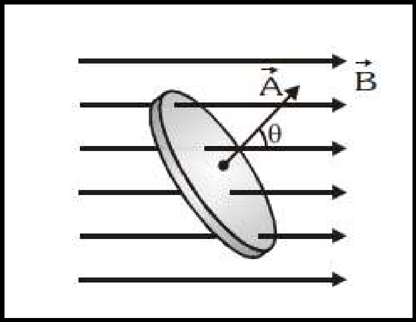
MAGNETIC FLUX

The magnetic flux (ϕ) linked with a surface held in a magnetic field (B) is defined as the number of magnetic lines of force crossing that area (A). If θ is the angle between the direction of the field and normal to the area, (area vector) then $\phi = \vec{B} \cdot \vec{A} = BA\cos\theta$



$$\boxed{\begin{gathered}\phi = \vec{B} \cdot \vec{Area} \\ \phi = |B|\,|A| \cos\theta\end{gathered}}$$

FLUX LINKAGE

If a coil has more than one turn, then the flux through the whole coil is the sum of the fluxes through the individual turns. If the magnetic field is uniform, the flux through one turn is

$\phi = \mathrm{BA}\cos\theta$ If the coil has N turns, the total flux linkage $\phi = \mathrm{NBA}\cos\theta$

Magnetic lines of force are imaginary, magnetic flux is a real scalar physical quantity with dimensions

$$[\phi] = \mathrm{B} \times \text{area} = \left[\frac{\mathrm{F}}{\mathrm{IL}}\right][\mathrm{L}^2] \because \mathrm{B} = \frac{\mathrm{F}}{\mathrm{IL}\sin\theta} [\because \mathrm{F} = \mathrm{B}I\,\mathrm{L}\sin\theta]$$

$$\therefore [\phi] = \left[\frac{MLT^2}{AL}\right][L^2] = [M^2T^{-2}A^{-1}]$$

SI UNIT of magnetic flux :

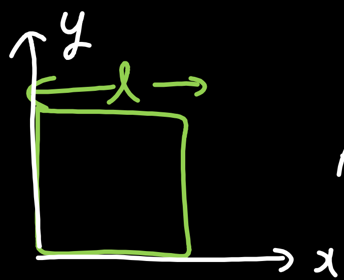
$\because$ $[ML^2 T^{-2}]$ corresponds to energy

$$\frac{\text{joule}}{\text{ampere}} = \frac{\text{joule} \times \text{second}}{\text{coulomb}} = \text{weber (Wb) or}$$

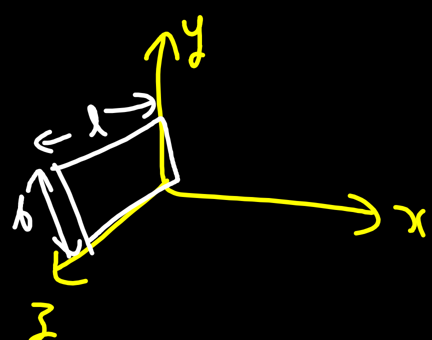
$$T - m^2 \text{ (as tesla } = Wb/m^2) \left[\text{ampere} = \frac{\text{coulomb}}{\text{sec ond}}\right]$$

$$\phi = \vec{B} \cdot \vec{A}$$

area $\rightarrow$ directed $\perp$'ur to the surface.



$$\vec{Area} = l^2 \hat{K}$$



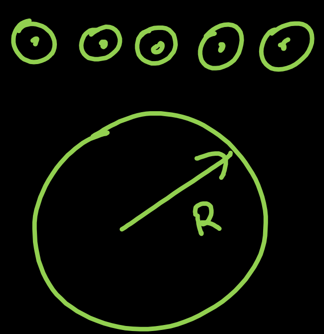
$$\vec{area} = l b \hat{i}$$

t.me/ajitlulla

#G.O.A.T
#BB 2.0

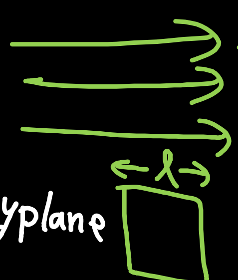
$\phi = \vec{B} \cdot \vec{A}$

Q. $B_0 \hat{k}$



$$\phi_{loop} = \vec{B} \cdot \vec{A}$$
$$= (B_0 \hat{k}) \cdot (\pi r^2 \hat{k})$$
$$\phi = B_0 \pi r^2$$

Q. $B_0 \hat{i}$



xyplane

$$\phi_{loop} = (B_0 \hat{i}) \cdot (l^2 \hat{k})$$
$$= 0$$

Extra

$\vec{a} \cdot \vec{b} = |a||b| \cos\theta$ <u>Scalar</u>

+ve	acute
0	90°
-ve	obtuse

$\hat{i} \cdot \hat{i} = 1$	$\hat{i} \cdot \hat{j} = 0$
$\hat{j} \cdot \hat{j} = 1$	$\hat{j} \cdot \hat{k} = 0$
$\hat{k} \cdot \hat{k} = 1$	$\hat{i} \cdot \hat{k} = 0$

Unit & dimension of Flux

$$\phi = \vec{B} \cdot \vec{A}$$

$$(\text{Tesla})\,m^2 = \text{Weber}$$

Area $= [L^2]$

$(\text{meter})^2$

$$F = BIl$$

$$\left[\frac{F}{Il}\right] = [B]$$

$F = MLT^{-2}$

$I = A$

$$[\phi] = [B][\text{Area}]$$

$$= \left[\frac{F}{Il}\right][\text{Area}]$$

$$= \left[\frac{MLT^{-2}}{AL}\right][L^2] = [ML^2T^{-2}A^{-1}]$$

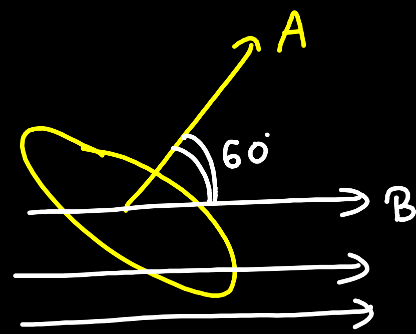
$$\left(\frac{\text{Joule}}{\text{Ampere}}\right)$$

$$\text{Flux} = \vec{B}\cdot\vec{A}$$
$$= |B|\,|A|\cos\theta$$

Scalar

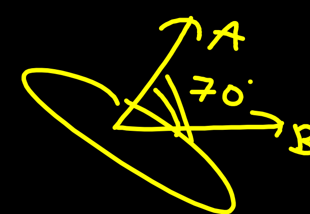
can be +ve, −ve or 0 when angle is acute, obtuse or 90°

represents no. of field lines crossing a surface ⊥'ly.



$$\phi = \vec{B}\cdot\vec{A}$$
$$= BA\cos 60$$
$$= \frac{BA}{2}$$

area = A



$$\phi = BA\cos\theta$$
$$= \underline{BA\cos 70^\circ}$$

C G S UNIT of magnetic flux : maxwell (Mx)

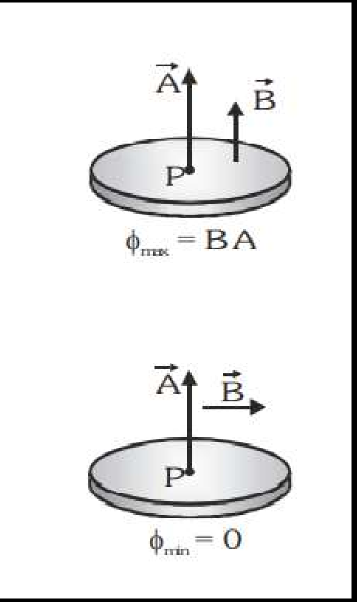
1 Wb = 10^8 Mx

For a given area flux will be maximum :

when magnetic field $\vec{B}$ is normal to the area $\theta = 0°$ => $\cos\theta$ = maximum = 1 Φ_{max} = B A

For a given area flux will be minimum when magnetic field B is parallel to the area

$\theta = 90°$ => $\cos\theta$ = minimum = 0 => $\boxed{\phi_{min} = 0}$



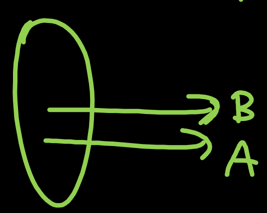
$\phi = BA\cos\theta$

for a given B & A

ϕ_{mag} maximum $\cos\theta = 1$

$\theta = 0^{\circ}$

B & A are parallel

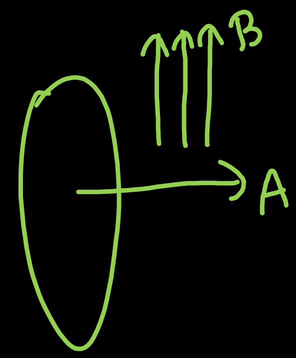


B & surface are $\perp$'ur.

ϕ_{mag} minimum $\cos\theta = 0$

$\theta = 90^{\circ}$

$\vec{B}$ & $\vec{A}$ $\perp$'ur

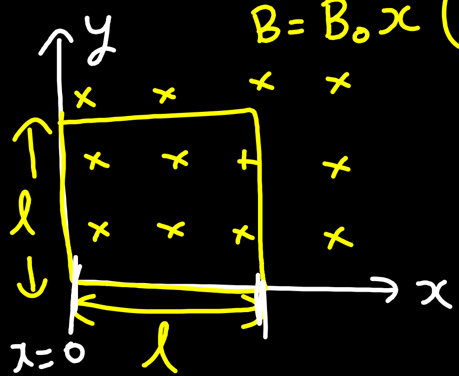


B is $\parallel$ to surface

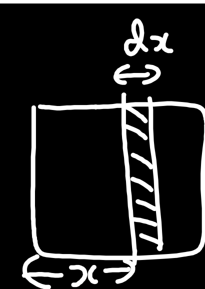
$$\boxed{\phi = \int \vec{B} \cdot d\vec{A}}$$

Q

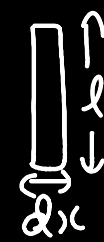
$B = B_0 x\,(-\hat{k})$



$\phi_{loop} = ??$



area $= l\,dx$



$$\int \vec{B} \cdot \vec{ds}$$

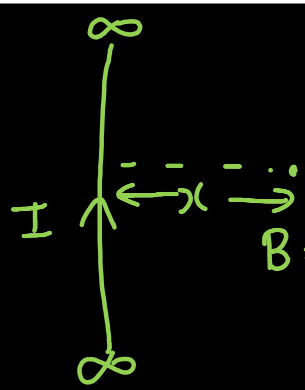
$$\int B\,\underline{\underline{ds}}\,\cos 0^\circ$$

$$\int (B_0 x)\, l\, dx$$

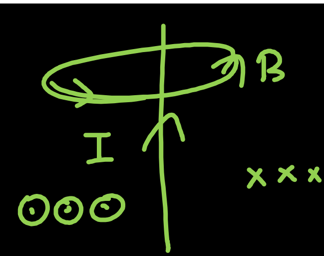
$$B_0 l \int x\,dx$$

$$B_0 l \left.\frac{x^2}{2}\right]^{l}$$

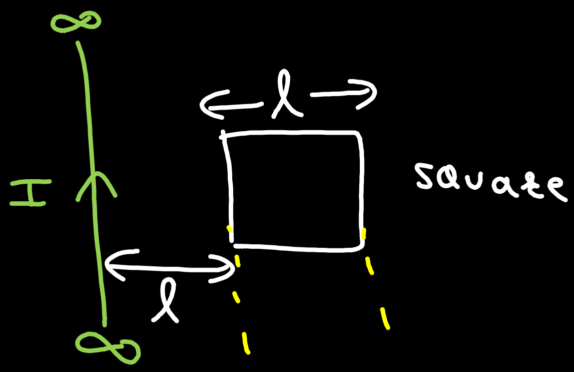
$$B_0 l \frac{l^2}{2} = \frac{B l^3}{2}$$



$$B = \frac{\mu_0 I}{2\pi x}$$



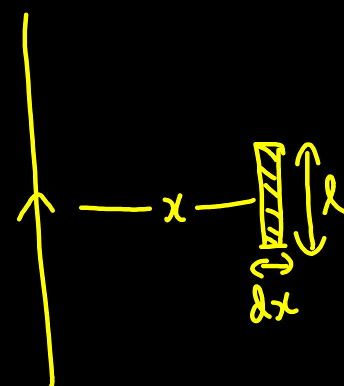
Q



$\phi_{loop} = ?$

l

$2l$



$$ds = l\,dx$$

$$\phi = \int B\,ds\cos 0$$

$$= \int \left(\frac{\mu_0 I}{2\pi x}\right) l\,dx$$

$$= \frac{\mu_0 I l}{2\pi} \int \frac{dx}{x}$$

$$= \frac{\mu_0 I l}{2\pi} \log_e(x) \Big]_{l}^{2l}$$

HL − LL

$$\ln(2l) - \ln(l)$$

$$\ln\left(\frac{2\cancel{l}}{\cancel{l}}\right) \quad \ln(2)$$

$$= \boxed{\frac{\mu_0 I l}{2\pi} \log_e(2)}$$

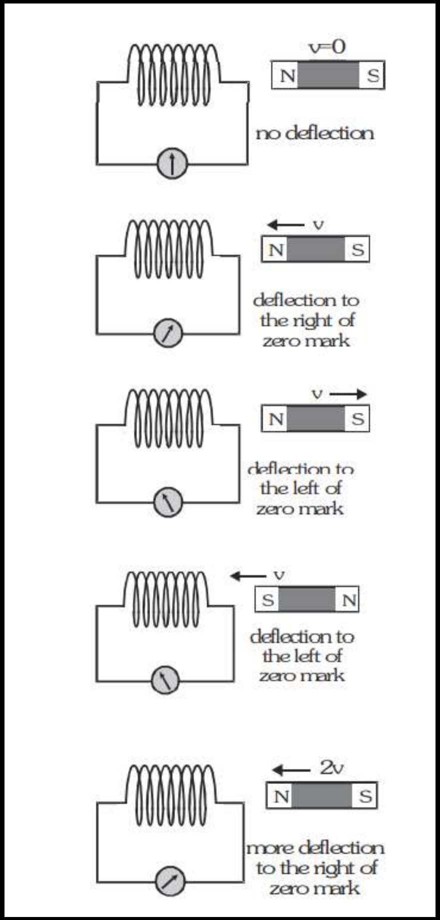
ELECTROMAGNETIC INDUCTION

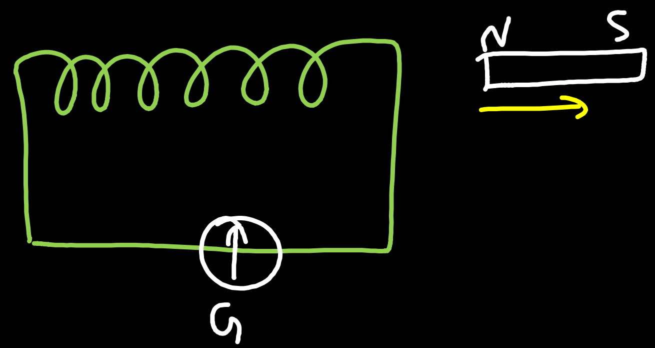
Michael Faraday explained the possibility of producing emf across the ends of a conductor when the magnetic flux linked with the conductor changes. This was termed as electromagnetic induction. The discovery of this phenomenon brought about a revolution in the field of electric power generation.

FARADAY'S EXPERIMENT

Faraday performed various experiments to discover and understand the phenomenon of electromagnetic induction. Some of them are : •

- When the magnet is held stationary anywhere near or inside the coil, galvanometer does not show any deflection.
- When the N-pole of a strong bar magnet is moved towards the coil, the galvanometer shows a deflection right to the zero mark.
- When the N-pole of a strong bar magnet is moved away from the coil the galvanometer shows a deflection left to the zero mark.
- If the above experiments are repeated by bringing the S-pole of the magnet towards or away from the coil, the direction of current in the coil is opposite to that obtained in the case of N-pole.
- The deflection in galvanometer is more when the magnet moves faster and less when the magnet moves slow




N
S
G

FARADAY'S LAWS OF ELECTROMAGNETIC INDUCTION

Based on his experimental studies on the phenomenon of electromagnetic induction, Faraday proposed the following two laws.

- First law : Whenever the amount of magnetic flux linked with a closed circuit changes, an emf is induced in the circuit. The induced emf lasts so long as the change in magnetic flux continues.

- Second law : The magnitude of emf induced in a closed circuit is directly proportional to rate of change of magnetic flux linked with the circuit. If the change in magnetic flux in a time dt is = d Φ then $e \propto \frac{d\phi}{dt}$

LENZ'S LAW

The Russian scientist H.F. Lenz in 1835 discovered a simple law giving the direction of the induced current produced in a circuit. Lenz's law states that the induced current produced in a circuit always flow in such a direction that it opposes the change or cause that produced it. If the coil has N number of turns and Φ is the magnetic flux linked with each turn of the coil then, the total magnetic flux linked with the coil at any time = N Φ

$$e = -\frac{d}{dt}(N\phi) \ - N\frac{d\phi}{dt} = - \frac{N(\phi_2-\phi_1)}{t}$$

(Coil face behaves as North pole to opposes the motion of magnet.)

(Coil face behaves as South pole to opposes the motion of magnet.)

$e = (-)\frac{d\phi}{dt}$, here negative sign indicates the concept of Lenz law.

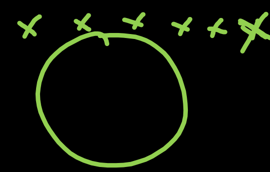
$\phi = \vec{B} \cdot \vec{A}$

$\phi = \underline{B}\,\underline{A}\cos\underline{\theta}$

- B
- A
- θ
- B & A
- A & θ
- B & θ
- B & A & θ

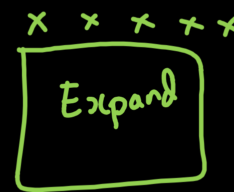
7 ways

B changing



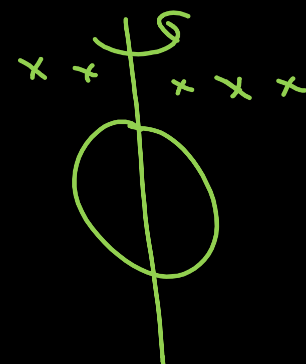
$B = (2t^2 + 5t)$

A changing



A = 5t

θ changing



Faraday & Lenz

ϕ constant — Faraday Khush
No emf generated

ϕ change with time — Faraday Naraz

→ Emf generates / induced in loop.

→ $|Emf| = \frac{d\phi}{dt}$

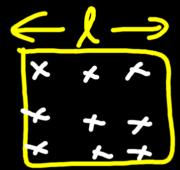
Lenz

$$\mathcal{E}mf = -\frac{d\phi}{dt}$$

ϕ constant Rahna Chahiye

Induced emf, it oppose the change of ϕ

× × × × × × × × × $B_{constant}$



$\longrightarrow$ move it vel $\underline{\underline{v}}$

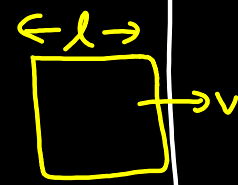
$\phi = BA$

$= \underline{\underline{B\ell^2}}$

$\phi \to$ constant / No change

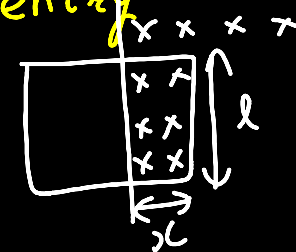
No B

× × × × × × × B_0 constant



$\phi_{initial} = 0$

entry



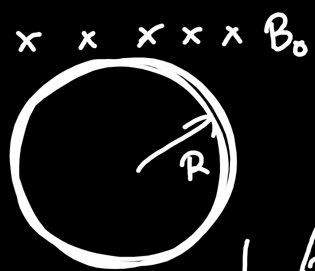
$\phi = B(\ell x)$

ϕ changing with time

$\hookrightarrow$ Emf / battery induced in loop

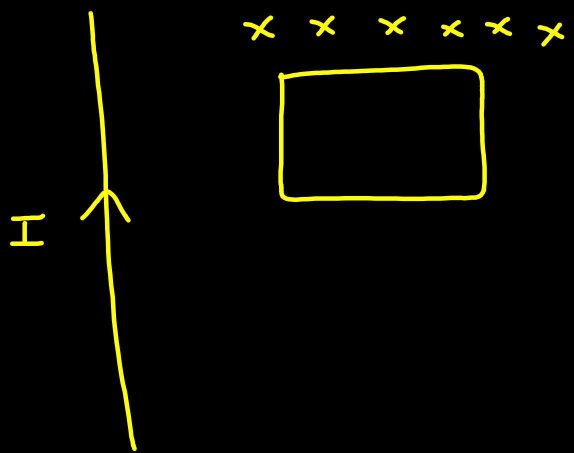
$$\text{Emf} = -\frac{d\phi}{dt}$$

If no. of turns = N



$$\phi_{total} = (B_0 \pi R^2) N$$

$$\phi = (BA) N$$

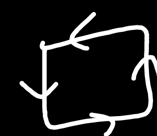


Find direction of induced current in loop

① I is constant — induced Emf = 0

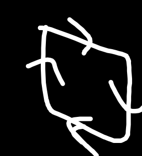
② I is increasing

ϕ inside increase $\Rightarrow$ induced Emf ACW

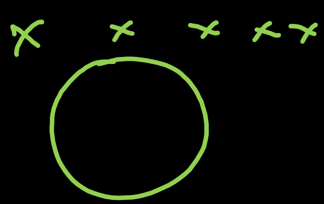


③ I is decreasing

ϕ inside decrease $\Rightarrow$ induced current CW



Q



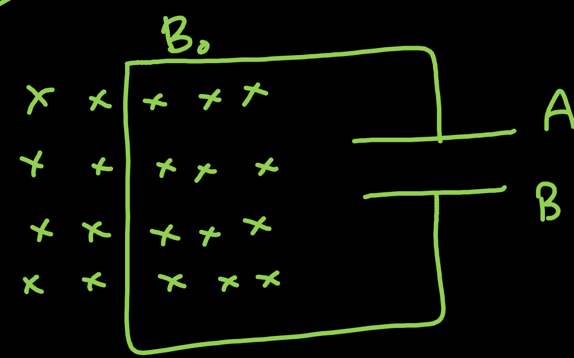
ϕ inside decreasing

area is contracting

induced emf direction =  CW

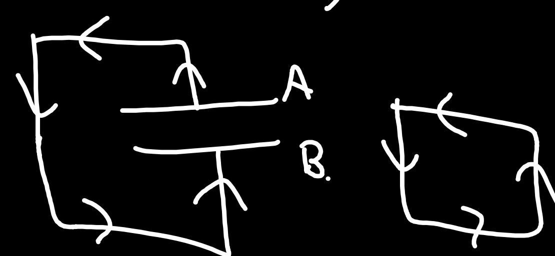
induced B inside

Q



which direction induced emf. if B is increasing ?

