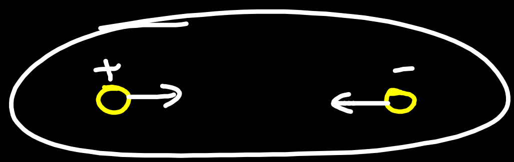
ELECTROSTATIC POTENTIAL ENERGY

Potential energy of a system of particles is defined only in conservative fields.

Potential energy of a system of particles we define as the work done in assembling the system in a given configuration against the interaction forces of particles.

Potential energy of a charge in electric field is defined as work done in bringing the charge from infinity to the given point in electric field

Electrostatic potential energy is defined in two ways.
(i) Interaction energy of charged particles of a system
(ii) Self energy of a charged object



PE is defined for a system

isolated particle

PE nahi hogi

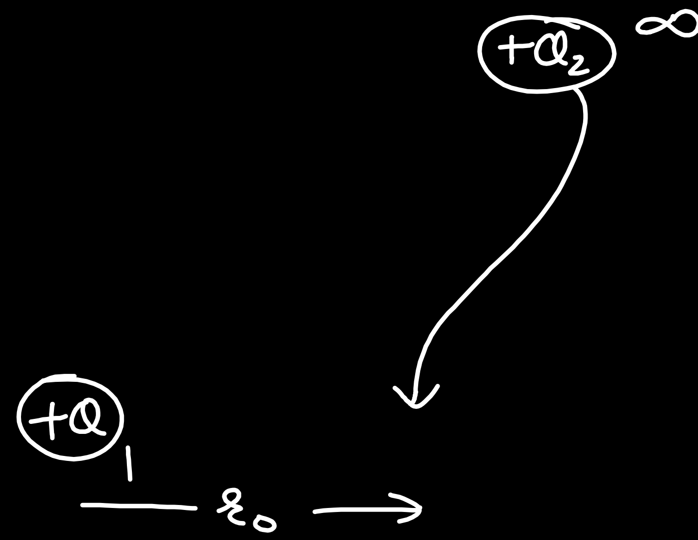
$$\Delta PE = -WD_{conservative}$$

$$PE_f - PE_i = -WD_{conservative}$$

Reference $= \infty$

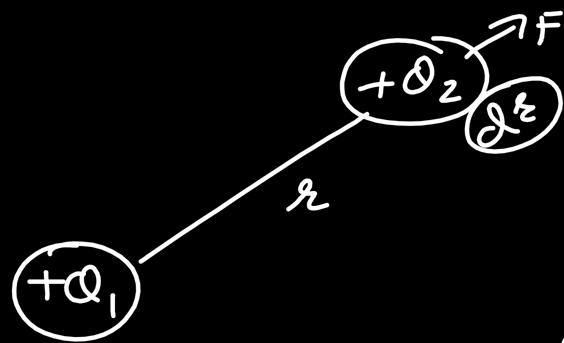
PE at $\infty = 0$

$PE_{reference} = 0.$



Variable

$$F = \frac{KQ_1Q_2}{r^2}$$



Small displ dr

$$WD = \int \vec{F}\cdot\vec{ds}$$

$$= \int \frac{KQ_1Q_2}{r^2}\,dr = KQ_1Q_2 \int r^{-2}\,dr = KQ_1Q_2 \frac{r^{-1}}{-1} = -KQ_1Q_2\left[\frac{1}{r}\right]_{\infty}^{r_0}$$

$\Rightarrow$ HL − LL

$$-KQ_1Q_2\left(\frac{1}{r_0} - \frac{1}{\infty}\right)$$

$$WD = \frac{-KQ_1Q_2}{r_0}$$

$$\Delta PE = -WD_{cons}$$

$$PE_f - \underbrace{\cancel{PE_i}}_{\text{reference}} \rightarrow 0 = -\left(-\frac{KQ_1Q_2}{r}\right)$$

$$PE = \frac{KQ_1Q_2}{r}$$

$$PE = \frac{KQ_1Q_2}{r}$$

→ denominator = r.

→ scalar

→ of a single Pair.

→ charge with Sign

$$PE = \frac{KQQ}{r} = +\frac{KQ^2}{r}$$

$Q \xleftarrow{\quad r \quad} \xrightarrow{} Q$

$+Q \xleftarrow{\quad r \quad} \xrightarrow{} -Q$

$$PE = \frac{K(Q)(-Q)}{r} = -\frac{KQ^2}{r}$$

Q

$Q \qquad Q \qquad Q$

$A \xleftrightarrow{r} B \xleftrightarrow{r} C$

$PE_{system} = ?$

$$\frac{KQQ}{r} + \frac{KQQ}{r} + \frac{KQQ}{2r}$$

$$\frac{KQ^2}{r}\left(1 + 1 + \frac{1}{2}\right) = \frac{5}{2}\frac{KQ^2}{r}$$

Q

$+Q \xleftrightarrow{r} -Q \xleftrightarrow{r} +Q$

$$PE = \frac{K(Q)(-Q)}{r} + \frac{K(Q)(-Q)}{r} + \frac{K(Q)(Q)}{2r}$$

$$= \frac{KQ^2}{r}\left(-1 - 1 + \frac{1}{2}\right) = \frac{KQ^2}{r}\left(-\frac{3}{2}\right)$$

$$-2 + \frac{1}{2}$$

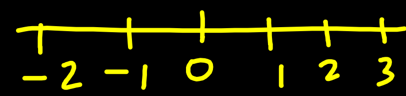
$$= -\frac{3}{2}\frac{KQ^2}{r}$$

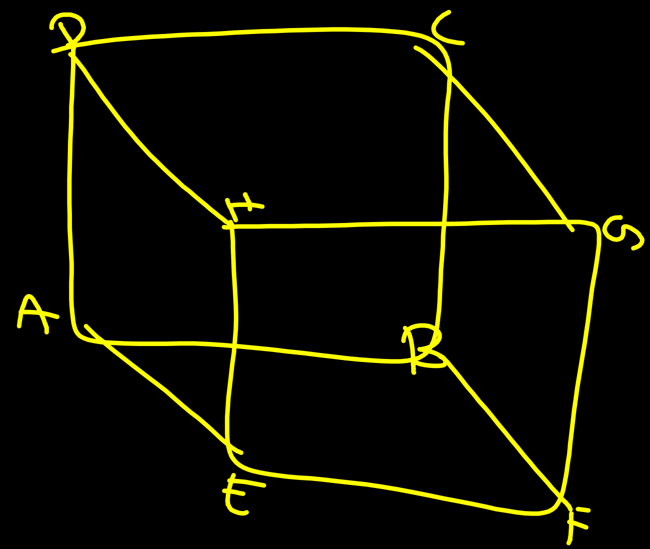
$PE_1 = \underbrace{-2}$

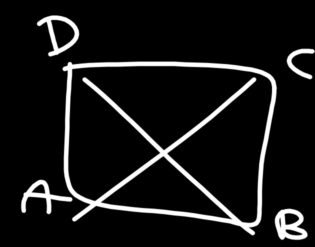
$PE_2 = \underbrace{-5}$

$PE_1 > PE_2$

PE -ve value -ve







Body

AG

DF

EC

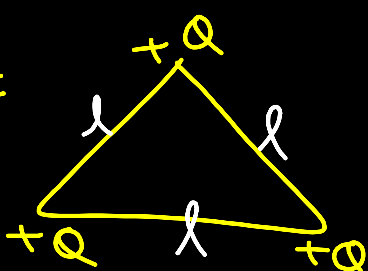
HB

Q

No. of Pairs (count) $= {}^nC_2 = \dfrac{n(n-1)}{2}$

Calculate length of each pair.

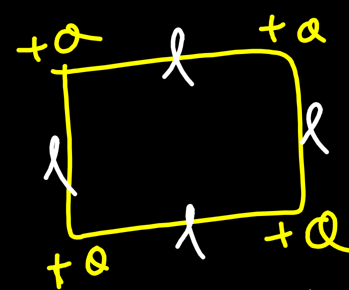
Q



no. of pairs $= \dfrac{3(3-1)}{2} = 3$

$$\frac{KQ^2}{l} + \frac{KQ^2}{l} + \frac{KQ^2}{l} = \frac{3KQ^2}{l}$$

Q



$\dfrac{4(4-1)}{2} = 6$

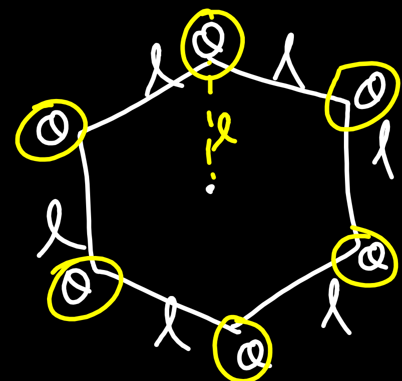
4 pair l length

2 pair $l\sqrt{2}$ "

$$4\left(\frac{KQ^2}{l}\right) + \left(\frac{KQQ}{l\sqrt{2}}\right)2$$

$$\frac{KQ^2}{l}\left[4 + \frac{2}{\sqrt{2}}\right]$$

Q



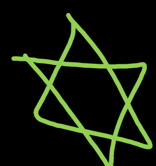
$$6\left(\frac{KQ^2}{l}\right) + \frac{3KQ^2}{2l} + 6\left(\frac{KQ^2}{l\sqrt{3}}\right)$$

$$\frac{6(6-1)}{2} = 15 \text{ pairs.}$$

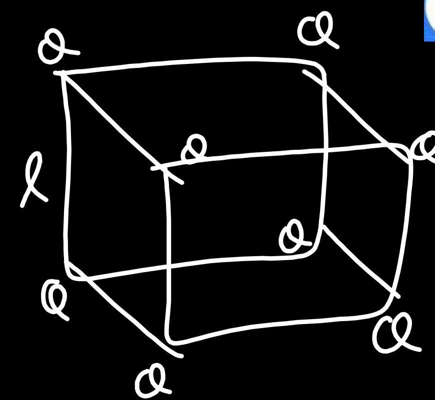
6 pair = l.

3 pair = $2l$ (opposite wale)

6 pairs $l\sqrt{3}$



Q



$$\frac{8(8-1)}{2} = 28_{\text{pairs}}.$$

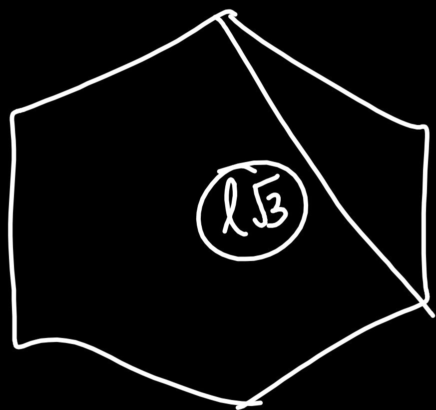
edges 12 = l

facediagonal = 12 = $l\sqrt{2}$

PE system = ?

