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## 1 Basic

### 1.1 .vimrc

```
syntax on
se ru nu ai
se ts=4 sts=4 sw=4 st=4 smarttab
inoremap {<ENTER> {}<LEFT><ENTER><ENTER><UP><TAB>
se mouse=a
se laststatus=2
"se expandtab
```

### 1.2 Check

```
for i in $(seq 1 10000);
do
    ./gen > input
    ./ac < input > out_ac
    ./wa < input > out_wa
    diff out_ac out_wa || break
done
```

### 1.3 Factor Count List

```
/*
(i, factor number of i)
10080 72, 50400 108, 110880 144,
221760 168, 332640 192, 498960 200,
554400 216, 665280 224, 720720 240,
1081080 256, 2162160 320, 3603600 360,
4324320 384, 6486480 400, 7207200 432,
8648640 448, 10810800 480, 21621600 576,
32432400 600, 43243200 672, 61261200 720,
73513440 768, 110270160 800, 245044800 1008,
367567200 1152, 551350800 1200, 698377680 1280,
735134400 1344, 1102701600 1440, 1396755360 1536
*/
```

### 1.4 Default

```
// Compile with "g++ -std=c++11 -Wall -Wextra -Wconversion -
Wshadow -Dlawfung"
#ifdef lawfung
#define debug(...) do {\
    fprintf(stderr, "%s - %d : (%s) = ", __PRETTY_FUNCTION__,
    __LINE__, #__VA_ARGS__); \
    _DO(__VA_ARGS__); \
}while(0)
template<typename I> void _DO(I&&x) {cerr << x << '\n';}
template<typename I, typename ...T> void _DO(I&&x,T&&...tail) {
    cerr << x << ", "; _DO(tail...);}
#define IOS
#else
#define debug(...)
#define IOS ios_base::sync_with_stdio(0);cin.tie(0)
#endif
```

### 1.5 Pragma

```
#pragma GCC optimize("Ofast", "no-stack-protector", "unroll-
loops")
#pragma GCC optimize("no-stack-protector")
#pragma GCC target("sse,sse2,sse3,ssse3,sse4,sse4.2,popcnt,abm,
mmx,avx,tune=native")
#pragma GCC diagnostic ignored "-W"
```

## 1.6 Random int

```
#include <random>
mt19937 rng(chrono::steady_clock::now().time_since_epoch().
    count());
int randint(int lb, int ub)
{ return uniform_int_distribution<int>(lb, ub)(rng); }
```

## 1.7 Increase Stack Size

```
const int size = 256 << 20;
register long rsp asm("rsp");
char *p = (char*)malloc(size) + size, *bak = (char*)rsp;
__asm__("movq %0, %%rsp\n":"r"(p));
// main
__asm__("movq %0, %%rsp\n":"r"(bak));
```

## 1.8 FasterIO

```
static inline char getRawChar() {
    static char buf[1 << 16], *p = buf, *end = buf;
    if (p == end) {
        if ((end = buf + fread_unlocked(buf, 1, 1 << 16, stdin)) ==
            buf) return '\0';
        p = buf;
    }
    return *p++;
}
while (c = getRawChar() && (unsigned)(c - '0') > 10U) n = n *
    10 + (c - '0');
```

# 2 Bitwise trick

## 2.1 Builtin Function

```
int __builtin_clz(unsigned int x)
int __builtin_clzll(unsigned long long x)
int __builtin_popcount(unsigned int x)
int __builtin_popcountll(unsigned long long x)
```

## 2.2 Subset Enumeration

```
int subset_enumeration(int s) {
    int now = s;
    while(now) {
        cout << now << ' ';
        now = (now - 1) & s;
    }
    cout << "\n";
}
```

## 2.3 Next Permutation on binary

```
ll next_perm(ll v) {
    ll t = v | (v - 1);
    return (t + 1) | (((~t & ~t) - 1) >> (__builtin_ctz(v) + 1))
        ;
}
```

## 2.4 SOS DP

```
// 0 is 0, 1 can be 1 or 0
for (int i = 0; i < n; ++i)
    for (int j = 0; j < (1 << n); ++j)
        if (j & (1 << i))
            a[j] += a[j ^ (1 << i)];
```

# 3 Theorem and Formula

- Pick's theorem  $A = i + \frac{b}{2} - 1$
- Laplacian matrix  $L = D - A$
- Derangement  $D_n = (n-1)(D_{n-1} + D_{n-2})$
- Möbius function  $\sum_{i|n} \mu(i) = [n=1]$
- Euler's totient function  $\sum_{i|n} \phi(i) = n$

- Inversion formula

$$f(n) = \sum_{i=0}^n \binom{n}{i} g(i), g(n) = \sum_{i=0}^n (-1)^{n-i} \binom{n}{i} f(i)$$

$$f(n) = \sum_{d|n} g(d), g(n) = \sum_{d|n} \mu\left(\frac{n}{d}\right) f(d)$$

- Sum of powers

$$\sum_{k=1}^n k^m = \frac{1}{m+1} \sum_{k=0}^m \binom{m+1}{k} B_k^+ n^{m+1-k}$$

$$\sum_{j=0}^m \binom{m+1}{j} B_j^- = 0$$

$$\text{note : } B_1^+ = -B_1^-, B_i^+ = B_i^-$$

- Cipolla's algorithm

$$\left(\frac{u}{p}\right) = u^{\frac{p-1}{2}}$$

$$1. \left(\frac{a^2 - n}{p}\right) = -1$$

$$2. x = (a + \sqrt{a^2 - n})^{\frac{p+1}{2}}$$

- High order residue

$$[d^{\frac{p-1}{n, p-1}} \equiv 1] \quad (p \text{ is odd prime and } p \nmid d)$$

- Packing and Covering

$$|\text{Maximum Independent Set}| + |\text{Minimum Vertex Cover}| = |V|$$

- König's theorem

$$|\text{Maximum matching}|(\text{easy}) = |\text{Minimum vertex cover}|$$

- Dilworth's theorem

$$\text{width} = |\text{smallest chain decomposition}| \quad (\text{vertex split and matching}) = |\text{largest antichain}| = |\text{maximum clique in Complement}| \quad (\text{easy})$$

- Mirsky's theorem

$$\text{height} = |\text{longest chain}|(\text{easy DP}) = |\text{smallest antichain decomposition}| = |\text{minimum anticlique partition}| \quad (\text{subset DP})$$

- Triangle center

$$- G : (1, 1, 1)$$

$$- O : (a^2(b^2 + c^2 - a^2),) = (\sin 2A, \sin 2B, \sin 2C)$$

$$- I : (a, b, c) = (\sin A, \sin B, \sin C)$$

$$- E : (-a, b, c) = (-\sin A, \sin B, \sin C)$$

$$- H : \left(\frac{1}{b^2 + c^2 - a^2},\right) = (\tan A, \tan B, \tan C)$$

- $\lfloor \frac{n}{i} \rfloor$  enumeration  $T_0 = 1, T_i = \lfloor \frac{n}{T_{i-1} + 1} \rfloor$

# 4 Data Structure

## 4.1 <ext/pb\_ds>

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/priority_queue.hpp>
#include <ext/rope>
using namespace __gnu_pbds;
using namespace __gnu_cxx;
using namespace std;

__gnu_pbds::priority_queue<int> pq;
__gnu_pbds::priority_queue<int>::point_iterator idx[10];
idx[0] = pq.push(1);

typedef tree<int, null_type, less<int>, rb_tree_tag,
    tree_order_statistics_node_update> TREE;
TREE name;
```

```
*name.find_by_order(0);
name.order_of_key(1);
name.insert(2);
name.delete(3);
name.split(v, b); /// value < v of a split to b
name.join(another TREE);
```

## 4.2 Unordered map hash

```
struct KeyHasher {
    size_t operator()(const Key& k) const {
        return k.first + k.second * 100000;
    }
};
typedef unordered_map<Key, int, KeyHasher> map_t;
```

## 4.3 Rope

```
#include <ext/rope>
using namespace __gnu_cxx;

rope<int> *p[N], *sz[N]; //use merge by size
int pp[N], szz[N];

int ret = p[ver]->at(x);
p[ver]->replace(x, ret);
p[0] = new rope<int>(pp, pp+n+1);
```

## 4.4 Disjoint set

```
struct DJS{
    int p[N], rk[N];
    vector<pair<int*, int*>> memo;
    vector<size_t> stk;
    void save(){
        stk.push_back(memo.size());
    }
    void undo(){
        while(memo.size() > stk.back()){
            *memo.back().first = memo.back().second;
            memo.pop_back();
        }
        stk.pop_back();
    }
    void assign(int *x, int v){
        memo.push_back({x, *x});
        *x=v;
    }
    //assign(&a, b); //a = b
} djs;
```

## 4.5 Persistent treap

```
#include <bits/stdc++.h>
using namespace std;

struct Treap {
    static Treap mem[P];
    Treap *lc, *rc;
    char c; int sz;
    Treap(){
        Treap(char _c) : lc(NULL), rc(NULL), sz(1), c(_c){}
    } Treap::mem[P], *ptr=Treap::mem;
    int Sz(Treap* t) {
        return t?t->sz:0;
    }
    void pull(Treap* t) {
        if (!t) return;
        t->sz = Sz(t->lc) + Sz(t->rc) + 1;
    }
    Treap* merge(Treap* a, Treap* b) {
        if (!a || !b) return a?a:b;
        Treap* ret;
        if (myRnd() % (Sz(a) + Sz(b)) < Sz(a)) {
            ret = new (ptr++) Treap(*a);
            ret->rc = merge(a->rc, b);
        }
        else {
```

```
ret = new (ptr++) Treap(*b);
ret->lc=merge(a, b->lc);
    }
    pull(ret);
    return ret;
}

void split(Treap* t, int k, Treap* &a, Treap* &b) {
    if (!t) a=b=NULL;
    else if (Sz(t->lc) + 1 <= k) {
        a = new (ptr++) Treap(*t);
        split(t->rc, k-Sz(t->lc)-1, a->rc, b);
        pull(a);
    }
    else {
        b=new (ptr++) Treap(*t);
        split(t->lc, k, a, b->lc);
        pull(b);
    }
}

int d;
char buf[M];
Treap* ver[N];

ptr = Treap::mem;
v_cnt++;
ver[v_cnt] = ver[v_cnt-1];
split(ver[v_cnt], p, tl, tr);
tl = merge(tl, new (ptr++) Treap(buf[j]));
```

## 4.6 Link Cut Tree

```
struct SplayNode {
    static SplayNode HOLE;
    SplayNode *ch[2], *par;
    bool rev;
    SplayNode(): par(&HOLE), rev(false) { ch[0] = ch[1] = &HOLE; }
    bool isRoot() {
        return (par->ch[0] != this && par->ch[1] != this);
    }
    void push() {
        if (rev) {
            if (ch[0]) ch[0]->rev ^= 1;
            if (ch[1]) ch[1]->rev ^= 1;
            swap(ch[0], ch[1]);
            rev ^= 1;
        }
    }
    void pushFromRoot() {
        if (!isRoot()) par->pushFromRoot();
        push();
    }
    void pull() {
        if (ch[0]) ch[0]->d = d + ch[0]->parLen;
        if (ch[1]) ch[1]->d = d + ch[1]->parLen;
    }
    void rotate() {
        SplayNode *p = par, *gp = p->par;
        bool dir = (p->ch[1] == this);
        p = gp;
        if (!p->isRoot()) gp->ch[gp->ch[1] == p] = this;
        p->ch[dir] = ch[dir ^ 1];
        p->ch[dir]->par = p;
        p->par = this;
        ch[dir ^ 1] = p;
        p->pull(), pull();
    }
    void splay() {
        pushFromRoot();
        while (!isRoot()) {
            if (!par->isRoot()) {
                SplayNode *gp = par->par;
                if ((gp->ch[0] == par) == (par->ch[0] == this))
                    rotate();
                else par->rotate();
            }
            rotate();
        }
    }
} SplayNode::HOLE;

namespace LCT {
    SplayNode *access(SplayNode *x) {
        SplayNode *last = &SplayNode::HOLE;
        while (x != &SplayNode::HOLE) {
            x->splay();
```

```

        x->ch[1] = last;
        x->pull();
        last = x;
        x = x->par;
    }
    return last;
}
void makeRoot(SplayNode *x) {
    access(x);
    x->splay();
    x->rev ^= 1;
}
void link(SplayNode *x, SplayNode *y) {
    makeRoot(x);
    x->par = y;
}
void cut(SplayNode *x, SplayNode *y) {
    makeRoot(x);
    access(y);
    y->splay();
    y->ch[0] = &SplayNode::HOLE;
    x->par = &SplayNode::HOLE;
}
void cutParent(SplayNode *x) {
    access(x);
    x->splay();
    x->ch[0]->par = &SplayNode::HOLE;
    x->ch[0] = &SplayNode::HOLE;
}
SplayNode *findRoot(SplayNode *x) {
    x = access(x);
    while (x->ch[0] != &SplayNode::HOLE) x = x->ch[0];
    x->splay();
    return x;
}
SplayNode *query(SplayNode *x, SplayNode *y) {
    makeRoot(x);
    return access(y);
}
SplayNode *queryLca(SplayNode *x, SplayNode *y) {
    access(x);
    auto lca = access(y);
    x->splay();
    return lca->data + lca->ch[1]->sum + (x == lca ? 0 : x->sum);
}
void modify(SplayNode *x, int data) {
    x->splay();
    x->data = data;
    x->pull();
}
}

```

## 4.7 Li Chao Tree

```

struct line {
    ll a, b;
    line(): a(0), b(0) {}
    line(ll a, ll b): a(a), b(b) {}
    ll operator()(ll x) const { return a * x + b; }
};

struct lichao {
    line st[NN];
    int sz, lc[NN], rc[NN];
    int gnode() {
        st[sz] = line(0, -1e18); //min: st[sz] = line(0, 1e18);
        lc[sz] = -1, rc[sz] = -1;
        return sz++;
    }
    void init() {
        sz = 0; gnode();
    }
    void add(int l, int r, line tl, int o) {
        // [l, r)
        bool lcp = st[o](l) < tl(l); //min: change < to >
        bool mcp = st[o]((l + r) / 2) < tl((l + r) / 2); //min:
            change < to >
        if (mcp) swap(st[o], tl);
        if (r - l == 1) return;
        if (lcp != mcp) {
            if (lc[o] == -1) lc[o] = gnode();
            add(l, (l + r) / 2, tl, lc[o]);
        } else {
            if (rc[o] == -1) rc[o] = gnode();

```

```

        add((l + r) / 2, r, tl, rc[o]);
    }
}
ll query(int l, int r, int x, int o) {
    if (r - l == 1) return st[o](x);
    if (x < (l + r) / 2) {
        if (lc[o] == -1) return st[o](x);
        return max(st[o](x), query(l, (l + r) / 2, x, lc[o]));
    } else {
        if (rc[o] == -1) return st[o](x);
        return max(st[o](x), query((l + r) / 2, r, x, rc[o]));
    }
}
} solver;

```

## 5 Flow

### 5.1 ISAP with bound

```

/*
Maximum density subgraph ( \sum W_e + \sum W_v ) / |V|
Binary search on answer:
For a fixed D, construct a Max flow model as follow:
Let S be Sum of all weight( or inf)
1. from source to each node with cap = S
2. For each (u,v,w) in E, (u->v, cap=w), (v->u, cap=w)
3. For each node v, from v to sink with cap = S + 2 * D - deg[v]
   ] - 2 * (W of v)
where deg[v] = \sum weight of edge associated with v
If maxflow < S * |V|, D is an answer.
Requiring subgraph: all vertex can be reached from source with
edge whose cap > 0.
*/

