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Binary Addition

i)

```
   1 1 0
 + 1 0 1
 1 0 1 1  ⇒ Sum
```

↓ carry ↘ sum

Summary of Binary addition :-

	Sum	Carry
0 + 0 ⇒	0	0
1 + 0 ⇒	1	0
0 + 1 ⇒	1	0
IMP 1 + 1 ⇒	0	1

IMP
जब तीन बार 1 आ जाए as like ① carry
1
1
then add two one's except carry & get result, then add this result with carry to get final result.

ii)

```
  ① ① ① ①
   1 1 0 1 1
 +   1 1 1 1
 1 0 1 0 1 0  Ans
```

iii)

```
  ①  ① ① ①
  1 0 1 0 1 1
  0 0 1 1 1 1
  1 1 1 0 1 0  Ans
```

Binary subtraction

Always Remember Borrow → ②

i)

```
 0 ② → Borrow
 1 1 0 1 1
-1 0 1 1 0
 0 0 1 0 1  Ans
```

ii)

```
    ②
 ⓪ ⓪ ⓪ ②
  1 1 1 0
-   1 1 1
  0 1 1 1  Ans
```

iii)

```
  ②
  ⓪ ②
  1 0 1 0 1
-   1 1 1 1
  0 0 1 1 0  Ans
```

Binary Multiplication :-

	Product
0 × 0	0
0×1 or 1×0	0
1 × 1	1

i)
```
   1010
 ×   11
 ------
   1010   } Apply addition
  1010×
 ------
  11110  Ans
```

ii)
```
    1101
 ×    11
 -------
    1101
   1101×
 -------
  100111  Ans
```

Binary Division :-

```
   016 → Ans
8 ) 128
   -0
    12
   - 8
    48
   -48
     0
```

1<110   10<110   101<110   1010>110   1001>110   110=110

```
       000111 Ans
110 ) 101010
     -0
      10
     -0
      101
     - 0
      1010
     - 110
      01001
     -  110
       00110
       - 110
           0
```

Q 1001 ÷ 11

Soln

$$\begin{array}{r} 0011 \\ 11\overline{)1001} \\ -0 \\ \hline 10 \\ -0 \\ \hline 100 \\ -11 \\ \hline 0011 \\ 11 \\ \hline 0 \end{array}$$

Q 111000 ÷ 111

Soln

$$\begin{array}{r} 000 \\ 111\overline{)111000} \\ 0 \\ \hline 11 \\ -0 \\ \hline 111 \\ -\cancel{111} \\ \hline 0 \end{array}$$

Octal Addition :-

Base (r) = 8 ⇒ 0 to 7

$$\begin{array}{rrr} A_1 & A_0 & 5 \\ + B_1 & B_0 & 4 \\ \hline & C_0 & 9 \end{array}$$

$\boxed{C_0 \le 7}$

Sum = C_0

Carry = 0

☆☆ $C_0 > 7$ 8, 9 ....

$\boxed{C_0 = C \times 8 + S}$

C → carry, 8 → base, S → sum

9 > 7

$9 = 1 \times 8 + 1$

1 → carry

i)

$$\begin{array}{rrr} 0 & 0 & \\ 2 & 4 & 3 \\ + 2 & 1 & 2 \\ \hline 4 & 5 & 5 \end{array}$$ Ans

ii)

$$\begin{array}{rrrr} 1 & 1 & 1 & \\ & 5 & 6 & 7 \\ + & 2 & 4 & 3 \\ \hline 1 & 0 & 3 & 2 \end{array}$$

10 > 7

$10 = 1 \times 8 + 2$

1 → carry, 2 → sum

$11 = 1 \times 8 + 3$

3 → sum

$8 = 1 \times 8 + 0$

0 → sum

iii)

$$\begin{array}{rrrr} 1 & 1 & 1 & \\ 1 & 7 & 7 & 6 \\ + & 3 & 4 & 5 \\ \hline 2 & 3 & 4 & 3 \end{array}$$ Ans

$11 = 1 \times 8 + 3$

$12 = 1 \times 8 + 4$

Octal Subtraction :- (Borrow → 8) → Base

①
$$\begin{array}{r} \overset{6\,8}{7} \;\overset{3\;8+3 \Rightarrow 11}{4\;\;3} \\ -\;5\;6\;4 \\ \hline 1\;5\;7 \end{array}$$
Ans

②
$$\begin{array}{r} 6\;2\;4 \\ -\;2\;6\;5 \\ \hline 3\;3\;7 \end{array}$$
Ans

③
$$\begin{array}{r} 5\;1\;1 \\ -\;\;\;3\;3 \\ \hline 4\;5\;6 \end{array}$$
Ans

④
$$\begin{array}{r} 6\;0\;0\;0 \\ -\;\;7\;7\;7 \\ \hline 5\;0\;0\;1 \end{array}$$
Ans

Octal Multiplication :- ($r = 8$, 0 to 7)

①

i)
$$\begin{array}{r} 7\;2 \\ \times\;1\;2 \\ \hline 1\;6\;4 \\ 7\;2\;\times \\ \hline 1\;1\;0\;4 \end{array}$$
Ans

$14 = 1 \times 8 + 6$

$8 = 1 \times 8 + 0$

$9 = 1 \times 8 + 1$

ii)
$$\begin{array}{r} 3\;5\;7 \\ \times\;2\;3 \\ \hline 1\;3\;1\;5 \\ 7\;3\;6\;\times \\ \hline 1\;0\;6\;7\;5 \end{array}$$
Ans

1 2 2

$21 = 2 \times 8 + 5$

$17 = 2 \times 8 + 1$

$11 = 1 \times 8 + 3$

iii)

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(2) (3)

iii)
$$\begin{array}{r} 635 \\ \times\ 55 \\ \hline 4021 \\ 4021\times \\ \hline 44231 \end{array}$$ Done

$25 = 3 \times 8 + 1$
$18 = 2 \times 8 + 2$
$32 = 4 \times 8 + 0$

(1)

iv)
$$\begin{array}{r} 63 \\ \times\ 24 \\ \hline 314 \\ 146\times \\ \hline 1774 \end{array}$$ Done

$12 = 1 \times 8 + 4$
$25 = 3 \times 8 + 1$

Hexadecimal Addition [Base (r) = 16]
0 to 15.

0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	A
11	B
12	C
13	D
14	E
15	F

Q.
$$\begin{array}{r} 5689 \\ +\ 4574 \\ \hline 9BFD \end{array}$$

(1) (1)

Q.
$$\begin{array}{r} A\ D\ D \\ +\ D\ A\ D \\ \hline 1\ 8\ 8\ A \end{array}$$

$26 \Rightarrow 1 \times 16 + x$
$x \Rightarrow 10$

$24 = 1 \times 16 + 8$

(1) (1)

Q.
$$\begin{array}{r} 899 \\ 189 \\ \hline A22 \end{array}$$

(1) (1)

Q.
$$\begin{array}{r} D\ A\ F \\ +\ B\ A\ F \\ \hline 1\ 9\ 5\ E \end{array}$$

Hexadecimal Subtraction :- [Borrow → 16]

①
$$\begin{array}{r} 9\ 6\ 5\ 4 \\ -\ 5\ 3\ 2\ 1 \\ \hline 4\ 3\ 3\ 3 \end{array}$$
Ans

②
$$\begin{array}{r} 9\ 7\ 4\ B \\ 5\ 8\ 7\ C \\ \hline 3\ E\ C\ F \end{array}$$

16

B → 11 C → 12

F → 15 25 − 14

③
$$\begin{array}{r} F\ A\ B \\ -\ A\ E\ F \\ \hline 4\ B\ C \end{array}$$

④
$$\begin{array}{r} 7\ 7\ 8\ 8 \\ -\ D\ E\ F \\ \hline 6\ 9\ 9\ 9 \end{array}$$
Ans

Hexadecimal Multiplication :-

①
$$\begin{array}{r} 9\ 4 \\ \times\ 1\ 2 \\ \hline 1\ 2\ 8 \\ 9\ 4\ \times \\ \hline A\ 6\ 8 \end{array}$$

18 = 1 × 16 + 2

A → 10
B → 11
C → 12

②
$$\begin{array}{r} 2\ \ 2\ \ \ \ \ 2 \\ 10\ \ 11\ \ \ \ \\ \cancel{A}\ \cancel{B}\ C \\ \times\ \ 2\ \ 3 \\ \hline 2\ 0\ 3\ 4 \\ 1\ 5\ 7\ 8\ \times \\ \hline 1\ 7\ 7\ B\ 4 \end{array}$$
Ans

36 ⇒ 2×16 + 4

35 = 2×16 + 3

32 = 2×16 + 0

24 = 1×16 + 8

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r's compliment :- [Radix complement]

Ex: complement of $(7)_{10}$

Sol: $10 - 7 \Rightarrow 3$ Ans

$N = 7$
$n = 1$
$r = 10$

Ex: complement of $(9)_{10}$

Sol: $10 - 9 \Rightarrow 1$

$\boxed{r^n - N}$ → General formula for r's compliment

Q. $N = 5690$
Determine 10's comp.

Sol: $r = 10$, $N = 5690$
$n = 4$

$10^4 - 5690 \Rightarrow 10000 - 5690 \Rightarrow 4310$

Q. $N = 1101$
find 2's comp.

Sol: $r = 2$, $N = 1101$, $n = 4$

$(2)^4 - 1101$
$16 - 1101$
$10000 - 1101$

$$\begin{array}{r} 10000 \\ -\ 1101 \\ \hline 00011 \end{array}$$

$\Rightarrow 11$ Ans

Q $N = 76895$
find 10's comp.

Sol: $N = 76895$
$r = 10$, $n = 5$

$10^5 - 76895$
$100000 - 76895$
$\Rightarrow 23105$ Ans.

Q $N = 11011$
find 2's comp.

Sol: $r = 2$, $n = 5$
$N = 11011$

$(2)^5 - 11011$
$32 - 11011$
101 Ans

(r-1)'s complement :-

	r's complement	(r-1)'s comp.
$r=10$	10's comp	9's comp
$r=2$	2's comp.	1's .
$r=8$	8's comp.	7's comp.
$r=16$	16's comp.	F's comp.