ϕ inside increases. $\Rightarrow$



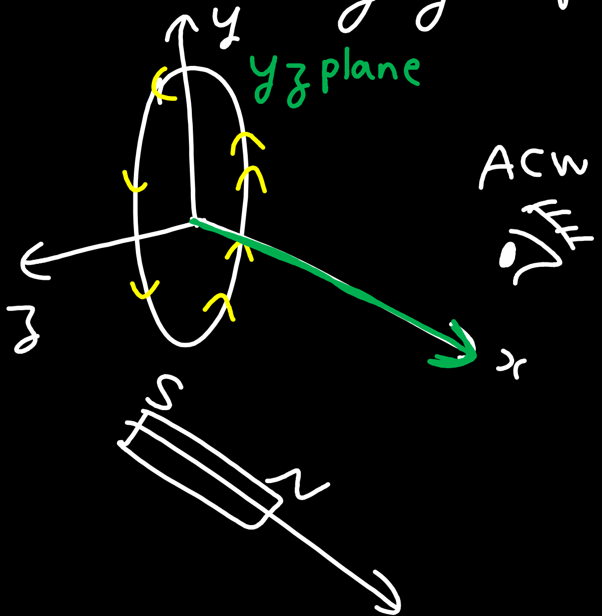
ACW

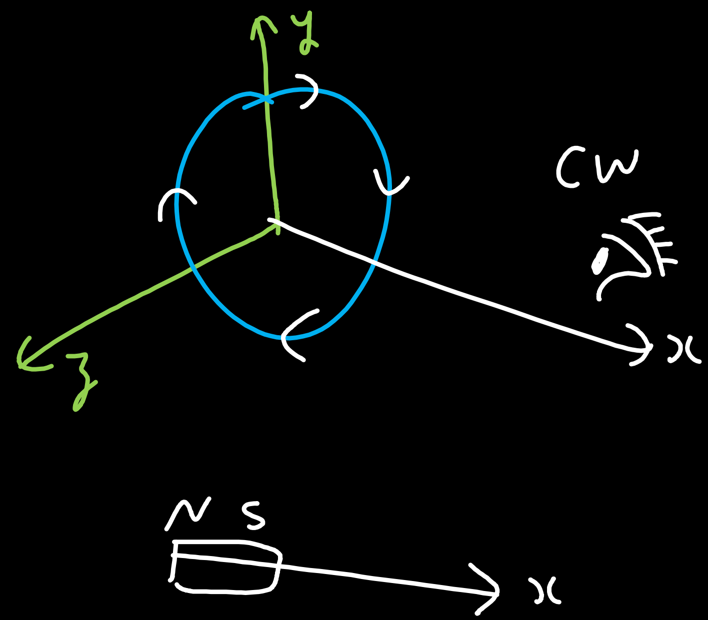
Ans

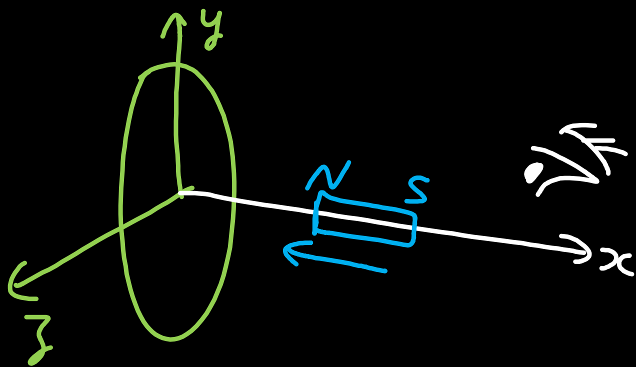
$$\phi = B\underline{\underline{A}}\cos\theta$$

Basics

Current Carrying loop ⇒ Magnet

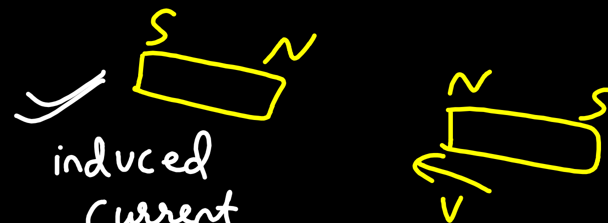




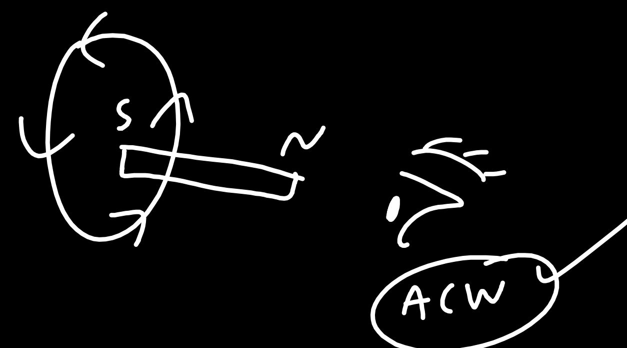


Direction of induced current as seen by eye??

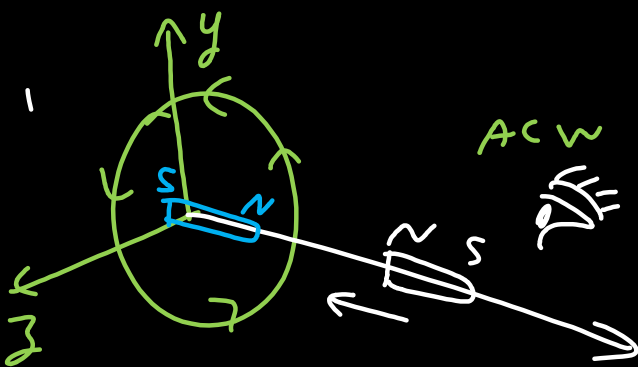
Lenz law



induced current



1



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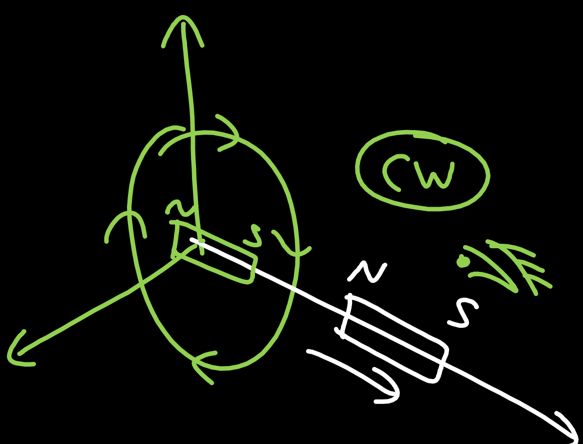

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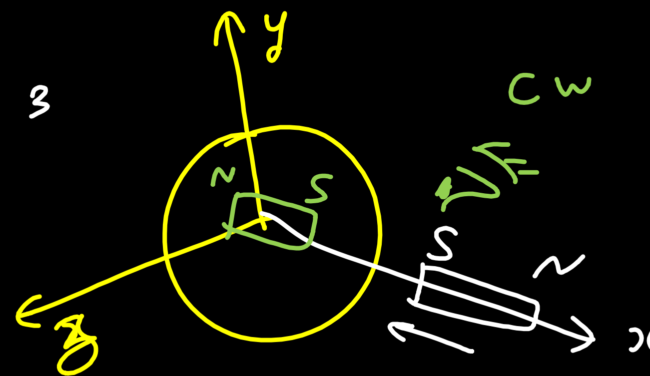

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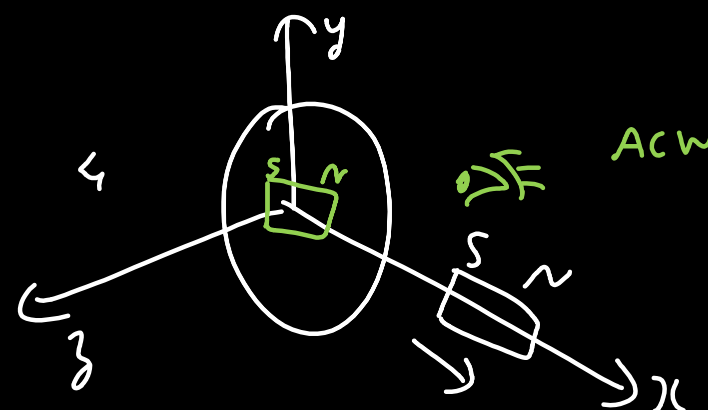
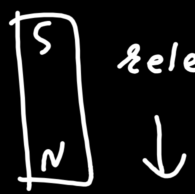
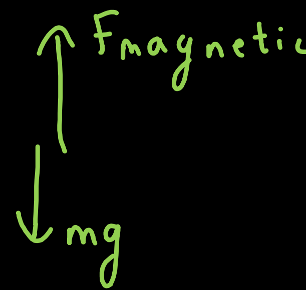
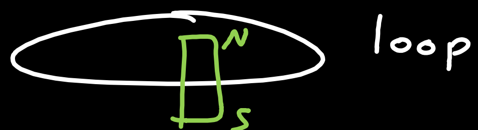


Q

$\downarrow g$







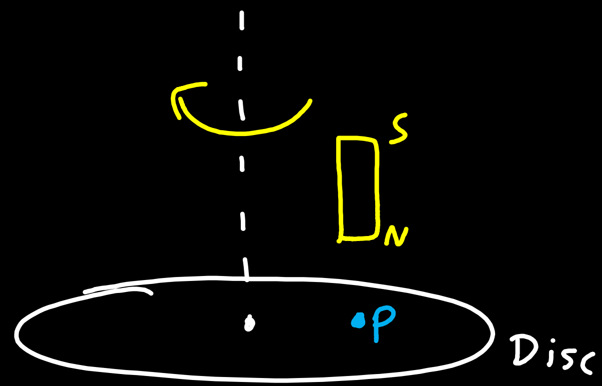
acc of magnet $= a$

(i) $a > g$

(ii) $a = g$

(iii) $a < g$

Jee
Advanced
2020



magnet initially rest.

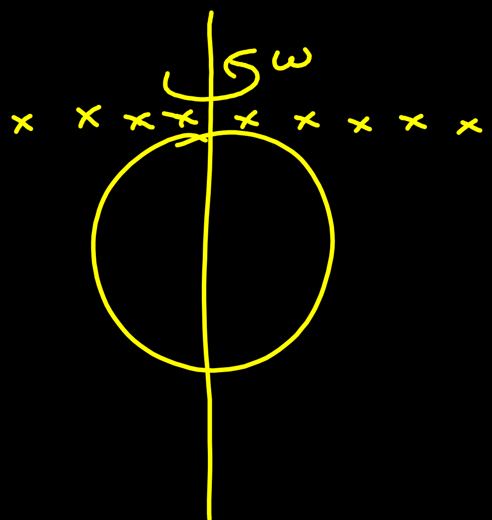
Magnet is rotated now about axis.

Disc free to rotate

(i) Disc rotate in direct of magnet

~~(ii)~~ " " " opp. " " "

Periodic Emf & AC generator



$$\theta = \omega t + \cancel{\frac{1}{2}\alpha t^2}^{\,0}$$

$$\boxed{\theta = \omega t}$$ constant ω

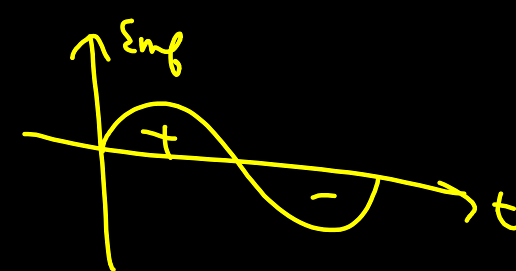
$$\phi = \vec{B} \cdot \vec{A}$$

$$\phi = BA\cos\theta$$

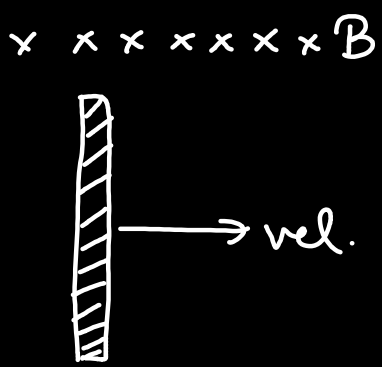
$$\phi = BA\cos(\omega t)$$

$$\text{Emf} = -\frac{d\phi}{dt} = -BA(-\sin(\omega t)) \times \omega$$

$$\text{Emf} = BA\omega \sin(\omega t)$$

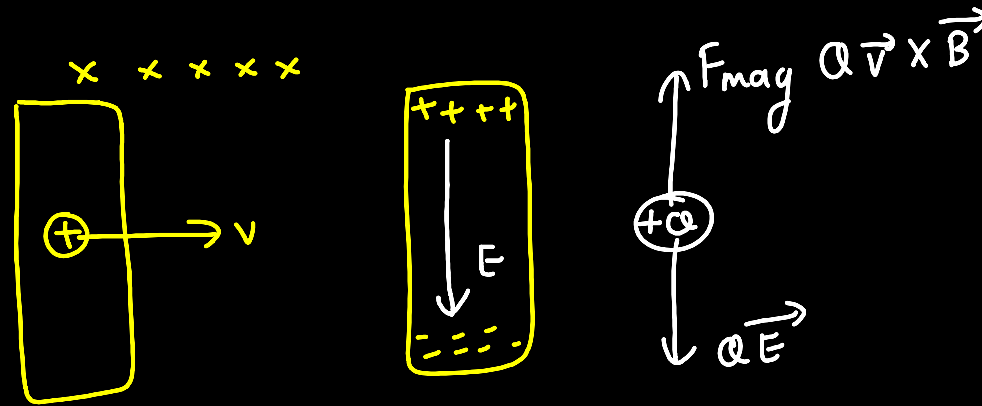


Motional Emf :



Conducting Rod

$$\vec{F} = Q\vec{v} \times \vec{B}$$



In equilibrium.

$$\vec{F}_{net} = 0$$

$$Q(\vec{v} \times \vec{B}) + Q\vec{E} = 0$$

$$\boxed{\vec{E}_{generated} = -(\vec{v} \times \vec{B})}$$

electrostatics formula

$$\underset{\text{potential diff}}{\Delta V} = -\int \vec{E} \cdot \vec{dl}$$

$$\mathcal{E}_{mf} = -\int \vec{E} \cdot \vec{dl}$$

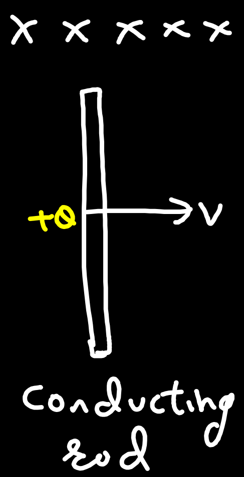
$$= -\int \left(-(\vec{V} \times \vec{B})\right) \cdot \vec{dl}$$

$$\mathcal{E}_{mf} = \int (\vec{V} \times \vec{B}) \cdot \vec{dl}$$

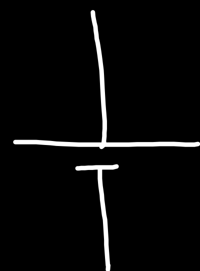
$$\boxed{\underset{\text{small}}{d(\mathcal{E}_{mf})} = (\vec{V} \times \vec{B}) \cdot \vec{dl}}$$

$$\mathcal{E}_{mf} = \int (\vec{V} \times \vec{B}) \cdot \vec{dl}$$

$$\boxed{\mathcal{E}_{mf} = (\vec{V} \times \vec{B}) \cdot \vec{l}}$$



$\Rightarrow$



$\mathcal{E}mf = \underline{\underline{Bl v}}$

$$\boxed{\mathcal{E}mf = \left(\vec{v} \times \vec{B}\right) \cdot \vec{l}}$$

$\vec{l}$ ⇓ effective length

displacement type

Conducting Rod moving in B, an Emf is generated in it.

Positive of battery decided by right hand rule.

