Assignment-3

Engineering Mechanics, Unit 3.

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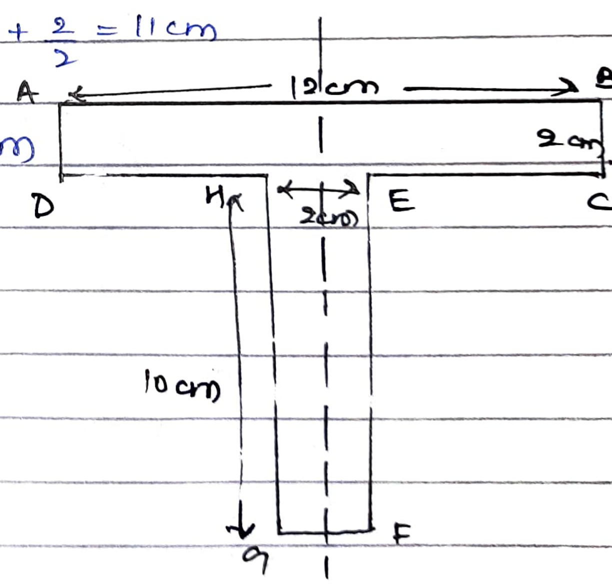
1) Find the centre of gravity of the T section.

The given section splits up into two rectangles ABCD and EFGH. It is symmetrical about Y-Y axis. Hence, the C.G. of the section will lie on the axis.

$\bar{y}$ = C.G. of the T section from G.F.
a_1 = Ar (ABCD) $= 12 \times 2 = 24\,cm^2$.
y_1 = C.G. of the area a_1 from GF $= 10 + \frac{2}{2} = 11\,cm$
a_2 = Ar (EFGH) $= 10 \times 2 = 20\,cm^2$
y_2 = C.G. of area a_2 from GF $= \frac{10}{2} = 5\,cm$



Now,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

$$\bar{y} = \frac{24 \times 11 + 20 \times 5}{24 + 20}$$

$$\bar{y} = 8.272\,cm.$$

2) Find centre of gravity of the I section.

The given I section splits into 3 rectangles ABCD, EFGH, IJKL. It is symmetrical about Y-Y axis.
Reference axis is taken as LK.

$\bar{y}$ = C.G. of the I- section from LK.

a_1 = area of ABCD = $8 \times 2 = 16 cm^2$

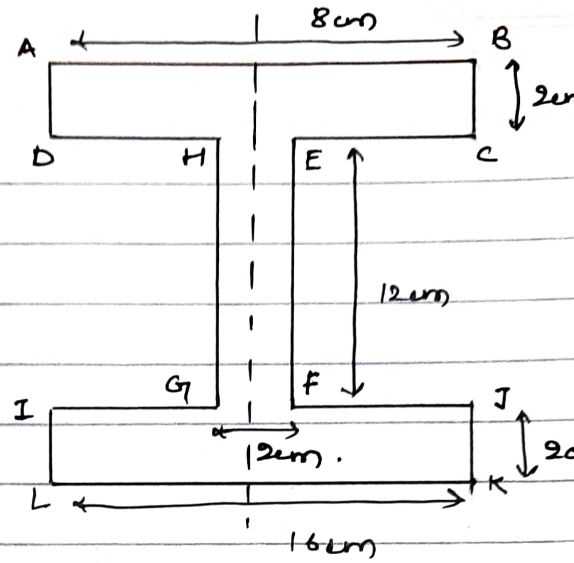
y_1 = C.G of a_1 from LK = $2 + 12 + \frac{2}{2} = 15 cm$

a_2 = area of EFGH = $2 \times 12 = 24 cm^2$

y_2 = C.G. of a_2 from LK = $2 + \frac{12}{2} = 8 cm$

a_3 = area of IJKL = $2 \times 16 = 32 cm^2$

y_3 = C.G. of a_3 from LK = $\frac{2}{2} = 1 cm$.



Now, $$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3} = \frac{16 \times 15 + 24 \times 8 + 32 \times 1}{16 + 24 + 32}$$

$$\bar{y} = 6.44 cm$$

3) Find the centre of gravity of the L- sections.

