Today's Targets

1. Introduction
2. Representation of Sets
3. Types of Sets
4. Operation on Sets
5. Laws on Algebra of Sets
6. Cardinal Number
7. Cartesian Product of Sets
8. PYQs

What is a Set

A well-defined collection of objects.

Examples

- The collection of vowels in English alphabets. ✓ {a, e, i, o, u}
- The collection of first five prime natural numbers. ~ {2, 3, 5, 7, 11}
- The collection of good cricket players of India. ✗
- The collection of **difficult** topics in mathematics. ✗
- The integers whose squares are less than 20. ~
- The set of most Naughty students of Arjuna ✗

Notations

$Z \subset N \rightarrow$ wrong
$N \subset Z \rightarrow$ correct

N	=	Set of all natural numbers = {1,2,3,4, ... }
W	=	Set of all whole numbers = {0,1,2,3, ...}
Z or I	=	set of all integers = {... − 3, −2, −1,0,1,2,3, ... }
Z^+	=	Set of all positive integers = {1,2,3, ... } = N
Z^-	=	Set of all negative integers = {−1, −2, −3, ... }
Q	=	The set of all rational numbers = $\left\{\frac{p}{q} : p, q \in I, q \neq 0\right\}$
R	=	The set of all real numbers
R − Q	=	The set of all irrational numbers

Representation of a Sets

Roster or Tabular Notation

Q- Write the Following Sets in Roaster form-

- *Set of all Natural numbers lying between 3 and 11* → {4, 5, 6, 7, 8, 9, 10}
- Set of all letters in the word '**INDEPENDENCE**'. {I, N, D, E, P, C}
- Set of all ODD natural numbers. {1, 3, 5, 7, 9, ---- ---}

Set Builder Form

Write the following sets in set builder form

1. $A = \{1, 4, 9, 16, 25, 36, 49\}$

Squares of first 7 natural Nos.

OR

$A = \{x : x = N^2$ where N is a natural no $1 \le N \le 7\}$

2. $B = \{1, 9, 25, 49, 81, 121\}$

$\{1^2, 3^2, 5^2, 7^2, 9^2, 11^2\}$, Squares of first 6 odd natural nos.

$A : x = (2N-1)^2$, $1 \le N \le 6$, $N \in N$

3. $C = \{2, 9, 28, 65, 126\}$

$\{1+1^3, 1+2^3, 1+3^3, 1+4^3, 1+5^3\}$

$x = 1 + N^3$, $1 \le N \le 5$

QUESTION

The set of Natural numbers which are multiples of 6 and less than 50 is written in set builder form as

$\{6, 12, 18, 24, 30, 36, 42, 48\}$

(A) $\{x: x \in N \text{ and } x \text{ is a multiple of } 6\}$ ✗

(B) $\{x: x = 6 \text{ and } 0 < x < 50\}$ ✗

(C) $\{x: x \text{ is a multiple of } 6 \text{ and } x \leq 50\}$ ✗

(D) $\{x: x \text{ is a multiple of } 6 \text{ and } 0 < x < 50, x \in N\}$ ✓

Types of Sets

Empty Set

ϕ

Null set

Singleton Set

Number of Elements = 1

Ex→ $A = \{a\}$

Finite Set

Infinite Set

(i) Let W be the set of the days of the week. (F)

(ii) Let S be the set of solutions of the equation $x^2 - 16 = 0$. $\Rightarrow x = \pm 4$

$\{-4, 4\}$

(iii) Let G be the set of points on a line. (I)

(iv) Set of Natural Numbers (I)

Equivalent Sets

$n(A) = n(B)$

$A = \{1, 2, 3, 4, 5\}$

$B = \{a, e, i, o, u\}$

Equal Sets

$A = \{1, 2, 3, 4, 5\}$

$B = \{2, 3, 5, 4, 1\}$

$C = \{2, 2, 3, 5, 1, 4\}$

Sub-Set & Super-Set

$A = \{1, 2, 3, 4, 5\}$

$B = \{1, 2, 3\}$, $X = \{1, 2, 3, 4, 5\}$

$X \subseteq A$ ✓

$B \subset A$

B is subset

A is super set

QUESTION

$A = \{x : x \text{ is a natural number and } x < 8\}$ and $B = \{1,2,3,4,5\}$. Which of the following is true?

(A) $A \subset B$

(B) $B \not\subset A$

(C) $B \in A$

(D) $B \subset A$ ✓

$A = \{1,2,3,4,5,6,7\}$

$B = \{1,2,3,4,5\}$

$B \subset A$

$1 \in B$

Difference Between $\in$ & $\subset$

$\in$ → belongs to / is an element

$\subset$ → Subset

$A = \{1, 2, 3, 4\}$

$1 \in A$ ✓

$2 \in A$ ✓

$3 \in A$ ✓

$4 \in A$ ✓

$\{1\} \subset A$ ✓

$\{1, 2\} \subset A$

$1, 2 \in A$

QUESTION

$\subset$ ✓ $\in$

Examine whether the following statements are true or false:

(i) $\{a, b\} \not\subset \{b, c, a\}$ (F)

(ii) $\{a, e\} \subset \{x : x \text{ is a vowel in the English alphabet}\}$ $\{a, e, i, o, u\} \rightarrow$ (T)

(iii) $\{1, 2, 3\} \subset \{1, 3, 5\}$ (F)

(iv) $\{a\} \subset \{a, b, c\}$ (T)

(v) $\{a\} \in \{a, b, c\}$ (F)

QUESTION

Element

$A\{1, 2, \{3,4\}, 5\}$

Let A = {1,2,{3,4},5}. State TRUE or FALSE?

(i) $\{3,4\} \subset A$ (F)

(ii) $\{3,4\} \in A$ (T)

(iii) $\{\{3,4\}\} \subset A$ (T)

(iv) $1 \in A$ (T)

(v) $1 \subset A$ (F)

(vi) $\{1,2,5\} \subset A$ (T)

(vii) $\{1,2,5\} \in A$ (F)

(viii) $\{1,2,3\} \subset A$ → (F)

(ix) $\phi \in A$ (F) ✓

(x) $\phi \subset A$ (T) ✓

(xi) $\{\phi\} \subset A$ (F)

"ϕ is a subset of Every set"

Number of Subsets

$A = \{1, 2\}$

Subsets $\rightarrow \{1\}, \{2\}, \{1, 2\}, \phi$

#

If a set A contains "n" distinct elements then number of Subsets is $= 2^n$

$${}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + \ldots + {}^nC_n = 2^n$$

$A = \{1, 2, 3, 4\}$

$\phi, \{1\}, \{2\}, \{3\}, \{4\}$

no of subsets : ${}^4C_0 + {}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4$

$= 2^4$

$A = \{a_1, a_2, a_3, \ldots a_n\}$

2^n

Proper Subset

$${}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_{n-1} = 2^n - 1$$

$A = \{1, 2\}$

Proper subsets $\rightarrow \phi, \{1\}, \{2\}$

QUESTION

If A = {a, b, c, d, e} then the number of proper subsets are?

