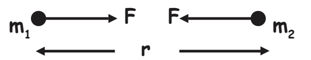
GRAVITATION

NEWTON'S LAW OF GRAVITATION



$$F = \frac{Gm_1m_2}{r^2}$$

G - Universal gravitational constant

Value of G

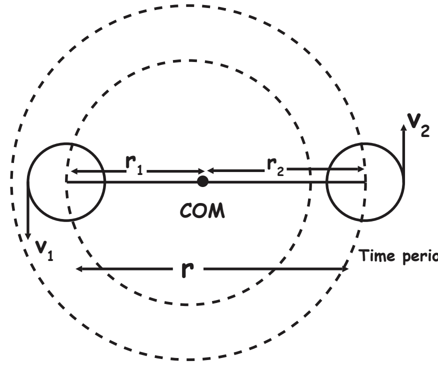
$6.67\times10^{-11}\ Nm^2Kg^{-2}$ (SI or MKS)

6.67×10^{-8} dyne cm^2g^{-2} (CGS)

Dimensional formula G

$[G]=[M^{-1}L^3T^{-2}]$

ROTATION OF 2 MASSES UNDER MUTUAL GRAVITATIONAL FORCE OF ATTRACTION



$$V_1 = \sqrt{\frac{Gm_2^2}{(m_1+m_2)\,r}}$$

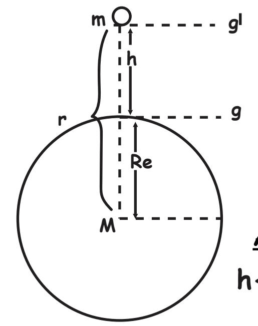
$$V_2 = \sqrt{\frac{Gm_1^2}{(m_1+m_2)\,r}}$$

Time period $T^2 = 4\pi^2 \dfrac{r^3}{G(m_1+m_2)}$

or $\boxed{T^2 \propto r^3}$

VARIATION IN THE VALUE OF ACCELERATION DUE TO GRAVITY

● Variation due to height 'h'



$$g' = \frac{GM}{(R+h)^2}$$

$$g' = \frac{gR^2}{(R+h)^2}$$

General Equation

Approximate equation

$h<<R$ ($h < 100$ km)

use, $g' = g\left(1-\frac{2h}{R}\right)$

Note the point

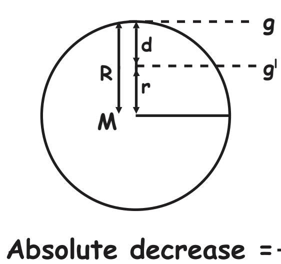
If $h<<R$, then decrease in the value of g with height

Absolute decrease $=\Delta g = g - g' = \frac{2hg}{R}$

Fractional decrease $= \frac{\Delta g}{g} = \frac{g-g'}{g} = \frac{2h}{R}$

Percentage decrease $= \frac{\Delta g}{g} = \frac{g-g'}{g}\times100 = \frac{2h}{R}\times 100$

● Variation due to depth 'd'



$$g' = g\left[1-\frac{d}{R}\right] = \frac{gr}{R}$$

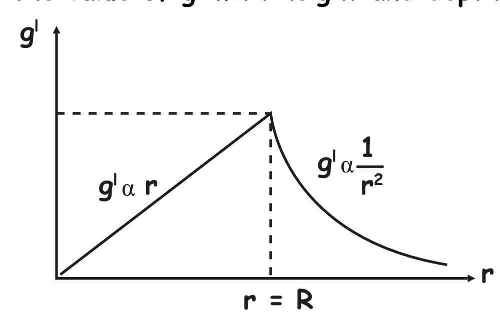
Absolute decrease $= \frac{\Delta g}{g} = g-g' = \frac{dg}{R}$

