PHYSICS

Date:
Page:

The Speed of light is universal constant & it is identical for all the observers of inertial frame

* Michle Mosley Exp :

- Whether speed of light get modified with in accordance with G.T

- ~~Matter~~ Aether (Matter) - material medium which was supposed to be present throughout the universe

(i) Perfectely Elastic
(ii) Transparent
(iii) Neglible density

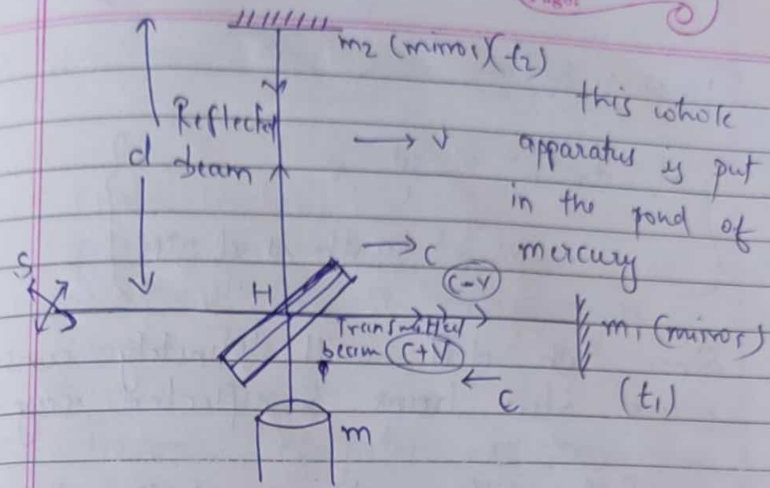
Existance of ~~matter~~ Aether was assumed as absolute frame of reference relative to the motion of bodies that can be detected.

- ~~To justify the hypothesis~~

If do not consider frame of reference, we can't detect motion of moving body

- To justify the Aether hypothesis

Date:
Page:



v : velocity of Earth / velocity of observer

we have done "c-v" and "c+v" because we are assuming that G.T is also applicable to the speed of light.

So, Reflected beam and transmitted beam both have covered '2d' distance

$\boxed{t_2 - t_1 = 0}$ and $\boxed{\text{Path difference} = 0}$

when earth is completely at rest in Aether medium

["2 beams are created from the single beam made by the source]

if considering velocity of earth / earth motion is considered

$\Delta t = t_2 - t_1 \neq 0$

$$\left\{ \Delta t = t_2 - t_1 = \frac{2d}{c} - \frac{2d}{c} = 0 \right\}$$

↳ Earth is at rest

Some Path diff. will definitely considered occur b/w trans & reflected ray.

Path diff $= \frac{dv^2}{c^2}$ ← if this is the fringe pattern should appear and visible on microscope.

when 90° rotated.

Path diff $= \frac{-dv^2}{dc^2}$

$\boxed{\text{No fringe shift was expt. observed}}$

⇒ $\boxed{v=0}$ → $\boxed{\text{motion of earth could not be detected relative to Aether}}$) when done many times the Expt.

⇒ before experiment it was assumed that earth was at rest in Aether but Expt. shows assumption is correct.

⇒ Experiment shows that Earth is absolutely at rest in Aether which is wrong

* So, Expt. has given $\boxed{\text{negative result}}$ $\boxed{\text{Null Result}}$

~~Conclo~~ Conclusion:

* Speed of light is universal const. and is identical for all the observers of inertial frame.

* Presence of Aether is meaningless.

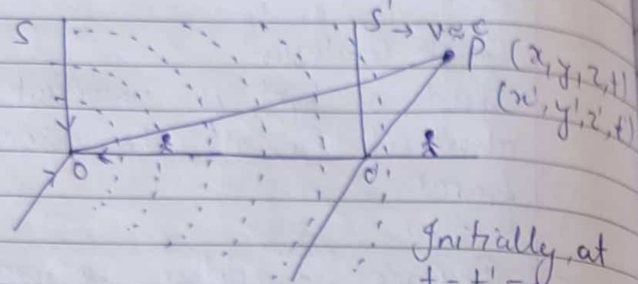
* So, Gallileon transformation is valid for velocities very less than speed of light.

Basic Postulates of Relativity

* All the laws of Physics are identical for all the observers of the inertial frame that move in a const velocity with one and another.

* The velocity of a light is a universal constant and is identical for all the observers of the inertial frame — conclusion drawn by Michelson–Morley Experiment
this is also known as constancy of velocity of light.

Lorentz Transformations



Initially, at $t = t' = 0$ origins are coinciding

Suppose a light signal is emmitted from origin O when S & S' are coinciding -

Suppose when light signal Reaches at P the coordinates of the arrival of light signal with respect to the rest frame observer is (x, y, z, t)
and similarly, with respect to moving frame observer, it is (x', y', z', t)

for rest frame observer
OP is the radius of spherical wavefront at P.

So,
time taken by the light signal to reach P for S-frame observer.

$$t = \frac{OP}{c} = \frac{(x^2+y^2+z^2)^{1/2}}{c}$$

$$\Rightarrow x^2+y^2+z^2 = c^2t^2 \quad \text{(1)}$$

for S' frame observer -

$$t' = \frac{O'P}{c} = \frac{((x')^2+(y')^2+(z')^2)^{1/2}}{c}$$

$$\therefore x'^2+y'^2+z'^2 = c^2t'^2 \quad \text{(2)}$$

$\because$ c is identical for all the observers

The new transformation will be such that equation (ii) transform to eqns (i).

let us apply G.T again.

$x' = x - vt, \quad y' = y, \quad z' = z, \quad t' = t$

$(x-vt)^2 + y^2 + z^2 = c^2t^2$

$$x^2 + v^2t^2 - 2vxt + y^2 + z^2 = c^2t^2 \quad \text{(3)}$$

Compairing eqns (i) & (3)

So, Extra term $\rightarrow v^2t^2 - 2vxt$
Gallileon Transformations failed or need some modification: -
Therefore, new transformations b/w (x, y, z, t) and (x', y', z', t) Should be such that eqn (3) transform eqn (i) and the extra

term $v^2t^2 - 2vxt$ should cancel.

Also new transformations should lead to G.T for $\frac{v}{c} \to 0$

let the modified eqns can be written as -

$x' = \alpha(x - vt)$, $t' = \alpha'(t + f(x))$

where, α, α' and f are const.

On subst these in eqn 2

$$\alpha^2(x-vt)^2 + y^2 + z^2 = c^2\alpha'^2(t+fx)^2$$

$$\alpha^2(x^2 + v^2t^2 - 2vxt) + y^2 + z^2 = c^2\alpha'^2(t^2 + f^2x^2 + 2fxt)$$

On equating coefficient of x^2, x and const. we get -

$$x^2[\alpha^2 - f^2\alpha'^2c^2] - 2xt[\alpha^2v + fc^2\alpha'^2] + y^2 + z^2$$

$$= c^2t^2\left[\alpha'^2 - \frac{\alpha^2v^2}{c^2}\right]$$

On comparing with eqn 2 -

$\alpha^2 - f^2\alpha'^2c^2 = 1$, $\alpha^2v + fc^2\alpha'^2 = 0$, $\alpha'^2 - \frac{\alpha^2v^2}{c^2} = 1$

on solving these eqn 2 -

$$\alpha = \alpha' = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} ; \quad f = \frac{-v}{c^2}$$

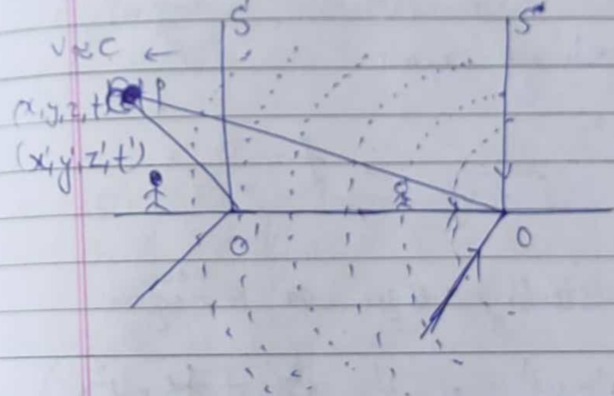
on subst. these in eqn above

$$\left\{ x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}, \; y' = y, \; z' = z, \; t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \right\}$$

Lorentz Transformation.

for small velocities - $\frac{v}{c^2} \to 0$

$x' = x - vt$; $y' = y$; $z' = z$, $t' = t$ } Galilean Transformation



$x = x' + vt'$, $y' = y$, $z' = z$, $t' = t$

$\because$ $x^2 + y^2 + z^2 = c^2t^2$

$\therefore$ $(x' + vt')^2 + y^2 + z^2 = c^2t^2$

$x = \alpha(x' + vt')$, $t' = \alpha'(t' + f(x))$

where, α, α' and f are const.