★ r's comp. $= r^n - N$

(r-1)'s comp $= \boxed{r^n - N} - 1$

(r-1)'s comp $=$ r's comp -1

★ $\boxed{\text{(r-1)'s comp} + 1 = \text{r's comp.}}$

No Borrow   Borrow

★ We have to avoid borrow operation

Q. 7's comp of octal no. 5674

Sol: ~~7=8~~ $7^4 - 5674$

$$\begin{array}{r} 7777 \\ -5674 \\ \hline 2103 \end{array}$$

Q. 8's comp of 5674

Sol: 7's comp of 5674 ⇒ 2103 (7's comp.)

$$\begin{array}{r} 2103 \\ +\ \ \ 1 \\ \hline 2104 \end{array}$$

2104 → 8's comp.

Q. 1's comp. of 1101

Sol:
$$\begin{array}{r} 1111 \\ -1101 \\ \hline 0010 \end{array}$$
0010 → 1's comp.

$$\begin{array}{r} 0010 \\ +\ \ \ 1 \\ \hline 0011 \end{array}$$
0011 → 2's comp

1's & 2's compliment :-

Q. obtain 1's compliment of $(1010)_2$

Soln

$$\begin{array}{r} 1111 \\ -\,1010 \\ \hline 0101 \end{array}$$ Ans

$(r-1)$'s comp $= r^4 - N - 1$

$= (r^4 - 1) - N$

If $r = 10$

9999

If $r = 8$

7777

If $r = 16$

FFFF

If $r = 2$

1111

comp of $1010 \Rightarrow 0101$

for 1's comp simply take compliment of number given in ques.

Q. obtain 2's comp of $(10111010)_2$

Soln 2's comp = 1's comp + 1

= 01000101 + 1

$$\Rightarrow \begin{array}{r} 01000101 \\ +\quad\quad 1 \\ \hline 01000110 \end{array}$$ → 2's comp

Q. 1's comp of 100111

Soln 011000 Ans

Q. 2's comp of 1100111

Soln 2's comp = 1's + 1

= 0011000+1

$$\Rightarrow \begin{array}{r} 0011000 \\ +\quad\quad 1 \\ \hline 0011001 \end{array}$$ Ans

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Shortcut for 2's compliment :-

Step ①: Write down given number

Step ② Starting from LSB, copy all the zeroes till the first 1.

Step ③ Copy the first 1.

Step ④ Complement all remaining bits.

Q. find 2's comp. of $(10111000)_2$

Sol. 10111000 → LSB

first 1

$\boxed{01001000}$ → 2's comp.

Q. $(101100)_2$

Sol. 101100 → LSB

first 1

2's comp → 010100 Ans

Data Representation using Signed Magnitude

- Data Representation
  - Magnitude
    - Unsigned — +ve binary no.
    - Signed — Both + & -ve
  - Complement
    - 1's compliment — + & −
    - 2's comp. — + & −

- In all 4 representation + ve no. is represent in same way.

Unsigned Mag. Representation :-

+6 = 110

−6 = Can't Repres.

Signed Mag. :-

$+6 \Rightarrow \underbrace{0}_{\text{Signed Bit}}\underbrace{110}_{\text{Mag}}$

extra bit add, if this bit is 0 then no. is +ve and if this bit is 1 then no. is −ve.

$-6 \Rightarrow \underbrace{1}_{\text{Signed bit or MSB}}\underbrace{110}_{\text{Magnitude}}$

☆
- Signed bit is 0 ⇒ No. is +ve
- " " " 1 ⇒ No. is −ve.

Q +13

Sol. 01101

Q -13

Sol. 11101

Range of Signed Mag.:

$$\boxed{-\left(2^{n-1}-1\right) \text{ to } +\left(2^{n-1}-1\right)}$$ $n \rightarrow$ no. of variables

$n = 4$

$-\left(2^{3}-1\right)$ to $\left(2^{3}-1\right)$

-7 to 7

Q i) +5 and -5
ii) +9 and -9
iii) +16 and -16

Use signed mag. rep.

Sol. i) $+5 \Rightarrow 0101$
$-5 \Rightarrow 1101$

ii) $+9 \Rightarrow 01001$
$-9 \Rightarrow 11001$

iii) $+16 \Rightarrow 010000$
$-16 \Rightarrow 110000$

Data Representation using 1's compliment :-

+6 ⇒ 0110   +9 ⇒ 1001
-6 ⇒ 1001   -9 ⇒ 0110

+0 ⇒ 0000 (positive zero)
-0 ⇒ 1111 (negative zero)

• write +ve no. rep. & compliment of +ve no. gives 1's comp. of negative no.

Range: $-(2^{n-1}-1)$ to $+(2^{n-1}-1)$

for 4 variables

$n = 4$

$-(2^3-1)$ to (2^3-1)

-7 to $+7$

Q i) +10 & -10 ii) +4 & -4 iii) +23 & -23
Rep. by using 1's comp.

Sol
i) +10 ⇒ 1010
-10 ⇒ 0101

ii) +4 ⇒ 100
-4 ⇒ 011

iii) +23 ⇒ 10111
-23 ⇒ 01000

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Data Representation using 2's compliment :

+6 = 0110
1's comp = 1001
= + 1

1010 → 2's comp

+4 ⇒ 0100
1's comp ⇒ 1011
2's comp = + 1

1100 → 2's comp.

→ Only one zero in case of 2's compliment.
0000

Range : -2^{n-1} to $+(2^{n-1} - 1)$
$n = 4$
-2^3 to $2^3 - 1$
-8 to $+7$

(8 -ve integers) (one zero) (7 +ve integers)

→ MSB indicates sign
if MSB is 1 → -ve no.

Binary Subtraction using 1's complement :-

Step ① Convert number to be subtracted to its 1's compliment form

Step ② Perform the addition

Step ③ If the final carry is 1, then add it to the result obtained in step ② If final carry is zero (0), result obtained in step ② is negative and in the 1's compliment form.

Imp

$$A-B = A+(-B)$$

$$\uparrow$$

[1's compliment of B = −B]

$$A-B = A+(-B)$$

$$A+(-B) = S \rightarrow \text{Final carry} = 1$$

$$+1 \quad \text{End around carry}$$

T Ans

$$A+(-B) = S \rightarrow (\text{Final carry} = 0)$$

−ve

Ex: Perform the subtraction: $(1100)_2 - (0101)_2$

Sol: $A = 1100 \quad B = 0101$

$B' = 1010 = -B$

$A+(-B')$

$$\begin{array}{r} 1100 \\ +\ 1010 \\ \hline 1\ 0110 \\ +1 \\ \hline 0110 \end{array}$$

Ans

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Q. Perform $(0101)_2 - (1100)_2$

Sol. $A = 0101$

$B = 1100 \qquad -B = 0011$

$$A+(-B) = \begin{array}{r} 0101 \\ +\ 0011 \\ \hline 01000 \end{array}$$

$01000 \rightarrow$ (-ve) $(-7)_{10}$

1's comp $\Rightarrow 0111 = (7)_{10}$ Ans

Q① $(1011)_2 - (0100)_2$ Q② $(0110)_2 - (1011)_2$

Q③ $(11001)_2 - (1111)_2$ Q④ $(10000)_2 - (11101)_2$

$2^3\ 2^2\ 2^1\ 2^0$

Sol① $A = 1011 \Rightarrow (11)_2$

$B = 0100 \qquad -B = 1011$

$$A+(-B) = \begin{array}{r} 1011 \\ +\ 1011 \\ \hline 10110 \\ +1 \\ \hline 0111 \end{array}$$ Ans

Sol② $A = 0110$

$B = 1011 \qquad -B = 0100$

$$A+(-B) = \begin{array}{r} 0110 \\ +\ 0100 \\ \hline 01010 \end{array}$$

1's comp

$\boxed{0101}$ Ans

Sol③ $A = 11001$

$B = 1111 \qquad -B = 0000$

$$A+(-B) = \begin{array}{r} 11001 \\ +\ 0000 \\ \hline 11001 \end{array}$$

Sol④ $A = 10000$

$B = 11101 \qquad -B = 00010$

$$A+(-B) = \begin{array}{r} 10000 \\ +\ 00010 \\ \hline 010010 \end{array}$$

comp.

$\boxed{01101}$ Ans

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Binary Subtraction using 2's compliment :-

Step ① find 2's comp. of number to be subtracted.

Step ② Perform the addition

Step ③ If final carry is generated (1) then the result is positive and in its true form.
If final carry is not produced (0), then the result is negative and in its 2's compliment form.

★ **Note:** we neglect the final carry in 2's compliment.

