Mathematical Reasoning & PMI

01 Contrapositive and Converse Statements

- The contrapositive of a statement $p \Rightarrow q$ is the statement $\sim q \Rightarrow \sim p$
- The converse of a statement $p \Rightarrow q$ is the statement $q \Rightarrow p$

02 Compound Statement

Many mathematical statements are obtained by combining one or more statements using some connecting words like "and", "or" etc. those statement are called a "Compound Statement."

03 Sentence

A sentence is called a mathematically acceptable statement if it is either true or false but not both.

04 Negation

A statement which is formed by changing the true value of a given statement by using the word like 'no', 'not' is called negation of given statement.

05 Implications

These are statements with word "if then", "only if" and "if and only if". if p then q is the same as following:

- p implies q is denoted by $p \Rightarrow q$, then symbol $\Rightarrow$ stands for implies
- p is a sufficient condition for q. then symbol $\Rightarrow$
- p only if q • q is a necessary condition for p
- $\sim q$ implies $\sim p$

06 Truth Table for Logical Operations

Conjunction operation:

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction operation:

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Negation operation:

p	$\sim p$
T	F
T	F
F	T
F	T

Implication operation:

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Biconditional operation:

p	q	$p \Leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

07 Additional Important Points

- $p \Rightarrow q = \sim p \vee q$
- $\sim (p \Rightarrow q) = \sim (\sim p \vee q) = p \wedge (\sim q)$
- $p \Leftrightarrow q = (p \Rightarrow q) \wedge (q \Rightarrow p)$
- $\sim (p \Leftrightarrow q) = (p \wedge \sim q) \vee (q \wedge \sim p)$
- $(p \Leftrightarrow q) \Leftrightarrow r = p \Leftrightarrow (q \Leftrightarrow r)$

08 General Logical Equivalences

It comprises the following laws:

Idempotent Law

1. $p \vee p \Leftrightarrow p$
2. $p \wedge p \Leftrightarrow p$

Associative Law

1. $(p \vee q) \vee r \Leftrightarrow p \vee (q \vee r)$
2. $(p \wedge q) \wedge r \Leftrightarrow p \wedge (q \wedge r)$

Commutative Law

1. $p \vee q \Leftrightarrow q \vee p$
2. $p \wedge q \Leftrightarrow q \wedge p$

Distributive Law

1. $p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$
2. $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$

Identity Laws

1. $p \wedge T \Leftrightarrow p$
2. $p \vee F \Leftrightarrow p$
3. $p \wedge F \Leftrightarrow F$

Complement Laws

1. $p \vee \sim p \Leftrightarrow T$
2. $p \wedge \sim p \Leftrightarrow F$
3. $\sim T \Leftrightarrow F$
4. $\sim F \Leftrightarrow T$

Absorption Law

1. $p \vee (p \wedge q) \Leftrightarrow p$
2. $p \wedge (p \vee q) \Leftrightarrow p$

Involution Law

1. $p \sim (\sim p) \Leftrightarrow p$

De-Morgan's Law

1. $q \sim (p \vee q) \Leftrightarrow \sim p \wedge \sim q$
2. $q \sim (p \wedge q) \Leftrightarrow \sim p \vee \sim q$

09 Tautology and Fallacy

- A tautology asserts that every possible interpretation has only one output, namely true.
- Fallacy implies an assertion of false in every possible interpretation.

NOTE: To evaluate tautology and fallacy, we can adapt the concept of the truth table that includes every possible valuation.

p	q	$p \Rightarrow q$	$q \Rightarrow p$	$(p \Rightarrow q) \vee (q \Rightarrow p)$	$\sim \{(p \Rightarrow q) \vee (q \Rightarrow p)\}$
T	T	T	T	T	F
T	F	F	T	T	F
F	T	T	F	T	F
F	F	T	T	T	F

10 Principle of Mathematical Induction

Base Case: The given statement is correct for first natural number that is, for n=1, p(1) is true.

Inductive Step: If the given statement is true for any natural number like n=k then it will be correct for n = k + 1 also that is, if p(k) is true then p(k + 1) will also be true.

The first principle of mathematical induction says that if both the above steps are proven then p(n) is true for all natural numbers.