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[Maths assignment Unit - 4]

1)

$$A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

$$A = AI$$

$$\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_1 \to R_1 - R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_2 \to R_2 - 2R_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_2 \to R_2 - 4R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_3 \to R_3 + R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -1 \\ -2 & 3 & -3 \end{bmatrix} A$$

$$A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$$

2)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$$

$$A = IA$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_2 \rightarrow R_2 - 2R_1$, $R_3 \rightarrow R_3 - 3R_1$

$$\cancel{\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & -1 \\ 1 & 2 & 3 \\ 0 & 0 & -1 \\ 3 & 5 & 6 \end{bmatrix}} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & -1 \\ 0 & -1 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} A$$

$R_2 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -3 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} A$$

$R_2 \leftrightarrow -R_2$, $R_3 \rightarrow -R_3$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 0 & -1 \\ 2 & -1 & 0 \end{bmatrix} A$$

$R_1 \rightarrow R_1 - 2R_2$

$$\begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 2 \\ 3 & 0 & -1 \\ 2 & -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -3 & 2 \\ -3 & 3 & -1 \\ 2 & -1 & 0 \end{bmatrix} A$$

$$A^{-1} = \begin{bmatrix} 1 & -3 & 2 \\ -3 & 3 & -1 \\ 2 & -1 & 0 \end{bmatrix}$$

3)

$$A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1+i}{\sqrt{3}} \\ \frac{1-i}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \end{bmatrix}$$

$$\bar{A} = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1-i}{\sqrt{3}} \\ \frac{1+i}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \end{bmatrix}$$

$$(\bar{A})' = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1+i}{\sqrt{3}} \\ \frac{1-i}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \end{bmatrix}$$

$$(\bar{A})' \cdot A = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1+i}{\sqrt{3}} \\ \frac{1-i}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1+i}{\sqrt{3}} \\ \frac{1-i}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{3} + \frac{1+1}{3} & \frac{1+i}{3} + \left(\frac{1+i}{3}\right) \\ \frac{1-i}{3} - \left(\frac{1-i}{3}\right) & \frac{1+1}{3} + \frac{1}{3} \end{bmatrix}$$

$$\begin{bmatrix} \frac{3}{3} & 0 \\ 0 & \frac{3}{3} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\boxed{(\bar{A})' \cdot A = I}$$

Hence A is a unitary matrix.

(4) $A = \begin{bmatrix} \alpha + i\gamma & -\beta + i\delta \\ \beta + i\delta & \alpha - i\gamma \end{bmatrix}$

$(\bar{A})' \begin{bmatrix} \alpha - i\gamma & \beta - i\delta \\ -\beta + i\delta & \alpha + i\gamma \end{bmatrix}$

For Matrix A to be Unitary

$(\bar{A})' \cdot A = I$

$$\begin{bmatrix} \alpha - i\gamma & \beta - i\delta \\ -\beta + i\delta & \alpha + i\gamma \end{bmatrix} \begin{bmatrix} \alpha + i\gamma & -\beta + i\delta \\ \beta + i\delta & \alpha - i\gamma \end{bmatrix}$$

$$\begin{bmatrix} \alpha^2 + \gamma^2 + \beta^2 + \delta^2 & (\alpha - i\gamma)(-\beta + i\delta) + (\beta - i\delta)(\alpha - i\gamma) \\ (-\beta + i\delta)(\alpha + i\gamma) + (\alpha + i\gamma)(\beta + i\delta) & \alpha^2 + \gamma^2 + \beta^2 + \delta^2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\boxed{\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = 1}$$

Hence Prove

(5)

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

Characteristic Matrix

$[A - \lambda I]$ -

$$\begin{bmatrix} 2-\lambda & -1 & 1 \\ -1 & 2-\lambda & -1 \\ - & -1 & 2-\lambda \end{bmatrix}$$

$L.H.S = A^3 - 6A^2 + 9A - 4I$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - 6\begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} + 9\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

R.H.S

Hence varified

6) $x + y + z = 6$

$x + 2y + 3z = 10$

$x + 2y + \lambda z = \mu$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{bmatrix}, \quad B = \begin{bmatrix} 6 \\ 10 \\ \mu \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$[A;B] = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{array}\right]$$

$R_2 \to R_2 - R_1, \quad R_3 \to R_3 - R_2$

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & \lambda - 3 & \mu - 10 \end{array}\right]$$