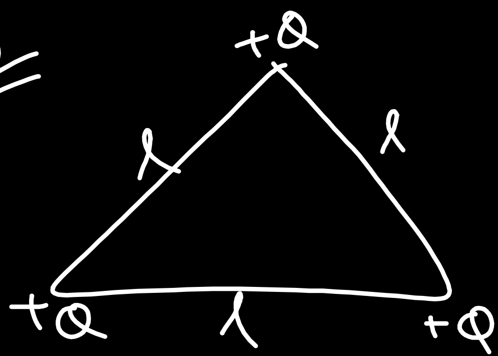
body " = 4 = $l\sqrt{3}$.

$$12\left(\frac{KQ^2}{l}\right) + \frac{12KQ^2}{l\sqrt{2}} + \frac{4KQ^2}{l\sqrt{3}}$$



PE can be Used
- → to find WD
- → to apply Co.E energy

Q



Find WD required to increase size from $l \rightarrow 2l$.

$$PE_i = 3\frac{kQ^2}{l}$$

$$PE_f = 3\frac{kQ^2}{2l}$$

$\Delta PE = -WD_{cons}$ → always Valid

$\Delta PE = +WD_{ext}$ → only valid if $\Delta KE = 0$

$$WD_{ext} = PE_f - PE_i = \frac{3kQ^2}{2l} - \frac{3kQ^2}{l} = \frac{-3}{2}\frac{kQ^2}{l}$$

$$WD_{\text{conservative}} + WD_{ext} = 0$$

(E)

Slowly $\Delta KE = 0$

W.E.T.

$$WD_{net} = \Delta KE \Rightarrow$$

Rakh Do $\Delta KE = 0$

$$\boxed{-WE_{cons} = WD_{ext}}$$

Q

$+Q$ $\quad$ $-Q$

initial $A \xleftarrow{\quad l \quad} \xrightarrow{} B$ $\quad PE_i = \dfrac{-KQ^2}{l}$ $\quad$ Find WD_{ext}

$\Delta PE = -WD_{cons}$

$\Delta PE = WD_{ext}$

increased dist

A $\quad$ B

$+Q \xleftarrow{\quad 2l \quad} \xrightarrow{} -Q$

$$PE_f = \frac{-KQ^2}{2l}$$

$$WD_{ext} = PE_f - PE_i = \left(-\frac{1}{2}\right) - (-1) = 1 - \frac{1}{2} = \frac{1}{2}\frac{KQ^2}{l}$$

Q

init at rest

A: $+2Q$, m

B: $+Q$, $2m$

ℓ

Find their speeds when they reach ∞

rest rest

m, v_1

$$PE_i = \frac{K(2Q)(Q)}{\ell} = \frac{2KQ^2}{\ell}$$

$$PE_f = \frac{KQ_1Q_2}{\infty} = 0$$

COM

$$P_{ini} = P_{final}$$

$$0 = 2mv_2 - mv_1$$

$2m$, v_2

$$\cancel{m}v_1 = 2\cancel{m}v_2$$

COE

$$PE_i + KE_i = PE_f + KE_f$$

$$\frac{2KQ^2}{\ell} + 0 = 0 + \frac{1}{2}mv_1^2 + \frac{1}{2}2mv_2^2$$

$$v_1 = 2v_2$$

$$\frac{v_1}{2} = v_2$$

$$\frac{2KQ^2}{l} = \frac{1}{2}mv^2 + \frac{1}{2}2m\left(\frac{v_1}{2}\right)^2$$

$$= \frac{1}{2}mv_1^2 + \frac{mv_1^2}{4}$$

$$= \frac{3}{4}$$

$$\sqrt{\frac{8KQ^2}{3l\,m}} = v_1$$

$$V_2 = \frac{V_1}{2}$$

$$V_2 = \frac{1}{2}\sqrt{\frac{8\,KQ^2}{3\,ml}}$$

ELECTRIC POTENTIAL (V)

Electric potential is a scalar property of every point in the region of electric field. At a point in electric field, potential is defined as the interaction energy of a unit positive charge.

PE = U

$V = \frac{U}{q_0}$ joule/coulomb

We can define electric potential as "work done in bringing a unit positive charge from infinity to the given point against the electric forces. So we can say that

$$V = -\int_{\infty}^{r} \vec{E} \cdot d\vec{r}$$

Mathematical representation :

If $\left(W_{\infty p}\right)_{ext}$ is the work required in moving a point charge q from infinity to a point P, the electric potential of the point P is $V_{\mathrm{p}} = \left.\frac{(\mathrm{W_{xp}})_{\mathrm{ett}}}{\mathrm{q}}\right]_{\mathrm{acc}-0}$

Properties :

(i) Potential is a scalar quantity, its value may be positive, negative or zero.

(ii) S.I. Unit of potential is volt = joule/coulomb and its dimensional formula is $[M^1 L^2 T^{-3} I^{-1}]$.

(iii) Electric potential due to a positive charge is always positive and due to negative charge it is always negative except at infinite. (taking $V_\infty = 0$).

(iv) Potential decreases in the direction of electric field.

Unacademy Atoms

$$V = \frac{PE}{charge}$$

$$PE = \frac{KQq}{r}$$

$$V = \frac{PE}{q}$$

$$V = \frac{KQ}{r}$$

change in potential

Potential diff

$$\Delta V = \frac{\Delta PE}{charge}$$

$$\Delta V = -\frac{WD_{cons}}{charge}$$

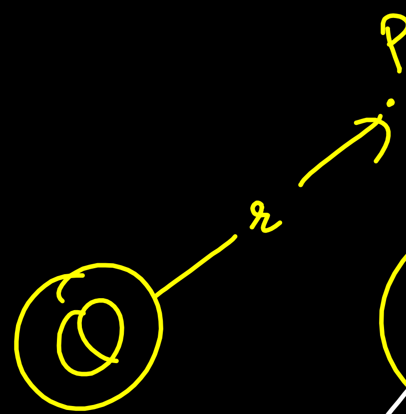
$$\Delta V = \frac{\Delta WD_{ext}}{charge}$$

Point P
+1c

+Q

$$\frac{KQ(1c)}{r}$$

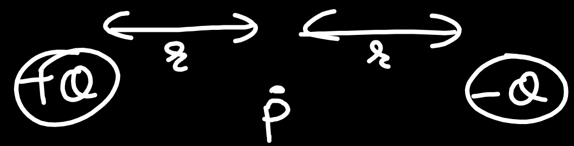
$$V \text{ at } P = \frac{KQ}{r}$$



$$V_P = \frac{KQ}{r}$$

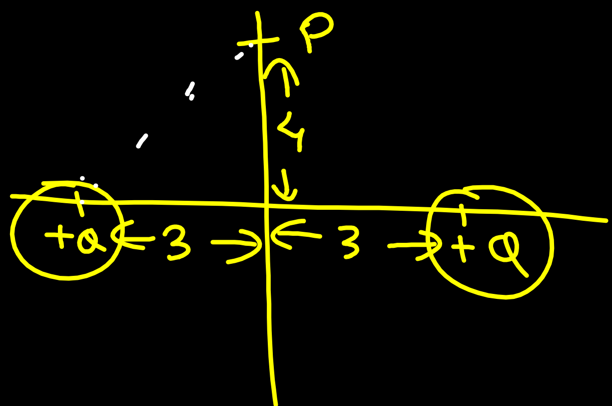
Charge with sign

Q



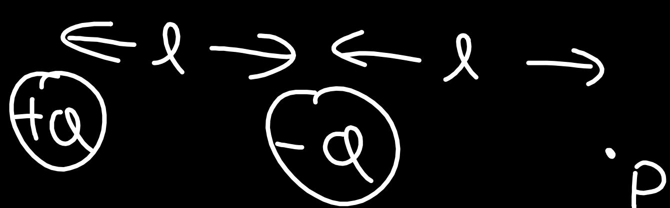
$$V_p = \frac{K(+Q)}{r} + \frac{K(-Q)}{r} = 0$$

Q



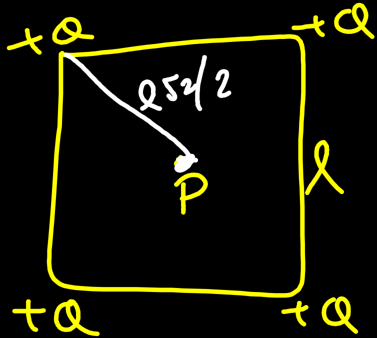
$$V_p = \frac{KQ}{5} + \frac{KQ}{5} = \frac{2KQ}{5}$$

Q



$$\frac{K(Q)}{2l} + \frac{K(-Q)}{l} = -\frac{1}{2}\frac{KQ}{l}$$

Q

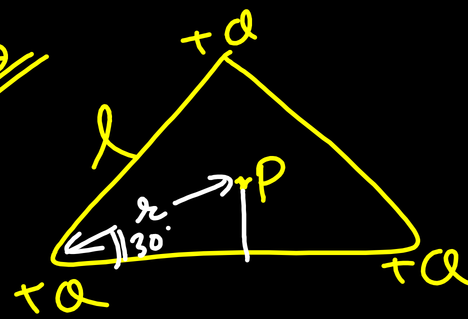


$$\frac{K(Q)}{\left(l\sqrt{2}/2\right)} \times 4$$

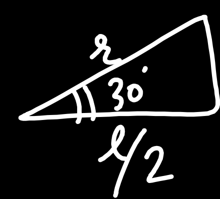
$$\frac{8KQ}{l\sqrt{2}}$$

$$\frac{4\sqrt{2}KQ}{l}$$

Q



P → centroid.



$$\cos 30 = \frac{l}{2r}$$

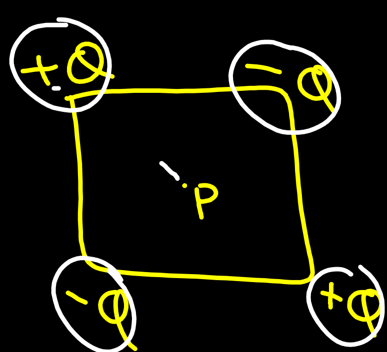
$$r\sqrt{3} = l$$

$$r = \frac{l}{\sqrt{3}}$$

$$\frac{KQ}{\left(l/\sqrt{3}\right)} \times 3$$

$$\frac{3\sqrt{3}KQ}{l}$$

Q



$V_P = ?$

$V = 0$

PE +Q +Q

Force

Potential =
- → PE
- → WD
- → field

•P

+Q

field
- → force
- → motion (speed, acc)
- → flux
- → gauss
- → potential

Point

+2c

$V_p = 3$ Volt

PE of charge = ?

$$V = \frac{PE}{Q}$$

$$PE = QV$$

$$PE = 2(3)$$

$$= +6 \text{ Joule}$$

20 Volt — A — final

30 Volt — B — ini

Find WD required to take −2c from B to A

$$\Delta V = \frac{\Delta WD_{ext}}{Q}$$

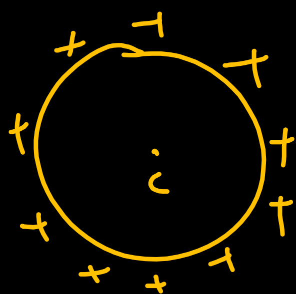
$$\Delta WD_{ext} = Q\,\Delta V$$

$$= -2\left[V_f - V_i\right]$$

$$= -2\left[20 - 30\right)$$

$$= -2(-10)$$

$$= +20$$



Vector

$E_c = 0$

$V_c = \frac{KQ}{R}$

Is it Possible That??

