- Measurements, Units and Dimension.
- Scalars & Vectors
- Kinematics
- Motion in 1 and 2 dimensions ??
- Fundamental Laws of mechanics
- Force and Momentum

$\frac{F\ell}{A\Delta l} = \frac{MLT^{-2}L}{L^2 \cdot L}$

$= \frac{ML^2T^{-2}}{L^3}$

$ML^{-1}T^{-2}$

m = L
kg = M
Time = T

Phy 101

3rd Part

UNITS, MEASUREMENT AND DIMENSION

UNITS AND MEASUREMENT

Physics is a science of observation of the world around us. It aims to give an understanding of this world in both observation and prediction of the way object behave. It is a science of measurement, but before any measurement can be made, we must define the units on which our measurement is made.

Therefore, any physical quantity must be associated with the units of measurement, otherwise such quantity will have no meaning.

Units of measurement are classified as Fundamental Units and Derived Units.

FUNDAMENTAL UNITS

Fundamental units are basics unit upon which other units depend. They are not form by combination of other units. They cannot be defined in terms of other quantities or derived from them.

The seven (7) fundamental units of the system international (S.I) Units are shown in the table 1.0 below:

TABLE 1.0

Quantity	Unit
Length	Metre (m)
Mass	Kilogram (kg)
Time	Second (s)
Electric Current	Ampere (A)
Temperature	Kelvin (k)
Luminous Intensity	Candela (cd)
Amount of Substance	Mole (mol.)

However, Fundamental units are also called Basic Units.

Accuracy
meter rule → 0.1cm
Vernier calliper diameter → 0.01cm
micrometer screw gauge very small distance → 0.00cm

BASIC UNITS

Time by stop-watch → 0.1 to 0.5 seconds.

Long distances are measured with tapes and metre rules, which graduated in cm or mm. The reading accuracy of a metre rule is about 0.1 cm or 1 mm. Smallest measurement can be estimated up to 0.05 cm or 0.5 mm. the Vernier Callipers are used to measure the diameter of a cylindrical object (E.g. rod), which has an accuracy of 0.01 cm or 0.1 mm, while very small distance such as thin of a wire or thickness of a paper are measured by micrometer screw gauge. The reading accuracy is about 0.001cm.

MASS AND WEIGHT

scalar

Mass is the quantity of matter contained in a body. It is always constant, it is usually measured with balance (i.e. beam, chemical, or lever). The accuracy depends on the sensitivity of the balance. It measured in kilogram (kg) and it is a scalar quantity.

vector

Weight is the force or pull with which the earth attracts the body towards the centre of the earth. It measured in Newton and it is a vector quantity.

Time can be defined as in which the events are distinguishable with reference to before and after. The most natural time unit is the solar day which is manifested by passing of day and night. Time measured in the laboratory with a stop-watch or a stop-clock. The reading accuracies vary from 0.1 to 0.5 seconds.

DERIVED UNITS

Derived units are those units formed by a combination of fundamental units. For example, the unit of acceleration is ms^{-2} and velocity is in ms^{-1}.

TABLE 1.1: Derived quantities and units

Quantity	Derivation	Units
Area (A)	Length × breath	m^2
Volume (v)	Length × breath × height	m^3
Density (ρ)	Mass per unit volume	kgm^{-3}
Speed (V)	Distance per unit time	ms^{-1}
Velocity (V)	Displacement per unit time	ms^{-1}
Acceleration (a)	Change velocity with time	ms^{-2}
Momentum (p)	Mass× velocity	$kgms^{-1}$

Impulse (I)	Force× $time$	$kgms^{-1}$ or NS
Pressure (p)	Force per unit area	Nm^{-1} or kgm^{-1}

DIMENSION OF PHYSICAL QUANTITIES

The way in which derived quantity is related to fundamental quantity or basic quantity can be shown by the dimension of the quantity. The dimension of a physical quantity tell how the quantity are related to the fundamental quantities of mass (M), length (L) and time (T), when we restrict ourselves to those used in mechanics and properties of matter only.

The dimension of any other physical quantity will involve one or more of these dimensions. For example, a measurement of force (N) or $kgms^{-2}$ are the product of mass and acceleration.

i.e. Force (F) = $mass \times acceleraton = kgms^{-2}$

Dimensionally = $M \times LT^{-2}$

$= MLT^{-2}$

Velocity = $\frac{Displacement}{time}$

$= LT^{-1}$

Power = $\frac{workdone}{time} = kgm^2s^{-3}$

Dimensionally = ML^2T^{-3}

Momentum (P) = $Mass \times Velocity$

Dimensionally = MLT^{-1}

Similarly,

Pressure = $\frac{force}{area}$

$= \frac{mass \times acceleration}{length \times breath} = M \times LT^{-2}/L^2$

Dimensionally = $ML^{-1}T^{-2}$

Work or Energy (F × s) = $kgm^2s^{-2} = ML^2T^{-2}$

APPLICATIONS OF DIMENSION IN PHYSICS

The most important applications of dimension in physics are:

i. To check or verify the correctness of a physical quantity i.e. to check whether the dimension of each term of the equation is the same.