A 32

B 33

C 31

D None of these

Ans $2^5 - 1$

$= 31$

Question (12th Jan 1st Shift 2019)

Let S = {1, 2, 3, ..., 100}. The number of non-empty subsets A of S such that the product of elements in A is even is [Ans. A]

(A) $2^{50}(2^{50}-1)$

(B) $2^{50}-1$

(C) $2^{50}+1$

(D) $2^{100}-1$

Total subsets − when no Even no is selected

$2^{100} - 2^{50}$

$2^{50}(2^{50}-1)$

$\{1, 3, 5, 7, \ldots 99\}$

2^{50}

Question (26th July 2nd Shift 2022)

Let A = {1, 2, 3, 4, 5, 6, 7} and B = {3, 6, 7, 9}. Then the number of elements in the set {C ⊆ A: C ∩ B ≠ ϕ} is _____. [Ans. 112]

$n(C)$ = Total Subsets of A − no of subsets when $C \cap B = \phi$

$2^7 - [2^4]$

$128 - 16.$

$= 112$

QUESTION [JEE Main 2020 (Sep.)]

Set A has m elements and Set B has n elements. If the total number of subsets of A is 112 more than total number of subsets of B, then the value of $m \cdot n$ is [Ans.

no. of subsets of A : 2^m

" " " B : 2^n

$2^m = 112 + 2^n$

$2^m - 2^n = 112$

M-1 HnT

$2^7 - 2^4 = 112$

$\Rightarrow$ $m = 7$ & $n = 4$

M-2

$2^m - 2^n = 112$

$2^n[2^{m-n} - 1] = 112$

$2^4[2^{m-n} - 1] = 2^4 \cdot 7$

$2^4[2^3 - 1]$

$n = 4$ & $m - n = 3$

$\frac{112}{4} = 28$

$4 \times 4 \times 7$

$2^2 \cdot 2^2 \cdot 7^1$

$2^4 \cdot 7$

QUESTION

Two finite sets have m and n elements respectively. The total number of subsets of first set is 992 more than the total number of subsets of the second set. Find the value of m & n.

$2^m = 992 + 2^n$

$2^m - 2^n = 992$

$2^{10} - 2^5 = 992$

$2^{10} = 1024$

$1024 - 32 = 992$

Ans:

$m = 10$

$n = 5$

Power Set

no of elements in P(A) is 2^n

Set of all subsets of A is called power set of A, It is denoted by P(A)

$A = \{a, b\}$

Power set of A $\{\phi, \{a\}, \{b\}, \{a, b\}\} = P(A)$

④

Find power set of the following sets

1) $A = \{a, b\}$ $\quad n(A) = 2 \rightarrow n(P(A)) = 2^2 = 4$

2) $A = \{a, \{b, c\}\}$

$P(A) = \{\phi, \{a\}, \{\{b, c\}\}, \{a, \{b, c\}\}\}$

QUESTION

If $A = \{1, 2, 4\}$ How many elements are present in

1) $P(A) = 2^3 = \boxed{8}$

2) $P(P(A)) = 2^8 = 256$

3) $PPP(A) = 2^{256}$

$A = \{a_1, a_2 \dots a_{10}\}$

$P(A) \longrightarrow 2^{10}$

$PP(A) \longrightarrow 2^{2^{10}}$

Operations of Sets

$A = \{1, 2, 3, 4\}$
$B = \{2, 3, 5, 6, 7\}$

1 Union of Sets

OR

$n(A \cup B) = \{1, 2, 3, 4, 5, 6, 7\}$

OR $n(A + B)$

2 Intersection of Sets

And

$A \cap B = \{2, 3\}$

QUESTION

$4^2 - 6 - 1$ $\quad 4^3 - 9 - 1$ $\quad 4^4 - 12 - 1$

$16 - 7 \quad 64 - 10 \quad 256 - 13 = 243$

If $X = \{4^n - 3n - 1; n \in \mathbb{N}\}$ and $Y = \{9(n-1): n \in \mathbb{N}\}$

Where $\mathbb{N}$ is a set of natural numbers. Then $X \cup Y$ is equal to:

$Y = \{0, 9, 18, 27, 36, \ldots\}$

$X = \{0, 9, 54, 243, \ldots\}$

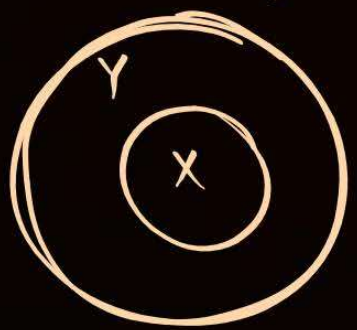
(A) X

(B) Y ✓

(C) N

(D) Y - X

$X \subset Y$



Difference of Sets

3 *Difference of Sets*

A – B = {all those elements of A which do not belong to B **}**

B – A = {all those elements of B which do not belong to A **}**

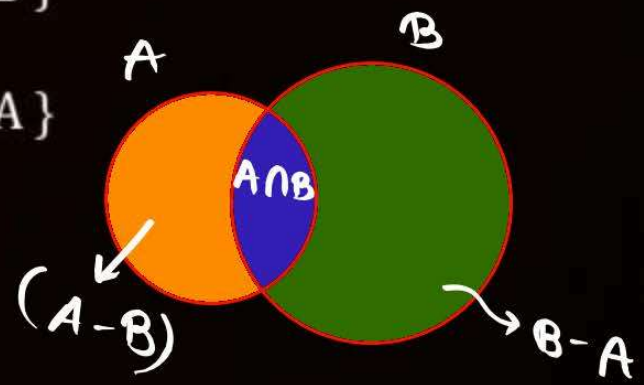
$(A-B)$

$A = \{1, 2, 3, 4\}$

$B = \{3, 4, 5, 6, 7\}$

$A - B = \{1, 2\}$

$B - A = \{5, 6, 7\}$



$(A-B) \Rightarrow A-(A \cap B)$

QUESTION

$A = \{x : x = 2n, n \leq 3, n \in \mathbb{N}\}$ $= \{2, 4, 6\}$

$B = \{x : x = 3n, n \leq 3, n \in \mathbb{N}\}$ $= \{3, 6, 9\}$

Find $A - B$: $\{2, 4\}$

Question (3rd Sept 1st Shift 2020)

Consider the two sets : A = {m ∈ R: both the roots of $x^2 - (m+1)x + m + 4 = 0$ are real} and B = [−3, 5). Which of the following is NOT true?

