Real Numbers

$\frac{1}{3} = 0.3333\ldots$

Irrational
$\pi = \frac{22}{7} \rightarrow$ It is not exact value this aprox value

Rational Numbers

co-prime → having only 1 as a common factor

ex → 2, 3

- A Number that can be expressed in that form of $\frac{p}{q}$ where 'p' and 'q' are co-prime integers and $q \neq 0$,Is called a rational number.

$3+\sqrt{2}$ → Irrational no

- Their decimal representation is either terminating or non-terminating and recurring (repeating)

Irrational Numbers

- A Number that cannot be expressed in the form $\frac{p}{q}$ where 'p' and 'q' are co-prime integers, is called an irrational number.
- Their decimal representation is non-terminating and non-recurring (non-repeating).
- (eg. $\sqrt{3}, \sqrt{2}+\sqrt{5}, \pi, 0.102003102 \ldots$)

Fundamental Theorem of Arithmetic

Every composite number can be written as a product of primes in one and only one ways, apart from the order in which the primes are written.

Note:→
1 is neither prime nor composite.

Prime number
↓
Having exactly 2 factors

2: 1, 2

3: 1, 3

Composite number
↓
Having more than 2 factors.

4: 1, 2, 4

6: 1, 2, 3, 6

3	15
5	5
	1

LCM And HCF

HCF Using Prime Factorization

HCF is the product of the smallest power of each common prime factor of the given numbers.

LCM Using Prime Factorization

LCM is the product of the greatest power of each prime factor of the given numbers.

HCF → common prime ki lowest power

LCM → Sabhi prime ki highest power

8, 24

$8 = 2^3$

$24 = 2^3 \times 3$

$$\begin{array}{r|l} 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$HCF = 2^3 = 8$

$LCM = 2^3 \times 3 = 24$

Relation between HCF & LCM of two numbers

If 'a' and 'b' are two numbers, then

15, 24 HCF(15, 24) = 3

LCM (15, 24) = 120

$$\boxed{\text{HCF }(a, b) \times \text{LCM }(a, b) = \text{Product of 'a' and 'b'}}$$

Note ⇨ HCF is always a factor of the LCM of two numbers

HCF × LCM = 15 × 24

3 × 120 = 360

360 = 360 HP

15 = 3 × 5

$24 = 2^3 \times 3$

Theorem

Let p be a prime number. If p divides a^2, then p divides a, where a is a positive integer.

$p = 3$, $a^2 = 36$, $a = 6$

$\frac{a^2}{p} = \frac{36}{3} = 12$

$\frac{a}{p} = \frac{6}{3} = 2$

QUESTION

Let a and b two positive integers such that $a = p^3q^4$ and $b = p^2q^3$, where p and q are prime numbers. If HCF $(a, b) = p^m q^n$ and LCM $(a, b) = p^r q^s$, then $(m+n)(r+s) =$

(A) 15

(B) 30

(C) 35

(D) 72

$a = p^3q^4$

$b = p^2q^3$

HCF $= p^2q^3$ LCM $= p^3q^4$

HCF $= p^m q^n$ LCM $= p^r q^s$

on comparing $\longrightarrow$ $r = 3, s = 4$

$m = 2, n = 3$

$(2+3)(3+4)$

5×7

35

QUESTION

The HCF of two numbers is 27 and their LCM is 162. If one of the numbers is 54, what is the other number ?

(A) 36

(B) 45

(C) 9

(D) 81

$$HCF \times LCM = a \times b$$

$$27 \times 162 = 54 \times b$$

$$b = \frac{\cancel{27} \times \cancel{162}^{\,81}}{\cancel{54}_{\,\cancel{2}}}$$

$$b = 81$$

QUESTION

If the HCF of 65 and 117 is expressible in the form $65m - 117$, then the value of m is

$65 = 5 \times 13$

$117 = 3^2 \times 13$

HCF = 13

$13 = 65m - 117$

$13 + 117 = 65m$

$65m = 130$

$m = \frac{130}{65}$

$m = 2$

(A) 4

(B) 2

(C) 1

(D) 3

QUESTION

The least number that is divisible by all the numbers from 1 to 10 (both inclusive)

(A) 10

(B) 100

(C) 504

(D) 2520 ✓

$1 \to 1$
$2 \to 2$
$3 \to 3$
$4 \to 2^2$
$5 \to 5$
$6 \to 2 \times 3$
$7 \to 7$
$8 \to 2^3$
$9 \to 3^2$
$10 = 2 \times 5$

$LCM(1 \to 10) = 2^3 \times 3^2 \times 5 \times 7$
$= 2520$

QUESTION

There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point ?

(A) 18 min

(B) 36 min

(C) 20 min

(D) 42 min

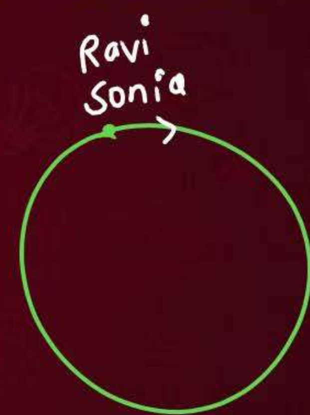
Sonia = 18 min

Ravi = 12 min

$18 = 2 \times 3^2$

$12 = 2^2 \times 3$

$LCM = 2^2 \times 3^2 = 36$ min



QUESTION

A Sweet seller has 420 kaju burfis and 150 badam burfis. He wants to stack them in such a way that each stack has the same number ,and they take up the least area of the tray. How many of these can be placed in each stack? How many stacks are formed ?

(A) 30 , 19

(B) 50 , 15

(C) 18 , 22

(D) None of these

$$\begin{array}{r} 420 \\ +150 \\ \hline 570 \end{array}$$

2	420
2	210
3	105
5	35
7	7
	1

3	150
5	50
5	10
2	2
	1

$420 = 2^2 \times 3 \times 5 \times 7$

$150 = 2 \times 3 \times 5^2$

HCF $= 2 \times 3 \times 5$

$= 30$ burfis in 1 stack

no of stack $= \dfrac{570}{30} = 19$

QUESTION

Prove that $5 + 3\sqrt{2}$ is an irrational number, it being given that $\sqrt{2}$ is irrational

let $5+3\sqrt{2}$ is rational

$5+3\sqrt{2} = a$ {a is any rational no.}

$3\sqrt{2} = a-5$

$\sqrt{2} = \frac{a-5}{3}$

as $\frac{a-5}{3}$ is rational

Hence, $\sqrt{2}$ is also rational

But this contradicts that $\sqrt{2}$ is irrational.

This has been arisen due to our wrong assumption.

So our assumption is wrong

$5+3\sqrt{2}$ is an Irrational number.