Date:
Page:

$$\alpha^2(x'+vt')^2+y'^2+z'^2 = c^2\alpha'^2(t'+fx')^2$$

$$\alpha^2(x'^2+v^2t'^2+2vx't')+y'^2+z'^2 = c^2\alpha'^2(t'^2+f^2x'^2+2t'fx')$$

$$x'^2(\alpha^2-c^2\alpha'^2f^2)+2x't'(\alpha^2v+f^2c^2\alpha'^2)+y'^2+z'^2 = c^2t'^2\left[\alpha'^2-\frac{\alpha^2v^2}{c^2}\right]$$

on comparing with eqn (2)

$$\alpha^2-c^2\alpha'^2f^2=1, \quad \alpha^2v+fc^2\alpha'^2=0$$

$$\alpha'^2-\frac{\alpha^2v^2}{c^2}=0$$

on solving eqns.

$$a=\alpha^2=\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} ; \quad f=\frac{-v}{c^2}$$

On substituting these in above eqn

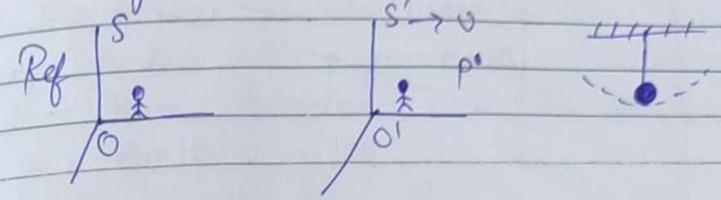
$$x=\frac{x'+vt'}{\sqrt{1-\frac{v^2}{c^2}}}, \quad t'=\left(\frac{t+\frac{v}{c^2}x'}{\sqrt{1-\frac{v^2}{c^2}}}\right)$$

$y'=y, \quad z'=z$

Date:
Page:

⇒ Consequences of lorentz transformation.

1) Time dilation → Slowing down of clock relative to stationary observer.



Proper time interval $(\Delta t')$ rest $= t_2'-t_1'$ — (1)

Improper observer time interval (Δt) motion $= t_2-t_1$ — (2)

On apply lorentz transformation :-

$$= \frac{t_2'+\frac{vx'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} - \frac{t_1'+\frac{vx'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Delta t = \frac{t_2'-t_1'}{\sqrt{1-\frac{v^2}{c^2}}}, \quad \Delta t = \frac{\Delta t'}{\sqrt{1-\frac{v^2}{c^2}}}$$

P.T.O

Time dilation is a Real Effect:

* Example from high energy physics (Elementry Particles)

μ-mesons are elementry particles which are produced in the upper atmosphere at high ~~attitudes~~ by the action of cosmic ray showers on π-mesons these are highly unstable and their life time in own frame of reference is 2.2×10^{-6} sec. So the distance travelled by μ-mesons in this life time

Travelled distⁿ in own frame

$d =$ life time $\times$ velocity

$= 2.2\times10^{-6} \times 0.998\,c$

for ease of we consider velocity as c only

$\Rightarrow d = 2.2\times10^{-6}\times3\times10^{8} = 660$ meters.

Therefore there is no expectation of finding these particles near to the surface of earth but experimentally these particles have been detected near the surface of Earth.

This can be ~~surfa~~ justified, consisting the Time dilation effect.

For laboratory frame observer :- $\Delta t = \dfrac{\Delta t'}{\sqrt{1-\frac{v^2}{c^2}}} = \dfrac{2.2\times10^{-6}}{\sqrt{1-\frac{(0.998c)^2}{c^2}}}$

$= 3.17\times10^{-5}$ sec.

Now lets,

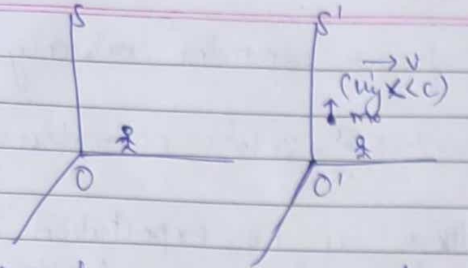
obs. by lab frame observer :-

$d' = 3.17\times10^{-5}\times3\times10^{8} = 9500$ m approx.

$\Rightarrow$ The presence of μ-mesons near the surface of Earth is justified.

$\Rightarrow$ Time Dilation is a real effect

* Mass of Particles $\rightarrow$ Mass undergoes variation with velocity when velocity become extremely high.



Can be concluded by momentum also

Momentum of particle for S' frame observer

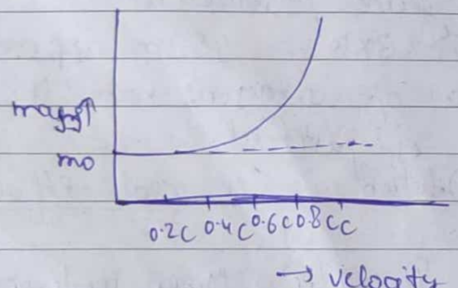
$$P_y' = m_0 u_y' = m_0 \frac{dy'}{dt'} \quad \text{--- (1)}$$

Momentum of same particle for S frame observer $dy' = dy$, $dt' = \frac{dt}{\sqrt{1-\frac{v^2}{c^2}}}$ → subst. in eq (1)

$$\frac{m_0 dy}{dt\sqrt{1-\frac{v^2}{c^2}}} = \frac{m_0 u_y}{\sqrt{1-\frac{v^2}{c^2}}} \; ; \; P_y = \frac{m_0 u_y}{\sqrt{1-\frac{v^2}{c^2}}} = m u_y$$

$$\boxed{m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}}$$

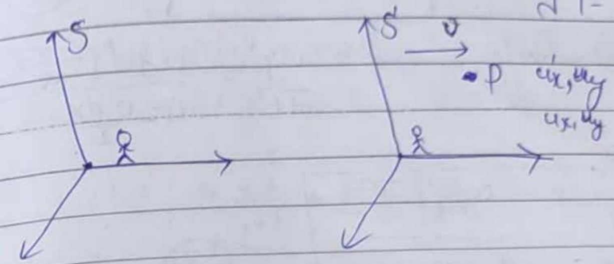
Graphical Representation :-



* Relativistic addition of velocities.

Lorentz transformations are :-

$$x' = \frac{x - vt}{\sqrt{1-\frac{v^2}{c^2}}} \; , \; y' = y \; , \; z' = z \; , \; t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$



velocity component $u_x' = \frac{dx'}{dt'}$, $u_y' = \frac{dy'}{dt'}$

$$u_z' = \frac{dz'}{dt'}$$

On differentiating Lorentz transformation equs →

$$dx' = \frac{dx - v\,dt}{\sqrt{1-\frac{v^2}{c^2}}} \; , \; dy' = dy \; , \; dz' = dz$$

$$dt' = \frac{dt - \frac{v\,dx}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$u_x' = \frac{dx'}{dt'} = \frac{dx - v\,dt}{\cancel{\sqrt{1-\frac{v^2}{c^2}}}} \times \frac{\cancel{\sqrt{1-\frac{v^2}{c^2}}}}{dt - \frac{v\,dx}{c^2}} = \frac{dx - v\,dt}{dt - \frac{v\,dx}{c^2}}$$

$$\frac{\cancel{\frac{dx}{dt}-v}}{\cancel{1-\frac{v^2}{c^2}}} \quad \frac{\frac{dx}{dt}-v}{1-\frac{v^2}{c^2}\frac{dx}{dt}} = \frac{u_x-v}{1-\frac{v^2}{c^2}u_x}$$

Therefore $\boxed{u_x' = \dfrac{u_x-v}{1-\frac{v u_x}{c^2}}}$ — (1)

Similarly, $u_y' = \dfrac{dy'}{dt'} = \dfrac{dy\sqrt{1-\frac{v^2}{c^2}}}{dt-\frac{v\,dx}{c^2}} = \dfrac{\frac{dy}{dt}\sqrt{1-\frac{v^2}{c^2}}}{1-\frac{v}{c^2}\frac{dx}{dt}}$

$$\boxed{u_y' = \frac{u_y\sqrt{1-\frac{v^2}{c^2}}}{1-\frac{v}{c^2}u_x}} \quad \text{— (2)}$$

$$u_z' = \frac{dz'}{dt'} = \frac{dz\sqrt{1-\frac{v^2}{c^2}}}{dt-\frac{v}{c^2}dx} = \frac{\frac{dz}{dt}\sqrt{1-\frac{v^2}{c^2}}}{1-\frac{v}{c^2}\frac{dx}{dt}}$$

$$\boxed{u_z' = \frac{u_z\sqrt{1-\frac{v^2}{c^2}}}{1-\frac{v}{c^2}u_x}} \quad \text{— (3)}$$

moving object moves with velocity

$$u_x' = \frac{c-v}{1-\frac{v}{c}} = c$$

which shows velocity of light is invariant.