//Be careful that it's zero base !!!!!!!
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
#define SZ(x) ((int)(x).size())
#define eb emplace_back
const ll INF = 0x3f3f3f3f3f3f3f3f;

const ll N = 5e2 + 5;
struct isap {
    struct edge {
        int t, r;
        ll c;
        edge(int _t, int _r, ll _c) : t(_t), r(_r), c(_c) {}
    };
    int n, S, T;
    vector<edge> adj[N];
    int dis[N], gap[N], ok;
    isap(int _n, int _s, int _t) : n(_n), S(_s), T(_t) {
        for(int i = 0; i < n + 2; ++i) adj[i].clear();
    }
    void add(int u, int v, ll c) {
        adj[u].eb( v, adj[v].size(), c );
        adj[v].eb( u, adj[u].size() - 1, 0 );
    }
    ll dfs(int now, ll f) {
        if(now == T) return f;
        int mi = n;
        for(edge &e : adj[now]) {
            if(e.c) {
                ll x;
                if( dis[now] == dis[e.t] + 1 && (x = dfs(e.t, min(f, e.c))) ) {
                    e.c -= x;
                    adj[e.t][e.r].c += x;
                    return x;
                }
                mi = min(mi, dis[e.t]);
            }
        }
        if( --gap[dis[now]] == 0 ) ok = 0;
        dis[now] = mi + 1;
        gap[ dis[now] ]++;
        return 0;
    }
    ll flow() {
        memset(dis, 0, n * 4);

```

```

memset(gap, 0, n * 4);
gap[0] = n;
ok = 1;
ll r = 0;
while(dis[S] < n && ok) r += dfs(S, INF);
return r;
}
// below for bounded only
ll D[N];
void bounded_init() {
    memset(D, 0, n * 8);
}
void add2(int u, int v, ll b, ll c) {
    add(u, v, c - b);
    D[u] -= b;
    D[v] += b;
}
ll bounded_flow() {
    int SS = n, TT = n + 1;
    ll base = 0;
    for(int i = 0; i < n; ++i) {
        if (D[i] > 0) base += D[i];
        if (D[i] > 0) add(SS, i, D[i]);
        if (D[i] < 0) add(i, TT, -D[i]);
    }
    add(T, S, INF);
    int tmps = S, tmpT = T;
    n += 2; S = SS, T = TT;
    ll f = flow();
    n -= 2; S = tmps; T = tmpT;
    return f == base ? flow() : -1LL;
}
};

```

## 5.2 Min Cost Max Flow

```

const ll N = 5e2 + 5;
struct MCFflow {
    struct edge {
        int t, r;
        ll cap, cos;
        edge(int _t, int _r, ll _cp, ll _co) : t(_t), r(_r), cap(_cp), cos(_co) {}
    };
    int n, S, T;
    vector<edge> adj[N];
    MCFflow(int _n, int _s, int _t) : n(_n), S(_s), T(_t) {
        for(int i = 0; i < n; ++i)
            adj[i].clear();
    }
    void add(int s, int t, ll cap, ll cos) {
        adj[s].eb(t, SZ(adj[t]), cap, cos);
        adj[t].eb(s, SZ(adj[s])-1, 0, -cos);
    }
    pll flow() {
        ll tc = 0, tf = 0, dis[N];
        int inq[N], pre[N], prE[N];
        while(1) {
            memset(dis, INF, n * 8);
            memset(inq, 0, n * 4);
            queue<int> qu;
            qu.push(S);
            inq[S] = 1;
            dis[S] = 0;
            while(SZ(qu)) {
                int now = qu.front();
                qu.pop();
                inq[now] = 0;
                for(int i = 0; i < SZ(adj[now]); ++i) {
                    auto e = adj[now][i];
                    if(e.cap && dis[now] + e.cos < dis[e.t]) {
                        dis[e.t] = dis[now] + e.cos;
                        pre[e.t] = now;
                        prE[e.t] = i;
                        if(!inq[e.t]) {
                            qu.push(e.t);
                            inq[e.t] = 1;
                        }
                    }
                }
            }
            if(dis[T] == INF) break;
            ll mi = INF;
            for(int now = T; now != S; now = pre[now])
                mi = min(mi, adj[pre[now]][prE[now]].cap);

```

```

        for(int now = T; now != S; now = pre[now]) {
            adj[pre[now]][prE[now]].cap -= mi;
            adj[now][adj[pre[now]][prE[now]].r].cap += mi;
        }
        tc += mi * dis[T];
        tf += mi;
    }
    return pll(tc, tf);
}
};

```

## 5.3 S-W Global Min Cut

```

struct SW {
    //find global min cut in O(V^3)
    //points are ZERO-BASE!!!
    static const int N = 506;
    int adj[N][N], wei[N], n;
    bool vis[N], del[N];
    void init(int _n) {
        n = _n;
        memset(adj, 0, sizeof(adj));
        memset(del, 0, sizeof(del));
    }
    void add_edge(int x, int y, int w) {
        adj[x][y] += w;
        adj[y][x] += w;
    }
    void search(int &s, int &t) {
        memset(wei, 0, sizeof(wei));
        memset(vis, 0, sizeof(vis));
        s = t = -1;
        while (true) {
            int mx = -1, mx_id = 0;
            for (int i = 0; i < n; ++i) {
                if (!del[i] && !vis[i] && mx < wei[i]) {
                    mx_id = i;
                    mx = wei[i];
                }
            }
            if (mx == -1) break;
            vis[mx_id] = true;
            s = t;
            t = mx_id;
            for (int i = 0; i < n; ++i)
                if (!vis[i] && !del[i])
                    wei[i] += adj[mx_id][i];
        }
    }
    int solve() {
        int ret = INF;
        for (int i = 0; i < n - 1; ++i) {
            int x, y;
            search(x, y);
            ret = min(ret, wei[y]);
            del[y] = true;
            for (int j = 0; j < n; ++j) {
                adj[x][j] += adj[y][j];
                adj[j][x] += adj[y][j];
            }
        }
        return ret;
    }
} SW;

```

## 5.4 Gomory Hu Tree

```

def cut(G,s,t) :
    return minimum s-t cut in G

def gomory_hu(G):
    T = {}
    p = [1] * IV(G)
    for s in [2,n] :
        t = p[s]
        C = cut(G, s, t)
        add(s, t, w(C)) to T
        for i in [s + 1, n] :
            if p[i] == t and s-i path exists in G\C :
                p[i] = s
    return T;

```

## 6 Graph

### 6.1 Biconnected Component

```
int low[N], dfn[N];
bool vis[N];
int cnt[N], e[N], x[N], y[N];
int stamp, bcc_no = 0;

vector<int> G[N], bcc[N];
stack<int> sta;

void dfs(int now, int par) {
    vis[now] = true;
    dfn[now] = low[now] = (++stamp);
    for (int i:G[now]) {
        int to = (e[i] ^ now);
        if (to == par) continue;
        if (!vis[to]) {
            sta.push(i); dfs(to, now);
            low[now] = min(low[now], low[to]);
            if (low[to] >= dfn[now]) {
                ++bcc_no; int p;
                do {
                    p = sta.top(); sta.pop();
                    bcc[bcc_no].push_back(p);
                } while (p != i);
            }
        }
        else if (dfn[to] < dfn[now]) {
            sta.push(i);
            low[now] = min(low[now], dfn[to]);
        }
    }
}
```

### 6.2 General graph matching

```
const int N = 100006, E = (2e5) * 2;
struct Graph {
    //1-index
    int to[E], bro[E], head[N], e;
    int lnk[N], vis[N], stp, n;
    int per[N];
    void init(int _n) {
        //remember to set every array to 0
        stp = 0; e = 1; n = _n;
        for (int i = 1; i <= n; i++)
            head[i] = lnk[i] = vis[i] = 0, per[i] = i;
        //random_shuffle(per+1, per+n+1);
    }
    void add_edge(int u, int v) {
        u = per[u], v = per[v];
        to[e] = v, bro[e] = head[u], head[u] = e++;
        to[e] = u, bro[e] = head[v], head[v] = e++;
    }
    bool dfs(int x) {
        vis[x] = stp;
        for (int i = head[x]; i; i = bro[i]) {
            int v = to[i];
            if (!lnk[v]) {
                lnk[x] = v, lnk[v] = x;
                return true;
            }
            else if (vis[lnk[v]] < stp) {
                int w = lnk[v];
                lnk[x] = v, lnk[v] = x, lnk[w] = 0;
                if (dfs(w)) {
                    return true;
                }
                lnk[w] = v, lnk[v] = w, lnk[x] = 0;
            }
        }
        return false;
    }
    int solve() {
        int ans = 0;
        for (int i = 1; i <= n; i++)
            if (!lnk[i])
                stp++; ans += dfs(i);
        return ans;
    }
} graph;
```

### 6.3 KM

```
const int INF = 0x3f3f3f3f;
const int maxn = 610;

int n, w[maxn][maxn], lx[maxn], ly[maxn], slk[maxn];
int s[maxn], t[maxn], good[maxn];

int match(int now) {
    s[now] = 1;
    for (int to = 1; to <= n; to++) {
        if (t[to]) continue;
        if (lx[now] + ly[to] == w[now][to]) {
            t[to] = 1;
            if (good[to] == 0 || match(good[to]))
                return good[to] = now, 1;
        }
        else slk[to] = min(slk[to], lx[now] + ly[to] - w[now][to]);
    }
    return 0;
}

void update() {
    int val = INF;
    for (int i = 1; i <= n; i++)
        if (t[i] == 0) val = min(val, slk[i]);
    for (int i = 1; i <= n; i++) {
        if (s[i]) lx[i] -= val;
        if (t[i]) ly[i] += val;
    }
}

void run_km() {
    for (int i = 1; i <= n; i++) {
        lx[i] = w[i][1];
        for (int j = 1; j <= n; j++)
            lx[i] = max(lx[i], w[i][j]);
    }
    for (int i = 1; i <= n; i++)
        ly[i] = 0, good[i] = 0;
    for (int i = 1; i <= n; i++) {
        for (int j = 1; j <= n; j++) slk[j] = INF;
        while (1) {
            for (int j = 1; j <= n; j++)
                s[j] = t[j] = 0;
            if (match(i)) break;
            else update();
        }
    }
}

/* how_to_use:
1. put edge in w[i][j]
2. run_km
3. match: (good[i], i)
*/
```

### 6.4 Maximum Weighted Matching (General Graph)

```
struct WeightGraph {
    static const int INF = INT_MAX;
    static const int N = 514;
    struct edge {
        int u, v, w; edge() {}
        edge(int ui, int vi, int wi) : u(ui), v(vi), w(wi) {}
    };
    int n, n_x;
    edge g[N*2][N*2];
    int lab[N*2];
    int match[N*2], slack[N*2], st[N*2], pa[N*2];
    int flo_from[N*2][N+1], S[N*2], vis[N*2];
    vector<int> flo[N*2];
    queue<int> q;
    int e_delta(const edge &e) {
        return lab[e.u] + lab[e.v] - g[e.u][e.v].w*2;
    }
    void update_slack(int u, int x) {
        if (!slack[x] || e_delta(g[u][x]) < e_delta(g[slack[x]][x]))
            slack[x] = u;
    }
    void set_slack(int x) {
        slack[x] = 0;
        for (int u = 1; u <= n; u++)
```



```

        if(g[u][x].w>0&&st[u]!=x&&S[st[u]]==0)
            update_slack(u,x);
    }
    void q_push(int x){
        if(x<=n)q.push(x);
        else for(size_t i=0;i<flo[x].size();i++)
            q_push(flo[x][i]);
    }
    void set_st(int x,int b){
        st[x]=b;
        if(x>n)for(size_t i=0;i<flo[x].size();++i)
            set_st(flo[x][i],b);
    }
    int get_pr(int b,int xr){
        int pr=find(flo[b].begin(),flo[b].end(),xr)-flo[b].begin();
        if(pr%2==1){
            reverse(flo[b].begin()+1,flo[b].end());
            return (int)flo[b].size()-pr;
        }else return pr;
    }
    void set_match(int u,int v){
        match[u]=g[u][v].v;
        if(u<=n) return;
        edge e=g[u][v];
        int xr=flo_from[u][e.u],pr=get_pr(u,xr);
        for(int i=0;i<pr;++i)set_match(flo[u][i],flo[u][i+1]);
        set_match(xr,v);
        rotate(flo[u].begin(),flo[u].begin()+pr,flo[u].end());
    }
    void augment(int u,int v){
        for(;;){
            int xnv=st[match[u]];
            set_match(u,v);
            if(!xnv) return;
            set_match(xnv,st[pa[xnv]]);
            u=st[pa[xnv]],v=xnv;
        }
    }
    int get_lca(int u,int v){
        static int t=0;
        for(++t;u!=v;swap(u,v)){
            if(u==0)continue;
            if(vis[u]==t) return u;
            vis[u]=t;
            u=st[match[u]];
            if(u)u=st[pa[u]];
        }
        return 0;
    }
    void add_blossom(int u,int lca,int v){
        int b=n+1;
        while(b<=n_x&&st[b]++b;
        if(b>n_x)++n_x;
        lab[b]=0,S[b]=0;
        match[b]=match[lca];
        flo[b].clear();
        flo[b].push_back(lca);
        for(int x=u,y; x!=lca; x=st[pa[y]])
            flo[b].push_back(x),flo[b].push_back(y=st[match[x]]),q_push(y);
        reverse(flo[b].begin()+1,flo[b].end());
        for(int x=v,y; x!=lca; x=st[pa[y]])
            flo[b].push_back(x),flo[b].push_back(y=st[match[x]]),q_push(y);
        set_st(b,b);
        for(int x=1;x<=n_x;++x)g[b][x].w=g[x][b].w=0;
        for(int x=1;x<=n;++x)flo_from[b][x]=0;
        for(size_t i=0;i<flo[b].size();++i){
            int xs=flo[b][i];
            for(int x=1;x<=n_x;++x)
                if(g[b][x].w==0||e_delta(g[xs][x])<e_delta(g[b][x]))
                    g[b][x]=g[xs][x],g[x][b]=g[x][xs];
            for(int x=1;x<=n;++x)
                if(flo_from[xs][x])flo_from[b][x]=xs;
        }
        set_slack(b);
    }
    void expand_blossom(int b){
        for(size_t i=0;i<flo[b].size();++i)
            set_st(flo[b][i],flo[b][i]);
        int xr=flo_from[b][g[b][pa[b]].u],pr=get_pr(b,xr);
        for(int i=0;i<pr;i+=2){
            int xs=flo[b][i],xns=flo[b][i+1];
            pa[xs]=g[xns][xs].u;

```

```

            S[xs]=1,S[xns]=0;
            slack[xs]=0,set_slack(xns);
            q_push(xns);
        }
        S[xr]=1,pa[xr]=pa[b];
        for(size_t i=pr+1;i<flo[b].size();++i){
            int xs=flo[b][i];
            S[xs]=-1,set_slack(xs);
        }
        st[b]=0;
    }
    bool on_found_edge(const edge &e){
        int u=st[e.u],v=st[e.v];
        if(S[v]==-1){
            pa[v]=e.u,S[v]=1;
            int nu=st[match[v]];
            slack[v]=slack[nu]=0;
            S[nu]=0,q_push(nu);
        }else if(S[v]==0){
            int lca=get_lca(u,v);
            if(!lca) return augment(u,v),augment(v,u),true;
            else add_blossom(u,lca,v);
        }
        return false;
    }
    bool matching(){
        memset(S+1,-1,sizeof(int)*n_x);
        memset(slack+1,0,sizeof(int)*n_x);
        q=queue<int>();
        for(int x=1;x<=n_x;++x)
            if(st[x]==x&&!match[x])pa[x]=0,S[x]=0,q_push(x);
        if(q.empty()) return false;
        for(;;){
            while(q.size()){
                int u=q.front();q.pop();
                if(S[st[u]]==1)continue;
                for(int v=1;v<=n;++v)
                    if(g[u][v].w>0&&st[u]!=st[v]){
                        if(e_delta(g[u][v])==0){
                            if(on_found_edge(g[u][v])) return true;
                        }else update_slack(u,st[v]);
                    }
            }
            int d=INF;
            for(int b=n+1;b<=n_x;++b)
                if(st[b]==b&&S[b]==1)d=min(d,lab[b]/2);
            for(int x=1;x<=n_x;++x)
                if(st[x]==x&&slack[x]){
                    if(S[x]==-1)d=min(d,e_delta(g[slack[x]][x]));
                    else if(S[x]==0)d=min(d,e_delta(g[slack[x]][x])/2);
                }
            for(int u=1;u<=n;++u){
                if(S[st[u]]==0){
                    if(lab[u]<=d) return 0;
                    lab[u]-=d;
                }else if(S[st[u]]==1)lab[u]+=d;
            }
            for(int b=n+1;b<=n_x;++b)
                if(st[b]==b){
                    if(S[st[b]]==0)lab[b]+=d*2;
                    else if(S[st[b]]==1)lab[b]-=d*2;
                }
            q=queue<int>();
            for(int x=1;x<=n_x;++x)
                if(st[x]==x&&slack[x]&&st[slack[x]]!=x&&e_delta(g[slack[x]][x])==0)
                    if(on_found_edge(g[slack[x]][x])) return true;
            for(int b=n+1;b<=n_x;++b)
                if(st[b]==b&&S[b]==1&&lab[b]==0) expand_blossom(b);
        }
        return false;
    }
    pair<long long,int> solve(){
        memset(match+1,0,sizeof(int)*n);
        n_x=n;
        int n_matches=0;
        long long tot_weight=0;
        for(int u=0;u<=n;++u)st[u]=u,flo[u].clear();
        int w_max=0;
        for(int u=1;u<=n;++u)
            for(int v=1;v<=n;++v){

```

