Unit-1 — Number System + Logic families will be done later :)

- Base Conversion

Base 3 — 7

convert to base 10

Q- $(121.2)_3$ — $(?)_7$

$(121.2)_3$

$3^{-1} \times 2$
$3^{0} \times 1 +$
$3^{1} \times 2 +$
$3^{2} \times 1 +$

$= (16.66)_{10}$

$(16.66)_{10}$ —

$7 \,|\, 16 \quad 2$
$\quad\,|\, 2$

$.66 \times 7 = 4.62 - 4$
$.62 \times 7 = 4.34 - 4$
$.34 \times 7 = 2.38 - 2$

$= (22.442)_7$

Base 2 to 4

Q $(0111011010.110110)_2 = (13122.312)_4$

* ~~Bases that can be written as power of bases that has to be converted can be ~~ob~~ converted using grouping acc. to power.~~

~~Base 3 to 9~~

$3 \rightarrow 9 \rightarrow 3^2$

can be paired as 2's groups and converted.

~~Base 4 to 8~~

10

2

~~Addita~~ ~~Any base to any base~~

10

2

check for powers (pairing / grouping)

— Addition

~~A~~

sum $(A \oplus B)$

carry (AB)

A	B	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

← Half adder

$1+1=2=(10)_2$

DATE
PAGE

- Substraction

$A - B = A + (-B)$

* Complement is used to represent −ve numbers

Signed binary numbers are those in which one bit is used to represent sign (0 = +ve, 1 = −ve)

Ex − $\underbrace{1}_{\text{sign bit}}\underbrace{101}_{\text{magnitude bit}} = -5$

unsigned binary numbers are only +ve

Ex Two complements in binary
→ 1's complement
→ 2's complement

1's complement is inverting all the 0's to 1 and 1's to 0.

2's complement is adding 1 to 1's complement

$(100010)_2 \xrightarrow{\text{1's complement}} (011101)_2$

$\downarrow$ 2's complement

$(011101 + 1)_2 = (011110)_2$

For writing 2's complement directly → Write as it is till 1st '1' for RHS and then flip like 1's complement.

$(100010)_2 \xrightarrow{\text{2's complement}} (011110)_2$

Substraction using 2's complement

A − B ⇒ A + (2's complement of B)

Observe the result

Carry = +ve
Result = don't consider carry, rest is result

no carry = -ve
Result = 2's complement of result.

Depending upon carry result will be +ve or -ve

04/10/21

Every base has 2 complements –

Base (r) → r's complement (radix)
→ (r-1)'s complement (diminished radix)

For example – base 10 $(1246)_{10}$

9's complement of $(1246)_{10}$ is 9999 – 1246 = 8753

10's complement is = 8752

Largest 4 digit num

This example helps us derive the formula for complement →

Base r →

$(r-1)$'s complement $= \boxed{((r^n - 1) - N)}$

r's complement $= (r^n - 1 - N) + 1 = \boxed{r^n - N}$

where n is the number of digits in the given number & N is the number

DATE
PAGE

— Understanding 2's complement

Purpose of find 2's (base 2) or 10's (base 10) or r's complement is it makes substraction simpler as compared to borrow system

(base 10)

For ex ① 10's complement of 7 is 6+1=7

② 7 − 3 = 4

7 + 7 = $\cancel{1}4$ or (drop the one)

③

2's complement represent −ve numbers in computer

— Boolean Algebra (0 & 1) (base 2)

Basic Operations

- → OR (+)
- → AND (.)
- → NOT (~/¯)

— Boolean Postulates & Theorems

- $x + 0 = x$
- $x \cdot 1 = x$
- $x + 1 = 1$
- $x \cdot 0 = 0$
- $x + \bar{x} = 1$
- $x \cdot \bar{x} = 0$
- $x + xy = x(1+y) = x$