Example 1

$$v = u + at$$

Now let us verify each term

$$v = LT^{-1}$$

$$u = LT^{-1}$$

$$at = LT^{-2}T^{1} = LT^{-1}$$

Therefore, the above equation is correct because the dimension of each term is the same.

Example 2

Impulse = ft = $c▯ange\ in\ momemtum = mv - mu$

Dimension of; $FT = MLT^{-1}$

$$MV = MLT^{-1}$$

$$MU = MLT^{-1}$$

$s = ut + \frac{1}{2}at^2$

$s = u^2 + 2as$

Hence the above equation is correct.

Example 3

$$s = ut + \frac{1}{2}at^2$$

Dimension of; $S = L$

$$UT = LT^{-1} \times T = L$$

Since numeric is dimensionless $at^2 = LT^{-2} \times T^2 = L$

Hence, the equation is correct because each the terms is the same.

ii. It helps to determining the appropriate unit of physical quantity. For example;

Volume = $lengt▯ \times breadt▯ \times ▯eig▯t = L^3$

$$\text{Density } (\rho) = \frac{mass}{volume} = ML^{-3}$$

iii. To derive the equation of physical quantities.

Example 1

Supposed that the period of simple pendulum depends on length (L) and acceleration due to gravity (g) only, what is the exact form of the relation?

<u>SOLUTION</u>

The relationship is written as:

$$T = KL^{x} g^{y} \quad \text{.......... (1)}$$

where k is dimensionless

Then, we have dimension for;

$$T = T$$

$$L = L$$

$$g = LT^{-2}$$

Substitute the above dimension into equation (1) above we have;

$$T = K \times L^{x} \times (LT^{-2})^{y} \quad \text{.......... (2)}$$

Equate the corresponding index power of L and T on both side we have;

T: $1 = -2y$ .... (3)

L: $0 = x + y$ .... (4)

Solve x and y simultaneously from equation (3) and (4) we have;

$$y = -1/2 \quad \text{and}$$

$$x = 1/2$$

Then, substitute x and y into equation (2) then, we have;

$$T = K\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{l}{g}}$$

$T = K L^{\frac{1}{2}} \times g^{-\frac{1}{2}}$

$= K\sqrt{L} \times \frac{1}{\sqrt{g}}$

$= K\frac{\sqrt{L}}{\sqrt{g}} = K\sqrt{\frac{L}{g}}$

Example 2

A wave is set up in a stretched string by flocking it. The velocity (v) of the wave depends on the tension (T) in string, its length and its mass. We therefore, supposed that:

$$V \alpha TLM$$

<u>SOLUTION</u>

Using, $V = KT^xL^yM^z$ ……………………………………………… (1)

where k is dimensionless constant

The dimension of the above equation can be written as;

$$V = LT^{-1}$$

$$T = MLT^{-2}$$

$$L = L$$

$$M = M$$

$\therefore$ *substituting the dimension then we obtained*

$$LT^{-1} = K(MLT^{-2})^xL^yM^z \quad \text{……………………………} (2)$$

Equating the corresponding power of M, L and T on both side it follows that

M: $0 = x + z$ ……………………………………………… (3)

L: $1 = x + y$ ……………………………………………… (4)

T: $-1 = -2x$ ……………………………………………… (5)

Solve the above equation simultaneously, it yields;

$$x = {}^1/_2, \quad y = {}^1/_2, \quad z = -{}^1/_2.$$

Inserting x, y, and z into equation (1)

$$V = K \times T^{\frac{1}{2}} \times L^{\frac{1}{2}} \times M^{-\frac{1}{2}}$$

$$V = K\sqrt{\frac{TL}{M}} \quad \textit{Hence, obtained}$$

$1 - \frac{1}{2}$ $2 - \frac{1}{2}$ $\frac{1}{2}$

PLAY QUESTIONS

1. Derived the dimension of the following physical quantities;

- Universal gravitational constant; $G = \frac{fr^2}{mm}$ $(ans: M^{-1}L^3T^{-2})$
- Plank constant (K) = $\frac{Energy}{Frequency}$ $(ans: ML^2T^{-1})$
- Surface Tension $(\gamma) = \frac{force}{Length}$ $(ans: MT^{-2})$
- Coefficient of Viscosity $(\eta) = \frac{Force}{Area \times Velocity\ gradient}$ $(ans: ML^{-1}T^{-1})$
- young's modulus (E)= $\frac{Tensile\ stress}{Tensile\ strain}$ $(ans: ML^{-1}T^{-2})$