$\left(q\vec{v} \times \vec{B}\right)$

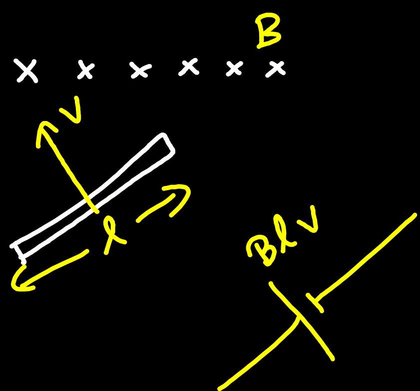
STP (Scalar triple product)

$(\vec{a} \times \vec{b}) \cdot \vec{c} \Rightarrow (\vec{b} \times \vec{c}) \cdot \vec{a} \Rightarrow (\vec{c} \times \vec{a}) \cdot \vec{b}$

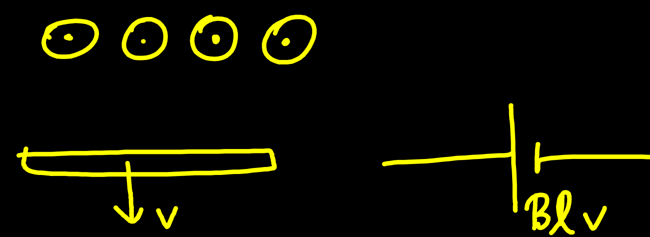
$\Rightarrow abc$ if all three are mutually $\perp$

$\Rightarrow 0$ if any two are $\parallel$

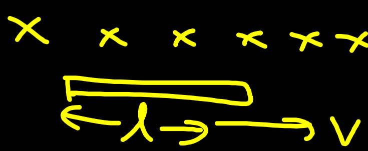
Q//



Q//

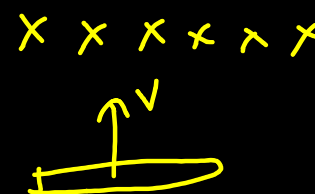


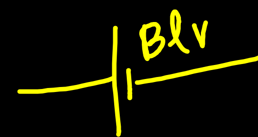
Q//



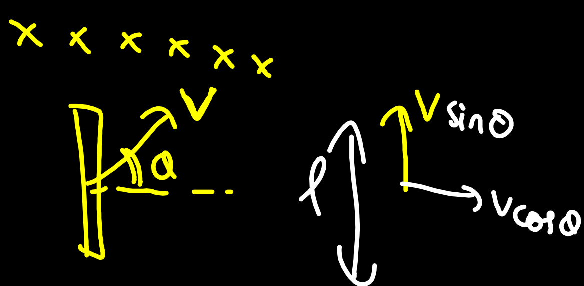
$\mathcal{E}mf = 0$

Q//





Q//

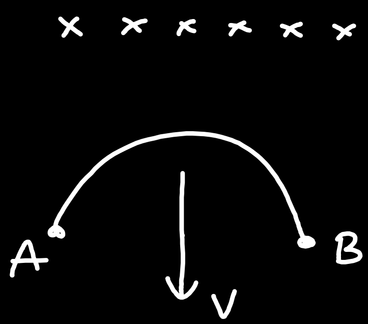


$Bl(v\cos\theta)$

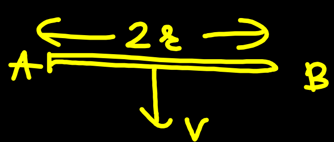
20 mins Break

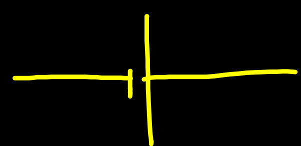
Resume 7:15pm

Motional Emf

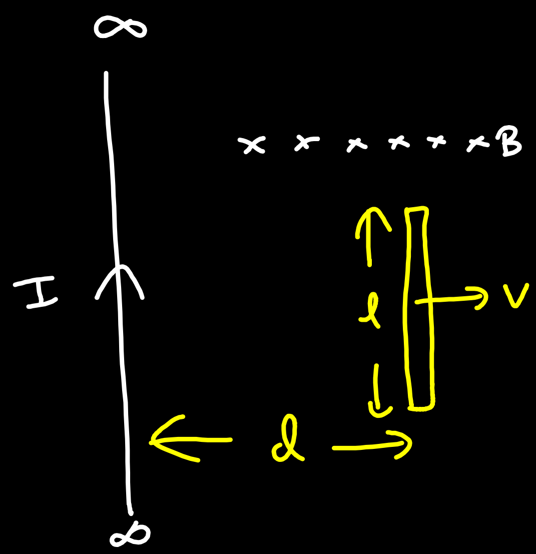


Semi circle
radius = r





$(B(2r)v)$



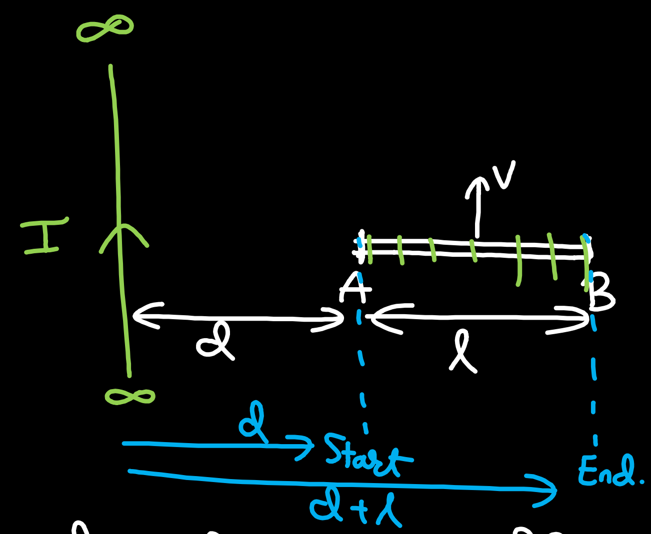
$$B = \frac{\mu_0 I}{2\pi x}$$

motional Emf in rod at this instant??

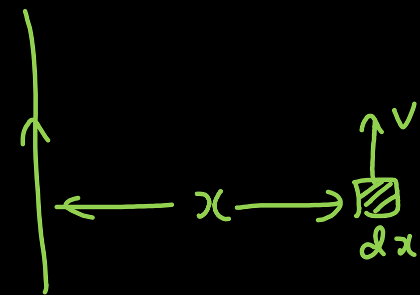
$$Blv$$

$$\left(\frac{\mu_0 I}{2\pi d}\right) l v$$

Q



Find emf induced in rod AB ??



Blv

$\frac{\mu_0 I}{2\pi x} dx \; v$

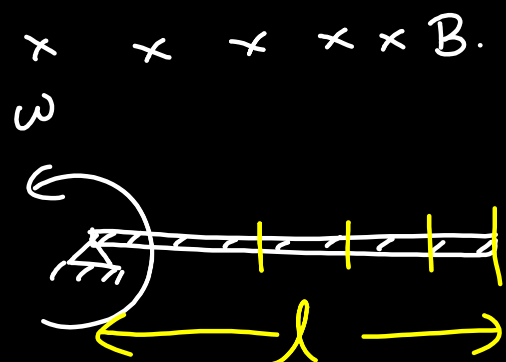
$\frac{\mu_0 I v}{2\pi} \int \frac{dx}{x}$

$\frac{\mu_0 I v}{2\pi} \log_e x \Big]_{d}^{d+l}$

$$\frac{\mu_0 I v}{2\pi} \log_e(l+d) - \log_e(d)$$

$$\frac{\mu_0 I v}{2\pi} \log_e\left(\frac{l+d}{d}\right)$$

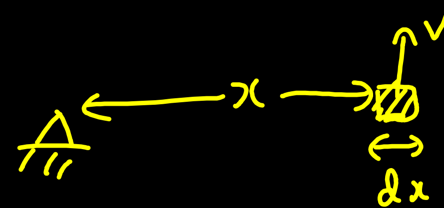
Rotating Rod



ω constant

$V = x\omega$

$B l \underline{v}$



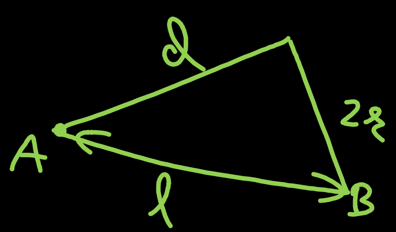
$$\begin{aligned} Emf &= B\, l\, v \\ &= B\,(dx)\,(x\omega) \\ &= B\omega \int x\, dx \\ &= B\omega \frac{x^2}{2}\Big]_0^{l} \end{aligned}$$

$$\Rightarrow \boxed{Emf = \frac{B\omega l^2}{2}}$$

motional Emf b/w A & B ??

$$\text{Emf} = \frac{B\omega l^2}{2}$$

$$= \frac{B\omega}{2}\left(\sqrt{d^2+4r^2}\right)^2$$



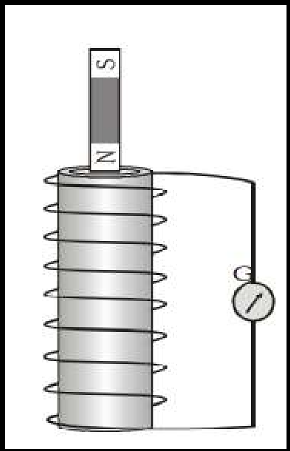
$$l^2 = d^2 + 4r^2$$

$$l = \sqrt{d^2+4r^2}$$

LENS'S LAW - A CONSEQUENCE OF CONSERVATION OF ENERGY

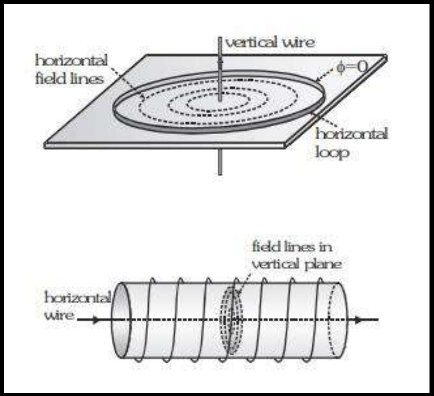
The direction of the induced current, according to Lenz's law is always to oppose the motion of the magnet.

The work done in moving the magnet is converted into electrical energy. This energy is dissipated as heat energy in the coil. Therefore, the induced current always flows in such a direction to oppose the cause. Thus it is proved that Lenz's law is the consequence of conservation of energy.



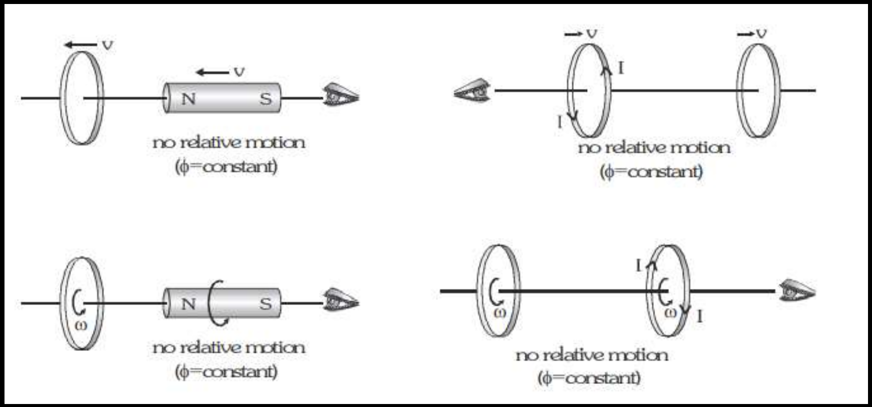
No E.M.I induced cases

Condition of EMI if :
No flux linkage through the coil Φ = 0
Or flux linkage through the coil Φ = constant



If current I increase with respect to time no induced current in loop because no flux associated with it as plane of circular field lines of straight conductor is parallel to plane of loop .

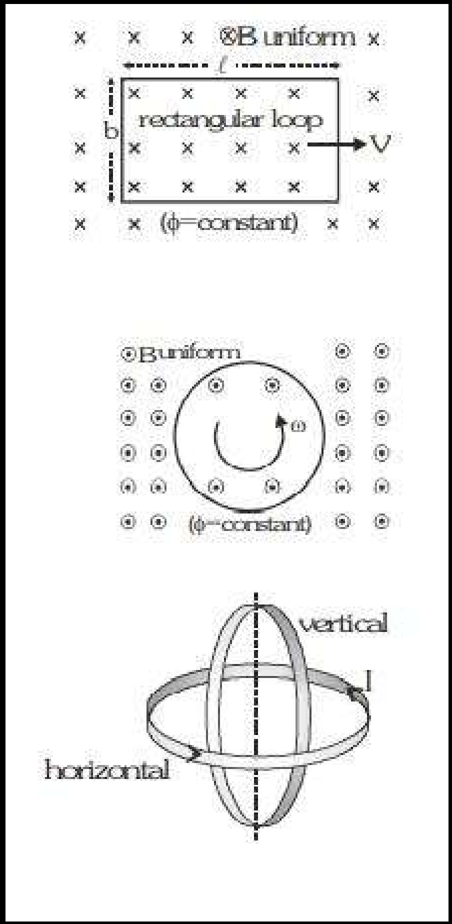
If current I increases with respect to time no induced parameter in solenoid because no flux associated with solenoid



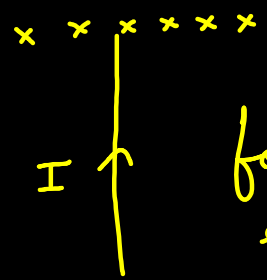
Any rectangle coil or loop translates within the uniform transverse magnetic field its flux remains constant.

Any coil or loop rotates about its geometrical axis in uniform transverse magnetic field its flux remains constant .

No flux associated for the coil or loop which are placed in mutually perpendicular planes. Hence if current of one either increase or decrease , there is no effect on flux of other



Basics



force on rod $= I\vec{l} \times \vec{B}$

$= |BIl\sin\theta|$

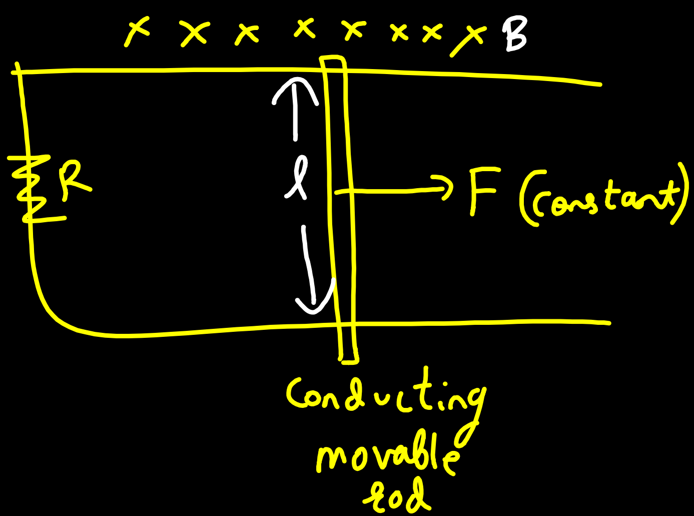
$\theta = 90^\circ$

$= \underline{\underline{BIl}}$

$WD = Fs$

$Power = Fv$

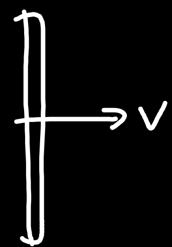
Power lost in resis $= I^2R.$

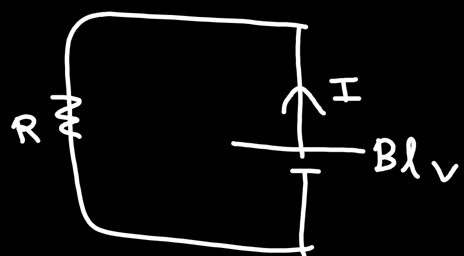


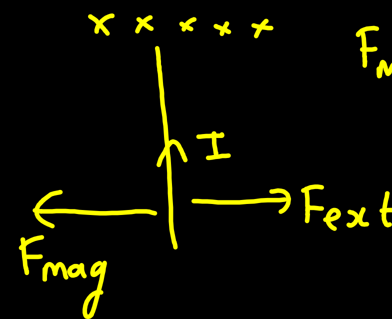
initial vel ⇒ v

Find ext force & power needed to move the rod with constant vel v ??

$F = ?$







$$F_{mag} = I\vec{l} \times \vec{B}$$

$$F_{ext} = F_{mag}$$

$$= BIl$$

$$= B\left(\frac{Bl v}{R}\right) l$$

$$\boxed{F = \frac{B^2 l^2 v}{R}}$$

$$I = \left(\frac{Blv}{R}\right)$$

$$\text{Power} = Fv$$

$$= \left(\frac{B^2 l^2 v}{R}\right) v$$

$$= \frac{B^2 l^2 v^2}{R}$$

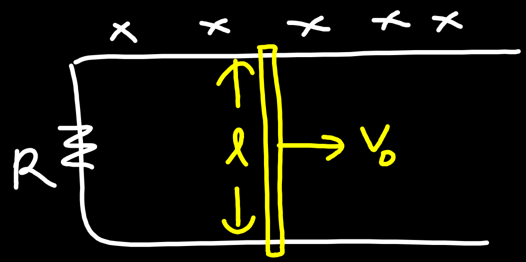
Note

Power lost in resistor $= I^2 R$

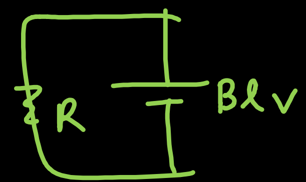
$$= \left(\frac{B l v}{R}\right)^2 R$$

$$= \frac{B^2 l^2 v^2}{R}$$

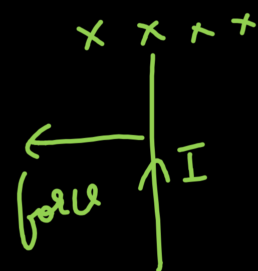
Q. Find Speed as function of time



inital vel $= v_0$



$$I = \frac{B\ell v}{R}$$



$$\text{force} = BI\ell$$

$$= B\left(\frac{B\ell v}{R}\right)\ell$$

$$\text{Force} = \frac{B^2\ell^2}{R}v$$

$$acc = \frac{B^2\ell^2}{mR}v$$

$u = V_0$

$$a = -\frac{B^2 l^2}{mR} v$$

$$\frac{dv}{dt} = -\frac{B^2 l^2}{mR} v$$

$$\int \frac{dv}{v} = -\frac{B^2 l^2}{mR} \int dt$$

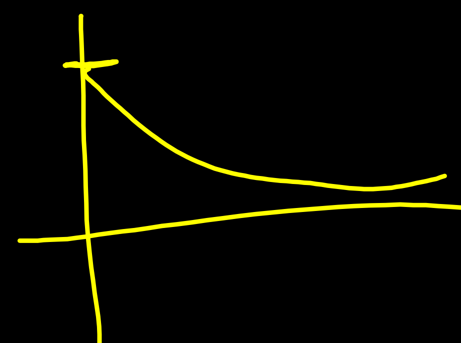
$$\log_e v \Big]_{V_0}^{V_f} = -\frac{B^2 l^2}{mR} t \Big]_0^t$$

$$\Rightarrow \log V_f - \log V_0$$

$$\log_e\left(\frac{V_f}{V_0}\right) = -\frac{B^2 l^2}{mR} t$$

$$\frac{V_f}{V_0} = e^{-\frac{B^2 l^2}{mR} t}$$

$$\boxed{V_f = V_0 e^{-\frac{B^2 l^2 t}{mR}}}$$



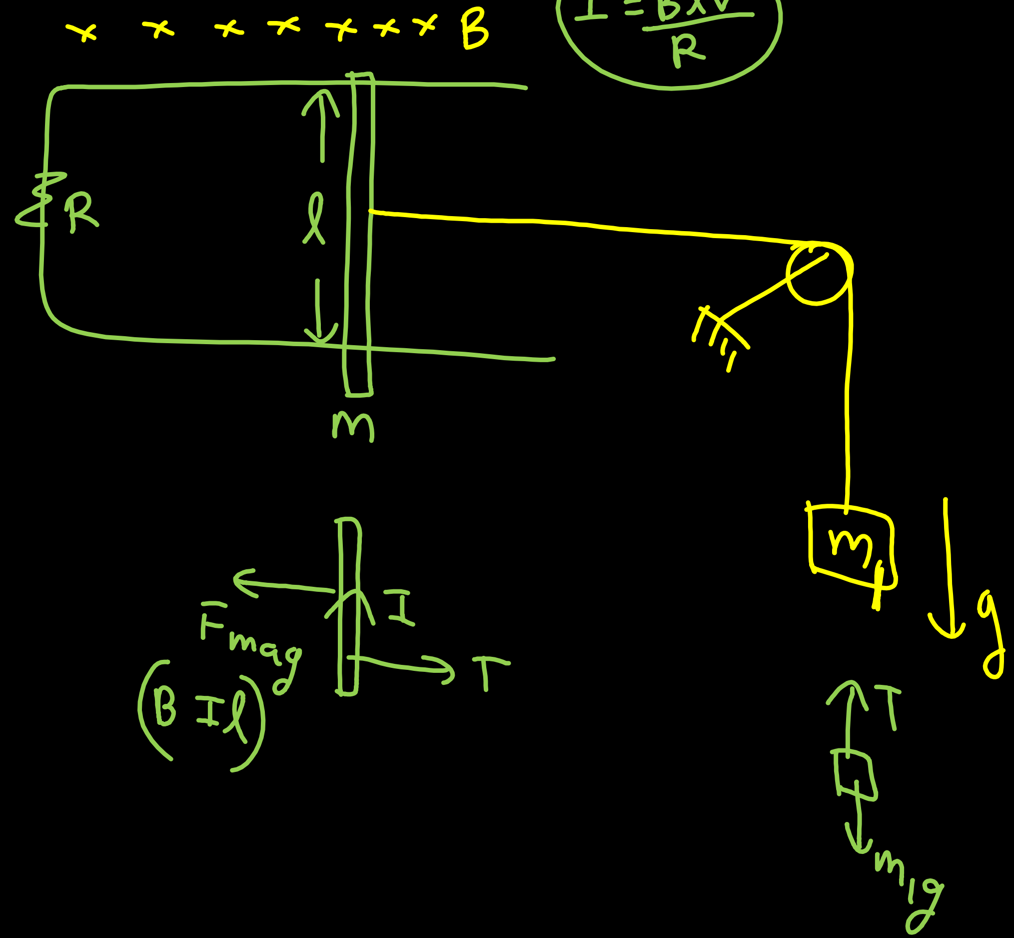
Terminal Velocity

→ if function with time is known
terminal velocity at $t = \infty$

→ if acceleration is known
terminal velocity when $acc = 0$

Q

$$I = \frac{Bl v}{R}$$



Find terminal vel of rod ?

$$m_1 g = T$$

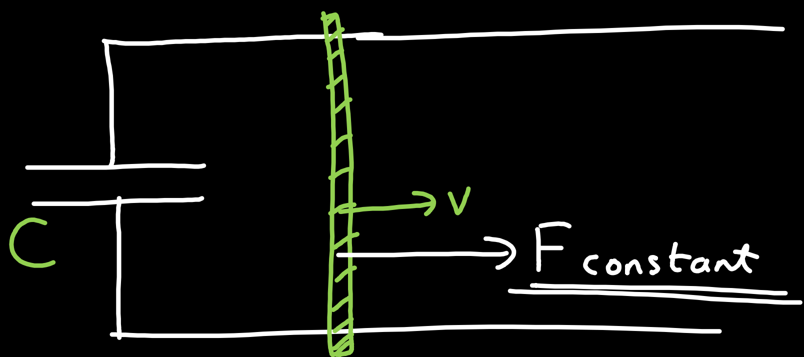
$$T = BIl$$

$$m_1 g = BIl$$

$$m_1 g = B\left(\frac{Blv}{R}\right)l$$

$$\frac{m_1 g R}{B^2 l^2} = v$$

Q

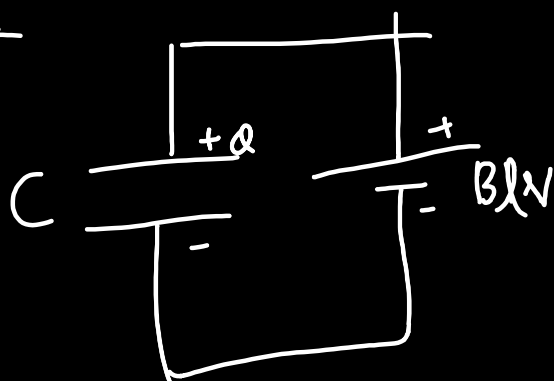


a constant force F is applied. Initial speed is 0.