Ex: $(1001)_2 - (0100)_2$

Sol: A = 1001 B = 0100

2's comp. of B = 1's comp. of B + 1
= 1011 + 1

$$\begin{array}{r} 1011 \\ +\quad 1 \\ \hline 1100 \end{array} \Rightarrow -B$$

A + (−B)

$$\begin{array}{r} 1001 \\ +\ 1100 \\ \hline 10101 \end{array}$$

final carry → neglect carry (overflow)

so ans is : $\boxed{0101}$

Condition for overflow :

$\bar{x} \cdot \bar{y} \cdot z + x \cdot y \cdot \bar{z} = 0$ [No overflow]
$= 1$ [overflow]

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Q. $(0110)_2 - (1011)_2$

Sol. $A = 0110$

$B = 1011$

$-B$ = 1's comp of B + 1

2's comp. of B $= 0100 + 1$

$= \begin{array}{r} 0100 \\ +\quad 1 \\ \hline 0101 \end{array}$

$A + (-B) = \begin{array}{r} 0110 \\ +0101 \\ \hline ⓪1011 \end{array}$

1's comp. of 1011 = 0100

2's comp. of 1011 = $\begin{array}{r} 0100 \\ +\quad 1 \\ \hline 0101 \end{array}$

Ans :- 0101

Q① $(0110)_2 - (0100)_2$

Q② $(0111)_2 - (1110)_2$

Use 2's comp. Method

Sol① $A = 0110$

$B = 0100$

$-B$ = 1's comp. of B + 1

$= \begin{array}{r} 1011 \\ +\quad 1 \\ \hline 1100 \end{array}$

$A + (-B)$,

$\begin{array}{r} 0110 \\ +1100 \\ \hline ①0010 \end{array}$

$\boxed{\text{Ans} \rightarrow 0010}$

Sol②

$A = 0111$ $B = 1110$

$-B = \begin{array}{r} 0001 \\ +\quad 1 \\ \hline 0010 \end{array}$

$A + (-B) = \begin{array}{r} 0111 \\ +0010 \\ \hline 1001 \end{array}$

Take 2's comp

$\begin{array}{r} 0110 \\ +\quad 1 \\ \hline 0111 \end{array}$ Ans

Classification of codes :-

$\boxed{\text{Codes} \Rightarrow \text{Group of symbols}}$

Codes

- Weighted codes
  - Ex: Binary, 3421, 2421, etc
- Non-Weighted codes
  - Ex: Excess-3 code, Gray code, etc
- Reflective codes (Self comp. code)
  - Ex: 2421, excess-3
- Sequential codes
  - Ex: 8421, excess-3
  - read as (Eight four minus 2 minus 1 code)
- Alphanumeric codes
  - Ex: ASCII
- Error Detecting & Correcting codes
  - Ex: Hamming code

$$\begin{array}{cccc} 8 & 4 & 2 & 1 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ a_3 & a_2 & a_1 & a_0 \end{array}$$

$$\begin{array}{cccc} 2 & 4 & 2 & 1 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ b_3 & b_2 & b_1 & b_0 \end{array}$$

Reflective codes :-

9 → comp. of 0

8 → comp. of 1

7 → comp. of 2

Decimal		2421 code
0	fixed first bit is (0) zero for first 5 numbers	0000
1		0001
2		0010
3		0011
4		0100
5	fixed first bit is one (1) for numbers (5→9)	1011
6		1100
7		1101
8		1110
9		1111

Complement

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Binary Coded Decimal (BCD) Code :-

- In this code each decimal digit is represented by a 4-bit binary number.

$r = 10$

- Positional weight are 8-4-2-1 $\quad$ 0 to $(r-1)$

0 to $(10-1)$

$\boxed{0 \text{ to } 9}$

Decimal	BCD (8421)
0	0 0 0 0
1	0 0 0 1
2	0 0 1 0
3	0 0 1 1
4	0 1 0 0
5	0 1 0 1
6	0 1 1 0
7	0 1 1 1
8	1 0 0 0
9	1 0 0 1

BCD code for decimal digits

$2^4 = 16$

10 out of 16

Decimal No.	
10	x x x x
11	x x x x
12	x x x x
13	x x x x
14	x x x x
15	x x x x

Invalid don't cares

② Conversion of decimal no. to BCD :-

① $(17)_{10} \longrightarrow (00010111) \rightarrow$ BCD

Sol:

0001 0111

ii) $(156)_{10} \longrightarrow \boxed{0001\ 0101\ 0110} \rightarrow$ BCD

Sol:

1 → 0001

5 → 0101

6 → 0110

③ Conversion of BCD to Decimal :-

i) 10100 to Decimal

- Make group of 4 bits from right.

Sol: $\underbrace{000 1}_{1}\underbrace{0100}_{4} \Rightarrow$ 14 Ans

ii) 1001001 to Decimal

Sol: $\underbrace{0100}_{4}\underbrace{1001}_{9} \Rightarrow$ 49 Ans

④ Comparison between Binary and BCD :-

★ BCD is less efficient than Binary because it requires more no. of bits.

Sol:

	Binary	BCD
$(10)_{10}$	1010	00010000
$(12)_{10}$	1100	00010010

(XS-3 or X3)

Excess - 3 code :

Decimal → 8-4-2-1 code (BCD code) $\xrightarrow[0011\ (3)]{Add}$ Excess 3

5 → 0101 $\xrightarrow[0011]{Add}$ 1000 (8)

$$\begin{array}{r} 0101 \\ 0011 \\ \hline 1000 \end{array}$$

★ XS-3 code is unweighted code

★ 4-bit code.

Decimal	BCD (8421)	XS-3
0	0000	0011
1	0001	0100
2	0010	0101
3	0011	0110
4	0100	0111
5	0101	1000
6	0110	1001
7	0111	1010
8	1000	1011
9	1001	1100

complement

XS-3 code for Decimal numbers:

Q) 24

0010	0100
0011	0011
0101	0111
5	7

0101 0111 → XS-3 code for 24

Ex: 2421 IMP

★ $W_3\ W_2\ W_1\ W_0$

(self comp.) $W_3+W_2+W_1+W_0 = 9$

(Not self comp) " $\neq 9$

IMP ★ XS-3 code is the only unweighted code which is self complementing

Gray Code :- [cyclic code]

- Also known as Reflected Binary code.
- We call it Gray code after Frank Gray.
- Unweighted code
- Unit distance code & Minimum error code.
- ★ Two successive value differ in only 1 bit.

IMP. Binary no. is converted to Gray code to reduce switching operation.

Decimal	Binary $b_3\ b_2\ b_1\ b_0$	Gray Code $g_3\ g_2\ g_1\ g_0$
0	0 0 0 0	
1	0 0 0 1	
2	0 0 1 0	
3	0 0 1 1	0 0 1 0
4	0 1 0 0	0 1 1 0
5	0 1 0 1	
6	0 1 1 0	
7	0 1 1 1	0 1 0 0
8	1 0 0 0	1 1 0 0
9	1 0 0 1	
10	1 0 1 0	
11	1 0 1 1	
12	1 1 0 0	
13	1 1 0 1	
14	1 1 1 0	
15	1 1 1 1	
16		

IMP ★

Binary to Gray Code Conversion :-

Step ① : Record MSB as it is

Step ② : Add MSB to the next bit, record the sum & neglect the carry. (XOR operation)

Step ③ : Repeat the process.

odd 1's detector (XOR)

A	B	$Y = A\bar{B} + \bar{A}B$
0	0	0
0	1	1
1	0	1
1	1	0

Ex: Convert 1011 to Gray code.

Sol: MSB $b_3\ b_2\ b_1\ b_0$ LSB

1 0 1 1

$\boxed{1\ 1\ 1\ 0}$ → Gray code

$g_3\ g_2\ g_1\ g_0$

$g_3 = b_3$

$g_2 = b_2 \oplus b_3$

$g_1 = b_1 \oplus b_2$

$g_0 = b_0 \oplus b_1$

Ex: 1 1 1 0 to Gray code

Sol: $\boxed{1\ 0\ 0\ 1}$ Gray code

Q① 1001 to Gray code

Q② 1010 to Gray code

Q③ 1111 to Gray code

Sol ①: 1 0 0 1

$\boxed{1\ 1\ 0\ 1}$ Gray code

Sol ②: 1 0 1 0

$\boxed{1\ 1\ 1\ 1}$ Ans

Sol ③: 1 1 1 1

$\boxed{1\ 0\ 0\ 0}$ Ans

Gray Code to Binary Conversion :-

b → g ⎫
g → b ⎭ MSB remains same.

Step ① Record MSB as it is.

Step ② Add MSB to the next bit of Gray code, record the sum & neglect the carry. (X-OR)

Step ③ Repeat the process.

Ex: ① 1110 (→ Gray code) to Binary

Sol: $\boxed{1011}$ → Binary

Ex: ② $g_3\, g_2\, g_1\, g_0$ → Gray code

1001 to Binary

Sol: $\boxed{1110}$ Binary

$b_3\; b_2\; b_1\; b_0$

$b_3 = g_3$

$b_2 = b_3 \oplus g_2$

$b_1 = b_2 \oplus g_1$

$b_0 = b_1 \oplus g_0$

Q 1100, 1011, 1000 to Binary

Sol:

1100 → $\boxed{1000}$

1011 → $\boxed{1101}$

1000 → $\boxed{1111}$

84-2-1 Code :-

Decimal	84-2-1
0	0 0 0 0
1	0 1 1 1
2	0 1 1 0
3	0 1 0 1
4	0 1 0 0
5	1 0 1 1
6	1 0 1 0
7	1 0 0 1
8	1 0 0 0
9	1 1 1 1

Self complement

BOOLEAN ALGEBRA :-

Rules : i) Compliment :-

A compliment = $\bar{A}$ or A' or (not A)

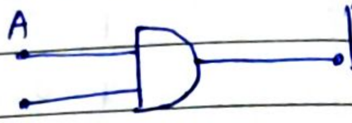
$$\boxed{\begin{array}{c} 0' = 1 \\ 1' = 0 \end{array}}$$

★ $(A')' = A$

ii) AND :-

I/P		O/P
A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

$2^2 = 4$



$\boxed{Y = A \cdot B}$

~~B~~

A

$A = 0 \longrightarrow 0 \cdot 0 \Rightarrow 0$

$A = 1 \longrightarrow 1 \cdot 1 \Rightarrow 1$

★ $\boxed{A \cdot A = A}$

$A = 0 \quad A' = 1$

$A \cdot A' = 0 \cdot 1 \Rightarrow 0$

★ $\boxed{A \cdot 0 = 0}$ $\quad 1 \cdot 0 = 0 \quad 0 \cdot 0 = 0$

$A = 1 \quad A' = 0$

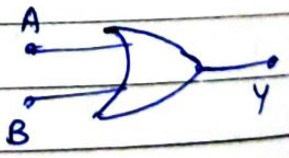
$A \cdot A' = 1 \cdot 0 \Rightarrow 0$

★ $\boxed{A \cdot 1 = A}$

★ $\boxed{A \cdot A' = 0}$

iii) OR :