Fractional decrease $= \frac{\Delta g}{g} = \frac{g-g'}{g} = \frac{d}{R}$

Percentage decrease $= \frac{\Delta g}{g}\times100 = \frac{d}{R}\times100$

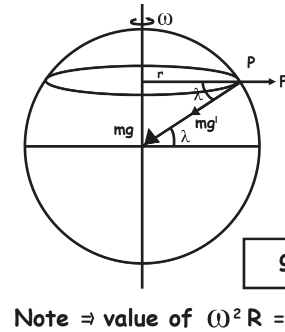
Very important graph

The graphical representation of change in the value of g' with height and depth



for $r \le R$, $g' = \frac{gr}{R}$ for $r \ge R$, $g' = \frac{gR^2}{r^2}$

● Variation of g due to rotation of earth



Latitude - Angle which the line joining the point to the centre of earth makes with the equitorial plane

$\boxed{g' = g - \omega^2 R\cos^2\lambda}$

Note ⇒ value of $\omega^2 R = 0.034$

For poles $\lambda = 90°$ $g' = g$

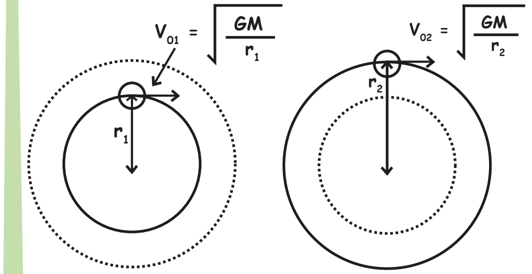
There is no effect of rotational motion of the earth on the value of g at poles.

For equator $\lambda = 0°$ $g' = g - \omega^2 R$

The effect of rotational motion of the earth on the value of g at the equator is maximum.

When a body of mass m is moved from equator to the poles, weight increases by an amount $m(g_p - g_e) = m\omega^2 R$

WORK DONE IN MOVING OBJECT FROM ONE ORBIT TO ANOTHER



$V_{01} = \sqrt{\frac{GM}{r_1}}$ $V_{02} = \sqrt{\frac{GM}{r_2}}$

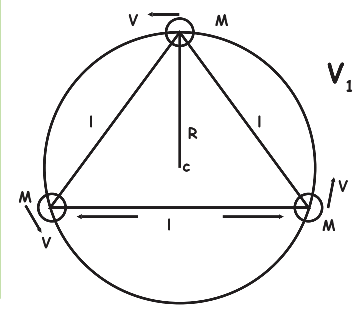
CONCEPT - WORK DONE BY EXTERNAL AGENT = CHANGE IN MECHANICAL ENERGY

$$W = E_2 - E_1 = \frac{GMm}{2}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

IMPORTANT POINTS ABOUT GRAVITATIONAL FORCE

1. Gravitational force is
   * Always attractive in nature
   * Independent of the nature of medium between masses
   * Independent of presence or absence of other bodies
2. Is a central force, acts along the line joining centre of gravity of two bodies.
3. Conservative force

