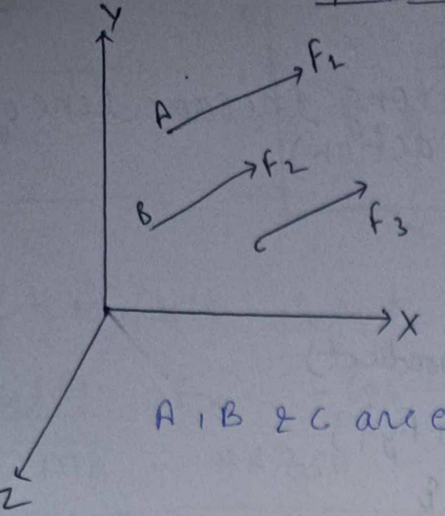
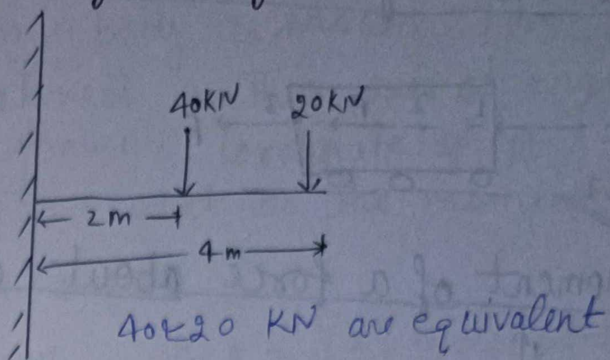
L1 Unit-1 Fundamental of Engineering Mechanics ①

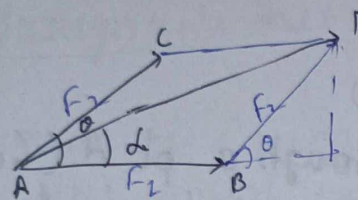


A, B & C are equal



40 & 20 KN are equivalent

Parallelogram Law



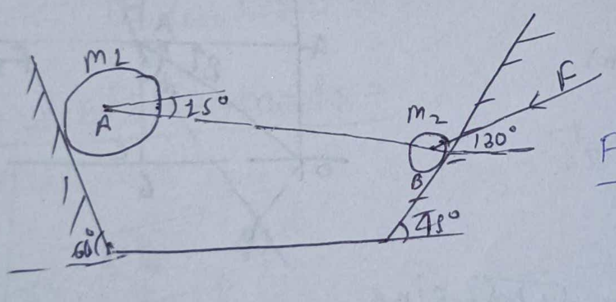
$$\boxed{\tan\alpha = \frac{F_2 \sin\theta}{F_1 + F_2\cos\theta}}$$

$$F_{net} = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

Several || Forces acting :-

$$\boxed{\Sigma F_i = 0}$$

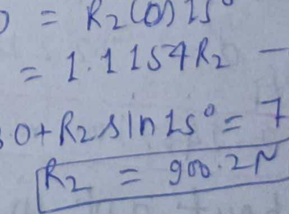
Ex →



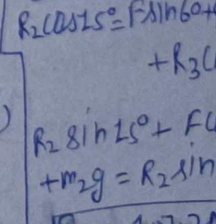
$m_1 = 75 \times 9.8 = 735N$

$m_2 = 50 \times 9.8 = 490N$

FBD of Cylinder A



FBD of B



From 'A' FBD

$R_1\cos 30 = R_2\cos 15°$

$R_1 = 1.1154R_2$ —(i)

$R_1\sin 30 + R_2\sin 15° = 735°$ —(ii)

$\boxed{R_2 = 900.2N}$

From B

$R_2\cos 15° = F\sin 60 + R_3\cos 45°$

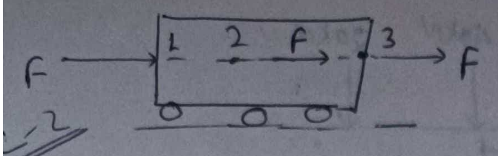
$R_2\sin 15° + F\cos 60 + m_2g = R_2\sin 45$

$\boxed{F = 107.2N}$

⇑

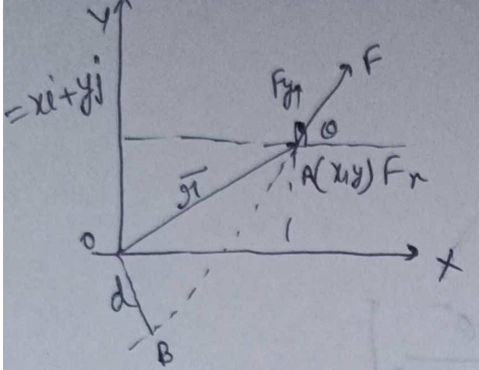
Que: Two Cylinder of mass m_1 & m_2 are Connected by riged bar A & B of ~~magnitude~~ negligible mass. Both are hinged at A & B. Cylinder are resting at inclined plane one at 60° & other at 45°. If $m_1 = 75kg$ & $m_2 = 5kg$. Determine the magnitude of F.

Principle of transmissibility for force :



$F_1 = F_2 = F_3$ (along the same line of action)

Moment of a force about origin :



$M_0 = \bar{r} \times F$ (Crossproduct)

$= (xi + yj) \times (F_x i + F_y j)$

$= (xF_y - yF_x)\hat{k}$

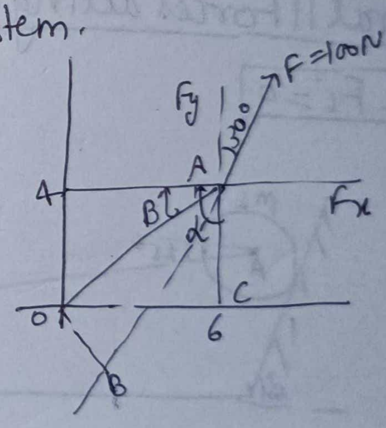
Also scalar method

$M_0 = OB \times F = d \times F$

(direcn R.H.T. Rule)

+ve (CCW), −ve (CW)

ex → A force of magnitude 100N is passing through a pt A (6,4)m on a body as shown in figure. Force 100 N is inclined at angle 30° with the verticle. Determine the moment of force about 'O' of co-ordinate system.



Soln, $F_x = 50N$

$F_y = 86.6 N$

$M_0 = xF_y - yF_x$; by direcn

$= 6 \times 86.6 + 4 \times 50$

$\boxed{M_0 = 314.6 \text{ CCW (N-m)}}$

IInd approach

Scalar Method

$OB = r\sin\beta = \sqrt{6^2+4^2} \sin\beta$

$\angle OAC = \alpha = \tan^{-1}\frac{6}{4} = 56.31$

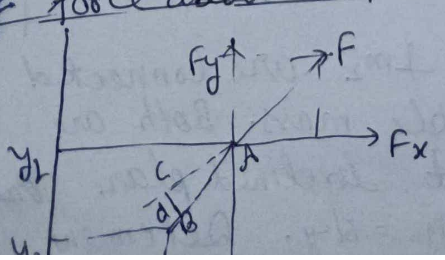
$\angle OAB = \beta = 56.31 - 30 \Rightarrow 26.31$

$d = 7.221 \times \sin 26.31 \Rightarrow 3.146$

$M_0 = d \times F$

$\boxed{M_0 = 3.146 \times 100 = 314.6 \text{ N-m}}$ Ans

Moment of force about a pt in space :



$m_B = \vec{r}_{BA} \times F$

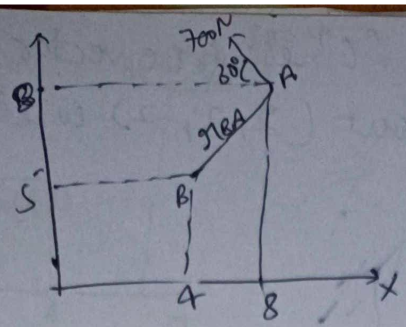
$\bar{r}_{BA} = \vec{r}_A - \bar{r}_B$

$= (x_1 - x_2)i + (y_2 - y_1)j$

$m_B = (x_1 - x_2)F_y - (y_1 - y_2)F_x \hat{k}$

Scalar Method $M_B = BC \times F$

Ex →



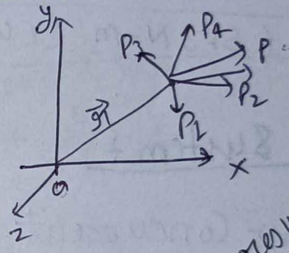
A force F 700 N is applied at a pt A on a body as shown. Force is inclined with x axis at angle 30 as shown. Cordinate of pt A are (8,8). What is the moment of this force about B(4,5) in the space.

Solⁿ → $F_x = -606.2\,N$, $F_y = 350\,N$ $r_{BA} = (8-4)\hat{i} + (8-5)\hat{j}$

$M_B = 4 \times 350 - 3 \times (-606.2)$ by using formula

$\boxed{M_B = 3218.6\,\hat{n} \text{ CCW N-m}}$ Ans

Varignon's theorem ÷ Sum of the moments about a pt. of a system of Concurrent forces is same as moment about the pt of the sum of forces.



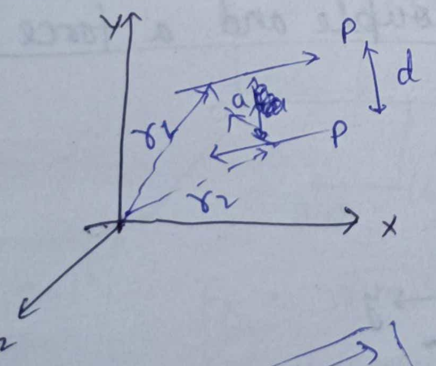
$M_O = \vec{r} \times P_1 + r \times \vec{P}_2 + \cdots + r \times \vec{P}_n$

$M_O = \vec{r} \times (P_1 + P_2 \cdots + P_n)$

$\boxed{M_O = r \times P_r}$ → Resultant force

resultant moment at O = sum of all the force moment

Couple & its Moments ÷

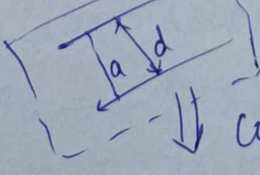


$M = \vec{r}_1 \times P + \vec{r}_2(-P) \Rightarrow (\vec{r}_1 - \vec{r}_2)P$

$\because \vec{r}_2 - \vec{r}_1 = \bar{a}$

$\boxed{M = \bar{a} \times P}$

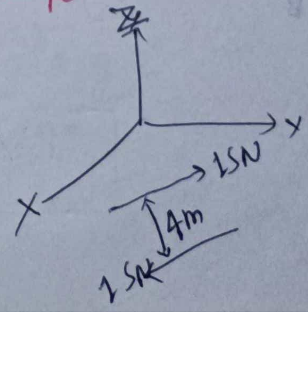
Couple is formed by equal & opposite force



Couple Vector

Couple has same moment about every pt in space, therefore Couple moment is free Vector.

Ex (i) Couple moment about origin
(ii) about (2,3,-2)



Sol^n (i) $P = 15N$

$d = 4m$

$M = |P| \times d$

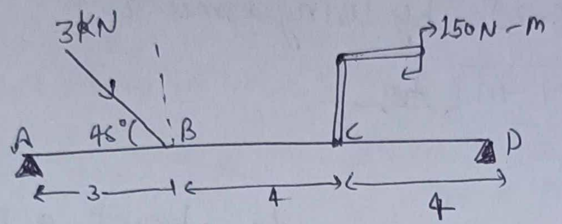
$\boxed{M = 60 N\text{-}m}$

right hand thumb rule $\boxed{M = -60 N.m}$

(ii) Since "C" is free vector so moment about $(2, 3, -2)$ will be same (4)

L-3, 18/12/2020

Ex →



Sol^n,

$M_A = 3 \sin 45 \times 3 \times 1000 + 150$

$\boxed{M_A = 6513 N\text{-}m \ (CCW)}$

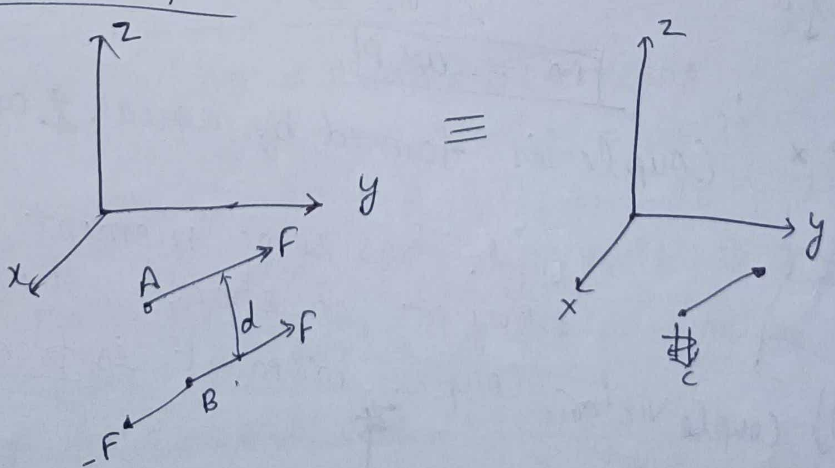
$M_D = 3 \sin 45 \times 8 \times 1000 - 150$