2. Use the method of dimensional analysis to check the validity of the following equations:

- $E = MC^2$
- $E = \frac{1/2 Ae^3}{L}$
- $V = \sqrt{2gr}$

3. Using the method of dimensional analysis, deduce the equation of frictional drags on sphere falling through a liquid. Suppose the frictional drag F on the sphere falling through the liquid depends on the radius of the sphere r, its terminal velocity V and fluid viscosity of the liquid η is given by;

$$F = Kr^x\eta^yV^z$$

4. Force (F), work (W) and velocity (V) are taken as fundamental quantities then the dimensional formula of time (T) is given by;

$$T \propto KFWV$$

Hint: $T = KF^aW^bV^c$. Form the exact equation

5. If pressure P, velocity V and time T are taken as fundamental physical quantities, the dimensional formula of the force is written as;

$$F = KP^xV^yT^z$$

Find the values of x, y, and z

(Best of luck)

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VECTORS

Physical quantities are divided into two main groups, scalar quantitres and vector quantities.

A scalar quantity is defined as the quantity that has only magnitude or size. E.g: length, area,,volume, mass, time etc.

A vector quantity is a quantity that has both magnitude and direction.

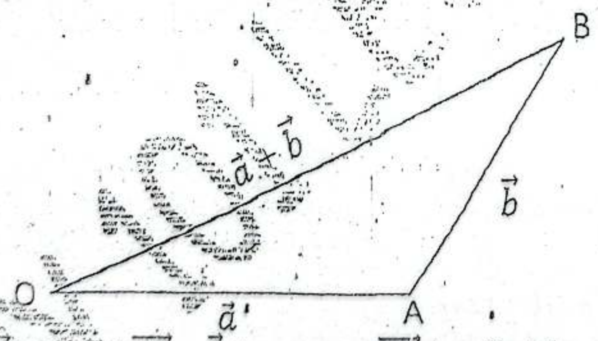
E.g. velocity, displacement, acceleration, force etc.

TYPES OF VECTORS

1. Equal vector. Two vectors having the same magnitude and the same direction.
2. Negative vector. A vector having the same magnitude but directed in the opposite sense to a given vector. It represented by minus sign.
3. Null or Zero vector. If the magnitude or modulus is zero. i.e. $|A| = 0$
4. Proper vector. A vector whose modulus or magnitude is not zero. i.e. $|A| \neq 0$.
5. Unit vector. A vector whose magnitude is unit. i.e. $\hat{A} = \frac{A}{|A|}$
6. Position vectors. The vectors which are used to specify the position of a point p with respect to some fixed point o is represented by $\overrightarrow{OA}$
7. Collinear vectors. Two or more vectors parallel or anti parallel to each other is called collinear vectors, etc.

ADDITION OF VECTORS

Let $\vec{a}$ and $\vec{b}$ be two given vectors as shown in the figure below:



$\overrightarrow{OA} = \vec{a}$ and $\overrightarrow{AB} = \vec{b}$ then vector $\overrightarrow{OB}$ is called the sum of $\vec{a}$ and $\vec{b}$

Symbolically,

$$\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}$$

$$\vec{a} + \vec{b} = \overrightarrow{OB}$$

RESOLUTION OF VECTOR

The process of splitting a vector into various parts or components is called "RESOLUTION OF VECTOR"

These parts of a vector may act in different directions and are called "components of vector"

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We can resolve a vector into a number of components. Generally there are three components of vector viz.

Component along X-axis called x-component

Component along Y-axis called Y-component

Component along Z-axis called Z-component

Here we will discuss only two components X-component & Y-component which are perpendicular to each other. These components are called rectangular components of vector.

METHODS OF RESOLVING A VECTOR INTO RECTANGULAR COMPONENTS

Consider a vector $\vec{V}$ acting at a point making an angle θ with positive X-axis. Vector $\vec{V}$ is represented by a line OA. From point A draw a perpendicular AB on X-axis as shown in the below figure 02. Suppose OB and BA represents two vectors. Vector OB is parallel to X-axis and vector BA is parallel to Y-axis. Magnitude of theses vectors are V_x and V_y respectively. By method of head to tail we notice that the sum of these vectors is equal to vector $\vec{V}$. Thus V_x and V_y are the rectangular components of vector $\vec{V}$.

V_x = Horizontal component of $\vec{V}$.

V_y = Vertical component of $\vec{V}$.

