Math - II Assignment.

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Q1- Find the fourier series of following periodic function of period 2π

$$f(x) = \begin{cases} 0 & -\pi < x < 0 \\ x^2 & 0 \le x < \pi \end{cases}$$

Hence, show that

$$\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} - - - - - \infty \ \&$$

$$\frac{\pi^2}{12} = 1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + - - - - \infty$$

↳ If x^2 ; $0 \le x < \pi$ is the half range-cosine series -

$$f(x) = \left(\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx \right)$$

$$a_0 = \frac{2}{\pi} \int_0^{\pi} f(x)dx = \frac{2}{\pi} \int_0^{\pi} x^2 dx = \frac{2\pi^2}{3}$$

$$\left[a_0 = \frac{2\pi^2}{3} \right]$$

$$a_n = \frac{2}{\pi} \int_0^{\pi} x^2 \cos nx \, dx$$

$$a_n = \frac{2}{\pi} \left[\frac{x^2 \sin nx}{n} - \int \frac{2x}{n} \sin nx \, dx \right]$$

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$$a_n = \frac{2}{\pi}\left[\frac{x^2 \sin nx}{n} - \frac{2}{n}\left\{-\frac{x\cos nx}{n} + \int\frac{\cos nx}{n}\,dx\right\}\right]$$

$$a_n = \frac{2}{\pi}\left[\frac{x^2 \sin nx}{n} - \frac{2}{n}\left\{-\frac{x}{n}\cos nx + \frac{\sin nx}{n^2}\right\}\right]_0^\pi$$

$$a_n = \frac{2}{\pi}\left[\frac{x^2 \sin nx}{n} + \frac{2x}{n^2}\cos nx - \frac{2}{n^3}\sin nx\right]_0^\pi$$

$$a_n = \frac{2}{\pi}\left[\left(\frac{\pi^2 \sin n\pi}{n} + \frac{2\pi}{n^2}\cos n\pi - \frac{2}{n^3}\sin n\pi\right) - 0\right]$$

$$= \frac{2}{\pi}\left[\frac{2\pi}{n^2}(-1)^n\right]$$

$$\boxed{a_n = \frac{4}{n^2}(-1)^n}$$

$$f(x) = \frac{\pi^2}{3} + 4\left(-\cos x + \frac{1}{2^2}\cos 2x - \frac{1}{3^2}\cos 3x + \frac{1}{4^2}\cos 4x + \cdots\right)$$

$$x^2 = \frac{\pi^2}{3} + 4\left(-\cos x + \frac{1}{2^2}\cos 2x - \frac{1}{3^2}\cos 3x + \cdots\right)$$

put $x = 0$.

$$\frac{\pi^2}{3} = -4\left(1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots \infty\right)$$

$$\left(1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots \infty\right) = \frac{\pi^2}{12}$$ Proved

Again put $x = \pi$

$$\pi^2 - \frac{\pi^2}{3} = 4\left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \cdots \infty\right)$$

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$$2\left(1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\frac{1}{5^2}+\cdots\infty\right)=\frac{\pi^2}{3}.$$

$$\left(1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\frac{1}{5^2}+\cdots\infty\right)=\frac{\pi^2}{6}$$ Proved.

Q2 Find the fourier expansion of $f(x) = x^2$, $-2 \le x \le 2$

$(-c \le x \le c)$ $c = 2$

$\hookrightarrow$ $a_0 = \frac{1}{c}\int_0^{2c} f(x)\,dx.$

$= \frac{1}{2}\int_0^4 x^2\,dx$

$= \frac{1}{2}\left[\frac{x^3}{3}\right]_0^4 = \frac{32}{3}.$

$a_n = \frac{1}{c}\int_0^{2c} f(x)\cos\frac{n\pi x}{c}\,dx = \frac{1}{2}\int_0^4 x^2\cos\left(\frac{n\pi x}{2}\right)dx$

$= \frac{1}{2}\left[\left\{\frac{2x^2}{n\pi}\sin\left(\frac{n\pi x}{2}\right)\right\}_0^4 - \int_0^4 2x\sin\left(\frac{n\pi x}{2}\right)\times 2\right]$

