

The Supplementary Materials

I. PROOF OF THE CALCULATION OF Θ

Proof. We first list the following axioms that are assumed to be true.

- $\pi \models \neg\mu \implies \pi \not\models \mu$
- if $\Phi = x \wedge y, \pi \models \neg x \implies \pi \not\models \Phi, \pi \models \neg y \implies \pi \not\models \Phi$
- if $\Phi = x \vee y, \pi \models (\neg x \wedge \neg y) \implies \pi \not\models \Phi$
- $N(\neg\theta_1) = \Theta(\theta_1), \Theta(\neg\theta_1) = N(\theta_1)$

If Φ is μ , we have $\pi \models \neg\mu \implies \pi \not\models \mu$, so $\Theta(\mu) = \{\neg\mu\}$ is reasonable.

If Φ is $x \wedge y$, we have $\pi \models \neg x \implies \pi \not\models \Phi, \pi \models \neg y \implies \pi \not\models \Phi$, so $\Theta(x \wedge y) = \Theta(x) \cup \Theta(y)$ is reasonable.

If Φ is $x \vee y$, we have $\pi \models (\neg x \wedge \neg y) \implies \pi \not\models \Phi$, so $\Theta(x \vee y) = \{x \wedge y \mid x \in \Theta(a) \wedge y \in \Theta(b)\}$ is reasonable.

If Φ is $\bigcirc x$, we have $(\pi, t) \models \bigcirc\Phi \iff (\pi, t+1) \models \Phi$. Given $\pi \models \neg\mu \implies \pi \not\models \mu$, we can get $(\pi, t) \models \bigcirc\neg\Phi \iff (\pi, t+1) \not\models \Phi$. Hence, $\Theta(\bigcirc x) = \{\bigcirc x' \mid x' \in \Theta(x)\}$ is reasonable.

If Φ is $x \mathcal{U}_{\mathcal{I}} y$, in our definition, there are two parts of $\Theta(x \mathcal{U}_{\mathcal{I}} y)$. The first part is set $\Theta_1: \{x' \wedge y' \mid x' \in \Theta(x) \wedge y' \in \Theta(y)\}$ which implies these equations are satisfies: $\pi \models x' \implies \pi \not\models x, \pi \models y' \implies \pi \not\models y$. Given the definition of $\mathcal{U}_{\mathcal{I}}$: $(\pi, t) \models x \mathcal{U}_{\mathcal{I}} y \iff \exists t' \in t+\mathcal{I}$ such that $(\pi, t') \models y \wedge \forall t'' \in [t, t'], (x, t'') \models x$, we can easily get:

$$\forall \xi \in \Theta_1. \pi \models \xi \implies \pi \not\models x \mathcal{U}_{\mathcal{I}} y$$

The second part is set $\Theta_2: \{x' \mathcal{U}_{\mathcal{I}} y' \mid x' \in \Theta(\neg x \vee y) \wedge y' \in \Theta(x \vee y)\}$. In order to obtain a contradiction, assume that there is an element ξ of set Θ_2 that satisfies $\pi \models \xi \implies \pi \models \Phi$. Then, for $\pi \models \Phi$, we get $\exists t' \in t+\mathcal{I}$ such that $(\pi, t') \models y \wedge \forall t'' \in [t, t'], (x, t'') \models x$, which means $(x \wedge \neg y)$ is satisfied until y is satisfied. For $\pi \models \xi$, we get $\exists t' \in t+\mathcal{I}$ such that $(\pi, t') \models (\neg x \wedge \neg y) \wedge \forall t'' \in [t, t'], (x, t'') \models (x \wedge \neg y)$, which means $(x \wedge \neg y)$ is satisfied until $(\neg x \wedge \neg y)$ is satisfied. Hence, at time step t , if x is satisfied in $[t, t']$ and violated at time step t'' after t' , we should get that y is satisfied before t'' and y is violated before t'' at the same time. This is a contradiction, and so the assumption that there is an element ξ of set Θ_2 satisfies $\pi \models \xi \implies \pi \models \Phi$ must be false. We can get:

$$\forall \xi \in \Theta_2. \pi \models \xi \implies \pi \not\models x \mathcal{U}_{\mathcal{I}} y$$