```

        flo_from[u][v]=(u==v?0);
        w_max=max(w_max,g[u][v].w);
    }
    for(int u=1;u<=n;++u)lab[u]=w_max;
    while(matching())++n_matches;
    for(int u=1;u<=n;++u)
        if(match[u]&&match[u]<u)
            tot_weight+=g[u][match[u]].w;
    return make_pair(tot_weight,n_matches);
}
void add_edge( int ui , int vi , int wi ){
    g[ui][vi].w = g[vi][ui].w = wi;
}
void init( int _n ){
    n = _n;
    for(int u=1;u<=n;++u)
        for(int v=1;v<=n;++v)
            g[u][v]=edge(u,v,0);
}
} graph;

```

## 6.5 Minimum mean cycle

```

/* minimum mean cycle O(VE) */
struct MMC{
    struct Edge { int v,u; double c; };
    int n, m, prv[V][V], prve[V][V], vst[V];
    Edge e[E];
    vector<int> edgeID, cycle, rho;
    double d[V][V];
    void init( int _n )
    { n = _n; m = 0; }
    // WARNING: TYPE matters
    void addEdge( int vi , int ui , double ci )
    { e[ m ++ ] = { vi , ui , ci }; }
    void bellman_ford() {
        for(int i=0; i<n; i++) d[0][i]=0;
        for(int i=0; i<n; i++) {
            fill(d[i+1], d[i+1]+n, inf);
            for(int j=0; j<m; j++) {
                int v = e[j].v, u = e[j].u;
                if(d[i][v]<inf && d[i+1][u]>d[i][v]+e[j].c) {
                    d[i+1][u] = d[i][v]+e[j].c;
                    prv[i+1][u] = v;
                    prve[i+1][u] = j;
                }
            }
        }
    }
    double solve(){
        // returns inf if no cycle, mmc otherwise
        double mmc=inf;
        int st = -1;
        bellman_ford();
        for(int i=0; i<n; i++) {
            double avg=-inf;
            for(int k=0; k<n; k++) {
                if(d[n][i]<inf-eps) avg=max(avg,(d[n][i]-d[k][i])/(n-k));
                else avg=max(avg,inf);
            }
            if (avg < mmc) tie(mmc, st) = tie(avg, i);
        }
        FZ(vst); edgeID.clear(); cycle.clear(); rho.clear();
        for (int i=n; !vst[st]; st=prv[i--][st]) {
            vst[st]++;
            edgeID.pb(prve[i][st]);
            rho.pb(st);
        }
        while (vst[st] != 2) {
            int v = rho.back(); rho.pop_back();
            cycle.pb(v);
            vst[v]++;
        }
        reverse(ALL(edgeID));
        edgeID.resize(SZ(cycle));
        return mmc;
    }
} mmc;

```

## 6.6 Heavy-Light decomposition

```

int siz[MAX] , son[MAX] , dep[MAX] , ffa[MAX];
int top[MAX] , idx[MAX] , idpo = 0;
int n , m;
int e[MAX][3];
vector<int> v[MAX];
struct node{ int big , sml; } st[MAX * 4];
void init(){
    REP(i , 0 , MAX) v[i].clear();
    MEM(siz , 0) , MEM(son , 0) , MEM(dep , 0) , MEM(ffa , 0);
    MEM(top , 0) , MEM(idx , 0) , idpo = 0;
}
void DFS1(int now , int fa , int deep){
    siz[now] = 1;
    dep[now] = deep;
    ffa[now] = fa;
    int big = 0;
    REP(i , 0 , v[now].size()){
        int to = v[now][i];
        if(to != fa){
            DFS1(to , now , deep + 1);
            siz[now] += siz[to];
            if(siz[to] > big) big = siz[to] , son[now] = to;
        }
    }
}
void DFS2(int now , int fa , int root){
    top[now] = root;
    idx[now] = ++idpo;
    if(son[now] != 0) DFS2(son[now] , now , root);
    REP(i , 0 , v[now].size()){
        int to = v[now][i];
        if(to != fa && to != son[now]) DFS2(to , now , to);
    }
}
void solveinit(){
    DFS1(1 , 0 , 0);
    DFS2(1 , 0 , 1);
    REP(i , 2 , n + 1){
        int a = e[i][0] , b = e[i][1] , c = e[i][2];
        if(dep[a] < dep[b]) swap(a , b);
        update(1 , 1 , n , idx[a] , c);
    }
}
void query(int a , int b){
    node ans;
    ans.big = -INF , ans.sml = INF;
    int t1 = top[a] , t2 = top[b];
    while(t1 != t2){
        if(dep[t1] < dep[t2]) swap(t1 , t2) , swap(a , b);
        ans = pull(ans , query(1 , 1 , n , idx[t1] , idx[a]));
        a = ffa[t1] , t1 = top[a];
    }
    if(dep[a] > dep[b]) swap(a , b);
    if(a != b) ans = pull(ans , query(1 , 1 , n , idx[son[a]] , idx[b]));
    return cout << ans.sml << " " << ans.big << endl , void();
}
init();
REP(i , 2 , n + 1){
    int a , b , c; cin >> a >> b >> c;
    e[i][0] = a , e[i][1] = b , e[i][2] = c;
    v[a].pb(b); v[b].pb(a);
}
solveinit();
query(a , b);

```

## 6.7 Centroid Decomposition

```

#include <iostream>
#include <stdio.h>
#include <vector>
#include <cstring>
using namespace std;

typedef long long Meruru;
const int MAX_N = 1e5 + 6;
const int MAX_P = 18;

struct Edge {
    int to,weight;
};

Edge MP(int _to,int _weight) {
    return Edge{_to,_weight};
}

```



```

}
vector<Edge> edg[MAX_N];
Meruru dis[MAX_P][MAX_N];
bool visit[MAX_N];
int sz[MAX_N];
int mx[MAX_N];

struct Cen {
    Meruru minus;
    Meruru val;
    int p;
    int sz;
    int depth;
} cen[MAX_N];

Cen MP_cen(int _p, int _depth) {
    return Cen{0, 0, _p, 0, _depth};
}

vector<int> v;

void get_cen(int id) {
    visit[id]=1;
    v.push_back(id);
    sz[id]=1;
    mx[id]=0;
    for (Edge i:edg[id]) {
        if (!visit[i.to]) {
            get_cen(i.to);
            mx[id] = max(mx[id], sz[i.to]);
            sz[id] += sz[i.to];
        }
    }
}

void get_dis(int id, int cen_depth, Meruru weight) {
    dis[cen_depth][id] = weight;
    visit[id]=1;
    for (Edge i:edg[id]) {
        if (!visit[i.to]) {
            get_dis(i.to, cen_depth, weight+i.weight);
        }
    }
}

void dfs(int id, int cen_depth, int p) {
    get_cen(id);
    int nn=v.size();
    int ccen=-1;
    for (int i:v) {
        if (max(nn-sz[i], mx[i]) <= nn/2) {
            ccen=i;
        }
        visit[i]=0;
    }
    get_dis(ccen, cen_depth, 0);
    for (int i:v) {
        visit[i]=0;
    }
    v.clear();
    visit[ccen]=1;
    cen[ccen] = MP_cen(p, cen_depth);
    for (Edge i:edg[ccen]) {
        if (!visit[i.to]) {
            dfs(i.to, cen_depth+1, ccen);
        }
    }
}

void add(int id) {
    int p=id;
    while (p!=-1) {
        cen[p].val += dis[cen[p].depth][id];
        cen[p].sz++;
        cen[p].minus += dis[cen[p].depth-1][id];
        p=cen[p].p;
    }
}

Meruru query(int id) {
    int p=id;
    Meruru ret=0;
    int szz=0;
    while (p!=-1) {
        ret += (cen[p].val - cen[p].minus);
        ret += (cen[p].sz - szz)*dis[cen[p].depth][id];

```

```

        szz = cen[p].szz;
        p=cen[p].p;
    }
    return ret;
}

int main () {
    int n,q;
    scanf("%d %d",&n,&q);
    for (int i=1;n>i;i++) {
        int a,b,c;
        scanf("%d %d %d",&a,&b,&c);
        a++;
        b++;
        edg[a].push_back(MP(b,c));
        edg[b].push_back(MP(a,c));
    }
    dfs(1,1,-1);
    memset(visit,0,sizeof(visit));
    while (q--) {
        int a,b;
        scanf("%d %d",&a,&b);
        b++;
        if (a==1 && !visit[b]) {
            add(b);
            visit[b]=1;
        }
        else if (a==2)printf("%lld\n",query(b));
    }
}

```

## 6.8 Dynamic MST

```

/* Dynamic MST  $O(Q \lg^2 Q)$ 
(qx[i], qy[i]) -> chg weight of edge No.qx[i] to qy[i]
delete an edge: (i, \infty)
add an edge: change from \infty to specific value
*/
const int SZ=M+3*MXQ;
int a[N],*tz;
int find(int xx){
    int root=xx; while(a[root]) root=a[root];
    int next; while((next=a[xx])){a[xx]=root; xx=next; }
    return root;
}
bool cmp(int aa,int bb){ return tz[aa]<tz[bb]; }
int kx[N],ky[N],kt, vd[N],id[M], app[M];
bool extra[M];
void solve(int *qx,int *qy,int Q,int n,int *x,int *y,int *z,int
    m1,long long ans){
    if(Q==1){
        for(int i=1;i<=n;i++) a[i]=0;
        z[qx[0]]=qy[0]; tz = z;
        for(int i=0;i<m1;i++) id[i]=i;
        sort(id,id+m1,cmp); int ri,rj;
        for(int i=0;i<m1;i++){
            ri=find(x[id[i]]); rj=find(y[id[i]]);
            if(ri!=rj){ ans+=z[id[i]]; a[ri]=rj; }
        }
        printf("%lld\n",ans);
        return;
    }
    int ri,rj;
    //contract
    kt=0;
    for(int i=1;i<=n;i++) a[i]=0;
    for(int i=0;i<Q;i++){
        ri=find(x[qx[i]]); rj=find(y[qx[i]]); if(ri!=rj) a[ri]=rj;
    }
    int tm=0;
    for(int i=0;i<m1;i++) extra[i]=true;
    for(int i=0;i<Q;i++) extra[qx[i]]=false;
    for(int i=0;i<m1;i++) if(extra[i]) id[tm++]=i;
    tz=z; sort(id,id+tm,cmp);
    for(int i=0;i<tm;i++){
        ri=find(x[id[i]]); rj=find(y[id[i]]);
        if(ri!=rj){
            a[ri]=rj; ans += z[id[i]];
            kx[kt]=x[id[i]]; ky[kt]=y[id[i]]; kt++;
        }
    }
    for(int i=1;i<=n;i++) a[i]=0;
    for(int i=0;i<kt;i++) a[ find(kx[i]) ]=find(ky[i]);
    int n2=0;
    for(int i=1;i<=n;i++) if(a[i]==0)

```

```

vd[i]=++n2;
for(int i=1;i<=n;i++) if(a[i])
vd[i]=vd[find(i)];
int m2=0, *Nx=x+m1, *Ny=y+m1, *Nz=z+m1;
for(int i=0;i<m1;i++) app[i]=-1;
for(int i=0;i<Q;i++) if(app[qx[i]]==-1){
    Nx[m2]=vd[ x[ qx[i] ] ]; Ny[m2]=vd[ y[ qx[i] ] ]; Nz[m2]=z[
        qx[i] ];
    app[qx[i]]=m2; m2++;
}
for(int i=0;i<Q;i++){ z[ qx[i] ]=qy[i]; qx[i]=app[qx[i]]; }
for(int i=1;i<=n2;i++) a[i]=0;
for(int i=0;i<tm;i++){
    ri=find(vd[ x[id[i]] ]); rj=find(vd[ y[id[i]] ]);
    if(ri!=rj){
        a[ri]=rj; Nx[m2]=vd[ x[id[i]] ];
        Ny[m2]=vd[ y[id[i]] ]; Nz[m2]=z[id[i]]; m2++;
    }
}
int mid=Q/2;
solve(qx,qy,mid,n2,Nx,Ny,Nz,m2,ans);
solve(qx+mid,qy+mid,Q-mid,n2,Nx,Ny,Nz,m2,ans);
}
int x[SZ],y[SZ],z[SZ],qx[MXQ],qy[MXQ],n,m,Q;
void init(){
    scanf("%d%d",&n,&m);
    for(int i=0;i<m;i++) scanf("%d%d%d",x+i,y+i,z+i);
    scanf("%d",&Q);
    for(int i=0;i<Q;i++){ scanf("%d%d",qx+i,qy+i); qx[i]--; }
}
void work(){ if(Q) solve(qx,qy,Q,n,x,y,z,m,0); }
int main(){init(); work(); }

```

## 6.9 Minimum Steiner Tree

```

// Minimum Steiner Tree
// O(V^3AT + V^2 2^AT)
struct SteinerTree{
#define V 33
#define T 8
#define INF 1023456789
    int n, dst[V][V], dp[1<<T][V], tdst[V];
    void init( int _n ){
        n = _n;
        for( int i = 0 ; i < n ; i ++ ){
            for( int j = 0 ; j < n ; j ++ )
                dst[ i ][ j ] = INF;
            dst[ i ][ i ] = 0;
        }
    }
    void add_edge( int ui , int vi , int wi ){
        dst[ ui ][ vi ] = min( dst[ ui ][ vi ] , wi );
        dst[ vi ][ ui ] = min( dst[ vi ][ ui ] , wi );
    }
    void shortest_path(){
        for( int k = 0 ; k < n ; k ++ )
            for( int i = 0 ; i < n ; i ++ )
                for( int j = 0 ; j < n ; j ++ )
                    dst[ i ][ j ] = min( dst[ i ][ j ],
                        dst[ i ][ k ] + dst[ k ][ j ] );
    }
    int solve( const vector<int>& ter ){
        int t = (int)ter.size();
        for( int i = 0 ; i < ( 1 << t ) ; i ++ )
            for( int j = 0 ; j < n ; j ++ )
                dp[ i ][ j ] = INF;
        for( int i = 0 ; i < n ; i ++ )
            dp[ 0 ][ i ] = 0;
        for( int msk = 1 ; msk < ( 1 << t ) ; msk ++ ){
            if( msk == ( msk & (-msk) ) ){
                int who = __lg( msk );
                for( int i = 0 ; i < n ; i ++ )
                    dp[ msk ][ i ] = dst[ ter[ who ] ][ i ];
                continue;
            }
            for( int i = 0 ; i < n ; i ++ )
                for( int submsk = ( msk - 1 ) & msk ; submsk ;
                    submsk = ( submsk - 1 ) & msk )
                    dp[ msk ][ i ] = min( dp[ msk ][ i ],
                        dp[ submsk ][ i ] +
                        dp[ msk ^ submsk ][ i ] );
            for( int i = 0 ; i < n ; i ++ ){
                tdst[ i ] = INF;
                for( int j = 0 ; j < n ; j ++ )

```

```

                    tdst[ i ] = min( tdst[ i ],
                        dp[ msk ][ j ] + dst[ j ][ i ] );
            }
            for( int i = 0 ; i < n ; i ++ )
                dp[ msk ][ i ] = tdst[ i ];
        }
        int ans = INF;
        for( int i = 0 ; i < n ; i ++ )
            ans = min( ans , dp[ ( 1 << t ) - 1 ][ i ] );
        return ans;
    }
} solver;

```

## 6.10 Maximum Clique

