Basic Integration Formulas

Indefinite Integration

1. $\int \boxed{x^n}.dx = \boxed{\frac{x^{n+1}}{n+1}} + c,\ n \neq -1$

2. $\int \frac{1}{x} dx = \underbrace{\log_e|x|} + c$

3. $\int e^x dx = e^x + c$

4. $\int a^x dx = \frac{a^x}{\underline{\underline{\log_e a}}} + c$

$$\int x^{-3} dx = \frac{x^{-3+1}}{-3+1}$$

$$\int x^{1/3} dx = \frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1}$$

$$\int x^{-1} dx = \int \frac{1}{x} dx = \underline{\ln|x| + c}$$

$$\int 2^x dx = \frac{2^x}{\ln 2} + c$$

$$\frac{d}{dx}(2^x) = \underline{2^x \ln 2}$$

Indefinite Integration

5. $\int \sin x \, dx = -\cos x + c$

6. $\int \cos x \, dx = \sin x + c$

7. $\int \sec^2 x \, dx = \tan x + c$

8. $\int \operatorname{cosec}^2 x \, dx = -\cot x + c$

9. $\int \sec x \tan x \, dx = \sec x + c$

10. $\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + c$

'C' wale negative

Indefinite Integration

11. $$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}x + c$$

12. $$\int \frac{dx}{1+x^2} = \tan^{-1}x + c$$

13. $$\int \frac{dx}{|x|\sqrt{x^2-1}} = \sec^{-1}x + c$$

$\sin^{-1}x \longrightarrow \frac{1}{\sqrt{1-x^2}}$

$\tan^{-1}x \longrightarrow \frac{1}{1+x^2}$

Techniques of Integration

Techniques of Integration

- Substitution
- By part (product rule)
- Partial (fraction)
- Miscellaneous

Substitution

i. $\int \tan x\, dx = \ln(\sec x) + C$ OR $-\ln(\cos x) + C$

ii. $\int \cot x\, dx = \ln(\sin x) + C$

iii. $\int \sec x\, dx = \ln|\sec x + \tan x| + C$ OR $\ell n\left|\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right| + C$

iv. $\int \operatorname{cosec} x\, dx = \ln|\operatorname{cosec} x - \cot x| + C$ OR $\ell n\left|\tan\frac{x}{2}\right|$

Unacademy Atoms

Q

JEE M 2022

The integral $\displaystyle\int \frac{\left(1-\frac{1}{\sqrt{3}}\right)(\cos x-\sin x)}{\left(1+\frac{2}{\sqrt{3}}\sin 2x\right)}dx$ is equal to

(A) $\frac{1}{2}\log_e\left|\frac{\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)}{\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)}\right|+C$

(C) $\log_e\left|\frac{\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)}{\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)}\right|+C$

(B) $\frac{1}{2}\log_e\left|\frac{\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)}{\left(\frac{x}{2}+\frac{\pi}{3}\right)}\right|+C$

(D) $\frac{1}{2}\log_e\left|\frac{\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)}{\tan\left(\frac{x}{2}-\frac{\pi}{6}\right)}\right|+C$

$$\frac{\left(1-\frac{1}{\sqrt{3}}\right)}{\frac{2}{\sqrt{3}}}\int\frac{(\cos x-\sin x)\,dx}{\left(\sin\left(\frac{\pi}{3}\right)+\sin 2x\right)}$$

$$\Rightarrow\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)\int\frac{(\cos x-\sin x)\quad dx}{2\sin\left(x+\frac{\pi}{6}\right)\cos\left(x-\frac{\pi}{6}\right)}$$

$$\Rightarrow\int\frac{\overset{\cos\frac{\pi}{6}}{\boxed{\frac{\sqrt{3}}{2}}}\cos x-\overset{\cos\frac{\pi}{6}}{\boxed{\frac{\sqrt{3}}{2}}}\sin x-\overset{\sin\frac{\pi}{6}}{\boxed{\frac{1}{2}}}\cos x+\overset{\sin\frac{\pi}{6}}{\boxed{\frac{1}{2}}}\sin x}{2\sin\left(x+\frac{\pi}{6}\right)\cos\left(x-\frac{\pi}{6}\right)}$$

$$\int \frac{\cos\left(x-\frac{\pi}{6}\right) - \sin\left(x+\frac{\pi}{6}\right)}{2\cos\left(x-\frac{\pi}{6}\right)\sin\left(x+\frac{\pi}{6}\right)}$$

$$\Rightarrow \frac{1}{2}\left[\int \operatorname{cosec}\left(x+\frac{\pi}{6}\right)dx - \int \sec\left(x-\frac{\pi}{6}\right)dx\right]$$

$$\Rightarrow \frac{1}{2}\left[\ln\tan\left(\frac{x}{2}+\frac{\pi}{12}\right) - \ln\tan\left(\frac{\pi}{4}+\frac{x}{2}-\frac{\pi}{12}\right)\right]$$

$$\Rightarrow \frac{1}{2}\ln\left(\frac{\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)}{\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)}\right) + c$$

General Substitution

Root Hatao!

$\Rightarrow$ $\sqrt{a^2 - x^2}$; $\boxed{x = a \sin \theta}$

$\Rightarrow$ $\sqrt{a^2 + x^2}$; $x = a \tan \theta$

$\Rightarrow$ $\sqrt{x^2 - a^2}$; $x = a \sec \theta$

$\Rightarrow$ $\sqrt{\dfrac{a^2 - x^2}{a^2 + x^2}}$; $x^2 = a^2 \cos 2\theta$

$\sqrt{a^2 - x^2}$

$\boxed{\square \rightarrow \cos 2\theta}$

$\sqrt{\dfrac{1 - \square}{1 + \square}}$

$\sqrt{\dfrac{1 - \cos 2\theta}{1 + \cos 2\theta}} = \tan \theta$

Loving Integrals

sin = paapi

$$\frac{1}{\sqrt{1-x^2}}$$

1. $$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c$$

2. $$\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$$

3. $$\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + c$$

4. $$\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln\left(x + \sqrt{x^2 - a^2}\right) + c$$

5. $$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln(x + \sqrt{x^2 + a^2})$$

Note

$\boxed{\frac{1}{Q} \,/\, \frac{1}{\sqrt{Q}}}$

For integration of type $\int \frac{dx}{ax^2 + bx + c}$ and $\int \frac{dx}{\sqrt{ax^2 + bx + c}}$

make **ax² + bx + c** as **perfect square**

For integration of type $\int \frac{px + q}{ax^2 + bx + c}$ and $\int \frac{px + q}{\sqrt{ax^2 + bx + c}}$

write $\boxed{px + q} = \lambda(2ax + b) + \mu$

$\boxed{L = AQ' + B}$

$\frac{L}{Q} \,\Big|\, \frac{L}{\sqrt{Q}}$

Q

$$\int \frac{5x+4}{\sqrt{x^2+2x+5}}\,dx$$

$\frac{L}{\sqrt{Q}}$ #NVStyle

$5x+4 = A(2x+2)+B$

$5 = 2A \Rightarrow \boxed{A = 5/2}$

$4 = 2A + B$

$4 = 5 + B \Rightarrow \boxed{B = -1}$

$$\int \frac{\frac{5}{2}(2x+2) - 1}{\sqrt{x^2+2x+5}}\,dx = \frac{5}{2}\int \frac{(2x+2)\,dx}{\sqrt{x^2+2x+5}} - \int \frac{dx}{\sqrt{x^2+2x+5}}$$