(A) $A - B = (-\infty, -3) \cup (5, \infty)$ (X)

(B) $A \cap B = \{-3\}$

(C) $B - A = (-3, 5)$

(D) $A \cup B = R$

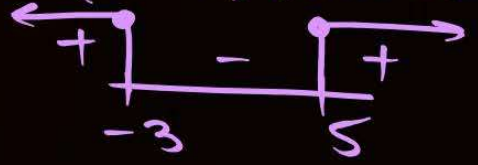
$A: D \geq 0$

$(m+1)^2 - 4(m+4) \geq 0$ [Ans. A]

$m^2 + 1 + 2m - 4m - 16 \geq 0$

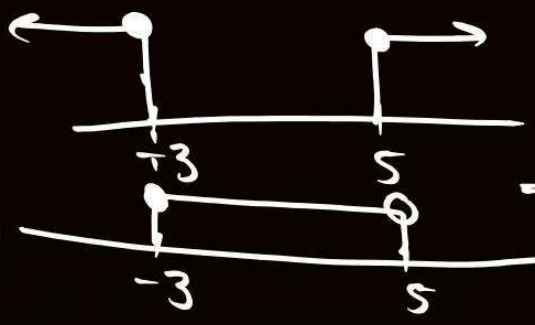
$m^2 - 2m - 15 \geq 0$

$(m-5)(m+3) \geq 0$



$A: m \in (-\infty, -3] \cup [5, \infty)$

$B: [-3, 5)$



$B - A = (-3, 5)$

$\Rightarrow A \cap B = \{-3\}$

$A - B: (-\infty, -3) \cup [5, \infty)$

QUESTION [JEE Main 2022 (June-II)]

[Ans. B]

Let $A = \{x \in R : |x + 1| < 2\}$ and $B = \{x \in R : |x - 1| \geq 2\}$. Then which one of the following statements is **NOT true**?

(A) $A - B = (-1, 1)$

(B) $B - A = R - (-3, 1)$

(C) $A \cap B = (-3, -1]$

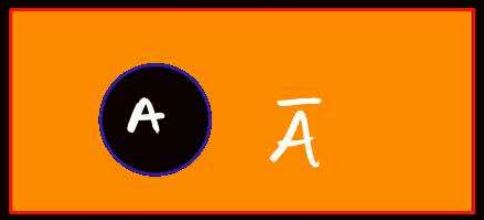
(D) $A \cup B = R - [1, 3)$

Complement of a Set

A^c or A' or $\bar{A}$

Ex: A $(-\infty, 4]$

$\bar{A}$ $(4, \infty)$



$A^c \rightarrow$ not A

Complement of a set

- $U^c = \phi$
- $\phi^c = U$
- $(A^c)^c = A$
- $A \cup A^c = U$
- $A \cap A^c = \phi$
- $A \cap B^c$
- $B \cap A^c$

Question (27th Aug I st Shift 2021)

$(A \cap B \cap C)^c \cap Z = \{-2, 1, 0, 1, 2, 3, 4, 5\}$

(8)

$2^8 = 256$

[Ans. 256]

If $A = \{x \in R: |x-2| > 1\}$. $B = \{x \in R: \sqrt{x^2-3} > 1\}$. $C = \{x \in R: |x-4| \geq 2\}$ and Z is the set of all integers, then the number of subsets of the set $(A \cap B \cap C)^c \cap Z$ is _____.

