



## Trigonometric Ratios



Note

- At 10<sup>th</sup> level, the trigonometry we study is only for right angled  $\triangle$ .
- The angles discussed (represented by  $\angle A, \angle X, \theta, \alpha, \beta, \gamma, \delta$ ) are  $0^\circ \leq \text{Angle} \leq 90^\circ$  → mostly acute.
- Trigo is the only concept in maths that connects sides to angles both.



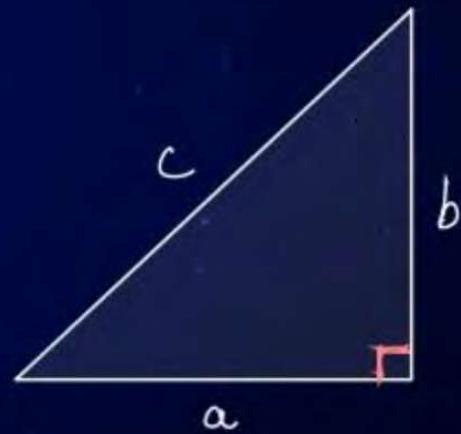
## Trigonometric Ratios

✓ As the name suggests Trigonometric Ratios (Tratio) is the ratio of sides of a right angled  $\Delta$ .

$$\left( \frac{a}{b}, \frac{b}{a}, \frac{b}{c}, \frac{c}{b}, \frac{a}{c}, \frac{c}{a} \right)$$

Since, 6 possibilities, hence 6 Tratio.

→ Tratio are unitless.



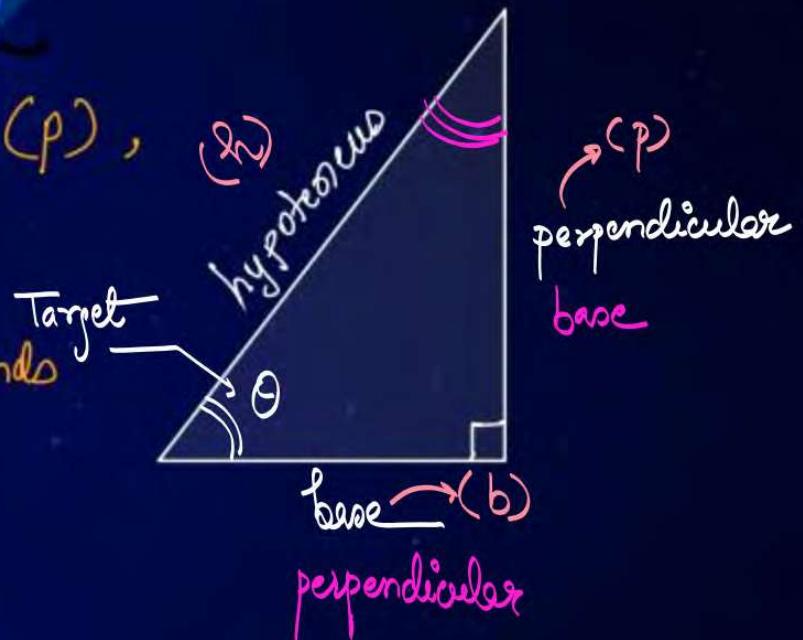


## Trigonometric Ratios



→ The sides are called perpendicular (p), base (b) & hypotenuse (h).

→ 'h' is fixed, but 'p' & 'b' depends on the selection of ' $\theta$ '.





## Trigonometric Ratios



\* first we decide target angle & then we work :-

reciprocal

sine	$\checkmark \sin \theta$	$p/h$
cosine	$\checkmark \cos \theta$	$b/h$
tangent	$\checkmark \tan \theta$	$p/b$
cotangent	$\checkmark \cot \theta$	$b/p$
cosecant	$\checkmark \csc \theta$	$h/p$
secant	$\checkmark \sec \theta$	$h/b$

(P) pandit	(b) badm	(P) prasad
(h)	(h)	(b)
(h)	(h)	(b)
sin	cos	tan
Ob	Qup	Then



## Trigonometric Ratios

\* Some important points

- \*  $\sin \theta$   $\leftrightarrow$  Sine of angle ' $\theta$ ' ✓  
 $\sin \theta \times \theta \times$

\*

$$\begin{aligned}(\sin \theta)^3 &= \sin^3 \theta \\ (\sin \theta)^2 &= \sin^2 \theta \\ (\sin \theta)^m &= \sin^m \theta\end{aligned}$$

→ This is how powers are tackled.



## Trigonometric Ratios



$\sin \rightleftharpoons \text{cosec}$   
 $\cos \rightleftharpoons \sec$   
 $\tan \rightleftharpoons \cot$



\* Some important formulas

a)  $\sin \theta = \frac{1}{\text{cosec } \theta}$

b)  $\text{cosec } \theta = \frac{1}{\sin \theta}$

c)  $\cos \theta = \frac{1}{\sec \theta}$

d)  $\sec \theta = \frac{1}{\cos \theta}$

e)  $\tan \theta = \frac{1}{\cot \theta}$

f)  $\cot \theta = \frac{1}{\tan \theta}$

g)  $\sin \theta \times \text{cosec } \theta = 1$

h)  $\cos \theta \times \sec \theta = 1$

i)  $\tan \theta \times \cot \theta = 1$



$$\sin \angle A = \frac{4}{5}$$

$$\sin \angle P = \frac{8}{10} = \frac{4}{5}$$

## Trigonometric Ratios

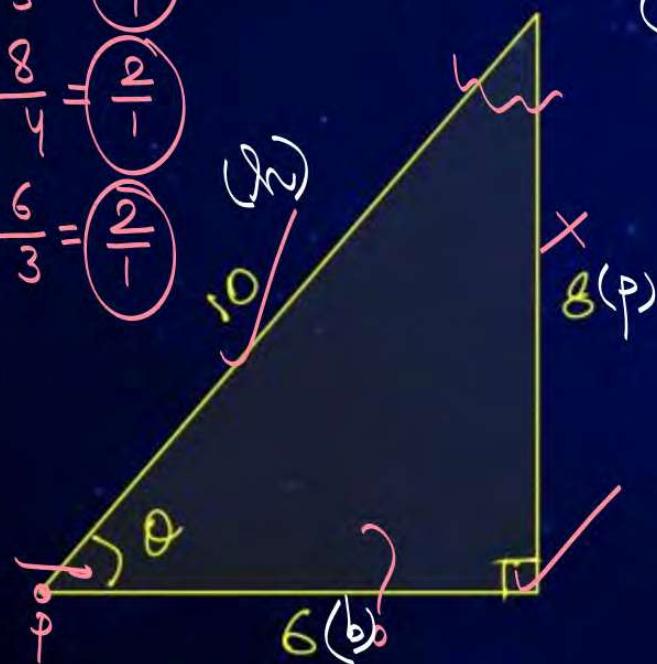


\* For similar  $\triangle$ s, value of T-ratios will be same.

$$\frac{10}{5} = \frac{2}{1}$$

$$\frac{8}{4} = \frac{2}{1}$$

$$\frac{6}{3} = \frac{2}{1}$$

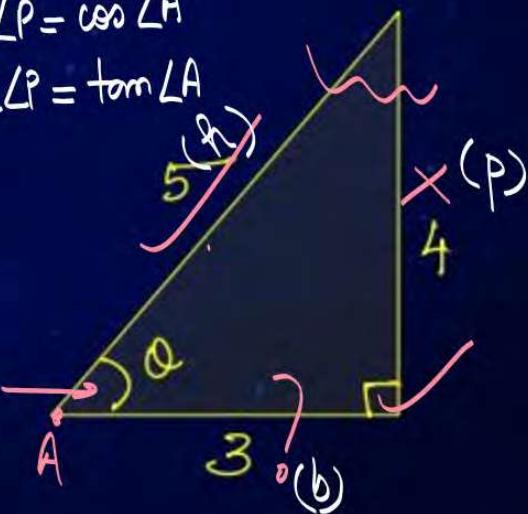


$$\angle P = \angle A$$

$$\sin \angle P = \sin \angle A$$

$$\cos \angle P = \cos \angle A$$

$$\tan \angle P = \tan \angle A$$



## QUESTION

In  $\triangle ABC$ , right-angled at B, AB = 24 cm, BC = 7 cm. Determine:

- (i)  $\sin A \cos A$
- (ii)  $\sin C \cos C$

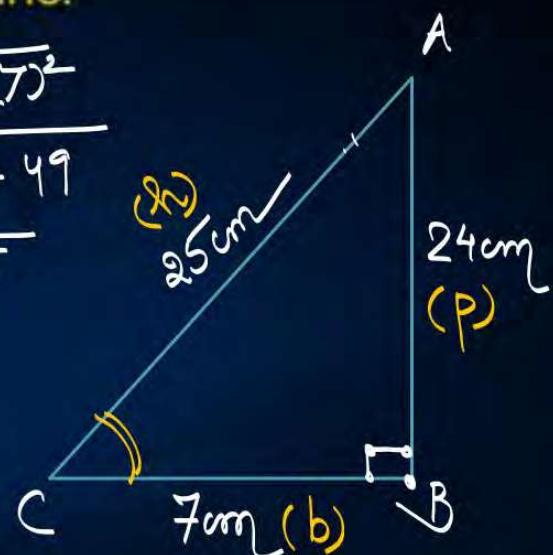
$$\text{Sol}^n \quad \text{i)} \quad \sin A = \frac{P}{R} = \frac{7\text{cm}}{25\text{cm}} = \frac{7}{25}$$

$$\quad \quad \quad \cos A = \frac{b}{R} = \frac{24\text{cm}}{25\text{cm}} = \frac{24}{25}$$

$$\text{ii)} \quad \sin C = \frac{24}{25}$$

$$\quad \quad \quad \cos C = \frac{7}{25}$$

$$\begin{aligned} AC &= \sqrt{(24)^2 + (7)^2} \\ &= \sqrt{576 + 49} \\ &= \sqrt{625} \\ &= 25 \end{aligned}$$



**QUESTION**

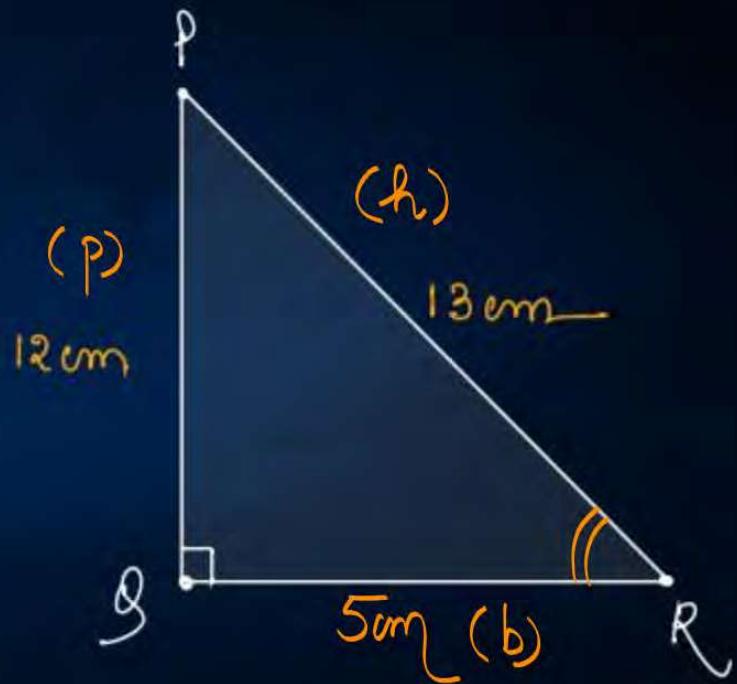
In Fig. find  $\tan P - \cot R$ .  $\rightarrow \frac{5}{12} - \frac{5}{12}$

Sol<sup>n</sup>

$$\tan P = \frac{5}{12}$$

$$= \boxed{O}$$

$$\cot R = \frac{5}{12}$$



**QUESTION**

Given  $\sec \theta = 13/12$ , calculate all other trigonometric ratios.

(5)

Sol<sup>n</sup>

$$\Rightarrow \boxed{\frac{h}{b} = \frac{13}{12}}$$

$h = 13x \text{ units}$   
 $b = 12x \text{ units}$

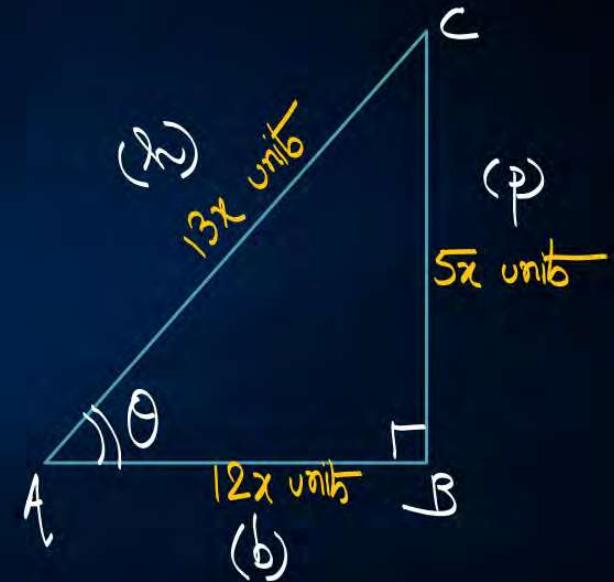
$$\sin \theta = \frac{5}{13}$$

$$\cos \theta = \frac{12}{13}$$

$$\tan \theta = \frac{5}{12}$$

$$\cot \theta = \frac{12}{5}$$

$$\operatorname{cosec} \theta = \frac{13}{5}$$



## QUESTION

Given  $15 \cot A = 8$ , find  $\sin A$  and  $\sec A$ .

Sol<sup>n</sup>

$$\cot A = \frac{8}{15}$$

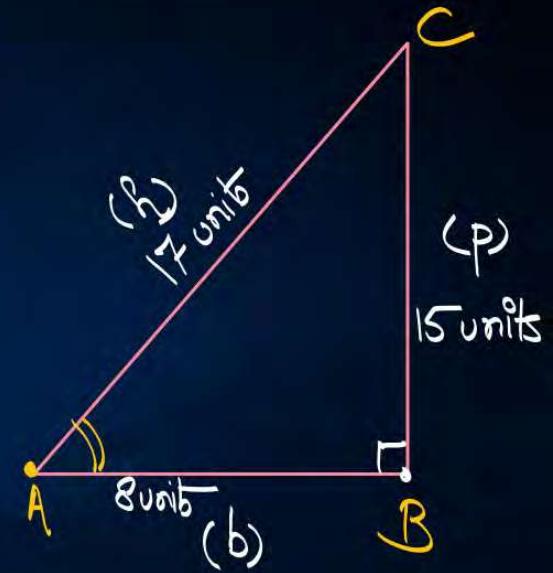
$$\frac{b}{p} = \frac{8}{15}$$

$$b = 8 \text{ units}$$

$$p = 15 \text{ units}$$

$$\sin A = \frac{15}{17}$$

$$\sec A = \frac{17}{8}$$



## QUESTION

$$k = \frac{YX}{BQ}$$

\*

If  $\angle A$  and  $\angle B$  are acute angles such that  $\cos A = \cos B$ , then show that  $\angle A = \angle B$ .

Sol<sup>n</sup>

$$\cos A = \cos B$$

$$\frac{AX}{AY} = \frac{BP}{BQ}$$

$$\left[ \frac{YX}{BQ} = \frac{AX}{BP} = \frac{AY}{BQ} \right] = k$$

$$\Rightarrow AX = k BP$$

$$\Rightarrow AY = k BQ$$

$$YX = \sqrt{(AY)^2 - (AX)^2}$$

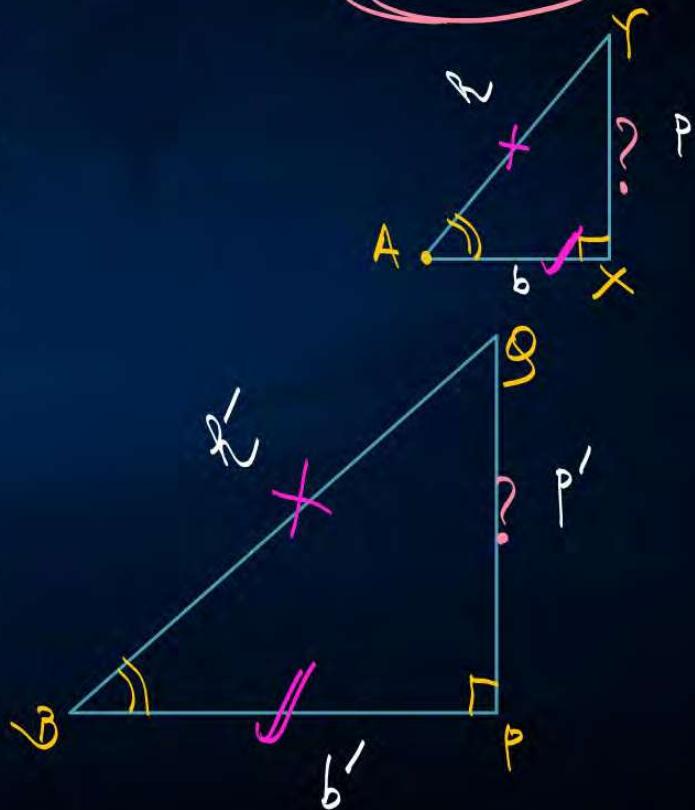
$$= \sqrt{(kBQ)^2 - (kBP)^2}$$

$$= \sqrt{k^2 BQ^2 - k^2 BP^2}$$

$$= \sqrt{k^2(BQ^2 - BP^2)}$$

$$= k \sqrt{BQ^2 - BP^2} \rightarrow QP$$

$$\Rightarrow YX = k QP$$



## QUESTION

If  $\cot \theta = 7/8$ , evaluate:

$$\sin \theta = \frac{8}{\sqrt{113}}$$

$$\cos \theta = \frac{7}{\sqrt{113}}$$

$$\left. \begin{aligned} & \sqrt{8^2 + 7^2} \\ & \sqrt{64 + 49} \\ & \sqrt{113} \end{aligned} \right\} \text{i)} (\omega t \theta)^2 \Rightarrow \left( \frac{7}{8} \right)^2 = \boxed{\frac{49}{64}}$$



$$\text{(i)} [(1 + \sin \theta)(1 - \sin \theta)] / [(1 + \cos \theta)(1 - \cos \theta)], *$$

$$\text{(ii)} \cot^2 \theta$$

**Soln**

$$\cot \theta = \frac{7}{8}$$

$$\Rightarrow \frac{b}{P} = \frac{7}{8}$$

$$\Rightarrow b = 7$$

$$\Rightarrow P = 8$$

$$\frac{(1 + \sin \theta)(1 - \sin \theta)}{(1 + \cos \theta)(1 - \cos \theta)}$$

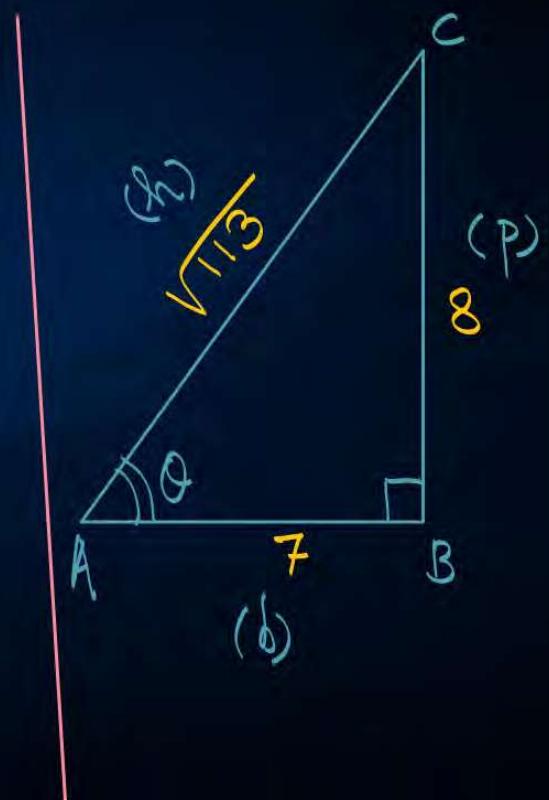
$$\Rightarrow \frac{1 - (\sin \theta)^2}{1 - (\cos \theta)^2}$$

$$\Rightarrow \frac{1 - \left(\frac{8}{\sqrt{113}}\right)^2}{1 - \left(\frac{7}{\sqrt{113}}\right)^2}$$

$$\left(1 - \frac{64}{113}\right)$$

$$\left(1 - \frac{49}{113}\right)$$

$$\boxed{\frac{49}{64}} *$$



## QUESTION

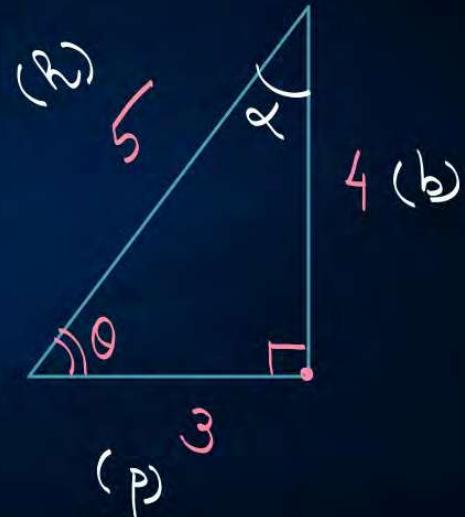
If  $3 \cot A = 4$ , check whether  $(1 - \tan^2 A)/(1 + \tan^2 A) = (\cos^2 A - \sin^2 A)$  or not.

\*  $\frac{4}{3} / \frac{3}{4}$  3, 4, 5

*concept*

$$\begin{aligned} \sin \theta &= \frac{4}{5} \\ \cos \theta &= \frac{3}{5} \\ \tan \theta &= \frac{4}{3} \end{aligned}$$

$$\begin{aligned} \sin \alpha &= \frac{3}{5} \\ \cos \alpha &= \frac{4}{5} \\ \tan \alpha &= \frac{3}{4} \end{aligned}$$



## QUESTION

If  $3 \cot A = 4$ , check whether  $(1 - \tan^2 A)/(1 + \tan^2 A) = (\cos^2 A - \sin^2 A)$  or not.

\*  $\frac{4}{3} / \frac{3}{4}$  concept

**Soln**  $3 \cot A = 4$

$$\cot A = \frac{4}{3}$$

$$\tan A = \frac{3}{4}$$

$$\sin A = \frac{3}{5}$$

$$\cos A = \frac{4}{5}$$

$$LHS = \frac{1 - \tan^2 A}{1 + \tan^2 A}$$

$$= \frac{1 - (\tan A)^2}{1 + (\tan A)^2}$$

$$= \frac{1 - \left(\frac{3}{4}\right)^2}{1 + \left(\frac{3}{4}\right)^2}$$

$$= \frac{\left(1 - \frac{9}{16}\right)}{\left(1 + \frac{9}{16}\right)}$$

$$\frac{7}{25} - \textcircled{i}$$

$$RHS = \cos^2 A - \sin^2 A$$

$$= (\cos A)^2 - (\sin A)^2$$

$$= \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$$

$$= \frac{16}{25} - \frac{9}{25} = \left(\frac{7}{25}\right) \textcircled{ii}$$



## QUESTION

$$\sqrt{1+3} \\ \sqrt{4} = 2$$



In triangle ABC, right-angled at B if  $\tan A = 1/\sqrt{3}$ , find the value of:

- (i)  $\sin A \cos C + \cos A \sin C$  → 1
- (ii)  $\cos A \cos C - \sin A \sin C$

**Sol**

$$\tan A = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{P}{b} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow P = 1$$

$$\Rightarrow b = \sqrt{3}$$

$$\sin A = \frac{1}{2}$$

$$\cos A = \frac{\sqrt{3}}{2}$$

$$\sin C = \frac{\sqrt{3}}{2}$$

$$\cos C = \frac{1}{2}$$

$$i) \sin A \cdot \cos C + \cos A \cdot \sin C$$

$$= \left( \frac{1}{2} \times \frac{1}{2} \right) + \left( \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} \right)$$

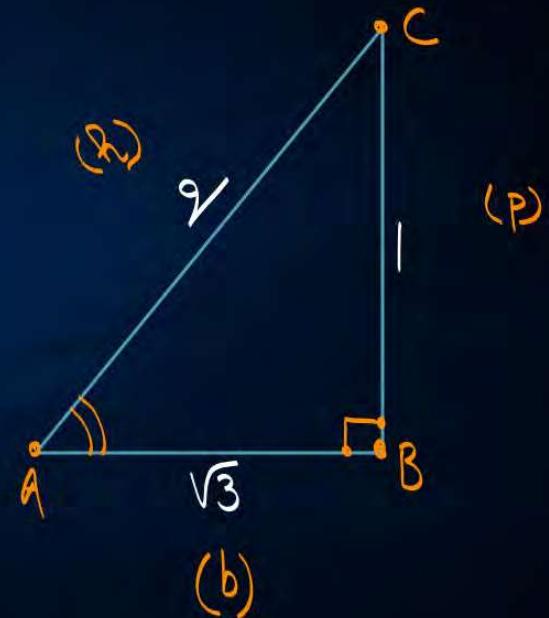
$$= \frac{1}{4} + \frac{3}{4}$$

$$= 1$$

$$ii) \cos A \cdot \cos C - \sin A \cdot \sin C$$

$$= \left( \frac{\sqrt{3}}{2} \times \frac{1}{2} \right) - \left( \frac{1}{2} \times \frac{\sqrt{3}}{2} \right)$$

$$= 0$$



**QUESTION**

In  $\triangle PQR$ , right-angled at Q.  $PR + QR = 25 \text{ cm}$  and  $PQ = 5 \text{ cm}$ . Determine the values of  $\sin P$ ,  $\cos P$  and  $\tan P$ .