$$I_1 = \int \frac{(2x+2)\,dx}{\sqrt{x^2+2x+5}}$$

$$x^2+2x+5 = t^2$$

$$(2x+2)\,dx = 2t\,dt$$

$$= \int \frac{2\cancel{t}\,dt}{\cancel{t}} = 2t + c = \underline{2\sqrt{x^2+2x+5}}$$

$$I_2 = \int \frac{dx}{\sqrt{(x+1)^2+2^2}} = \ln\left(x+1+\sqrt{(x+1)^2+2^2}\right)$$

$$= \ln\left(x+1+\sqrt{x^2+2x+5}\right)$$

$$\int \frac{dx}{\sqrt{x^2+a^2}}$$

Q

If $\int \frac{1}{x}\sqrt{\frac{1-x}{1+x}}\,dx = g(x) + c,\ g(1) = 0$, then $g\left(\frac{1}{2}\right)$ is equal to :

JEE M 2022

$2-\sqrt{3}$

$\frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{4-2\sqrt{3}}{2} = 2-\sqrt{3}$

(A) $\log_e\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)+\frac{\pi}{3}$

(B) $\log_e\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)+\frac{\pi}{3}$

A

(C) $\log_e\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)-\frac{\pi}{3}$

(D) $\frac{1}{2}\log_e\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)-\frac{\pi}{6}$

$$\int \frac{1}{x}\sqrt{\frac{1-x}{1+x}}\,dx$$

Put

$x = \cos 2\theta$

$dx = -2\sin 2\theta \, d\theta$

$$\Rightarrow \int \frac{1}{\cos 2\theta}\,\frac{\sin\theta}{\cancel{\cos\theta}} \cdot -2\left(2\sin\theta\,\cancel{\cos\theta}\right) d\theta$$

$$\Rightarrow \frac{-4}{2}\int \frac{1-\cos 2\theta}{\cos 2\theta}\,d\theta$$

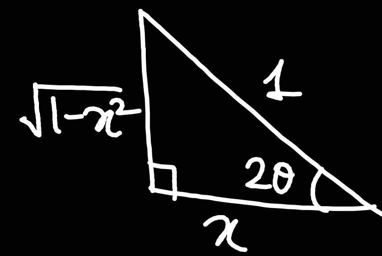
$$\Rightarrow -2\left\{\int \sec 2\theta - \int d\theta\right\}$$

$$\Rightarrow -2\left(\frac{1}{2}\ln\left(\sec 2\theta + \tan 2\theta\right) - \theta\right)$$

$\frac{x}{1} = \cos(2\theta)$

$\cos^{-1} x = 2\theta$

$$\therefore \boxed{\frac{\cos^{-1} x}{2} = \theta}$$



$$-\ln\left(\underline{\underline{\sec 2\theta}} + \underline{\underline{\tan 2\theta}}\right) + 2\theta$$

$$g(x) = -\ln\left(\frac{1}{x} + \frac{\sqrt{1-x^2}}{x}\right) + \cos^{-1}x + \cancel{c}$$

$$\cancel{g(1)} = -\ln\left(\cancel{\frac{1}{1} + \frac{0}{1}}\right) + \cancel{\cos^{-1}(1)} + c \qquad \therefore \boxed{c = 0}$$

$$g(x) = -\ln\left(\frac{1}{x} + \frac{\sqrt{1-x^2}}{x}\right) + \cos^{-1}x$$

$$g\left(\frac{1}{2}\right) = -\ln\left(2 + 2\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(\frac{1}{2}\right)$$

$$= \underline{\ln(2+\sqrt{3})^{-1} + \frac{\pi}{3}} = \ln(2-\sqrt{3}) + \frac{\pi}{3}$$

The integral $\int \frac{dx}{(x+4)^{8/7}(x-3)^{6/7}}$ is equal to :

(where C is a constant of integration)

[JEE Main 2020]

A. $\left(\frac{x-3}{x+4}\right)^{1/7} + C$

B. $-\left(\frac{x-3}{x+4}\right)^{-1/7} + C$

C. $\frac{1}{2}\left(\frac{x-3}{x+4}\right)^{3/7} + C$

D. $-\frac{1}{13}\left(\frac{x-3}{x+4}\right)^{-13/7} + C$

A

$$\int \frac{dx}{(x+4)^{\frac{8}{7}+\frac{6}{7}}\frac{(x-3)^{\frac{6}{7}}}{(x+4)^{6/7}}}$$

$$\int \frac{dx}{(x+4)^2\left(\frac{x-3}{x+4}\right)^{6/7}}$$

$$\left(\frac{x-3}{x+4}\right) = t^7$$

$$\frac{1(x+4)-1(x-3)}{(x+4)^2}\,dx = 7t^6\,dt$$

$$\boxed{\frac{dx}{(x+4)^2} = t^6\,dt}$$

$$\int \frac{t^6\,dt}{(t^7)^{6/7}}$$

$$\int \frac{t^6\,dt}{t^6}$$

$$\int dt = t + c$$

$$= \left(\frac{x-3}{x+4}\right)^{1/7} + c$$

The integral $\int \frac{1}{\sqrt[4]{(x-1)^3(x+2)^5}}dx$ is equal to:

(where C is a constant of integration)

[JEE Main 2021]

A. $\frac{3}{4}\left(\frac{x+2}{x-1}\right)^{\frac{1}{4}}+C$

B. $\frac{3}{4}\left(\frac{x+2}{x-1}\right)^{\frac{5}{4}}+C$

C. $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{\frac{1}{4}}+C$

D. $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{\frac{5}{4}}+C$

C

$$\int \frac{dx}{\frac{(x-1)^{\frac{3}{4}}}{(x+2)^{3/4}}(x+2)^{5/4+\frac{3}{4}}}$$

$$\int \frac{dx}{\left(\frac{x-1}{x+2}\right)^{3/4}(x+2)^2}$$

$$\frac{x-1}{x+2}=t^4$$

$$\frac{1(x+2)-1(x-1)}{(x+2)^2}dx=4t^3dt$$

$$\boxed{\frac{dx}{(x+2)^2}=\frac{4t^3dt}{3}}$$

$$\frac{4}{3}\int \frac{\cancel{t^3}\,dt}{\cancel{t^3}}$$

$$\Rightarrow \frac{4}{3}t + c$$

$$\Rightarrow \frac{4}{3}\left(\frac{x-1}{x+2}\right)^{1/4} + c$$

Q

If $\int \frac{dx}{x^3(1+x^6)^{2/3}} = \boxed{xf(x)}(1+x^6)^{\frac{1}{3}} + C$

where, C is a constant of integration, then the function $f(x)$ is equal to **(2019 Main, 8 April II)**

A. $-\frac{1}{6x^3}$

B. $-\frac{1}{2x^3}$ ✓

C. $-\frac{1}{2x^2}$

D. $\frac{3}{x^2}$

$$\int \frac{dx}{x^3(1+x^6)^{2/3}}$$

$\boxed{\text{Take highest power out}}$

$$\int \frac{dx}{x^3(x^6)^{2/3}(x^{-6}+1)^{2/3}} = \int \frac{x^{-7}\,dx}{(x^{-6}+1)^{2/3}}$$