Einstien Mass Energy relation

$E=mc^2$

consider a body move with velocity v under the action of force F

Therefore,

$$F = \frac{dP}{dt} = \frac{d(mv)}{dt} \quad \text{— (1)}$$

In relativistic mechanics both the mass and velocity are variable —

$$F = m\frac{dv}{dt} + v\frac{dm}{dt} \quad \text{— (2)}$$

workdone on the body $dW = dK = F \cdot ds$

$dK = m\frac{dv}{dt}\cdot ds + v\frac{dm}{dt}ds$

$$dK = mv\,dv + v^2 dm \quad \text{— (3)}$$

mass variation can be expressed as —

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow m^2\left[1-\frac{v^2}{c^2}\right] = m_0^2$$

$\Rightarrow m^2[c^2-v^2] = m_0^2c^2$

$m^2c^2 - m^2v^2 = m_0^2c^2$ — (4)

$\Rightarrow$ on diff. (4) $\rightarrow$

$\Rightarrow 2m\,dm\cdot c^2 - [m^2\,2v\,dv + v^2\,2m\,dm] = 0$

$\Rightarrow mv\,dv + v^2dm = c^2dm$

on subs. in eqn (3) —

$$dK = c^2 dm \quad \text{— (5)}$$

let K.E be change K.E of the body (doing movement $m_0 \to m$)

$$K = \int_{m_0}^{m} c^2 dm = (m - m_0)c^2 \quad \text{—— (6)}$$

Total Energy E = Rest mass Energy + Kinetic Energy

$$= m_0c^2 + (m-m_0)c^2$$

$$\Rightarrow \boxed{E = mc^2}$$

* Relativistic Kinetic Energy →

$$K = (m-m_0)c^2, \quad E = mc^2$$

chemical value of Kinetic Energy

$$= \frac{1}{2}m_0v^2$$

$$= m_0c^2\left[1 + \frac{v}{2c^2} + \cdots\right]$$

for smaller velocities, higher powers can be neglected.

$$K.E = \frac{1}{2}m_0v^2$$

Rel. Momentum & Energy →

$$E^2 = p^2c^2 + m_0^2c^2$$

$$E^2 - p^2c^2 = m_0^2c^2$$

$$m^2c^2 - m^2v^2c^2$$

$$\frac{m_0^2c^2}{1-\frac{v^2}{c^2}} - m_0^2c^2$$

de Broglie Hypothesis: em spectrum:

γ-rays - Hartizon Waves

Prof louis de Broglie (1914)

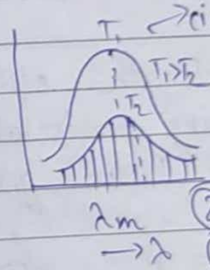
nature of Radiation (Light)

Wave nature

1. Reflection
2. Refraction
3. Interference
4. Diffraction
5. Polarisation

Particle nature

1. Black body Radiation
   (i) Weins displacement law
   (ii) Planck's law
   (iii) Stefan's law
2. Photo Electric Effect
3. compton Effect



$$\lambda_m \propto \frac{1}{T} \Rightarrow \boxed{\lambda_m T = \text{const}}$$

Planck's Radiation formula

$$\boxed{E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5}\frac{d\lambda}{e^{hc/\lambda k}}}$$

* **De Broglie Hypothesis (matter wave concept)**

The matter and radiation both exhibit the dual character i.e. particle and wave character. so In order to stablise a correlation (link) b/w these two entirely different character, Prof Louis de Broglie proposed that, " There is associated a wave with each moving material particle".

OR

A moving material particle under certain circumtancy present the wave nature.

The wavelength of the wave, associated with moving material particle (de Broglie wavelength) in given

$$\lambda = \frac{h}{mv} \quad \text{— (1)}$$

If electron enriching in H-atom presents the wave nature, the ~~the~~ circumference of the orbit of H-atom

$$2\pi r = n\lambda \quad \text{— (2)}$$

2nd Bohr's Postulates-

$$mvr = \frac{nh}{2\pi}$$

$$\Rightarrow 2\pi r = \frac{nh}{mv} \Rightarrow n\lambda = \frac{nh}{mv}$$

$$\boxed{\lambda = \frac{h}{mv}}$$

Different forms of de Broglie wavelength

(a) Non-relativistic case:

The Kinetic Energy of the particle,

$$E_K = \frac{p^2}{2m} \rightarrow p = \sqrt{2mE_K}$$

$$\boxed{\lambda = \frac{h}{\sqrt{2mE_K}}}$$

(b) Relativistic case:

In terms Relativistic K.E

$$E^2 = p^2c^2 + m_0^2c^4$$

$$\Rightarrow p^2c^2 = E^2 - m_0^2c^4 = (E_K + m_0c^2)^2 - m_0^2c^4$$

$$= E_K^2 + \cancel{m_0^2c^4} + 2E_Km_0c^2 - \cancel{m_0^2c^4}$$

$$p^2c^2 = E_K[E_K + 2m_0c^2]$$

$$p = \frac{\sqrt{E_K(E_K + 2m_0c^2)}}{c}$$

$$\Rightarrow \lambda = \frac{h}{p} = \frac{hc}{\sqrt{E_K(E_K + 2m_0c^2)}}$$

(b) In terms of accelerating potential-
$E_K = eV$; $\boxed{\lambda = \frac{h}{\sqrt{2meV}}}$

<u>de-Broglie wave velocity</u>: w

$$w = \nu\lambda = \frac{E}{\cancel{h}} \times \frac{\cancel{h}}{mv} \qquad \nu \rightarrow \text{freq}$$

$$= \frac{E}{p} = \frac{\cancel{m}c^2}{\cancel{m}v} \Rightarrow \frac{c^2}{v}$$

from relativistic mechanics-

$v << c \Rightarrow \boxed{w >> c}$

Not acceptable
$\boxed{\text{ABSURD}}$ controversial result

<u>Schrodinger's</u> <u>Solution</u>: [wave velocity (phase velocity v_p) & group velocity v_g]

To resolve the above controversial result schrodinger suggested that there may be associated a number of wave rather than single wave with a moving material particle.
He imagined the situation as the production of beats in sounds where ~~two~~ waves more than one wave superpose each-other to give rise to resultant wave and we receive the sounds in regular intervals.

for mathematical simplicity schrodinger assumed two wave having slightly different in angular frequency and propagation vector. which give rise to resultant wave.

The equation first wave constituting
$y_1 = A\cos(\omega t - kx)$ — (1)

similarily, equation of other wave

$$y_2 = A\cos[(\omega t + d\omega)t - (k + dk)x]$$

equation of resultant wave (wave group) -

$$y = y_1 + y_2 =$$
$$A\cos(\omega t - kx) + A\cos[(\omega t + d\omega) + -(k + dk)x]$$

$$= 2A\left[\cancel{\cos(2\omega t - kx)} + \cos(\omega t + d\omega)\right.$$

$$= 2A\cos\frac{1}{2}[(2\omega + d\omega)t - (2k + dk)x]\cos\frac{1}{2}[d\omega t - dkx]$$

Since $d\omega$ and dk are very small,
$2\omega + d\omega \approx 2\omega$, $2k + dk \approx 2k$

using these approximations above -

$$\boxed{y = 2A \underbrace{\cos(\omega t - kx)}_{\text{original wave}} \cos\left[\left(\frac{d\omega}{2}\right)t - \left(\frac{dk}{2}\right)x\right]}$$

Equation of Resultant wave (wave group)

The equation of resultant wave contains the original wave having angular frequency ω and propagation vector k superimposed upon a modulation having angular frequency $\frac{d\omega}{2}$ and propagation vector $\frac{dk}{2}$,

The effect of which is to produce the successive wave group.

