Determinant

Determinants

- Determinants are always **Square**
- Scalar Value

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{\underline{2\times 3}}$$

$$R \longrightarrow \begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix}_{2\times 2}$$

Representation

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix}_{2\times 2} = \det(A)$$

$$|A| = \det(A)$$

$$\boxed{\begin{matrix} 1\times 1 \\ 2\times 2 \\ 3\times 3 \end{matrix}}$$

4×4

Determinant value of 1 × 1 & 2 × 2

$$A = \begin{vmatrix} -2 \end{vmatrix}_{1\times 1} = (-2)$$

$$A = \begin{vmatrix} a & b \\ c & d \end{vmatrix}_{2\times 2} = \boxed{ad - bc}$$

The **value of** $\begin{vmatrix} a+1 & a-2 \\ a+2 & a-1 \end{vmatrix}$ is

A. $2a^2$

B. 0

C. -3

D. 3

$(a+1)(a-1) - (a-2)(a+2)$

$= (\cancel{a^2} - 1) - (\cancel{a^2} - 4)$

$= 3$

The **value of** $\begin{vmatrix} 1+\cos\theta & \sin\theta \\ \sin\theta & 1-\cos\theta \end{vmatrix}$ is

A. 2

B. -1

C. 0

D. $\cos 2\theta$

$\Rightarrow (1+\cos\theta)(1-\cos\theta) - \sin^2\theta$

$\Rightarrow (1-\cos^2\theta) - \sin^2\theta$

$\Rightarrow 0$

Minor & Cofactor

Minors

$$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}_{3\times 3}$$

$$M_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} \qquad M_{32} = \begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix}$$

$$M_{21} = \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix}$$

Cofactor :

$$C_{ij} = (-1)^{i+j} M_{ij}$$

$$D = \begin{vmatrix} a_{11} & \boxed{a_{12}} & a_{13} \\ \boxed{a_{21}} & a_{22} & \boxed{a_{23}} \\ a_{31} & \boxed{a_{32}} & a_{33} \end{vmatrix}$$

$C_{11} = (-1)^{1+1} M_{11} = M_{11}$

$C_{12} = (-1)^{1+2} M_{12} = -M_{12}$

$C_{13} = (-1)^{1+3} M_{13} = M_{13}$

$$\boxed{C_{ij} = (-1)^{i+j} M_{ij}}$$

$\underline{i+j = \text{odd}}$

$C_{12} = -M_{12}$

$C_{21} = -M_{21}$

$C_{32} = -M_{32}$

$C_{23} = -M_{23}$

Remember :

$$C_{ij} = (-1)^{i+j} M_{ij}$$

$$\mathbf{D} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$D = \begin{vmatrix} + & \boxed{-} & + \\ \boxed{-} & + & \boxed{-} \\ + & \boxed{-} & + \end{vmatrix}$$

Expanding/Opening Determinant

Expanding w.r.t R1

$$D = \begin{vmatrix} \mathbf{a_{11}} & \mathbf{a_{12}} & \mathbf{a_{13}} \\ \mathbf{a_{21}} & \mathbf{a_{22}} & \mathbf{a_{23}} \\ \mathbf{a_{31}} & \mathbf{a_{32}} & \mathbf{a_{33}} \end{vmatrix}$$

w.r.t R1

$$\Rightarrow \boxed{a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}}$$

w.r.t C_2

$$\Rightarrow \boxed{a_{12}C_{12} + a_{22}C_{22} + a_{32}C_{32}}$$

Determinant value of 3×3

$$A = \begin{vmatrix} 2 & -3 & 1 \\ 2 & 0 & -1 \\ 1 & 4 & 5 \end{vmatrix}$$

Minor

$$\begin{vmatrix} 4 & 11 & 8 \\ -19 & 9 & 11 \\ 3 & -4 & 6 \end{vmatrix}$$

Cofactor

$$\begin{vmatrix} 4 & -11 & 8 \\ 19 & 9 & -11 \\ 3 & 4 & 6 \end{vmatrix}$$

w.r.t R1

$$(2)(4) + (-3)(-11) + (1)(8) = \boxed{49}$$

w.r.t C_2

$$(-3)(-11) + (0)9 + (4)4 = \boxed{49}$$

Determinant value of 3×3

$$\begin{vmatrix} 2 & -3 & 1 \\ 2 & 0 & -1 \\ 1 & 4 & 5 \end{vmatrix}$$

$(2)(3) + (-3)(4) + (1)(6) \Rightarrow \boxed{0}$

$(-3)(8) + (0)(-11) + (4)(6) = \boxed{0}$

Cofactor property

In a determinant **the sum of the product's** of the element's of any row (column) with their corresponding cofactor's is **equal to the value of determinant**.

$$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Cofactor property

$$\begin{vmatrix} 2 & -3 & 1 \\ 2 & 0 & -1 \\ 1 & 4 & 5 \end{vmatrix}$$

Shortcut to find value of determinant

#Shortcut (Rule of Sarrus)

$$
\begin{vmatrix} a & b & c \\ e & f & g \\ h & i & j \end{vmatrix}
=
\begin{array}{ccc|cc}
+ & + & + & & \\
\hline
a & b & c & a & b \\
e & f & g & e & f \\
h & i & j & h & i \\
\hline
- & - & - & &
\end{array}
= afj + bgh + cei - hfc - iga - jeb
$$

#Shortcut (Rule of Sarrus)

$$\begin{array}{|ccc|cc} 2 & -3 & 1 & 2 & -3 \\ 2 & 0 & -1 & 2 & 0 \\ 1 & 4 & 5 & 1 & 4 \end{array}$$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

$$ad - bc$$

$$(0+3+8) - (0-8-30)$$

$$= \boxed{49}$$

#Shortcut (Rule of Sarrus)

$$\begin{bmatrix} 2 & 3 & 1 \\ 1 & 1 & 1 \\ 2 & 2 & 1 \end{bmatrix} \begin{array}{cc} 2 & 3 \\ 1 & 1 \\ 2 & 2 \end{array}$$

$$(2+6+\cancel{2}) - (\cancel{2}+4+3)$$

$$= \boxed{1}$$

Properties of Determinant

Properties of Determinants

① $|A^T| = |A|$

② $|I| = 1$

1. $|A^T| = |A|$

Note: $|I| = 1$

Ex $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} 2 & 4 \\ 3 & 5 \end{vmatrix}$

$10 - 12 = 10 - 12$

Properties of Determinants

2. If any two rows (or columns) of a determinant **be interchanged,** the value of determinant is **changed in sign only.**

$$\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix} = -\begin{vmatrix} p & q & r \\ a & b & c \\ x & y & z \end{vmatrix}$$

Properties of Determinants

3. If row or columns are rotated in cyclic order then value of determinant is $\underbrace{\text{unchanged}}_{\text{same}}$

$$\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix} = \begin{vmatrix} x & y & z \\ a & b & c \\ p & q & r \end{vmatrix}$$

$$-\begin{vmatrix} p & q & r \\ a & b & c \\ x & y & z \end{vmatrix}$$

$\boxed{1 \text{ Cycle} = 2 \text{ swaps}}$

Properties of Determinants

4. If a determinant has **any two rows (or columns) identical**, then its **value is zero**.

$$\begin{vmatrix} a & b & c \\ a & b & c \\ \boxed{x \quad y \quad z} \end{vmatrix} = 0$$

$1:2$

$$\begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ x & y & z \end{vmatrix} = 0$$

$$x(0) + y(0) + z(0) = 0$$

$$2\begin{vmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ x & y & z \end{vmatrix} = 0$$

Properties of Determinants

5. Scalar multiplication: Scalar will be multiplied in any one row (or column)

e.g. If $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ then kD $= \begin{vmatrix} \boxed{ka_1 \quad kb_1 \quad kc_1} \\ a_2 \quad b_2 \quad c_2 \\ a_3 \quad b_3 \quad c_3 \end{vmatrix}$

$$k\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix}$$

Properties of Determinants

6. $|kA| = k^n |A|$, where n is order of A.

$$|kA| = k^n |A|$$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$kA = \begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix}$$

$$|kA| = \begin{vmatrix} \boxed{ka \quad kb} \\ \boxed{kc \quad kd} \end{vmatrix}$$

$$= k^2 \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

Evaluate $\begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 36 \\ 17 & 3 & 6 \end{vmatrix} = 6 \begin{vmatrix} 17 & 3 & 6 \\ 1 & 3 & 36 \\ 17 & 3 & 6 \end{vmatrix} = 0$

$6 : 1$

Let A = [a_{ij}] and B = [b_{ij}] be two 3 x 3 real matrices such that $b_{ij} = (3)^{(i+j-2)} a_{ij}$, where i, j = 1, 2, 3. If the determinant of B is 81, then the determinant of A is :

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(Given)

$|B| = 81$

$|A| = ?$

$$b_{ij} = 3^{i+j-2} a_{ij}$$

A. 1/3

B. 3

C. 1/81

D. 1/9

$$B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}_{3\times 3}$$

$$|B| = \begin{vmatrix} a_{11} & 3a_{12} & 3^2 a_{13} \\ 3a_{21} & 3^2 a_{22} & 3^3 a_{23} \\ 3^2 a_{31} & 3^3 a_{32} & 3^4 a_{33} \end{vmatrix} = 81$$

$$3^3 \begin{vmatrix} a_{11} & \boxed{\cancel{3}\, a_{12}} & \boxed{\cancel{3^2}\, a_{13}} \\ a_{21} & \cancel{3}\, a_{22} & \cancel{3^2}\, a_{23} \\ a_{31} & \cancel{3}\, a_{32} & \cancel{3^2}\, a_{33} \end{vmatrix} = 81$$

$$3^3 \cdot 3^3 \, |A| = 81$$

$$\boxed{|A| = \frac{1}{9}}$$

Let A be a 3 x 3 matrix with det(A) = 4. Let R_i denote the i^{th} row of A. If a matrix B is obtained by performing the operation $\boxed{R_2 \to 2R_2 + 5R_3}$ on $\boxed{2A}$, then det(B) is equal to