① $E = 0$ at point but $V \neq 0$

Yes

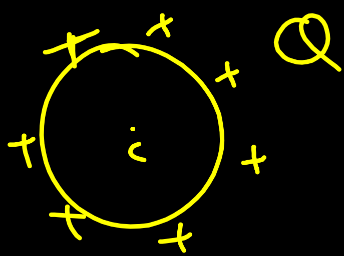
② $V = 0$ at point but $E \neq 0$.

Yes

$V = 0$

$E \neq 0$

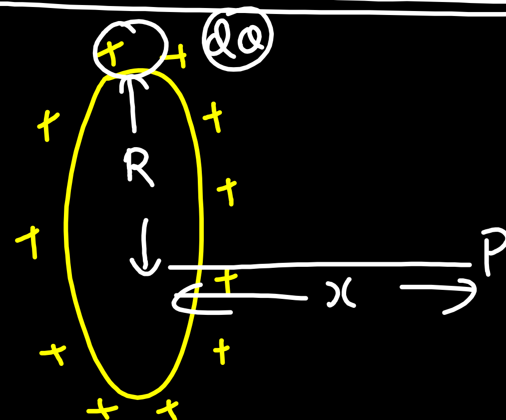
Ring



$$V_c = \frac{KQ}{R}$$

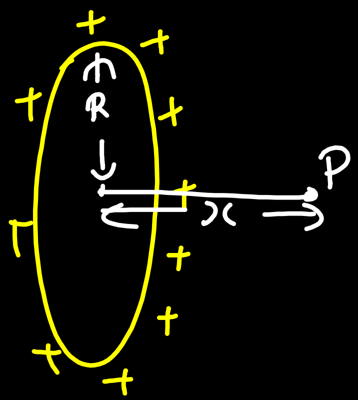
$$V_c = \frac{KQ}{R}$$

Point on Axis of Ring



$$V = \int \frac{K\,(dQ)}{\sqrt{R^2 + x^2}}$$

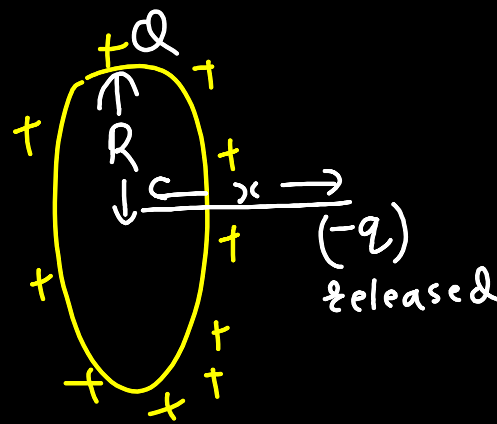
$$V = \frac{KQ}{\sqrt{R^2 + x^2}}$$



$$E_P = \frac{KQx}{(R^2+x^2)^{3/2}}$$

$$V_P = \frac{KQ}{\sqrt{R^2+x^2}}$$

Q//

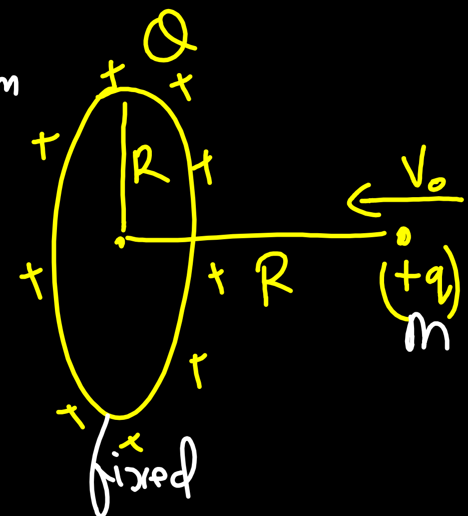


$x <<< R$

Find T of SHM?

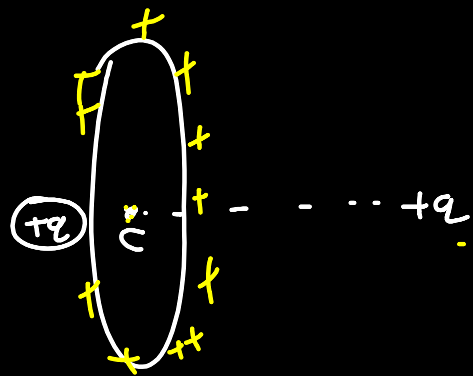
Bounceback L1 ~~previous lec~~

Q// Find minimum V_0, so that q reaches ∞ on other side.



$$V_i = \frac{KQ}{\sqrt{R^2+R^2}}$$

$$V_f = \frac{KQ}{R}$$



Just reaches Center

f is at C

$$PE_i + KE_i = PE_f + KE_f$$

$$\frac{KQq}{R\sqrt{2}} + \frac{1}{2}mv_0^2 = \frac{KQq}{R} \qquad 0$$

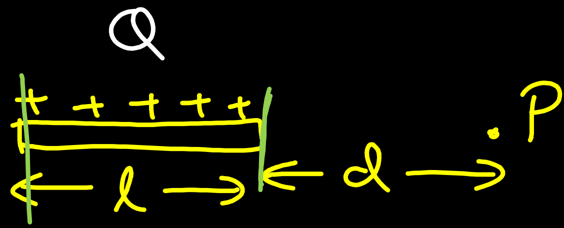
$$\boxed{v_0 = \sqrt{\frac{2KQq}{mR}\left(1 - \frac{1}{\sqrt{2}}\right)}}$$

$$V = \frac{PE}{q} \qquad PE = qV$$

$$V_c = \frac{KQ}{R} \qquad PE = \frac{KQq}{R}$$

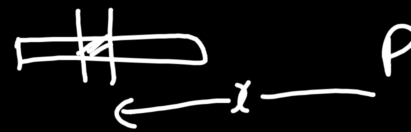
$$V_i = \frac{KQ}{\sqrt{R^2+R^2}} = \frac{KQ}{R\sqrt{2}} \qquad PE_i = \frac{KQq}{R\sqrt{2}}$$

Q



linear charge density = λ

Find potential at P?



$dQ = \lambda dx$

dQ

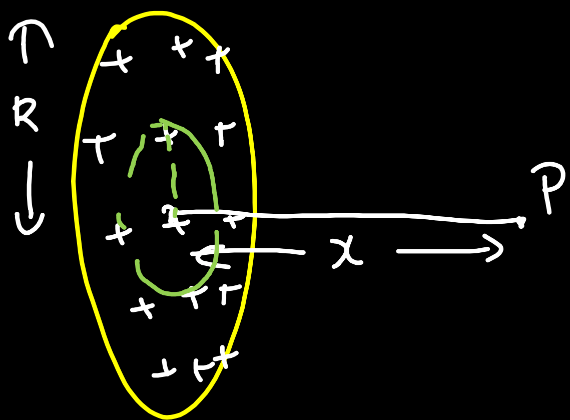
$$V = \frac{K(dQ)}{x} = K\lambda \int_{d}^{d+l} \frac{dx}{x}$$

$$= K\lambda \left[\ln(d+l) - \ln(d)\right]$$

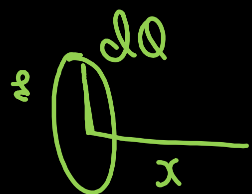
$$= \boxed{K\lambda \ln\left(\frac{d+l}{d}\right)}$$

~~Use of potential :~~

Potential at Axis of Disc

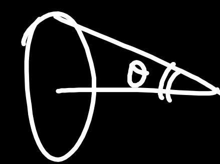


$$dQ = \sigma \, dA$$
$$= \sigma \, 2\pi r \, dr$$



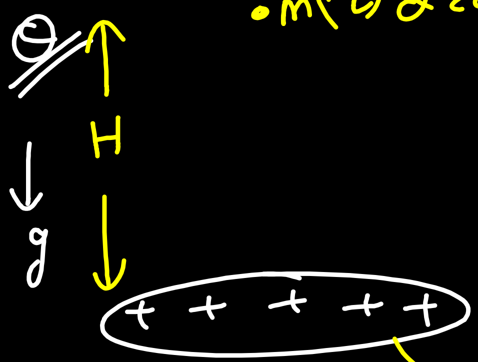
$$V = \int \frac{K \, dQ}{\sqrt{x^2 + r^2}}$$

$$E = \frac{\sigma}{2\varepsilon_0}\left(1 - \cos\theta\right)$$



$$V = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{x^2 + R^2} - x\right)$$

• m(+q) drop



radius = a

charge = σ
density

given

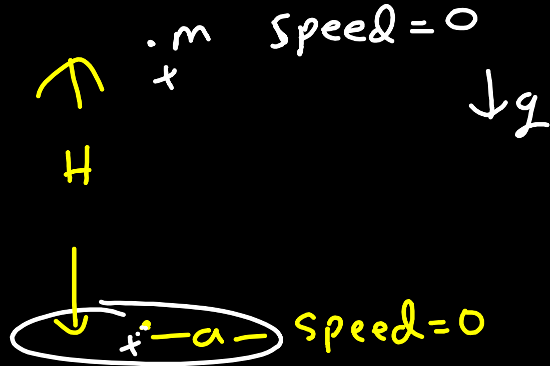
$$\frac{q}{m} = \frac{4\varepsilon_0 g}{\sigma}$$

$$\frac{\sigma q}{4\varepsilon_0 g} = m$$

Find H so that particle Just reaches the disc??

a) $H = \frac{3}{4}a$ b) $H = a/2$

c) $H = \frac{4}{3}a$ ✓ d) None.



