Boolean Algebra

It is a set of rules used to simplify the given logic explanation without changing its functionality.

eg. $f = \underbrace{\overset{\text{NOT}}{\bar{A}}B}_{\text{AND}} \overset{\text{OR}}{\oplus} \underbrace{BC}_{\text{AND}} \oplus \underbrace{ABC}_{\text{AN}}$ (5) gates.

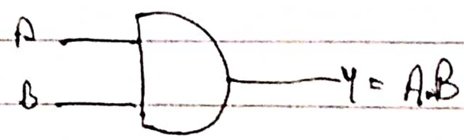
$= \underbrace{\overset{\text{NOT}}{\bar{A}}B}_{\text{AND}} \underset{\text{OR}}{\oplus} \underbrace{BC}_{\text{AND}}$

↗ Compliment Rule- ($\bar{A}$ or A') or (Not A)

$0 \longrightarrow 1$

$1 \longrightarrow 0$

↗ AND operator :-



A	B	Y
0	0	0
1	0	0
0	1	0
1	1	1

eg. $A \cdot A = 0$, $A.0 = 0$, $A.1 = 1$

If $A = 0$.
$A \cdot A = 0$.

If $A = 1$.
$A.A = 1.1 = 1$

$A.1$: $0 = 0$, $1 = 1$

Priority → NOT > AND > OR

→ $A \cdot A' = 0$

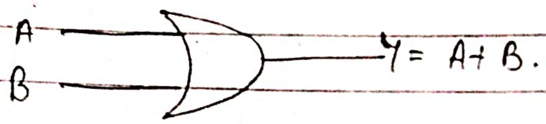
if $A = 0$.

$0 \cdot 1 = 0$

if $A = 1$.

$1 \cdot 0 = 0$.

↗ **OR Operators :-**



A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1.

$(A')' = A$

$\boxed{A + A = A}$, $\boxed{A + 0 = A}$, $\boxed{A + 1 = 1}$

$\boxed{A + A' = 1}$

Q. $F = A + AB + ABC$

$= A(1 + B + BC)$

$= A$.

↗ **Distributive Law :-**

$A \cdot (B + C) = AB + AC$.

$A + (B \cdot C) = (A + B) \cdot (A + C)$

Q. $Y = BA\bar{C} + BA\bar{C} + B\bar{C}$

$= A\bar{C}(B + \bar{B}) + B\bar{C}$

$= A\bar{C} + B\bar{C} = \bar{C}(A + B)$

- Commutative Law :-

$$A + B = B + A$$
$$A \cdot B = B \cdot A$$

- Associative law :-

$$A \cdot (B \cdot C) = (A \cdot B) \cdot C$$

- De - morgan's law :- (break the line, change sign)

$$\overline{(A + B)} = \cancel{\bar{A}+\bar{B}}\ \bar{A} \cdot \bar{B}$$
$$\overline{A \cdot B} = \bar{A} + \bar{B}$$

- Any function can be expressed in 2 ways :-

SOP form
POS form

1. SOP form (Sum of Product)

$2^3 = 8$

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

$F(A,B,C) = \Sigma m(m_2 + m_4 + m_5 + m_6 + m_7).$

$\Sigma m(2,4,5,6,7).$

0's written with (‾) and '1' is without (‾)

$= \bar{A}B\bar{C} + A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C} + ABC.$

$= A + \bar{A}B\bar{C}$

~~A~~ $A + \bar{A}$ $(A+\bar{A})(A+B\bar{C})$

$1.(A+B\bar{C}).$

Q. Simplify the expression.

$Y(A,B) = \Sigma m(0,2,3)$

$Y = \bar{A}\bar{B} + A\bar{B} + AB$

$= \bar{B}(A+\bar{A}) + AB$

$= \bar{B} + AB$

$= (\bar{B}+A).(\bar{B}+B)$ (By distrib. Law).

$= A+\bar{B}$

Digital Electronics.

