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Exercise 2: Programming Distributed Systems (Summer 2025)

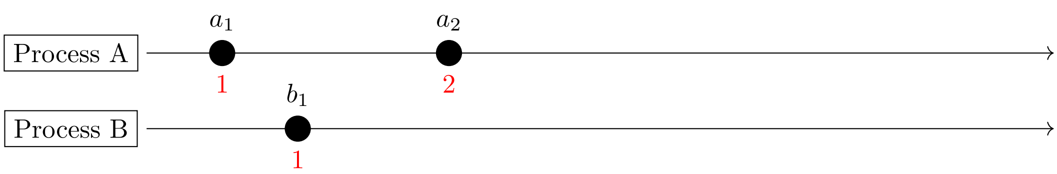
Deadline: 13.05.25 AOE

Submission

- You need a team and a Gitlab repository for this exercise sheet.
- In your Git repository, create a branch for this exercise sheet (for example with `git checkout -b ex2`)
- Create a folder named "ex2" in your repository and add your solutions to this folder.
- Test your submission with the provided test cases[1]. Feel free to add more tests, but do not change the existing test cases.
- Make use of Dialyzer annotations to improve your code quality

1 Time and causality

- Give an example execution that shows that for two timestamps from Lamport clocks, $C(e_1) < C(e_2)$ does not imply that $e_1 \rightarrow e_2$.
- Given a configuration of three servers S^1, S^2, S^3 and their current corresponding vector-clocks $S^1_C = [1, 3, 2]$ $S^2_C = [1, 4, 2]$ $S^3_C = [1, 0, 3]$. Is such a configuration possible? If yes, draw an example execution. If no, explain why.



$t(b_1) = 1 < 2 = t(a_2)$, but $b_1 \not\rightarrow a_2$.

Not possible. S_3 receives an event from S_1, but S_1 also receives at least one event from S_3. Locally, the vector clock increases on sending and on receiving events . Therefore, the frist component of S_1 cannot be 1. The configuration is impossible.

2 Implementing Vector Clocks

A vector clock is a mapping from processes to positive integers[2]. Implement a module named `Vectorclock` with the following functions:

- `new()` creates a new vector clock, where all processes have value 0.
- `increment(vc, p)` increments the entry of process `p` by 1.

[1] You can use `mix test` for executing the tests

[2] In the literature it is often assumed that processes are numbered which allows to write down clocks like $[4, 7, 3]$ or $\begin{pmatrix}4\\7\\3\end{pmatrix}$ instead of the longer $\{p_1 \mapsto 4, p_2 \mapsto 7, p_3 \mapsto 3\}$. However, in this exercise we do not assume that the number of processes is known and arbitrary terms can be used as process names.

- `get(vc, p)` returns the value for process `p`.
- `leq(vc1, vc2)` checks, whether `vc1` is less than or equal to `vc2`. This is the case, iff $\forall p.\ get(vc_1, p) \leq get(vc_2, p)$.
- `merge(vc1, vc2)` merges two vector clocks by computing their least upper bound (the smallest vector clock `v`, such that $vc_1 \leq v$ and $vc_2 \leq v$).

Feel free to add more tests, but do not change the existing test cases.

3 Testing Vector Clocks

Specify at least 3 invariants that should hold for your vector clock implementation. The tests should be using these invariants, derive at least 2 test cases for each invariant and for the following example invariant:

1. $new() === new()$

Example invariants:

- $\forall v, p$: $v \leq increment(v, p)$
  - Increment by one strictly increases size
- $\forall v$: $new() \leq v$
  - A new vectorclock is smaller or equal to any other vectorclock
- $\forall v, v_2$: $v \leq merge(v, v_2)$
  - Merging a vectorclock produces a bigger or equal vectorclock
- $\forall v$: $v \leq v$
  - $\leq -$reflexivity
- $\forall v_1, v_2, v_3$: $v_1 \leq v_2 \wedge v_2 \leq v_3 \rightarrow v_1 \leq v_3$
  - $\leq -$transitivity
- $\forall v_1, v_2$: $v_1 \leq v_2 \wedge v_2 \leq v_1 \rightarrow v_1 === v_2$
  - $\leq -$antisymmetry
- $\forall v_1, v_2, v_3$: $merge(merge(v_1, v_2), v3) === merge(v_1, merge(v_2, v_3))$
  - Merge associativity
- $\forall v_1, v_2, \forall (p, i) \in merge(v_1, v_2) : (p, i) \in v_1 \vee (p, i) \in v_2$
  - Merge by components
- $\forall v_1, v_2 . v' = merge(v_1, v_2) : v_1 \leq v_2 \rightarrow v' === v_2$
  - Merge subsumption
- $\forall v_1, v_2 . merge(v_1, v_2) === merge(v_1, merge(v_1, v_2))$
  - Merge is idempotent

Be careful with using structural equality ===! Sometimes, data structures have metadata attached that should be ignored in the equality check.