Find ① acc

② charge on capacitor as a function of time.

initially uncharged

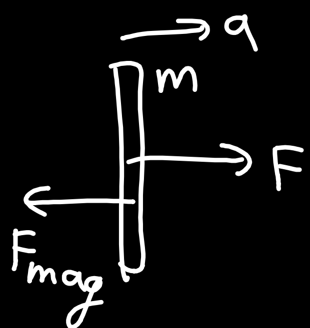


$$\frac{Q}{C} = Blv$$

$$Q = BlCv$$

$$I = \frac{dQ}{dt} = BlC\frac{dv}{dt} = BlCa$$

$$F_{mag} = BIl$$
$$= B(BlCa)l$$
$$= B^2l^2Ca$$



$$F_{net} = ma$$
$$F - F_{mag} = ma$$
$$F - B^2l^2Ca = ma$$
$$F = ma + B^2l^2Ca$$

$$\Rightarrow \quad a = \frac{F}{m + B^2l^2C}$$

$$V = u + at \qquad u = 0$$

$$\boxed{V = \left(\frac{F}{m + B^2l^2C}\right)t}$$

$$Q = BlCv$$

$$Q = \left(\frac{BlCF}{m + B^2l^2C}\right)t$$

METHODS OF PRODUCING INDUCED EMF (TYPES OF EMI)

Emf can be induced in a closed loop by changing the magnetic flux linked with a circuit.
The magnetic flux is $\phi = BA\cos\theta$ Magnetic flux can be changed by one of the following methods :

(i) Changing the magnetic field B. (Static emi)
(ii) Changing the area A of the coil and (dynamic emi)
(iii) Changing the relative orientation θ of $\vec{B}$ and $\vec{A}$ (Periodic emi)

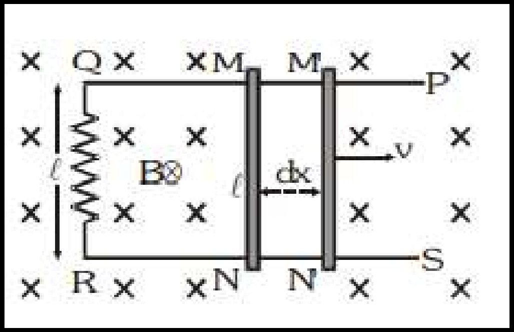
Induced emf by changing the magnetic field B

When there is a relative motion between the magnet and a closed loop, the magnetic lines of force passing through it changes, which results in change in magnetic flux. The changing magnetic flux produces induced emf in the loop.

Induced emf by changing the area of the coil

A U shaped frame of wire, PQRS is placed in a uniform magnetic field B perpendicular to the plane and vertically inward. A wire MN of length ℓ is placed on this frame. The wire MN moves with a speed v in the direction shown. After time dt the wire reaches to the position $\mathrm{M'N'}$ and distance covered = dx. The change in area $\Delta\mathrm{A}$ = Length area = ℓdx Change in the magnetic flux linked with the loop in the dt is $\mathrm{d}\phi = \mathrm{B}\,\Delta\mathrm{A} = \mathrm{B}\,\ell\mathrm{dx}$ induced emf $e = \frac{\mathrm{d}\phi}{\mathrm{dt}} = \mathrm{B}\ell\frac{\mathrm{dx}}{\mathrm{dt}} = \mathrm{B}\ell\mathrm{v}$

$$\because \left[\mathrm{v} = \frac{\mathrm{dx}}{\mathrm{dt}}\right]$$



If the resistance of circuit is R and the circuit is closed then the current through the circuit $I = \frac{e}{R} \Rightarrow I = \frac{Bv\ell}{R}$ A magnetic force acts on the conductor in opposite direction of velocity is

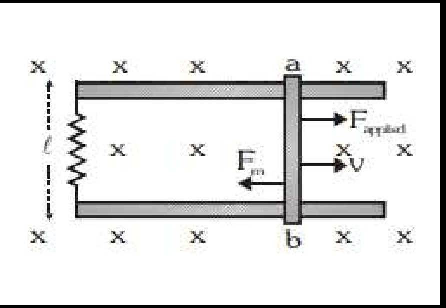
$$F_m = i\ell B = \frac{B^2\ell^2 v}{R}.$$

So, to move the conductor with a constant velocity v an equal and opposite force F has to be applied in the conductor. $F = F_m = \frac{B^2\ell^2 v}{R}$

The rate at which work is done by the applied force is, $P_{\text{applied}} = Fv = \frac{B^2\ell^2 v^2}{R}$

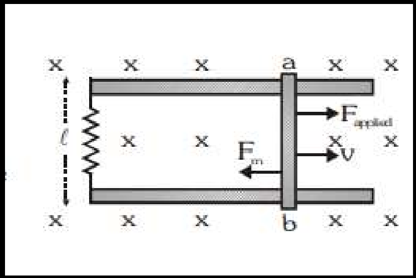
and the rate at which energy is dissipated in the circuit is,

$$P_{\text{dissipated}} = i^2 R = \left[\frac{Bv\ell}{R}\right]^2 R = \frac{B^2\ell^2 v^2}{R}$$



This is just equal to the rate at which work is done by the applied force.

A magnetic force acts on the conductor in opposite direction of velocity is



$$F_m = i\ell B = \frac{B^2\ell^2 v}{R}.$$

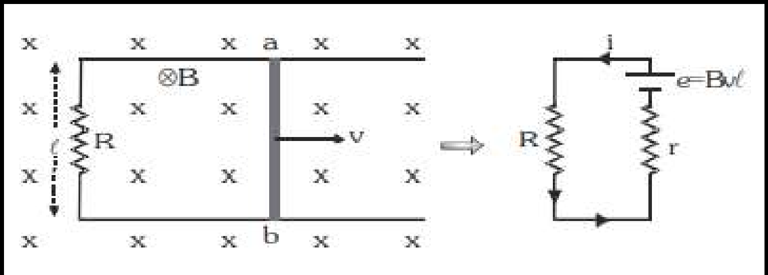
So, to move the conductor with a constant velocity v an equal and opposite

force F has to be applied in the conductor. $F = F_m = \frac{B^2\ell^2 v}{R}$

The rate at which work is done by the applied force is, $P_{\text{applied}} = Fv = \frac{B^2\ell^2v^2}{R}$

and the rate at which energy is dissipated in the circuit is, $P_{\text{dissipated}} = i^2R = \left[\frac{Bv\ell}{R}\right]^2 R = \frac{B^2\ell^2v^2}{R}$

This is just equal to the rate at which work is done by the applied force.
In the figure shown, we can replaced the moving rod ab by a battery of emf $Bv\ell$ with the positive terminal at a and the negative terminal at b.



The resistance r of the rod ab may be treated as the internal resistance of the battery.

Hence, the current in the circuit is $i = \frac{e}{R+r} = \frac{Bv\ell}{R+r}$

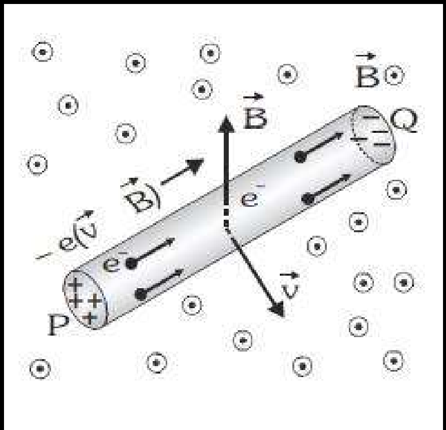
MOTIONAL EMF FROM LORENTZ FORCE

A conductor PQ is placed in a uniform magnetic field B, directed normal to the plane of paper outwards. PQ is moved with a velocity v, the free electrons of PQ also move with the same velocity. The electrons experience a magnetic Lorentz force, $\vec{F}_m = (\vec{v} \times \vec{B})$. According to Fleming's left hand rule, this force acts in the direction PQ and hence the free electrons will move towards Q.

A negative charge accumulates at Q and a positive charge at P.

An electric field E is setup in the conductor from P to Q.

Force exerted by electric field on the free electrons is, $\vec{F}_e = e\vec{E}$



The accumulation of charge at the two ends continues till these two forces balance each other. so $\vec{F}_m = -\vec{F}_e \Rightarrow e(\vec{v} \times \vec{B}) = -e\vec{E} \Rightarrow \vec{E} = -(\vec{v} \times \vec{B})$

The potential difference between the ends P and Q is $V = \vec{E} \cdot \vec{\ell} = (\vec{v} \times \vec{B}) \cdot \vec{\ell}$.It is the magnetic force on the moving free electrons that maintains the potential difference and produces the emf $\mathcal{E} = B\ell v$(for $\vec{B} \perp \vec{v} \perp \vec{\ell}$)

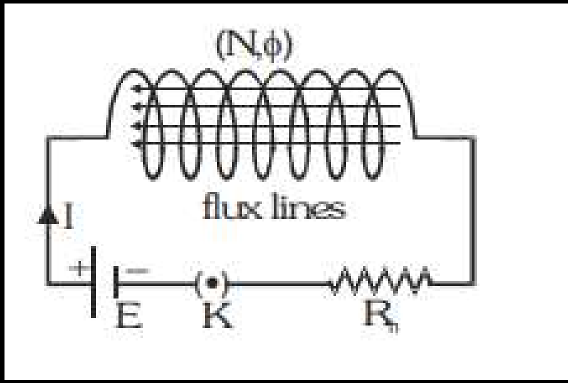
As this emf is produced due to the motion of a conductor, so it is called a motional emf.

The concept of motional emf for a conductor can be generalized for any shape moving in any magnetic field uniform or not. For an element $\overrightarrow{d\ell}$ of conductor the contribution de to the emf is the magnitude $d\ell$ multiplied by the component of $\vec{v} \times \vec{B}$ parallel to $\overrightarrow{d\ell}$,

that is $de = (\vec{v} \times \vec{B}) \cdot \overrightarrow{d\ell}$

SELF INDUCTION

When the current through the coil changes, the magnetic flux linked with the coil also changes. Due to this change of flux a current induced in the coil itself according to lenz concept it opposes the change in magnetic flux. This phenomenon is called self induction and a factor by virtue of coil shows opposition for change in magnetic flux called coefficient of self inductance of coil.



SELF-INDUCTANCE OF A PLANE COIL

Total magnetic flux linked with N turns,

$$\phi = \mathrm{NBA} = \mathrm{N}\left(\frac{\mu_0 \mathrm{NI}}{2\mathrm{r}}\right)\mathrm{A} - \frac{\mu_0\ \mathrm{N}^2\mathrm{I}}{2\mathrm{r}}\mathrm{A} - \frac{\mu_0\ \mathrm{N}^2\mathrm{I}}{2\mathrm{r}}\ \pi\mathrm{r} = \frac{\mu_0 \pi \mathrm{N}^2\mathrm{r}}{2}\mathrm{I}\ \ \text{But } \phi = \mathrm{LI} \therefore \mathrm{L} - \frac{\mu_0 \pi \mathrm{N}^2\mathrm{r}}{2}$$

SELF-INDUCTANCE OF A SOLENOID

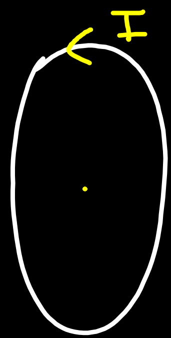
Let cross-sectional area of solenoid=A, Current flowing through it=I

Length of the solenoid $= \ell$, then $\phi = \mathrm{NBA} = \mathrm{N}\frac{\mu_0 \mathrm{NI}}{\ell}\mathrm{A} = \frac{\mu_0\ \mathrm{N}^2\ \mathrm{A}}{\ell}\mathrm{I}$

But $\phi = \mathrm{LI} \therefore \mathrm{L} = \frac{\mu_0\ \mathrm{N}^2\ \mathrm{A}}{\ell}$ or $\mathrm{L_m} = \frac{\mu_0 \mu_\mathrm{r} \mathrm{N}^2\ \mathrm{A}}{\ell}$

If no iron or similar material is nearby, then the value of self-inductance depends only on the geometrical factors (length, cross-sectional area, number of turns).

Self Inductance



B depends on I

ϕ depends on I

$\phi \propto I$

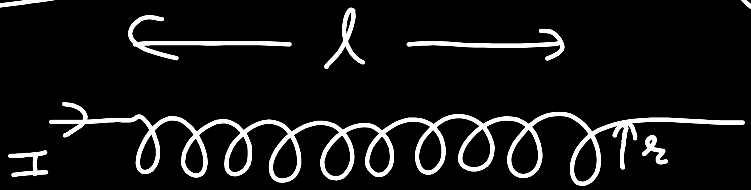
flux due to self current

$$\boxed{\phi = L I}$$

$L \rightarrow$ self inductance

$L \rightarrow$ unit Henry

Solenoid



$$A = \pi r^2$$

$$n = \frac{N}{l}$$

$$n = \frac{\text{no. of turns}}{\text{length}}$$

$$nl = N$$

$$B = \mu_0 n I$$

$$\phi = (BA)N$$

$$= (\mu_0 n I) A\, nl$$

$$\boxed{\phi = \mu_0 n^2 A l I}$$

$$\Rightarrow \phi = LI = \mu_0 n^2 A l I$$

$$\boxed{L = \mu_0 n^2 A l}$$

$$\phi = LI$$

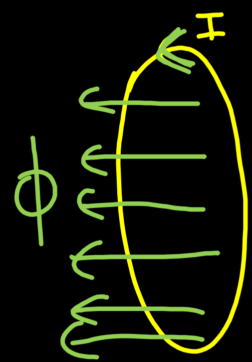
$L \rightarrow$ self inductance

$\downarrow$

depends on

$\rightarrow$ geometry

$\rightarrow$ material.



I change

$\downarrow$

ϕ change

$\downarrow$

Emf generate

$$|Emf| = \frac{d\phi}{dt} = \frac{d(LI)}{dt}$$

$$Emf = L\frac{dI}{dt}$$

electrical interia

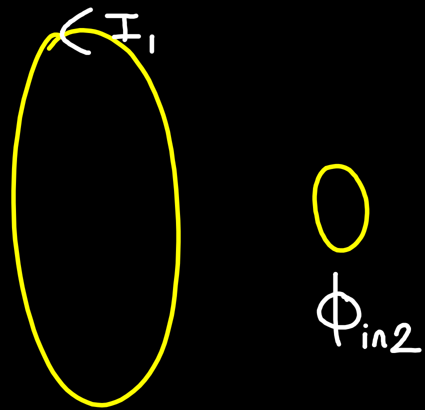
I constant Faraday Khush

I vary Faraday Naraz

$$\mathcal{E}_{mf} = L\frac{dI}{dt}$$

$$\mathcal{E}_{mf} = \frac{d\phi}{dt}$$

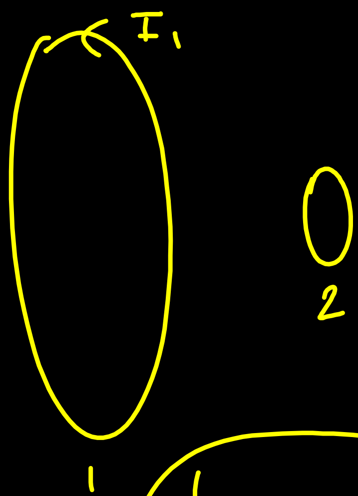
Mutual Inductance



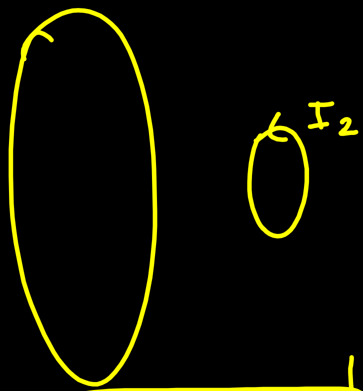
ϕ_{in2} depends on I in 1^{st} loop

$$\phi_2 \propto I_1$$

$$\boxed{\phi_2 = M I_1}$$



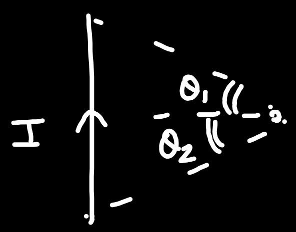
$\phi_2 = M I_1$



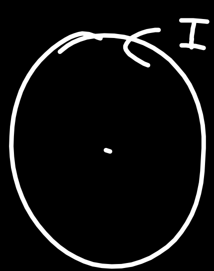
$\boxed{\phi_1 = M I_2}$

SI unit of $M \rightarrow$ henry

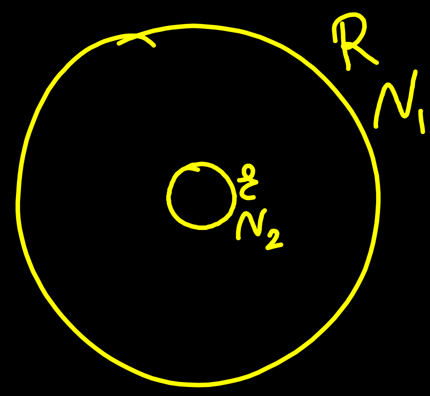
Basics



$$\frac{\mu_0 I}{4\pi r}\left(\sin\theta_1 + \sin\theta_2\right)$$



$$\boxed{B = \frac{\mu_0 I}{2r}}$$



$R \ggg r$

Find Mutual Inductance b/w them.

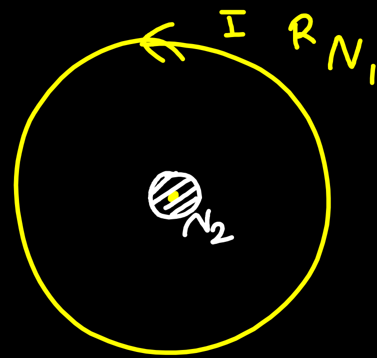
Steps

Assume current in any one loop

Find ϕ in another loop

$\phi_2 = M I_1$

Find M



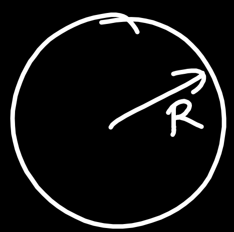
$$B = \left(\frac{\mu_0 I}{2R}\right) N_1$$

$$\phi_2 = (BA) N_2$$

$$\phi = \frac{\mu_0 I N_1 \pi r^2}{2R} N_2 = M I$$

$$M = \frac{\mu_0 N_1 N_2 \pi r^2}{2R}$$

Q. Find self inductance



N turns.