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- **A.** 64
- **B.** 16
- **C.** 80
- **D.** 128

$\underbrace{|A| = 4}$ $\quad |B| = ?$

$$2A = \begin{bmatrix} 2a & 2b & 2c \\ 2x & 2y & 2z \\ 2p & 2q & 2r \end{bmatrix}$$

$2(R_2) + 5R_3$

$$|B| = \begin{vmatrix} 2a & 2b & 2c \\ 4x+10p & 4y+10q & 4z+10r \\ 2p & 2q & 2r \end{vmatrix}$$

$$|B| = \begin{vmatrix} 2a & 2b & 2c \\ 4x & 4y & 4z \\ 2p & 2q & 2r \end{vmatrix} + \cancel{\begin{vmatrix} 2a & 2b & 2c \\ 10p & 10q & 10r \\ 2p & 2q & 2r \end{vmatrix}} \rightarrow 0$$

$$= 16 \begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix}$$

$$= 16 \times |A|$$

$$= 16 \times 4$$

$$= 64$$

Let p and p + 2 be prime numbers and let

$$\Delta = \begin{vmatrix} p! & (p+1)! & (p+2)! \\ (p+1)! & (p+2)! & (p+3)! \\ (p+2)! & (p+3)! & (p+4)! \end{vmatrix}$$

$\alpha_{max} + \beta_{max}$

Then the sum of the maximum values of α and β, such that p^{α} and $(p + 2)^{\beta}$ divide Δ, is____. 4

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$$\Delta = p!\,(p+1)!\,(p+2)! \begin{vmatrix} 1 & p+1 & (p+2)(p+1) \\ 1 & p+2 & (p+3)(p+2) \\ 1 & p+3 & (p+4)(p+3) \end{vmatrix}$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_2$

$$\Delta = p!\,(p+1)!\,(p+2)! \begin{vmatrix} 1 & p+1 & (p+2)(p+1) \\ 0 & 1 & 2p+4 \\ 0 & 1 & 2p+6 \end{vmatrix}$$

$$p^{\alpha=3}\,(p+2)^{\beta=1}$$

$$\Delta = p!\,(p+1)!\,(p+2)! \times 2$$

$$= p\,(p-1)!\,(p+1)(p)(p-1)!\,(p+2)(p+1)\,p\,(p-1)! \times 2$$

Properties of Determinants

Note: The value of a skew symmetric determinant of odd order is zero.

$$\begin{vmatrix} 0 & a & -b \\ -a & 0 & c \\ b & -c & 0 \end{vmatrix}$$

★★★★★

$$\boxed{\text{Skew-Sym.} + \text{odd order} \quad |A| = 0}$$

① all diagonal elements must be "0"

② Mirror Image ⇒ Sign will be opp.

Properties of Determinants

7. Adding Determinants

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} x & y & z \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 + x & b_1 + y & c_1 + z \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

One at a time

$$\begin{vmatrix} - & - & 1 \\ - & - & 2 \\ - & - & 3 \end{vmatrix} + \begin{vmatrix} - & - & 4 \\ - & - & 5 \\ - & - & 6 \end{vmatrix}$$

Properties of Determinants

8. Splitting Determinants

$$\begin{vmatrix} a_1 + x & b_1 + y & c_1 + z \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} x & y & z \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

One at a time

Find $\begin{vmatrix} a & b & c \\ a+2x & b+2y & c+2z \\ x & y & z \end{vmatrix}$

$$= \begin{vmatrix} a & b & c \\ a & b & c \\ x & y & z \end{vmatrix} + 2\begin{vmatrix} a & b & c \\ x & y & z \\ x & y & z \end{vmatrix}$$

$$= 0 + 0$$

$$= \boxed{0}$$

Properties of Determinants

$4 = 1 \times 4$

9. $|AB| = |A|\,|B|$

$\underline{|AB| = |A| \cdot |B|}$

$|A| = \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}$ and $B = \begin{vmatrix} 1 & 0 \\ 1 & 4 \end{vmatrix}$ Then $AB = \begin{vmatrix} 2 & 4 \\ 3 & 8 \end{vmatrix}$

$\underline{|A| = 1}$

$\underline{|B| = 4}$

$AB = \begin{vmatrix} \boxed{1 \quad 1} \\ \boxed{1 \quad 2} \end{vmatrix} \cdot \begin{vmatrix} \boxed{\begin{matrix} 1 \\ 1 \end{matrix}} & \begin{matrix} 0 \\ 4 \end{matrix} \end{vmatrix}$

$\underline{|ABC| = |A|\,|B|\,|C|}$

$|AB| = \begin{vmatrix} 2 & 4 \\ 3 & 8 \end{vmatrix} = 16 - 12 = 4$

Properties of Determinants

10. If det(A) = 0, then A is known as singular matrix.

$$eg. \ \underbrace{\begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix}}_{\text{Singular Matrix}} = 0 \qquad \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$$

11) $\boxed{|A^n| = |A|^n}$

Let β be a real number. Consider the matrix

$$I + A = \begin{pmatrix} \beta & 0 & 1 \\ 2 & 1 & -2 \\ 3 & 1 & -2 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

If $\boxed{A^7 - (\beta - 1)A^6 - \beta A^5}$ is a singular matrix, then the value of 9β is ________.

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$$\left| A^7 - (\beta-1)A^6 - \beta A^5 \right| = 0$$

$$\left| A^5 \left(A^2 - (\beta-1)A - \beta I \right) \right| = 0$$

$$\left| A^5 \left(\underline{A^2 - \beta A} + \underline{A - \beta I} \right) \right| = 0$$

$$\left| A^5 (A+I)(A-\beta I) \right| = 0$$

$$|A| = \beta(0) + 1(-1)$$

$$|A| = -1$$

$$|A+I| = \begin{vmatrix} \cancel{\beta+1} & \cancel{0} & \cancel{1} \\ 2 & 2 & -2 \\ 3 & 1 & -1 \end{vmatrix}$$

$$= (\beta+1)(0) + 1(2-6)$$

$$= -4$$

$$\underbrace{|A|^5}_{\text{non-zero}} \cdot \underbrace{|A+I|}_{\text{non-zero}} \cdot |A-\beta I| = 0$$

$$A-\beta I = \begin{bmatrix} \beta & 0 & 1 \\ 2 & 1 & -2 \\ 3 & 1 & -2 \end{bmatrix} - \begin{bmatrix} \beta & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & \beta \end{bmatrix}$$

$$|A-\beta I| = \begin{vmatrix} 0 & 0 & 1 \\ 2 & 1-\beta & -2 \\ 3 & 1 & -2-\beta \end{vmatrix} = 0$$

$$2-3(1-\beta) = 0$$

$$\beta = \frac{1}{3}$$

$$\boxed{9\beta = 3}$$ Ans

Elementary Transformation

~~11.~~ The value of determinant remains same if we apply elementary transformation $R_1 \longrightarrow R_1 + kR_2 + mR_3$ or $C_1 \longrightarrow C_1 + kC_2 + mC_3$

Row transform

$R_1 \rightarrow 1R_1 + aR_2 + bR_3$

$a, b \in$ Constants.

$R_3 \rightarrow 1R_3 + \underbrace{2R_1 - 3R_2}$

Prove that $\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3$

Objective. (1) O O [redacted]

(2) 1 1 1

$3a+\cancel{2b}-2(a+\cancel{b})$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 3R_1$

$$\begin{vmatrix} a & a+b & a+b+c \\ 0 & a & 2a+b \\ 0 & 3a & 7a+3b \end{vmatrix} = a(7a^2+3\cancel{ab}-6a^2-3\cancel{ab})$$

$= a^3$

The maximum value of $f(x)=\begin{vmatrix} \sin^2 x & 1+\cos^2 x & \cos 2x \\ 1+\sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & \sin 2x \end{vmatrix}, x \in R$ is

16th Mar, 2021 (shift 2)

A. √7

B. 3/4

C. √5

D. 5

$C_1 \to C_1 + C_2$

$$f(x)=\begin{vmatrix} 2 & 1+\cos^2 x & \cos 2x \\ 2 & \cos^2 x & \cos 2x \\ 1 & \cos^2 x & \sin 2x \end{vmatrix}$$

$R_1 \to R_1 - R_2$

$$=\begin{vmatrix} 0 & 1 & 0 \\ 2 & \cos^2 x & \cos 2x \\ 1 & \cos^2 x & \sin 2x \end{vmatrix} = -1\left(2\sin 2x - \cos 2x\right)$$

$$\boxed{f(x) = 1\cos 2x - 2\sin 2x}$$

$$\sqrt{1^2+2^2} = \sqrt{5}$$

$$-\sqrt{a^2+b^2} \leq a\sin\theta + b\cos\theta \leq \sqrt{a^2+b^2}$$

The solutions of the equation $\begin{vmatrix} 1+\sin^2 x & \sin^2 x & \sin^2 x \\ \cos^2 x & 1+\cos^2 x & \cos^2 x \\ 4\sin 2x & 4\sin 2x & 1+4\sin 2x \end{vmatrix}=0$

