Real Number

* **Basics of Number System**

i) **Natural Numbers** — Counting Numbers $(1, 2, 3, 4, \ldots)$

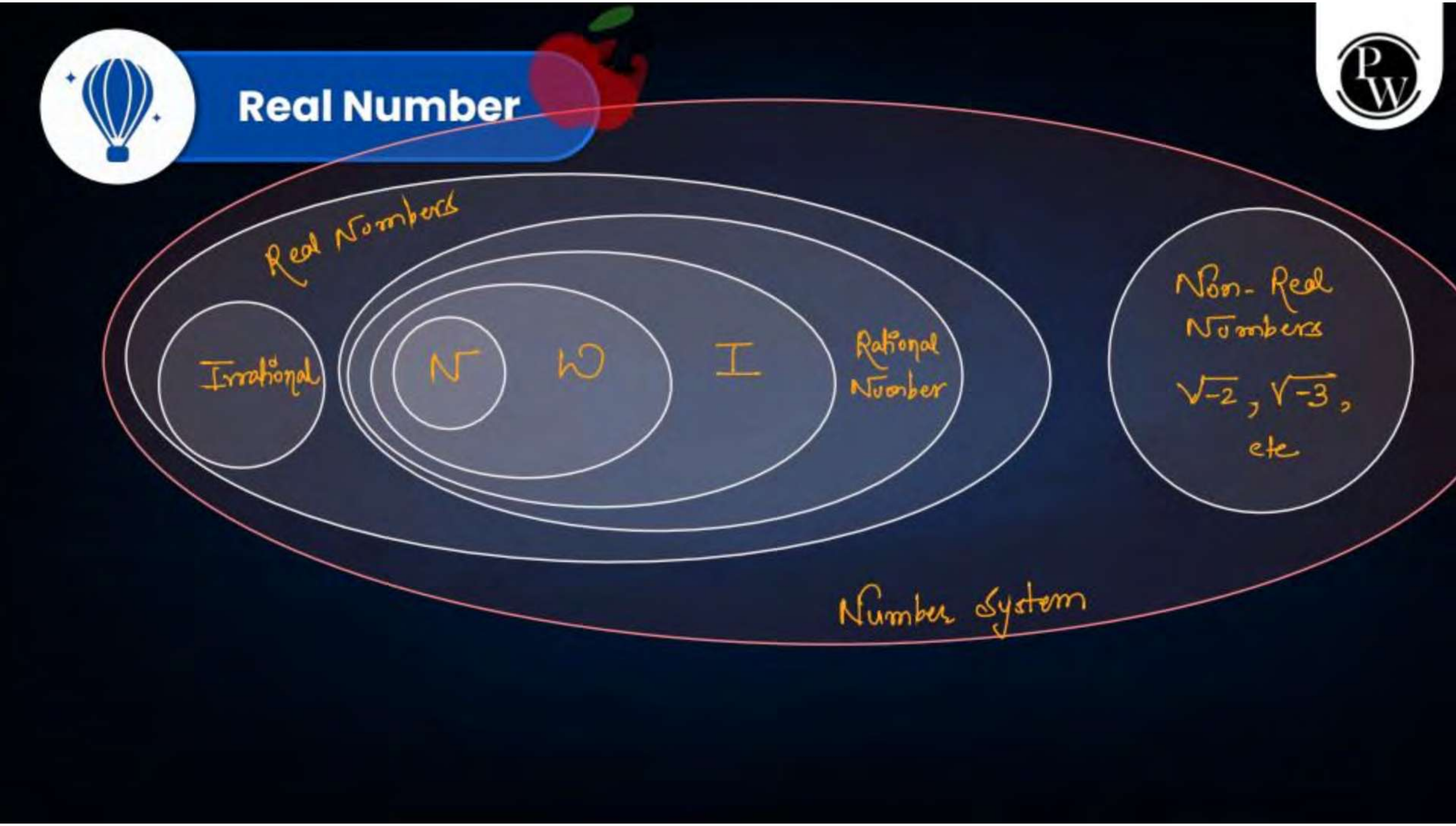
ii) **Whole Numbers** — $0 +$ Natural Numbers

iii) **Integers** — $\underset{(-ve\ Int)}{-\text{Natural Numbers}} + \overbrace{0 + \underset{(+ve\ Int)}{\text{Natural Numbers}}}^{\text{whole numbers}}$

iv) **Rational Numbers** — $\boxed{\dfrac{Num}{Den}}$ $(Den \neq 0)$ $\left.\begin{matrix} Num \\ Den \end{matrix}\right]$ Integers

v) **Irrational Numbers** — Not rational (Non Terminating & Non Repeating)