```

struct BKB{
    static const int MAX_N = 50;
    typedef bitset<MAX_N> bst;
    bst N[MAX_N];
    int n;
    ll wei[MAX_N], ans, cc;
    BKB(int _n = 0): n(_n), ans(0), cc(0){
        for(int i = 0; i < _n; ++ i)
            N[i].reset();
    }
    void add_edge(int a, int b) {
        N[a][b] = N[b][a] = 1;
    }
    void set_wei(int a, ll w) {
        wei[a] = w;
    }
    ll CNT(bst P) {
        //if vertices have no weight: return P.count();
        ll rt = 0;
        for(int i = P._Find_first(); i < n; i = P._Find_next(i))
            rt += wei[i];
        return rt;
    }
    void pro(bst P, ll cnt = 0) {
        if (!P.any()){
            if(cnt == ans)
                ++ cc;
            else if(cnt > ans) {
                ans = cnt;
                cc = 1;
            }
            return;
        }
        // "<" can be change to "<=" if we don't need to count
        if ( CNT(P) + cnt < ans )
            return;
        int u = P._Find_first();
        bst now = P & ~N[u];
        for( int i = now._Find_first(); i < n; i = now._Find_next(i) ) {
            pro(P & N[i], cnt + wei[i]);
            P[i] = 0;
        }
        return;
    }
    pll solve() {
        bst tmp;
        tmp.reset();
        for(int i = 0; i < n; ++ i)
            tmp[i] = 1;
        pro(tmp);
        return pll(ans, cc);
    }
} ss(0);

```

## 6.11 Zhu Liu Algo

```

struct ZL{
    //1 base edge and vertex
    static const int N=556,M=2660, MM = M * 10,inf=1e9;
    //MM = M * log N
    struct bian{
        int u,v,w,use,id;
    }b[M],a[MM];
    int n,m=0,ans,pre[N],id[N],vis[N],root,In[N],h[N],len,way[M];
    void init(int _n,int _root){
        for( int i = 0 ; i < MM; ++i) {

```

```

        a[i] = {0, 0, 0, 0, 0};
    }
    n=_n; m=0; b[0].w=1e9; root=_root;
}
void add(int u,int v,int w){
    b[+m]=(bian){u,v,w,0,m};
    a[m]=b[m];
}
int work(){
    len=m;
    for (;;){
        for (int i=1;i<=n;i++){pre[i]=0; In[i]=inf; id[i]=0;
            vis[i]=0; h[i]=0;}
        for (int i=1;i<=m;i++)
            if (b[i].u!=b[i].v&&b[i].w<In[b[i].v]){
                pre[b[i].v]=b[i].u; In[b[i].v]=b[i].w; h[b[i].v]=b[i].id;
            }
        for (int i=1;i<=n;i++) if (pre[i]==0&&i!=root) return 0;
        int cnt=0; In[root]=0;
        for (int i=1;i<=n;i++){
            if (i!=root) a[h[i]].use++;
            int now=i; ans+=In[i];
            while (vis[now]==0&&now!=root){
                vis[now]=i; now=pre[now];
            }
            if (now!=root&&vis[now]==i){
                cnt++; int kk=now;
                while (1){
                    id[now]=cnt; now=pre[now];
                    if (now==kk) break;
                }
            }
        }
        if (cnt==0) return 1;
        for (int i=1;i<=n;i++) if (id[i]==0) id[i]=++cnt;
        for (int i=1;i<=m;i++){
            int k1=In[b[i].v]; int k2=b[i].v;
            b[i].u=id[b[i].u]; b[i].v=id[b[i].v];
            if (b[i].u!=b[i].v){
                b[i].w=k1; a[+len].u=b[i].id; a[len].v=h[k2];
                b[i].id=len;
            }
        }
        n=cnt;
        root=id[root];
    }
    return 1;
}
int getway(){
    for (int i=1;i<=m;i++) way[i]=0;
    for (int i=len;i>m;i--){
        a[a[i].u].use+=a[i].use; a[a[i].v].use-=a[i].use;
    }
    for (int i=1;i<=m;i++) way[i]=a[i].use;
    int ret = 0;
    for (int i = 1; i <= m; ++i){
        if (way[i] == 1) {
            ret += a[i].w;
        }
    }
    return ret;
}
}
} z1;
//if z1.work() == 0, then it is not connected
//otherwise, use z1.getway() to check bian is selected or not

```

## 7 Math

### 7.1 Fast Power

```

ll power(ll a, ll x, ll mod) {
    if(x == 0) return 1;
    if(x & 1) return power(a * a % mod, x >> 1, mod) * a % mod;
    else return power(a * a % mod, x >> 1, mod);
}

```

### 7.2 Extended Euclidean

```

// ax + by = gcd(a, b)
ll exgcd(ll a, ll b, ll &x, ll &y){
    if(a == 0) return x = 0, y = 1, b;
    ll g = exgcd(b % a, a, y, x);
    x -= b / a * y;
    return g;
}

```

### 7.3 Big Integer

```

struct Bigint{
    static const int LEN = 60;
    static const int BIGMOD = 10000;
    int s;
    int vl, v[LEN];
    // vector<int> v;
    Bigint() : s(1) { vl = 0; }
    Bigint(long long a) {
        s = 1; vl = 0;
        if (a < 0) { s = -1; a = -a; }
        while (a) {
            push_back(a % BIGMOD);
            a /= BIGMOD;
        }
    }
    Bigint(string str) {
        s = 1; vl = 0;
        int stPos = 0, num = 0;
        if (!str.empty() && str[0] == '-') {
            stPos = 1;
            s = -1;
        }
        for (int i=SZ(str)-1, q=1; i>=stPos; i--) {
            num += (str[i] - '0') * q;
            if ((q *= 10) >= BIGMOD) {
                push_back(num);
                num = 0; q = 1;
            }
        }
        if (num) push_back(num);
        n();
    }
    int len() const {
        return vl; //return SZ(v);
    }
    bool empty() const { return len() == 0; }
    void push_back(int x) {
        v[vl++] = x; //v.PB(x);
    }
    void pop_back() {
        vl--; //v.pop_back();
    }
    int back() const {
        return v[vl-1]; //return v.back();
    }
    void n() {
        while (!empty() && !back()) pop_back();
    }
    void resize(int nl) {
        vl = nl; //v.resize(nl);
        fill(v, v+vl, 0); //fill(ALL(v), 0);
    }
    void print() const {
        if (empty()) { putchar('0'); return; }
        if (s == -1) putchar('-');
        printf("%d", back());
        for (int i=len()-2; i>=0; i--) printf("%.4d", v[i]);
    }
    friend std::ostream& operator << (std::ostream& out, const
        Bigint &a) {
        if (a.empty()) { out << "0"; return out; }
        if (a.s == -1) out << "-";
        out << a.back();
        for (int i=a.len()-2; i>=0; i--) {
            char str[10];
            snprintf(str, 5, "%.4d", a.v[i]);
            out << str;
        }
        return out;
    }
    int cp3(const Bigint &b) const {
        if (s != b.s) return s - b.s;
        if (s == -1) return -(*this).cp3(-b);
        if (len() != b.len()) return len()-b.len(); //int
        for (int i=len()-1; i>=0; i--)

```

```

        if (v[i]!=b.v[i]) return v[i]-b.v[i];
        return 0;
    }
    bool operator<(const Bigint &b)const
    { return cp3(b)<0; }
    bool operator<=(const Bigint &b)const
    { return cp3(b)<=0; }
    bool operator==(const Bigint &b)const
    { return cp3(b)==0; }
    bool operator!=(const Bigint &b)const
    { return cp3(b)!=0; }
    bool operator>(const Bigint &b)const
    { return cp3(b)>0; }
    bool operator>=(const Bigint &b)const
    { return cp3(b)>=0; }
    Bigint operator - () const {
        Bigint r = (*this);
        r.s = -r.s;
        return r;
    }
    Bigint operator + (const Bigint &b) const {
        if (s == -1) return -(*this)+(-b);
        if (b.s == -1) return (*this)-(-b);
        Bigint r;
        int nl = max(len(), b.len());
        r.resize(nl + 1);
        for (int i=0; i<nl; i++) {
            if (i < len()) r.v[i] += v[i];
            if (i < b.len()) r.v[i] += b.v[i];
            if (r.v[i] >= BIGMOD) {
                r.v[i+1] += r.v[i] / BIGMOD;
                r.v[i] %= BIGMOD;
            }
        }
        r.n();
        return r;
    }
    Bigint operator - (const Bigint &b) const {
        if (s == -1) return -(*this)-(-b);
        if (b.s == -1) return (*this)+(-b);
        if ((*this) < b) return -(b-(*this));
        Bigint r;
        r.resize(len());
        for (int i=0; i<len(); i++) {
            r.v[i] += v[i];
            if (i < b.len()) r.v[i] -= b.v[i];
            if (r.v[i] < 0) {
                r.v[i] += BIGMOD;
                r.v[i+1]--;
            }
        }
        r.n();
        return r;
    }
    Bigint operator * (const Bigint &b) {
        Bigint r;
        r.resize(len() + b.len() + 1);
        r.s = s * b.s;
        for (int i=0; i<len(); i++) {
            for (int j=0; j<b.len(); j++) {
                r.v[i+j] += v[i] * b.v[j];
                if (r.v[i+j] >= BIGMOD) {
                    r.v[i+j+1] += r.v[i+j] / BIGMOD;
                    r.v[i+j] %= BIGMOD;
                }
            }
        }
        r.n();
        return r;
    }
    Bigint operator / (const Bigint &b) {
        Bigint r;
        r.resize(max(1, len()-b.len()+1));
        int oriS = s;
        Bigint b2 = b; // b2 = abs(b)
        s = b2.s = r.s = 1;
        for (int i=r.len()-1; i>=0; i--) {
            int d=0, u=BIGMOD-1;
            while(d<u) {
                int m = (d+u+1)>>1;
                r.v[i] = m;
                if((r*b2) > (*this)) u = m-1;
                else d = m;
            }
            r.v[i] = d;
        }
        s = oriS;
    }

```

```

        r.s = s * b.s;
        r.n();
        return r;
    }
    Bigint operator % (const Bigint &b) {
        return (*this)-(*this)/b*b;
    }
};

```

## 7.4 FFT

```

#include <bits/stdc++.h>
using namespace std;

const int MAXN = 2*262144;
typedef long double ld;
typedef complex<ld> cplx;
const ld PI = acos(-1);
const cplx I(0,1);
cplx omega[MAXN+1];
void pre_fft() {
    for (int i=0; i<=MAXN; i++) {
        omega[i] = exp(i*2*PI/MAXN*I);
    }
}
void fft(int n, cplx a[], bool inv=false) {
    int basic=MAXN/n;
    int theta=basic;
    for (int m=n; m>=2; m>=1) {
        int mh=m>>1;
        for (int i=0; i<mh; i++) {
            cplx w=omega[inv?MAXN-(i*theta%MAXN):i*theta%MAXN];
            for (int j=i; j<n; j+=m) {
                int k=j+mh;
                cplx x=a[j]-a[k];
                a[j] += a[k];
                a[k] = w*x;
            }
            theta = (theta*2)%MAXN;
        }
        int i=0;
        for (int j=1; j<n-1; j++) {
            for (int k=n>>1; k>(i^=k); k>=1) ;
            if (j<i) swap(a[i], a[j]);
        }
        if (inv) {
            for (int i=0; i<n; i++) a[i]/=n;
        }
    }
    cplx a[MAXN], b[MAXN], c[MAXN];
    //how to use :
    /*
    pre_fft();
    fft(n, a);
    fft(n, b);
    for (int i=0; n>i; i++) {
        c[i] = a[i]*b[i];
    }
    fft(n, c, 1);
    */
}

```

## 7.5 NTT

```

// Remember coefficient are mod P
/*
(mod, root)
(65537, 3)
(23068673, 3)
(998244353, 3)
(1107296257, 10)
(2013265921, 31)
(2885681153, 3)
*/
typedef long long ll;
const int maxn = 65536;

struct NTT {
    ll mod = 2013265921, root = 31;
    ll omega[maxn+1];
    void prentt() {
        ll x=fpow(root, (mod-1)/maxn);
        omega[0] = 1;
    }
}

```

```

    for (int i=1;i<=maxn;++i) {
        omega[i] = omega[i-1] * x % mod;
    }
}
void real_init(ll _mod,ll _root) {
    mod = _mod;
    root = _root;
    prentt();
}
ll fpow(ll a,ll n) {
    (n += mod-1) %= mod - 1;
    ll r = 1;
    for (; n; n>>=1) {
        if (n&1) (r*=a)%=mod;
        (a*=a)%=mod;
    }
    return r;
}
void bitrev(vector<ll> &v,int n) {
    int z = __builtin_ctz(n)-1;
    for (int i=0;i<n;++i) {
        int x=i;
        for (int j=0;j<=z;++j) x ^= ((i>>j&1) << (z-j));
        if (x>i) swap(v[x],v[i]);
    }
}
void ntt(vector<ll> &v,int n) {
    bitrev(v,n);
    for (int s=2;s<=n;s<=1) {
        int z = s>>1;
        for (int i=0;i<n;i+=s) {
            for (int k=0;k<z;++k) {
                ll x = v[i+k+z] * omega[maxn/s * k] % mod;
                v[i+k+z] = (v[i+k] + mod - x)%mod;
                (v[i+k] += x) %= mod;
            }
        }
    }
}
void intt(vector<ll> &v,int n) {
    ntt(v,n);
    reverse(v.begin()+1,v.end());
    ll inv = fpow(n,mod-2);
    for (int i=0;i<n;++i) {
        (v[i] *= inv) %= mod;
    }
}
vector<ll> conv(vector<ll> a,vector<ll> b) {
    int sz=1;
    while (sz < a.size() + b.size() - 1) sz <= 1;
    vector<ll> c(sz);
    while (a.size() < sz) a.push_back(0);
    while (b.size() < sz) b.push_back(0);
    ntt(a,sz), ntt(b,sz);
    for (int i=0;i<sz;++i) c[i] = (a[i] * b[i]) % mod;
    intt(c,sz);
    while (c.size() && c.back() == 0) c.pop_back();
    return c;
}
};
ll chinese(ll b1, ll m1, ll b2, ll m2) {
    ll a1 = bigpow(m2,m1-2,m1)*b1 % m1;
    ll a2 = bigpow(m1,m2-2,m2)*b2 % m2;
    ll ret= (a1*m2 + a2*m1)%(m1*m2);
    assert(ret%m1 == b1 && ret%m2 == b2);
    return ret;
}
}

```

## 7.6 FWT

```

void FWT(ll a[],int n){
    for(int d = 1 ;d < n; d <= 1) // d = half of block size
        for(int i = 0; i < n; i += d + d) // every block
            for(int j = i; j < i + d; j++){ //processing
                ll x = a[j], y = a[j + d];
                a[j] = x + y; a[j + d] = x - y; //FWT XOR
                a[j] = x + y; //FWT AND
                a[j + d] = y + x; //FWT OR
                a[j] = (x + y) / 2; a[j + d] = (x - y) / 2;
                //IFWT XOR
                a[j] = x - y; //IFWT AND
                a[j + d] = y - x; //IFWT OR
            }
}
}

```

## 7.7 Subset Convolution

```

for(int i = 0; i <= n; ++ i) {
    // f[__builtin_popcount(s)][s] = s, otherwise = 0. So is g[i]
    FWT(f[i], n) // OR
    FWT(g[i], n) // OR
    for(int s = 0; s < (1 << n); ++ s)
        for(int j = 0; j <= i; ++ j)
            h[i][s] += f[j][s] * g[i - j][s]
    IFWT(h[i], n) // OR
    for(int s = 0; i < (1 << n); ++ s)
        h[__builtin_popcount(s)][s] // is the real answer
}

```

## 7.8 Gaussian Elimination