$|x-2| > 1$

$\Rightarrow x-2 > 1$ or $x-2 < -1$

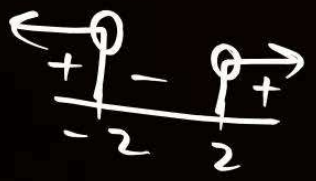
A $\boxed{x > 3 \text{ or } x < 1}$

B: $\sqrt{x^2-3} > 1$

$\Rightarrow x^2-3 > 1$

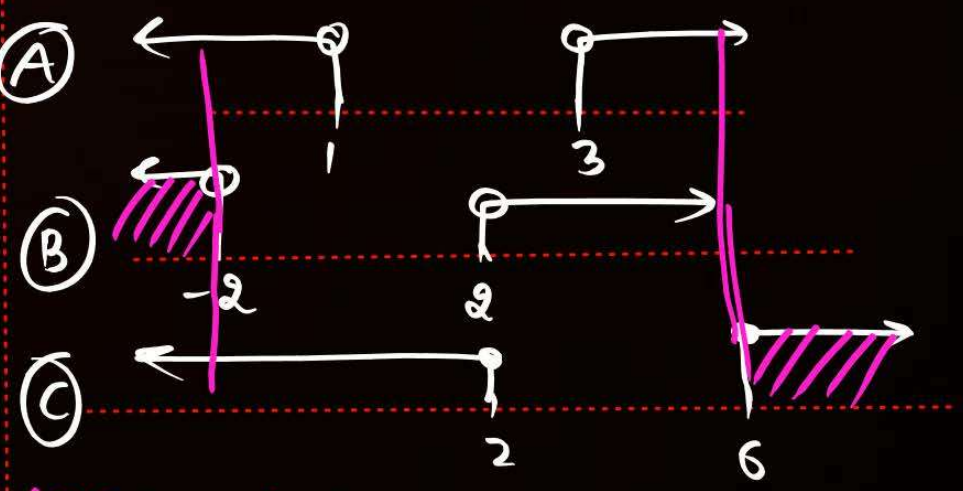
$x^2-4 > 0$

$(x-2)(x+2) > 0$



C: $x-4 \geq 2$ or $x-4 \leq -2$

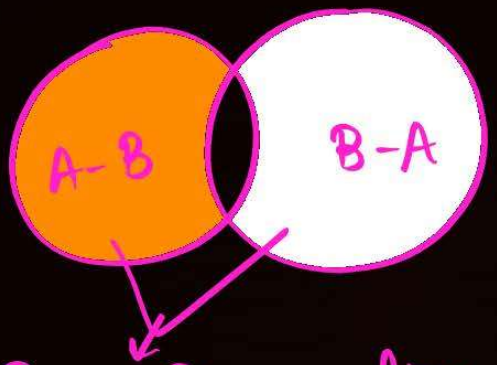
$x \geq 6$ or $x \leq 2$



$A \cap B \cap C = (-\infty, -2) \cup [6, \infty)$

$(A \cap B \cap C)^c = [-2, 6)$

Symmetric Difference



$A \Delta B$ = Symm. diff

$$n(A \Delta B) = n(A \cup B) - n(A \cap B)$$

Laws of Algebra of Sets

Commutative Law

$A \cup B = B \cup A$

$A \cap B = B \cap A$

Associative Law

$(A \cup B) \cup C = A \cup (B \cup C)$

$(A \cap B) \cap C = A \cap (B \cap C)$

Identity Law

$A \cap U = A$

$A \cup \phi = A$

Complement Law

$A \cup A' = U$

$A \cap A' = \phi$

$(A')' = A$

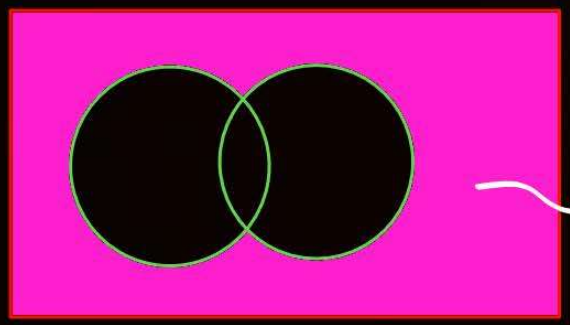
Laws of Algebra of Sets

De-Morgan's Law

1) $\overline{(A \cup B)} = (\bar{A} \cap \bar{B})$

$\overline{(A \cap B)} = \bar{A} \cup \bar{B}$

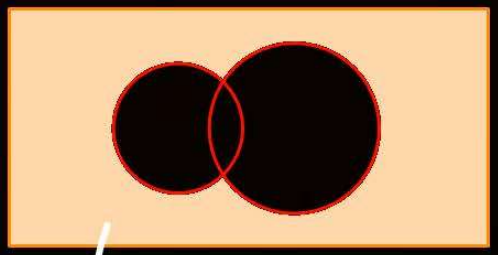
LHS



LHS

$\overline{A \cup B}$

RHS



$\bar{A} \cap \bar{B}$

Laws of Algebra of Sets

Distributive Law

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
i.e. union is distributive over intersection.

(ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
i.e. Intersection is distributive over Union.

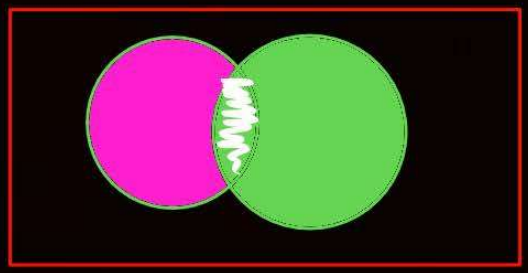
Cardinal Number

$A = \{1, 2, 3, 4, 5\}$

$n(A) = 5$

For 2 Sets

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$



QUESTION

Let A and B be two sets such that $n(A) = 24, n(A \cup B) = 46$ and $n(A \cap B) = 8$. Find

(i) $n(B)$ $= 30$

(ii) $n(A - B) = 16$

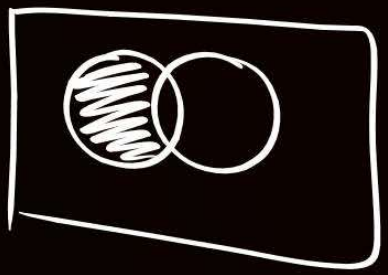
(iii) $n(B - A) = 30 - 8$
$= 22$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$46 = 24 + n(B) - 8$

$46 = 16 + n(B)$

$n(B) = 30$



QUESTION

$n(M \cup C) = 20$

$n(M) = 12$

In a school there are 20 teachers who teach math's or chemistry. Out of these 12 teach math and 4 teach both math and chemistry. How many teach only chemistry?

$n(M \cap C) = 4$

$n(C) = ?$

A) 10

B) 12

C) 8 ✓

D) None of these

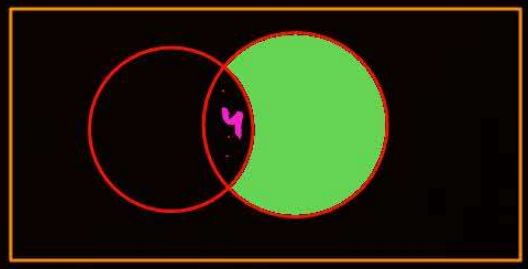
$n(M \cup C) = n(M) + n(C) - n(M \cap C)$

$20 = 12 + n(C) - 4.$

$20 = 8 + n(C)$

$\boxed{n(C) = 12}$

$n(\text{only chem}) = 12 - 4 = (8)$



2nd Method

$\underbrace{n(M \cup C) - n(M)}_{20 - 12 = (8)}$

QUESTION [Main April, 2019]

[Ans. D]

The are two newspapers which are published in a city, namely A and B. It is known that 25% of the city population reads A and 20% reads B while 8% reads both A and B. Further 30% of those who read A but not B, read advertisement and 40% of those who read B but not A also read advertisements, while 50% of those who read both A and B read advertisements. What is the percentage of the population who read the advertisements?

(A) 13.7 %

(B) 14.9 %

(C) 14.7 %

(D) 13.9 % ✓

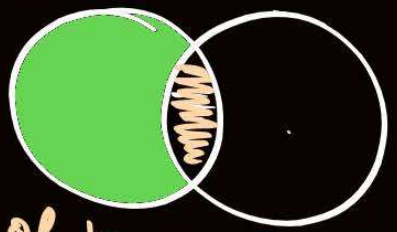
$n(A) = 25$

$n(B) = 20$

$n(A \cap B) = 8$

Only $A = 17$

Only $B = 12$



Ad: 30% of 17 + 40% of 12 + 50% of 8

$= \frac{30}{100} \times 17 + \frac{40}{100} \times 12 + \frac{50}{100} \times 8$

$\frac{51 + 48 + 40}{10} = \frac{139}{10} = 13.9\%$

QUESTION [JEE Main 2015 (Online)]

HW

In a certain town, 25% of the families own a phone and 15% own a car; 65% families own neither a phone nor a car and 2,000 families own both a car and a phone. Consider the following three statements : [Ans. C]

(A) 5% families own both a car and a phone

(B) 35% families own either a car or a phone

(C) 40,000 families live in the town

Then,

A. Only (A) and (C) are correct.

B. Only (B) and (C) are correct.

C. All (A), (B) and (C) are correct.

D. Only (A) and (B) are correct.

QUESTION [JEE Main 2020 (Jan.)]