I/P		O/P
A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1



$Y = A + B$

★ $A + A = A$

★ $A + 0 = A$

★★ $\boxed{A + 1 = 1}$

★ $\boxed{A + A' = 1}$

$A = 0$

$Y = 0 + 0 = 0$

$A = 1$

$Y = 1 + 1 = 1$

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Distributive law :-

$A \cdot (B + C) \Rightarrow A \cdot B + A \cdot C$

★★ $A + (B \cdot C) \Rightarrow (A + B) \cdot (A + C)$

$A + \bar{A}B = (A + \bar{A}) \cdot (A + B)$

$= (1) \cdot (A + B)$

$\boxed{A + \bar{A}B = A + B}$

★ $\boxed{\bar{A} + AB = \bar{A} + B}$

$\bar{A} + AB$

$\cancel{(1-A)}$ $\bar{A} + (1 - \bar{A})B$

$\bar{A} + B - \bar{A}B$

$\bar{A}(1 - B) + B$

v) Commutative law :-

$A + B = B + A$

$A \cdot B = B \cdot A$

vi) Associative law :-

$\boxed{(A \cdot B) \cdot C = A \cdot (B \cdot C)}$

★ **Priority :-**

NOT
AND
OR

$Y = BAC' + B'AC' + BC'$

$Y = AC'(B + B') + BC'$

$Y = AC'(1) + BC'$

$\boxed{Y = C'(A + B)}$

vii) De Morgans law :-

★ $(\overline{A + B}) = \bar{A} \cdot \bar{B}$

★ $(\overline{A \cdot B}) = \bar{A} + \bar{B}$

Q $AB + AB'$

Sol→
$$F = AB + AB'$$
$$F = A(B + B')$$
$$F = A \cdot 1$$
$$F = A$$
$$\boxed{AB + AB' = A}$$

Q $G = (A+B)(A+B')(A'+B)(A'+B')$

Sol→
$$G = (A + B \cdot B') \cdot (A' + B \cdot B')$$
$$G = (A+0) \cdot (A'+0)$$
$$G = A \cdot A'$$
$$\boxed{G = 0} \text{ Ans}$$

Q $AB + AB'C + AB'C'$

Sol→
$$AB + AB'(C + C')$$
$$AB + AB'(1)$$
$$AB + AB'$$
$$A(B + B')$$
$$A \cdot 1 \Rightarrow A$$
$$\boxed{AB + AB'C + AB'C' = A}$$

Q $F = (A+B+C) \cdot (A+B'+C) \cdot (A+B+C')$

Sol→
Let $x = A+B$
$$F = (x+C) \cdot (A+B'+C) \cdot (x+C')$$
$$F = (x+C) \cdot (x+C')(A+B'+C)$$
$$F = (x + C \cdot C') \cdot (A+B'C)$$
$$F = (x+0) \cdot (A+B'C)$$
$$F = x \cdot (A+B'C)$$
$$F = (A+B) \cdot (A+B'C)$$
$$F = A + B \cdot (B'+C)$$
$$F = A + \cancel{BB'}^{\,0} + B \cdot C$$
$$F = A + B \cdot C \text{ Ans}$$

Redundancy Theorem :- [Consensus theorem]

- Three variables
- Each variable is repeated twice
- One variable is complement
- Take the complemented variable

$Y = AB + A'C + BC$ → redundant term

→ $Y = AB + A'C$

$Y = AB + A'C + BC \cdot 1$

$Y = AB + A'C + B \cdot C \cdot (A + A')$

$Y = AB + A' \cdot C + ABC + A'BC$

$Y = AB\,[1 + C] + A'C\,[1 + B]$

$Y = AB + A' \cdot C$

Q. $F = A B + B\bar{C} + AC$

Sol → $F = AB + B\bar{C} + AC$ ← Omitted term

$F = B\bar{C} + AC$ Ans

Q. $F = A\bar{B} + BC + AC$ → omitted

Sol → $F = A\bar{B} + BC$

Q. $G = (A+B) \cdot (\bar{B}+C) \cdot (A+C)$ → Omitted

Sol → $G = (A+B) \cdot (\bar{B}+C)$

Q. $F = (A+B) \cdot (\bar{A}+C) \cdot (B+C)$ → omitted

Sol → $F = (A+B) \cdot (\bar{A}+C)$

Different Approach

Q. $F = \bar{A}\bar{B} + A\bar{C} + \bar{B}\bar{C}$ → omitted

Sol → $F = \bar{A}\bar{B} + A\bar{C}$

Sum of Products [SOP Form]

no. of variables

Total no. of combination $\Rightarrow 2^n$

$\Rightarrow 2^3 \Rightarrow 8$

Decimal		A	B	C	F
0	m_0	0	0	0	0
1	m_1	0	0	1	0
2	m_2	0	1	0	1
3	m_3	0	1	1	0
4	m_4	1	0	0	1
5	m_5	1	0	1	1
6	m_6	1	1	0	1
7	m_7	1	1	1	1

F → SOP Form

F → POS form

$$\boxed{\begin{array}{l} A \rightarrow 1 \\ \bar{A} \rightarrow 0 \end{array}}$$

IMP

→ SOP form is written only for high output values ["1"]

$$F = \underbrace{\overset{(m_2)}{\bar{A}\cdot B\cdot \bar{C}}}_{\text{AND op}} \underbrace{+}_{\text{OR op.}} \overset{(m_4)}{A\cdot\bar{B}\cdot\bar{C}} + \overset{(m_5)}{A\cdot\bar{B}\cdot C} + \overset{(m_6)}{A\cdot B\cdot\bar{C}} + \overset{(m_7)}{A\cdot B\cdot C}$$

SOP form

Standard or Canonical SOP form

(m) → Min term → $\bar{A}\cdot B\cdot C$, ABC, $A\cdot\bar{B}C$, ... etc

$$F(A,B,C) = m_2 + m_4 + m_5 + m_6 + m_7$$

$$\boxed{F(A,B,C) = \sum m(2,4,5,6,7)}$$

$$F = \bar{A}\cdot B\cdot\bar{C} + A\bar{B}(\cancel{\bar{C}+C})^{1} + AB(\cancel{\bar{C}+C})^{1}$$

$$= \bar{A}\cdot B\cdot\bar{C} + A\bar{B} + AB$$

$$= \bar{A}B\bar{C} + A(\cancel{\bar{B}+B})^{1}$$

$$= \cancel{\bar{A}}B\bar{C} + A$$

$$\boxed{F = A + B\bar{C}} \quad (SOP)$$

$$\boxed{\text{Minimal SOP form}}$$

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- **Canonical / standard SOP form :-** Each min term is having all the variables in $\underbrace{\text{normal}}_{A \text{ or } B}$ or $\underbrace{\text{complimented}}_{A' \text{ or } B'}$ form.

Ex:- $F = \underbrace{\bar{A}B}_{m_1} + \underbrace{AB}_{m_3} + \underbrace{\bar{A}\bar{B}}_{m_0}$

- **Minimal SOP form :-** Each min term does not have all the variables in normal or complimented form.

Ex: $G = A + \bar{B}C$

Q. for the given truth table, minimize the SOP form

	A	B	Y
m_0	0	0	0
m_1	0	1	1
m_2	1	0	0
m_3	1	1	1

$1 \rightarrow A$

$0 \rightarrow A'$

$\boxed{Y = \bar{A} \cdot B + A \cdot B} \rightarrow$ Canonical SOP form

$Y = B(\bar{A} + A)$

$Y = B \cdot 1$

$\boxed{Y = B} \rightarrow$ minimal SOP form

Q. simplify the expression for
$Y(A, B) = \Sigma m(0, 2, 3)$

Sol:

$$Y = m_0 + m_2 + m_3$$
$$= \bar{A}\bar{B} + A\bar{B} + AB \rightarrow \text{Canonical SOP form}$$
$$= \bar{B}(\bar{A} + A) + AB$$
$$= \bar{B} + AB$$
$$Y = \bar{B} + A$$

$\boxed{Y = \bar{B} + A} \rightarrow$ Minimal SOP form

PRODUCT OF SUM (POS form)

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$2^3 = 8$

IMP

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	0	0	1
1	1	1	1

★ POS form is used when the output value is low ("0").

★★ $\boxed{\begin{array}{l} 0 \rightarrow A \\ 1 \rightarrow \bar{A} \end{array}}$

$$Y = (A+B+C)\cdot(A+B+\bar{C})\cdot(A+\bar{B}+\bar{C})$$

Max term

Canonical POS form

$$\bar{Y} = \bar{A}\cdot\bar{B}\cdot\bar{C} + \bar{A}\cdot\bar{B}\cdot C + \bar{A}\,B\cdot C$$

compliment of both sides

$$(\bar{\bar{Y}}) = \left[\overline{\bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}BC}\right]$$

$$Y = (\bar{A}\bar{B}\bar{C})'\cdot(\bar{A}\bar{B}C)'\cdot(\bar{A}BC)'$$

$$Y = (A+B+C)\cdot(A+B+\bar{C})\cdot(A+\bar{B}+\bar{C})$$

$$Y = (\underbrace{A+B}_{X}+C)\cdot(\underbrace{A+B}_{X}+\bar{C})\cdot(A+\bar{B}+\bar{C})$$

$$Y = (X + \underbrace{C\cdot\bar{C}}_{0})\cdot(A+\bar{B}+\bar{C})$$

$$Y = (A+B)\cdot(A+\bar{B}+\bar{C})$$

$$Y = A + B\cdot(\bar{B}+\bar{C})$$

$$Y = A + \underbrace{B\cdot\bar{B}}_{0} + B\cdot\bar{C}$$

$$Y = A + B\cdot\bar{C}$$

$$\boxed{Y = (A+B)\cdot(A+\bar{C})}$$

Minimal POS form

Q. for given truth table minimize the POS form:

	A	B	Y
M_0	0	0	1
M_1	0	1	0
M_2	1	0	1
M_3	1	1	0

$0 \rightarrow A$

$1 \rightarrow \bar{A}$

Max term (M)

$$\boxed{Y = (A+\bar{B}) \cdot (\bar{A}+\bar{B})}$$

Canonical POS form

$$\boxed{Y = \pi(M_1, M_3)}$$

$Y = \bar{B} + \underbrace{A \cdot \bar{A}}_{0}$

$\boxed{Y = \bar{B}} \rightarrow$ Minimal form

or

$$\boxed{Y = \pi M(1,3)}$$

(POS) Max term

$$\boxed{Y = \Sigma m(0,2)}$$

(SOP) Min term

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SOP form & POS form Examples :-