$\sqrt{2}, \sqrt{5}, 2+\sqrt{3}$, etc


Real Number
Real Numbers
Irrational
N
W
I
Rational Number
Non-Real Numbers
$\sqrt{-2}$, $\sqrt{-3}$, etc
Number System

Real Number

* Basics of divisibility

$$\boxed{\text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder}}$$

Divisor) Dividend (Quotient

Remainder

When $\boxed{\text{Remainder} = 0}$, then

Dividend is completely divisible by divisor.

Hence, Divisor | Dividend →

4 | 24

Real Number

2 multiples 2, 4, 6, 8, 10, 12, ...

* Factors & Multiples

* Factor : The numbers that completely divide a number are called its factors.

Eg : 8 → 1, 2, 4, 8
11 → 1, 11
→ Factors

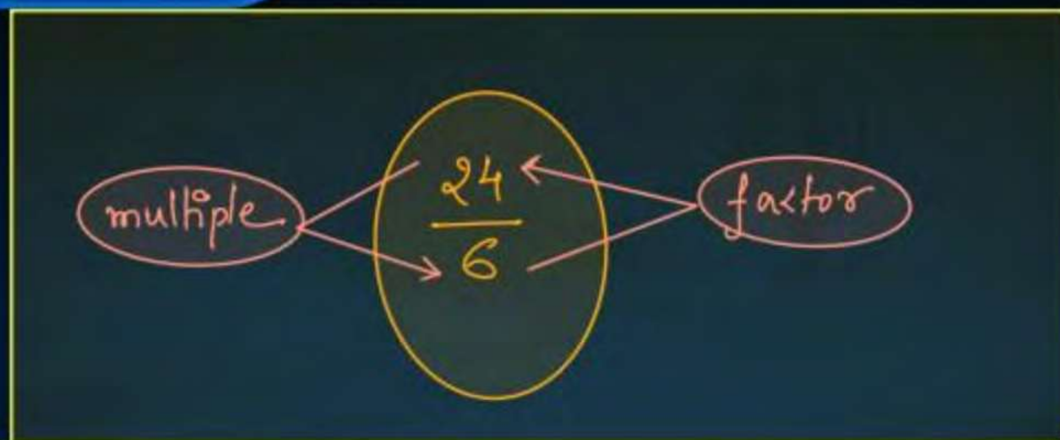
* Multiples : The number divides its multiples. (Table of the number)

Eg : 3 → 3, 6, 9, 12, ...
→ Multiples

Real Number

i.e.,

$$\begin{array}{r|l} 6 \,) \; 24 & (4 \\ \underline{24} & \\ \underline{\times} & \end{array}$$



Real Number

Nat

* **Prime Numbers** — Only 2 factors 1 & itself. Eg → 2, 3, 5, 7, ...
* **Composite Numbers** — More than 2 factors. Eg → 4, 6, 8, 9, 10, 12, ..

1 ×

2: 1, 2
3: 1, 3
5: 1, 5
4: 1, 2, 4
6: 1, 2, 3, 6

1 → 1

N → 1, Prime, Composite

Real Number

* Fundamental Theorem of Arithmetic

* Every composite number can be expressed (factorised) as product of primes & this factorisation is $\boxed{\text{unique}}$ except for the order in which the prime factors occur.

1
2
5
10

$10 = 2 \times 5$

QUESTION

2, 3, 5, 7, 11, 13, ...