THREE EQUAL MASSES REVOLVING UNDER MUTUAL GRAVITATIONAL FORCE

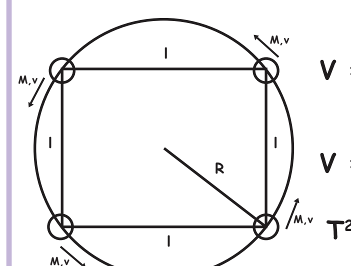


$$V_1 = \sqrt{\frac{GM}{l}} = \sqrt{\frac{GM}{R\sqrt3}}$$

Time period

$T^2 \propto l^3$

FOUR EQUAL MASSES UNDER MUTUAL GRAVITATIONAL FORCE

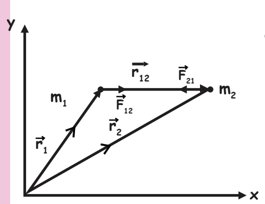


$$V = \sqrt{\frac{GM}{2\sqrt2\,l}(2\sqrt2+1)}$$

$$V = \frac{1}{2}\sqrt{\frac{GM}{R}(2\sqrt2+1)}$$

$T^2 \propto l^3$ Here $R = \frac{l}{\sqrt2}$

VECTOR FORM



$\vec F_{21}$ = force on 2 due to 1

$\vec r_{12}$ = vector from m_1 to m_2

$$\vec F_{21} = \frac{G\,m_1m_2}{r_{12}^2}(-\hat r_{12})$$

or $\vec F_{21} = -\frac{G\,m_1m_2}{r_{12}^3}\vec r_{12}$

Similarly

$\vec F_{12}$ = force on 1 due to 2

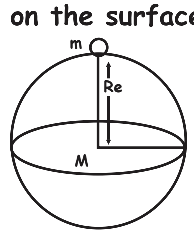
$\vec F_{12} = \frac{G\,m_1m_2}{r_{12}^2}(\hat r_{12})$ or $\vec F_{12} = \frac{G\,m_1m_2}{r_{12}^3}\vec r_{12}$

Clearly, Gravitational force follows:

Newtons third law $\vec F_{21} = -\vec F_{12}$

GRAVITY

Acceleration due to gravity on the surface of earth, $g = \frac{GM_e}{R_e^2}$



M - mass of earth

R - Radius of earth

[Put $GM_e = g R_e^2$ to solve problems easily]

Gravitational force is a two body interaction. Force between two particles does not depend on the presence or absence of other particles. The principle of superposition is valid here. "Force on a particle due to a no. of particles is the resultant of forces due to individual particles."

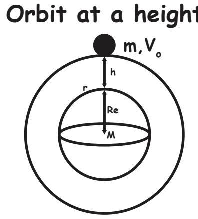
g IN TERMS OF DENSITY OF EARTH

$g = \frac{4}{3}\pi G\rho R_e$ $g \propto \rho R_e$

"If density is mentioned use the above equation"

ORBITAL VELOCITY OF A SATELLITE

Orbit at a height 'h' from the surface



$$V_o = \sqrt{\frac{GM}{(R+h)}} = \sqrt{\frac{gR^2}{(R+h)}}$$

If orbit is closer to earth's surface(neglect 'h') $V_o = \sqrt{\frac{GM}{R}} = \sqrt{gR}$

(called first cosmic velocity)

Note - for easy calculations

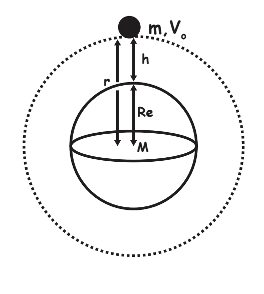
$\sqrt{gR} = 8$ km/s or $\sqrt{\frac{GM}{R}} = 8$ km/s $= 8\times10^3$ m/s

or $\frac{GM}{R} = 64\times10^6$

K.E, P.E AND T.E FOR AN ORBITING SATELLITE

$KE = \frac{GMm}{2r}$, $U = -\frac{GMm}{r}$

$TE = -\frac{GMm}{2r}$

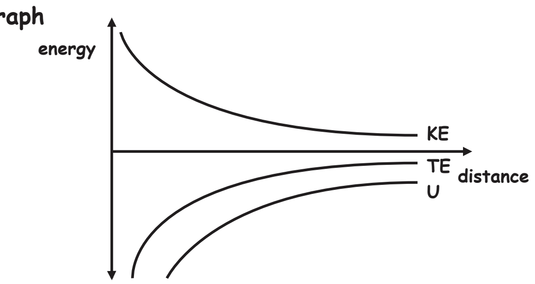


Relation between K.E, U & T.E

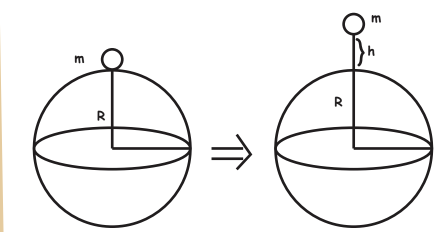
$U = 2\times T.E$

$K.E = -T.E$

Graph



WORK DONE IN MOVING AN OBJECT FROM SURFACE OF EARTH TO HEIGHT h ABOVE SURFACE



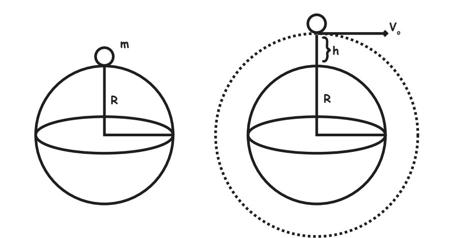
$W = (U_f - U_i) = \frac{-GMm}{R} - \left[\frac{-GMm}{R+h}\right]$