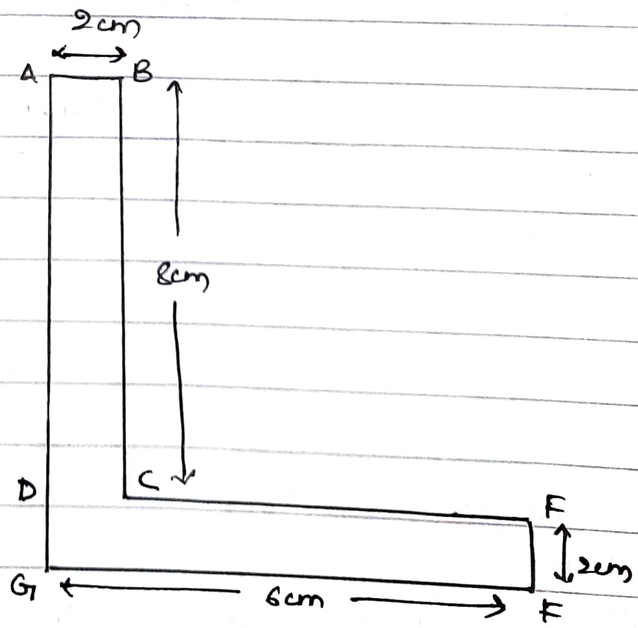
Given L- section is not symmetrical, Hence, there will be two axis of references. G.F. is reference axis for calculating $\bar{y}$ and AG for calculating $\bar{x}$. L section splits into two rectangles ABCD and DEFG.

a_1 = area of ABCD = $2 \times 8 = 16 cm^2$

y_1 = C.G of a_1 from GF = $2 + \frac{8}{2} = 6 cm$

a_2 = area of DEFG = $2 \times 6 = 12 cm^2$

y_2 = C.G of a_2 from G.F. = $\frac{2}{2} = 1 cm$



Now,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{16 \times 6 + 12 \times 1}{16 + 12}$$

$$\bar{y} = \frac{96 + 12}{16 + 12}$$

$$\bar{y} = 3.857 cm.$$

To find $\bar{x}$,

x_1 = C.G. of ABCD from AG = $\frac{2}{2} = 1 cm$

x_2 = C.G. of DEFG from AG = $\frac{6}{2} = 3 cm$

$$\bar{x} = \frac{a_1x_1 + a_2x_2}{a_1+a_2} = \frac{16\times 1 + 12\times 2}{16+12}.$$

$\bar{x} = 1.857\,cm.$

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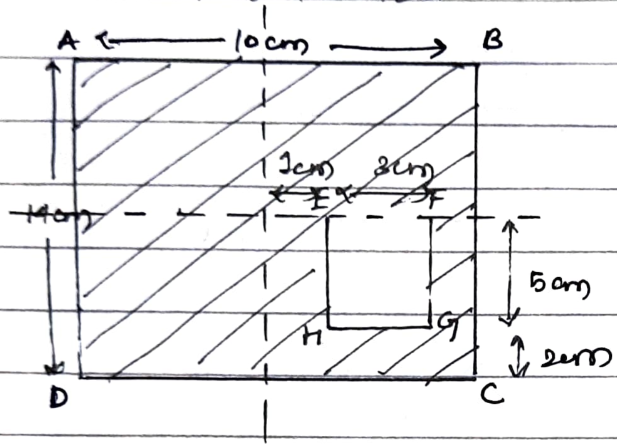
4) from rectangular lamina ABCD a rectangular pole is cut. find C.G. of remainder lamina.



Let $\bar{y}$ be the C.G. of the remained system.