$= \frac{1}{2}\left[0 - \frac{4}{n\pi}\left[\left\{\frac{-2x\cos\left(\frac{n\pi x}{2}\right)}{n\pi}\right\}_0^4 + \left\{\frac{4\sin\left(\frac{n\pi x}{2}\right)}{n^2\pi^2}\right\}_0^4\right]\right]$

$= \frac{1}{2}\left[0 + \frac{4}{n^2\pi^2}\left[4\cos(\underbrace{2n\pi}_{1}) - 0 - \frac{16}{n^3\pi^3}(0)\right]\right]$

$= \frac{1}{2}\left[\frac{32}{n^2\pi^2}\right] = \frac{16}{n^2\pi^2}$

$b_n = \frac{1}{c}\int_0^{2c} \underbrace{f(x)}_{\text{even}}\,\underbrace{\sin\frac{n\pi x}{c}}_{\text{odd.}}\,dx$

$[b_n = 0]$

$f(x) = \frac{a_0}{2} + \sum_{n>1}^{\infty} a_n\cos\left(\frac{n\pi x}{c}\right)dx.$

$f(x) = \frac{16}{3} + \frac{16}{\pi^2}\cos\frac{\pi x}{2} + \frac{4}{\pi^2}\cos\pi x + \frac{16}{9\pi^2}\cos\frac{3\pi x}{2} + \cdots \infty$

Q3- Obtain the fourier cosine series of the following function

$$f(x) = \begin{cases} x^2 & 0 < x < 2 \\ 4 & 2 \le x \le 4 \end{cases}$$

here $c = 4$, because it is half range series

$\hookrightarrow$ $f(x) = \dfrac{a_0}{2} + a_1 \cos\dfrac{\pi x}{c} + a_2 \cos\dfrac{2\pi x}{c} + \cdots \infty$

$\hookrightarrow$ $a_0 = \dfrac{2}{c}\displaystyle\int_0^c f(x)\,dx = \dfrac{2}{4}\left[\int_0^{2} x^2 dx + \int_2^4 4\,dx\right]$

$$= \frac{1}{2}\left[\left[\frac{x^3}{3}\right]_0^2 + \left[4x\right]_2^4\right]$$

$$= \frac{1}{2}\left[\frac{8}{3} + 8\right] = \frac{16}{3}.$$

$\hookrightarrow$ $a_n = \dfrac{2}{c}\displaystyle\int_0^c f(x)\cos\frac{n\pi x}{c}\,dx$

$$= \frac{2}{4}\left[\int_0^2 x^2 \cos\frac{n\pi x}{4}\,dx + \int_2^4 4\cdot\cos\frac{n\pi x}{4}\,dx\right]$$

$$= \frac{1}{2}\left[0 - \int_0^2 \frac{8x}{n\pi}\sin\left(\frac{n\pi x}{4}\right)dx\right] + 4\left[\sin\left(\frac{n\pi x}{4}\right)\left(\frac{4}{n\pi}\right)\right]_2^4$$

$$= \frac{1}{2}\left[-\frac{8}{n\pi}\left\{\frac{-4x\cos\left(\frac{n\pi x}{4}\right)}{n\pi}\right]_0^2 + 0\right] + 4\left[0 - \frac{4}{4n\pi}\sin\left(\frac{n\pi}{2}\right)\right]$$

$$= \frac{16}{n^2\pi^2}\left\{2\cos\left(\frac{n\pi}{2}\right) - 0\right\} - \frac{16}{n\pi}\sin\left(\frac{n\pi}{2}\right)$$

$$= \frac{-16}{n\pi}\sin\left(\frac{n\pi}{2}\right) = \begin{cases} 0 & , n = \text{even} \\ \dfrac{-16}{n\pi}\sin\left(\dfrac{n\pi}{2}\right) & , n = \text{odd} \end{cases}$$

$$\left[f(x) = \frac{8}{3} - \frac{16}{\pi}\cos\left(\frac{\pi x}{4}\right) + \frac{16}{3\pi}\cos\left(\frac{3\pi x}{4}\right) - - - - \right]$$

Q4. Find the fourier series expansion of x^2 in $(-\pi, \pi)$. Use parseval's identity to prove that

$$1 + \frac{1}{2^4} + \frac{1}{3^4} + - - - = \frac{\pi^4}{90}$$

$\hookrightarrow$ $$f(x) = x^2 = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