$x^{-6}+1 = t^3$

$-6x^{-7}dx = 3t^2dt$

$\boxed{x^{-7}dx = \frac{t^2dt}{-2}}$

$$\frac{1}{-2}\int \frac{\cancel{t^2}\,dt}{\cancel{t^2}}$$

$$\frac{-1}{2}\,t + c$$

$$\frac{-1}{2}\left(x^{-6}+1\right)^{1/3} + c$$

$$\frac{-1}{2}\left(\frac{1+x^6}{x^6}\right)^{1/3} + c = \frac{-1}{2x^2}\left(1+x^6\right)^{1/3} + c$$

$$x\,f(x) = \frac{-1}{2x^2}$$

$$\boxed{f(x) = \frac{-1}{2x^3}}$$

If $\int \frac{\cos x\, dx}{\sin^3 x(1+\sin^6 x)^{2/3}} = f(x)(1+\sin^6 x)^{1/\lambda} + c$

a constant of integration, then $\lambda f(\pi/3)$ is equal to :

A. -9/8

B. 2

C. 9/8

D. -2

[JEE Main 2020]

$\sin x = t$

$\cos x\, dx = dt$

$\int \frac{dt}{t^3(1+t^6)^{2/3}} \longrightarrow$ Homework

Q

If $f(x)=\int \dfrac{\dfrac{5x^8+7x^6}{x^{14}}}{\left(x^2+1+2x^7\right)^2}\,dx,\ (x\ge 0)$,

and f(0) = 0, then the value of f(1) is :

[JEE Main 2019]

A. -1/2

B. -1/4

C. 1/2

D. 1/4

$$\int \frac{\left(5x^{-6}+7x^{-8}\right)dx}{\left(x^{-5}+x^{-7}+2\right)^2}$$

$$x^{-5}+x^{-7}+2=t$$

$$\left(-5x^{-6}-7x^{-8}\right)dx=dt$$

$$\int \frac{-dt}{t^2}=\frac{1}{t}+c$$

$$=\frac{1}{x^{-5}+x^{-7}+2}+c$$

$$f(x) \Rightarrow \frac{x^7}{x^2+1+2x^7}$$

$$\boxed{f(1) = \frac{1}{4}}$$

If $f(x)=\int \frac{5x^8+7x^6}{\left(x^2+1+2x^7\right)^2}dx,(x\geq 0),f(0)=0$ and f(1) = 1/k, then the value of K is

4

[JEE Main 2021]

$$f(1)=\frac{1}{k}=\frac{1}{4}$$

Unacademy Atoms

Q The integral $\int \frac{3x^{13}+2x^{11}}{(2x^4+3x^2+1)^4}dx$ is equal to :

(where C is a constant of integration)

$$\int \frac{(3x^{-3}+2x^{-5})dx}{(2+3x^{-2}+x^{-4})^4}$$

A. $\frac{x^4}{6(2x^4+3x^2+1)^3}+C$

B. $\frac{x^{12}}{6(2x^4+3x^2+1)^3}+C$ ✓

[JEE Main 2019]

C. $\frac{x^4}{(2x^4+3x^2+1)^3}+C$

D. $\frac{x^{12}}{(2x^4+3x^2+1)^3}+C$

$$2+3x^{-2}+x^{-4}=t$$

$$(-6x^{-3}-4x^{-5})dx=dt$$

$$-2\underline{(3x^{-3}+2x^{-5})dx}=dt$$

$$\int \frac{dt}{-2\, t^4}$$

$$= \frac{1}{-2} \frac{t^{-4+1}}{-4+1} + c$$

$$= \frac{1}{6} \frac{1}{(2+3x^{-2}+x^{-4})^3} + c \rightarrow \frac{1}{6} \frac{x^{12}}{(2x^4+3x^2+1)^3} + c$$

If $\int \frac{\sqrt{1-x^2}}{x^4}dx = A(x)\left(\sqrt{1-x^2}\right)^m + C$, for a suitably chosen integer m and function A(x), where C is a constant of integration, then $(A(x))^m$ equals :

[JEE Main 2019]

A. $-1/27x^9$

B. $-1/3x^3$

C. $1/27x^6$

D. $1/9x^4$

$$\int \frac{\sqrt{x^{-2}-1}}{x^3}\,dx$$

$$\int \sqrt{x^{-2}-1}\;\underbrace{x^{-3}dx}$$

$$-\int t \cdot t\,dt$$

$$-\frac{t^3}{3}+C$$

$$x^{-2}-1=t^2$$

$$-\cancel{2}x^{-3}dx=\cancel{2}t\,dt$$

$$x^{-3}dx=-t\,dt$$

$$\frac{-1}{3}\left(x^{-2}-1\right)^{3/2}+c$$

$$\frac{-1}{3}\,\frac{\left(1-x^{2}\right)^{3/2}}{x^{3}}$$

$$\frac{-1}{3x^{3}}\left(\sqrt{1-x^{2}}\right)^{3}$$

$$A(x)\left(\sqrt{1-x^{2}}\right)^{m}$$

$$A(x)=\left(\frac{-1}{3x^{3}}\right)^{3}=\frac{-1}{27x^{9}}$$

$$m=3$$

The integral $\int \frac{(2x-1)\cos\sqrt{(2x-1)^2+5}}{\sqrt{4x^2-4x+6}}dx$ is equal to

(where c is a constant of integration)

[JEE Main 2021]

A. $\frac{1}{2}\sin\sqrt{(2x-1)^2+5}+c$

B. $\frac{1}{2}\cos\sqrt{(2x+1)^2+5}+c$

C. $\frac{1}{2}\cos\sqrt{(2x-1)^2+5}+c$

D. $\frac{1}{2}\sin\sqrt{(2x+1)^2+5}+c$

$$\int \frac{(2x-1)\cos\sqrt{(2x-1)^2+5}}{\sqrt{(2x-1)^2+5}}\,dx$$

$$(2x-1)^2+5=t^2$$

$$2(2x-1)(\cancel{2})\,dx=\cancel{2}t\,dt$$

$$\boxed{(2x-1)\,dx=\frac{t\,dt}{2}}$$

$$\frac{1}{2}\int \frac{\cancel{t}\,dt\,\cos t}{\cancel{t}}$$

$$\frac{1}{2}\int \cos t\,dt$$

$$\Rightarrow \frac{1}{2}\sin\sqrt{(2x-1)^2+5}+C$$

Q

If $\int \frac{dx}{(x^2-2x+10)^2}$

$= A\left(\tan^{-1}\left(\frac{x-1}{3}\right)+\frac{\boxed{f(x)}}{x^2-2x+10}\right)+C$ where C is a

constant of integration, then :

[JEE Main 2019]