Or, $\boxed{W = mgh\frac{R}{R+h}}$

Work done to move object to a height h = R $\boxed{W = \frac{mgR}{2}}$

Work done to move object to a height h = R/2 $\boxed{W = \frac{mgR}{3}}$

WORK DONE IN MOVING OBJECT FROM SURFACE TO CIRCULAR ORBIT



$W = E_f - E_i$

$W = E_{total} - U_i$

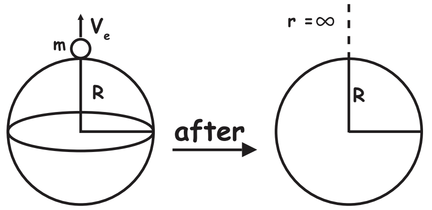
$W = \frac{-GMm}{2(R+h)} + \frac{GMm}{R}$

PHYSICS WALLAH

GRAVITATION

ESCAPE VELOCITY

"Minimum velocity given to an object such that it escapes out of Earth's gravitational field" $v=0$ m



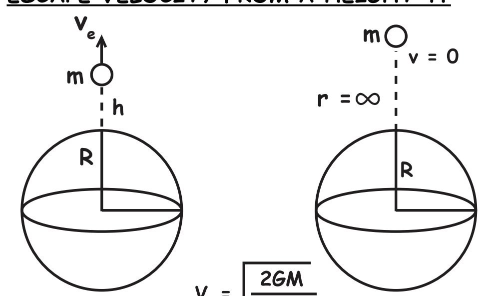
$V_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$ (from the surface of earth)

$V_e = \sqrt{2}\, V_o$

(for closer orbits)

$V_e = \sqrt{2 \times 8} = 8\sqrt{2}$ km/s

ESCAPE VELOCITY FROM A HEIGHT 'H'

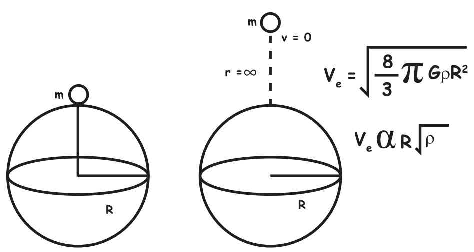


$V_e = \sqrt{\frac{2GM}{(R+h)}}$

ESCAPE ENERGY FOR ORBITING BODY

ΔE = Escape energy = $\frac{GMm}{2(R+h)}$

ESCAPE VELOCITY FROM SURFACE OF EARTH AND RELATION WITH DENSITY



$V_e = \sqrt{\frac{8}{3}\pi G\rho R^2}$

$V_e \propto R\sqrt{\rho}$

TRICK TO SOLVE PROBLEMS

Given speed greater than escape speed(Hint)

find the final speed after escaping (question)

short trick

if $V_{given} = nV_e$ (when $n>1$)

final speed, $V = V_e\sqrt{n^2-1}$

Given speed less than escape speed(Hint)

find the maximum height it reached (question)

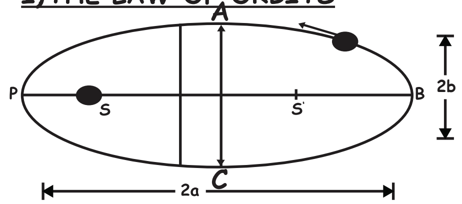
short trick

if $V_{given} = nV_e$ $(n<1)$

maximum height, $h = \frac{n^2R}{1-n^2}$

KEPLER'S LAWS OF PLANETARY MOTION

I)THE LAW OF ORBITS

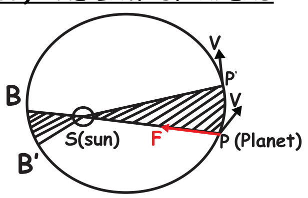


Every planet moves around the sun in an elliptical orbit with sun at one of the foci.