$A + 0 = A$	OR
$A.1 = A$	AND
$A + \bar{A} = 1$	NOT
$A\bar{A} = 0$	NAND
	NOR

min term → $0 \to \bar{A}$

$1 \to A$

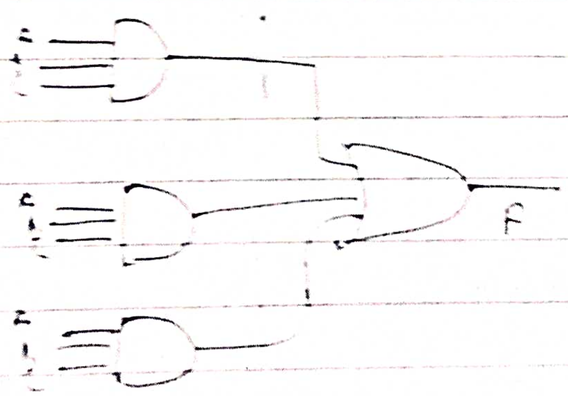
max term → $0 \to A$

$1 \to \bar{A}$

	A	B	C	AB	$B\bar{C}$	$AB + B\bar{C}$
0 →	0	0	0	0	0	0
1 →	0	0	1	0	0	0
2 →	0	1	0	0	1	1
3 →	0	1	1	0	0	0
4 →	1	0	0	0	0	0
5 →	1	0	1	0	0	0
6 →	1	1	0	1	1	1
7 →	1	1	1	1	1.	1

$F = \Sigma(m_2, m_6, m_7)$ or $\Sigma(2, 6, 7)$

$F = \bar{A}B\bar{C} + AB\bar{C} + ABC$.



1st phase
AND

2nd phase
OR

Q. $\overline{AB + B\bar{C}} = \overline{AB} . \overline{B\bar{C}}$

$(\bar{A} + \bar{B}) . (\bar{B} + \bar{\bar{C}})$

$(\bar{A} + \bar{B}) , (\bar{B} + C)$

Q.

A	B	Y	$\bar{Y}$
0	0	0	1
0	1	0	1
1	0	1	0
1	1	0	1

$Y = A\bar{B}$

$Y = \Sigma(2)$

$\bar{Y} = \Sigma(0, 1, 3)$

$= m_0 + m_1 + m_3$

$\bar{Y} = \bar{A}\bar{B} + \bar{A}B + AB.$

$\bar{Y} = \bar{A}(\bar{B} + B) + AB$

$= \bar{A} + AB$

$\bar{Y} = \bar{A} + B$

Q. $Y = AB + A\bar{B}$

Convert SOP $\rightarrow$ POS.

$\underbrace{AB}_{x} + \underbrace{A}_{y}\underbrace{\bar{B}}_{z}$ (use distributive law)

$(AB + A) . (AB + \bar{B})$

$(A + A)(B + A) . (A + \bar{B})(B + \bar{B})$

$A . (B + A) . (A + \bar{B})$.

Q. $Y = ABC + \bar{A}\bar{B}C + AB\bar{C}$ Convert it to POS.

$Y = \underbrace{ABC + \bar{A}\bar{B}C}_{x} + \underbrace{AB}_{y}\underbrace{\bar{C}}_{z}$

~~$(ABC + \bar{A}\bar{B}C) . ABC + \bar{A}\bar{B}C +$~~

$(ABC + \bar{A}\bar{B}C + AB) . (ABC + \bar{A}\bar{B}C + \bar{C})$

$(ABC + \bar{A}\bar{B}C + A) . (ABC + \bar{A}\bar{B}C + B) . (ABC + \bar{A}\bar{B}C + \bar{C})$

K - map (Karnaugh map)

2^n

$n \rightarrow$ no. of variables.

eg. A, B

Cells $= 2^2 = 4$.

Minimise

$f(A, B, C) = \Sigma m\,(1, 3, 5, 7)$.

Number of Cells $= 2^3 = 8$.