$A \cap B = \{14, 28, 42\}$

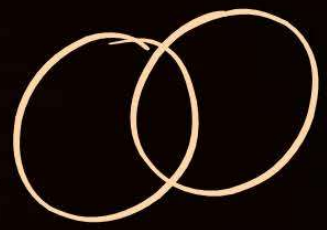
Let $X = \{n \in N: 1 \leq n \leq 50\}$. If $A = \{n \in X: n$ is a multiple of $2\}$; $B = \{n \in X: n$ is a multiple of $7\}$, then the number of elements in the smallest subset of X containing both A and B is

[Ans.29]

$X = \{1, 2, 3, \ldots 50\}$

$A = \{2, 4, 6, \ldots 50\} \Rightarrow n(A) = 25$

$B = \{7, 14, 21, \ldots 49\} \Rightarrow n(B) = 7$



$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$25 + 7 - 3 = 25 + 4 = 29$

QUESTION [JEE Main 2023 (Jan.-I)]

Number of 4-digit numbers that are less than or equal to 2800 and either divisible by 3 or by 11, is equal to _____. [Ans. 710]

$n(3) = \left[\frac{2800}{3}\right] - \left[\frac{999}{3}\right] =$

$n(11) = \left[\frac{2800}{11}\right] - \left[\frac{999}{11}\right] =$

$n(3 \cap 11) = \left[\frac{2800}{33}\right] - \left[\frac{999}{33}\right] =$

$n(3 \cup 11) = n(3) + n(11) - n(3 \cap 11)$

1 to 100 $\longrightarrow$ $n(2) = 50$

1 to 100 $\longrightarrow$ $n(3) = 33$

1 to 999 $\longrightarrow$ $n(3) = \left[\frac{999}{3}\right]$

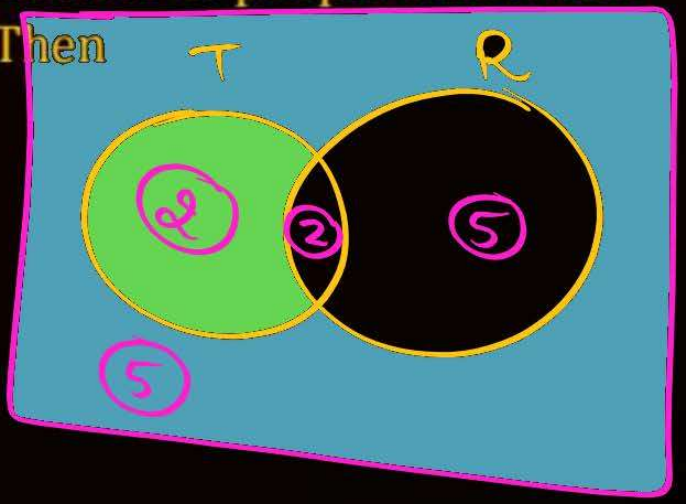
1 to 2800 $\longrightarrow$ $n(3) = \left[\frac{2800}{3}\right]$

QUESTION

$T \cup R = 14 - 5 = 9$

Out of 14 people, twelve said that it was not the case that they watched T.V. but did not listen to the radio. Also, for nine people it is ~~not~~ the case that they do not watch T.V. and do not listen to the radio. Finally, seven people either watch T.V. or listen to the radio but do not do both. Now let A be number of people that watch T.V. and B be the number of people that listen radio. Then [Ans.

$(\bar{T} \cap \bar{R})$



(A) A = 2

(B) B = 5

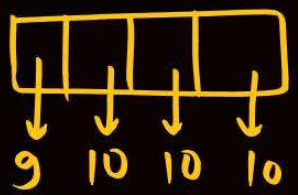
(C) A = 4 ✓

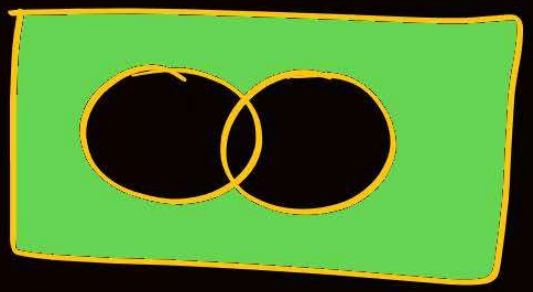
(D) B = 7 ✓

Question [JEE Main 2021)]

The number of 4-digit numbers which are neither multiple of 7 nor multiple of 3 is .

[Ans. 5143]





$9000 - n(3 \cup 7)$

$$n(3) = \left[\frac{9999}{3}\right] - \left[\frac{999}{3}\right]$$

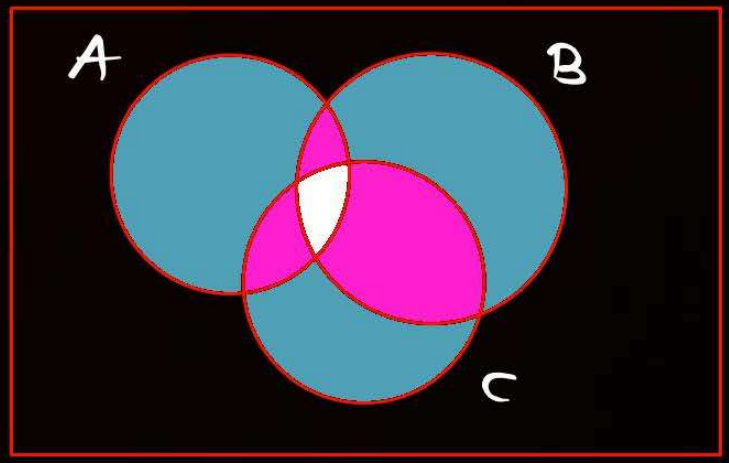
$$n(7) = \left[\frac{9999}{7}\right] - \left[\frac{999}{7}\right]$$

$$n(21) = \left[\frac{9999}{21}\right] - \left[\frac{999}{21}\right]$$

Cardinal Number

For 3 Sets

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$



QUESTION

In a survey of 60 people, it was found that 25 people read newspaper H, 26 read newspaper T, 26 read newspaper I, 9 read both H and I, 11 read both H and T, 8 read both T and I, 3 read all three newspapers. Find:

(i) the number of people who read at least one of the newspapers.