Put $b_n = 0$ as $f(x)$ is even

$\hookrightarrow$ $$\left[f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx \right]$$

$\hookrightarrow$ $$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx = \frac{2}{\pi}\int_0^{\pi} x^2\,dx = \frac{2}{3}\pi^2$$

$\hookrightarrow$ $$a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos x\,dx = \frac{2}{\pi}\int_0^{\pi} x^2 \cos nx\,dx$$

$\hookrightarrow$ $$a_n = \frac{4}{x^2}(-1)^n$$

$\hookrightarrow$ $$f(x) = \frac{\pi^2}{3} + \left[-\frac{4}{1}\cos x + \cos 2x - \frac{4}{9}\cos 3x \right]$$

$\hookrightarrow$ $$f(x) = \frac{\pi^3}{3} + 4\left[-\frac{\cos x}{1} + \frac{\cos 2x}{2^2} - \frac{\cos 3x}{3^2} + \cdots \right]$$

$\hookrightarrow$ from Parseval's formula $\rightarrow$

$$\int_{-c}^{c} [f(x)]^2\,dx = c\left[\frac{a_0^2}{2} + \sum_{n=1}^{\infty} (a_n^2 + b_n^2) \right]$$

$$\Rightarrow \int_{-\pi}^{\pi} x^4\,dx = \pi\left\{\frac{2}{9}\pi^4 + \sum_{n=1}^{\infty}\frac{16}{n^4}\right\}$$

$$\Rightarrow \frac{2\pi^5}{5} = \pi\left\{\frac{2}{9}\pi^4 + \sum_{n=1}^{\infty}\frac{16}{n^4}\right\}$$

$$\Rightarrow \frac{2}{5}\pi^4 - \frac{2}{9}\pi^4 = \sum_{n=1}^{\infty}\frac{16}{n^4}.$$

$$\Rightarrow \frac{\cancel{8}\pi^4}{45} = \cancel{16}^{\,2}\sum_{n=1}^{\infty}\frac{1}{n^4}$$

$$\Rightarrow \frac{\pi^4}{90} = \sum_{n=1}^{\infty}\frac{1}{n^4}.$$

$$\Rightarrow \left[\frac{1}{1} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \cdots = \frac{\pi^4}{90}\right]$$

Q5 Analyze harmonically the data given below and express $y = f(x)$ in fourier series upto third harmonic.

x :	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
y :	1.0	1.4	1.9	1.7	1.5	1.2	1.0

$\Rightarrow$

x	$\sin x$	$\sin 2x$	$\sin 3x$	$\cos x$	$\cos 2x$	$\cos 3x$	$f(x)$	$f(x)\sin x$	$f(x)\sin 2x$	$f(x)\sin 3x$
0	0	0	0	1	1	1	1	0	0	0
$\pi/3$	0.866	0.866	0	0.5	-0.5	-1	1.4	1.2124	1.2124	0
$2\pi/3$	0.866	-0.866	0	-0.5	-0.5	1	1.9	1.6454	-1.6454	0
π	0	0	0	-1	1	-1	1.7	0	0	0
$4\pi/3$	-0.866	0.866	0	-0.5	-0.5	1	1.5	-1.299	1.299	0
$5\pi/3$	-0.866	-0.866	0	0.5	-0.5	-1	1.2	-1.0392	-1.0392	0
2π	0	0	0	1	1	1	1.0	0	0	0

	Sum	mean
$\Sigma f(x)$	9.7	1.386
$\Sigma f(x)\sin x$	0.5196	0.07423
$\Sigma f(x)\sin 2x$	-0.1732	-0.02474
$\Sigma f(x)\sin 3x$	0	0
$\Sigma f(x)\cos x$	-0.1	-0.01428
$\Sigma f(x)\cos 2x$	0.7	0.1
$\Sigma f(x)\cos 3x$.	1.1	0.1571

$f(x)\cos x$	$f(x)\cos 2x$	$f(x)\cos 3x$
1	1	1
0.7	-0.7	-1.4
-0.95	-0.95	1.9
-1.7	1.7	-1.7
-0.75	-0.75	1.5
0.6	-0.6	-1.2
1	1	1.0