**Sol<sup>n</sup>**

$$x^2 = (25-x)^2 + 25$$

$$\Rightarrow x^2 - (25-x)^2 = 25$$

~~$$\Rightarrow 25x(2x-25) = 25$$~~

$$\Rightarrow 2x-25 = 1$$

~~$$\Rightarrow x = 13$$~~

$$\Rightarrow x = 13$$

$$QR = 25 - x$$

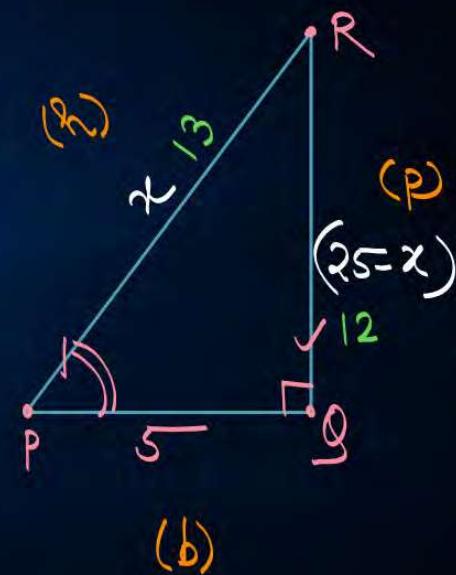
$$\sin P = \frac{12}{13}$$

$$\cos P = \frac{5}{13}$$

$$\tan P = \frac{12}{5}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$



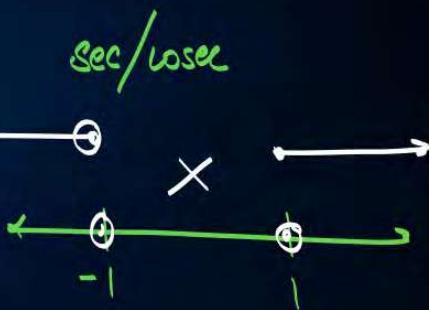
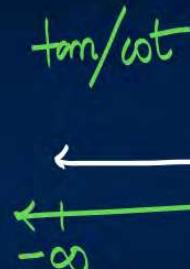
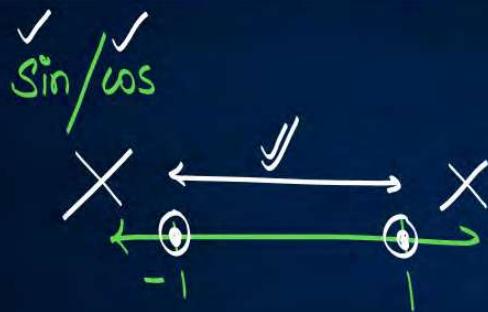
**QUESTION**

State whether the following are true or false. Justify your answer.

(i) The value of  $\tan A$  is always less than 1. (X)

\* Range of  $\sin/\cos/\tan/\sec/\csc/\cot$

Sol<sup>n</sup>



**QUESTION**

State whether the following are true or false. Justify your answer.

(ii)  $\sec A = 12/5$  for some value of angle A. (✓)

Sol<sup>n</sup>

$$\sec A = 2.4$$

**QUESTION**

State whether the following are true or false. Justify your answer.

(iii)  $\cos A$  is the abbreviation used for the cosecant of angle A. (X)

cosec A

Sol

**QUESTION**

State whether the following are true or false. Justify your answer.

(iv)  $\cot A$  is the product of  $\cot$  and  $A$ . (X)

Sol<sup>n</sup>

$$\cot A = \cot \times A \quad X$$

**QUESTION**

State whether the following are true or false. Justify your answer.  
(v)  $\sin \theta = 4/3$  for some angle  $\theta$ . (x)

Soln  $\boxed{\sin \theta = 1.3}$



## Trigonometric Ratios

$0^\circ$   $30^\circ$   $45^\circ$   $60^\circ$   $90^\circ$

\* When we write our Trig ratios for any angle ' $\theta$ '

$$0^\circ \leq \theta \leq 90^\circ$$

- $\sin\theta$   
 $\cos\theta$   
 $\tan\theta$   
 $\sec\theta$   
 $\csc\theta$   
 $\cot\theta$
- }       $\theta \rightarrow$  acute  $\overset{??}{\rightarrow} (30^\circ, 45^\circ, 60^\circ)$
- ✓  $\theta \rightarrow 0^\circ$  ✓
- ✓  $\theta \rightarrow 90^\circ$  ✓



## Trigonometric Ratios



$$\begin{array}{c|c|c|c|c|c} \theta & 0^\circ & 30^\circ & 45^\circ & 60^\circ & 90^\circ \\ \hline \sin \theta & 0 & \frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{\sqrt{3}}{2} & 1 \\ \hline \cos \theta & 1 & \frac{\sqrt{3}}{2} & \frac{1}{\sqrt{2}} & \frac{1}{2} & 0 \\ \hline \tan \theta & 0 & \frac{1}{\sqrt{3}} & 1 & \sqrt{3} & \text{undefined } (\frac{1}{0}) \\ \hline \sec \theta & 1 & \frac{2}{\sqrt{3}} & \sqrt{2} & 2 & \text{undefined } (\frac{1}{0}) \\ \hline \cot \theta & \text{undefined } (\frac{1}{0}) & \sqrt{3} & 1 & \frac{1}{\sqrt{3}} & 0 \end{array}$$



## QUESTION

Evaluate the following:

$$2 \tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$$

~~soln~~ 
$$2(\tan 45^\circ)^2 + (\cos 30^\circ)^2 - (\sin 60^\circ)^2$$

$$= 2 \times (1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= [2]$$

## QUESTION



Evaluate the following:

$$(\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ) / (\sec 30^\circ + \cos 60^\circ + \cot 45^\circ)$$

$$\begin{aligned} & \boxed{98\%} \quad \frac{(\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ)}{(\sec 30^\circ + \cos 60^\circ + \cot 45^\circ)} \\ &= \frac{\left(\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}\right)}{\left(\frac{2}{\sqrt{3}} + \frac{1}{2} + 1\right)} \\ &= \frac{\left(\frac{3}{2} - \frac{2}{\sqrt{3}}\right)}{\left(\frac{3}{2} + \frac{2}{\sqrt{3}}\right)} \end{aligned}$$

$$\begin{aligned} & \rightarrow \frac{\left(\frac{3\sqrt{3}-4}{2\sqrt{3}}\right)}{\left(\frac{3\sqrt{3}+4}{2\sqrt{3}}\right)} \\ &= \frac{(3\sqrt{3}-4)}{(3\sqrt{3}+4)} \times \frac{(3\sqrt{3}-4)}{(3\sqrt{3}-4)} \\ &= \frac{(3\sqrt{3}-4)^2}{(27-16)} \end{aligned}$$

$$\begin{aligned} & \frac{(27+16-24\sqrt{3})}{11} \\ & \boxed{\frac{43-24\sqrt{3}}{11}} \end{aligned}$$

**QUESTION**

Evaluate the following:

$$(5 \cos^2 60^\circ + 4 \sec^2 30^\circ - \tan^2 45^\circ) / (\sin^2 30^\circ + \cos^2 30^\circ)$$

~~Quesn~~

$$\frac{5 \cdot (\cos 60^\circ)^2 + 4 \cdot (\sec 30^\circ)^2 - (\tan 45^\circ)^2}{(\sin 30^\circ)^2 + (\cos 30^\circ)^2}$$

$$= \frac{5 \times \left(\frac{1}{2}\right)^2 + 4 \times \left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \left( \frac{5}{4} + \frac{16}{3} - 1 \right)$$

$$= \frac{15 + 64 - 12}{12}$$

$$= \frac{15 + 52}{12}$$

$$= \left( \frac{67}{12} \right) \text{Ans}$$

## QUESTION

Choose the correct option and justify your choice:

(i)  $2 \tan 30^\circ / (1 + \tan^2 30^\circ) =$

A ✓  $\sin 60^\circ$

B ✗  $\cos 60^\circ$

C ✗  $\tan 60^\circ$

D ✗  $\sin 30^\circ$

Soln

$$\begin{aligned} & \frac{2 \tan 30^\circ}{(1 + (\tan 30^\circ)^2)} \\ &= \frac{\left(2 \times \frac{1}{\sqrt{3}}\right)}{\left(1 + \left(\frac{1}{\sqrt{3}}\right)^2\right)} \\ &= \frac{\left(\frac{2}{\sqrt{3}}\right)}{\left(\frac{4}{3}\right)} \end{aligned}$$

$\nearrow \cancel{2} \times \cancel{\frac{1}{\sqrt{3}}} \quad \searrow \cancel{4} \times \cancel{\frac{1}{2}}$   $= \boxed{\frac{\sqrt{3}}{2}}$