Express each number as a product of its prime factors: → Normal

→ Factor Tree

(i) 140

$2 \times 5 \times 3$

$2 \times 3 \times 5$

$5 \times 3 \times 2$

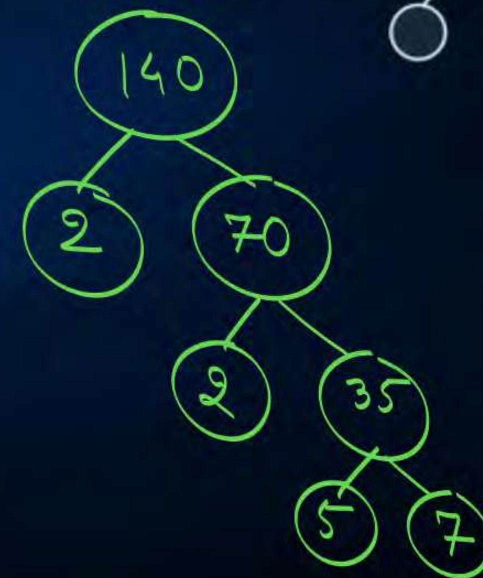
$$140 = 2^2 \times 5^1 \times 7^1$$

$$= 5^1 \times 2^2 \times 7^1$$

$$= 7^1 \times 2^2 \times 5^1$$

2	140
2	70
5	35
	7

Factor Tree

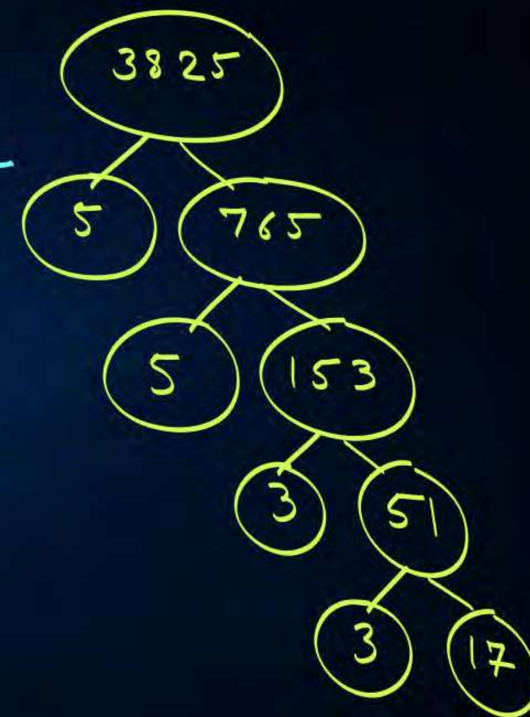


QUESTION

Express each number as a product of its prime factors:
(ii) 3825

$$3825 = 3^2 \times 5^2 \times 17^1$$
$$= 5^2 \times 3^2 \times 17^1$$
$$= 17^1 \times 3^2 \times 5^2$$
$$= 5^2 \times 17^1 \times 3^2$$

$$\begin{array}{r|l} 5 & 3825 \\ \hline 5 & 765 \\ \hline 3 & 153 \\ \hline 3 & 51 \\ \hline & 17 \end{array}$$



Real Number

* **What is HCF ?**

→ Its the greatest common factor of the given numbers.

Eg: $4 \rightarrow 1, 2, 4$

$8 \rightarrow 1, 2, 4, 8$

$HCF(4, 8) = 4$

Real Number

* What is LCM ?

→ Its the lowest common multiple of the given numbers.

Eg : $4 \rightarrow 4, 8, 12, 16, \ldots$

$12 \rightarrow 12, 24, 36, \ldots$

$$\text{LCM}(4, 12) = 12$$

QUESTION

2, 3, 101

Highest
HCF
LCM
Lowest

Find the HCF of 96 and 404 by the prime factorisation method. Hence, find their LCM.

$$96 = 2^5 \times 3^1$$
$$404 = 2^2 \times 101^1$$

$$\begin{array}{r} 303 \\ \times\ 32 \\ \hline 606 \\ 909\times \\ \hline 9696 \end{array}$$

$$\begin{array}{r|l} 2 & 96 \\ \hline 2 & 48 \\ \hline 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline & 3 \end{array}$$

$$\begin{array}{r|l} 2 & 404 \\ \hline 2 & 202 \\ \hline & 101 \end{array}$$

$$\text{HCF}(96, 404) = 2^2 = \boxed{4}$$
$$\text{LCM}(96, 404) = 2^5 \times 3^1 \times 101^1$$
$$= 303 \times 32$$
$$= \boxed{9696}$$

QUESTION

2, 3, 5

Find the HCF and LCM of 6, 72 and 120, using the prime factorisation method.

$$\begin{array}{r|l} 2 & 6 \\ \hline & 3 \end{array} \qquad \begin{array}{r|l} 2 & 72 \\ \hline 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline & 3 \end{array} \qquad \begin{array}{r|l} 2 & 120 \\ \hline 2 & 60 \\ \hline 2 & 30 \\ \hline 3 & 15 \\ \hline & 5 \end{array}$$

