CHAPTER 10

System of Particles and Centre of Mass

Centre of Mass of a System of 'N' Discrete Particles

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots + m_n\vec{r}_n}{m_1 + m_2 + \cdots + m_n}\ ;\ \vec{r}_{cm} = \frac{\sum_{i=1}^{n} m_i\vec{r}_i}{\sum_{i=1}^{n} m_i} = \frac{1}{M}\sum_{i=1}^{n} m_i\vec{r}_i$$

Centre of Mass of a Continuous Mass Distribution

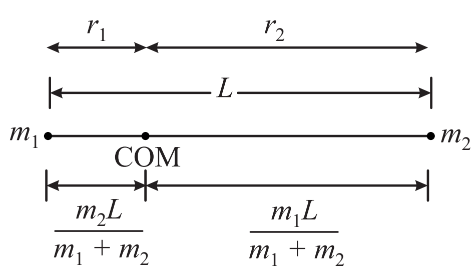
$$x_{\text{cm}} = \frac{\int x\,dm}{\int dm},\quad y_{\text{cm}} = \frac{\int y\,dm}{\int dm},\quad z_{\text{cm}} = \frac{\int z\,dm}{\int dm}$$

$$\int dm = M\ (\text{mass of the body})$$

Centre of Mass of Some Common Systems

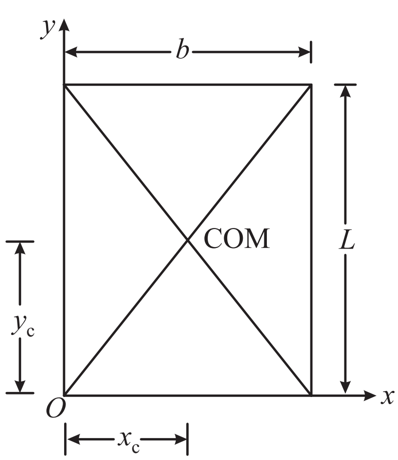
❖ **System of two point masses.**

$m_1 r_1 = m_2 r_2$; The centre of mass lies closer to the heavier mass.



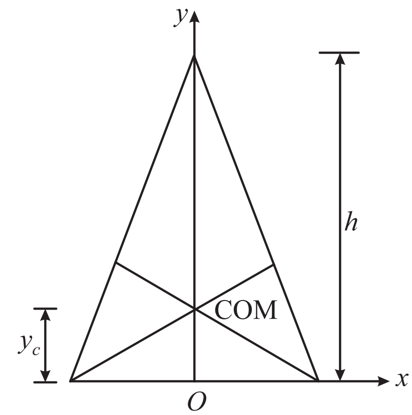
❖ **Rectangular plate**

$$x_c = \frac{b}{2};\ y_c = \frac{L}{2}$$



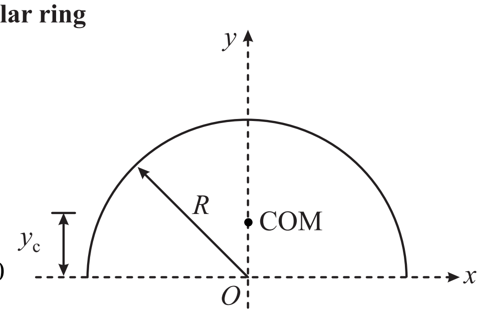
❖ **A triangular plate at the centroid**