	A	B	C	Y
m_0	0	0	0	1
M_1	0	0	1	$\boxed{0}$
m_2	0	1	0	1
m_3	0	1	1	1
M_4	1	0	0	$\boxed{0}$
M_5	1	0	1	$\boxed{0}$
m_6	1	1	0	1
m_7	1	1	1	1

or $\boxed{Y(A,B,C) = \Sigma m(0,2,3,6,7)}$

or $\boxed{Y(A,B,C) = \pi M(1,4,5)}$

SOP form :- $0 \rightarrow \bar{A}$
$1 \rightarrow A$

$$\boxed{Y = \bar{A}\cdot\bar{B}\cdot\bar{C} + \bar{A}\cdot B\cdot\bar{C} + \bar{A}\cdot B\cdot C + AB\bar{C} + ABC}$$

Canonical SOP form

$$Y = \bar{A}\cdot\bar{B}\cdot\bar{C} + \bar{A}B\left[\bar{C}+C\right] + AB\left[\bar{C}+C\right]$$

$$Y = \bar{A}\bar{B}\bar{C} + \bar{A}\cdot B\cdot 1 + AB\cdot 1$$

$$Y = \bar{A}\bar{B}\bar{C} + B\left(\bar{A}+A\right)$$

$$Y = \bar{A}\bar{B}\bar{C} + B$$

$$\boxed{Y = \bar{A}\bar{C} + B}$$

A & C are absent

Minimal SOP form

B is absent

POS form :-

$0 \rightarrow A$

$\bar{A} \rightarrow 1$

$$\boxed{Y = (A+B+\bar{C})\cdot(\bar{A}+B+C)\cdot(\bar{A}+B+\bar{C})}$$

Canonical POS form

$$y = (A+B+\bar{C})\cdot(\bar{A}+B + C\cdot\bar{C})$$

$$y = (A+B+\bar{C})\cdot(\bar{A}+B)$$

$$y = B + (A+\bar{C})\cdot\bar{A}$$

$$y = B + \cancel{A\cdot\bar{A}}^{0} + \bar{A}\cdot\bar{C}$$

$$y = B + \bar{A}\cdot\bar{C}$$

$$\boxed{y = (\bar{A}+B)\cdot(B+\bar{C})} \rightarrow \text{Minimal POS form}$$

Minimal to Canonical form Conversion :- [• SOP]

$$y = A + B'C$$

Step ① :- 3 variables [A, B & C]

$$y = A\cdot\underbrace{1}_{(B+B')}\cdot\underbrace{1}_{(C+C')} + B'C\cdot\underbrace{1}_{(A+A')}$$

Step ② :- Variables absent in each min term

m_1 : A ✓, B X, C X

m_2 : A X, B ✓, C ✓

$$y = A(B+B')\cdot(C+C') + B'C\cdot(A+A')$$

$$y = (A\cdot B + AB')\cdot(C+C') + B'C\cdot A + B'C\cdot A'$$

Step ③:

$$y = A\cdot B\cdot C + AB\cdot C' + AB'\cdot C + AB'\cdot C' + \cancel{\boxed{B'C\cdot A}}_{\text{repeated}} + B'C\cdot A'$$

$$\boxed{y = A\cdot B\cdot C + A\cdot B\cdot C' + AB'C + AB'\cdot C}$$ Ans

Minimal to Canonical form [POS]

$F = (A+B+C')(A'+C)$

$F = (A+B+C')(A'+C+0)$

$F = (A+B+C')(A'+C+B \cdot B')$

$F = (A+B+C') \cdot (A'+C+B) \cdot (A'+C+B')$

$$\boxed{F = (A+B+C') \cdot (A'+C+B) \cdot (A'+C+B')}$$

Canonical POS form

i) A, B & C

ii) Term ① A ✓ B ✓ C ✓

Term ② A ✓ B ✗ C ✓

K'MAP [Karnaugh Map]

$F = A + B\bar{C}$

	A	B	C	F
m_0	0	0	0	0
m_1	0	0	1	0
m_2	0	1	0	1
m_3	0	1	1	0
m_4	1	0	0	1
m_5	1	0	1	1
m_6	1	1	0	1
m_7	1	1	1	1

$F = \bar{A}B\bar{C} + A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C} + ABC$

$F = \bar{A}B\bar{C} + A\bar{B}[\bar{C}+C] + AB[\bar{C}+C]$

$F \Rightarrow \bar{A}B\bar{C} + A\bar{B} + AB$

$F \Rightarrow \bar{A}B\bar{C} + A[\cancel{\bar{B}+B}]^{1}$

$F \Rightarrow \bar{A}B\bar{C} + A$

$F \Rightarrow A + B\bar{C}$

A	BC	
0	00	m_0
0	01	m_1
0	10	m_2
0	11	m_3
1	00	m_4
1	01	m_5
1	10	m_6
1	11	m_7

	BC 00	BC 01	BC 11	BC 10
A = 0	m_0	m_1	m_3	m_2
A = 1	m_4	m_5	m_7	m_6

Map rolling

$(A + \bar{A}) = 1$

A \ BC	00	01	11	10
0	0 m_0	0 m_1	0 m_3	1 m_2
1	1 m_4	1 m_5	1 m_7	1 m_6

I

II

Pairing:

↓

2 1's

↓

4 1's

↓

8 1's

↓

16 1's

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F = I + II

$\downarrow$

group of 1's [Implicants]

$F = A \cdot 1 \cdot 1 + B\bar{C}$

$F = A + B\bar{C}$

Ex! $f(A, B, C) = \Sigma m(1, 3, 5, 7)$ — $m_1\ m_3\ m_5\ m_7$

A → MSB, C → LSB, Σm → $\boxed{SOP}$

i) find out no. of variables

$n = 3 (A, B, C)$

ii) find out no. of cells in K' map

$2^3 = 8$ [cells]

A \ BC	00	01	11	10
0	0 m_0	1 m_1	1 m_3	0 m_2
1	0 m_4	1 m_5	1 m_7	0 m_6

→ Implicant (I)

F = I

$\boxed{F = C}$ Ans

ii) $F(A, B, C) = \Sigma m(0, 1, 2, 4, 7)$

A \ BC	00	01	11	10
0	1	1		1
1	1		1	

I, II, III, IV

F = I + II + III + IV

$F = \bar{B}\bar{C} + \bar{A}\bar{B} + ABC + \bar{A}\bar{C}$

Ans

K' Map :- [Examples]

Ex: $F(A,B,C) = \Sigma m(1,3,6,7)$

A \ BC	00	01	11	10
0	0	1	1	0
1	0	0	1	1

$F = I + II + III$

$F = AB + \bar{A}C + BC$ → neglected — By Redundancy Theorem

$$\boxed{F = AB + \bar{A}C}$$

Ex: $F(A,B,C) = \Sigma m(0,1,5,6,7)$

A \ BC	00	01	11	10
0	1	1		
1		1	1	1

$F = I + II + III$ $\qquad$ $F = I + II + III'$

$F = AB + \bar{A}\bar{B} + AC$ $\qquad$ $F = AB + \bar{A}\bar{B} + \bar{B}C$

Note: The result is minimum but may not be same or unique.

★ Combine 2 1's ⇒ 1 literal reduced
- 4 1's ⇒ 2 literal reduced
- 8 1's ⇒ 3 " "
- 16 1's ⇒ 4 " "

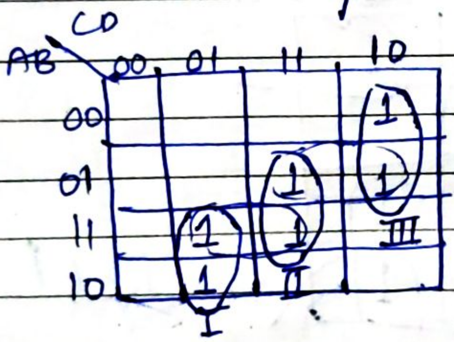
★ **Implicants :-** The group of 1's called as implicants

Eg :- 1, 2, 4, 8, 16, ... etc

→ **Prime implicants :-** It is the largest possible group of 1

→ **Essential Prime Implicant :-** At least there is single 1 which can not be combined in any other way.