$a_0 = 2 \times$ meanvalue of $f(x) = 2.772$

$a_1 = 2 \times$ " " " " $= -0.02056$

$a_2 = 2 \times$ " " " " $= 0.2$

$a_3 = 2 \times$ " " " " $= 0.3142$

$b_1 = 2 \times$ " " " " $= 0.14846$

$b_2 = 2 \times$ " " " " $= -0.04948$

$b_3 = 2 \times$ " " " " $= 0$

$$f(x) = \frac{a_0}{2} + a_1\cos x + a_2\cos 2x - - - b_1\sin x + b_2\sin 2x - - - -$$

$$f(x) = 1.386 - 0.02056\cos x + 0.2\cos 2x - - - + 0.14846\sin x - 0.04948\sin 2x - - -$$

Q⑥ Obtain a PDE from $f(xy+z^2, x+y+z)=0$ by eliminating the arbitrary function f

↳ $f(u,v)=0$

↳ $\frac{\partial f}{\partial u}\left(\frac{\partial u}{\partial x}+\frac{\partial u}{\partial z}p\right)+\frac{\partial f}{\partial v}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial z}q\right)=0$

↳ (Let $u=xy+z^2$, $v=x+y+z$)

↳ $\frac{\partial f}{\partial u}(y+2zp)+\frac{\partial f}{\partial v}(1+p)=0$ — (I)

similarly
↳ $\frac{\partial f}{\partial u}\left(\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}q\right)+\frac{\partial f}{\partial v}\left(\frac{\partial v}{\partial y}+\frac{\partial v}{\partial z}q\right)=0$

↳ $\frac{\partial f}{\partial u}(x+2zq)+\frac{\partial f}{\partial v}(1+q)=0$ — (II)

Eliminating $\partial f/\partial u$ and $\partial f/\partial v$ from eqn (I) & (II)

$$\begin{vmatrix} y+2zp & 1+p \\ x+2zq & 1+q \end{vmatrix}=0$$

$(y+2zp)(1+q)-(x+2zq)(1+p)=0.$

$$\left[y(1+q)-x(1+p)+2z(p-q)=0\right]$$

Q7- Solve $xz(z^2+xy)p - zy(z^2+xy)q = x^4$

$Pp+Qq=R$

$P = xz(z^2+xy)$, $Q=-zy(x^2+zy)$, $R=x^4$

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then, $\dfrac{dx}{xz(z^2+xy)} = \dfrac{dy}{-zy(z^2+xy)} = \dfrac{dz}{x^4}$

taking first two –

$\dfrac{dx}{x} = \dfrac{dy}{-y} \rightarrow \dfrac{dx}{x} + \dfrac{dy}{y} = 0$

$\log(xy) = \log c_1$

$xy = c_1$

taking first and third.

$$\frac{dx}{xz(z^2+xy)} = \frac{dz}{x^4}$$

$$\frac{dz}{z(z^2+c_1)} = \frac{dz}{x^3}$$

$x^3 dx = z(z^2+c_1)\,dz$

$\dfrac{x^4}{4} = \dfrac{z^4}{4} + \dfrac{z^2}{2}c_1 + c_2 \rightarrow c_2 = \dfrac{x^4}{4} - \dfrac{z^4}{4} - \dfrac{z^2}{2}xy$

$$\left[xy = f\left(\frac{x^4}{4} - \frac{z^4}{4} - \frac{xyz^2}{2} \right) \right]$$

(8) solve $(x^2 - y^2 - yz)p + (x^2 - y^2 - xz)q = (x-z)z$

$$\frac{dx}{x^2-y^2-yz} = \frac{dy}{x^2-y^2-xz} = \frac{dz}{(x-z)z}$$

$$\frac{dx-dy}{z(x-y)} = \frac{dz}{(x-y)z}$$

$dx - dy = dz$

$x - y = z + c_1$

$$\frac{x\,dx - y\,dy}{(x+y)(x-y)z} = \frac{dx-dy}{z(x-y)}$$

$$\frac{x\,dx - y\,dy}{x+y} = \frac{dx-dy}{z}$$

$$\int \frac{2(x\,dx - y\,dy)}{x^2-y^2} = \int \frac{2(dx-dy)}{z}$$

$$\log c_2 + \log(x^2-y^2) = \frac{2}{z}(x-y)$$

$$c_2 = \frac{e^{2/z(x-y)}}{x^2-y^2}$$

$$f(c_1, c_2) = 0.$$

$$\left[f\left(x-y-z, \frac{e^{2/z(x-y)}}{x^2-y^2}\right) = 0. \right]$$

(9) solve $(y-px)(p-1) = p$.

