KINETIC THEORY OF GASES

01 — DEGREES OF FREEDOM

- For monoatomic gas, $f = 3$
- For diatomic gas,
  - (a) at room temperature, $f = 5$
  - (b) at high temperature, $f = 7$
- For triatomic gas,
  - (a) Linear $f = 5$
  - (b) Non-linear $f = 6$
- For each vibrational mode, $f = 2$

Q1 Ideal gas is composed of polyatomic molecule that has 4 vibrational modes. Total degrees of freedom is
a) 12   b) 14   c) 8   d) 6

02 — SPECIFIC HEAT CAPACITY

a) $C_P - C_V = R$

b) $C_P - C_V = \dfrac{R}{M}$ (specific heat per unit mass)

c) $C_V = \dfrac{R}{\gamma-1} = \dfrac{f}{2}R$

d) $C_P = \dfrac{\gamma R}{\gamma-1} = \left(1 + \dfrac{f}{2}\right)R$

e) $\gamma = \dfrac{C_P}{C_V} = 1 + \dfrac{2}{f}$

Mono- $\gamma = \dfrac{5}{3}$

Dia- $\gamma = \dfrac{7}{5}$

Tri- $\gamma = \dfrac{4}{3}$

Q2 If C_p and C_v denote the specific heats of unit mass of nitrogen at constant pressure and volume respectively, then
a) $C_p - C_v = \dfrac{R}{28}$ b) $C_p - C_v = \dfrac{R}{14}$ c) $C_p - C_v = \dfrac{R}{7}$ d) $C_p - C_v = R$

03 — MIXING OF GASES

$$C_{Vmix} = \frac{n_1 c_{v_1} + n_2 c_{v_2} + \ldots}{n_1 + n_2 + \ldots}$$

$$C_{Pmix} = \frac{n_1 c_{P_1} + n_2 c_{P_2} + \ldots}{n_1 + n_2 + \ldots}$$

$$\gamma_{mix} = \frac{C_{P\,mix}}{C_{V\,mix}}$$

Q3 Consider a mixture of n moles of helium gas and $2n$ moles of oxygen gas (molecules taken to be rigid) as an ideal gas. It's C_p/C_v value will be:
a) 19/13   b) 67/45   c) 40/27   d) 23/15

04 — LAW OF EQUIPARTITION OF ENERGY

Energy for each molecule per $f = \dfrac{1}{2} K_B T$

Total energy for molecule $= \dfrac{f}{2} K_B T$

Monoatomic Molecule $= \dfrac{3}{2} K_B T$

Total energy for a mole $= \dfrac{f}{2} R T$

Total energy for n moles $= \dfrac{nfRT}{2}$

Monoatomic $= \dfrac{3}{2} R T$ (1 mole)

Diatomic $= \dfrac{5}{2} R T$ (1 mole)

Translatory Kinetic energy $= \dfrac{3}{2} R T$ (1 mole, f = 3)

Q4 A gas mixture consists of 2 moles of O_2 and 4 moles of Ar at temperature T. Neglecting all vibrational modes, the total internal energy of the system is
a) 4RT   b) 15RT   c) 9RT   d) 11RT

06 — VELOCITY OF GAS

$V_{mp} : V_{avg} : V_{rms} = 1 : 1.13 : 1.225$

Root Mean square speed:
Square root of mean of square of speeds of different molecules,
$$v_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + \ldots + v_n^2}{n}}$$
$$v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3k_BT}{m}}$$

Average Speed:
Arithmetic mean of speed of molecules of gas at given temperature.
$$v_{avg} = \frac{|\overline{v_1}| + |\overline{v_2}| + \ldots + |\overline{v_n}|}{n}$$
$$v_{avg} = \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{8P}{\pi\rho}}$$

Most probable speed:
Speed possessed by maximum number of molecules of gas.
$$V_{mp} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2P}{\rho}} = \sqrt{\frac{2k_BT}{m}}$$