Characteristic eqn-

$$|A-\lambda I| = 0$$

$$(2-\lambda)[(2-\lambda)^2-1] + 1(\lambda-2+1) + 1(1-2+\lambda) = 0$$

$$(2-\lambda)[\lambda^2-4\lambda+3] + 2(\lambda-1) = 0$$

$$(\lambda-3)(\lambda-1)(2-\lambda) + 2(\lambda-1) = 0$$

$$(\lambda-1)[(\lambda-3)(2-\lambda)+2] = 0$$

$$(\lambda-1)[5\lambda-\lambda^2-4] = 0$$

$$(\lambda-1)(\lambda^2-5\lambda+4] = 0$$

$$(\lambda-1)(\lambda-1)(\lambda-4) = 0$$

$$\boxed{\lambda = 1, 1, 4}$$

Characteristics eqn-

$$\lambda^3 - 5\lambda^2 + 4\lambda - \lambda^2 + 5\lambda - 4 = 0$$

$$\lambda^3 - 6\lambda^2 + 9\lambda - 4 = 0$$

Cayley Hamilton theorem-

$$A^3 - 6A^2 + 9A - 4I = 0$$

$$A^2 = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 4+1+1 & -2-2-1 & 2+1+2 \\ -2-2-1 & 1+4+1 & -1-2-2 \\ 2+1+2 & -1-2-2 & 1+1+4 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} \qquad A^3 = \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix}$$

(i) For No soln

$\lambda - 3 = 0$ $\mu - 10 \neq 0$

$\boxed{\lambda = 3}$ $\boxed{\mu \neq 10}$

(ii) For unique soln

$\lambda - 3 \neq 0$ $\boxed{\mu \in R}$

(iii) For infinite soln

$\lambda - 3 = 0$ $\mu - 10 = 0$

$\boxed{\lambda = 3}$ $\boxed{\mu = 10}$

(7) $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - 2R_1$

$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & -1 \\ 0 & 0 & 0 \end{bmatrix}$ $\boxed{\text{Rank}(A) = 2}$

8) $A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -2 & 1 \\ 1 & -1 & 4 & 0 \\ -2 & 2 & 8 & 0 \end{bmatrix}$

$R_3 \to R_3 - R_1$, $R_4 \to R_4 + 2R_1$

$\begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -2 & 1 \\ 0 & -1 & 2 & -1 \\ 0 & 2 & 12 & 2 \end{bmatrix}$

$R_3 \to R_3 + R_2$

$R_4 \to R_4 + 2R_1$

$$A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 16 & 0 \end{bmatrix}$$

$$R_4 \to \frac{1}{16} R_4$$

$$A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$R_3 \leftrightarrow R_4$$

$$A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C_3 \to C_3 - 2C_1 + 2C_2 \quad \cancel{C_3 \to C_3}$$

$$C_4 \to C_4 - C_1 - C_2$$

$$A = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$A \left[\begin{array}{c|c} I_3 & 0 \end{array}\right]$$

$$\boxed{\text{Rank}(A) = 3}$$

9)

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}$$

$$A = I_3 \cdot A \cdot I_3$$

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_2 \to R_2 - R_1$$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_3 \to R_3 + R_2$$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$C_2 \to C_2 - C_1, \quad C_3 \to C_3 - (C_1 + C_2)$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix} A \begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \qquad Q = \begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

(10)

$$A = \begin{bmatrix} 1 & -2 & 1 & -1 \\ 1 & 1 & -2 & 3 \\ 4 & 1 & -5 & 8 \\ 5 & -7 & 2 & -1 \end{bmatrix}$$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - 4R_1$, $R_4 \to R_4 - 5R_1$

$$A = \begin{bmatrix} 1 & -2 & 1 & -1 \\ 0 & 3 & -3 & 4 \\ 0 & 9 & -9 & 12 \\ 0 & 3 & -3 & 4 \end{bmatrix}$$

$R_4 \to R_4 - R_2$, $R_3 \to R_3 - 3R_2$

$$A = \begin{bmatrix} 1 & -2 & 1 & -1 \\ 0 & 3 & -3 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\boxed{\text{Rank}(A) = 2}$

(11)

$$A = \begin{bmatrix} 1 & 5 & 4 \\ 0 & 3 & 2 \\ b & 13 & 10 \end{bmatrix}$$

$R_3 \to R_3 - (2R_1 + R_2)$

$$A = \begin{bmatrix} 1 & 5 & 4 \\ 0 & 3 & 2 \\ b-1 & 0 & 0 \end{bmatrix}$$

Rank of Matrix is 2

$b - 2 = 0$

$\boxed{b = 2}$

12)