```

const int GAUSS_MOD = 100000007LL;
struct GAUSS{
    int n;
    vector<vector<int>> v;
    int ppow(int a , int k){
        if(k == 0) return 1;
        if(k % 2 == 0) return ppow(a * a % GAUSS_MOD , k >> 1);
        if(k % 2 == 1) return ppow(a * a % GAUSS_MOD , k >> 1)
            * a % GAUSS_MOD;
    }
    vector<int> solve(){
        vector<int> ans(n);
        REP(now , 0 , n){
            REP(i , now , n) if(v[now][now] == 0 && v[i][now]
                != 0)
                swap(v[i] , v[now]); // det = -det;
            if(v[now][now] == 0) return ans;
            int inv = ppow(v[now][now] , GAUSS_MOD - 2);
            REP(i , 0 , n) if(i != now){
                int tmp = v[i][now] * inv % GAUSS_MOD;
                REP(j , now , n + 1) (v[i][j] += GAUSS_MOD -
                    tmp * v[now][j] % GAUSS_MOD) %= GAUSS_MOD;
            }
            REP(i , 0 , n) ans[i] = v[i][n + 1] * ppow(v[i][i] ,
                GAUSS_MOD - 2) % GAUSS_MOD;
            return ans;
        }
        // gs.v.clear() , gs.v.resize(n , vector<int>(n + 1 , 0));
    }
} gs;

```

## 7.9 Miller Rabin

```

ll mul(ll a,ll b,ll mod) {
    //calculate a*b % mod
    ll r=0; a%=mod; b%=mod;
    while (b) {
        if (b&1) r=(a+r>=mod?a+r-mod:a+r);
        a=(a+a>=mod?a+a-mod:a+a);
        b>>=1;
    }
    return r;
}
ll power(ll a,ll n,ll mod) {
    if (n==0) return 1ll;
    else if (n==1) return a%mod;
    return mul( power(mul(a,a,mod),n/2,mod),n%2?a:1,mod );
}
const bool PRIME = 1, COMPOSITE = 0;
bool miller_robin(ll n,ll a) {
    if (__gcd(a,n) == n) return PRIME;
    if (__gcd(a,n) != 1) return COMPOSITE;
    ll d=n-1,r=0,ret;
    while (d%2==0) {
        r++; d/=2;
    }
    ret = power(a,d,n);
    if (ret==1 || ret==n-1) return PRIME;
    while (r-->0) {
        ret = mul(ret,ret,n);
        if (ret==n-1) return PRIME;
    }
    return COMPOSITE;
}
bool isPrime(ll n) {
    //for int: 2,7,61
}

```

```

ll as[7] = {2,325,9375,28178,450775,9780504,1795265022};
for (int i=0;7>i;i++) {
    if (miller_robin(n,as[i]) == COMPOSITE) return COMPOSITE;
}
return PRIME;
}

```

## 7.10 Pollard Rho

```

map<ll, int> cnt;
void PollardRho(ll n) {
    if (n == 1) return;
    if (isPrime(n)) return ++cnt[n], void();
    if (n % 2 == 0) return PollardRho(n / 2), ++cnt[2], void();
    ll x = 2, y = 2, d = 1, p = 1;
    auto f = [&](auto x, auto n, int p) { return (mul(x, x, n) +
        p) % n; };
    while (true) {
        if (d != n && d != 1) {
            PollardRho(n / d);
            PollardRho(d);
            return;
        }
        if (d == n) ++p;
        x = f(x, n, p); y = f(f(y, n, p), n, p);
        d = __gcd(abs(x - y), n);
    }
}

```

## 7.11 Build phi and mu

```

void build_phi(int ax[], int n){
    for(int i = 1; i <= n; ++i)
        ax[i] = i;
    for(int i = 1; i <= n; ++i)
        for(int j = i + i; j <= n; j += i)
            ax[j] -= ax[i];
}
void build_mu(int ax[], int n){
    for(int i = 1; i <= n; ++i)
        ax[i] = 0;
    ax[1] = 1;
    for(int i = 1; i <= n; ++i)
        for(int j = i + i; j <= n; j += i)
            ax[j] -= ax[i];
}

```

## 7.12 Primitive root

```

#define int int_fast64_t
// build_phi, power, eb
// M has primitive root when M = 2, 4, p^n, 2p^n
ll Primitive_root(ll n) {
    if(n == 2) return 1;
    vector<ll> sol;
    ll val = phi[n];
    for(ll i = 2; i * i <= val; ++i){
        if(val % i == 0){
            sol.eb(i);
            while(val % i == 0) val /= i;
        }
    }
    if(val != 1) sol.eb(val);
    for(ll i = 2; i < n; ++i){
        if(__gcd(i, n) != 1) continue;
        ll ok = 1;
        for(auto to : sol){
            if(power(i, phi[n] / to, n) == 1){
                ok = 0;
                break;
            }
        }
        if(ok)
            return i;
    }
    return -1;
}

```

## 7.13 Cipolla's algorithm

```

struct Cipolla {
    ll p, n, a, w;
    Cipolla(ll _p, ll _n) : p(_p), n(_n){
        n %= p;
        a = -1;
    }
    ll power(ll a, ll x) {
        if(x == 0) return 1;
        return power(a * a % p, x >> 1) * (x & 1 ? a : 1) % p;
    }
    inline int lgd(ll x) {
        return power(x, (p - 1) / 2);
    }
    ll rnd() {
        return ((ll)rand() << 28) + rand();
    }
    pll mul(pll a, pll b) {
        return pll( (a.F * b.F + a.S * b.S % p * w) % p,
            (a.F * b.S + a.S * b.F) % p );
    }
    pll power(pll ii, ll x) {
        if(x == 0) return pll(1, 0);
        return mul(power(mul(ii, ii), x >> 1), (x & 1 ? ii :
            pll(1, 0)));
    }
    ll solve() {
        if(p == 2)
            return n & 1;
        if(lgd(n) == p - 1) return -1;
        if(n == 0) return 0;
        while(a = rnd() % p, lgd((a * a - n + p) % p) == 1);
        w = (a * a - n + p) % p;
        pll ii = power(pll(a, 1), (p + 1) / 2);
        assert(ii.S == 0);
        return ii.F;
    }
};

```

## 7.14 Discrete Log

```

// power
int DiscreteLog_with_s(int s, int x, int y, int m) {
    int kStep = max((int)sqrt(m), 10); // 32000
    unordered_map<int, int> p;
    int b = 1;
    for (int i = 0; i < kStep; ++i) {
        p[y] = i;
        y = 1LL * y * x % m;
        b = 1LL * b * x % m;
    }
    for (int i = 0; i < m + 10; i += kStep) {
        s = 1LL * s * b % m;
        if (p.find(s) != p.end()) return i + kStep - p[s];
    }
    return -1;
}
int DiscreteLog(int x, int y, int m) {
    if (m == 1) return 0;
    // y %= m;
    int s = 1;
    for (int i = 0; i < 70; ++i) {
        if (s == y) return i;
        s = 1LL * s * x % m;
    }
    if (s == y) return 70;
    int p = 70 + DiscreteLog_with_s(s, x, y, m);
    if (power(x, p, m) != y) return -1;
    return p;
}

```

## 7.15 Integer partition

```

void build_partition(int _dp[], int n, int mod){
    _dp[0] = 1;
    for(int i = 1; i <= n; ++i){
        for(int j = 1; j <= n; ++j){
            int tmp = j * (j * 3 - 1) / 2;
            if(tmp > i) break;

```



```

        else if(j % 2 == 1) _dp[i] = (_dp[i] + _dp[i - tmp
        ]) % mod;
        else if(j % 2 == 0) _dp[i] = (_dp[i] - _dp[i - tmp
        + mod) % mod;
    }
    for(int j = 1; j <= n; ++j){
        int tmp = j * (j * 3 + 1) / 2;
        if(tmp > i) break;
        else if(j % 2 == 1) _dp[i] = (_dp[i] + _dp[i - tmp
        ]) % mod;
        else if(j % 2 == 0) _dp[i] = (_dp[i] - _dp[i - tmp
        + mod) % mod;
    }
}
return;
}
}

```

## 7.16 Meissel-Lehmer Algorithm

```

#define MEM1(a) memset( (a) , 0 , sizeof( (a) ) );
const int N = 320000 + 6;
const int C = 10005;
const int D = 306;
LL pi_form[N];
LL phi_form[C][D];
LL p2_form[C][D];
LL p[N];
bool prime[N];
void init() {
    MEM1(phi_form);
    MEM1(p2_form);
    prime[0] = prime[1] = 1;
    int id=1;
    for (int i=2;N>i;i++) {
        if (!prime[i]) {
            for (LL j=i*1LL*i;N>j;j+=i) prime[j] = 1;
            p[id++] = i;
        }
        pi_form[i] = pi_form[i-1] + (!prime[i]);
    }
}
LL pi(LL m);
LL p2(LL m,LL n) {
    //cout<<"p2 = "<<p2_form[m][n]<<endl;
    if (m<C && n<D && p2_form[m][n] != -1) return p2_form[m][n];
    if (p[n] == 0) return 0;
    LL ret = 0, tmp=sqrt(m);
    for (LL i=n+1;p[i] <= tmp;i++) ret += pi(m/p[i]) - pi(p[i])
    + 1;
    if (m < C && n < D) p2_form[m][n] = ret;
    return ret;
}
LL phi2(LL m,LL n) {
    if (m < C && n < D && phi_form[m][n] != -1) return phi_form
    [m][n];
    if (!n) return m;
    if (p[n] >= m) return 1;
    if (m<C && n<D) return phi_form[m][n] = phi2(m,n-1) - phi2
    (m/p[n],n-1);
    return phi2(m,n-1) - phi2(m/p[n],n-1);
}
LL pi(LL m) {
    if (m < N) return pi_form[m];
    else {
        LL n=ceil(cbrt(m));
        return phi2(m,n) + n - 1 - p2(m,n);
    }
}
//init(); cin >> n; cout << pi(n); (n <= 10^11)
...

```

## 7.17 De Bruijn

```

int res[maxn], aux[maxn], a[maxn], sz;
void db(int t, int p, int n, int k) {
    if (sz >= tg) return;
    if (t > n) {
        if (n % p == 0) {
            for (int i = 1; i <= p && sz < tg; ++i) res[sz++] =
            aux[i];
        }
    }
}

```

```

    }
} else {
    aux[t] = aux[t - p];
    db(t + 1, p, n, k);
    for (int i = aux[t - p] + 1; i < k; ++i) {
        aux[t] = i;
        db(t + 1, t, n, k);
    }
}
}
int de_bruijn(int k, int n) {
    // return cyclic string of length k^n such that every
    // string of length n using k character appears as a
    // substring.
    if (k == 1) {
        res[0] = 0;
        return 1;
    }
    for (int i = 0; i < k * n; i++) aux[i] = 0;
    sz = 0;
    db(1, 1, n, k);
    return sz;
}

```

## 7.18 Simplex Algorithm

```

/*
maximize Cx under
Ax <=b
x >= 0
b >= 0
n variables
m constraints
A is m by n
*/
const int MAX = 45;
int n, m;
double arr[MAX][MAX];
bool pro(){
    double mi = 0;
    int x = 1;
    for(int i = 1; i <= n + m; i++) if(arr[0][i] < mi){
        mi = arr[0][i];
        x = i;
    }
    if(abs(mi) < eps) return 0; // sigma <= 0
    mi = INF; // theta
    int y = 0;
    for(int i = 1; i <= m; i++){
        if(arr[i][x] > eps && arr[i][n + m + 1] / arr[i][x] <
        mi) {
            mi = arr[i][n + m + 1] / arr[i][x];
            y = i;
        }
    }
    assert(y);
    double weed = arr[y][x];
    for(int i = 1; i <= n + m + 1; ++i)
        arr[y][i] /= weed;
    // now arr[y][n + m + 1] == theta
    for(int i = 0; i <= m; i++){
        if(i == y) continue;
        double f = arr[i][x];
        for(int j = 1; j <= m + n + 1; j++)
            arr[i][j] -= f * arr[y][j];
    }
    return 1;
}
int main(){
    cin >> n;
    cin >> m;
    memset(arr, 0, sizeof arr);
    // input C
    for(int i = 1; i <= n; i++){
        cin >> arr[0][i];
        arr[0][i] = - arr[0][i];
    }
    for(int i = 1; i <= m; i++){
        // input A
        for(int j = 1; j <= n; j++)
            cin >> arr[i][j];
        arr[i][n + i] = 1;
        // input b
        cin >> arr[i][n + m + 1];
    }
}

```

```

    }
    while(pro());
    cout << arr[0][n + m + 1] << "\n";
    return 0;
}

```

## 8 String

### 8.1 string tools

```

const KMP_SIZE = ;
struct KMP{
    string s;
    int f[KMP_SIZE] , pos;
    void solve(){
        f[0] = pos = -1;
        REP(i , 1 , s.size()){
            while(pos != -1 && s[pos + 1] != s[i]) pos = f[pos];
            if(s[pos + 1] == s[i]) pos ++;
            f[i] = pos;
        }
    }
};
const int ZVALUE_SIZE = ;
struct Z_VALUE{
    string s;
    int l = 0 , r = 0 , z[ZVALUE_SIZE];
    void solve(){
        REP(i , 0 , s.size()){
            z[i] = max(min(z[i - 1] , r - i) , 0LL);
            while(i + z[i] < s.size() && s[z[i]] == s[i + z[i]]){
                l = i , r = i + z[i];
                z[i] ++;
            }
        }
    }
};
const int PALINDROME_MAX = 2 * ;
struct Palindrome{
    string s , ss; // ss = input
    int z[PALINDROME_MAX];
    void solve(){
        s.resize(ss.size() + ss.size() + 1 , '.');
        REP(i , 0 , ss.size()) s[i + i + 1] = ss[i];
        int l = 0 , r = 0;
        REP(i , 0 , s.size()){
            z[i] = max(min(z[l + l - i] , r - i) , 1);
            while(i - z[i] >= 0 && i + z[i] < s.size() && s[i - z[i]] == s[i + z[i]]){
                l = i , r = i + z[i];
                z[i] ++;
            }
        }
    }
};

```

### 8.2 Aho–Corasick algorithm

```

struct AC_Automata {
    static const int N = 2e4 + 6;
    static const int SIGMA = 26;
    int ch[N][SIGMA] , val[N] , sz;
    int last[N] , fail[N];
    int que[N] , qs , qe , cnt[N];
    void init() {
        sz = 1;
        memset(ch[0] , 0 , sizeof(ch[0]));
        qs = qe = 0;
        memset(cnt , 0 , sizeof(cnt)); memset(val , 0 , sizeof(val));
        memset(last , 0 , sizeof(last));
    }
    int idx(char c) {
        return c - 'a';
    }
    int insert(string s , int v) {
        int now = 0;
        int n = s.size();
        for (int i = 0; i < n; ++i) {

```

```

            int c = idx(s[i]);
            if (!ch[now][c]) {
                memset(ch[sz] , 0 , sizeof(ch[sz]));
                val[sz] = 0; ch[now][c] = sz++;
            }
            now = ch[now][c];
        }
        val[now] = v;
        return now;
    }
    void print(int j) {
        if (j) {
            //now we match string v[j]
            print(last[j]); //may match multiple strings
        }
    }
    void getFail() {
        qs = 0 , qe = 0; fail[0] = 0;
        for (int c = 0; c < SIGMA; c++) {
            int now = ch[0][c];
            if (now) {
                fail[now] = 0;
                que[qe++] = now;
                last[now] = 0;
            }
        }
        while (qs != qe) {
            int t = que[qe++];
            for (int c = 0; c < SIGMA; c++) {
                int now = ch[t][c];
                if (!now) continue;
                que[qe++] = now;
                int v = fail[t];
                while (v && !ch[v][c]) v = fail[v];
                fail[now] = ch[v][c];
                last[now] = val[ fail[now] ]? fail[now]:last[ fail[now] ];
            }
        }
    }
    void Find(string s) {
        getFail();
        int n = s.size() , now = 0;
        for (int i = 0; i < n; i++) {
            int c = idx(s[i]);
            while (now && !ch[now][c]) now = fail[now];
            now = ch[now][c];
            cnt[now]++;
        }
        for (int i = qe - 1; i >= 0; i--) {
            cnt[ fail[que[i]] ] += cnt[ que[i] ];
        }
    }
    void AC_evolution() {
        for (qs = 1; qs != qe; ) {
            int now = que[qs++];
            for (int i = 0; i < SIGMA; i++) {
                if (ch[now][i] == 0) ch[now][i] = ch[fail[now]][i];
            }
        }
    }
} ac;

```

```

const int N = 156;
string s[N];
int ed[N];

ac.init();
ac.insert(s[i] , i);
ac.Find();
ac.cnt[ ac.insert(s[i] , i) ];

```

### 8.3 Suffix array