$PE = qV$

$$V = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{x^2+R^2} - x\right)$$

$$V_{ini} = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{H^2+a^2} - H\right)$$

$$V_{fin} = \frac{\sigma}{2\varepsilon_0}a$$

COE

$$PE_i + \overset{0}{\cancel{KE_i}} = PE_f + \overset{0}{\cancel{KE_f}}$$

$$q\frac{\sigma}{2\varepsilon_0}\left(\sqrt{H^2+a^2} - H\right) + mgH = \frac{\sigma}{2\varepsilon_0}a\,q$$

$$mgH = \frac{q\sigma}{2\varepsilon_0}\left[a + H - \sqrt{H^2+a^2}\right]$$

$$\frac{\cancel{q\sigma}}{\cancel{4\varepsilon_0 g}}\cancel{g}H = \frac{\cancel{\sigma q}}{\cancel{2\varepsilon_0}}$$

$$\frac{H}{2} = a + H - \sqrt{H^2+a^2}$$

$$\frac{H}{2} = a + H - \sqrt{}$$

$$\sqrt{} = a + H - \frac{H}{2}$$

$$\sqrt{H^2 + a^2} = a + \frac{H}{2}$$

$$H^2 + \cancel{a^2} = \cancel{a^2} + \frac{H^2}{4} + aH$$

$$\frac{3H^2}{4} = aH$$

$$H = \frac{4a}{3}$$

Relation b/w V & E

$U = PE$

$\Delta PE = -WD_{con}$

$$\boxed{\Delta PE = -\int \vec{F} \cdot \vec{ds}}$$

$$\boxed{\Delta V = -\int \vec{E} \cdot \vec{ds}}$$

$\vec{F_c} = -$ partial diff of PE

$$\vec{F_c} = -\frac{\partial U}{\partial x}\hat{i} - \frac{\partial U}{\partial y}\hat{j} - \frac{\partial U}{\partial z}\hat{k}$$

$$\vec{E} = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}$$

Unacademy Atoms

Q. $\vec{E} = x\hat{i} + y\hat{j}$

Find $\Delta V = ?$

$$\vec{ds} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$$

$$-\int \vec{E} \cdot \vec{ds}$$

$$-\int (x\hat{i} + y\hat{j}) \cdot (dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k})$$

$$-\int (x\,dx + y\,dy)$$

$$\Delta V = -\left(\frac{x^2}{2} + \frac{y^2}{2}\right)$$

Q $V = \frac{5}{r}$

Find E function with r

$$E = -\frac{dV}{dr}$$

$$E = -\frac{dV}{dx}$$

$$V = 5r^{-1}$$

$$E = -\frac{dV}{dr}$$

$$\frac{dV}{dr} = 5(-1)r^{-1-1}$$

$$E = +\frac{5}{r^2}$$

$$\frac{dV}{dr} = \frac{-5}{r^2}$$

Q

$$V = x + y + z$$

$$\vec{E} = ?$$

(only x treat variable) $\dfrac{\partial V}{\partial x} = (1) + 0 + 0$

(y " ") $\dfrac{\partial V}{\partial y} = 0 + (1) + 0$

(z " ") $\dfrac{\partial V}{\partial z} = 0 + 0 + 1$

$$\vec{E} = -1\hat{i} - 1\hat{j} - 1\hat{k}$$

$$V = xy$$

$$5x \quad \xrightarrow{\text{diff.}} \quad 5(1)$$

only x variable

$$\frac{\partial V}{\partial x} = (1)y$$

$$\frac{\partial V}{\partial x} = y$$

$$\frac{\partial V}{\partial y} = x(1)$$

$$\vec{E} = -y\hat{i} - x\hat{j}$$

$$V = xyz$$

$$\vec{E} = ?$$

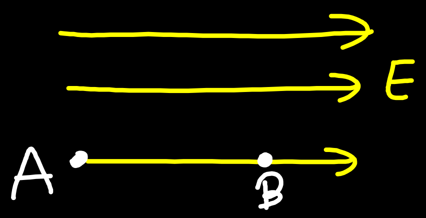
$$\frac{\partial V}{\partial x} = (1)yz$$

$$\frac{\partial V}{\partial y} = x(1)z$$

$$\frac{\partial V}{\partial z} = xy(1)$$

$$\vec{E} = -yz\hat{i} - xz\hat{j} - xy\hat{k}$$

In Uniform Field



(high potential to low potential)

$V_A > V_B$

$$\Delta V = -\int \vec{E}\cdot\vec{ds}$$

$$\Delta V = -\vec{E}\cdot\vec{\Delta s}$$

$$\boxed{|\Delta V| = Ed}$$

$$\boxed{\frac{|\Delta V|}{d} = E}$$

distance along field direction

Equipotential surfaces

For a given charge distribution, locus of all points having same potential is called 'equipotential surface'.

Equipotential surfaces can never cross each other

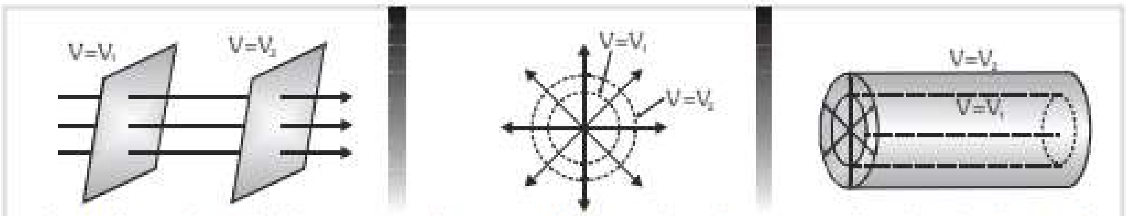
Equipotential surfaces are always perpendicular to direction of electric field.

If a charge is moved from one point to the other over an equipotential surface then work done is 0

$\frac{WD}{\text{charge}} = \Delta V$ $WD = Q\Delta V$

$WD = 0$

Shapes of equipotential surfaces.

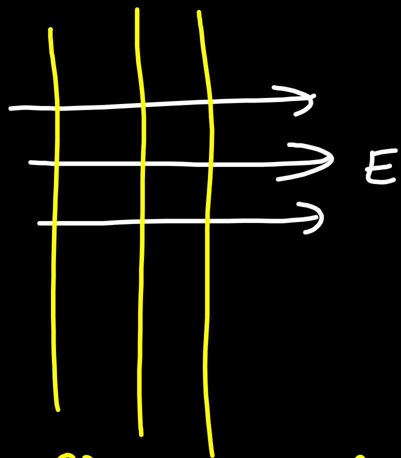


for uniform electric field equipotential surfaces are parallel plane

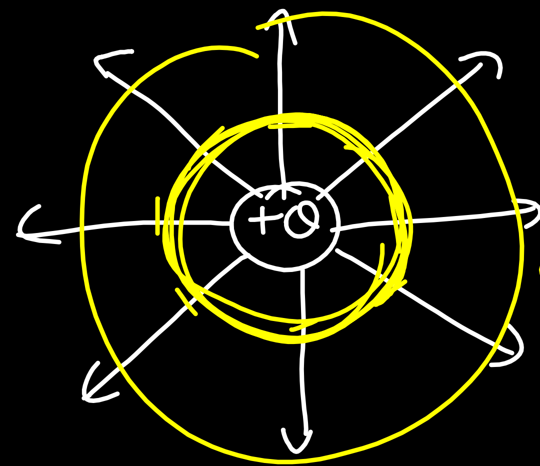
for a point charge, spherical conductor equipotential surfaces are spherical

for a line distribution of charge equipotential surfaces are cylinderical

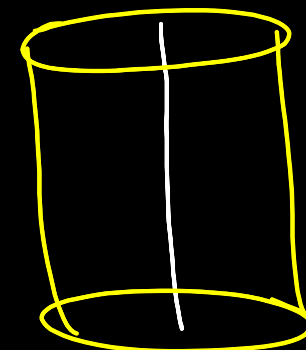
The intensity of electric field along an equipotential surface is always zero



equipotential
planar



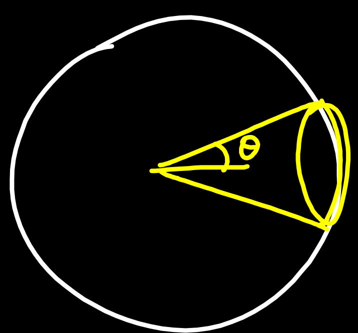
equipotential



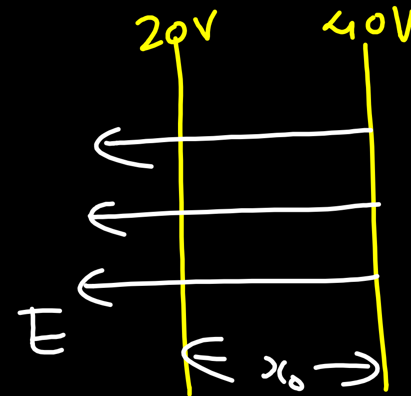
(E is always ⊥'ar to equipotential)