$x + y + z = -3$

$2x + 4y + 7z = 7$

$3x + y - 2z = -2$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 7 \\ 3 & 1 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} -3 \\ 7 \\ -2 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$[A:B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & -3 \\ 2 & 4 & 7 & 7 \\ 3 & 1 & -2 & -2 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & -3 \\ 0 & 2 & 5 & 13 \\ 0 & -2 & -5 & 7 \end{array}\right]$$

$R_3 \to R_3 + R_2$

$$[A:B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & -3 \\ 0 & 2 & 5 & 13 \\ 0 & 0 & 0 & 20 \end{array}\right]$$

$\text{Rank}[A:B] = 3$

$\text{Rank}(A) = 2$

$\text{Rank}[A:B] \neq \text{Rank}(A)$

Hence given eqn are inconsistent and has no soln

(13) $x + 2y = 4$

$3x + 3y + 2z = 1$

$10y + 3z = -2$

$2x - 3y - z = 5$

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 3 & 2 \\ 0 & 10 & 3 \\ 2 & -3 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 4 \\ 1 \\ -2 \\ 5 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$[A:B] = \left[\begin{array}{ccc|c} 1 & 2 & 0 & 4 \\ 3 & 3 & 2 & 1 \\ 0 & 10 & 3 & -2 \\ 2 & -3 & -1 & 5 \end{array}\right]$$

$R_2 \to R_2 - 3R_1$, $R_4 \to R_4 - 2R_2$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 0 & 4 \\ 0 & -3 & 2 & -11 \\ 0 & 10 & 3 & -2 \\ 0 & -7 & -1 & -3 \end{array}\right]$$

$R_3 \to R_3 + R_2 + R_4$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 0 & 4 \\ 0 & -3 & 2 & -11 \\ 0 & 0 & 4 & -16 \\ 0 & -7 & -1 & -3 \end{array}\right]$$

$R_4 \to R_4 - \frac{7}{3} R_2$

$$\begin{bmatrix} 1 & 2 & 0 & : & 4 \\ 0 & -3 & 3 & : & -11 \\ 0 & 0 & 4 & : & -16 \\ 0 & 0 & -17/3 & : & -6/3 \end{bmatrix}$$

$R_4 \to R_4 - \frac{1}{4} R_3$

$$= \begin{bmatrix} 1 & 2 & 0 & : & 4 \\ 0 & -3 & 2 & : & -11 \\ 0 & 0 & 4 & : & -16 \\ 0 & 0 & 0 & : & 0 \end{bmatrix}$$

Rank(A) = rank[A : B] = 3 = no of variables

Hence given system has unique solⁿ

$4z = -16$

$\boxed{z = -4}$

$-3y + 2z = -11$

$\boxed{y = 1}$

$x + 2y = 4$

$\boxed{x = 2}$

(14) $2x + 3y + 5z = 9$

$7x + 3y - 2z = 8$

$2x + 3y + \lambda z = \mu$

$$A = \begin{bmatrix} 2 & 3 & 5 \\ 7 & 3 & -2 \\ 2 & 3 & \lambda \end{bmatrix}, \quad B = \begin{bmatrix} 9 \\ 8 \\ \mu \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$[A : B] = \begin{bmatrix} 2 & 3 & 5 & : & 9 \\ 7 & 3 & -2 & : & 8 \\ 2 & 3 & \lambda & : & \mu \end{bmatrix}$$

$R_2 \to R_2 - 3R_1$

$$\begin{bmatrix} 2 & 3 & 5 & : & 9 \\ 1 & -6 & -17 & : & -19 \\ 2 & 3 & \lambda & : & \mu \end{bmatrix}$$

$R_2 \leftrightarrow R_1$

$$\begin{bmatrix} 1 & -6 & -17 & : & -19 \\ 2 & 3 & 5 & : & 9 \\ 2 & 3 & \lambda & : & \mu \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$\left[\begin{array}{ccc:c} 1 & -6 & -17 & -11 \\ 2 & 3 & 5 & 9 \\ 0 & 0 & \lambda-5 & \mu-9 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$

$$\left[\begin{array}{ccc:c} 1 & -6 & -17 & -19 \\ 0 & 15 & 99 & 47 \\ 0 & 0 & \lambda-5 & \mu-9 \end{array}\right]$$