$\boxed{M_D = 16818 N\text{-}m \ (CCW)}$

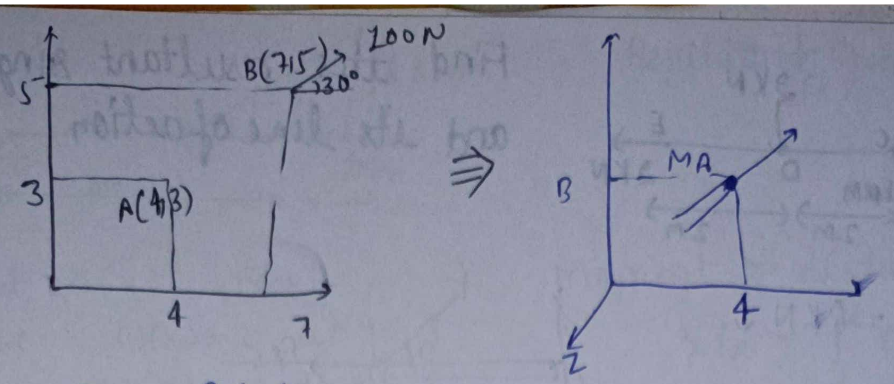
Resultant force system :

(i) Sum of ~~comp~~ Concurrent forces is a single force.
(ii) Force may be moved along line of action.
(iii) Effect of a couple is only a Couple

Replacement of a force by equivalent couple and a force at any other pt :



Ex→



force B shifts at A with couple ⑤

Sol^n

$F_x = 86.6\,N$

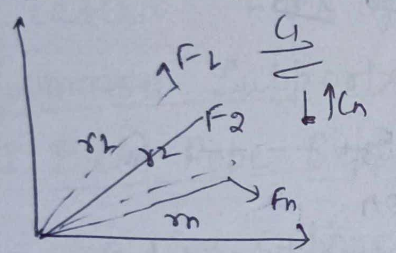
$F_y = 50\,N$

$F_A = 86.6\hat{i} + 50\hat{j}\,N$

$M_A = (7-4)\,F_y - (5-3)\,F_x$

$$\boxed{M_A = 150 - 173.2 = -23.2\,N\text{-}m}$$

Resultant of a force System:



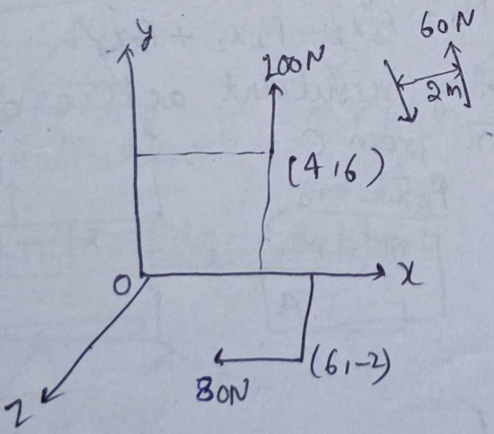
$F_R = F_1 + F_2 + \dots + F_N$

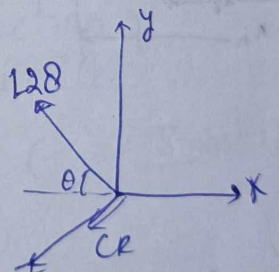
$= \sum_{i=1}^{n} F_{xi} + \sum_{i=1}^{n} F_{yi}$

$C_R = r_1 \times F_1 + r_2 \times F_2 + \dots + r_n \times F_n + C_1 + C_2 \dots C_n$

single force F_R & single couple C_R

Ex





Sol^n $F_R = \sqrt{(100)^2 + (80)^2} = 128\,N$

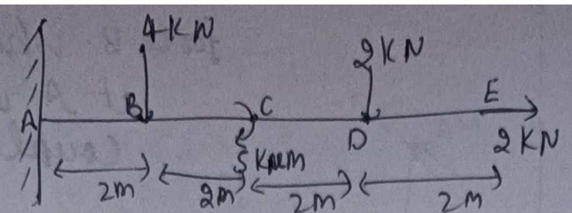
Resultant Couple at 'O'

$M_O = -80\times2 + 100\times4 + 60\times2$

$M_O = 360\,N\text{-}m$ ccw or along z

$$\boxed{\theta = \tan^{-1}\frac{100}{80} = 51.34°}$$

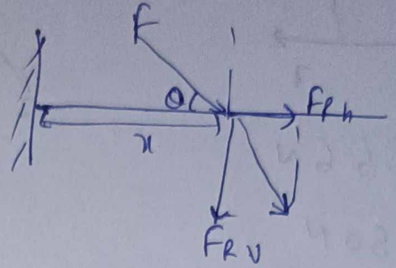
Qus



Find the resultant single force and its line of action

Sol^n,
$F_{RV} = 4+2 = 6$ KN ↓

$F_{RH} = 2$ KN →



$$F_{RV} \times x + F_{RH} \times 0 = 4\times 2 + 2\times 6 + 5$$
$$= 25 \text{ KN-m}$$

$$\boxed{x = \frac{25}{6} = 4.16\,m}$$

$$F_R = \sqrt{6^2+2^2} = 6.32\,N$$

$$\theta = \tan^{-1}\frac{F_{RV}}{F_{Rh}}$$

$$\theta = \tan^{-1}\frac{6}{2}$$

$$\boxed{\theta = \tan^{-1} 3}$$

Ans

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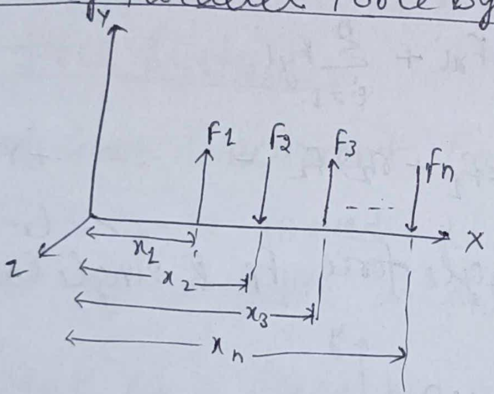
Resultant of Parallel Force System →



$F_R = F_1 - F_2 + F_3 + \text{----} - F_N$

Line of action

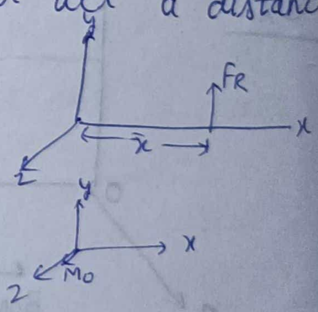
Taking moment about o

$M_o = F_1 x_1 - F_2 x_2 + F_3 x_3 + \text{---} - F_N x_n$

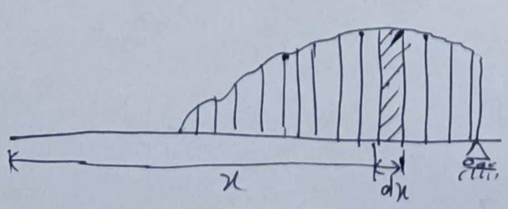
Say resultant act a distance $\bar{x}$ from O

$F_R \bar{x} = M_o$

$$\boxed{\bar{x} = \frac{M_o}{F_R}}$$



Distributed Force System →



$df_x = w(x)dx$

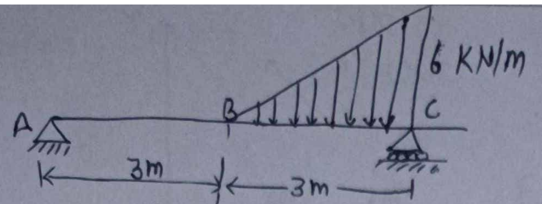
where $w(x)$ is intensity of loading

Resultant Load on the beam

$$F_R = -\int w(x)dx$$

Position
$$\bar{x} = \frac{-\int x(w(x))dx}{-\int w(x)dx}$$

Ex →



Resultant force $(F_R) = \int_3^6 w(x)\,dx$

$\boxed{F_R = 9KN}$

Sol^n → Let $w(x) = ax + b$

B.C. $w(x) = 0$ at $x = 3$

$w(x) = 6$ at $x = 6$

$a = 2 \quad b = -6$

$\boxed{w(x) = 2x - 6}$

Moment of distributed load again

$\bar{x} F_R = \int_3^6 x\,(w(x))\,dx$

$= \int_3^6 x(2x-6)\,dx$

$= 45$

$\boxed{\bar{x} = 5m}$ Ans

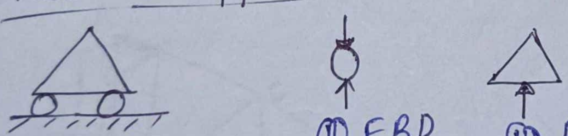
*** Free body Diagram (F.B.D.) →**

A Sketch of the object showing all the actions and reactions at proper places is called F.B.D of the object.

Common Supports & their Reactions ÷

(a) Belts Rope Support : A force directed away from the body

(b) Roller Support :



① Roller support ② FBD of Roller ③ Reaction of Roller

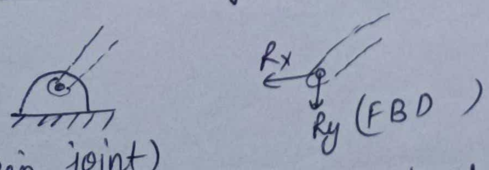
(c) knife Edge Support ÷

① knife Edge

(f) Smooth Surface ÷

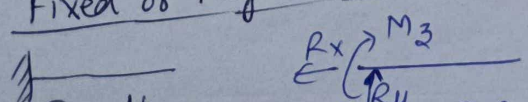
(g) friction Surface ÷

(d) Pen or Hinged Joint ÷



(Pen joint)

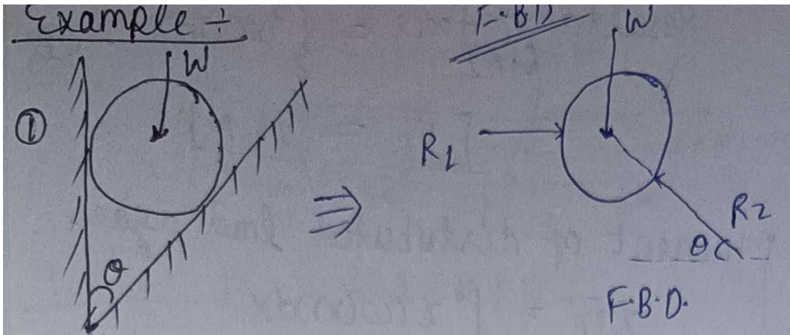
(e) Fixed or Rigid Support ÷



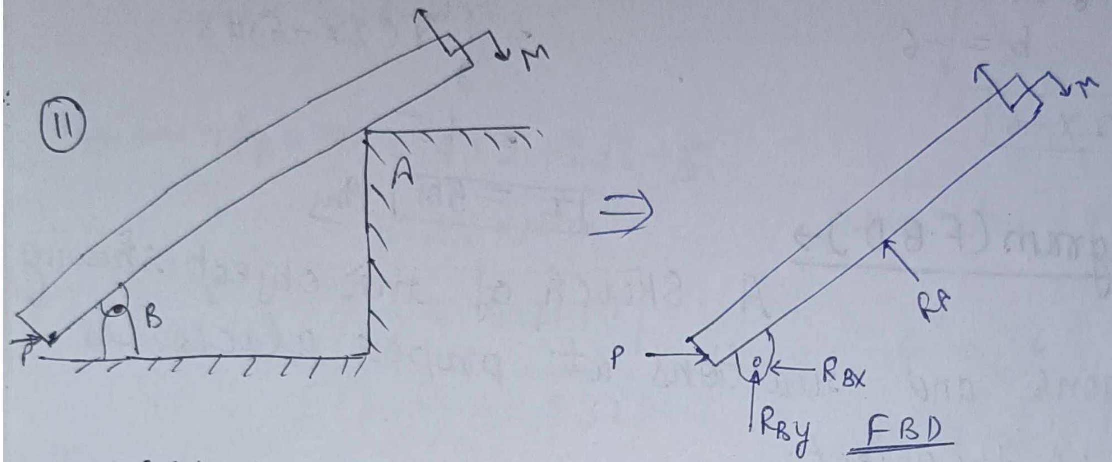
Fixed beam

Example ÷

(I)



(II)



Equilibrium of Riged Bocher ÷

When the Resultant forces or couple become zero the body is said to be in equilibrium.

For equilibibrium ÷

$$F_R = 0 \quad \& \quad C_R = 0$$

For 3D ÷

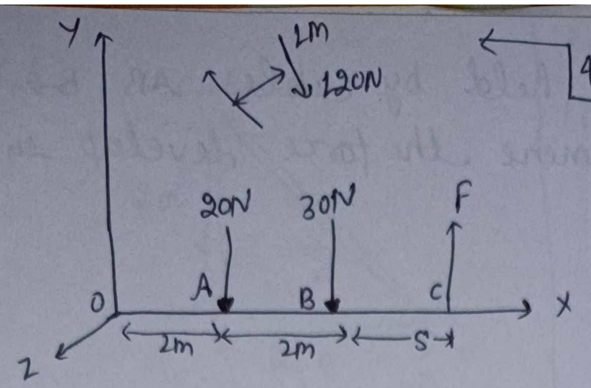
$\Sigma F_x = 0$ — (i) $\quad \Sigma F_y = 0$ — (ii) $\quad F_z = 0$ — (iii)

$\Sigma M_x = 0$ — (iv) $\quad \Sigma M_y = 0$ — (v) $\quad \Sigma M_z = 0$ — (vi)

For 2D Coplaner ÷

$$\left.\begin{array}{ll} \Sigma F_x = 0 & \text{— (i)} \\ \Sigma F_y = 0 & \text{— (ii)} \\ \Sigma M_z = 0 & \text{— (iii)} \end{array}\right\} \text{eq}^n \text{ of equilibrium}$$

Ex →



Find F & S, check equilibrium about B.