```

const int SA_SIZE = ;
const int logn = 1 + ;
string s;
int sa[SA_SIZE] , rk[SA_SIZE] , lcp[SA_SIZE];
int tma[2][SA_SIZE] , c[SA_SIZE] , sp[SA_SIZE][logn];

int getsa(){
    -> update m = ? // how many char
    int *x = tma[0] , *y = tma[1] , n = s.size() , m = 200;

```

```

REP(i , 0 , m) c[i] = 0;
REP(i , 0 , n) c[x[i]] = s[i] ++;
REP(i , 1 , m) c[i] += c[i - 1];
RREP(i , n - 1 , 0) sa[--c[x[i]]] = i;
for(int k = 1 ; k <= n ; k <= 1){
    REP(i , 0 , m) c[i] = 0;
    REP(i , 0 , n) c[x[i]] ++;
    REP(i , 1 , m) c[i] += c[i - 1];
    int p = 0;
    REP(i , n - k , n) y[p++] = i;
    REP(i , 0 , n) if(sa[i] >= k) y[p++] = sa[i] - k;
    RREP(i , n - 1 , 0) sa[--c[x[y[i]]]] = y[i];
    y[sa[0]] = p = 0;
    REP(i , 1 , n) {
        if( x[sa[i]] == x[sa[i - 1]] && sa[i] + k < n && sa
            [i - 1] + k < n &&
            x[sa[i] + k] == x[sa[i - 1] + k] );
        else p ++;
        y[sa[i]] = p;
    }
    swap(x , y);
    if(p + 1 == n) break;
    m = p + 1;
}
}
void getlcp(){
    int tmp = 0 , n = s.size();
    REP(i , 0 , n) rk[sa[i]] = i;
    REP(i , 0 , n){
        if(rk[i] == 0) lcp[0] = 0;
        else {
            if(tmp) tmp --;
            int po = sa[rk[i] - 1];
            while(tmp + po < n && tmp + i < n && s[tmp + i] ==
                s[tmp + po]) tmp ++;
            lcp[rk[i]] = tmp;
        }
    }
}
void getsp(){
    int n = s.size();
    REP(i , 0 , n) sp[rk[i]][0] = s.size() - i;
    REP(i , 1 , n) sp[i - 1][1] = lcp[i];
    REP(i , 2 , logn){
        REP(j , 0 , n){
            if(j + (1 << (i - 2)) >= s.size()) continue;
            sp[j][i] = min(sp[j][i - 1] , sp[j + (1 << (i - 2))
                ][i - 1]);
        }
    }
}
int Query(int L , int R){
    int tmp = (L == R) ? 0 : 32 - __builtin_clz(R - L);
    if(tmp == 0) return sp[L][0];
    else return min(sp[L][tmp] , sp[R - (1 << (tmp - 1))][tmp]);
}
int Find(string ss){
    int L = 0 , R = s.size() , now;
    while(R - L > 1){
        now = (L + R) / 2;
        if(s[sa[now]] == ss[0]) break;
        else if(s[sa[now]] > ss[0]) R = now;
        else if(s[sa[now]] < ss[0]) L = now;
    }
    if(s[sa[now]] != ss[0]) return 0;
    REP(i , 1 , ss.size()){
        int pre = now , ty = 0;
        if(sa[now] + i >= s.size()) L = now , ty = 0;
        else if(s[sa[now] + i] == ss[i]) continue;
        else if(s[sa[now] + i] > ss[i]) R = now , ty = 1;
        else if(s[sa[now] + i] < ss[i]) L = now , ty = 0;

        while(R - L > 1){
            now = (L + R) / 2;
            if(sa[now] + i >= s.size()){
                if(ty == 0) R = now;
                if(ty == 1) L = now;
            }
            else if(ty == 0 && Query(pre , now) < i) R = now;
            else if(ty == 1 && Query(now , pre) < i) L = now;
            else if(s[sa[now] + i] == ss[i]) break;
            else if(s[sa[now] + i] > ss[i]) R = now;
            else if(s[sa[now] + i] < ss[i]) L = now;
        }
        if(sa[now] + i >= s.size()) return 0;
    }
}

```

```

        if(s[sa[now] + i] != ss[i]) return 0;
    }
    L = now , R = now;
    RREP(i , 19 , 0){
        if(R + (1 << i) >= s.size()) continue;
        else if(Query(L , R + (1 << i)) >= ss.size()) R += (1
            << i);
    }
    RREP(i , 19 , 0){
        if(L - (1 << i) < 0) continue;
        else if(Query(L - (1 << i) , R) >= ss.size()) L -= (1
            << i);
    }
    return R - L + 1;
}
/*
how to use :
1. cin >> s;
2. getsa() , getlcp() , getsp();
3. string ss;
4. cin >> ss;
5. cout << Find(ss) << endl;
*/

```

## 8.4 Lexicographically Smallest Rotation

```

string s;
const int N = 4000006;
int f[N];
void solve() {
    s = s + s;
    int n = (int)s.size();
    for (int i=0;i<n;++i) f[i] = -1;
    int k=0;
    for (int j=1;j<n;++j) {
        char sj = s[j];
        int i = f[j-k-1];
        while (i != -1 && sj != s[k+i+1]) {
            if (sj < s[k+i+1]) {
                k = j-i-1;
            }
            i = f[i];
        }
        if (sj != s[k+i+1]) {
            if (sj < s[k]) {
                k = j;
            }
            f[j-k] = -1;
        }
        else f[j-k] = i+1;
    }
    n>>=1;
    if (k >= n) k-= n;
    for (int i=k;i<k+n;++i) {
        cout << s[i];
    }
    cout << endl;
}

```

## 9 Geometry

### 9.1 Circle

```

//Note that this code will crash if circle A and B are the same
typedef pair<double, double> pdd;
pdd rtcw(pdd p){return pdd(p.Y, -p.X); }
vector<pdd> circlesintersect(pdd A, pdd B, double r1, double r2
    ){
    vector<pdd> ret;
    double d = dis(A, B);
    if(d > r1 + r2 || d + min(r1, r2) < max(r1, r2))
        return ret;
    double x = (d * d + r1 * r1 - r2 * r2) / (2 * d);
    double y = sqrt(r1 * r1 - x * x);
    pdd v = (B - A) / d;
    ret.eb(A + v * x + rtcw(v) * y);
    if(y > 0)
        ret.eb(A + v * x - rtcw(v) * y);
    return ret;
}

```

## 9.2 Half Plane Intersection

```

Pt interPnt( Line l1, Line l2, bool &res ){
    Pt p1, p2, q1, q2;
    tie(p1, p2) = l1; tie(q1, q2) = l2;
    double f1 = (p2 - p1) ^ (q1 - p1);
    double f2 = (p2 - p1) ^ (q1 - p2);
    double f = (f1 + f2);
    if( fabs(f) < eps){ res=0; return {0, 0}; }
    res = true;
    return q1 * (f2 / f) + q2 * (f1 / f);
}

bool isin( Line l0, Line l1, Line l2 ){
    // Check inter(l1, l2) in l0
    bool res; Pt p = interPnt(l1, l2, res);
    return ( (l0.SE - l0.FI) ^ (p - l0.FI) ) > eps;
}

/* If no solution, check: 1. ret.size() < 3
 * Or more precisely, 2. interPnt(ret[0], ret[1])
 * in all the lines. (use (l.S - l.F) ^ (p - l.F) > 0
 */
/* --^-- Line.FI --^-- Line.SE --^-- */
vector<Line> halfPlaneInter( vector<Line> lines ){
    int sz = lines.size();
    vector<double> ata(sz), ord(sz);
    for( int i=0; i<sz; i++) {
        ord[i] = i;
        Pt d = lines[i].SE - lines[i].FI;
        ata[i] = atan2(d.Y, d.X);
    }
    sort( ord.begin(), ord.end(), [&](int i, int j) {
        if( fabs(ata[i] - ata[j]) < eps )
            return ( (lines[i].SE - lines[i].FI) ^
                    (lines[j].SE - lines[j].FI) ) < 0;
        return ata[i] < ata[j];
    });
    vector<Line> fin;
    for( int i=0; i<sz; i++)
        if (!i or fabs(ata[ord[i]] - ata[ord[i-1]]) > eps)
            fin.pb(lines[ord[i]]);
    deque<Line> dq;
    for( int i=0; i<(int)(fin.size()); i++) {
        while((int)(dq.size()) >= 2 and
            not isin(fin[i], dq[(int)(dq.size()-2)],
                dq[(int)(dq.size()-1)]))
            dq.pop_back();
        while((int)(dq.size()) >= 2 and
            not isin(fin[i], dq[0], dq[1]))
            dq.pop_front();
        dq.push_back(fin[i]);
    }
    while( (int)(dq.size()) >= 3 and
        not isin(dq[0], dq[(int)(dq.size()-2)],
            dq[(int)(dq.size()-1)]))
        dq.pop_back();
    while( (int)(dq.size()) >= 3 and
        not isin(dq[(int)(dq.size()-1)], dq[0], dq[1]))
        dq.pop_front();
    vector<Line> res(dq.begin(), dq.end());
    return res;
}

```

## 9.3 Convex Hull 3D

```

#define SIZE(X) (int(X.size()))
#define PI 3.14159265358979323846264338327950288
struct Pt{
    Pt cross(const Pt &p) const
    { return Pt(y * p.z - z * p.y, z * p.x - x * p.z, x * p.y -
        y * p.x); }
} info[N];
int mark[N][N], n, cnt;;
double mix(const Pt &a, const Pt &b, const Pt &c)
{ return a * (b ^ c); }
double area(int a, int b, int c)
{ return norm((info[b] - info[a]) ^ (info[c] - info[a])); }
double volume(int a, int b, int c, int d)
{ return mix(info[b] - info[a], info[c] - info[a], info[d] -
    info[a]); }
struct Face{
    int a, b, c; Face(){
        Face(int a, int b, int c): a(a), b(b), c(c) {}
        int &operator [] (int k)
        { if (k == 0) return a; if (k == 1) return b; return c; }
    }
}

```

```

};
vector<Face> face;
void insert(int a, int b, int c)
{ face.push_back(Face(a, b, c)); }
void add(int v) {
    vector<Face> tmp; int a, b, c; cnt++;
    for( int i = 0; i < SIZE(face); i++) {
        a = face[i][0]; b = face[i][1]; c = face[i][2];
        if(Sign(volume(v, a, b, c)) < 0)
            mark[a][b] = mark[b][a] = mark[b][c] = mark[c][b] =
                mark[c][a] = mark[a][c] = cnt;
        else tmp.push_back(face[i]);
    } face = tmp;
    for( int i = 0; i < SIZE(tmp); i++) {
        a = face[i][0]; b = face[i][1]; c = face[i][2];
        if (mark[a][b] == cnt) insert(b, a, v);
        if (mark[b][c] == cnt) insert(c, b, v);
        if (mark[c][a] == cnt) insert(a, c, v);
    }
}
int Find(){
    for( int i = 2; i < n; i++) {
        Pt ndir = (info[0] - info[i]) ^ (info[1] - info[i]);
        if (ndir == Pt()) continue; swap(info[i], info[2]);
        for( int j = i + 1; j < n; j++) if (Sign(volume(0, 1,
            2, j)) != 0) {
            swap(info[j], info[3]); insert(0, 1, 2); insert(0,
                2, 1); return 1;
        } } return 0; }
int main() {
    for( ; scanf("%d", &n) == 1; ) {
        for( int i = 0; i < n; i++) info[i].Input();
        sort(info, info + n); n = unique(info, info + n) - info;
        face.clear(); random_shuffle(info, info + n);
        if (Find()) { memset(mark, 0, sizeof(mark)); cnt = 0;
            for( int i = 3; i < n; i++) add(i); vector<Pt> Ndir;
            for( int i = 0; i < SIZE(face); ++i) {
                Pt p = (info[face[i][0]] - info[face[i][1]]) ^
                    (info[face[i][2]] - info[face[i][1]]);
                p = p / norm(p); Ndir.push_back(p);
            } sort(Ndir.begin(), Ndir.end());
            int ans = unique(Ndir.begin(), Ndir.end()) - Ndir.
                begin();
            printf("%d\n", ans);
        } else printf("1\n");
    } }
double calcDist(const Pt &p, int a, int b, int c)
{ return fabs(mix(info[a] - p, info[b] - p, info[c] - p) / area
    (a, b, c)); }
//compute the minimal distance of center of any faces
double findDist() { //compute center of mass
    double totalWeight = 0; Pt center(.0, .0, .0);
    Pt first = info[face[0][0]];
    for( int i = 0; i < SIZE(face); ++i) {
        Pt p = (info[face[i][0]] + info[face[i][1]] + info[face[i]
            ][2]] + first) * .25;
        double weight = mix(info[face[i][0]] - first, info[face
            ][i][1]
            - first, info[face[i][2]] - first);
        totalWeight += weight; center = center + p * weight;
    } center = center / totalWeight;
    double res = 1e100; //compute distance
    for( int i = 0; i < SIZE(face); ++i)
        res = min(res, calcDist(center, face[i][0], face[i][1],
            face[i][2]));
    return res; }

```

## 9.4 Convex Hull

```

/* Given a convexhull, answer queries in O(\lg N)
 * CH should not contain identical points, the area should
 * be > 0, min pair(x, y) should be listed first */
double det( const Pt& p1 , const Pt& p2 )
{ return p1.X * p2.Y - p1.Y * p2.X; }
struct Conv{
    int n;
    vector<Pt> a;
    vector<Pt> upper, lower;
    Conv(vector<Pt> _a) : a(_a){
        n = a.size();
        int ptr = 0;
        for(int i=1; i<n; ++i) if (a[ptr] < a[i]) ptr = i;
        for(int i=0; i<=ptr; ++i) lower.push_back(a[i]);
    }
}

```

```

    for(int i=ptr; i<n; ++i) upper.push_back(a[i]);
    upper.push_back(a[0]);
}
int sign(LL x){ // fixed when changed to double
    return x < 0 ? -1 : x > 0; }
pair<LL,int> get_tang(vector<Pt> &conv, Pt vec){
    int l = 0, r = (int)conv.size() - 2;
    for(; l + 1 < r; ){
        int mid = (l + r) / 2;
        if(sign(det(conv[mid+1]-conv[mid],vec))>0)r=mid;
        else l = mid;
    }
    return max(make_pair(det(vec, conv[r]), r),
        make_pair(det(vec, conv[0]), 0));
}
void upd_tang(const Pt &p, int id, int &i0, int &i1){
    if(det(a[i0] - p, a[id] - p) > 0) i0 = id;
    if(det(a[i1] - p, a[id] - p) < 0) i1 = id;
}
void bi_search(int l, int r, Pt p, int &i0, int &i1){
    if(l == r) return;
    upd_tang(p, l % n, i0, i1);
    int sl=sign(det(a[l % n] - p, a[(l + 1) % n] - p));
    for(; l + 1 < r; ){
        int mid = (l + r) / 2;
        int smid=sign(det(a[mid%n]-p, a[(mid+1)%n]-p));
        if (smid == sl) l = mid;
        else r = mid;
    }
    upd_tang(p, r % n, i0, i1);
}
int bi_search(Pt u, Pt v, int l, int r) {
    int sl = sign(det(v - u, a[l % n] - u));
    for(; l + 1 < r; ){
        int mid = (l + r) / 2;
        int smid = sign(det(v - u, a[mid % n] - u));
        if (smid == sl) l = mid;
        else r = mid;
    }
    return l % n;
}
// 1. whether a given point is inside the CH
bool contain(Pt p) {
    if (p.X < lower[0].X || p.X > lower.back().X) return 0;
    int id = lower_bound(lower.begin(), lower.end(), Pt(p.X,
        -INF)) - lower.begin();
    if (lower[id].X == p.X) {
        if (lower[id].Y > p.Y) return 0;
    }else if(det(lower[id]-p,lower[id-1]-p)<0)return 0;
    id = lower_bound(upper.begin(), upper.end(), Pt(p.X,
        INF), greater<Pt>()) - upper.begin();
    if (upper[id].X == p.X) {
        if (upper[id].Y < p.Y) return 0;
    }else if(det(upper[id]-p,upper[id-1]-p)<0)return 0;
    return 1;
}
// 2. Find 2 tang pts on CH of a given outside point
// return true with i0, i1 as index of tangent points
// return false if inside CH
bool get_tang(Pt p, int &i0, int &i1) {
    if (contain(p)) return false;
    i0 = i1 = 0;
    int id = lower_bound(lower.begin(), lower.end(), p) -
        lower.begin();
    bi_search(0, id, p, i0, i1);
    bi_search(id, (int)lower.size(), p, i0, i1);
    id = lower_bound(upper.begin(), upper.end(), p, greater
        <Pt>()) - upper.begin();
    bi_search((int)lower.size() - 1, (int)lower.size() - 1
        + id, p, i0, i1);
    bi_search((int)lower.size() - 1 + id, (int)lower.size()
        - 1 + (int)upper.size(), p, i0, i1);
    return true;
}
// 3. Find tangent points of a given vector
// ret the idx of vertex has max cross value with vec
int get_tang(Pt vec){
    pair<LL, int> ret = get_tang(upper, vec);
    ret.second = (ret.second+(int)lower.size()-1)%n;
    ret = max(ret, get_tang(lower, vec));
    return ret.second;
}
// 4. Find intersection point of a given line
// return 1 and intersection is on edge (i, next(i))
// return 0 if no strictly intersection
bool get_intersection(Pt u, Pt v, int &i0, int &i1){

```

