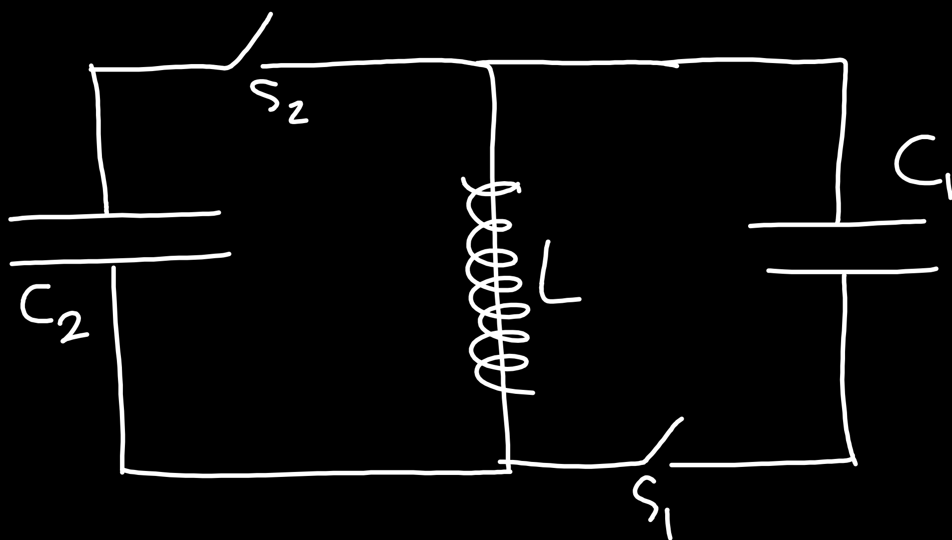
$$= Q_0 \frac{1}{\sqrt{2}}$$

$$E_{cap} = E_{ind}$$

$$\cos^2 = \sin^2 \qquad 45^\circ$$

$$\theta = \omega t = \pi/4$$

Q



$C_1 = 900\,\mu F$

$C_2 = 100\,\mu F$

$L = 10\,H$

Initially C_1 was charged with 100 Volt battery & C_2 was uncharged.

Now S_1 is closed & S_2 is open for sometime (t_1). And after that S_1 is open & S_2 is closed for time t_2. Finally ΔV across C_2 was 300 Volt

Find minimum value of t_1 & t_2

Hint

$t=0$ C_1 energy $\longrightarrow \frac{1}{2}C_1V_1^2 = \frac{1}{2}(900\mu)(100)^2 = \frac{9}{2}\,J$

finally C_2 energy $\longrightarrow \frac{1}{2}C_2V_2^2 = \frac{1}{2}(100\mu)(300)^2 = \frac{9}{2}\,J$

minimum $t_1 \longrightarrow \frac{T_1}{4} = \frac{2\pi\sqrt{LC_1}}{4} = \frac{\pi}{2}\sqrt{(10)(900\,\mu)} = \frac{3}{20}$ sec

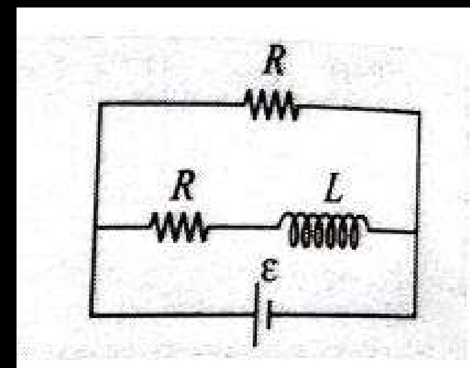
minimum $t_2 \longrightarrow \frac{T_2}{4} = \frac{2\pi\sqrt{LC_2}}{4} = \frac{2\pi}{4}\sqrt{(10)(100\,\mu)} = \frac{1}{20}$ sec