(ii) the number of people who read exactly one newspaper.

$n(H \cup T \cup I)$

$= 25+26+26-(9+11+8)+3$

$= 25+24+3 = 25+25+2 = 52$

$n(H) = 25$

$n(T) = 26$

$n(I) = 26$

$n(H \cap I) = 9$

$n(H \cap T) = 11$

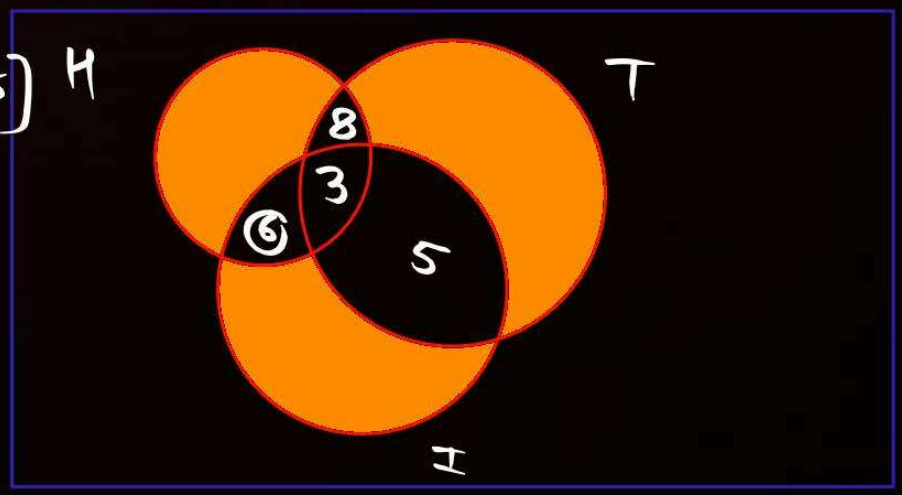
$n(T \cap I) = 8$

$n(H \cap T \cap I) = 3$

Ans $52-(8+3+6+5)$

$52-[22]$

$= \boxed{30}$



QUESTION (11th April Shift 2023)

[Ans. C]

An organization awarded 48 medals in event 'A', 25 in event 'B' and 18 in event 'C'. If these medals went to total 60 men and only five men got medals in all the three events, then, how many received medals in exactly two of the three events?

(A) 10

(B) 9

(C) 21 ✓

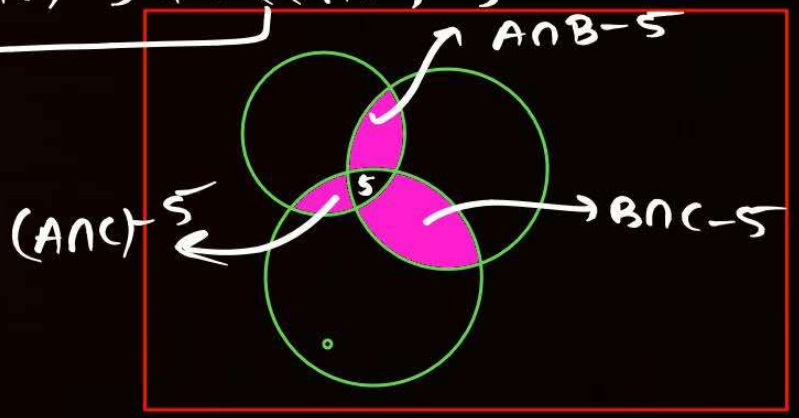
(D) 15

$n(A)=48$
$n(B)=25$
$n(C)=18$
$n(A\cap B\cap C)=5$
$n(A\cup B\cup C)=60$

$n(A\cap B)-5+n(B\cap C)-5+n(C\cap A)-5$

$36-15$

$=21$



$60=48+25+18-(n(A\cap B)+n(B\cap C)+n(C\cap A))+5$

$n(A\cap B)+n(B\cap C)+n(C\cap A)=48+25+18+5-60$

$=18+18=36$

QUESTION

From 50 students taking examinations in Mathematics, Physics and Chemistry, each of the student has passed in at least one of the subject, 37 passed Mathematics, 24 Physics and 43 Chemistry. At most 19 passed Mathematics and Physics, at most 29 Mathematics and Chemistry and at most 20 Physics and Chemistry. What is the largest possible number that could have passed all three examination? [Ans. 14]

$x = n(M \cap P \cap C)_{max} = ?$

$n(M \cup P \cup C) = 50$

$n(M) = 37$

$n(P) = 24$

$n(C) = 43$

$n(M \cap P) \leq 19$

$n(M \cap C) \leq 29$

$n(P \cap C) \leq 20$

$50 = 37 + 24 + 43 - [n(P \cap C) + n(C \cap M) + (M \cap P)] + x$

K

$x = (50 - 37 - 24 - 43) + K$

$x = K - 54$

$x_{max} \Rightarrow K_{max} - 54$

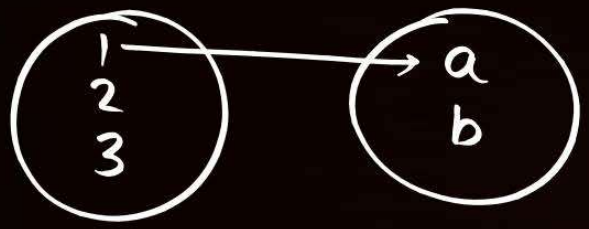
$\Rightarrow x_{max} = 19 + 29 + 20 - 54$

$= 68 - 54$

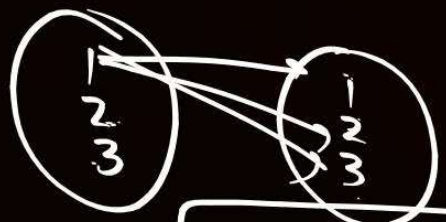
$= 14$

Cartesian Product of 2 Sets

$A \times B$



$A \times B = \{(1,a), (1,b), (2,a), (2,b), (3,a), (3,b)\}$



$n(A \times B) = n(A) \cdot n(B)$

Cartesian Product of 2 Sets

Property -1

In general $A \times B \neq B \times A$

Property -2

$n(A \times B) = n(A).n(B)$.