DATE PAGE

$A + B = B + A$

$AB = BA$

$A + (B + C) = (A + B) + C$

$A \cdot (B + C) = AB + AC$

$A + BC = (A + B)(A + C)$

De Morgan's Law

$\overline{(A + B)} = \overline{A} \cdot \overline{B}$

$\overline{A \cdot B} = \overline{A} + \overline{B}$

– Dual of an Expression

$A + \overline{A} = 1 \xleftrightarrow{\text{dual}} A \cdot \overline{A} = 0$

For dual change →

$+ \leftrightarrow \cdot$

$1 \leftrightarrow 0$

$A + \overline{A}B = (A + \overline{A})(A + B)$

$= A + B$

$A \cdot (\overline{A} + B) = A\overline{A} + AB = AB$ dual

05/10/21 <u>Input / Output</u>

Variable → {n-bit → 2^n combinations of input for output

For example ~~2-bit~~ 2bit variables → $2^2 = 4$ combinations for output

Half Adder table →

A	B	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

To make circuits (from design perspective) -

1) Identify the number of input & output
2) Write output for every combo (make truth table)
3) Use gates to derive it.

— Solving / Simplifying Boolean Expression

Half Adder →

	A	B	C
0	0	0	0
1	0	0	1
2	0	1	0
3	0	1	1
4	1	0	0
5	1	0	1
6	1	1	0
7	1	1	1

Repeats same pattern

SOP → ($0 = \overline{A}, \overline{B}$; $1 = A, B$)
POS → ($0 = A, B$; $1 = \overline{A}, \overline{B}$)

A	B	S	SOP	POS
0	0	0	$\overline{A}.\overline{B}$	$A+B$
0	1	1	$\overline{A}.B$	$A+\overline{B}$
1	0	1	$A.\overline{B}$	$\overline{A}+B$
1	1	0	$A.B$	$\overline{A}+\overline{B}$

SOP → ~~[illegible]~~ min terms (m)

=> collect all ones

=> $(\overline{A}B + A\overline{B}) = y$ (function)

=> $\Sigma(1, 2)$

POS → maxterms (M)
⇒ collect 0's
⇒ $(A+B)\cdot(\bar{A}+\bar{B}) = y$ (function)
⇒ ~~Σ~~ $\Pi(0,3)$

- Canonical & Standard form conversion

Q $Y = AB + B\bar{C}$ ⇒ $\Sigma(2,6,7)$
(standard) (canonical)

To convert $AB + B\bar{C}$ to $\Sigma(2,6,7)$
we need C in $AB + B\bar{C}$. (multiply 1)
$\therefore$ $AB(C+\bar{C}) + (A+\bar{A})(B\bar{C})$
$= ABC + AB\bar{C} + AB\bar{C} + \bar{A}B\bar{C}$
$= ABC + AB\bar{C} + \bar{A}B\bar{C}$
$\quad\;\; 7 \qquad\;\; 6 \qquad\;\; 2$
$= \Sigma(2,6,7)$

Q $Y = (A+B)\cdot(\bar{B}+C)$ ⇒ $\pi(0,1,2,6)$
To convert add 0

$(A+B+C\bar{C})\;(A\bar{A}+\bar{B}+C)$
$= (A+B+C)\cdot(A+B+\bar{C})\cdot(A+\bar{B}+C)\;(\bar{A}+\bar{B}+C)$
~~A+B+C~~ $\quad 0 \qquad 1 \qquad 2 \qquad 6$
$= \pi(0,1,2,6)$

Q Convert (i) $AB + \bar{B}C$ to POS
(ii) $(A+B)(B+\bar{C})$ to SOP

(i) $AB + \bar{B}C$ ⇒ POS
$AB + \bar{B}C = \Sigma(2,6,7)$ ~~$= \pi(0,1,3,$~~

DATE
PAGE

$= \pi(0, 1, 3, 4, 5)$

(ii) $(A+B)(B+\bar{C}) \Rightarrow$ SOP

$= (A+B+C\bar{C})\ (A\bar{A}+B+\bar{C})$

$= (A+B+C).(A+B+\bar{C}).(A+B+\bar{C})(\bar{A}+B+\bar{C})$

$\quad\ 0 \qquad\qquad 1 \qquad\qquad 1 \qquad\qquad 5$

$= \pi(0, 1, 5) = \Sigma(2, 3, 4, 6, 7)$

Complement ($\bar{\ }$) of a function

$X = AB + BC$

$\bar{X} = \overline{AB + BC} = \cancel{(\overline{AB})(\overline{BC})}\quad \overline{AB} + \overline{BC}$

$= (\bar{A}+\bar{B}).(\bar{B}+\bar{C})$

$= \bar{A}\bar{B} + \bar{B} + \bar{A}\bar{C} + \bar{B}\bar{C}$

$= \bar{B}(\bar{A}+1) + \bar{A}\bar{C} + \bar{B}\bar{C}$

$= \bar{B} + \bar{B}\bar{C} + \bar{A}\bar{C} = \bar{B} + \bar{A}\bar{C}$

To write complement of an expression, write the dual and complement each term.