$2\pi(1-\cos\theta)$

Solid angle = $\dfrac{\text{Surface area}}{(\text{Radius})^2}$



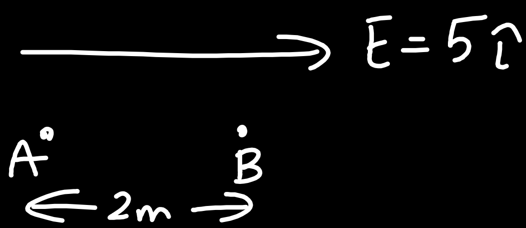
two equipotential surface



in case of uniform E

$E = \left|\dfrac{\Delta V}{d}\right|$ along field

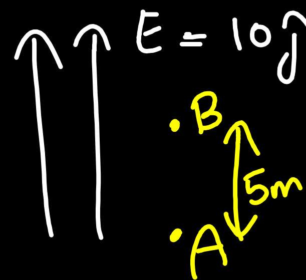
$$E = \frac{40-20}{x_0}$$



$$|\Delta V| = Ed$$

$$\boxed{V_A - V_B = 10 \text{ Volt}}$$

Q

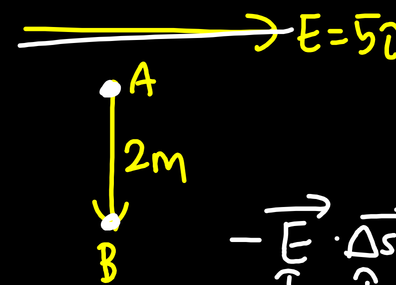


$$|\Delta V| = |Ed|$$

$$= 10 \cdot 5$$

$$V_A - V_B = 50$$

Q

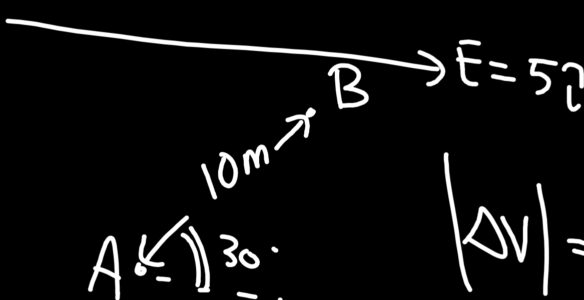


$$-\vec{E} \cdot \vec{\Delta s}$$

$$\hat{i} \cdot \hat{j}$$

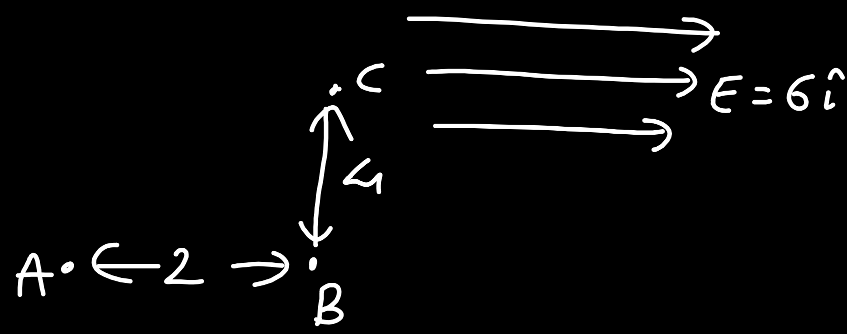
$$|\Delta V| = 0$$

Q



$$|\Delta V| = ? \quad Ed = 5(10\cos 30) = 50\frac{\sqrt{3}}{2} = 25\sqrt{3}$$

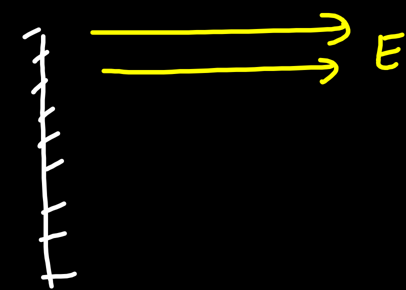
Q.



$$V_A - V_B = Ed = 12$$

$$V_A - V_C = 12$$

$$V_B - V_C = 0$$



all have same pot.

$$\Delta V = -\int \vec{E} \cdot \vec{ds}$$

when $\vec{ds}$ is $\perp$'ar to $\vec{E}$

$$\Delta V = 0$$

change in pot = 0

Pot. constant

512 identical drops of mercury are charged to a potential of 2 V each.

The drops are joined to form a single drop. The potential of this drop is ..128..... V.

Jee 2021

$$\frac{KQ}{R} = 2V$$

512 drops.

$512Q$

$$\frac{4}{3}\pi R'^3 = 512\,\frac{4}{3}\pi R^3$$

$$R'^3 = 512 R^3$$

$$R' = 8R$$

$$V' = \frac{K(512Q)}{8R}$$

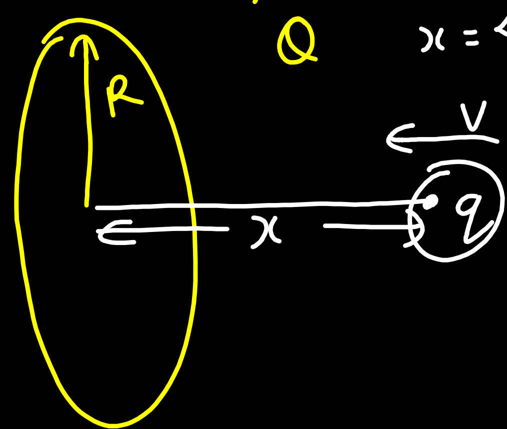
$$= 64\frac{KQ}{R} = 64 \times 2 = 128V$$

H.W.

A uniformly charged ring of radius $3a$ and total charge q is placed in xy-plane centered at origin. A point charge q is moving towards the ring along the z-axis and has speed v at $z = 4a$. The minimum value of v such that it crosses the origin is :

$R = 3a$

$Q \qquad x = 4a$ [10 April 2019]



(a) $\sqrt{\frac{2}{m}}\left(\frac{4}{15}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$

(b) $\sqrt{\frac{2}{m}}\left(\frac{1}{5}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$

(c) $\sqrt{\frac{2}{m}}\left(\frac{2}{15}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$ check.

(d) $\sqrt{\frac{2}{m}}\left(\frac{1}{15}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$

In a certain region of space, the potential is given by $V = k(2x^2 - y^2 + z^2)$. The electric field at the point (1,1,1) has magnitude :

(A) $k\sqrt{6}$

(B) $2k\sqrt{6}$

(C) $2k\sqrt{3}$

(D) $4k\sqrt{3}$

$$\frac{\partial V}{\partial x} = K(4x)$$

$$\frac{\partial V}{\partial y} = K(-2y)$$

$$\frac{\partial V}{\partial z} = K(2z)$$

$$\vec{E} = -4kx\hat{i} + 2ky\hat{j} - 2kz\hat{k}$$

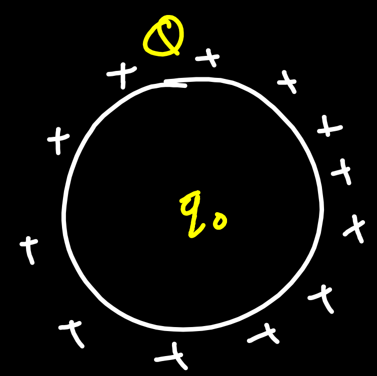
$$\vec{E} = -4k\hat{i} + 2k\hat{j} - 2k\hat{k}$$

$$\sqrt{16K^2 + 4K^2 + 4K^2} = \sqrt{24}\,K$$

$$= 2\sqrt{6}\,K$$

A thin wire ring of radius r has an electric charge q. What will be the increment of the force stretching the wire if a point charge q_0 is placed at the ring's center?

field force concept



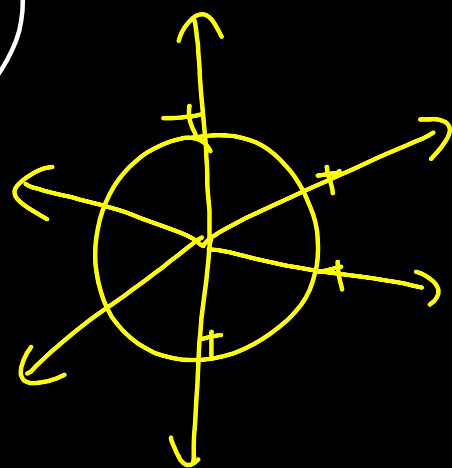
Q

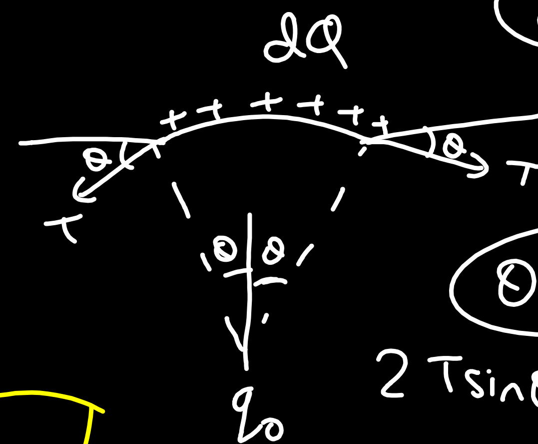
q_0

$$\text{Tension} = \frac{(\text{Scalar addition of force})}{2\pi}$$

$$= \frac{KQq_0}{R^2\, 2\pi}$$

BouceBack
G.O.A.T



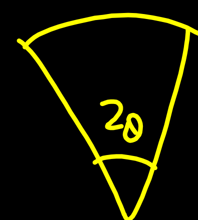


$dQ = \lambda dx$

dx = arc length

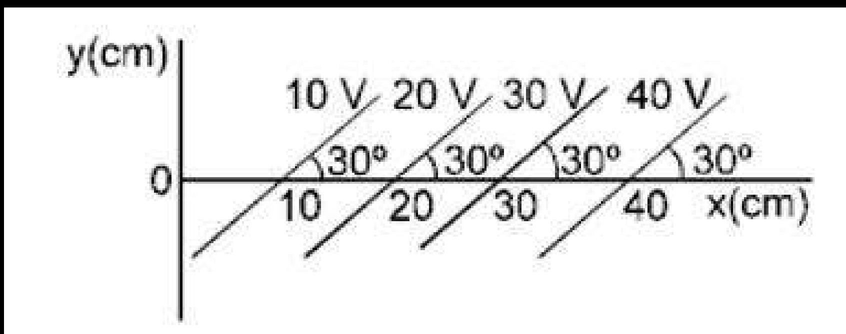
θ small

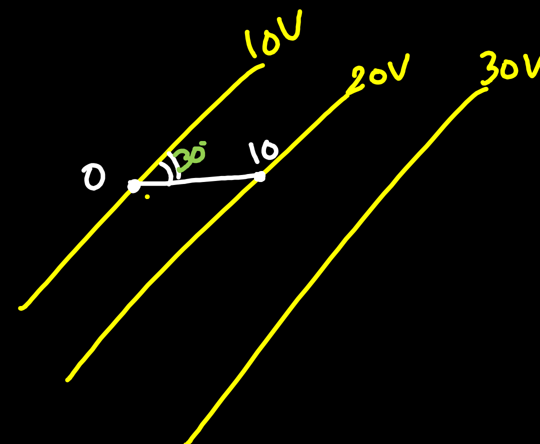
$$2T\sin\theta = \frac{k(q_0)(dQ)}{R^2}$$



$dx = R(2\theta)$

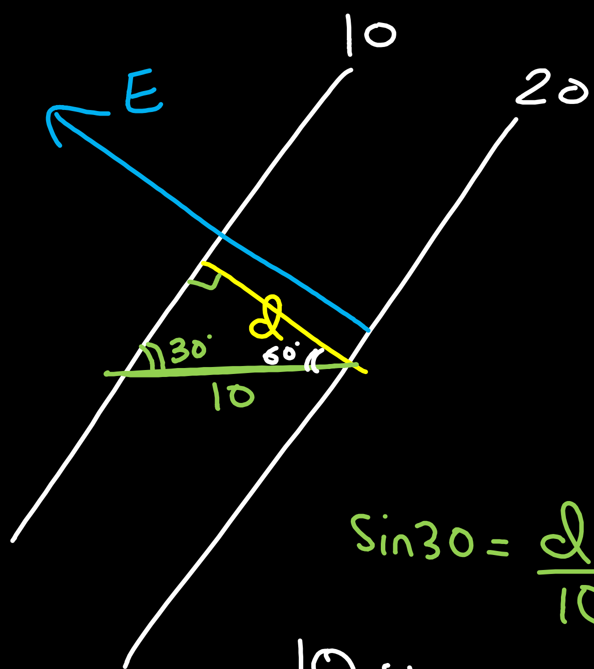
Some equipotential surfaces are shown in figure. What can you say about the magnitude of the electric field?