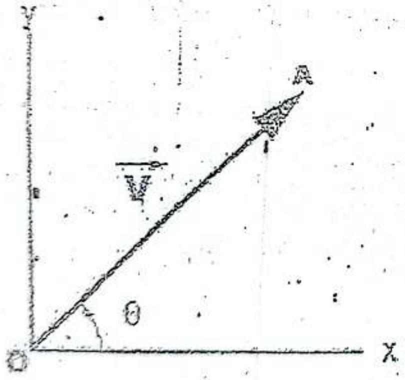

figure 01

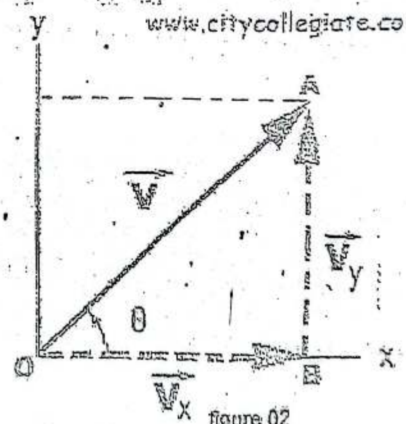

figure 02

MAGNITUDE OF HORIZONTAL COMPONENT

Consider right angled triangle ΔOAB

$$\cos\theta = \frac{\overline{OB}}{\overline{OA}}$$

$$\overline{OB} = \overline{OA}\cos\theta$$

$$V_x = V\cos\theta$$

MAGNITUDE OF VERTICAL COMPONENT

Consider right angled triangle ΔOAB

$$\sin\theta = \frac{\overline{AB}}{\overline{OA}}$$

$$\overline{AB} = \overline{OA}\sin\theta$$

$$V_y = V\sin\theta$$

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UNIT OF A VECTOR

Let a vector be $x\hat{i} + y\hat{j} + z\hat{k}$

$$\boxed{\text{Unit vector} = \frac{x\hat{i} + y\hat{j} + z\hat{k}}{\sqrt{x^2 + y^2 + z^2}}}$$

PRODUCT OF TWO VECTORS

The product of two vectors results in two different ways, the one is a number and the other is vector. So, there are two types of product of two vectors, namely;

i. Scalar or Dot product

ii. Vector or Cross Product

They are written as; $\vec{a} \cdot \vec{b}$ and $\vec{a} \times \vec{b}$

x axis: $a\cos\theta$

y axis: $a\sin\theta$

1. SCALAR OR DOT PRODUCT

The scalar or Dot product of two vectors $\vec{a}$ and $\vec{b}$ is defined as; $|\vec{a}||\vec{b}|\cos\theta$.

Where θ is the angle between $\vec{a}$ and $\vec{b}$

Symbolically, $\boxed{\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta}$

$\cos 90 = 0$

Due to the Dot between $\vec{a}$ and $\vec{b}$ it is called Dot product.

Note that:

$\hat{i} \cdot \hat{i} = 1$ and $\hat{i} \cdot \hat{j} = 0$

$\hat{j} \cdot \hat{j} = 1$ $\quad \hat{i} \cdot \hat{k} = 0$

$\hat{k} \cdot \hat{k} = 1$ $\quad \hat{j} \cdot \hat{k} = 0$

radius in a circle

dependent on time

$\vec{r} = r(t)$

direction of vector in a circle.

$\frac{dr}{dt} =$

2. VECTOR OR CROSS PRODUCT

This can be defined as; $\boxed{\vec{a} \times \vec{b} = |\vec{a}||\vec{b}|\sin\theta}$

$\sin 90 = 1$

Note that:

$\hat{i} \times \hat{i} = 0$ and $\hat{i} \times \hat{j} = 1$

$\hat{j} \times \hat{j} = 0$ $\quad \hat{i} \times \hat{k} = 1$

$\hat{k} \times \hat{k} = 0$ $\quad \hat{j} \times \hat{k} = 1$

find the unit vector of

$4i - 3$

$\boxed{\text{unit vector } \hat{a} = \frac{a}{|a|}}$

VECTOR PRODUCT EXPRESSED AS A DETERMINANT

If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, then,

$\vec{a} \times \vec{b} = (a_1\hat{i} + a_2\hat{j} + a_3\hat{k}) \times (b_1\hat{i} + b_2\hat{j} + b_3\hat{k})$

$\boxed{\text{modulus of } |a| = \sqrt{a^2}}$

$\boxed{\text{modulus of } |a+b| = \sqrt{a^2+b^2}}$

Thus,

$$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ a & a & a \\ b & b & b \end{vmatrix}$$

Example 1

Particle of mass 3 kg moves under a force of $4\hat{\imath} + 8\hat{\jmath} + 10\hat{k}$ N. Calculate the acceleration, if the particle starts from rest, calculate its coordinate S after 3 seconds.