Q5 Consider a gas of triatomic molecules. The molecules are assumed to be triangular, made up of massless rigid rods whose vertices are occupied by atoms. The internal energy of a mole of the gas at temperature T is:
a) $\dfrac{5}{2}RT$ b) $\dfrac{3}{2}RT$ c) $\dfrac{9}{2}RT$ d) $3RT$

Q6 The rms speeds of the molecules of Hydrogen, Oxygen & Carbon dioxide at the same temperature are V_H, V_O and V_c respectively, then:
a) $V_H > V_O > V_c$ b) $V_c > V_O > V_H$
c) $V_H = V_O > V_c$ d) $V_H = V_O = V_c$

07 — MEAN FREE PATH

Average distance travelled by molecules between two successive collisions
$$\lambda_{mean} = \frac{1}{\sqrt{2}\,\pi d^2 n}$$
d = diameter of molecules.
n = no. of molecules per unit volume

$\lambda \propto \dfrac{1}{d^2}$

$\lambda \propto \dfrac{1}{r^2}$

$\lambda \propto \dfrac{T}{P}$

Q7 The mean free path of molecules of gas, (radius r) is inversely proportional to
a) r^3 b) r^2
c) r d) $\sqrt{r}$
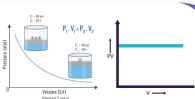
05 — FIRST LAW OF THERMODYNAMICS

$Q_P = \triangle U + W$

$\triangle U = nc_V \triangle T$

$W = \int Pdv$

$\dfrac{\triangle U}{Q_P} = \dfrac{1}{\gamma}$

$\dfrac{W}{Q_P} = 1 - \dfrac{1}{\gamma}$
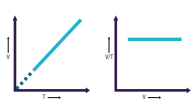
BOYLE'S LAW

- PV = constant, if T = Constant
- $P_1V_1 = P_2V_2$, when gas changes it's state under constant temperature.

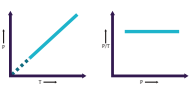## CHARLE'S LAW

- $V \propto T$; $\dfrac{V}{T}$ = constant; P = constant.
- $\dfrac{V_1}{T_1} = \dfrac{V_2}{T_2}$, when gas change its state under constant pressure.

GAY LUSSAC'S LAW

- $P \propto T$; $\dfrac{P}{T}$ = constant; V = constant.
- $\dfrac{P_1}{T_1} = \dfrac{P_2}{T_2}$, when gas changes its state under constant Volume.

PRESSURE OF GAS

$PV = \dfrac{1}{3}mn\,V_{rms}^2 = \dfrac{1}{3}mn\,\overline{V}^2$

Relation between pressure and Kinetic Energy.
$$E = \frac{3}{2}PV$$

IDEAL GAS LAW

$PV = nRT$

$R = 8.314\ JK^{-1}mol^{-1}$

$\rho = \dfrac{PM}{RT}$
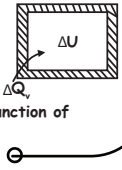
Specific heat of Solids = 3R
WATER = 9R

THERMODYNAMICS

Internal Energy(U)

(1) $\Delta U = \Delta Q_v = nC_v\Delta T = n\frac{f}{2}R\Delta T$

$= \frac{nR\Delta T}{\gamma-1} = \frac{\Delta(PV)}{\gamma-1}$
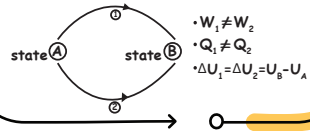
$= \frac{P_fV_f - P_iV_i}{\gamma-1}$

ΔU

ΔQ_v

(Internal Energy is only the function of temperature of the gas)

- First law of T.D $\Longrightarrow Q = \Delta U + W$
- Q, W $\Longrightarrow$ path functions
- $\Delta U \Longrightarrow$ state function

state (A) state (B)

- $W_1 \neq W_2$
- $Q_1 \neq Q_2$
- $\Delta U_1 = \Delta U_2 = U_B - U_A$

Isothermal process $\Rightarrow \Delta T = 0 \longrightarrow \Delta U = 0$

Cyclic process
$A \longrightarrow B \longrightarrow C \longrightarrow D \longrightarrow A$