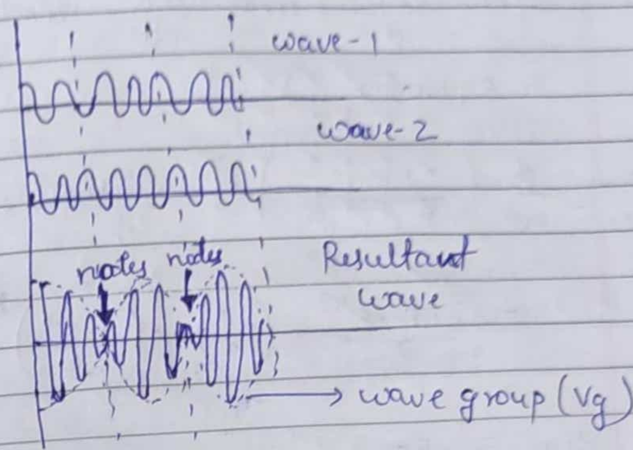
He defined the phase velocity as the ratio of angular frequency and propagation vector of the original wave. Which comes out equal to wave velocity.

phase velocity $V_p = \frac{\omega}{k} = 2\pi\nu \times \frac{\lambda}{2\pi} = \omega$

(wave velocity)

Similarily, Group velocity $v_g = \frac{d\omega/2}{dk/2} = \frac{d\omega}{dk}$

Graphical Representation of wave group →



Velocity of wave Group : V_g

The energy of the particle $E = mc^2$

$$= \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

The momentum of the particle

$$p = m_0 v = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\omega = 2\pi\nu = \frac{2\pi \times E}{h} = \frac{2\pi \times m_0 c^2}{h\sqrt{1 - \frac{v^2}{c^2}}} \quad (3)$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi p}{h} = \frac{2\pi m_0 v}{h\sqrt{1 - \frac{v^2}{c^2}}} \quad (4)$$

Group velocity $v_g = \frac{d\omega}{dk} = \frac{d\omega/dv}{dk/dv}$ — (5)

find $\frac{d\omega}{dv}$ and $\frac{dk}{dv}$ and substitute in equation (3)

$$\frac{d\omega}{dv} = \frac{d}{dv}\left(\frac{2\pi m_0 c^2}{h\sqrt{1-\frac{v^2}{c^2}}}\right) = \frac{2\pi m_0 c^2}{h} \times \frac{d}{dv}\left(\left(1-\frac{v^2}{c^2}\right)^{-1/2}\right)$$

$$= \frac{2\pi m_0 c^2}{h} \times \left(\frac{-1}{2}\right) \times \left(1-\frac{v^2}{c^2}\right)^{-3/2} \times \left(\frac{-2v}{c^2}\right)$$

$$\frac{d\omega}{dv} = \frac{2\pi m_0}{h} \times \frac{(v)}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}$$

$$\frac{dk}{dv} = \frac{d}{dv}\left(\frac{2\pi m_0 v}{h\sqrt{1-\frac{v^2}{c^2}}}\right) = \frac{2\pi m_0}{h}\frac{d}{dv}\left(\frac{v}{\sqrt{1-\frac{v^2}{c^2}}}\right)$$

$$= \frac{2\pi m_0}{h} \times \left\{\frac{\sqrt{1-\frac{v^2}{c^2}} - \frac{v}{2\sqrt{1-\frac{v^2}{c^2}}} \times \frac{-2v}{c^2}}{\left(1-\frac{v^2}{c^2}\right)}\right\} =$$

$$= \frac{2\pi m_0}{h}\frac{\left(1-\frac{v^2}{c^2}+\frac{v^2}{c^2}\right)}{\left(1-\frac{v^2}{c^2}\right)^{3/2}},$$

$$\frac{d\omega}{dk} \Rightarrow \boxed{v_g = v}$$ $\boxed{\text{acceptable}}$

Therefore, a moving material particle (microscopic particle) is associated with a wave group which moving with the same velocity as that of the particle.

OR

A microscopic moving material particle presents itself as a wave group.

* <u>Experimental confirmation of matter waves</u>:

- Electron microscopic (diffraction phenomena)
- Davisson - Germer Experiment (diffraction)
- GP Thomson Experiment

* Davisson Germer Expt: 1926

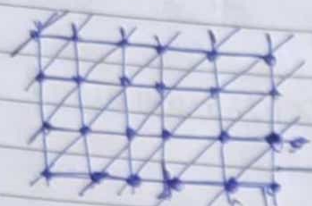
Suppose an electron is accelerated by 100 volts. The wavelength associated with this electron will

$$\lambda = \frac{h}{\sqrt{2meV}} = \frac{6.64\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times1.6\times10^{-19}\times100V}} = 1.23\,Å$$

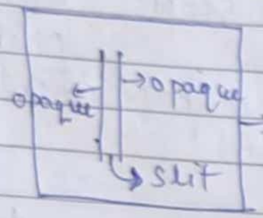
$$\frac{6.64\times10^{-34}}{}$$

Therefore, we need such a slit system is which slit spacing should be nearly of this order.

- Bragg's suggested that we should use crystal lattice insted of slit



eg →



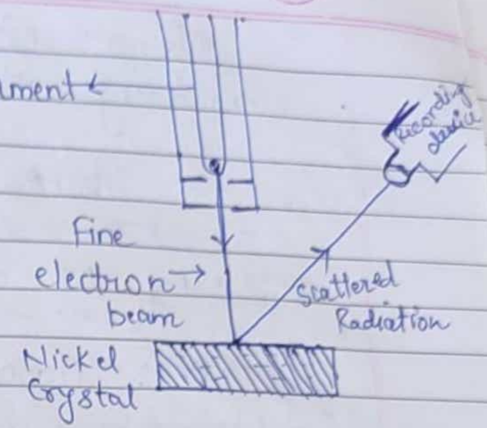
Slit System → Nickel crystal (Spacing ≈ 1.23Å)

Major Components:

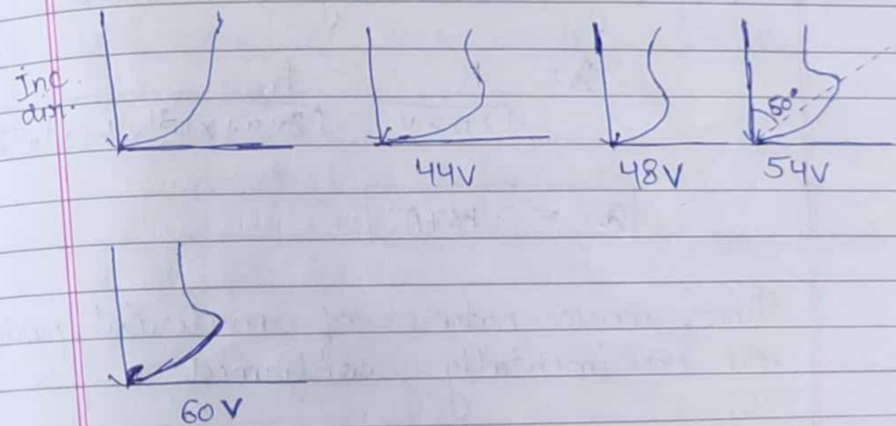
a - Electron Gun → to produce electron beam, collimate & accelerate

b - Slit system → upon which fine beam of electron is allowed to incident (Ni-crystal)

c - Recording Device - scattered intensity into current



The Experiment was performed at different voltages varying from 40v to 100 volts and the polar graph ware plotted.



If electron beam is behaving (presenting) wave it must undergo diffraction similar to x-radiation And therefore one can use Bragg's law undergo this situation

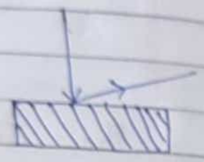
$2d\sin\theta = n\lambda$

Considering 1st order diffraction-

$2d\sin\theta = n\lambda$

Considering 1st order diffraction

$\theta \rightarrow$ Glancing Angle



$2 \times 1.23 A^\circ \times \sin 65^\circ = 1 \times \lambda$

$\boxed{\lambda = 1.65 \mathring{A}}$

Considering the de Broglie's formula

$$\lambda = \frac{h}{\sqrt{2meV}} = \frac{h}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 54}}$$

$$\boxed{\lambda = 1.67 A^\circ}$$

Hence, wave nature of material particle was experimentally confirmed.

Heisenberg's Uncertainty Principle:

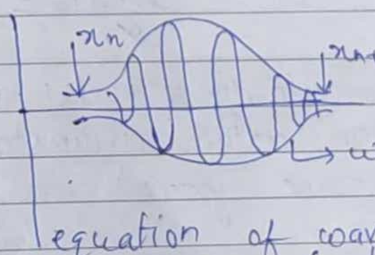
Werner Heisenberg

- dual nature of matter and radiation is responsible for this.

$$\left\{ \begin{array}{ll} \Delta P \Delta x \geq \frac{\hbar}{2} & \text{- Momentum-Position uncertainty} \\ \Delta J \Delta \theta \geq \frac{\hbar}{2} & \text{- Ang. momentum-Position uncertainty} \\ \Delta E \Delta t \geq \frac{\hbar}{2} & \text{- Energy time - uncertainty} \end{array} \right.$$

$$\boxed{\hbar = \frac{h}{2\pi}}$$

* Derivation of Uncertainty Principle:



The equation of wave group -

$$y = 2A\cos(\omega t - kx) \cdot \cos\left(\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x\right)$$

at nodes $y = 0$

$$\Rightarrow \cos\left[\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x\right] = 0$$

$$\Rightarrow \frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x_n = \frac{(2n+1)\pi}{2} \quad \text{— ①}$$

$$\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x_{n+1} = \frac{(2n+3)\pi}{2} \quad \text{— ②}$$