Sol^n : eq^n (1) $-20-30+F=0$

$\boxed{F=50N}$

(3) $-20\times2-30\times4+F(4+S)-120\times1+40=0$

$\boxed{S=0.8m}$

Check eq^n about B : $F_R=0$

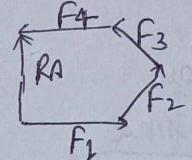
29/12/2020

$C_{RB}=20\times2+50\times0.8-120+40$

L-5

$\boxed{C_{RB}=0}$ equilibirium

Law of Polygon :

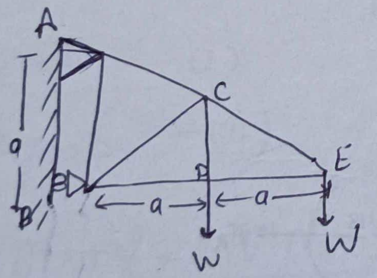


Non Concurrent Coplanner forces

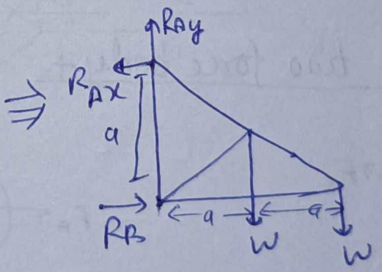
$F_1+F_2+\dots F_N=0$ Vectorially

$(r_1\times F_1+r_2\times F_2+\dots+r_n\times F_n)$
$+C_1+C_2\dots C_n=0$

Ex



⇒



$R_{Ay}=W+W$
$=2W\uparrow$

$\Sigma M_B=0$

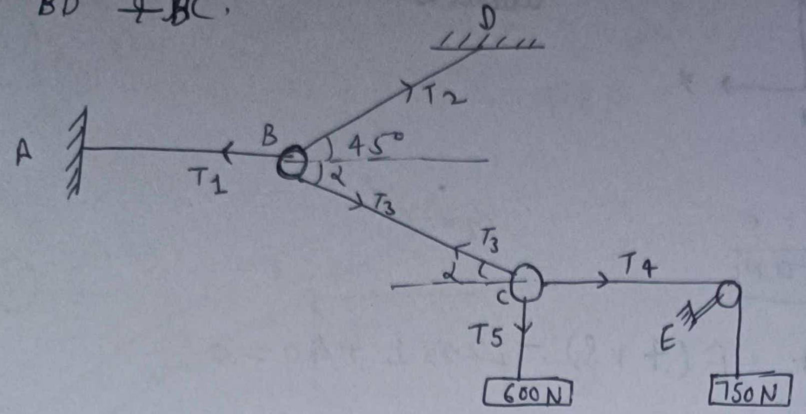
$W\times a+W\times2a=R_{Ax}\times a$

$\boxed{R_{Ax}=3W}$

$\boxed{R_B=3W}$

~~Ex~~

Ex2 — Two ring B & C are held by cables AB, BD, BC & C as shown in figure. Determine the force develop in cable BD & BC.

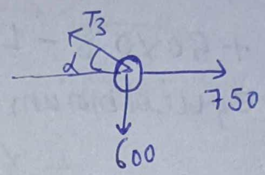


Sol^n →

$T_4 = 750N$

$T_5 = 600 N$

F.B.D of C

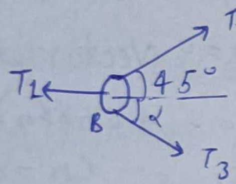


$T_3 \cos\alpha = 750$

$T_3 \sin\alpha = 600$

$\begin{cases} \alpha = 38.66 \\ T_3 = 961.5 \end{cases}$

F.BD of B

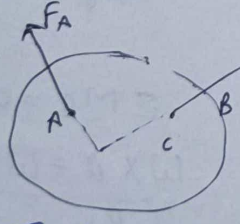


$T_1 = T_2 \cos 45° + T_3 \cos\alpha$

$T_2 \sin 45° = T_3 \sin\alpha$

$\boxed{T_2 = 849.6 N}$

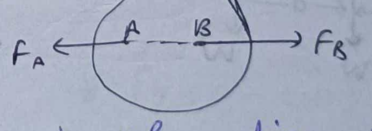
Equilibrium of a two force body →



$\Sigma M_c = 0$

But $\Sigma F_x \neq 0$

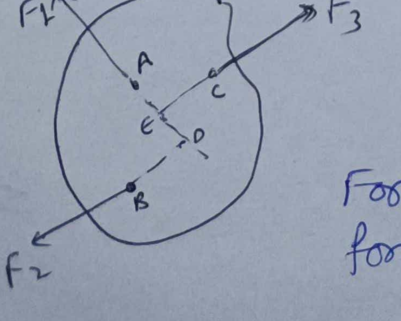
$\Sigma F_y \neq 0$



Line of action should be same

same magnitude & opposite direc^n

Equilibrium of three force body →



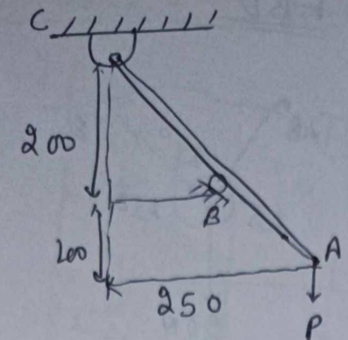
$\Sigma M_D \neq 0$

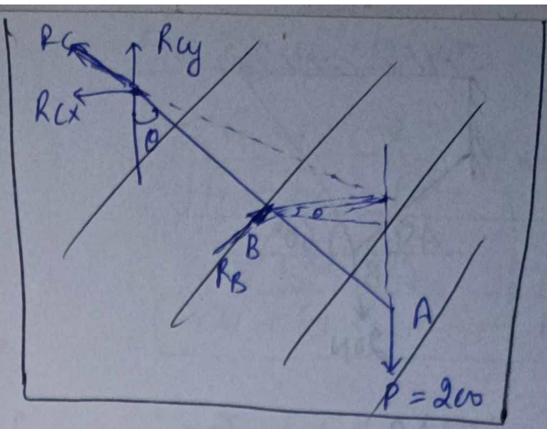
Untill line of action of F_3 passes through D

For Equilibrium the line of action of three forces must be concurrent.

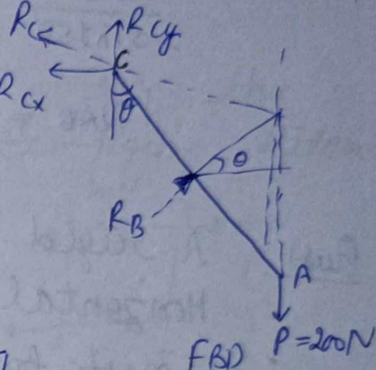
only for Non // forces.

Example:





Sol^n: $CB = \sqrt{(200)^2 + (125)^2} = 235.8\,mm$



$\Sigma M_c = 0 \Rightarrow R_B \times CB - 200 \times 250 = 0$

$\boxed{R_B = 212N}$

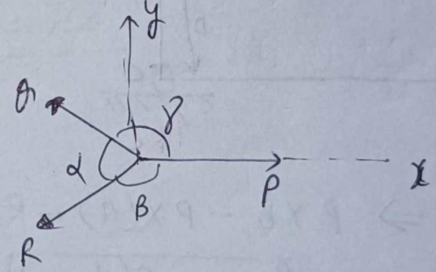
$\tan\theta = \frac{250}{400} \Rightarrow \boxed{\theta = 32°}$

$\Sigma F_x = 0 \quad R_B \sin\theta - 200 + R_{cy} = 0$

$\boxed{R_{cy} = 87.6N}$ $\boxed{R_{cx} = 179.8N}$

$\boxed{R_c = 200N}$

Lami's Theorem:



$\Sigma F_x = 0$ — (i)

$\Sigma F_y = 0$ — (ii)

(2) $\Rightarrow$ $Q \sin\gamma = R \sin\beta$

$$\boxed{\frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}}$$

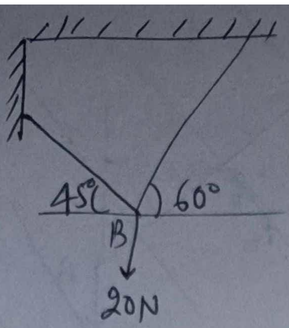
Similary had we selected the X-axis coinciding with θ, the result would have been

$$\frac{P}{\sin\alpha} = \frac{R}{\sin\gamma}$$

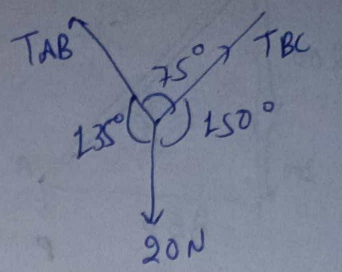
$$\boxed{\frac{P}{\sin\alpha} = \frac{R}{\sin\gamma} = \frac{Q}{\sin\beta}}$$

The ratio of a force and sine of angle b/w other two forces is Constant.

Example :-



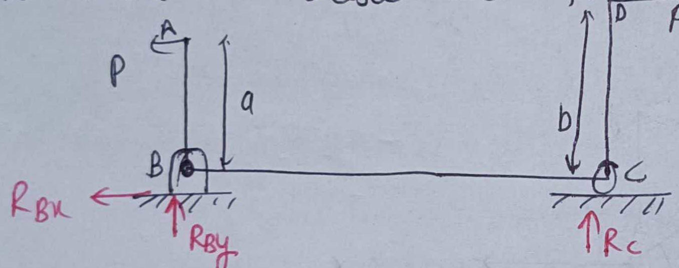
FBD



$$\frac{T_{AB}}{\sin 150} = \frac{T_{BC}}{\sin 135°} = \frac{20}{\sin 75}$$

$$\boxed{T_{AB} = 10.35\,N} \qquad \boxed{T_{BC} = 14.64\,N}$$

Ques: A rigid ~~body~~ bar ABCD is supported of B & C & Horizontal force are applied A, B and D. two equal & opposite force is applied at D & A of magnitude P. Determine the resultent force at support.



Sol^n:-

$$\Sigma M_B = 0 \Rightarrow P \times b - P \times (a) - R_C \times l = 0$$

$$\boxed{R_C = \frac{P(b-a)}{l}}$$

$\Sigma F_y = 0$

$$R_{By} + R_C = 0$$

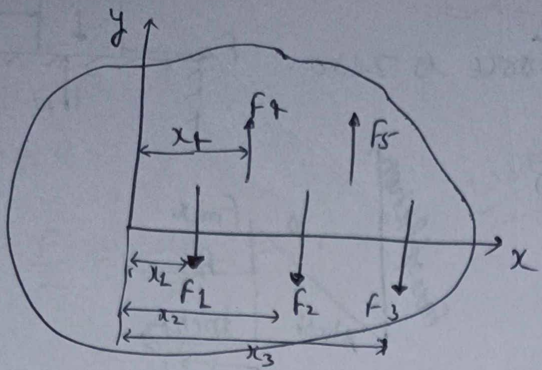
$$\boxed{R_{By} = -\frac{P(b-a)}{l}}$$

→ opposite direc^n

$\Sigma F_x = 0$

$$\boxed{R_{Bx} = 0}$$

Several ∥ force Acting :-



For Equilibrium

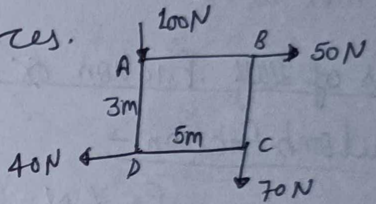
$\Sigma F_i = 0 \Rightarrow$

$$\boxed{F_1 + F_2 + F_3 + \text{---} = 0}$$

$\Sigma F_i x_i = 0$

$$\boxed{F_1 x_1 + F_2 x_2 + \text{---} F_n x_n = 0}$$

Example :- 4 System of force is acting on the rectangular block as shown in figure. Determine the magnitude & Direction of forces.



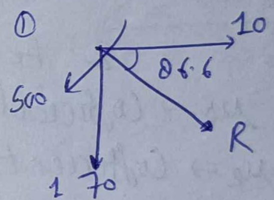
Sol^n :- $F_A = -100\hat{j}$, $F_B = 50\hat{i}$, $F_C = -70\hat{j}$

$F_D = -40\hat{i}$

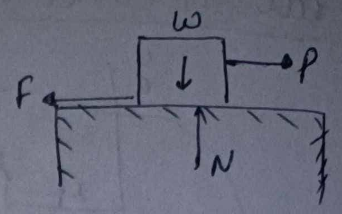
$R = (-100\hat{j} - 70\hat{j} + 50\hat{i} - 40\hat{i})\,N$

$$\boxed{R = (10\hat{i} - 170\hat{j})\,N}$$

$$\boxed{M_D = 50\times3 + 70\times5 = 500\ N\cdot m}$$

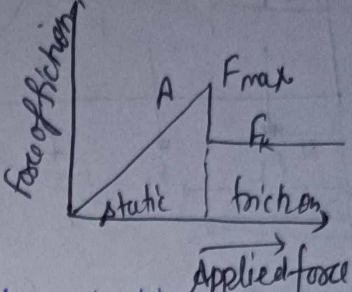


Friction



For smooth surface friction force is zero.