$$py - p^2x - y + px = p$$

$$p(y+x-1) - p^2x - y = 0$$

$$p^2x - p(y+x-1) + y = 0$$

$$p = \frac{(y+x-1) \pm \sqrt{(y+x-1)^2 - 4xy}}{2x}$$

$$p = \frac{(x+y-1) \pm (x-y-1)}{2x}$$

$$\left(\frac{\partial z}{\partial x}\right) = \frac{2x-2}{2x} \text{ or } y/x$$

$$\frac{\partial z}{\partial x} = 1-\frac{1}{x}$$

$$z = x - \log x + c_1$$

$$\frac{\partial z}{\partial x} = \frac{y}{x}$$

$$z = y\log x + c_2$$

Now $\left[\begin{array}{l} z = x - \log x + c_1 \\ z = y\log x + c_2 \end{array}\right]$

(10) $x^2p^2 + y^2q^2 = z^2$

$$\frac{x^2}{z^2}p^2 + \frac{y^2}{z^2}q^2 = 1$$

$$\left(\frac{x}{z}\frac{\partial z}{\partial x}\right)^2 + \left(\frac{y}{z}\frac{\partial z}{\partial y}\right)^2 = 1$$

$$\left(\frac{\frac{\partial z}{z}}{\frac{\partial x}{x}}\right)^2 + \left(\frac{\frac{\partial z}{z}}{\frac{\partial y}{y}}\right)^2 = 1 \quad \text{— (i)}$$

let $\frac{\partial z}{z} = \partial Z$, $\frac{\partial x}{x} = \partial X$, $\frac{\partial y}{y} = \partial Y$

$\log z = Z$, $\log x = X$, $\log y = Y$

$$\left(\frac{\partial Z}{\partial X}\right)^2 + \left(\frac{\partial Z}{\partial Y}\right)^2 = 1$$

$$P^2 + Q^2 = 1$$

Let the required solⁿ be :-

$$Z = aX + bY + c \quad \text{— (ii)}$$

$$P = \frac{\partial Z}{\partial X} = a, \quad Q = \frac{\partial Z}{\partial Y} = b$$

From eqⁿ (ii)

$$a^2 + b^2 = 1 \Rightarrow b = \sqrt{1-a^2}$$

$$Z = aX + \sqrt{1-a^2}\,Y + c$$

$$\left[\log z = a\log x + \sqrt{1-a^2}\log y + c\right]$$

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(11) $p - q = x^2 + y^2$

$p - x^2 = y^2 + q = a$ (let)

$p - x^2 = a$

$p = a + x^2$

$q = a - y^2$

we know, $dz = p\,dx + q\,dy$

substitute value of p & q

$dz = (a + x^2)dx + (a - y^2)dy$

$$\left[z = ax + \frac{x^3}{3} + ay - \frac{y^3}{3} + b \right]$$

(12) $(D_x^2 + D_y^2)z = x^2y^2$

put $D_x = m$ & $D_y = 1$

for C.F = $(D_x^2 + D_y^2)z = 0$

A.E $= m^2 + 1 = 0$

$m = \pm i$

$$\left[C.F = f_1(y + ix) + f_2(y - ix) \right]$$

for P.I

$$PI = \frac{1}{(D_x^2 + D_y^2)}(x^2y^2) = \frac{1}{D_x^2\left(1 + \frac{D_y^2}{D_x^2}\right)}(x^2y^2)$$

$$= \frac{1}{D_x^2}(x^2y^2) - \frac{D_y^2}{D_x^2}(x^2y^2)$$

$$= \frac{x^4y^2}{12} - \frac{1}{D_x^4}(2x^2)$$

$\Rightarrow$ $PI = \frac{1}{180}(15x^4y^2 - x^6)$

complete solⁿ → $f_1(y + ix) + f_2(y - ix) + \frac{1}{180}(15x^4y^2 - x^6)$

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(13) $(D_x^2 - D_y^2)z = \sin x \cos 2y$