② – ①

$$\frac{\Delta k}{2}\left(x_n - x_{n+1}\right) = \pi$$

$$\frac{\Delta k}{2}\Delta x = \pi$$

or $\Delta k\,\Delta x = 2\pi$

$$\frac{2\pi}{h}\Delta P\cdot\Delta x = 2\pi$$

$\Rightarrow$ $\boxed{\Delta P\,\Delta x = h}$

$$K = \frac{2\pi}{\lambda} = \frac{2\pi}{h}\Delta P$$

$$\Delta K = \frac{2\pi}{h}\Delta P$$

<u>Energy time uncertainity</u> $\rightarrow$

Kinetic Energy, $E = \frac{p^2}{2m}$,

$$\Delta E = \frac{\cancel{2}P\Delta P}{\cancel{2}m} = \frac{P\Delta P}{m}$$

($\frac{P}{m}$ is velocity of packet which under certain circumstances behave as wave group)

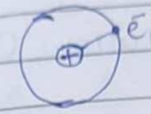
$$P = mv = \frac{m\Delta x}{\Delta t} \Rightarrow \frac{P}{m} = \frac{\Delta x}{\Delta t}$$

$$\Rightarrow \Delta E = \frac{\Delta x}{\Delta t}\cdot\Delta P \Rightarrow \Delta E\cdot\Delta t = \Delta x\cdot\Delta P$$

$$\boxed{\Delta x\cdot\Delta P = \Delta E\cdot\Delta t = h}$$

<u>Consequences of uncertainity Principle:</u>

<u>Ground State Energy & Radius of H-atom:</u>



Total Energy of H-atom –

$$E = \frac{P^2}{2m} - \frac{e^2}{4\pi\epsilon_0 r} \quad \text{— ①}$$

Error $\Delta E = \frac{\Delta P^2}{2m} - \frac{e^2}{4\pi\epsilon_0(\Delta r)}$ — ②

using uncertainity principle –

$\Delta P\Delta x \sim \hbar$ * $\Delta P\Delta r \sim \hbar$

$$\Delta P \sim \frac{\hbar}{\Delta r}$$

Assuming $\Delta P \sim P$, $\Delta r \sim r$

$$E \sim \Delta E = \frac{(\hbar)^2}{2m(\Delta r)^2} - \frac{e^2}{4\pi\epsilon_0(\Delta r)} \quad \text{— ③}$$

The system will be in the ground state (minimum energy state)

$$\therefore \frac{d(\Delta E)}{d(\Delta r)} = 0$$

On differentiating eqn ③ –

$$\frac{d(\Delta E)}{d(\Delta r)} = -\frac{\hbar^2}{\cancel{2}m}\cdot\frac{\cancel{2}}{(\Delta r)^3} + \frac{e^2}{4\pi\epsilon_0}\times\frac{1}{(\Delta r)^2} = 0$$

$$\frac{\hbar^2}{m\Delta r} = \frac{e^2}{4\pi\epsilon_0} \Rightarrow \boxed{\Delta r = \frac{4\pi\epsilon_0\hbar^2}{me^2}}$$

Date:
Page:

$$\boxed{r_0 \sim \Delta r = \frac{4\pi\epsilon_0 \hbar^2}{me^2}} = 0.53 A^\circ$$

Ground state Energy
on subst. Δr in equation (3) -

$$E_{min} \sim \Delta E = \frac{\hbar^2}{2m \cdot \left[\frac{4\pi\epsilon_0 \hbar^2}{me^2}\right]^2} - \frac{e^2}{4\pi\epsilon_0 \left[\frac{4\pi\epsilon_0 \hbar^2}{me^2}\right]}$$

$$= \frac{\cancel{h}^2 (6.63\times10^{-34})^2}{4\pi^2 \times 2\times 9.1\times10^{-31} \times (0.53\times10^{-10})^2} - \frac{(1.6\times10^{-19})^2 \times 10^9}{0.53\times10^{-10}}$$

$$= -13.6\,eV$$

* Zero Point Energy of Harmonic Oscillator:

Total Energy of Harmonic oscillator -

$$E = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$$

$$\Delta E = \frac{(\Delta p)^2}{2m} + \frac{1}{2} m\omega^2 (\Delta x)^2 \text{ — (1)}$$

using uncertainity Principle -

$$\Delta p \Delta x \sim \hbar \Rightarrow \Delta p \sim \frac{\hbar}{\Delta x}$$

Assuming $\Delta E \sim E$, $\Delta x \sim x$

Date:
Page:

The system will be in the lower energy state -

$$\frac{d(\Delta E)}{d(\Delta x)} = 0$$

$$-\frac{\hbar^2}{\cancel{2}m} \times \frac{\cancel{2}}{(\Delta x)^3} + \frac{1}{\cancel{2}} m\omega^2 (\Delta x)\cdot \cancel{2} = 0$$

$$\frac{\hbar^2}{m(\Delta x)^3} = m\omega^2 (\Delta x)$$

$$(\Delta x)^4 = \frac{\hbar^2}{m^2\omega^2}$$

$$(\Delta x)^2 = \frac{\hbar}{m\omega} \; ; \; \Delta x = \sqrt{\frac{\hbar}{m\omega}}$$

on substituting. Δx in eqn (1) & simplifying

$$\Delta E = \frac{(\Delta p)^2}{2m} \; \frac{\hbar^2}{2m(\Delta x)^2} + \frac{1}{2} m\omega^2 (\Delta x)^2$$

$$= \frac{\hbar^2}{2m \cdot \frac{\hbar}{m\omega}} + \frac{1}{2} \cancel{m}\omega^2 \cdot \frac{\hbar}{\cancel{m}\omega}$$

$$= \frac{\hbar\omega}{2} + \frac{\hbar\omega}{2} = \hbar\omega$$

$$\boxed{E_{min} \sim \Delta E = \boxed{\hbar\omega_0}} \rightarrow \text{most probable value}$$

8

Date:
Page:

3. Non-Existance of Electron in the nucleus:-

Assume that electron can reside in the nucleus

Nuclear diameter $\sim 10^{-14}$ m

$\Delta P \Delta x \sim \hbar$

Assume that $\Delta P \sim P$, $\Delta x \sim x$

$P \sim \Delta P \sim \frac{\hbar}{\Delta x} = \frac{1.05 \times 10^{-34}}{10^{-14}} = 1.05 \times 10^{-20}$ J-sec.

An electron whose momentum is uncertain by 1.05×10^{-20} this much order of magnitude, must have energy many times to this value using relativistic mom.-energy relation:

$$E^2 = p^2c^2 + m_0^2c^4$$

$$E = \sqrt{p^2c^2 + m_0^2c^4}$$

$$E = \sqrt{(1.05 \times 10^{-20})^2 * (3.0 \times 10^8)^2 + (9.1 \times 10^{-31})^2 \times (3 \times 10^8)^4}$$

$$E = \frac{1.05 \times 10^{-20} \times 3 \times 10^8}{1.6 \times 10^{-19}} = 2.0 \times 10^7 \text{ eV} = 20 \text{ MeV}$$

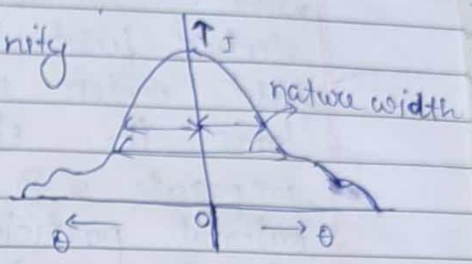
but experimentally using β-ray ≈ 2-3 MeV

$\Rightarrow$ electron can't reside in the nucleus.

Date:
Page:

4. Finite width of spectral lines:

Energy time uncertainity relation -



$\Delta E \Delta t \sim \hbar$

$\hbar \Delta \nu \cdot \Delta t \sim \hbar$

$\Delta \nu = \frac{1}{\Delta t} = \frac{1}{10^{-8}}$

$\Delta \nu \approx 10^8$ Hz Spectral lines can never be infinitely sharp

E is const. {by Bohr's 1st law}

$\therefore \Delta E = 0$

$\Delta E \cdot \Delta t \sim \hbar$; $\Delta t \sim \frac{\hbar}{\Delta E} \sim \infty$

Actually it is only 10^{-8} s

Matter Wave function and its physical Significance 'ψ'

* Basic postulates of Quantum Mechanics (Wave Mechanics)

When we associate a wave with a material particle this definetely indicate that certain physical quantity will we varying in space and time for example:- The variation of pressure of in space and time gives rise to the propagation of mechanical wave! Similarly the variation of electric field vector and magnetic field vector in space and time leads the electromagnetic wave to propagate. So if the microscopic material particle exhibit as a wave (matter wave). it must have then some wave equation in which some physical quantity will definitely be there which will vary in space and time. At variable quantity characterising a matter wave is known as wave function represented by 'ψ'. This 'ψ' alone has no physical significance since it is linked with the probability of finding the particle, and the probability can never be negative. It is either zero or one.