(i) for unique soln —

$\lambda - 5 \neq 0$

$\boxed{\lambda \neq 5}$ $\boxed{\mu \in R}$

(ii) No soln —

$\lambda - 5 = 0$ $\mu - 9 \neq 0$

$\boxed{\lambda = 5}$ $\boxed{\mu \neq 9}$

(15) $x + y + 3z = 0$
$2x + y + 2z = 0$
$4x + 3y + \lambda z = 0$

$A = \begin{bmatrix} 1 & 1 & 3 \\ 2 & 1 & 2 \\ 4 & 3 & \lambda \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$A = \begin{bmatrix} 1 & 1 & 3 \\ 2 & 1 & 2 \\ 4 & 3 & \lambda \end{bmatrix}$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 4R_2$

$A = \begin{bmatrix} 1 & 1 & 3 \\ 0 & -1 & -4 \\ 0 & -1 & \lambda-12 \end{bmatrix}$

$R_3 \to R_3 - R_2$

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 0 & -1 & -4 \\ 0 & 0 & \lambda-8 \end{bmatrix}$$

For non zero soln $Rank(A) < 3$

$Rank(A) <$ no of variables

$\lambda - 8 = 0$

$\boxed{\lambda = 8}$

(6)

$$A = \begin{bmatrix} 3 & 4 & 1 \\ 2 & 1 & 6 \\ -1 & 4 & 7 \end{bmatrix}$$

Characteristic matrix $= [A - \lambda I] = \begin{bmatrix} 3-\lambda & 4 & 1 \\ 2 & 1-\lambda & 6 \\ -1 & 4 & 7-\lambda \end{bmatrix}$

characteristic eqn –

$$|A - \lambda I| = 0$$

$$(3-\lambda)[(1-\lambda)(7-\lambda) - 24] - 4[2(7-\lambda)+6] + 1[8+1-\lambda]$$

$$(3-\lambda)[7 - 8\lambda + \lambda^2 - 24] - 4[14 - 2\lambda + 6] + 9 - \lambda = 0$$

$$(3-\lambda)[\lambda^2 - 8\lambda - 21] - 4(20 - 2\lambda) + 9 - \lambda = 0$$

$$-\lambda^3 + 11\lambda^2 - 2\lambda - 113 = 0$$

$$\lambda^3 - 11\lambda^2 + 2\lambda + 113 = 0$$

Cayley Hamilton theorem –

$$A^3 - 11A^2 + 2A + 113I = 0$$

$$A^2 - 11A + 2I + 113A^{-1} = 0$$

$$A^{-1} = \frac{1}{113}[11A - A^2 - 2I]$$

$$A^2 = \begin{bmatrix} 3 & 4 & 1 \\ 2 & 1 & 6 \\ -1 & 4 & 7 \end{bmatrix} \begin{bmatrix} 3 & 4 & 1 \\ 2 & 1 & 6 \\ 1 & 4 & 7 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 16 & 20 & 34 \\ 2 & 33 & 50 \\ -2 & 28 & 72 \end{bmatrix}$$

$$A^{-1} = \frac{1}{113}\left\{\begin{bmatrix} 83 & 44 & 11 \\ 22 & 11 & 60 \\ 11 & 44 & 77 \end{bmatrix} - \begin{bmatrix} 14 & 70 & 34 \\ 2 & 33 & 50 \\ -2 & 18 & 72 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}\right\}$$

$$A^{-1} = \frac{1}{113}\begin{bmatrix} 15 & 27 & -23 \\ 20 & -24 & 16 \\ -9 & 16 & 3 \end{bmatrix}$$

(17)

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$

Characteristics eqn matrix.

$$[A - \lambda I] = \begin{bmatrix} 2-\lambda & 1 & 1 \\ 0 & 1-\lambda & 0 \\ 1 & 1 & 2-\lambda \end{bmatrix}$$

Characteristics eqn

$$|A - \lambda I| = 0$$

$$-\lambda^3 + 3\lambda^2 - 2\lambda + 2\lambda^2 - 6\lambda + 4 + \lambda - 1 = 0$$

$$\lambda^3 - 5\lambda^2 + 7\lambda - 3 = 0$$

$$(\lambda - 1)(1 - \lambda)(\lambda - 3) = 0$$

$$\boxed{\lambda = 1, 1, 3}$$

Characteristics eqn —

$$\lambda^3 - 5\lambda^2 + 7\lambda - 3 = 0$$

Carley Hermilton theorem —

$$A^3 - 5A^2 + 7A - 3I = 0$$

$$A^2 = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 4 & 4 \\ 0 & 1 & 0 \\ 4 & 4 & 5 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 5 & 4 & 4 \\ 0 & 1 & 0 \\ 4 & 4 & 5 \end{bmatrix}\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 14 & 13 & 13 \\ 0 & 1 & 0 \\ 13 & 13 & 14 \end{bmatrix}$$