Hence, $\Theta(x \mathcal{U}_{\mathcal{I}} y) = \{x' \mathcal{U}_{\mathcal{I}} y' \mid x' \in \Theta(\neg x \vee y) \wedge y' \in \Theta(x \vee y)\} \cup \{x' \wedge y' \mid x' \in \Theta(x) \wedge y' \in \Theta(y)\}$ is reasonable.

Since the temporal operators $\mathcal{U}_{\mathcal{I}}$ and \bigcirc are functionally complete, we omit the proof of the remaining temporal operators. \square

TABLE I: Road Traffic Rules of Singapore

Rules	Content	D	Reason
Rule 1	Definitions	×	Irrelevant
Rule 2	Give Signals	✓	–
Rule 3	Drive on Left	✓	–
Rule 4	Overtake	✓	–
Rule 5	Left-hand	✓	–
Rule 6	No Obstruction	✓	–
Rule 7	Lane Direction	✓	–
Rule 8	Right Turn	✓	–
Rule 9	Left Turn	✓	–
Rule 10	No Stop Area	✓	–
Rule 11	Uncontrolled Area	✓	–
Rule 12	Priority	✓	–
Rule 13	U-turn	✓	–
Rule 14-21	Irrelevant	×	Irrelevant
Rule 22	Park	✓	–
Rule 23	Repair on Road	×	Irrelevant
Rule 24	No Stop Area	✓	–
Rule 25	Load Goods	×	Irrelevant
Rule 26	No Obstruction	✓	–
Rule 27	Road Works	×	Irrelevant
Rule 28	Footway	✓	–
Rule 29-37	Irrelevant	×	Irrelevant

II. EXTENSION TO LAWS OF OTHER COUNTRIES

LawBreaker can also describe traffic laws of other countries. We go through the *Road Traffic Rules of Singapore* [1] to illustrate this. The results are summarised in Table I. In the table, *Content* briefly summarises the content of the specific traffic rule, *D* checks whether the specific rule can be described by **LawBreaker** and *Reason* summarises the reason why **LawBreaker** cannot support the specific rule. As can be seen from Table I, all the relevant traffic rules can be supported by **LawBreaker**. In the following, we present one example of the translation of *Road Traffic Rules of Singapore*.

Rule6: No Obstruction. Rule6 of the *Road Traffic Rules of Singapore* stipulates that vehicles should not obstruct other vehicles when crossing a traffic light controlled intersection or junction of roads. The original text of Article #52 is shown below:

Vehicle not to obstruct other vehicles at road intersection or junction 6. Every vehicle arriving at a controlled intersection or junction of roads which does not intend to turn right or left at the intersection or junction shall, on being released, move forward straight across such intersection or junction in such a manner as not to obstruct any other vehicle proceeding in the same direction on either side of such vehicle.

Making use of **LawBreaker**, we can describe this rule in the following way:

```
1 sub1 = trafficLightAhead.color == green |  
    trafficLightAhead.direction.color == green;  
2 sub2 = direction == forward & NPCAhead.speed > 0.5;  
3 sub3 = F[0,2](speed>0.5);  
4 rule6 = G((sub1 & sub2) -> sub3);
```

The above translation describes when the traffic light ahead shows a signal to release vehicles that go forward, the ego vehicle is expected to follow the NPC vehicle ahead to avoid obstructing other vehicles. Note that the constants are examples values, which can be customized easily.

REFERENCES

- [1] Singapore Government, “Road traffic rules,” <https://sso.agc.gov.sg/SL/RTA1961-R20?DocDate=20191025>, 2021, online; accessed Nov 2021.