A , condition of impending motion



$F \propto P$ upto limit F_{max}

As the motion starts, there is a slight reduction in friction force F_{max} to F_K $\quad F_K \rightarrow$ kinetic friction

Laws of Dry Friction or Coulomb Friction ÷

Coulomb friction ÷

$$F_s \propto N$$

$$F_R \propto N$$

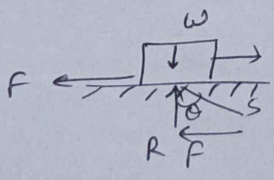
$F_s = \mu_s N$ & $F_K = \mu_K N$

$\mu_s \rightarrow$ Cofficient of static friction

$\mu_K \rightarrow$ Cofficient of kinetic friction

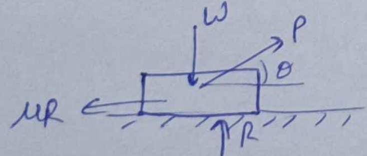
μ_K is 20 - 30% less than μ_s

Angle of friction ÷



$\tan\theta = \frac{F}{R} = \frac{\mu R}{R}$

$\boxed{\tan\theta = \mu}$



If ω, P & θ known

R can be find out

μ " " "

$F = P\cos\theta$

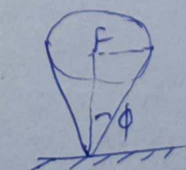
$\mu R = P\cos\theta$

$R + P\sin\theta = \omega$

$R = \omega - P\sin\theta$

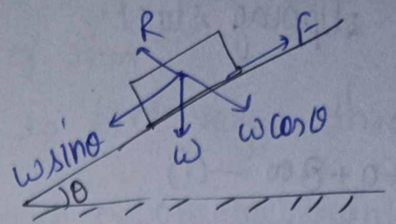
Cone of friction ÷

θ is angle of friction



(Pt of Contact)

Angle of Repose :



$w \sin\theta = F$

$w \cos\theta = R$

$\tan\theta = \frac{F}{R} = \frac{\mu \cancel{R}}{\cancel{R}} = \mu$

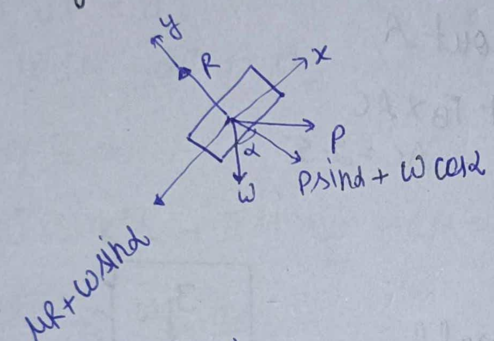
$= \tan\phi$

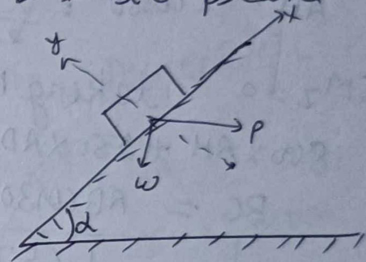
$\boxed{\theta = \phi}$

Example,

(a) Resolving the force along the plane & ⊥ to plane

Sol^n,





$w \sin\alpha + \mu R = P\cos\alpha$ — (i)

$w\cos\alpha + P\sin\alpha = R$ — (ii)

substituting value of R in eq^n (ii)

$w\sin\alpha + \mu(w\cos\alpha + P\sin\alpha) = P\cos\alpha$

$P(\cos\alpha - \mu\sin\alpha) = w(\sin\alpha + \mu\cos\alpha)$

$P = \frac{w(\sin\alpha + \mu\cos\alpha)}{(\cos\alpha - \mu\sin\alpha)}$ $\mu = \tan\phi$

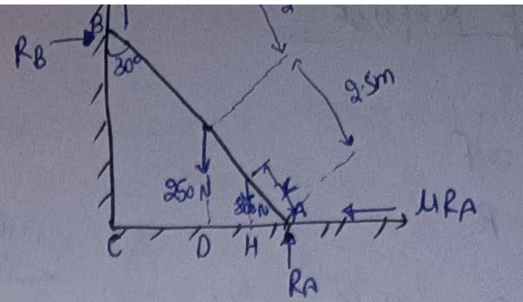
$\boxed{P = w\tan(\alpha + \phi)}$

(b) Going down

$\boxed{P = w\tan(\alpha - \phi)}$

Qus, A ladder 5m long & 250 N weight is placed against a vertical wall in a position where the inclination to vertical is 30°. A man weighting 800 N clambing the ladder. At what position will he induced sliping surface rough. & coff^n of friction is 0.2.

Sol: Let x be distance climbed by man when slipping start.



② ⇒ $\Sigma F_y = 0$

$R_A + \mu R_B = 250 + 800$ — (i)

$\Sigma F_x = 0$

$R_B - \mu R_A = 0$

$R_B = 0.2 R_A$ — (ii)

① + ② gives $R_A = 1009.6\,N$ + $R_B = 201.9\,N$

$AD = 2.5\cos 60° = 1.25$

$AH = x\cos 60° = \frac{x}{2}$

$\Sigma M_z = 0$ taking moment about A

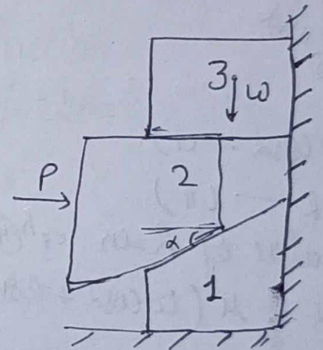
$800 \times AH + 250 \times AD = R_B \cdot BC + F_B \times AC$

$BC = AB\cos 30° = 4.33$ $AC = 2.5$

$\boxed{x = 1.657}$ Ans

Wedge:

Wedge is useful for small adjustment in position of body



From FBD ①

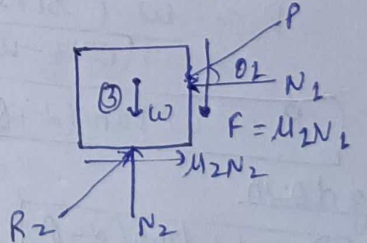
$\Sigma F_y = 0$

$W = N_2 - \mu_1 N_1$ — (i)

$\Sigma F_x = 0$

$\mu_2 N_2 = N_1$ — (ii)

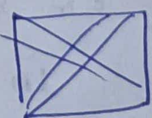
FBD of ③

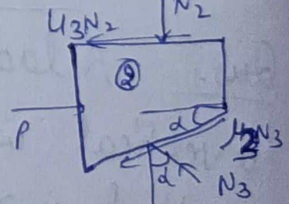


from ① & ②

$N_2 = \frac{W}{1 - \mu_1\mu_2}$

$N_1 = \frac{\mu_2 W}{1 - \mu_1\mu_2}$





From FBD ②

$\Sigma F_y = 0$

$N_2 = N_3\cos\alpha - \mu_3 N_3 \sin\alpha$ — (iii)

$P = \mu_2 N_2 + \mu_3 N_3 \cos\alpha + N_3 \sin\alpha$ — (iv)

α is known N_3 & P can be calculated

Belt & Rope Drive :-

Power transmission depends upon the frictional resistance b/w belt & surface of rim.

for rough surface → tension in belt vary throughout.

Frictional resistance → increases exponential maner.

belt is in contanct over angle θ (Angled leb)

$T_1 > T_2$

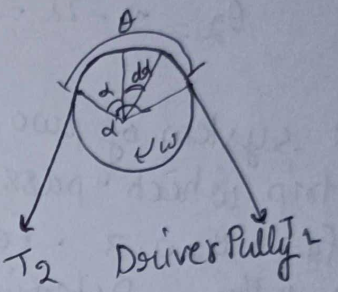
For Equilibrium $\Sigma F_x = 0$

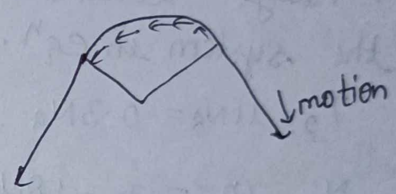
$(T+\Delta T)\cos\frac{d\alpha}{2} - T\cos\frac{d\alpha}{2} - df = 0$

$d\alpha \to 0 \qquad \cos\frac{\theta}{2} \to 1$

$\Rightarrow df = dT$

$\mu dN = dT$ —— (i)





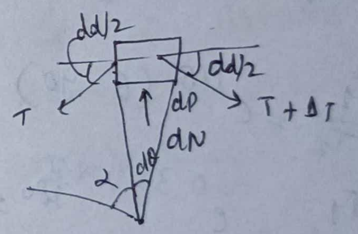
$\Sigma F_y = 0$

$(T+dT)\sin\frac{d\alpha}{2} + T\sin\frac{d\alpha}{2} - dN = 0$

$\sin\frac{d\alpha}{2} \simeq \frac{d\alpha}{2}$

$\Rightarrow dN = 2T\frac{d\alpha}{2} + dT\frac{d\alpha}{2}$

$= Td\alpha$ —— (ii)



① & ②

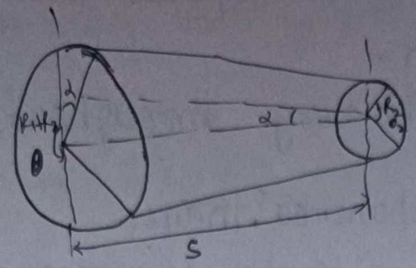
$\frac{dT}{\mu} = Td\alpha$

$\int_{T_2}^{T_1}\frac{dT}{T} = \int_0^\theta \mu d\alpha$

$$\boxed{\frac{T_1}{T_2} = e^{\mu\theta}}$$

θ must be in radians

Types Belt Drive :- (I) open belt & (II) crossed belt

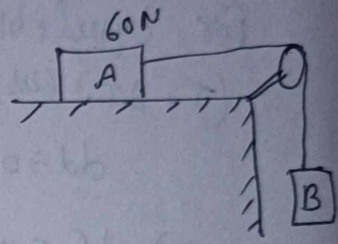


Torque

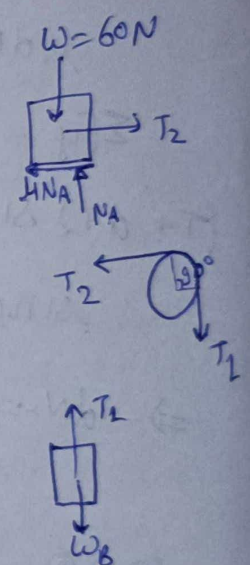
$\tau = (T_1 - T_2) R$

$\theta_1 = \pi + 2\alpha = \pi + 2\sin^{-1}\frac{R_1 - R_2}{s}$

$\theta_2 = \pi - 2\alpha = \pi - 2\sin^{-1}\frac{R_1 - R_2}{s}$



Ex A system of two places connected by strip which passes over pully as shown fig. $\mu = 0.3$. For block & plane & belts & Pully. Determine min weight of B to keep the system in eqn.



Soln, $T_2 = \mu N_A = 0.3 N_A$

$N_A = 60 \Rightarrow T_2 = 18N$

$\frac{T_1}{T_2} = e^{\mu\theta} \quad (\theta = 90)$

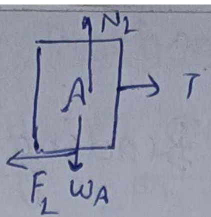
$\frac{T_1}{T_2} = e^{0.3 \times 90 \times \frac{\pi}{180}} = 1.6$

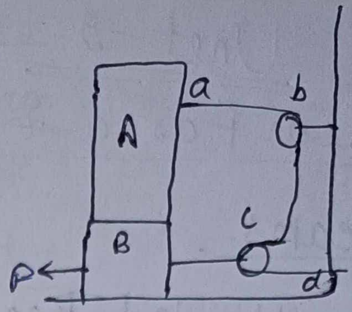
$T_1 = 1.6 T_2 = 28.8 N$

$\boxed{W_B = T_1 = 28.8 \text{ Ans}}$

Ques: Two blocks A & B of mass 60kg and 90kg respectively are placed on horizontle plane. Both are connected to string abcd as shown fig. The cofficient of friction for all contacting plane is 0.3. Determine the value of the largest force P that can be applied without moving the block A & B.

Sol^n, Block A

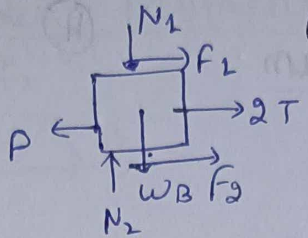




$N_1 = W_A = 588.6$

$\boxed{T = F_1 = 0.3 N_1 = 176.6} N$

Block B



$N_1 + W_B = N_2$

$\boxed{N_2 = 1471.5} N$

$F_2 = 0.3 N_2 = 441.45 N$

$2T = 2 \times 176.6 = 353.2 N$

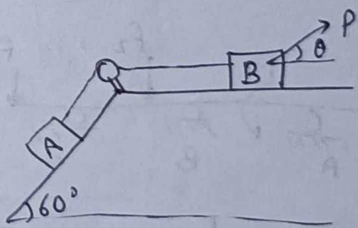
$\Sigma F_x = 0$

$P = F_1 + F_2 + 2T$

$P = 176.6 + 441.45 + 355.2$

$\boxed{P = 970} N$ Ans

Ques: what is the least value of P required to cause the motion impend in the arrangement shown below. Assume the cofficient of friction on all Contanct surface as 0.2. weight of block A & B are 840 N & 560 N respectivly.