Ex :-



I → Essential prime implicant

II → Not ess. prime implicant

III → Essential prime implicant

K'Map with 4 variables :-

Ex: i) $F(A,B,C,D) = \Sigma m(0,2,3,7,11,13,14,15)$

A → MSB, D → LSB

$m_0 = 1$

$m_2 = 1$

$m_3 = 1$

$m_7 = 1$

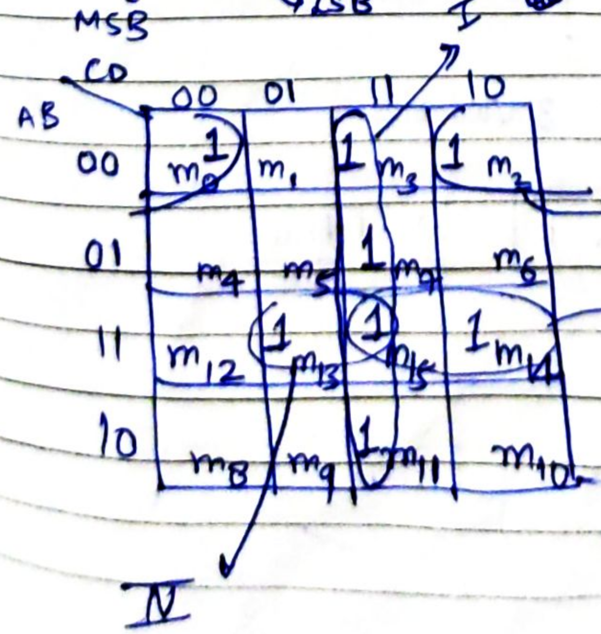
$m_{11} = 1$

$m_{13} = 1$

$m_{14} = 1$

$m_{15} = 1$

Sol →



0100
1100
12

$F = I + II + III + IV$

$$\boxed{F = CD + \bar{A}\bar{B}\bar{D} + ABC + ABD}$$

Q. $F(A, B, C, D) = \Sigma m(0, 2, 3, 5, 7, 8, 10, 11, 14, 15)$

$m_0 = 1$ $\quad m_8 = 1$

$m_2 = 1$ $\quad m_{10} = 1$

$m_3 = 1$ $\quad m_{11} = 1$

$m_5 = 1$ $\quad m_{14} = 1$

$m_7 = 1$ $\quad m_{15} = 1$

AB \ CD	00	01	11	10
00	1		1	1
01		1	1	
11			1	1
10	1		1	1

IV I III II

1011

$F = I + II + III + IV$

$F = CD + \bar{B}\bar{D} + AC + \bar{A}BD$ Ans

IMP case ★

A \ BC				
	1	1	1	1
	1	1	1	1

8 1's → 3 variables

$\boxed{1 \cdot 1 \cdot 1 \cdot 1}$ ⇒ 1 Ans

→ for signals

→ Don't care

Don't Care in K' Map

don't care

Ex: $F(A,B,C) = \Sigma m(2,3,4,5) + \Sigma d(6,7)$
↳ ϕ

A \ BC	00	01	11	10
0			1	1
1	1	1	X	X

$F = \bar{A}B + A\bar{B}$
AND, OR, AND

Don't care

★ Imp

d → 1 → [Min term]

d → 0 → [Max term]

A \ BC	00	01	11	10
0			1	1
1	1	1	1	1

→ quad 1

quad 2

F = quad 1 + quad 2

$\boxed{F = B + A}$ Ans
OR

[Grouping of zeroes]

K'Map using Max term :-

Ex: ①

A \ BC	00	01	11	10
0	0	0	0	1
1	1	1	1	1

$$\boxed{F = A + BC'}$$

Because F gives false output: $(\bar{F}) = \mathrm{I} + \mathrm{II}$

$\bar{F} = \bar{A}\bar{B} + \bar{A}C$

De Morgans law :-

$(\bar{F})' = (\bar{A}\bar{B} + \bar{A}C)'$

$F = (\bar{A}\bar{B})' \cdot (\bar{A}C)'$

$F = (A+B) \cdot (A+C')$ $\boxed{POS}$

$$\boxed{F = A + BC'}$$

Ex: ② $F(A,B,C,D) = \Sigma m(1,3,4,5,9,11,14,15)$ ⟶ min term

Sol: $F(A,B,C,D) = \Pi M(0,2,6,7,8,10,12,13)$ ⟶ max term

$2^4 = 16$

AB \ CD	00	01	11	10
00	0			0
01			0	0
11	0	0		
10	0			0

$\bar{F} = \mathrm{I} + \mathrm{II} + \mathrm{III}$

$\bar{F} = \bar{B}\bar{D} + \bar{A}BC + AB\bar{C}$

De Morgan's Law

~~$F = (B+B)(A\bar{B}+\bar{C}) \cdot (AB+C)$~~

$F = (B+D) \cdot (A+\bar{B}+\bar{C}) \cdot (\bar{A}+\bar{B}+C)$

Ans

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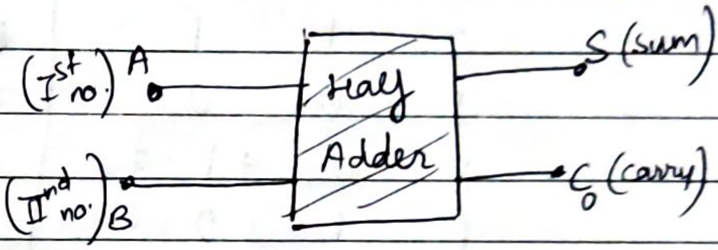
Half Adder :-

→ It is used to add single bit number.
→ It does not takes carry from previous sum.

A	B	(S) Sum	C_0 (carry)
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

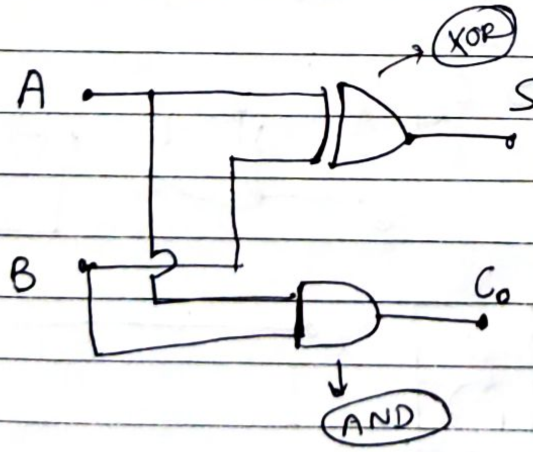


$\boxed{S = A \oplus B}$ → XOR → output is 1 when only odd no. of input equal to 1 otherwise output is zero.

or

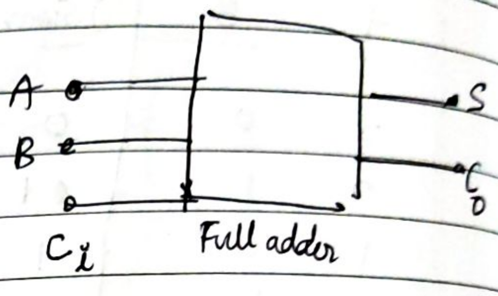
$\boxed{S = \bar{A}B + A\bar{B}}$

$\boxed{C_0 = A \cdot B}$ → AND



Full Adder :-

A	B	C_i	S	C_0
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0=1 + 1 = 0		0	1
1	1=0 + 0 = 0		0	1
1	1=0 + 1 = 1		1	1



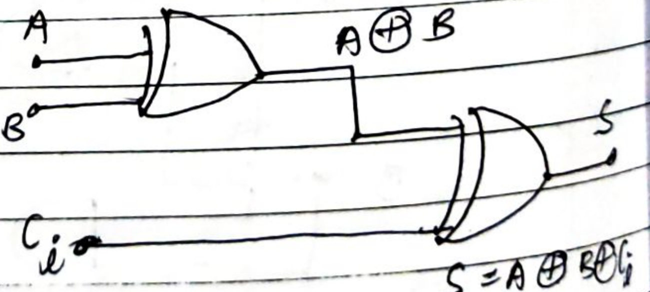
BC_i [for Sum] :

A \ BC_i	00	01	11	10
0	0	1	0	1
1	1	0	1	0

No combination possible.

☆ [check board configuration]

$$\boxed{S = A \oplus B \oplus C_i}$$



for C_0 :-

A \ BC_i	00	01	11	10
0	0	0	1	0
1	0	1	1	1

$$\boxed{C_0 = BC_i + AB + AC_i}$$

or

$$\boxed{C_0 = AB + C_i [A \oplus B]}$$

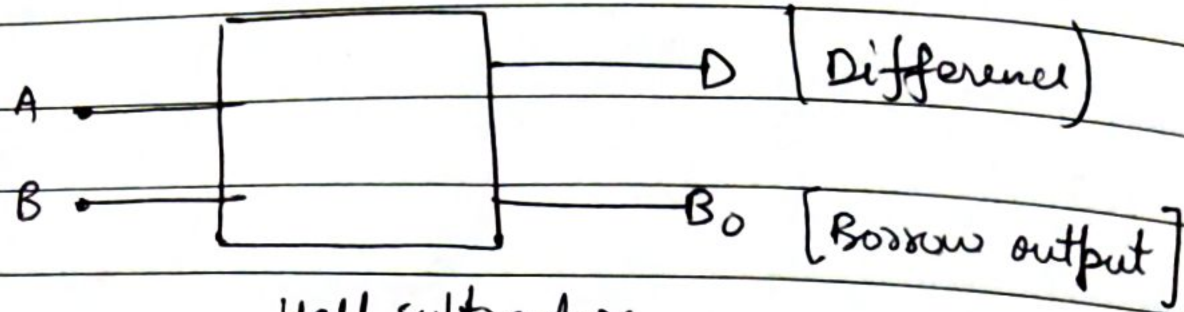
A \ BC_i	00			
0	0	0	1	0
1	0	1	1	1

$C_0 = AB + A\bar{B}C_i + \bar{A}BC_i$

$C_0 = AB + C_i [A\bar{B} + \bar{A}B]$

$C_0 = AB + C_i [A \oplus B]$

HALF SUBTRACTOR :-



Half subtractor

$2^1 \quad 2^0$

$0 \quad \cancel{1} \quad \overset{2}{0} \quad 2^1 \rightarrow 2$

$- \quad 0 \quad 1$

$0 \quad 1$ Ans

A	B	D	B_0
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

$$\boxed{D = A \oplus B}$$

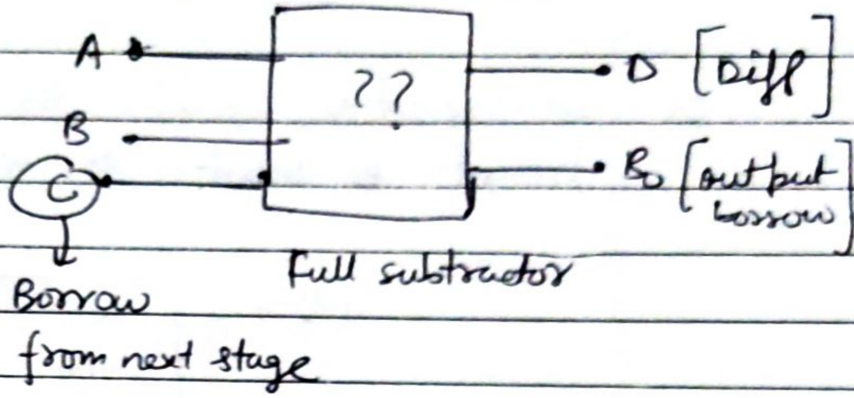
↓

XOR

$$\boxed{B_0 = \bar{A} B}$$

Full subtractor :-

A	B	C	D	B_o
0 − 0 = 0 − 0 = 0			0	0
0 − 0 = 0 − 1			1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1



for D :-

A \ BC	00	01	11	10
0	0	1	0	1
1	1	0	1	0

→ ☆★ check board config.

$$\boxed{D = A \oplus B \oplus C}$$

for B_o :-

A \ BC	00	01	11	10
0	0	1	1	1
1	0	0	1	0

$$B_o = BC + \bar{A}C + \bar{A}B$$

Sequential Circuit

Till now, we have seen combinational circuits where output was a function of current inputs only.

