CHAPTER

9 Circular Motion

Circular Motion

Definition of Circular Motion

When a particle moves in a plane such that its distance from a fixed (or moving) point remains constant then its motion is called as circular motion with respect to that fixed point. That fixed point is called centre and the distance is called radius of circular path.

Radius Vector

The vector joining the centre of the circle and the particle performing circular motion is called radius vector. It has constant magnitude and variable direction. It is directed outwards.

Frequency (n or f)

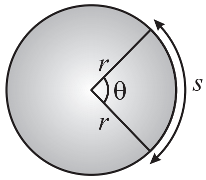
Number of revolutions described by particle per sec. Its unit is revolutions per second (r.p.s.) or revolutions per minute (r.p.m.).

Time Period (T)

It is time taken by particle to complete one revolution.

$$T = \frac{1}{n}$$

- Angle $\theta = \frac{\text{arc length}}{\text{radius}} = \frac{s}{r}$ (Unit $\to$ radian)
- Average angular velocity $\omega = \frac{\Delta\theta}{\Delta t}$ (unit $\to$ rad/sec)
- Instantaneous angular velocity $\omega = \frac{d\theta}{dt}$ (a vector) unit $\to$ rad/sec



- For uniform angular velocity $\omega = \frac{2\pi}{T} = 2\pi f$ or $2\pi n$
- Angular displacement $\theta = \omega t$
- Relation between ω (uniform) and v, $\omega = \frac{v}{r}$
- In vector from velocity $\vec{v} = \vec{\omega} \times \vec{r}$
- Acceleration $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(\vec{\omega} \times \vec{r}) = \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \frac{d\vec{r}}{dt}$

$$= \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v} = \vec{a}_t + \vec{a}_C$$

- Tangential acceleration: $a_t = \frac{dv}{dt} = \alpha r$
- Centripetal acceleration: $a_C = \omega v = \frac{v^2}{r}$

$$= \omega^2 r$$

- Magnitude of net acceleration:

$$a = \sqrt{a_C^2 + a_t^2} = \sqrt{\left(\frac{v^2}{r}\right)^2 + \left(\frac{dv}{dt}\right)^2}$$

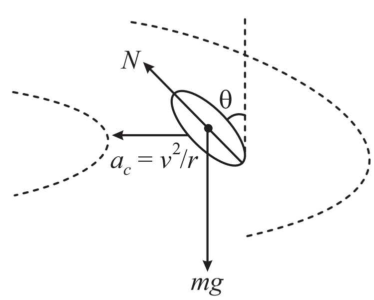
Maximum and Minimum Speed in Circular Motion

- On unbanked road: $v_{\max} = \sqrt{\mu_s Rg}$
- On banked road: $v_{\max} = \sqrt{\left(\frac{\mu_s + \tan\theta}{1 - \mu_s \tan\theta}\right) Rg}$

$$v_{\min} = \sqrt{\frac{(\tan\theta - \mu_S) Rg}{1 + \mu_S \tan\theta}} \qquad v_{\min} \le v_{car} \le v_{\max}$$

where ϕ = angle of friction = $\tan^{-1}\mu_s$; θ = angle of banking.

- Bending of cyclist: $\tan\theta = \frac{v^2}{rg}$



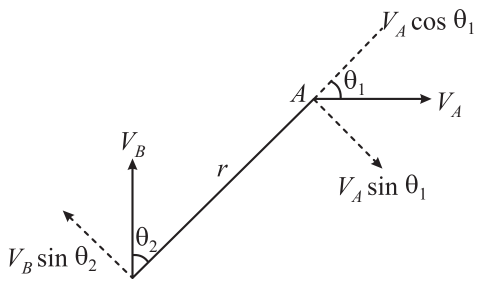
Key Tips

- Average angular velocity is a scalar physical quantity whereas instantaneous angular velocity is a vector physical quantity.
- Small Angular displacement $d\theta$ is a vector quantity, but large angular displacement $\vec{\theta}$ is scalar quantity.

$$d\vec{\theta}_1 + d\vec{\theta}_2 = d\vec{\theta}_2 + d\vec{\theta}_1 \quad \text{But } \vec{\theta}_1 + \vec{\theta}_2 \neq \vec{\theta}_2 + \vec{\theta}_1$$

Relative Angular Velocity

Relative angular velocity of a particle 'A' w.r.t. other moving particle B is the angular velocity of the position vector of A w.r.t. B.



That means it is the rate at which position vector of 'A' w.r.t. B rotates at that instant

$$\omega_{AB} = \frac{(v_{AB})_\perp}{r_{AB}}$$

$$= \frac{\text{Relative velocity of } A \text{ w.r.t. } B \text{ perpendicular to line } AB}{\text{separation between } A \text{ and } B}$$

here $(v_{AB})_\perp = v_A \sin\theta_1 + v_B \sin\theta_2$

$$\therefore \qquad \omega_{AB} = \frac{v_A \sin\theta_1 + v_B \sin\theta_2}{r}$$