L.H.S = $\begin{bmatrix} 14 & 13 & 13 \\ 0 & 1 & 0 \\ 13 & 13 & 14 \end{bmatrix} - 5\begin{bmatrix} 5 & 4 & 4 \\ 0 & 1 & 0 \\ 4 & 4 & 5 \end{bmatrix} + 7\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 28 & 20 & 20 \\ 0 & 8 & 0 \\ 20 & 20 & 28 \end{bmatrix} - \begin{bmatrix} 28 & 20 & 20 \\ 0 & 8 & 0 \\ 20 & 20 & 28 \end{bmatrix}$

$= 0$ R.H.S Hence Varified

$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$

$A^5[A^3 - 5A^2 + 7A - 3I] + A[A^3 - 5A^2 + 7A - 3I] + A^2 + A + I$

$= A^2 + A + I$

$= \begin{bmatrix} 5 & 4 & 4 \\ 0 & 1 & 0 \\ 4 & 4 & 5 \end{bmatrix} + \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 8 & 5 & 5 \\ 0 & 3 & 0 \\ 5 & 5 & 8 \end{bmatrix}$

(18)

$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

Characteristics Matrix —

$[A - \lambda I] = \begin{bmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{bmatrix}$

characteristics eqn —

$|A - \lambda I| = 0$

$(8-\lambda)[\lambda^2 - 10\lambda + 5] + 6[6\lambda - 10] + 2[2\lambda + 10] = 0$

$8\lambda^2 - 80\lambda + 40 - \lambda^3 + 10\lambda^2 - 5\lambda + 36\lambda + 60 + 4\lambda + 20 = 0$

$x_1 = 2x_2 = k_2 \Rightarrow x_1 = 2k_2 ,\quad x_1 = k_2 ,\ x_3 = -2k_2$

$$X_2 = k_2 \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$$

$[A - 15I] X_3 = 0$

$$\begin{bmatrix} -7 & -6 & 2 \\ -6 & -8 & -4 \\ 2 & -4 & -12 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$-7x_1 - 6x_2 + 2x_3 = 0$

$-6x_1 - 8x_2 - 4x_3 = 0$

$2x_2 - 4x_2 - 12x_3 = 0$

$10x_1 + 10x_2 = 0$

$x_1 = -x_2 = k_3$

$-7k_3 + 6k_3 + 2x_3 = 0$

$x_3 = \frac{k_3}{2}$

$$X_3 = k_3 \begin{bmatrix} 1 \\ -1 \\ 1/2 \end{bmatrix}$$

eigen's vector are —

$$k_1 \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix},\ k_2 \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix},\ k_3 \begin{bmatrix} 1 \\ -1 \\ 1/2 \end{bmatrix}$$

(19) $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ 1 & -2 & 0 \end{bmatrix}$

characteristic matrix

$$[A - I\lambda] = \begin{bmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ 1 & -2 & -\lambda \end{bmatrix} = 0$$

characteristics eqn —

$|A - I\lambda| = 0$

$-\lambda^3 + 18\lambda^2 - 45\lambda = 0$

$-\lambda[\lambda^2 - 18\lambda + 45] = 0$

$\lambda(\lambda-15)(\lambda-3) = 0$

$\boxed{\lambda = 0, 15, 3}$

$\boxed{\text{eigen values} = 0, 15, 3}$

eigen's vector ~~ents~~

$[A - \lambda I]x_1 = 0$

$[A - 0]x_1 = 0$

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$4x_1 - 3x_2 + x_3 = 0$

$-6x_1 + 7x_2 - 4x_3 = 0$

$2x_1 - 4x_2 + 3x_3 = 0$

$$\frac{x_1}{\begin{vmatrix} -3 & 1 \\ 7 & -4 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 4 & 1 \\ -6 & -4 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 4 & -3 \\ -6 & 7 \end{vmatrix}}$$

$$\frac{x_1}{5} = \frac{x_2}{10} = \frac{x_3}{10} = k_1$$

$\boxed{x_1 = k_1}$, $x_2 = x_3 = 2k_1$

$$x_1 = k_1\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

$[A - 3I]x_2 = 0$

$$\begin{bmatrix} 5 & -6 & 2 \\ -6 & 4 & -4 \\ 2 & -4 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$5x_1 - 6x_2 + 2x_3 = 0$