A. $A=\frac{1}{54}$ and $f(x) = 3(x-1)$

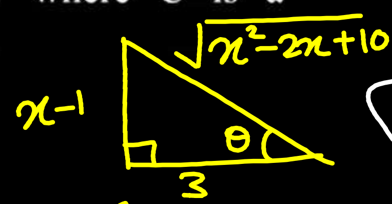
B. $A=\frac{1}{81}$ and $f(x) = 3(x-1)$

C. $A=\frac{1}{27}$ and $f(x) = 9(x-1)$

D. $A=\frac{1}{54}$ and $f(x) = 9(x-1)^2$

$x^2+a^2 \rightarrow x = a\tan\theta$

$$\boxed{\tan\theta = \frac{x-1}{3}}$$



$$\int \frac{dx}{(x^2-2x+10)^2}$$

$$\int \frac{dx}{\left((x-1)^2+3^2\right)^2}$$

$$\boxed{x-1 = 3\tan\theta}$$

$\therefore dx = 3\sec^2\theta\, d\theta$

$$\int \frac{3\sec^2\theta\, d\theta}{\left(3^2\tan^2\theta + 3^2\right)^2}$$

$$\int \frac{3\sec^2\theta\, d\theta}{3^4\sec^4\theta}$$

$$\frac{1}{3^3}\int \cos^2\theta\, d\theta$$

$$\frac{1}{2(27)}\int (1+\cos 2\theta)\, d\theta$$

$$\frac{1}{54}\left[\theta + \sin\theta\cos\theta\right] + c$$

$$\frac{1}{54}\left[\tan^{-1}\left(\frac{x-1}{3}\right) + \frac{(x-1)\,3}{x^2-2x+10}\right] + c$$

$$\frac{1}{54}\left[\tan^{-1}\left(\frac{x-1}{3}\right) + \frac{3(x-1)}{x^2-2x+10}\right] + c$$

If $\int \frac{dx}{(x^2+x+1)^2} = \underline{\underline{a}}\underbrace{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)} + \underline{\underline{b}}\left(\underbrace{\frac{2x+1}{x^2+x+1}}\right) + C\ x > 0$

where C is the constant of integration, then the value of $9(\sqrt{3}a + b)$ is equal to. 15

[JEE Main 2021]

$$\int \frac{dx}{(x^2+x+1)^2}$$

$$\Rightarrow \int \frac{dx}{(\underbrace{x^2+x+\tfrac{1}{4}}+\tfrac{3}{4})^2}$$

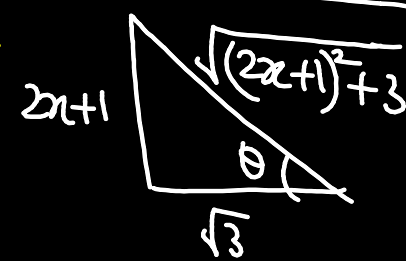
$$\Rightarrow \int \frac{dx}{\left(\left(x+\tfrac{1}{2}\right)^2+\left(\tfrac{\sqrt{3}}{2}\right)^2\right)^2}$$

$$x+\frac{1}{2} = \frac{\sqrt{3}}{2}\tan\theta$$

$$\therefore\ dx = \frac{\sqrt{3}}{2}\sec^2\theta\, d\theta$$

$$\int \frac{\frac{\sqrt{3}}{2}\sec^2\theta\, d\theta}{\frac{9}{16}\sec^4\theta}$$

$$\boxed{\tan\theta = \frac{2x+1}{\sqrt{3}}}$$



$$4\,\frac{\cancel{8}\sqrt{3}}{9}\int\frac{1+\cos 2\theta}{\cancel{2}}\,d\theta$$

$$\frac{4\sqrt{3}}{9}\left(\theta+\sin\theta\cos\theta\right)+c$$

$$\frac{4\sqrt{3}}{9}\left(\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+\frac{(2x+1)\sqrt{3}}{4(x^2+x+1)}\right)+c$$

$$\sqrt{3}a=\frac{4\sqrt{3}}{9}\cdot\sqrt{3}$$

$$b=\frac{4\sqrt{3}}{9}\times\frac{\sqrt{3}}{4}=\frac{3}{9}$$

$$9(\sqrt{3}a+b)$$

$$9\left(\frac{4}{3}+\frac{1}{3}\right)$$

15

Q

Let $\underline{f(x)} = \int \frac{\sqrt{x}}{(1+x)^2} dx \, (x \geq 0)$.

dx (x ≥ 0). Then $\underline{f(3) - f(1)}$ is equal to :

$\sqrt{x} = \tan\theta$

$x = \tan^2\theta$

[JEE Main 2020]

A. $-\frac{\pi}{12}+\frac{1}{2}+\frac{\sqrt{3}}{4}$

B. $\frac{\pi}{6}+\frac{1}{2}-\frac{\sqrt{3}}{4}$

D

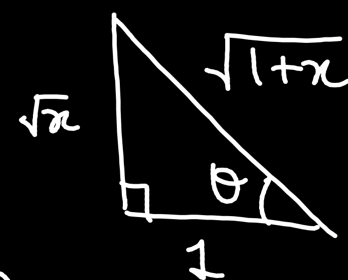
C. $-\frac{\pi}{6}+\frac{1}{2}+\frac{\sqrt{3}}{4}$

D. $\frac{\pi}{12}+\frac{1}{2}-\frac{\sqrt{3}}{4}$

$$\int \frac{\sqrt{x}}{(1+x)^2} dx$$

$$\int \frac{\tan\theta \; 2\tan\theta \; \sec^2\theta \, d\theta}{\sec^4\theta}$$

$$2\int \frac{\tan^2\theta}{\sec^2\theta} d\theta = 2\int \sin^2\theta \, d\theta$$



$$\frac{2}{2}\int (1-\cos 2\theta)\,d\theta$$

$$\Rightarrow \left(\theta - \frac{\sin 2\theta}{2}\right) + c$$

$$\Rightarrow \left(\theta - \sin\theta\cos\theta\right) + c$$

$$f(x) \Rightarrow \tan^{-1}\sqrt{x} - \frac{\sqrt{x}}{1+x} + c$$

$$f(3) = \frac{\pi}{3} - \frac{\sqrt{3}}{4} + c$$

$$f(1) = \frac{\pi}{4} - \frac{1}{2} + c$$

$$f(3) - f(1)$$

$$\Rightarrow \underline{\frac{\pi}{12} - \frac{\sqrt{3}}{4} + \frac{1}{2}}$$

If $\int \frac{d\theta}{\cos^2\theta(\tan 2\theta + \sec 2\theta)} = \lambda \tan\theta + 2\log_e|f(\theta)| + C$

where C is a constant of integration, then the ordered pair $(\lambda, f(\theta))$ is equal to :

[JEE Main 2020]

A. $(1, 1 - \tan\theta)$

Homework

B. $(-1, 1 - \tan\theta)$

C. $(-1, 1 + \tan\theta)$

D. $(1, 1 + \tan\theta)$

The integral $\int \frac{2x^3-1}{x^4+x}dx$ is equal to :

(Here C is a constant of integration)

[JEE Main 2019]