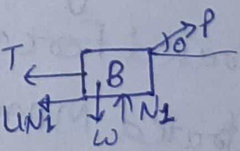


sol^n, Block B

$T + u N_1 = P \cos\theta$ — (i)

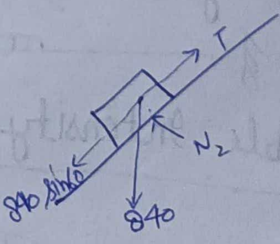
$N_1 = 500 - P\sin\theta$ — (ii)

$T + u(500 - P\sin\theta) = P\cos\theta$ — (iii)



Block A

$N_2 = 840 \cos 60° = 420 N.$



$840 \sin 60° + u N_2 = T$ — (4)

Putting value of T in (4) from (3)

$T + 112 - 0.2 P\sin\theta = P\cos\theta$

after Solving

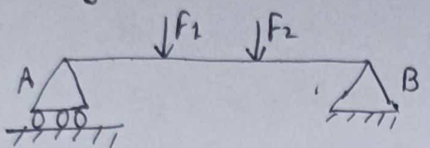
$\boxed{P_{min} = 905.5 N}$

Unit - 2

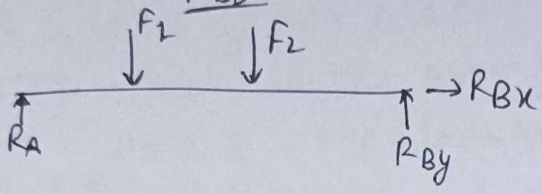
Shear Force & Bending Moment

Types of Beams

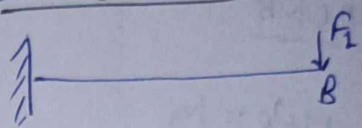
(I) Simply supported Beam

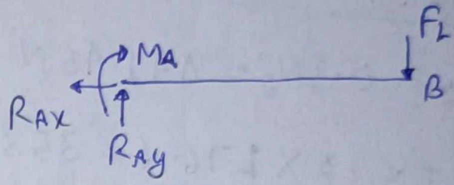


FBD

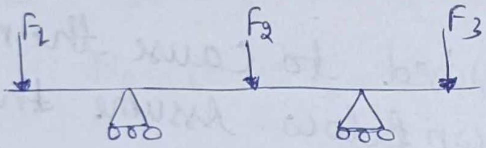


(II) Cantilever

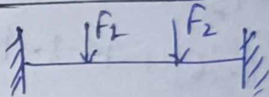




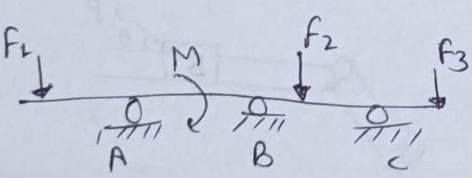
(III) Over hanging Beam



(IV) Fixed Beam →



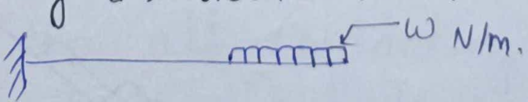
(V) Continuous Beam



Type of Loading

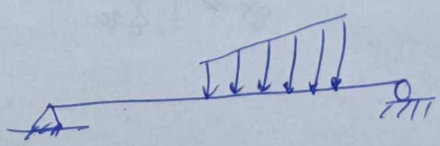
1. Concentrated Force
2. Concentrated Moment
3. Uniformly Distributed Load

Ex



4. Variable Intensity Loading (UVL)

Ex

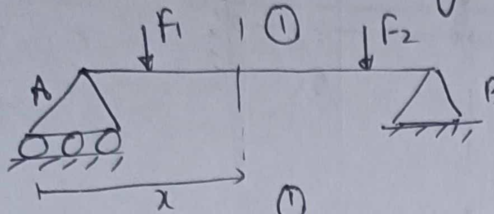


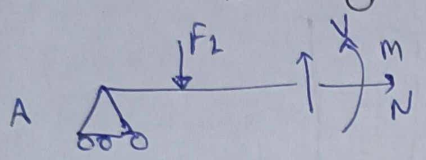
Determining of Support Reaction ÷

For statically determination beam

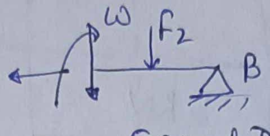
$$\Sigma F_x = 0 \ , \ \Sigma F_y = 0 \ , \ \Sigma M_z = 0$$

Shear Force & Bending Moment ÷





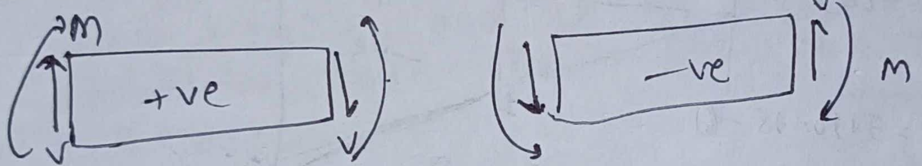
FBD of LHS①



FBD of RHS①

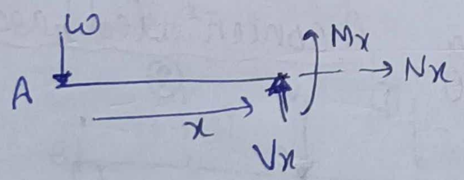
According to newtons 3rd Law

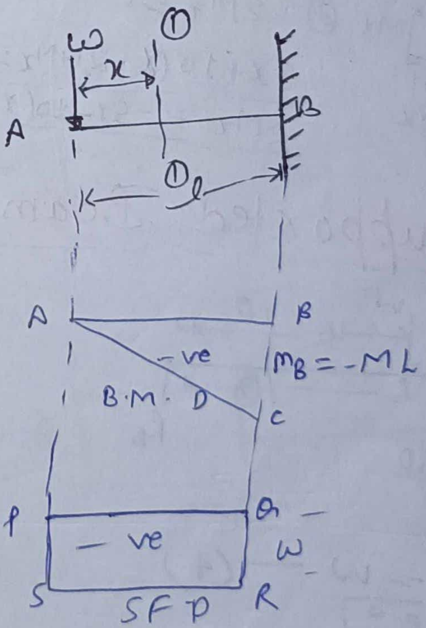
Sign Convention →



Example $0 \le x \le L$

FBD of L.H.S.





① $\Sigma F_x = 0$

$N_x = 0 \quad —(4)$

② $\Sigma F_y = 0$

$\omega - V_x = 0$

$V_x = \omega \quad —(5)$

③ $\Sigma M_x = 0$

$\omega x + M_x = 0$

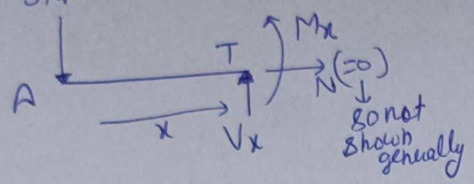
$M_x = -Mx \quad —(6)$

$$\frac{dM_x}{dx} = -\omega \qquad \frac{dM_x}{dx} = 0$$

Example A cantilever ABC 5m long fixed at C carry a constrated load 5KN at A & 10KN at B at a distance of 2m from A. Draw the shear force & Bending moment diagram.

Sol^n:

$0 \le x \le 2$



② ⇒ $V_x - 5 = 0$

$V_x = 5$ — (4)

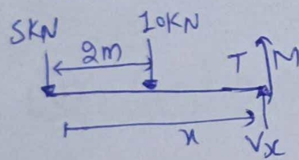
③ $\Sigma M_T = 0$

$5x + M_x = 0$

$M_x = -5x$ — (5)

section ② - ②

$2 \le x \le 5$



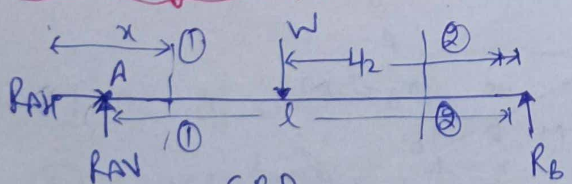
② $V_x = 5 + 10 = 15$ — ⑥

③ $\Sigma M_T = 0$

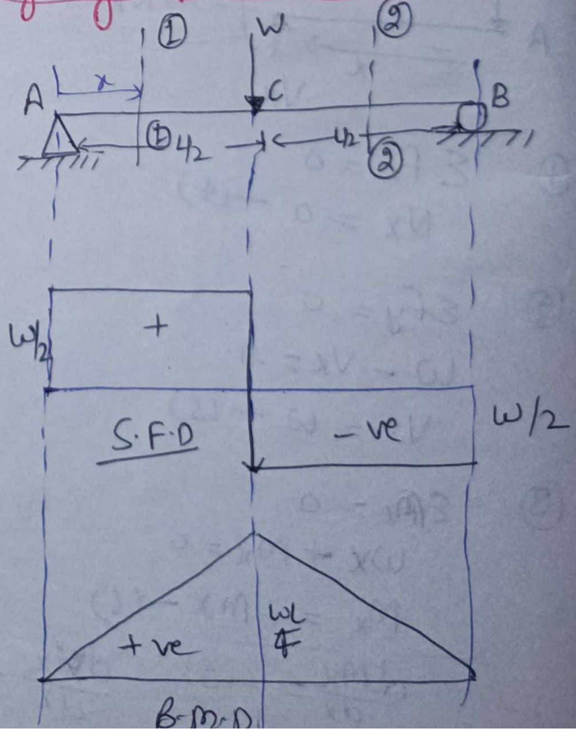
$5x + 10(x-2) + M_x = 0$

$\boxed{M_x = -5x - 10(x-2)}$ — (7)

Simply Supported Beam carrying A concentrated load



⇐



① **Reaction**

$R_{AV} + R_B = W$ — (4)

$\boxed{R_{AH} = 0}$

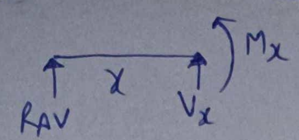
② $\Sigma M_A = 0$

$R_B \times l - W \times \frac{l}{2} = 0$

$\boxed{R_B = \frac{W}{2}}$

$\boxed{R_{AV} = W/2}$

section ① - ①



$0 \le x \le L/2$

$\Sigma M_{①-①} = 0$

$R_{AV}\, x - M_x = 0$

$\boxed{M_x = \frac{W}{2} x}$ — (5)

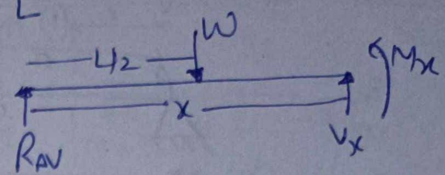
$F_y = 0$

$R_{AV} + V_x = 0$

$\boxed{V_x = -\frac{W}{2}}$ — ⑥

section ② - ②

$\frac{L}{2} \le x \le L$



$\Sigma F_y = 0$

$W = R_{AV} + V_x$

$V_x = \frac{W}{2}$

$\Sigma M_{②-②} = 0$

$R_{AV} \times x - W\left(x - \frac{L}{2}\right) - M_x = 0$

$M_x = \frac{W}{2}(L - x)$

at $x = L/2$, $M_x = \frac{WL}{4}$

at $x = L$, $M_x = 0$

Ques → A beam AB 6m long simply supported at end carries 6 KN & 12 KN load at the distance 2m & 4m from A as shown in figure. Draw S.F.D & B.M.D.

Sol^n → $\Sigma M_B = 0$

$R_A \times 6 - 6 \times 4 - 12 \times 2 = 0$

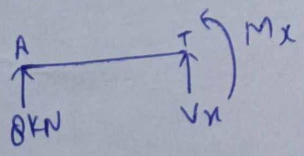
$\boxed{R_A = 8}$ KN

$\Sigma F_y = 0$

$R_A + R_B = 6 + 12$

$\boxed{R_B = 10}$ KN

Section ① - ①



$\Sigma F_y = 0$

$8 + V_x = 0$

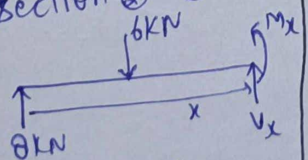
$\boxed{V_x = -8\text{ KN}}$

$\Sigma M_{①-①} = 0$

$8x - M_x = 0$

$\boxed{M_x = 8x}$

section ② - ②



② ⇒ $8 + V_x - 6 = 0$

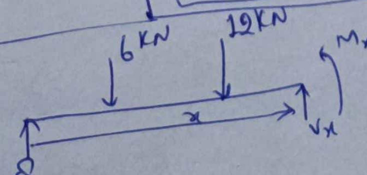
$\boxed{V_x = -2}$ KN

③ ⇒ $\Sigma M_x = 0$

$8x - 6(x-2) - M_x = 0$

$\boxed{M_x = 2x + 12}$

Section ③ - ③



$4 \le x \le 6$

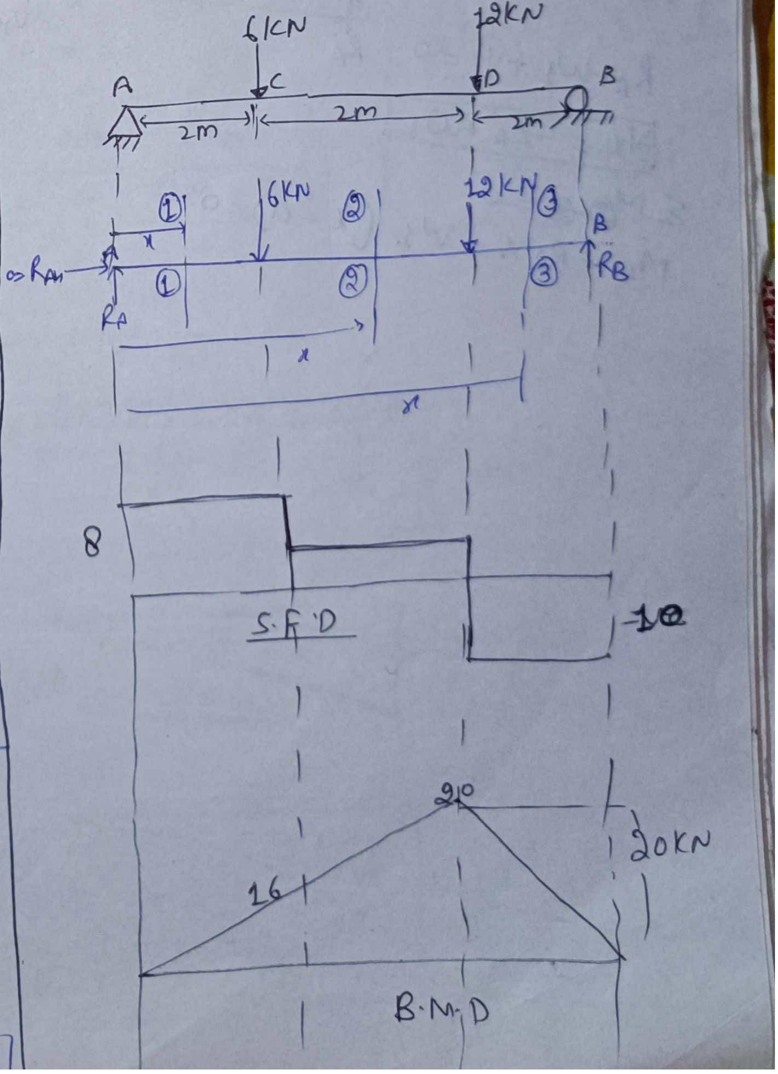
$8 + V_x - 6 - 12 = 0$

$\boxed{V_x = 10}$ (-ve)