## QUESTION

(ii)  $(1 - \tan^2 45^\circ) / (1 + \tan^2 45^\circ) =$

- A   $\tan 90^\circ$
- B  1
- C   $\sin 45^\circ$
- D  0

**Soln**

$$\begin{aligned}& \frac{(1 - (\tan 45^\circ)^2)}{(1 + (\tan 45^\circ)^2)} \\&= \frac{1 - (1)^2}{1 + (1)^2} \\&= \frac{0}{2} = 0\end{aligned}$$

## QUESTION

(iii)  $\sin 2A = 2 \sin A$  is true when  $A =$ 

- A  $0^\circ$
- B  $30^\circ$
- C  $45^\circ$
- D  $60^\circ$

**Sol<sup>n</sup>**

$$\begin{aligned}\sin 2A &= 2 \sin A \\&= \sin 0^\circ && 2 \times \sin 0^\circ \\&= 0 && 2 \times 0 \\&\hline \sin 60^\circ &= \frac{\sqrt{3}}{2} && 2 \times \sin 30^\circ \\&= \frac{\sqrt{3}}{2} && 2 \times \frac{1}{2} = 1\end{aligned}$$

**QUESTION**

$$(iv) \frac{2 \tan 30^\circ}{(1 - \tan^2 30^\circ)} =$$

A  $\cos 60^\circ$

B  $\sin 60^\circ$

C  $\tan 60^\circ$

D  $\sin 30^\circ$

Sol

$$\begin{aligned}
 & \frac{2 \tan 30^\circ}{(1 - \tan^2 30^\circ)} \\
 &= \frac{\left(2 \times \frac{1}{\sqrt{3}}\right)}{1 - \left(\frac{1}{\sqrt{3}}\right)^2} \\
 &= \frac{\left(\frac{2}{\sqrt{3}}\right)}{\left(\frac{2}{3}\right)}
 \end{aligned}$$

$$\begin{aligned}
 & \frac{2 \times \sqrt{3}}{\sqrt{3} - 2} \\
 &= \sqrt{3}
 \end{aligned}$$

**QUESTION**

If  $\tan(A + B) = \sqrt{3}$  and  $\tan(A - B) = 1/\sqrt{3}$ ;  $0^\circ < A + B \leq 90^\circ$ ,  $A > B$ , find A and B

**Sol:**

$$\left. \begin{array}{l} \tan(A+B) = \sqrt{3} \\ A+B = 60^\circ \end{array} \right\} \left. \begin{array}{l} \tan(A-B) = \frac{1}{\sqrt{3}} \\ A-B = 30^\circ \end{array} \right.$$

$$\begin{aligned} A &= 90^\circ - 60^\circ \\ A &= 45^\circ \end{aligned}$$

$$\begin{aligned} B &= 60^\circ - 45^\circ \\ B &= 15^\circ \end{aligned}$$

**QUESTION**

$$\left. \begin{array}{l} A = 0^\circ \\ B = 90^\circ \end{array} \right\} \quad \left. \begin{array}{l} 60^\circ, 30^\circ \end{array} \right\}$$

$$\left. \begin{array}{l} A = 60^\circ \\ B = 30^\circ \end{array} \right.$$



State whether the following are true or false. Justify your answer.

(i)  $\sin(A + B) = \sin A + \sin B$ .  $(\times)$

Soln

$$\begin{aligned} L.H.S &= \sin(A + B) \\ &= \sin(0^\circ + 90^\circ) \\ &= \sin 90^\circ \\ &= 1 \end{aligned}$$

$$\begin{aligned} R.H.S &= \sin A + \sin B \\ &= \sin 0^\circ + \sin 90^\circ \\ &= 0 + 1 \\ &= 1 \end{aligned}$$

$$\begin{aligned} L.H.S &= \sin(A + B) \\ &= \sin(60^\circ + 30^\circ) \\ &= \sin 90^\circ \\ &= 1 \end{aligned} \quad (\text{ii}) \quad \times$$

$$\begin{aligned} R.H.S &\Rightarrow \sin A + \sin B \\ &= \sin 60^\circ + \sin 30^\circ \\ &= \frac{\sqrt{3}}{2} + \frac{1}{2} \\ &= \left(\frac{\sqrt{3}+1}{2}\right) \end{aligned} \quad (\text{iii}) \quad \times$$

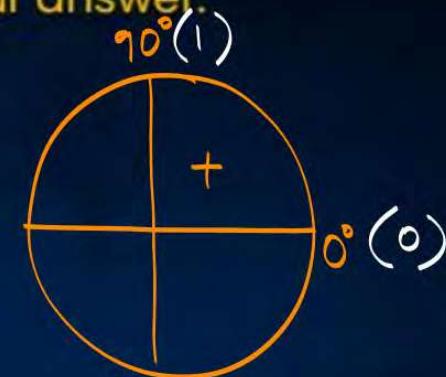
**QUESTION**

State whether the following are true or false. Justify your answer.

(ii) The value of  $\sin \theta$  increases as  $\theta$  increases.

\* Variation of basic T-ratios

Sol





## Trigonometric Ratios

$\theta \uparrow, \sin \theta \uparrow$

Ratio	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$
$\sin \theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan \theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined
$\sec \theta$	undefined	2	$\sqrt{2}$	$2/\sqrt{3}$	1
$\csc \theta$	1	$2/\sqrt{3}$	$\sqrt{2}$	2	undefined
$\cot \theta$	undefined	$\sqrt{3}$	1	$1/\sqrt{3}$	0



## QUESTION

State whether the following are true or false. Justify your answer.

(ii) The value of  $\sin \theta$  increases as  $\theta$  increases. (✓)

\* Variation of basic T-ratios

Sol

**QUESTION**

State whether the following are true or false. Justify your answer.

(iii) The value of  $\cos \theta$  increases as  $\theta$  increases. (X)

Sol<sup>n</sup>



## QUESTION

State whether the following are true or false. Justify your answer.

(iv)  $\sin\theta = \cos\theta$  for all values of  $\theta$ .

*Q(X)*

*Sol<sup>n</sup>*



## QUESTION

State whether the following are true or false. Justify your answer.

(v)  $\cot A$  is not defined for  $A = 0^\circ$ . (✓)

Sol<sup>n</sup>



## Trigonometric Identities



(Related to Trigonometry)

(& pl. type of equation)

\* **Identity** :

→ It's a type of equation which is true for any value of variable/variables involved.

$$\checkmark (x^2 - 16) = (x+4)(x-4)$$

$$\checkmark (x+y)^2 = x^2 + y^2 + 2xy$$

True for  
any 'x'.

(Hence, these eq's are  
called identities.)



## Trigonometric Identities



Whereas,

$$x^2 - 5 = 20$$

Its true for some 'x', not for all 'x'.  
Hence, its an equation but not an identity.

Hence, every identity is an eq<sup>n</sup> but every equation is not an identity.

## Trigonometric Identities



\* Trigonometric Identity :-

→ An equation involving T-Ratios for some angle ( $\theta, \alpha, \beta, \dots$ ) is called a trigonometric Identity if its satisfied for all values of angles for which occurring Trigonometric Ratios are defined.

Eg :-  $\sin^2\theta + \cos^2\theta = 1$       defined for all ' $\theta$ '.

$$\left(\cos^2\theta - \frac{1}{4}\cos\theta\right) = \cos\theta\left(\cos\theta - \frac{1}{4}\right)$$



## Trigonometric Identities



$$1 + \tan^2\theta = \sec^2\theta \rightarrow \text{defined for all } \theta \text{ except } 90^\circ$$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta} \rightarrow \text{defined for all } \theta \text{ except } 0^\circ.$$

\* Hence, these are all identities.

~~$2\tan^2x - 3\tan x = -1$~~  → ??