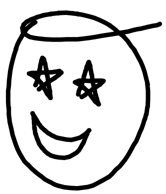
18th Mar, 2021 (shift 1)

A. π/12, π/6

B. π/6, 5π/6

C. 5π/12, 7π/12

D. 7π/12, 11π/12



$R_1 \to R_1 + R_2$

$$\begin{vmatrix} 2 & 2 & 1 \\ \cos^2 x & 1+\cos^2 x & \cos^2 x \\ 4\sin 2x & 4\sin 2x & 1+4\sin 2x \end{vmatrix} = 0$$

$C_1 \to C_1 - C_2$

$$\begin{vmatrix} 0 & 2 & 1 \\ 1 & 1+\cos^2 x & \cos^2 x \\ 0 & 4\sin 2x & 1+4\sin 2x \end{vmatrix} = 0$$

$$1\left(2+8\sin 2x-4\sin 2x\right)=0$$

$$2+4\sin 2x=0$$

$$\sin 2x=\frac{-1}{2}$$

$$2x=\frac{7\pi}{6},\ \frac{11\pi}{6}$$

$$\boxed{x=\frac{7\pi}{12},\ \frac{11\pi}{12}}$$

S-1 $\quad \sin(\alpha)=\frac{1}{2}$

$$\alpha=\frac{\pi}{6}$$

S-2

S	A
T	C

S-3

$\pi-\alpha$	α
$\pi+\alpha$	$2\pi-\alpha$

$$\pi+\frac{\pi}{6} \qquad 2\pi-\frac{\pi}{6}$$

$$\text{Let } f(x) = \begin{vmatrix} \sin^2 x & -2+\cos^2 x & \cos 2x \\ 2+\sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & 1+\cos 2x \end{vmatrix}, x \in [0, \pi]$$

Then the maximum value of f(x) is equal to

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HOMEWORK

The total number of distinct $x \in R$ for which $\begin{vmatrix} x & x^2 & 1+x^3 \\ 2x & 4x^2 & 1+8x^3 \\ 3x & 9x^2 & 1+27x^3 \end{vmatrix} = 10$ is

2 Sol^n

[JEE Adv 2016]

$$x^3 \begin{vmatrix} 1 & 1 & 1+x^3 \\ 2 & 4 & 1+8x^3 \\ 3 & 9 & 1+27x^3 \end{vmatrix} = 10$$

$$x^3 \left[\begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & 1 \\ 3 & 9 & 1 \end{vmatrix} + x^3 \begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & 8 \\ 3 & 9 & 27 \end{vmatrix} \right] = 10$$

$$x^3 (2 + 12x^3) = 10$$

$$2x^3 + 12x^6 = 10$$

$$x^3 = t$$

$$2t + 12t^2 = 10$$

$$6t^2 + t - 5 = 0$$

$$6t^2 + 6t - 5t - 5 = 0$$

$$(6t-5)(t+1) = 0$$

$$\boxed{t = \frac{5}{6}, -1}$$

$$x^3 = \frac{5}{6}, -1$$

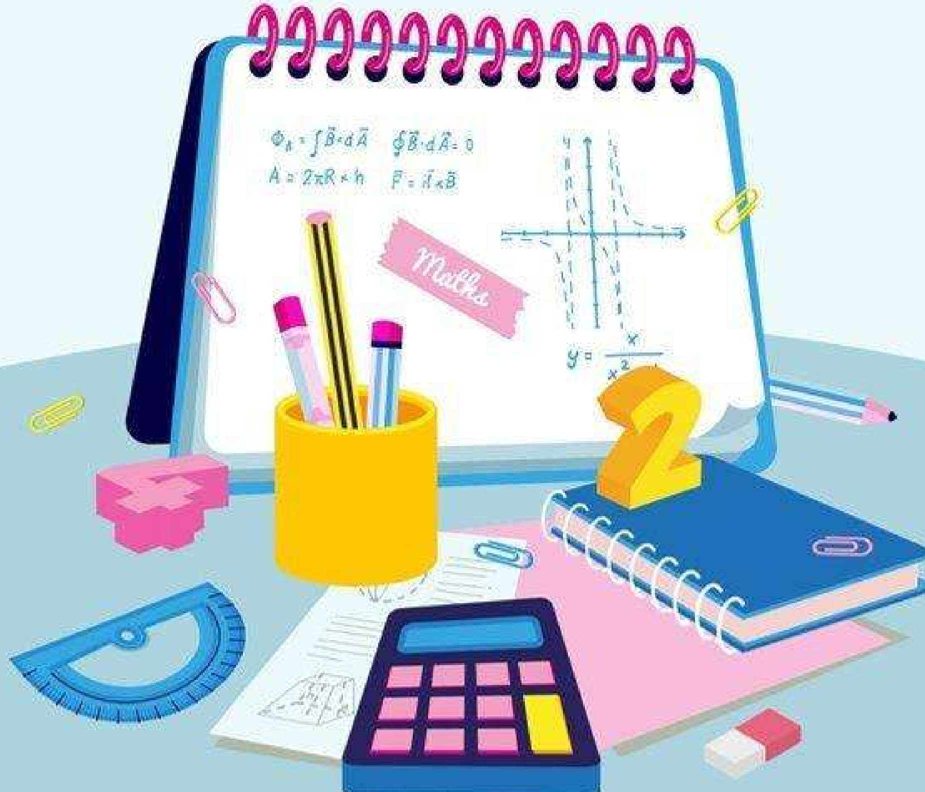
$$\boxed{x = \left(\frac{5}{6}\right)^{1/3}, -1}$$

$\boxed{x^3 = 1}$ — 1, ω, ω^2

★ $\omega^3 = 1$

★ $1 + \omega + \omega^2 = 0$

W.F.E $\boxed{x^2 + x + 1 = 0}$ — ω, ω^2



Advance Ques

(Determinant + Complex Numbers)

$$x^3 - 1^3 = 0$$

$$\underbrace{(x-1)}_{x=1}\underbrace{(x^2+x+1)}_{x=\omega,\ x=\omega^2} = 0$$

$x=1$ $x=\omega$

$x=\omega^2$

Let ω be the complex number $\boxed{\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}}$. Then the number of distinct complex numbers z satisfying $\underbrace{\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix}}$ $= 0$ is equal to

$\to$ 1

JEE Adv 2010

$\Rightarrow z^3 = 0$

$\Rightarrow \boxed{z = 0}$ only Solⁿ

Let α and β be the roots of the equation $x^2 + x + 1 = 0$. Then for

$\omega = \alpha$

$\omega^2 = \beta$

$y \neq 0$ in R, $\begin{vmatrix} y+1 & \alpha & \beta \\ \alpha & y+\beta & 1 \\ \beta & 1 & y+\alpha \end{vmatrix}$ is equal to:

[JEE M 2019]

A. $y(y^2 - 1)$

B. $y(y^2 - 3)$

C. y^3

D. $y^3 - 1$

$$\begin{vmatrix} y+1 & \omega & \omega^2 \\ \omega & y+\omega^2 & 1 \\ \omega^2 & 1 & y+\omega \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$

$$\Rightarrow y \begin{vmatrix} 1 & 1 & 1 \\ \omega & y+\omega^2 & 1 \\ \omega^2 & 1 & y+\omega \end{vmatrix}$$

$$\Rightarrow y\left\{1\left((y+\omega)(y+\omega^2)-\cancel{y}\right)-1(y\omega)+1(-\omega^2 y)\right\}$$

$$\Rightarrow y\left\{y^2+\cancel{\omega^2 y}+\cancel{\omega y}-\cancel{y\omega}-\cancel{\omega^2 y}\right\}$$

$$\Rightarrow y^3$$

Let ω be the complex cube root of unity with $\omega \neq 1$ and $P = [P_{ij}]$ be a n x n matrix with $p_{ij} = \omega^{i+j}$. Then $P^2 \neq O$, when n =

[JEE Adv 2013]

A. 57

B. 55

C. 58

D. 56

BCD

$$p_{ij} = \omega^{i+j} \qquad P^2 \neq O \qquad n = ?$$

for $n=2$

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}_{2\times2} = \begin{bmatrix} \omega^2 & 1 \\ 1 & \omega \end{bmatrix}$$

$$P^2 = \begin{bmatrix} \omega^2 & 1 \\ 1 & \omega \end{bmatrix} \cdot \begin{bmatrix} \omega^2 & 1 \\ 1 & \omega \end{bmatrix} = \begin{bmatrix} \omega+1 & — \\ — & — \end{bmatrix} \neq O$$

for $n=3$

$$p=\begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix}_{3\times 3} = \begin{bmatrix} \omega^2 & 1 & \omega \\ 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \end{bmatrix}$$

$$p^2=\begin{bmatrix} \boxed{\omega^2 \quad 1 \quad \omega} \\ 1 \quad \omega \quad \omega^2 \\ \omega \quad \omega^2 \quad 1 \end{bmatrix} \cdot \begin{bmatrix} \omega^2 & 1 & \omega \\ 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$n \to$ multi of 3 $\quad P^2 = 0$

$n \to$ not " " " $\quad P^2 \neq 0$

Let $\omega \neq 1$ be the cube root of unity and S be the set of all non-singular matrices of the form $\begin{bmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{bmatrix}$

where each of a, b and c is either ω or ω^2. Then the number of distinct matrices in the set S is

[JEE Adv 2011]