$(m^2-1)=0, \quad m=\pm 1$ $\quad (D, D')$

$CF = f_1(y+x) + f_2(y-x)$

$PI = \frac{1}{D^2 - D'^2} \sin x \cos 2y = \frac{1}{2(D^2-D'^2)} [\sin(x+2y) + \sin(x-2y)]$

$= \frac{1}{2}\left[\frac{\sin(x+2y)}{(-1+4)} + \frac{\sin(x-2y)}{(-1+4}\right)\Big]$

(15)

$= \frac{1}{2}\left[\frac{\sin(x+2y)}{3} + \frac{\sin(x-2y)}{3}\right]$

complete solⁿ $(z) = f_1(y+x) + f_2(y-x)$

$+ \frac{1}{6}[\sin(x+2y) + \sin(x-2y)]$

(14) $(D_x - D_y - 1)(D_x - D_y - 2)z = e^{2x-y} + x$

$f(D, D') = f(x,y)$

Let form be $(\alpha D + \beta D' + \gamma)z = 0$

then C.F

$z = e^{-\frac{\gamma}{\alpha}x} \cdot \phi(\alpha y - \beta x)$

$CF = e^{-\left(\frac{-1}{1}\right)x} \phi(1 \cdot y - (-1)x) + e^{-\left(\frac{-2}{1}\right)x} \phi_2(1 \cdot y + $

$CF = e^{x} \phi_1(y+x) + e^{2x} \phi_2(y+x)$

$PI = \frac{e^{2x-y} + x}{(D^2 - 5DD' + 2D'^2)} = \frac{e^{2x-y}}{4+10+2} + \frac{x}{(D^2+5DD'+2D'^2}$

let $1 \cdot x + 0 \cdot y = u.$

$$= \frac{e^{2x-y}}{16} + \frac{1}{(1-0)} \iint u\,du\cdot du$$

$$= \frac{e^{2x-y}}{16} + \frac{u^3}{6}$$

$$PI = \frac{e^{2x-y}}{16} + \frac{x^3}{6}$$

complete solⁿ → $e^{x}\phi_1(y+x) + e^{2x}\phi_2(y+x) + \frac{e^{2x-y}}{16} + \frac{x^3}{6}$

(15) $(x^2 D_{xx} - 4xy D_{xy} + 4y^2 D_{yy} + 6y D_y) = x^3 y^4$

$(x^2 D_{xx} - 4xy D_{xy} + 4y^2 D_{yy} + 6y D_y) z = 0$

Let $x = e^p$ $\quad p = \log_e^x$

$y = e^q$ $\quad q = \log_e^y$

$(D(D-1) - 4DD' + 4D'(D'-1) + 6D') P = 0$

$(D^2 - D + 2D') P = 0$ $\quad$ Let $D = m$, $D' = 1$

$m^2 - m + 2m = 0$

$$m = \frac{1 \pm i\sqrt{7}}{2}$$

$$CF = f_1\left[q + \left(\frac{1+i\sqrt{7}}{2}\right)p\right] + f_2\left[q + \left(\frac{1-i\sqrt{7}}{2}\right)p\right]$$

$$= f_1\left[\log y + \left(\frac{1+i\sqrt{7}}{2}\right)\log x\right] + f_2\left[\log y\left(\frac{1-i\sqrt{7}}{2}\right)\log x\right]$$

$$PI = \frac{e^{3p+4q}}{(D^2 - D + 2D')} = \frac{e^{3p+4q}}{(D^2 - D + 2D')}$$

$$PI = \frac{x^3 y^4}{9-3+8} = \frac{x^3 y^4}{14}$$

complete solⁿ → $f_1\left[\log y + \left(\frac{1+i\sqrt{7}}{2}\right)\log x\right] + f_2\left(\log y + \left(\frac{1-\sqrt{7}i}{2}\right)\log x\right)$

$$+ \frac{x^3 y^4}{14}$$