## Trigonometric Identities

\* So, there can be many T-identities but out of them the 3 main ones in class 10<sup>th</sup> are :-

$$\sin^2\theta + \cos^2\theta = 1$$

King of Trigonometry

$$1 + \tan^2\theta = \sec^2\theta$$

Queen of Trigonometry

$$1 + \cot^2\theta = \operatorname{cosec}^2\theta$$



## Trigonometric Identities



\* Proof :-

$$\sin^2 \theta + \cos^2 \theta = 1$$

~~Proof~~

$$\sin^2 \theta + \cos^2 \theta = 1$$

Case I  $p^2 + b^2 = h^2$

$$LHS = \sin^2 \theta + \cos^2 \theta$$

$$= (\sin \theta)^2 + (\cos \theta)^2$$

$$= \left(\frac{p}{h}\right)^2 + \left(\frac{b}{h}\right)^2$$

$$= \frac{p^2}{h^2} + \frac{b^2}{h^2} = \frac{p^2 + b^2}{h^2} = \frac{h^2}{h^2} = 1$$

RHS = 1



Case II

$$\theta = 0^\circ$$

$$LHS = \sin^2 \theta + \cos^2 \theta$$

$$= (\sin 0^\circ)^2 + (\cos 0^\circ)^2$$

$$= 0 + 1 = 1$$

RHS = 1

Case III

$$\theta = 90^\circ$$

$$LHS = \sin^2 \theta + \cos^2 \theta$$

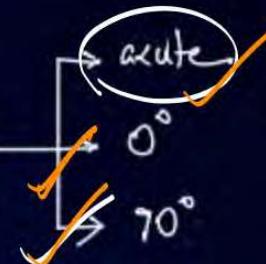
$$= (\sin 90^\circ)^2 + (\cos 90^\circ)^2$$

$$= 1 + 0$$

$$= 1$$

RHS = 1

$$0^\circ \leq \theta \leq 90^\circ$$





## Trigonometric Identities

Note:

$$\sin^2 \theta \rightarrow (\sin \theta)^2$$



$$\sin^2 \times \theta$$

\* Likewise for other T-ratio



## Trigonometric Identities

KOT

$$\sin^2\theta + \cos^2\theta = 1$$



$$*\cos^2\theta = 1 - \sin^2\theta$$

$$\cos\theta \times \cos\theta = (1)^2 - (\sin\theta)^2$$

$$\cos\theta \times \cos\theta = (1 + \sin\theta)(1 - \sin\theta)$$

$$\frac{1}{\cos} \frac{\sin}{\cos} = \frac{\cos}{1} \frac{\sin}{\sin}$$

$$\sin\theta \times \sin\theta = (1)^2 - (\cos\theta)^2$$

$$\sin\theta \times \sin\theta = (1 + \cos\theta)(1 - \cos\theta)$$

$$\frac{\cos\theta}{1 + \sin\theta} = \frac{1 - \sin\theta}{\cos\theta}$$

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\frac{1}{\sin} \frac{\cos}{\sin} = \frac{\sin}{1} \frac{\cos}{\cos}$$

$$\frac{\sin\theta}{1 + \cos\theta} = \frac{1 - \cos\theta}{\sin\theta}$$

$$\frac{1 + \cos\theta}{\sin\theta} = \frac{\sin\theta}{1 - \cos\theta}$$

$$\frac{1 + \sin\theta}{\cos\theta} = \frac{\cos\theta}{1 - \sin\theta}$$



## Trigonometric Identities

\* Queen

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$$



## Trigonometric Identities



$$* 1 + \tan^2 \theta = \sec^2 \theta$$

Proof :-

Case - I



$0^\circ$        $90^\circ$

Case - II

$\theta = 0^\circ$

Case - III





## Trigonometric Identities



$$\frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$\checkmark \quad \tan^2\theta = (\sec^2\theta - 1)$$

$$\frac{\sec 1}{\tan} = \frac{\tan}{\sec 1}$$

$$\checkmark \quad (\sec^2\theta - \tan^2\theta) = 1$$

$$(\sec\theta + 1)(\sec\theta - 1) = \tan^2\theta$$

$$\frac{\tan\theta}{(\sec\theta + 1)} = \frac{(\sec\theta - 1)}{\tan\theta}$$

$$\frac{\tan\theta}{(\sec\theta - 1)} = \frac{(\sec\theta + 1)}{\tan\theta}$$

$$(\sec + \tan) \Leftrightarrow (\sec - \tan)$$

\*

$$(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$$

$$(\sec\theta + \tan\theta) = \frac{1}{(\sec\theta - \tan\theta)}$$

$$(\sec\theta - \tan\theta) = \frac{1}{(\sec\theta + \tan\theta)}$$



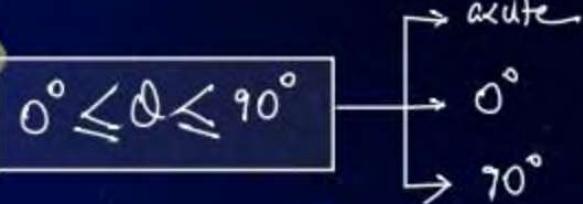
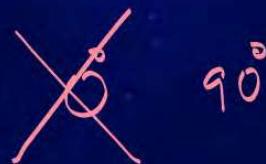
## Trigonometric Identities



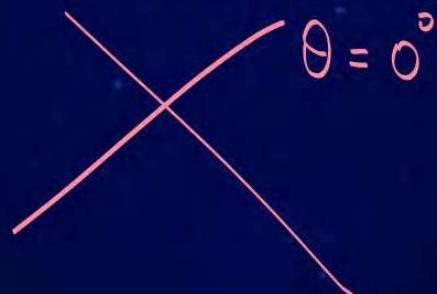
\* Queen :  $\frac{\pi}{2}$

$$1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$$

\* Proof :- Case - I



Case - II



Case - III  
 $\theta = 90^\circ$



## Trigonometric Identities



$$1 + \cot^2 \theta = \csc^2 \theta$$

$$\cot^2 \theta = (\csc^2 \theta - 1)$$

$$\frac{\csc 1}{\cot} = \frac{\cot}{\csc 1}$$

$$\csc^2 \theta - \cot^2 \theta = 1$$

$$\cot^2 \theta = (\csc \theta + 1)(\csc \theta - 1)$$

$$\frac{\cot \theta}{(\csc \theta + 1)} = \frac{(\csc \theta - 1)}{\cot \theta}$$

$$\frac{\cot \theta}{(\csc \theta - 1)} = \frac{(\csc \theta + 1)}{\cot \theta}$$

$$(\csc + \cot)(\csc - \cot)$$

\*

$$(\csc \theta + \cot \theta)(\csc \theta - \cot \theta) = 1$$

$$\csc \theta + \cot \theta = \frac{1}{(\csc \theta - \cot \theta)}$$

$$\csc \theta - \cot \theta = \frac{1}{(\csc \theta + \cot \theta)}$$



## Trigonometric Identities

\* How to approach Qs of T-identities :-

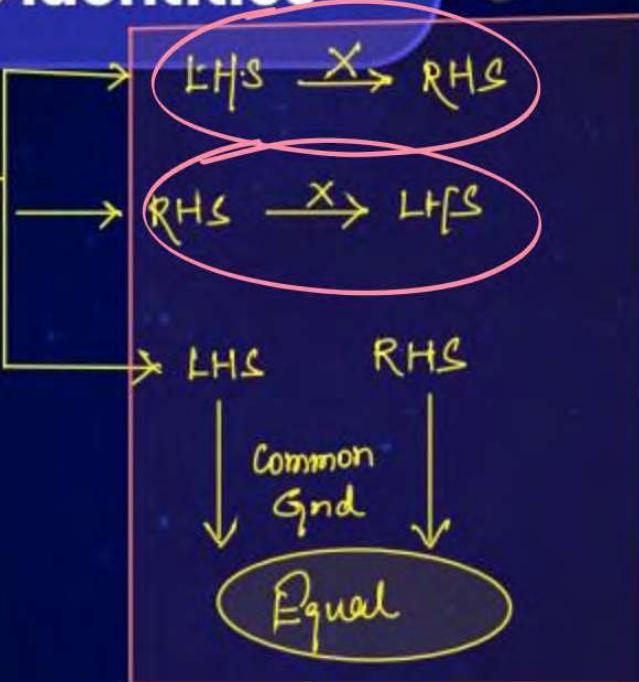
- i) If one observes the use of any T-identity / algebraic formula , do apply it .
- ii) Taking common usually helps
- iii) When proving follow  

Complicated Side	Easy Side
------------------	-----------



## Trigonometric Identities

\* In some situations





## Trigonometric Identities



$$\sqrt{\frac{N}{D}} \rightarrow \frac{a+b}{\cancel{(a-b)}}, \frac{a-b}{a+b}$$



- \* In situations of  $\left(\frac{a \pm b}{a \mp b}\right)$  format, dividing/Multiplying Num & Den by same thing helps.
- \* In  $\sqrt{\frac{N}{D}}$  format or  $\frac{N}{(a \pm b)}$  format  $\rightarrow$  Rationalisation helps.  
(as  $(a+b)(a-b) = a^2 - b^2$  helps)



## Trigonometric Identities



BRAHMASTRA



\* Kuch samay me nai aa saka ho to puri duniya sin/cos me  
badal do aur simplify kerte jao.

\* Kaun si identity kahan lyegi , uska jo logic seekha hai , usko  
hmara dimag me rakho .

✓ ✓ sin/cos ke saath aur bhi different identities hain to puri duniya  
sin/cos me badal do.

## QUESTION

Express the ratios  $\cos A$ ,  $\tan A$  and  $\sec A$  in terms of  $\sin A$ .