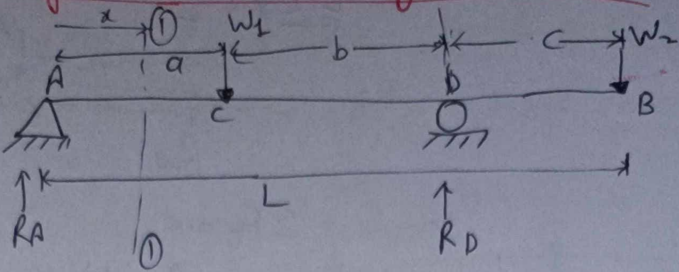
$\Sigma M_{③-③} = 0$

$8x - 6(x-2) - 12(x-4) - M_x = 0$

$\boxed{M_x = -10x + 60}$



SFD & BMD of over hangeng Beam



$\Sigma M_A = 0$

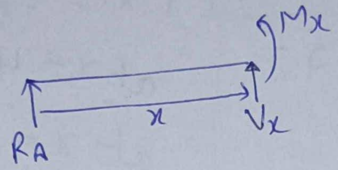
$W_1 a + W_2 L = R_D(a+b)$

$$R_D = \frac{W_1 a + W_2 L}{a+b} \quad \text{--- (i)}$$

$$R_A = W_1 + W_2 - R_D \quad \text{--- (ii)}$$

section ① - ①

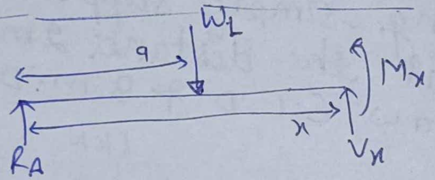
$0 \le x \le a$



$V_x = -R_A x$

$M_x = R_A x$

For sec ② - ②

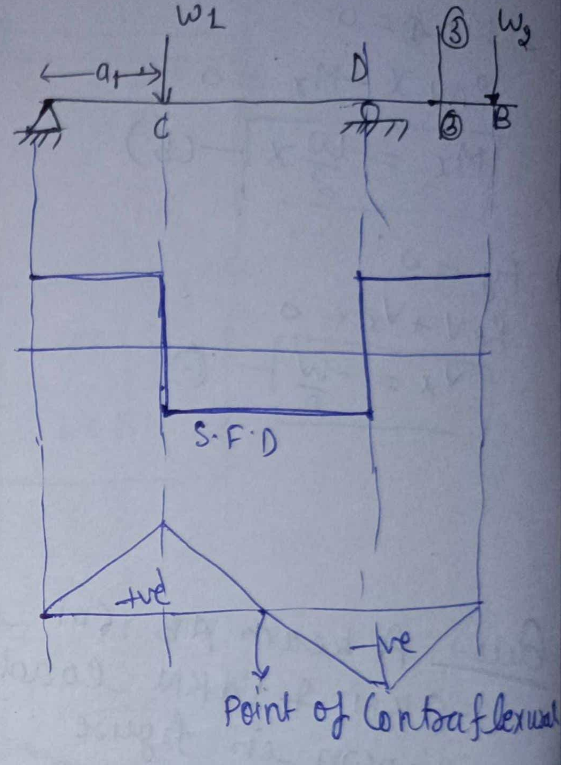


$R_A - W_1 + V_x = 0$

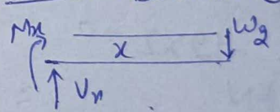
$\boxed{V_x = -R_A + W_1}$

$\Sigma M_{②-②} = 0$

$M_x - R_A x + W_1 (x - a) = 0$



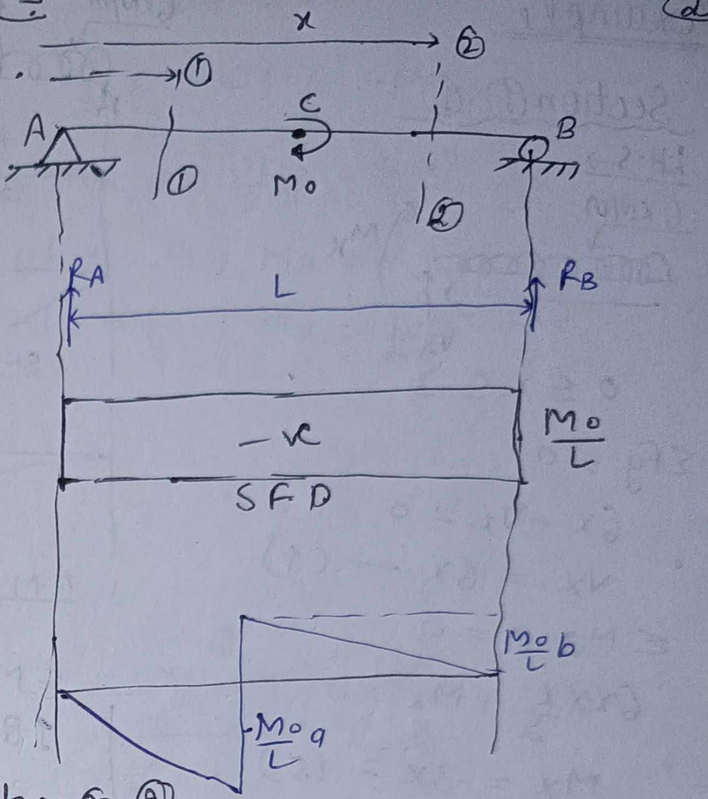
RHS section ③ ③



$\boxed{V_x = W_2}$

$\boxed{M_x = W_2 C}$

Beam Subjected to Couple :-



$\Sigma M_B = 0$

$R_A L + M_B = 0$

$R_A = -\frac{M_0}{L}$

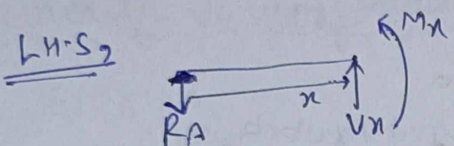
&

$R_A + R_B = 0$

$R_B = \frac{M_0}{L}$

section ① ①

LH.S,



$V_x - R_A = 0$

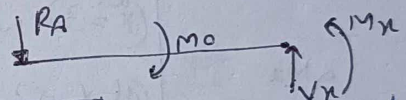
$\boxed{V_x = R_A}$

$\Sigma M_x = 0$

$R_A x + M_x = 0$

$\boxed{M_x = -R_A x}$

section ② ②



$\boxed{V_x = R_A}$

$\Sigma M_x = 0$

$R_A x + M_x - M_0 = 0$

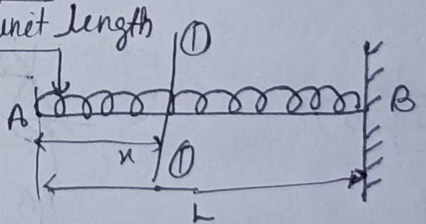
$\boxed{M_x = -R_A x + M_0}$

at $x = L$, $M_x = 0$

at $x = a$

$\boxed{M_x = \frac{M_0 a}{L}}$

Uniformly Distributed Load →

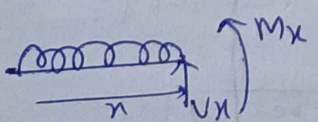


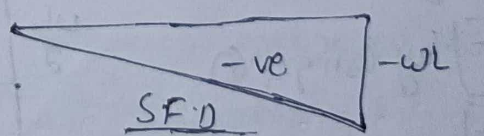
section ① - ①

LH.S of beam

FBD

$0 \leq x \leq L$

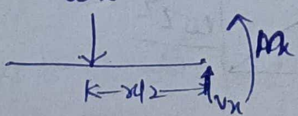




$V_x - wx = 0$

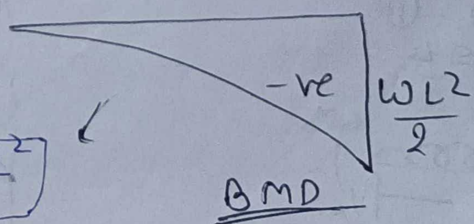
$V_x = wx \, (-ve)$

wx ~~(-ve)~~

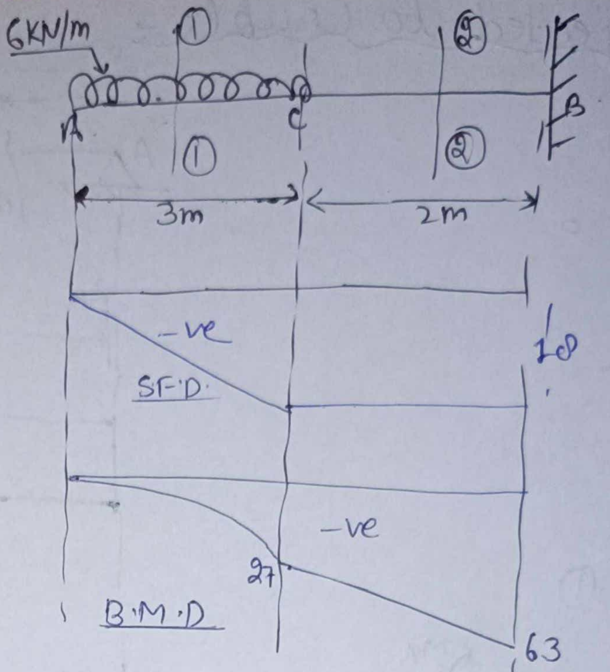


$\Rightarrow \quad wx \cdot \frac{x}{2} + M_x = 0$

$\boxed{M_x = -\frac{wx^2}{2}}$



Example

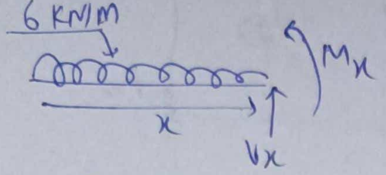


{left & downward side then V_x is negative & +Ve upward

{Concave up -ve moment

Section ① - ①

L.H.S →



$0 \le x \le 3$

$\Sigma F_y = 0$

$6x - V_x = 0$

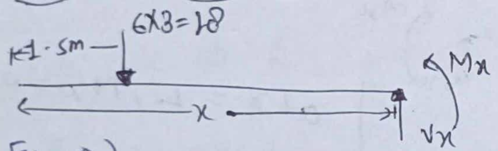
$V_x = 6x$ — (4)

$\Sigma M_{①-①} = 0$

$6x \times \frac{x}{2} + M_x = 0$

$M_x = -3x^2$ — (5)

Section ② - ②



($F_y = 0$) $18 - V_x = 0$

$V_x = 18$ (-ve)

$\Sigma M_{x-②-②} = 0$

$18(x - 1.5) + M_x = 0$

$M_x = -18(x - 1.5)$

(at $x = 3$) $M_x = -27$ N/m

(at $x = 5$) $M_x = -63$ N/m

Simply Supported Beam Carrying UDL

Reaction

$\because F_y = 0$

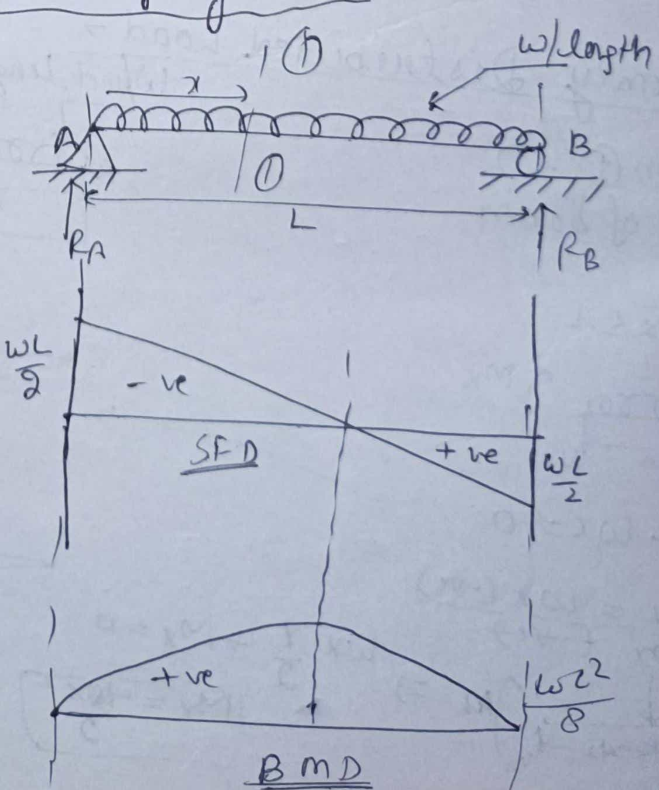
$\boxed{R_A + R_B = WL}$ — (i)