P ⟶ Perihelion (perigee) (nearst point)

B ⟶ apogee or aphelion (farthest point)

II) THE LAW OF AREAS



"The line joining the sun to the planet sweeps out equal areas in equal interval of time"

"i.e. areal velocity is constant"

"According to this law, planet will move slowly when it is farthest from sun & rapidly when is nearest to sun."

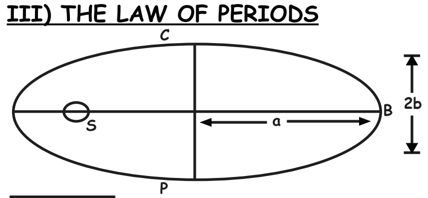
"Law of areas is due to law of conservation of angular momentum"

Areal velocity= $\frac{L}{2m}$

$\frac{\Delta A}{\Delta t} = \frac{L}{2m}$ L ⟶ Angular constant momentum

⇒ Areal velocity is constant

III) THE LAW OF PERIODS

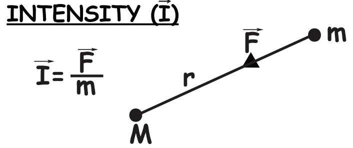


$\boxed{T^2 \propto a^3}$ a ⟶ semi-major axis of elliptical orbit

$\boxed{T^2 = \frac{4\pi^2}{GM_s}a^3}$ M_s=Mass of sun

GRAVITATIONAL FIELD INTENSITY ($\vec{I}$)

$\vec{I} = \frac{\vec{F}}{m}$

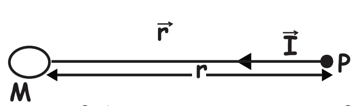


vector quantity

direction: same as that of gravitational force

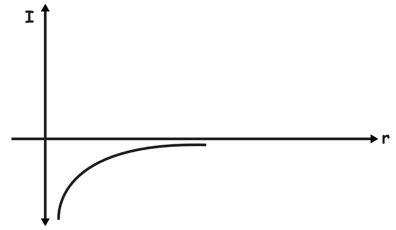
SI unit- N/kg Dimensions-M^0LT^{-2}

Due to point mass

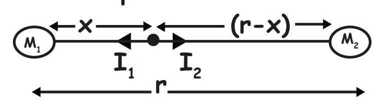


$\vec{I} = -\frac{GM}{r^3}\vec{r}$ Magnitude, $I = \frac{GM}{r^2}$

Graph



Neutral point



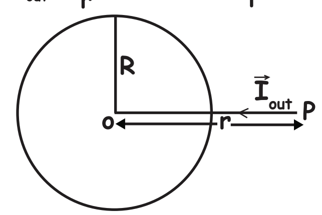
$I_1 = I_2$

$\frac{GM_1}{x^2} = \frac{GM_2}{(r-x)^2}$ or $x = \frac{\sqrt{M_1}\, r}{\sqrt{M_1}+\sqrt{M_2}}$

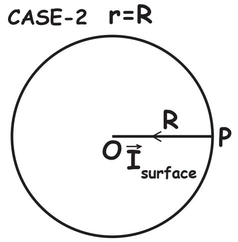
GRAVITATIONAL FIELD INTENSITY DUE TO A SPHERICAL SHELL