A \ BC	00 $\bar{B}\bar{C}$	01 $\bar{B}C$	11 BC	10 $B\bar{C}$
$0 \leftarrow \bar{A}$	000	001	011	010
$1 \leftarrow A$	100	101	111	110

A \ BC	00	01	11	10
0	0	1	3	2
1	4	5	7	6

$\Sigma m(1, 3, 5, 7)$

A \ BC	00	01	11	10
		1	1	
		1	1	

Ans. $f = C$.

Q $f(A, BC) = 0, 1, 2, 4, 7$

No. of cells: $2^3 = 8$

$A \backslash BC$	00	01	11	10
0	1	1		1
1	1		1	

~~F =~~ ~~$A\bar{B}\bar{C}$ +~~ ~~$A\bar{B}C$~~

$F = ABC + \bar{B}\bar{C} + \bar{A}\bar{B} + \bar{A}\bar{C}$

Q $f(A,B,C,D) = \Sigma m(0, 2, 3, 7, 11, 13, 14, 15)$

No. of cells $2^4 = 16$

	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}\bar{D}$	0	4	12	8
$\bar{C}D$	1	5	13	9
CD	3	7	15	11
$C\bar{D}$	2	6	14	10

or

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	1	3	2
$\bar{A}B$	4	5	7	6
AB	12	13	15	14
$A\bar{B}$	8	9	11	10

$\Sigma m(0, 2, 3, 7, 11, 13, 14, 15) \rightarrow$

	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}\bar{D}$	1 (0)	(4)	(12)	(8)
$\bar{C}D$	(1)	(5)	1 (13)	(9)
CD	1 (3)	1 (7)	1 (15)	1 (11)
$C\bar{D}$	1 (2)	(6)	1 (14)	(10)

$f = ACD + ABC + ABD + \bar{A}\bar{B}\bar{D}$

Q $f(A,B,C,D) = \Sigma m(2,3,4,5) + \Sigma d(6,7)$

$X \begin{cases} \to 0 \\ \to 1. \end{cases}$

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$			1	1
$\bar{A}B$	1	1	X	X
AB				
$A\bar{B}$				

$f = \bar{A}B + C\bar{A}$

Q: Simplify the boolean function

$f(A,B,C,D,E) = \Sigma m(0,2,4,6,9,13,21,23,25,29,31)$

No. of cells $= 2^5 = 32$.

first overlap & take common.

	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{D}\bar{E}$	1	1		
$\bar{D}E$			1	1
DE				
$D\bar{E}$	1	1		

$\bar{A} = 0$

	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{D}\bar{E}$				
$\bar{D}E$		1	1	1
DE		1	1	
$D\bar{E}$				

$A = 1.$

$f = B\bar{D}E + AEC + \bar{A}\bar{B}\bar{E}$

→ Redundancy term.

$F(A, B, C) = \Sigma m(1,3,6,7)$

$2^3 = 8$ cells.

A \ BC	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A} \to 0$		1	1	
$A \to 1$			1	1

$$f = \bar{A}C + \underbrace{ABC}_{\downarrow} + AB$$

Redundancy term.

$$f = \bar{A}C + AB$$

→ According to redundancy theorem.

1.) Three variables must be present in ~~example~~ ans.
2.) each variable is repeated twice.
3.) One variable must be in compliment form.
4.) Result is complimented variable.

eg. ~~Q~~. $A\bar{B} + BC + AC$.
Ans → $A\bar{B} + BC$

eg. $(A+B).(\bar{A}+C).(B+C)$
Ans → $(A+B).(\bar{A}+C)$

$\Rightarrow \bar{A}C + AB + BC(A+\bar{A})$

$\Rightarrow \bar{A}C + AB + ABC + \bar{A}BC$

$\Rightarrow \bar{A}C(1+B) + AB(1+C)$

$\to \bar{A}C + AB$

Minimal

Q. $f = AB + A\bar{B}C + A\bar{B}\bar{C}$

$f = A$

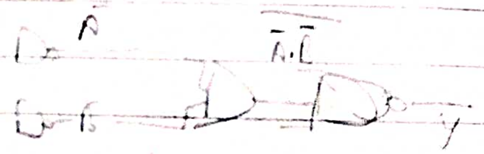
	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A} \to 0$				
$A \to 1.$	1	1	1	1

↗ LOGIC GATE

Basic Gates → AND, OR, NOT
Universal Gates → NAND, NOR
Arithmetic Gates → XOR, X-NOR.