$y = a\sin(\omega t + \phi)$

$|\psi|^2$ will always be positive and therefore it is known as probability density, if it is considered over unit volume of space

$|\psi|^2 \propto$ Prob. of finding the particle in a given region at a ~~given~~ instant.

$\int_{-\infty}^{\infty} |\psi|^2 dv = 1 \rightarrow$ containing of finding the particle

In many situations, the wavefunction is a complex quantity, i.e,

$\psi = a + ib$, $\psi^* = a - ib$

$\psi\psi^* = a^2 + b^2 =$ a real quantity

$\int_{-\infty}^{\infty} \psi(x,t)\, \psi^*(x,t)\, dv = 1 \rightarrow$ wavefunction is said to be normalized.

$= 0 \rightarrow$ absence of particle in a given region at a given instant (wave function is said to be orthogonal)

Basic requirement of $\psi \rightarrow$

1. for all values of (x,y,z), ψ should be single value.

2. for all values of (x,y,z) ψ should be continuous.

3. for all values of (x,y,z) ψ should be finite. (probability always finite)

4. $\frac{\partial \psi}{\partial x}, \frac{\partial \psi}{\partial y}, \frac{\partial \psi}{\partial z}$ (partial derivatives) must exist and continuous)

Now, if ψ is not normalized, then it is to be multiply by a constant 'A'.

$$\int_{-\infty}^{\infty} A\,\psi(x,t)\,\psi'(x,t) = 1$$

$$A^2 = \int_{\infty}^{\infty} \psi(x,t)\,\psi^*(x,t)\,dV = 1$$

$$\Rightarrow \boxed{A^2 = \frac{1}{\int_{\infty}^{\infty} \psi(x,t)\,\psi(x,t)\,dV}}$$ → Normalized condition (presence of particle)

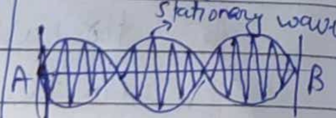
{dx dy dz}

* Schrodinger's Wave Equation:

The differential equation of wave motion

$$\boxed{\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}}$$ ← (1) Classical knowledge

$$y = A e^{-i\omega(t - \frac{x}{v})}$$



$y = A\sin(\omega t - \phi)$, $\frac{dy}{dt} = 0$

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2}$$

$$\psi(x,t) = A e^{-i\omega(t - \frac{x}{v})} = A e^{-2\pi i(\nu t - \frac{\nu x}{v})}$$

$$= A e^{-2\pi i(\nu t - \frac{x}{\lambda})} \quad \text{— (2)}$$

using $\lambda = \frac{h}{p} = \frac{2\pi\hbar}{2\pi p} = \frac{2\pi\hbar}{p}$ } de-Broglie Concept

$$E = h\nu = \frac{2\pi\hbar\nu}{2\pi} = 2\pi\hbar\nu$$ } de-Broglie concept

$$\boxed{\psi(x,t) = A e^{-\frac{i}{\hbar}(Et - px)}} \quad \text{— (3)}$$

eqn of wave particle (of that microscopic material particle) which under certain circumstances present wave character

* Total energy of the Particle -

$$E = \frac{p^2}{2m} + V \quad \text{— (4)}$$

on opening this by $\psi(x,t)$ -

$$E\psi(x,t) = \frac{p^2}{2m}\psi(x,t) + V\psi(x,t)$$

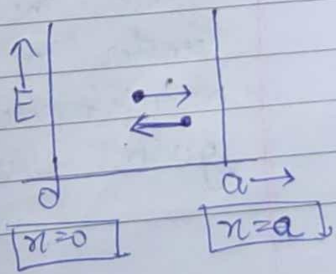
On differentiating partially eqn (3) w.r.t t -

$$\frac{\partial \psi(x,t)}{\partial t} = \frac{-iE}{\hbar}\psi(x,t) \Rightarrow \boxed{E}\psi(x,t) = \boxed{\frac{-\hbar}{i}\frac{\partial \psi(x,t)}{\partial t}}$$

Application of Schrodinger's Equations :-

Particle confined in one-dimensional box:

Let us assume a box which is having the side a and the walls of the box are



infinitely rigid in this box a particle is trapped which is free to move inside and can not cross the walls as the walls are infinitely rigid. During the motion this particle hits the walls and the walls of the box compell the particle to bounce back. In this way the particle execute a to and fro motion which within the box. In several situation we find at the microscopic level where the particle's motion is restricted

⇒ for example :-

A gas molecule inside the cylinder, free electron inside a metal piece, e^- revolving in an atom. In all the above situation the particle's motion is subject to certain restrictions.

* The basic objective of the above problem is to see ⇒

① How the Schrodinger's eqn is solve under a situation where the particle's motion is restricted to a given region

② To find out the particle's energy which it can have when its motion is restricted and to see whether such a particle can have all possible energies or not.

③ To compare the new result with those well known classical results.

* Schrodinger's equation can be written as -

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2}(E-V)\psi(x) = 0 \quad \text{— ①}$$

for a free particle, $V=0$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2} E\psi(x) = 0$$

$$\frac{\partial^2 \psi}{\partial x^2} + K^2\psi(x) = 0 \quad \text{— ②}$$

where $K^2 = \frac{2mE}{\hbar^2}$

The solution can be written as

$$\psi(x) = Ae^{ikx} + Be^{-ikx} \quad \text{— ③}$$

where A & B are constants

The Boundary conditions are — $[x=0$ & $x=a]$

$$0 = A+B \Rightarrow \boxed{B=-A}$$

$$\psi(x) = Ae^{ikx} - Ae^{-ikx}$$
$$= 2iA\sin kx = C\sin kx$$

Applying boundary condition at $x=a$

$$0 = C\sin ka$$
$$\sin kx = 0 \quad , \quad ka = n\pi \; \therefore K = \frac{n\pi}{a}$$

The particle's momentum :-
$$P_n = \hbar K = \frac{n\pi\hbar}{a}$$

Energy of the particle $E_n = \frac{P_n^2}{2m}$
$$= \frac{\pi^2 n^2 \hbar^2}{2ma^2} \quad \text{--- (4)}$$

where $n = \cancel{0}, 1, 2, 3, \ldots$

$\cancel{E_0}$ - $E_1 = \frac{\pi^2\hbar^2}{2ma^2} \rightarrow$ lowest energy $\rightarrow n=1$

1st Excited state, $E_2 = \frac{4\pi^2\hbar^2}{2ma^2} = 4E_1 \rightarrow n=2$

$E_3 = \frac{9\pi^2\hbar^2}{2ma^2} = 9E_1 \rightarrow n=3$

When the particle's motion is restricted it's energy is quantised

Energy Level Diagram :-

$E_3 = 9E_1$ ——— $n=3$

$E_2 = 4E_1$ ——— $n=2$

$E_1 = \frac{\pi^2\hbar^2}{2ma^2}$ ——— $n=1$

not permitted $\times$ - - - - - - - - $n=0$

i.e the particle can exhibit certain discrete values of the energy (when its motion is restricted) and not any arbitrary value.

The wave function: $\psi_n(x)$

$$\psi(x) = C\sin kx = C\sin\frac{n\pi x}{a} \quad \text{--- (5)}$$

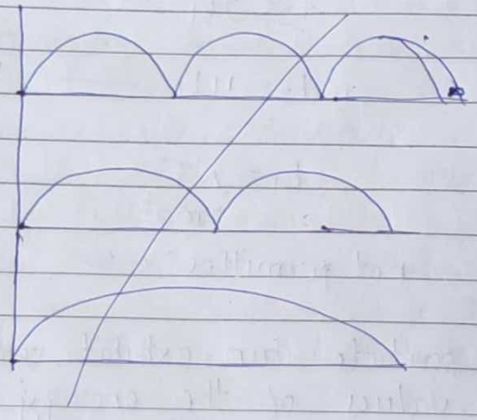
W.f should be normalised-

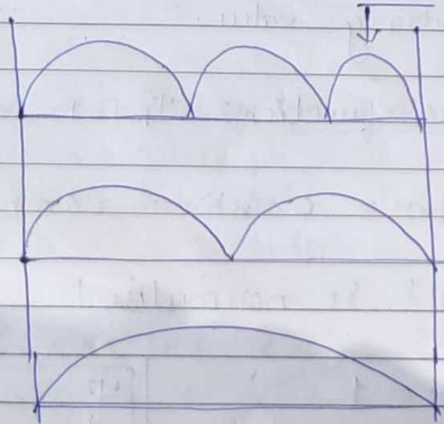
$$\int_0^a |\psi(x)|^2 dx = 1$$
$$c^2 \int_0^a \sin^2\left(\frac{n\pi x}{a}\right) dx = 1$$

on solving $\rightarrow$ $C = \sqrt{\frac{2}{a}}$

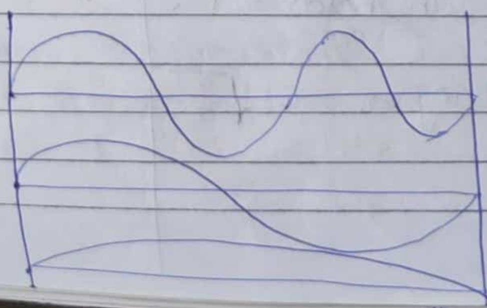
$$\boxed{\psi_n(x) = \sqrt{\frac{2}{a}} \sin\frac{n\pi x}{x}}$$