A. $\frac{1}{2}\log_e \frac{|x^3+1|}{x^2}+C$

B. $\frac{1}{2}\log_e \frac{(x^3+1)^2}{|x^3|}+C$

C. $\log_e \left|\frac{x^3+1}{x}\right|+C$

D. $\log_e \frac{|x^3+1|}{x^2}+C$

Hint :-

$$\int \frac{2x^3-1}{x^4+x}dx$$

$$\Rightarrow \int \frac{(2x-x^{-2})dx}{(x^2+x^{-1})} = \ln(x^2+x^{-1})+c$$

Integration By Parts

Integration by Parts

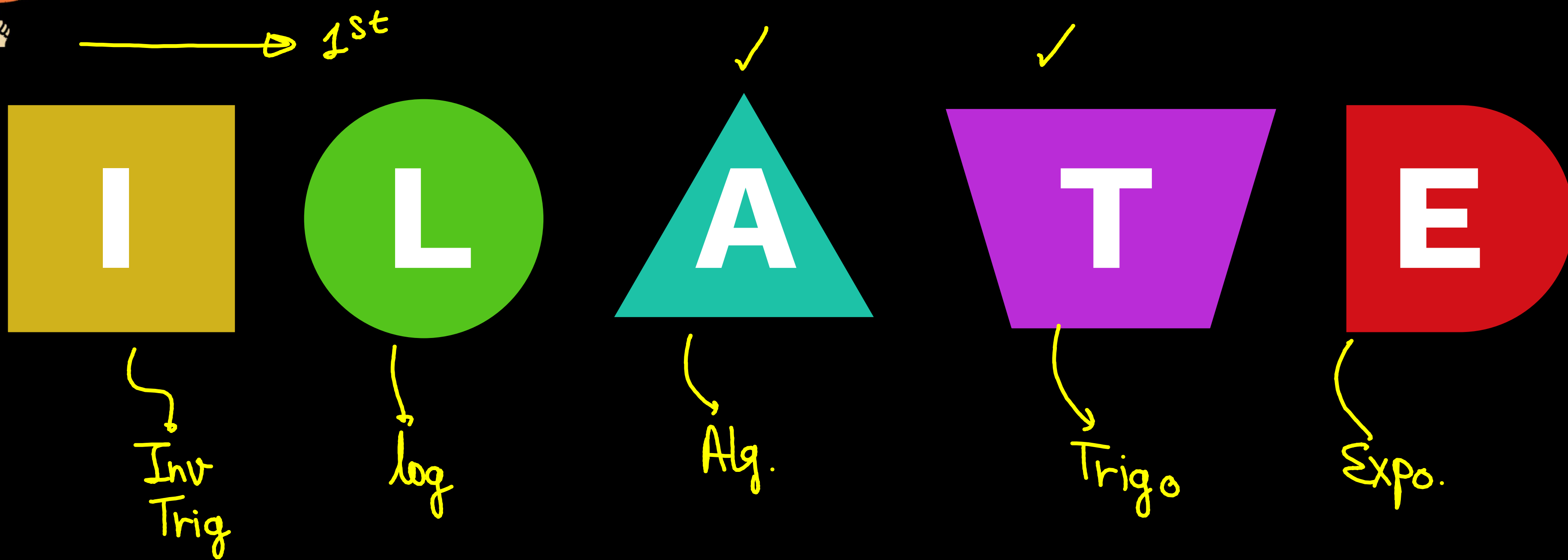
$$\int (uv)dx = u\int vdx - \int\left(\frac{du}{dx}\int vdx\right)dx$$

$$\int \mathrm{I}\cdot\mathrm{II}\,dx \Rightarrow \underbrace{\mathrm{I}\cdot\int\mathrm{II} - \int\left(\mathrm{I}'\cdot\int\mathrm{II}\right)dx}$$

✓ $\int \underset{①}{x}\ \underset{②}{\sin x}\,dx$ $\qquad \int \underset{②}{1}\ \underset{①}{\ln x}$

✓ $\int \underset{①}{x}\ \underset{②}{e^x}\,dx$ $\qquad \int \underset{②}{1}\ \underset{①}{\tan^{-1}x}$

Integration by Parts



CRACK in SECONDS!

$$\int x^n f(x)\, dx$$

DI Method: Shortcut

$$\int x^2 \cos 2x \, dx = \underbrace{\frac{x^2 \sin 2x}{2} + \frac{2x \cos 2x}{4} - \frac{\sin 2x}{4} + c}$$

	Diff	Int
$+$	x^2	$\cos 2x$
$-$	$2x$	$\frac{\sin 2x}{2}$
$+$	2	$-\frac{\cos 2x}{4}$
$-$	0	$-\frac{\sin 2x}{8}$

DI Method: Shortcut

$$\int \underbrace{x^2}_{1} \underbrace{e^x}_{2} dx = x^2 e^x - 2x e^x + 2e^x + C$$

	D	I
$+$	x^2	e^x
$-$	$2x$	e^x
$+$	2	e^x
$-$	0	e^x

If $\int x^5 e^{-x^2}\,dx = \underline{\underline{g(x)}}\underline{\underline{e^{-x^2}}} + c$,

where c is a constant of integration, then g(-1) is equal to :

[JEE Main 2019]

A. -1

B. 1

C. -5/2

D. -1/2

$$\int x^5 e^{-x^2}\,dx$$

$$\int (x^2)^2 e^{-x^2}\,\underline{x\,dx}$$

$x^2 = t$

$2x\,dx = dt$

$$\underbrace{\frac{1}{2}\int t^2 e^{-t}}\,dt$$

$$\frac{1}{2}\left(-t^2 e^{-t} - 2t e^{-t} - 2e^{-t}\right) + c$$

D	I
$+\ t^2$	e^{-t}
$-2t$	$-e^{-t}$
$+\ 2$	e^{-t}
$-\ 0$	$-e^{-t}$

$$-\frac{e^{-t}}{2}\left(t^2+2t+2\right)+c$$

$$-\frac{e^{-x^2}}{2}\left(x^4+2x^2+2\right)+c$$

$$g(x)=\frac{-1}{2}\left(x^4+2x^2+2\right)$$

$$g(-1)=\frac{-1}{2}(5)=-5/2$$

If $\int x^5 e^{-4x^3}\, dx = \frac{1}{48} e^{-4x^3} f(x) + C,$

where C is a constant of integration, then f (x) is equal to :

A. $-2x^3 - 1$

B. $-4x^3 - 1$

C. $-2x^3 + 1$

D. $4x^3 + 1$

[JEE Main 2019]

$$\int x^3 \cdot e^{-4x^3} \underline{x^2 dx}$$

$$4x^3 = t$$

$$12x^2 dx = dt$$

$$\frac{1}{12}\int \frac{t}{4} \cdot e^{-t}\, dt$$

$$\frac{1}{48}\int t \cdot e^{-t}\, dt$$

D	I
$+t$	e^{-t}
-1	$-e^{-t}$
$+0$	e^{-t}

$$\frac{1}{48}\left(-te^{-t}-e^{-t}\right)+c$$

$$-\frac{1}{48}e^{-t}(t+1)+c$$

Ho na Ho, Woh na Ho!

a. $\int e^x (f(x) + f'(x))\, dx = e^x f(x) + C$

b. $\int (f(x) + xf'(x))\, dx = x f(x) + C$

☆ $\int e^x \left(\underline{f(x) + f'(x)} \right) = e^x f(x)$

☆ $\int \left(x f'(x) + f(x) \right) dx = x f(x)$

Ho na Ho, Woh na Ho!