$|E| = ?$

$|E| = \left|\frac{\Delta V}{d}\right|$

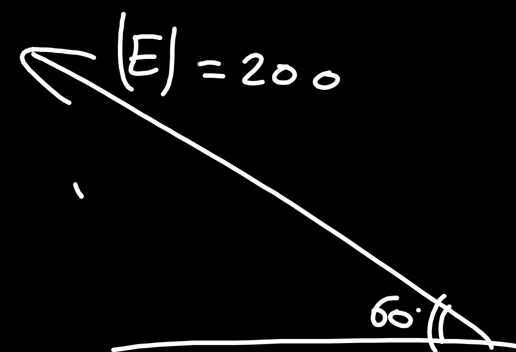


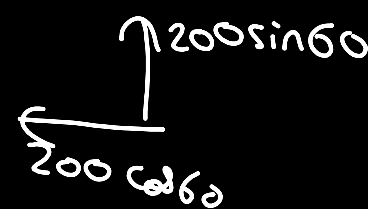
$$\sin 30 = \frac{d}{10}$$

$$10 \sin 30 = d$$

$$5 = d.$$

$$d = \frac{5}{100}\,m$$





$$E = \frac{\Delta V}{d} = \frac{20 - 10}{\left(5/100\right)}$$

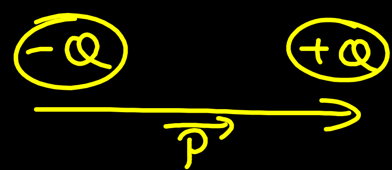
$$= \frac{1000}{5} = 200\ V/m$$

$$\vec{E} = -200\cos 60\,\hat{i} + 200 \sin 60\,\hat{j}$$

<u>**ELECTRIC DIPOLE**</u>

A system of two equal and opposite charges separated by a certain distance is called electric dipole, shown in figure. Every dipole has a characteristic property called dipole moment.

dipole moment = $\vec{P}$



In some molecules, the centers of positive and negative charges do not coincide. This results in the formation of electric dipole.

Atom is non-polar because in it the centers of positive and negative charges coincide.

Dipole Moment : Dipole moment $\vec{p} = q\vec{d}$

(i) Vector quantity, directed from negative to positive charge

(ii) **Dimension** : [LTA], **Units** : coulomb × meter (or C – m)

$$\xleftarrow{\quad} l \xrightarrow{\quad}$$

$-Q$ $\qquad\qquad$ $+Q$

$$|\vec{P}| = (Q)\ l$$

$$= (\text{charge})(\text{dist b/w them})$$

direct^n $\Rightarrow$ $-$ve to $+$ve

$$\xleftarrow{\quad} 2d \xrightarrow{\quad}$$

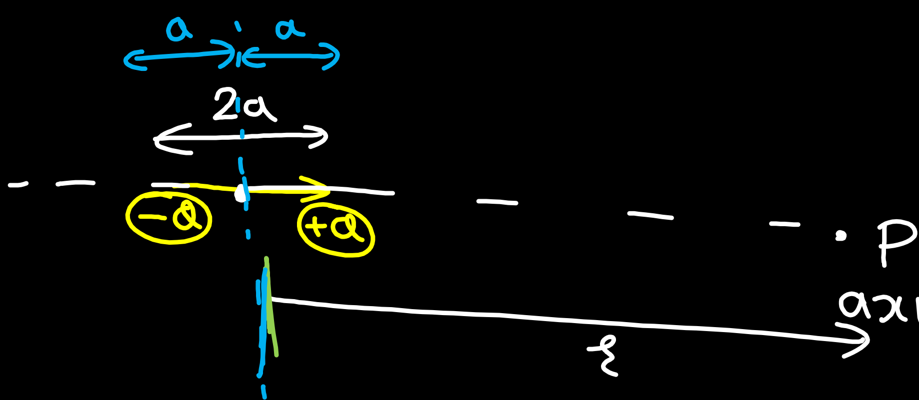
$-Q$ $\qquad\qquad$ $+Q$

$$\vec{P} = Q(2d)\hat{i}$$

Q. E due to Dipole

$\overleftarrow{E_-} \cdot \overrightarrow{E_+}$

$\frac{KQ}{(r+a)^2}$ $\qquad$ $\frac{KQ}{(r-a)^2}$



axial/end on

$$E = \frac{KQ}{(r-a)^2} - \frac{KQ}{(r+a)^2}.$$

$r >>> a.$

$$\frac{KQ}{(r-\cancel{a})^2(r+\cancel{a})^2}\left[(r+a)^2-(r-a)^2\right]$$

$$P = Q(2a)$$

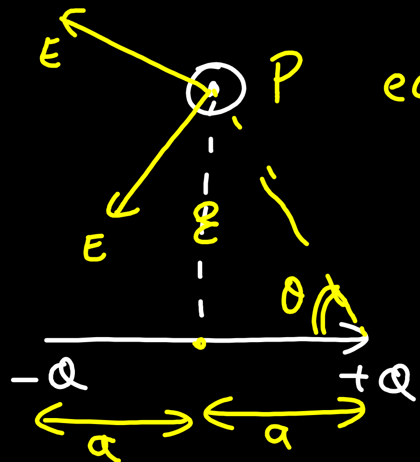
$$r >>> a$$

$$\cancel{r^2}+\cancel{a^2}+2ar-\cancel{r^2}-\cancel{a^2}+2ar$$

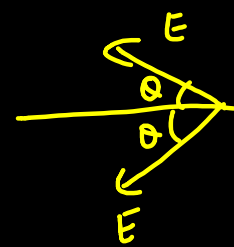
$$\boxed{\vec{E}_{axis} = \frac{2K\vec{P}}{r^3}}$$

$$\frac{KQ\,4a\cancel{r}}{(r^2)(\cancel{r^2})}$$

$$\frac{2K(Q2a)}{r^3}$$



equitorial



$\longleftarrow$

$2E\cos\theta$

$$2 \times \frac{KQ}{\left(\sqrt{r^2+a^2}\right)^2} \times \frac{a}{\sqrt{r^2+a^2}}$$

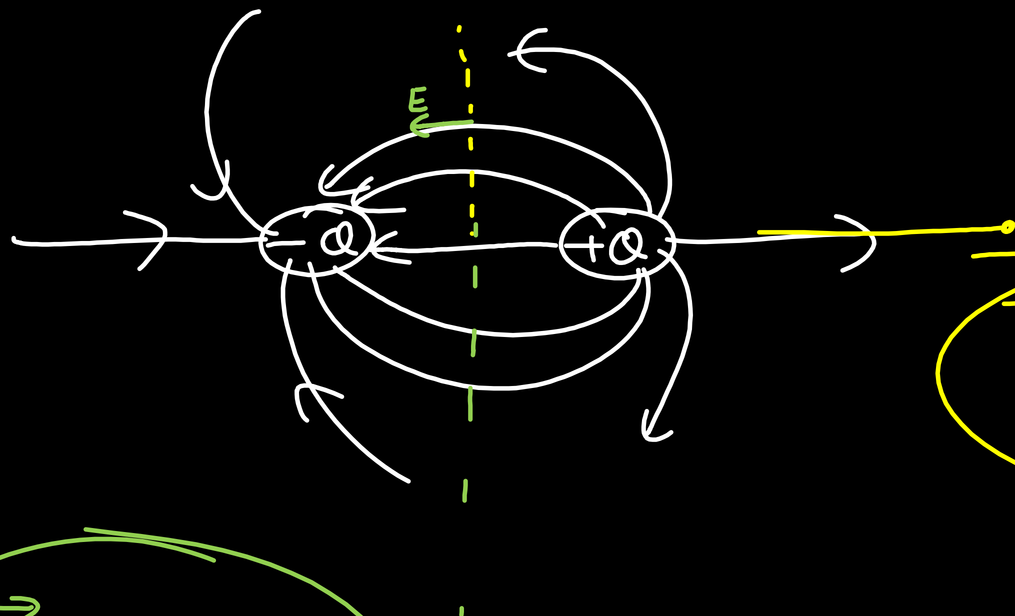
$$\frac{2KQa}{(r^2+a^2)^{3/2}}$$

$r \gg a$

$$\frac{2KQa}{r^3}$$

$$\vec{E} = -\frac{K\vec{p}}{r^3}$$

equitorial.