$X = AB + BC$

Dual of $X = (A+B).(B+C)$

$= AB + B + BC + AC$

complement each term $\Rightarrow \bar{A}\bar{B} + \bar{B} + \bar{B}\bar{C} + \bar{A}\bar{C}$

$\therefore$ Complement of $(AB+BC) = \bar{A}\bar{B} + \bar{B} + \bar{B}\bar{C} + \bar{A}\bar{C}$

$= \cancel{\bar{B}\bar{C}\bar{A}}$

$= \bar{B} + \bar{A}\bar{C}$

Q Complement of $y = \cancel{ABC}\ AB\bar{C} + \bar{A}BC + ABC$

Dual of $y = (A+B+\bar{C})(B+\bar{A}+C)(A+B+C)$

$= (AB + \cancel{A\bar{A}} + AC + B + \bar{A}B + BC + B\bar{C} + \bar{A}\bar{C} + \cancel{C\bar{C}})(A+B+C)$

$= \cancel{AB} + AB\cancel{C} + \cancel{AC} + A\cancel{BC} + A\cancel{B} + B + \cancel{BC} + \cancel{\bar{A}B} + \bar{A}BC$
$AB\bar{C} + \cancel{B\bar{C}} + \bar{A}B\bar{C}$

$= AB + AC + BC + \bar{A}B + B\bar{C} + ABC + \bar{A}BC$
$+ \bar{A}B\bar{C} + AB\bar{C} + B$

Complement of $= \cancel{\bar{A}\bar{B}} + \bar{A}\bar{C} + \cancel{\bar{B}\bar{C}} + \cancel{A\bar{B}} + \cancel{\bar{B}C} + \cancel{\bar{A}\bar{B}}$
$+ A\bar{B}\bar{C} + A\bar{B}C + \bar{A}\bar{B}C$

$= \bar{B} + \bar{A}\bar{C} + A\bar{B}\bar{C} + A\bar{B}C$

Complement of $y = \bar{B} + \bar{A}\bar{C} + A\bar{B}C + A\bar{B}\bar{C}$

6/10/21 ✿ SOP is used when too much product exists & POS is used when too much sum exists.

✿ Consensus Theorem

$$\boxed{xy + x'z + yz = xy + x'z}$$

Proof:-

$xy + x'z + (x+x')yz = xy + x'z + xyz + x'yz$
$= xy(1+z) + x'z(1+y)$
$= xy + x'z$

Q Simplify $xy' + x'z + y'z$

$$xy' + x'z + y'z = xy' + x'z \quad \text{(by CT)}$$

Q Derive dual of consensus theorem

$xy + x'z$

$$\Rightarrow xy + x'z + yz \xrightarrow{dual} (x+y)(x'+z)(y+z)$$

dual changed position of z & y.

$$= (x+y)(x'+z)(y+z+xx')$$

$$= (xx' + xz + x'y + yz)(y+z+x)(y+z+x')$$

$$= (xz + x'y)(y + yz + yx' + z + zx' + xy + xz + \cancel{xx'})$$

$$= (xz + x'y)(z + y + xz + x'y + zx' + xy)$$

$$= (xz + x'y)(z + y)$$

$$= xz + xyz + x'yz + x'y$$

$$= xz + x'y + yz$$

$$\boxed{= xz + x'y}$$

Karnaugh Map (K-map)