Q. Design NOR Gate with the help of NAND Gate.

$Y = \overline{A+B}$



$= \overline{\bar{A}\cdot\bar{B}} = \overline{\bar{\bar{A}} + \bar{\bar{B}}}$

$= \overline{A+B}$

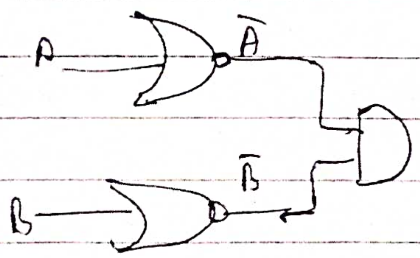
Design NAND gate with the help of NOR gate

$Y = \overline{\overline{\bar{A}\cdot\bar{B}}} = \overline{\bar{\bar{A}} + \bar{\bar{B}}}$

$= \overline{\cancel{AB}}$

NOR

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0



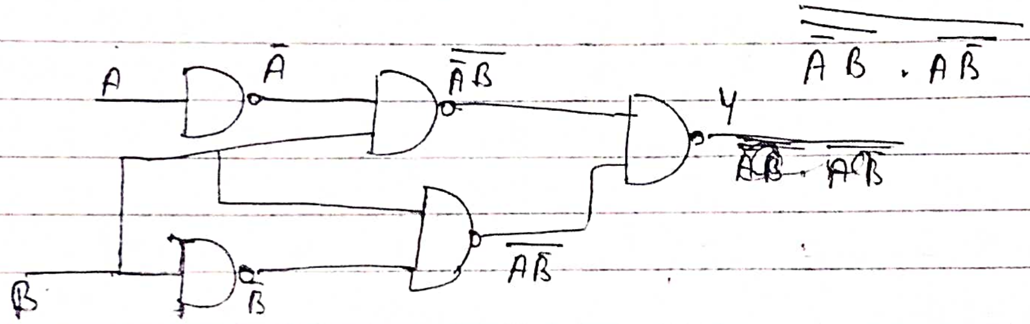
Q) Design a XOR gate using NAND gate.

Input → NAND gate → $\overline{A \cdot B} = Y$.

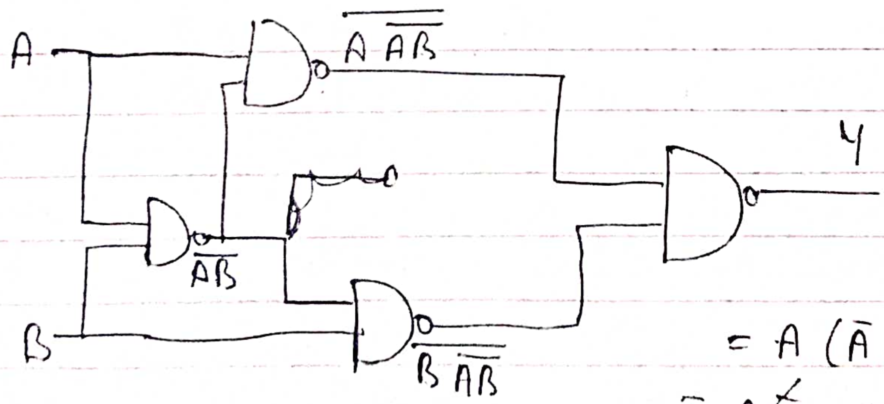
Output → X-OR gate → $Y = \bar{A}B + A\bar{B}$

$$Y = \overline{\overline{\bar{A}B + A\bar{B}}}$$

$$Y = \overline{\overline{\bar{A}B} \cdot \overline{A\bar{B}}} = \bar{A}B + A\bar{B}$$



Or



$$Y = \overline{A \cdot \overline{AB} \cdot B\overline{AB}}$$

$$= A(\bar{A} + \bar{B}) + B(\bar{A} + \bar{B})$$

$$= \cancel{A\bar{A}} + A\bar{B} + B\bar{A} + \cancel{B\bar{B}}$$

$$= A\bar{B} + B\bar{A}$$

↗ Basic Combinational Circuits

→ Adder
→ Subtractor.