Property -3

$A \times A \times A = \{a, b, c) : a, b, c \in A\}$.

Here (a,b,c) is called an ordered triplet.

QUESTION [JEE Main 2019]

Let Z be the set of integers. If $A = \{x \in Z: 2^{(x+2)(x^2-5x+6)} = 1\}$ and $B = \{x \in Z: -3 < 2x - 1 < 9\}$, then the number of subsets of the set A × B, is:

(A) 2^{18}

(B) 2^{15}

(C) 2^{12}

(D) 2^{10}

$-3 < 2x-1 < 9$

$-2 < 2x < 10$

$-1 < x < 5$

$B = \{0, 1, 2, 3, 4\}$

$2^{(x+2)(x^2-5x+6)} = 2^0$

$(x+2)(x-3)(x-2) = 0$

$\Rightarrow x = -2, 2 \text{ or } 3$

$A = \{-2, 2, 3\}$

$n(A \times B) = n(A) \cdot n(B)$

$= 3 \times 5 = \boxed{15}$

Subsets $= 2^{15}$

QUESTION [2023 (08 April Ist Shift]

[Ans. C]

Let the number of elements in sets A and B be five and two respectively. Then the number of subsets of A × B each having at least 3 and at most 6 elements is

$n(A) = 5$

$n(B) = 2$

$n(A \times B) = 10$

$^{10}C_3 + {}^{10}C_4 + {}^{10}C_5 + {}^{10}C_6$

(A) 752

(B) 782

(C) 792

(D) 772

Range Problems on Cardinal Number

↳ For 2 sets → $n(A \cup B)$ / $n(A \cap B)$ → Range.

↳ For 3 sets → $n(A \cap B \cap C)$.

QUESTION

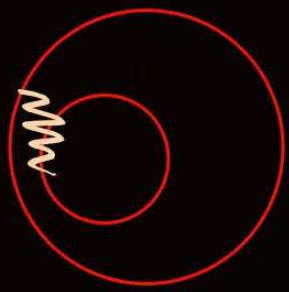
$n(A \cap B)_{max} = \min\{n(A), n(B)\}$

In a survey it was found that, 63% Indians like mangoes whereas 76% Like apples. If x % of Indians like both Apples and Mangoes then identify the correct Statement

$n(M) = 63$

$n(A) = 76$

$n(M \cap A) = x$



$n(M \cup A) = 63 + 76 - x$.

$x = 139 - n(M \cup A)$

$x_{min} \Rightarrow n(M \cup A) \Rightarrow Max$

$\Rightarrow x_{min} = 39$

$x_{max} = 63$

(A) $x = 39$

(B) $x = 63$

(C) $39 \le x \le 63$ ✓

(D) Cannot be determined

Question (5th Sept 1st Shift 2020)

A survey shows that 73% of the persons working in an office like coffee, whereas 65% in tea, If x denotes the percentage of them, who like both coffee and tea, then x cannot be

[Ans. B]

$x \in [38, 65]$

(A) 63

(B) 36 ✓

(C) 54

(D) 38

$n(C) = 73$

$n(T) = 65$

$x = n(C \cap T)$

$x_{min} = 73 + 65 - 100$

$= 138 - 100$

$x_{min} = 38$

$x_{max} = 65$

Question (26th Aug 1st Shift 2021)

[Ans. A]

Out of all the patients in a hospital 89% are found to be suffering from heart ailment and 98% are suffering from lungs infection. If K % of them are suffering from both ailments, then K can not belong to the set

(A) {79, 81, 83, 85} ✓

(B) {84, 87, 90, 93}

(C) {80, 83, 86, 89}

(D) {84, 86, 88, 90}

$K_{min} = 89 + 98 - 100 = 87$

$K_{max} = 89$

$K \in [87, 89]$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A \cup B) = 9 - n(A \cap B)$

QUESTION

If A and B be two sets containing 3 and 6 elements respectively, what can be the maximum number of elements in A ∪ B? $n(A \cup B)_{max} = 9$

(A) 9 ✓ (B) 6 (C) 3 (D) Cant say

$n(A \cap B)_{max} = 3$

$\Rightarrow n(A \cup B)_{min} = 9 - 3 = 6$

QUESTION

Find also, the minimum number of elements in A ∪ B.

(A) 9 (B) 6 ✓ (C) 3 (D) Cant say

QUESTION

In a Class, 70% of the Students Like Apples, 80% like bananas, 85% like Mangoes. If x % like all the Three fruits, then minimum value of x

[Ans. D]

$n(A) = 70\%$
$n(B) = 80\%$
$n(M) = 85\%$

$\longrightarrow n(A \cap B)_{min} = 80 + 70 - 100 = 50$

$n(A \cap B \cap M)_{min}$

$(A \cap B) \cap C$

$Ans = 50 + 85 - 100$
$= 35\%$

(A) 25%

(B) 20%

(C) 30%

(D) 35%

QUESTION

In a Class, 70% of the Students Like Apples, 80% like bananas, 85% like Mangoes, 90% Like Oranges. If x % like all the four fruits, then minimum value of x

[Ans. A]

$n(A \cap B \cap M \cap O)_{min} = ?$

$n(A) = 70$
$n(B) = 80$
$n(M) = 85$
$n(O) = 90$

$\rightarrow$ (35)

$(90 + 35) - 100 = 25\%$

(A) 25%

(B) 20%

(C) 30%

(D) 35%


PYQs

QUESTION [2023 (06 April 2nd Shift]

[Ans. A]

In a group of 100 persons 75 speak English and 40 speak Hindi. Each person speaks at least one of the two languages. If the number of persons who speak only English is α and the number of persons who speak only Hindi is β, then the eccentricity of the ellipse $25(\beta^2x^2 + \alpha^2y^2) = \alpha^2\beta^2$ is

(A) $\frac{\sqrt{119}}{12}$

(B) $\frac{\sqrt{117}}{12}$

(C) $\frac{3\sqrt{15}}{12}$

(D) $\frac{\sqrt{129}}{12}$

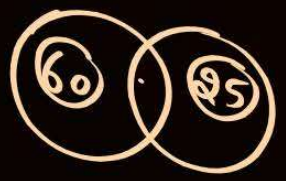
$n(E) = 75$

$n(H) = 40$

$n(E \cup H) = 100$

$100 = 75 + 40 - n(E \cap H)$

$\Rightarrow n(E \cap H) = 15$



$\alpha = 60, \beta = 25$

$25\left[\frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2}\right] = 1$

$25\left[\frac{x^2}{60^2} + \frac{y^2}{25^2}\right] = 1$

$\frac{x^2}{12^2} + \frac{y^2}{25} = 1$

$a^2 = 144, b^2 = 25$

$b^2 = a^2(1 - e^2)$

$25 = 144(1 - e^2)$

$\Rightarrow 1 - e^2 = 25/144$

$e^2 = 119/144$

$e = \frac{\sqrt{119}}{12}$

QUESTION [JEE Main 2019 (Jan.)]