While discussing combinational circuits, we have seen that there are practical scenarios where memory is also involved.

Take one more example of Digital watch.

If it is showing 10:00, next output should be 10:01, then 10:02 and so on.

We can observe that output depends on previous output also.

Whenever memory is involved, circuit is called sequential.

Combinational circuit → o/p is f(current inputs)

Sequential circuit → o/p is f(current inputs, <u>past outputs</u>)

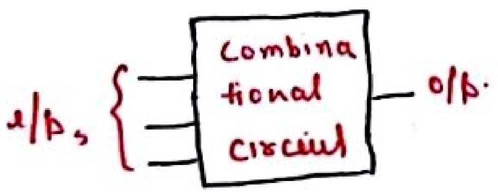
Past outputs → stored as memory in memory element and is called <u>state</u> of the memory element

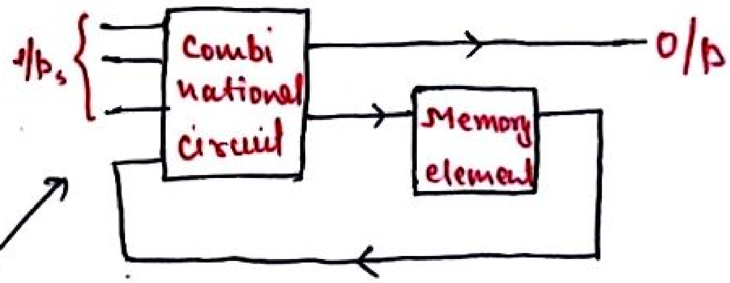
State of Memory elements :

Let us suppose, memory element can store 1 bit only.

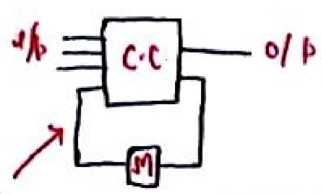
There are 2 possible states - 0 or 1.

If memory element can store 2 bits, there are $2^2 = 4$ possible states - 00, 01, 10, 11. With 3 bits, $2^3 = 8$ states & so on...





sequential → Combination of Combinational & Memory or in other words combinational circuit with feedback



Types of sequential circuits

↳ 1. Synchronous sequential circuit
↳ 2. Asynchronous sequential circuit.

In synchronous circuits, all circuit elements are synchronized.
Synchronized means → they all change their state simultaneously.
how ? → By the use of a master clock, which is connected to every element of circuit.

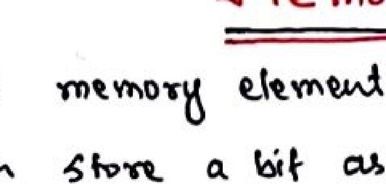
* State of all the circuit elements change only at discrete point of time, which is governed by a master clock.
(sometimes also called universal clock).
In Nutshell, we say that all different elements are synchronized by a universal clock.

* In Asynchronous circuit, there is no universal clock.
We will not detail out this topic further.
We will be studying synchronous circuits only in our course.

Memory Element

- Most basic memory element is a latch.
- Latch can store a bit as long as it is not disturbed by any external input.
- Latch can be prepared by Nand or Nor gate.

inputs



outputs { complementary outputs }
O/P should be complement of each other for proper operation

Fig 2

Both inputs are zero

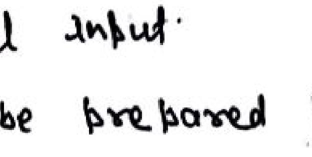


Fig 3

Both inputs are one

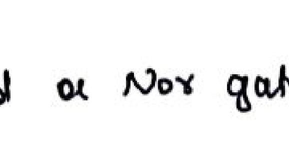


Fig 4

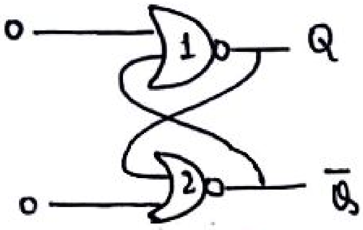
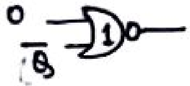


Fig 5

See NO. 1 NOR Gate



$0+\bar{Q}=\bar{Q}$ & $\bar{\bar{Q}}=Q$.

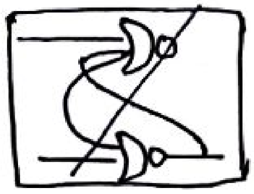
$0+Q=Q$.

In this way, Q & $\bar{Q}$ will be maintained.

Q. You can verify it for Nand Gate, whether 1 1 input maintains the output or not?

Q. You can also check what happens when 0,0 input is supplied to NAND latch and 1,1 input is supplied to NOR latch. Can we say that outputs will be maintained in this case also?

What happens when other possible input combinations are provided at input of the latch.



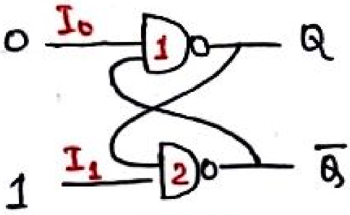
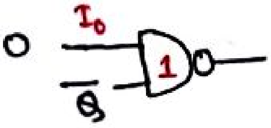


Fig 6

This is the Nand latch & let our state is Q in 1^{st} Nand Gate & $\bar{Q}$ in 2^{nd}. what will happen if 01 is provided at inputs. I_0 & I_1 respectively.



$0\cdot\bar{Q}=0$ & $\bar{0}=1$. next $Q=1$

$1\cdot Q=Q$ & $\bar{Q}=\bar{Q}$. next $\bar{Q}$ will remain $\bar{Q}$ only

but as soon as Q becomes 1, $\bar{Q}$ will become 0.

once, $Q=1$, $\bar{Q}=0$ After that there will be no change.

(*) If we want to make our output 1, (at Q terminal), irrespective of my previous Q, I can provide the input 0,1 to this latch.

Similarly, we can check for 1, 0 (1 at I_0 & 0 at I_1)
for this output $Q = 0$, & $\overline{Q} = 1$.

What will happen when 0, 0 is provided

Till now, you must have observed that whenever we make an input equal to 0, Nand Gate output will become 1., irrespective of other input

$$I_0 = 0 \rightarrow Q = 1$$

$$I_1 = 0 \rightarrow \overline{Q} = 1$$

If we make $I_0 = 0$ & $I_1 = 0$ Simultaneously, both outputs will become same i.e. 1. in this case.

This is not a desired Condition in latch.

Therefore, we never provide 0,0 input to this latch.

Summary of All Possible Input Configurations

1 1 → As Memory, Q_{t+1} remains Q_t

0 1 → Q_{t+1} becomes 1

1 0 → Q_{t+1} becomes 0

0 0 → Avoidable state.

Drawing the characteristic Table (Truth Table of Combinational circuit is now called characteristic Table)

output will depend on current inputs & past output.

Next output $Q(t+1)$ will depend on Inputs I_0, I_1 & $Q(t)$

As output is a function of 3 variables . 8 possible combination

Clue to draw characteristic Table

I_0	I_1	Q	$Q(t+1)$
⋮	⋮	⋮	

Q can have value 0 or 1. for each of the four possible $I_0 I_1$ pair. For ex- For 1,1, $Q(t+1) = Q(t)$ It means 0 will remain 0 1 will remain 1.

Qualitative / Intuitive Characteristic Table

I_0	I_1	$Q(t+1)$
0	0	Avoidable state
0	1	~~Rese~~ 1
1	0	0
1	1	Q

For $I_0 I_1 = 01$:
If $Q(t)=0 \rightarrow Q(t+1)=1$
If $Q(t)=1 \rightarrow Q(t+1)=1$

For $I_0 I_1 = 10$:
If $Q(t)=0 \rightarrow Q(t+1)=0$
If $Q(t)=1 \rightarrow Q(t+1)=0$

For $I_0 I_1 = 11$:
If $Q(t)=0 \rightarrow Q(t+1)=0$
If $Q(t)=1 \rightarrow Q(t+1)=1$

In all these cases, o/p is fixed & doesn't depend on previous o/p

In 1st two case o/p is 1 whether previous o/p was 0 or 1 & same for 2nd case.

Detailed Characteristic Table

$\boxed{\text{This table is not for Nand latch}}$

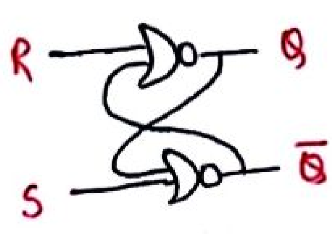
I_0	I_1	Q	$Q(t+1)$	
0	0	0	0	No change / Memory
0	0	1	1	
0	1	0	1	o/p is set
0	1	1	1	
1	0	0	0	o/p is Reset
1	0	1	0	
1	1	0	X	Avoidable
1	1	1	X	

Nand latch table will be like this

I_0	I_1	Q	$Q(t+1)$	
0	0	0	X	undesirable
0	0	1	X	
0	1	0	1	o/p is set
0	1	1	1	
1	0	0	0	o/p is Reset
1	0	1	0	
1	1	0	0	Q
1	1	1	1	

Now we will formally Define One of the basic latch i.e. SR latch (S- set, R- Reset) Set-Reset latch.