$$\vec{E} = \frac{2k\vec{p}}{r^3}$$

$$\vec{E}_{eq} = -\frac{k\vec{p}}{r^3}$$

t.me/ajitlulla

Dipole

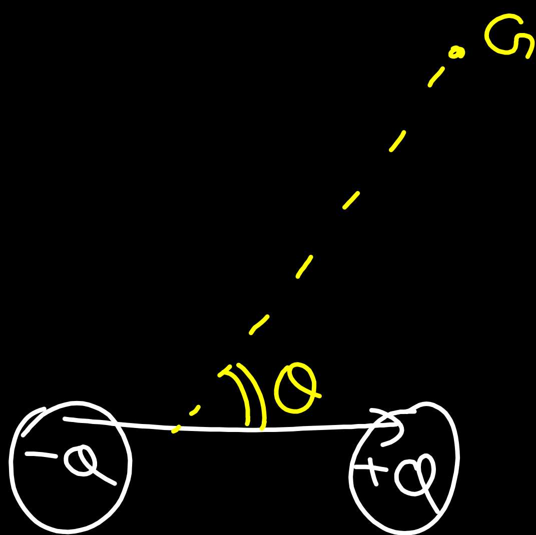
$$\vec{E} = -\frac{k\vec{p}}{r^3}$$

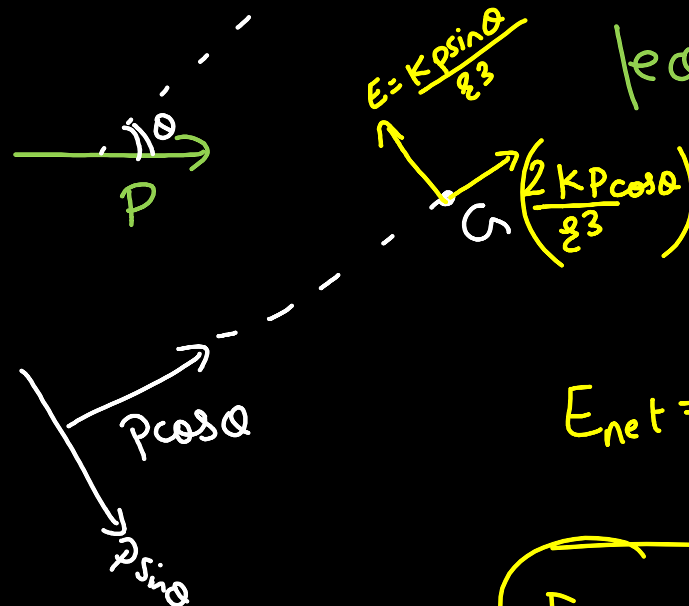
$$\vec{E} = \frac{2k\vec{p}}{r^3}$$

$$\vec{p} = q(d)$$

$$E \propto \frac{1}{r^3}$$

E at any general Point



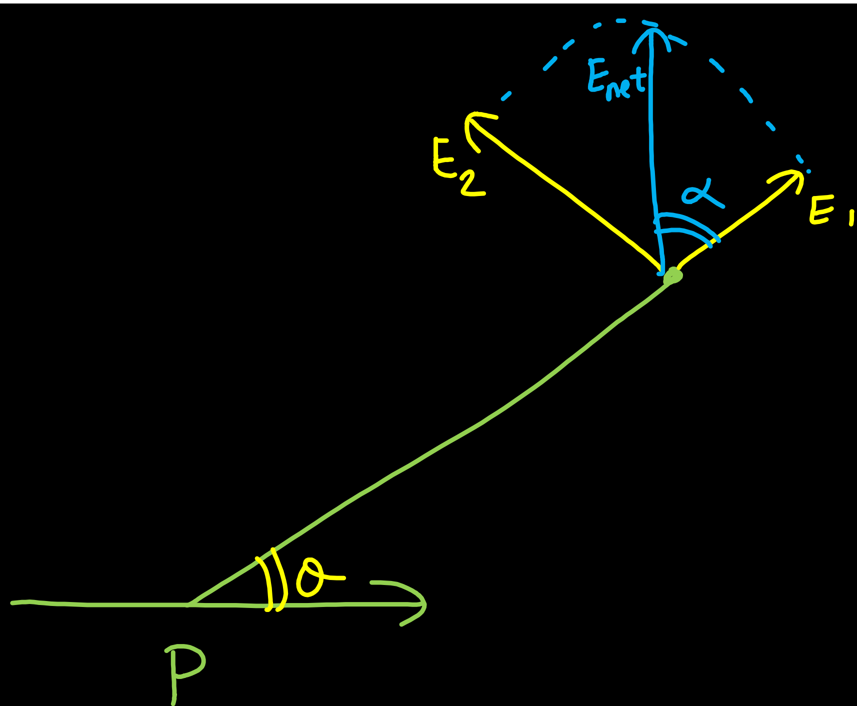


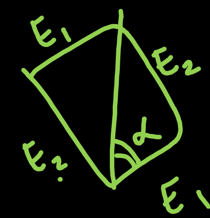
$|\text{axial}| = \frac{2Kp}{r^3}$

$|\text{eq}| = \frac{Kp}{r^3}$

$$E_{net} = \sqrt{\left(\frac{2Kp\cos\theta}{r^3}\right)^2 + \left(\frac{Kp\sin\theta}{r^3}\right)^2}$$

$$E_{net} = \frac{Kp}{r^3}\sqrt{1 + 3\cos^2\theta}$$



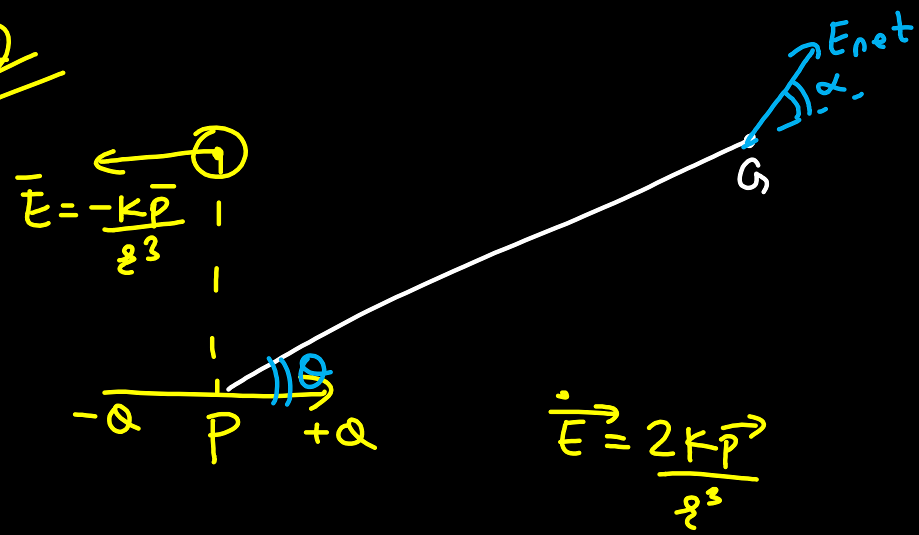


$$\tan\alpha = \frac{E_2}{E_1}$$

$$= \frac{\sin\theta}{2\cos\theta}$$

$$\tan\alpha = \frac{\tan\theta}{2}$$

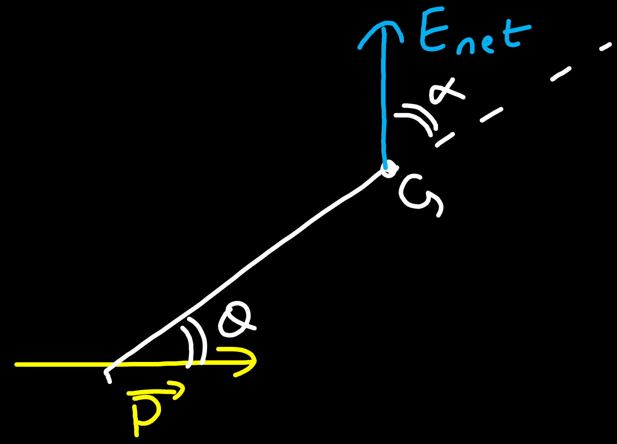
$\underline{\underline{\theta}}$



$$\boxed{E_{\substack{net \\ at\ G}} = \frac{kp}{r^3}\sqrt{1+3\cos^2\theta}}$$

$$\boxed{\tan\alpha = \frac{\tan\theta}{2}}$$

Q. Find angle θ for general point G where E_{net} is $\perp$'ur to $\vec{P}$?



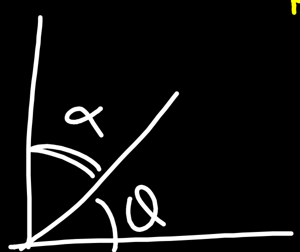
a) $\tan^{-1}(\sqrt{3})$

b) $\tan^{-1}(3)$

c) $\tan^{-1}(\sqrt{2})$

$$\tan\alpha = \frac{\tan\theta}{2}$$

$$\tan(90-\theta)$$



$$\alpha + \theta = 90$$

$$\alpha = (90-\theta)$$

$$\cot\theta = \frac{\tan\theta}{2}$$

$$\frac{1}{\tan\theta} = \frac{\tan\theta}{2}$$

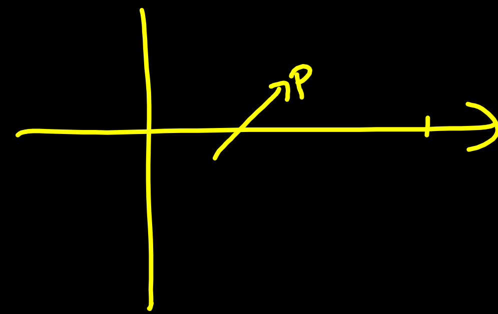
$$2 = \tan^2\theta$$

$$\tan\theta = \sqrt{2}$$

$$\theta = \tan^{-1}(\sqrt{2})$$

Q. Dipole kept at (2,0,0)

$\vec{P} = \hat{i} + \sqrt{3}\,\hat{j}$



Find $|E| = ?$ at point (4,0,0)

$$|P| = \sqrt{(1)^2 + (\sqrt{3})^2} = \sqrt{4} = 2$$

$$E = \frac{KP}{r^3}\sqrt{1+3\cos^2\theta}$$

$$= \frac{K\,2}{(2)^3}\sqrt{1+3\left(\frac{1}{4}\right)}$$

$$= \frac{K}{4}\sqrt{\frac{7}{4}} = \frac{\sqrt{7}\,K}{8}$$

$$\vec{P} = \hat{i} + \sqrt{3}\,\hat{j}$$

$$\vec{r} = 2\hat{i}$$

$$\vec{P}\cdot\vec{r} = |P||r|\cos\theta$$

$$2 = 2\;2\cos\theta$$

$$1/2 = \cos\theta$$

$\vec{r}$ = Position vector of Point w.r.t. center of dipole

$$\vec{r}_{point\ C} = \vec{r}_{P} - \vec{r}_{C}$$

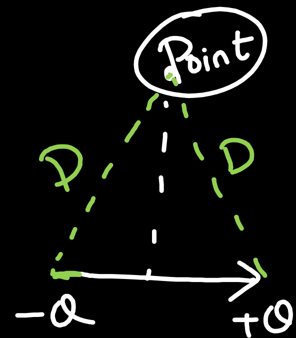
$$= (4,0,0) - (2,0,0)$$

$$\vec{r} = 2\hat{i}$$

$$|r| = 2$$

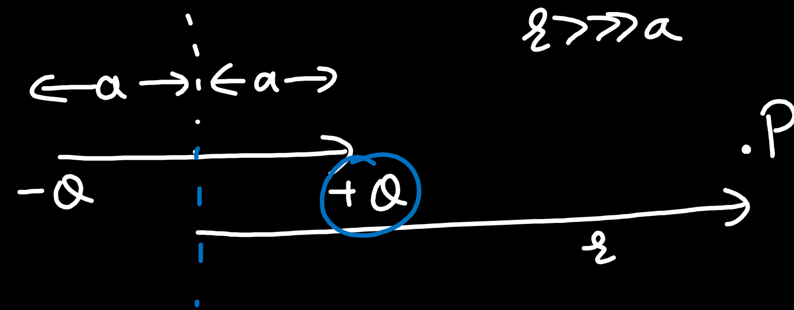
Pontential Due to Dipole

$$V = \frac{KQ}{D} - \frac{KQ}{D}$$



$$V_{equitorial} = 0$$

$r >>> a$



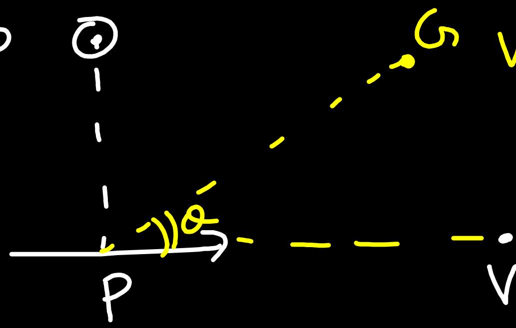
$$V_p = \frac{KQ}{r-a} - \frac{KQ}{(r+a)} = \frac{KQ\left[r+a-(r-a)\right]}{(r-a)(r+a)}$$