$a, b, c \in \{\omega, \omega^2\}$

A. 2

B. 6

C. 4

D. 8

$$\begin{vmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{vmatrix} \neq 0$$

$$1(1 - c\omega) - a(\omega - c\omega^2) + b(0) \neq 0$$

$$(1 - c\omega) - a\omega(1 - c\omega) \neq 0$$

$$(1 - a\omega)(1 - c\omega) \neq 0$$

$$(1 - a\omega)(1 - c\omega) \neq 0$$

$\boxed{a \neq \omega^2}$ $\boxed{c \neq \omega^2}$

	a	b	c
1	ω	ω	ω
2	ω	ω^2	ω

Ans :- 2

If $A = \begin{pmatrix} 0 & \sin\alpha \\ \sin\alpha & 0 \end{pmatrix}$ and $\det\left(A^2 - \frac{1}{2}I\right) = 0$, then a possible value of α is

17th Mar, 2021 (shift 1)

A. $\pi/2$

B. $\pi/3$

C. $\pi/4$

D. $\pi/6$

$$A^2 = \begin{bmatrix} 0 & \sin\alpha \\ \sin\alpha & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & \sin\alpha \\ \sin\alpha & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} \sin^2\alpha & 0 \\ 0 & \sin^2\alpha \end{bmatrix}$$

$$\left|A^2 - \frac{1}{2}I\right| = \begin{vmatrix} \sin^2\alpha - \frac{1}{2} & 0 \\ 0 & \sin^2\alpha - \frac{1}{2} \end{vmatrix} = 0$$

$$\left(\sin^2\alpha - \frac{1}{2}\right)^2 = 0 \Rightarrow \boxed{\sin^2\alpha = \frac{1}{2}}$$

Let A be a 2×2 matrix with $\det(A) = -1$ and $\det((A + I)(\operatorname{Adj}(A) + I)) = 4$. Then the sum of the diagonal elements of A can be :

(A) -1

(B) 2

(C) 1

(D) $-\sqrt{2}$

JEE Main 2022

$a+d = 2/-2$

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \boxed{ad - bc = -1}$

$|A+I| = \begin{bmatrix} a+1 & b \\ c & d+1 \end{bmatrix}$

$= (a+1)(d+1) - bc$

$= \cancel{ad} + a + d + \cancel{1} - \cancel{bc}$

$= a+d$

$|(A+I)(\operatorname{adj}A + I)| = 4$

$|A+I|\,|\operatorname{adj}A + I| = 4$

$(a+d)(a+d) = 4$

$(a+d)^2 = 4$

$a+d = 2$ OR -2

2×2

$(a+d)^2 = 4$

$$\text{adj}A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$|\text{adj}A + I| = \begin{vmatrix} d+1 & -b \\ -c & a+1 \end{vmatrix}$$

$$= (d+1)(a+1) - bc$$

$$= a + d + \cancel{1} + \cancel{ad} - \cancel{bc}$$

$$= a+d$$

$$A\,\text{adj}A = |A|\,I$$

$$= \underline{-I}$$

$$|\cancel{A\,\text{adj}A} + A + \text{adj}A + \cancel{I}| = 4$$

$$|A + \text{adj}A| = 4$$

$$\left|\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}\right| = 4$$

$$\begin{vmatrix} a+d & 0 \\ 0 & a+d \end{vmatrix} = 4$$

Domain

Let $|M|$ denote the determinant of a square matrix M. Let $g:\left[0, \frac{\pi}{2}\right] \to \mathbb{R}$ be the function defined by

AC

$$g(\theta) = \sqrt{f(\theta) - 1} + \sqrt{f\left(\frac{\pi}{2} - \theta\right) - 1}$$

where

$$f(\theta) = \frac{1}{2}\begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix} + \begin{vmatrix} \sin\pi & \cos\left(\theta + \frac{\pi}{4}\right) & \tan\left(\theta - \frac{\pi}{4}\right) \\ \sin\left(\theta - \frac{\pi}{4}\right) & -\cos\frac{\pi}{2} & \log_e\left(\frac{4}{\pi}\right) \\ \cot\left(\theta + \frac{\pi}{4}\right) & \log_e\left(\frac{\pi}{4}\right) & \boxed{\tan\pi} \end{vmatrix}.$$

$\to 0$

JEE Adv. 2022

$$\begin{vmatrix} 0 & k & - \\ -k & 0 & - \\ - & - & 0 \end{vmatrix}$$

Skew Sym + odd order

$|A| = 0$

Let $p(x)$ be a quadratic polynomial whose roots are the maximum and minimum values of the function $g(\theta)$, and $p(2) = 2 - \sqrt{2}$. Then, which of the following is/are TRUE ?

(A) $p\left(\frac{3+\sqrt{2}}{4}\right) < 0$ $\quad \frac{3+1.4}{4} = \boxed{1.1}$

(B) $p\left(\frac{1+3\sqrt{2}}{4}\right) > 0$ $\quad \frac{1+3(1.4)}{4} = \boxed{1.3}$

(C) $p\left(\frac{5\sqrt{2}-1}{4}\right) > 0$ $\quad \frac{5(1.4)-1}{4} = \boxed{1.5}$

(D) $p\left(\frac{5-\sqrt{2}}{4}\right) < 0$ $\quad \frac{5-1.4}{4} = \boxed{0.9}$

$\cos\left(\theta + \frac{\pi}{4}\right)$

$= \sin\left(\frac{\pi}{2} - \theta - \frac{\pi}{4}\right)$

$= \sin\left(\frac{\pi}{4} - \theta\right)$

$= -\sin\left(\theta - \frac{\pi}{4}\right)$

$$f(\theta) = \frac{1}{2}\begin{vmatrix} 2 & \sin\theta & 1 \\ 0 & 1 & \sin\theta \\ 0 & -\sin\theta & 1 \end{vmatrix}$$

$$= \frac{1}{\cancel{2}}\,\cancel{2}\,(1 + \sin^2\theta)$$

$$\boxed{f(\theta) = 1 + \sin^2\theta}$$

$$g(\theta) = \sqrt{f(\theta) - 1} + \sqrt{f\left(\tfrac{\pi}{2} - \theta\right) - 1}$$

$$= \sqrt{1 + \sin^2\theta - 1} + \sqrt{1 + \cos^2\theta - 1}$$

$$= |\sin\theta| + |\cos\theta| = \underline{\sin\theta + \cos\theta}$$

$g(0) = 1 \qquad g\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}$

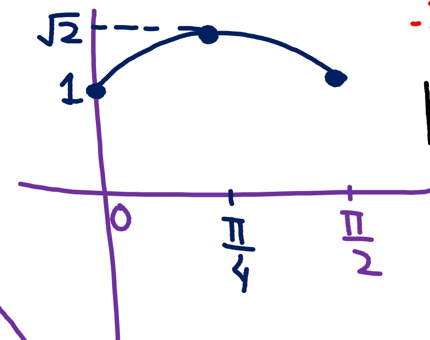
$g\left(\frac{\pi}{2}\right) = 1$

$$\boxed{g(\theta) = \sin\theta + \cos\theta}$$

max $= \sqrt{2}$

min $= 1$

GRAPH



$$p(x) = K(x - \sqrt{2})(x - 1)$$

$$\cancel{2 - \sqrt{2}} = K\,\cancel{(2 - \sqrt{2})}\,(2 - 1)$$

$\therefore$ $k = 1$

$$\underline{p(x) = (x - \sqrt{2})(x - 1)}$$

$$p(x) = (x - 1.414)(x - 1)$$

$$p(1.1) = (1.1 - 1.4)(1.1 - 1) = -$$

$$p(1.3) = (1.3 - 1.4)(1.3 - 1) = -$$

$$p(1.5) = (1.5 - 1.4)(1.5 - 1) = +$$

$$p(0.9) = (0.9 - 1.4)(0.9 - 1) = +$$

Special Determinants

Special Determinants

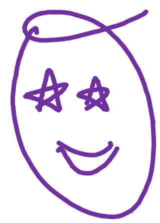
$$\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = (x-y)(y-z)(z-x)$$

$$\begin{vmatrix} 1 & x & x^3 \\ 1 & y & y^3 \\ 1 & z & z^3 \end{vmatrix} = (x-y)(y-z)(z-x)(x+y+z)$$

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = -\left(a^3 + b^3 + c^3 - 3abc\right)$$

Special Determinants

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = (x-y)(y-z)(z-x)$$



$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$\begin{vmatrix} 1 & x & x^2 \\ 0 & y-x & (y-x)(y+x) \\ 0 & z-x & (z-x)(z+x) \end{vmatrix} = (y-x)(z-x)\begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & y+x \\ 0 & 1 & z+x \end{vmatrix}$$

$$= (y-x)(z-x)(z-y)$$

$$= (x-y)(y-z)(z-x)$$

Prove that $\begin{vmatrix} b+c & c & b \\ c & c+a & a \\ b & a & a+b \end{vmatrix} = \underline{4abc}$

$\checkmark\ 1 \quad 1 \quad 1$

$\checkmark\ 0 \quad 0$

$R_1 \longrightarrow R_1 - (R_2 + R_3)$

$$\begin{vmatrix} \boxed{0 \quad -2a \quad -2a} & & \\ c & c+a & a \\ b & a & a+b \end{vmatrix} = -2a \begin{vmatrix} 0 & 1 & 1 \\ c & c+a & a \\ b & a & a+b \end{vmatrix}$$

$$= -2a \begin{vmatrix} 0 & 0 & 1 \\ c & c & a \\ b & -b & a+b \end{vmatrix}$$

$$= -2a(-2bc)$$

$$= \underline{4abc}$$

Show that $\begin{vmatrix} b+c & a+b & a \\ c+a & b+c & b \\ a+b & c+a & c \end{vmatrix} = a^3 + b^3 + c^3 - 3abc.$

H.W.