$6 = 2^1 \times 3^1$

$72 = 2^3 \times 3^2$

$120 = 2^3 \times 3^1 \times 5^1$

$\text{HCF}(6, 72 \,\&\, 120) = 2^1 \times 3^1 = \boxed{6}$

$\text{LCM}(6, 72 \,\&\, 120) = 2^3 \times 3^2 \times 5^1$

$= 40 \times 9$

$= \boxed{360}$

QUESTION

Find the LCM and HCF of the following integers by applying the prime factorisation method.
17,23 and 29

$17 = 17^1$
$23 = 23^1$
$29 = 29^1$

$HCF(17, 23 \& 29) = 1$

$LCM(17, 23 \& 29) = 17 \times 23 \times 29$

$= \underline{\underline{11,339}}$

$$\begin{array}{r} 29 \\ \times 23 \\ \hline 87 \\ 58\times \\ \hline 667 \\ \times 17 \\ \hline 4669 \\ 667\times \\ \hline 11339 \end{array}$$

Real Number

* Some important properties of HCF & LCM

i) HCF can never be greater than the numbers.

ii) LCM can never be smaller than the numbers.

iii) HCF divides LCM completely.

iv) For 2 numbers, $\boxed{HCF \times LCM = \text{product of the 2 numbers}}$

v) HCF of numbers divide the numbers completely.

vi) LCM is completely divisible by the numbers.

QUESTION

2, 7, 13

HCF × LCM = product

Find the LCM and HCF of the following pairs of integers and verify that LCM x HCF = product of the two numbers.

(i) 26 and 91

$26 = 2^1 \times 13^1$

$91 = 7^1 \times 13^1$

$$\begin{array}{r} 182 \\ \times 13 \\ \hline 546 \\ 182\times \\ \hline 2366 \end{array}$$

$$\begin{array}{r|l} 2 & 26 \\ \hline & 13 \end{array} \qquad \begin{array}{r|l} 7 & 91 \\ \hline & 13 \end{array}$$

$$\begin{array}{r} 91 \\ \times 26 \\ \hline 546 \\ 182\times \\ \hline 2366 \end{array}$$

$HCF(26, 91) = 13^1 = \boxed{13}$

$LCM(26, 91) = 2^1 \times 7^1 \times 13^1$

$= 26 \times 7$

$= \boxed{182}$

L.H.S = HCF × LCM

$= 13 \times 182$

$= 2366$ —— (i)

R.H.S = 26×91

$= 2366$ —— (ii)

QUESTION

Given that HCF (306, 657) = 9, find LCM (306, 657). → ?

x

Sol^n: HCF × LCM = product of the 2 numbers

$\Rightarrow \cancel{9}^{3} x = 306 \times \cancel{657}\ \cancel{219}\ 73$

$\Rightarrow x = 306 \times 73$

LCM = $\boxed{22338}$ Ans

$$\begin{array}{r} 306 \\ \times\ 73 \\ \hline 918 \\ 2142\times \\ \hline 22338 \end{array}$$

Real Number

$(\square)^n \rightarrow \_\ \_\ \_\ 0$

$(\square)^n = \_\ \_\ \_\ 5$

* Concept of numbers ending in 0 / 5

- For a number to end in '0', it must have both $\boxed{2 \;\&\; 5}$ as its prime factors.
- For a number to end in '5', it must have 5 as its prime factor & mustn't contain 2 simultaneously.

→ If factors of (p) → a, b, c
then factors of $(p)^n$ → a, b, c.

$6 \longrightarrow 2, 3$

$(6)^2 = 2, 3$

$(6)^5 = 2, 3$

$(6)^{\text{lcrore}} = 2, 3$

QUESTION

$\underline{285}$ $(\boxed{\ })^n \longrightarrow \_\ \_\ \_\ 0$

5 × 2

Consider the numbers 4^n, where n is a natural number. Check whether there is any value of n for which 4^n ends with the digit zero.

$(4)^n \xrightarrow[\text{factors}]{\text{prime}} 2$ ✓

$4 \xrightarrow[\text{factor}]{\text{prime}} 2$

$(4)^n \xrightarrow[\text{digit}]{\text{end}} 0$ ✗

$\xrightarrow[\text{digit}]{\text{end}} 5$ ✗

$$\begin{array}{c|c} 2 & 4 \\ \hline & 2 \end{array}$$

QUESTION

$$\frac{0}{2 \times 5} \quad \frac{5}{5 \quad \cancel{2}}$$

Check whether 6^n can end with the digit 0 for any natural number n.