$a_1 =$ are(ABCD) $= 14\times 10 = 140\,cm^2$

$y_1 =$ C.G. of ABCD from DC $= \frac{14}{2} = 7\,cm$

$a_2 =$ are(EFGH) $= 3\times 5 = 15\,cm^2$

$y_2 =$ C.G. of EFGH from DC $= 2 + \frac{5}{2} = 4.5\,cm.$

Now,

$$\bar{y} = \frac{a_1y_1 + a_2y_2}{a_1+a_2} = \frac{140\times 7 - 15\times 4.5}{140-15}$$

$$\bar{y} = 7.3\,cm$$

To find $\bar{x}$,

$x_1 =$ C.G. of ABCD from AD $= \frac{10}{2} = 5\,cm$

$x_2 =$ C.G. of EFGH from AD $= \cancel{4}5 + 1 + \frac{3}{2} = 7.5\,cm$

$$\bar{x} = \frac{a_1x_1 - a_2x_2}{a_1 - a_2} = \frac{140\times 5 - 15\times 7.5}{140-15}$$

$\bar{x} = 4.7\,cm.$

5) for the T- system, determine the moment of inertia of the section about the horizontal and vertical axis passing through C.G. of the section.

The given section is symmitrical about y-axis Hence C.G of the section will lie on Y-Y axis.

Given section splits up in the two rectangles ABCD and EFGH.

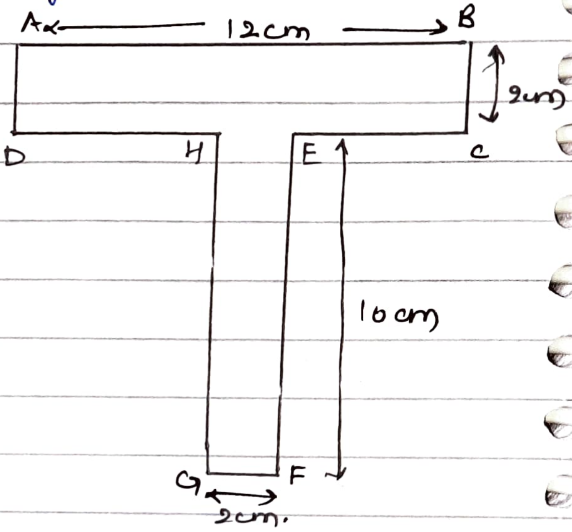
$\bar{y}$ = C.G. of the section from the G.F.

$a_1 = ar(ABCD) = 12\times2 = 24\,cm^2$

y_1 = C.G of the a_2 from line GF $= 10+\frac{2}{2} = 11\,cm$

$a_2 = ar(EFGH) = 10\times2 = 20\,cm^2$

y_2 = C.G. of area a_2 from line G.F $= 10/2 = 5\,cm$.



Now,

$$\bar{y} = \frac{a_1y_1+a_2y_2}{a_1+a_2} = \frac{24\times11+20\times5}{24+20}$$

$$\bar{y} = 8.272\,cm.$$

for moment of inertia of section T,

IG_1 = moment of inertia of are 1 about horizontal axis passing through its C.G.

IG_2 = moment of inertia of are 2 about horizontal axis passing through its C.G.

h_2 = distance b/w C.G. of T section and C.G. of ar 2

$= \bar{y}-y_2 = 8.272-5$

$= 3.272\,cm$

h_1 = distance b/w C.G. of T-section and C.G. of ar 1

$= y_1-\bar{y} = 11-8.277$

$= 2.728\,cm.$

$$IG_1 = \frac{bd^3}{12} = \frac{\cancel{12}\times2^3}{\cancel{12}} = 8\,cm^4$$

$$IG_2 = \frac{bd^3}{12} = \frac{2\times10^3}{12} = 166.67\,cm^4.$$

from the theorem of parallel axis, the moment of inertia of rectangle ABCD about horizontal axis passing through C.G. of the given section is —

$$= I_{G_1} + a_1 h_1^2$$

$$= 8 + 24 \times (2.728)^2$$

$$= 186.56 \text{ cm}^4.$$

Similarly, moment of inertia of area EFGH about horizontal axis passing through C.G. of the given section is —

$$= I_{G_2} + a_2 h_2^2$$

$$= 166.67 + 20 \times (3.272)^2$$

$$= 380.87 \text{ cm}^4.$$

∴ The moment of inertia of the given section but the horizontal axis passing through C.G. of the section —

$$= 186.56 + 380.87$$

$$= 567.43 \text{ cm}^4.$$

The moment of inertia of the given section about vertical axis passing through the C.G. of the section -

$$= \frac{d_1 b_1^3}{12} + \frac{d_2 b_2^3}{12}$$

$$= \frac{2 \times 12^3}{12} + \frac{10 \times 2^3}{12}$$

$$= 294.67 \text{ cm}^4.$$

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6) For the I section, find the moment of inertia about centeroid axis X-X per pendicular to the web.