Show that $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3.$

H.W.

Show that $\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$

Concept

#chalaki

$$\begin{array}{c} \times x \\ \times y \\ \times z \end{array} \begin{vmatrix} x^2 & x^3 & yzx \\ y^2 & y^3 & zxy \\ z^2 & z^3 & xyz \end{vmatrix} = \begin{vmatrix} x^2 & x^3 & 1 \\ y^2 & y^3 & 1 \\ z^2 & z^3 & 1 \end{vmatrix}$$

xyz

$$\begin{vmatrix} x^2 & x^3 & 1 \\ y^2-x^2 & y^3-x^3 & 0 \\ z^2-x^2 & z^3-x^3 & 0 \end{vmatrix}$$

$$(y-x)(z-x)\begin{vmatrix} x^2 & x^3 & 1 \\ y+x & y^2+x^2+xy & 0 \\ z+x & z^2+x^2+xy & 0 \end{vmatrix}$$

Show that $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix} = 1 + a^2 + b^2 + c^2.$

#chalaki

$$\begin{array}{c c} a \rightarrow & \\ b \rightarrow & \\ c \rightarrow & \end{array} \begin{vmatrix} (a^2+1)\cancel{a} & a\cancel{b}^2 & a\cancel{c}^2 \\ \cancel{a}b^2 & (b^2+1)\cancel{b} & b\cancel{c}^2 \\ \cancel{a}c^2 & \cancel{b}c^2 & \cancel{c}(c^2+1) \end{vmatrix} = \begin{vmatrix} a^2+1 & a^2 & a^2 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix}$$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ a & b & c \end{array}$$

$R_1 \rightarrow R_1 + R_2 + R_3$

$$(1+a^2+b^2+c^2)\begin{vmatrix} 1 & 1 & 1 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix}$$

$$(1+a^2+b^2+c^2)\begin{vmatrix} 1 & 0 & 0 \\ b^2 & 1 & 0 \\ c^2 & 0 & 1 \end{vmatrix}$$

$$= \underline{1+a^2+b^2+c^2}$$

Which of the following values of α satisfy the equation

$$\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ (2+\alpha)^2 & (2+2\alpha)^2 & (2+3\alpha)^2 \\ (3+\alpha)^2 & (3+2\alpha)^2 & (3+3\alpha)^2 \end{vmatrix} = -648\alpha \ ?$$

(JEE Adv. 2015)

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_2$

A. -4

B. 9

C. -9

D. 4

$$\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ 2\alpha+3 & 4\alpha+3 & 6\alpha+3 \\ 2\alpha+5 & 4\alpha+5 & 6\alpha+5 \end{vmatrix} = -648\alpha$$

$R_3 \to R_3 - R_2$

$$2\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ 2\alpha+3 & 4\alpha+3 & 6\alpha+3 \\ 1 & 1 & 1 \end{vmatrix} = -648\alpha$$

$$2\begin{vmatrix} (1+\alpha)^2 & 3\alpha^2+2\alpha & 5\alpha^2+2\alpha \\ 2\alpha+3 & 2\alpha & 2\alpha \\ 1 & 0 & 0 \end{vmatrix} = -648\alpha$$

$$\cancel{2}\left(+\cancel{4}\,\alpha^3\right) = +\cancel{648}^{\,81}\alpha$$

$$\alpha^3 - 81\alpha = 0$$

$$\alpha(\alpha-9)(\alpha+9) = 0$$

$$\underline{\alpha = 0, 9, -9}$$

$$\boxed{a^2+b^2+c^2+2=0}$$

If $\boxed{a^2 + b^2 + c^2 = -2}$ and

$$f(x) = \begin{vmatrix} 1+a^2x & (1+b^2)x & (1+c^2)x \\ (1+a^2)x & 1+b^2x & (1+c^2)x \\ (1+a^2)x & (1+b^2)x & 1+c^2x \end{vmatrix}$$

Hint

then f(x) is a polynomial degree

(JEE Adv. 2005)

A. 1

B. 0

C. 3

D. 2

$C_1 \to C_1 + C_2 + C_3$

$$\begin{vmatrix} 1+(\cancel{a^2+b^2+c^2+2})x & - & - \\ 1+(\cancel{a^2+b^2+c^2+2})x & - & - \\ 1+(\cancel{a^2+b^2+c^2+2})x & - & - \end{vmatrix}$$

$$f(x) = \begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 1 & 1+b^2x & (1+c^2)x \\ 1 & (1+b^2)x & 1+c^2x \end{vmatrix}$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$f(x) = \begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 0 & 1-x & 0 \\ 0 & 0 & 1-x \end{vmatrix}$$

$$f(x) = (1-x)^2 = 1 - 2x + x^2$$

Find **values of**

$$\begin{vmatrix} \sin\theta & \cos\theta & \sin 2\theta \\ \sin\left(\theta+\frac{2\pi}{3}\right) & \cos\left(\theta+\frac{2\pi}{3}\right) & \sin\left(2\theta+\frac{4\pi}{3}\right) \\ \sin\left(\theta-\frac{2\pi}{3}\right) & \cos\left(\theta-\frac{2\pi}{3}\right) & \sin\left(2\theta-\frac{4\pi}{3}\right) \end{vmatrix}$$

(2000 - 3 marks)

$R_3 \longrightarrow R_3 + R_2$

$$\begin{vmatrix} \boxed{\sin\theta \quad \cos\theta \quad \sin 2\theta} \\ \sin\left(\theta+\frac{2\pi}{3}\right) \quad — \quad — \\ \boxed{\cancel{2}\sin\theta\left(\frac{-1}{\cancel{2}}\right) \quad \cancel{2}\cos\theta\left(\frac{-1}{\cancel{2}}\right) \quad \cancel{2}\sin 2\theta\left(\frac{-1}{\cancel{2}}\right)} \end{vmatrix} = 0$$

$$\cos\frac{2\pi}{3} = \cos\left(\pi - \frac{\pi}{3}\right) = -\cos\frac{\pi}{3} = \frac{-1}{2}$$

$$\cos\left(\frac{4\pi}{3}\right) = \cos\left(\pi + \frac{\pi}{3}\right) = \frac{-1}{2}$$

Method to Solve System of Linear Equations

System Linear Equations

1. Determinant Method (Cramer's Rule)
2. Matrix Method (Gauss- Jordan Method)

Cramer’s Rule

Cramer's Rule

Const → Right side

$$\begin{cases} a_1x + b_1y + c_1z = d_1 \\ a_2x + b_2y + c_2z = d_2 \\ a_3x + b_3y + c_3z = d_3 \end{cases}$$

System of L.E.

$$\boxed{x = \frac{D_1}{D}} \quad \boxed{y = \frac{D_2}{D}} \quad \boxed{z = \frac{D_3}{D}}$$

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$D = 0$

No $\quad \infty$

$D \neq 0$

$x =$
$y =$
$z =$

Unique Sol^n

$$D_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix} \quad D_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix} \quad D_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

Cramer's Rule

If $D \neq 0$
Unique solution

If $D = 0$

- If atleast one of $D_1, D_2, D_3 \neq 0$
  No solution
- If $D_1 = D_2 = D_3 = 0$
  Infinite Solution Or No Solution

planes are parallel

$x = \frac{D_1}{D}$ $\quad \frac{0}{0}$

∞

$D \neq 0$
Unique

$D = 0$

$D_1\ D_2\ D_3 \neq 0$
No Soln

$D_1 = D_2 = D_3 = 0$

~~No (parallel)~~ $\quad \infty$ (∞)

$D_1\ D_2\ D_3$ Koi ek $\neq 0$
No Soln

Important terms

i. **Consistent:** solution exists (unique or infinite solution)
ii. **Inconsistent:** solution does not exist (No solution)
iii. **Homogeneous equations:** constant terms are zero
iv. **Trivial solution:** all variables = zero i.e., x = 0, y = 0, z = 0.

$x = 0 \quad y = 0 \quad z = 0$

$\boxed{\text{all Var} = 0}$

Cramer's Rule (for Homogeneous Equations)

Homogeneous Linear Equations

$$\begin{bmatrix} a_1x + b_1y + c_1z = 0 \\ a_2x + b_2y + c_2z = 0 \\ a_3x + b_3y + c_3z = 0 \end{bmatrix}$$

Homo Linear Eqn.