Now Inputs are called S & R



Logic Diagram of NOR SR latch

Q What will be $Q(t+1)$ when $R=0, S=0$

Q What will be $Q(t+1)$ when $R=0, S=1$

Q What will be $Q(t+1)$ when $R=1, S=0$

Q What will be $Q(t+1)$ when $R=1, S=1$

If You have thought on these questions., you must have got this table

S	R	Q(t+1)
0	0	No change → $Q_{t+1} = 0 + \bar{Q} = \bar{\bar{Q}} = Q$
0	1	Reset (i.e. 0) → $R=1$, $Q(t+1) = \overline{1+\bar{Q}} = \bar{1} = 0$
1	0	~~Re~~ set (i.e 1) → $S=1$, $\bar{Q}(t+1) = \overline{S+Q} = \overline{1+Q} = \bar{1} = 0$, $\bar{Q} = 0 \Rightarrow Q = 1$. $\because Q(t+1) = \overline{R+\bar{Q}} = \overline{0+0} = 1$
1	1	X

Characteristic Table

S	R	Q(t)	Q(t+1)	
0	0	0	0	No change
0	0	1	1	
0	1	0	0	Reset
0	1	1	0	
1	0	0	1	set
1	0	1	1	
1	1	0	X	Undesirable
1	1	1	X	

Characteristic Equation

⇓

Equation of $Q(t+1)$ in terms of $S, R, \& Q$.

Q Find $Q(t+1)$ using K-Map

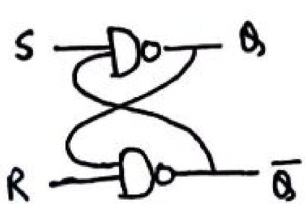
	$\bar{R}\bar{Q}$	$\bar{R}Q$	RQ	$R\bar{Q}$
$\bar{S}$	0	1	0	0
S	1	1	X	X

→ S

$\bar{R}Q$

$$\boxed{Q(t+1) = S + \bar{R}Q}$$

S-R latch using NAND Gate



Q Show that Characteristic Table of this latch will be like this →

SR	Q(t+1)
00	X
01	
10	
11	... change.

*** Observations**

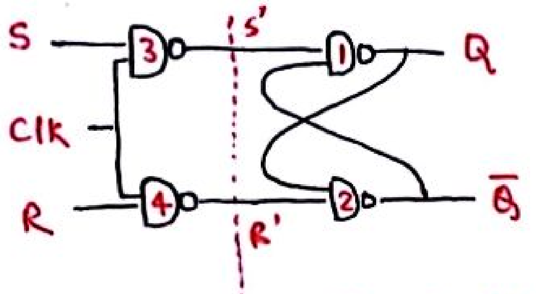
Q Will the table change if input name S & R are interchanged?

Q In nor latch, S was linked to $\bar{Q}$ directly & to $\bar{Q}$ by feedback
In Nand ", S is linked to Q directly & to $\bar{Q}$ by feedback.
Table will changed according to our logic diagram, location of S & R. There can be different table depending upon S & R location. But, location & Table should match.

Scanned with CamScanner

Add a clock pulse in the latch to make it Flip-Flop

Clock provide an additional control. The device will work only when a particular condition of clock is met.



A common clock has been added to Nand Gates 3 & 4.

Clock portion | SR Nand latch. → no change on 1 1

(Logic Diagram of SR Flip flop)

As we have used SR as the input on Nand Gates 3 & 4, input of 1 & 2 can be considered to be S', R'.

when ~~S=0, R=0~~ Clk=0

In clock is zero, o/p of Nand Gates 3 & 4 must be 1.

S' & R' will surely be 1 & 1.

we know that for SR latch 1 1 i/p brings no change.

Hence, overall there will be No change when clk = 0 irrespective of input S & R.

when clk = 1

Now output of Nand gate 3 will be $\overline{1 \cdot S} = \overline{S}$ &

output of Nand gate 4 will be $\overline{R}$

our basic SR latch.

S	R	Q(t+1)
0	0	X
0	1	1
1	0	0
1	1	No change

SR flip flop, clk = 1

S	R	S'	R'	Q(t+1)
0	0	1	1	No change.
0	1	1	0	0
1	0	0	1	1
1	1	0	0	X

SR flip-flop

S	R	Q(t+1)
0	0	No change
0	1	Clear (Reset)
1	0	Set
1	1	X

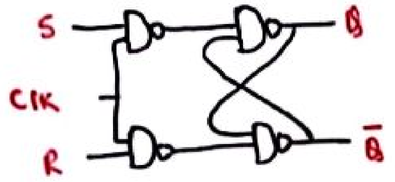
Problem with SR flip flop

1.1 condition should be avoided in SR f.f. as both outputs becomes same (which is not desired for proper operation of FF.)

For Any Flip-Flop, we will see these points

1. logic diagram.
2. characteristic Table
3. characteristic Equation
4. logic symbol

1. logic Diagram



2. characteristic Table

S	R	Q	Q(t+1)	
0	0	0	0	no change
0	0	1	1	
0	1	0	0	clear/ Reset.
0	1	1	0	
1	0	0	1	set
1	0	1	1	
1	1	0	x	
1	1	1	x	

compact form of characteristic Table

S	R	Q(t+1)
0	0	Q(t)
0	1	clear/Reset
1	0	set
1	1	x

3. characteristic Eq^n

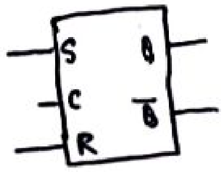
	$\bar{R}\bar{Q}$	$\bar{R}Q$	RQ	$R\bar{Q}$
$\bar{S}$	0	1	0	0
S	1	1	x	x

→ S

↓ $\bar{R}Q$

$$Q(t+1) = S + \bar{R}Q$$

4. logic symbol

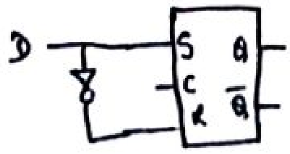


NOTE - we have not written clk in any of characteristic Table, but for FF to work, clk must be high. If it is low, FF will not show any change.

Solving the Problem of SR FF by D-flip Flop.

Problem was input 11. we make such an arrangement that 11 will never appear.

Q what we do? we have to do something such that 11 will never appear.



we have made only a flip flop of one input

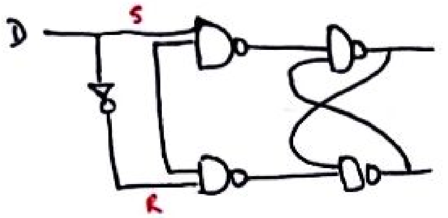
if D=0 → S=0, R=1 , if D=1 → S=1, R=0

S=D & R=$\bar{D}$, therefore S & R will never be same.

we will not get 00 or 11 this.

- Q Draw logic Diagram
- Q Draw characteristic Table
- Q find characteristic Equation
- Q Draw logic symbol

logic Diagram



Charateristic Table

D	S R	Q_{n+1}
0	0 1	0
1	1 0	1

only for clarity

characteristic Table

D	Q_{n+1}
0	0
1	1

D	Q	Q_{n+1}
0	0	0
0	1	0
1	0	1
1	1	1

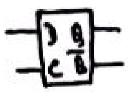
characteristic eq^n

	$\bar{Q}$	Q
$\bar{D}$	0	0
D	1	1

→ D

$Q(t+1) = D$

logic symbol



D- Flip - Delay FF
- Transparent F.F.

Problem with D flip flop

Q Do you really feel that D has solved the problem of 11 ?

or

It has bypassed the problem.

JK flip- Flop to solve the problem of 1,1.

We will first see what JK does, then we will look into how it does that

Characteristic Table

J K	Q (t+1)
0 0	No change ; Q(t)
0 1	Reset ; 0
1 0	set ; 1
1 1	Toggle { complement the previous output } : $\overline{Q(t)}$

At 1,1 it toggles.

Till now, we have seen only 3 operations in flip flop

- No change (as memory)
- set
- Reset.

Now, one more operation has been introduced i.e toggle.

Q Draw detailed characteristic table and find characteristic eqⁿ for JK FF.

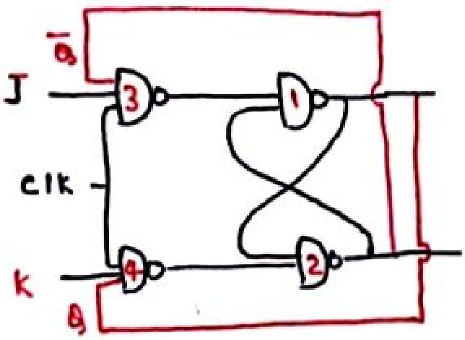
J	K	Q	Q(t+1)	
0	0	0	0	no change
0	0	1	1	
0	1	0	0	Reset
0	1	1	0	
1	0	0	1	set
1	0	1	1	
1	1	0	1	Toggle
1	1	1	0	

	$\bar{K}\bar{Q}$	$\bar{K}Q$	KQ	$K\bar{Q}$
$\bar{J}$	0	1	0	0
J	1	1	0	1

$\downarrow$ $\bar{K}Q$ $\quad$ $\downarrow$ $J\bar{Q}$

$$\boxed{Q(t+1) = J\bar{Q} + \bar{K}Q}$$

Q Verify that circuit shown below has a characteristic table equivalent to JK FF.



- Notice the interlocking of inputs. $\bar{Q}$ is going as i/p of Nand 3. & Q is going as i/p of Nand 4.

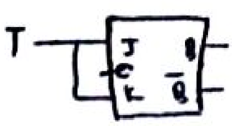
Q Hint → Break the circuit into two parts.
1st half - Combinatorial circuit $\overline{J\bar{Q}}$, $\overline{KQ}$
2nd half - SR Nand latch - use the Truth Table of SR nand latch.

T- Flip flop

As we have made D from SR, similarly T is one input FF made from JK. were T represent Toggle.

Q Draw logic diagram, characteristic Table, characteristic Equation & logic symbol

Hint -



Observe - Same T is going in both J & K. (not inverted one)