$\pi \simeq \sqrt{10}$

Find the current flowing in the battery at

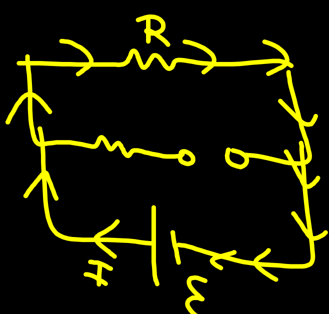
(i) $t = 0$

(ii) $t = \infty$



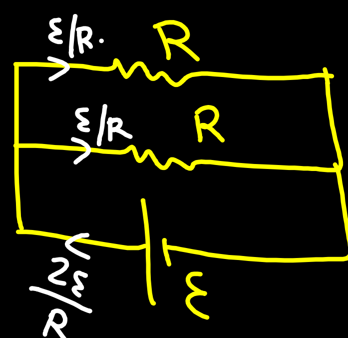
$t = 0$

 $\Rightarrow$ open

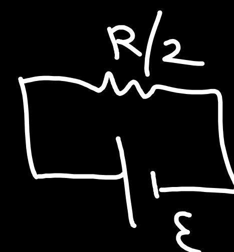


$I = \frac{\varepsilon}{R}$

$t = \infty$



$=$



$I = \frac{\varepsilon}{R/2}$

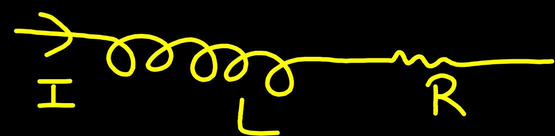
$I = \frac{2\varepsilon}{R}$

An inductor coil stores 64 J of magnetic field energy and dissipates energy at the rate of 640 W when a current of 8A is passed through it. If this coil is joined across an ideal battery, find the time constant of the circuit in seconds.

a. 0.4 **b.** 0.8 **c.** 0.125 **d.** 0.2

Jee

Inductor Coil $\Rightarrow$



$I = 8A$

$$\left(\frac{1}{2} L I^2\right) = 64$$

Powerloss

$$I^2 R = 640$$

$$\tau = \frac{L}{R}$$

$$\frac{1}{2} L I^2 = 64$$

$$I = 8$$

$$\frac{1}{2} L (8)^2 = 64$$

$$L = 2$$

$$I^2 R = 640$$

$$(8)^2 R = 640$$

$$R = 10$$

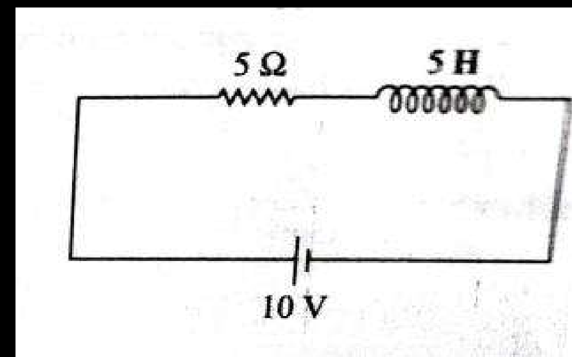
$$\tau = \frac{L}{R}$$

$$= \frac{2}{10}$$

$$\tau = 0.2 \text{ sec}$$

At $t = \tau$

At one time constant, find the



(i) current through the resistor $I = ?$

$I^2 R = ?$

(ii) rate at which energy is dissipated across the resistor

$I_0 = \frac{\varepsilon}{R} = \frac{10}{5} = 2$

(iii) rate at which energy is stored in the inductor $\left(L\frac{dI}{dt}\right) I$

$\tau = \frac{L}{R} = \frac{5}{5} = 1$

(iv) power is delivered by the battery.

$P = (\varepsilon) I$

$$I = I_0\left(1 - e^{-t/\tau}\right)$$

$$\boxed{I = 2\left(1 - e^{-t}\right)}$$

① at $t = 1\,\text{sec}$

$$I = 2\left(1 - e^{-1}\right)$$

$$= 2\left(1 - \frac{1}{e}\right)$$

$$= 2\left(1 - \frac{1}{2.7}\right)$$

$$= 2\,(0.63)$$

$$I = 1.26\,A$$

② $I^2 R$

$$(1.26)^2 (5)$$

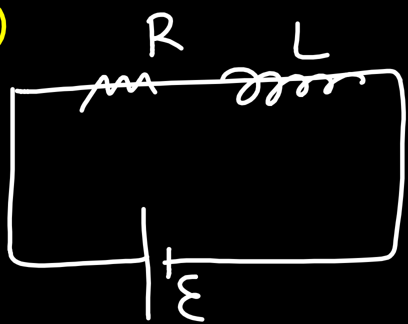
$$\underline{\underline{7.938\ \text{Watt}}}$$