Sol?

$$\cos A = \sqrt{1 - \sin^2 A}$$

$$\tan A = \frac{\sin A}{\cos A} = \frac{\sin A}{\sqrt{1 - \sin^2 A}}$$

$$\sec A = \frac{1}{\sqrt{1 - \sin^2 A}}$$

$$(\ )^2 = \boxed{\phantom{00}}$$

$$(\ ) = \pm \sqrt{\boxed{\phantom{00}}}$$

$$\sin^2 A + \cos^2 A = 1$$

$$\cos^2 A = 1 - \sin^2 A$$

$$\cos A = \sqrt{1 - \sin^2 A}$$

## QUESTION

Prove that  $\sec A (1 - \sin A)(\sec A + \tan A) = 1$ .

Soln

$$\text{LHS} = \underline{\sec A} \underline{(1 - \sin A)} \underline{(\sec A + \tan A)}$$

$$\Rightarrow \frac{(1 - \sin A)}{\cos A} \times (\sec A + \tan A)$$

$$\Rightarrow (\sec A - \tan A)(\sec A + \tan A)$$

$$\Rightarrow \underline{\sec^2 A - \tan^2 A}$$

$$\Rightarrow \underline{1 = \text{R.H.S}}$$

$$\begin{aligned} \text{LHS} &= \frac{1}{\cos A} \times (1 - \sin A) \times \left( \frac{1}{\cos A} + \frac{\sin A}{\cos A} \right) \\ &= \frac{1}{\cos A} \times (1 - \sin A) \times \frac{(1 + \sin A)}{\cos A} \\ &= \frac{(1 - \sin^2 A)}{\cos^2 A} \\ &= \frac{\cos^2 A}{\cos^2 A} = 1 = \text{R.H.S} \end{aligned}$$

**QUESTION**

Prove that  $(\cot A - \cos A) / (\cot A + \cos A) = (\cosec A - 1) / (\cosec A + 1)$

**Sol<sup>n</sup>**

$$LHS = \frac{(\cot A - \cos A)}{(\cot A + \cos A)}$$

$$= \frac{\left( \frac{\cos A}{\sin A} - \cos A \right)}{\left( \frac{\cos A}{\sin A} + \cos A \right)}$$

$$= \frac{\cancel{\cos A} \left( \frac{1}{\sin A} - 1 \right)}{\cancel{\cos A} \left( \frac{1}{\sin A} + 1 \right)}$$

$$\frac{\left( \frac{1}{\sin A} - 1 \right)}{\left( \frac{1}{\sin A} + 1 \right)}$$

$$\frac{(\cosec A - 1)}{(\cosec A + 1)} = RHS$$

**QUESTION**

Prove that  $(\sin\theta - \cos\theta + 1) / (\sin\theta + \cos\theta - 1) = 1 / (\sec\theta - \tan\theta)$ , using the identity  
 $\sec^2\theta = 1 + \tan^2\theta$ .

**Sol<sup>n</sup>**

$$\begin{aligned} \text{LHS} &= \frac{(\sin\theta - \cos\theta + 1)}{(\sin\theta + \cos\theta - 1)} \\ &= \frac{\sin\theta - (\cos\theta - 1)}{\sin\theta + (\cos\theta - 1)} \\ &= \frac{[\sin\theta - (\cos\theta - 1)] \times [\sin\theta - (\cos\theta - 1)]}{[\sin\theta + (\cos\theta - 1)] \times [\sin\theta - (\cos\theta - 1)]} \end{aligned}$$

$$\begin{aligned} &\frac{[\sin\theta - (\cos\theta - 1)]^2}{\sin^2\theta - (\cos\theta - 1)^2} \\ &\frac{\sin^2\theta + (\cos\theta - 1)^2 - 2\sin\theta(\cos\theta - 1)}{(1 - \cos^2\theta) - (\cos\theta - 1)^2} \\ &\frac{(1 - \cos^2\theta) + (\cos\theta - 1)^2 - 2\sin\theta(\cos\theta - 1)}{(1 + \cos\theta)(1 - \cos\theta) - (\cos\theta - 1)^2} \\ &\frac{(\cancel{\cos\theta - 1})[-1 - \cos\theta + \cancel{\cos\theta - 1} - 2\sin\theta]}{(\cancel{\cos\theta - 1})[1 - \cos\theta - \cos\theta + \cancel{1}]} \end{aligned}$$

$$\frac{1 - \sin}{\cos} \quad \frac{\cos}{1 + \sin}$$

$$\Rightarrow \frac{\cancel{1}(1 + \sin \theta)}{\cancel{1}\cos \theta}$$

$$(\sec \theta + \tan \theta) \leftarrow \boxed{\frac{1 + \sin \theta}{\cos \theta}}$$

$$\Rightarrow \frac{\cos \theta}{1 - \sin \theta}$$

$$\Rightarrow \frac{1}{\frac{(1 - \sin \theta)}{\cos \theta}}$$

$$(\sec \theta - \tan \theta)$$

**QUESTION**

Write all the other trigonometric ratios of  $\angle A$  in terms of  $\sec A$ .

**Sol<sup>n</sup>**

$$\begin{aligned} \sin A &= \sqrt{1 - \cos^2 A} &= \sqrt{1 - \frac{1}{\sec^2 A}} \\ \cos A &= \frac{1}{\sec A} &= \sqrt{\frac{\sec^2 A - 1}{\sec^2 A}} \\ \tan A &= \sqrt{\sec^2 A - 1} &= \sqrt{\frac{\sec^2 A - 1}{\sec A}} \\ \cot A &= \frac{1}{\sqrt{\sec^2 A - 1}} \\ \cosec A &= \frac{\sec A}{\sqrt{\sec^2 A - 1}} \end{aligned}$$



$$1 + \tan^2 A = \sec^2 A$$

$$\tan^2 A = (\sec^2 A - 1)$$

$$\tan A = \sqrt{\sec^2 A - 1}$$

**QUESTION**

Choose the correct option. Justify your choice.

(i)  $9\sec^2 A - 9\tan^2 A =$

A

1

B

9

C

8

D

0

$$\begin{aligned}\text{Soln} &= 9\sec^2 A - 9\tan^2 A \\ &= 9(\sec^2 A - \tan^2 A) \\ &= 9 \times 1 \\ &= 9\end{aligned}$$

**QUESTION**

$$(ii) (1 + \tan\theta + \sec\theta) (1 + \cot\theta - \operatorname{cosec}\theta) =$$

**Sol<sup>n</sup>**

$$\begin{aligned}
 & \left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right) \times \left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right) \\
 & \frac{(\cos\theta + \sin\theta + 1)}{\cos\theta} \times \frac{(\sin\theta + \cos\theta - 1)}{\sin\theta} \\
 & = \frac{(\sin\theta + \cos\theta)^2 - 1}{\cos\theta \cdot \sin\theta} \\
 & = \frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta} \\
 & = 2
 \end{aligned}$$

- A** 0
- B** 1
- C** 2
- D** -1

**QUESTION**

$$(iii) (\sec A + \tan A)(1 - \sin A) =$$

- A**  $\sec A$
- B**  $\sin A$
- C**  $\operatorname{cosec} A$
- D**  $\cos A$

Sol<sup>n</sup>

$$\begin{aligned}
 & (\sec A + \tan A)(1 - \sin A) \\
 &= \left( \frac{1}{\cos A} + \frac{\sin A}{\cos A} \right)(1 - \sin A) \\
 &= \frac{(1 + \sin A)}{\cos A} \times (1 - \sin A) \\
 &= \frac{1 - \sin^2 A}{\cos A} \\
 &= \frac{\cos^2 A}{\cos A} = \cancel{\cos A}
 \end{aligned}$$

## QUESTION

(iv)  $(1 + \tan^2 A) / (1 + \cot^2 A) =$

- A  $\sec^2 A$
- B -1
- C  $\cot^2 A$
- D  $\tan^2 A$

**Sol<sup>n</sup>**

$$\frac{(1 + \tan^2 A)}{(1 + \cot^2 A)}$$

$$= \frac{\sec^2 A}{\cosec^2 A}$$

$$= \tan^2 A$$

$$\begin{aligned} \tan A &\rightarrow \frac{\sin}{\cos} \rightarrow \frac{\sec}{\cosec} \\ \cosec &\rightarrow \sec \end{aligned}$$

## QUESTION

Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

(i)  $(\csc \theta - \cot \theta)^2 = (1 - \cos \theta)/(1 + \cos \theta)$

Sol<sup>n</sup>

$$\begin{aligned}
 \text{LHS} &= (\csc \theta - \cot \theta)^2 \\
 &= \left( \frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta} \right)^2 \\
 &= \frac{(1 - \cos \theta)^2}{\sin^2 \theta} \\
 &= \frac{(1 - \cos \theta)^2}{(1 - \cos^2 \theta)} \\
 &\quad \left. \begin{array}{l} \nearrow \\ \nearrow \end{array} \right. \frac{(1 - \cos \theta)}{(1 + \cos \theta)(1 - \cos \theta)} \\
 &\quad \left. \begin{array}{l} \nearrow \\ \nearrow \end{array} \right. \frac{(1 - \cos \theta)}{(1 + \cos \theta)} = R
 \end{aligned}$$