Evaluate $\int \frac{xe^x}{(1+x)^2}dx$

$$= \int e^x \frac{\overbrace{x+1}-1}{(1+x)^2}dx$$

$$= \int e^x\left(\underbrace{\frac{1}{1+x}} + \underbrace{\frac{-1}{(1+x)^2}}\right)dx$$

$$= e^x\left(\frac{1}{1+x}\right) + c$$

Unacademy Atoms

Q

$$\int \frac{(x^2+1)e^x}{(x+1)^2}dx = f(x)e^x + C,$$ Where C

JEE M 2022

constant, then $\frac{d^3 f}{dx^3}$ at x = 1 is equal to :

(A) $-\frac{3}{4}$

(B) $\frac{3}{4}$

(C) $-\frac{3}{2}$

(D) $\frac{3}{2}$

B

$$\int \frac{e^x(x^2+1)}{(x+1)^2}dx$$

$$\int e^x\left(\frac{x^2-1+2}{(x+1)^2}\right)dx$$

$$\int e^{x}\left(\underbrace{\frac{x-1}{x+1}}_{f(x)}+\underbrace{\frac{2}{(x+1)^2}}_{f'(x)}\right)$$

$$\frac{x-1}{x+1} \rightarrow \frac{1(x+1)-1(x-1)}{(x+1)^2}$$

$$\rightarrow \frac{2}{(x+1)^2}$$

$$\Rightarrow \underline{e^{x}}\left(\frac{x-1}{x+1}\right)+c$$

$$f(x)=\frac{x-1}{x+1}$$

$$f'(x)=\frac{2}{(x+1)^2}$$

$$f''(x)=\frac{-4}{(x+1)^3}$$

$$f'''(x)=\frac{12}{(x+1)^4}$$

$$f'''(1)=\frac{12}{16}=3/4$$

JEE M 2022

For $I(x)=\int \frac{\sec^2 x-2022}{\sin^{2022} x}dx$, if $I\left(\frac{\pi}{4}\right)=2^{1011}$, then

(A) $3^{1010} I\left(\frac{\pi}{3}\right)-I\left(\frac{\pi}{6}\right)=0$

(B) $3^{1010} I\left(\frac{\pi}{6}\right)-I\left(\frac{\pi}{3}\right)=0$

(C) $3^{1011} I\left(\frac{\pi}{3}\right)-I\left(\frac{\pi}{6}\right)=0$

(D) $3^{1011} I\left(\frac{\pi}{6}\right)-I\left(\frac{\pi}{3}\right)=0$

$$I(x)=\int \underbrace{(\sin x)^{-2022}}_{①}\ \underbrace{\sec^2 x}_{②} - \int 2022(\sin x)^{-2022}dx$$

$$I\left(\frac{\pi}{3}\right)=\frac{2^{2022}}{3^{1010}}\cdot\frac{1}{\sqrt{3}}$$

$$I\left(\frac{\pi}{6}\right)=\frac{2^{2022}}{\sqrt{3}}$$

$$3^{1010} I\left(\frac{\pi}{3}\right)=I\left(\frac{\pi}{6}\right)$$

$$I(x) = (\sin x)^{-2022} \cdot \tan x + \cancel{\int 2022 (\sin x)^{-2023} \cancel{\cos x} \cdot \frac{\sin x}{\cancel{\cos x}}\, dx - 2022 \int (\sin x)^{-2022}\, dx}$$

$$I(x) = \frac{\tan x}{(\sin x)^{2022}} + \cancel{c}$$

$$\cancel{2^{1011}} = \cancel{\frac{1}{\left(\frac{1}{\sqrt{2}}\right)^{2022}}} + c \qquad \therefore \boxed{c = 0}$$

$$I(x) = \frac{\tan x}{(\sin x)^{2022}}$$

$$I\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{\left(\frac{\sqrt{3}}{2}\right)^{2022}} = \frac{2^{2022}}{3^{1010}\sqrt{3}}$$

$$I\left(\frac{\pi}{6}\right) = \frac{\frac{1}{\sqrt{3}}}{\left(\frac{1}{2}\right)^{2022}} = \frac{2^{2022}}{\sqrt{3}}$$

The integral $\int \cos(\log_e x)dx$

is equal to : (Where C is a constant of integration)

[JEE Main 2019]

A. $\frac{x}{2}[\sin(\log_e x)-\cos(\log_e x)]+C$

B. $x[\cos(\log_e x)+\sin(\log_e x)]+C$

C. $\frac{x}{2}[\cos(\log_e x)+\sin(\log_e x)]+C$

D. $x[\cos(\log_e x)-\sin(\log_e x)]+C$

$\ln x = t$

$x = e^t$

$dx = e^t dt$

$\int \cos(\ln x)\, dx$

$I = \int \cos t \cdot e^t\, dt$

	D	I
$+$	$\cos t$	e^t
$+$	$\sin t$	e^t
$-$	$\cos t$	e^t

$$I = \cos t \, e^t + \sin t \, e^t - I$$

$$\therefore I = \frac{1}{2} e^t (\cos t + \sin t)$$

$$I = \frac{1}{2} x \left(\cos(\ln x) + \sin(\ln x) \right)$$

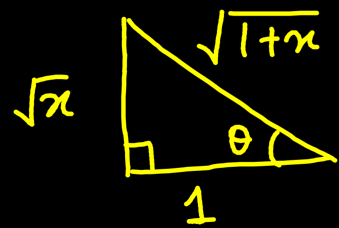
Q

If $\int \underbrace{\sin^{-1}\left(\sqrt{\frac{x}{1+x}}\right)} dx = \underline{A(x)}\tan^{-1}(\sqrt{x}) + \underline{B(x)} + C,$

$B(x)$

where C is a constant of integration, then the ordered pair (A(x), B(x)) can be :

[JEE Main 2020]



A. $(\underline{x+1}, \underline{-\sqrt{x}})$ ✓

B. $(\underline{x+1}, \sqrt{x})$

~~C.~~ $(x-1, -\sqrt{x})$

~~D.~~ $(x-1, \sqrt{x})$

(A)

$$\int \underset{\text{II}}{1} \cdot \underset{\text{I}}{\boxed{\tan^{-1}\sqrt{x}}}\, dx$$

$$\Rightarrow \tan^{-1}\sqrt{x} \cdot x - \int \frac{1}{1+x} \cdot \frac{1}{2\sqrt{x}} \cdot x\, dx$$

Unacademy Atoms

$$\Rightarrow x\tan^{-1}\sqrt{x} - \frac{1}{2}\int \frac{\sqrt{x}}{1+x}\,dx$$

$$\Rightarrow x\tan^{-1}\sqrt{x} - \frac{1}{\cancel{2}}\left(\cancel{2}\left(\sqrt{x} - \tan^{-1}(\sqrt{x})\right)\right)$$

$$\Rightarrow x\,\underbrace{\tan^{-1}\sqrt{x}} - \sqrt{x} + \underbrace{\tan^{-1}\sqrt{x}}$$

$$\Rightarrow \underline{(x+1)}\tan^{-1}\sqrt{x}\ \underbrace{-\ \sqrt{x}} + C$$

$$\int \frac{\sqrt{x}}{1+x}\,dx$$

$$x = t^2$$

$$\int \frac{t\,(2t\,dt)}{1+t^2}$$

$$2\int \frac{t^2+1-1}{t^2+1}\,dt$$

$$2\left(\int\left(1 - \frac{1}{t^2+1}\right)dt\right)$$

$$2\left(t - \tan^{-1}(t)\right)$$

Partial Fraction

Type 1: Linear Factors

$$\int \frac{dx}{(2x+3)(x+5)}$$

$$\frac{1}{(2x+3)(x+5)} = \frac{A}{2x+3} + \frac{B}{x+5}$$

Type 2: Repeated Linear Factors

Find $\int \frac{3x-2}{(x+1)^2(x+3)}dx$

$$\frac{3x-2}{(x+1)^2(x+3)} = \frac{A}{(x+1)^1} + \frac{B}{(x+1)^2} + \frac{C}{x+3}$$

$$\frac{5x+4}{(x+1)^3(x+3)} \quad \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3} + \frac{D}{x+3}$$