SOLUTION

$f = ma$

$\therefore a = \dfrac{f}{m}$

$$a = \frac{4\hat{\imath} + 8\hat{\jmath} + 10\hat{k}\ \text{N}}{3}\ \text{ms}^{-2}$$

thus, displacement after 3 seconds:

$$s = ut + \tfrac{1}{2}at^2$$

But, initial velocity is zero;

$$\therefore s = \tfrac{1}{2}at^2$$

$$s = \frac{1}{2} \times \frac{4\hat{\imath} + 8\hat{\jmath} + 10\hat{k}\ \text{N}}{3} \times 3^2$$

$$s = 6\hat{\imath} + 12\hat{\jmath} + 15\hat{k}\ \text{m}$$

Example 2

Find the unit vector of (A+B) if A = $2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$ and B = $\hat{\imath} + 4\hat{\jmath} + 5\hat{k}$.

SOLUTION

$(A+B) = (2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}) + (\hat{\imath} + 4\hat{\jmath} + 5\hat{k})$

$= 3\hat{\imath} + 7\hat{\jmath} + 9\hat{k}$

scalar magnitude is therefore, $|A + B| = \sqrt{3^2 + 7^2 + 9^2}$

$= \sqrt{139}$

$\therefore unit\ vector = \frac{A+B}{|A+B|} = \frac{3\hat{\imath} + 7\hat{\jmath} + 9\hat{k}}{\sqrt{139}}$

Example 3

If $a = 2\hat{\imath} + 4\hat{\jmath} - 3\hat{k}$

$b = \hat{\imath} + 3\hat{\jmath} + 2\hat{k}$.

Determine the scalar and vector products and the angle between the two given vector.

<u>SOLUTION</u>.

Scalar product $\vec{a}.\vec{b} = (2\hat{\imath} + 4\hat{\jmath} - 3\hat{k}).(\hat{\imath} + 3\hat{\jmath} + 2\hat{k})$

$= (2 + 12 - 6)$

$= 8$

Magnitude of $|\hat{a}| = \sqrt{2^2 + 4^2 - 3^2}$

$= \sqrt{29}$

$= 5.39$

Magnitude of $|\hat{b}| = \sqrt{1^2 + 3^2 + 2^2}$

$= \sqrt{14}$

$= 3.74$

Angle between the two vectors;

$$\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

$$\theta = \cos^{-1}\frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$$

$$= \cos^{-1}\frac{8}{5.39 \times 3.74}$$

$= 66.6184$

$\approx 67^\circ$

For a vector product;

$$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ a & a & a \\ b & b & b \end{vmatrix}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 2 & 4 & -3 \\ 1 & 3 & 2 \end{vmatrix}$$

$= (8+9)\,I - (4+3)\,j + (6-4)\,k$

12

$= 17i - 7j + 2k$

KINEMATICS

Kinematics is a branch of classical mechanic that deal with the study of a pure motion without reference to the masses or forces involved in it.

DERIVATION OF KINEMATICS EQUATIONS UNDER UNIFORM ACCELERATION

Suppose an object move in a straight line with initial velocity u (ms^{-1}) and attained final velocity V (ms^{-1}), if it covered a distance of S (m) at a time t (second) with a uniform acceleration.

Then the acceleration of a body is given by;

$$a = \frac{v-u}{t}$$

But the average velocity $= \frac{v+u}{2}$

Therefore, the distance covered, $s = v_{average} \times time$ ....(1)

$$from.\,a = \frac{v-u}{t}$$

$$v = u + at \text{ .... (2)}$$

Since, $S = \left(\frac{v+u}{2}\right) \times t$

Eliminate V by putting it into equation (1) we have;

$$s = \frac{u + at + u}{2} \times t$$

$$s = \left(\frac{2u + at}{2}\right) \times t$$

$$= \frac{2ut + at^2}{2}$$

$$S = ut + {}^{1}\!/_{2}\, at^2 \text{ .... (3)}$$

To obtained equation (4)

Thus, $a = \frac{v-u}{t}$

By making t the subject we've;

$$t = \frac{v-u}{a}$$

Recall that,

$$S = \left(\frac{v+u}{2}\right) \times t$$

Substituting t now we've;

$$S = \frac{v+u}{2} \times \frac{v-u}{a}$$

$$S = \frac{v^2 - uv + uv - u^2}{2a}$$

When rearrange; $v^2 - u^2 = 2as$

$\therefore v^2 = u^2 - 2as$ .... (4)

Examples

A car moving with a velocity of 10 ms^{-1} accelerate uniformly at the rate of 2 ms^{-2} until it reaches a velocity of 15 ms^{-1}.

Calculate:

i. The time taken.

ii. Distance travelling during the acceleration.

iii. The velocity reached 100 m from the place where the acceleration began.

