CHAPTER-2

Wave Particle Duality and Heisenberg Uncertainty Principle Contents

"There is not the radiation only which sometimes appear to behave like wave and some times exhibit particle like characteristics; The matter also presents itself both ways. This is known as wave particle duality of radiation"

2.1 de-Broglie Hypothesis: Matter wave concept or Wave particle duality

A wave is associated with a moving material body or a moving material boady present itself wave like similar to the radiation. This hypothetical wave was named as 'matter wave' or 'de-Broglie wave' and the wave length of the wave associated was named as de-Broglie wavelength

We know that $E = mc^2$ -①

$E = h\nu$ -② (Plank's Quantum theory)

$E = \frac{hc}{\lambda}$ -③ [$\nu = c/\lambda$]

From eqn ① & ③ $E = mc^2 = \frac{hc}{\lambda}$

$$\lambda = \frac{h}{mc}$$

This is the velocity of Photon for matter particle c can be replaced by v

$$\lambda = \frac{h}{mv} \quad \text{--- (4)}$$

2.2 Different forms of de Broglie Wave-length

* Non-relativistic Case

a) ~~The~~ In terms of Kinetic energy

$$K = \frac{P^2}{2m}$$

$$P = \sqrt{2mK}$$

$$\lambda = \frac{h}{P} = \frac{h}{\sqrt{2mK}}$$

$$\lambda = \frac{h}{\sqrt{2mK}} \quad \text{--- (5)}$$

b) In terms of Accelerating Potential

$$K = qV \qquad \lambda = \frac{h}{\sqrt{2mqV}}$$

For an electron) $\lambda = \dfrac{6.625 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times V}} = \dfrac{12.3}{\sqrt{V}} \, A^{\circ}$ --- (6)

* Relative Case:-

a) In terms of K.E.

$$E^2 = p^2c^2 + m_0^2 c^4 \quad \text{—(7)} \quad (m_0 = \text{rest mass})$$

$$p = \frac{1}{c}\sqrt{E^2 - m_0^2 c^4} \quad \text{—(8)}$$

Total energy as per Eisten's relation

$$E = (m - m_0)c^2 + m_0 c^2$$

$$E = K + m_0 c^2 \quad \text{—(9)}$$

$$p = \frac{1}{c}\sqrt{(K + m_0 c^2)^2 - m_0^2 c^4}$$

$$p = \frac{1}{c}\sqrt{K(K + 2m_0 c^2)}$$

$$\lambda = \frac{hc}{\sqrt{K(K + 2m_0 c^2)}} \quad \text{—(10)}$$

b) In terms of accelerating potential

$$\lambda = \frac{hc}{\sqrt{qV(2m_0 c^2 + qV)}} \quad \text{—(11)}$$

$$\lambda = \frac{h}{\sqrt{2m_0 c^2 qV\left(1 + \frac{qV}{2m_0 c^2}\right)}} \quad \text{—(12)}$$

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2.3 de-Broglie wave-velocity:

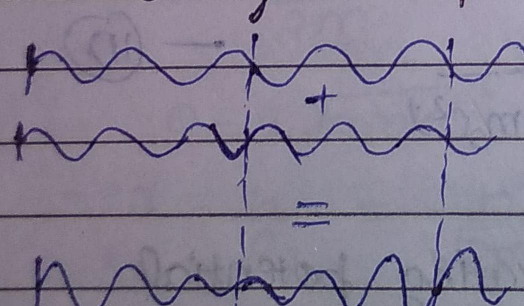
de-Broglie wave velocity $w = \nu\lambda$

$$= \frac{E}{h} \times \frac{h}{p} = \frac{E}{p} = \frac{mc^2}{mv} = \frac{c^2}{v}$$

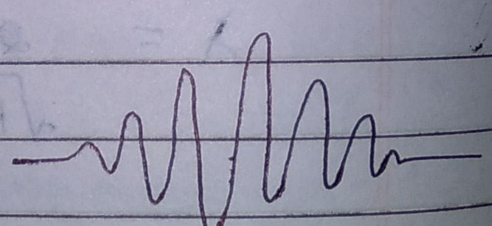
$$w >>> c$$

The result seems absurd and therefore is unacceptable. This controvery Erwin Shrödinger to think in some other way and to modify the hypothesis praposed by Louis de Broglie

2.4 Schrödinger's Explanation



Production of Beats in sound



de-Broglie wave group

Let the eqn. of the wave forming a wave-group be

$$y_1 = A\cos(\omega t - kx) \quad -①$$

$$y_2 = A\cos[(\omega + d\omega)t - (k + dk)x] \quad -②$$

eq. of resultant wave

$$y = y_1 + y_2 \quad \text{—③}$$
$$= A\,[\cos(\omega t - kx) + \cos[(\omega + d\omega)t - (k+dk)x]]$$
$$= 2A\cos\tfrac{1}{2}[(2\omega + d\omega)t - (2k+dk)x]\cos\tfrac{1}{2}(d\omega t - dk.x)$$

assuming $d\omega$ & dk very small compared to ω & K

so $d\omega + 2\omega = 2\omega$ $\qquad dk + 2K = 2K$

$$y = 2A\cos(\omega t - kx)\cos\left(\frac{d\omega}{2}t - \frac{dk}{2}x\right) \quad \text{—④}$$