R is very small

$$B = \left(\frac{\mu_0 I}{2R}\right) N$$

$$\phi = (BA)N$$

$$\phi = \left(\frac{\mu_0 I N}{2R}\right) \pi R^2 N = L I$$

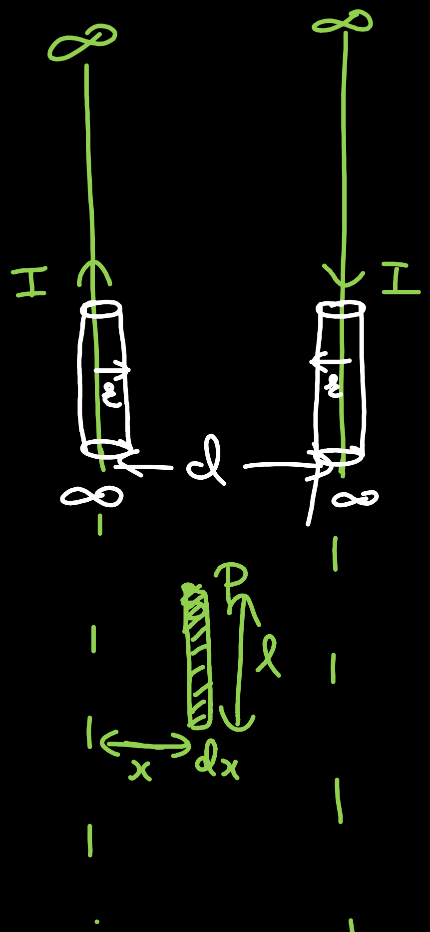
$$L = \frac{\mu_0 N^2 \pi R}{2}$$

Steps

1. assume Current
2. find flux
3. $\phi = L I$

Q/

Find self inductance of this system?



$$B = \frac{\mu_0 I}{2\pi x} + \frac{\mu_0 I}{2\pi(d-x)}$$

$$= \frac{\mu_0 I}{2\pi}\left[\frac{1}{x} + \frac{1}{d-x}\right]$$

$$= \frac{\mu_0 I}{2\pi}\left[\frac{d-x+x}{d-x}\right] = \frac{\mu_0 I}{2\pi}\left(\frac{d}{d-x}\right)$$

$$\phi = BA$$

$$= \frac{\mu_0 I}{2\pi}\left(\frac{d}{d-x}\right) l\,dx$$

$$= \frac{\mu_0 I d l}{2\pi} \int \frac{dx}{d-x}$$

$$= \frac{\mu_0 I d}{2\pi} l \ln(d-x)\Big]_{r}^{d-r}$$

$$\phi = \frac{\mu_0 I d l}{2\pi} \ln\left[\frac{r}{d-r}\right]$$

$$\phi = L I$$

$$L = \frac{\mu_0 d l}{2\pi} \ln\left(\frac{r}{d-r}\right)$$

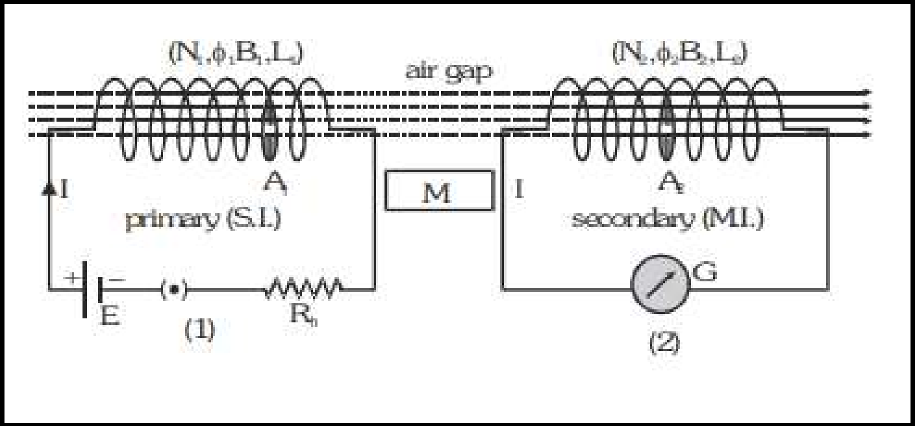
$$\frac{L}{l} = \frac{\mu_0 d}{2\pi} \ln\left(\frac{r}{d-r}\right)$$

MUTUAL INDUCTION

Whenever the current passing through primary coil or circuit change then magnetic flux neighboring secondary coil or circuit will also change. Acc. to Lenz for opposition of flux change, so an emf induced in the neighboring coil or circuit.

This phenomenon called as 'Mutual induction'. In case of mutual inductance for two coils situated close to each other, flux linked with the secondary due to current in primary.

Due to Air gap always $\phi_2 < \phi_1$ and $\phi_2 = B_1 A_2$ $(\theta = 0)$.



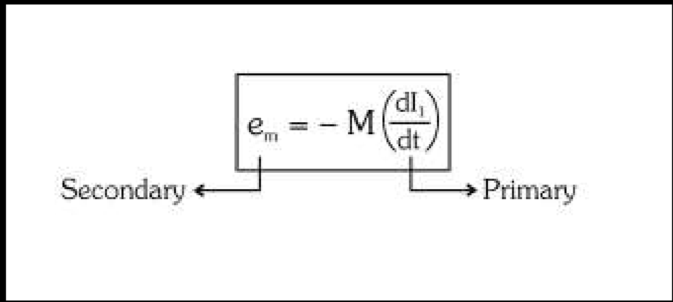
Case - I When current through primary is constant

Total flux of secondary is directly proportional to current flow through the primary coil $N_2\phi_2 \propto I_1 \Rightarrow N_2\phi_2 = MI_1, M = \frac{N_2\phi_2}{I_1} = \frac{N_2\ B_1\ A_2}{I_1} = \frac{(\phi_T)_s}{I_p}$ where M : is coefficient of mutual induction.

Case - II When current through primary changes with respect to time

If $\frac{dI_1}{dt} \rightarrow \frac{dR_1}{dt} \rightarrow \frac{dd\phi_1}{dt} \rightarrow \frac{dd\phi_2}{dt} \Rightarrow$ Static EMI

$$N_2\phi_2 = MI_1 - N_2\frac{d\phi_2}{dt} = -M\frac{dI}{dt}, \left[-N_2\frac{d\phi}{dt}\right]$$



called total mutual induced emf of secondary coil e_m.

- The units and dimension of M are same as ' L '.
- The mutual inductance does not depends upon current through the primary and it is constant for circuit system.

ENERGY STORED IN INDUCTOR

The energy of a capacitor is stored in the electric field between its plates. Similarly, an inductor has the capability of storing energy in its magnetic field .

An increasing current in an inductor causes an emf between its terminals.

Power P $=$ The work done per unit time $= \frac{dW}{dt} = -ei = -\left[L\frac{di}{dt}\right] i = -Li\frac{di}{dt}$

here i $=$ instantaneous current and L $=$ inductance of the coil

From $dW = -dU$ (energy stored) so $\frac{dW}{dt} = -\frac{dU}{dt} \therefore \frac{dU}{dt} = Li\frac{di}{dt} \Rightarrow dU = Lidi$

The total energy U supplied while the current increases from zero to final value i is,

$$U = L\int_0^1 idi = \frac{1}{2}L(i^2)_0^1 \;\therefore U = \frac{1}{2}LII^2$$

the energy stored in the magnetic field of an inductor when a current I is $= \frac{1}{2}LI^2$.

MAGNETIC ENERGY PER UNIT VOLUME OR ENERGY DENSITY

- The energy is an inductor is actually stored in the magnetic field within the coil. For a long solenoid its magnetic field can be assumed completely within the solenoid.
  The energy U stored in the solenoid when a current I is,

$$\mathrm{U} = \frac{1}{2}\mathrm{LI}^2 = \frac{1}{2}(\mu_0 \mathrm{n}^2\ \mathrm{V})\mathrm{I}^2\ (\mathrm{L} = \mu_0 \mathrm{n}^2\ \mathrm{V})(\mathrm{V} = \text{ Volume } = \mathrm{A}\ell)$$

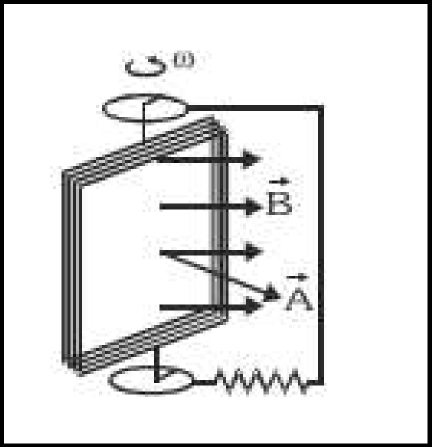
The energy per unit volume $\mathrm{u} = \frac{\mathrm{U}}{\mathrm{V}} = \frac{1}{2}\mu_0 \mathrm{n}^2 \mathrm{I}^2 = \frac{(\mu_0 \mathrm{nI})^2}{2\mu_0} = \frac{\mathrm{B}^3}{2\mu_0}\ (\mathrm{B} = \mu_0 \mathrm{nI})$

$$\therefore \mathrm{u} = \frac{1}{2}\frac{\mathrm{B}^3}{\mu_0}$$

PERIODIC EMI

Let a coil initially placed perpendicular to uniform magnetic field. Now this coil starts rotation about an axis that the flux linked with the coil change due to change in orientation of area vector $\vec{A}$ with respect to magnetic field $\vec{B}$

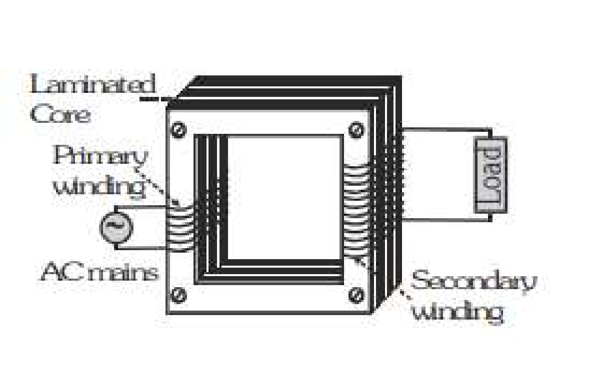
Angle in between area vector $\vec{\mathrm{A}}$ and magnetic field $\vec{\mathrm{B}}$ is θ then



TRANSFORMER

Working principle

Mutual induction



Transformer has basic two section

(a) Shell :

Consist of primary and secondary coil of copper.
The effective resistance between primary and secondary coil is infinite because electric circuit between two is open $\left(R_{ps} = \infty\right)$

(b) Core :

Which is between two coil and magnetically coupled two coils. Two coils of transformer would on the same core. The alternating current passing through the primary creates a continuously changing flux through the core. This changing flux induces alternating emf in secondary.

Work

It regulates AC voltage and transfer the electrical power without change in frequency of input supply. (The alternating current changes itself.)

Special Points

- It can't work with D.C. supply, and if a hattery is connected to its primary, then output is across secondary is always zero i.e. No working of transformer.
- It can't called 'Amplifier' as it has no power gain like transistor.
- It has no moving part, hence there are no mech. losses in transformer.

Types : According to voltage regulation it has two –

(i) Step up transformer : $N_s > N_p$
(ii) Step down transformer $N_s < N_p$

Step up transformer: Converts low voltage high current in to High voltage low current

Step up transformer: Converts low voltage high current in to High voltage low current

Step down transformer : Converts High voltage low current into low voltage high current.
Power transmission is carried out always at "High voltage low current" so that voltage drop and power losses are minimum in transmission line.
voltage drop $= I_L R_L, I_L =$ line current $R_L =$ total line resistance,
$\mathrm{I_L} = \dfrac{\text{power to be transmission}}{\text{line voltage}}$ power losses $= \mathrm{I_L^2 R_L}$

High voltage coil having more number of turns and always made of thin wire and high current coil having less number of turns and always made of thick wires.

Ideal Transformer : ($\eta = 100\%$)

(a) No flux leakage $\phi_\mathrm{s} = \phi_\mathrm{p} \Rightarrow \dfrac{-\mathrm{d}\phi_\mathrm{s}}{\mathrm{dt}} = \dfrac{-\mathrm{d}\phi_\mathrm{p}}{\mathrm{dt}}$
$e_s = e_p = e$ induced emf per turn of each coil is also same.
total induced emf for secondary $E_s = N_s e$ total induced emf for primary $E_p = N_p e$
$\dfrac{E_s}{E_p} = \dfrac{N_s}{N_p} = n$ or p where n : turn ratio, p : transformation ratio

(b) No load condition

$V_p = E_p$ and $E_S = V_s$ $\frac{V_S}{V_p} = \frac{N_s}{N_p}$ from (i) and (ii) $\frac{V_S}{V_p} = \frac{N_s}{N_p} = n$ or p

(c) No power loss

$P_{\text{out}} = P_{\text{in}}$ and $V_S I_S = V_P I_P$ $\frac{V_S}{V_p} = \frac{I_P}{I_S}$ valid only for ideal transformer from equation (iii) and (iv)

$$\frac{V_S}{V_p} = \frac{I_p}{I_s} = \frac{N_s}{N_p} = n \text{ or } p$$

Note : Generally transformers deals in ideal condition i.e. $\mathrm{P_{tn}} = \mathrm{P_{out}}$, if other information are not given.

Real transformer ($\neq 100\%$)

Some power is always lost due to flux leakage, hysteresis, eddy currents, and heating of coils.

hence $P_{\text{out}} < P_{\text{in}}$ always. efficiency of transformer $\eta = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{V_s}{V_P} \cdot \frac{I_s}{I_P} \times 100$

LOSSES OF TRANSFORMER

(a) Copper or joule heating losses

Where : There losses occurs in both coils of shell part

Reason : Due to heating effect of current ($H = I^2Rt$)

Remedy : To minimize these losses, high current coil always made up with thick wire and for removal of produced heat circulation of mineral oil should be used.

(b) Flux leakage losses

Where : There losses occurs in between both the coil of shell part.

Cause : Due to air gap between both the coils.

Remedy : To minimize there losses both coils are tightly wound over a common soft iron core (high magnetic permeability) so a closed path of magnetic field lines formed itself within the core and tries to makes coupling factor $K \rightarrow 1$

(c) Iron losses
Where : There losses occurs in core part.

Types : (i) Hysteresis losses (ii) Eddy currents losses

(i) Hysteresis losses

Cause : Transformer core always present in the effect of alternating magnetic field ($B = B_0 \sin \omega t$) so it will magnetized & demagnetized with very high frequency ($f = 50$ Hz).During its demagnetization a part of magnetic energy left inside core part in form of residual magnetic field. Finally this residual energy waste as heat.

Remedy : To minimize these losses material of transformer core should be such that it can be easily magnetized & demagnetized. For this purpose soft ferromagnetic material should be used. For example soft iron (low retentivity and low coercivity)

Step Up

Voltage output > Voltage input

$N_2 > N_1$

Step Down

Voltage output < Voltage input

$N_2 < N_1$

Non ideal Transformer

Power loss

$$\frac{Emf_1}{Emf_2} = \frac{N_1}{N_2}$$

Suppose 80% effeciency

$$P_{output} = 80\% \ P_{input}$$

$$V_2 I_2 = \frac{80}{100} V_1 I_1$$

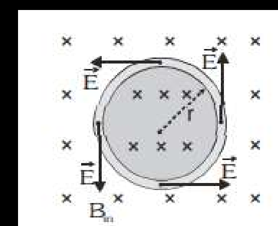
$$\frac{V_1}{V_2} = \frac{10}{8} \frac{I_2}{I_1}$$

INDUCED ELECTRIC FIELD

When the magnetic field changes with time (let it increases with time) there is an induced electric field in the conductor caused by the changing magnetic flux. Important properties of induced electric field :

(i) It is non conservative in nature. The line integral of E around a closed path is not zero. When a charge q goes once around the loop, the total work done on it by the electric field is equal to q times the emf. Hence

$$\oint \vec{E} \cdot \vec{d}\, l = e \;=\; - d\Phi / dt$$



(i) This equation is valid only if the path around which we integrate is stationary.

(ii) Due to of symmetry, the electric field E has the same magnitude at every point on the circle and it is tangential at each point (figure).

(iii) Being nonconservative field, so the concept of potential has no meaning for such a field.

(iv) This field is different from the conservative electrostatic field produced by stationary charges.

(v) The relation $\vec{F} = q\vec{E}$ is still valid for this field.

(vi) This field can vary with time.

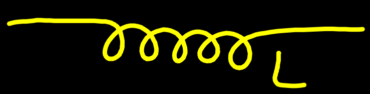
For symmetrical situations $El = \left|\frac{d\phi}{dt}\right| = A\left|\frac{dB}{dt}\right|$

l = the length of closed loop in which electric field is to be calculated

A = the area in which magnetic field is changing.

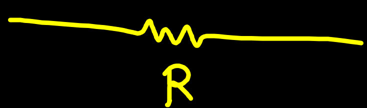
(iii) Being nonconservative field, so the concept of potential has no meaning for such a field.
(iv) This field is different from the conservative electrostatic field produced by stationary charges.
(v) The relation is still valid for this field.
(vi) This field can vary with time.
For symmetrical situations $El = \left|\frac{d\phi}{dt}\right| = A\left|\frac{dB}{dt}\right|$
l = the length of closed loop in which electric field is to be calculated
A = the area in which magnetic field is changing.

Energy Stored in Inductor



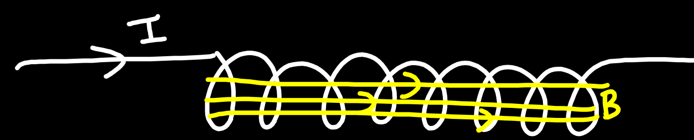
⇒ energy stored in B

⇒ " " " E



⇒ energy lost in heat

$$\phi = LI$$



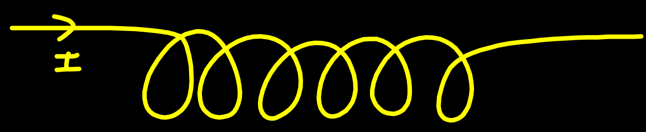
$$Emf = L\frac{dI}{dt}$$

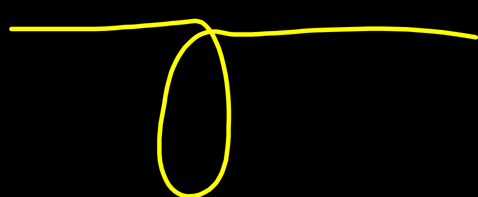
$$\text{Power} = (Emf)\, I$$

$$P = L\frac{dI}{dt}\, I$$

$$\text{energy} = \int P\,dt = \int L\,(dI)\,I = \frac{1}{2}LI^2$$

$$E = \frac{1}{2} L I^2$$



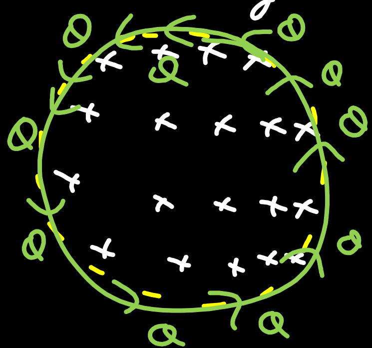


$$\boxed{\frac{\text{Energy}}{\text{Volume}} = \text{Energy density} = \frac{1}{2}\frac{B^2}{\mu_0}}$$

$$\text{energy} = \left(\frac{B^2}{2\mu_0}\right) \text{Volume}$$

Induced Electric Field

B vary



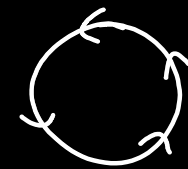
B vary
↓
ϕ vary
↓
Emf aayega
↓
I current
↓
charge flow ⇒ force applied by induced E

change in electric flux ⟶ magnetic field.