$(6)^n \xrightarrow[\text{factors}]{\text{prime}} 2, 3$

$6 \xrightarrow[\text{factors}]{\text{prime}} 2, 3$

$(6)^n \xrightarrow[\text{digit}]{\text{end}} \cancel{\ } \; 0$

$(6)^n \xrightarrow[\text{digit}]{\text{end}} \cancel{\ } \; 5$

$$\begin{array}{c|c} 2 & 6 \\ \hline & 3 \end{array}$$

QUESTION

factors 13, 78, 1

Explain why 7 × 11 × 13 + 13 and 7 × 6 × 5 × 4 × 3 × 2 × 1 + 5 are composite numbers.

$$\begin{array}{r} 84 \\ \times 12 \\ \hline 168 \\ 84\times \\ \hline 1008 \end{array}$$

Sol^n

$$\boxed{7\times 11\times 13+13}$$

$$= 13\cdot(7\times 11+1)$$

$$= 13\times 78\times 1$$

$$7\times 6\times 5\times 4\times 3\times 2\times 1+5$$

$$= 5(7\times 6\times 4\times 3\times 2\times 1+1)$$

$$= 5\times 1009\times 1$$

factors

Real Number

* HCF/LCM Word Problems

i) HCF (Gadhas) = Gadha ✓ Ghoda ✗

LCM (Aam) = Aam ✓ Amrud ✗

ii) Check if my ans can come greater/less than the given data to determine HCF/LCM.

iii) Use basic Maths & common sense, wherever required.

QUESTION

H, L

A sweet seller has 420 kaju barfis & 130 badam barfis. She wants to stack them in such a way that each stack has the same number, and they take up the least area of the tray. What is the number of barfis that can be placed in each stack for this purpose ?

10 barfis

$$420 = 2^2 \times 3^1 \times 5^1 \times 7^1$$

$$130 = 2^1 \times 5^1 \times 13^1$$

2	420
3	210
2	70
5	35
	7

13	130
2	10
	5

$$HCF(420 \,\&\, 130) = 2^1 \times 5^1 = 10 \text{ barfis}$$

Kaju Barfi $\longrightarrow \frac{420}{10} \rightarrow$ 42 stacks

Badam Barfi $\longrightarrow \frac{130}{10} \rightarrow$ 13 stacks

$\rightarrow (42+13) = 55$ stacks

QUESTION

E → 96
H → 240
M → 336 350 H Total no of stacks = 14

Three sets of English, Hindi & Mathematics books have to be stacked in such a way that all the books are stored topic wise & the height of each stack is the same. The number of English, Hindi & Maths books are 96, 240 & 336 respectively. Assuming that the books are of the same thickness, determine the number of books in each stack & number of stacks of English, Hindi & Mathematics books.

$$96 = 2^5 \times 3^1$$
$$240 = 2^4 \times 3^1 \times 5^1$$
$$336 = 2^4 \times 3^1 \times 7^1$$
$$HCF(96, 240 \,\&\, 336) = 2^4 \times 3^1$$
$$= 48 \text{ books}$$

$E \rightarrow \frac{96}{48} \rightarrow 2$

$H = \frac{240}{48} \rightarrow 5$

$M = \frac{336}{48} \rightarrow 7$

2	96
2	48
2	24
2	12
2	6
	3

2	240
2	120
2	60
2	30
3	15
	5

2	336
2	168
2	84
2	42
3	21
	7

QUESTION

Sec A → 32 students
Sec B → 36 students → L / H

$$\begin{array}{r} 32 \\ \times 9 \\ \hline 288 \end{array} \qquad \begin{array}{r} 36 \\ \times 8 \\ \hline 288 \end{array}$$

In a school there are 2 sections – section A & section B of Class 10th. There are 32 students in section A & 36 students in section B . Determine the minimum number of books required for their class library so that they can be distributed equally among students of section A or section B.