SOLUTION

U= $10ms^{-1}$ $\quad v = 15ms^{-1}$

$a = 2ms^{-2}$

i. The time taken

from equation (2) above; $v = u + at$

$$15 = 10 + 2t$$

$$15 - 10 = 2t \qquad \therefore t = \frac{5}{2} = 2.5\ second$$

ii. The distance travelled

Recall that, $S = ut + \frac{1}{2}at^2$

$$= 10 \times 2.5 + \frac{1}{2} \times 2 \times 2.5^2$$

$$= 25 + 6.25$$

$$\therefore S = 31.25\ m$$

iii. The velocity V

But, $v^2 = u^2 + 2as$

14

$$= 10^2 + 2 \times 2 \times 100$$
$$= 100 + 400$$
$$= 500$$
$$V = \sqrt{500}$$
$$\therefore v = 22.36ms^{-1}$$

Example 2

An airplane lands on the run way with a velocity of $50ms^{-1}$ and decelerates at 10 ms^{-2} *to a velocity of* 20 ms^{-1}. Calculate the distance travelled on the run way.

SOLUTION

$$u = 50ms^{-1} \quad and \quad v = 20ms^{-1}$$

$$a = -10ms^{-2}$$
$$s = ?$$

Also, from equation (4)

$$v^2 = u^2 - 2as$$
$$20^2 = 50^2 + 2(-10) \times s$$
$$400 = 2500 - 20s$$
$$s = \frac{-2100}{-20}$$
$$\therefore s = 105$$

Example 3

A particle moving along the x-axis has a velocity given by $v = 4t - 3t^2\ cms^{-1}$ for t second. Find its acceleration (a) t = 0.5 *second* and (*b*) $t = 3.0$ *seconds*.

SOLUTION

$$a = \frac{dv}{dt} = 4 - 6t\ cms^{-2}$$

(a) At $t = 0.5$ *second*

$\Rightarrow$ $a = 4 - 6 \times 0.5 = 1\ cms^{-2}$ or $0.01ms^{-2}$

(b) At $t = 3$ *second*

$$\therefore a = 4 - 6 \times 3$$
$$= 4 - 18$$

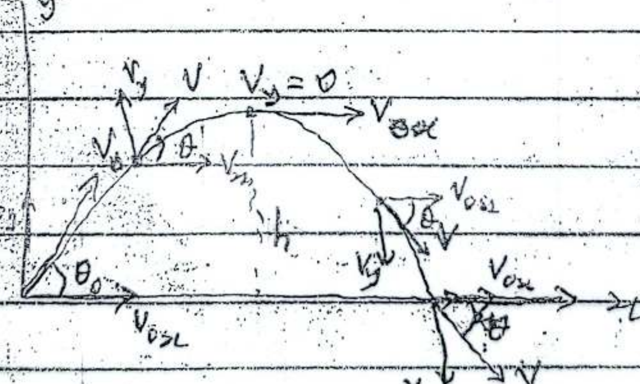
Hence, $a = -14\ cms^{-2}$ or $0.14\ ms^{-2}$

The negative sign implies that the particle decelerates at t = 3.0 seconds.

Two Dimensional Motion

So far we have considered objects moving along strai line path, rectilinear motion, such as the x-axis. In an object moves in a plane, we say that the object in both the x and y directions simultaneously, mean moves in two dimensions. A particular form of tw dimensional motion that is of interest is called proje motion.

A projectile is an object launched into the air and allow move on its own or freely under gravity; e.g. a baseba a catapult, etc. The path of a projectile is curved, meaning projectile motion is motion in a curved path, called a parabola, as shown in Figure below.



Note: Neglect effect of air res the rotation and time of Earth

Consider an object/projectile moving upward so that the is vertical, with initial velocity V_0 at time $t = 0$ inclined at angle θ_0 the horizontal, x-axis, called the projection angle. The acceleration in the vertical y-direction is $-g$, just as in fall, and the acceleration in the horizontal x-direction 0 (since air resistance is neglected).

The initial velocity V_0 is resolved into two compone horizontally and vertically,

$V_{0x} = V_0 \cos\theta_0$ and $V_{0y} = V_0 \sin\theta_0$

Do not write in either margin

Question No.

Write on both sides of the paper

The equations of projectile motion after time t seconds

$$V = V_0 \sin\theta_0 - gt \quad \text{--- (1)}$$

$$S = V_0 t \sin\theta_0 - \frac{1}{2}gt^2 \quad \text{--- (2)}$$

$$V^2 = V_0^2 \sin^2\theta_0 - 2gS \quad \text{--- (3)}$$

Time to reach maximum height, knowing that at max height, $V = 0$, so using eqn (1) gives

$$t = \frac{V_0 \sin\theta_0}{g}$$

Maximum height, h, reached, knowing at max height ~~V_0~~ $V = 0$, then from eqn (3),