change in magnetic flux ⟶ electric field

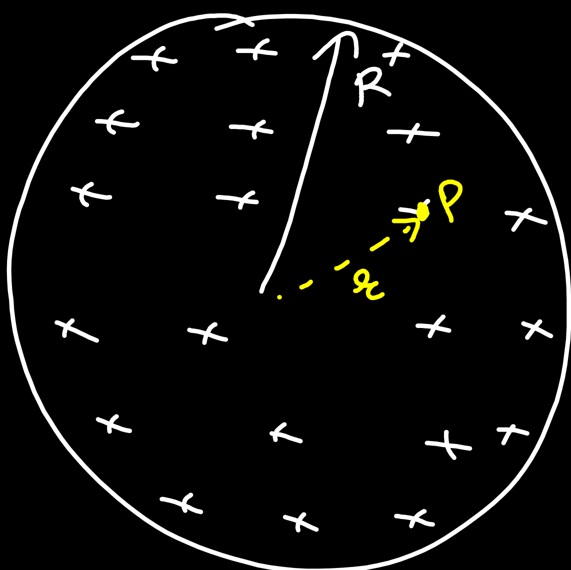
induced E

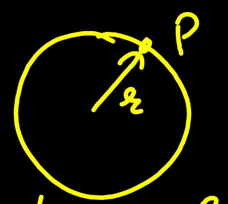
- → closed curve lines



- → same magnitude of E at same radius.
- → non conservative E field

B vary with time





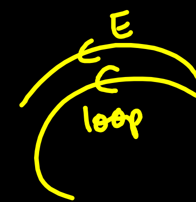
$$\phi = B\pi r^2$$

$$\mathcal{E}mf = \frac{d\phi}{dt}$$

$$\mathcal{E}mf = \pi r^2 \frac{dB}{dt}$$

$$|\mathcal{E}mf| = \left|\oint \vec{E}\cdot\vec{dl}\right|$$

$$\mathcal{E}mf = E\, 2\pi r$$

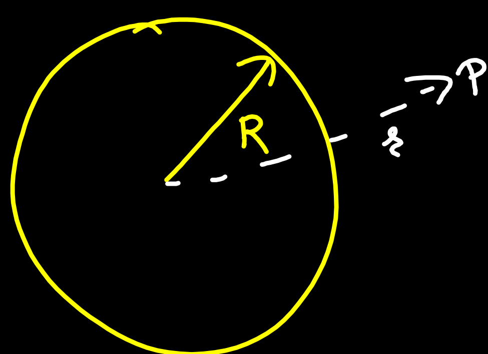


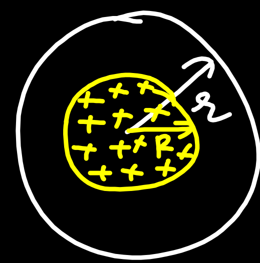
Result

$$E\, 2\cancel{\pi} r = \cancel{\pi} r^2 \frac{dB}{dt}$$

$$E_{inside} = \frac{r}{2}\frac{dB}{dt}$$

$$E_{inside} = \frac{r}{2}\frac{dB}{dt}$$





$$\mathcal{E}mf = \left|\oint \vec{E}\cdot\vec{dl}\right| = E\,2\pi r$$

$$\phi = B\pi R^2$$

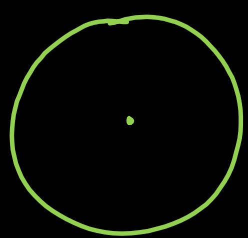
$$\frac{d\phi}{dt} = \pi R^2\frac{dB}{dt}$$

$$\mathcal{E}mf = \pi R^2\frac{dB}{dt}$$

Result

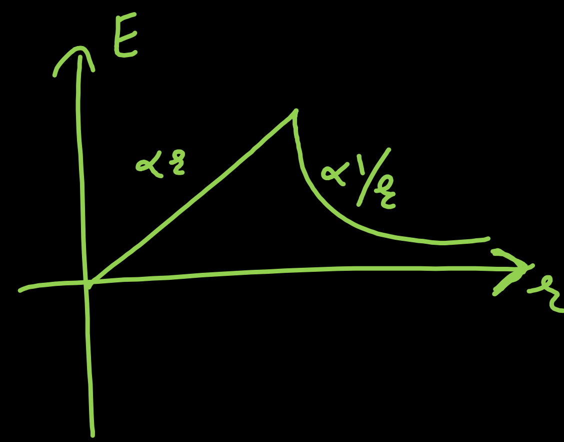
$$E\,2\pi r = \pi R^2\frac{dB}{dt}$$

$$E_{outside} = \frac{R^2}{2r}\frac{dB}{dt}$$



$$E_{inside} = \frac{r}{2}\frac{dB}{dt}$$

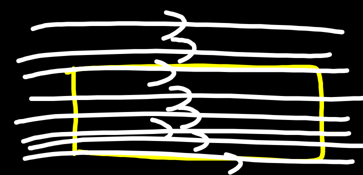
$$E_{outside} = \frac{R^2}{2r}\frac{dB}{dt}$$

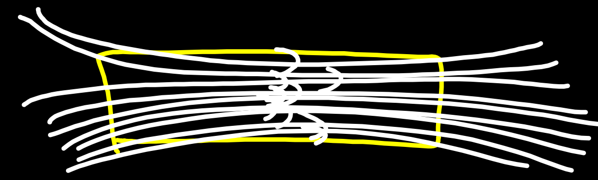


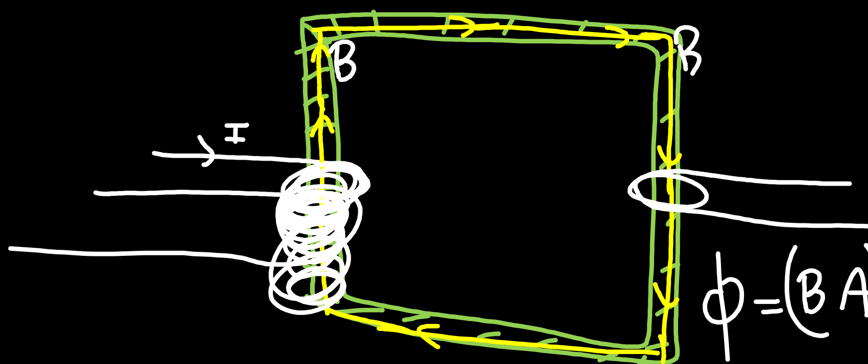
$r \rightarrow$ distance from center.

Transformer

soft iron

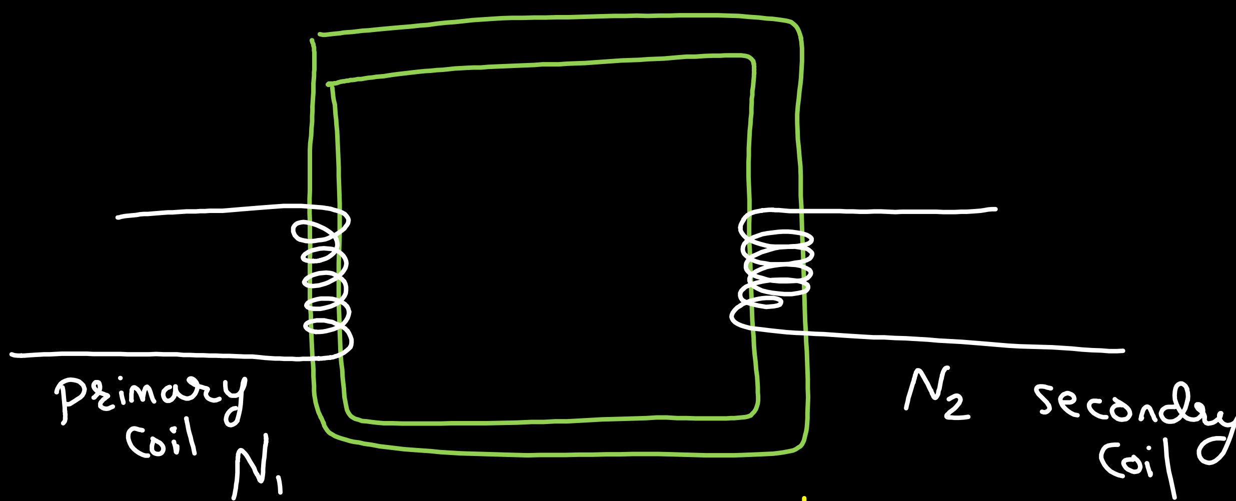






$$\phi = (BA)$$

$$\phi = (BA)\,1000$$



$\phi_{total} \propto N_1$

$Emf_1 \propto N_1$

$\phi_{total} \propto N_2$

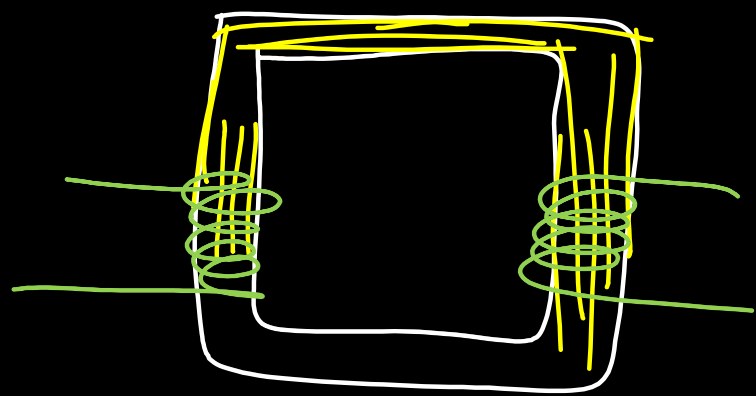
$Emf_2 \propto N_2$

$$\frac{Emf_1}{Emf_2} = \frac{N_1}{N_2}$$

ideal case

No energy loss

$\text{Power} = VI$



$$\text{Power}_{\text{primary}} = \text{Power}_{\text{secondry}}$$

$$\text{Emf}_1 I_1 = \text{Emf}_2 I_2$$

$$\frac{\text{Emf}_1}{\text{Emf}_2} = \frac{I_2}{I_1}$$

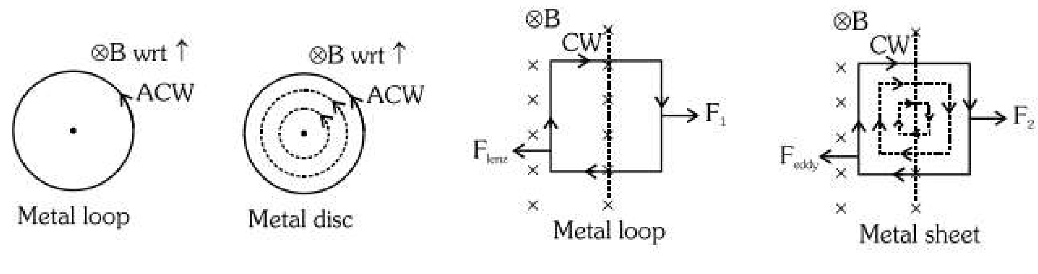
$$\frac{Emf_1}{Emf_2} = \frac{N_1}{N_2} = \frac{I_2}{I_1}$$

$Emf \propto N$ (flux linkage)

$$Power_{input} = Power_{output}$$

$$V_1 I_1 = V_2 I_2$$

EDDY CURRENTS (or Focalt's currents)



$(F_1 <<< F_2)$

Eddy current are basically the induced currents set up inside the body of conductor whenever the magnetic flux linked with it changes .

Eddy currents tends to follow the path of least resistance inside a conductor . So they from irregularly shaped loops . However , their directions are not random , but guide by Lenz's Law .

Eddy currents have both undesirable effects and practically useful applications.

Application of eddy currents:

(i) Induction furnace

(ii) Electric brakes

(iii) Electromagnetic damping

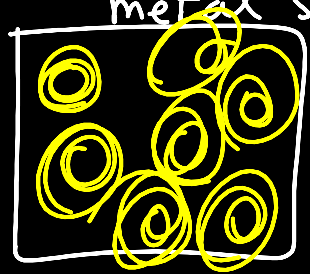
(iv) Speedometers

(v) Induction motor

(vi) Energy meters

(vii) Electromagnetic shielding

metal solid



ϕ magnetic change

$\downarrow$

Emf induce

$\downarrow$

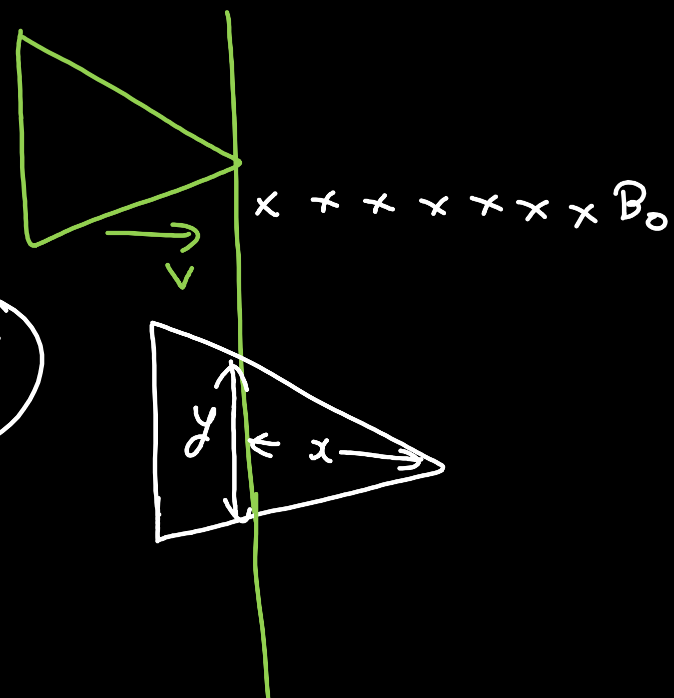
I induce

$\downarrow$

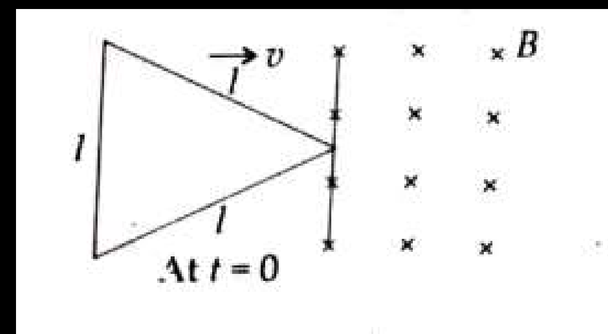
- These currents are produced only in closed path within the entire volume and on the surface of metal body. Therefore their measurement is impossible.
- Circulation plane of these currents is always perpendicular to the external field direction.
- Generally resistance of metal bodies is low so magnitude of these currents is very high.
- These currents heat up the metal body and some time body will melt out (Application : Induction furnace)
- Due to these induced currents a strong eddy force (or torque) acts on metal body which always apposes the translatory (or rotatory) motion of metal body, according to lenz.
- Transformer
  Cause : Transformer core is always present in the effect of alternating magnetic field $(B = B_0 \sin t)$. Due to this eddy currents are produced in its volume, so a part of magnetic energy of core is wasted as heat.

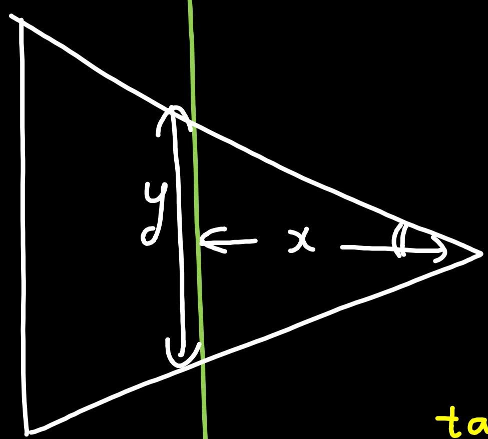
Remedy : To minimize these losses transformer core should be laminated. with the help of lamination process, circulation path of eddy current is greatly reduced & net resistance of system is greatly increased. So these currents become

An equilateral loop of side length l is moved with constant velocity v as shown. Find the emf induced and current through the loop as a function of time. Also draw emf vs time graph. The resistance of the loop is R.



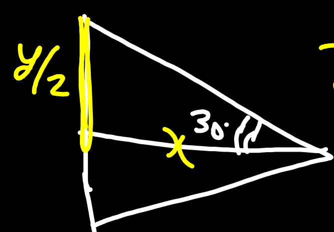
$$\frac{dx}{dt} = V$$





$$\text{area} = \frac{1}{2} y x$$

$$\tan 30 = \frac{y/2}{x}$$



$$\frac{1}{\sqrt{3}} = \frac{y}{2x}$$

$$\frac{2x}{\sqrt{3}} = y$$

$$\text{area} = \frac{1}{2}\left(\frac{2x}{\sqrt{3}}\right) x = x^2/\sqrt{3}$$

$$\phi = BA$$

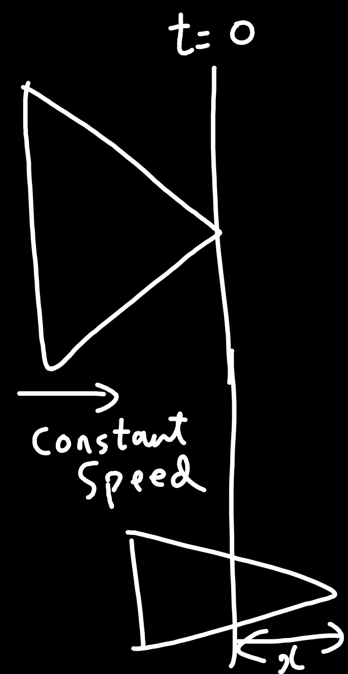
$$= B\left(\frac{x^2}{\sqrt{3}}\right)$$

$$\phi = \frac{Bx^2}{\sqrt{3}}$$

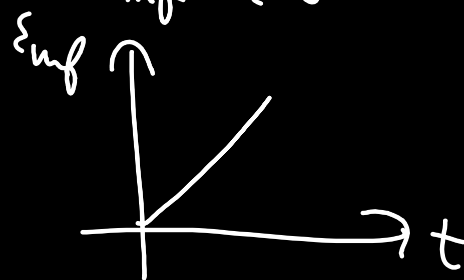
$$\phi = \frac{Bx^2}{\sqrt{3}}$$

$$\frac{d\phi}{dt} = \frac{B}{\sqrt{3}}\left(2x\,\frac{dx}{dt}\right)$$

$$|\mathcal{E}mf| = \frac{B}{\sqrt{3}}(2x)(v)$$



$$\mathcal{E}mf \propto t$$



Dist = Speed time

$$x = vt$$

$$\phi = \frac{B(vt)^2}{\sqrt{3}} = \frac{Bv^2t^2}{\sqrt{3}}$$

$$\frac{d\phi}{dt} = \frac{Bv^2\,2t}{\sqrt{3}}$$

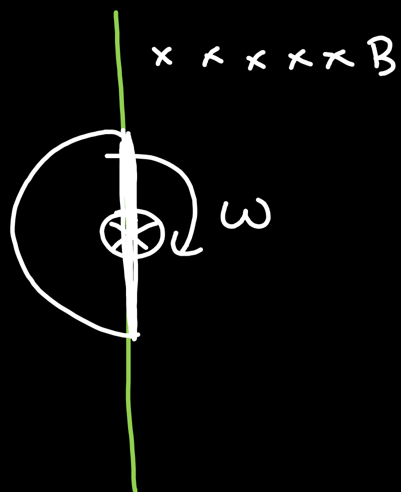
$$\boxed{|\mathcal{E}mf| = \frac{2Bv^2}{\sqrt{3}}\,t}$$

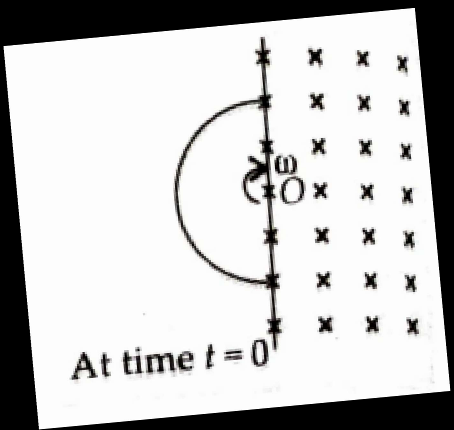
If const acc   if $u = 0$

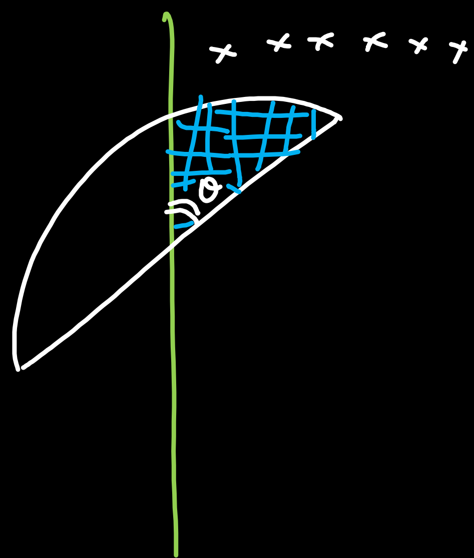
$$x = \frac{1}{2}at^2$$

$$V = at$$

A semi-circular loop of radius R is made to rotate with constant angular velocity ω as shown. Find the emf induced as a function of time. Draw ε vs t graph for one complete rotation of the loop assuming anticlockwise flow of current as positive.







area of sector $= \frac{\theta}{2} R^2$

(θ in radian)

$$\phi = \left(\frac{\theta R^2}{2}\right) B$$

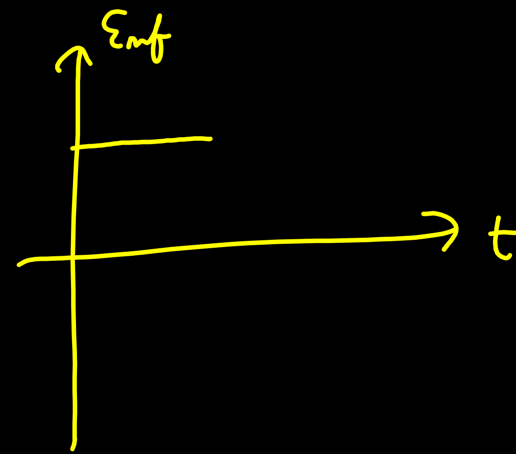
$$\phi = \frac{BR^2}{2}\,\theta$$

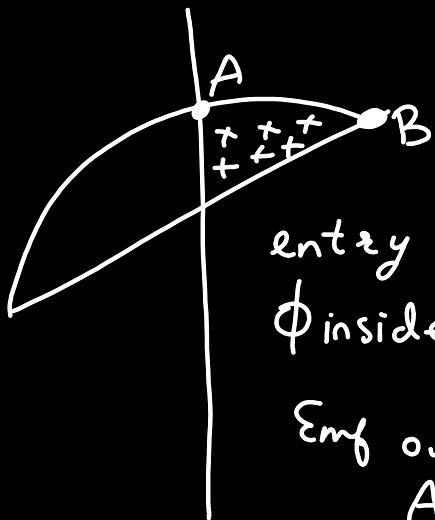
$$\frac{d\phi}{dt} = \frac{BR^2}{2}\,\frac{d\theta}{dt}$$

$$\frac{d\phi}{dt} = \frac{BR^2}{2}\,\omega$$

$$\mathcal{E}mf = \frac{BR^2}{2}\,\omega$$

$$\omega = \frac{d\theta}{dt}$$





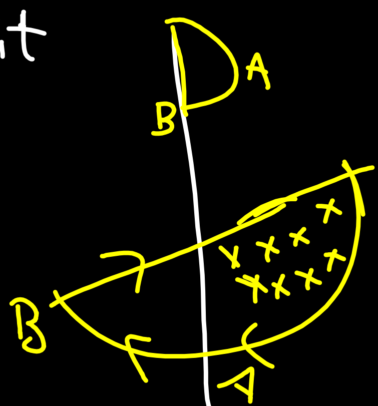
entry

ϕ inside increase

Emf outside increase
ACW

B to A

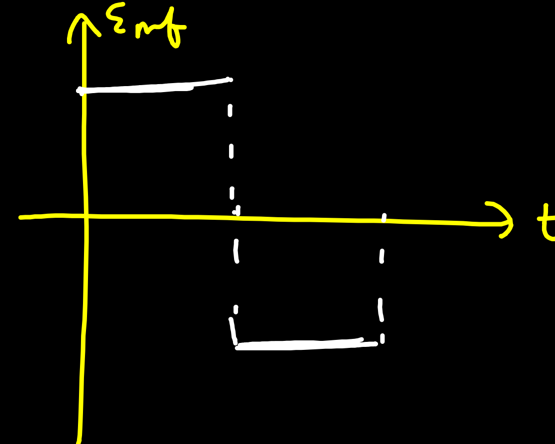
Exit



ϕ inside decreasing

Emf CW

A to B



The flux linked with a coil at any instant ' t ' is given by $\phi = 10t^2 - 50t + 250$.