The given system is symmetrical about Y-Y axis. Hence, ~~the~~ C.G. of the section will lie on Y-axis.

The given section splits up into three rectangles ABCD, EFGH and KLMN.

Now,

y_1 = C.G. of section 1 from line ML = $2 + 12 + \frac{2}{2} = 15$ cm

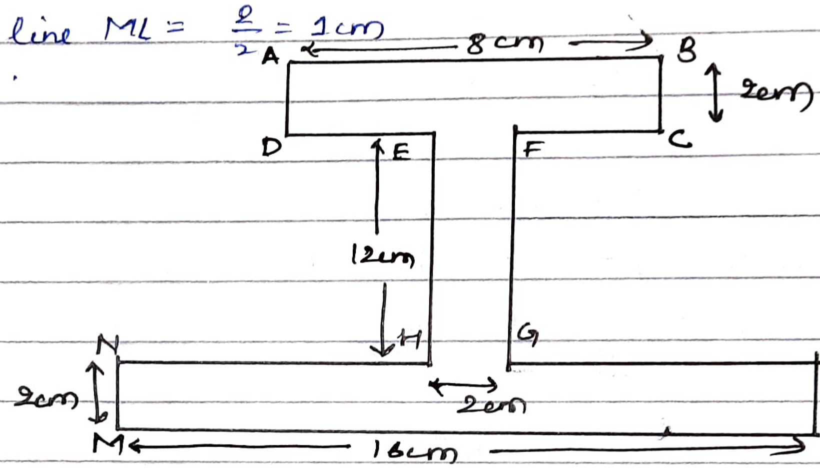
a_1 = ar(ABCD) = $8 \times 2 = 16\text{cm}^2$.

y_2 = C.G. of section 2 from line ML = $2 + \frac{12}{2} = 8$cm.

a_2 = ar(EFGH) = $2 \times 12 = 24\text{cm}^2$

y_3 = C.G. of section 3 from line ML = $\frac{2}{2} = 1$cm

a_3 = ar(KLMN) = $16 \times 2 = 32\text{cm}^2$.



$$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1 + a_2 + a_3}$$

$$\bar{y} = \frac{16 \times 15 + 24 \times 8 + 32 \times 1}{16 + 24 + 32}$$

$\bar{y}$ = 6.44cm

∴ C.G. of the given section lies at a distance 6.44cm cm from line ML.

for calculating moment of inertia of given section

I_{G_1} = moment of inertia of ar 1 about the horizontal axis passing through its C.G.

I_{G_2} = M.I. of ar 2 about the horizontal axis passing through its C.G.

I_{G_3} = M.I. of area 3 about the horizontal axis passing through its C.G.

h_1 = Distance b/w C.G. of ar 1 and C.G. of the given system.

$y_1 - \bar{y}$ = 15 − 6.44 = 8.56cm.

h_2 = Distance b/w C.G. of ar 2 and C.G. of given section

$y_2 - \bar{y}$ = (8 − 6.44)cm = 1.56 cm.

h_3 = distance b/w C.G of ar 3 and C.G of given section

$\bar{y} - y_3$ = 6.44 − 1 = 5.44 cm.