$$0 = V_0^2 \sin^2\theta_0 - 2gS$$

$$S \text{ or } h = \frac{V_0^2 \sin^2\theta_0}{2g}$$

OR

Using eqn (2) and substituting $t = \frac{V_0 \sin\theta_0}{g}$

$$S \text{ or } h = V_0 \sin\theta_0 \left(\frac{V_0 \sin\theta_0}{g}\right) - \frac{1}{2}g\left(\frac{V_0 \sin\theta}{g}\right)$$

$$= \frac{V_0^2 \sin^2\theta_0}{2g}$$

For time of flight, from $S = V_0 t \sin\theta_0 - \frac{1}{2}gt^2$, for time of flight $S = 0$, $t = T$,

$$T = \frac{2V_0 \sin\theta_0}{g} = 2t$$

Range is the maximum horizontal displacement that the motion covered in time of flight

$$R = V_x T = V_0 \cos\theta_0 \times \frac{2V_0 \sin\theta_0}{g} = \frac{2V_0^2 \sin\theta \cos}{} = V_0

FUNDAMENTAL LAWS OF MECHANICS

DYNAMICS

In dynamics we deal with forces that gives rise to motion, classical dynamics is based on Newton's laws of motion.

NEWTON'S LAWS OF MOTION

Newton's laws of motion are only applicable if an object moves with a velocity which is negligible compared to the speed of light (i.e. $3 \times 10^8\ ms^{-1}$).

FIRST NEWTON'S LAWS OF MOTION

According to this law, a body continues to be in a state of rest or uniform motion along a straight line, unless it's acted upon by some external force to change the state.

This law consists of three parts:

i. Without the application of an external force a body at rest continues to be at rest.

ii. Without the application of an external force a body in uniform motion continues uniformly. It's slightly difficult to realize, in everyday life, we found that a ball rolling on the ground does not stop after some time. Those bodies are being opposed by air resistance and by force of friction between the body and the ground.

If we remove these forces, a body in uniform motion shall never stop on its own.

iii. Without the application of an external force a body cannot change its direction of motion i.e. it continues to move in a straight line.

INERTIA

According to the Newton's first law of motion a body continues to be in a state of rest or in a state of uniform motion along a straight line, unless acted upon by external force to change its state. The ability of an object or body to change its self in its state of rest or in uniform state of motion along a straight line is called INERTIA. Hence, that is why Newton's first law of motion is often referred to as law of inertia.

TYPES OF INERTIA

There are three types of inertia which are:

i. INERTIA OF REST: it is the ability of a body to change by itself, its state of rest. A body at rest remains at rest and cannot start moving on its own.

E.g. a person in a car falls backward, when car suddenly starts. It is because the lower portion in contact with the car comes in motion whereas, the upper part try to remain at rest.

Another example, when we shake the branch of a tree the leave or fruits fall down because the branches come in motion whereas, the leaves or fruits tend to remain at rest.

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ii. **INERTIA OF MOTION**: The inability of a body to change by itself in its state of uniform motion is known as inertia of motion.

E.g. when a moving car suddenly stop the person sitting in the car falls forward because the lower portion of the body contacts with the car come to rest whereas, the upper part tends to remain in motion due to inertia of motion.

iii. **INERTIA OF DIRECTION**: the inability of a body to change itself its direction of motion.

Example: when a car moves round a curve the person setting inside is thrown outward in order to maintain his direction of motion due to inertia of motion.

NEWTON'S SECOND LAW OF MOTION

States that the rate of change of linear momentum of a body is directly proportional to the external force applied on the body and this change take place always in the direction of the force applied.

To understand this further. Suppose;

M= $mass\ of\ the\ body$

$\vec{v} =\ velocity\ of\ the\ body$

The linear momentum of the body p is thus;

$\vec{p} = m\vec{v}$ .... (i)

Let $\vec{f} = external\ force\ applied\ on\ the\ body$

$\overrightarrow{dp}$ = a small change in the linear momentum in a small time dt

Rate of change of linear momentum of the body, $\frac{dp}{dt}$

According to the Newton's second law:

$f \propto \frac{dp}{dt}$, $f = k\frac{dp}{dt}$ .... (ii)

Where k is a constant of proportionality.

Substitute equation (i) and (ii) yields;

$$f = k\frac{d(mv)}{dt} = km\frac{dv}{dt}$$

$f = km\vec{a}$ .... (iii)

Where $\vec{a} = \frac{dv}{dt}$ represent the acceleration of the body.

Where k is depending on the unit adopted for measuring the force, selected in such a way that k =1

Put the value of k into (iii) we've;

$\vec{f} = m\vec{a}$ .... (iv)

IMPULSE the force which act on a body for a short time are called impulsive forces.

Impulse of a force is a measure of total effect of the force. It is given by the product of a force and the time for which the force acts on the body.