$$y_c = \frac{h}{3}$$



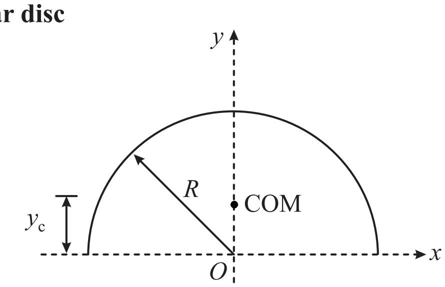
❖ **A semi-circular ring**

$$y_c = \frac{2R}{\pi};\ x_c = 0$$



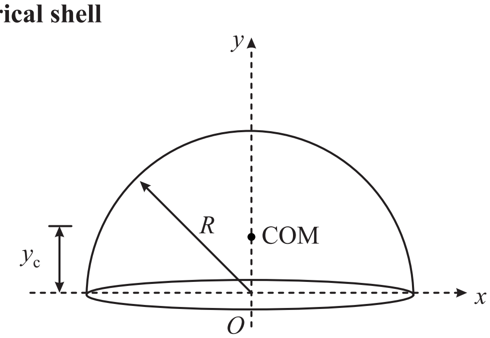
❖ **A semi-circular disc**

$$y_c = \frac{4R}{3\pi};\ x_c = 0$$

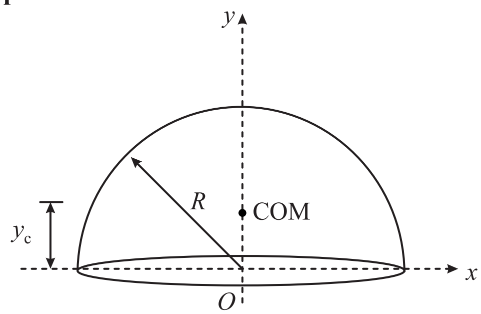


❖ **A hemispherical shell**

$$y_c = \frac{R}{2};\ x_c = 0$$

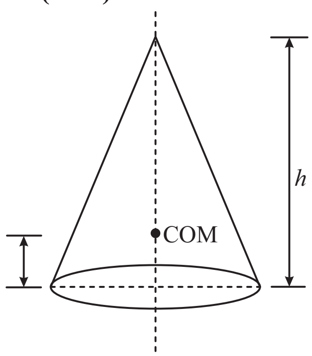


❖ **A solid hemisphere**



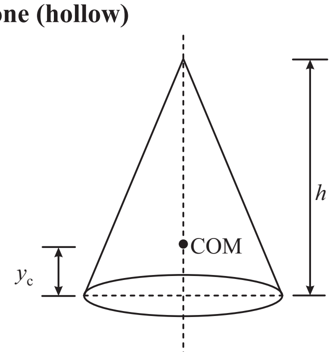
$y_c = \frac{3R}{8}$; $x_c = 0$

❖ **A circular cone (solid)**



$y_c = \frac{h}{4}$

❖ **A circular cone (hollow)**



$y_c = \frac{h}{3}$

Motion of Centre of Mass

Velocity of Centre of Mass of System

$$\vec{v}_{cm} = \frac{m_1\frac{\overrightarrow{dr_1}}{dt} + m_2\frac{\overrightarrow{dr_2}}{dt} + m_3\frac{\overrightarrow{dr_3}}{dt} \cdots + m_n\frac{\overrightarrow{dr_n}}{dt}}{M} = \frac{m_1\overrightarrow{v_1} + m_2\overrightarrow{v_2} + m_3\overrightarrow{v_3} \ldots + m_n\overrightarrow{v_n}}{M}$$

$$\vec{P}_{sys} = M\vec{v}_{cm}$$

Acceleration of Centre of Mass of System

$$\vec{a}_{cm} = \frac{m_1\frac{\overrightarrow{dv_1}}{dt} + m_2\frac{\overrightarrow{dv_2}}{dt} + m_3\frac{\overrightarrow{dv_3}}{dt} \cdots + m_n\frac{\overrightarrow{dv_n}}{dt}}{M}$$

$$= \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 \ldots + m_n\vec{a}_n}{M}$$

$$= \frac{\text{Net force on system}}{M}$$

$$= \frac{\text{Net external force} + \text{Net internal force}}{M}$$

$$= \frac{\text{Net External Force}}{M} \qquad (\because \quad \Sigma \text{ Internal force} = 0)$$

$$\vec{F}_{ext} = M\vec{a}_{cm}$$

Impulse

❖ Impulse of a force $\vec{F}$ on a body is defined as

$$I = \int_{t_i}^{t_f} \vec{F}\,dt = \int_{t_i}^{t_f} d\vec{P} = \Delta\vec{P}$$

❖ Area under the Force vs time curve gives the impulse

❖ Impulse – momentum theorem

$$\vec{I} = \Delta\vec{P}$$

Principle of Conservation of Linear Momentum

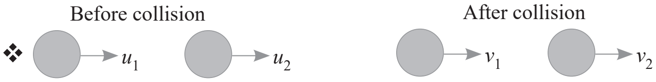
❖ If, $\left(\sum \vec{F}_{ext}\right)_{system} = 0 \Rightarrow (\overrightarrow{P_i})_{system} = (\overrightarrow{P_f})_{system}$

❖ $(KE)_{system} = \frac{1}{2}(m_1v_1^2 + m_2v_2^2 + \ldots m_nv_n^2) \neq \frac{1}{2}MV_{com}^2$

$(KE)_{system} = \frac{1}{2}Mv_{com}^2 + (KE)_{rel/com}$

Coefficient of Restitution (e)

❖ Before collision / After collision



$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

❖ $e = \frac{\text{Impulse of reformation}}{\text{Impulse of deformation}} = \frac{\int F_r dt}{\int F_d dt}$

$= \frac{\text{Velocity of separation along line of impact}}{\text{Velocity of approach along line of impact}}$

$v_1 = \frac{P_i + m_2e(u_2 - u_1)}{m_1 + m_2}, v_2 = \frac{P_i + m_1e(u_1 - u_2)}{m_1 + m_2}$

❖ $e = 1$: Impulse of Reformation = Impulse of Deformation
Velocity of separation = Velocity of approach
Kinetic Energy is conserved (before and after collision)
Elastic collision.

❖ $e = 0$: Impulse of Reformation = 0
Velocity of separation = 0
Kinetic Energy is not conserved
Perfectly Inelastic collision.

❖ $0 < e < 1$: Impulse of Reformation < Impulse of Deformation
Velocity of seapration < Velocity of approach
Kinetic Energy is not conserved
Inelastic collision.

Variable Mass System

If a mass is added or ejected from a system, at rate μ kg/s and relative velocity v_{rel} (w.r.t. the system), then the force exerted by this mass on the system has magnitude $\mu\,|\,v_{rel}\,|$.

Thrust Force (F_t)

$$F_t = v_{rel}\frac{dm}{dt}$$