Now,

$$I_{G_1} = \frac{b_1 d_1^3}{12} = \frac{8 \times 2^3}{12} = 5.33\ cm^4$$

$$I_{G_2} = \frac{b_2 d_2^3}{12} = \frac{12^3 \times 2}{12} = 288\ cm^4$$

$$I_{G_3} = \frac{b_3 d_3^3}{12} = \frac{16 \times 2^3}{12} = 10.67\ cm^4.$$

From the theorem of ∥ axis moment of inertia of rectangle 1 about horizontal axis passing through C.G. of section $= I_{G_1} + a_1 h_1^2$

$= 5.33 + 16 \times (8.56)^2$

$= 1177.7\ cm^4$

Similarily,

moment of inertia of rectangle 2 $= I_{G_2} + a_2 h_2^2$

$= 346.32\ cm^4$

moment of inertia of rectangle 3 $= I_{G_3} + a_3 h_3^2$

$= 957.55\ cm^4.$

Now,

M.I. of sections $= 1177.7 + 346.32 + 957.55$

$= 2481.57\ cm^4.$

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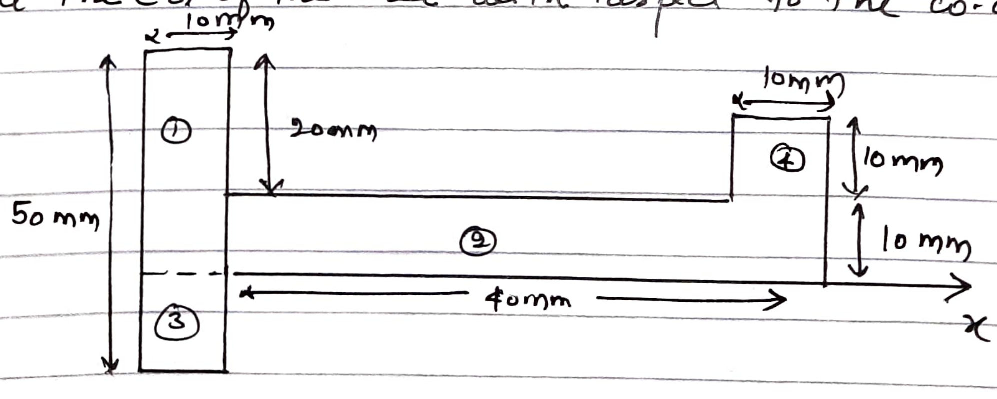
7) Locate the C.G of the area with respect to the co-ordinate axes.



There are two axis of references OX for calculating $\bar{y}$ and OY for calculating $\bar{x}$.

The given area splits up into 4 rectangles.

For calculating $\bar{y}$,

y_1 = C.G. of area 1 from OX = $30/2 = 15mm$

a_1 = area of region 1 = $30 \times 10 = 300mm^2$

y_2 = C.G. of area 2 from OX = $10/2 = 5mm$

a_2 = area of region 2 = $10 \times 40 = 400mm^2$

y_3 = C.G of area 3 from OX = $-20/2 = -10mm$

a_3 = area of region 3 = $10 \times 20 = 200m^2$

y_4 = C.G. of area 4 from OX = $10 + \frac{10}{2} = 15mm$

a_4 = area of region 4 = $10 \times 10 = 100m^2$.

Now

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3 + a_4 y_4}{a_1 + a_2 + a_3 + a_4}.$$

$$\bar{y} = \frac{300 \times 15 + 400 \times 5 + 200(-10) + 100 \times 15}{300 + 400 + 200 + 100}$$

$$\bar{y} = 6mm$$

for calculating $\bar{x}$,

x_1 = C.G. of ar 1 from OY = $10/2 = 5mm$

x_2 = C.G. of ar 2 from OY = $10 + \frac{40}{2} = 30mm$

x_3 = C.G. of ar 3 from OY = $10/2 = 5mm$

x_4 = C.G. of ar 4 from OY = $10 + 30 + 10/2 = 45mm$

So,

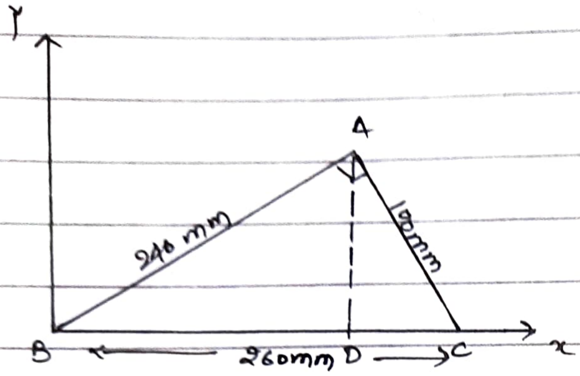
$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3 + a_4 x_4}{a_1 + a_2 + a_3 + a_4}$$

$$\bar{x} = \frac{300 \times 5 + 400 \times 30 + 200 \times 5 + 45 \times 100}{300 + 400 + 200 + 100}$$

$$\bar{x} = 19mm.$$

$\therefore$ coordinate of C.G = (19, 6).