$-6x_1 + 4x_2 - 4x_3 = 0$

$-7k_3 + 2x_2 + 3k_3 = 0$

$x_2 = 2k_3$

$x_3 = k_3 \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$

Eigen's vectors are

$k_3 \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, k_1 \begin{bmatrix} 1 \\ 0 \\ 1/3 \end{bmatrix} \& k_2 \begin{bmatrix} 0 \\ 1 \\ 2/3 \end{bmatrix}$

(20)

$A = \begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$

Characteristic matrix

$[A - \lambda I] = \begin{bmatrix} -1-\lambda & 2 & -2 \\ 1 & 2-\lambda & 1 \\ -1 & -1 & -\lambda \end{bmatrix}$

Characteristic eqn –

$|A - \lambda I| = 0$

$(-1-\lambda)[\lambda^2 - 2\lambda + 1] + 2(\lambda - 1) + 2(\lambda - 1) = 0$

$-(1+\lambda)(\lambda-1)^2 + 4(\lambda-1) = 0$

$(\lambda-1)[1 - \lambda^2 + 4] = 0$

$(\lambda-1)(-\lambda^2 + 5) = 0$

$\lambda = 1, \sqrt{5}, -\sqrt{5}$

Eigen's values are $1, \sqrt{5}, -\sqrt{5}$

Eigen's vectors –

$[A - I]x_1 = 0$

$\begin{bmatrix} -2 & 2 & -2 \\ 1 & 1 & 1 \\ -1 & -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$

$$\begin{vmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & -\lambda \end{vmatrix} = 0$$

$$(-2-\lambda)[\lambda(\lambda-1)-12] - 2[-2\lambda-6] - 3(-4+1-\lambda) = 0$$

$$(\lambda+3)(\lambda-5)(\lambda+3) = 0$$

$$\lambda = -3, -3, 5$$

$$\boxed{\text{eigen's values} = -3, -3, 5}$$

Eigen's vectors $\Rightarrow [A+3I]\,X_2 = 0$

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 4 & -6 \\ -1 & -2 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$x_1 + 2x_2 - 3x_3 = 0$$
$$2x_1 + 4x_2 - 6x_3 = 0$$
$$-x_1 - 2x_2 + 3x_3 = 0$$

Let $x_1 = k_1$, $x_2 = k_2$, $x_3 = \frac{k_1 + 2k_2}{3}$

$$X_1 = X_2 = \begin{bmatrix} k_1 \\ k_2 \\ \frac{k_1+2k_2}{3} \end{bmatrix} = k_1\begin{bmatrix} 1 \\ 0 \\ 1/3 \end{bmatrix} + k_2\begin{bmatrix} 0 \\ 1 \\ 2/3 \end{bmatrix}$$

$$[A-5I]\,X_3 = 0$$

$$\begin{bmatrix} -7 & 2 & -3 \\ 2 & -4 & -6 \\ -1 & -2 & -5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$-7x_1 + 2x_2 - 3x_3 = 0$$
$$2x_1 - 4x_2 - 6x_3 = 0$$
$$-x_1 - 2x_2 - 5x_3 = 0$$

$$-12x_1 - 12x_3 \Rightarrow x_1 = -x_3 = k_3$$

$$x_1 = +k_3, \quad x_3 = -k_3$$

$$= \begin{bmatrix} 0 & -1 & -1 \\ 1/2\sqrt{5} & -\frac{1}{\sqrt{5}} - \frac{1}{2} & \frac{1}{2\sqrt{5}} \\ -1/2\sqrt{5} & \frac{1}{2} - \frac{1}{\sqrt{5}} & -\frac{1}{2\sqrt{5}} \end{bmatrix} \begin{bmatrix} 1 & 5-\sqrt{5} & 5+\sqrt{5} \\ 0 & \sqrt{5} & -\sqrt{5} \\ -1 & -\sqrt{5} & \sqrt{5} \end{bmatrix}$$

Digonal form : $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -\sqrt{5} \end{bmatrix}$

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$$A = \begin{bmatrix} 2 & 3-4i \\ 3+4i & 5 \end{bmatrix}$$

$$\bar{A} = \begin{bmatrix} 2 & 3+4i \\ 3-4i & 5 \end{bmatrix}$$

$$(\bar{A})' = \begin{bmatrix} 2 & 3-4i \\ 3+4i & 5 \end{bmatrix}$$

$\boxed{(\bar{A})' = A}$ Hence given matrix is Hermitian