

# Mathematical Reasoning & PMI

#### **Contrapositive and Converse Statements**

- The contrapositive of a statement  $p \Rightarrow q$  is the statement  $q \Rightarrow p$
- The converse of a statement  $p \Rightarrow q$  is the statement  $q \Rightarrow p$

# **Compound Statement**

Many mathematical statements are obtained by combining one or more statements using some connecting words like "and", "or" etc. those statement are called a "Compound Statement."

#### Sentence

A sentence is called a mathematically acceptable statement if it is either true or false but not both.

#### Negation

A statement which is formed by changing the true value of a given statement by using the word like 'no', 'not' is called negation of given statement.

# **Implications**

These are statements with word "if then", "only if" and "if and 05 only if". if p then q is the same as following:

- p implies q is denoted by p⇒ q, then symbol⇒ stands for implies
  - p is a sufficient condition for q. then symbol⇒
    - p only if q q is a necessary condition for p
      - · ~q implies ~p

# **Truth Table for Logical Operations**

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Conjunction operation:

-		-		
р	σ	p∧q		
Т	T	Т		
Т	F	F		
F	Τ	F		
F	F	F		

Disjunction operation:				
р	q	p∨q		
Т	Т	Т		
Т	F	Т		
F	T	Т		
F	F	F		

gation	n: I	mplic	
р	~p		р
Т	F		Т
Т	F		Т
F	Т		F
Е	т		F

ation operation:

p⇒q

В	Biconditional operation:					
	р	q	p⇔q			
	Т	Т	Т			
	Т	F	F			
	F	Т	F			
	F	F	Т			

# **Additional Important Points**

•  $p \Rightarrow q = \sim p \vee q$ 

 $\cdot \sim (p \Rightarrow q) = \sim (\sim p \lor q) = p \land (\sim q)$ 

•  $p \Leftrightarrow q = (p \Rightarrow q) \land (q \Rightarrow p)$ 

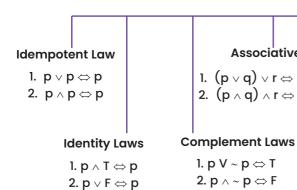
 $\bullet \sim (p \Leftrightarrow q) = (p \land \sim q) \lor (q \land \sim p)$ 

•  $(p \Leftrightarrow q) \Leftrightarrow r = p \Leftrightarrow (q \Leftrightarrow r)$ 

# **General Logical Equivalences**

It comprises the following laws:

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3.  $p \land F \Leftrightarrow F$ 

**Associative Law** 

1.  $(p \lor q) \lor r \Leftrightarrow p \lor (q \lor r)$ 2.  $(p \land q) \land r \Leftrightarrow p \land (q \land r)$ 

1.  $p V \sim p \Leftrightarrow T$ 

2.  $p \land \sim p \Leftrightarrow F$ 

3.  $\sim$  T  $\Leftrightarrow$  F

**4.** ∼ **F** ⇔ **T** 

2.  $p \land q \Leftrightarrow q \land p$ 

2.  $p \land (p \lor q) \Leftrightarrow p$ 

**Absorption Law** 1.  $p \lor (p \land q) \Leftrightarrow p$ 

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Commutative Law

1.  $p \lor q \Leftrightarrow q \lor p$ 

**Involution Law** 

1.  $p \sim (\sim p) \Leftrightarrow p$ 

Distributive Law

1.  $p \lor (q \land r) \Leftrightarrow (p \lor q) \land (p \lor r)$ 2.  $p \land (q \lor r) \Leftrightarrow (p \land q) \lor (p \land r)$ 

De-Morgan's Law

1.  $q \sim (p \vee q) \Leftrightarrow \sim p \wedge \sim q$ 2.  $q \sim (p \wedge q) \Leftrightarrow \sim p \vee \sim q$ 

#### **Tautology and Fallacy**

- A tautology asserts that every possible interpretation has only one output, namely true.
- Fallacy implies an assertion of false in every possible interpretation.

NOTE: To evaluate tautology and fallacy, we can adapt the concept of the truth table that includes every possible valuation.

р	q	$p \Rightarrow q$	$q \Rightarrow p$	$(p \Rightarrow q) \lor (q \Rightarrow p)$	$\sim \{(p \Rightarrow q) \lor (q \Rightarrow p)\}$
Т	Т	Т	Т	Т	F
Т	F	F	Т	Т	F
F	Т	T	F	Т	F
F	F	T	T	T	F

#### **Principle of Mathematical Induction**

**Base Case:** The given statement is correct for first natural number that is, for n=1, p(1) is true.

**Inductive Step:** If the given statement is true for any natural number like n=k then it will be correct for n=k+1 also that is, if p(k) is true then p(k + 1) will also be true.

The first principle of mathematical induction says that if both the above steps are proven then p(n) is true for all natural numbers.