8) A thin homogeneous wire is bent into a triangular shape ABC such that AB = 240 mm. Locate C.G.



AB = 240 mm.

The wire ABC splits into two wires AB and AC. BX is the reference axis to determine $\bar{y}$ and BY be the reference axis for determining $\bar{x}$.

Applying sine rule in ΔABC,

$$\frac{BC}{\sin 90^\circ} = \frac{AC}{\sin\alpha} = \frac{AB}{\sin B}$$

$$\frac{260}{1} = \frac{100}{\sin\alpha} = \frac{240}{\sin\beta}$$

$\sin\alpha = \frac{100}{260} = \frac{5}{13}$ and $\sin\beta = \frac{\cancel{240}^{12}}{\cancel{260}_{13}} = \frac{12}{13}$

$\alpha = \sin^{-1}\frac{5}{13}$ $\qquad$ $\beta = \sin^{-1}\frac{12}{13}$

for calculating $\bar{y}$,

y_1 = C.G. of wire AB measured from BX $= \frac{240}{2} \times \sin\alpha$

$= 46.15$ mm

L_1 = length of wire AB = 240 mm

y_2 = C.G. of AC measured from BX = $\frac{100}{2}\sin\beta = 46.154$

L_2 = length of wire AC = 100 mm

y_3 = C.G. of wire AC measured from BX = 0 mm

L_3 = length of wire BC = 260 mm

Now, $\bar{y} = \frac{L_1y_1 + L_2y_2 + L_3y_3}{L_1 + L_2 + L_3} = \frac{240 \times 46.154 + 100 \times 46.154 + 0 \times 260}{240 + 100 + 260}$

$\bar{y} = 26.154$ mm

for calculating $\bar{x}$,

x_1 = C.G. of wire AB from BY axis

$= \frac{240}{2} \times \cos\alpha = 110.769\,mm.$

x_2 = C.G of wire AC from BY axis = BD + CD

$= 240\cos\alpha + 100/2 \cos\beta = 240.77\,mm.$

x_3 = C.G. of wire BC from BY axis

$= \frac{260}{2} = 130\,mm.$

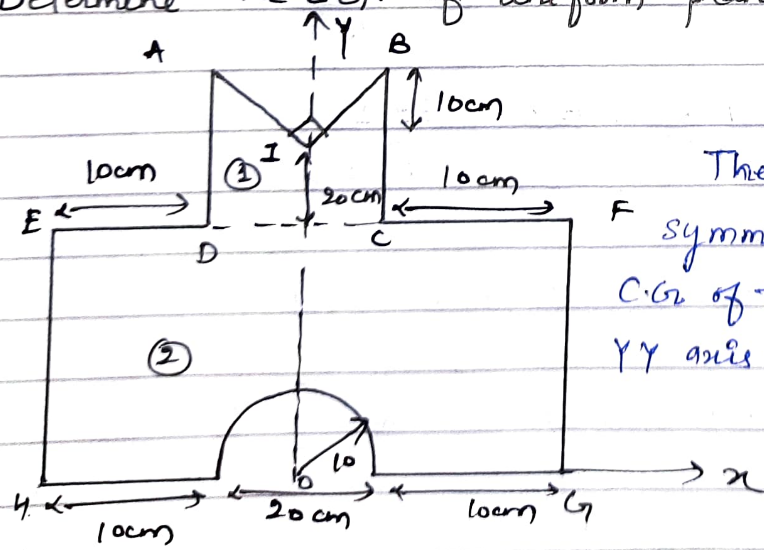
Now,

$$\bar{x} = \frac{x_1 L_1 + x_2 L_2 + x_3 L_3}{L_1 + L_2 + L_3}$$

$$\bar{x} = \frac{110.769 \times 240 + 240.77 \times 100 + 260 \times 130}{240 + 260 + 100}$$

$$\bar{x} = 140.77\,mm$$

9) Determine the C.G. of uniform plane lamina shown below.