Hence, Impulse = $force\ (f) \times time(t)$ ……………………………………………(v)

According to the Newton's second law of motion;

$$f = \frac{dp}{dt} \qquad fdt = dp$$

By taking the integral of the both side within the limit we've;

$$\int_o^t fdt = \int_{p_1}^{p_2} dp \quad \text{(vi)}$$

Where p_1 is the initial linear momentum at t = 0 and p_2 is final linear momentum at t = t

$$f\int_0^t dt = \int_{p_1}^{p_2} dp.$$

$$f \times [t]_0^t = [P]_{p_1}^{p_2}$$

$$f[t - o] = [p_2 - p_1]$$

$$ft = p_2 - p_1$$

$$\text{Impulse } I = F \times t = p_2 - p_1 \quad \text{(vii)}$$

NEWTON'S THIRD LAW OF MOTION

The law states that to every action there is an equal and opposite reaction.

The term action refers to the force exerted by one body m_1 to another body m_2. While the term reaction means the force exerted by second body on the first.

If f_{ab} is the force exerted on the body A by body B and f_{ba} is the force exerted on the body B by body A.

Then, according to the Newton's third law of motion;

$$f_{ab} = -f_{ba} \quad \text{(viii)}$$

Newton's third law signifies that forces in nature always occur in pairs. Force of action and reaction acts always in different bodies. But they are always equal and opposite.

Example 1

A constant retarding force of 50 N is applied to a body of mass 20 kg moving initially with a speed of 15 ms^{-1}. How long does it take to stop?

<u>**SOLUTION**</u>

$f = -50\ N$ $\qquad$ $u = 15\ ms^{-1}$ $\qquad$ $m = 20kg$

$v = 0\ ms^{-1}$ $\qquad$ $t = ?$

Recall that;

$F = ma$

$\therefore a = {}^{f}/_{m}$

$a = \frac{-50}{20} \quad = -2.5ms^{-2}$

From equation (2) $\quad v = u + at$

$$0 = 15 + (-2.5)t$$
$$t = \frac{15}{2.5}$$
$$\therefore t = 6\ seconds$$

Example 2

A force acts for 8 seconds on a body of mass 10 kg. The body then stop acting and the body describes 80 m in the next 5 seconds. Find the force applied.

SOLUTION

M = 10 kg

T = 8 seconds

After the force stop acting, the body moves with a uniform velocity by;

$$v = \frac{distance\ travelled}{time\ taken} = \frac{80}{5} = 16\ ms^{-1}.$$

Initial velocity; u = 0

From equation (2) $\quad v = u + at$

$$16 = 0 + a \times 8$$
$$16 = 8a$$
$$a = \frac{16}{8} = 2\ ms^{-2}$$

But from Newton's second law of motion $F = ma$

$\therefore F = 10 \times 2 = 20\ N$

CONSERVATION OF LINEAR MOMENTUM

It states that, the total momentum of a system remains constant (same) irrespective of their direction, provided that no external force except collision acts on them.

Thus, the conservation can be stated as a system in which the total momentum before collision is equal to the total momentum after collision.

The principle of the conservation of linear momentum follows Newton's third and second laws of motion.

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Force and Momentum

FORCE

Newton's first law explains what happened to an object when no force acts on it; the object either remains at rest or continues moving in a straight line with constant speed.

Now, when ~~an object~~ the exert a horizontal force on an object, it has been found that the acceleration of the object is directly proportional to the NET force acting on it, and also pushing objects has its acceleration affected by the mass and conclude that acceleration of an object is inversely proportional to it mass. These two observation also constitute the Newton's second law which can be defined as: The acceleration of an object is directly proportional to the NET force acting on it and inversely proportional to its mass.

$$\bar{a} \propto \frac{\sum \bar{F}}{m}$$

In vector form, we can state Newton's second law as

$$\bar{a} \propto \frac{\sum \bar{F}}{m} \quad \text{or} \quad \sum \bar{F} = m\bar{a}$$

where $\bar{a}$ is the acceleration of the object, m is its mass, and $\sum \bar{F}$ represents the vector sum of all forces acting on the object.

Note

$$\sum F_x = ma_x, \quad \sum F_y = ma_y; \quad \sum F_z = ma_z$$

This IT

MOMENTUM

The momentum of an object of mass m moving with a velocity $\vec{v}$ is defined as the product of the mass and the velocity.

$$\vec{P} = m\vec{v}$$

Momentum is a vector quantity, with the direction matching that of the velocity. Momentum has dimension ML/T, and its units are Kg.m/s.

Sometimes we find it necessary to work with the components of momentum. For two-dimensional motion, these are

$$P_x = mv_x, \quad P_y = mv_y$$

where P_x represent the momentum of an object in the x-direction;

* In order to change the momentum of an object, a force must be applied to it. The Newton's second law of motion becomes

$$F_{net} = \frac{\text{change in momentum}}{\text{Time interval}} = \frac{\Delta \vec{P}}{\Delta t}$$

as stated earlier!