Type 3: Quadratic Factors (non-factorizable)

$$\int \frac{1}{(x+1)(x^2+1)}\,dx$$

$$\frac{1}{\cancel{(x+1)(x^2+1)}} = \int \frac{A}{x+1} + \int \frac{Bx+C}{x^2+1}$$

$$1 = A(x^2+1) + (Bx+C)(x+1)$$

$x=-1$: $1 = A(2) \longrightarrow \boxed{A = 1/2}$

$x=0$: $1 = A + C \rightarrow \boxed{C = 1/2}$

$x=1$: $1 = 2A + 2B + 2C$

$1 = 1 + 2B + 1 \longrightarrow \boxed{B = -1/2}$

Partial Fraction (Concept)

$$\frac{1}{(x+1)^3(x+2)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3} + \frac{D}{x+2}$$

$$\frac{1}{(x^2+1)^2(x-1)} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{(x^2+1)^2} + \frac{E}{x-1}$$

Partial Fraction (Concept)

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln\left(\frac{\boxed{a +} x}{a - x}\right)$$

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln\left(\frac{\boxed{x -} a}{x + a}\right)$$

Ex → hatana

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln\left(\frac{a+x}{a-x}\right)$$

Remember

$N^r < D^r$

Partial Fraction can be applied only if $\boxed{\text{Deg }(N^r) < \text{Deg }(D^r)}$

$$\int \frac{2x-1}{(x+1)(x+2)} \checkmark$$

If Deg (Nr) ≥ Deg (D^r) then Long Division

$$\int \frac{x^2+x+1}{x^2-3x+2}dx \quad deg(N^r) \geq deg(D^r)$$

Partial Fractions Involving Even Powers of x only:

★ $\int \frac{x^2+1}{(x^2+2)(x^2+3)}dx$

Thori der K liye.

$x^2 = y$

$$\frac{x^2+1}{(x^2+2)(x^2+3)}$$

$$\frac{y+1}{(y+2)(y+3)} = \boxed{\frac{A}{y+2} + \frac{B}{y+3}}$$

$$y+1 = A(y+3) + B(y+2)$$

$y=-2$: $\boxed{-1 = A}$

$y=-3$: $-2 = -B \rightarrow \boxed{B=2}$

$$\int \frac{-1}{x^2+2} + \int \frac{2}{x^2+3}$$

If $\int \frac{\sin x}{\sin^3 x+\cos^3 x} dx =$

$\alpha \log_c |1 + \tan x| + \beta \log_e | 1 - \tan x + \tan^2 x | + \gamma \tan^{-1} \left(\frac{2\tan x-1}{\sqrt{3}}\right) + C,$

when C is constant of integration, then the value of $18(\alpha + \beta + \gamma^2)$ is.

[JEE Main 2021]

$$\int \frac{\frac{\sin x}{\cos x}\,\frac{1}{\cos^2 x}\,dx}{\frac{\sin^3 x+\cos^3 x}{\cos^3 x}}$$

$\tan x = t$

$\sec^2 x\, dx = dt$

$$\Rightarrow \int \frac{\tan x\,\boxed{\sec^2 x\,dx}}{\tan^3 x+1}$$

$$\int \frac{t\ dt}{t^3+1}$$

$\alpha = \frac{-1}{3}$

$\beta = \frac{1}{6}$

$\gamma = \frac{1}{\sqrt{3}}$

$18\left(\cancel{\frac{-1}{3}} + \frac{1}{6} + \cancel{\frac{1}{3}}\right)$

$= 3$

$$\int \frac{t\,dt}{(t+1)(t^2-t+1)}$$

$$\frac{t}{(t+1)(t^2-t+1)} = \frac{A}{t+1} + \frac{Bt+C}{t^2-t+1}$$

$$t = A(t^2-t+1) + (Bt+C)(t+1)$$

$t=-1$: $-1 = 3A \Rightarrow \boxed{A = -1/3}$

$t=0$: $0 = \frac{-1}{3} + C \Rightarrow \boxed{C = 1/3}$

$t=1$: $1 = \frac{-1}{3} + 2B + \frac{2}{3}$ $\boxed{B = \frac{1}{3}}$

$$\int \frac{\frac{-1}{3}}{t+1} + \int \frac{\frac{1}{3}t + \frac{1}{3}}{t^2-t+1}$$

$$\Rightarrow \frac{-1}{3} \ln|t+1| + \frac{1}{3} \int \frac{\boxed{(t+1)}\,dt}{t^2-t+1}$$

$$+ \frac{1}{3} \int \frac{\frac{1}{2}(2t-1) + \frac{3}{2}}{t^2-t+1}$$

$$+ \frac{1}{6} \int \frac{2t-1}{t^2-t+1} + \frac{1}{2} \int \frac{dt}{\left(t-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$+ \frac{1}{6} \ln|t^2-t+1| + \frac{1}{2} \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2t-1}{\sqrt{3}}\right)$$

$$t+1 = A(2t-1) + B$$

$1 = 2A \qquad A = 1/2$

$1 = -A + B$

$$\boxed{3/2 = B}$$

Integration Of Trigonometric Functions

Integrals of Trigonometric Functions

Type - 1

$$\int \frac{dx}{a + b\sin^2 x} \Big/ \int \frac{dx}{a + b\cos^2 x} \Big/ \int \frac{dx}{a\sin^2 x + b\cos^2 x + c\sin x\cos x} \Big/ \frac{dx}{(a\cos x + b\sin x)^2}$$

Multiply N^r and D^r by $\sec^2 x$ or $\operatorname{cosec}^2 x$ and proceed

$$\int \frac{\boxed{dx\ \sec^2 x}}{(a + b\sin^2 x)\sec^2 x}$$

$$\int \frac{\sec^2 x\, dx}{a(1+\tan^2 x) + b\tan^2 x} \qquad \boxed{\tan x = t}$$

Integrals of Trigonometric Functions

Type - 2

$$\int \frac{dx}{a + b\sin x} \Big/ \int \frac{dx}{a + b\cos x} \Big/ \int \frac{dx}{a + b\sin x + c\cos x}$$

$$\int \frac{dx}{3 + 4\sin x}$$

$$\int \frac{2dt}{3(1+t^2) + 4(2t)}$$

Convert sinx and cosx into their corresponding tangent to half the angles and put $\tan\frac{x}{2} = t$

