CHAPTER

8 Indefinite Integration

1. If f & g are functions of x such that $g'(x) = f(x)$ then,

$\int f(x)dx = g(x) + c \Leftrightarrow \frac{d}{dx}\{g(x)+c\} = f(x)$, where c is called the constant of integration.

2. Standard Formula:

(i) $\int (ax+b)^n\, dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c, n \neq -1$

(ii) $\int \frac{dx}{ax+b} = \frac{1}{a} ln\,|ax+b| + c$

(iii) $\int e^{ax+b}\, dx = \frac{1}{a} e^{ax+b} + c$

(iv) $\int a^{px+q} dx = \frac{1}{p}\frac{a^{px+q}}{ln\, a} + c; a > 0$

(v) $\int \sin(ax+b)dx = -\frac{1}{a}\cos(ax+b) + c$

(vi) $\int \cos(ax+b)dx = \frac{1}{a}\sin(ax+b) + c$

(vii) $\int \tan(ax+b)dx = \frac{1}{a} ln \sec|(ax+b)| + c$

(viii) $\int \cot\,(ax+b)dx = \frac{1}{a} ln\,|\sin(ax+b)| + c$

(ix) $\int \sec^2(ax+b)dx = \frac{1}{a}\tan(ax+b) + c$

(x) $\int \text{cosec}^2\,(ax+b)dx = -\frac{1}{a}\cot(ax+b) + c$

(xi) $\int \sec x\, dx = ln\,|(\sec x + \tan x)| + c$

Or $ln\left|\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right| + c$

(xii) $\int \text{cosec}\, x\, dx = ln\,|(\text{cosec}\, x - \cot x)| + c$

Or $ln\left|\tan\frac{x}{2}\right| + c$

Or $-ln\,|(\text{cosec}\, x + \cot x)| + c$

(xiii) $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + c$

(xiv) $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

(xv) $\int \frac{dx}{|x|\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\frac{x}{a} + c$

(xvi) $\int \frac{dx}{\sqrt{x^2+a^2}} = ln\left|\,x + \sqrt{x^2+a^2}\,\right| + c$

(xvii) $\int \frac{dx}{\sqrt{x^2-a^2}} = ln\left|\,x + \sqrt{x^2-a^2}\,\right| + c$

(xviii) $\int \frac{dx}{(a^2-x^2)} = \frac{1}{2a} ln\left|\frac{a+x}{a-x}\right| + c$

(xix) $\int \frac{dx}{x^2-a^2} = \frac{1}{2a} ln\left|\frac{x-a}{x+a}\right| + c$

(xx) $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + c$

(xxi) $\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2} ln\left|\frac{x+\sqrt{x^2+a^2}}{a}\right| + c$

(xxii) $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2} ln\left|\frac{x+\sqrt{x^2-a^2}}{a}\right| + c$

3. Integration by substitutions:

If we substitute $f(x) = t$, then $f'(x)\, dx = dt$

4. Integration by part:

$$\int (f(x)g(x)dx = f(x))\int (g(x))dx - \int\left(\frac{d}{dx}(f(x))\int (g(x))dx\right)dx$$

5. Integration of type:

$\int \frac{dx}{ax^2+bx+c}, \int \frac{dx}{\sqrt{ax^2+bx+c}}, \int \sqrt{ax^2+bx+c}\, bx$

Make the substitution $x + \frac{b}{2a} = t$

6. Integration of type:

$\int \frac{px+q}{ax^2+bx+c}dx, \int \frac{px+q}{\sqrt{ax^2+bx+c}}dx,$

$\int (px+q)\sqrt{ax^2+bx+c}\, dx$

Make the substitution $x + \dfrac{b}{2a} = t$, then split the integral as sum of two integrals one containing the linear term and the other containing constant term.

7. Integration of trigonometric functions:

(*i*) $\displaystyle\int \frac{dx}{a+b\sin^2 x}$ Or, $\displaystyle\int \frac{dx}{a+b\cos^2 x}$ Or,

$\displaystyle\int \frac{dx}{a\sin^2 x + b\sin x\cos x + c\cos^2 x}$, put $\tan x = t$

(*ii*) $\displaystyle\int \frac{dx}{a+b\sin x}$ Or, $\displaystyle\int \frac{dx}{a+b\cos x}$ Or

$\displaystyle\int \frac{dx}{a+b\sin x + c\cos x}$, put $\tan\dfrac{x}{2} = t$.

(*iii*) $\displaystyle\int \frac{a.\cos x + b.\sin x + c}{l.\cos x + m.\sin x + n}dx$.Express

$N^r \equiv A(D^r) + B\dfrac{d}{dx}(D^r) + c$ & proceed.

8. Integration of type:

$\displaystyle\int \frac{x^2 \pm 1}{x^4 + Kx^2 + 1}dx$ where K is any constant.

Divide N^r & D^r by x^2 & put $x \mp \dfrac{1}{x} = t$

9. Integration of type:

$\displaystyle\int \frac{dx}{(ax+b)\sqrt{px+q}}$ Or $\displaystyle\int \frac{dx}{(ax^2+bx+c)\sqrt{px+q}}$;

put $px + q = t^2$.

10. Integration of type:

$\displaystyle\int \frac{dx}{(ax+b)\sqrt{px^2+qx+r}}$, put $ax + b = \dfrac{1}{t}$;

$\displaystyle\int \frac{dx}{(ax^2+b)\sqrt{px^2+q}}$ put $x = \dfrac{1}{t}$

Some Standard Substitution

1. $\displaystyle\int f(x)^n f'(x)dx$ Or $\displaystyle\int \frac{f'(x)}{[f(x)]^n}dx$ put $f(x) = t$ & proceed.

2. $\displaystyle\int \frac{dx}{ax^2+bx+c}, \int \frac{dx}{\sqrt{ax^2+bx+c}}, \int \sqrt{ax^2+bx+c}\,dx$

Express $ax^2 + bx + c$ in the form of perfect square & then apply the standard results.

3. $\displaystyle\int \frac{(px+q)dx}{ax^2+bx+c}, \int \frac{(px+q)}{\sqrt{ax^2+bx+c}}dx$

Express $px + q = A$ (differential coefficient of denominator) $+ B$.

4. $\displaystyle\int e^x[f(x) + f'(x)]dx = e^x.f(x) + c$

5. $\displaystyle\int [f(x) + xf'(x)]dx = xf(x) + c$

6. $\displaystyle\int \frac{dx}{x(x^n+1)}$, $n \in N$, take x^n common & put $1 + x^{-n} = t$.

7. $\displaystyle\int \frac{dx}{x^2(x^n+1)^{(n-1)/n}}$, $n \in N$, take x^n common & put $1 + x^{-n} = t^n$.

8. $\displaystyle\int \frac{dx}{x^n(1\quad x^n)^{1/n}}$, take x^n common and put $1 + x^{-n} = t$.

9. $\displaystyle\int \sqrt{\frac{x-\alpha}{\beta-x}}dx$ Or $\displaystyle\int \sqrt{(x-\alpha)(\beta-x)}$; put $x = \alpha\cos^2\theta + \beta\sin^2\theta$

$\displaystyle\int \sqrt{\frac{x-\alpha}{x-\beta}}dx$ Or $\displaystyle\int \sqrt{(x-\alpha)(x-\beta)}$; put $x = \alpha\sec^2\theta - \beta\tan^2\theta$

$\displaystyle\int \frac{dx}{\sqrt{(x-\alpha)(x-\beta)}}$; put $x - \alpha = t^2$ or $x - \beta = t^2$.