Sol^n)

$32 = 2^5$

$36 = 2^2 \times 3^2$

$\text{LCM}(32, 36) = 2^5 \times 3^2$

$= 32 \times 9$

$= 288$ books

Sec A → $\frac{288}{32}$ → 9

Sec B → $\frac{288}{36}$ → 8

$$\begin{array}{r|l} 2 & 32 \\ \hline 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline & 2 \end{array} \qquad \begin{array}{r|l} 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

QUESTION

(slower) Sonia → 18 min
Ravi → 12 min → H/L ✓
(faster)

There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

$18 = 2^1 \times 3^2$

$12 = 2^2 \times 3^1$

$\text{LCM}(18, 12) = 2^2 \times 3^2$

$= 4 \times 9$

$= 36 \text{ min}$

2	18
3	9
	3

3	12
2	4
	2

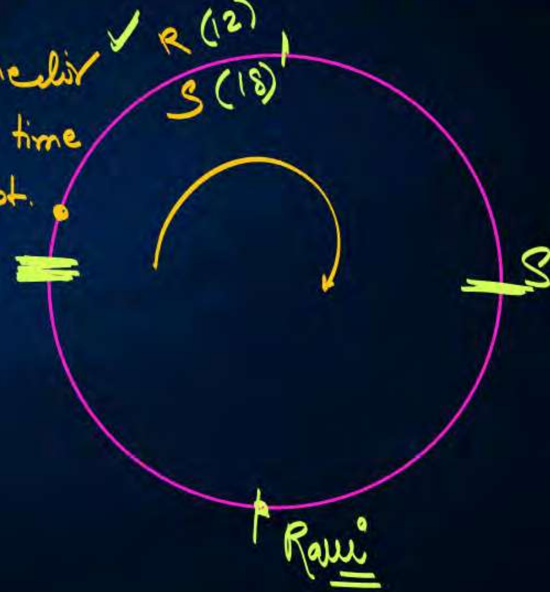
$R \rightarrow \frac{\cancel{36}}{\cancel{12}} \rightarrow 3$

$S \rightarrow \frac{\cancel{36}}{\cancel{18}} \rightarrow 2$

same dir ✓

same time

st pt.



QUESTION

Let p be a prime number . If p divides a^2 , then p divides a, where a is a positive integer.

$(9)^2$

3) 9 (3) 81 (

$(8)^2$

2) 8 (2) 64 (

Real Number

* **Irrational Numbers**

→ A number whose decimal form is non-terminating (NT) & non repeating. (NR)

→ Eg :- $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, etc ....

Real Number

* How to prove irrationality ?

i) Method of contradiction.

ii) $p | a^2 \Rightarrow p | a$

iii) If something is a rational number $\left(\frac{p}{q}\right)$ form, then it must be in simplest form.

QUESTION

Prove that $\sqrt{2}$ is irrational.

$2\overline{)a}($ c, remainder 0 $\longrightarrow$ $a = 2c + 0$

$a = 2c$

$2\overline{)b}($, remainder 0

$\sqrt{\text{prime}}$

Let $\sqrt{2}$ be rational.

$\Rightarrow \sqrt{2} = \frac{a}{b}$ [where a & $b \rightarrow$ int, $b \neq 0$, a & b doesn't have any other common factor than 1]

$\Rightarrow a = \sqrt{2}\, b$

$\Rightarrow a^2 = 2 b^2$

$\Rightarrow 2 \mid a^2$

$\Rightarrow 2 \mid a$

$\Rightarrow$ 2 is a factor of 'a' — (i)

$a = 2c$

$a^2 = 4c^2$

$\cancel{2}b^2 = {}^{2}\cancel{4}c^2$

$b^2 = 2c^2$

$\Rightarrow 2 \mid b^2$

$\Rightarrow 2 \mid b$

2 is also a factor of 'b'. — (ii)

From (i) & (ii)

2 is a common-factor of both a & b

But it contradicts the fact that

QUESTION

$\frac{1}{\sqrt{2}}$ ✓, $3+\sqrt{2}$ ✓ $\sqrt{\text{prime}}$ ✓

Show that $3\sqrt{2}$ is irrational.

Let $3\sqrt{2}$ be rational.

$3\sqrt{2} = \frac{a}{b}$ [where a & b are int, $b \neq 0$, a & b de______]

$\Rightarrow \sqrt{2} = \boxed{\frac{a}{3b}}$ $\frac{a \to I}{3b \to I} \to I / R$

$\sqrt{2} \to IR$

$\therefore$