$\Sigma M_B = 0$

$R_A L - WL \cdot \frac{L}{2} = 0$

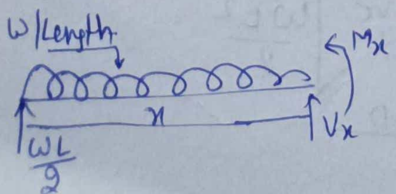
$\boxed{R_A = \frac{WL}{2}}$ by using eqn ①

$\boxed{R_B = \frac{WL}{2}}$



Section ① - ①

$0 \le x \le L$



$\sum F_y = 0$

$\frac{wL}{2} - wx + V_x = 0$

$V_x = wx - \frac{wL}{2}$

at $x = 0$

$V_x = -\frac{wL}{2}$ (+ve)

at $x = L$

$V_x = \frac{wL}{2}$ (−ve)

$\sum M_{1-1} = 0$

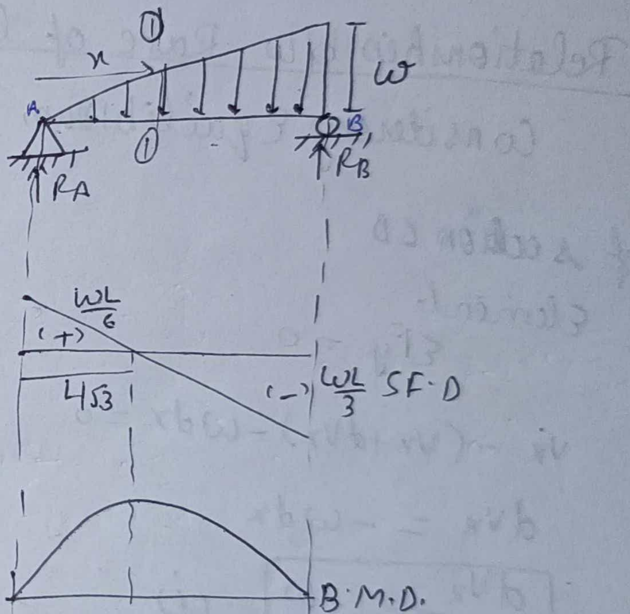
$\frac{wL}{2}x - wx \cdot \frac{x}{2} - M_x = 0$

$M_x = \frac{wL}{2}x - \frac{wx^2}{2}$

at $x = 0$, $M_x = 0$

at $x = \frac{L}{2}$, $M_x = \frac{wL^2}{8}$

Linearly varying load →



$\sum M_B = 0$

weight x length from B

$R_A L - (\text{Area})\left(L - \frac{2L}{3}\right) = 0$

$R_A L - \frac{wL}{2} \times \frac{L}{3} = 0$

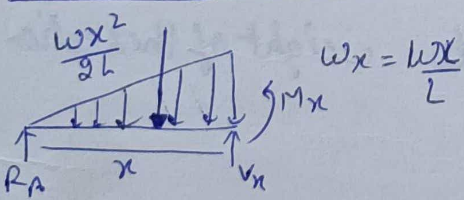
$R_A = \frac{wL}{6}$ —(4)

$\sum F_y = 0$

$R_A + R_B = \frac{wL}{2}$

$R_B = \frac{wL}{3}$ —(5)

section ① − ①



② ⇒ $R_A + V_x - \frac{wx^2}{2L} = 0$

$V_x = \frac{wx^2}{2L} - \frac{wL}{6}$

at $x = 0$, $V_x = -\frac{wL}{6}$ (+ve)

at $x = L$, $V_x = \frac{wL}{3}$

$V_x = 0 \Rightarrow x = \frac{L}{\sqrt{3}}$

$\sum M_x = 0$

③ ⇒ $M_x + \frac{wx^2}{2L} \times \frac{x}{3} - \frac{wL}{6}x = 0$

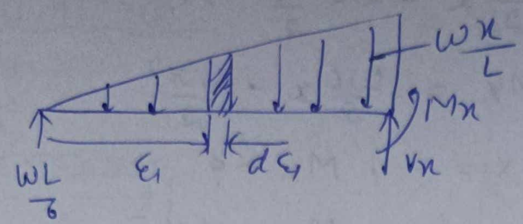
$M_x = \frac{wL}{6}x - \frac{wx^2}{6L}$

at $x = 0, L$

$M_x = 0$

at $x = L/\sqrt{3}$

$M_x = \frac{wL^2}{9\sqrt{3}}$

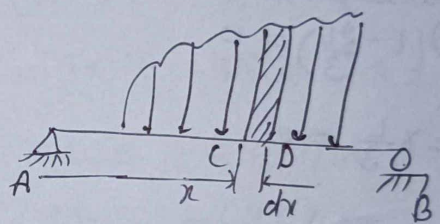


$$R_A x - \int_0^x \frac{w\xi_1}{L} d\xi_1 (x-\xi_1) - M_x = 0$$

$$\boxed{M_x = \frac{wL}{6}x - \frac{wx^3}{6L}}$$

Relationship b/w Rate of loading, SF & BM

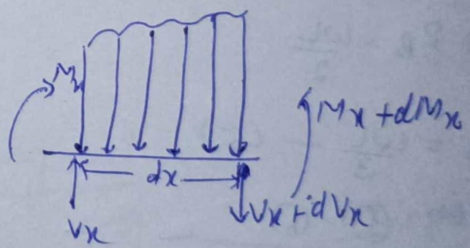
Considering Equilibrium of section CD Element



$$\Sigma F_y = 0$$

$$V_x - (V_x + dV_x) - w\,dx = 0$$

$$dV_x = -w\,dx$$



$$\boxed{\frac{dV_x}{dx} = -w} \quad \text{(i)}$$

Rate of change of S.F. of section is equal to weight of the section

$$\Sigma M_D = 0$$

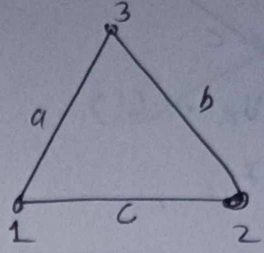
$$M_x + dM_x - M_x - V_x dx + w(x)\cancel{dx}\frac{dx}{2} = 0$$

$$\boxed{\frac{dM_x}{dx} = V_x} \quad \text{(ii)}$$

Rate of change of B.M of section is equal to S.F of the section

Trusses

Plane Trusses



m = number of members
n = number of joints

$m = 3 \quad n = 3$

$3 = 3 \times 2 - 3$

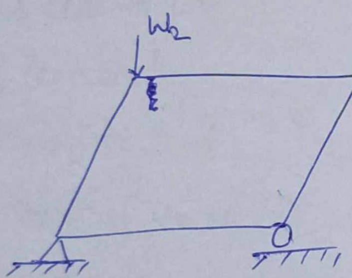
Perfect Trusses.

For Perfect truss

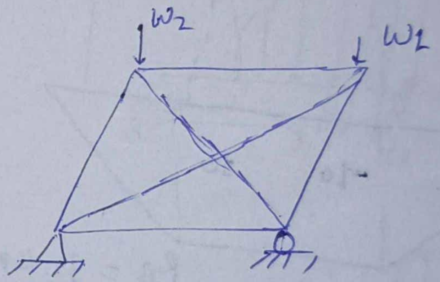
① $\boxed{m = 2n - 3}$

(ii) $m < (2n - 3)$
collapsible truss

(iii) $m > (2n - 3)$
redundant truss



Collapsible truss



Redundant truss

Analysis of Frame

① Analytical Method — Method of Joints / Method of section

② Graphical Method

Method of Joints

$\Sigma F_x = 0 \quad , \quad \& \ \Sigma F_y = 0$

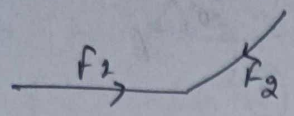
sign convention

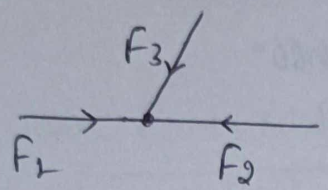
Tension
Pulling of each end

Pushing at each end

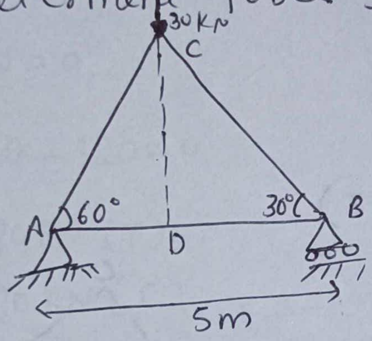
Identify the member with zero force

(I)  $F_1 = 0$

(II)  $F_1 = F_2 = 0$

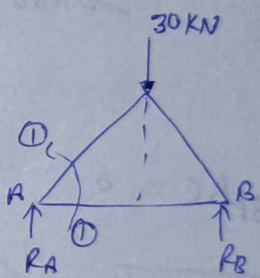
(III)  $F_3 = 0$

Example Determine force in all members



$AC = AB\cos 60 = 2.5$

$AD = AC\cos 60 = 1.25$



sol^n^: (1)

$\Sigma M_A = 0$

$R_B \times 5 = 30 \times 1.25$

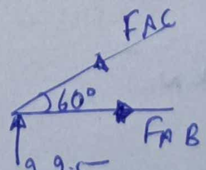
$\boxed{R_B = 7.5 KN}$

$R_A + R_B = 30$

$\boxed{R_A = 22.5}$ KN

from F.B.D of Joint A

(2) $F_{AC}\sin 60 - 22.5 = 0$



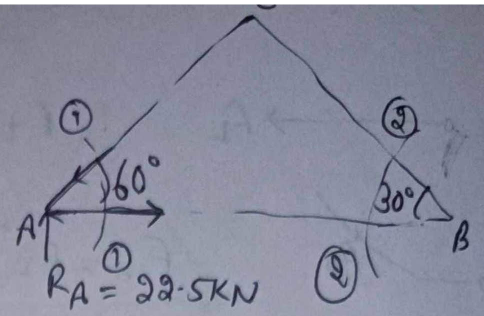
$\boxed{F_{AC} = 25.97}$ (compressive (C)) FBD of Joint A

(1) ⇒ $F_{AB} - F_{AC}\cos 60° = 0$

$F_{AB} = \frac{25.97}{2} = 12.98 N$ (T)

~~$F_{AC} = 25.97$ (T) Tensile~~

② Method of Section
or
Method of motion



sol^n: $\Sigma M_B = 0$

$R_A \times AB - F_{AC} \times BC = 0$

$F_{AC} = \frac{R_A \times 5}{4.33}$

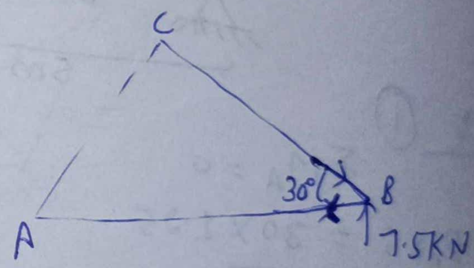
$F_{AC} = 25.98 (C)$

$BC = AC \sin 60°$
$= 4.33$

$\Sigma M_C = 0$

$R_A \times AC \cos 60° - F_{AB} AC \sin 60° = 0$

$F_{AB} = 22.5 \frac{\cos 60°}{\sin 60°} = 12.99 (T)$



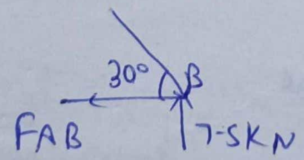
$\Sigma M_A = 0$

$R_B \times 5 - F_{BC} AC = 0$

$\boxed{F_{BC} = 15KN}$

FBD of joint B

Checking

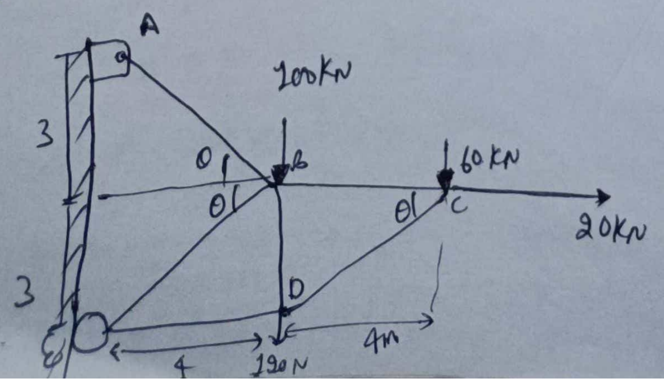


$\Sigma F_x = 0$

$F_{BC} \cos 30 - F_{AB} = 0$

$F_{BC} = 15KN (C)$

Example Analyse the truss shown in Fig.