Half Adder :- (2-bit)

A	B	Sum	Carry.
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

Sum.

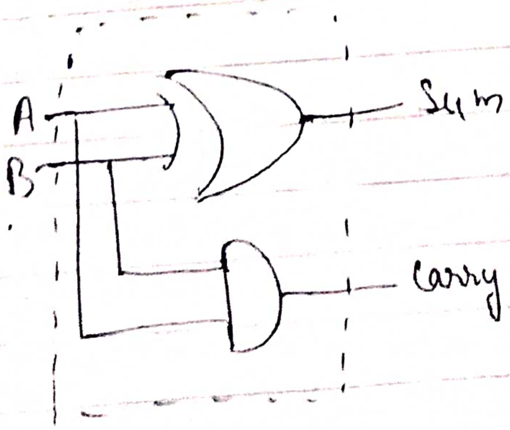
	$\bar{A}$	A
$\bar{B}$		1
B	1	

$$S = \bar{A}B + A\bar{B}$$
$$= A \oplus B$$

Carry.

	$\bar{A}$	A
$\bar{B}$		
B		1

$$C = AB.$$



Full Adder :- (3 bit)

A	B	Cin	Sum	Carry
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

No. of cells $= 2^3 = 8$.

	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	(0)	1 (1)	(3)	1 (2)
A	1 (4)	(5)	1 (7)	(6)