2 Variable →

A \ B	$\bar{B}$	B
$\bar{A}$	$\bar{A}\bar{B}$ 0	$\bar{A}B$ 1
A	$A\bar{B}$ 2	AB 3

B \ A	$\bar{A}$	A
$\bar{B}$	$\bar{A}\bar{B}$ 0	$A\bar{B}$ 2
B	$\bar{A}B$ 1	AB 3

3 Variable →

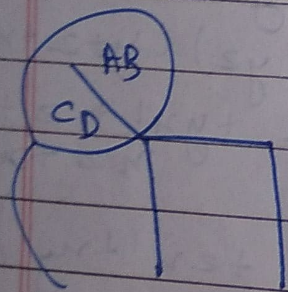
AB \ C	$\bar{C}$	C
$\bar{A}\bar{B}$	000 0	001 1
$\bar{A}B$	010 2	011 3
AB	110 6	111 7
$A\bar{B}$	100 4	101 5

A \ BC	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	000 0	001 1	011 3	010 2
A	100 4	101 5	111 7	110 6

* **Grey Code** - Every successive number differs by 1 bit (Ordering of binary numbers)

Binary	Gray Code
00	00
01	01
10	11
11	10

4 Variable -



AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	1	3	2
$\bar{A}B$	4	5	7	6
AB	12	13	15	14
$A\bar{B}$	8	9	11	10

Rows and columns interchange

BD \ AC	$\bar{A}\bar{C}$	$\bar{A}C$	AC	$A\bar{C}$
$\bar{B}\bar{D}$	0	2	10	8
$\bar{B}D$	1	3	11	9
BD	5	7	15	13
$B\bar{D}$	4	6	14	12

DATE
PAGE

5 Variable -

For 5 input variables, there will be $2^5 = 32$ combinations.

There could be 8x4, 4x8 tables or we can make two maps, put A = 0 and make a 4x4 table and put A = 1 and make a 4x4 table which will give us ~~simi~~ the same expression as 8x4 or 4x8.

BC \ DE	$\bar{D}\bar{E}$	$\bar{D}E$	DE	$D\bar{E}$
$\bar{B}\bar{C}$	0	1	3	2
$\bar{B}C$	4	5	7	6
BC	12	13	15	14
$B\bar{C}$	8	9	11	10

A = 0

BC \ DE	$\bar{D}\bar{E}$	$\bar{D}E$	DE	$D\bar{E}$
$\bar{B}\bar{C}$	16	17	19	18
$\bar{B}C$	20	21	23	22
BC	28	29	31	30
$B\bar{C}$	24	25	27	26

A = 1

We can do this because in first 16 entries A is 0 and 1 in next 16.

Grouping

We make pairs, quadrants, octets....

Only horizontal and vertical groups are allowed. No diagonal grouping.

Make largest possible groups.

Groups of 2^n reduces to n variable

* Prime Implicants are elements covered only once in grouping. These can't be removed. Groups which have prime implicants can't be removed.

Ex- Map $\Sigma(1, 2, 4, 5, 10, 11, 14, 15)$ (SOP expression)

pi

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0_{0}	1_{1}	0_{3}	1_{2}
$\bar{A}B$	1_{4}	1_{5}	0_{7}	0_{6}
AB	0_{12}	0_{13}	1_{15}	1_{14}
$A\bar{B}$	0_{8}	0_{9}	1_{11}	1_{10}

pi

pi

The map is not rectangle, it's like toroid

$\Rightarrow \bar{A}B\bar{C} + \bar{A}\bar{C}D + AC + \bar{B}C\bar{D}$

$\bar{A} \rightarrow A$ vanishes

Don't Care Condition

Q $Y = \Sigma(0, 2, 5) + d(6, 7)$. Solve.

	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	1_{0}	0_{1}	0_{3}	1_{2}
A	0_{4}	1_{5}	$1 = X_{7}$	X_{6}

don't care condition means the circuit doesn't get affected by 0 or 1 at these places. So we can put 0 or 1 according to the conveinence of grouping

To make pair, we assume x at 7 = 1

$\therefore \Rightarrow \bar{A}\bar{C} + AC$

A \ B	$\bar{B}$	B
$\bar{A}$	0_{0}	1_{1}
A	1_{2}	0_{3}

$\Sigma(1, 2)$

$\Rightarrow \bar{A}B + A\bar{B} = A \oplus B$

Diagonal elements can be implemented using XOR or XNOR gate.