$$\vec{P} = Q(2a)$$

$$= \frac{KQ\,2a}{r^2-a^2}$$

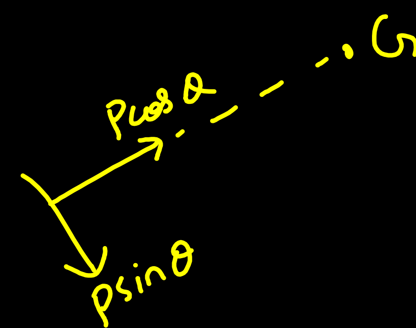
$$= \frac{KQ2a}{r^2} = \frac{Kp}{r^2}$$

$V = 0$



$$V = \frac{Kp\cos\theta}{r^2}$$

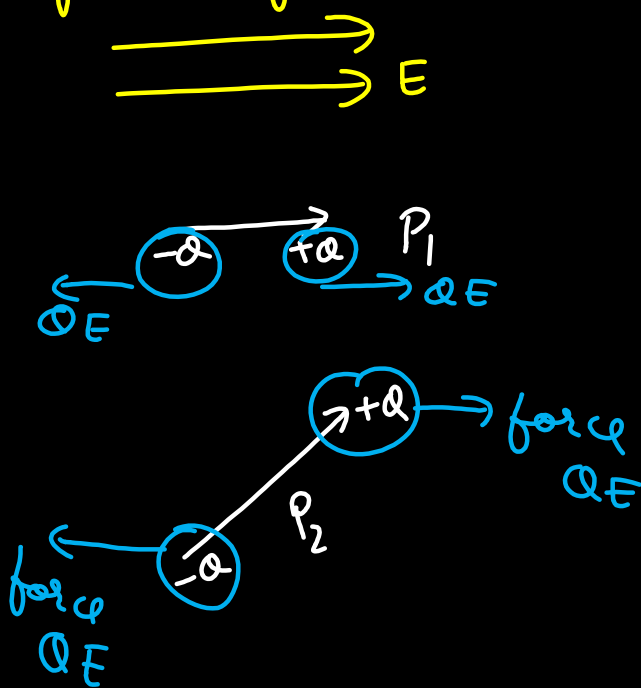
$$V = \frac{Kp}{r^2}$$



$$V \propto \frac{1}{r^2}$$

Dipole in external E field

① uniform E field



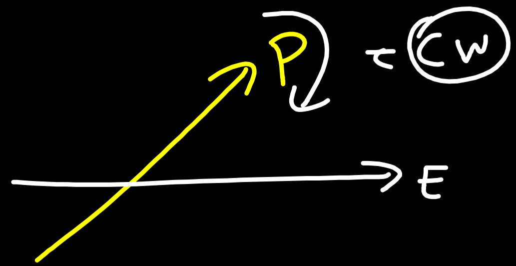
Net force on dipole = 0

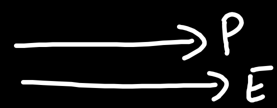
But there can be Torque acting on it

$$\boxed{\vec{\tau} = \vec{P} \times \vec{E}}$$

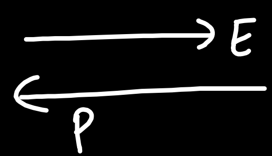
$$\tau = PE \sin\theta$$

TorQue $\longrightarrow$ Chahat $\longrightarrow$ wants to align dipole along E lines

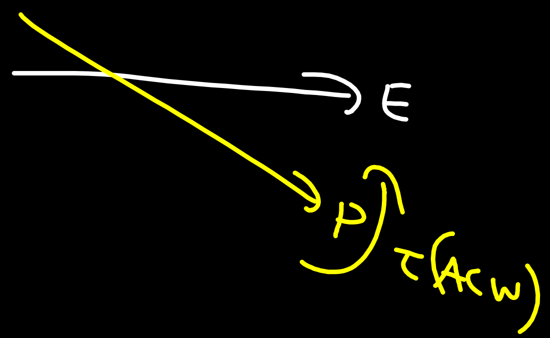


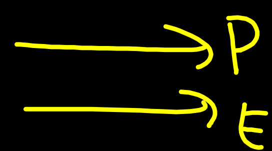


$\tau = \vec{p} \times \vec{E} = pE\sin\theta = 0$

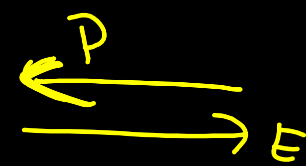


$\tau = pE\sin 180 = 0$



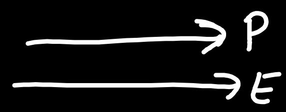


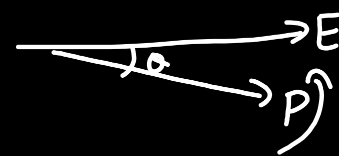
Stable



Unstable

SHM





θ small $\sin\theta \approx \theta$

Find T of small oscillations.

$\tau = PE\sin\theta$

$$\tau = I\alpha$$

moment of inertia

$I\alpha = PE\theta$

SHM

SHM

$a = -\omega^2 x$

$\alpha = -\omega^2\theta$

$F = -kx$

$$\alpha = -\omega^2\theta$$

$$\alpha = -\frac{PE}{I}\theta$$

$\omega = \sqrt{\frac{PE}{I}}$

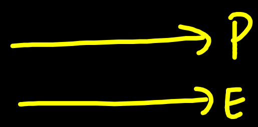
$T = \frac{2\pi}{\omega}$

$$T = 2\pi\sqrt{\frac{I}{PE}}$$

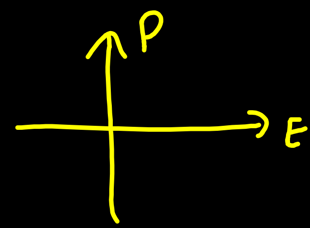
PE of Dipole in ext. field

$$U = -\vec{P}\cdot\vec{E}$$

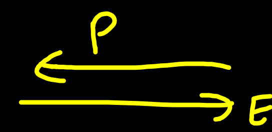
$$= -PE\cos\theta$$



$$U = -PE$$



$$U = 0$$



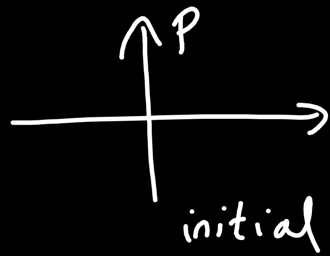
$$U = -PE\cos 180$$

$$\underline{U = +PE}$$

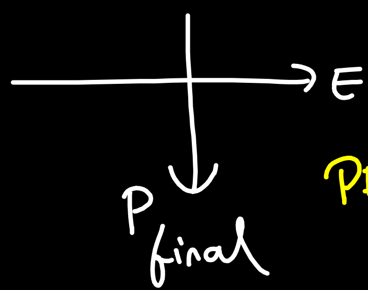
WD required to turn dipole?

$WD_{ext} = \Delta PE$

Q



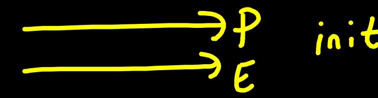
$PE_i = 0$



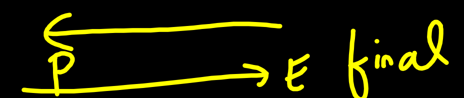
$PE_f = 0$

$WD = 0 - 0 = 0$

Q



$PE_i = -PE$



$P_{final} = +PE$

$$WD_{ext} = P_f - P_i$$
$$= PE - (-PE)$$
$$= +2PE$$

Dipole in Non Uniform E

$\longrightarrow P$

$\longrightarrow E$

t.me/ajitlulla

$\vec{E} = E_0 x \hat{i}$

$-Q \qquad +Q$

$\vec{\text{Force}} = Q(\vec{E})$

$E \qquad (E + dE)$

$\longleftarrow dx \longrightarrow$

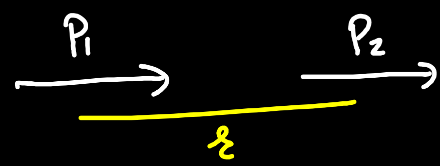
$QE \qquad QE + QdE$

$$F = Q\,dE$$

$$F = (Q\,dx)\frac{dE}{dx}$$

$$F = P\frac{dE}{dx}$$

$$\vec{F} = \vec{p}\,\frac{d\vec{E}}{dx}$$



Find force of attraction b/w two shown dipoles.

field due to p_1 $= \dfrac{2Kp_1}{r^3}$

$$E = 2Kp_1 r^{-3}$$

$$\frac{dE}{dr} = 2Kp_1(-3)r^{-3-1}$$

$$= \frac{-6Kp_1}{r^4}$$

$$|F| = p_2\frac{dE}{dr}$$

$$|F| = p_2\left(\frac{6Kp_1}{r^4}\right)$$

$$= \frac{6Kp_1p_2}{r^4}.$$

$\dot{Q}$ $\quad$ $\dot{q}$ $\quad$ $F = \frac{KQ_1Q_2}{r^2}$ $\quad \propto 1/r^2$

$\vec{P}$ $\quad$ $\dot{q}$ $\quad$ $F = QE = Q\frac{2KP}{r^3}$ $\quad \propto 1/r^3$

$\vec{P}$ $\quad$ $\vec{P}$ $\quad$ $F = \frac{6KP_1P_2}{r^4}$ $\quad \propto 1/r^4$

→ Conductors

→ V due to Sphere & E in Cavity

→ Self Energy

CONDUCTOR AND IT'S PROPERTIES
[FOR ELECTROSTATIC CONDITION]

(i) Conductors are materials which contains large number of free electrons which can move freely inside the conductor.
(ii) In electrostatics conductors are always equipotential surfaces.
(iii) Charge always resides on outer surface of conductor.
(iv) If there is a cavity inside the conductor having no charge then charge will always reside only on outer surface of conductor.
(v) Electric field is always perpendicular to conducting surface.
(vi) Electric lines of force never enter into conductors.

→ Conductors

→ V due to Sphere & E in Cavity

→ Self Energy