The induced emf at $t = 3s$ is [2006]

(a) -190 V

(b) -10 V

(c) 10 V

(d) 190 V

$$\varepsilon_{mf} = -\frac{d\phi}{dt}$$

$$= -(20t - 50)$$

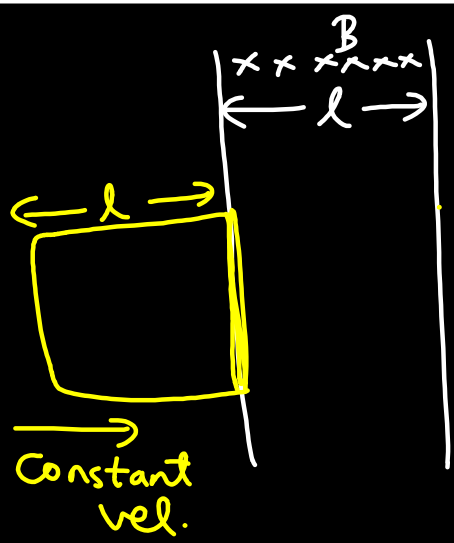
$$= -(60 - 50)$$

$$= -10$$

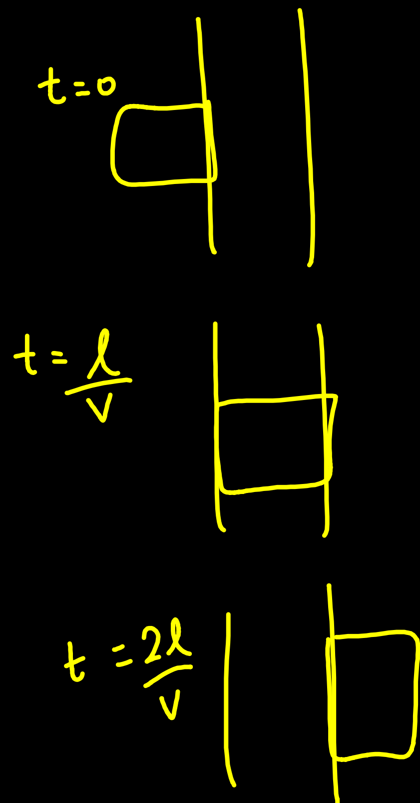
$$\phi = \frac{BR^2}{2}\theta \qquad \theta = \omega t + \cancelto{0}{\frac{1}{2}\alpha t^2}$$

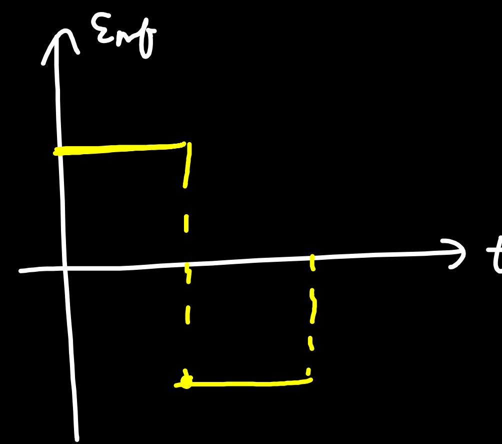
$$\phi = \frac{BR^2}{2}\omega t$$

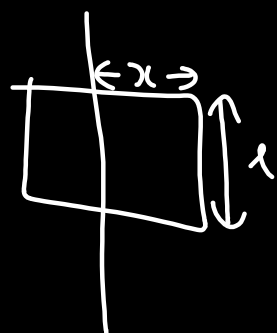
$$\frac{d\phi}{dt} = \frac{BR^2}{2}\omega.$$



Emf v/s time graph



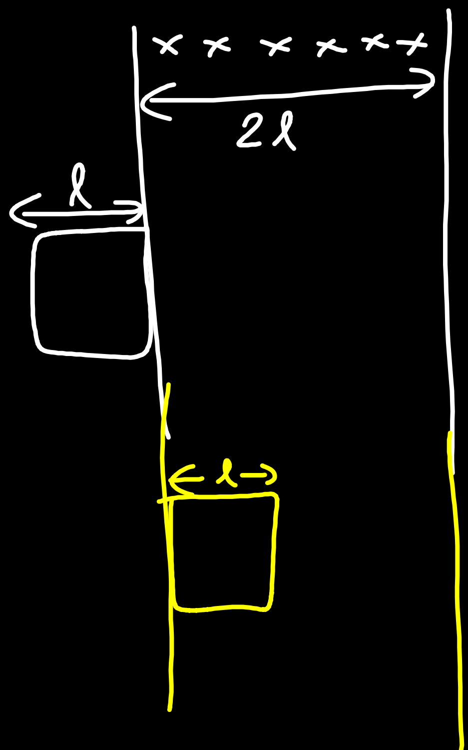


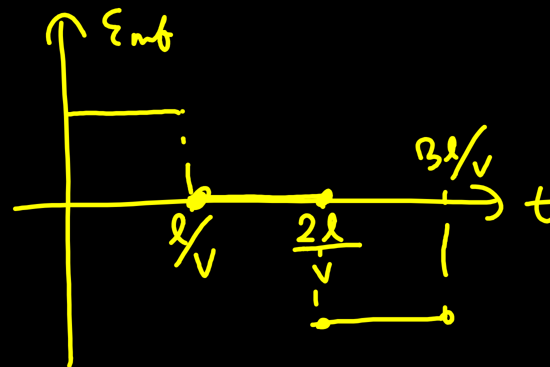


$$\phi = Blx$$

$$\frac{d\phi}{dt} = Bl\frac{dx}{dt}$$

$$\underline{Emf = Blv}$$





when here Completely inside

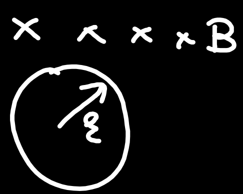
ϕ = constant

Emf = 0

H.W.

A circular coil of radius 10 cm is placed in a uniform magnetic field of 3.0×10^{-5} T with its plane perpendicular to the field initially. It is rotated at constant angular speed about an axis along the diameter of coil and perpendicular to magnetic field so that it undergoes half of rotation in 0.2 s. The maximum value of EMF induced (in μV) in the coil will be close to the integer ______ 15 μV ______

[NA Sep. 02, 2020 (I)]



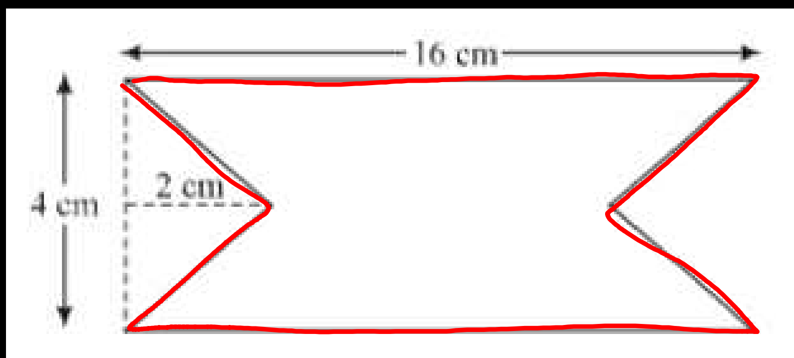
$$\mathcal{E}_{mf} = BA\omega \sin(\omega t)$$

$$\max \mathcal{E}_{mf} = BA\omega$$

$$= B(\pi r^2)\omega$$

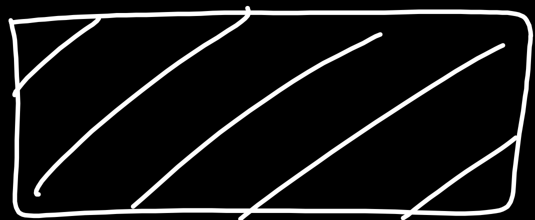
At time t = 0 magnetic field of 1000 gauss is passing perpendicularly through the area defined by closed loop shown in the figure. If the magnetic field reduces linearly to 500 gauss, in the next 5 s , then induced EMF in the loop is :

[NA 8 jan .2020 I]



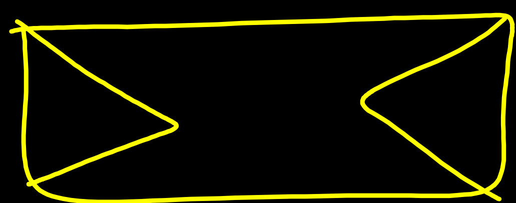
$$\varepsilon_{mf} = \frac{\Delta\phi}{\Delta t}$$

(a) 56 μ V (b) 28 μV (c) 48μV (d) 36 μV



$$Area = (16)(4) - 2\left[\frac{1}{\cancel{2}}(4)(2)\right]$$

$$= 56\,cm^2$$



Rectangle − 2Δ

$$\Delta\phi = \frac{(1000\ gauss\ 56\,cm^2) - (500\ gauss)\ 56\,cm^2}{5\,s}$$

$$= \frac{(56 \times 10^{-4}\,m^2)(500 \times 10^{-4})}{5}$$

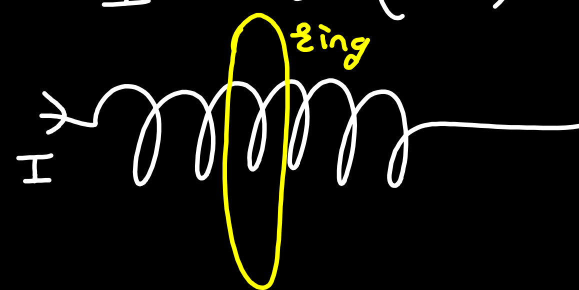
A long solenoid of radius R carries a time (t) - dependent current $I(t) = I_0 t(1-t)$. A ring of radius $2R$ is placed coaxially near its middle. During the time interval $\boxed{0 \le t \le 1}$, the induced current (I_R) and the induced $EMF(V_R)$ in the ring change as:

[7 Jan. 2020 I]

(a) Direction of I_R remains unchanged and V_R is maximum at $t = 0.5$
(b) At $t = 0.25$ direction of I_R reverses and V_R is maximum
(c) Direction of I_R remains unchanged and V_R is zero at $t = 0.25$
(d) At $t = 0.5$ direction of I_R reverses and V_R is zero

$B = (\mu_0 n I)$

$I = I_0 t(1-t)$



$$B = \mu_0 n I$$

$$= \mu_0 n I_0 t(1-t)$$

$$B = \mu_0 n I_0 (t - t^2)$$

$$\phi = B \text{ area}$$

$$\phi = \mu_0 n I_0 A (t - t^2)$$

$$|\mathcal{E}_{mf}| = \frac{d\phi}{dt} = \mu_0 n I_0 A (1-2t)$$

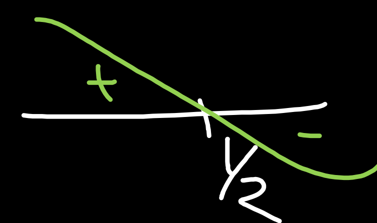
V_R $\boxed{\mathcal{E}_{mf} = \mu_0 n I_0 A (1-2t)}$

$$I_R = \frac{\mathcal{E}_{mf}}{R} = \frac{\mu_0 n I_0 A (1-2t)}{R}$$

$t = \frac{1}{2}$ $\mathcal{E}_{mf} = 0$

$t > 1/2$ $\mathcal{E}_{mf}$ -ve

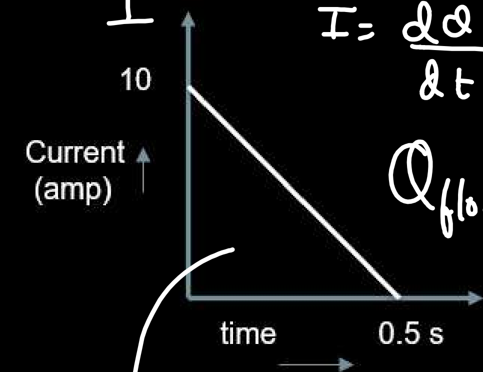
$t < 1/2$ $\mathcal{E}_{mf}$ +ve.



In a coil of resistance 100 Ω , a current is induced by changing the magnetic flux through it as shown in the figure .The magnitude of change in the flux through the coil is [2017]

~~(a)~~ 250 Wb (b) 275 Wb (c) 200Wb (d) 225 Wb

I



t

$I = \frac{dQ}{dt}$

$Q_{flow} = \int I\,dt$

$\text{area} = \Delta Q_{flow}$

$\Delta\phi = ??$

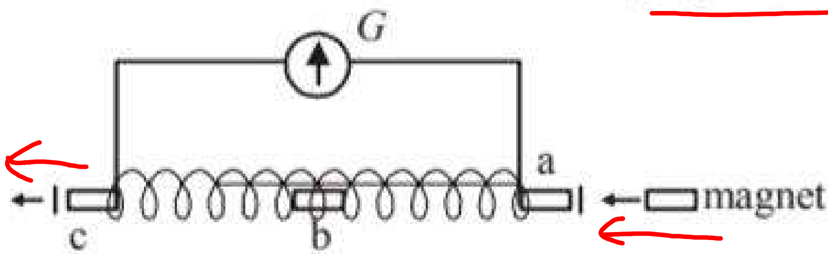
$I = \frac{\mathcal{E}_{mf}}{R} = \frac{\Delta\phi}{\Delta t\ R}$

$\Delta Q_{flow} = \frac{\Delta\phi}{R}$

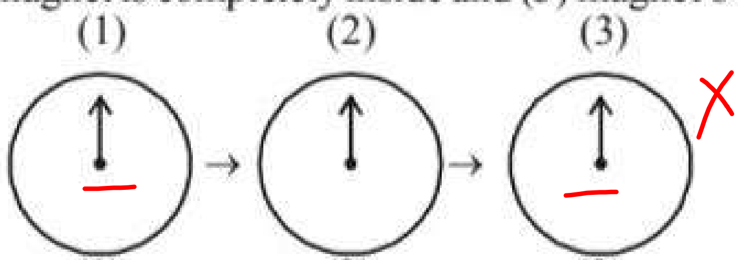
$\frac{1}{2}(0.5)(10) = \frac{\Delta\phi}{100}$

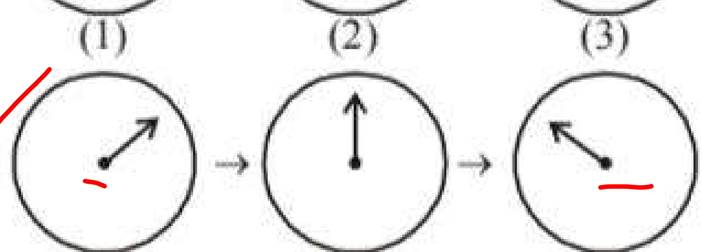
$250 = \Delta\phi$

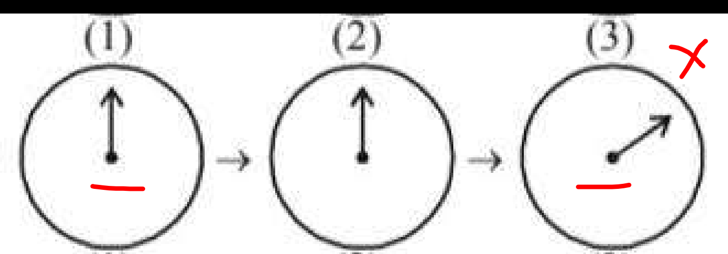
A small bar magnet is moved through a coil at constant speed from one end to the other. Which of the following series of observations will be seen on the galvanometer G attached across the coil ? **[Sep. 04, 2020 (I)]**

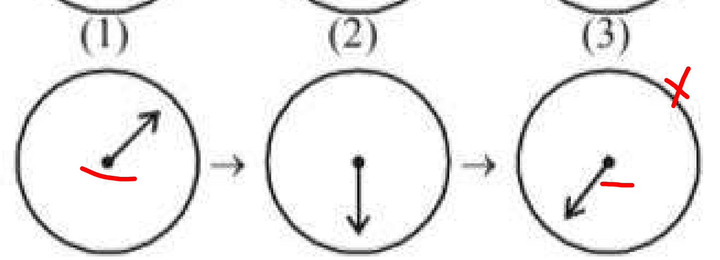


Three positions shown describe : (1) the magnet's entry (2) magnet is completely inside and (3) magnet's exit.