$$Sum = \bar{A}\bar{B}C + A\bar{B}\bar{C} + \bar{A}B\bar{C} + ABC$$

$$= \bar{A}[\bar{B}C + B\bar{C}] + A[\bar{B}\bar{C} + BC]$$

$$= \bar{A}\underbrace{(B \oplus C)}_{X} + A(B \odot C)$$

$$= \bar{A}X + A\bar{X}$$

$$= A \oplus X$$

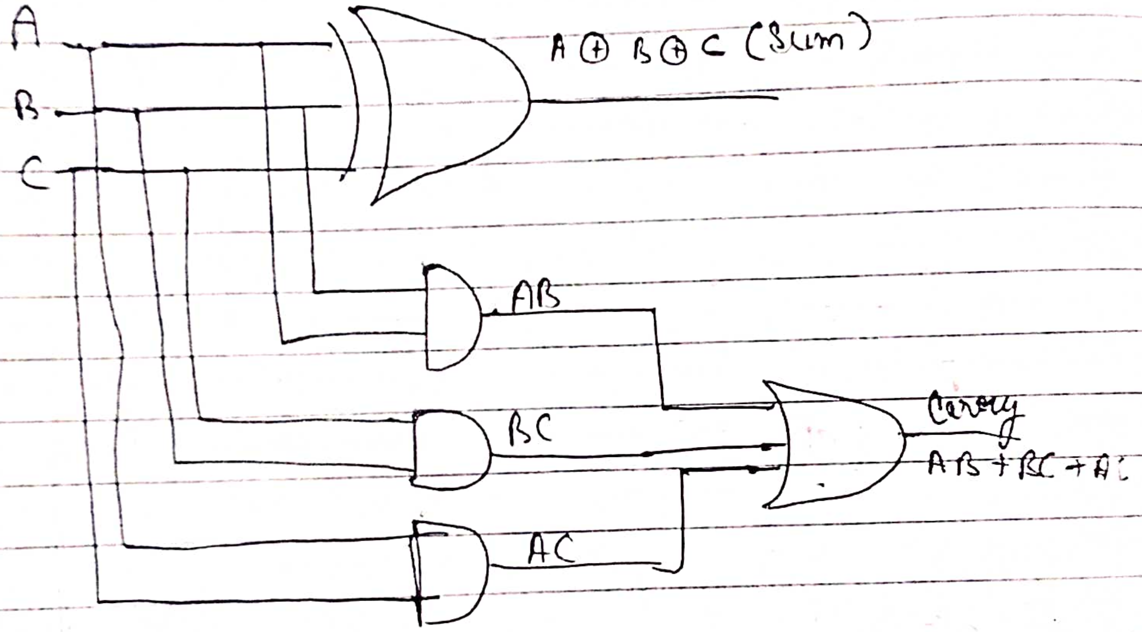
$$Sum = A \oplus B \oplus C$$

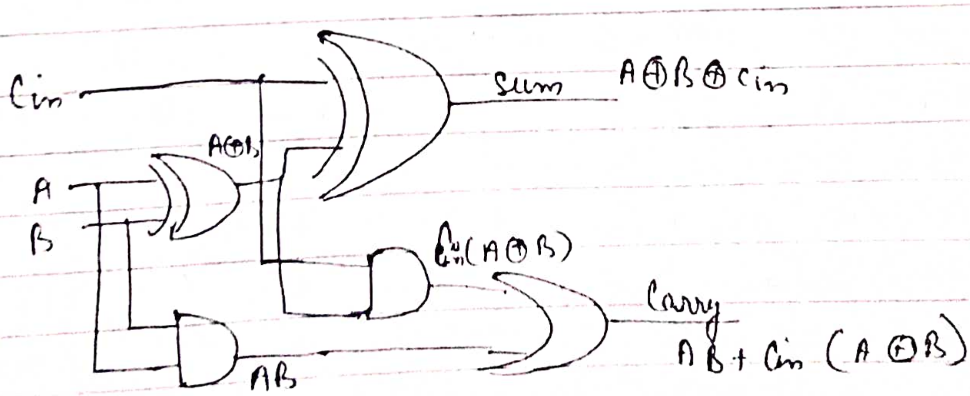
Carry :

	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	0	0	1	0
A	0	1	1	1

Carry = $AC + ABC + \cancel{ABC}\ AB$

Carry = $AB + BC + AC$.
$= AB + Cin(A \oplus B)$





Output by adding 2 half Adders.

Half Subtractor :-

(A-B)

A	B	Difference	Borrow
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

Difference →

	$\bar{A}$	A
$\bar{B}$	(0)	1 (2)
B	1 (1)	(3)

$$D = A\bar{B} + \bar{A}B$$
$$= A \oplus B$$

Borrow

	$\bar{A}$	A
$\bar{B}$	(0)	(2)
$\bar{B}$	1 (1)	(3)

Full Subtractor :- (A-B) - C.