$\Delta U = U_A - U_A = 0$
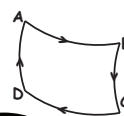
WORK
Work done: path function
$W = F.dx = \int Pdv$
- Unit: Joule(J)

Sign Convention

$\xrightarrow{W}$ System $\xrightarrow{W}$

-ve (compressive) work done on the gas

+ve (expansion) work done by the gas

Work

System

+ve $\uparrow Q$

-ve $\downarrow Q$
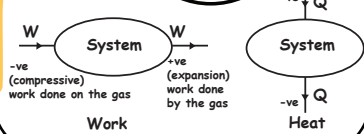
Heat

HEAT

Heat: path function $\Rightarrow$ unit:calorie/Joule

For any process,
$Q = nC\Delta T$
where, C= Specific heat capacity for the process

Adiabatic process $\Rightarrow \Delta Q = 0$ [No heat transfer]

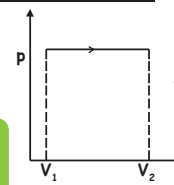At constant volume $\Rightarrow Q_v = \Delta U = nC_v\Delta T$

At constant pressure $\Rightarrow Q_p = \Delta U + W = nC_p\Delta T$
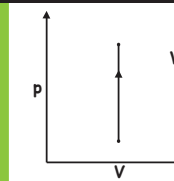

Work done from P-V Graph

Area=W
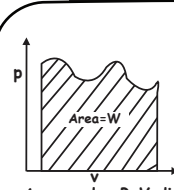
Area under P-V diagram gives work done by the gas
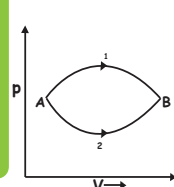
Compression

Adiabatic, Isothermal, Isochoric, Isobaric

$|W_{adiabatic}| > |W_{isothermal}| > |W_{isobaric}| > |W_{isochoric}|$

Isobaric process

$W = P(V_2 - V_1)$

Isochoric process

$W = P\Delta V = 0$

Expansion

Isobaric, Isothermal, Adiabatic, Isochoric
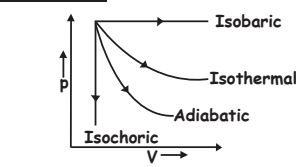
$W_{isobaric} > W_{isothermal} > W_{adiabatic} > W_{Isochoric}$

Cyclic process

- W=are inside the graph
- For clockwise process, W=-ve
- For anti-clockwise process, W=+ve

$W = \frac{\pi}{4}(P_2 - P_1)(V_2 - V_1)$

$W = \frac{1}{2}(P_2 - P_1)(V_2 - V_1)$

$W = (P_2 - P_1)(V_2 - V_1)$
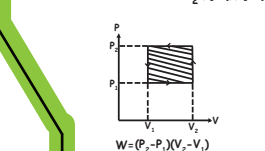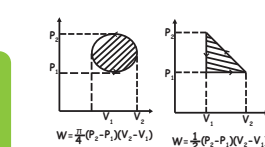
Thermodynamic processes

01

Adiabatic process

- Q=0 [no exchange of heat]

- Rapid or spontaneous process/insulated vessel

$Q = \Delta U + W$

$Q = 0 \quad \Delta U = -W$

Compression

$W = -ve \quad \Delta U = +ve$

$\Delta U \uparrow \Longrightarrow$ Temperature$\uparrow$
$\Rightarrow$ Pressure$\uparrow$

Expansion

$W = -ve \quad \Delta U = +ve$

$\Delta U \downarrow \Longrightarrow$ Temperature$\downarrow$
$\Rightarrow$ Pressure$\downarrow$

Equation of state
$PV^\gamma = $ constant
$TV^{\gamma-1} = $ constant

$PT^{\frac{\gamma-1}{\gamma}} = $ constant

Work done by the gas

$W = -\Delta U = nC_v(T_1 - T_2)$

$= n\frac{f}{2}R(T_i - T_f)$

$W = \frac{nR}{\gamma-1}(T_i - T_f)$

$W = \frac{P_iV_i - P_fV_f}{\gamma-1}$

Slope of adiabatic process
= $\gamma \times$ slope of isothermal process specific heat of gas $\Rightarrow C = 0$