Phase velocity $V_p = \dfrac{\omega}{K}$
(or wave velocity)

Group velocity $V_g = \dfrac{d\omega/2}{dK/2} = \dfrac{d\omega}{dK}$

2.5 Wave velocity (Phase velocity) or Group velocity

$(\omega t - kx)$ is phase of the motion in the eqn. of wave it means this is the velocity of that wave which come into existence when the particles withe constant phase travels: i.e

$$\omega t - kx = \text{constant} \quad \text{—①}$$

$$\frac{d}{dt}(\omega t - kx) = 0 \quad \text{—②}$$

$$\omega - K\frac{dx}{dt} = 0$$

$$V_p = \frac{dx}{dt} = \frac{\omega}{K} = \frac{2\pi\nu}{2\pi/\lambda} = \nu\lambda \quad \text{—③}$$

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$$V_g = \frac{d\omega}{dK} = \frac{2\pi d\nu}{2\pi d\left(\frac{1}{\lambda}\right)} = -\lambda^2 \frac{d\nu}{d\lambda} \quad -④$$

2.6 Relation between Phase velocity and Group veloc

a) In dispersive Medium

$$V_p = \frac{\omega}{K} \quad \text{or} \quad \omega = KV_p$$

$$V_g = \frac{d\omega}{dK} = \frac{d(KV_p)}{dK} \quad -⑤$$

$$= V_p + K \cdot \frac{dV_p}{dK}$$

$$V_g = V_p + \frac{2\pi}{\lambda} \frac{dV_p}{d\left(\frac{2\pi}{\lambda}\right)}$$

$$V_g = V_p - \frac{2\pi\lambda \, dV_p}{2\pi \, d\lambda}$$

$$V_g = V_p - \lambda \frac{dV_p}{d\lambda} \quad -⑥$$

from the above we can conclude that $V_g < V_p$ in dipersive medium

b) In Non-dispersive Medium:

For a non-dispersive medium V_p is independent of λ

i.e. $\frac{dV_p}{d\lambda} = 0$

so $V_g = V_p$

* velocity of wave-group (wave paket)

$$E = mc^2 = \frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}} \quad -(7)$$

$$P = mv = \frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}} \quad -(8)$$

$$\nu = \frac{E}{h} = \frac{m_0 c^2}{h\sqrt{1-\frac{v^2}{c^2}}} \quad -(9)$$

$$\omega = 2\pi\nu = \frac{2\pi m_0 c^2}{h\sqrt{1-v^2/c^2}} \quad -(10)$$

$$\frac{d\omega}{dv} = \frac{2\pi m_0 c^2}{h\left(1-\frac{v^2}{c^2}\right)^{3/2}} \times -\frac{1}{2} \times \frac{-2v}{c^2})$$

$$\cancel{\frac{d\omega}{dv}} = \cancel{2\pi m_0 c^2} \quad \cancel{\frac{2\pi m_0 v}{h\sqrt{1-\frac{v^2}{c^2}}}}$$

$$\frac{d\omega}{dv} = \frac{2\pi m_0 v}{h\left(1-\frac{v^2}{c^2}\right)^{3/2}}$$

$$\lambda = \frac{h}{p} = \frac{h\left(1-\frac{v^2}{c^2}\right)^{1/2}}{m_0 v}$$

propogating vector $K = \frac{2\pi}{\lambda} = \frac{2\pi m_0 v}{h\left(1-\frac{v^2}{c^2}\right)^{1/2}}$

$$\frac{dK}{dv} = \frac{2 m_0}{h}\left[\left\{\left(1-\frac{v^2}{c^2}\right)^{-1/2} + \frac{v^2}{c^2}\left\{1-\frac{v^2}{c^2}\right\}\right]\right.$$

$$\frac{dk}{dv} = \frac{2\pi m_0}{h\left(1-\frac{v^2}{c^2}\right)^{3/2}}$$

$$V_g = \frac{d\omega}{dk} = \frac{d\omega/dv}{dk/dv} \quad \text{— (11)}$$

$$V_g = \frac{2\pi m_0 v}{h\left(1-\frac{v^2}{c^2}\right)^{3/2}} \times \frac{h\left(1-\frac{v^2}{c^2}\right)^{3/2}}{2\pi m_0}$$

$$V_g = v$$

"A moving material boady presents itself as a wave - group or wave - packet associated with a moving particle travel with the velocity of moving material body. Therefore the de - Broglie hypothesis was slightly modified as -