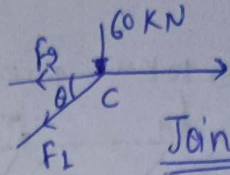
$DC = 5$

$\sin\theta = 3/5$

$\cos\theta = 4/5$

sol^n : we can not apply formula at pt D because three unknown

so Joint C



Joint C
FBD

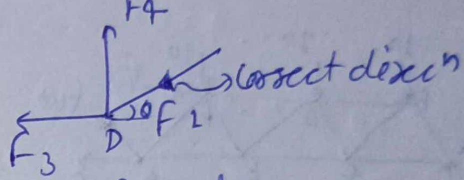
② ⇒ $-60 - F_1 \sin\theta = 0$

$F_1 = -100\ KN$

↑ opposite dirn

① ⇒ $20 - F_2 - F_1 \cos\theta = 0$

$\boxed{F_2 = 100\ KN}$



FBD of D

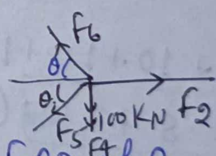
① ⇒ $F_3 + F_1 \cos\theta = 0$

② ⇒ $F_4 - F_1 \sin\theta - 120 = 0$

$F_4 = 180\ KN\ (T)$

$F_3 = -80\ KN\ (C)$

~~AB = 2~~



FBD of B

① ⇒ $F_2 = F_5 \cos\theta + F_6 \cos\theta$

$\frac{F_2}{\cos\theta} = F_5 + F_6$

$F_6 + F_5 = \frac{100 \times 5}{4} = 125\ N$ —(i)

② ⇒ $100 + F_4 + F_5 \sin\theta = F_6 \sin\theta$

$\frac{100 + 180}{\sin\theta} = F_6 - F_5$

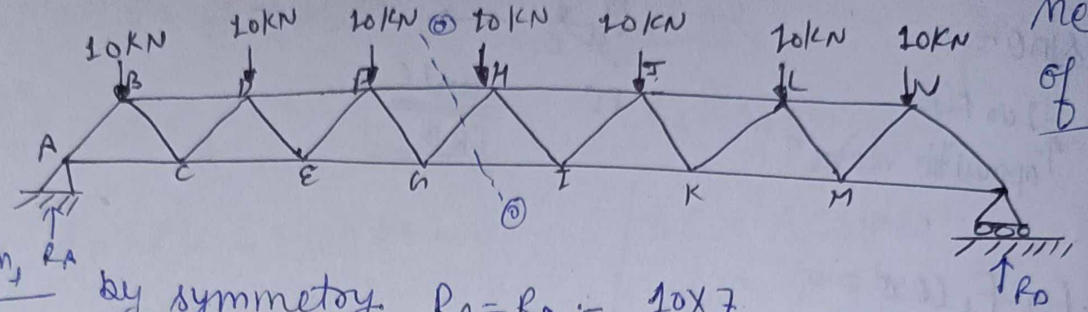
$\frac{280 \times 5}{3} = F_6 - F_5$ —(ii)

from (I) + (II)

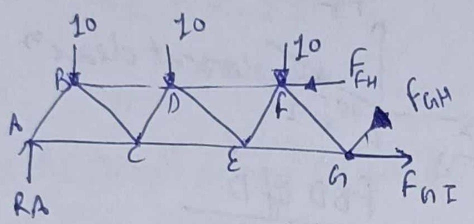
$\left.\begin{aligned} &(AB)\ F_6 = 295.8\ (T) \\ &(BE)\ F_5 = 170.8\ (C) \end{aligned}\right\}$ Ans

Ex A equilateral triangle of length 4m side find the force in FH, GH & GI. If A force 10 KN is acting on each triangle

Method of section



soln, by symmetry $R_A = R_B = \frac{10 \times 7}{2} = 35 KN$



(F.B.D of L.H.S.)

$\Sigma M_G = 0$

$-R_A \times 12 + F_{FH} \times 4 \sin 60° + 10 \times 10 + 10 \times 6 + 10 \times 2 = 0$

$\boxed{F_{FH} = 69.28 kN} (C)$

② ⇒ $F_{GH} \sin 60 + 10 + 10 + 10 - 35 = 0$

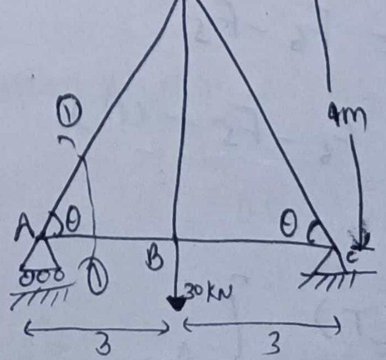
$\boxed{F_{GH} = 5.77 KN} (C)$

① ⇒ $F_{GI} - F_{FH} - F_{GH} \cos 60° = 0$

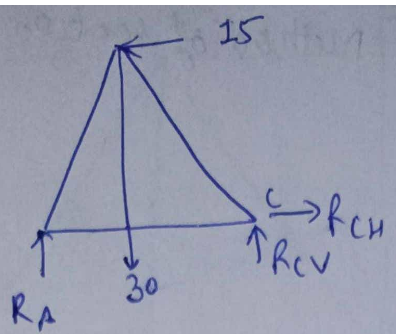
$\boxed{F_{GI} = 72.16 KN} (T)$

Ex $\sin\theta = 0.8$

$\cos\theta = 0.6$



sol^n :



$\Sigma M_C = 0$

$R_A \times 6 - 30 \times 3 - 15 \times 4 = 0$

$\boxed{R_A = 25\,KN}$

② ⇒ $R_A + R_{CV} = 30$

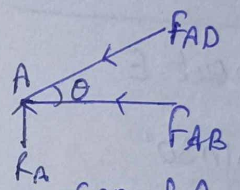
$\boxed{R_{CV} = 5}$

① ⇒ $\boxed{R_{CH} = 15}$

① Method of Joints

$F_{AD} \sin\theta = R_A$

$F_{AD} = 31.25\,KN (C)$



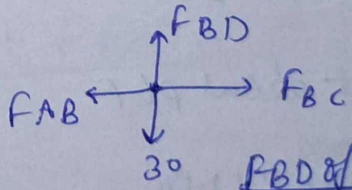
FBD of A

$F_{AD} \cos\theta + F_{AB} = 0$

$\boxed{F_{AB} = -18.75\,KN (T)}$

$F_{BD} = 30\,KN (T)$

$F_{BC} = 18.75\,KN (T)$

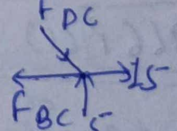


FBD of Joint B

Joint C

$F_{DC} \sin\theta = 5$

$\boxed{F_{DC} = 6.125}$ (C)

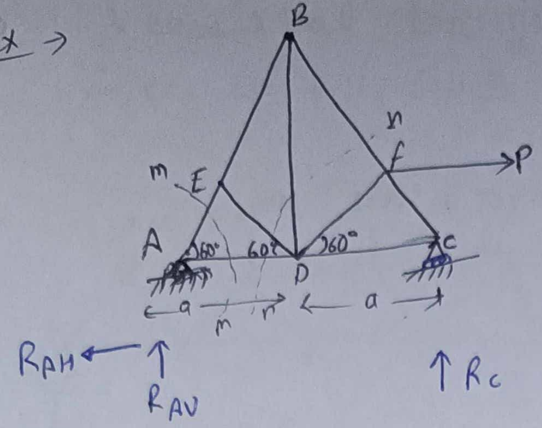


Joint C

so

Member	Load	Nature
DA	-31.25	C
AB	+18.75	T
BC	+18.75	T
BD	+30.0	T
CD	-6.25	C

Ex →



Soln →

$\Sigma M_A = 0$

$P a \sin 60 = 2a R_C$

$\boxed{R_C = 0.433P} \uparrow$

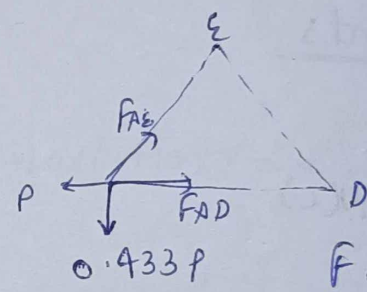
(2) ⇒ $\boxed{R_{AV} = -0.433P \downarrow}$

(1) ⇒ $\boxed{R_{AH} = P}$

Taking moment about E

$P \times a \sin 60 = 0.433 P a \sin 30° + F_{AD}\, a \sin 60°$

$\boxed{F_{AD} = 0.75P}$ (T)



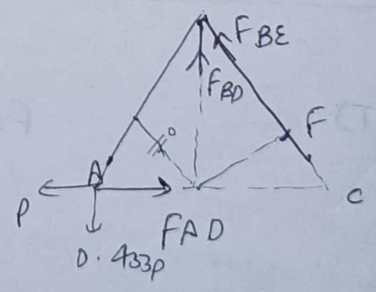
FBD of (m-m) LHS part

Force in member DE = 0

Taking moment about pt F

$F_{AD} \times a \sin 60 + 0.433 P \times 1.5a - P a \sin 60 - F_{BD} \times \frac{a}{2} = 0$

$\boxed{F_{BD} = 0.866P}$ (C)



FBD of n-n section

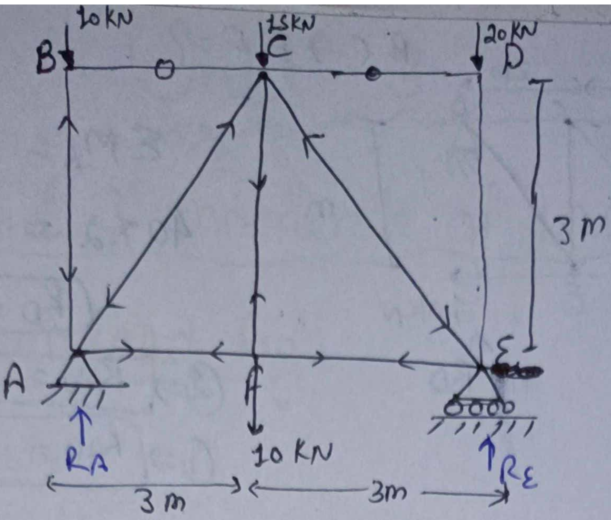
or

Taking moment about C

$R_{BD} \times a = R_{AV} \times 2a$

$\boxed{F_{BD} = 0.866C}$

Example :-



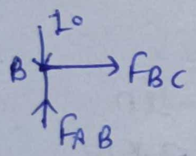
Sol^n:- **Reaction** $\Sigma M_A = 0$

$$15 \times 3 + 20 \times 6 + 10 \times 3 - R_E \times 6 = 0$$

$$\boxed{R_E = 32.5\ kN}$$

$$R_A = 10 + 15 + 20 + 10 - 32.5 = 22.5\ kN$$

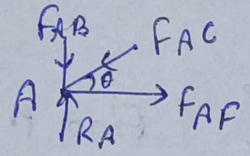
Joint B FBD



$F_{BC} = 0$

$\boxed{F_{AB} = 10\ kN\ (C)}$

Joint A FBD



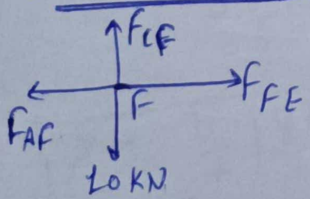
$\cos\theta = \frac{1}{\sqrt{2}}$

$\boxed{\theta = 45^\circ}$

$F_{AF} = F_{AC} \cos 45$

$\boxed{F_{AF} = 12.5\ (T)}$

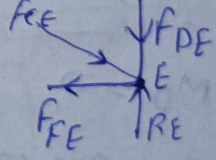
Joint F FBD



$\boxed{F_{CF} = 10\ kN\ (T)}$

$\boxed{F_{AF} = F_{FE} = 12.5\ (T)}$

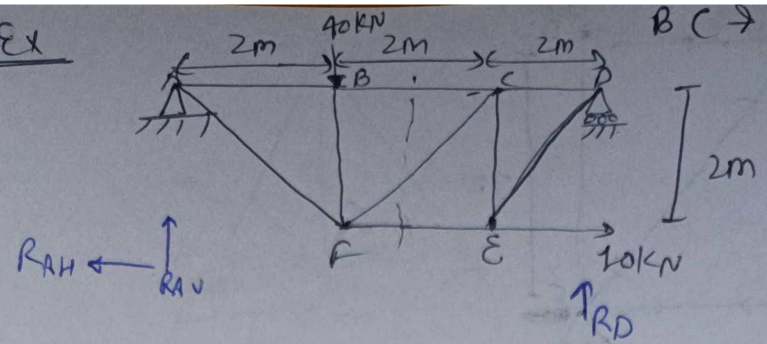
Joint E FBD



$F_{CE} \sin 45 + 20 = 32.5$

$\boxed{F_{CE} = 17.7\ kN\ (C)}$

Ex



B C → E F = (

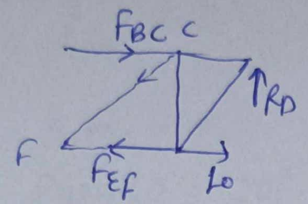
$\Sigma M_A = 0$

$40 \times 2 = R_D \times 6 + 10 \times 2$

$\boxed{R_D = 10KN}$

② ⇒ $\boxed{R_{AV} = 30 KN}$

① ⇒ $\boxed{R_{AH} = 10 KN}$



$\Sigma M_F = 0$

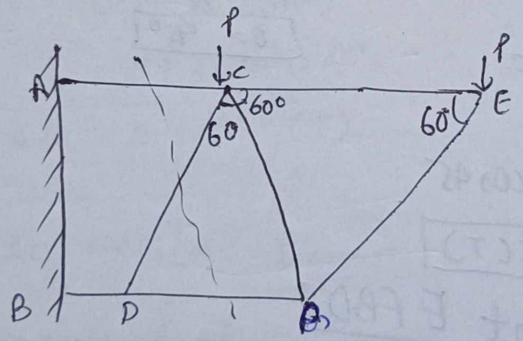
$10 \times 4 = F_{BC} \times 2 \Rightarrow \boxed{F_{BC} = 20KN(C)}$

$\Sigma M_C = 0$

$F_{EF} \times 2 = 10 \times 2 + 10 \times 2$

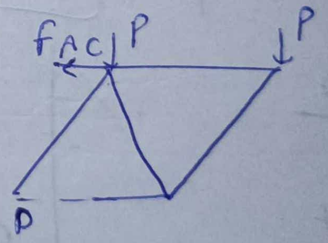
$\boxed{F_{EF} = 20KN(T)}$

Ex



Find P. s.t. Force in AC is 3 KN

sol^n,



$M_D = 0$

$F_{AC} AB = P \times 1.5 + P \times 4.5$

$\boxed{F_{AC} = 3}$ $\quad$ $\therefore AB = 3 \sin 60°$

② ⇒ $\boxed{P = 1.29}$ KN Ans