CASE-1 $r>R$

$\vec{I}_{out} = \frac{GM}{r^2}(-\hat{r}) \Rightarrow I \propto \frac{1}{r^2}$



CASE-2 $r=R$

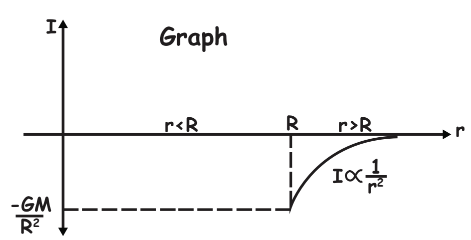


$\vec{I}_{surface} = \frac{GM}{R^2}(-\hat{r})$

CASE-3 $r<R$

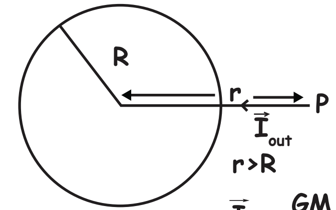
The point is inside then I=0

Graph



SOLID SPHERE

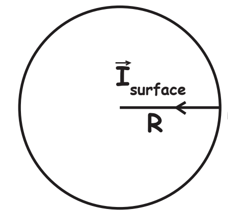
Case-1



$r>R$

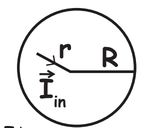
$\vec{I}_{out} = \frac{GM}{r^2}(-\hat{r})$

Case-2



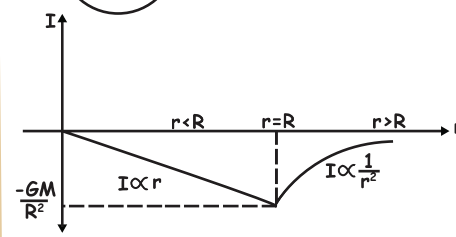
$\vec{I}_{surface} = \frac{GM}{R^2}(-\hat{r})$

Case-3



$r<R$ $\vec{I}_{in} = \frac{GMr}{R^3}(-\hat{r})$

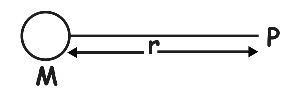
$\vec{I}_{in} \propto r$



GRAVITATIONAL POTENTIAL

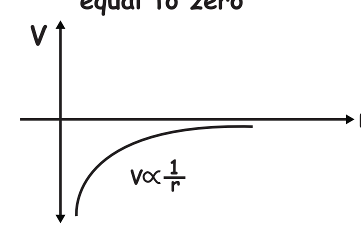
$V = \frac{W_{net}}{m}$ W_{net}-Work done

GRAVITATIONAL POTENTIAL FOR POINT MASS



$V_p = -\frac{GM}{r}$

at $r=\infty$, value of V_p- maximum, equal to zero



RELATION BETWEEN FIELD AND POTENTIAL

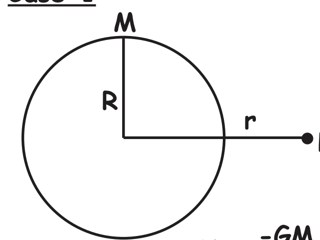
$\vec{I} = -\frac{dV}{dr}$ & $\Delta V = -\int \vec{I} \cdot \vec{dr}$

$\vec{I} = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}$

GRAVITATIONAL POTENTIAL DUE TO OTHER BODIES

SOLID SPHERE

Case-1



$V_{out} = \frac{-GM}{r}$

Case-2

$r=R$

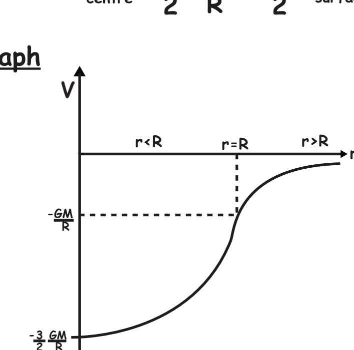
$V_{surface} = \frac{-GM}{R}$

Case-3

$V_{in} = -\frac{GM}{2R^3}(3R^2-r^2)$

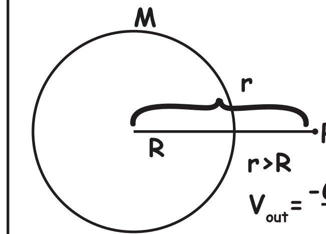
At centre, $V_{centre} = -\frac{3}{2}\frac{GM}{R} = -\frac{3}{2}V_{surface}$

Graph



HOLLOW SPHERE

Case-1



$r>R$

$V_{out} = \frac{-GM}{r}$

Case-2

$r=R$

$V_{surface} = \frac{-GM}{R}$

Case-3

$V_{in} = V_{surface}$=same everywhere

Graph