$D_1 = D_2 = D_3 = 0$

$D_1 = 0 = D_2 = D_3$

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$D = 0$ OR $D \neq 0$

$$D_1 = \begin{vmatrix} 0 & - & - \\ 0 & - & - \\ 0 & - & - \end{vmatrix} = 0 \qquad D_2 = \begin{vmatrix} & 0 & \\ & 0 & \\ & 0 & \end{vmatrix} = 0 \qquad D_3 = \begin{vmatrix} & & 0 \\ & & 0 \\ & & 0 \end{vmatrix} = 0$$

Homogeneous Linear Equations

If D ≠ 0
Trivial solution

✓ Unique Solⁿ

$\boxed{D \neq 0}$

Homo
+
Non-Trivial

D = 0

If D = 0
Non - Trivial solution
or
Infinite Solution

✓ Non-Trivial
✓ ∞ Solⁿ
✓ Non-Zero

Gauss–Jordan Method

Matrix Method (Gauss-Jordan Method)

x + y + z = 6 ✓
x - y + z = 2 ✓
2x + y - z = 1

$x = \frac{D_1}{D}$ $\quad y = \frac{D_2}{D}$ $\quad z = \frac{D_3}{D}$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \\ 1 \end{bmatrix}$$

$$AX = B$$

$$\underbrace{A^{-1}A}X = A^{-1}B$$

$$\boxed{X = A^{-1}.B}$$

$$\boxed{X = \frac{(\text{adj } A).B}{|A|}}$$

$\frac{0}{0}$ $\quad \infty$

$|A| \neq 0$

\# Unique

$|A| = 0$

$(\text{adj}A).B = 0$ (infinite)

$(\text{adj}A).B \neq 0$ (No Sol^n)

Matrix Method (Gauss-Jordan Method)

- If $|A| \neq 0$
  Unique solution
- If $|A| = 0$
  - $(\text{adj } A).B \neq 0$
    No solution
  - $(\text{adj } A).B = 0$
    Infinite solution

Questions

For what values of p and q the system of equations has

$$2x + py + 6z = 8$$
$$x + 2y + qz = 5$$
$$x + y + 3z = 4$$

i. Unique solution ii. No solution iii. Infinite solutions

i) Unique Sol^n

$D \neq 0$

$(p-2)(q-3) \neq 0$

$\boxed{p \neq 2 \text{ and } q \neq 3}$

ii) No Sol^n $D = 0$ (D_1, D_2, D_3) Koi ek $\neq 0$

$\boxed{p \neq 2 \quad q = 3}$

iii) ∞ Sol^n $D = D_1 = D_2 = D_3 = 0$

$\boxed{p = 2;\ q \in R}$

$$D = \begin{vmatrix} 2 & p & 6 \\ 1 & 2 & q \\ 1 & 1 & 3 \end{vmatrix} = (p - 2)(q - 3)$$

$$D_1 = \begin{vmatrix} 8 & p & 6 \\ 5 & 2 & q \\ 4 & 1 & 3 \end{vmatrix} = (p - 2)(4q - 15)$$

$$D_2 = \begin{vmatrix} 2 & 8 & 6 \\ 1 & 5 & q \\ 1 & 4 & 3 \end{vmatrix} = 0$$

$$D_3 = \begin{vmatrix} 2 & P & 8 \\ 1 & 2 & 5 \\ 1 & 1 & 4 \end{vmatrix} = (p - 2)$$

The system of equations

$kx + y + z = 1$
$x + ky + z = k$
$x + y + zk = k^2$

has no solution, if k is equal to :

17th March 2021, Shift 1

$D = 0$

(D_1, D_2, D_3) atleast one $\neq 0$

A. ~~0~~

B. 1

C. ~~-1~~

D. -2 ✓

$$D = \begin{vmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{vmatrix} = (k+2)(k-1)^2$$

$D = 0 \quad K = -2 / 1$

✓ ✗

$$D_1 = \begin{vmatrix} 1 & 1 & 1 \\ k & k & 1 \\ k^2 & 1 & k \end{vmatrix} = -(k-1)^2(k+1)$$

$k = 1$

if $K = (-2)$ $\quad D = 0$ ✓ $\quad D_1 \neq 0$ ✓

Let the system of linear equations

$4x + \lambda y + 2z = 0$
$2x - y + z = 0$
$\mu x + 2y + 3k = 0, \lambda, \mu \in R$

has non-trivial solution, then which of the following is true?

Homo + Non-trivial

$D = 0$

$$D = \begin{vmatrix} 4 & \lambda & 2 \\ 2 & -1 & 1 \\ \mu & 2 & 3 \end{vmatrix} = (\lambda + 2)(\mu - 6)$$

18th March 2021, Shift 2

A. $\mu = 6, \lambda \in R$

B. $\lambda = 2, \mu \in R$

C. $\lambda = 3, \mu \in R$

D. $\mu = -6, \lambda \in R$

For the system of linear equations

$$\boxed{\begin{aligned} &x - 2y = 1 \\ &x - y + kz = -2 \\ &ky + 4z = 6,\ k \in R \end{aligned}}$$

24th February 2021, Shift 2

Consider the following statements:

1. The system has unique solution if $k \neq 2$, $k \neq -2$
2. The system has unique solution if $k = -2$
3. The system has unique solution if $k = 2$
4. The system has no solution if $k = 2$
5. The system has infinite number of solutions if $k \neq -2$

Which of the following statements are correct?

A. 2 & 5 only

B. 3 & 4 only

C. 1 & 4 only

D. 1 & 5 only

$k = -2 \longrightarrow \infty$ soln

$$D = \begin{vmatrix} 1 & -2 & 0 \\ 1 & -1 & k \\ 0 & k & 4 \end{vmatrix} = (2-k)(2+k)$$

$$D_1 = \begin{vmatrix} 1 & -2 & 0 \\ -2 & -1 & k \\ 6 & k & 4 \end{vmatrix} = -(k+2)(k+10)$$

$$D_2 = \begin{vmatrix} 1 & 1 & 0 \\ 1 & -2 & k \\ 0 & 6 & 4 \end{vmatrix} = -6(k+2)$$

$$D_3 = \begin{vmatrix} 1 & -2 & 1 \\ 1 & -1 & -2 \\ 0 & k & 6 \end{vmatrix} = 3(k+2)$$

if $k=2$

$D=0$

$D_3 \neq 0$

Unique Sol^n

$D \neq 0$

$\boxed{k \neq 2, -2}$

∞ sol^n

$\boxed{k = -2}$

If the system of equations

kx + y + 2z = 1
3x - y - 2z = 2
-2x - 2y - 4z = 3

has infinitely many solutions, then k is equal to -

25th February 2021, Shift 1

$$D = \begin{vmatrix} k & 1 & 2 \\ 3 & -1 & -2 \\ -2 & -2 & -4 \end{vmatrix} = 0$$

$$D_1 = \begin{vmatrix} 1 & 1 & 2 \\ 2 & -1 & -2 \\ 3 & -2 & -4 \end{vmatrix} = 0$$

$$D_2 = \begin{vmatrix} k & 1 & 2 \\ 3 & 2 & -2 \\ -2 & 3 & -4 \end{vmatrix} = -2(k - 21)$$

$$D_3 = \begin{vmatrix} k & 1 & 1 \\ 3 & -1 & 2 \\ -2 & -2 & 3 \end{vmatrix} = (k - 21)$$

$$\boxed{K = 21}$$

The following system of linear equations

$2x + 3y + 2z = 9$
$3x + 2y + 2z = 9$
$x - y + 4z = 8$

25th February 2021, Shift 2

A. ~~does not have any solution~~

B. has a unique solution

C. has a solution (α, β, γ) satisfying $\alpha + \beta^2 + \gamma^3 = 12$

$0 + 1^2 + 2^3 \neq 12$

D. ~~has infinitely many solutions~~

$$D = \begin{vmatrix} 2 & 3 & 2 \\ 3 & 2 & 2 \\ 1 & -1 & 4 \end{vmatrix} = -20$$

$$D_1 = \begin{vmatrix} 9 & 3 & 2 \\ 9 & 2 & 2 \\ 8 & -1 & 4 \end{vmatrix} = 0$$

$$D_2 = \begin{vmatrix} 2 & 9 & 2 \\ 3 & 9 & 2 \\ 1 & 8 & 4 \end{vmatrix} = -20$$

$$D_3 = \begin{vmatrix} 2 & 3 & 9 \\ 3 & 2 & 9 \\ 1 & -1 & 8 \end{vmatrix} = -40$$

$D \neq 0$

unique

$x = \frac{D_1}{D} = 0$

$y = \frac{D_2}{D} = 1$

$z = \frac{D_3}{D} = 2$

$(0, 1, 2)$

Soln

If the following system of linear equations

$2x + y + z = 5$

$x - y + z = 3$

$x + y + az = b$

has no solution, then :

31 Aug 2021, Shift 1

$a = \frac{1}{3}$

$\boxed{D = 0}$ $D_1, D_2, D_3 \neq 0$

$$D = \begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & a \end{vmatrix} = 1 - 3a$$

$a = \frac{1}{3}$ $b \neq \frac{7}{3}$

$\boxed{D = 0 \quad D_3 \neq 0}$

No Solⁿ

~~A.~~ $a = -\frac{1}{3}$, $b \neq \frac{7}{3}$

~~B.~~ $a \neq \frac{1}{3}, b = \frac{7}{3}$

~~C.~~ $a \neq -\frac{1}{3}$, $b = \frac{7}{3}$

D. $a = \frac{1}{3}$, $b \neq \frac{7}{3}$

$$D_3 = \begin{vmatrix} 2 & 1 & 5 \\ 1 & -1 & 3 \\ 1 & 1 & b \end{vmatrix} = 7 - 3b$$

The value of a and b, for which the system of equations

$2x + 3y + 6z = 8$

$x + 2y + az = 5$

$3x + 5y + 9z = b$

has no solution, are :

25 July 2021, Shift 1

$D = 0$

A. $a = 3, b \neq 13$

B. $a \neq 3, b \neq 13$

C. $a \neq 3, b = 3$

D. $a = 3, b = 13$

$$D = \begin{vmatrix} 2 & 3 & 6 \\ 1 & 2 & a \\ 3 & 5 & 9 \end{vmatrix} = 3 - a = 0$$

$$D_3 = \begin{vmatrix} 2 & 3 & 8 \\ 1 & 2 & 5 \\ 3 & 5 & b \end{vmatrix} = b - 13 \neq 0$$