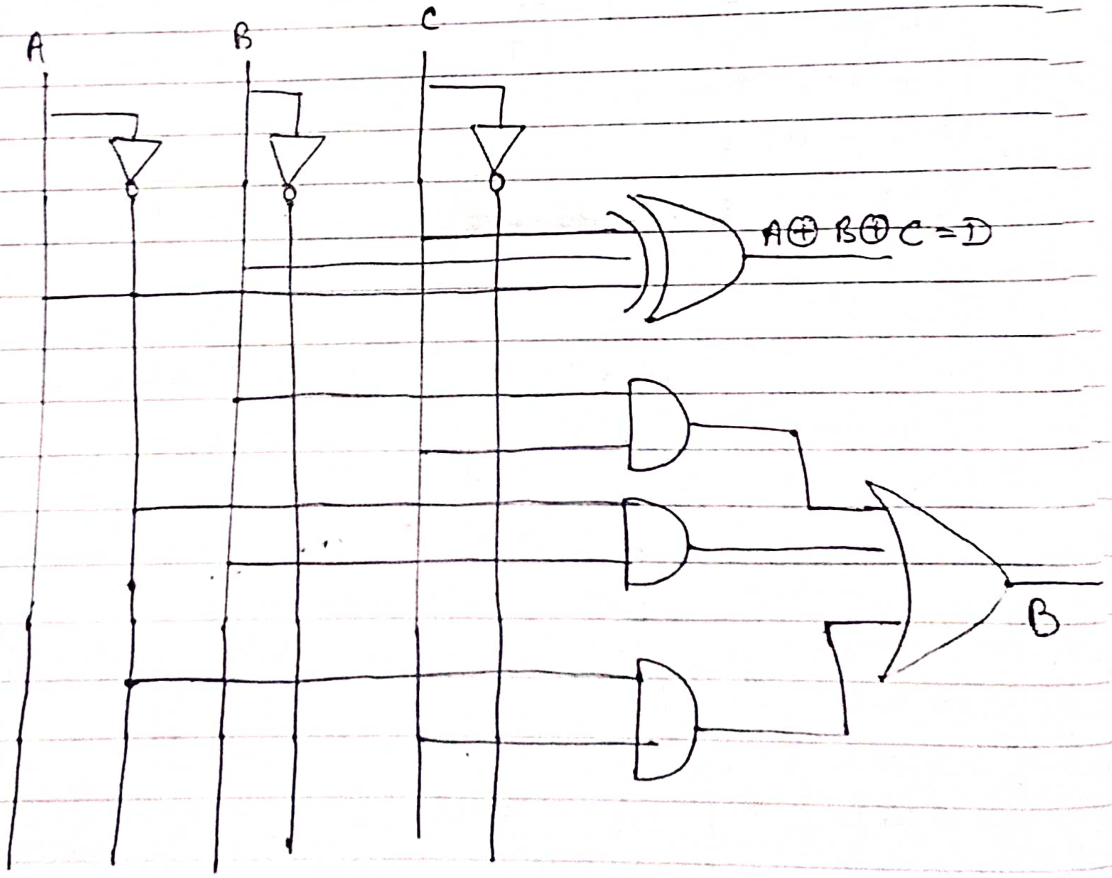
A	B	C	Difference	Borrow.
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

$D = A \oplus B \oplus C$

	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	(0)	1 (1)	(3)	1 (2)
A	1 (4)	(5)	1 (7)	(6)

	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	(0)	1 (1)	1 (3)	1 (2)
A	(4)	(5)	1 (7)	(6)