## QUESTION

$$\frac{\cos}{1 - \sin}$$



Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

(ii)  $\cos A / (1 + \sin A) + (1 + \sin A) / \cos A = 2 \sec A$

Sol<sup>n</sup>

$$\begin{aligned}
 \text{LHS} &= \frac{\cos A}{(1 + \sin A)} + \frac{1 + \sin A}{\cos A} \\
 &= \frac{\cos^2 A + (1 + \sin A)^2}{\cos A (1 + \sin A)} \\
 &= \frac{\cos^2 A + 1 + \sin^2 A + 2 \sin A}{\cos A (1 + \sin A)} \\
 &= \frac{2(1 + \sin A)}{\cos A (1 + \sin A)} = \frac{2 \sec A}{1}
 \end{aligned}$$

$$\begin{aligned}
 &\frac{(1 - \sin A)}{\cos A} + \frac{(1 + \sin A)}{\cos A} \\
 &\frac{1 - \sin A + 1 + \sin A}{\cos A} \\
 &\frac{2}{\cos A} \\
 &2 \sec A
 \end{aligned}$$

**QUESTION**

Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

$$(iii) \tan \theta / (1 - \cot \theta) + \cot \theta / (1 - \tan \theta) = (1 + \sec \theta \cosec \theta)$$

**Sol<sup>n</sup>**

$$\begin{aligned} \text{LHS} &= \frac{\tan \theta}{(1 - \cot \theta)} + \frac{\cot \theta}{(1 - \tan \theta)} \\ &= \left( \frac{\sin \theta}{\cos \theta} \right) + \left( \frac{\cos \theta}{\sin \theta} \right) \\ &= \frac{\sin \theta}{(1 - \frac{\cos \theta}{\sin \theta})} + \frac{\cos \theta}{(1 - \frac{\sin \theta}{\cos \theta})} \\ &= \frac{\sin \theta}{\cos \theta} \times \frac{\sin \theta}{(\sin \theta - \cos \theta)} + \frac{\cos \theta}{\sin \theta} \times \frac{\cos \theta}{(\sin \theta - \cos \theta)} \end{aligned}$$

$$\begin{aligned} &\frac{1}{(\sin \theta - \cos \theta)} \left[ \frac{\sin^2 \theta}{\cos \theta} - \frac{\cos^2 \theta}{\sin \theta} \right] \\ &\frac{1}{(\sin \theta - \cos \theta)} \left[ \frac{(\sin \theta)^3 - (\cos \theta)^3}{\sin \theta \cdot \cos \theta} \right] \\ &\frac{(\sin \theta - \cos \theta)(\sin^2 \theta + \sin \theta \cos \theta + \cos^2 \theta)}{\sin \theta \cos \theta (\sin \theta - \cos \theta)} \\ &\frac{1 + \sin \theta \cos \theta}{\sin \theta \cos \theta} = \boxed{1 + \cosec \theta \sec \theta} \end{aligned}$$

## QUESTION

Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

(iv)  $\underbrace{(1 + \sec A)}_{\text{LHS}} / \sec A = \sin^2 A / (1 - \cos A)$

Sol LHS

$$\begin{aligned} &= \frac{1 + \sec A}{\sec A} \\ &= \frac{\left(1 + \frac{1}{\cos A}\right)}{\left(\frac{1}{\cos A}\right)} \\ &= (\cos A + 1) \end{aligned}$$

$$\begin{aligned} \text{R.H.S} &= \frac{\sin^2 A}{(1 - \cos A)} \\ &= \frac{1 - \cos^2 A}{(1 - \cos A)} \\ &= (1 + \cos A) \end{aligned}$$

## QUESTION

Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

$$(v) (\cos A - \sin A + 1)/(\cos A + \sin A - 1) = (\cosec A + \cot A)$$

Soln

$$\begin{aligned} \text{LHS} &= \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} \\ &= \frac{[\cos A - (\sin A - 1)]^2}{[\cos A + (\sin A - 1)]} \times \frac{[\cos A - (\sin A - 1)]}{[\cos A - (\sin A - 1)]} \\ &= \frac{\cos^2 A + (\sin A - 1)^2 - 2 \cos A (\sin A - 1)}{\cos^2 A - (\sin A - 1)^2} \end{aligned}$$

$$\begin{aligned} &\frac{(\sin A - 1)[-1 - \sin A + \sin A - 1 - 2 \cos A]}{(\sin A - 1)[-1 - \sin A - \sin A + 1]} \\ &\frac{-2(1 + \cos A)}{-2 \sin A} \\ &= \frac{1 + \cos A}{\sin A} \\ &= \cosec A + \cot A \end{aligned}$$

## QUESTION

Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

$$(vi) \sqrt{[(1 + \sin A)/(1 - \sin A)]} = (\sec A + \tan A)$$

Sol

$$\begin{aligned}
 \text{LHS} &= \sqrt{\frac{1 + \sin A}{1 - \sin A}} \\
 &= \sqrt{\frac{(1 + \sin A)}{(1 - \sin A)} \times \frac{(1 + \sin A)}{(1 + \sin A)}} \\
 &= \sqrt{\frac{(1 + \sin A)^2}{1 - \sin^2 A}}
 \end{aligned}$$

$$\begin{aligned}
 &\sqrt{\frac{(1 + \sin A)^2}{(\cos A)^2}} \\
 &\frac{1 + \sin A}{\cos A} \\
 &\text{Sec A} + \tan A
 \end{aligned}$$

## QUESTION

Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

$$(vii) (\sin \theta - 2 \sin^3 \theta) / (2 \cos^3 \theta - \cos \theta) = \tan \theta$$

L.H.S

$$\begin{aligned} L.H.S &= \frac{\sin \theta - 2 \sin^3 \theta}{2 \cos^3 \theta - \cos \theta} \\ &= \frac{\sin \theta (1 - 2 \sin^2 \theta)}{\cos \theta (2 \cos^2 \theta - 1)} \\ &= \underline{\tan \theta} \end{aligned}$$

$$\cos^2 \theta - \sin^2 \theta$$

$$= 2 \cos^2 \theta - 1$$

$$= 1 - 2 \sin^2 \theta$$

## QUESTION

Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

$$(viii) (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = (1 + \tan^2 A + \cot^2 A)$$

Sol:

$$\text{LHS} \Rightarrow (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$$
$$\Rightarrow \cancel{\sin^2 A} + \cancel{\operatorname{cosec}^2 A} + 2\cancel{\sin A \operatorname{cosec} A} + \cancel{\cos^2 A} + \cancel{\sec^2 A} + 2\cancel{\cos A \sec A}$$

$$\Rightarrow 1 + 2 + 2 + \operatorname{cosec}^2 A + \sec^2 A$$

$$\Rightarrow 1 + 1 + \tan^2 A + 1 + \cot^2 A$$

$$\Rightarrow 1 + \tan^2 A + \cot^2 A$$

## QUESTION

Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

$$(ix) (\csc A - \sin A)(\sec A - \cos A) = 1/(\tan A + \cot A)$$

Soln

$$\begin{aligned} \text{LHS} &\Rightarrow (\csc A - \sin A)(\sec A - \cos A) \\ &\Rightarrow \left( \frac{1}{\sin A} - \sin A \right) \left( \frac{1}{\cos A} - \cos A \right) \\ &\Rightarrow \frac{(1 - \sin^2 A)}{\sin A} \times \frac{(1 - \cos^2 A)}{\cos A} \\ &\Rightarrow \frac{\cancel{\cos^2 A}}{\cancel{\sin A}} \times \frac{\cancel{\sin^2 A}}{\cancel{\cos A}} \\ &\Rightarrow \sin A \cdot \cos A \end{aligned}$$

$$\begin{aligned} \text{RHS} &\Rightarrow \frac{1}{(\tan A + \cot A)} \\ &\Rightarrow \frac{1}{\left( \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A} \right)} \\ &\Rightarrow \frac{1}{\cancel{\left( \frac{\sin^2 A + \cos^2 A}{\sin A \cos A} \right)}} \\ &= \underline{\underline{\sin A \cos A}} \end{aligned}$$



## QUESTION

Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

$$(x) \left( \frac{1 + \tan^2 A}{1 + \cot^2 A} \right) = \left( \frac{1 - \tan A}{1 - \cot A} \right)^2 = \tan^2 A$$

Sol<sup>n</sup>

✓ H.w