The value of λ and μ such that the system of equations $x + y + z = 6$, $3x + 5y + 5z = 26$, $x + 2y + \lambda z = \mu$ has no solutions, are :

22 July 2021, shift 1

A. $\lambda = 3, \mu = 5$

B. $\lambda = 3, \mu \neq 10$

C. $\lambda \neq 2, \mu = 10$

D. $\lambda = 2, \mu \neq 10$

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 3 & 5 & 5 \\ 1 & 2 & \lambda \end{vmatrix} = 2\lambda - 4$$

$D = 0$

$\boxed{\lambda = 2}$

The value of k ∈ R, for which the following system of linear equations

$3x - y + 4z = 3$

$x + 2y - 3z = -2$

$6x + 5y + kz = -3$

Has infinitely many solutions is :

20 July 2021, shift 2

A. 3

B. -5

C. 5

D. -3

$D = D_1 = D_2 = D_3 = 0$

$$D = \begin{vmatrix} 3 & -1 & 4 \\ 1 & 2 & -3 \\ 6 & 5 & k \end{vmatrix} = 7k + 35 = 0$$

$K = -5$

Cramer’s Rule
(JEE Main 2022)

If the system of equations

$x + y + z = 6$

$2x + 5y + \alpha z = \beta$

$x + 2y + 3z = 14$

Has infinitely many solutions, then $\alpha + \beta$ is equal to :

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A. 8

B. 36

C. 44

D. 48

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 5 & \alpha \\ 1 & 2 & 3 \end{vmatrix} = 8 - \alpha = 0$$

$$D_3 = \begin{vmatrix} 1 & 1 & 6 \\ 2 & 5 & \beta \\ 1 & 2 & 14 \end{vmatrix} = 36 - \beta = 0$$

$\alpha = 8$

$+ \beta = 36$

$\alpha + \beta = 44$

The number of values of α for which the system of equations :

$x + y + z = \alpha$
$\alpha x + 2\alpha y + 3z = -1$
$x + 3\alpha y + 5z = 4$

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Is inconsistent, is

No soln

D = 0

A. 0

B. 1

C. 2

D. 3

$$\begin{vmatrix} 1 & 1 & 1 \\ \alpha & 2\alpha & 3 \\ 1 & 3\alpha & 5 \end{vmatrix} = 3(\alpha - 1)^2 = 0$$

$$\begin{vmatrix} \alpha & 1 & 1 \\ -1 & 2\alpha & 3 \\ 4 & 3\alpha & 5 \end{vmatrix} = \alpha^2 - 11\alpha + 7 \neq 0$$

Let the system of linear equations

$x + y + \alpha z = 2$
$3x + y + z = 4$
$x + 2z = 1$

have a unique solution (x^*, y^*, z^*). If (α, x^*), (y^*, α) and $(x^*, -y^*)$ are collinear points, then the sum of absolute values of all possible values of α is:

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A. 4

B. 3

C. 2

D. 1

$|1| + |-1|$

$\Rightarrow 2$

$$\underbrace{(\alpha, 1),\ (1, \alpha) \text{ and } (1, -1)}_{\text{Collinear}}$$

$$\begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & -1 & 1 \end{vmatrix} = 0$$

$\alpha(\alpha+1) - 1(0) + 1(-1-\alpha) = 0$

$\alpha^2 + \cancel{\alpha} - 1 - \cancel{\alpha} = 0$

$\boxed{\alpha = \pm 1}$

$$\Delta = \begin{vmatrix} 1 & 1 & \alpha \\ 3 & 1 & 1 \\ 1 & 0 & 2 \end{vmatrix} = -(\alpha + 3)$$

$$\Delta_1 = \begin{vmatrix} 2 & 1 & \alpha \\ 4 & 1 & 1 \\ 1 & 0 & 2 \end{vmatrix} = -(3 + \alpha)$$

$$\Delta_2 = \begin{vmatrix} 1 & 2 & \alpha \\ 3 & 4 & 1 \\ 1 & 1 & 2 \end{vmatrix} = -(\alpha + 3)$$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 2 \\ 3 & 1 & 4 \\ 1 & 0 & 1 \end{vmatrix} = 0$$

$x^* = 1 \quad y^* = 1 \quad z^* = 0$

The ordered pair (a, b), for which the system of linear equations

$3x - 2y + z = b$

$5x - 8y + 9z = 3$

$2x + y + az = -1$

Has no solution, is :

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A. $\left(3, \frac{1}{3}\right)$

B. $\left(-3, \frac{1}{3}\right)$

C. $\left(-3, -\frac{1}{3}\right)$ ✓

D. $\left(3, -\frac{1}{3}\right)$

$$D = \begin{vmatrix} 3 & -2 & 1 \\ 5 & -8 & 9 \\ 2 & 1 & a \end{vmatrix} = -14(a+3) = 0$$

$a = -3$

$$D_3 = \begin{vmatrix} 3 & -2 & b \\ 5 & -8 & 3 \\ 2 & 1 & -1 \end{vmatrix} = 7(3b-1) \neq 0$$

$b \neq \frac{1}{3}$

If the system of equations $\alpha x + y + z = 5$, $x + 2y + 3z = 4$, $x + 3y + 5z = \beta$, has infinitely many solutions, then the ordered pair (α, β) is equal to:

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A. (1, -3)

B. (-1, 3)

C. (1, 3)

D. (-1, -3)

$$D = \begin{vmatrix} \alpha & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{vmatrix} = \alpha - 1 = 0$$

$$D_1 = \begin{vmatrix} 5 & 1 & 1 \\ 4 & 2 & 3 \\ \beta & 3 & 5 \end{vmatrix} = \beta - 3 = 0$$

$\underline{D = D_1 = D_2 = D_3 = 0}$

Let the system of linear equations x + 2y + z = 2, αx + 3y - z = α, -αx + y + 2z = -α be inconsistent. Then α is equal to :

↳ No Solⁿ

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A. 5/2

B. -5/2

C. 7/2

D. -7/2

$$D = \begin{vmatrix} 1 & 2 & 1 \\ \alpha & 3 & -1 \\ -\alpha & 1 & 2 \end{vmatrix} = 7 + 2\alpha = 0$$

$$\alpha = -7/2$$

$$D_1 = \begin{vmatrix} 2 & 2 & 1 \\ \alpha & 3 & -1 \\ -\alpha & 1 & 2 \end{vmatrix} = 14 + 2\alpha \neq 0$$

If the system of linear equations

$2x + y - z = 7$

$x - 3y + 2z = 1$

$x + 4y + \delta z = k$, where $\delta, k \in R$

has infinitely many solutions, then $\delta + k$ is equal to :

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A. -3

B. 3

C. 6

D. 9

$$D = \begin{vmatrix} 2 & 1 & -1 \\ 1 & -3 & 2 \\ 1 & 4 & \delta \end{vmatrix} = -7(\delta + 3) = 0$$

$$D_3 = \begin{vmatrix} 2 & 1 & 7 \\ 1 & -3 & 1 \\ 1 & 4 & k \end{vmatrix} = -7(k - 6) = 0$$

$\delta = -3$

$k = 6$

$\delta + k = 3$

Let p, q, r be nonzero real numbers that are, respectively, the 10^{th}, 100^{th} and 1000^{th} terms of a harmonic progression. Consider the system of linear equations

$\frac{1}{p}, \frac{1}{q}, \frac{1}{r} \to$ AP.

$$\begin{gathered} x+y+z=1 \\ 10x+100y+1000z=0 \\ qr\,x+pr\,y+pq\,z=0 \end{gathered}$$

pqr

$$\frac{1}{p} = A+9D$$
$$\frac{1}{q} = A+99D$$
$$\frac{1}{r} = A+999D$$

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$$\frac{p}{q} = \frac{A+99D}{A+9D}$$

List-I	**List-II**
(I) If $\frac{q}{r}=10$, then the system of linear equations has	(P) $x=0,\ y=\frac{10}{9},\ z=-\frac{1}{9}$ as a solution
(II) If $\frac{p}{r}\neq 100$, then the system of linear equations has	(Q) $x=\frac{10}{9},\ y=-\frac{1}{9},\ z=0$ as a solution
(III) If $\frac{p}{q}\neq 10$, then the system of linear equations has	(R) infinitely many solutions
(IV) If $\frac{p}{q}=10$, then the system of linear equations has	(S) no solution
	(T) at least one solution

The correct option is:

(A) (I) → (T); (II) → (R); (III) → (S); (IV) → (T)

(B) (I) → (Q); (II) → (S); (III) → (S); (IV) → (R)

(C) (I) → (Q); (II) → (R); (III) → (P); (IV) → (R)