"There is associated wave group or a wave packet with a moving material body"

2.7 Relation between V_p and V_g for a non-relativ

$$\lambda = \frac{h}{mv}$$

$$V_p = v$$

(Total energy) $E = \frac{1}{2}mv^2$

$$E = h\nu$$

$$\nu = \frac{E}{h} = \frac{1}{2}\frac{mv^2}{h}$$

$$v_p = \nu\lambda = \frac{1}{2}\frac{mv^2}{h} \times \frac{h}{mv} = \frac{v}{2}$$

$$v_p = \frac{v}{2} = \frac{v_g}{2}$$

2.8 Experimental confirmation of Matter waves

~~Our next~~

The following given experiments are sufficient enough to justify that fast moving material particles exhibit a wave - character

- Electron Microscope
- Davisson - Germer Experiment
- GP Thomson Experiment

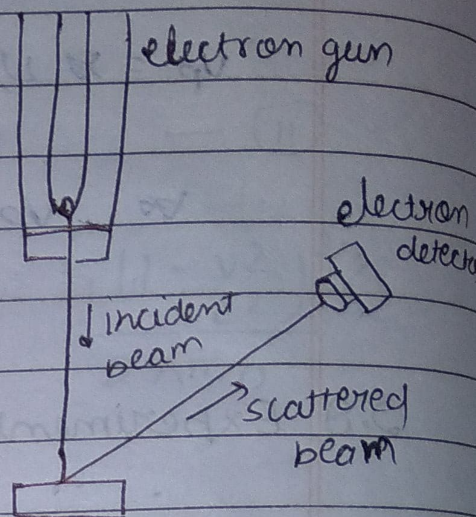
All the above experiment utilize accelerated electron beam which ultimately undergo diffraction and hence confirm the wave nature

2.9 Davisson Germer Experiment:

* To provide experiment confirmation of Matter-Wave
* Fast moving electrons can undergo diffraction ~~and hence confirm the wave~~ which is essential wave nature confirmation

Experimental set-up:

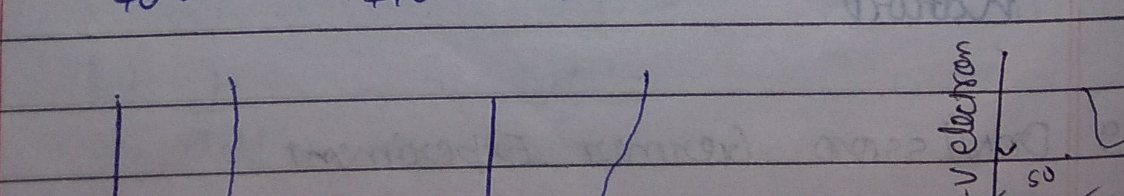
1. Electron Gun:- The purpose of e gun is to produce collimate and accelerated the e beam

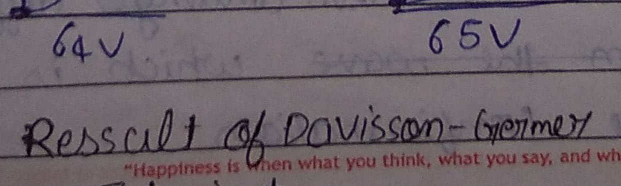


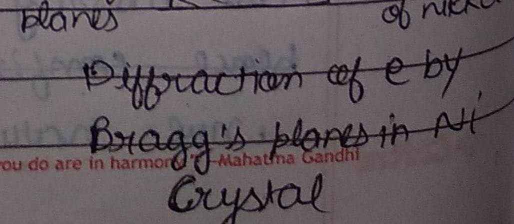
2. Nickel Crystal:- Inter planer separation of this crystal matches roughly with the wave-length of electron waves and therefore Ni- crystal can act as slit system (grating)

3. Detector - This is the device which collect the scattered radiation and convert it into the current which is measure of intensity

Experimental observation:-







Resscult of Davisson-Germer Experinet

Diffraction of e by Bragg's planes in Ni Crystal

* Explanation:

well defined bump started to appear with a more pronounced peak appeared at 54 volts at an angle of 50°. If fast moving electron beam behave wave-like it must undergo diffraction and then Bragg's Law can be applied

Bragg's equation -

$$2d\sin\theta = n\lambda \quad \text{—— (1)}$$

For Ni crystal $d = 0.91$ A and $\theta = 50°$ (but this becomes 65° w.r.t. Bragg's Planes)

Now For $n = 1$

$$\lambda = 2 \times 0.91 \times \sin 65°$$

$$= 1.65 \text{ A}°$$

(wavelength of electron) $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE_K}} = \frac{6.64 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-3} \times 54 \times 1.6 \times 10^{-19}}}$

$\lambda = 1.66$ A°

Hence the material particle also exhibit wave like nature under certain circumstances similar to the radiation