(a)


(b)


(c)


(d)


When current in a coil changes from 5 A to 2 A in 0.1 s, average voltage of 50 V is produced. The self - inductance of the coil is:

[Online April 10, 2015]

(a) 6H

(b) 0.67H

(c) 3H

(d) 1.67H

$\phi = LI$

$I \Rightarrow 5$ to 2

$t = 0.1s$

50 Volt

$Emf = L\frac{dI}{dt}$

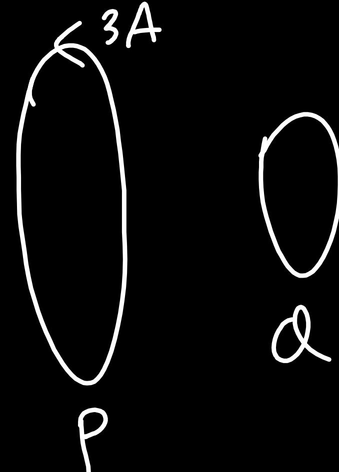
$50 = L\frac{(5-2)}{0.1}$

$L \Rightarrow \frac{5}{3} = \underline{1.67\,H}$

Two coils ' P ' and ' Q ' are separated by some distance. When a current of 3 A flows through coil ' P ', a magnetic flux of 10^{-3} Wb passes through ' Q '. No current is passed through ' Q '. When no current passes through ' P ' and a current of 2A passes through ' Q ', the flux through ' P ' is:

[9 Apr. 2019 II]

(a) 6.67×10^{-4} Wb
(b) 3.67×10^{-3} Wb
(c) 6.67×10^{-3} Wb
(d) 3.67×10^{-4} Wb

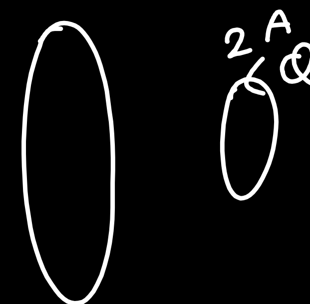


$\phi_Q = 10^{-3}$

$\phi_2 = M I_1$

$10^{-3} = M(3)$

$M = \frac{10^{-3}}{3}$



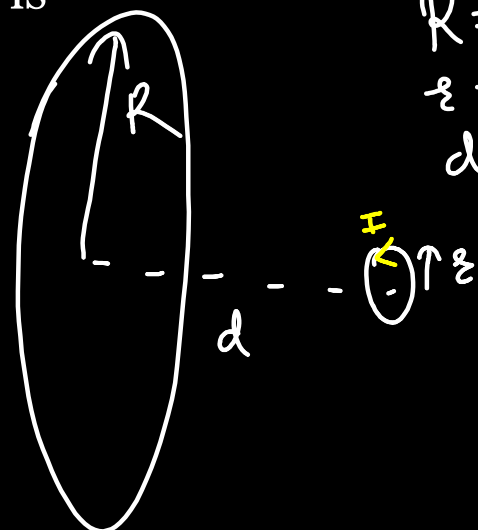
$\phi_P = M I_2$

$\phi = \frac{10^{-3}}{3} \times 2 = \underline{6.67 \times 10^{-4}}$

A circular loop of radius 0.3 cm lies parallel to a much bigger circular loop of radius 20 cm. The center of the small loop is on the axis of the bigger loop. The distance between their centers is 15 cm. If a current of 2.0 A flows through the smaller loop, then the flux linked with bigger loop is

[2013]

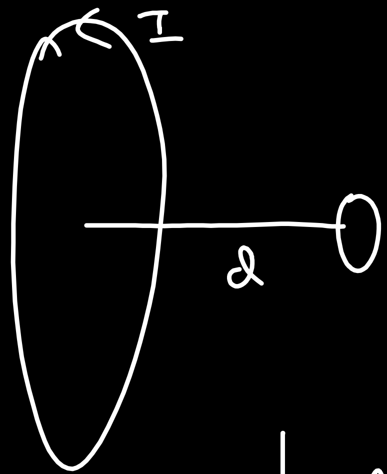
(a) 9.1×10^{-11} weber
(b) 6×10^{-11} weber
(c) 3.3×10^{-11} weber
(d) 6.6×10^{-9} weber



$R = 20$ cm
$r = 0.3$ cm
$d = 15$ cm

$I = 2$ A

$\phi_{\text{Big loop}} = ?$



$$B = \frac{\mu_0 I R^2}{2(R^2+d^2)^{3/2}}$$

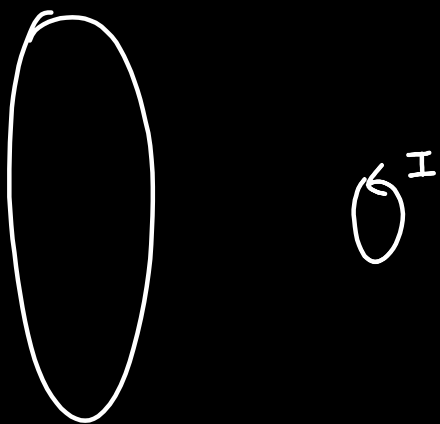
$$\phi = BA$$

$$\phi = MI$$

$$\phi = \frac{\mu_0 I R^2}{2(R^2+d^2)^{3/2}} \pi r^2$$

$$MI =$$

$$M = \frac{\mu_0 R^2 \pi r^2}{2(R^2+d^2)^{3/2}}$$

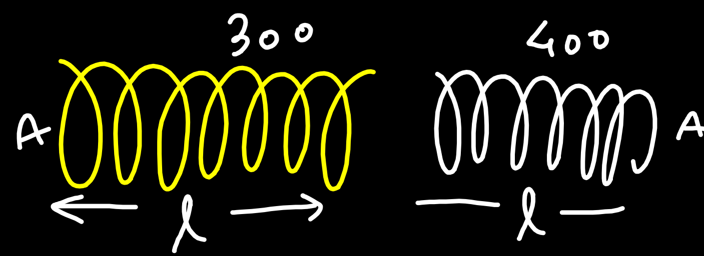


$$\phi = MI$$

$$\phi = \left[\frac{\mu_0 I R^2 \pi r^2}{2\left(R^2+d^2\right)^{3/2}}\right]_2$$

Two coaxial solenoids are made by winding thin insulated wire over a pipe of cross-sectional area $A = 10 \text{ cm}^2$ and length $= 20$ cm. If one of the solenoid has 300 turns and the other 400 turns, their mutual inductance is [2008]

$(\mu_0 = 4\pi \times 10^{-7} \text{TmA}^{-1})$

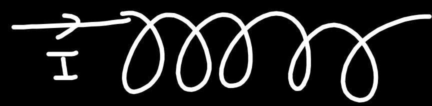


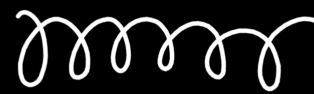
(a) $2.4\pi \times 10^{-5}$H

(b) $4.8\pi \times 10^{-4}$H

(c) $4.8\pi \times 10^{-5}$H

(d) $2.4\pi \times 10^{-4}$H





$$n_1 = \frac{N_1}{l}$$

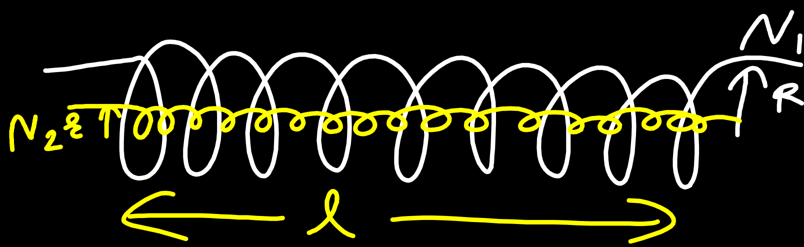
$$B = (\mu_0 n_1 I)$$

$$\phi = (BA) N_2$$

$$\phi = (\mu_0 n_1 \cancel{I}) A N_2 = M \cancel{I}$$

$$= \mu_0 n_1 N_2 A$$

$$\boxed{M = \frac{\mu_0 N_1 N_2 A}{l}}$$



$\xrightarrow[I]{}$

$$B = (\mu_0 n_1 I)$$

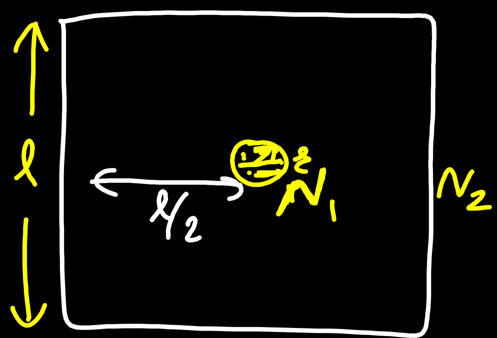
$$\phi_{\substack{\text{inside} \\ \text{small}}} = (B)(\pi r^2) N_2$$

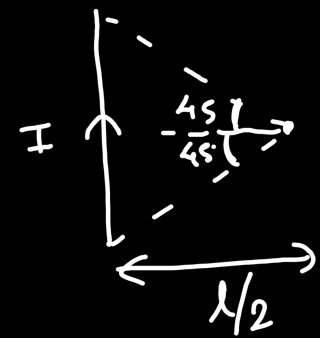
$$= \mu_0 n_1 I \pi r^2 N_2$$

$$\phi = \frac{\mu_0 N_1 I}{l} \pi r^2 N_2 = MI$$

$$M = \frac{\mu_0 N_1 N_2 \pi r^2}{l}$$

chote ka area

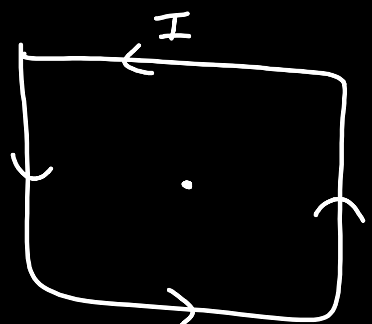




$$\frac{\mu_0 I}{4\pi\left(\frac{l}{2}\right)}\left(\sin 45 + \sin 45\right) \times 4$$

$$\frac{2\mu_0 I \sqrt{2}}{\pi l}$$

$$\frac{2\sqrt{2}\,\mu_0 I}{\pi l}$$



$$B = \left(\frac{2\sqrt{2}}{\pi l}\mu_0 I N_2\right)$$

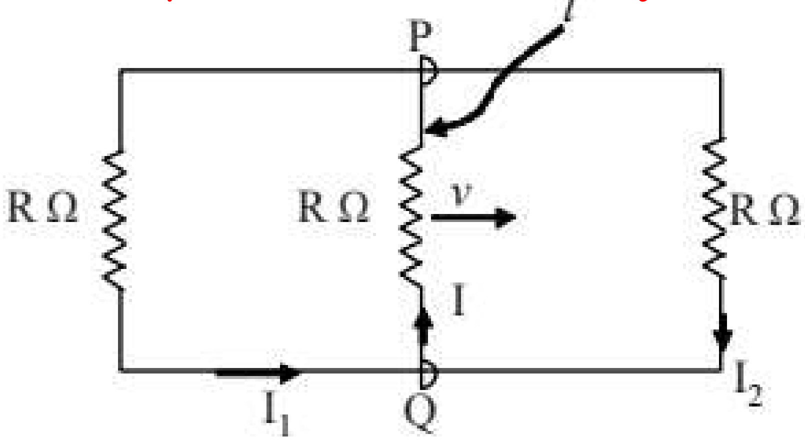
$$\phi_{small} = (BA)\,N_1$$

$$MI = \frac{2\sqrt{2}\,\mu_0 I N_2 \pi l^2 N_2}{\pi l}$$

$$M = \frac{2\sqrt{2}\,\mu_0 N_1 N_2}{l}\, l^2$$

A rectangular loop has a sliding connector PQ of length l and resistance R Ω and it is moving with a speed v as shown. The set-up is placed in a uniform magnetic field going into the plane of the paper. The three currents I_1, I_2 and I are **[2010]**

B_0

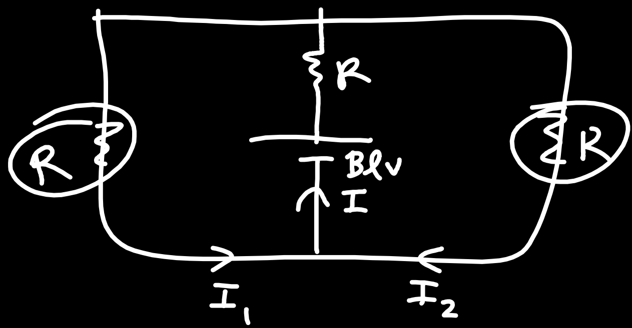


(a) $I_1 = -I_2 = \frac{Blv}{6R},\ I = \frac{2Blv}{6R}$

(b) $I_1 = I_2 = \frac{Blv}{3R}, I = \frac{2Blv}{3R}$

(c) $I_1 = I_2 = I = \frac{Blv}{R}$

(d) $I_1 = I_2 = \frac{Blv}{6R},\ I = \frac{Blv}{3R}$



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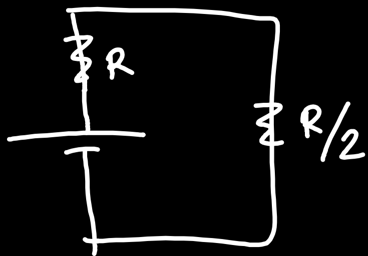


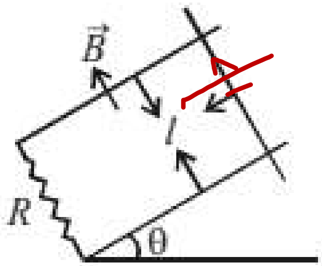
$$R_{eq} = R + \frac{R}{2}$$

$$R_{eq} = \frac{3R}{2}$$

$$I = \frac{Emf}{R_{eq}} = \frac{Blv}{3R/2}$$

$$= \frac{2Blv}{3R}.$$

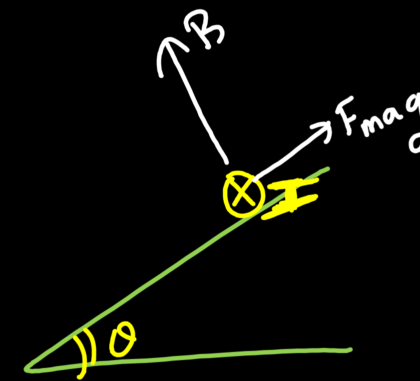
A copper rod of mass m slides under gravity on two smooth parallel rails, with separation l and set at an angle of θ with the horizontal. At the bottom, rails are joined by a resistance R. There is a uniform magnetic field B normal to the plane of the rails, as shown in the figure. The terminal speed of the copper rod is: **[Online April 15, 2018]**

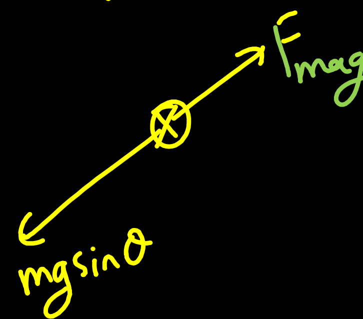


$I = \frac{B l v}{R}$

(a) $\frac{mgR\cos\theta}{B^2 l^2}$ (b) $\frac{mgR\sin\theta}{B^2 l^2}$

(c) $\frac{mgR\tan\theta}{B^2 l^2}$ (d) $\frac{mgR\cot\theta}{B^2 l^2}$



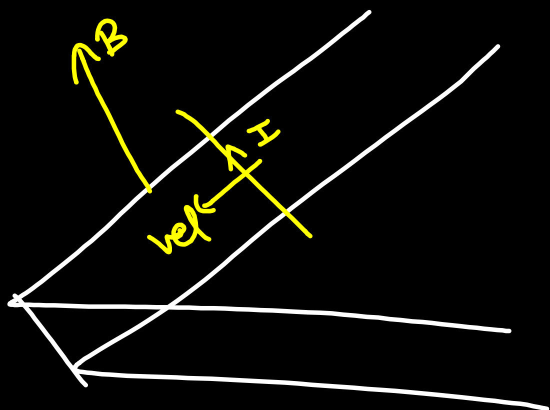


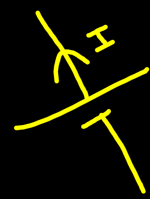
$mg\sin\theta = BIl$

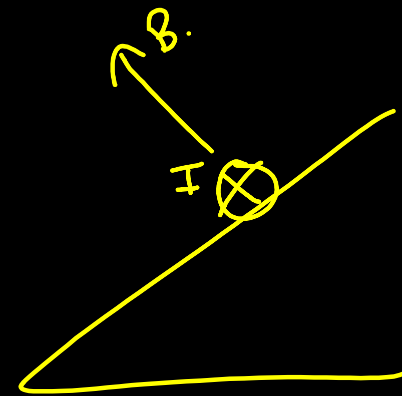
$mg\sin\theta = B\frac{Blv}{R}l$

$\frac{mgR\sin\theta}{B^2 l^2} = v$

$Q\vec{v} \times B$







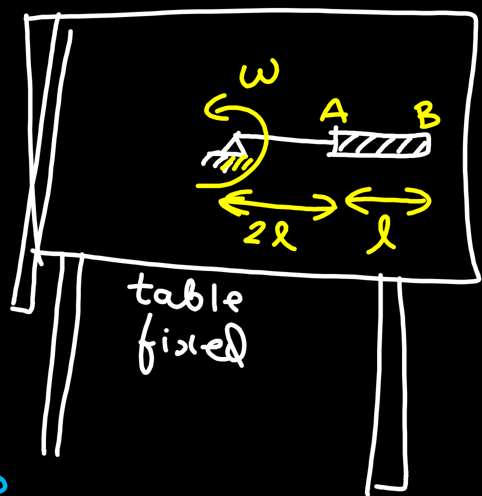
Q//

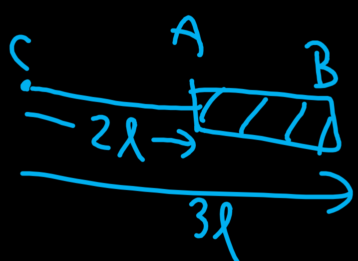
string $= 2l$

rod $= l$

Motional Emf A to B ??

(Jee 2013)



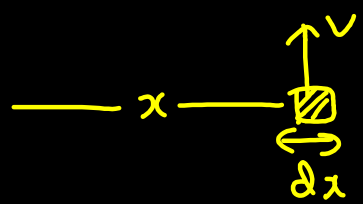


a) $\dfrac{2B\omega l^2}{2}$

b) $\dfrac{3B\omega l^2}{2}$

c) $\dfrac{4B\omega l^2}{2}$

d) $\dfrac{5B\omega l^2}{2}$



$$\mathcal{E}_{mf} = B\,l\,v$$

$$= B(dx)(x\,\omega)$$

$$= B\omega \int_{2l}^{3l} x\,dx$$

$$= B\omega \left[\frac{x^2}{2}\right]_{2l}^{3l}$$

$$\Rightarrow \frac{B\omega}{2}\left(9l^2 - 4l^2\right)$$

$$\frac{5B\omega l^2}{2}$$

Q. Power transmission line feeds input power at 2300 V to a step down transformer. $N_{primary} = 4000$

$N_{secondary} = ?$ for output voltage $= 230$ Volt.

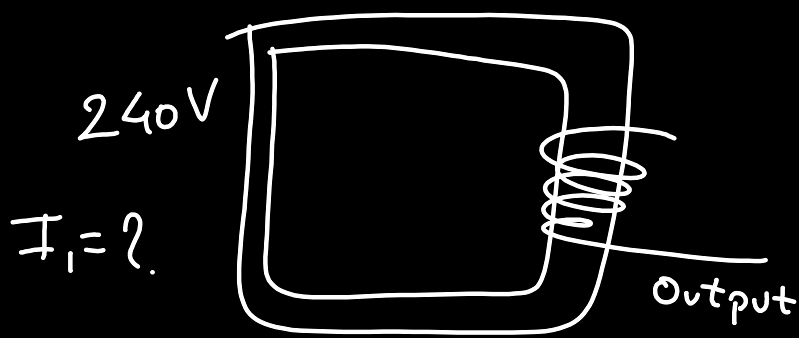
$$\frac{Emf_1}{Emf_2} = \frac{N_1}{N_2}$$

$$\frac{2300}{230} = \frac{4000}{N_2}$$

$$N_2 = 400$$

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{I_2}{I_1}$$

The output voltage of an ideal transformer, connected to a 240 V a .c. mains is 24 V. When this transformer is used to light a bulb with rating 24 V, 24 W calculate the current in the primary coil of the circuit?



24V Voltage

24Watt Power

$$P_2 = V_2 I_2$$

$$24 = 24 I_2$$

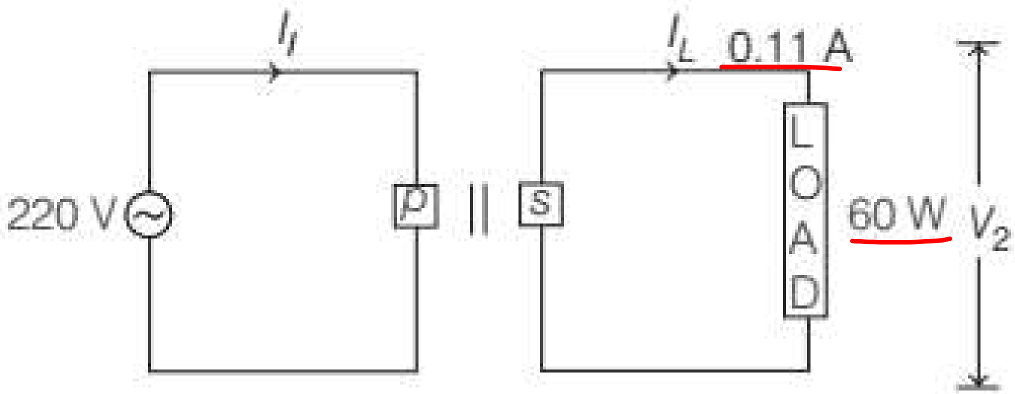
$$1 = I_2$$

$$\frac{V_1}{V_2} = \frac{I_2}{I_1}$$

$$\frac{240}{24} = \frac{1}{I_1}$$

$$I_1 = \frac{1}{10} = 0.1A$$

For the given circuit, comment on the type of transformer used.



a. Auxilliary transformer
b. Auto transformer
c. Step-up transformer
d. Step down transformer

2021 Jee

Secondry

$I_2 = 0.11$

$P_2 = 60 = V_2 I_2$

$V_2 = \frac{60}{0.11} = \frac{60 \times 100}{11}$

$V_2 \approx 545$

A common transistor radio set required 12 V (DC) for its operation . The DC source is constructed by using a transformer and a rectifier circuit , which are operated at 220 V (AC) on standard domestic AC supply . The number of turns of secondary coil are 24 , then the number of turns of primary are………

JEE 2021

$N_1 = ?$

$V_1 = 220V$

$N_2 = 24$

$V_2 = 12$

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} \Rightarrow \frac{220}{12} = \frac{N_1}{24}$$

$N_1 = 440$