④ $P = \mathcal{E} I$

$$= (10)(1.26)$$

$$P = 12.6$$

3



Power given by Battery = Power lost from R + Power stored in L

$12.6 = 7.938 + $ Power stored in L

$4.662 =$ Power of Inductor

or

③ Power of $L = L\left(\frac{dI}{dt}\right) I = L\left[2e^{-t}\right]\left[2(1-e^{-t})\right]$

$I = 2(1-e^{-t})$

$\frac{dI}{dt} = 2\left(0 - e^{-t}(-1)\right)$

$\frac{dI}{dt} = 2e^{-t}$

$L = 5$

$t = 1$

$= 5\left[2e^{-1}\right] 2\left(1-e^{-1}\right)$

$= \frac{20}{e}\left(1-\frac{1}{e}\right)$

$= 20\,\frac{(0.63)}{2.7}$

$= \underline{4.662}$

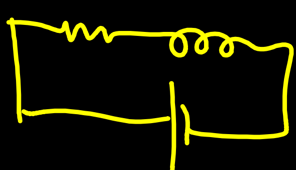
An inductor of 10 mH is connected to a 20 V battery through a resistor of 10 kΩ and a switch. After a long time, when maximum current is set up in the circuit, the current is switched off. The current in the circuit after 1 μs is $\frac{x}{100}$ mA. Then, x is equal to ...74... . (Take, $e^{-1} = 0.37$)

Jee 2021

$L = 10\,mH$

$\mathcal{E}_{mf} = 20\,Volt$

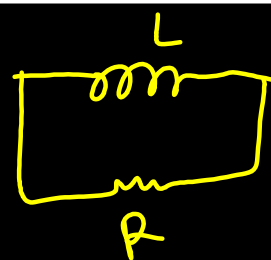
$R = 10{,}000\,\Omega$



I_0 ✓

$I_0 = \frac{20}{10000}$

$I_0 = 2\,mA$



decay

after 1μs find I

$I = I_0\, e^{-t/\tau}$

$$I = \left(2\, e^{-t/\tau}\right) mA \qquad \tau = \frac{L}{R} = \frac{10m}{10k} = 10^{-6}$$

At $t = 1\mu s$

$$I = 2\, e^{-\frac{1\mu s}{1\mu}}$$

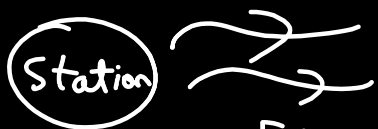
$$= 2e^{-1}$$

$$= 2\frac{1}{e} = \frac{2}{2.7} = 0.74\ mA = \frac{x}{100} mA$$

$$74 = x$$

A transmitting station releases waves of wavelength 960 m. A capacitor of 2.56 μF is used in the resonant circuit. The self-inductance of coil necessary for resonance is ...10... $\times 10^{-8}$ H.

Jee 2021



EMwaves

$\lambda = 960m$

$$f = \frac{c}{\lambda}$$

$C = 2.56\mu$

$L = ?$

$$f = \frac{1}{2\pi\sqrt{LC}}$$

Resonance

$\downarrow$

frequency match karna

$$\frac{c}{\lambda} = \frac{1}{2\pi\sqrt{LC}}$$

$$\frac{3\times 10^{8}}{960} = \frac{1}{2\pi\sqrt{L(2.56)\times 10^{-6}}}$$

$$L(2.56\times 10^{-6}) = \frac{1}{4\pi^2}\,\frac{960}{3\times 10^{8}}\times\frac{960}{3\times 10^{8}}$$

$$L = \frac{96\cancel{0}\times 96\cancel{0}}{4\times 3\times 3\times \pi^2\times 10^{8}\times \cancel{10^{8}}\times 2.56\times \cancel{10^{-6}}}$$

$$= \frac{\overset{32}{\cancel{96}}\times \overset{32}{\cancel{96}}}{\cancel{4}\times\cancel{9}\times\cancel{\pi^2}\times\cancel{256}}\;10^{-8} = \underline{10\times 10^{-8}}$$