(D) (I) → (T); (II) → (S); (III) → (P); (IV) → (T)

$$\left.\begin{aligned} x+y+z&=1\\ 10x+100y+1000z&=0\\ \frac{x}{p}+\frac{y}{q}+\frac{z}{r}&=0 \end{aligned}\right\}$$

$$D=\begin{vmatrix} 1 & 1 & 1 \\ 10 & 100 & 1000 \\ A+9D & A+99D & A+999D \end{vmatrix}=\begin{vmatrix} 1 & 0 & 0 \\ 10 & 90 & 900 \\ A+9D & 90D & 900D \end{vmatrix}=0$$

$$D_1=\begin{vmatrix} 1 & 1 & 1 \\ 0 & 100 & 1000 \\ 0 & A+99D & A+999D \end{vmatrix}=900(D-A)$$

$$D_2=\begin{vmatrix} 1 & 1 & 1 \\ 10 & 0 & 1000 \\ A+9D & 0 & A+999D \end{vmatrix}=990(D-A)$$

$$D = 0$$
$$D_1 = 900(D-A)$$
$$D_2 = 990(D-A)$$
$$D_3 = 90(D-A)$$

∞ Sol^n

$$\underline{\underline{D = A}}$$

No Sol^n

$$\underline{\underline{D \neq A}}$$

III

$$\frac{A+99D}{A+9D} \neq 10$$
$$A + 99D \neq 10A + 90D$$
$$9D \neq 9A$$
$$\underline{D \neq A}$$

IV

$$\underline{D = A}$$

Question Stem

Let α, β and γ be real numbers such that the system of linear equations

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$$x + 2y + 3z = \alpha$$
$$4x + 5y + 6z = \beta$$
$$7x + 8y + 9z = \gamma - 1$$

$D = 0$ ✓

$D_1 = D_2 = D_3 = 0$

is consistent. Let $|M|$ represent the determinant of the matrix

(/ ∞ Sol^n)

$$M = \begin{bmatrix} \alpha & 2 & \gamma \\ \beta & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

3D

Let P be the plane containing all those (α, β, γ) for which the above system of linear equations is consistent, and D be the **square** of the distance of the point $(0, 1, 0)$ from the plane P.

$|M| = \alpha(1) - \beta(2) - 1(-\gamma)$

$= \alpha - 2\beta + \gamma = 1$

Q.7 The value of |M| is 1.

Q.8 The value of D is $\boxed{1.5}$.

$$(0,1,0) \rightarrow \boxed{x-2y+z=1}$$

$$D_1 = \begin{vmatrix} \alpha & 2 & 3 \\ \beta & 5 & 6 \\ \gamma-1 & 8 & 9 \end{vmatrix} = 0$$

$$\alpha(45-48) - \beta(18-24) + (\gamma-1)(12-15) = 0$$

$$-3\alpha + 6\beta - 3(\gamma-1) = 0$$

$$\alpha - 2\beta + \gamma - 1 = 0$$

$$\underline{\alpha - 2\beta + \gamma = 1}$$

$$\left|\frac{0-2+0-1}{\sqrt{1^2+2^2+1^2}}\right|^2$$

$$\left(\frac{3}{\sqrt{6}}\right)^2$$

$$\frac{9}{6} = 1.5$$

How to Differentiate a Determinant?

Differentiation of Determinants

$$F(x) = \begin{vmatrix} f_1(x) & f_2(x) & f_3(x) \\ g_1(x) & g_2(x) & g_3(x) \\ h_1(x) & h_2(x) & h_3(x) \end{vmatrix}$$

$$F'(x) = \begin{vmatrix} f_1'(x) & f_2'(x) & f_3'(x) \\ g_1(x) & g_2(x) & g_3(x) \\ h_1(x) & h_2(x) & h_3(x) \end{vmatrix} + \begin{vmatrix} - & - & - \\ g_1'(x) & g_2'(x) & g_3'(x) \\ - & - & - \end{vmatrix} + \begin{vmatrix} - & - & - \\ - & - & - \\ h_1'(x) & h_2'(x) & h_3'(x) \end{vmatrix}$$

If $f(x) = \begin{vmatrix} x+a^2 & ab & ac \\ ab & x+b^2 & bc \\ ac & bc & x+c^2 \end{vmatrix}$ find $f'(x)$

~~f'(x)~~

$$f'(x) = \begin{vmatrix} 1 & 0 & 0 \\ ab & x+b^2 & bc \\ ac & bc & x+c^2 \end{vmatrix} + \begin{vmatrix} x+a^2 & ab & ac \\ 0 & 1 & 0 \\ ac & bc & x+c^2 \end{vmatrix} + \begin{vmatrix} x+a^2 & ab & ac \\ ab & x+b^2 & bc \\ 0 & 0 & 1 \end{vmatrix}$$

If $\cancel{a}x^4 + \cancel{b}x^3 + \cancel{c}x^2 + \cancel{d}x + e = \begin{vmatrix} 2x & x-1 & x+1 \\ x+1 & x^2-x & x-1 \\ x-1 & x+1 & 3x \end{vmatrix}$

then the **value of e,** is

A. 0

B. -2

C. 3

D. 2

$x = 0$ both side

$$e = \begin{vmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{vmatrix}$$

If $ax^4 + bx^3 + cx^2 + dx + e = \begin{vmatrix} 2x & x-1 & x+1 \\ x+1 & x^2-x & x-1 \\ x-1 & x+1 & 3x \end{vmatrix}$

then the **value of d,** is

$$4a\cancel{x^3} + 3b\cancel{x^2} + 2c\cancel{x} + d = \begin{vmatrix} 2 & 1 & 1 \\ x+1 & x^2-x & x-1 \\ x-1 & x+1 & 3x \end{vmatrix} + \begin{vmatrix} - & - & - \\ 1 & 2x-1 & 1 \\ - & - & - \end{vmatrix} + \begin{vmatrix} - & - & - \\ - & - & - \\ 1 & 1 & 3 \end{vmatrix}$$

$$x = 0$$

$$d = \underbrace{\begin{vmatrix} \quad \end{vmatrix} + \begin{vmatrix} \quad \end{vmatrix} + \begin{vmatrix} \quad \end{vmatrix}}$$

Cayley – Hamilton Theorem

Cayley - Hamilton Theorem

Every square matrix satisfies a specific polynomial equation known as characteristic equation.

$$P(\lambda) = |A - \lambda I|$$

$$P(A) = 0$$

$|A - \lambda I| = 0$

$A \rightarrow$ any Sq. matrix $\Longrightarrow$ characteristic Eqn

Every Sq. matrix will satisfy in its Char. Eqn.

Using **Cayley Hamilton theorem**, find A^{-1} and A^4

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & 2 & 0 \\ 2 & -1-\lambda & 0 \\ 0 & 0 & 1-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)\{(-1-\lambda)(1-\lambda)\} - 2(2-2\lambda) = 0$$

$$(1-\lambda)(-1-\cancel{\lambda}+\cancel{\lambda}+\lambda^2) - 4 + 4\lambda = 0$$

$$(1-\lambda)(-1+\lambda^2) - 4 + 4\lambda = 0$$

$$-1 + \lambda^2 + \underline{\lambda} - \lambda^3 - 4 + \underline{4\lambda} = 0$$

$$\underline{\lambda^3 - \lambda^2 - 5\lambda + 5 = 0}$$

char. Eqn.

CH :- $A^3 - A^2 - 5A + 5I = 0$

$A^5 \quad A^{10} \quad A^{20}$ — ?

CH

Kab?

$A^{-1} = f(A)$

$$A^3 = A^2 + 5A - 5I$$

$$\begin{aligned} A^4 &= A \cdot A^3 \\ &= A(A^2 + 5A - 5I) \\ &= A^3 + 5A^2 - 5A \\ &= A^2 + \cancel{5A} - 5I + 5A^2 - \cancel{5A} \end{aligned}$$

$$\boxed{A^4 = 6A^2 - 5I}$$

$\underline{A\,A}\,\underline{A\,A^{-1}}$

$(A^3 - A^2 - 5A + 5I = 0)\,A^{-1}$

$A^2 - A - 5I + 5A^{-1} = 0$

$$\boxed{A^{-1} = \frac{-A^2 + A + 5I}{5}}$$

Let $A = \begin{pmatrix} 4 & -2 \\ \alpha & \beta \end{pmatrix}$

$|A| = 4\beta + 2\alpha = 18$

If $A^2 + \gamma A + 18I = O$, then det (A) is equal to

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A. -18

B. 18

C. -50

D. 50

$$\begin{vmatrix} 4-\lambda & -2 \\ \alpha & \beta-\lambda \end{vmatrix} = 0$$

$$(4-\lambda)(\beta-\lambda) + 2\alpha = 0$$

$$4\beta - 4\lambda - \lambda\beta + \lambda^2 + 2\alpha = 0$$

$$\lambda^2 + (-4-\beta)\lambda + (2\alpha + 4\beta) = 0$$

$-4-\beta = \gamma$

$2\alpha + 4\beta = 18$

CH: $A^2 + (-4-\beta)A + (2\alpha+4\beta)I = 0$

$A^2 + \gamma A + 18I = 0$

Let $A = \begin{pmatrix} 1 & 2 \\ -2 & -5 \end{pmatrix}$. Let $\alpha, \beta \in \mathbb{R}$ be such that $\alpha A^2 + \beta A = 2I$. Then $\alpha + \beta$ is equal to -

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A. -10

B. -6

C. 6

D. 10

$$\begin{vmatrix} 1-\lambda & 2 \\ -2 & -5-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)(-5-\lambda) + 4 = 0$$

$$\lambda^2 + 4\lambda - 1 = 0$$

$$2(A^2 + 4A - I = 0)$$

$$2A^2 + 8A = 2I$$

$$\alpha A^2 + \beta A = 2I$$

$$\alpha + \beta = 10$$

$$\left(A^2 + 4A = I\right) A^{-1}$$

$$A + 4I = A^{-1}$$

Let $A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$ If $A^{-1} = \alpha I + \beta A$, $\alpha, \beta \in R$, I is a 2 x 2 identity matrix, then $4(\alpha - \beta)$ is equal to :

A. 5

B. 8/3

C. 2

D. 4

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