```

    int p0 = get_tang(u - v), p1 = get_tang(v - u);
    if(sign(det(v-u,a[p0]-u))*sign(det(v-u,a[p1]-u))<0){
        if (p0 > p1) swap(p0, p1);
        i0 = bi_search(u, v, p0, p1);
        i1 = bi_search(u, v, p1, p0 + n);
        return 1;
    }
    return 0;
}
};
};

```

## 9.5 Polar Angle Sort

```

bool cmp(vec a,vec b){
    if((a.Y>0||(a.Y==0&&a.X>0))&&(b.Y<0||(b.Y==0&&b.X<0)))
        return 1;
    if((b.Y>0||(b.Y==0&&b.X>0))&&(a.Y<0||(a.Y==0&&a.X<0)))
        return 0;
    return (a^b)>0;
}

```

## 9.6 Circle and Polygon intersection

```

struct Circle_and_Segment_Intersection {
    const ld eps = 1e-9;
    vector<pdd> solve(pdd p1, pdd p2, pdd cen, ld r) {
        //please notice that p1 != p2
        //condiser p = p2 + (p1 - p2) * t, 0 <= t <= 1
        vector<pdd> ret;
        p1 = p1 - cen; p2 = p2 - cen;
        ld a = (p1 - p2) * (p1 - p2);
        ld b = 2 * (p2 * (p1 - p2));
        ld c = p2 * p2 - r * r;
        ld bb4ac = b * b - 4 * a * c;
        if (bb4ac < -eps) return ret; //no intersection
        vector<ld> ts;
        if ( (bb4ac) <= eps) {
            ts.push_back(-b / 2 / a);
        }
        else {
            ts.push_back( (-b + sqrt(bb4ac)) / (a * 2) );
            ts.push_back( (-b - sqrt(bb4ac)) / (a * 2) );
        }
        sort(ts.begin(), ts.end());
        for (ld t: ts) {
            if (-eps <= t && t <= 1 + eps) {
                t = max(t, 0.0);
                t = min(t, 1.0);
                pdd pt = p2 + t * (p1 - p2);
                pt = pt + cen;
                ret.push_back(pt);
            }
        }
        return ret;
    }
} solver;

double f(ld a, ld b) {
    ld ret = b - a;
    while (ret <= -pi - eps) ret += 2 * pi;
    while (ret >= pi + eps) ret -= 2 * pi;
    return ret;
}

ld solve_small(pdd cen, ld r, pdd p1, pdd p2) {
    p1 = p1 - cen, p2 = p2 - cen;
    cen = {0, 0};
    vector<pdd> inter = solver.solve(p1, p2, cen, r);
    ld ret = 0.0;
    if ((int)inter.size() == 0) {
        if (in_cir(cen, r, p1)) {
            ret = (p1 ^ p2) / 2;
        }
        else {
            ret = (r * r * f(atan2(p1.Y, p1.X), atan2(p2.Y, p2.X))) / 2;
        }
    }
    else if ( (int)inter.size() == 1) {
        if (!in_cir(cen, r, p1) && !in_cir(cen, r, p2)) {
            //outside cut
            ret = (r * r * f(atan2(p1.Y, p1.X), atan2(p2.Y, p2.X))) / 2;
        }
    }
}

```

```

    else if (!in_cir(cen, r, p1)) {
        pdd _p1 = inter[0];
        ret += ((_p1 ^ p2) / 2);
        ret += (r * r * f(atan2(p1.Y, p1.X), atan2(_p1.Y,
            _p1.X))) / 2;
    }
    else if (!in_cir(cen, r, p2)) {
        pdd _p2 = inter[0];
        ret += ((p1 ^ _p2) / 2);
        ret += (r * r * f(atan2(_p2.Y, _p2.X), atan2(p2.Y,
            p2.X))) / 2;
    }
}
else if ((int)inter.size() == 2) {
    pdd _p2 = inter[0], _p1 = inter[1];
    ret += ((_p1 ^ _p2) / 2);
    ret += (r * r * f(atan2(_p2.Y, _p2.X), atan2(p2.Y, p2.
        X))) / 2;
    ret += (r * r * f(atan2(p1.Y, p1.X), atan2(_p1.Y, _p1.
        X))) / 2;
}
return ret;
}
}
ld solve(pdd cen, ld r, vector<pdd> pts) {
    ld ret = 0;
    for (int i = 0; i < (int)pts.size(); ++i) {
        ret += solve_small(cen, r, pts[i], pts[(i + 1) % int(
            pts.size())]);
    }
    ret = max(ret, -ret);
    return ret;
}
}

```

## 9.7 Line Intersection

```

int intersect(PII a, PII b, PII c, PII d){
    if(max(a.F, b.F) < min(c.F, d.F)) return 0;
    if(max(c.F, d.F) < min(a.F, b.F)) return 0;
    if(max(a.S, b.S) < min(c.S, d.S)) return 0;
    if(max(c.S, d.S) < min(a.S, b.S)) return 0;
    if(cross(b - a, c - a) * cross(b - a, d - a) == 1) return
        0;
    if(cross(d - c, a - c) * cross(d - c, b - c) == 1) return
        0;
    return 1;
}
}

```

## 9.8 Line Intersection Point

```

pdd intersect(pdd p1, pdd p2, pdd q1, pdd q2) {
    //make sure that p1p2 is not parallel to q1q2
    return p1 + ((q1 - p1) ^ (q2 - q1)) / ((p2 - p1) ^ (q2 - q1
        )) * (p2 - p1);
}
}

```

## 9.9 Rotating Calipers

```

#define NXT(x) ((x + 1) % m)
int main () {
    vector<pii> v; // v is the input points
    sort(v.begin(), v.end());
    vector<pii> up, down;
    for (pii p: v) {
        while (SZ(down) >= 2 && sgn((p - down[SZ(down) - 2]) ^
            (p - down.back())) >= 0) {
            down.pop_back();
        }
        down.push_back(p);
    }
    reverse(v.begin(), v.end());
    for (pii p: v) {
        while (SZ(up) >= 2 && sgn((p - up[SZ(up) - 2]) ^ (p -
            up.back())) >= 0) {
            up.pop_back();
        }
        up.push_back(p);
    }
    vector<pii> all;
    for (pii p: down) { all.push_back(p); } all.pop_back();
}

```

```

for (pii p: up) { all.push_back(p); }
all.pop_back();
int m = all.size();
int ptr = (int)down.size() - 1;
for (int i = 0; i < m; ++i) {
    while (((all[NXT(ptr)] - all[ptr]) ^ (all[NXT(i)] - all
        [i])) > 0) {
        ptr = NXT(ptr);
    }
}
}
}

```

# 10 Boook

## 10.1 Block Tree

//Query on Tree 1, SPOJ

```

int t, n, m, N = 100;
vector<int> v[MAX], g[MAX];
int pa[MAX], dep[MAX], val[MAX];
int siz[MAX], id[MAX], mm[MAX];
void init(){
    REP(i, 0, n + 1) id[i] = 0;
    REP(i, 0, n + 1) v[i].clear();
    REP(i, 0, n + 1) g[i].clear();
}
void DFS(int now, int fa, int deep){
    pa[now] = fa, dep[now] = deep;
    if(id[now] == 0) siz[id[now] = now] = 1;
    for(auto to : v[now]){
        if(to == fa) continue;
        if(siz[id[now]] + 1 < N){
            g[now].pb(to);
            siz[id[to] = id[now]] ++;
        }
        DFS(to, now, deep + 1);
    }
}
void build(int now, int v){
    mm[now] = max(v, val[now]);
    for(auto to : g[now]){
        build(to, mm[now]);
    }
}
int query(int a, int b){
    int res = 0;
    while(a != b){
        if(id[a] == id[b]){
            if(dep[a] < dep[b]) swap(a, b);
            res = max(res, val[a]);
            a = pa[a];
        }
        else {
            if(dep[id[a]] < dep[id[b]]) swap(a, b);
            res = max(res, mm[a]);
            a = pa[id[a]];
        }
    }
    return res;
}
int x[MAX][3];
char c[MAX];
int32_t main(){
    scanf("%d", &t);
    REP(times, 0, t){
        scanf("%d", &n);
        init();
        REP(i, 1, n){
            REP(j, 0, 3) scanf("%d", &x[i][j]);
            v[x[i][0]].pb(x[i][1]);
            v[x[i][1]].pb(x[i][0]);
        }
        DFS(1, 0, 0);
        REP(i, 1, n){
            if(dep[x[i][0]] > dep[x[i][1]]) val[x[i][0]] = x[i
                ][2];
            else val[x[i][1]] = x[i][2];
        }
        REP(i, 1, n + 1){
            if(id[i] == i) build(i, -INF);
        }
        int q, w, tmp;
    }
}

```



```

while(scanf("%s",c) == 1){
    if(c[0] == 'D') break;
    scanf("%d%d", &q , &w);
    if(c[0] == 'C'){
        if(dep[x[q][0]] > dep[x[q][1]]) val[x[q][0]] =
            w , tmp = x[q][0];
        else val[x[q][1]] = w , tmp = x[q][1];
        if(tmp == id[tmp]) build(tmp , -INF);
        else build(tmp , mm[pa[tmp]]);
    }
    else if(c[0] == 'Q'){
        printf("%d\n", query(q , w));
    }
}
}
return 0;
}

```

## 10.2 Dancing Link

```

#define MAX 1050
#define INF 0x3f3f3f3f
struct DLX{
    int n , sz , s[MAX];
    int row[MAX * 100] , col[MAX * 100];
    int l[MAX * 100] , r[MAX * 100] , u[MAX * 100] , d[MAX * 100];
    int ans;
    void init(int n){
        this -> n = n;
        ans = INF;
        REP(i , 0 , n + 1){
            u[i] = d[i] = i;
            l[i] = i - 1;
            r[i] = i + 1;
        }
        r[n] = 0 , l[0] = n;
        sz = n + 1;
        MEM(s , 0);
    }
    void AddRow(int rr , vector<int> sol){
        int tmp = sz;
        for(auto to : sol){
            l[sz] = sz - 1;
            r[sz] = sz + 1;
            d[sz] = to;
            u[sz] = u[to];
            d[u[to]] = sz , u[to] = sz;
            row[sz] = rr , col[sz] = to;
            s[to] ++ , sz ++;
        }
        r[sz - 1] = tmp , l[tmp] = sz - 1;
    }
#define FOR(i , way , to) for(int i = way[to] ; i != to ; i = way[i])
    void remove(int c){
        l[r[c]] = l[c];
        r[l[c]] = r[c];
        FOR(i , d , c) FOR(j , r , i){
            u[d[j]] = u[j];
            d[u[j]] = d[j];
            --s[col[j]];
        }
    }
    int restore(int c){
        FOR(i , u , c) FOR(j , l , i){
            ++s[col[j]];
            u[d[j]] = j;
            d[u[j]] = j;
        }
        l[r[c]] = c;
        r[l[c]] = c;
    }
    void DFS(int floor){
        if(r[0] == 0){
            ans = min(ans , floor);
            return;
        }
        if(floor >= ans) return;
        int c = r[0];
        FOR(i , r , 0) if(s[i] < s[c]) c = i;
        remove(c);
        FOR(i , d , c){
            FOR(j , r , i) remove(col[j]);

```

```

DFS(floor + 1);
        FOR(j , l , i) restore(col[j]);
    }
    restore(c);
}
} solver;
int n , m;
int32_t main(){
    IOS;
    while(cin >> n >> m){
        solver.init(m);
        REP(i , 0 , n){
            int nn , in;
            cin >> nn;
            vector<int> sol;
            REP(j , 0 , nn) cin >> in , sol.pb(in);
            solver.AddRow(i , sol);
        }
        solver.DFS(0);
        if(solver.ans == INF) cout << "No" << endl;
        else cout << solver.ans << endl;
    }
    return 0;
}

```

## 10.3 Joseph Problem

```

int main() {
    long long n , k , i , x = 0 , y;
    scanf( "%I64d%I64d", &n, &k );
    for( i = 2; i <= k && i <= n; ++i ) x = ( x + k ) % i;
    for( ; i <= n; ++i ) {
        y = ( i - x - 1 ) / k;
        if( i + y > n ) y = n - i;
        i += y;
        x = ( x + ( y + 1 ) % i * k ) % i;
    }
    printf( "%I64d\n", x + 1 );
    return 0;
}

```

## 10.4 Middle Speed Linear Recursion

```

#define MAX 100000
#define INF 0x3f3f3f3f
#define mod 10000
int n , k , x[MAX] , c[MAX];
vector<int> mul(vector<int> a , vector<int> b){
    vector<int> ans(n + n + 1);
    REP(i , 1 , n + 1) REP(j , 1 , n + 1)
        ans[i + j] = (ans[i + j] + (a[i] * b[j])) % mod;
    RREP(i , n + n , n + 1){
        REP(j , 1 , n + 1) ans[i - j] = (ans[i - j] + ans[i] * c[j]) % mod;
        ans[i] = 0;
    }
    return ans;
}
vector<int> ppow(vector<int> a , int k){
    if(k == 1) return a;
    if(k % 2 == 0) return ppow(mul(a , a) , k >> 1);
    if(k % 2 == 1) return mul(ppow(mul(a , a) , k >> 1) , a);
}
int main(){
    IOS;
    while(cin >> n && n){
        REP(i , 1 , n + 1) cin >> x[i];
        REP(i , 1 , n + 1) cin >> c[i];
        vector<int> v(n + n + 1);
        v[1] = 1;
        cin >> k , k ++;
        v = ppow(v , k);
        int ans = 0;
        REP(i , 1 , n + 1) ans = (ans + x[i] * v[i]) % mod;
        cout << ans << endl;
    }
    return 0;
}

```

## 10.5 Segment Max segment sum

```

int n , m , x[MAX];
class N{
public: int tag , sml , sum , none;
} b[MAX * 4];
void Pull(int now , int l , int r){
    if(l == r){
        if(b[now].tag){
            b[now].sum = b[now].tag;
            b[now].none = 0;
            b[now].sml = b[now].tag;
        }
        else{
            b[now].sum = 0;
            b[now].none = 1;
            b[now].sml = INF;
        }
    }
    else {
        b[now].sml = min(b[ls].sml , b[rs].sml);
        if(b[now].tag) b[now].sml = min(b[now].sml , b[now].tag);

        b[now].sum = b[ls].sum + b[rs].sum;
        b[now].none = b[ls].none + b[rs].none;
        if(b[now].tag) b[now].sum += b[now].tag * b[now].none ,
            b[now].none = 0;
    }
}
void take_tag(int now , int l , int r , int val){
    if(b[now].tag && b[now].tag < val) b[now].tag = 0;
    if(l != r && b[ls].sml < val) take_tag(ls , l , mid , val);
    if(l != r && b[rs].sml < val) take_tag(rs , mid + 1 , r ,
        val);
    Pull(now , l , r);
}
void Build(int now , int l , int r){
    b[now].none = 0;
    if(l == r) b[now].tag = b[now].sml = b[now].sum = x[l];
    else {
        Build(ls , l , mid) , Build(rs , mid + 1 , r);
        Pull(now , l , r);
    }
}
void update(int now , int l , int r , int ql , int qr , int val)
){
    if(b[now].tag >= val) return ;
    if(ql <= l && r <= qr){
        take_tag(now , l , r , val);
        b[now].tag = val;
        Pull(now , l , r);
    }
    else{
        if(qr <= mid) update(ls , l , mid , ql , qr , val);
        else if(mid + 1 <= ql) update(rs , mid + 1 , r , ql ,
            qr , val);
        else update(ls , l , mid , ql , qr , val) , update(rs ,
            mid + 1 , r , ql , qr , val);
        Pull(now , l , r);
    }
}
PII query(int now , int l , int r , int ql , int qr){
    if(ql <= l && r <= qr) return mp(b[now].sum , b[now].none);
    else {
        PII ans = mp(0 , 0);
        if(qr <= mid) ans = query(ls , l , mid , ql , qr);
        else if(mid + 1 <= ql) ans = query(rs , mid + 1 , r ,
            ql , qr);
        else {
            PII a = query(ls , l , mid , ql , qr);
            PII b = query(rs , mid + 1 , r , ql , qr);
            ans = mp(a.A + b.A , a.B + b.B);
        }
        if(b[now].tag != 0) ans.A += ans.B * b[now].tag , ans.B
            = 0;
        return ans;
    }
}
REP(i , 1 , n + 1) cin >> x[i];
Build(1 , 1 , n);
update(1 , 1 , n , l , r , v);
cout << query(1 , 1 , n , l , r).A << endl;

```

## 10.6 Chinese Remainder Theorem