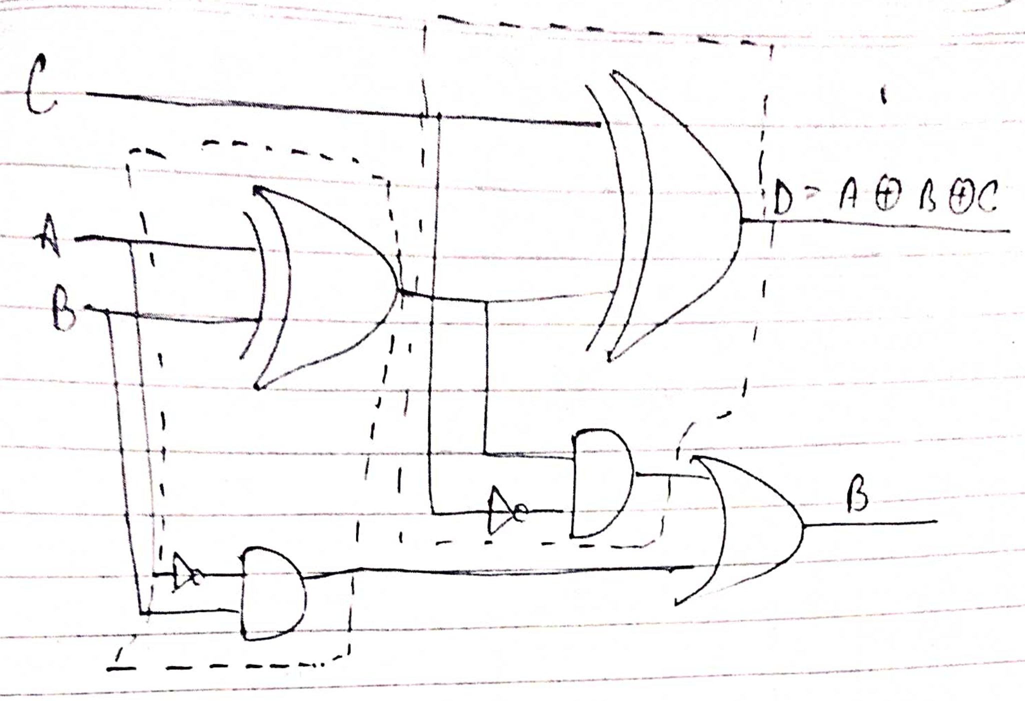
Borrow $= \bar{A}C + B\bar{A} + BC$



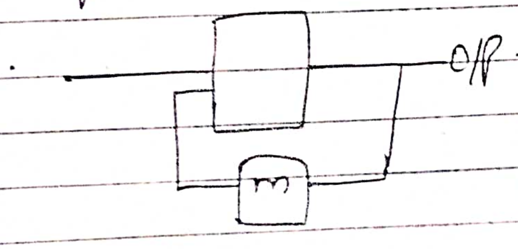
Borrow $= BC + \bar{A}C + B\bar{A}$

$= \bar{A}B + \bar{A}\bar{B}C + ABC$

$= \bar{A}B + [A \odot B]C$



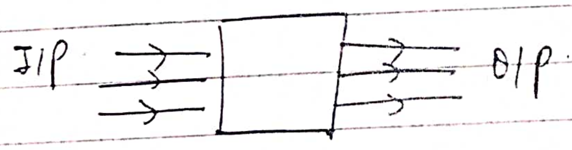
↗ Sequencial Circuits ⇒



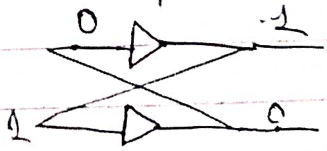
present output depends on the present input as well as past output.

(previous output is stored in memory block).

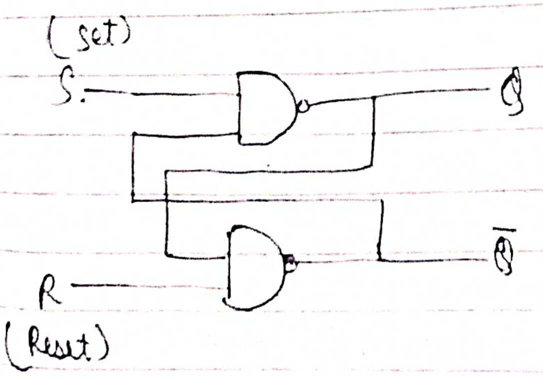
↗ Combinational Circuit



present output depends only on present input



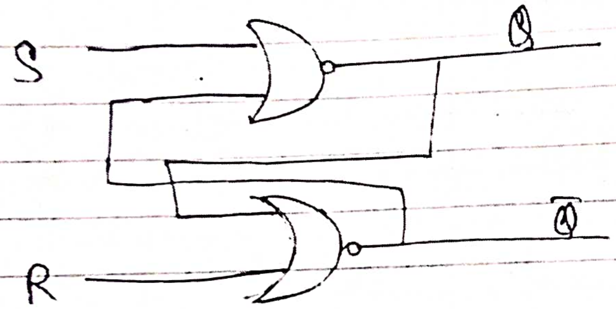
↗ Latch and flip-flop.



A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

NAND

S	R	Q	$\bar{Q}$
1	0	0	1
1	1	0	1
0	1	1	0
1	1	1	0
0	0	undefined	



NOR

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

S	R	Q	$\bar{Q}$
1	0	0	1
0	0	0	1
0	1	1	0
0	0	1	0
1	1	undefined	