Prob. ~~Density~~ Density





Prob. density



$\psi_3(x)$

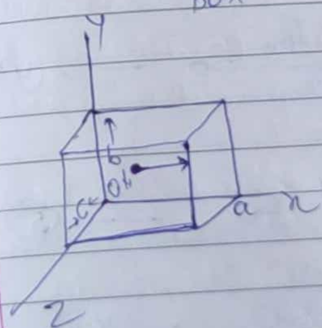
$\psi_2(x)$

$\psi_1(x)$

Particle in three dimensional box:

Objective:

Identical to one-dimensional box



Side of the box are: a, b, c

Mathematically it can be expressed

$V(x,y,z) = 0 \; ; \; 0 < x < a$

$V(x,y,z) = 0 \; ; \; 0 < y < b$

$V(x,y,z) = 0 \; ; \; 0 < z < c$

$V(x,y,z) = \infty \rightarrow$ outside the box

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} + \frac{2mE\psi}{\hbar^2} = 0 \quad \text{--- (1)}$$

$$\psi(x,y,z) = X(x) \cdot Y(y) \cdot Z(z) \quad \text{--- (2)}$$

Putting this in eqn (1) -

$$YZ\frac{\partial^2 X}{\partial x^2} + XZ\frac{\partial^2 Y}{\partial y^2} + XY\frac{\partial^2 Z}{\partial z^2} + \frac{2mEXYZ}{\hbar^2} = 0$$

on dividing by XYZ,

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} + \frac{1}{Z}\frac{\partial^2 Z}{\partial z^2} + \frac{2m}{\hbar^2}E_{(n_x,n_y,n_z)} = 0$$

Where $E_{n_x,n_y,n_z} = E_{n_x} + E_{n_y} + E_{n_z}$ — (3)

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + \frac{2mE_{n_x}}{\hbar^2} = 0 \quad \text{— (4)}$$

$$\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} + \frac{2mE_{n_y}}{\hbar^2} = 0 \quad \text{— (5)}$$

$$\frac{1}{Z}\frac{\partial^2 Z}{\partial z^2} + \frac{2m}{\hbar^2}E_{n_z} = 0 \quad \text{— (6)}$$

$$\frac{\partial^2 X}{\partial x^2} + \frac{2mE_{n_x}}{\hbar^2}X(x) = 0 \quad \text{— (7)}$$

$$\frac{\partial^2 Y}{\partial y^2} + \frac{2mE_{n_y}Y(y)}{\hbar^2} = 0 \quad \text{— (8)}$$

$$\frac{\partial^2 Z}{\partial z^2} + \frac{2m}{\hbar^2}E_{n_z}Z(z) = 0 \quad \text{— (9)}$$

$$E_{n_x} = \frac{\pi^2\hbar^2 n_x^2}{2ma^2}, \quad E_{n_y} = \frac{\pi^2\hbar^2 n_y^2}{2mb^2}, \quad E_{n_z} = \frac{\pi^2\hbar^2 n_z^2}{2mc^2}$$

$$E_{n_x,n_y,n_z} = \frac{\pi^2\hbar^2}{2m}\left[\frac{n_x^2}{a^2} + \frac{n_y^2}{b^2} + \frac{n_z^2}{c^2}\right] \quad \text{— (10)}$$

for a cubical box -> $a = b = c = l$

$$E_{n_x,n_y,n_z} = \frac{\pi^2\hbar^2}{2ml^2}\left[n_x^2 + n_y^2 + n_z^2\right] \quad \text{— (11)}$$

$n_x = 1,2,3,\ldots$
$n_y = 1,2,3,\ldots$
$n_z = 1,2,3,\ldots$

Wave function-

$$X(x) = \sqrt{\frac{2}{a}}\sin\frac{n_x\pi x}{a}$$

$$Y(y) = \sqrt{\frac{2}{b}}\sin\frac{n_y\pi y}{b}$$

$$Z(z) = \sqrt{\frac{2}{c}}\sin\frac{n_z\pi z}{c}$$

The complete wave function

$$\psi_{(n_x,n_y,n_z)} = X(x)\,Y(y)\,Z(z)$$

$$= \sqrt{\frac{8}{abc}}\sin\frac{n_x\pi x}{a}\sin\frac{n_y\pi y}{b}\sin\frac{n_z\pi z}{c}$$

Date:
Page:

For cubical box, $= \sqrt{\frac{8}{l^3}} \sin \frac{n_x \pi x}{l} \cdot \sin \frac{n_y \pi y}{l} \cdot \sin \frac{n_z \pi z}{l}$

Conclusion :-

① Minimum 3 integers n_x, n_y, n_z which are also known as quantum no. are needed to describe each stationary state any change in the sign of these quantum numbers does not affect the total energy therefore all the stationary states are assigned by positive and integral values

② Non of these quantum can be zero because that would mean the particle's absence in the region ~~that~~ under consideration.

③ The minimum energy of the particle, $n_x=1, n_y=1, n_z=1$

$$E_{n_x,n_y,n_z} = \frac{3\pi^2 \hbar^2}{2ml^2} \longrightarrow \psi_{(1,1,1)} = \sqrt{\frac{8}{l^3}}$$

$$\psi_{111} = \sqrt{\frac{8}{l^3}} \sin \frac{\pi x}{l} \sin \frac{\pi y}{l} \sin \frac{\pi z}{l}$$

$\boxed{\text{Non- degerate state}}$ ← Groundstate Energy level

Page:

1st Excited state :-

$n_x=2, n_y=1, n_z=1$

$$E_{n_x,n_y,n_z} = \frac{6\pi^2 \hbar^2}{2ml^2} \begin{cases} \psi_{211} \\ \psi_{121} \\ \psi_{112} \end{cases} \Rightarrow \text{Triply degenerate state}$$

n_x	n_y	n_z
2	1	1
1	2	1
1	1	2

fourth Excited state

$n_x n_y \longrightarrow n$

$n_x = n_y = n_z = 2$

$$E_{n_x,n_y,n_z} = \frac{12\pi^2 h^2}{2ml^2} - \psi_{222} - \text{non degenerate state}$$

Third excited state $n_x = 3, n_y = 1, n_z = 1$

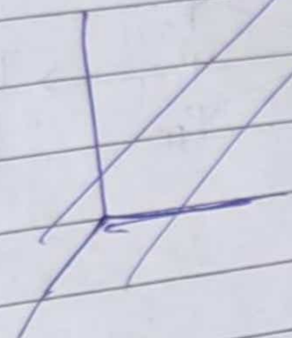
Electromagnetism

formation → for fundamental laws of electric and magnetic

1st gauss law ⇒ $\oint E \cdot ds = \frac{q}{\epsilon_0}$

$= 0$ { when closed surface does not enclose any charge }

~~Time dilation with the help of Lorentz transformation~~



Gauss law of magnetism

2nd :- ~~mag~~ monopole does not exist

$$\oint \vec{B} \cdot \vec{ds} = 0$$

Since, magnetic field lines form a closed path.

3rd :- Faraday law

→ Rate of change of magnetic flux i.e equal to induced emf

$$\varepsilon = -\frac{d\phi_B}{dt}$$ → time varying field

4th

3. Faraday law of e.m. induction.

$\oint_c \vec{E}\cdot d\vec{l} = e = -\frac{d\phi_B}{dt}$ = time ~~lagging~~ varying magnetic field

$\oint_2 \vec{H}\cdot d\vec{l} = I$ } - Ampere's law

$\oint_2 \vec{B}\cdot d\vec{l} = \mu_0 I$ } valid for static field

or

it is not const. fig time varying field (is not include time varying field)

↓

Incomplete

1. Stoke's theorem →

Surface integral → time integral

2. Gauss's Divergence Theorem →

Surface integral → volume integral

Maxwell's Equations (differential equation)

$D = \rho$ → Relates the electric flax with change each by hypothetical surface.

$div.B = 0$ → Monopole does not exist in magnetism, Magnetic flux always ~~from~~ form a closed path

$\bar{\nabla} \times \bar{E}$ (curl E)

$\oint e = -\frac{d}{dt} \oint \bar{B} \cdot d\bar{s} \rightarrow$ Relates time changing magnetic field with electric field.