```

#define INF 0x3f3f3f3f
void extgcd(ll a , ll b , ll &d , ll &x , ll &y){
    if(b == 0) d = a , x = 1 , y = 0;
    else extgcd(b , a % b , d , y , x) , y -= (a / b) * x;
}
ll n;
vector<ll> v , m;
int main(){
    while(cin >> n){
        v.clear() , m.clear();
        ll ans , mod , d , x , y;
        REP(i , 0 , n) cin >> mod >> ans , m.pb(mod) , v.pb(ans);

        mod = m[0] , ans = v[0];
        REP(i , 1 , n){
            ll res = ((v[i] - ans) % m[i] + m[i]) % m[i];
            extgcd(mod , m[i] , d , x , y);
            if(res % d != 0){ ans = -1; break; }

            res = (res / d * x % m[i] + m[i]) % m[i];
            ans = ans + res * mod;
            mod = mod * m[i] / d;
        }
        if(ans == -1) cout << ans << endl;
        else cout << ans % mod << endl;
    }
    return 0;
}

```

## 10.7 Stone merge

```

int n , x[MAX] , ans = 0;
vector<int> v;
int DFS(int now){
    int val = v[now] + v[now + 1];
    ans += val;
    v.erase(v.begin() + now);
    v.erase(v.begin() + now);
    int id = 0;
    RREP(i , now - 1 , 0) if(v[i] >= val) { id = i + 1; break; }
    v.insert(v.begin() + id , val);
    while(id >= 2 && v[id - 2] <= v[id]){
        int dis = v.size() - id;
        DFS(id - 2);
        id = v.size() - dis;
    }
}
int32_t main(){
    IOS;
    cin >> n;
    REP(i , 0 , n) cin >> x[i];
    REP(i , 0 , n){
        v.pb(x[i]);
        while(v.size() >= 3 && v[v.size() - 3] <= v[v.size() -
            1])
            DFS(v.size() - 3);
    }
    while(v.size() > 1) DFS(v.size() - 2);
    cout << ans << endl;
    return 0;
}

```

## 10.8 Range modify and query BIT

```

int n , m , k;
int bit[4][MAX][MAX];
void update(int c[MAX][MAX] , int a , int b , int val){
    for(int i = a + 10 ; i < MAX ; i += i & -i)
        for(int j = b + 10 ; j < MAX ; j += j & -j)
            c[i][j] += val;
}
int update(int x , int y , int val){
    update(bit[0] , x , y , val);
    update(bit[1] , x , y , -val * x);
    update(bit[2] , x , y , -val * y);
    update(bit[3] , x , y , val * x * y);
}
void update(int a , int b , int x , int y , int val){
    update(a , b , val);
    update(a , y + 1 , -val);
    update(x + 1 , b , -val);
    update(x + 1 , y + 1 , val);
}

```

```

}
int query(int c[MAX][MAX] , int a , int b){
    int cnt = 0;
    for(int i = a + 10 ; i > 0 ; i -= i & -i)
        for(int j = b + 10 ; j > 0 ; j -= j & -j)
            cnt += c[i][j];
    return cnt;
}
int query(int x , int y){
    int cnt = 0;
    cnt += query(bit[0] , x , y) * (x + 1) * (y + 1);
    cnt += query(bit[1] , x , y) * (y + 1);
    cnt += query(bit[2] , x , y) * (x + 1);
    cnt += query(bit[3] , x , y);
    return cnt;
}
int query(int a , int b , int x , int y){
    int cnt = 0;
    cnt += query(a - 1 , b - 1);
    cnt -= query(a - 1 , y);
    cnt -= query(x , b - 1);
    cnt += query(x , y);
    return cnt;
}
int32_t main(){
    IOS;
    cin >> n >> m >> k;
    int tmp;
    REP(i , 1 , n + 1) REP(j , 1 , m + 1){
        cin >> tmp;
        update(i , j , i , j , tmp);
    }
    REP(i , 1 , k + 1){
        int a , b , x , y , val , add;
        cin >> a >> b >> x >> y >> val >> add;
        int sum = query(b , a , y , x);
        if(sum < val * (x - a + 1) * (y - b + 1)){
            update(b , a , y , x , add);
        }
    }
    REP(i , 1 , n + 1){
        REP(j , 1 , m + 1) cout << query(i , j , i , j) << " ";
        cout << endl;
    }
    return 0;
}

```

## 10.9 Manhattan Spanning Tree

```

#define edge pair<int , PII>
int n , sol[MAX];
PII x[MAX];
vector<edge> v;
class djs{
public:
    int x[MAX];
    void init(){ REP(i , 0 , MAX) x[i] = i; }
    int Find(int now){ return x[now] == now ? now : x[now] = Find(x[now]); }
    void Union(int a , int b){ x[Find(a)] = Find(b); }
    int operator[](int now){ return Find(now); }
} ds;
PII bit[MAX];
void update(int from , int val , int id){
    for(int i = from ; i < MAX ; i += i & -i)
        bit[i] = max(bit[i] , mp(val , id));
}
int query(int from){
    PII res = bit[from];
    for(int i = from ; i > 0 ; i -= i & -i)
        res = max(res , bit[i]);
    return res.B;
}
int cmp(int a , int b){
    return x[a] < x[b];
}
int DIS(int q , int w){
    return abs(x[q].A - x[w].A) + abs(x[q].B - x[w].B);
}
void BuildEdge(){
    vector<int> uni;
    REP(i , 0 , MAX) bit[i] = mp(-INF , -1);
    REP(i , 0 , n) sol[i] = i;
    REP(i , 0 , n) uni.pb(x[i].B - x[i].A);
    sort(ALL(uni));

```

```

    uni.resize(unique(ALL(uni)) - uni.begin());
    sort(sol , sol + n , cmp);
    REP(i , 0 , n){
        int now = sol[i];
        int tmp = x[sol[i]].B - x[sol[i]].A;
        int po = lower_bound(ALL(uni)) , tmp) - uni.begin() + 1;
        int id = query(po);
        if(id >= 0) v.pb(mp(DIS(id , now) , mp(id , now)));
        update(po , x[now].A + x[now].B , now);
    }
}
void Build(){
    BuildEdge();
    REP(i , 0 , n) swap(x[i].A , x[i].B);
    BuildEdge();
    REP(i , 0 , n) x[i].A *= -1;
    BuildEdge();
    REP(i , 0 , n) swap(x[i].A , x[i].B);
    BuildEdge();
}
int solveKruskal(){
    ds.init();
    sort(ALL(v));
    int res = 0;
    REP(i , 0 , v.size()){
        int dis = v[i].A;
        PII tmp = v[i].B;
        if(ds[tmp.A] != ds[tmp.B]){
            ds.Union(tmp.A , tmp.B);
            res += dis;
        }
    }
    return res;
}
int32_t main(){
    IOS;
    cin >> n;
    REP(i , 0 , n) cin >> x[i].A >> x[i].B;
    Build();
    int ans = solveKruskal();
    cout << ans << endl;
    return 0;
}

```

## 10.10 K Cover Tree

```

int n , k , dp[MAX] , ans;
vector<int> v[MAX];
void DFS(int now , int fa){
    if(v[now].size() == 1 && v[now][0] == fa)
        return dp[now] = -1 , void();
    int sml = INF , big = -INF;
    for(auto to : v[now]) if(to != fa){
        DFS(to , now);
        sml = min(sml , dp[to]);
        big = max(big , dp[to]);
    }
    if(sml == -k) dp[now] = k , ans ++;
    else if(big - 1 >= abs(sml)) dp[now] = big - 1;
    else dp[now] = sml - 1;
}
int32_t main(){
    IOS;
    cin >> n >> k;
    REP(i , 2 , n + 1){
        int a , b; cin >> a >> b;
        v[a].pb(b); v[b].pb(a);
    }
    if(k == 0) cout << n << endl;
    else {
        DFS(0 , 0) , ans += dp[0] < 0;
        cout << ans << endl;
    }
    return 0;
}

```

## 10.11 M Segments' Maximum Sum

```

-----Greedy-----
int n , m , fr[MAX] , ba[MAX];
int v[MAX] , idx = 1;
set<PII> cc;

```

```

void erase(int id){
    if(id == 0) return;
    int f = fr[id] , b = ba[id];
    ba[fr[id]] = b , fr[ba[id]] = f;
    cc.erase(mp(abs(v[id]) , id));
}

int32_t main(){
    cin >> n >> m;
    int sum = 0 , pos = 0 , ans = 0;
    REP(i , 0 , n){
        int tmp; cin >> tmp;
        if(tmp == 0) continue;
        if((tmp >= 0 && sum >= 0) || (tmp <= 0 && sum <= 0)){
            sum += tmp;
        }
        else {
            if(sum > 0) ans += sum , pos ++;
            v[idx ++] = sum , sum = tmp;
        }
    }
    if(sum) v[idx ++] = sum;
    if(sum > 0) ans += sum , pos ++;
    REP(i , 0 , idx){
        fr[i + 1] = i;
        ba[i] = i + 1;
        if(i) cc.insert(mp(abs(v[i]) , i));
    }
    ba[idx - 1] = 0;
    while(pos > m){
        auto tmp = cc.begin();
        int val = (*tmp).A , id = (*tmp).B;
        cc.erase(tmp);
        if(v[id] < 0 && (fr[id] == 0 || ba[id] == 0)) continue;
        if(v[id] == 0) continue;
        ans -= val , pos --;
        v[id] = v[fr[id]] + v[id] + v[ba[id]];
        cc.insert(mp(abs(v[id]) , id));
        erase(fr[id]) , erase(ba[id]);
    }
    cout << ans << endl;
    return 0;
}

```

-----Aliens-----

```

int n , k , x[MAX];
PII dp[MAX] , rd[MAX]; // max value , times , can be buy , times
int judge(int now){
    dp[1] = mp(0 , 0) , rd[1] = mp(-x[1] , 0);
    REP(i , 2 , n + 1){
        dp[i] = max(dp[i - 1] , mp(rd[i - 1].A + x[i] - now , rd[i - 1].B + 1));
        rd[i] = max(rd[i - 1] , mp(dp[i - 1].A - x[i] , dp[i - 1].B));
    }
    return dp[n].B;
}

int32_t main(){
    IOS;
    cin >> n >> k;
    n ++;
    REP(i , 2 , n + 2) cin >> x[i];
    REP(i , 1 , n + 1) x[i] += x[i - 1];
    if(judge(0) <= k) cout << dp[n].A << endl;
    else {
        int l = 0 , r = 1000000000000LL;
        while(r - l > 1){
            int mid = l + ((r - l) >> 1) , res = judge(mid);
            if(res == k) return cout << dp[n].A + dp[n].B * mid << endl , 0;
            else if(res < k) r = mid;
            else if(res > k) l = mid;
        }
        judge(l);
        cout << dp[n].A + k * l << endl;
    }
    return 0;
}

```

## 10.12 Minimum Enclosing Cycle

```

pdd arr[MAX], cen;
double r;
inline double dis(pdd a, pdd b){ return hypot(a.X-b.X, a.Y-b.Y); }
int n, m;
inline double sq(double x){ return x*x; }

```

```

pdd external(pdd p1, pdd p2, pdd p3){
    double a1=p1.X-p2.X, a2=p1.X-p3.X;
    double b1=p1.Y-p2.Y, b2=p1.Y-p3.Y;
    double c1=( sq(p1.X)-sq(p2.X)+sq(p1.Y)-sq(p2.Y) )/2;
    double c2=( sq(p1.X)-sq(p3.X)+sq(p1.Y)-sq(p3.Y) )/2;
    double dd=a1*b2-a2*b1;
    return pdd( (c1*b2-c2*b1)/dd , (a1*c2-a2*c1)/dd );
}

int main(){
    IOS
    srand(time(0));
    while(cin >> n >> m){
        if(n+m==0) return 0;
        for(int i=0; i<m; i++){
            cin >> arr[i].X >> arr[i].Y;
        }
        random_shuffle(arr, arr+m);
        r=0;
        for(int i=0; i<m; i++){
            if(dis(cen, arr[i])>r){
                cen=arr[i]; r=0;
                for(int j=0; j<i; j++){
                    if(dis(cen, arr[j])>r){
                        cen=pdd( (arr[i].X+arr[j].X)/2 , (arr[i].Y+arr[j].Y)/2 );
                        r=dis(cen, arr[j]);
                        for(int k=0; k<j; k++){
                            if(dis(cen, arr[k])>r){
                                cen=external(arr[i], arr[j], arr[k]);
                                r=dis(cen, arr[j]);
                            }
                        }
                    }
                }
            }
        }
        cout << stp << r << '\n';
    }
    return 0;
}

```

## 10.13 Rotating Sweep Line

```

PII p[MAX];
int n , idx[MAX] , pos[MAX];
long long wnt;
vector<PII> v;
inline PII operator + (PII x , PII y){ return mp(x.A + y.A , x.B + y.B); }
inline PII operator - (PII x , PII y){ return mp(x.A - y.A , x.B - y.B); }
inline long long cross(PII x , PII y){ return 1ll * x.A * y.B - 1ll * x.B * y.A; }
inline long long calcArea(PII x , PII y , PII z){
    long long val = abs(cross(y - x , z - x));
    return val;
}

inline int cmp1(PII x , PII y){
    x = p[x.B] - p[x.A];
    y = p[y.B] - p[y.A];
    return cross(x , y) > 0;
}

int32_t main(){
    cin >> n >> wnt , wnt += wnt;
    REP(i , 1 , n + 1) cin >> p[i].A >> p[i].B;
    sort(p + 1 , p + 1 + n);
    REP(i , 1 , n + 1) idx[i] = i , pos[i] = i;
    REP(i , 1 , n + 1) REP(j , i + 1 , n + 1) v.pb(mp(i , j));
    sort(ALL(v) , cmp1);
    for(auto line : v){
        int fr = pos[line.A] , ba = pos[line.B] , now;
        if(fr > ba) swap(fr , ba);
        now = fr;
        RREP(i , 10 , 0){
            int to = now - (1 << i);
            if(to >= 1 && calcArea(p[idx[fr]] , p[idx[ba]] , p[idx[to]]) <= wnt) now = to;
        }
        now = ba;
        RREP(i , 10 , 0){
            int to = now + (1 << i);
            if(to <= n && calcArea(p[idx[fr]] , p[idx[ba]] , p[idx[to]]) <= wnt) now = to;
        }
    }
}

```

```
        swap(idx[fr] , idx[ba]) , swap(pos[line.A] , pos[line.B
    });
}
cout << "No" << endl;
return 0;
}
```

## 10.14 Hilbert Curve

```
//soring Mo's with hilbert(L, R) can be faster!!!
ll hilbert(int n, int x, int y) {
    ll res = 0;
    for (int s = n / 2; s; s >>= 1) {
        int rx = (x & s) > 0;
        int ry = (y & s) > 0;
        res += s * 1ll * s * ((3 * rx) ^ ry);
        if (ry == 0) {
            if (rx == 1) {
                x = s - 1 - x;
                y = s - 1 - y;
            }
            swap(x, y);
        }
    }
    return res;
}
```