Ans = 140 − n(M ∪ C ∪ P)

140 − 102 = 38

In a class of 140 students numbered 1 to 140, all even numbered students opted Mathematics course, those whose number is divisible by 3 opted Physics course and those whose number is divisible by 5 opted Chemistry course. Then the number of students who did not opt for any of the three courses is

[Ans. D]

(A) 102

(B) 42

(C) 1

(D) 38 ✓

$n(M) = \left[\frac{140}{2}\right] = 70$

$n(P) = \left[\frac{140}{3}\right] = 46$

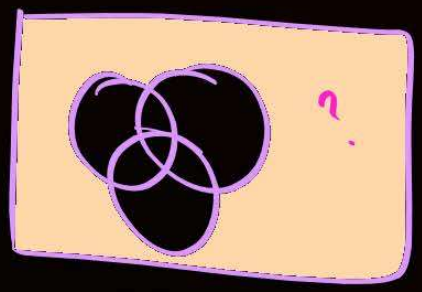
$n(C) = \left[\frac{140}{5}\right] = 28$

$n(M \cap P) = \left[\frac{140}{6}\right] = \left[\frac{70}{3}\right] = 23$

$n(P \cap C) = \left[\frac{140}{15}\right] = 9$

$n(M \cap C) = \left[\frac{140}{10}\right] = 14$

$n(M \cap P \cap C) = \left[\frac{140}{30}\right] = 4$



70 + 46 + 28 − (23 + 9 + 14) + 4

70 + 32 = 102

QUESTION [JEE ADV 2022]

$900 - (x+y+z+30) = 720$ ✓

In a study about a pandemic, data of 900 persons was collected. It was found that [Ans. 0.80]

190 persons had symptom of fever, $= n(F)$

220 persons had symptom of cough, $= n(C)$

220 persons had symptom of breathing problem, $= n(B)$

330 persons had symptom of fever or cough or both, $\Rightarrow n(F \cup C) = 330$

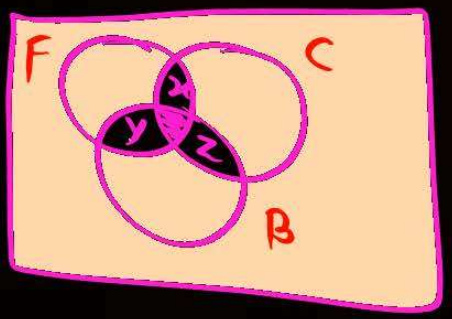
350 persons had symptom of cough or breathing problem or both, $= n(C \cup B)$

340 persons had symptom of fever or breathing problem or both, $= n(F \cup B)$

30 persons had all three symptoms (fever, cough and breathing problem). $= n(F \cap C \cap B)$

If a person is chosen randomly from these 900 persons, then the probability that the person has at most one symptom is _____

$P = \frac{720}{900} = \frac{8}{10} = 0.8$



$n(F \cup C) = 190 + 220 - (x+30) = 330 \Rightarrow$

$n(C \cup B) = 220 + 220 - (z+30) = 350$

$n(F \cup B) = 190 + 220 - (y+30) = 340$

Question (15th April 1st Shift 2023)

The number of elements in the set $\{n \in \mathbb{N} : 10 \leq n \leq 100$ and $3^n - 3$ is a multiple of $7\}$ is _____. [Ans. 15]

$3^n - 3 = 7K$

$\boxed{3^n = 7K+3}$

$3^n/7 \rightarrow$ Remainder $\neq 3$

13, 19, 25, - - - - T_n

$T_n \leq 100$

$13 + (n-1)6 \leq 100$

$6n + 7 \leq 100$

$6n \leq 93$

$n \leq 31/2$

	R
$3^1 \rightarrow 3/7$	3
$9/7$	2
$27/7$	6
$81/7$	4
$243/7$	5
$729/7$	1
$3^7 \leftarrow 2187/7$	3
3^{13}	

Question (24th June 2nd Shift 2022)

Total sum − n(A∪B)
5050 − 51×67
1633

$2^3 \cdot 3$

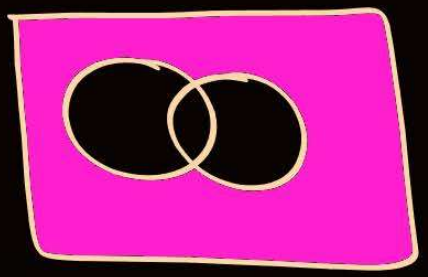
The sum of all the elements of the set $\{\alpha \in \{1,2,\ldots.100\}: \text{HCF}(\alpha, 24) = 1$ is [Ans. 1633]

"$\alpha \rightarrow$ is neither div by 2 nor by 3"

A : div by 2 $n(A) = (2+4+6+\ldots+100) =$

B : div by 3 ; $n(B) = (3+6+9+\ldots-99)$

$n(A \cap B) = (6+12+18+\ldots+96)$



$n(A) = \frac{50}{2}(2+100) = 50 \times 102$

$n(B) = \frac{33}{2}[3+99] = 33 \times 102$

$n(A \cap B) = \frac{16}{2}[6+96] = 16 \times 102$

$n(A \cup B) = 50 \times 51 + 33 \times 51 - 16 \times 51$

$= 51[50+33-16] = 51 \times (50+17) = 51 \times 67$

Question (26th June 1st Shift 2022)

$45 = 5 \times 3^2$

Let $A = \{n \in N : \text{H.C.F. } (n, 45) = 1\}$ and
Let $B = \{2k : k \in 1, 2, \ldots, 100\}\}$. Then the sum of all the elements of $A \cap B$ is

[Ans. 5264]

Question (22nd July 2nd Shift 2021)

HW

The sum of all the elements in the set {n ∈ { 1, 2, ..., 100}| H.C.F. of n and 2040 is 1} is equal to _____ .

[Ans. 1251]