$\nabla \times \bar{H} = J + \frac{\partial D}{\partial t} \rightarrow$ Relates time changing electric field with magnetic field. (modified ampere's law)

* Conservation of charge : Equation of continuity

Total charge contained in a given volume remains conserved. The decrease of change in a given volume constitutes a current

$$I = -\frac{dq}{dt} = -\frac{d}{dt} \oint_V \rho \, dV$$

$\rho \rightarrow$ volume density of charge

$$\oint_S \bar{J} \cdot d\bar{s} = -\oint_V \frac{\partial \rho}{\partial t} dV$$

$\oint_S \bar{J} \cdot d\bar{s}$ using Gauss's divergence theorem-

$$\oint_S (\text{div } J) \, dV = -\oint_V \frac{\partial \rho}{\partial t} dV$$

$\Rightarrow$ $\text{div } J = -\frac{\partial \rho}{\partial t}$ | $\nabla \cdot J = -\frac{\partial \rho}{\partial t}$

or $\boxed{\text{div } J + \frac{\partial \rho}{\partial t} = 0}$ - Equation of continuity (Conservation of charge)

For stationary charge (static field)

$\rho = \text{const.}$ & $\frac{\partial \rho}{\partial t} = 0$

$\Rightarrow$ $\boxed{\text{div } J = 0}$

<u>Derivation of Maxwell's Equations:</u>

1. $\text{div } D = \rho$
2. $\text{div } B = 0$
3. $\nabla \times \bar{E} = -\frac{\partial B}{\partial t}$
4. $\nabla \times \bar{H} = I + \frac{\partial D}{\partial t} \rightarrow$ {Time ~~time~~ changing ~~electrostatics~~ electric field}

$\text{div } D = \rho$

1. From Gauss's law of electrostatic-

$$\oint_S \bar{E} \cdot d\bar{s} = \frac{q}{\epsilon_0} = \frac{1}{\epsilon_0} \oint_V \rho \, dV$$

$$\oint_S ((\epsilon_0 \bar{E}) \cdot d\bar{s}) = \oint_V \rho \, dV$$

Using Gauss's divergence then-

$$\oint_V (\text{div } D) \, dv = \oint_V \rho \, dV$$

$\Rightarrow \boxed{\text{div } D = \rho}$

2. div $B = 0$

$\phi_B = \oint_S \bar{B} \cdot d\bar{s} = 0$

Applying Gauss's divergence theorem

$\oint_V (\text{div } B)\, dv = 0$

$\Rightarrow \boxed{\text{div } B = 0}$

3. $\bar{\nabla} \times \bar{E} = -\dfrac{\partial B}{\partial t}$

From ~~for~~ Faraday's law of e.m induction.

$e = -\dfrac{d\phi_B}{dt} = -\dfrac{d}{dt} \oint_S \bar{B} \cdot d\bar{s}$

$\oint \bar{E} \cdot d\bar{l} = -\oint \dfrac{\partial \bar{B}}{\partial t}\, \partial \bar{s}$

Using stoke's theorem -

$\oint (\nabla \times \bar{E})\, ds = -\oint_S \dfrac{\partial \bar{B}}{dt}\, \partial \bar{s}$

$\Rightarrow \nabla \times \bar{E} = -\dfrac{\partial B}{\partial t}$

(4) $\nabla \times \bar{H} = J + \dfrac{\partial D}{\partial t}$

using ampere's law -

$\oint_C \bar{H} \cdot d\bar{l} = I = \oint_S \bar{J} \cdot d\bar{s}$

Using stoke's theorem

$\oint_S (\nabla \times \bar{H})\, ds = \oint_S \bar{J} \cdot d\bar{s}$

$\Rightarrow \nabla \times \bar{H} = J$ ...... (1)

On taking divergence

$\bar{\nabla} \cdot (\nabla \times \bar{H}) = \bar{\nabla} \cdot \bar{J}$

$\Rightarrow \bar{\nabla} \cdot \bar{J} = 0 \Rightarrow$ ~~Stati~~ valid for static field

Maxwell found it incomplete as it does not include time varying electric field. So he suggested that something is missing in eqn -(1). And let this missing term similar to current density J_d.

$\nabla \times \bar{H} = J + J_d$ ...... (2)

Again on taking divergence

$\bar{\nabla} \cdot (\nabla \times \bar{H}) = \bar{\nabla} \cdot \bar{J} + \bar{\nabla} \cdot \bar{J_d}$

$0 = \nabla \cdot \bar{J} + \bar{\nabla} \cdot \bar{J_d}$

$\Rightarrow \bar{\nabla} \cdot \bar{J_d} = -\nabla \cdot \bar{J} = \dfrac{\partial \rho}{\partial t} = \dfrac{\partial}{\partial t}(\nabla \cdot D)$

$$\bar{\nabla}\cdot\bar{J_d} = \bar{\nabla}\cdot\frac{\partial D}{\partial t}$$

$$\Rightarrow J_d = \frac{\partial D}{\partial t}$$

on subst in eqn (2) - modified form of Ampere's law

$$\oint \bar{H}\cdot d\bar{l} = \boxed{\nabla \times \bar{H} = \underset{\text{current}}{J} + \underset{\text{(Displacement current)}}{\frac{\partial D}{\partial t}}}$$

→ magneto motive force (mmf)

$\boxed{\text{magnetic circuit}}$

↓

Path of magnetic flux line

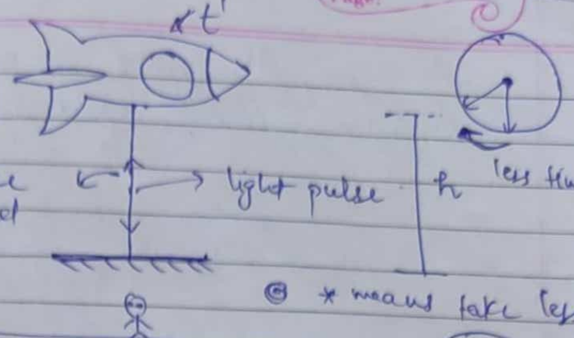
Magneto motive force (mmf) is defined as workdone by isolated north pole once surround the closed magnetic circuit if it is able to do so

ohm's law of magnetism -

$e = iR$

$\boxed{mmf = \phi R_e}$ → ohm's law of magnetism

Time dilation

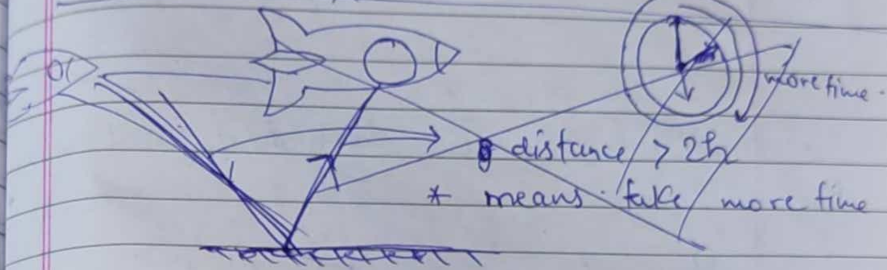


$2h = ct'$

$\boxed{t' = \frac{2h}{c}}$

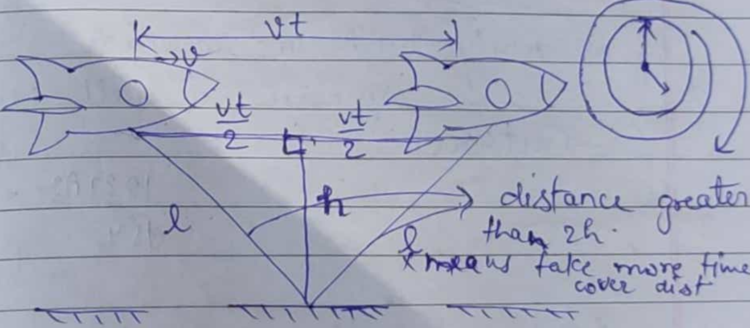
distance travelled → $2h$

* means take less time



distance > $2h$

* means take more time



→ distance greater than $2h$.

& means take more time to cover dist

$2l = ct$

$$t = \frac{2h}{c\sqrt{1-\frac{v^2}{c^2}}}$$

$$\boxed{\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}}$$

$$l^2 = \frac{v^2t^2}{4} + h^2$$

$$\frac{c^2t^2}{4} = \frac{v^2t^2}{4} + h^2$$

$$t^2\left(\frac{c^2-v^2}{4}\right) = h^2$$

$$\boxed{t = \frac{t'}{\sqrt{1-\frac{v^2}{c^2}}}}$$

$$t^2 = \frac{4h^2}{\ldots} \;;\; \boxed{t = \frac{2h}{\ldots}}$$