$dx \to 2dt$

$c \to c(1+t^2)$

$\sin x \to 2t$

$\cos x \to (1-t^2)$

$$\boxed{t = \tan\left(\frac{x}{2}\right)}$$

Integrals of Trigonometric Functions

Type - 3

$\frac{L}{L}$

$$\int \frac{a\sin x + b\cos x + c}{l\sin x + m\cos x + n}dx; \qquad N^r = A(D^r) + B\left(\frac{d}{dx}D^r\right) + C$$

$$\boxed{N = A(D) + B(D') + C}$$

$$\int \frac{3\sin x + 4\cos x - 5}{\sin x + \cos x - 1}$$

$$3S + 4C - 5 = A(S + C - 1) + B(C - S) + C$$

$3 = A - B$

$4 = A + B$

$-5 = -A + C$

$A =$

$B =$

$C =$

Integrals of Trigonometric Functions

Type - 4

$$\int \frac{x^2+1}{x^4+kx^2+1}dx \quad \text{or} \int \frac{x^2-1}{x^4+kx^2+1}dx$$

Divide N^r and D^r by x^2 and take suitable substitution

$$\int \frac{(x^2+1)\,dx}{x^4+kx^2+1}$$

$$\int \frac{\left(1+\frac{1}{x^2}\right)dx}{\left(x-\frac{1}{x}\right)^2+k+2}$$

$x-\frac{1}{x}=t$

$\left(1+\frac{1}{x^2}\right)dx=dt$

Integrals of Trigonometric Functions

Type - 5

$$\int \sin^m x \cdot \cos^n x \, dx$$

Jiski power Even hai $= t$

Approach:

$$\int (\sin x)^4 (\cos x)^2 \boxed{dx \ \cos x}$$

$\sin x = t$

$$\int t^4 (1-t^2)\, dt$$

$$\int t^4 dt - \int t^6 dt$$

$$\frac{t^5}{5} - \frac{t^7}{7} + C$$

(a) Substitute sin x = t, is n is odd;

(b) Substitute cos x = t, is m is odd

Integrals of Trigonometric Functions

Type - 5

$$\boxed{\int \sin^m x \cdot \cos^n x \, dx}$$

Approach:

$$\frac{(\sin x)^m (\cos x)^{n+m}}{(\cos x)^m}$$

(c) Substitute tan x = t, if m + n is a negative even integer.

Integrals of Trigonometric Functions

Type - 5

$$\int \sin^m x \cdot \cos^n x \, dx$$

Approach:

(d) If m and n are rational numbers and $\left(\frac{m+n-2}{2}\right)$ is a negative integer, then cot x = t or tan x = t

Integrals of Trigonometric Functions

Type - 6

$\sin x \pm \cos x = t$

$c - s$ ✓

$c \times s$ ✓

$$\int \frac{\cos x - \sin x}{\boxed{\sin x \cos x} + 2}\,dx$$

$(\cos x + \sin x)^2 = t^2$

$(-\sin x + \cos x)\,dx = dt$

$1 + 2\sin x \cos x = t^2$

$$\int \frac{dt}{\frac{t^2-1}{2} + 2}$$

$$\int \frac{2\,dt}{t^2+3}$$

If $\int \frac{\cos x-\sin x}{\sqrt{8-\sin 2x}}dx = a\sin^{-1}\left(\frac{\sin x+\cos x}{b}\right)+c,$

where c is a constant of integration, then the ordered pair (a,b) is equal to :

$(\sin x+\cos x)^2 = t^2 \longrightarrow 1+\sin 2x = t^2$

$(\cos x-\sin x)dx = dt$

[JEE Main 2021]

A. (1, -3)

B. (1, 3)

C. (-1, 3)

D. (3, 1)

$$\int \frac{dt}{\sqrt{8-(t^2-1)}} = \int \frac{dt}{\sqrt{9-t^2}}$$

$$= \sin^{-1}\left(\frac{t}{3}\right)+c$$

$$= 1\sin^{-1}\left(\frac{\sin x+\cos x}{3}\right)+c$$

The integral $\int \sec^{2/3} x \operatorname{cosec}^{4/3} x \, dx$ is equal to :

A. $-3 (\tan x)^{-1/3} + C$

B. $-3/4 \tan^{-4/3} x + C$

C. $-3 \cot^{-1/3} x + C$

D. $3 (\tan x)^{-1/3} + C$

(Here C is a constant of integration)

[JEE Main 2019]

$$\int \frac{dx}{\dfrac{(\cos x)^{2/3+\frac{4}{3}}(\sin x)^{4/3}}{(\cos x)^{4/3}}}$$

$$\int \frac{\sec^2 x\, dx}{(\tan x)^{4/3}} = \int \frac{dt}{t^{4/3}}$$

$$= \frac{t^{-\frac{4}{3}+1}}{-\frac{4}{3}+1} = \underline{-3 (\tan x)^{-1/3} + c}$$

Integrals of $\sqrt{\text{Quadratic}}$

(i) $\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left| x + \sqrt{x^2 - a^2} \right| + C$

(ii) $\int \sqrt{x^2 + a^2}\, dx = \frac{1}{2}x\sqrt{x^2 + a^2} + \frac{a^2}{2}\log\left| x + \sqrt{x^2 + a^2} \right| + C$

(iii) $\int \sqrt{a^2 - x^2}\, dx = \frac{1}{2}x\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

Integrals of Irrational Algebraic Function

Integrals of Irrational Algebraic Function

Unacademy Atoms

$L = t^2$

Type - 1 $\dfrac{1}{L\sqrt{L}}$

Type - 2 $\dfrac{1}{Q\sqrt{L}}$

Type - 3 $\dfrac{1}{L\sqrt{Q}}$

$\boxed{L = \frac{1}{t}}$

Type - 4 $\dfrac{1}{Q\sqrt{Q}}$

$\boxed{x = \frac{1}{t}}$

Summary

	Integral	Substitution
$\frac{1}{L\sqrt{L}}$ ✓	$\int \frac{dx}{(ax+b)\sqrt{px+q}}$	$Px + q = t^2$
$\frac{1}{Q\sqrt{L}}$	$\int \frac{dx}{(ax^2+bx+c)\sqrt{px+q}}$	$Px + q = t^2$
$\frac{1}{L\sqrt{Q}}$	$\int \frac{dx}{(ax+b)\sqrt{px^2+qx+r}}$;	$L = \frac{1}{t}$ $ax + b = 1/t$
$\frac{1}{Q\sqrt{Q}}$	$\int \frac{dx}{(ax^2+b)\sqrt{cx^2+d}}$	$x = 1/t$

Reduction formula

$$\int \sin^n x \, dx = \frac{-\cos x \cdot \sin^{n-1} x}{n} + \frac{n-1}{n} \int \sin^{n-2} x \, dx$$

$$\int \cos^n x \, dx = \frac{\sin x \cdot \cos^{n-1} x}{n} + \frac{n-1}{n} \int \cos^{n-2} x \, dx$$

Reduction formula

Reduction formula of $\int \tan^n x\, dx$

$$\boxed{I_n = \frac{\tan^{n-1}x}{n-1} - I_{n-2}, n \geq 2}$$

Reduction formula of $\int \cot^n x\, dx$

$$\boxed{I_n = -\frac{\cot^{n-1}x}{n-1} - I_{n-2}, n \geq 2}$$

Reduction formula

Reduction formula of $\int \sec^n x \, dx$

$$\boxed{I_n = \frac{\tan x \sec^{n-2} x}{n-1} + \frac{n-2}{n-1} I_{n-2}}$$

Reduction formula of $\int \operatorname{cosec}^n x \, dx$

$$\boxed{I_n = \frac{\cot x \operatorname{cosec}^{n-2} x}{-(n-1)} + \frac{n-2}{n-1} I_{n-2}}$$