The given plane lamina is symmetric about Y-Y axis Hence C.G of the lamina will lie on YY axis.

The C.O.G. of the given section is determined by first as complete one and then sub traiting the area of the cut-out hole.

for calculating $\bar{y}$.

$\bar{y}$ = C.G. of the given lamina with cut-out hole at region-3 and 4.

y_1 = C.G. of the region 1 from base OX. $= 30 + \frac{30}{2} = 45$ cm

$a_1 = 30 \times 20 = 600\ cm^2$

y_2 = C.G of the region 2 from base OX $= \frac{30}{2} = 15$ cm.

$a_2 = 40 \times 30 = 1200\ cm^2$

y_3 = C.G. of region 3 from base OX $= \frac{4r}{3\pi} = \frac{4 \times 10}{3\pi} = \frac{40}{3\pi}$ cm.

$a_3 = -\frac{\pi r^2}{2} = -\frac{\pi (10)^2}{2} = -50\pi\ cm^2$

y_4 = C.G. of region 4 from base OX $= 30 + 30 - \frac{h}{3} = 60 - \frac{10}{3} = \frac{170}{3}$ cm.

$a_4 = -\frac{1}{2} \times 20 \times 10 = -100\ cm^2$.

Now $\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3 + a_4 y_4}{a_1 + a_2 + a_3 + a_4}$

$\bar{y} = 25.06$ cm.

C.G. of given plane lamina is at a distance of 25.06 cm from point O on y axis.

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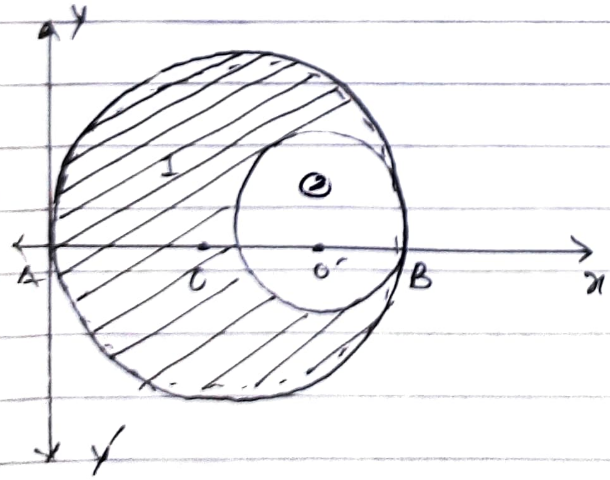
10) From a circular plate of diameter 100 mm, a circular part of diameter 50 mm is cut, find centroid of remainder.



AB = 100 mm, OA = 50 mm

OB = 50 mm, OB = 25 mm.

The given circular plate is symmetrical about x-axis Hence the centeroid lies on x. axis

The C.G of given circular plate is determined first by complete plate and then substracting the area of the cut-out circular plate.

The reference axis is taken as Y axis. for calculating $\bar{x}$.

x_1 = C.G. of plate 1 measured from Y axis $= \frac{100}{2} = 50mm$.

a_1 = area of plate 1 $= \pi r^2 = 2500\pi \ mm^2$

x_2 = C.G. of cut out plate $= 75mm$

a_2 = area of plate $= -\pi (25)^2 = mm^2$.

Now,

$$\bar{x} = \frac{a_1x_1 - a_2x_2}{a_1 - a_2} = \frac{\pi(50)^2 \times 50 - \pi(25)^2 \times 75}{\pi(50)^2 - \pi(25)^2}$$

$$\bar{x} = 41.67mm.$$

Centeroid lies at a distance of 41.67 mm from point A on the x-axis.

Coordinates are $(41.67, 0)$.