$C = \frac{Q}{\Delta t} \longrightarrow Q = 0$

$C = 0$

02

Isothermal process

$\Rightarrow \Delta T = 0 \Rightarrow \Delta U = 0$
eg:- perfectly conducting slow process

$Q = \Delta U + W$

$Q = W$

equation of states $\Rightarrow PV = Constant$

$P_1V_1 = P_2V_2$

Workdone by the gas

$W = 2.303 \, nRT \log\left(\frac{V_2}{V_1}\right)$

$W = 2.303 \, nRT \log\left(\frac{P_1}{P_2}\right)$

Slope:
Slope of adiabatic process
= $\gamma \times$ slope of isothermal process specific heat $C = \infty$

03

Isobaric process

$\Delta P = 0$

$Q = \Delta U + W$

equation of state $\Rightarrow V \propto T$

$\frac{V_1}{V_2} = \frac{T_1}{T_2}$

Work done by the gas

$W = P\Delta V = P(V_2 - V_1) = nR(T_2 - T_1)$
specific heat

$\Rightarrow C_p = \left(1 + \frac{f}{2}\right)R$

$= \frac{\gamma R}{\gamma-1}$

04

Isochoric process

$\Delta V = 0$ or $V = $ constant
equation of state $\Rightarrow P \propto T \Rightarrow \frac{P_1}{P_2} = \frac{T_1}{T_2}$
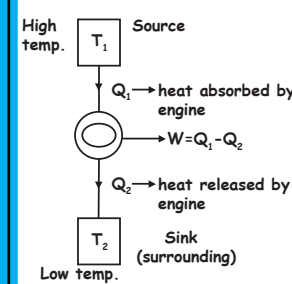
Work done by the gas

$\Delta V = 0 \Rightarrow W = 0$
specific heat

$\Rightarrow C_v = \frac{f}{2}R$

$= \frac{R}{\gamma-1}$

Heat Engine

'Device that converts heat into work'

High temp. T_1 Source

$Q_1 \longrightarrow$ heat absorbed by engine

$\longrightarrow W = Q_1 - Q_2$

$Q_2 \longrightarrow$ heat released by engine

T_2 Sink (surrounding)

Low temp.

efficiency(η)

$\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$

$\eta_{carnot} = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1}$

$\eta_{max} \Longrightarrow$ When $Q_2 = 0$ or $T_2 = 0K$ (not possible)

Carnot Engine

$\longrightarrow$ Ideal engine

$\longrightarrow \eta = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1}$

pressure

Isothermal expansion at temperature T_A

Adiabatic compression

Adiabatic expansion

Isothermal compression at temperature T_C

volume

Refrigerator and heat pump

Source T_1 (High Temp.)

Q_1

$\longleftarrow$ W (work done)

Q_2

Sink T_2 (Low Temp.)
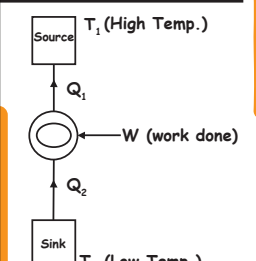
Coefficient of performance of refgeration(β)

$\beta = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} \qquad \beta = \frac{T_2}{T_1 - T_2}$

Coefficient of performance of heat pump

$= \frac{Q_1}{W} = \frac{Q_1}{Q_1 - Q_2} = \frac{T_1}{T_1 - T_2}$

Relationship between

$\beta = \frac{1-\eta}{\eta}$

$(COP)_{heat \, pump} = 1 + (COP)_{refrigerator}$

Cascaded engine

T_1

$E_1 \longrightarrow W_1$

T

$E_2 \longrightarrow W_2$

T_3

Same work output
$W_1 = W_2$

$T_1 - T = T - T_3$

$2T = T_1 + T_3$

$T = \frac{T_1 + T_3}{2}$

Same efficiency

$1 - \frac{T}{T_1} = 1 - \frac{T_3}{T}$

$T^2 = T_1 T_3$

$T = \sqrt{T_1 T_3}$